

PFC/JA-95-19

**RF WAVE EFFECTS ON THE NEOCLASSICAL
ELECTRON DISTRIBUTION FUNCTION
IN TOKAMAKS**

S. D. Schultz, A. Bers, and A. K. Ram

September 1995

Plasma Fusion Center
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 USA

This work was supported by DOE Grant No. DE-FG02-91ER-54109 and NSF Grant No. ECS-94-42438; and in part by the Magnetic Fusion Science Fellowship Program. Reproduction, translation, publication, use and disposal, in whole or part, by or for the United States Government is permitted.

To be published in *Proceedings of the 11th Topical Conference on Radio Frequency Power in Plasmas*, Palm Springs, California, May 17-19, 1995.

**RF WAVE EFFECTS ON THE NEOCLASSICAL
ELECTRON DISTRIBUTION FUNCTION
IN TOKAMAKS**

S. D. Schultz, A. Bers, and A. K. Ram

TABLE OF CONTENTS

Abstract	1
Introduction	1
Drift Kinetic Equation	2
Approximate Solutions	2
Analysis	4
Acknowledgements	4
References	4

RF WAVE EFFECTS ON THE NEOCLASSICAL ELECTRON DISTRIBUTION FUNCTION IN TOKAMAKS

S. D. Schultz, A. Bers, and A. K. Ram
Plasma Fusion Center, M.I.T., Cambridge, MA 02139 U.S.A.

ABSTRACT

In advanced steady-state tokamak scenarios, a substantial fraction of the necessary plasma current will be provided by the bootstrap current. We consider the possibility of enhancing the bootstrap current using radio frequency (RF) waves. The velocity-space distribution function for electrons satisfies the drift kinetic equation which includes radial drifts due to magnetic field gradient and curvature. RF waves are then included by adding a quasilinear diffusion operator to the drift kinetic equation. The resulting bootstrap current is then found by taking the usual velocity-space moment of the RF-modified electron distribution. Analytic solutions using several techniques, including an expansion in small parameters, will be considered for different RF waves (lower hybrid, fast Alfvén waves, electron cyclotron). Numerical methods for solving the drift kinetic equation will be discussed.

INTRODUCTION

In advanced steady-state tokamak scenarios, a substantial fraction of the necessary plasma current will be provided by the bootstrap current. The bootstrap current was initially predicted by the neoclassical theory of transport in toroidally shaped plasmas [1]. If we consider a plasma in the shape of a cylindrical torus, then the confinement of this plasma by a magnetic field results in a gradient of plasma pressure inwards along the minor radius. The diffusion of particles against this gradient drives a current in the toroidal direction. Quantitatively, this current has a value

$$J_{\parallel} \sim \left(\frac{r}{R}\right)^{1/2} \frac{1}{B_{\theta}} \frac{dp}{dr}.$$

Neoclassical kinetic theory, which includes electron guiding center drifts, gives an electron distribution function which is asymmetric with respect to the direction of motion along the magnetic field, which results in the bootstrap current.

Radio frequency waves can be used to generate the remainder of the current in steady state tokamaks. In current drive, RF waves interact primarily with those electrons which move in resonance with the phase velocity of the waves. In this particular resonant region of velocity space, the diffusion of electrons can be modelled by a quasilinear operator [2]. In the absence of the bootstrap current, it is the balance of this quasilinear diffusion with collisions which gives the steady-state distribution of electrons in velocity space. Usually RF waves interact with tail electrons; however, they can be used to modify the bulk distribution.

In studying the effect of RF waves on neoclassical theory, we hope to show that by using RF to selectively diffuse electrons in velocity space, we can increase the asymmetry of the electron distribution along the magnetic field and enhance the bootstrap current. In particular, parallel diffusion of trapped electrons so they become untrapped in the direction of the bootstrap current may give current enhancement [3].

DRIFT KINETIC EQUATION

The steady-state distribution function f of electrons is assumed to be averaged over the magnetic gyromotion, and, for an axisymmetric tokamak, is independent of the toroidal angle ϕ . Under these assumptions, f can be written as a function of the guiding center coordinates r and θ and two constants of the motion, the electron's energy E and magnetic moment μ . Then f will satisfy the steady-state form ($\partial f/\partial t = 0$) of the drift kinetic equation (DKE)

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial f}{\partial \theta} + v_{Dr} \frac{\partial f}{\partial r} = C(f) + Q(f). \quad (1)$$

In terms of the coordinates (E, μ, r, θ) , the parallel velocity is given by

$$v_{\parallel}(E, \mu, r, \theta; \sigma) = \sigma \left[\frac{2}{m} (E - \mu B(r, \theta)) \right]^{1/2} \quad (2)$$

where $\sigma \equiv v_{\parallel} / |v_{\parallel}| = +1$ or -1 , and the radial drift velocity is

$$v_{Dr}(E, \mu, r, \theta) = \frac{m}{eB_o} \frac{v_{\parallel}}{r} \frac{\partial}{\partial \theta} (h v_{\parallel}), \quad (3)$$

where $h = R/R_o = 1 + (r/R_o) \cos \theta$. The current is found by taking the first-order velocity space moment of f , $J_{\parallel} = -e \int d^3v v_{\parallel} f$.

APPROXIMATE SOLUTIONS

The DKE (1) is a partial differential equation in four dimensions, which makes finding a general solution for $f(E, \mu, r, \theta)$ complicated. One way to reduce the number of dimensions is to average over one bounce orbit in θ . This bounce average is defined as

$$\{A\} \equiv \int \frac{d\theta}{v_{\parallel}} A / \int \frac{d\theta}{v_{\parallel}} \quad (4)$$

where the integral is taken from $\theta = -\pi$ to $+\pi$ for passing particles and from $\theta = -\theta_c$ to θ_c for trapped particles.

An approximate solution for f can be obtained by expanding in terms of two small parameters. The first of these expansion parameters is the ratio of the width of a banana orbit to the tokamak's minor radius, $\Delta_b/a \equiv \delta \ll 1$.

The second parameter is the ratio of the electron collision frequency (as evaluated in the collision operator C) to the frequency of bounce orbits, $\nu_c/\omega_b \equiv \hat{\nu} \ll 1$. Expanding in these parameters gives $f = f_o^{(o)} + \delta f_1^{(o)} + \hat{\nu} f_o^{(1)} + \delta \hat{\nu} f_1^{(1)} + \dots$. Substituting this into (1) and equating terms of the same order, we get

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial f_o^{(o)}}{\partial \theta} = 0 \quad (5)$$

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial f_o^{(1)}}{\partial \theta} = C(f_o^{(o)}) + Q(f_o^{(o)}) \quad (6)$$

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial f_1^{(o)}}{\partial \theta} = -v_{Dr} \frac{\partial f_o^{(o)}}{\partial r} \quad (7)$$

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial f_1^{(1)}}{\partial \theta} = C(f_1^{(o)}) + Q(f_1^{(o)}). \quad (8)$$

The bounce average of (6) gives an equation for $f_o^{(o)}$:

$$0 = \{C(f_o^{(o)}) + Q(f_o^{(o)})\}. \quad (9)$$

This is the equation for the modified electron distribution due to RF effects. The bootstrap current is contained in the $f_1^{(o)}$ term, so we only solve for $f_o^{(o)}$ and $f_1^{(o)}$. From (7) and (3), we find the θ -dependent part of $f_1^{(o)}$:

$$f_1^{(o)} = -\frac{m}{eB_o} v_{\parallel} \frac{\partial f_o^{(o)}}{\partial r} + \bar{f}_1^{(o)}. \quad (10)$$

Finally, to find the θ -independent quantity $\bar{f}_1^{(o)}$, we must bounce average (8).

Alternatively, we can use the adjoint technique [4,5], which may allow us to solve for the plasma current without finding all the details of the electron distribution. This technique makes use of the self-adjoint property of the linearized collision operator C in the DKE (1). For any two distribution functions f and g which are slightly perturbed from a Maxwellian f_M , the adjoint property can be written as

$$\int d^3v g f_M^{-1} C(f) = \int d^3v f f_M^{-1} C(g). \quad (11)$$

If f is the solution of (1) and we let g be the toroidal generalization of the Spitzer-Härm distribution function [6], then we can express the parallel current in terms of g :

$$J_{\parallel} = -\frac{eB}{2\pi q} \int_0^{2\pi} \frac{rd\theta}{B_{\theta}} \int d^3v [\vec{S}_{RF} \cdot \frac{\partial}{\partial \vec{v}} + \frac{m}{eB_o} \frac{v_{\parallel}}{r} v_{\parallel} h \frac{\partial f}{\partial r} \frac{\partial}{\partial \theta}] g e^{-E/T_e} \quad (12)$$

Although originally applied only to RF and beam driven currents, the adjoint technique has been used to predict the bootstrap current in the absence of RF waves [7]. RF waves can be included as shown in (12).

ANALYSIS

The equations discussed above can be solved analytically and numerically to study the net current produced by bootstrap in conjunction with several different types of RF waves, including lower hybrid waves, fast Alfvén waves, and electron cyclotron waves. This will allow us to predict an RF scheme which can modify the electron distribution to provide an enhanced bootstrap current.

Numerically, the distribution function is defined on a set of grid points in four dimensions, and the conjugate gradient method can be used to relax the distribution to a value which satisfies the steady-state condition $\frac{1}{2}\left(\frac{\partial f}{\partial t}\right)^2 = 0$. Because a grid in four dimensions requires a very large number of points, we are adapting our algorithms toward solving the four-dimensional DKE in its complete form on a parallel processing machine.

Solving the equations for a reduced number of dimensions is possible on the NERSC Crays. Using the approximations described in the last section, the grid is reduced to two dimensions. The collision term is replaced by a linear operator, taken from the model of a system where electron collisions are dominated by a fixed Maxwellian background of ions with a large value of Z .

Our current study of the RF-bootstrap interaction is focused on the creation and optimization of the numerical solvers described above to produce enough significant data to suggest an analytic formulation.

ACKNOWLEDGEMENTS

This work is supported by DoE Grant No. DE-FG02-91ER-54109 and NSF Grant No. ECS-94-42438; and in part by the Magnetic Fusion Science Fellowship Program.

REFERENCES

- [1] Rosenbluth, M.N., Hazeltine, R.D. and Hinton, F.L., Phys. Fluids **13**, 116 (1972).
- [2] Kennel, C.F. and Engelmann, F., Phys. Fluids **9**, 2377 (1966).
- [3] Politzer, P.A. and Chan, V.S., Bull. Am. Phys. Soc. **37**, 1514, 6Q30 (1992).
- [4] Hirshman, S.P., Phys. Fluids **23**, 1238 (1980).
- [5] Antonsen, T.M. Jr. and Chu, K.R., Phys. Fluids **25**, 1295 (1982).
- [6] Spitzer, L., and Härm, R., Phys. Rev. **89**, 977 (1953).
- [7] Lin-Liu, Y.R., Hinton, F.L., Karney, C.F.F. and Hirschman, S.P., International Sherwood Fusion Theory Conference, Dallas, Texas, 1994; Sauter, O., Lin-Liu, Y.R., Hinton, F.L. and Vaclavik, V., CRPP Report LRP 503/94, p.41 (1994).
- [8] Harvey, R.W., McCoy, M.G., Politzer, P.A., Chan, V.S. and Kupfer, K., International Sherwood Fusion Theory Conference, Newport, Rhode Island, (1993).