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The Influence of Anomalous Diffusion on Parallel Ion Transport in Edge Plasmas

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The influence of anomalous diffusion on parallel ion transport in edge plasmas

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Abstract

In edge plasmas, such as the tokamak scrape-off layer, transport parallel and perpendicular to the magnetic field are strongly coupled. It is shown that this leads to an unconventional form of the parallel transport laws. In particular, if the radial transport is governed by anomalous diffusion, the parallel ion transport cannot be entirely classical. This general phenomenon is demonstrated explicitly within a simple model where the anomalous diffusion is driven by weak electrostatic turbulence. Since the parallel and perpendicular flows balance each other in the plasma edge, anomalous terms proportional to the radial shear of the parallel velocity are shown to enter into, and modify, the usual Spitzer problem for parallel ion transport. As a result, parallel ion transport is driven not only by parallel gradients, but also by radial gradients, which is of importance for the flow of impurities in the tokamak edge. A new type of parallel thermal force is found resulting from the combined action of anomalous diffusion, parallel velocity shear, and a radial temperature gradient.

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I. INTRODUCTION

The edge plasma in the tokamak scrape-off layer (SOL) is not confined by the magnetic field. Indeed, the plasma flows freely along the magnetic field lines towards the limiter or the divertor collector plates, and the loss is balanced by cross-field transport. In the continuity equation,

$$\nabla_{\|}(nV_{\|}) + \partial\Gamma_r/\partial r = 0,$$

where n is the density, $V_{\|}$ the mean parallel velocity, r the radius (or any flux surface label) and Γ_r the radial flux, both terms are thus of the same magnitude. For this reason the parallel and perpendicular transport are strongly coupled. The conventional estimate of the SOL width W following from this picture is [1]

$$v_T/L_{\|} \sim D/W^2 \qquad \Rightarrow \qquad W \sim \sqrt{DL_{\|}/v_{Ti}}, \tag{1}$$

where $v_T \equiv (2T/m)^{1/2}$ is the thermal ion speed characterizing the parallel flow, $L_{\|}$ the connection length, and D the radial diffusion coefficient, determining the flux by $\Gamma_r \sim -D\partial n/\partial r$.

Conventional classical transport theory [2, 3] does not take into account the strong perpendicular transport implied by (1). The usual parallel transport coefficients are therefore not necessarily applicable to edge plasmas. A recent paper [4] rederived classical transport laws for an impure plasma using the edge orderings corresponding to (1), and found novel contributions to parallel transport from the perpendicular diffusion. It turns out that the parallel fluxes of particles and energy are driven not only by parallel gradients in the electrostatic potential, pressure and temperature, but also by radial gradients. The new terms describing this effect in the expressions for parallel particle and heat fluxes are important whenever radial gradients are large enough, that is, as large as predicted by Eq. (1) if the classical diffusion coefficient for ion-impurity collisions is used for D.

In practice, radial diffusion at the edge is thought to be anomalous rather than classical. In fact, the relation (1) is sometimes used to estimate the diffusion coefficient from measurements of the SOL width, and the inferred radial transport is usually strongly anomalous [1]. On the other hand, the parallel transport is generally taken to be purely classical. However, because of the nature of the SOL as indicated by (1), the parallel and perpendicular dynamics are not independent. Any anomalous radial diffusion affects the parallel transport, which therefore cannot be entirely classical. It is the purpose of the present paper to bring attention to this important effect. The point we wish to make is perfectly general, relying only on the balance between parallel and perpendicular transport, independently of the nature of the latter. For definiteness, however, we specialize to the case of anomalous diffusion by electrostatic turbulence, like, e.g., Hinton and Kim [5] in a recent fluid formulation of turbulent edge transport theory.

It is important to realize that the balance (1) is realistic only for the ions. The electrons move much faster along the field lines, but are lost at the same rate as the ions to maintain global ambipolarity. Parallel streaming therefore dominates over, rather than balances, perpendicular diffusion in the electron kinetic equation. (The SOL width may still be comparable for the two species since the boundary conditions are different. Most electrons are reflected by the Debye sheaths at the walls, whereas the ions are absorbed and recycled.) The parallel electron dynamics is therefore more likely to be conventionally classical, and accordingly, we focus on the ions in the present work.

In Sec. II, we adopt orderings, relevant to the edge, exhibiting the desired balance between ion streaming and diffusion on a kinetic level. In the next section, we proceed to solve the drift-kinetic equation in a quasilinear approximation. As expected, the diffusion across the field affects the parallel dynamics, and brings in new terms to the usual (parallel) Spitzer problem. Our conclusions are summarized in Sec. IV.

II. EDGE ORDERINGS

It is our ambition to adopt orderings which are realistic for the SOL in present and future tokamaks, and yet allow the ion kinetic equation to be simplified and solved. Cross-field transport is taken to occur as a result of fluctuating electric fields giving rise to random $\mathbf{E} \times \mathbf{B}$ drifts. The fluctuating part of quantities such as the ion distribution function, $\tilde{f}_i$, is assumed to be small in comparison with the averaged part $\bar{f}_i$ by a small parameter δ,

$$\frac{\tilde{f}_i}{\bar{f}_i} \sim \delta \ll 1, \tag{2}$$

which is our basic expansion parameter. This weak-turbulence assumption enables a quasilinear analysis of the kinetic equation. In order to be able to treat the kinetics on the drift-kinetic, rather than on the gyrokinetic, level, we take the perpendicular wavelength $2\pi/k_\perp$ of the fluctuations to be long in comparison with the (ion) Larmor radius $\rho \equiv v_T/\Omega$,

$$k_\perp \rho \sim \delta.$$

The fluctuating $\mathbf{E} \times \mathbf{B}$ velocity should then be of the order

$$\tilde{V}_E \sim k_\perp \tilde{\Phi}/B \sim \delta^2 v_T, \tag{3}$$

since the perturbed electrostatic potential is expected to be $\tilde{\Phi} \sim \delta T/e$. The SOL is magnetized but narrow in comparison with the minor radius a,

$$\rho \ll W \ll a,$$

in all tokamaks. There is no need to quantify these inequalities in terms of δ; we need them only for establishing that the diamagnetic drift in the poloidal direction,

$$\mathbf{V}_{dia} = \frac{\mathbf{B} \times \nabla p}{neB^2} \sim \frac{\rho v_T}{W},$$

is smaller than the thermal speed, but larger than magnetic drift velocity, which is of the order of $\rho v_T/R$, where $R > a$ is the major radius. The frequency of the fluctuations is expected to be of the order of the diamagnetic frequency,

$$\omega \sim \omega_* \equiv k_\perp V_{dia} \sim \delta v_T/W, \tag{4}$$

which is the conventional estimate for electrostatic turbulence. Finally, anticipating that the quasilinear diffusion coefficient is of the order

$$D \sim \tilde{V}_E^2/\omega \sim \delta^3 v_T W, \tag{5}$$

we can then conclude that parallel streaming balances radial diffusion, as in Eq. (1), if the SOL width is

$$W \sim \delta^3 L_\parallel. \tag{6}$$

Let us briefly discuss the relevance of these orderings. Our strongest assumption is that of weak turbulence (2), which may not always be satisfied at the edge, as indicated, e.g., by probe measurements in the SOL of the Texas Experimental Tokamak (TEXT) [6]. Fluctuations of about 10% are, however, not uncommon. Indeed, taking $\delta \sim 0.1$ appears realistic, since the most unstable drift-modes typically have $k_\perp \rho \sim 0.1$, and with a typical SOL width of $W \sim 1cm$ and connection length of the order $L_\| \sim 10m$, the ordering $W \sim \delta^3 L_\|$ is quite reasonable. Moreover, if the temperature is 100 eV, the diffusion coefficient becomes $D \sim 1m^2/s$, a value that is often quoted from experiments and is frequently used in numerical edge computations.

Although it is not crucial for our general argument, we shall assume that the collisional mean-free path is short in comparison with the connection length, but not necessarily in comparison with the fluctuation wavelength. The mean-free path is usually about 1/10 or less of the connection length in the SOL of most tokamaks.

III. SOLUTION OF THE DRIFT-KINETIC EQUATION

The drift-kinetic equation for ions has the form [7]

$$\frac{\partial f_i}{\partial t} + (\mathbf{v}_\| + \mathbf{V}_d + \mathbf{V}_E) \cdot \nabla f_i + eE_\| v_\| \frac{\partial f_i}{\partial \epsilon} = C_i \qquad (7)$$

where $\epsilon \equiv m_i v^2/2$ is the kinetic energy, $\mathbf{V}_d$ is the magnetic drift velocity, and $E_\|$ is the parallel electric field. Ions typically experience collisions, described by the operator C_i, with both impurities (Z) and other ions (i),

$$C_i = C_{iZ} + C_{ii}.$$

The distribution function and the $\mathbf{E} \times \mathbf{B}$ drift consist of averaged and fluctuating pieces,

$$f = \bar{f}_i + \tilde{f}_i,$$

$$\mathbf{V}_E = \bar{\mathbf{V}}_E + \tilde{\mathbf{V}}_E,$$

and (7) is split into similar parts,

$$\frac{\partial \bar{f}_i}{\partial t} + (\mathbf{v}_\| + \mathbf{V}_d + \bar{\mathbf{V}}_E) \cdot \nabla \bar{f}_i + \langle \tilde{\mathbf{V}}_E \cdot \nabla \tilde{f}_i \rangle + eE_\| v_\| \frac{\partial \bar{f}_i}{\partial \epsilon} = \bar{C}_i, \qquad (8)$$

$$\frac{\partial \tilde{f}_i}{\partial t} + (\mathbf{v}_\parallel + \mathbf{V}_d + \bar{\mathbf{V}}_E)\cdot\nabla\tilde{f}_i + \tilde{\mathbf{V}}_E\cdot\nabla\bar{f}_i + eE_\parallel v_\parallel \frac{\partial \tilde{f}_i}{\partial \epsilon} = \tilde{C}_i, \tag{9}$$

where $\langle\cdots\rangle$ denotes an average over fluctuations, and nonlinear terms have been neglected in the second equation, as well as the fluctuating parallel electric field $\tilde{E}_\parallel$, which will be justified presently. Note that the fluctuationg part of the collision operator, $\tilde{C}_i$, involves fluctuations in both the ion and the impurity distribution functions.

In the first equation (8), collisions dominate since the mean-free path is short. The solution $\bar{f}_i = \bar{f}_{i0} + \bar{f}_{i1} + \ldots$, becomes Maxwellian to the lowest order in the mean-free path,

$$\bar{f}_{i0} = f_{Mi} \equiv n\left(\frac{m_i}{2\pi T}\right)^{3/2}\exp\left(-\frac{m_i v_\perp^2}{2T} - \frac{m_i(v_\parallel - V_\parallel)^2}{2T}\right), \tag{10}$$

with a parallel flow velocity $V_\parallel$ equal to the the mean impurity velocity. The perpendicular flow is assumed to be small in comparison with the ion thermal speed. In the SOL, the parallel plasma flow may be large, so we permit it to be of the order of the ion thermal speed,

$$V_\parallel \sim v_T,$$

as in Eq. (1). In the next order we have

$$C_i(\bar{f}_{i1}) = (\mathbf{v}_\parallel + \mathbf{V}_d + \bar{\mathbf{V}}_E)\cdot\nabla f_{Mi} + eE_\parallel v_\parallel\frac{\partial f_{Mi}}{\partial\epsilon} + \langle\tilde{\mathbf{V}}_E\cdot\nabla\tilde{f}_i\rangle, \tag{11}$$

where the quasilinear term $\langle\tilde{\mathbf{V}}_E\cdot\nabla\tilde{f}_i\rangle$ representing cross-field transport is comparable to parallel streaming $v_\parallel\nabla_\parallel f_{Mi}$ because of the orderings developed in the last Section. We must therefore turn our attention to Eq. (9) and solve for the fluctuations before proceeding with the analysis of (11).

A. Fluctuations

Our edge orderings enable us to significantly simplify the fluctuating part (9) of the drift-kinetic equation because the terms containing parallel streaming, magnetic drift $V_d \sim (\rho/R)v_T$, and the parallel electric field $E_\parallel \sim T/eL_\parallel$ are all small in comparison with the inertial term since

$$\frac{v_\parallel\nabla_\parallel\tilde{f}_i}{\partial\tilde{f}/\partial t} \sim \frac{v_T k_\parallel}{\omega} \sim \delta^2 k_\parallel L_\parallel \ll 1,$$

$$\frac{\mathbf{V}_d \cdot \nabla \tilde{f}_i}{\omega \tilde{f}_i} \sim \frac{(\rho/R) v_T k_\perp}{\delta v_T / W} \sim \frac{W}{R} \ll 1,$$

$$\frac{eE_\| v_\| \partial \tilde{f}_i / \partial \epsilon}{\omega \tilde{f}_i} \sim \delta^2 \ll 1.$$

In the first of these estimates, we require $k_\| L_\| \ll 1/\delta^2$, ruling out unrealistically short parallel wavelengths. We also note that we were justified in neglecting the fluctuating parallel electric field in (9) since $e\tilde{E}_\| v_\| \partial \bar{f}_i / \partial \epsilon$ is of the same order as the small term $v_\| \nabla \tilde{f}_i$.

In the equation (9) for $\tilde{f}_i$, the nonlinear terms were neglected. This (quasilinear) approximation is justified if

$$\frac{\tilde{\mathbf{V}}_E \cdot \nabla \tilde{f}_i}{\tilde{\mathbf{V}}_E \cdot \nabla \bar{f}_i} \sim \delta k_\perp W \ll 1. \tag{12}$$

On the other hand, to facilitate the solution of the drift-kinetic equation, it is convenient to take a Fourier transform of fluctuating quantities in time and in the direction perpendicular to the magnetic field,

$$[\tilde{f}_i, \tilde{\mathbf{V}}_E] = \sum_{\omega, \mathbf{k}_\perp} [\hat{f}_i(\mathbf{k}_\perp, \omega), \hat{\mathbf{V}}_E(\mathbf{k}_\perp, \omega)] e^{i(\mathbf{k}_\perp \cdot \mathbf{x} - \omega t)}.$$

This procedure is worthwhile only if the wavelength is short in comparison with the width of the SOL, $k_\perp W \gg 1$, since averaged and fluctuating quantities then vary on different perpendicular length scales. Consequently we require $\delta \ll \delta k_\perp W \ll 1$; we may, for example, take $k_\perp W \sim \delta^{-1/2}$.

Taking the Fourier transform and neglecting small terms in (9) gives the lowest-order result

$$\hat{C}_i + i\omega_k \hat{f}_i = \hat{\mathbf{V}}_E \cdot \nabla \bar{f}_i \tag{13}$$
$$\simeq \hat{\mathbf{V}}_E \cdot \left[\nabla \ln p + \left(\frac{m_i u^2}{2T} - \frac{5}{2} \right) \nabla \ln T + \frac{m_i u_\|}{T} \nabla V_\| \right] f_{Mi},$$

where we have written $\omega_k \equiv \omega - \mathbf{k}_\perp \cdot \bar{\mathbf{V}}_E$ for the Doppler shifted frequency, $p \equiv nT$ is the pressure, and $\mathbf{u} \equiv \mathbf{v} - \mathbf{V}_\|$ is the velocity relative to $\mathbf{V}_\|$. The driving term on the right-hand side of (13) contains the averaged distribution function $\bar{f}_i$, which is given by the Maxwellian (10) to lowest order. The Fourier transformed, fluctuating part of the collision operator $\hat{C}_i$ contains fluctuations in both the ion and the impurity distribution functions, so it is necessary to determine the latter from the corresponding equation for the impurities,

$$\hat{C}_Z + i\omega_k \hat{f}_Z = \hat{\mathbf{V}}_E \cdot \nabla \bar{f}_Z, \tag{14}$$

where $\bar{f}_Z$ is a Maxwellian identical to (10), but with the the impurity mass m_Z instead of m_i. The equations (13) and (14) are remarkably easily solved. The lowest-order solutions are, in fact, independent of the collision frequency, and are thus equal to those of the corresponding equation without collisions,

$$\hat{f}_a = \frac{\hat{\mathbf{V}}_E \cdot \nabla \bar{f}_{Ma}}{i\omega_k}, \quad a = i, Z. \tag{15}$$

To verify these solutions, we need only demonstrate that the collision operators vanish when operating on them. This is, however, already clear from the fact that the distribution functions corresponding to (15) are simply equally displaced Maxwellians

$$f_a(\mathbf{x}) = f_{Ma}(\mathbf{x}) + \hat{f}_a(\mathbf{x}) \simeq f_{Ma}(\mathbf{x} + \hat{\mathbf{V}}_E / i\omega_k)$$

to the lowest order. The linearized collision operators $\hat{C}_i$ and $\hat{C}_Z$ in Eqs. (13) and (14) therefore operate on Maxwellians with the same velocity and temperature, and must therefore vanish. This completes the solution of the fluctuating part (9) of the drift-kinetic equation.

B. Transport laws

We are now ready to return to Eq.(11), from which edge transport laws can be deduced. Taking the strongest perpendicular gradients to be in the radial (r) direction in the quasilinear term, we can write the equation as

$$C_i(\bar{f}_{i1}) = u_\| (\nabla_\| \ln f_{Mi} - eE_\| / T) f_{Mi} + \frac{\partial}{\partial r} \langle \tilde{V}_{Er} \tilde{f}_i \rangle + (\mathbf{V}_\| + \bar{\mathbf{V}}_E) \cdot \nabla f_{Mi}. \tag{16}$$

The magnetic drift has been neglected since

$$\frac{\mathbf{V}_d \cdot \nabla \bar{f}_i}{v_\| \nabla_\| \bar{f}_i} \sim \frac{\rho L_\|}{WR} \ll 1,$$

because the connection length $L_\|$ is comparable to the major radius R. Anomalous fluxes arise from the quasilinear term

$$\langle \tilde{V}_{Er} \tilde{f} \rangle = -D \left[\frac{\partial \ln p}{\partial r} + \left(\frac{m_i u^2}{2T} - \frac{5}{2} \right) \frac{\partial \ln T}{\partial r} + \frac{m_i u_\|}{T} \frac{\partial V_\|}{\partial r} \right] f_{Mi},$$

where

$$D \equiv \pi \sum_{\omega,\mathbf{k}_\perp} |\hat{\mathbf{V}}_{Er}(\mathbf{k}_\perp,\omega)|^2 \delta(\omega - \mathbf{k}_\perp \cdot \bar{\mathbf{V}}_E), \tag{17}$$

and we have used Landau's rule $1/i\omega_k \to -\pi\delta(\omega_k)$. The anomalous diffusion coefficient D scales as anticipated in (5), and determines the diffusion of particles, momentum, and heat,

$$\Gamma_r \equiv \int \langle \tilde{V}_{Er} \tilde{f} \rangle d^3v = -D\frac{\partial n}{\partial r}, \tag{18}$$

$$\Pi_{r\|} \equiv \int \langle \tilde{V}_{Er} \tilde{f} \rangle m u_\| d^3v = -m_i n D \frac{\partial V_\|}{\partial r}, \tag{19}$$

$$q_r \equiv \int \langle \tilde{V}_{Er} \tilde{f} \rangle \left(\frac{m_i u^2}{2} - \frac{5T}{2} \right) d^3v = -D \left(\frac{3n}{2}\frac{\partial T}{\partial r} - T\frac{\partial n}{\partial r} \right), \tag{20}$$

in a conventional way. These relations contain no surprises, and are Onsager symmetric when written in terms of the pressure and temperature gradients.

More interesting are the parallel transport laws that follow from (16). To determine parallel fluxes (relative to the lowest-order velocity $V_\|$), we need only the part of $\bar{f}_{i1}$ that is proportional to $u_\|$, which we denote by $\bar{F}_{i1}$. Isolating the terms in (16) that are proportional to $u_\|$ and keeping in mind that the spatial derivatives are to be taken at fixed $\mathbf{v}$ rather than at fixed $\mathbf{u}$ gives

$$C(\bar{F}_{i1}) = u_\| \left\{ \nabla_\| \ln p - \frac{eE_\|^*}{T} + \left(\frac{m_i u^2}{2T} - \frac{5}{2} \right) \nabla_\| \ln T - \frac{m_i}{T}\frac{\partial}{\partial r}\left(D \frac{\partial V_\|}{\partial r} \right) \right.$$
$$\left. - \frac{2 m_i D}{T}\frac{\partial V_\|}{\partial r} \left[\frac{\partial \ln n}{\partial r} + \left(\frac{m_i u^2}{2T} - \frac{5}{2} \right) \frac{\partial \ln T}{\partial r} \right] \right\} f_M, \tag{21}$$

where we have introduced $E_\|^* \equiv E_\| - (m_i/e)(\mathbf{V}_\| + \bar{\mathbf{V}}_E) \cdot \nabla V_\|$. The equation (21) has the form of a Spitzer problem [2] modified by anomalous diffusion. It is well known that the usual electron-ion Spitzer problem carries over exactly to the corresponding ion-impurity problem [8] if the ion charge number in the electron-ion problem is replaced by the impurity strength $\alpha \equiv n_Z Z^2/n_i$, where n_Z is the impurity density, and the impurities are assumed to be massive, $m_Z \gg m_i$. In the present situation there are additional terms proportional to $D\,\partial V_\|/\partial r$ that add to the usual thermodynamic forces, but since the diffusion coefficient D is independent of the velocity u, the equation is still mathematically equivalent to the Spitzer problem. By analogy, we can thus immediately write down the ion particle and heat fluxes relative to the impurities as

$$\Gamma_\| \equiv \int \bar{f}_i u_\| d^3u = n(V_{i\|} - V_\|) = -\frac{nT\tau_{iZ}}{m_i}(\lambda_{11}A_1 + \lambda_{12}A_2), \tag{22}$$

$$q_\| \equiv \int \bar{f}_i \left(\frac{m_i u^2}{2} - \frac{5T}{2}\right) u_\| d^3v = -\frac{nT\tau_{iZ}}{m_i}T(\lambda_{21}A_1 + \lambda_{22}A_2), \tag{23}$$

where

$$\tau_{iZ} \equiv \frac{3}{4(2\pi)^{1/2}}\frac{m_i^{1/2}T_i^{3/2}}{n_Z Z^2 e^4 \ln\Lambda}$$

is the ion-impurity collision time,

$$A_1 \equiv \nabla_\| \ln p - \frac{eE_\|^*}{T} - \frac{m_i}{T}\frac{\partial}{\partial r}\left(D\frac{\partial V_\|}{\partial r}\right) - \frac{2m_i D}{T}\frac{\partial V_\|}{\partial r}\frac{\partial \ln n}{\partial r},$$

$$A_2 \equiv \nabla_\| \ln T - \frac{2m_i D}{T}\frac{\partial V_\|}{\partial r}\frac{\partial \ln T}{\partial r},$$

are parallel thermodynamic forces modified by anomalous transport through the terms proportional to D, and λ_{jk} are the usual parallel transport coefficients, satisfying Onsager symmetry, $\lambda_{12} = \lambda_{21}$. They depend on the impurity strength α and on the mass ratio m_Z/m_i. For massive impurities, $m_Z \gg m_i$, the coefficients are equal to those in the usual electron-ion Spitzer problem, and can be looked up in Ref.[9]. For example, if $\alpha = 1$ (i.e. $Z_{eff} \equiv (n_Z Z^2 + n_i)/(n_Z Z + n_i) = 2$ in the limit $Z \gg 1$), we have $\lambda_{11} = 1.975$, $\lambda_{12} = \lambda_{21} = 1.389$, and $\lambda_{22} = 4.174$.

To conclude this section, let us use the result (22) to evaluate the ion-impurity force $R_\|$ in the parallel ion momentum equation

$$m_i n[(\mathbf{V}_i \cdot \nabla)\mathbf{V}_i]_\| = neE_\| - \nabla_\| p - (\nabla \cdot \Pi)_\| + R_\|, \tag{24}$$

where $\mathbf{V}_i$ is the mean ion velocity. In the present situation, $\mathbf{V}_i$ consists of the parallel velocity, the $\mathbf{E} \times \mathbf{B}$ drift and the radial diffusion velocity,

$$\mathbf{V}_i = \mathbf{V}_\| + \bar{\mathbf{V}}_E - D\frac{\partial \ln n}{\partial r}\nabla r,$$

where the small transport correction $\Gamma_\|/n$ to the parallel velocity has been neglected. Recalling that the ions flow mainly in the parallel direction, $V_{i\|} \gg V_{i\perp}$, we may write

$$neE_\| - m_i n[(\mathbf{V_i} \cdot \nabla)\mathbf{V_i}]_\| = neE_\|^* + m_i D\frac{\partial n}{\partial r}\frac{\partial V_\|}{\partial r},$$

and with the dominant piece of the viscosity tensor given by (19) to the lowest order, the viscous force becomes

$$(\nabla \cdot \Pi)_\parallel \simeq \frac{\partial \Pi_{r\parallel}}{\partial r} \simeq -\frac{\partial}{\partial r}\left(m_i n D \frac{\partial V_\parallel}{\partial r}\right).$$

Using these results and (22) it is now straightforward to solve for the parallel force in the momentum equation (24), with the result

$$R_\parallel = -\frac{m_i \Gamma_\parallel}{\lambda_{11}\tau_i Z} - \frac{\lambda_{12}}{\lambda_{11}}\left(n \nabla_\parallel T - \frac{2 m_i n D}{T}\frac{\partial V_\parallel}{\partial r}\frac{\partial T}{\partial r}\right). \tag{25}$$

The first two terms in this expression, proportional to the relative velocity and the parallel temperature gradient, respectively, are conventional. In addition, there is a third term of an unusual form, involving radial, but no parallel, derivatives. In the next Section, we shall give it a physical interpretation. It represents an unconventional form of thermal force, and its presence indicates that the anomalous radial diffusion changes the parallel dynamics already on the kinetic level. It is, in other words, not possible to obtain the parallel flux (22) simply by inserting an anomalous cross-field flux and a corresponding anomalous viscosity in the parallel momentum equation (24) while keeping the classical expression for the parallel force $R_\parallel$.

IV. CONCLUSIONS

At the edge, parallel and perpendicular transport of ions are very strongly coupled. In particular, the parallel dynamics cannot be entirely classical if the radial transport is anomalous. We have constructed a simple transport model exhibiting this feature in a realistic way. To make the argument as transparent as possible, the turbulence is assumed to be weak, enabling a quasilinear treatment. In addition, the wavelength is taken to be larger than the ion Larmor radius, so that the drift-kinetic, rather than the gyrokinetic, equation applies. It must be emphasized that these simplifications are made solely for the sake of transparency and analytic tractability. The basic point we wish to make is independent of the mechanism underlying the anomalous transport. Also for this reason, we have not addressed the instability mechanism driving the fluctuations. To do so, it is necessary to analyze the electron dynamics (and sometimes also the impurity dynamics) in detail, which is beyond the scope of the present paper. The edge orderings adopted here permit a variety of instabilities, such as Kelvin-Helmholtz instabilities and drift instabilities modified by parallel-velocity shear [10] or by the presence of impurities [11]. It is interesting to note that when combining the very rough, but customary, mixing-length estimate for the diffusion coeffient

$$D \sim \gamma / k_\perp^2$$

with (5) and our other edge orderings (3), (4), (6), and (12), one obtains an instability growth rate γ that is small in comparison with the the real part of the frequency,

$$\gamma / \omega \sim (\delta k_\perp W)^2 \ll 1,$$

commensurate with drift instabilities.

The fluxes (22) and (23), differ from classical results by the addition of several terms proportional to the anomalous diffusion coefficient. These terms are of the same magnitude as the conventional ones, and some, but not all, of them can be traced back to radial inertia and anomalous viscosity in the ion momentum equation. They contribute essentially to the parallel dynamics of the ion-impurity mixture. For example, since the temperature drops along field lines from the main SOL to the divertor, the thermal force (the $\nabla_\parallel \ln T$ term) tends to drive impurities from the divertor towards the core, which is of concern for tokamak operation since Z_{eff} must be kept low in the core. Our analysis shows that the thermal force may be counteracted not only by friction and the electric field, but also by a term proportional to $D(\partial V_\parallel/\partial r)(\partial T/\partial r)$, as in Eq.(25). This modification of the parallel dynamics should be included in any numerical edge code attempting to keep track of impurities.

To understand the physical meaning of the new force [the last term in (25)], let us consider a situation where there is no relative flow between the ions and the impurities, $\Gamma_\parallel = 0$. The usual thermal force, proportional to $\nabla_\parallel T$, arises since ions travelling in the direction of $\nabla_\parallel T$ originate from a colder region than the ones moving in the opposite direction. The former are therefore more collisional than the latter, and exert a force on the impurities in the direction of $\nabla_\parallel T$. Consequently, the ions experience a force in the opposite direction. In other words, the thermal force arises since the distribution is *asymmetric* in parallel velocity, even if the average parallel velocity vanishes. The new force arises in a similar manner. If, for instance, $\partial V_\| / \partial r$ and $\partial T / \partial r$ are both negative, anomalous diffusion transports hot ions with large $V_\|$ radially outwards in the SOL. Again, this causes the ion distribution to be asymmetric since faster ions replace cooler, slower ones at the radius of interest, causing the distribution function to asymmetrically depart from Maxwellian with an elevated tail for $u_\| > 0$. If the relative velocity vanishes, $\Gamma_\| = 0$, the colder, more collisional, ions with $u_\| < 0$, exert a force on the impurities in the direction of $-(\partial V_\| / \partial r)(\partial T / \partial r)$ along the field lines, and the ions are pushed in the opposite direction. Note that the distortion of the distribution function is asymmetric only if both $\partial V_\| / \partial r \neq 0$ and $\partial T / \partial r \neq 0$.

In a conventional weak-gradient expansion, the new force is higher order than the usual thermal force, since it is involves two derivatives, $(\partial V_\| / \partial r)(\partial T / \partial r)$, rather than one, $\nabla_\| T$. However, in the tokamak edge, the relation (1) implies exactly that two radial derivatives balance one parallel one.

For edge computations, it is also of interest to note that, in the present quasilinear formulation of edge turbulence, one single diffusion coefficient determines the anomalous particle diffusivity, viscosity, and heat conduction coefficient for ions across the magnetic field, according to Eqs. (18)-(20). The perpendicular particle and heat fluxes are also shown to obey Onsager symmetry in a way not always accounted for in numerical edge codes. These features have been discussed extensively in the theory of neoclassical transport in the presence of fluctuations [12].

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