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**RF EFFECTS ON NEOCLASSICAL THEORY
AND THE BOOTSTRAP CURRENT IN TOKAMAKS**

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RF Effects on Neoclassical Theory and the Bootstrap Current in Tokamaks

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Abstract. We consider the possibility of synergistic effects in combining bootstrap and RF current drive in tokamaks [1]. In neoclassical theory, the bootstrap current is derived from an electron velocity-space distribution satisfying the drift kinetic equation (DKE), with radial drifts due to magnetic field gradient and curvature. Numerical solutions to the DKE are found with KT3D-NC, a Fokker-Planck code with quasilinear diffusion [2] which calculates perturbations to the electron distribution due to the drift term of the DKE. The resulting bootstrap current is then found by taking the parallel velocity-space moment of the RF-modified electron distribution. A quasilinear diffusion operator is chosen to represent several RF wave scenarios (lower hybrid, fast Alfvén waves, electron cyclotron). The numerical results are compared to analytic solutions using several approximate techniques.

INTRODUCTION

With the goal of producing a steady-state tokamak in mind, there is a great deal of interest in the bootstrap current and in the use of radio frequency waves for heating and current drive. However, there has been little consideration of the interaction between these two important topics. Calculation of the bootstrap current in the presence of RF waves is complicated by the fact that RF may cause the distribution of electrons to become highly non-Maxwellian, while existing neoclassical theory [3,4] uses the approximation of a nearly Maxwellian distribution.

It is clear that RF heating will affect the bootstrap current by altering the pressure gradients which drive this current. More generally, it may be possible to choose an RF scheme which enhances the bootstrap current on a kinetic level. This would allow us to use RF to drive current indirectly. For example, it has been suggested that parallel diffusion of trapped electrons to transfer momentum to untrapped electrons may enhance the current [5]. We consider this and other uses of RF to alter the bootstrap current.

NEOCLASSICAL KINETIC THEORY WITH RF

Neoclassical kinetic theory, which includes electron guiding center drifts, gives an electron distribution function which is asymmetric with respect to

the direction of motion along the magnetic field, producing the bootstrap current. RF waves interact primarily with those electrons which move in resonance with the phase velocity of the waves. In the resonant region of velocity space, the diffusion of electrons can be modelled by a quasilinear operator [6].

Drift Kinetic Equation

The steady-state distribution function f of electrons is assumed to be averaged over the magnetic gyromotion, and, for an axisymmetric tokamak, is independent of the toroidal angle. Under these assumptions, f can be written as a function of the guiding center coordinates r and θ and two constants of the motion, the electron's energy E and magnetic moment μ . Then f will satisfy the steady-state form ($\partial f / \partial t = 0$) of the drift kinetic equation (DKE)

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial f}{\partial \theta} + v_{Dr} \frac{\partial f}{\partial r} = C(f) + Q(f). \quad (1)$$

In terms of the coordinates (E, μ, r, θ) , the parallel velocity is given by

$$v_{\parallel} = \sigma \left[\frac{2}{m} (E - \mu B(r, \theta)) \right]^{1/2} \quad (2)$$

where $\sigma \equiv v_{\parallel} / |v_{\parallel}| = +1$ or -1 , and the radial drift velocity is

$$v_{Dr} = \frac{m v_{\parallel}}{e r} \frac{\partial}{\partial \theta} \left(\frac{v_{\parallel}}{B} \right). \quad (3)$$

C is the collision operator, and Q is the RF quasilinear operator. The current is found by taking the first-order velocity space moment of f , $J_{\parallel} = -e \int d^3v v_{\parallel} f$.

Approximate Solutions

The DKE (1) is a partial differential equation in four dimensions, which makes finding a general solution for $f(E, \mu, r, \theta)$ complicated. One way to reduce the number of dimensions is to average over one bounce orbit in θ . This bounce average is defined as

$$\langle A \rangle \equiv \frac{1}{\tau_b} q R_0 \int \frac{d\theta}{v_{\parallel}} A \quad (4)$$

where the integral is taken from $\theta = -\pi$ to $+\pi$ for passing particles and from $\theta = -\theta_c$ to θ_c for trapped particles. τ_b is the time for the particle to make one bounce orbit or transit.

An approximate solution for f can be obtained by small parameter expansion. The first expansion parameter is the ratio of the width of a banana

orbit to the tokamak's minor radius, $\Delta_b/a \ll 1$. A second parameter is the ratio of the electron collision frequency (as derived with the collision operator C) to the frequency of bounce orbits, $\nu_c/\omega_b \ll 1$. Without loss of generality, we take both these parameters to be the same order of magnitude, $\sim \delta$.

Let $F \equiv \langle f \rangle$, and $\tilde{f} \equiv f - F$. Then (1) becomes

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial \tilde{f}}{\partial \theta} = C(F + \tilde{f}) + Q(F + \tilde{f}) - v_{Dr} \frac{\partial (F + \tilde{f})}{\partial r}. \quad (5)$$

Expanding $F = F_0 + \delta F_1 + \dots$, and $\tilde{f} = \tilde{f}_0 + \delta \tilde{f}_1 + \dots$, one can quickly see that $\tilde{f}_0 = 0$. The expanded drift kinetic equation is

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial \tilde{f}_1}{\partial \theta} = C(F_0) + Q(F_0) - v_{Dr} \frac{\partial F_0}{\partial r} \quad (6)$$

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial \tilde{f}_2}{\partial \theta} = C(F_1 + \tilde{f}_1) + Q(F_1 + \tilde{f}_1) - v_{Dr} \frac{\partial (F_1 + \tilde{f}_1)}{\partial r} \quad (7)$$

The bounce average of (6) gives an equation for F_0 :

$$0 = \langle C(F_0) + Q(F_0) \rangle \quad (8)$$

This is the equation for the modified electron distribution due to RF effects. Having found F_0 , we then use (6) to find \tilde{f}_1 : with v_{Dr} from (3),

$$v_{\parallel} \frac{B_{\theta}}{B} \frac{1}{r} \frac{\partial}{\partial \theta} \left(\tilde{f}_1 + \frac{mv_{\parallel}}{eB_{\theta}} \frac{\partial F_0}{\partial r} \right) = C(F_0) + Q(F_0) \quad (9)$$

Finally we bounce average (7) to get an equation for F_1 .

$$0 = \langle C(F_1 + \tilde{f}_1) + Q(F_1 + \tilde{f}_1) \rangle - \left\langle v_{Dr} \frac{\partial \tilde{f}_1}{\partial r} \right\rangle \quad (10)$$

Often a further simplification can be made by ignoring the last term in (10), when the banana width parameter is much smaller than the collisional parameter. The bootstrap current is contained in the F_1 and \tilde{f}_1 terms.

NUMERICAL ANALYSIS

The KT3D-NC code is a Fokker-Planck code which includes a quasilinear diffusion operator, similar to codes described in Ref. [2]. KT3D-NC is specifically designed to solve the approximate forms of the DKE in (8) and (10), to find the neoclassical components of the distribution function f_1 and F_1 and integrate to find the bootstrap current.

The functions are found on a given radial surface r using imposed density and temperature gradients, which do not change on the timescales of interest.

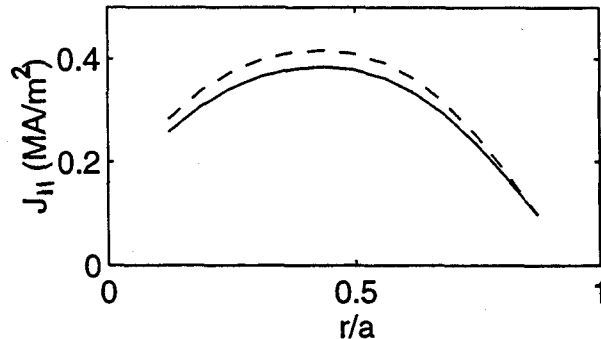


FIGURE 1. Bootstrap current J_{\parallel}^{BS} in MA/m² for a Maxwellian plasma from KT3D-NC (solid line) and Ref. 3 (dashed line).

Early benchmarking of KT3D-NC has been completed which shows that the bootstrap current can be calculated accurately in the absence of RF waves. Figure 1 shows a comparison of the values of J_{\parallel}^{BS} as calculated by KT3D-NC and analytically in Ref. [3], showing good agreement in the results.

KT3D-NC also contains a simple quasilinear operator for parallel diffusion in velocity space, which results from the damping of lower hybrid and fast Alfvén waves. Future modifications will allow us to model perpendicular diffusion due to electron cyclotron waves.

Currently, we are conducting more tests to show that KT3D-NC can accurately evaluate the RF modified distribution function F_0 . Then the code will be used to predict the bootstrap current in a variety of RF scenarios.

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