A coupling between cross-field density and temperature gradient scale lengths in high-recycling scrape-off layers

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Abstract

Energy transport in a high-recycling scrape-off layer is considered for the specific case when anomalous cross-field transport coefficients are nearly constant in space and volumetric power losses are small. In the case when upstream density and temperature profiles approximately follow an exponential variation across magnetic field lines, energy balance imposes a constraint on these profiles such that $\lambda_n \quad 0.4\,\lambda_T$, where λ_n and λ_T are respective characteristic gradient scale lengths. Contrary to intuition, this relationship approximately holds over a wide range of particle and energy transport coefficients and is simply a consequence of the transport physics: cross-field heat flux proportional to density and temperature, parallel heat transport independent of density and proportional to $T_e^{7/2}$.

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1. Introduction

Detailed measurements and modeling of cross-field density and temperature profiles in the scrape-off layer (SOL) of tokamak experiments are now being routinely performed (e.g., [1-3]). One important goal is to determine the magnitude of anomalous cross-field heat and particle transport in the SOL and its scaling with fundamental plasma parameters. Typically, numerical transport models employ adjustable coefficients for cross-field particle and heat transport which are determined from a best-fit to the experimental data.

If one were allowed free choice of the energy and particle source distributions and the transport coefficients in numerical simulations of SOL plasmas then one could produce cross-field density and temperature profiles of arbitrary shape. However, in many SOL plasmas of interest, volumetric heat sources/sinks are small and the heat transport is dominated by a balance between anomalous cross-field transport and classical parallel electron conduction to the divertor target. This regime often exists in "high-recycling divertors" in which the electron-ion mean free path is much shorter than the distance between divertor surfaces along magnetic field lines.

In this specific regime, one still has some "freedom" in specifying the particle source distribution and the magnitude of the anomalous cross-field transport coefficients. However, the energy equation enforces an interesting constraint on the problem: Since anomalous cross-field heat transport is (most likely) proportional to the local density while classical parallel heat transport is independent of density (proportional to $T_e^{7/2}$), there must exist a tight coupling between the shapes of the cross-field density and temperature profiles. Only within this constraint is one able to "independently" adjust density and temperature profiles via "external" fitting parameters (particle sources, transport coefficients). In the case when the cross-field profiles are well approximated by exponential functions (similar to many experimental situations), these "external" parameters offer little or no separate control over the density and temperature gradient scale lengths. Here, energy conservation simply enforces a relationship between gradient scale lengths, λ_n 0.4 λ_T.

The idea of a close coupling between λ_n and λ_T is not new. Previous analyses (e.g., [4,5]) of SOL transport physics have concluded that λ_n and λ_T must be of similar size. However, these arguments have been based on particle and energy balance considerations within a single flux tube (e.g., [4]) or on a dimensional analysis of cross-field gradient scale lengths (e.g., [5]). Although the latter analysis does yield the scaling, λ_n 0.4 λ_T, the underlying physics and its consequences are not immediately apparent in this kind of analysis. In examining a large set of discharges on JET and Alcator C-Mod, one does indeed find a tendency for λ_n 0.5 λ_T in high-recycling regimes, prompting a more detailed look at possible reasons for this behavior [6] and motivating the discussion in this paper.

The following two sections examine the coupling between λ_n and λ_T arising from the physics of energy transport in a high-recycling regime by considering two example cases. Section 2 considers SOL plasmas with constant anomalous cross-field transport coefficients and exponential cross-field T_e profiles. Section 3 uses a slightly different approach and considers SOL plasmas with D = 0.4 χ , V = 0, and with electron pressure profiles having a local gradient scale length that varies linearly with cross-field coordinate. The principal points of this analysis are summarized in Sec. 4.

2. Constant Cross-Field Transport Coefficients

Consider the specific case when the parallel heat flux in the scrape-off layer, $q_{//}$, is dominated by classical electron conduction such that

$$q_{//} = -\frac{2}{7}\,\kappa_{0//}\quad {}_{//}T_e^{7/2}, \tag{1}$$

where the Spitzer coefficient, $\kappa_{0//}$, is approximately 2800 W m^{-1} eV$^{-7/2}$. (Standard SI units are used throughout this paper with T_e in units of eV, e = 1.6x10^{-19} J eV^{-1}.) This situation can arise when the electron-ion mean free path (λ_{ei}) is much shorter than the distance along field lines to the divertor surfaces (L),

$$\frac{\lambda_{ei}}{L} \quad 1.5\mathrm{x}10^{16} \frac{T_e^2}{n\,L} << 1,$$

or equivalently, the plasma is in a "high-recycling" transport regime, i.e., significant temperature gradients exist along a magnetic field line. In this regime, one also typically finds strong collisional energy transfer between electrons and ions such that $T_i \sim T_e$.

Anomalous cross-field heat transport may be considered as some combination of heat conduction ($n\,\chi$) and heat convection arising from particle diffusion (D) and convection or "pinch" (V). For this analysis it is assumed that the anomalous cross-field heat flux can be approximated as

$$q \;= q^e + q^i \approx -2\,e\,n\,\chi \qquad T - 5\,e\,T\,D \qquad n + 5\,e\,T\,n\,V \;, \tag{2}$$

where the coefficients χ , D , and V are taken as constants. Assuming volumetric heat losses are small, the electron and ion energy conservation equations simply become

$$q \;+\; {}_{//}\,q_{//} = 0\,, \tag{3}$$

$$T_i = T_e = T\,. \tag{3b}$$

Integrating (3) along a field line,

$$q_{//}(s) \approx -\langle \quad q \;\rangle\, s\,, \tag{4}$$

where s is the distance from the symmetry point (s = 0) towards the divertor plate and $\langle\;\rangle$ indicates an average over the length of the field line. For $s = L$, the heat flux must match the heat through the divertor sheath,

$$q_{//}(L) = \gamma e\, n_{sh} C_s T_{sh} = -\langle \quad q \; \rangle L\,, \tag{5}$$

and be consistent with the values of density (n_{sh}), sound speed (C_s), electron temperature (T_{sh}), and heat transmission factor (γ) at the sheath edge. Integrating (4) again from $s = 0$ to L using (1),

$$T_{sym}^{7/2} - T_{sh}^{7/2} \approx -\frac{7L^2}{4\kappa_{0//}} \langle \quad q \; \rangle\,. \tag{6}$$

From (5) and (6) one can show that $T_{sym}^{7/2} >> T_{sh}^{7/2}$ when $\frac{\lambda_{ei}}{L} << 1$, i.e., in the high-recycling regime. From (2) and (6),

$$\frac{4\kappa_{0//}}{e\,7L^2} T_{sym}^{7/2} \approx \langle \quad [2n\chi \qquad T - 5TD \qquad n + 5TnV\;]\rangle\,. \tag{7}$$

As is customarily done in simple two-point analyses of energy transport in the scrape-off layer, the right hand side of (7) is now approximated by its value "upstream" at the symmetry location,

$$\frac{4\kappa_{0//}}{e\,7L^2} T_{sym}^{7/2} \approx \quad [2n_{sym}\chi \qquad T_{sym} - 5T_{sym}D \qquad n_{sym} + 5T_{sym}n_{sym}V\;]\,. \tag{8}$$

Now suppose experimental measurements of the upstream electron temperature profile indicate that it is accurately represented by an exponential function of cross-field coordinate (ρ),

$$T_{sym} = T_0 \exp[-\rho/\lambda_T]\,.$$

In this case, (8) can be written in the form (dropping the "sym" notation),

$$^{2}\, n - \alpha \qquad n + \beta n = \delta \exp[-5\rho/2\lambda_T]\,, \tag{9}$$

with the definitions:

$$\alpha = \frac{2\chi \quad \lambda_T}{5D} + \frac{1}{\lambda_T} + \frac{V}{D}\,, \tag{9a}$$

$$\beta = \frac{\alpha}{\lambda_T} - \frac{1}{\lambda_T^2}\,, \tag{9b}$$

$$\delta = \frac{4\,\kappa_{0//}\,T_0^{5/2}}{35\,e\,D\;\;L^2}\;. \tag{9c}$$

The solution to (9) is the sum of homogeneous and particular solutions,

$$n(\rho) = n_H + n_p\,.$$

The particular solution is,

$$n_p = n_{p0}\,\exp[-5\,\rho/2\,\lambda_T]\,, \tag{10}$$

with

$$n_{p0} = \frac{4\,\kappa_{0//}\,T_0^{5/2}\,\lambda_T{}^2}{35\,e\,D\;\;L^2}\left(\frac{35}{4} + \frac{7\,\chi}{5\,D} + \frac{7\,V\;\;\lambda_T}{2\,D}\right)^{-1}. \tag{10a}$$

The homogeneous solution is of the form,

$$n_H = A\,\exp\!\left[\rho\left(\frac{2\,\chi}{5\,D\;\;\lambda_T} + \frac{V}{D}\right)\right] + B\,\exp\!\left[\frac{\rho}{\lambda_T}\right], \tag{11}$$

with coefficients A and B determined by two boundary conditions such as the total plasma flux to the wall and the plasma density at the wall.

Note that in the absence of a strong inward pinch, the homogeneous solution is an exponentially increasing function of ρ. In this case, one would require $A \to 0$ and $B \to 0$ in order to have bounded densities at a distant wall. The density profile would therefore be determined entirely by the particular solution in this case.

Although direct information on the pinch velocity and/or appropriate boundary conditions is often lacking, the density profile shape must be consistent with experimental measurements. When measurements show that the cross-field density profile closely follows an exponential behavior,

$$n = n_0\,\exp[-\rho/\lambda_n]\;,$$

then again $A \to 0$ (excepting a special case below), $B \to 0$, and the particular solution alone (n_p) determines the density gradient scale length. Thus, one would expect the e-folding lengths of the temperature and density profiles to be simply related by $\lambda_n = 0.4\ \lambda_T$. In this case, the particular solution also sets the density at the separatrix. Taking values near the separatrix in Alcator C-Mod of D = 0.03 m^2 s^{-1}, χ = 0.1 m^2 s^{-1}, V = 0 m s^{-1}, λ_T = 0.01 m, L = 10 m, and T = 65 eV, eq. (10a) yields the density n_p = 1.7x10^{20} m^{-3} which is consistent with measured values - although one should be aware that there is tremendous leeway here in the choice of the transport coefficients! For the special case when

$$\frac{2\chi}{5D\ \ \lambda_T} + \frac{V}{D} = -\frac{5}{2},$$

coefficient A may not be zero. However the relationship between e-folding lengths is the same.

This result is a bit counter-intuitive. Intuitively one might expect the cross-field density profile to be somewhat independent of the temperature profile and set by the magnitude of particle sources (boundary conditions) and anomalous particle transport. However, these results show that in a regime where classical parallel heat conduction dominates the energy equation, there exists a tendency for the density profile to be set more by the requirements of energy balance. (Or one may consider that the reverse statement is more physically correct: the power flow profile in the SOL, $T_e^{7/2}$, is constrained by the cross-field density profile.) In actuality, plasma profiles are not exactly exponential and transport coefficients may not be constant in space, admitting a richer set of density and temperature profile "solutions". Yet, the tendency for λ_n ~ 0.4 λ_T exists.

Finally, one can more fully appreciate the underlying physics of this coupling by performing the following thought experiment: Let the parallel heat flux not have the Spitzer relationship but have the proportionality,

$$q_{//} \quad -n \quad {}_{//}T\,. \tag{1'}$$

That is, take the parallel conductivity to be independent of temperature and proportional to the local density. In this case, eq. (9) becomes a homogeneous equation with constant coefficients,

$$^2 n - \alpha \qquad n + \beta' n = 0. \qquad (9')$$

In stark contrast to actual plasmas, the density gradient scale lengths in this fictitious plasma could now be adjusted arbitrarily for virtually any value of λ_T by the "external" fitting parameters, i.e., the density and/or particle flux boundary conditions and the magnitude of the cross-field transport coefficients.

3. Special Case: $D_\perp = 0.4\,\chi_\perp$, $V_\perp = 0$

An interesting situation arises when the cross-field diffusivities have the approximate relationship, $D \quad \approx 0.4\,\chi$. In this case, nearly exact analytic expressions for the 2-D temperature and density profiles in the SOL can be constructed. Again, assume that volumetric losses are small. With $D \quad = 0.4\,\chi$, eqs. (2) and (6) yield

$$T(s)^{7/2} - T_{sh}^{7/2} \approx \frac{7e\left(L^2 - s^2\right)}{4\kappa_{0//}} \left\langle 2\chi \qquad {}^2 nT - 5V \qquad nT \right\rangle . \qquad (12)$$

If the magnetic field lines in the SOL were straight (i.e., no poloidal flux expansion or compression) and acceleration to sonic flows occurred close to the divertor plate such that $nT \approx$ constant on a field line, then eq. (12) would approach the exact 2-D relationship (with the implicit assumptions of small volumetric energy sources/sinks, classical parallel transport, etc.),

$$T(s,\rho)^{7/2} = T_{sh}(\rho)^{7/2} + \frac{7e\left(L^2 - s^2\right)}{4\kappa_{0//}} \left(2\chi \quad P'' - 5V \quad P'\right), \qquad (13)$$

where $P(\rho) = nT$ is defined as the stagnation electron pressure on a field line passing through major radius $R = R_{sep} + \rho$ at the outer midplane (R_{sep} is the major radius at the separatrix). The prime superscript (') indicates differentiation with respect to ρ. The sheath boundary condition, eq. (5), sets the electron temperature profile at the divertor sheath for a specified pressure profile,

$$T_{sh}(\rho) \ = \ \frac{2\, e\, m_i\, L^2}{\gamma^2}\left(2\,\chi \ \ \frac{P''}{P} - 5\,V \ \ \frac{P'}{P}\right)^2 . \qquad (14)$$

Here the definition of sound speed, $C_s^2 \ = \ \frac{2\, e\, T}{m_i}$, has been used.

In experiments, it is often observed that the cross-field electron pressure profile in the SOL approximates an exponential behavior with a local gradient scale length that increases as function of ρ [1]. Consider the case when the midplane pressure profile varies across field lines as,

$$P(\rho) \ = \ P_0\left(1 + \frac{\varepsilon_p\, \rho}{\lambda_{p0}}\right)^{-1/\varepsilon_p} . \qquad (15)$$

This function becomes an exponential in the limit ε_p -> 0,

$$\mathrm{Lim}_{\varepsilon_p \to 0}\left\{ P_0\left(1 + \frac{\varepsilon_p\, \rho}{\lambda_{p0}}\right)^{-1/\varepsilon_p}\right\} = \ P_0 \exp\left(- \frac{\rho}{\lambda_{p0}}\right),$$

and has a "local e-folding length", λ_p, that varies linearly with cross-field coordinate according to the parameter ε_p,

$$\lambda_p(\rho) \ \equiv \ -\frac{P}{P'} \ = \ \lambda_{p0} + \varepsilon_p\, \rho\, .$$

With this pressure profile (and setting V = 0 for algebraic simplicity), eqs. (13) and (14) become,

$$T(s,\rho)^{7/2} = T_{sh}(\rho)^{7/2} + \frac{7\, e\left(L^2 - s^2\right)}{2\, \kappa_{0\,//}} \ \frac{\chi \ \ (1+\varepsilon_p)\, P_0}{\lambda_{p0}^2\left(1 + \frac{\varepsilon_p\, \rho}{\lambda_{p0}}\right)^{1/\varepsilon_p + 2}} , \qquad (16)$$

$$T_{sh}(\rho) \ = \ \frac{8\, e\, m_i\, L^2 \chi^2}{\gamma^2}\frac{(1+\varepsilon_p)^2}{(\lambda_{p0} + \varepsilon_p\, \rho)^4} \ . \qquad (17)$$

Now consider the temperature profile at the symmetry point (s=0) from eq. (16). In high-recycling flux tubes where $T_{sym}^{7/2} >> T_{sh}^{7/2}$, the cross-field temperature profile at the symmetry point has the form,

$$T_{sym}(\rho) \ = \ T_{sym}(0)\left(1 + \frac{\varepsilon_p\, \rho}{\lambda_{p0}}\right)^{-2/7\varepsilon_p - \ 4/7} .$$

From eq. (15), the density profile must have the form,

$$n_{sym}(\rho) = n_{sym}(0)\left(1 + \frac{\varepsilon_p \rho}{\lambda_{p0}}\right)^{-5/7\varepsilon_p + 4/7},$$

yielding local e-folding lengths for temperature,

$$\lambda_T(\rho) = \frac{7\lambda_p(\rho)}{2 + 4\varepsilon_p}$$

and density,

$$\lambda_n(\rho) = \frac{7\lambda_p(\rho)}{5 - 4\varepsilon_p} .$$

In this case, the energy equation imposes a relationship between local density and temperature gradient scale lengths of

$$\lambda_n(\rho) = \left(\frac{2 + 4\varepsilon_p}{5 - 4\varepsilon_p}\right)\lambda_T(\rho) . \tag{18}$$

As before, the relationship $\lambda_n \sim 0.4\ \lambda_T$ is obtained for purely exponential electron pressure profiles. Larger values of λ_n / λ_T result when the local pressure gradient scale length increases with ρ (see Table I).

Since one expects the kinetic energy density (i.e., pressure) in all SOL plasmas to decay across the magnetic field more or less exponentially (independent of ionization source locations, wall boundary conditions, flows, etc.), then eq. (18) indicates that one should also expect that the temperature and density profiles decay more or less exponentially with $\lambda_n \sim 0.4\ \lambda_T$ in high-recycling regimes (with implicit assumptions of low volumetric power losses, Spitzer parallel conduction, etc.). Note that this relationship is not a consequence of the assumption made here of $D = 0.4\ \chi$ (recall Sec. 2). The reason $\lambda_n \sim 0.4\ \lambda_T$ is because the cross-field heat flux is proportional to density and temperature while parallel heat transport is independent of density and proportional to $T_e^{7/2}$.

Finally, it is worth commenting on the cross-field temperature profile at the divertor sheath edge. For purely exponential pressure profiles ($\varepsilon_p = 0$), eq. (17) yields a flat temperature profile at the sheath edge. Thus, if the SOL plasma is in the high-recycling regime at some ρ coordinate (yielding an exponential cross-field T_e profile at the symmetry point), then a transition to a sheath-limited regime ($T_{sym} \quad T_{sh}$) can occur at some larger ρ value in the SOL. This behavior is seen in experiments [1].

4. Summary

The physics of energy transport in a high-recycling scrape-off layer imposes a strong coupling between the shapes of the cross-field density and temperature profiles. In scrape-off layers where volumetric power losses are small and upstream density and temperature profiles approximately follow an exponential variation across magnetic field lines, energy balance requires that $\lambda_n \quad 0.4\ \lambda_T$, where λ_n and λ_T are respective characteristic gradient scale lengths. The relationship is a consequence of the assumptions that cross-field heat flux is locally proportional to density and temperature and that parallel heat flux is independent of density and proportional to $T_e^{7/2}$. Contrary to intuition, the trend holds over a wide range of cross-field particle and energy transport coefficients and is insentive to boundary conditions of density and/or particle flux. Thus, for the purposes of modeling exponential density and temperature data in high-recycling regimes, the relative magnitudes of spatially constant, cross-field particle and energy transport coefficients offers little or no control over the ratio of temperature and density gradient scale lengths. Although plasma profiles are not exactly exponential and transport coefficients may not be constant in space, with the exception of extreme cases, the underlying transport physics still imposes a tendency for a local gradient scale length relationship, $\lambda_n \sim 0.4\ \lambda_T$.

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Table I - Values of λ_n/λ_T from eq. (18)

ε_p	0	0.083	0.25	0.375
λ_n/λ_T	0.4	0.5	0.75	1