THE EFFECTS OF SECONDARY FLOWS ON THE HEAT TRANSFER TO TURBINE NOZZLE ENDWALL AND ROTOR SHROUD

by

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Chairman, Departmental Committee on Graduate Students.

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Submitted to the Department of Mechanical Engineering in October 1978 in partial fulfillment of the requirements for the degree of Doctor of Science in Mechanical Engineering.

ABSTRACT

The improvement of the efficiency of gas turbines is one of the primary goals of the gas turbine industry. The efficiency can be increased by increasing the gas inlet temperature, for a given pressure ratio; but material considerations limit the gas temperature that can be employed. In order to overcome the overheating problems experienced by certain turbine components, a better understanding of the factors that influence heat transfer to the components is required.

In the present work, the effects of secondary flows on turbine heat transfer are investigated. The current secondary-flow equation was solved, taking into account the acceleration of the flow and the variations of the turning of the different streamlines. It was found that even for small inlet shear-flow, strong secondary vortices develop in the blade passage because of the large turning the flow undergoes.

Experiments were carried out over a wide range of flow profiles and the average heat-transfer rates for the nozzle endwall and the rotor shroud measured. Empirical formulae (incorporating the effects of secondary vorticity) were found that gave good predictions of the Nusselt-number distributions for the nozzle endwall and the rotor shroud.

Thin-film heat-transfer gauges, capable of detecting temperature changes as small as a fraction of a degree and having a frequency of the order of MHz, were built. Such a gauge was used to measure the temperature changes on a cascade wall due to the compressibility of the air at the tip of an air turbine. Because of the large tip clearance and the insufficiently clean environment in the T64 turbine, no temperature variation was detected on the rotor shroud.

DOCTORAL COMMITTEE

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Title: Professor of Mechanical Engineering.

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Title: Professor of Mechanical Engineering.

Dr. Okon Amana
Title: Research Associate.
TO MY BELOVED LATE PARENTS
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a Measure of steepness of inlet-flow profile (equation 2-38 and figure 2-5).

b Rotor-blade axial chord (equation C-8).
Distance measured in the blade spanwise direction.
Distance measured in the binormal direction.

b Unit vector in the binormal direction.

b' Unit vector in the relative binormal direction.
c Specific heat.
Blade chord (equation 2-7).
c\_l Constant for thin-film heat-transfer gauge (equation D-1).
c\_g Specific heat of copper (equation C-3).
c\_p Specific heat at constant pressure.
dL Streamline length (equation 2-6).

f(x_1,x_2,...) Numerator in equation B-1.
g\_c Gravitational conversion factor, 32.2 lbm-ft/lbf-sec^2.
h Heat transfer coefficient.

h_{air} Heat transfer coefficient at the copper gauge/air interface (equation C-2).

h_{air_x} Local gas heat transfer coefficient (equation C-8).
k Designation of potential surface positions (figures 2-2, 2-4, 2-6).
Thermal conductivity.

k_{air_x} Local gas thermal conductivity (equation C-8).
k_{ins} Thermal conductivity of copper-gauge insulating material (nylon), (equation C-4).
\( m_g \) Mass of copper slug (gauge), (equation C-3).
\( \dot{m} \) Injection mass-flow rate.
\( \dot{m}_{ac} \) Actual mass-flow rate.
\( \dot{m}_{ex} \) Experimental mass-flow rate.
\( n \) Streamline principal normal direction.
\( \mathbf{n} \) Blade pitchwise direction.
\( \mathbf{R} \) Unit vector in the principal normal direction.
\( \mathbf{n}' \) Direction of the principal normal to the relative streamline.
\( q \) Magnitude of absolute velocity vector.
\( q_1, q_2 \) Magnitudes of absolute velocities on a streamline, location 1 being upstream of location 2.
\( \dot{q}_{lg} \) Rate of heat loss from the copper gauge (equation C-1).
\( \dot{q}_{sg} \) Rate of heat storage in the copper gauge (equation C-1).
\( \dot{q}_{tgr} \) Rate of heat transfer to the copper gauge (equation C-1).
\( r \) Distance measured in the blade spanwise (radial) direction.
\( r_h \) Blade hub radius = 7.17 inches.
\( r_T \) Blade tip radius = 8.16 inches.
\( s \) Streamline direction.
\( \mathbf{s} \) Unit vector in the streamline direction.
\( s' \) Relative streamline direction.
\( \mathbf{s}' \) Unit vector in the relative streamline direction.
\( s_1 \) Constant for thin-film heat-transfer gauge (equation D-2).
\( t \) Time.
\( v \) Magnitude of velocity vector.
\( v_1 \) Velocity of stream 1.
\[ v_2 \] Velocity of stream 2.

\[ v_b \] Spanwise component of secondary vorticity.

\[ v_{b2} \] Value of \( v_b \) at downstream end of a streamline segment.

\[ \underline{\underline{v}_2} \] Absolute velocity vector at downstream end of a streamline segment.

\[ v_n \] Pitchwise component of secondary velocity.

\[ v_{n2} \] Value of \( v_n \) at downstream end of a streamline segment.

\[ v_r \] Radial component of absolute velocity.

\[ x \] Streamline length, measured from the leading edge of the endwall or the shroud.

\[ x_1 \] Streamline length from the leading edge of the endwall to the position of gauge 1.

\[ y \] Distance measured perpendicular to the shroud (equation D-3).

\[ z \] Axial direction of the turbine (equation 2-22).

Distance measured in the blade spanwise direction (equations 2-38 and 4-5).

\[ 1,2 \] Positions on a streamline, 1 being upstream of 2.

\[ A \] Constant in equation 2-9.

\[ A_1 \] Flow area at location of gauge 1 (equation E-2).

\[ A_g \] Cross-sectional area of copper gauge (equation C-2).

(Paint) area of thin-film heat-transfer gauge (equation D-2).

\[ A_{\text{ins}1} \] Cross-sectional area of copper gauge (equation C-4).

\[ A_{\text{ins}2} \] Lateral area of copper gauge (equation C-4).

\[ B \] Rotor-blade axial chord.

\[ B_{\text{noz}} \] Measure of influence of passage geometry on the nozzle secondary vorticity (equation 2-14).

\[ B_{\text{rot}} \] Measure of influence of passage geometry on the rotor secondary vorticity (equation 2-35).
D  Position of oscillograph pen (equation C-12).
D_i Initial position of oscillograph pen (equation C-12).
H  Distance separating free streams 1 and 2.
I  Equivalent total pressure relative to a rotating frame,
   \( I = P + \frac{1}{2} \rho W^2 - \frac{1}{2} \rho (\omega r)^2 \), (equation 2-18).
   Current applied to thin-film heat-transfer gauge (equation D-2).
   Spanwise numbering of potential streamline locations (figure 2-9a).
J  Energy conversion factor, 778 ft-lbf/Btu.
K_p  Tip-leakage parameter (equation 4-10).
L  Reference length in vorticity number (equation 4-3).
   Turbine reference length (equation 3-4).
M  Main mass-flow rate.
\( \dot{M}_T \)  Total mass-flow rate, \( \dot{M}_T = \dot{m} + \dot{M} \).
\( \dot{m}/\dot{M} \)  Ratio of injection to main mass-flow rates.
Nu  Nusselt number.
\( \text{Nu}_x \)  Nusselt number based on streamline length, \( x \).
P  Fluid static pressure.
P_o  Fluid stagnation pressure.
P_{pr}  Pressure on the rotor-blade pressure surface.
P_{s1}  Tip static pressure at the inlet to the nozzle.
P_{s2}  Tip static pressure at the inlet to the rotor.
P_{s3}  Tip static pressure at the exit from the rotor.
P_{suc}  Pressure on the rotor-blade suction surface.
P_T  Fluid stagnation pressure.
P_T1 Stagnation pressure at the inlet to the nozzle.
$P_{T2}$ Stagnation pressure at the inlet to the rotor.

$P_{T3}$ Stagnation pressure at the exit from the rotor.

$R$ Gas constant.

$R$ Radius of curvature of absolute streamline.

$R'$ Radius of curvature of relative streamline.

$R_g$ Electrical resistance of thin-film heat-transfer gauge (equation D-2).

$Re_x$ Reynolds number based on streamline length, $x$.

$Re_{x1}$ Reynolds number at location of gauge 1, based on streamline length.

$S$ Turbine scaling factor (equation 3-1).

$T$ Temperature (equation D-3).

$T_{ac}$ Temperature in actual turbine.

$T_{air}$ Air temperature (equation C-2).

$T_{ex}$ Temperature in experimental turbine.

$T_g$ Temperature of copper gauge (equation C-2).

$T_i$ Copper-gauge temperature at the start of a test (equation C-12).

$T_{inj}$ Injection-flow static temperature.

$T_{tl}$ Stagnation temperature at the inlet to the nozzle.

$T_{t2}$ Stagnation temperature at the inlet to the rotor.

$T_{t3}$ Stagnation temperature at the exit from the rotor.

$T_{th}$ Main-flow static temperature.

$T_{wl}$ Copper-gauge temperature just before the diaphragm burst (equation C-4).

$T_{w2}$ Average temperature of copper-gauge insulation, $T_{w2} = 1/2(T_{wl}+T_g)$.

$U$ Blade circumferential velocity.

$U_T$ Rotor-tip speed.

$V$ Magnitude of fluid velocity.
Velocity vector.

$V_1$ Absolute fluid velocity at the inlet to the rotor.

Fluid velocity at location of gauge 1 (equation E-1).

$V_z$ Fluid axial velocity.

$V_{z1}$ Fluid axial velocity at inlet to the rotor.

$V_\theta$ Fluid circumferential velocity.

$V_{\theta1}$ Fluid circumferential velocity at inlet to the rotor.

$V_\infty$ Magnitude of free-stream velocity (equation 2-38).

$W$ Magnitude of fluid relative velocity.

$W$ Fluid relative velocity vector.

$W_1$ Fluid relative velocity at inlet to the rotor.

Fluid relative velocity at the upstream end of a streamline segment.

$W_2$ Fluid relative velocity at downstream end of a streamline segment.

$W_r$ Radial component of fluid relative velocity.

$W_z$ Axial component of fluid relative velocity.

$W_{z1}$ Value of $W_z$ at inlet to the rotor.

$W_\theta$ Circumferential component of fluid relative velocity.

$W_{\theta1}$ Value of $W_\theta$ at inlet to the rotor.

$X$ Measurement in rotor-blade axial direction, starting from the blade leading edge.

$\alpha$ Flow-turning angle.

Thermal diffusivity.

$\alpha_1$ Flow angle at upstream end of an absolute streamline segment.

$\alpha_2$ Flow angle at downstream end of an absolute streamline segment.

$\alpha_0$ Value of $\alpha$ far upstream of the blade row.
\( \alpha_1 \) Relative flow angle at upstream end of a streamline segment.

\( \alpha_2 \) Relative flow angle at downstream end of a streamline segment.

\( \beta \) Mixing parameter (equation 4-6).

\( \gamma \) Ratio of specific heats, \( c_p/c_v \).

\( \gamma_1 \) Relative flow angle (measured from axial direction) at rotor inlet.

\( \theta \) Circumferential direction.

\( \theta_1 \) Absolute flow angle (measured from axial direction) at rotor inlet.

\( \varphi \) Angle between the normal to the Bernoulli (constant-total-pressure) surface and the binormal direction.

\( \varphi_0 \) Value of \( \varphi \) far upstream of the blade row.

\( \varphi' \) Angle between the normal to the relative Bernoulli plane and the relative binormal direction.

\( \phi \) Angle between the normal to the relative Bernoulli plane and the rotation vector.

\( \rho \) Fluid density.

\( \rho_1 \) Fluid density at the location of gauge 1.

\( \varepsilon \) Flow total-turning angle through the passage.

\( \omega \) Angular speed of rotating frame.

\( \psi \) Secondary-flow stream function.

\( \delta \) Boundary-layer thickness.

Extent of cosine profile at blade inlet (equation 2-38 and fig. 2-5).

\( \mu \) Fluid dynamic viscosity.

\( \nu \) Fluid kinematic viscosity.

\( \Omega \) Magnitude of absolute vorticity vector.

\( \Omega \) Absolute vorticity vector.

\( \Omega_{nl} \) Pitchwise component of vorticity at inlet to the passage.
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<td>$\Omega_\theta$</td>
<td>Circumferential component of vorticity.</td>
</tr>
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<td>$\psi$</td>
<td>Vorticity number, defined in equations 4-3 and 4-5.</td>
</tr>
<tr>
<td>$\phi_T$</td>
<td>Tip-flow coefficient.</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Kinetic energy parameter (equation 4-9).</td>
</tr>
<tr>
<td>$\Delta D$</td>
<td>Oscillograph-pen deflection.</td>
</tr>
<tr>
<td>$\Delta R_g$</td>
<td>Change in electrical resistance of thin-film heat-transfer gauge.</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time difference.</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>Total-temperature drop across rotor stage (equation 4-9).</td>
</tr>
<tr>
<td>$\Delta T_g$</td>
<td>Change in temperature of copper gauge (equation C-3).</td>
</tr>
<tr>
<td>$\Delta x_1$</td>
<td>Thickness of insulation at the bottom of copper gauge.</td>
</tr>
<tr>
<td>$\Delta x_2$</td>
<td>Thickness of insulation at the sides of copper gauge.</td>
</tr>
<tr>
<td>$\Delta V_g$</td>
<td>Change in voltage from thin-film heat-transfer gauge.</td>
</tr>
</tbody>
</table>
1.1 INTRODUCTION

One of the primary concerns of gas turbine designers is the improvement of the efficiency of gas turbines. The recent increase in the price of fuels and the realization that the present commercially-available fuels are not inexhaustible have heightened this need.

Thermodynamics considerations show that the efficiency of the gas turbine cycle depends on the compressor inlet temperature, the turbine inlet temperature and the pressure ratio. For a given pressure ratio, the thermal efficiency increases with increasing turbine inlet temperature. The use of higher gas inlet temperature will result in higher specific energy per unit mass of air. This will in turn result in smaller turbines, for a given work output, with consequent gain in payload (if the turbine is used to power aircrafts, say). The theoretical upper limit of the turbine inlet temperature is that achieved at stoichiometric fuel-air ratios. In practice, however, the thermal constraints of the materials used in building the turbine components limit the gas inlet temperature to well below the fuel stoichiometric temperature.

Certain critical parts of the gas turbine suffer from over-heating and the thermal failure of these parts indicate that secondary flows may influence the over-heating. [Secondary flows are produced when an originally-sheared flow is turned (by turbomachine blades, say). At the inlet to the flow pa-
ssage, the shear flow will contain vortex lines that are perpendicular to the streamlines. The turning of the flow aligns the vorticity vector in such a way that it has a component in the streamline direction. The streamline component of vorticity (or secondary vorticity) induces a flow pattern which is perpendicular to the main flow direction. This induced flow is called secondary flow]. Secondary flows also affect the aerodynamic performance of the turbine by changing the blade loading and the gas exit angles. It is thus apparent that a better understanding of secondary flows would aid the overall effort to improve the turbine efficiency.

1.2 CLASSIFICATION OF SECONDARY FLOWS

In a gas turbine, secondary flows can be brought about by one or more of several factors, as given by Griepentrog [1] and Salvage [2].

(i) The turning of flow of non-uniform velocity or total-pressure profile, as it passes through the blade passage. This results in the formation of passage secondary flows.

(ii) The leakage of fluid at the tip and/or hub of a blade, from the pressure side to the suction side of the blade, giving rise to tip-leakage vortices.

(iii) The scraping of the shroud boundary layer by the rotor blades, resulting in scraping vortices.

(iv) Spanwise variation of circulation, giving rise to trailing shed vorticity.

Numbers in square brackets indicate references at the end of the thesis.
Figure 1-1 depicts the different types of secondary flows and vortices in a turbine.

PASSAGE SECONDARY FLOW

Consider a shear flow upstream of a blade row. On encountering the blades, the streamlines are deflected by the blades and a circumferential pressure gradient develops within the blade passage, the pressure increasing from the suction to the pressure surface. If there is a balance between pressure gradient and centrifugal forces, we can write:

\[
\frac{\partial p}{\partial n} = \frac{\rho v^2}{R}
\]

where

\[
\frac{\partial p}{\partial n} \equiv \text{pressure gradient normal to the streamlines.}
\]

\[
\rho \equiv \text{fluid density.}
\]

\[
v \equiv \text{local fluid velocity.}
\]

\[
R \equiv \text{local radius of curvature of the streamline.}
\]

The pressure impressed at the edge of the boundary layer is the same inside the layer and in the inviscid region outside the boundary layer, for a given circumferential and axial position. However, the velocity in the boundary layer is smaller than that in the main flow. Therefore, for constant \(\frac{\partial p}{\partial n}\), \(R\) must decrease as \(v\) decreases. Hence the fluid in the boundary layer experiences more turning than the main flow. The low-momentum boundary-layer fluid moves towards the corner formed by the blade suction surface and the endwall (or hub). At the suction surface, the boundary-layer fluid turns
and continues its movement along the blade span, but it encounters the main flow and rolls up into a spiral to form a corner vortex.

**LEAKAGE VORTEX**

For non-rotating shrouds, there is a clearance between the tip of the rotor blade and the shroud. Since there is a pressure gradient across this gap, there is the tendency for the boundary-layer fluid near the gap to leak from the pressure to the suction side of the blade. The fluid that so leaks through the tip clearance does not take part in useful energy transfer across the blade. Instead the fluid rolls up into a tip-leakage vortex, the direction of which is opposite to that of the passage vortex.

**SCRAPING VORTEX**

Boundary layers are formed on both the casing and the hub. For a sufficiently small tip clearance, the blade-passing time may be longer than the time it takes the boundary layer to grow thicker than the clearance. The result is that each blade that passes over a given point on the casing scrapes off the boundary layer. In a turbine, the scraping vortex so formed has the same direction as the passage vortex. It has an opposite direction in a compressor. For a given fluid axial velocity, the effect of the scraping action becomes more pronounced at very low flow coefficients.

**TRAILING SHED VORTICITY**

For a blade (even in uniform flow), if there is a spanwise variation of circulation, a vortex sheet is formed at the trailing edge of the blade.
Since a vortex sheet is generally unstable, it rolls up into a spiral to form a trailing-shed vortex.

RADIAL FLOWS

If there is a spanwise pressure gradient, the boundary-layer fluid on the blade may start moving in the spanwise direction along the blade. (Recall the Radial Equilibrium Equation, $\frac{\partial p}{\partial r} = \frac{\rho v_r}{r}$). In general, radial movement is to the hub for the stator and to the tip for the rotor.

1.3 REVIEW OF SECONDARY FLOWS

As early as 1876, Professor J. Thomson, in a paper to The Royal Society [3], explained that it is the cross-channel (secondary) flow in rivers that is responsible for the deposition on and the erosion of river banks. It was not until about three quarters of a century later that the first significant quantitative description of secondary flow appeared. Squire and Winter [4] showed that the change of secondary vorticity along a streamline is equal to twice the product of the inlet normal vorticity and the angle turned by the flow. In their analysis, they assumed that the flow was steady, incompressible, inviscid, with no body force and that the magnitude of the flow velocity was constant along a streamline.

Since the early 1950s, a great deal of work has been done in trying to understand the influence of secondary flows under different flow conditions. An area of particular interest was flow through turbomachine blades, especially compressor blades. Preston [5], using semi-geometrical reasoning,
obtained a result similar to Squire and Winter's. It was obvious that the Squire and Winter's relation was very restrictive in its application and could not adequately explain the experimentally-observed secondary flows.

Hawthorne [6] used vector analysis and the equations of fluid flow to derive a more general relation for the generation of secondary vorticity. Hawthorne's secondary-flow equation, as it has come to be called, is the cornerstone of most of the secondary-flow analyses done up to date. The equation should strictly be called the passage secondary-flow equation. Because of its importance, its implications will be discussed here. The equation is given below.

$$\left(\frac{\Omega_2}{q}\right)_2 - \left(\frac{\Omega_1}{q}\right)_1 = 2\int_1^2 \nabla \left(\frac{P}{\rho}\right) \cos \varphi \left(\frac{\partial \alpha}{\partial \varphi}\right)_2$$

where

1, 2 = positions on a streamline, 1 being upstream of 2.

$\Omega_s$ = streamwise component of vorticity.

$P_o$ = fluid total pressure.

$\rho$ = fluid density.

$\varphi$ = angle between the binormal and the normal to the Bernoulli (constant total-pressure) surface.

$q$ = magnitude of fluid velocity.

$\alpha$ = angle turned through by the fluid.

The equation assumes that the flow is steady, incompressible, inviscid and with no body force. Like the Squire and Winter's relation, it assumes that secondary flow is a small perturbation of the main flow. According to Hawthorne's equation, three conditions are necessary for the existence of
secondary vorticity.

1. There has to be a gradient in the fluid total pressure.

2. The normal to the Bernoulli surface is not parallel to the principal normal.

3. The flow undergoes a turning.

Hawthorne has, since the original paper, applied the equation to different flow conditions [7-11].

The accuracy attained with Hawthorne's equation depends (in part) on how appropriate the underlying assumptions are for any given situation. Lakshminarayana and Horlock [12] obtained expressions for secondary vorticity, which incorporates the effects of compressibility and viscosity, while in reference [13], Lakshminarayana demonstrated that total-temperature gradients (as exist at the exit of combustion chambers), in the presence of gradients in total pressure, will also generate secondary flows in the blade passages. Smith [14] derived a relation for the development of secondary vorticity in rotating passages; it is pointed out in reference [12] that rotation gives rise to additional secondary vorticity only when the cross product of the absolute vorticity and the rotation vectors has a component in the relative streamline direction.

Considerable effort has been put into improving the accuracy of the solution of the secondary-flow equation. Papailiou et al [15] states that the equation has been remarkably successful in predicting the compressor cascade fluid-outlet angle, but not so successful in predicting the cascade losses
due to secondary flows. The presently-available computational methods do not permit the solution of the equation without first making simplifying assumptions. A common simplification is to assume that the flow is incompressible and inviscid and that the Bernoulli surface does not distort as the fluid goes through the passage. The results obtained from such a solution overestimate the strength of the secondary vorticity. Dean [16] included the effects of the Bernoulli surface rotation in the solution while Louis [17] included the effects of viscosity. Each of these methods was an improvement on the original assumption. To the author's knowledge, Lakshminarayana and Horlock's work [18] is one of the best for predicting compressor-cascade outlet-flow angles. They took into account the effects of secondary flow, Bernoulli-surface rotation, viscosity and spanwise displacement due to tangential vorticity. The agreement between the measured and the predicted values is remarkably good. They used their analysis to predict the changes in the gas outlet angles for a compressor cascade having a total turning of 20 degrees 53 minutes, a blade chord of 6 inches and a blade height of 29 inches. The maximum prediction error is 64%.

The present theory treats secondary flow as a small perturbation of the main flow, in which the secondary vorticity is transported by the main flow. Flow-visualization studies done at the NACA Lewis Laboratory [19], however, indicate that in certain cases, the secondary velocity is of the same order of magnitude as the primary velocity. For a flow turning of 60°, it was noticed that the endwall boundary layer next to the blade-pressure surface moved across the passage to the blade-suction surface, thus showing
(as Bardon et al [20] found) the unsuitability of using a linear model to represent flows of large turning.

In reference [19], it was found that the trailing vortex was negligible compared to the passage vortex, while tip-leakage flow had a significant influence on the overall flow picture. Dean [21], Khabbaz [22] and Yokoyama [23] also found that tip-clearance effects are important. A survey of the literature, however, shows apparently contradictory results. In reference [21], Dean reported that the scraping effects were negligible compared to the tip-leakage effects, for a tip-flow coefficient above 0.43, while in reference [19], it was reported that the scraping effect dominates over both the tip-leakage and the passage-vortex effects. Lack of adequate definition of flow conditions in reference [19], however, makes comparison difficult.

The larger the tip-stagger angle, the weaker is the scraping vortex because more of the casing boundary-layer fluid has a component parallel to the blade. As shown in figure 1-1, the tip-leakage vortex and the scraping vortex have opposite senses in a turbine. It is therefore possible to design a turbine such that the effects of tip leakage and boundary-layer scraping essentially nullify each other [19].

Because of the difficulty encountered in compressor development (due to the existence of adverse pressure gradients), more effort has been put into compressor than into turbine research. In the area of secondary flows, an
overwhelming proportion of the work done so far has concerned itself with compressor cascades; in some cases, it is difficult to verify the reported good agreement between prediction and experiment. In reference [15], Papailiou used Horlock's cross-flow profile to predict the exit velocity profile for a compressor cascade and found that: "although the trends are there, one cannot claim even a fair agreement." For large flow turnings (as in turbines), agreement between theory and experiment is even more difficult to obtain [20] because the secondary-flow equation is valid for "small" turning only. However, there does not appear to be a precise definition of "small" because Squire and Winter [4] applied their relation to a flow after a 94.8° turning and reported obtaining good agreement between their theoretical prediction and experiment. This is an interesting result, judging from the large cross-flows observed in a cascade with a flow turning of 60° [19]. Ungar [24] also reported that he had good agreement between theory and experiment for flow through turbine blades with total turning of over 100°.

There is considerable published material on heat transfer in gas turbines. Most experimental work has been directed towards measuring the heat-transfer rate to the blades and the casings, while theoretical analyses have relied heavily on the flat-plate analogy. Wilson and Pope [25] measured the heat-transfer distribution on the surface of a turbine in a cascade, and found that the heat transfer depends not only on the Reynolds number but also on the angle of incidence. They listed a number of correlations suitable for predicting heat-transfer rates at different positions on the blade surface and noted that while the correlations were fair for regions of strongly
adverse pressure gradient, they (correlations) were poor in regions of weakly adverse or strongly favourable pressure gradient. Walker and Markland [26] carried out experiments similar to those of reference [25], but with artificially-induced strong secondary vortices. Their report did not show what influence (if any) that secondary flow has on the heat transfer to the blade surface. The results were basically the same as those of reference [25]. Blair [27] used a fixed cascade to study the heat transfer to the turbine endwalls, and obtained a map of heat-transfer distribution on the endwall. His results (like the other investigators') showed that heat transfer rate increased with flow turning, in the same way as secondary flows increase in strength with flow turning. He explained the measured distribution in terms of secondary vorticity.

In tests carried out at the M.I.T. gas-turbine laboratory [28-30], the heat transfer to a turbine endwall and shroud was measured. These tests show that in an actual turbine (as in the cascade), heat-transfer rate generally increases with flow turning, indicating that this increase may, to a degree, depend on the secondary flows generated in the blade passage.

1.4 SCOPE OF PRESENT WORK

The investigation reported here can be divided into two broad sections. The first section entails the solution of the secondary-flow equation for both a stationary and for a rotating co-ordinate frames. As pointed out in section 1.3, the equation has been good in predicting the gas outlet angle for compressor cascades. Here the equation is used to describe the flow through
the T64 turbine nozzle and rotor blades. The primary-flow velocity variation (both in the streamline and in the pitchwise directions) is taken into consideration. This is done by employing several primary-flow streamlines to cover the blade channel (instead of one streamline as is sometimes used in compressor analysis). The results of the solution are confirmed by experimental investigation.

The second section of the investigation is experiment which can further be sub-divided into two parts. The first part involves measuring the total-pressure and the total-temperature profiles at the entrance and the exit of the blade rows, and measuring the average heat-transfer rates to the nozzle endwall and the rotor shroud. The second part involves measuring the pressure and the heat-transfer fluctuations on the rotor shroud due to the relative movement of the shroud and the blade tips.

Finally, expressions are obtained for predicting the average heat-transfer distribution on the endwall and the shroud. The significance and application of the expressions are examined in relation to the objective of designing better blade cooling schemes, which will permit the use of higher turbine inlet temperature with the consequent increase in the turbine efficiency.
CHAPTER 2

SOLUTION OF SECONDARY-FLOW EQUATIONS

2.0 INTRODUCTION

Hawthorne [6], and Lakshminarayana and Horlock [12] derived expressions for the generation of secondary vorticity in a stationary frame, while Smith [14] and Lakshminarayana and Horlock [12] obtained secondary-flow expressions for a rotating co-ordinate frame. Since all the authors come up with essentially the same expressions, it is not intended to reproduce the derivations here. Instead, the equations will be solved and, from the results, the suitability or otherwise of the theory to the present case will be determined. It is assumed that the flow is steady, incompressible, inviscid and without body force.

2.1 SOLUTION FOR THE NOZZLE

Hawthorne's secondary-flow equation (reproduced below) is applicable to a nozzle.

\[
\left( \frac{\Omega_s}{q} \right)_2 - \left( \frac{\Omega_s}{q} \right)_1 = 2 \int_1^2 \frac{|\sigma_{\rho} - \cos \varphi|}{\rho q^2 R} ds
\]

where

1, 2 \equiv positions on a streamline, 1 being upstream of 2.

\( \Omega_s \equiv \) streamwise component of vorticity.

\( P_o \equiv \) fluid total pressure.

\( \rho \equiv \) fluid density.
\( \gamma \) \equiv angle between the binormal and the normal to the Bernoulli (constant-total-pressure) surface.

q \equiv magnitude of fluid velocity.

\( \alpha \) \equiv angle turned through by the fluid.

The effect of the rotation of the Bernoulli surfaces (increase in the angle \( \gamma \)), is to decrease the rate of change of the secondary vorticity along a streamline.

For incompressible, inviscid flow, the total-pressure gradient can be taken outside the integral sign in equation 2-1. If the total-pressure variation is in the spanwise, b, direction only, then \( \nabla \left( \frac{p}{\rho} \right) = \frac{dq_1}{db} \), ie, the total-pressure gradient at station 1. If it is further assumed that there is no streamwise component of vorticity at the entry to the passage (location 1), equation 2-1 becomes:

\[
\Omega_{s2} = 2 q_1 q_2 \frac{dq_1}{db} \int_{\gamma_1}^{\gamma_2} \cos \gamma \, d\gamma
\]

where \( d\gamma = \frac{ds}{R} = angle turned by the streamline. \)

Employing continuity relations and recalling that \( \frac{dq_1}{db} = \Omega_{n1} \), we can write equation 2-2 in the alternative form:

\[
\frac{\Omega_{s2}}{\Omega_{n1}} = \frac{2}{\cos \gamma_1 \cos \gamma_2} \int_{\gamma_1}^{\gamma_2} \cos \gamma \cos^2 \alpha \, d\alpha \]

The secondary-flow equations 2-2 and 2-3 are strictly applicable to small flow turnings. The latter equation shows that if the angle, \( \alpha_2 \), becomes 90°, the secondary vorticity, \( \Omega_{s2} \), becomes indeterminate.
For a turbine blade, both the flow angle and the velocity changes are considerable. In the solution, each streamline is divided into many segments and the equation is applied to each segment. The average velocity for a streamline segment is taken as the arithmetic mean velocity, given by:
\[
q = \frac{1}{2}(q_1 + q_2),
\]
where the subscripts 1 and 2 now apply to a streamline segment, which can be approximated by a circular arc. If there is no rotation of the Bernoulli surface, the angle \( \Psi \) remains constant at 0°. Therefore equation 2-2 integrates to:
\[
\Omega_{s2} = \frac{8 q_1 q_2 \Omega n_1}{(q_1 + q_2)^2} \alpha
\]

Now let's consider the rotation of the Bernoulli surfaces. It is assumed that the surfaces rotate without distortion and, following Dean [16], that each Bernoulli surface rotates in such a way that its trace on any pitch-span plane is a straight line. Under the above conditions, the rate of rotation of the Bernoulli surfaces is given by:
\[
\frac{d\Psi}{dt} = \frac{1}{2} \Omega_{s}
\]
(since vorticity = twice angular velocity). With the approximation that streamlines follow circular arcs, we get
\[
dt = \frac{dL}{q} = \frac{2 R d\alpha}{(q_1 + q_2)}
\]
where
\[
dL \equiv \text{streamline length.}
\]
\[
R \equiv \text{radius of curvature of streamline.}
\]
The radius of curvature, $R$, is related to the blade chord, $c$, and the total turning angle, $\varepsilon$.

$$ R = \frac{c}{\varepsilon} \Rightarrow dt = \frac{2c d\alpha}{(q_1 + q_2)\varepsilon} \quad 2-7 $$

Combining equations 2-4, 2-5 and 2-7, we get:

$$ d\varphi = \frac{8 q_1 q_2 c \sum_{n_1} \varphi d\alpha}{(q_1 + q_2)^3 \varepsilon} \quad 2-8 $$

or

$$ \int_{\varphi_0}^{\varphi} d\varphi = \frac{8 q_1 q_2 c \sum_{n_1} \varphi d\alpha}{(q_1 + q_2)^3 \varepsilon} \int_{\alpha_0}^{\alpha} d\alpha \quad 2-8 $$

At $\alpha_0 = 0$, $\varphi_0 = 0$, where the subscript zero denotes position far upstream of the blade row. Therefore,

$$ \varphi = \frac{4 q_1 q_2 c \sum_{n_1} \varphi^2}{(q_1 + q_2)^3 \varepsilon} = A \varphi^2 \quad 2-9 $$

where

$$ A \equiv \frac{4 q_1 q_2 c \sum_{n_1}}{(q_1 + q_2)^3 \varepsilon} $$

Substitute equation 2-9 into equation 2-3.

$$ \frac{\Omega_s x}{\sum_{n_1}} = \frac{2}{\cos \alpha_1 \cos \alpha_2} \int_{\alpha_1}^{\alpha_2} \cos^2 \alpha \cos(A \alpha^2) \, d\alpha \quad 2-10 $$

For small values of $A \alpha^2$, equation 2-10 can be written in the form:
The second term on the right-hand side of equation 2-11 represents the effect of rotation of the Bernoulli surface. It is seen that this rotation results in a reduction in the development of secondary vorticity. Integration of equation 2-11 gives:

\[
\frac{\Omega_{s2}}{\Omega_{n1}} = \frac{2}{\cos q_1 \cos q_2} \left\{ \int_{q_1}^{q_2} \cos^2 \theta \, d\theta - \frac{1}{2} A^2 \int_{q_1}^{q_2} \cos^2 \theta \, d\theta \right\} 2-11
\]

The secondary vorticity, \(\Omega_{s2}\), can be regarded as composed of two parts. The first part, \(\Omega_{s21}\), represented by the terms outside the square brackets, \([\,]\), in equation 2-12, is the secondary vorticity when there is no Bernoulli-surface rotation. The second part, \(\Omega_{s22}\), represented by the terms inside the square brackets, \([\,]\), in equation 2-12, is the contribution to the secondary vorticity due to Bernoulli-surface rotation. Therefore,

\[
\Omega_{s2} = \Omega_{s21} + \Omega_{s22} 2-13
\]

A look at the right-hand side of equation 2-12 reveals that apart from the
inlet normal vorticity, $\Omega_{n1}$, every other thing is essentially a property of the flow geometry. In order to see the effect of flow-turning on the development of secondary vorticity, we non-dimensionalize everything in equation 2-12 and divide $\Omega_{s21}$ by $\Omega_{n1}$ and $\Omega_{s22}$ by $(\Omega_{n1})^3$. This is equivalent to eliminating the inlet vorticity, $\Omega_{n1}$, from the right-hand side of equation 2-12. We can then define a new term,

$$B_{noz} = \frac{\Omega_{s21}}{\Omega_{n1}} + \frac{\Omega_{s22}}{(\Omega_{n1})^3}$$  2-14

It is to be recalled that the flow is assumed to be steady, incompressible, inviscid and with no body force. The variation of $B_{noz}$ with flow-turning angle, $\alpha$, is shown plotted in figure 2-1. In the calculation, the inlet-flow angle, $\alpha_1$, is assumed to be zero and the flow net shown in figure 2-2 is used in computing the main-flow velocities, $q_1$ and $q_2$ and in getting the flow-turning angle, $\alpha$. The computed values of $\Omega_{s21}/\Omega_{n1}$ and $\Omega_{s22}/(\Omega_{n1})^3$ suggest that the rotation of the Bernoulli surface may have only very little effect on the strength of the secondary vorticity. The ratio $\frac{\Omega_{s22}}{(\Omega_{n1})^3}/\frac{\Omega_{s21}}{\Omega_{n1}}$ increases with flow turning but even at a turning angle of $71^\circ$, the ratio is only 0.45%. Lakshminarayana and Horlock [18] also found that the strength of the secondary vorticity is not altered much by Bernoulli surface rotation.

The parameter, $B_{noz}$, is purely a geometrical parameter. As indicated in the preceding paragraph, the rotation of the Bernoulli surface does not change the strength of the secondary vorticity by a significant amount. Therefore, the terms representing the rotation of the Bernoulli surface in equation
2-12 can be ignored. The equation then reduces to:

\[- \Omega s_2 = - \frac{\Omega n_1}{\cos \alpha_1 \cos \alpha_2} \left\{ (\xi_2 - \varphi_1) + \frac{\sin 2\varphi_2 - \sin 2\varphi_1}{2} \right\} \quad 2-15\]

and \( B_{noz} \) is redefined as

\[ B_{noz} \equiv \frac{\Omega s_2}{\Omega n_1} = \frac{1}{\cos \alpha_1 \cos \alpha_2} \left\{ (\xi_2 - \varphi_1) + \frac{\sin 2\varphi_2 - \sin 2\varphi_1}{2} \right\} \quad 2-16\]

Therefore

\[ - \Omega s_2 = \Omega n_1 B_{noz} \quad 2-17\]

If the inlet normal vorticity, \( \Omega n_1 \), is constant, then \( B_{noz} \) is directly proportional to the strength of the secondary vorticity. From the plot of \( B_{noz} \) vs \( \alpha \) (cf figure 2-1), \( B_{noz} \) (and by inference, \( \Omega s_2 \)) is linearly proportional to \( \alpha \) up to a flow turning of 30°. Beyond this angle, \( B_{noz} \) increases exponentially with flow turning. As shown in section 2-4, the actual flow is highly non-linear and the use of the theory in predicting fluid particle trajectories is questionable.

### 2.2 SOLUTION FOR THE ROTOR

The variation of secondary vorticity in a rotating co-ordinate frame is given by [12]:

\[ \frac{\partial}{\partial s} \left( \frac{n s'}{W} \right) = \frac{2}{W^2 R} \left| \frac{\nabla \mathbf{I}}{\rho} \right| \cos \varphi' + \frac{2\omega}{W^3 \rho} \left| \frac{\nabla \mathbf{I}}{\rho} \right| \cos \Theta \quad 2-18\]

where

- \( s' \equiv \) relative streamline direction.
- \( W \equiv \) fluid relative velocity.
\( \Omega_s \equiv \) magnitude of absolute secondary vorticity in the relative streamline direction.

\( R' \equiv \) radius of curvature of the relative streamline.

\( I \equiv P + \frac{1}{2} \rho W^2 - \frac{1}{2} \rho (\omega r)^2 = \) equivalent total pressure relative to a rotating frame.

\( \gamma' \equiv \) angle between the normal to the relative Bernoulli plane and the relative binormal direction.

\( \omega \equiv \) angular speed of the rotating frame.

\( \theta \equiv \) angle between the normal to the relative Bernoulli plane and the rotation vector.

Consider an axial-flow turbomachine rotor where the rotation vector, \( \omega \), is along the axis of the rotor. If it is assumed that the relative Bernoulli surfaces are cylindrical through the rotor blades, then the normal to the surfaces will lie in the radial (or \( b' \)) direction. With this assumption the angle, \( \theta \), is equal to 90° and hence the second term on the right-hand side of equation 2-18 is zero. The equation then reduces to

\[
\frac{\partial}{\partial s'} \left( \frac{n_{s'}}{W} \right) = \frac{2}{W^2 R'} \left| \frac{\nabla 'I}{p} \right| \cos \gamma' \tag{2-19}
\]

With the above simplification, the distortion of the Bernoulli surfaces has been restricted to pure rotation about the rotor axis. Upstream of the rotor blades, if the effects of the neighbouring blade rows are negligible, the angle \( \gamma' = 0 \). If it is further assumed that this value of \( \gamma' \) remains unchanged through the rotor passage, equation 2-19 integrates to:

\[
\left( \frac{n_{s'}}{W} \right)_2 - \left( \frac{n_{s'}}{W} \right)_1 = 2 \int_{\alpha'}^{\gamma'} \left| \frac{\nabla 'I}{p} \right| d\gamma' \tag{2-20}
\]
where \( \frac{d\alpha'}{R} = \frac{ds'}{R} \).

Equation 2-20 is used in describing the flow through the rotor passage. For incompressible flow, both \( I \) and \( \frac{\mathbf{V}'}{\rho} \) are invariant along a relative streamline. By employing continuity conditions, we can write equation 2-20 as

\[
\left( \frac{\Omega s'}{W} \right)_2 - \left( \frac{\Omega s'}{W} \right)_1 = \frac{\left| \frac{\mathbf{V}'}{\rho} \right|}{w_1 w_c \cos \alpha'_1 \cos \alpha'_2} \left[ (\alpha'_2 - \alpha'_1) \right. \\
+ \frac{1}{2} \left( \sin 2\alpha'_2 - \sin 2\alpha'_1 \right) \right]  \tag{2-21}
\]

Because of the nozzle blades ahead of the rotor blades, there will be a streamwise component of vorticity, \( \Omega_{s1}' \), at the inlet to the rotor blades. This inlet vorticity can be estimated by using the rotor-inlet velocity triangle. Consider

\[
\frac{\Omega s'}{W} = \frac{-\Omega \cdot W}{W^2} = \frac{-\Omega_r W_r + \Omega_\theta W_\theta + \Omega_2 W_2}{W^2}  \tag{2-22}
\]

where \( r, \theta, z \), denote radial, circumferential and axial directions respectively. For axially symmetric flow, if there is radial equilibrium, the components of vorticity are given by:

\[
\begin{align*}
\Omega_r &= 0  \tag{2-23} \\
\Omega_\theta &= -\frac{\partial W_\theta}{\partial r}  \tag{2-24} \\
\Omega_2 &= \frac{1}{r} \frac{\partial (r W_\theta)}{\partial r}  \tag{2-25}
\end{align*}
\]

Consider an inlet-velocity triangle as shown on the next page. In the figure, \( V_1 \equiv \text{inlet absolute velocity} \).
$W_1$ = inlet relative velocity.
$U$ = blade circumferential velocity.
$V_{z1}$ = axial component of $V_1$.
$W_{z1}$ = axial component of $W_1$.
$V_{\theta 1}$ = circumferential component of $V_1$.
$W_{\theta 1}$ = circumferential component of $W_1$.
$\theta_1$ = angle between the absolute velocity and the axial direction.
$\gamma_1$ = angle between the relative velocity and the axial direction.

From the figure, the following relations are obtained.

\begin{align*}
W_{\theta 1} &= W_1 \sin \gamma_1 \quad 2-26 \\
W_{z1} &= V_{z1} = W_1 \cos \gamma_1 \quad 2-27 \\
V_{\theta 1} &= W_1 \cos \gamma_1 \tan \theta_1 \quad 2-28
\end{align*}

Substitute equations 2-23 to 2-28 into equation 2-22, which is now applied to the inlet to the rotor.
\[
\left( \frac{-\Omega s}{W} \right) = \frac{-\sin \gamma \frac{\partial (W \cos \gamma \tan \theta)}{\partial y} + \cos \gamma \frac{\partial (W \cos \gamma \tan \theta)}{\partial y}}{W_1} \quad 2-29
\]

Equation 2-21 then becomes

\[
\left( \frac{-\Omega s'}{W} \right) = \frac{1}{W_1} \left[ \cos \gamma \frac{\partial (W \cos \gamma \tan \theta)}{\partial y} - \sin \gamma \frac{\partial (W \cos \gamma \tan \theta)}{\partial y} \right]
+ \left[ \frac{\sqrt{I'}}{\rho} \left[ \left( \alpha'_2 - \alpha'_1 \right) + \frac{\sigma}{2} (\sin 2\gamma'_2 - \sin 2\gamma'_1) \right] \right] \frac{W_1 W_2 \cos \gamma_1 \cos \gamma_2}{W_1 W_2} \quad 2-30
\]

In equation 2-30, the terms in the first square brackets represent the rotor inlet secondary vorticity while the second set of terms represents the change of secondary vorticity within the blade passage.

Utilizing the definition of I and recalling that \( \frac{\sqrt{I'}}{\rho} \) is invariant along a relative streamline, we can re-arrange equation 2-30 in the following way:

\[
-\Omega s_2' = \left[ \cos \gamma_1 W_1 \frac{\partial (W \cos \gamma_1 \tan \theta)}{\partial y} - \sin \gamma_1 W_1 \frac{\partial (W \cos \gamma_1 \tan \theta)}{\partial y} \right] \left( \frac{W_2}{W_1} \right)
+ \left[ \left( \alpha'_2 - \alpha'_1 \right) + \frac{\sigma}{2} (\sin 2\gamma'_2 - \sin 2\gamma'_1) \right] \frac{W_2}{W_1} \cos \gamma_1 \tan \theta_1 \left( \frac{W_2}{W_1} \right)
- \sin \gamma_1 \cos \gamma_1 \left( \frac{W_2}{W_1} \right) \left| \frac{\partial W_1}{\partial y} \right| \quad 2-31
\]

Again as for the nozzle, equation 2-31 is non-dimensionalized. The secon-
dary vorticity, \( \Omega_{s2}' \), can be thought of as being composed of two parts, ie

\[
\Omega_{s2}' = \Omega_{s21}' + \Omega_{s22}'
\]

where

\[
\Omega_{s21}' = \left[ \cos \gamma_1 W_1 \frac{d}{d \gamma} \left( \cos \gamma_1 \tan \theta_1 \right) \right] \left( \frac{W_2}{W_1} \right) \tag{2-33}
\]

\[
\Omega_{s22}' = \left[ \frac{\left( \alpha'_2 - \alpha'_1 \right) + \frac{1}{2} \left( \sin 2 \gamma'_2 - \sin 2 \gamma'_1 \right)}{\cos \gamma'_1 \cos \gamma'_2} + \cos^2 \gamma_1 \tan \theta_1 \left( \frac{W_2}{W_1} \right) \right]
\]

\[
- \sin \gamma_1 \tan \theta_1 \left( \frac{W_2}{W_1} \right) \left| \frac{d W_1}{d \gamma} \right|
\]

From equations 2-33 and 2-34, it is clear that \( \Omega_{s21}' \) is independent of the inlet velocity gradient while \( \Omega_{s22}' \) is dependent on it. In order to study the effect of the passage geometry on the development of secondary vorticity, \( \Omega_{s22}' \) is divided by \( \left| \frac{d W_1}{d \gamma} \right| \) and a new parameter, \( B_{rot} \), is defined such that,

\[
B_{rot} \equiv \Omega_{s21}' + \frac{\Omega_{s22}'}{\left| \frac{d W_1}{d \gamma} \right|} \tag{2-35}
\]

The variation of \( B_{rot} \) with flow turning is shown in figure 2-3; the flow net shown in figure 2-4 is used in obtaining the primary-flow angles, \( \alpha'_1 \) and \( \alpha'_2 \) and the velocities \( W_1 \) and \( W_2 \). Once more, it is worth remembering that we are considering steady, incompressible, inviscid flow, with no body forces, and that the relative Bernoulli surfaces rotate about the rotor axis. The observations made in section 3.1 concerning \( B_{noz} \) are equally app-
licable to $B_{rot}$.

### 2.3 NUMERICAL SOLUTION

In preparation for the numerical solution of the secondary-flow equation, each streamline in the T64 blade channels (figures 2-2 and 2-4) is divided into many segments. The upstream end of the segment is denoted by 1 and the downstream end denoted by 2. If at location 2 it is assumed that the streamline component of velocity gradient is much smaller than the components in the normal and the binormal directions, then a secondary-flow stream function, $\psi(n,b)$, can be defined which satisfies the incompressible continuity equation.

By definition,

$$v_n = \frac{\partial \psi}{\partial b} ; \quad v_b = -\frac{\partial \psi}{\partial n}$$

Equation 2.36

By definition,

$$\Omega_{s2} = \mathbf{\Sigma} \cdot \nabla \times \mathbf{V}_2 = \frac{\partial v_{b2}}{\partial n} - \frac{\partial v_{n2}}{\partial b} = -\frac{\partial^2 \psi}{\partial n^2} - \frac{\partial^2 \psi}{\partial b^2}$$

Therefore,

$$\frac{\partial^2 \psi}{\partial n^2} + \frac{\partial^2 \psi}{\partial b^2} = -\Omega_{s2}$$

Equation 2.37

For the nozzle, $\Omega_{s2}$ is given by equation 2-12 while for the rotor it is given by equation 2-31.

The solution of the secondary-flow equation involves integration along a streamline. Therefore one of the first steps in the solution is to assume a shape for the primary-flow streamlines. In the present case, potential
flow is assumed and the resulting streamlines through the T64 blade channels (figures 2-2 and 2-4) are generated using the Pratt and Whitney (M122) [33] computer program.

Only one half of the channel (from mid-span to the tip) is analysed, since the solution is symmetrical about the blade mid-span. The one-half channel is divided into sixteen equally-spaced sections by planes perpendicular to the blade span [24]. Each plane has a flow net of streamlines and potential lines drawn on it. Figures 2-2 and 2-4 are the nets for the nozzle and the rotor respectively.

Cosine velocity profiles (figure 2-5) are assumed at the inlet to the blade passages. Three profiles of different steepness are used. The use of this inlet profile is an attempt to simulate the jet profile used in the experimental investigation which will be reported in section 3.3. This representation is not seen as restrictive since any profile can be represented by a cosine series using Fourier analysis. The assumed velocity profile is given by:

\[
V = V_\infty \left[1 + a (1 + \cos \pi \frac{z}{z_0})\right]
\]

where

\[V\] = velocity at any point.

\[V_\infty\] = undisturbed free-stream velocity.

\[a\] = a factor that determines the steepness of the flow gradient.

\[z\] = distance in the spanwise direction, measured from the mid-span.
The extent of the cosine profile in the spanwise direction [taken to be \( \frac{1}{8}(\text{span}) \) here].

The details of the computer program (a copy of which can be found in Appendix A) used in solving the secondary-flow equation are given below. (The flow chart for the program is also shown in Appendix A). The turning angles and the pitchwise spacing of the streamlines, as well as the spacing of the potential lines are obtained around each node of the flow net. Equation 2-37 is solved iteratively using a finite difference scheme.

The solution starts from the first potential (pitch-span) surface upstream of the blade row and then progresses downstream to the exit of the passage. The iteration is stopped when a desired level of accuracy of the stream function values is achieved or if the number of iterations exceeds a set value. With known values of secondary-flow stream function, the associated velocity components \( v_n \) and \( v_b \) are obtained by differentiation as shown in equation 2-36. Next, the paths of the fluid particles through the channel are traced. By secondary-flow theory, a fluid particle is carried along the principal streamline by the main flow. At the same time, the particle is acted upon by the secondary-flow velocities. The particle, in going from one potential surface to the next, deviates in both the pitchwise and the spanwise directions, thus ending up at a position away from the primary streamline location. A summation of these deviations from inlet to any potential surface farther downstream gives the position of the fluid particle on the potential surface.
2.4 NUMERICAL RESULTS

Reference has already been made to figures 2-1 and 2-3 in which it was pointed out that the secondary vorticity increases exponentially with flow turning when the turning exceeds 30°. Figures 2-6 to 2-8 are plots of constant secondary-flow stream functions for the three inlet-flow profiles shown in figure 2-5. For each figure, the value of "a" indicates which velocity profile is used in the solution while the value of "k" shows which potential surface (figure 2-2) the plot is drawn for. The higher the value of "k", the farther downstream the potential surface.

The dependence of secondary vorticity on the flow turning is seen from any one of the sets of plots. Consider figure 2-8, for example. In figure 2-8a, the secondary-flow streamline with $\psi = 0.05$ has a complete loop in the plot. In figure 2-8c, the streamline with $\psi = 0.05$ does not form a complete loop due to the concentration of secondary vortices at the base of the figure. Because of this increased concentration, the spacing between consecutive streamlines becomes progressively smaller. The influence of the inlet-flow gradient is seen by considering plots on the same potential ("k") surface but with different ("a") flow profiles. The higher the value of "a", the larger the secondary vorticity.

Figures 2-9 and 2-10 are plots of fluid-particle flow paths through the nozzle blade passage, obtained as described earlier. The primary streamlines are on an axial-pitch plane, 1/32 of the blade span from mid-span. The position of a fluid particle is projected onto a pitch-span plane as
it (particle) moves through the passage. The pitchwise spacing of the streamlines in figure 2-2 is normalized in such a way that any two adjacent streamlines are separated by a non-dimensional distance of 0.1. Similarly the spanwise spacing of any two adjacent streamlines is a non-dimensional distance of 0.05 (equivalent to 1/32 of the blade span).

In figure 2-9a for example, the origin for the curve represents a point on the original potential-flow streamline, far upstream of the blade row. The numbers 2 to 5 on the curves identify the corresponding streamlines in figure 2-2. Consider curve number 2 in figure 2-9a. The point (0.1, -0.0375) indicates that streamline 2 in figure 2-2 is at a position (in the pitch direction) originally occupied by the adjacent streamline (3) and in the spanwise direction it (streamline 2) is 3/4 of the way to the next axial-pitch plane. Similarly for curve 3 in figure 2-9a, the point (0.35, -0.025) indicates that streamline 3 in figure 2-2 has crossed the positions of three streamlines (ie 4, 5, 6) and is half way between streamlines 6 and 7 (in the pitchwise direction); in the spanwise direction, streamline 3 is half way to the next axial-pitch plane. The curves (figures 2-9 and 2-10) show that there is an appreciable distortion of the flow due to secondary flows. Consider streamline 4 in figure 2-9a. After 68° turning, the streamline has crossed the positions of seven other streamlines and is at the position of streamline 11 (in figure 2-2), which corresponds to the blade suction surface. In figure 2-10a (drawn for a steeper inlet-flow profile than for figure 2-9a), the same streamline, 4, deviates to the blade surface after 55° turning, pointing out once more the dependence of secondary vorticity
As pointed out earlier, the current secondary-flow theory is a linear theory which treats secondary flow as a small perturbation of the main flow. The ratio of secondary velocity to the primary velocity is assumed to be very small. Where this ratio becomes appreciable, the assumption of linearity is no longer justified and the theory becomes invalid. In the plot of $B_{noz}$ vs flow turning (figure 2-1), it was found that the curve became non-linear after 30° turning. Since $B_{noz}$ is dependent on the flow geometry only, it can be said that as the flow turning gets larger than 30°, non-linear terms become important in the calculation of the secondary vorticity. From figures 2-9 and 2-10, it is seen that with the large flow deviations, the secondary velocity cannot (in this case) be considered very small compared to the primary velocity.

Figures 2-9 and 2-10 should be taken as qualitative descriptions of secondary flows because as Hawthorne pointed out [44], the current secondary-flow theory (being a linear theory) should not be expected to yield the trajectory of a fluid particle, since the displacement is the product of two small quantities: time and velocity. Figures 2-11, 2-12 and 2-13 are photographs of flow-visualization studies of reference [19]. Figure 2-11 shows the movement of the endwall boundary-layer fluid towards the blade suction surface. Figure 2-12 is a downstream view of a 45° channel showing the formation of passage vortex due to the roll-up of the endwall boundary layer. In figure 2-13 (which shows a 60° channel), the endwall boundary-layer fluid is seen
to move from the pressure surface to the suction surface. From these figures, it is seen that the large turning in turbine blades results in such large secondary flows that the small-perturbation assumption is no longer valid. Consequently, the current secondary-flow theory is not applicable to the large flow turning in turbine blades. The same conclusion has been reached by Bardon [20] and Sjolander [47]. Since the theory (at its present level of development) is invalid for large flow turning, it (theory) cannot be used to predict the heat-transfer distribution on the turbine casings.

The influence of flow-turning on the development of secondary vorticity is examined further in Appendix B, where it is shown that the problem associated with flow-turning cannot be minimized by taking either viscosity or Bernoulli-surface rotation into consideration.
CHAPTER 3

EXPERIMENTAL EVALUATION

3.0 INTRODUCTION
The objective of the experimental evaluation was to measure the effect of secondary flows on the heat transfer to the T64 turbine nozzle endwall and rotor shroud. From chapter 2, it is clear that the strength of the secondary vorticity is dependent on the shear in the inlet flow and the turning that the flow undergoes (cf equations 2-2 and 2-20). The experiment was therefore designed to study the effect of these two parameters: the inlet shear and the flow-turning angle. The inlet shear flow was obtained by injecting colder fluid (using the injection ring shown in figures 3-1 to 3-3) into the main flow, upstream of the first-stage nozzle, and by varying the ratio of the injection to the main flows, the inlet shear was varied. The flow was of course turned by the blades, the turning increasing as one moved downstream. It was therefore possible to vary the strength of the secondary vorticity and thus study its influence on the heat-transfer rate to the nozzle endwall and the rotor shroud. The experiments were carried out in the M.I.T. turbine blowdown facility which is described next.

3.1 TURBINE BLOWDOWN FACILITY
The blowdown facility is a short-duration test facility with running time of approximately one second. Turbine modelling and similarity laws are employed to scale down the operating conditions of an actual turbine [48].
In order to have similar conditions in the experimental and the actual turbines, the following dimensionless numbers should have the same values in both cases: Reynolds, Mach, Prandtl and Nusselt numbers. A scaling factor, $S$, is defined such that

$$S = \frac{T_{\text{ex}}}{T_{\text{ac}}}$$  \hspace{1cm} 3-1

where

$T_{\text{ex}} \equiv$ temperature of the experimental turbine.

$T_{\text{ac}} \equiv$ temperature of the actual turbine.

The equality of Mach number in the experimental and the actual turbines leads to the following relation:

$$\left(\frac{V}{\sqrt{\gamma R T}}\right)_{\text{ex}} = \left(\frac{V}{\sqrt{\gamma R T}}\right)_{\text{ac}}$$  \hspace{1cm} 3-2

where

$\left(\quad\right)_{\text{ex}} \equiv$ experimental conditions.

$\left(\quad\right)_{\text{ac}} \equiv$ actual conditions.

$V \equiv$ velocity.

$R \equiv$ gas constant.

$\gamma \equiv$ ratio of specific heats at constant pressure and constant volume.

$T \equiv$ temperature.

If $R$ and $\gamma$ are assumed to have the same values in the experimental and in the actual turbines, then

$$\frac{V_{\text{ex}}}{V_{\text{ac}}} = \left(\frac{T_{\text{ex}}}{T_{\text{ac}}}\right)^{\frac{1}{2}} = S^{\frac{1}{2}}$$  \hspace{1cm} 3-3
Therefore the velocities are scaled down by the square root of the scaling factor. The equality of Reynolds number in both cases means that

\[
\left(\frac{\rho V L}{\mu}\right)_e = \left(\frac{\rho V L}{\mu}\right)_a
\]

where

\(\rho\) \equiv \text{fluid density.}

\(L\) \equiv \text{turbine reference length (blade chord, say).}

\(\mu\) \equiv \text{fluid dynamic viscosity.}

For the same turbine, \(L\) is the same in both cases and since the viscosity varies as the square root of the temperature, the densities are equal.

\[
\therefore \quad \frac{\rho_e}{\rho_a} = 1
\]

From the ideal-gas relations, we get:

\[
\frac{P_e}{P_a} = \frac{(\rho RT)_e}{(\rho RT)_a}
\]

where

\(P\) \equiv \text{fluid pressure.}

Substitution of equations 3-1 and 3-5 into equation 3-6 yields

\[
\frac{P_e}{P_a} = S
\]

The ratio of mass-flow rates is given by:

\[
\frac{\dot{m}_e}{\dot{m}_a} = \frac{(\rho VA)_e}{(\rho VA)_a}
\]

where
\( \dot{m}_{\text{ex}} \equiv \) experimental mass-flow rate.

\( \dot{m}_{\text{ac}} \equiv \) actual mass-flow rate.

\( A \equiv \) flow area through the turbine.

For the same turbine, the area is unchanged and using equations 3-3 and 3-5, we get,

\[
\frac{\dot{m}_{\text{ex}}}{\dot{m}_{\text{ac}}} = S^{\frac{1}{2}} \quad 3.9
\]

Because of the short duration of the test, the turbine-wall temperature will remain constant at room temperature. Hence for given conditions of the actual turbine, the test conditions are determined.

Below we give a brief description of the turbine blowdown facility (figure 3-1); a more detailed description will be found in reference [34].

(1) **STORAGE CYLINDERS**: These are high-pressure cylinders that supply air for the experiment.

(2) **PRESSURE REGULATORS**: These are dome-type regulators that maintain a maximum flow rate while regulating the pressure from the cylinders to a pre-set value.

(3) **PEBBLE-BED HEATER**: A 6ft-by-1-1/2ft-diameter vertical tank that is filled with 1-inch stainless-steel balls. The heated balls provide a means of heating the experimental air from the storage bottles.

(4) **DIAPHRAGM SECTION**: An aluminium diaphragm of known thickness is punctured here to let the hot pressurized air from the heater flow to the turbine section during the test.
(5) **THROAT SECTION:** This is a 1-1/2-inch-diameter throat which is choked during the test. The main-flow conditions are monitored at the throat.

(6) **SUPERSONIC NOZZLE AND DIFFUSER:** The nozzle is a diverging 7° cone downstream of the throat while the diffuser consists of a bundle of tubes that reduces the pressure of the air from the pebble-bed heater.

(7) **AUXILIARY AIR-DRIVE SYSTEM:** By this means the turbine is brought up to speed, using cold compressed air, before the diaphragm is punctured.

(8) **SLOTTED INJECTION RING:** This (figures 3-2 and 3-3), located 2 inches upstream of the first-stage nozzle, is used to vary the inlet-flow profile by injecting fluid into the main flow.

(9) **TURBINE SECTION:** The two-stage gas-generator section of the General Electric T64 helicopter engine is used as the test bed. The turbine is instrumented to measure the flow conditions and the heat transfer to the endwall and the shroud.

(10) **REAR SUBSONIC DIFFUSER:** This is located downstream of the turbine and expands the exhaust air to atmospheric pressure. Part of the magnetic pickup, for measuring the turbine rotational speed, is attached to the rear diffuser.

### 3.2 INSTRUMENTATION

Total-temperature and total-pressure probes were used to measure the flow profiles at the inlet to and the exit from the first-stage nozzle and rotor. Static pressures were also measured at the same locations. The main-flow conditions were measured at the throat by means of total-temperature and
total-pressure probes, while the injection-flow conditions were measured at an orifice section in the injection line. The sensing element of the total-temperature probe was a copper-constantan thermocouple bead.

For heat-transfer measurements, two types of gauges were used. The first was a copper calorimetric gauge for measuring the average heat transfer to the nozzle endwall and the rotor shroud. The positions of the copper gauges are shown in figures 3-4 and 3-5, numbered 1 to 12, for the nozzle endwall and in figures 3-6 and 3-7, numbered 13 to 20, for the rotor shroud. The second type of gauge was a thin-film heat-transfer gauge, located in one of the tapped holes in figure 3-6. In the second tapped hole was installed a miniature Kistler pressure transducer. The two gauges in the tapped holes were used to monitor the heat transfer and the pressure changes on the rotor shroud due to the blade-end effects, i.e., tip leakage and/or blade-scraping.

The signals from the gauges were fed into Bell and Howell oscillograph machines (which had optical galvanometers and light-sensitive papers), a storage oscilloscope and a CAMAC crate (containing LeCroy waveform digitizers).

3.3 THE TEST

Short-duration tests were performed, employing the gas-generator section of the General Electric T64 engine as the test bed. The test consisted of pressurizing the system from the hot pebble bed to the diaphragm section. Then using the auxiliary line, the turbine was brought up to speed using cold compressed air. When the desired turbine speed was attained, the auxiliary-
air flow was stopped and the valve on the injection line was opened, thus letting air flow through the injection ring. (The injection-air temperature could be varied from room temperature to the pebble-bed-air temperature). While the injection flow was on, the diaphragm was punctured thus letting the main, hot, air from the pebble-bed heater flow through the turbine. The flow of hot air lasted for one to two seconds and during this time, conditions in the turbine were measured by the gauges described in section 3.2.

3.4 EXPERIMENTAL RESULTS

(i) TOTAL AND STATIC PRESSURES
Figures 3-8a to 3-8j give the stagnation pressure profiles at the inlet to and the exit from the first-stage nozzle and rotor. In each figure, the curve labelled $P_{T1}$ was the stagnation-pressure profile at the inlet to the nozzle; the curve labelled $P_{T2}$ was the profile at the inlet to the rotor; and the curve labelled $P_{T3}$ was the profile at the exit from the rotor. The dashed lines labelled $P_{S1}$, $P_{S2}$ and $P_{S3}$ were the tip-static pressures measured at the same axial locations as $P_{T1}$, $P_{T2}$ and $P_{T3}$ respectively.

At the entrance to the nozzle, the jet and the wake from the injection ring had opposite effects on the flow profile. The resulting profile depended on whether the jet-effect or the wake-effect was stronger. A look at the profiles reveals that the jet-effect was stronger than the wake effect, since the mid-span total pressure was larger than the value at 3/4 span, say. Because of frictional losses, lower stagnation pressure was measured at the tip. The introduction of shear-flow at the mid-span led to higher losses at
mid-span than at 3/4 span, as the flow went through the nozzle; hence the shape of the profile measured downstream of the nozzle.

(ii) **TOTAL TEMPERATURE**

The total-temperature profiles, as measured by the total-temperature probes, are shown in figures 3-9a to 3-9j. For each figure, the plots labelled $T_{t1}$, $T_{t2}$ and $T_{t3}$ were the profiles at the inlet to the nozzle, the inlet to the rotor and the exit from the rotor respectively. There was a significant change in the $T_{t1}$ profile from the case of no injection to that of injection (cf figure 3-9a to the other figures). Because of the large heat loss to the cold turbine walls, the tip temperature at the nozzle exit was considerably below that at the inlet. The injection fluid was cooler than the main flow and at the inlet to the nozzle the mixing of the two fluids was not complete. More of the injection fluid than the main flow got to the gauge at mid-span. By the time the flow went through the nozzle, there was a better mixing of the two streams. Therefore, at the higher injection rates (figures 3-9e, i and j) there was the apparently anomalous condition where the measured total temperature was lower at the inlet to the nozzle than at the exit.

(iii) **AVERAGE HEAT TRANSFER**

The average heat transfer to the nozzle endwall and the rotor shroud was measured by means of the calorimetric copper gauges. The results from these gauges are presented in the form of Nusselt numbers. Two methods of calculating the Nusselt numbers were used in the presentation here. As shown in
Appendix C, the gas temperature was required in the calculation of the Nusselt number. The first method of calculation ignored the work done by the gas as it flowed through the rotor. The gas temperature measured at the inlet to the rotor was used in calculating the Nusselt numbers throughout the rotor shroud. This method was simple to use and was based on actually-measured temperatures. However, near the rotor-blade trailing edge, the actual gas temperature was considerably lower than the value measured at the rotor inlet. Thus the calculated Nusselt number was smaller than the value based on the actual gas temperature, the difference between the two getting larger with increasing work done per turbine stage. The Nusselt numbers based on this first method are shown in figure 3-10a to 3-10j.

The second method of calculating the Nusselt numbers estimated the work done by the gas from the blade leading edge to any pint of interest, hence giving an estimate of the local gas temperature at the gauge location. In the present case, the potential-flow pressure difference across the blade (figure 3-11) was obtained and from this, the gas total temperature at any location in the rotor-blade channel could be estimated. The Nusselt numbers based on the second method are shown in figures 3-12a to 3-12j.

The numbers shown in figures 3-10 and 3-12 refer to the calorimetric copper gauges, the locations of which are shown in figures 3-4 to 3-7. The two curves shown in each of the Nusselt-number plots are the tip-flow coefficients, $\phi_T$, shown on each figure. Because of the nature of the experiments, the values of the test flow coefficients could not be controlled rigidly.
Neither the air mass-flow rate nor the turbine speed (both of which determine the flow coefficient) was known precisely until after the test. For a given axial velocity, the flow coefficient decreases with increasing blade speed. Since a higher rotational speed means stronger centrifuging of the fluid to the shroud, heat-transfer rate to the rotor shroud increases with decreasing tip-flow coefficient, as shown in the figures.

As can be seen from figures 3-10 and 3-12, there was a general increase in the value of the nozzle Nusselt number from the inlet to gauge number 9. Then there was a sharp increase from gauge 9 to gauge 10, followed by a sharp drop from gauge 10 to gauges 11 and 12. This trend in the Nusselt-number distribution can be explained in terms of the secondary vorticity and the acceleration effects through the nozzle. For a given inlet profile, the strength of the secondary vorticity increases with the flow turning. The increasing secondary vorticity (from inlet to gauge 9) gave rise to increasing heat-transfer rate. Around the nozzle throat, the flow velocity was very high, leading to the thinning down of the boundary layer and the flow started to relaminarize [35, 36]. In flow visualization studies, Herzig [19] found that the pattern of passage vortex formation did not change with varying inlet-flow Mach number. The consistently-high heat-transfer rates measured at the location of gauge 10 suggested that the secondary vortex impinged on the wall around gauge 10. Therefore, the combined effects of secondary vorticity and flow acceleration increased the heat-transfer rate from gauge 9 to gauge 10. As had been found elsewhere [37 to 43] in analysing heat transfer in rocket nozzles, the rapid rise in heat transfer was followed by an equally
rapid drop in the heat-transfer rate. The rapid drop occurred because the passage vortex detached from the corner of the blade-suction surface and the endwall, just downstream of gauge 10.

In the rotor shroud, there was a noticeable increase in the value of the Nusselt number at gauge 13 (as compared to gauge 12). As gauge 13 was located within the gap between the nozzle and the rotor, the increase in the Nusselt-number value was due to the centrifugal effects of the rotating turbine disc. As for the nozzle, the highest Nusselt-number values in the shroud were close to the blade trailing edge (gauges 18 and 19) while gauge 20 gave values lower than those for gauges 18 and 19. An interpretation of the rotor-shroud Nusselt-number variation is similar to that given for the nozzle endwall. In figure 3-12, it is noticed that the maximum shroud-Nusselt-number values were higher than the endwall values (gauges 18 and 19 vs gauge 10). This is not surprising because the flow-turning was higher in the rotor than in the nozzle. This, probably coupled with rotational and blade-end (tip leakage and/or blade scraping) effects, as well as the inlet streamwise vorticity and the wake from the nozzle, would result in increased values of the Nusselt number.

As the injection rate, \( \frac{m}{m} \), was increased from zero, the mid-span shear of the inlet flow was first decreased because the jet reduced the effect of the wake from the injection ring. At a certain value of the injection rate (below the minimum value attainable in the present work), there was no mid-span shear at the inlet the nozzle row. Increasing the injection rate above this value
resulted in increased mid-span inlet shear, whose effect complemented the effect of the boundary-layer shear.

Since the injection fluid in the present work was at a lower temperature than the main flow, the injected air tended to have two opposite effects on the heat transfer to the nozzle endwall and the rotor shroud. First, by increasing the shear of the inlet flow, it increased the heat-transfer rate due to the increased secondary vorticity. Secondly, by lowering the bulk temperature of the main flow, it reduced the heat-transfer rate. As the injection rate was increased, therefore, there was first an increase in the heat-transfer rate, followed by a decrease (if the injection rate was increased further). This trend in the heat-transfer rate is shown in figure 3-13, in which the ratio of the Nusselt number with injection and that without injection is plotted against the injection rate.

(iv) **BLADE-END EFFECTS**

As mentioned earlier, the Kistler miniature pressure transducer and the thin-film heat-transfer gauge were used to measure the effects of tip leakage and/or blade-passing on the rotor shroud. As the blades swept over a particular point on the shroud, the point was alternately exposed to relatively high and low pressures, depending on whether it was exposed to the pressure or the suction side of the blade. The Kistler gauge was to measure this periodic pressure variation. For a sufficiently small tip-clearance, each blade that passed over the point on the shroud scraped off the boundary layer and thus exposed the point momentarily to the hotter gas in the core of the flow,
before the boundary layer built up again. Figure 4-14 gives the oscilloscope trace of the periodic pressure variation on the shroud, as measured by the Kistler gauge. In spite of exhaustive sets of tests, no periodic temperature variation was detected by the thin-film heat-transfer gauge. As shown in Appendix D, the gauge was quite capable of measuring temperatures below 0.1°C and for blade-passing time as small as one micro-second. No temperature variation was detected because the rotor shroud was coated with a thin layer of the lubricating oil, which increased the thermal resistance around the gauge, thereby dampening out whatever temperature variation that might exist. Even in the absence of the oil film (which film does in fact occur in actual turbines), the tip leakage and/or the blade-scraping would not have resulted in temperature variations on the shroud. In Appendix D it is shown that the tip clearance was too large for the blade-passing frequency, to allow any temperature variation (due to tip leakage and/or blade-passing) to reach the shroud before the next blade came along.

3.5 COMPARISON WITH OTHER INVESTIGATIONS
The main difficulty in comparing an investigation of this nature with similar works is that very often investigators do not present a good description of the blade geometry and the measuring stations. Furthermore, the test conditions vary from one investigator to another and an exhaustive search did not turn up any suitable work that can be compared to the present work, quantitatively. Therefore, only qualitative comparison will be given here.

The present results for the nozzle have the same trends as the results obtai-
ed by Blair in reference [27]. He found that for the first 1/3 of the end-wall, the change in the heat-transfer rate was not much (cf gauges 4 to 8). There was an increase of heat transfer from the pressure to the suction side of the blade (cf gauges 5 and 6). Just downstream of the throat, he noticed a sharp rise followed by a drop in the heat-transfer rate (cf gauges 9 to 12). The same rapid changes in the heat-transfer rate near the throat had been observed by Wilson and Pope [25], Walker and Markland [26] and in heat-transfer analyses of rocket nozzles [37 to 43].

3.6 ESTIMATION OF EXPERIMENTAL ERROR

Every effort was made to ensure that the measurements were as accurate as possible. The first major step in this direction was to establish a new heating procedure for the facility. In this procedure, the exhaust air from the pebble-bed heater was used to heat up the system piping, thereby reducing the large drop that used to occur in the air temperature from the pebble-bed heater to the turbine entrance.

Because of the large number of thermocouple gauges used, a new thermocouple ice-box was built, in order to isolate the thermocouples from one another. This eliminated the errors associated with using one cold bath for the junctions of different thermocouples. (See the A.S.M.E. Code on the use of thermocouples for heat-transfer measurements).

The main errors associated with the calculated Nusselt numbers can be divided into two parts. The first was the error introduced by the data-acquisition instruments, i.e., the oscillograph machine and its galvanometers. The second
part was the error involved in measuring the traces on the oscillograph paper. In the analysis here, we take the instrument errors to be the same as that quoted by the manufacturers. These errors would cause the pen deflection to be different from the expected (ideal) deflection. Because of the width of the oscillograph paper (7 inches) and the need to keep all the eighteen pens on the paper, it was possible to make an error of 1 mm in measuring the deflection of a pen. [This error will hopefully be significantly reduced when the new computer-based data-acquisition system becomes operational]. The two types of errors (instrument and measurement) are assumed to be additive. There would also be errors introduced by the thermal contact resistance at the copper-slug-thermocouple interface and the dissipative losses in the transmission line from the turbine to the oscillograph machines, but these errors are considered negligible.

From the manufacturer's specifications, there would be three types of errors involved in using the oscillograph machine.

(i) The non-linearity of the galvanometers has an error of about 1%. This however, would cause a negligible error in the calculated Nusselt numbers.

(ii) The timing of the oscillograph machine has an error of about 1%.

(iii) The paper-speed-regulating mechanism has an error of about 3%

Errors (ii) and (iii) mean that the timing in the experiment could be wrong by about 4%. This would introduce an error of 3.2% in the calculated Nusselt numbers.
A look at equation C-11 shows that errors in the calculated Nusselt numbers could be due to errors in measuring any or all of the temperatures, $T_{\text{air}}$, $T_g$, $T_{wl}$, and $\Delta T_g$. Here, we assume an error of 1 mm in reading the deflections of the oscillograph pens. Below, we tabulate the errors associated with the temperatures given above.

<table>
<thead>
<tr>
<th>TEMPERATURE</th>
<th>% ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{air}}$</td>
<td>3.4</td>
</tr>
<tr>
<td>$T_g$</td>
<td>5.0</td>
</tr>
<tr>
<td>$T_{wl}$</td>
<td>1.2</td>
</tr>
<tr>
<td>$\Delta T_g$</td>
<td>12.3</td>
</tr>
</tbody>
</table>

If the errors are assumed to be random, then the error in the calculated Nusselt number is given by

$$\text{Error} = \left[ \sum (\text{error})^2 \right]^{1/2}$$

\[3-10\]

\[\therefore \text{Error} = (3.2^2 + 3.4^2 + 5.0^2 + 1.2^2 + 12.3^2)^{1/2} = 14.1\%\]

Hence, there could be an error of about 14% in the values of the Nusselt numbers.
4.1 PRELIMINARY

In chapter 2, it was pointed out that the streamline deviations (due to secondary flows) were so large that the small-perturbation assumption of the secondary-flow theory was no longer valid. Moreover, as Professor Hawthorne pointed out [44], the current theory is a linear theory and should not be expected to give the flow deviation (which is the product of two small terms: velocity and time). In view of the inapplicability of the theory to the present case, empirical formulae will be obtained to correlate the data for both the nozzle endwall and the rotor shroud.

The formulations of these formulae are given in this chapter. In order to use them, the following are assumed to be given.

(i) the turbine-blade geometry and setting.
(ii) the potential-flow net through the blade channel.
(iii) the total-pressure profiles at the inlet to the channel.

The correlations apply to points within the blade passage where flow-turning has occurred and there is therefore secondary vorticity.

4.2 NOZZLE CORRELATION

In the nozzle, the main factors that are likely to influence the heat transfer to the endwall are the Reynolds number, the inlet (normal) vorticity, the flow turning and the mixing parameter as given by equation 4-6 on page 72.

[Note that what is important, here, is the combination of inlet vorticity...}
and the flow turning, which combination gives rise to secondary vorticity].

A relation of the form,

$$\text{Nu}_\infty = f(R_{e_x}, \psi, \alpha, \beta)$$  \hspace{1cm}  (4-1)

can be written, where

$R_{e_x}$ \equiv Reynolds number based on the streamline length, $x$, and in which the fluid properties are evaluated at the nozzle stagnation temperature and static pressure.

$\psi$ \equiv a vorticity number as suggested by Shapiro [45].

$\alpha$ \equiv flow-turning angle, radians.

$\beta$ \equiv mixing parameter.

**REYNOLDS NUMBER, $R_{e_x}$**

Reynolds number is the ratio of inertia to viscous forces. It is given by the relation:

$$R_{e_x} = \frac{\rho V x}{\mu}$$  \hspace{1cm}  (4-2)

where

$\rho$ \equiv fluid density evaluated at the nozzle total temperature and nozzle-inlet static pressure.

$V$ \equiv fluid velocity.

$x$ \equiv streamline length, measured from the leading edge of the endwall.

$\mu$ \equiv fluid dynamic viscosity, evaluated at the nozzle total temperature.

A dominant parameter in the heat transfer in rocket nozzles is the mass-flow rate per unit area. Therefore, in order to account for the acceleration of the flow through the turbine nozzle, local values of mass-flow rate per
unit area ($\rho V$) are used in computing the Reynolds numbers.

**VORTICITY NUMBER, $\psi$**

The vorticity number, $\psi$, as given by Shapiro [45], is defined by the relation:

$$\psi = \frac{\frac{\partial V}{\partial z} L^2}{\nu} \quad 4-3$$

where

$$\frac{\partial V}{\partial z} \equiv \text{inlet velocity gradient.}$$

$$\nu \equiv \text{fluid kinematic viscosity, evaluated at the nozzle total temperature.}$$

$$L \equiv \text{reference length, determined as explained below.}$$

In the present study, there are two sources of inlet vorticity: the endwall boundary layer and the wake and/or the jet from the injection ring. (The injection ring takes the place of the combustion cans in an actual turbine).

If the boundary-layer growth up to the blade leading edge is over a short distance, then the boundary-layer velocity gradient is approximately equal to the free-stream velocity divided by the boundary-layer thickness, $\delta$.

The reference length, $L$, in the vorticity number, is equal to the corresponding boundary-layer momentum thickness.

As shown in figures 2-6, 2-7 and 2-8, the secondary-flow streamlines are more crowded near the plane of the inlet vorticity than farther from the plane. The sketch on the next page represents a typical plot of constant secondary-flow stream functions where the inlet vorticity is along the pitch
of the channel.

The distance, $L$, in the above figure, is the reference length used in the vorticity number for the ring and in cases where the growth of the boundary layer, up to the blade leading edge, is over a long distance. The vorticity number can be written in the alternative form,

$$
\psi = \frac{3V}{g_c} \frac{L^2}{v} = \frac{3V}{g_c} \frac{dP}{dz} \frac{L^2}{v}
$$

4-4

where

- $g_c = \text{gravitational conversion factor, 32.2 lbm-ft/lbf-sec}^2$.
- $V = \text{average main-flow velocity at the leading edge of the blade}$.
- $\frac{dP}{dz} = \text{total-pressure gradient, read off from the total-pressure profile at the blade leading edge}$.

For the definitions of $\rho$, $v$, and $L$ see equations 4-2 and 4-3. The vorticity number, as defined here, is a normal vorticity number. What determines the strength of secondary flow is the secondary vorticity which is dependent on the normal vorticity and the flow turning. We therefore define a
secondary-vorticity number as

\[ \Psi_s \equiv \Psi \cdot \alpha \]

where

\( \Psi \) \equiv normal vorticity number as defined in equations 4-3 and 4-4.
\( \alpha \) \equiv flow-turning angle.

The secondary-vorticity number, \( \Psi_s \), according to Shapiro [49], can be considered a Reynolds number. It is the ratio of inertia transport due to secondary vorticity and the viscous diffusion of secondary vorticity. The secondary-vorticity number is different from a Reynolds number based on a reference velocity. While Reynolds number can be defined (and has a non-zero value) for any flow, the secondary-vorticity number is defined only when a velocity gradient exists, and there is a flow turning. Consider the situations depicted in the figures below where two streams of velocities \( v_1 \) and \( v_2 \) are separated by a distance, \( H \). Suppose that the separation dis-

![Figure (a)](image-a)
![Figure (b)](image-b)
tance is decreased from \( H_1 \) in figure (a) to \( H_2 \) in figure (b) while the velocities, \( v_1 \) and \( v_2 \), remain unchanged. The vorticity is stronger in figure (b) than in figure (a), and the vorticity number (as defined here) has a higher value for figure (b) than for figure (a). On the other hand, the Reynolds number (whether based on \( v_1 \), \( v_2 \) or \( |v_1 - v_2| \)) remains unchanged.

The pitchline vorticity will have the same sense as the boundary-layer vorticity if there is a jet at the mid-span, but an opposite sense if there is a wake at mid-span. In the present study, a jet is introduced at mid-span; therefore the two vorticity numbers add together. The secondary-vorticity number is obtained by multiplying the normal-vorticity number (as defined in equations 4-3 and 4-4) by the flow-turning angle.

**MIXING PARAMETER, \( \beta \)**

The mixing parameter accounts for the fact that at the inlet to the nozzle blades, the injection and the main flows are not well mixed and the two streams have different temperatures. The mixing parameter is given by the relation:

\[
\beta = \frac{\dot{m} T_{inj}}{\dot{M} T_{th}} \tag{4-6}
\]

where

- \( \dot{m} \equiv \) injection mass-flow rate, lbm/sec.
- \( \dot{M} \equiv \) main mass-flow rate, lbm/sec.
- \( T_{inj} \equiv \) injection-flow static temperature, °R.
- \( T_{th} \equiv \) main-flow static temperature, °R.
The Nusselt number increases with both the Reynolds number and the secondary vorticity but decreases with the injection ratio. Therefore, the following functional relation can be written:

\[ Nu_x = f \left( \frac{Re_x \Omega \alpha}{\beta} \right) \]  \hspace{1cm} 4-7

A plot of \( Nu_x \) vs \( Re_x \Omega \alpha \) is shown in figure 4-1, from which the following relation is obtained:

\[ Nu_x = \frac{0.023}{\beta} (Re_x \Omega \alpha)^{0.4589} \]  \hspace{1cm} 4-8

\[ 2 \times 10^6 \leq (Re_x \Omega \alpha) \leq 10^9 \]

\[ 0.02 \leq \beta \leq 0.08 \]

4.3 ROTOR CORRELATION

The heat transfer to the rotor shroud will be influenced by the flow Reynolds number, the secondary vorticity, the blade-imparted kinetic energy and the tip-clearance effects. The tip-flow coefficient did not vary very much in the tests and Wilson and Pope's work [25] shows that the influence of the tip-flow coefficient is not very strong. Only the kinetic energy term and the tip-clearance effect are the additional terms not found in the nozzle correlation.

Tip-leakage vortex and scraping vortex occur at the blade tip, the latter being important only at very high rotational speeds (cf Salvage [2] and
Dean [21]). For the speeds used in the tests, tip-leakage vortex will be more important than scraping vortex. The rotating blades will centrifuge the gases towards the shroud, thereby increasing the heat transfer to the shroud. Therefore, the following functional relationship can be written for the rotor-shroud Nusselt number.

\[ \text{Nu}_x = f(\text{Re}_x, \Psi, \alpha, \Phi, K_p) \quad 4-9 \]

where

- \( \text{Re}_x \): Reynolds number based on streamline length, \( x \), and the local stagnation temperature and static pressure.
- \( \Psi \): normal vorticity number (equations 4-3 and 4-4).
- \( \alpha \): flow-turning angle.
- \( \Phi \): kinetic energy parameter (defined below).
- \( K_p \): tip-leakage parameter (defined below).
- \( \text{Nu}_x \): Nusselt number based on the central relative streamline length, measured from the shroud leading edge.

**KINETIC-ENERGY PARAMETER**

Weidhopf [28] and Louis, et al [20] used an expression similar to the following:

\[ \Phi \equiv \frac{U_T^2}{29.8 \sqrt{C_p(\Delta \text{T})}} \quad 4-10 \]

as a measure of the kinetic energy imparted to the gas by the blades. In the above expression,

- \( U_T \): blade-tip speed.
- \( g_c \): gravitational conversion factor, 32.2 lbm-ft/lbf-sec².
J \equiv \text{energy conversion factor, 778 ft-lbf/Btu.}

c_p \equiv \text{gas specific heat.}

\Delta T \equiv \text{total temperature drop across the rotor stage.}

The kinetic-energy parameter is the ratio of the rotational energy to the enthalpy drop of a given mass of fluid.

**TIP-LEAKAGE PARAMETER, } K_p

For a given tip clearance, the amount of tip leakage depends on the pressure difference between the pressure and the suction surfaces of the blade. The strength of the tip-leakage vortex is thus proportional to the difference between the pressures on the pressure and the suction sides of the blade. We define the tip-leakage parameter by the relation,

\[
K_p \equiv \frac{P_{pv} - P_{suc}}{P_T}
\]

where

- \( P_{pr} \) \equiv \text{static pressure on the pressure surface.}
- \( P_{suc} \) \equiv \text{static pressure on the suction surface.}
- \( P_T \) \equiv \text{total pressure.}

Dean [21] and Lakshminarayana [46] show that tip leakage has only a small effect in the first 1/4 to 1/3 of the blade chord. Therefore, we consider the tip-leakage vortex as influencing the heat transfer in the last 2/3 of the blade chord. It will be recalled that the tip-leakage vortex acts in a direction opposite to the passage vortex.
The rotor-shroud Nusselt number correlation may be written as

\[ \text{Nu}_x = f\left(\frac{\text{Re}_x \Phi}{1 - \frac{sK_p}{1}}\right) \]

where \(sK_p\) is the value at 1/3(chord) obtained from the differential-pressure curve of figure 3-11. The plot of \(\text{Nu}_x(1 - \frac{sK_p}{1})\) vs \(\text{Re}_x \Phi\) is shown in figure 4-2, from which the following relation is obtained:

\[ \text{Nu}_x = \frac{0.859}{(1 - \frac{sK_p}{1})} (\text{Re}_x \Phi)^{0.363} \]

\[ 4 \times 10^6 \leq \text{Re}_x \Phi \leq 2 \times 10^9 \]

Predictions using equations 4-8 and 4-13 are compared with measured values and the results are shown in figure 4-3. The ordinate denotes the error, given by: \((\text{Measured value}) - (\text{Predicted value})\). The figure shows that 60% of the entire data are predicted with less than 23% error and 90% of the entire data are predicted with less than 55% error. A plot of measured vs predicted Nusselt numbers is shown in figure 4-4. The 45° line represents the data with 0% error.

A look at figures 4-1 and 4-2 reveals that the rotor-shroud correlation gives better predictions than the nozzle endwall correlation. This is the case because the big jump in the measured Nusselt numbers on the rotor shroud (gauges 18 and 19) was reduced by incorporating the tip-leakage
parameter into the correlation. On the other hand, the big jump in the measured Nusselt numbers on the nozzle endwall could not be reduced as much as the jump in the rotor shroud. As was explained earlier, the rapid changes in the Nusselt number values around gauge 10 were probably caused by the secondary vorticity impinging on and detaching from the wall. Since there is, as yet, no way of estimating the trajectory of the secondary vortex through the turbine nozzle passage, the influence of the trajectory of the secondary vortex was not incorporated into the nozzle-endwall Nusselt-number correlation. The incorporation of the effect of this trajectory would probably have resulted in a better correlation formula for the nozzle endwall.

A sample calculation using equations 4-8 and 4-13 is presented in Appendix E.

4.4 SCOPE AND LIMITATIONS OF THE CORRELATIONS

Although the correlations were obtained using the data from this investigation, because of the large number of experimental data involved, the correlations are expected to apply to similar situations. The formulae, however, are not claimed to be universally applicable to all situations. As was pointed out in chapter 3, the blade-mid-span flow profile (at the inlet) depends on the injection rate employed. For a wake profile, the mid-span vorticity and the boundary-layer vorticity are opposed, while for the jet profile, the vortices complement each other. The experimental facility can give only one case (no injection) of wake profile. The correlations therefore apply to cases where the velocity (at inlet) decreases from mid-span
to the casings. The requirement is not restrictive because it is the condition that obtains in practice.

The mixing parameter, $\beta$, appears explicitly in the nozzle-endwall correlation because the injection and the main flows are not well mixed at the inlet to the nozzle. If there were a more thorough mixing (as would occur in practice), the correlation will likely not involve the mixing parameter explicitly (cf the correlation for the rotor shroud). In view of the absence of other suitable experimental data at the present time, confirmation of the applicability of the correlations to other data must await the availability of such data.

In order to be able to apply the correlations to any other case, the conditions stated in section 4-1 have to be met. The nozzle total-turning angle should not exceed $76^\circ$ while the Reynolds number (based on streamline length) lies between $8 \times 10^4$ and $7.5 \times 10^5$. The corresponding values for the rotor shroud are $109^\circ$ for the flow turning and between $3.3 \times 10^4$ and $2.9 \times 10^5$ for the Reynolds number. A sample calculation, using the correlations, is presented in Appendix E.

The correlations will be of practical use in the design of cooling systems for turbine casings. Using equations 4-8 and 4-13 and the methods outlined in this chapter and Appendix E, one calculates the heat-transfer distribution on the endwall and the shroud. From film-cooling studies by Amana [50], Liess [51], Ramette [52], Muska [53] and others, it is known that at the
point of injection the coolant jet separates from the surface to be cooled and then re-attaches to the surface farther downstream. The distance from the injection hole to the point where the jet re-attaches to the wall depends on the ratio of the coolant to the mainstream momentum and the angle of injection. The highest cooling effectiveness occurs at the point where the jet re-attaches to the surface. In designing a film-cooling scheme, one should place the injection holes at such a location that the maximum cooling effectiveness will occur at the point where the heat transfer is highest. Furthermore, since streamwise vortices are generated when coolant flow is injected into the mainstream during film cooling, one can probably use these vortices to counter the effect of the vortices generated in the mainstream. This will result in a more uniform heating of the casings, with the consequent prolongation of the life of the turbine casing.
5.1 CONCLUSIONS

The following is a summary of the significant results of this work.

1) The current secondary-flow theory is suitable for flow turning below 30°. Above this value, the predictions from the theory become increasingly unreliable with increasing flow turning.

2) Even for small shear-flow at the inlet to the channel, if the flow turning is large, strong secondary vortices are developed in the flow channel. This strong dependence of secondary vorticity on the flow turning is a property of the flow channel and cannot be significantly changed by taking either the Bernoulli-surface rotation or the fluid viscosity into consideration.

3) The trend of the nozzle Nusselt-number distribution (increasing from the leading edge to the trailing edge, and from the pressure surface to the suction surface) shows that secondary flows affect the heat transfer to the endwall. At 28% of the axial chord from the blade leading edge (where the flow has not been significantly accelerated), the heat transfer near the suction surface is about 140% of the value near the blade pressure surface and 170% of the value measured upstream of the blade row. At 57% of the axial chord, the measured heat-transfer rate is over 200% of the value upstream of the blade. The maximum measured value near the throat is over 400% of the value.
measured upstream of the blade row.

(4) Around the nozzle throat, the flow velocity is high and a combination of high velocity and secondary vorticity results in a rapid increase in the heat-transfer rate. This rapid increase is followed by a rapid decrease, suggesting impingement on and then detachment from the wall by the secondary vorticity, and the relaminarization of the flow.

(5) For moderate injection rates, the measured heat-transfer rates were much higher than the values when there was no injection. This is another evidence of the influence of secondary vorticity on the heat transfer to the casings. An injection profile at the inlet to the blade row means that the secondary vorticity from the jet will reinforce the boundary-layer secondary vorticity. The strong resultant secondary vorticity pushes up the rate of heat transfer.

(6) The rotor, because of its larger turning angle, develops stronger secondary vorticity than the nozzle. In addition to the large turning angle, centrifugal and tip-leakage effects, as well as the inlet vorticity and the wakes from the nozzle blades, gives rise to even higher heat-transfer rates in the rotor than in the nozzle, in spite of the work done by the gas as it flows through the rotor. The maximum heat-transfer rate in the rotor is about 160% of the maximum rate in the nozzle.

(7) The fact that the heat-transfer rate is higher near the trailing edge than near the leading edge means that over-heating of the trailing edge of the nozzle endwall and the rotor shroud may occur if
there is no adequate cooling of these parts of the casing.

(8) In the correlation for the rotor-shroud Nusselt numbers, the rapid changes in the Nusselt-number values near the shroud trailing edge are well accounted for by including the tip-leakage effect in the correlation.

(9) The trajectory of the passage vortex through the nozzle-blade channel has a significant influence on the distribution of heat transfer on the nozzle endwall. At present, there is no reliable way of estimating this trajectory. Incorporation of the trajectory in the nozzle-endwall correlation will lead to a correlation that can predict the rapid changes in the Nusselt-number values better than is now possible.

5.2 RECOMMENDATIONS

One of the main difficulties in turbomachine analyses is that very many variables are at play simultaneously and it is virtually impossible to isolate and study just one variable while the machine is running. Therefore, simplified models are used in studying specific variables. A set of experiments is listed below which if carried out would probably give a better insight into the mechanisms that influence heat transfer to the turbine casings.

(1) Heat-transfer measurement on the walls of a straight constant-area rectangular duct, with no flow acceleration.

(2) Heat-transfer measurement on the walls of a straight, convergent, rectangular duct.
(3) Heat-transfer measurement on the walls of a curved, constant-area, rectangular duct.

(4) Heat-transfer measurement on the walls of a stationary cascade with variable tip clearance.

A rectangular channel is a better model than a flat plate, for turbomachine work. Tests (1) and (2) will yield information on the effect of flow acceleration on the heat transfer to the walls. Tests (1) and (3) will be used to determine the influence of passage secondary flow alone on the heat transfer to the walls while test (4) will yield information on the effects of tip clearance on the heat transfer to the walls.

If measurements are made on a real turbine casing, it will be easy to estimate what fraction of the observed value is due to passage secondary flows, flow acceleration, tip-leakage, etc.

In film-cooling schemes, it will be beneficial to move the injection holes five to ten hole-diameters upstream of the location of the highest heat-transfer rate (as determined using the correlations of chapter 4). It is also desirable to determine what combination of coolant-injection rate and injection angle will be the best for neutralizing the effect of the secondary vorticity due to the non-uniformity of the main flow.
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FLOW CHART FOR SECONDARY FLOW CALCULATION

Start

Read input values

Calculate potential velocities

Initialize stream function

Calculate secondary vorticity

Set iteration limit

Solve the Poisson equation

Is accuracy acceptable?

Yes

Print stream function values

Compute and print secondary velocities

END

No

Has iteration limit been exceeded?

Yes

Compute and print particle deviations

STOP

No
C ASSIGN VALUES TO THE STREAM FUNCTION, SIGN.
0022  DO 60  K = 1,5
0023  DO 60  I = 1,17
0024  60  CONTINUE
0025  SIGM(I,J,K) = 0,9
0026  CONTINUE
0027  DO 70  K = 1,5
0028  DO 70  I = 2,10
0029  SIGM(I,J,K) = 1,0
0030  CONTINUE
0031  70  CONTINUE
C READ IN VALUES INTO 2-D ARRAY OF POTENTIAL SURFACE SEPARATION.
0032  READ(6,10) (DSEP(I,J), J=1,17, I=1,17)
C ASSIGN VALUES TO 3-D ARRAY FOR POTENTIAL SURFACE SPACING.
0033  DO 90  K = 1,5
0034  DO 90  I = 1,17
0035  DSEP(I,J,K) = DSEP(I,K)
0036  90  CONTINUE
C ASSIGN VALUES TO DIMENSIONLESS POTENTIAL VELOCITY DISTRIBUTION, PDTV.
0037  CONV = 3.70274 * 0.15
0038  TSEP = 0.164*CONV
0039  DO 99  K = 1,5
0040  DO 99  I = 2,10
0041  DSEP(I,J,K) = CONV*(TSEP(I,J-1,K) + TSEP(I,J,K)) / 2.0
0042  99  CONTINUE
0043  DO 100  K = 1,5
0044  DO 100  I = 1,17
0045  PDTV(I,J,K) = CONV*(TSEP(I,J,K))
0046  100  CONTINUE
C ASSIGN VALUES TO DISTANCE OF POTENTIAL STREAMLINES FROM BLADE PRESSURE SURFACE
0047  DO 105  K = 1,43
0048  DO 105  I = 1,17
0049  XSEP(I,J,K) = I1 + 0.1
0050  105  CONTINUE
C DETERMINATION OF THE RHS OF THE POISSON EQUATION.
0051  DO 110  K = 1,43
0052  DO 110  I = 1,17
0053  110  CONTINUE
C PRETEND THE CONSTANT, E, ON THE RHS OF THE POISSON EQUATION.
0054  E = TSEP(I,J,K)
0055  DS2 = (SIN(2.54*ALPHA(I,J,K))) / 2.0
0056  PCO = CONV(TSEP(I,J,K)) / DS2
0057  ETA = (SIN(2.54*ALPHA(I,J,K))) / DS2
C EVALUATE THE RHS OF THE POISSON EQUATION.

\[ D_{ij} = (\Delta x \cdot \Delta y) \cdot \left( \frac{\partial^2 u}{\partial x^2} \right)_{ij} + (\Delta x \cdot \Delta y) \cdot \left( \frac{\partial^2 u}{\partial y^2} \right)_{ij} \]

C EVALUATE THE RHS ON THE RHS OF THE POISSON EQUATION.

\[ D_{ij} = (\Delta x \cdot \Delta y) \cdot \left( \frac{\partial^2 u}{\partial x^2} \right)_{ij} + (\Delta x \cdot \Delta y) \cdot \left( \frac{\partial^2 u}{\partial y^2} \right)_{ij} \]

C REDUCE FOR USP IN FINITE DIFFERENCE FORM.

\[ F_{ij} = \text{DESFHS} \]

C START THE ITERATION PROCES.

\[ r = 1.0 \]

C SP LIMIT ON THE NUMBER OF ITERATIONS.

\[ r = 10 \]

C TEST RESULT FOR ACCURACY.

\[ r = \text{DESFHS} \]
DO 210  
X = 1.67
DO 200  J = 2, 10
DO 200  T = 2, 16
G = I

VPM(I, J, P) = SIGM(T+1, J, P) - SIGM(T-1, J, P) / (2.0001 * 50.1)

VPM(I, J, 0) = SIGM(T, J-1, K) - SIGM(T, J+1, K) / ((TDP(I, J, P) + 1) * TDP(I, J-1, K) * 50.1)

C CALCULATE TRAVels FROM THE POTENTIAL SURFACE TO THE NEXT.

DTS = ((SIGT(I, J, K) + (POTV(I, J, K))

C CALCULATE COMPONENT DEVIATIONS, VPM AND VPP.

TP(I, J, P) = DPM(I, J, P) * (DPV(I, J, K) * POTV(I, J, K) / TCDY)

1 = TDP(I, J-1, K) + (DPV(I, J-1, K))

TP(I, J, P) = DPM(I, J, P) / TDP(I, J-1, K)

TP(I, J, 0) = DPM(I, J, 0) / TDP(I, J-1, K)

CONTINUE

WRITE(*, 175) (KEV(I, J, K), J = 2, 10, I = 2, 16)
WRITE(*, 176) X
WRITE(*, 176) K
CONTINUE

CONTINUE

FORMAT(12, 100) 1
CONTINUE

220 FORMAT(12, 100) 1
CONTINUE

210 FORMAT(12, 100) 1
CONTINUE

240 FORMAT(12, 100) 1
CONTINUE

290 FORMAT(12, 100) 1
CONTINUE

300 FORMAT(12, 100) 1
CONTINUE

STOP
END
C SECONDARY FLOW CALCULATIONS FOR T64 1ST STAGE ROTOR.
C
C ALPHA = TURBINE ANGLE AT DIFFERENT POINTS ON STREAMLINES LYING ON A PLANE
C NORMAL TO THE BLADE SPAN.
C TALPHA = TURBINE ANGLE AT DIFFERENT POINTS ON ALL STREAMLINES IN THE BLADE
C PASSAGE.
C DP = PITCHWISE SPACING OF STREAMLINES ON A PLANE NORMAL TO BLADE SPAN.
C TD = PITCHWISE SPACING OF ALL STREAMLINES IN THE BLADE CHANNEL.
C POTV = DIMENSIONLESS POTENTIAL VELOCITY.

DIMENSION ALPHA(11,37), TALPHA(17,11,37), DP(10,37),
        1 TDP(17,10,37), SIGH(17,11,37), DST(11,37), TDST(17,11,37),
        1 KDEV(17,11,37), ZDEV(17,11,37), XP(17,10,37)

C READ IN VALUES INTO THE 2-D ARRAY OF TURBINE ANGLES.
READ(5,13) ((ALPHA(J,K),J=1,11), K=1, 37)
R8F10.5/3F10.5
FANG = 0.01745 * 1.0
C ASSIGN VALUES IN RAZIANS TO THE 3-D ARRAY OF TURBINE ANGLES.
DO 20 J=1,11
    DO 20 K=1,17
    TALPHA(I,J,K) = (ALPHA(J,K)) * 0.01745 + FANG
20 CONTINUE

C READ IN VALUES INTO 2-D ARRAY OF PITCHWISE SPACING OF STREAMLINES.
READ(5,40) ((DP(J,K), J=1,10), K=1, 37)
40 F8.5/2F10.5
C ASSIGN VALUES TO THE 3-D ARRAY OF PITCHWISE SPACING OF STREAMLINES.
DO 30 J=1,10
    DO 30 K=1,37
        TDP(I,J,K) = DP(J,K)
30 CONTINUE

C READ IN VALUES INTO THE 2-D ARRAY OF POTENTIAL SURFACE SEPARATION.
READ(5,10) ((DST(J,K), J=1,11), K=1, 37)
10 F8.5/2F10.5
C ASSIGN VALUES TO 3-D ARRAY FOR POTENTIAL SURFACE SPACING.
DO 40 K=1,17
    J=1,11
    DO 40 I=1,17
        DST(I,J,K) = DST(J,K)
40 CONTINUE

C ASSIGN VALUES TO THE STREAM FUNCTION, SIGH.
DO 50 K=1, 37
    J=1,11
    DO 50 I=1,17
        SIGH(I,J,K) = 0.0
50 CONTINUE

C ASSIGN VALUES TO DIMENSIONLESS RELATIVE POTENTIAL VELOCITY DISTRIBUTION, POTV.
DO 60 K=1, 37
    J=1,11
    DO 60 I=1,17
        POTV(I,J,K) = POTV(I,J,K)
60 CONTINUE
DO 90 K = 1, 37
DO 90 J = 2, 10
POTV(I,JK) = CON/((TDP(I,J-1,K)+TDP(I,J,K))/2.0)

DO 130 K = 1, 37
DO 100 I = 1, 17
POTV(I,1,K) = CON/(TDP(I,1,K))
POTV(I,11,K) = CON/(TDP(I,10,K))

H = 0.45/0.6
DS = 16.0
DZ = H/DS
DZSQ = DZ*DZ

C AXILL VALUES TO DISTANCE OF POTENTIAL STREAMLINES FROM THE BLADE

C PRESSURE SURFACE.

C DISTURBANCE OF THE RHS OF THE POISSON EQUATION.

VS = POTV(I,J,K)/POTV(I,J,1)
SUB11 = (SUB11 - SUB12)*VR
SUB21 = SUB1*SUB2*DW1DR
SUB22 = SUB11*SUB2*DW1DR

FHS = (-SUB11*SUB2*DW1DR)

C REDUCE RHS FOR USE IN FINITE DIFFERENCE FORM.

FHS(I,J,K) = 0.25*FHS
C START THE ITERATION PROCESS.

DO 190 K = 1,36
ICOUNT = 0

C SET LIMIT ON THE NUMBER OF ITERATIONS.

DO 130 N = 1,300
ICHECK = 0
ICOUNT = ICOUNT+1

DO 120 J = 1,10
DO 120 I = 2,16
DN = (TDP(I,J,K)+TDP(I,J-1,K))/2.0
DNSQ = D*N

C APPLY CENTRAL DIFFERENCE ITERATION FORMULA TO POISSON EQUATION.

STORE = SIGH(I,J,K)

SIGH(I,J,K) = (SIGH(I-1,J,K)+SIGH(I+1,J,K)+
(/SIGH(I,J-1,K)+
1 SIGH(I,J+1,K)+PSIGh(I,J,K)))/(2.0*1.0*DNSQ))

C TEST RESULT FOR ACCURACY.

TEST = ABS(SIGH(I,J,K)-STORE)

IF (TEST .GT. 0.001) ICHECK = 1

CONTINUE

IF(ICHECK) 130, 170, 130

130 CONTINUE

WRITE(6,140) ICOUNT, K

140 FORMAT(///'DID NOT CONVERGE AFTER ',1X,I3,'ITERATIONS BUT','
1' STEEP FUNCTION DISTRIBUTION FOR K = ',12,'FOLLOWS:'//)

145 FORMAT ///'VELOCITY DISTRIBUTION FOR K = ',12,'FOLLOWS:'///)

150 FORMAT(6,155) ((SIGH(I,J,K), J = 1,9), I = 1,17)
155 FORMAT(6,155) ((SIGH(I,J,K), J = 10,11), I = 1,17)
160 FORMAT(2(2,F13.7,2X))
165 FORMAT(2(2,F13.7,2X))
170 FORMAT(1X,1P2E15.6)
180 FORMAT(///'STEEP FUNCTION DISTRIBUTION AFTER ',12,'ITERATIONS FOR K = ',12,'FOLLOWS:'//)

185 FORMAT ///'VELOCITY DISTRIBUTION FOR K = ',12,'FOLLOWS:'///)

190 CONTINUE

C CALCULATE PERFORMED VELOCITIES, VPN AND VPZ.

DO 210 J = 1,36
210 CONTINUE

DO 230 I = 2,16
230 CONTINUE

RT = ((3-1.0)*D2
VPN(I,J,K) = (SIGH(I+1,J,K)-SIGH(I-1,J,K))/(2.0*D2)*14.5
VPZ(I,J,K) = (SIGH(I,J-1,K)-SIGH(I,J+1,K))/(2.0*D2)*
1 TDP(I,J-1,K))*14.5

C CALCULATE TIME TO GO FROM ONE POTENTIAL SURFACE TO THE NEXT.

DTIME = (TDST(I,J,K))/(PSIGh(I,J,K))
C CALCULATE COMPONENT DEVIATIONS, XDEV AND ZDEV.
0126 IF(K.EQ.1) XDEV(I,J,K) = (DTIME*VPN(I,J,K)*POTV(I,J,K))/TCON
0127 IF(K.GT.1) XDEV(I,J,K) = (DTIME*VPN(I,J,K)*POTV(I,J,K))/
0128   TCON + (XDEV(I,J,K-1))
0129 IF(K.GT.1) ZDEV(I,J,K) = DTIME*VPZ(I,J,K)
0129 IF(K.GT.1) ZDEV(I,J,K) = DTIME*VPZ(I,J,K) + ZDEV(I,J,K-1)
0130 CONTINUE
0131 FORMAT(///' COMPONENTS OF PERTURBATION VELOCITY, VPN, FOR K=',
0132   12, ' FOLLOW:/')
0133 FORMAT(///' COMPONENTS OF PERTURBATION VELOCITY, VPZ, FOR K = ',
0134   12, ' FOLLOW:/')
0136 FORMAT(///' FLOW DEVIATIONS IN X-DIRECTION FOR K = ',12,
0137   1 ' FOLLOW:/')
0139 FORMAT(///' FLOW DEVIATIONS IN Z-DIRECTION FOR K = ',12,
0140   1 ' FOLLOW:/')
0140 STOP
0141 END
As was pointed out in section 1.3, the current secondary-flow theory treats secondary flow as a small perturbation of the main flow. Therefore in situations where the secondary-flow effect is comparable to the main-flow effect, the theory will be inapplicable.

The solution of the secondary-flow equation can be written in the following way:

\[ \Omega_{s2} = \frac{f(x_1, x_2, x_3, \ldots)}{\cos \alpha_1 \cos \alpha_2} \]  

B-1

where

\( \Omega_{s2} \) = secondary vorticity at location 2 on a streamline.

\( \alpha_1, \alpha_2 \) = flow angles at locations 1 and 2 along a streamline, 1 being upstream of 2.

\( f(x_1, x_2, x_3, \ldots) \) = function of inlet vorticity, flow turning, primary velocity, etc.

For a given situation, \( \alpha_1 \) is fixed and \( \alpha_2 \) varies from \( \alpha_1 \) to the flow angle at the blade exit; \( f(x_1, x_2, x_3, \ldots) \) usually does not vary very much. For small values of \( \alpha_2 - \alpha_1 \), \( \cos \alpha_1 \cos \alpha_2 \) does not vary over a wide range. Therefore the secondary vorticity, \( \Omega_{s2} \), has a small value and the associated secondary velocities are small also. In compressor blades, the total flow-turning angle is small (about 20° to 30°) and the secondary-flow equation
in many cases gives good predictions.

For large flow turning, as in turbines (over 60°), the angle \( \alpha_2 \) is large, making \( \cos \alpha_2 \) small. Therefore as \( \alpha_2 \) gets larger (\( \cos \alpha_2 \) smaller), the secondary vorticity gets larger. At \( \alpha_2 = 90^\circ \), the solution is indeterminate. This partly explains why predictions for large flow turnings have been unsatisfactory.

Below, we give a pictorial description of what happens as the streamlines are turned by the blades. Figure B-1 shows how potential surfaces deform due to flow turning. The plane ABCD corresponds to a (pitch-streamline) plane of the primary flow. At an arbitrary location in the blade channel, another plane, abcd (pitch-span), is drawn perpendicular to ABCD. In the solution for the secondary-flow velocities, the derivatives of the stream function are obtained in the abcd plane.

Suppose that the line 1-2 on ABCD represents a primary streamline (perpendicular to abcd) which intersects abcd at the point 2. Because of secondary-flow effects, the actual streamline will intersect the plane abcd at point 2'. The primary streamline may be thought of having deflected by amounts 2 to 3 and 3 to 2', giving a new flow path, 1-2'. This new path is no longer perpendicular to the plane abcd. The appropriate potential surface for the streamline 1-2' is not abcd but a plane perpendicular to 1-2'.

All the primary streamlines do not have the same turning for a give position
downstream of the blade-leading edge. Hence at any location in the passage, there will be countless potential surfaces (one for each streamline), all having different orientations. For small flow-turning, the real potential surfaces do not deviate very much from the original potential surface, abcd. Therefore, calculations based on the original surface give fairly good predictions.

For large turning, the situation is different. Since the strength of the secondary vorticity and the associated velocities increase with flow turning, the actual potential surfaces deviate very much from the original surface. Therefore the calculations based on the original surface do not reflect the actual situation. This problem, associated with large flow turning, is not solved by taking viscosity and Bernoulli-surface rotation into account in the solution.
SAMPLE DATA REDUCTION AND HEAT TRANSFER CALCULATION.

Below, the relation for the heat transfer rate and the Nusselt number (using the calorimetric copper gauges) is derived.

The gauge is a small slug of copper encased in a nylon cup and the face of the slug in the cup has thermocouple wires spot-welded to it. The gauge is then embedded in the nozzle endwall and the rotor shroud.

The following assumptions are made in the analysis:

(i) The copper slug is so small that at any instant, it has a uniform temperature.

(ii) During the period of the run (approximately one second) the temperature of the material surrounding the gauge remains unchanged from the temperature at the initiation of the run (diaphragm burst).

Energy balance for the gauge is

$$q_{trg} = q_{sg} + q_{lg}$$

C-1
where

\( q_{\text{trg}} = \text{rate of heat transfer from the gas to the gauge.} \)

\( q_{\text{sg}} = \text{rate at which the gauge stores heat.} \)

\( q_{\text{lg}} = \text{rate at which the gauge loses heat through the insulation.} \)

\[
q_{\text{trg}} = h_{\text{air}} A_g (T_{\text{air}} - T_g) \tag{C-2}
\]

\[
q_{\text{sg}} = (mc)_g \frac{dT_g}{dt} = (mc)_g \frac{\Delta T}{\Delta t} \tag{C-3}
\]

\[
q_{\text{lg}} = k_{\text{ins}} A_{\text{ins1}} \frac{T_g - T_{\text{wl}}}{\Delta x_1} + k_{\text{ins}} A_{\text{ins2}} \frac{T_g - T_{\text{w2}}}{\Delta x_2} \tag{C-4}
\]

where

\( h_{\text{air}} = \text{heat transfer coefficient at the gauge/air interface.} \)

\( A_g = \text{gauge area exposed to gas.} \)

\( T_{\text{air}} = \text{air temperature.} \)

\( T_g = \text{gauge temperature.} \)

\( m_g = \text{mass of copper slug.} \)

\( c_g = \text{specific heat of copper.} \)

\( t = \text{time.} \)

\( k_{\text{ins}} = \text{thermal conductivity of nylon.} \)

\( A_{\text{ins1}} = \text{cross-sectional area of gauge.} \)

\( A_{\text{ins2}} = \text{lateral area of gauge.} \)

\( \Delta x_1 = \text{thickness of insulation at the bottom.} \)

\( \Delta x_2 = \text{thickness of the insulation at the sides.} \)

\( T_{\text{w1}} = \text{gauge temperature just before diaphragm burst.} \)
\[ T_{w2} = \frac{1}{2}(T_{w1} + T_g) \]

\[
k_{\text{ins}}A_{\text{ins}2} \frac{T_g - T_{w2}}{\Delta x_2} = (kA_2)_{\text{ins}} \frac{T_g - \frac{1}{2}(T_{w1} + T_g)}{\Delta x_2} = (kA_2)_{\text{ins}} \frac{1/2(T_g - T_{w1})}{\Delta x_2} \]

C-5

Substitute in equation C-4 to get:

\[
q_{lg} = (kA_1)_{\text{ins}} \frac{T_g - T_{w1}}{\Delta x_1} + (kA_2)_{\text{ins}} \frac{T_g - T_{w1}}{2\Delta x_2} = k_{\text{ins}}(T_g - T_{w1}) \left[ \frac{A_{\text{ins}1}}{\Delta x_1} + \frac{A_{\text{ins}2}}{2\Delta x_2} \right] \]

C-6

Substitute equations C-2, C-3, and C-6 into equation C-1.

\[
h_{\text{air}}g(T_{\text{air}} - T_g) = (mc) \frac{\Delta T_g}{\Delta t} + k_{\text{ins}}(T_g - T_{w1}) \left[ \frac{A_{\text{ins}1}}{\Delta x_1} + \frac{A_{\text{ins}2}}{2\Delta x_2} \right] \]

C-7

Define Nusselt number,

\[
Nu_x = \frac{h_{\text{air}}}{k_{\text{air}}} \]

C-8

where

- \( b \equiv \text{rotor blade axial chord.} \)
- \( h_{\text{air}} \equiv \text{local gas heat transfer coefficient.} \)
- \( k_{\text{air}} \equiv \text{local gas thermal conductivity.} \)

Therefore, \( h_{\text{air}} = \frac{Nu_x k_{\text{air}}}{b} \)

C-9

Substitute for \( h_{\text{air}} \) in equation C-7.

\[
\frac{Nu_x k_{\text{air}}}{b} \frac{A_{\text{air}}}{A_{\text{g}}}g(T_{\text{air}} - T_g) = (mc) \frac{\Delta T_g}{\Delta t} + k_{\text{ins}}(T_g - T_{w1}) \left[ \frac{A_{\text{ins}1}}{\Delta x_1} + \frac{A_{\text{ins}2}}{2\Delta x_2} \right] \]

C-10
Re-arranging, we get:

\[
\text{Nu}_x = \frac{b}{k_{\text{air}} x} \frac{\Delta T}{g} + A_{\text{ins}} (T_g - T_{W}) \frac{\Delta T}{A x_1} + \frac{A_{\text{ins}}}{2 A x_2} \tag{C-11}
\]

Equation C-11 is used in calculating the Nusselt numbers for the copper gauges. It may be observed that the quantity in the curly brackets, \{\}, is the heat-transfer rate to the gauges. The terms in the square brackets, [ ], are known from the physical dimensions of the nylon cup. The blade axial chord, \(b\), is known, so also are the quantities \(A_g\), \((mc)\), and \(k_{\text{ins}}\). From the experiment, \(T_{W1}\), \(T_g\), and \(\frac{\Delta T}{\Delta t}\) are known. The only term to be determined on the right-hand side of equation C-11 is the air temperature. This is known at the locations of the total-temperature probes. The air temperature at the inlet to the rotor blade may be used for calculating the Nusselt numbers for all the gauges. This is one of the two methods used here. An alternative method (also used here) estimates the work done by the gas as it flows through the rotor and hence the gas temperature at each gauge location is estimated. The difference between the two methods is that the first method gives Nusselt number values (near the rotor-trailing edge) smaller than the real values.

A brief description of the method used in obtaining the temperatures needed in solving equation C-11 is given below. Before each experiment, the position of each oscillograph-galvanometer pen is noted, as well as the corresponding gauge temperature. After the experiment, each pen deflection, due to a known temperature change (usually between gauge temperature and ice...
point) is measured. Since the temperature vs pen-deflection plot is linear 
for the temperature range of interest, the temperature/deflection slope, 
\( \Delta T/\Delta D \), is known for each gauge.

During the experiment, the pen position is known at each instant. Suppose 
that the initial pen position is denoted by \( D_i \) and the corresponding tem-
perature, \( T_i \). If during the experiment the pen is at a position \( D \), then the 
gauge temperature corresponding to \( D \) is given by:

\[
T_g = (D - D_i) \frac{\Delta T}{\Delta D} + T_i
\]  

C-12

In this way, \( T_{\text{w1}} \), \( T_g \), and \( T_{\text{air}} \) (see equation C-11) are determined. After 
about 0.2 second of the experiment, the pens for the copper gauges have 
linear displacement with time. This gives constant \( \Delta D/\Delta t \) for each gauge. 
Therefore,

\[
\frac{\Delta T_g}{\Delta t} = \left( \frac{\Delta D}{\Delta t} \right) \left( \frac{\Delta T_g}{\Delta D} \right)
\]  

C-13

Hence, all the quantities needed to solve equation C-11 are known either 
from the physical properties of the gauge or from experimental measurements.
In chapter 3, it was stated that the thin-film heat-transfer gauge did not measure any periodic temperature change on the rotor shroud. Here more details are given about the experiments with the gauge and reasons are given why no periodic temperature variations were detected.

The gauge was exposed to the temperature behind a shock in a shock tube and its (gauge) response was noted. Next, by means of a pulse generator, constant currents were passed through the gauge. Whether the pulse duration was 1 ms or 0.1 μs, the gauge response (which was almost instantaneous) was the same. These sets of tests showed that the gauge could respond to high frequency (of the order of micro-seconds) signals.

In order to use the gauge for heat-transfer measurements, two constants were needed. The first constant was obtained by immersing the gauge in a bath of varying temperature and measuring the changes in the gauge resistance with changes in temperature. The constant, $c_1$, is given by:

$$c_1 = \frac{\Delta R_g}{\Delta T_g}$$

where

$\Delta R_g =$ change in gauge resistance.

$\Delta T_g =$ change in gauge temperature.
The second constant was needed for calculating the heat flux to the gauge. It was obtained by passing a known current through the gauge and measuring the change in the gauge voltage with time. The constant, \( s_1 \), is given by:

\[
 s_1 = \frac{A_g \Delta V_g(t)}{I^2 c_1 R_g \sqrt{t}} 
\]

where

- \( A_g \) = gauge (paint) surface area.
- \( I \) = electrical current.
- \( R_g \) = gauge resistance (ohm).
- \( \Delta V_g(t) \) = change in gauge voltage.
- \( t \) = time.

In the next set of tests, the gauge was installed in a cascade under an air-turbine in order to measure the temperature variations due to the compression of the air by the turbine blades. Figure D-1 shows the measured periodic variation for a turbine speed of 60,000 rpm. Peak-to-peak voltage in the figure corresponds to a temperature variation of 0.1°C. For a known pressure variation at the above turbine speed, isentropic relations predict a temperature variation of 4.2°C. Hence, the measured temperature change is only about 2% of the predicted value.

The gauge was transferred from the cascade to the T64 turbine and tested with both cold air (as for the cascade) and hot air. In none of the tests was any periodic changes detected, although the pressure variation was twice as large as for the cascade test.
Below, we give reasons why very small temperature variation was measured for the air turbine and none for the T64 turbine.

(i) The blade-passing frequency for the T64 turbine was about five times that for the air turbine.

(ii) The tip clearance for the T64 turbine was about two times as large as that for the air turbine.

From reason (i), it is seen that heat had about five times as much time to diffuse through a given layer of air in the air turbine, as it (heat) has in the T64 turbine. From reason (ii), it can be said that a thicker layer of air would cover the gauge in the T64 turbine than in the air turbine. The two reasons together made it more difficult for periodic temperature changes to occur in the T64 turbine than in the air turbine.

The use of the heat diffusion model would apply to the case of tip-leakage and blade-passing. But the temperature variation associated with the pressure change should get to the shroud since the pressure change got to it periodically (figure 3-14). The fact that a very low value was measured in the cascade and none in the turbine suggested that either something was interfering with the gauge or the ideal-gas assumption was invalid in the neighbourhood of the gauge. The two effects seemed to be occurring simultaneously.

The air for the air-turbine test came straight from the storage cylinders and was not contaminated by combustion products. However, the turbine-lubrication oil left a film of oil on the cascade wall, even though the cascade
was well ventillated. The oil film increased the thermal resistance close
to the cascade wall and this tended to dampen out any temperature vari-
tations before they reached the gauge. That explained why only a small frac-
tion of the predicted temperature was measured. The situation was worse for
the T64 turbine. The test air got contaminated in flowing through the pebble-
bed heater and the system piping. Moreover, the gauge was in a confined space
so that any lubrication-oil mist in the turbine could not be vented out as
fast as in the cascade. Here again we can explain the fact that no tempera-
ture variation was measured on the shroud.

Therefore, a combination of tip clearance, turbine speed and environmental
conditions made it impossible for measurable periodic temperature to reach
the T64 rotor shroud.

Below, we give a short order-of-magnitude analysis of heat diffusion through
the shroud boundary layer. The one-dimensional heat-transfer equation app-
lied to the boundary layer is,

$$\frac{dT}{dt} = \frac{2}{\alpha} \frac{a \cdot T}{\alpha y^2}$$

D-3

where
T = temperature
t = time
\(\alpha\) = thermal diffusivity
\(y\) = distance measured away from the shroud.
Order-of-magnitude analysis:

\[
\frac{T}{t} \sim \alpha \frac{T}{\bar{c}} \; ; \; \; \; t \sim \frac{\delta^2}{\alpha} \quad D-4
\]

where \( \delta \) = boundary-layer thickness. For the turbine, the minimum design clearance is 0.025" and the maximum is 0.035". The gauge was recessed a bit to protect it from damage. Let us assume that when a blade was over the gauge, the thickness of the boundary layer was 0.025". For air at 200°F, \( \alpha = 0.05 \) in²/sec. Therefore, substitution in equation D-1 gives:

\[
t \sim (0.025)^2/0.05 = 0.0125 \text{ sec.}
\]

\[
t \sim 1.25 \times 10^{-2} \text{ sec}
\]

Blade-passing time = 68 μs = \( o(10^{-4} \text{ sec}) \).

Therefore \( \frac{t_{\text{temp}}}{t_{\text{blade}}} \) = \( 10^{-2}/10^{-4} = 100 \) \quad D-5

The above analysis indicates that the turbine was too fast for the temperature variation to be felt periodically on the shroud. A solution of the one-dimensional equation (even with a smaller boundary-layer thickness) leads to the same conclusion as was reached in the order-of-magnitude analysis.
APPENDIX E

SAMPLE CALCULATION USING CORRELATION FORMULAE

The use of equations 4-8 and 4-13 is demonstrated by applying them to a specific test (number 4).

NOZZLE

\[ \text{Nu}_x = \frac{0.023}{\beta} (\text{Re}_x \cdot \Psi \alpha)^{0.4589} \]  
\[ 2 \times 10^6 \leq (\text{Re} \cdot \Psi \alpha) \leq 10^9 \]  
\[ 0.02 \leq \beta \leq 0.08 \]

where

\( \text{Nu}_x \) = nozzle endwall Nusselt number based on streamline length, measured from the leading edge of the endwall.

\( \beta \) = mixing parameter.

\( \text{Re}_x \) = Reynolds number based on streamline length.

\( \Psi \equiv \frac{\partial v}{\partial z} \frac{L^2}{v} \) = vorticity number (see chapter 4).

\( \alpha \) = flow-turning angle, obtained from the potential-flow net (fig. 2-2). Recall that the secondary vorticity number, \( \Psi_s \equiv \Psi \alpha \).

In calculating the Reynolds numbers, the local values of the mass-flow rate per unit area, \( \rho v \), are used. These are obtained from the potential-flow
net of figure 2-2. At the location of gauge 1, the mass-flow rate per unit area is given by,

\[ (\rho V)_1 = \frac{M_T}{A_1} \tag{E-1} \]

where \( M_T \) is the total mass-flow rate and \( A_1 \) is the flow area at gauge 1 location.

\[ A_1 = \pi (r_T^2 - r_h^2) \tag{E-2} \]

where

\[ r_T \equiv \text{blade tip radius} = 8.16 \text{ ins.} \]
\[ r_h \equiv \text{blade hub radius} = 7.17 \text{ ins.} \]

The dynamic viscosity (based on the gas total temperature) is constant through the nozzle passage.

\[ \Re_x = \frac{(\rho V)_x}{\mu} \cdot \frac{x_1}{x_1} \frac{(\rho V)_1}{(\rho V)_1} = \frac{(\rho V)_1 x_1}{\mu} \frac{x}{x_1} \frac{(\rho V)_1}{(\rho V)_x} \]

\[ \therefore \ Re_x = Re_x \left( \frac{x}{x_1} \right) \frac{(\rho V)_1}{(\rho V)_1} \tag{E-3} \]

Hence, once the Reynolds number for gauge 1 is calculated, the values for the other gauges are obtained by multiplying the gauge 1 value by the ratio,

\[ \left( \frac{x}{x_1} \right) \frac{(\rho V)_1}{(\rho V)_1} \],

where the streamline lengths are taken from the potential-flow net of figure 2-2.

The reference length for the boundary-layer vorticity number is taken as
(an equivalent flat-plate) momentum thickness at the blade leading edge. For the mid-span vorticity number, the reference length was found as in section 4-2. For test number 4, the boundary-layer vorticity number was 20.71 while the mid-span vorticity number was 288.76. The net vorticity number was therefore 309.47. The pertinent values for test number 4 are listed in the following table.

\[ \psi = 309.47 \quad \beta = 0.0756 \]

| Gauge | \( x \) in | \( |\alpha^o| \) | \( \frac{(\rho V)}{(\rho V)^1} \) | \( Re_x \) | \( Re_x \psi\alpha \) |
|-------|------------|-----------------|-----------------|-------------|------------------|
| 1     | 0.29       | 0               | 1               | 4.38 \times 10^4 | -                |
| 4     | 0.465      | 1.08            | 1.30303         | 9.16 \times 10^4 | 5.34 \times 10^5 |
| 5     | 0.64       | 21.90           | 1.075           | 1.04 \times 10^5 | 1.23 \times 10^7 |
| 6     | 0.63       | 6.40            | 1.72            | 1.64 \times 10^5 | 5.67 \times 10^6 |
| 7     | 0.65       | 22.96           | 1.30303         | 1.28 \times 10^5 | 1.59 \times 10^7 |
| 8     | 0.87       | 35.315          | 1.5357          | 2.02 \times 10^5 | 3.85 \times 10^7 |
| 9     | 1.67       | 69.00           | 4.3             | 1.08 \times 10^6 | 4.02 \times 10^8 |
| 10    | 1.86       | 73.67           | 4.3             | 1.21 \times 10^6 | 4.81 \times 10^8 |
| 11    | 2.06       | 76.25           | 3.91            | 1.22 \times 10^6 | 5.02 \times 10^8 |
| 12    | 2.235      | 74.67           | 3.07            | 1.04 \times 10^6 | 4.19 \times 10^8 |

It will be recalled that equation 4-8 applies to points within the blade channel, ie, gauges 5 to 12. With the equation and the values in the above table, we predict the Nusselt numbers at the gauge locations. The predicted and the measured values as well as the errors are shown on the next page.
The calculations for the rotor are similar to those for the nozzle. Here, however, the flow net of figure 2-4 is used. In calculating the Reynolds numbers, the dynamic viscosity is based on the static temperature at the inlet to the rotor passage. For both the boundary-layer and the mid-span vorticity numbers, the reference length is found as in in section 4-2 and the flow gradient is read from the total-pressure profile at the inlet to the rotor. The net vorticity number for test 4 is 407.74.

\[ \phi = \frac{U_T^2}{2g_c J_c \rho \Delta T} = 0.602 \]  

\[ \text{E-4} \]
The multiplying factor, due to tip-leakage vortex, ie equal to

\[ 1 - \frac{f_K}{p_p} = 0.34 \]

The information needed to use equation 4-13 for test 4 are shown in the following table.

\[ \psi = 407.74; \quad \phi = 0.602; \quad 1 - \frac{f_K}{p_p} = 0.34 \]

<table>
<thead>
<tr>
<th>Gauge</th>
<th>x in</th>
<th>( \alpha^\circ )</th>
<th>( \frac{(\rho V)}{(\rho V)_{13}} )</th>
<th>Re_x</th>
<th>Re_x \psi \cdot \alpha \phi</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>0.1175</td>
<td>1.75</td>
<td>1</td>
<td>4.90 \times 10^4</td>
<td>3.67 \times 10^5</td>
</tr>
<tr>
<td>14</td>
<td>0.23</td>
<td>10.23</td>
<td>1.027</td>
<td>9.86 \times 10^4</td>
<td>4.32 \times 10^6</td>
</tr>
<tr>
<td>15</td>
<td>0.325</td>
<td>21.50</td>
<td>1.0555</td>
<td>1.43 \times 10^5</td>
<td>1.32 \times 10^7</td>
</tr>
<tr>
<td>16</td>
<td>0.415</td>
<td>33.78</td>
<td>1.0411</td>
<td>1.80 \times 10^5</td>
<td>2.60 \times 10^7</td>
</tr>
<tr>
<td>17</td>
<td>0.5275</td>
<td>50.32</td>
<td>1.0704</td>
<td>2.36 \times 10^5</td>
<td>5.09 \times 10^7</td>
</tr>
<tr>
<td>18</td>
<td>0.59</td>
<td>67.92</td>
<td>1.1604</td>
<td>2.86 \times 10^5</td>
<td>8.31 \times 10^7</td>
</tr>
<tr>
<td>19</td>
<td>0.69</td>
<td>84.08</td>
<td>1.2882</td>
<td>3.71 \times 10^5</td>
<td>1.34 \times 10^8</td>
</tr>
<tr>
<td>20</td>
<td>0.805</td>
<td>97.33</td>
<td>1.4615</td>
<td>4.91 \times 10^5</td>
<td>2.05 \times 10^8</td>
</tr>
</tbody>
</table>

\[ \text{Nu}_x = \frac{0.859}{(1 - \frac{f_K}{p_p})} (\text{Re}_x \psi \cdot \alpha \phi)^{0.363} \] 4-13

With equation 4-13 and the values in the above table, the Nusselt-number distribution on the rotor shroud is predicted. As pointed out earlier, equation 4-13 is applicable to points within the blade passage. It will be recalled that the tip-leakage factor, \( (1 - \frac{f_K}{p_p}) \), applies to only the last 2/3 of the blade chord.
The predicted and the measured Nusselt numbers, as well as the associated errors are shown in the table below.

<table>
<thead>
<tr>
<th>Gauge</th>
<th>$N_{\text{me}}$</th>
<th>$N_{\text{pre}}$</th>
<th>$\frac{N_{\text{me}} - N_{\text{pre}}}{N_{\text{me}}} %$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>147</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>249</td>
<td>220</td>
<td>+ 11.6</td>
</tr>
<tr>
<td>15</td>
<td>406</td>
<td>330</td>
<td>+ 18.7</td>
</tr>
<tr>
<td>16</td>
<td>467</td>
<td>422</td>
<td>+ 9.6</td>
</tr>
<tr>
<td>17</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>18</td>
<td>2090</td>
<td>1896</td>
<td>+ 9.3</td>
</tr>
<tr>
<td>19</td>
<td>2760</td>
<td>2256</td>
<td>+ 18.3</td>
</tr>
<tr>
<td>20</td>
<td>2683</td>
<td>2632</td>
<td>+ 1.9</td>
</tr>
</tbody>
</table>
Fig. 1-1  Turbomachinery Secondary Flow Classification.
The analysis (for the curve) is based on the assumption of steady, incompressible, inviscid flow, with no body force but Bernoulli-surface rotation and flow acceleration are taken into account.

Figure 2-1  Effect of Flow Turning on The Nozzle Secondary Vorticity.
Figure 2-2  Nozzle Flow Net (Used for The Solution of The Secondary-Flow Equation).
Flow Turning Angle

The analysis (for the curve) is based on the assumption of steady, incompressible, inviscid flow, with no body force; the rotation of the Bernoulli surfaces is about the rotor axis.

Figure 2-3  Effect of Flow Turning on The Rotor Secondary Vorticity.
Figure 2-4
Rotor Flow Net
(Used for the Solution of the Secondary-Flow Eqn.).
The assumed cosine velocity profiles in the solution of the secondary-flow equation. The cosine profile is assumed to extend to 1/8 of the blade span.

\[ V = V_0 \left[ 1 + a \left( 1 + \cos \frac{Z}{\delta} \right) \right] \]

Figure 2-5 Nozzle Inlet Velocity Profile.
Tip

\[ \psi = 0.0 \]

0.005
0.01
0.03
0.05

\[ \psi = \text{secondary-flow stream function} \]

For the meaning of "a" see figure 2-5 and for the meaning of "k" see figure 2-2.

Figure 2-6a Secondary Flow Stream Functions

for \( a = \frac{1}{18} \; \; \text{K} = 5 \)
Figure 2-6b  Secondary Flow Stream Functions
for $a = 1/18 \ K = 10$

$\psi$ = secondary-flow stream function

Tips
Pressure Surface
Pitch
Suction Surface
Tip

$\psi = 0.0$

$0.001$

$0.003$

$0.005$

$0.01$

$0.03$

$0.05$

$0.1$

$\psi = \text{secondary-flow stream function}$

Figure 2-6c Nozzle Secondary Flow Stream Functions

for $a = 1/18$ $K = 20$
Figure 2-6d  Nozzle Secondary Flow Stream Functions
for $a = 1/18, K = 30$

$\psi = \text{secondary-flow stream function}$
Figure 2-6e  Nozzle Secondary Flow Stream Functions
for $a = 1/18 \ K = 40$
Figure 2-7a  Nozzle Secondary Flow Stream Functions
for \( a = 1/4 \)  \( K = 5 \)

\( \psi = \text{secondary-flow stream function} \)
Figure 2-7b  Nozzle Secondary Flow Stream Functions
for $a = 1/4 \ K = 10$
Figure 2-7c  Nozzle Secondary Flow Stream Functions
for $a = 1/4$  $K = 20$

$\psi = \text{secondary-flow stream function}$
Figure 2-7d  Nozzle Secondary Flow Stream Functions for $a = 1/4$  $K = 30$

$\psi = \text{secondary-flow stream function}$
Figure 2-8a  Nozzle Flow Streams Functions
for $a = 1/2$  $K = 5$
Figure 2-8b

Nozzle Secondary Flow Stream Functions

for $a = 1/2 \ K = 10$

Tip

Pressure Surface

Pitch

Suction Surface

$\psi = 0.0$

$\psi = \text{secondary-flow stream function}$

Levels: 0.01, 0.03, 0.05, 0.1, 0.2
Figure 2-8c Nozzle Secondary Flow Streams Functions
for $a = 1/2$  $K = 20$

$\psi = \text{secondary-flow stream function}$
Figure 2-8d  Nozzle Secondary Flow Stream Functions
for a = 1/2  K = 30

\[ \psi = \text{secondary-flow stream function} \]

Tip

Pressure Surface

Pitch

Suction Surface
Figure 2-8e Nozzle Secondary Flow Stream Functions
for \(a = 1/2, \ K = 40\)

\(\psi = \text{secondary-flow stream function}\)
Tip

\[ \psi = 0.0 \]

\[ 0.002 \]

\[ 0.005 \]

\[ 0.01 \]

\[ 0.03 \]

\[ 0.05 \]

\[ 0.1 \]

\[ 0.2 \]

\[ \psi = \text{secondary-flow stream function} \]

Figure 2-8f  Nozzle Secondary Flow Stream Functions

for \( a = 1/2 \)  \( K = 50 \)
"a" = measure of steepness of velocity profile at nozzle inlet.
(Figure 2-5).

Figure 2-9a  Nozzle Flow Paths For \( a = \frac{1}{4} \), \( I = 2 \)
Figure 2-9b  Nozzle Flow Paths For $a = \frac{1}{4}$, $I = 2$
Figure 2-10a  Nozzle Flow Paths for a = 1/2, I=2
Figure 2-10b: Nozzle Flow Paths for $a=1/2, I=2$
(Insert is the schematic of the entire streamline pattern).

Figure 2-1: Overturning in Endwall Boundary Layer (Ref. 19).
Roll-up of endwall boundary layer.

Figure 2-12  Formation of Passage Vortex in a 45° Channel (Reference 19).
Movement of boundary-layer fluid across the channel.

Note that the boundary-layer fluid moves from the pressure to the suction side of the channel.

Figure 2-13  Deflection of Boundary-Layer Fluid in a 60° Channel (Reference 19).
Figure 3-1  Schematic of Turbine Blowdown Facility
Flow in Injection slot

Flow in

Injection Ring.

Figure 3-2

Section 'XX'
Static-pressure probe.

NOTE: Fluid jet is directed to mid-span of blade.

Hub radius = 7.17"
Tip radius = 8.16"
Blade height (span) = 0.99"
Rotor blade axial chord = 0.6"

Figure 3–3a Position of Injection Ring in Relation to The First-Stage Blades (Pictorial View).
Figure 3-3b  Position of Injection Ring in Relation to The 1st Stage Blades (Schematic View).
Figure 3-4  Nozzle Gauge Locations.

<table>
<thead>
<tr>
<th>Hole</th>
<th>X in.</th>
<th>Y in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.346</td>
<td>1.200</td>
</tr>
<tr>
<td>2</td>
<td>0.355</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>0.520</td>
<td>0.900</td>
</tr>
<tr>
<td>4</td>
<td>0.173</td>
<td>1.100</td>
</tr>
<tr>
<td>5</td>
<td>0.370</td>
<td>0.800</td>
</tr>
<tr>
<td>6</td>
<td>0.800</td>
<td>0.960</td>
</tr>
<tr>
<td>7</td>
<td>0.520</td>
<td>0.700</td>
</tr>
<tr>
<td>8</td>
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<tr>
<td>9</td>
<td>0.104</td>
<td>0.210</td>
</tr>
<tr>
<td>10</td>
<td>0.277</td>
<td>0.110</td>
</tr>
<tr>
<td>11</td>
<td>0.450</td>
<td>0.010</td>
</tr>
<tr>
<td>12</td>
<td>0.600</td>
<td>-0.09</td>
</tr>
</tbody>
</table>
Figure 3-5  NOZZLE ENDWALL GAUGE LOCATIONS.
HOLE No. 13 14 15 16 17 18 19 20
X in. 0.08 0.168 0.256 0.344 0.432 0.520 0.608 0.696

Hole Co-ordinates

Figure 3-6 Shroud Piece and Gauge Locations.
Figure 3-7

ROTOR SHROUD GAUGE LOCATIONS.
Figure 3-8a  Total Pressure Profiles for $\frac{\dot{m}}{M} = 0\%$

- $P_{T1}$ = Measured total-pressure profile at inlet to the nozzle
- $P_{T2}$ = Measured total-pressure profile at inlet to the rotor
- $P_{T3}$ = Measured total pressure profile at exit from the rotor
- $P_{S1}$, $P_{S2}$, $P_{S3}$ = Measured tip-static pressures at locations for $P_{T1}$, $P_{T2}$, and $P_{T3}$ respectively
Figure 3-8b  Total Pressure Profiles for $\dot{m} = 6\%$ at $70^\circ F$
Figure 3-8c Total Pressure Profiles for $\dot{m} = 9\%$ at $70^\circ F$
Figure 3-8d  Total Pressure Profiles for $\frac{\hat{m}}{\hat{M}} = 11\%$ at 70°F
Figure 3-8e  Total Pressure Profiles for $\frac{\dot{m}}{\dot{M}} = 12\%$ at 70°F
Figure 3-8f  Total Pressure Profiles for $\frac{\hat{m}}{M} = 4.5\%$ at $130^\circ F$
Total Pressure, psia

Figure 3-8g  Total Pressure Profiles for $\frac{\dot{m}}{M} = 5\%$ at 150°F
Figure 3-8h  Total Pressure Profiles for $\frac{\dot{m}}{\dot{m}_0}$ = 8% at 200°F
Figure 3-8i  Total Pressure Profiles for $\frac{\dot{m}}{M} = 11\%$ at $200^\circ F$
Figure 3-8j  Total Pressure Profiles for $\hat{\text{n}} = 12\%$ at 200°F
Figure 3-9a Total Temperature Profiles (measured).

\[ \frac{\dot{m}}{M} = 0\% \]
Figure 3-9b  Total Temperature Profiles (measured).

for \( \frac{\dot{m}}{\dot{M}} = 6\% \) at 70°F

Legend

Symbol \( \phi_T \)

\( \Delta \) -------- 0.583

\( \cdot \) -------- 0.529

\( \phi_T \) = tip-flow coefficient.
Figure 3-9c Total Temperature Profiles (measured)

\[
\frac{\dot{m}}{\dot{m}} = 9\% \text{ at } 70^\circ F
\]

\[\phi_T = \text{tip-flow coefficient.}\]
Figure 3-9d  Total Temperature Profiles (measured), for \( \frac{\dot{m}}{M} = 11\% \) at 70°F
Figure 3-9e  Total Temperature Profiles (measured).

for \( \frac{\dot{m}}{M} = 12\% \) at 70°F
Legend
Symbol $\phi_T$

- $\cdots$ 0.578
- $\triangle$ 0.540

$\phi_T$ = tip-flow coefficient.

Figure 3-9f  Total Temperature Profiles (measured).

for $\frac{\dot{m}}{M} = 4.5\%$ at 130°F
Figure 3-9g  Total Temperature Profiles (measured).

for $\frac{\dot{m}}{M} = 5\%$ at 150°F
Inlet to 1st stage nozzle

Legend
Symbol \( \phi_T \)
- \( \phi_T \) = 0.602
\( \Delta \) \( \phi_T \) = 0.559

\( \phi_T = \) tip-flow coefficient.

Inlet to 1st stage rotor

Exit from 1st stage rotor

Figure 3-9h Total Temperature Profiles (measured).
for \( \frac{\dot{m}}{\dot{M}} = 8\% \) at 200°F
Figure 3-9i Total Temperature Profiles (measured).
for $\frac{\dot{m}}{\dot{m}_0} = 11\%$ at 200°F
Figure 3-9j  Total Temperature Profiles (measured).

\[
\frac{\dot{m}}{M} = 12\% \text{ at } 200^\circ\text{F}
\]
Figure 3-10a  Nu Distribution for $\frac{\dot{m}}{\dot{M}} = 0\%$

Legend

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\phi_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>0.646</td>
</tr>
<tr>
<td>Δ</td>
<td>0.565</td>
</tr>
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</table>

$\phi_T =$ tip-flow coefficient
Figure 3-10b  Nu. Distribution for $\frac{n}{w} = 6\%$ at 70°F
(Based on $T_{t2tip}$)

Legend
Symbol  $\phi_T$

- - - - - 0.583
$\Delta$ - - - 0.529

$\phi_T = \text{tip-flow coefficient}$
**Legend**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\phi_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>0.616</td>
</tr>
<tr>
<td>Δ</td>
<td>0.577</td>
</tr>
</tbody>
</table>

$\phi_T =$ tip-flow coefficient

Figure 3-10c  Nu. Distribution for $\frac{m}{\dot{m}} = 9\%$ at 70°F
(Based on $T_{t2tip}$)
Figure 3-10d  Nu Distribution for $\frac{\dot{m}}{M} = 11\%$ at 70°F  
(based on $T_{t2\text{tip}}$)

Legend

Symbol  $\phi_T$

- $\Delta \quad 0.584$
- $\bullet \quad 0.681$

$\phi_T = \text{tip-flow coefficient}$
Figure 3-10e  Nu Distribution for $\frac{\dot{m}}{\dot{m}} = 12\%$ at 70°F
(based on $T_{t2TIP}$)

$\phi_T = \text{tip-flow coefficient}$

Legend

Symbol  $\phi_T$

- - - - 0.642

$\Delta$ - - - 0.550
Figure 3-10f  Nu Distribution for $\frac{\dot{m}}{\dot{M}} = 4.5\%$ at 130°F
(based on $T_{t2TIP}$)

Legend

Symbol  $\phi_T$

$\bullet$  0.578
$\Delta$  0.540

$\phi_T$ = tip-flow coefficient
Figure 3-10g  Nu. Distribution for $\frac{\dot{m}}{\dot{m}_0} = 5\%$ at $150^\circ F$

(Based on $T_{t2tip}$)
Figure 3-10h  
Nu Distribution for $\frac{\dot{m}}{M} = 8\%$ at 200°F  
(based on $T_{2\text{tip}}$)
Figure 3-101  Nu Distribution for $\frac{\dot{m}}{\dot{m}_0} = 11\%$ at $200^\circ F$

Legend

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<thead>
<tr>
<th>Symbol</th>
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</tr>
</thead>
<tbody>
<tr>
<td>•</td>
<td>0.713</td>
</tr>
<tr>
<td>△</td>
<td>0.561</td>
</tr>
</tbody>
</table>

$\phi_T$ = tip-flow coefficient

Based on $T_{t2\text{tip}}$

Based on $T_{t3\text{tip}}$
Figure 3-10j  Nu Distribution for $\frac{\dot{m}}{\dot{M}} = 12\%$ at 200°F

Legend

<table>
<thead>
<tr>
<th>Symbol</th>
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</tr>
</thead>
<tbody>
<tr>
<td>•</td>
<td>0.681</td>
</tr>
<tr>
<td>△</td>
<td>0.584</td>
</tr>
</tbody>
</table>

$\phi_T = \text{tip-flow coefficient}$

Gauge Axial Location.

Nusselt Number

Based on $T_{t2\text{tip}}$

Based on $T_{t3\text{tip}}$
Figure 3-11 Potential-Flow Differential Pressure Across The Rotor Blade.
Figure 3-12a  Nu Distribution for $\frac{\dot{m}}{\dot{m}_0} = 0\%$

Legend

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>•</td>
<td>0.646</td>
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<tr>
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$\phi_T$ = tip-flow coefficient
Figure 3-12b  Nu Distribution for $\frac{\dot{m}}{\dot{M}} = 6\%$ at 70°F
Symbol $\phi_T$

- $\cdots$ 0.616
- $\Delta$ $\cdots$ 0.577

$\phi_T$ = tip-flow coefficient

Figure 3-12c Nu Distribution for $\frac{\dot{m}}{M} = 9\%$ at 70°F
Figure 3-12d  Nu Distribution for $\frac{\dot{m}}{M} = 11\%$ at 70°F
Figure 3-12e  Nu Distribution for $\frac{\dot{m}}{M} = 12\%$ at $70^\circ F$
Figure 3-12f  Nu Distribution for $\frac{\dot{m}}{M} = 4.5\%$ at $130^\circ F$

Symbol  \( \phi_T \)

-  \( \phi_T \) = tip-flow coefficient

-  \( \bullet \) 0.578

-  \( \Delta \) 0.540
Figure 3-12g  Nu Distribution for $\frac{\dot{m}}{\dot{m}} = 5\%$ at 150°F

Symbol $\phi_T$

- $\cdots$ 0.596
- $\triangle$ 0.553

$\phi_T = \text{tip-flow coefficient}$
Figure 3-12h  Nu Distribution for $\frac{\dot{m}}{M} = 8\%$ at 200°F.
Figure 3-12i  Nu Distribution for $\frac{\dot{m}}{\dot{m}_0} = 11\%$ at 200°F

$\phi_T = \text{tip-flow coefficient}$

Symbol | $\phi_T$
---|---
• | 0.713
Δ | 0.561

$\phi_T = \text{tip-flow coefficient}$
Figure 3-12j  Nu Distribution for \( \frac{\dot{m}}{\bar{m}} = 12\% \) at 200°F

Legend

Symbol  \( \phi_T \)

- \( \phi_T = 0.681 \)
- \( \Delta = 0.584 \)

\( \phi_T \) = tip-flow coefficient
At low injection rates, secondary-flow effects dominate but at high injection rates, the cooling effects of the injection fluid dominate.

Figure 3-13  Effect of Injection Rate on The Nozzle Endwall Nusselt Number.
Peak-to peak variation represents 2 psi pressure variation.

Figure 3-14  Oscilloscope Trace of Shroud Pressure Variation Due to T64 Blade-passing.
Figure 4-1  Nozzle-Endwall Nusselt Number Plot.
Figure 4-2  Rotor-Shroud Nusselt Number Plot.
Figure 4-3  Prediction Error in Using the Correlation Equations.
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Predicted Nusselt Numbers (Thousands).

Figure 4-4 Measured vs Predicted Nusselt Numbers Based on Streamline Legth.
Figure B-1  Deformation of Original Potential Surfaces Due to Secondary Flows.
Peak-to-peak variation represents a temperature variation of $0.1^\circ$C.

Figure D-1 Oscilloscope Trace of Cascade-Wall Temperature Variation Due to Air-Turbine Blade-passing.
The author was educated at the Government Technical Institute, Enugu (Nigeria) and the University of Nigeria, Nsukka. He graduated from the latter institution in 1972 with B.Sc. (1st Class Honours) in mechanical engineering. He was the best graduate of the university in 1972, and won the following prizes: Departmental Prize, Faculty Prize, University Prize, Foundation Scholarship, Nigerian Society of Engineers' Prize, West African Professional Engineers' Prize, etc.

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He has one publication titled "Construction and Preliminary Test of Fiber-Reinforced Concrete Pump Casing." (ASME Paper No. 76-WA/PVP-11).