

18.05. Practice test 1.

(1) Suppose that 10 cards, of which five are red and five are green, are placed at random in 10 envelopes, of which five are red and five are green. Determine the probability that exactly two envelopes will contain a card with a matching color.

(2) Suppose that a box contains one fair coin and one coin with a head on each side. Suppose that a coin is selected at random and that when it is tossed three times, a head is obtained three times. Determine the probability that the coin is the fair coin.

(3) Suppose that either of two instruments might be used for making a certain measurement. Instrument 1 yields a measurement whose p.d.f. is

$$f_1(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Instrument 2 yields a measurement whose p.d.f. is

$$f_2(x) = \begin{cases} 3x^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Suppose that one of the two instruments is chosen at random and a measurement X is made with it.

(a) Determine the marginal p.d.f. of X .

(b) If $X = 1/4$ what is the probability that instrument 1 was used?

(4) Let Z be the rate at which customers are served in a queue. Assume that Z has p.d.f.

$$f(z) = \begin{cases} 2e^{-2z}, & z > 0, \\ 0, & \text{otherwise} \end{cases}$$

Find the p.d.f. of average waiting time $T = \frac{1}{Z}$.

(5) Suppose that X and Y are independent random variables with the following p.d.f.

$$f(x) = \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{otherwise} \end{cases}$$

Determine the joint p.d.f. of the following random variables:

$$U = \frac{X}{X+Y} \text{ and } V = X+Y.$$