18.05 Spring 2005 Lecture Notes

18.05 Lecture 1
February 2, 2005

Required Textbook - DeGroot & Schervish, "Probability and Statistics," Third Edition
Recommended Introduction to Probability Text - Feller, Vol. 1

§1.2-1.4. Probability, Set Operations.
What is probability?

- Classical Interpretation: all outcomes have equal probability (coin, dice)
- Subjective Interpretation (nature of problem): uses a model, randomness involved (such as weather)
  - ex. drop of paint falls into a glass of water, model can describe $\mathbb{P}$(hit bottom before sides)
  - or, $\mathbb{P}$(survival after surgery)- "subjective," estimated by the doctor.
- Frequency Interpretation: probability based on history
  - $\mathbb{P}$(make a free shot) is based on history of shots made.

Experiment $\rightarrow$ has a random outcome.
1. Sample Space - set of all possible outcomes.
coin: S={H, T}, die: S={1, 2, 3, 4, 5, 6}
two dice: S={(i, j), i, j=1, 2, ..., 6}

2. Events - any subset of sample space
ex. A $\subseteq$ S, $\mathcal{A}$ - collection of all events.

3. Probability Distribution - $\mathbb{P}$: $\mathcal{A} \rightarrow [0, 1]$
Event A $\subseteq$ S, $\mathbb{P}$(A) or Pr(A) - probability of A

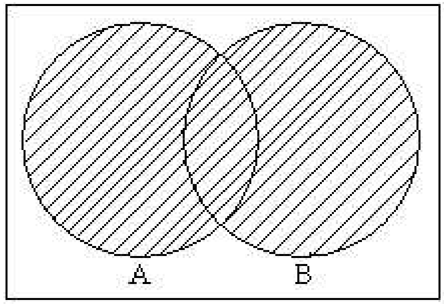
Properties of Probability:
1. $0 \leq \mathbb{P}(A) \leq 1$
2. $\mathbb{P}(S) = 1$
3. For disjoint (mutually exclusive) events A, B (definition $\rightarrow$ A $\cap$ B = $\emptyset$)
P(A or B) = P(A) + P(B) - this can be written for any number of events.
For a sequence of events A_1, ..., A_n, ... all disjoint ($A_i \cap A_j = \emptyset$, i $\neq$ j):

$$\mathbb{P}(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} \mathbb{P}(A_i)$$

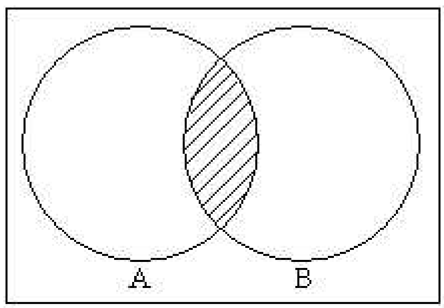
which is called "countably additive."
If continuous, can't talk about $\mathbb{P}$(outcome), need to consider $\mathbb{P}$(set)
Example: $S = [0, 1], 0 < a < b < 1$.
$\mathbb{P}([a, b]) = b - a, \mathbb{P}(a) = \mathbb{P}(b) = 0.$

Need to group outcomes, not sum up individual points since they all have $\mathbb{P} = 0$.

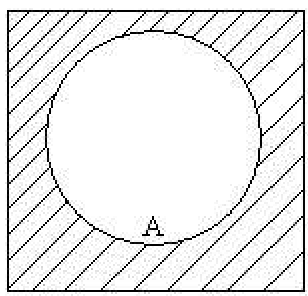
§1.3 Events, Set Operations



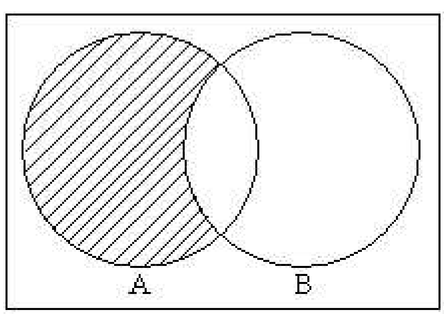
Union of Sets: $A \cup B = \{s \in S : s \in A \text{ or } s \in B\}$



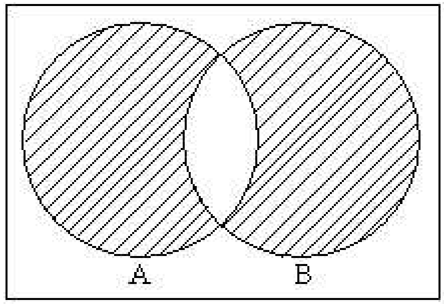
Intersection: $A \cap B = AB = \{s \in S : s \in A \text{ and } s \in B\}$



Complement: $A^c = \{s \in S : s \notin A\}$



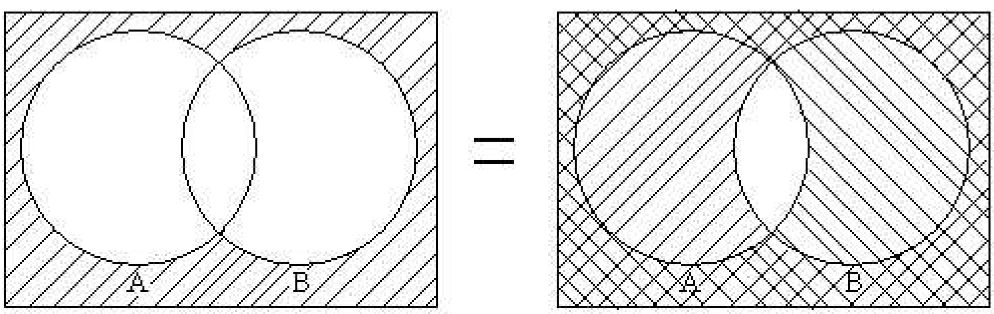
Set Difference: $A \setminus B = A - B = \{s \in S : s \in A \text{ and } s \notin B\} = A \cap B^c$



Symmetric Difference: $(A \cap B^c) \cup (B \cap A^c)$

Summary of Set Operations:
1. Union of Sets: $A \cup B = \{s \in S : s \in A \text{ or } s \in B\}$
2. Intersection: $A \cap B = AB = \{s \in S : s \in A \text{ and } s \in B\}$
3. Complement: $A^c = \{s \in S : s \notin A\}$
4. Set Difference: $A \setminus B = A - B = \{s \in S : s \in A \text{ and } s \notin B\} = A \cap B^c$
5. Symmetric Difference:
$A \triangle B = \{s \in S : (s \in A \text{ and } s \notin B) \text{ or } (s \in B \text{ and } s \notin A)\} =$
$(A \cap B^c) \cup (B \cap A^c)$

Properties of Set Operations:
1. $A \cup B = B \cup A$
2. $(A \cup B) \cup C = A \cup (B \cup C)$
Note that 1. and 2. are also valid for intersections.
3. For mixed operations, associativity matters:
$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
think of union as addition and intersection as multiplication: (A+B)C = AC + BC
4. $(A \cup B)^c = A^c \cap B^c$ - Can be proven by diagram below:



Both diagrams give the same shaded area of intersection.

5. $(A \cap B)^c = A^c \cup B^c$ - Prove by looking at a particular point:
$s \in (A \cap B)^c = s \notin (A \cap B)$
$s \notin A$ or $s \notin B = s \in A^c$ or $s \in B^c$
$s \in (A^c \cup B^c)$
QED

** End of Lecture 1