Massachusetts Institute of Technology

Physics 8.09 Fall 2006

Homework 1

due Wednesday, September 13, 2006 before 12:30 PM

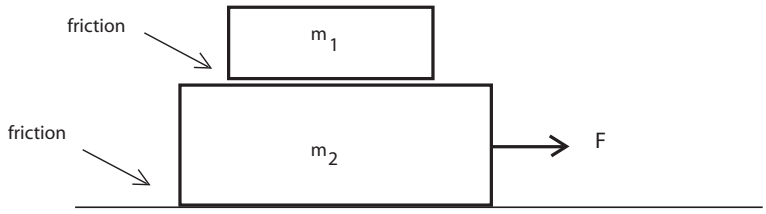
Suggested Reading

Refresh your mechanics knowledge by reading from your favorite Mechanics book. For example the book by Marion & Thornton is very helpful. Goldstein is a bit too dry for this initial "Newtonian mechanics refresher".

Problems

Problem 1.1 (20 pts)

A block of mass m_1 rests on top of a block of mass m_2. The static and kinetic coefficients of friction between the two blocks and the bottom block and the table are μ_s and μ_k. A fixed force F is applied to the bottom block.



Describe the motion and find the accelerations of the two blocks a_1 and a_2 for all possible values of F from $F = 0 \rightarrow \infty$.

Problem 1.2 (20 pts)

Consider a thin circular hoop of radius R and mass m attached to the wall with a frictionless screw. The hoop can swing freely but it cannot slip. Parametrize the position of the hoop in terms of the angular displacement from the vertical θ. Write the equations of motion in terms of θ and its derivatives. Find all solutions in the limit of small amplitude motion of the hoop. What is the period of oscillation of the hoop ? What is the length of a simple mathematical pendulum of length L and mass m with the same period of oscillation? Gravitational acceleration is g.

Problem 1.3 (20 pts)

Mass m slides down a frictionless incline as shown in Figure 2. The mass is released at a height h above the bottom of the loop.

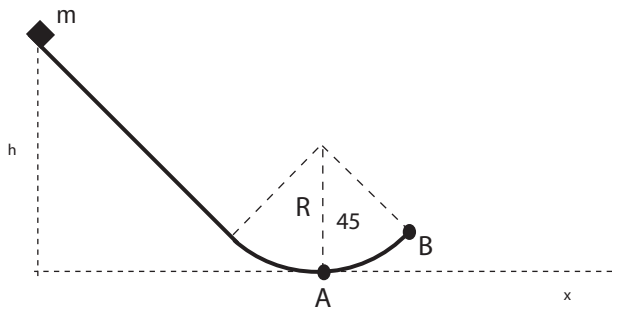
a. What is the magnitude and direction of the velocity and the force on the mass at the bottom of the incline (point A).

b. What is the magnitude and direction of the velocity and the force on the mass at point B, right before the mass leaves the track?

c. At what speed does the mass leave the track?

d. How far away from point A does the block land on the level ground?

e. Sketch the potential energy of the mass as a function of position $U(x)$.



Problem 1.4 (20 pts)

A particle moves in a two dimensional orbit defined by

$$x(t) = A(2\alpha t - \sin \alpha t)$$
$$y(t) = A(1 - \cos \alpha t)$$

a. Find the tangential acceleration a_t and normal acceleration a_n as a function of time where the tangential and normal components are taken with respect to the velocity.

b. Determine at what times in the orbit a_n has a maximum.

Problem 1.5 (20 pts)

A particle of mass m slides down an inclined plane, with inclination angle θ, under the influence of gravity. If the motion is resisted by a force $f = kmv^2$, show that the time required to move a distance d after starting from rest is

$$t = \frac{cosh^{-1}(e^{kd})}{\sqrt{kg \sin \theta}}$$