

18.440 PROBLEM SET 8

A. FROM TEXTBOOK CHAPTER SEVEN:

1. Problems: 51, 58, 61
2. Theoretical Exercises: 24, 27, 46, 50, 55

B. FROM TEXTBOOK CHAPTER EIGHT:

1. Problems: 1
2. Theoretical Exercises: 1

C. (Just for fun — not to hand in) Let $V = (V_1, V_2, \dots, V_n)$ be a random vector whose components V_i are independent, identically distributed normal random variables of mean zero, variance one. Note that the density function for V may be written as $f(v) = (2\pi)^{-n/2} e^{-|v|^2/2}$ where $v = (v_1, v_2, \dots, v_n)$ and $|v|^2 = v_1^2 + v_2^2 + \dots + v_n^2$.

1. Let M be an $n \times n$ matrix. Write $W = MV$ and compute the mean and covariance of W_i for each $1 \leq i \leq n$.
2. Write the probability density function for W .
3. Classify the set of matrices M for which MV has the same probability density function as V .
4. Is every n -dimensional mean zero multivariate normal distribution (as defined in Section 7.8 of the textbook) the distribution of MV for some choice of M ? If so, to what extent does the distribution uniquely determine M ?

D. (Just for fun — not to hand in) Try to formulate and prove a version of the central limit theorem that shows that sums of independent heavy-tailed random variables (divided by appropriate constants) converge in law to a stable random variable (instead of a normal random variable). See for example <http://eom.springer.de/A/a013920.htm> or the wikipedia articles on stable distributions for definitions and hints. You will need to use characteristic functions instead of the moment generating function.

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