Experiments on Surface Reconstruction for Partially Submerged Marine Structures

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<td><a href="http://dx.doi.org/10.1002/rob.21478">http://dx.doi.org/10.1002/rob.21478</a></td>
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<td>Publisher</td>
<td>Wiley Blackwell</td>
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<td>Version</td>
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Abstract

Over the last 10 years, significant scientific effort has been dedicated to the problem of 3-D surface reconstruction for structural systems. However, the critical area of marine structures remains insufficiently studied. The research presented here focuses on the problem of 3-D surface reconstruction in the marine environment. This paper summarizes our hardware, software, and experimental contributions on surface reconstruction over the last few years (2008-2011). We propose the use of off-the-shelf sensors and a robotic platform to scan marine structures both above and below the waterline, and we develop a method and software
system that uses the Ball Pivoting Algorithm (BPA) and the Poisson reconstruction algorithm to reconstruct 3-D surface models of marine structures from the scanned data. We have tested our hardware and software systems extensively in Singapore waters, including operating in rough waters, where water currents are around 1m/s to 2m/s. We present results on construction of various 3-D models of marine structures, including slowly moving structures such as floating platforms, moving boats and stationary jetties. Furthermore, the proposed surface reconstruction algorithm makes no use of any navigation sensor such as GPS, DVL or INS.

1 Introduction

Surface reconstruction of marine structures is an important problem with several applications in marine environments, including marine vehicle navigation, marine environment inspection, and harbor patrol and monitoring. A marine structure is a structure that is fully submerged or partially submerged in the sea. In this paper, we are interested in surface reconstruction of partially submerged marine structures. The surface reconstruction problem is an essential part of the inspection problem that we are interested in. Depending on the application, the required resolution can vary from 3in (e.g. mine detection tasks) to 20in (missing or broken parts) or even to 1m for navigation tasks.

Marine structures are exposed to challenging conditions such as water currents, corrosion, and several hurricanes per year. This exposure results in unpredictable damages to the marine structures, which could be life threatening for the people on and around the structures. Therefore, to ensure safety, thorough structural inspections are required regularly or after a platform is affected by natural disasters. For example, after hurricane Dennis in 2005, structural inspection of Thunder Horse oil platform (Fig. (1)) was needed as soon as possible to identify damages on the platform and re-evaluate platform’s condition. Currently, such inspections are performed by human workers through visual observation. They inspect the above-water part of the structures aboard small boats, and use diving equipment to inspect the submerged portions of structures. In both situations, inspectors often lack the time and comfort to inspect the structures and re-evaluate structures’ status properly. In addition, visual examination does not give inspectors the ability to track important structural changes. Scanning the structures, using robots, and automatically reconstructing 3-D models of the structures from the scanned data will give the inspectors the opportunity to inspect structures from the comfort of their offices, thereby reducing the safety risks of the inspectors and increasing the accuracy of the
inspection results.

Figure 1: Thunder Horse oil platform after the Hurricane Dennis in 2005.

There is a plethora of scientific work (Nüchter et al., 2007; Newman et al., 2009) dedicated to the reconstruction of 3-D models for land based structures, however very few works have been dedicated to marine environments. Unpredictable disturbances such as water currents, and the marine environment itself pose significantly more challenges compared to the challenges posed by terrestrial (shore) environments. Water currents generate disturbance forces on marine vehicles, resulting in large roll and pitch angles. Furthermore, many marine structures are floating platforms, which deflect under strong currents. On top of that, GPS access close to big marine structures is not reliable and other localization sensors developed for marine vehicles lag behind the ones developed for ground robots.

The 3-D surface reconstruction problem often requires gathering 3-D point clouds and registering them under the same coordinate frame. In the case of stationary structures, 2-D laser scanners and localization sensors (GPS, IMU, DVL or wheel encoders) can be used to gather 3-D data from stationary environments (Howard et al., 2004). In addition, advanced SLAM techniques can be used to optimize the estimated trajectory and the resulting map (Tong and Barfoot, 2012; Scherer et al., 2012). On the other hand, in the case of slowly moving structures special care should be taken in both gathering the 3-D point clouds and registering them under the same coordinate frame. In the absence of knowledge over the motion of the structure, data gathered using 2-D laser scanners combined with localization units cannot be used to reconstruct point cloud representations of structure views.

From our discussion above, it is clear that when dealing with moving structures, 3-D laser scanners with a
wide field of view that allows significant overlap between subsequent laser scans should be used to gather point clouds. Then, registration algorithms that are invariant to motion, such as the Iterative Closest Point (ICP) (Besl and McKay, 1992), can be used to register all data gathered under the same coordinate frame resulting in a point cloud representation of the structure of interest. In the case of stationary structures, researchers commonly use estimates of vehicle positions to initialize the ICP algorithm, (Nüchter et al., 2007), however this is not always feasible in the case of moving structures.

This paper presents the hardware and software designs for scanning and reconstructing 3-D surface models of marine structures. We alleviate the difficulty caused by disturbances, moving structures, and sensor errors by hardware and software design. In terms of hardware, we select sensors that would ease construction of 3-D models from scanned data when no positioning sensors are available. Furthermore, we develop a software system that constructs 3-D point cloud models of marine structures from the scanned data and by using known surface reconstruction techniques reconstructs 3-D surface models of marine structures. We have successfully tested our system in various conditions in Singapore waters between 2009 and 2011, including operating in the water with up to 2m/s water currents, and constructing 3-D models of slowly moving structures. All the structures are constructed without any information from positioning sensors, such as GPS and DVL.

The problem of reconstructing surfaces from registered point clouds still remains an open problem. There are several algorithms that reconstruct surfaces from point clouds, some of them reconstruct exact surfaces (interpolating surface) by a triangulation that uses a subset of the input point cloud as vertices (Bernardini et al., 1999), (Amenta and Bern, 1998). These approaches perform well in clean point clouds and present problems when the point clouds are noisy. Some other approaches reconstruct approximating surfaces by using best-fit techniques (Hoppe et al., 1993) (Kazhdan et al., 2006), (Alliez et al., 2007). These approaches are robust to noise in the point clouds, however they tend to over-smooth the surfaces and thus important part of the structure geometry may be lost. A comparison between several state-of-the-art surface reconstruction techniques can be found at (Berger et al., 2011).

In this paper, our main focus is on gathering, registering and cleaning real 3-D point clouds that can be used to reconstruct surfaces. We also reconstruct 3D surfaces using known techniques, such as the ball pivoting algorithm (Bernardini et al., 1999), and the Poisson surface reconstruction algorithm (Kazhdan et al., 2006). Developing a new algorithm that uses noisy point clouds and builds a surface model of structures is crucial but it is out of the scope of this paper.
The main contribution in this paper is on the experimental side of robotics. Using standard sensors such as the Velodyne sensor, the micro-bathymetry sonar and well known algorithms such as the ICP and the Poisson Surface Reconstruction algorithm, we were able to reconstruct real models of marine structures in the ocean (where 1-2m/s water currents substantially accentuate the control and sensing errors). The proposed framework can reconstruct surfaces from both parts (the above waterline part and the below waterline part) of marine structures. It is also important to say that the vehicle we assembled is a novel vehicle.

In the next section, Section (2) we describe the related work. In Section (3), we describe the vehicle and the sensors used in the experiments. Section (4) explains our surface reconstruction algorithms. In Section (5) we present our experimental results, and we close with conclusions and future work in Section (6).

2 Related Work

The 3D model reconstruction problem can be considered a special category of the Simultaneous Localization and Mapping (SLAM) problem (Thrun et al., 2005). SLAM was originally introduced by Smith et al. (Smith and Cheeseman, 1986) and solved using EKF approaches and feature based maps (Smith et al., 1990). Dissanayake et al. proves that the solution to the SLAM problem is possible and proposes another solution to the EKF-SLAM problem (Dissanayake et al., 2001). Thrun, Montemerlo et al. approach the problem from a probabilistic point of view (Thrun et al., 2004) (Montemerlo et al., 2003), which was the foundation of the modern approaches that followed: Batch smoothing least squares optimization methods (Dellaert and Kaess, 2006), its incremental equivalent (Kaess et al., 2008) and incorporating loop closures (Cummins and Newman, 2007) (Cummins and Newman, 2009).

2.1 3D Model Reconstruction using Ground Robots

The 3-D model reconstruction problem has attracted substantial research interest over the last 10 years. Although 3-D model reconstruction by marine robots has not been done sufficiently due to its difficulty, comparable processes have been researched using ground robots. Several robotic platforms were used and different mapping algorithms were proposed. Two different types of sensors (visual sensors and laser sensors) have been used, and each type has strengths and weaknesses. Visual sensors are less expensive, but they do not provide data in $\mathbb{R}^3$. However, by using machine learning techniques or vehicle motion or stereo vision, this method can obtain 3-D data, making reconstruction feasible (Newcombe and Davison, 2010) (Izadi et al., 2011). Currently, vision-based 3D model reconstruction can be mainly accomplished in indoor environments.
where the distances are small and the brightness is limited.

In the early stages of large-scale outdoor mapping research, 2-D laser scanners were used to gathered 3-D laser data. Howard et al. gathered 3-D data by mounting a 2-D laser scanner on a segway (Howard et al., 2004). They fused GPS and INS measurements to get estimates of vehicle trajectory and reconstructed 3-D point cloud representations of outdoor environments. Thrun et al. reconstructed 3-D point cloud maps of indoor environments using 2 laser scanners (2-D). The first laser scanner was mounted horizontally and was used, combined with odometry measurements, to localize the vehicle. The second laser scanner was mounted vertically to scan the environment (Thrun et al., 2000). Because the vertical scanner is not able to scan horizontal surfaces Zhao and Shibasaki used 2-D laser scanners mounted vertically and shifted 45 degrees to be able to capture horizontal surfaces. In their work, they used GPS and an expensive INS to localize the vehicle; estimates of vehicle trajectory and the laser scanner data were used to reconstruct 3-D maps of outdoor environments (Zhao and Shibasaki, 2001).

In later research, vehicles were capable of directly gathering 3-D point clouds by coupling a 2-D laser scanner with a low cost nodding actuator or by using a 2-D spinning laser scanner. Harrison and Newman developed a system able to gather 3-D point clouds by coupling a 2-D laser scanner with a low cost nodding actuator. They used odometry measurements and feature extraction algorithms to localize the robot and finally to reconstruct point cloud maps of outdoor environments (Harrison and Newman, 2008). A similar approach was taken by (Cole and Newman, 2006). Recently, Bosse and Zlot used a 2-D spinning laser scanner to reconstruct 3-D point cloud maps of outdoor environments. They used scan matching algorithms such as the ICP without using any other navigation sensor (Bosse and Zlot, 2009). A similar approach was taken by Holenstein et al. in reconstructing watertight surfaces of caves using 3-D laser data (Holenstein et al., 2011).

A similar approach was taken by Nuchter et al. They used a ground robot equipped with dead reckoning sensors and a rotating 2-D laser scanner to reconstruct occupancy grid based maps of outdoor environments. The algorithm proposed was based on the ICP algorithm and dead reckoning measurements obtained onboard that were used to initialize the ICP algorithm. In addition efficient data structures such as KD-trees, were used to accelerate the ICP algorithm (Nüchter et al., 2007). In another work, Nuchter et al. proposed an algorithm that does not use any navigation sensor. This new approach is based on the existence of skyline features. The skyline features were extracted from panoramic 3-D scans and encoded as strings enabling the use of string matching algorithms for merging the scans. Initial results of the proposed method in the old city center of Bremen are presented. They also use the ICP algorithm for fine registration (Nuchter et al.,
Significant progress on 3D model reconstruction was done by Newman and his group (Newman et al., 2009). Newman and his group utilized a combination of stereo vision and laser data to first localize the vehicle and then reconstruct 3-D maps of the environment. They employed the Sliding Window Filter developed by Sibley (Sibley, 2006) (that marginalizes poses that are further away as opposed to full pose batch optimization methods) to approximate the locally optimal trajectory. The proposed system also identifies and utilizes loop closures; their loop closure system, FAB-MAP is described in work by (Cummins and Newman, 2007) (Cummins and Newman, 2009). After estimating a good trajectory they use the laser scanner data obtained and the estimated trajectory to produce a 3-D point cloud map of the environment. In a different contribution, they managed to illustrate good localization results using stereo cameras and loop closures for missions over 142km (Sibley et al., 2010).

The majority of the works presented in the previous paragraphs reconstruct point cloud maps. Konolige et al., instead of using point cloud representation of the environment, proposed the use of occupancy grid based maps. They developed a ground vehicle equipped with stereo cameras, GPS and wheel encoders. Using feature extraction methods they computed visual odometry. The visual odometry, the GPS and dead reckoning were used to localize the vehicle and reconstruct occupancy grid maps (Konolige et al., 2006) of outdoor environments. In an expansion of their work they incorporated loop closures and formulated the problem as a Bayes graph estimation problem which was solved using non-linear least squares techniques which computes the trajectory that minimizes the squares error (Konolige and Agrawal, 2008).

In recent research, Tong et al. designed a vehicle and algorithms for mapping planetary work site environments. The vehicle used was a rover equipped with a 2-D laser scanner mounted on a pan-tilt mechanism to utilize 3-D data capture, stereo camera to provide visual odometry measurements and IMU. Advanced feature extraction algorithms were used to extract features. Extracted features and IMU measurements were used to localize the vehicle using advanced full pose non-linear least squares SLAM techniques (Tong and Barfoot, 2012).

### 2.2 3D Model Reconstruction using Marine Vehicles

What little research has been done on 3D model reconstruction by marine robots deals mostly with underwater surfaces and reconstruction of bathymetric maps. In the early years of bathymetric mapping, Singh et al. presented results (Singh et al., 2000) taken from deep seas in Italy. Their approach utilized a Long Base
Line (LBL) system that localized the underwater vehicle and a set of sonar sensors (side-scan sonar and an actuated pencil profiling sonar) to scan the seabed. At the same time, Bebbett and Leonard reconstructed bathymetric maps from the Charles River using the AUV Odyssey II. They use dead reckoning sensors on board and a single altimeter sonar (Bennett and Leonard, 2000).

In more recent years of bathymetric research, researchers have been using SLAM algorithms to localize the vehicle. Ruiz et al. used a side-scan sonar and onboard sensors to reconstruct bathymetric maps. They manually extracted features out of the sonar data and they used SLAM techniques to localize the vehicle and get better estimates of bathymetric maps (Tena Ruiz et al., 2004). A similar approach was taken by Shkurti et al. (Shkurti et al., 2011). They designed a new vehicle called The AQUA Underwater Robot, which is equipped with IMU and cameras. By extracting features on the seabed and using IMU measurements they are able to reconstruct the bathymetric map of small areas. Their approach utilizes an EKF SLAM approach. Roman et al. reconstructed bathymetric maps by fusing navigation measurements and relative poses given by the ICP algorithm (Roman and Singh, 2007).

Johnson et al., using an underwater vehicle, presented results on large scale 3-D model reconstruction and visualization of bathymetric maps from Australian coastal waters. The proposed vehicle was equipped with stereo cameras, sonars, a DVL, accelerometers, depth sensors, GPS (it surfaces to acquire GPS fixes) and was also using Ultra-short Base Line (USBL). Using stereo cameras they extracted features that are used to compute visual odometry. The visual odometry was combined with readings from other sensors on board to estimate vehicle trajectory and then the vehicle trajectory was used to reconstruct the map of the seafloor using reading from the cameras (Johnson-Roberson et al., 2010). They also used meshing algorithms to reconstruct mesh representations of the seafloor.

In a different direction of the surface reconstruction problem using marine vehicles, Fairfield et al. used a hovering underwater vehicle to reconstruct surfaces of caves and tunnels. Their method consisted of a Rao-Blackwellized particle filter with a 3-D evidence grid map representation. In addition, they used occupancy based grid representation of the environment (Fairfield et al., 2007) and (Fairfield and Wettergreen, 2008). More recently, significant progress has been achieved on surface reconstruction for ship hull inspection tasks by Hover et al. (Hover et al., 2012).

The closest work to that presented in this paper is Leedekerken’s recent work (Leedekerken et al., 2010). In Leedekerken’s work, a set of 2-D laser scanners is combined with a high-accuracy localization unit that combines GPS-IMU and DVL. In contrast, our work uses a powerful LiDAR Velodyne laser scanner and
assumes a GPS-denied environment without using any other localization sensors such as IMU or DVL. In addition, Leedekerken uses a forward-looking sonar that mostly provides bathymetry data rather than data on the submerged part of the structure, whereas we scan the below-water part of the marine structure of interest with a side-looking sonar.

Another key difference between our approach and other surface reconstruction approaches (including (Leedekerken et al., 2010)) is that our approach can reconstruct 3-D models of moving structures such as floating platforms and boats. Results, presented later in the paper, indicate that the proposed algorithm can reconstruct 3-D models of slowly moving structures. Slowly moving structures are commonly found in oil industry (e.g. Fig. (20)). Reconstructing models of large 3-D moving structures is not easily done using known SLAM techniques that first compute vehicle trajectory and then project laser scanners or cameras data using vehicle trajectory. In the case of moving structures this cannot be done because vehicle trajectory and laser data obtained from different views are not sufficient to describe the structure. In this work, we are not interested in vehicle trajectory, instead we compute an equivalent trajectory (set of transformation matrices) that minimizes registration error between different scans. In the case of stationary structures, the equivalent trajectory obtained should be close to the actual one.

3 Proposed Robotic Platform

Our goal is to generate 3-D models for marine structures. Given the complexity of the marine environment, we would like to use a small but powerful vehicle with high maneuverability that will be able to access hidden places of the structure without crashing into the structure (due to water currents). For this purpose, we use a SCOUT Autonomous Surface Vehicle (ASV) – a kayak with a 3m length, 0.5m width, and 90kg mass. The ASV is equipped with a 245N thruster for propulsion, as well as a steering servo (Curcio et al., 2005) (Fig. (2(a))).

No localization sensors such as GPS, INS, or DVL are used in the current paper, but the vehicle is nevertheless equipped with a GPS (Garmin GPS-18) and a compass (Ocean Server OS5000) so that, in the future, it will be able to expand our current work in all possible directions.

To facilitate data capturing and autonomous control capability, the main compartment of the ASV is equipped with a Main Vehicle Computer (MVC). The MVC consists of a pair of single-board computers connected through an Ethernet cable. Each single-board computer is equipped with 1GB RAM. In addition,
one of the single-board computers is equipped with a 120GB hard drive to facilitate large data capturing capability. The ASV can be controlled remotely using a remote control or autonomously using the well-known autonomy software MOOS (Newman, 2003),(Benjamin et al., 2009).

![Figure 2: (a) The ASV with Velodyne LiDAR mounted on top of it. An additional pontoon is attached to the ASV to improve stability of the ASV. (b) The Velodyne LiDAR and the mounting platform for placing the LiDAR in an inverted configuration on the ASV. (c) The BlueView MB2250 micro-bathymetry sonar mounted in a sideways configuration.](image)

Registration algorithms frequently fail because the two point clouds to be combined have no common features. Given that our goal is to perform surface reconstruction without using any navigation sensors, we use a laser scanner with a wide field of view that allows significant overlap between subsequent scans. In addition, we use a laser scanner that completes a scanning action much faster than the vehicle and structure motion, so that we do not need to incorporate vehicle and structure motion within a single scan. This makes the procedure simpler and reduces the computational cost of the algorithm. One sensor that meets the desired specifications is the Velodyne HDL-64E S2 shown in Fig. (2(b)). The Velodyne HDL-64E S2 is a 3-D LiDAR (Light Detection and Ranging) which completes each scanning action in 0.1 second (scanning frequency =10 Hz).

The Velodyne LiDAR was initially developed for the 2007 Urban DARPA Grand Challenge. Its original configuration was supposed to be mounted on car roofs to perform scanning actions in which a full 360-degree horizontal picture with vertical arc of 26.80° (from 2° to -24.80°) is captured. The Velodyne is mounted on the kayak in an inverse configuration to maximize the scanning surface. To reduce the amplitude of the rolling motion caused by the marine environment, we installed pontoons (see Fig. (2(a))).

To scan the below-water part of the marine structure of interest, we use a 3-D Micro-bathymetry sonar (BlueView MB2250 micro-bathymetry sonar). The MB2250-45 sonar uses 256 beams with one degree beam
width in elevation. Since we are interested in mapping marine structures, instead of mounting the sonar in a forward-looking configuration such as (Leedekerken et al., 2010), we mount it sideways on the vehicle (Fig.(2(c))).

4 Surface Reconstruction Algorithms

The goal of this paper is to illustrate that a 3-D model of marine slow-moving structures in a sea environment can be constructed through the use of a single LiDAR without using any other scanning or navigation sensor. Since this work is an early one to be done on surface reconstruction of marine structures that are partially submerged in GPS-denied environment, we want to keep things simple; thus, we do not use advanced techniques like feature extraction and loop closures. In addition, as explained in Section (1), advanced SLAM techniques that optimally localize the agent and reconstruct maps, in their current state, are not easily used to reconstruct the shape of moving structures.

Based on the vehicle design described in the previous section, we propose algorithms to construct the 3-D model of partially submerged marine structures. We propose 3 different algorithms, the first one (Algorithm(1)), is used to reconstruct point cloud representations of the above-water part of marine structures, the second one, (Algorithm(2)), is used to reconstruct mesh models of the above-water part of marine structures. The third algorithm (Algorithm(3)) is used to reconstruct 3-D models (point cloud representations, mesh representations and occupancy grid based maps) of both parts (the above- and below-waterline parts) of partially submerged marine structures.

We use scan matching techniques to construct the 3-D model for above-water part. We then use the transformations, computed by the scan matching algorithm for the above-water part, to construct the 3-D model from a sequence of 2-D sonar data from the below-water part. We then combine the 3-D model of above- and below-water parts to construct a complete 3-D model of the partially submerged marine structure. We construct two types of maps: a low quality and a high quality map. The low quality map can be constructed on-line and would be useful for navigation purposes. The high quality map is constructed off-line and can be used for inspection purposes.
4.1 Registration Algorithm

To scan a marine structure, the vehicle is driven around the structure of interest, gathering 3-D laser data. The data is logged and saved in the ASV’s computer in data structures called point clouds. Each point cloud represents a set of points in $\mathbb{R}^3$ gathered by a complete $360^\circ$ rotation of the LiDAR. Since the scanning action is performed much faster than the vehicle’s speed, all points within a point cloud are expressed within a common orthogonal reference frame that is aligned to the center of the LiDAR at the starting point of the scanning cycle.

Given 2 point clouds $M_i$ in $\mathbb{R}^{M \times 3}$ and $M_j$ in $\mathbb{R}^{D \times 3}$ that include common features, we need to compute the transformation $i^j T_j$ that transforms each point $m_k$ of $M_i$ to each point $d_l$ of $M_j$. This problem was originally proposed and solved by Besl & McKay in 1992 using the ICP algorithm (Besl and McKay, 1992), by minimizing the following metric:

$$E(i^j R_j, i^j b_j) = \sum_{k=1}^{M} \sum_{l=1}^{D} (w_{k,l} |\| m_k - (i^j R_j d_l + i^j b_j) |\|^2)$$

(1)

Here, $w_{k,l}$ is a binary variable as follows: $w_{k,l}=1$ if $m_k$ is the closest point to $d_l$ within a close limit and is equal to zero otherwise. $i^j R_j$ and $i^j b_j$ are the rotation matrix and the translation vector defined in (Nüchter et al., 2007). The minimization is done using non-linear optimization tools such as Levenberg-Marquardt.

4.2 Above-Water Surface Reconstruction Algorithm

The vehicle gathers point clouds ($M_1, M_2, ..., M_{n-1}, M_n$) with frequency of 10 Hz. We sequentially merge these 3-D point clouds, each in their respective coordinate systems, into a single coordinate system. The transformations between sequential point clouds ($^0 T_1, ^1 T_2, ..., ^{n-2} T_{n-1}, ^{n-1} T_n$) is given by the ICP algorithm described in the above section. The first point cloud in the sequence is transformed into the coordinate system of the second point cloud. The union of the first two point clouds is then transformed into the coordinate system of the third point cloud, and so on. This process continues until we transform all the point clouds into the coordinate system of the last point cloud in the sequence.

The Velodyne LiDAR generates around 8 MB of data per second (250,000 points per scanning cycle and 10 scanning cycles per second). The computational cost and memory requirements to deal with this amount of data are huge, making ICP impossible to run on-line. The time for a single merging process in the worst case is $O(N \log N)$ where $N$ is the number of points in the current map. This complexity is dominated.
by searching for correspondence points. For online-mapping, we speed up the search process by using the ASV maximum speed to bound the maximum possible displacement $d$, and hence limit our search space. Furthermore, we fix a maximum number of possible iterations to ensure termination within the required time.

In addition, we reduce these computational demands in two other ways. First, instead of using the raw data as gathered, we use data from scanning actions performed every $\Delta t$ seconds. Second, we perform spatial sub-sampling on each point cloud by discretizing the bounding box of the point cloud into a regular grid with user-specified resolution; thus, all the points inside a single grid cell are represented by a single point. By limiting cells to a given size (resolution), we both reduce the amount of data to a reasonable quantity and also cancel out the errors (assuming zero mean noise).

To get the on-line map, we simplify the data as described above using a large simplification cell size and large $\Delta t$, as shown in Fig. (3) and Algorithm (1). This on-line map is a low-resolution map that can be used for navigation.

![Figure 3: The online map: we use big simplification cell size.](image)

**Algorithm 1** Construct3DModel($P$, $s$, $t$) Construct a 3-D model from a sequence of point clouds $P$. The input $s$ and $t$ are the user-specified spatial and temporal resolution, respectively.

1. $\text{MergedData} = \text{SpatialSubSampling}(P[1], s)$.
2. for $i = t$ to $|P|$ step $t$ do
3. $P' = \text{SpatialSubSampling}(P[i], s)$.
4. Let $T0$ be the identity transformation matrix.
5. $T = \text{ICP}(\text{MergedData}, P', T0)$.
6. $\text{MergedData} = \text{Transform} (\text{MergedData}, T)$.
7. $\text{MergedData} = \text{MergedData} \cup P'$.
8. Return $\text{MergedData}$.

To get a higher quality map, we want to use as much data as we can handle. To do so, we generate an occupancy grid-based map under a probabilistic framework such as OctoMap (Wurm et al., 2010). Because the LiDAR generates a huge amount of data, we still use simplified data (albeit less simplified than above), rather than raw data, in order to get the transformation matrices. We then use the transformation matrices to merge the raw data, resulting in a single, dense 3-D point cloud. An occupancy grid-based map is then generated using OctoMap. The resulting grid-based map is used to get the mesh of the structure using the
ball-pivoting algorithm (Bernardini et al., 1999). This process is shown in Fig. (4) and in Algorithm (2). This high-resolution map must be generated off-line, and can be used for inspection or for further analysis (depending on the application).

![Diagram of the off-line map process](image_url)

Figure 4: The off-line map: Less simplified data are used to get vehicle’s trajectory and then vehicles’ trajectory is used to project raw data resulting a dense point cloud.

**Algorithm 2** Construct3DModel($P, s, t$) Construct a 3-D high quality models (mesh representation, dense point clouds, occupancy grid) from a sequence of point clouds $P$. The input $s$ and $t$ are the user-specified spatial and temporal resolution, respectively.

1: $\text{MergedData} = \text{SpatialSubSampling}(P[1], s)$.
2: \textbf{for} $i = t$ to $|P|$ \textbf{step} $t$ \textbf{do}
3: \hspace{1em} $P' = \text{SpatialSubSampling}(P[i], s)$.
4: \hspace{1em} Let $T0$ be the identity transformation matrix.
5: \hspace{1em} $T = \text{ICP}($MergedData, $P', T0$).
6: \hspace{1em} $\text{DenseMergedData} = \text{Transform} (P[i], T)$.
7: \hspace{1em} $\text{MergedData} = \text{DenseMergedData} \bigcup P[i]$.
8: \hspace{1em} $\text{DenseMergedData} = \text{Transform} (\text{MergedData}, T)$.
9: \hspace{1em} $\text{MergedData} = \text{Transform} (\text{MergedData}, T)$.
10: $\text{OccupancyGrid} = \text{Octomap}(\text{DenseMergedData})$
11: $\text{MeshModel} = \text{BallPivoting}(\text{OccupancyGrid})$
12: Return $\text{MergedData}, \text{OccupancyGrid}, \text{MeshModel}$.

Two parameters drastically affect the computational cost of our method: $\Delta_t$ and cell size. A small $\Delta_t$ results in big computational costs, leaving time intervals between scans that are too small to solve the problem on-line. On the other hand, given that we are not using other localization sensors, a large $\Delta_t$ may result in the failure of the ICP algorithm, since the algorithm cannot merge point clouds gathered from sequential locations that are not sufficiently close to each other (i.e., get stuck in a local minimum).

In the Fig. (5) below, we can see the trajectories that the ICP algorithm yields for different values of $\Delta_t$ ($C_1, C_2, \ldots C_5$). We observe that trajectories corresponding to different values of $\Delta_t$ form a sequence with decreasing differences as $\Delta_t$ goes to zero (i.e., the sequence trajectories have the Cauchy property and thus there exists a limit). Therefore, for this particular dataset, the benefit of reducing $\Delta_t$ below 1 second is not
worth the computational cost for either the online mapping or the offline mapping.

![Figure 5: Trajectories generated during the first 100 seconds of one of the experiments: Trajectories corresponding to different $\Delta t$ form a sequence with decreasing deference.](image)

Regardless of cell size, as long as it is small enough to capture geometrical features that are important to the ICP algorithm, it does not affect the localization. For the off-line map, simplification cell size generally does not matter, as long as the localization works properly, since we are using vehicle’s trajectory to project dense raw data. However, for the on-line map, the cell size is bounded by the accuracy we want to have in the map.

### 4.3 Combined Map

In order to create a complete 3-D model of the entire partially submerged marine structure, we use the 3-D Micro-bathymetry sonar described in the previous section to get the below-water part of the marine structure, and then we combine this data with the model of the above-water part of the structure. The vehicle’s trajectory generated by our above-water mapping algorithm is used to register the 2-D sonar data into the global 3-D map.

---

1 In the offline map, the voxel size and the probability threshold given in the OctoMap algorithm are important and reflect the resolution we want to capture.
The vehicle’s sonar generates 2-D data in the polar coordinate system \((r, \phi, I)\), where \(r\) and \(\phi\) are the ranges and the angles of the returns and \(I\) is the intensity (see Fig. (7)). Since we want to project 2-D sonar data into the Cartesian global coordinate frame generated by the above-water surface reconstruction algorithm, we transform the 2-D sonar data to an equivalent local Cartesian frame using the equations below:

\[
{s_x} = r \cos \phi \quad \text{(2)}
\]

\[
{s_z} = r \sin \phi \quad \text{(3)}
\]

\[
{s_y} = 0 \quad \text{(4)}
\]

The sonar data is then transformed into the current LiDAR coordinate system using Equation (5). This allows the sonar data to be treated and propagated as equivalent to Velodyne data as described in section (4.2)

\[
{x_s} = [T_v][x_s] \quad \text{(5)}
\]

where \([T_v]\) is the transformation matrix from the sonar coordinate system to the Velodyne system, as shown
in Fig. (6) and \( \mathbf{s} = [s_{x}, s_{z}, y_{s}]^T \). At the present time, the registration between sonar and LiDAR data is done by manually measuring the transformation from sonar frame to the LiDAR frame. In the near future, we intend to implement a registration algorithm to do the registration between LiDAR and sonar data. The proposed algorithm for surface reconstruction of the combined model is shown in Fig. (8) and Algorithm(3).

Figure 7: Sonar coordinate system: Sonar gives returns up to 10 meters with an angle of 45 degrees.

**Algorithm 3** Construct3DModel(\( P, S, s, t, b_{v}, R_{v} \)) Construct a 3-D models (of the above- and below-water parts of marine structures) from a sequence of point clouds \( P \). The input \( s \) and \( t \) are the user-specified spatial and temporal resolution, respectively.

```plaintext
1: MergedData = SpatialSubSampling(\( P[1], s \)).
2: for \( i = t \) to \( |P| \) step \( t \) do
3: \( P' = \) SpatialSubSampling(\( P[i], s \)).
4: \( T0 \) be the identity transformation matrix.
5: \( T = \) ICP(\( MergedData, P', T0 \)).
6: \( MergedData = \) Transform(\( MergedData, T \)).
7: \( MergedData = MergedData \cup P' \).
8: \{\( S_{temp} \}=\) FindSonarLogs(\( t, t-1 \))
9: \( SonarMergedData = SonarRegistration(\{S_{temp}\}, T, b_{v}, R_{v}) \).
10: \( SonarMergedData = SonarMergedDatat \cup SonarMergedData \)
11: \( OccupancyGrid=Octomap(DenseMergedData) \)
12: \( OccupancyGridSonar=Octomap(SonarMergedData) \)
13: \( SonarAndVelodyneGrid= OccupancyGrid \cup OccupancyGridSonar \)
14: \( MeshModel=BallPivoting(SonarAndVelodyneGrid) \).
15: Return \( MeshModel, SonarAndVelodyneGrid \).
```

Sonar data is noisy, so to clean up the data, we extract objects from the raw sonar data using clustering filtering methods. The main concern is to separate the object from the noise. For this purpose, we use
Figure 8: Surface reconstruction of both parts of marine structures, above and below waterline. The ICP algorithm is used to find the transformation matrices between different poses (using the laser scanner data). Then we use the transformation matrices to register the laser scanner data under the same coordinate frame and register the unregistered 2D sonar data as 3D data to the same coordinate frame. Then we use a probabilistic framework such as the occupancy grid based maps to clean up the point clouds, and then we use known surface reconstruction algorithms (ball pivoting algorithm and the Poisson reconstruction algorithm) to reconstruct a 3D surface model of the marine structure.

simple background removal on the 2D intensity map. Initially, we capture a sonar reading when no objects are within the sonar’s range. The 2-D intensity map Fig. (9(b)) of this scan becomes the “background” intensity map. For robustness, we do not compare the object intensity map and background intensity map per pixel. Instead, we use the background map to find a good threshold to determine if an intensity at a particular pixel can be considered as object or just noise. This is done by dividing the background intensity map into 6 clusters, Fig. (9(b)), based on the background intensity map and then use the most frequent intensity in each region as the threshold. Given an object intensity map, we divide the map into 6 regions as in the background intensity map, and consider a point to be part of an object whenever its intensity is higher than the threshold for the region. Fig. (9(a)) compares the raw sonar data to the data that has been cleaned using clustering filtering methods.

5 Experimental Results

To evaluate our algorithms, we performed a set of experiments in the Singapore area between January of 2009 and August of 2010. Results presented in this section were initially presented in our ISOPE 2011 and
IROS 2011 conference contributions (Kurniawati et al., 2011) and (Papadopoulos et al., 2011). Results given here are post-processing results, since none of our surface reconstruction algorithms were running on-line.

Our goal was to test our system in rough water environments. However, before testing our system in rough water, we performed preliminary experiments in calm water to ensure the system is ready for rough water testing. To test the system in rough water, first we need to decide the location and the time of testing. We would like to find marine structures, such as jetties, that would be as close as possible to open waters. Therefore we chose a jetty in a small island (of size less than a square kilometer). To decide the time and the dates of the experiments we look at the tide and water current predictions to ensure that the environment is challenging enough but not too extreme that could possibly put our lives and equipment in danger (water currents with speeds greater than 4 m/s are considered, for our case, dangerous environments). In order to go to the operational area we had to consult a ship and a few boats. We ran the experiments onboard a boat (e.g. Pandan Reservoir experiment, or the boat reconstruction experiment) or from a workstation at the marine structure of interest (e.g Selat Pauh experiment).

The first experiment was performed in a calm water environment, in Pandan Reservoir, Singapore and we reconstructed point clouds and alpha shape representations of the above part of a jetty (see Fig. (10(a))). The vehicle was also equipped with a sonar micro-bathymetry sensor but for technical reasons we did not manage to gather reliable sonar data to reconstruct the below-water part of the structure.

To give an illustration of the difficulty in merging the scanned data, Fig (10(c)) shows the resulting 3-D point clouds scanned by the LiDAR, plotted in one coordinate system. Fig. (10(d)) shows the resulting point clouds representation of the structure when the coordinate systems are transformed to the first coordinate.
system based on the GPS and compass information alone. Fig. (10(b)) and Fig. (10(e)) show the results of our 3-D model reconstruction algorithm on the above data set.

Figure 10: Reconstruction of a jetty in Pandan Reservoir, Singapore. (a) The target jetty. (b) Side view of the jetty model constructed by our algorithm. (c) Top view of multiple frames of the 3-D LiDAR data before processing. (d) Top view of the constructed 3-D model based on GPS and compass information. (e) Top view of the constructed 3-D model generated by our algorithm.

The second experiment was performed in rough sea water environment in Selat Pauh at the Singapore Straits (Fig. (11(a))). The water currents in Selat Pauh are around 2m/s. In addition, Selat Pauh is a busy strait with a significant amount of ship traffic, causing high frequency water wakes that significantly disturbs the motion of small marine vehicles. In that particular experiment we reconstruct point clouds representation for a slowly moving boat, illustrated in Fig. (11(b)). Although the boat is moving slowly, the water currents and wakes cause the boat to drift and move up and down significantly. As an illustration of the effect of water currents and wakes on the scanned data, Fig. (11(c)) and Fig. (11(f)) show the 3-D point clouds scanned by the LiDAR over a 2-seconds period, plotted in the same coordinate system. Fig. (11(d)) and Fig. (11(g)) show the data when the coordinate systems are transformed to the first coordinate system based on GPS and compass information alone. Fig. (11(e)) and Fig. (11(h)) show the results of applying our 3-D model reconstruction algorithm to the above data set.

In our last experiment we present results from 2 missions performed with a jetty located at Pulau Hantu (a
small island few kilometers away from Singapore). We deployed our vehicle from a ship near Pulau Hantu and drove it about the jetty to gather data. We present two missions. The first mission lasted 3 minutes and gathered data for the above-water part of the jetty. In this mission, we drove the vehicle a distance of about 200m making sure that the vehicle approached the structure from different views to recover all the hidden parts of the structure. The second mission lasted 1 minute and gathered data from the above- and below-water parts of the floating platform that was located in front of the jetty (see Fig. (17(a)).

We present 3 different maps of the jetty and the floating platform. The first map is a low-quality point cloud-based map that could be generated online and can be used for navigational purposes (Fig. (12(b),12(c)) ). The second map is a higher quality mesh-based map (Fig. (13,14,15,16)). The third one combines both the above- and the below-water parts of a single marine structure (Fig. (17(b),17(c))).
In Fig. (12(b),12(c)) we can see the low-resolution point cloud-based maps of the jetty for different cell sizes. We can verify that the one that was generated with 30 cm cell size can be generated on-line. For both cases $\Delta t = 1$ second.

(a) The marine structure of interest.

(b) Low-resolution map, cell size= 30cm, the mission lasts 3mins, is generated in 3mins (3GHz CPU).

(c) Low-resolution map, cell size= 18cm, the mission lasts 3mins, is generated in 8mins (3GHz CPU).

Figure 12: Low resolution “on-line” model.

In Fig. (13,14) we present different views of the high-quality mesh-based maps of the jetty (the mesh was reconstructed using the ball pivoting surface reconstruction algorithm). The voxel size used in the occupancy grid generation was 8cm and $\Delta t =1$ second. Here we present two different high quality maps; the first one (Fig. (13)) was generated using a high probability threshold for occupied cells, and the second one (Fig. (14)) was generated using a low probability threshold for occupied cells resulting in a dense map.

In Fig. (15) we can see the mesh-based high quality map for the above-water part of the jetty, using a low
probability threshold for occupied cells and the Poisson surface reconstruction algorithm. From our results we can see that the Poisson surface reconstruction algorithm produces better results than the results the ball pivoting algorithm gives. In addition the computation time of the Poisson surface reconstruction algorithm is on the order of minutes for a point cloud that includes about 1 million points. On the other hand the ball pivoting algorithm took several hours to run. In Fig. (16) we can see the Poisson surface reconstruction results focused on certain parts of the structure. The upper part, shows the pillars of the structure, we can see that the algorithm can capture the pillars’ geometry pretty well. The below left part, shows a surface of a human body; during the experiments we have people sitting and possibly moving on the the jetty, thus the inside part of the structure presents some anomalies. In the below right part of Fig. (16), we can see 2 large buoys that are used to avoid direct collision of boats on the jetty.

Fig. (17) shows results from both parts of a marine structure. Specifically, Fig. (17(b)) and Fig. (17(c)) show the point based map for both the above- and below-water parts for the floating platform. Fig. (17(d)) is a zoomed-in view of the combined mesh-based map. We notice that the buoy that supports the floating platform is flattened due to regular contact with boats. In all cases, the mesh is generated using meshlab (a tool developed with the support of the 3D-CoForm project) (Visual Computing Lab ISTI - CNR, 2010).

Figure 13: The mesh-based high quality map for the above-water part of the jetty, using a high probability threshold for occupied cells and the ball pivoting surface reconstruction algorithm. Cell-size=8cm.

Results Validation

In this section we validate our experimental results. Probably, the best way to evaluate our results is to use a mesh comparison method, such as the one developed by Roy et al. that compares 3D models and produces statistics on mesh similarity (Roy et al., 2004). Unfortunately, we have no access to 3-D models of the jetties we reconstructed, however we infer our results quality by the following ways:
Figure 14: The mesh-based high quality map for the above-water part of the jetty, using a low probability threshold for occupied cells and the ball pivoting surface reconstruction algorithm. Cell-size=8cm.

Figure 15: The mesh-based high quality map for the above-water part of the jetty, using a low probability threshold for occupied cells and the Poisson surface reconstruction algorithm. Cell-size=8cm.
Figure 16: Zoom in of the mesh-based high quality map for the above-water part of the jetty, using a low probability threshold for occupied cells and the Poisson reconstruction algorithm. Upper part, shows the pillars of the structure. Below left part, shows a surface of a human body (during the experiments we have people sitting and possibly moving on the jetty). Below right, shows 2 large buoys that are used to avoid direct collision of boats on the jetty. Cell-size=8cm.
We compute the quality of all ICP registrations using as a metric the ICP “goodness” as defined in (Blanco-Claraco, 2009). For the cases we studied in this paper, the mean “goodness” for all ICP registrations is 96.5% with standard deviation about 2%. At the same time the ICP took on average 277 iterations to converge. Another way to capture a numerical quality of the accuracy of the registration is described by Douillard et al. (Douillard et al., 2012).

To show consistent reconstruction from both parts of the marine structure of interest we present an occupancy grid based map of the above- and below-water parts of the structure and the probability of each cell to be occupied. The color indicates the probability of the cells to be occupied, Fig. (18). Low probability gives blue colors and high probability gives red colors. From this figure we can clearly see the waterline and three areas: The above-water part of the structure, the below-water part of the structure and the interface area between the LiDAR and the sonar data.

The probability of above-water part of the structure cells to be occupied is close to uniform and less dense than the ones from the below water part of the structure. Generally, laser scanners produce denser point clouds than the sonars, however this is not the case here because to avoid huge data sets we simplify/regularize our raw data using very small simplification cell-size (e.g few millimeters to few centimeters). Still we can see that the above-water part of the structure is reconstructed
consistently.

The underwater part of the structure is also reconstructed consistently and the probability of the cells of the underwater part of the structure to be occupied is high (the cells with the highest probability are located in the far left side of the structure where the mission started from and the vehicle probably was sitting there accumulating sonar data from the same position for a few seconds). In the interface area between the LiDAR and sonar data the probability of a cell to be occupied, as expected, is lower than the below-water part of the structure but is high enough to achieve surface reconstruction in the interface area. In the far right area of the underwater part of the structure we do not get enough sonar returns. This is probably due to the fact that in this particular location the vehicle started turning towards the left and thus the side looking sonar (with a very narrow beam) instead of pointing next to the kayak was pointing towards to previously scanned areas (e.g. in the left side of the platform, behind the kayak) and, then we stopped the mission. This problem can be solved using a better sonar sensor such as the Didson sensor (used by Hover et al. (Hover et al., 2012).) or designing trajectories -that take into account vehicle dynamics- to be informative enough to reconstruct structures (Papadopoulos et al., 2013).

- We measure characteristic lengths of the jetty using google maps and compare them to the ones we get from the reconstructed jetty. To reduce the effect of constant errors such as conversion from “Velodyne units” to metric units and errors due to the direction the aerial picture was taken from, instead of comparing the actual lengths we reconstruct dimensionless numbers that characterize the structure, Fig. (19). In our case we have $L_1/W_1 = 4.9$, which is close to $L_2/W_2 = 4.8$ (Error 2%), the ones obtained using the google maps. In addition, $L_1/D_1 = 3.5$, which is also close to $L_2/D_2 = 3.4$ (Error 2.8%), the ones obtained using the google maps. Where $L_1, W_1, D_1$ are the characteristic lengths of the reconstructed jetty and $L_2, W_2, D_2$ are the characteristic lengths of the jetty as measured using Google maps.
Figure 18: Upper part: A point cloud representation of the above- and below-water parts of the structure. The color indicates the density of the points. Lower part: An occupancy grid based map of the above- and below-water parts of the structure. The color represents the probability of the cells to be occupied. Low probability gives blue colors and high probability gives red colors.

Figure 19: Comparison between our results and pictures of the actual jetty taken from google maps.
6 Conclusions and Future Work

In this paper, we present a hardware and software system to reconstruct 3-D surface models of marine structures that are partially submerged. In particular, this paper made the following contributions.

- Using off-the-shelf sensors and a commercial robotic kayak developed by Curcio et al. (Curcio et al., 2005), we assembled a novel surface vehicle that is capable of using a powerful 3-D laser scanner (Velodyne) and a side-looking sonar to scan marine structures both above and below the waterline.

- Using Velodyne and sonar data, without using any other navigation sensor such as GPS, DVL or INS, we propose a method to reconstruct 3-D models of partially submerged slowly moving marine structures.

- We present various surface reconstruction experimental results including results from sea environments with water currents around 1m/s–2m/s. More specifically, we present 3 experimental results of mapping the above-water part of marine structures and 1 experimental result of mapping both the above- and below-water parts of marine structures. To the best of our knowledge, this is the first results on 3-D surface reconstruction experiments from rough waters and slowly moving structures. Experiments presented here are as realistic as possible. We present results in rough waters with moving structures (floating platforms and boats) under tidal effects (1-3 meters) and water disturbances arising from big ships moving in the busy Singapore waters.

The results show that our robotic system for 3-D model construction of marine structures is reliable to operate in rough sea water environments. The resulting scanned data indicates that the LiDAR’s mounting platform does not pose significant degradation in the quality of the scanned data. Furthermore, because of the high scanning frequency of the Velodyne LiDAR and sufficient overlap between point clouds generated by different scanning cycles the simple merging algorithm we propose is sufficient to construct a rough 3-D model of marine structures.

We also show results indicating that the proposed algorithm can reconstruct 3-D models of slowly moving structures. Slowly moving structures are commonly found in oil industry (e.g. Fig. (20)). Reconstructing models of large 3-D moving structures is not easily done using standard SLAM techniques that first compute vehicle trajectory and then project laser scanners or cameras data using vehicle trajectory. In the case of moving structures this cannot be done because vehicle trajectory and laser data obtained from different views are not sufficient to describe the structure. In contrast to the SLAM problem, we are not interested in vehicle trajectory, instead we compute an equivalent trajectory (set of transformation matrices) that minimizes the
registration error between different scans. In the case of non-moving structures, the equivalent trajectory obtained should be close to the actual one.

One thing we notice is that extremely strong currents excite vehicle roll and pitch motions, causing some of the LiDAR scans empty (taken with the LiDAR looking at the sky).\footnote{2} If we try to merge one of these LiDAR scans, the ICP algorithm is likely to fail. To avoid this problem, before we call the surface reconstruction module we filter out LiDAR scans with extremely low amount of data.

Despite the above promising results, there is still plenty of room for improvement. In this work, we have assumed a GPS-denied environment, without using any other navigation sensors such as DVL or INS. Of course whenever possible, we are interested in using GPS and other navigation sensors. Furthermore, we would like to use more SLAM advanced techniques such as feature extraction and loop closures to bound localization accuracy, which crucially affects the quality of the map. However, as indicated above, special treatment should be considered in the case of moving structures such as floating platforms (Lin and Wang, 2010; Hsiao and Wang, 2011). In addition, more research needs to be done on integrating data from above and below water parts of marine structure. More specifically, we would like to use registration algorithms to better align the above water part of the structure with the below ones. Another possible direction of future work will be the combination of the ICP solver with randomization methods to find the global minimum in Equation (1). We believe that as the number of sample points increases, ICP solvers combined with randomization methods will converge to the globally optimal transformation matrix (given that the samples are chosen correctly).

![Different types of oil platforms](image)

Figure 20: Different types of oil platforms, the majority of the platforms are floating and slowly moving. The picture is from the Office of Ocean Exploration and Research.

\footnote{In our case we try to minimize this effect by installing pontoons on the kayak.}
Acknowledgments

This work was supported by the Singapore-MIT Alliance for Research and Technology (SMART) Center for Environmental Sensing and Modeling (CENSAM). We would also like to thank Dr. Leedekerken and Professor John Leonard for thoughtful discussions and Andrew Patrikalakis for his help in the initial vehicle design. We thank the anonymous reviewers for their input.

Appendix A: Index to multimedia Extensions

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<th>Media Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Video</td>
<td>It shows some of the experiments presented in this paper</td>
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</table>

References


