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18.311 Principles of Applied Mathematics
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18.311 Principles of Applied Mathematics, S2009. Aslan Kasimov.

Midterm Exam, 3/19/09.

No books, notes, or calculators are allowed.

Your name:_____

Problem M1 (15 pts). Find two-term regular perturbation expansion of the solution of the boundary-value problem:

$$y'' = \epsilon y^2, \;\; y(0) = 0, \, y(1) = 1, \;\; \epsilon \to 0.$$

Problem M2 (15 pts). Find two-term asymptotic expansion of the roots of the equation:

$$x^3 - x + \epsilon = 0, \quad \epsilon \to 0.$$

Problem M3 (15 pts). Solve the initial-boundary-value problem and sketch the characteristics:

$$u_t + (1+x)\,u_x = 0, \quad x > 0,\, t > 0, \quad u\,(0,t) = t, \quad u\,(x,0) = \sin x.$$

Problem M4 (15 pts). (a) Solve

$$u_t + uu_x = k^2 x, \quad u(x,0) = u_0(x), \quad x \in R,\ t > 0, \text{ and } k \geq 0.$$

(b) Under what conditions on $u_0(x)$ does the solution remain regular at all t?

(c) Suppose now that $u_0(x) = -2x$ and $k = 1$. What is $u(x,t)$ and at what $t = t_c$ does the gradient catastrophe occur?