MODEL INVESTIGATION OF EFFECTS
OF VEHICULAR VIBRATION
ON TWO-SPAN BRIDGES

by

George C. Tso

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Signature redacted

Signature of Author____
Department of Civil & Sanitary Engineering
Course in Building Engineering & Construction
January, 1956.

Signature of Professor in
charge of Research____ Signature redacted

Signature of Head of
Course____ Signature redacted

ABSTRACT

The purpose of this thesis has been to study experimentally the dynamic effects of a moving vehicle on two-span bridges. The primary objectives were to ascertain the effects of: (1) The natural frequency of the vehicle; (2) The vehicular velocity; (3) The magnitude of initial oscillation in the vehicle; (4) The damping characteristics of the vehicle.

A dynamically similar model was built of a generally representative highway bridge and of a standard type heavily laden two axle truck.

The bridge model consisted of two pieces of steel beam fastened together by screws in such a manner as to introduce a camber equal to the total dead load deflection. The required mass of the model was obtained by hanging cylindrical weights uniformly along the span. The vehicle model consisted of an aluminum chassis supported on two axles. The chassis in turn supported a cantilever beam of spring steel to simulate the springs and body of the prototype. The vehicle was pulled across the beam by means of a synchronous motor with a master speed control.

Runs were made at three different variations of vehicle velocity, frequency, damping and initial oscillation. They were so arranged that data could be analyzed for one parameter alone, while others were kept constant.

The mid-span motions were measured by use of a differential transformer and the oscillations of the vehicle spring were recorded by means of a strain gage. Tracings of each crossing were produced in a Sanborn Recorder.

From this study it was concluded that: (1) The magnitude of the initial oscillation has an almost linear effect on bridge amplitude and deflection; (2) The effects of changing frequency and velocity are noticeable, but no simple relationship existed; (3) The larger the damping in a vehicle, the smaller amplitude and deflection will exist in a bridge; (4) A resonant vehicular frequency will cause the bridge to vibrate predominantly at same frequency as the vehicle.

Cambridge 39, Massachusetts
January, 1956

Professor Leicester F. Hamilton
Secretary of The Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In partial fulfillment of the requirements for the degree of Master of Science, I herewith submit a thesis entitled, "Model Investigation of Effects of Vehicular Vibration on Two-Span Bridges".

Respectfully submitted,

Signature redacted

George C. Tso

ACKNOWLEDGMENTS

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NOTATION

E = Modulus of Elasticity

I = Moment of Inertia with Reference to Natural Axis

L = Span Length

g = Acceleration of Gravity

W = Weight per Unit Length of Span

p = Natural frequency of Bridge in Cycle per Second

ω = Natural Frequency of Vehicle in Cycle per Second

v = Velocity

l = Wheel Base of a Two-axle Vehicle

P = Load Applied

r = Subscript Indicating Ratio of Model to Prototype

A_{max} = Maximum Amplitude of Oscillation

Δ_{max} = Maximum Deflection of the Bridge at Mid-Span

Δ_{st} = Static Deflection of the Bridge at Mid-Span

Z = Damping Coefficient of Vehicle where $A_n = A_o e^{-2\pi n z}$

A_o = Initial Amplitude

A_n = Amplitude of the Nth Cycle

TABLE OF CONTENTS

I. INTRODUCTION

A. History

As early as 1847 the problem of vibration attracted attention, when a British Commission was appointed to study the dynamic effects in railway bridges. The differential equation of deflection was established by neglecting the inertia of the bridge and by considering the moving load as a concentrated mass[1]. An exact solution of the equation was obtained by Sir G.G. Stokes[2].

A later and more realistic approach was made by A.N. Kryloff[3], by assuming that the mass of the constant load was negligible in comparison with the mass of the beam. The next step was made by S.P. Timoshenko[4], who solved the problem of a pulsating force moving with uniform velocity along a beam. Other work has been done, including that of A.N. Lowan[5], who studied the case in which the velocity of the traversing force is not constant, but the most comprehensive study is that of Sir C.E. Inglis[6,7]. The foregoing investigations were all limited

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1. "Appendix to the Report of the Commissioners to Inquire into the Application of Iron to Railway Structures", by R. Willis, London, England, 1849.
2. "Discussion of a Differential Equation Relating to the Breaking of Railway Bridges", by Sir G.G. Stokes, Math. Physics Paper, Vol. 2, P. 179, 1849.
3. "Uber der erzwungenen Schwingungen von gleichformigen elastischen Staben", by A.N. Kryloff, Mathematische Annalen, Vol. 61, P. 211, 1905.
4. "On the Forced Vibration of Bridges", by S.P. Timoshenko, Philosophical Magazine, Vol. 43, P. 1018, 1922.
5. "On Transverse Oscillations of Beams Under the Action of Moving Variable Loads", by A.N. Lowan, Philosophical Magazine, Vol. 19, Series 7, P. 708, 1935.
6. "Theory of Transverse Oscillations in Girders and its Relation to Live Load and Impact Allowances", by Sir C.E. Inglis, Proceedings of the Institution of Civil Engineers, London, England, Vol. 218, P. 225, 1924.
7. "A Mathematical Treatise on Vibrations in Railway Bridges", by Sir C.E. Inglis, Cambridge University Press, 1934.

to single-span structures.

In the past few years the problem of bridge vibration has re-ignited the interest of engineers. This is, to a large extent, due to the increased use of more flexible structures in which the dynamic effects of moving loads are more important.

B. Theoretical Solutions

Today, solutions to the problem are still being sought, with attempts being made to correlate experimental and theoretical solutions. Professor J.M. Biggs, at the Massachusetts Institute of Technology has conducted an investigation of the problem of bridge vibration and so far has developed a formula. In his solution, the motion produced by an alternating load of negligible mass was considered. The solution for a simple span is represented by:

$$
\begin{aligned}
Y \doteq \frac{PL^3}{EI\pi^4}\frac{1}{\sqrt{1-S^2}}\left(e^{-Spt}\right)\Big\{&[A_1 + A_2 + A_3 + A_4]\sin\sqrt{1-S^2}\,pt \\
&+ [B_1 + B_2 + B_3 + B_4]\cos\sqrt{1-S^2}\,pt\Big\} \\
+\frac{PL^3}{EI\pi^4}\frac{1}{\sqrt{1-S^2}}\left(e^{-\omega Zt}\right)\Big\{&C\cos[(m+\omega)t+\phi] \\
&+ D\cos[(m-\omega)t-\phi] + F\sin[(m+\omega)t+\phi] \\
&+ G\sin[(m-\omega)t-\phi]\Big\}
\end{aligned}
$$

where P = total magnitude of alternating force

L = span length

t = time in seconds

S = damping coefficient of the bridge

Z = damping coefficient of alternating force

p = natural frequency of the bridge in radians/sec.

ω = natural frequency of the alternating force in radians/sec.

$m = \frac{\pi v}{L}$

ϕ = phase angle, where $\cos(\ t + \phi) = 1$ when P is a maximum

The constants are further defined as:

$$A_1 = \frac{-(\frac{\omega}{p} Z - S) \sin \phi - (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p}) \cos \phi}{(\frac{\omega}{p} Z - S)^2 + (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p})^2}$$

$$A_3 = \frac{-(\frac{\omega}{p} Z - S) \sin \phi - (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p}) \cos \phi}{(\frac{\omega}{p} Z - S)^2 + (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p})^2}$$

$$A_3 = \frac{+(\frac{\omega}{p} Z - S) \sin \phi - (\frac{m}{p} - \sqrt{1-S^2} - \frac{\omega}{p}) \cos \phi}{(\frac{\omega}{p} Z - S)^2 + (\frac{m}{p} - \sqrt{1-S^2} - \frac{\omega}{p})^2}$$

$$A_4 = \frac{-(\frac{\omega}{p} Z - S) \sin \phi - (\frac{m}{p} - \sqrt{1-S^2} + \frac{\omega}{p}) \cos \phi}{(\frac{\omega}{p} Z - S)^2 + (\frac{m}{p} - \sqrt{1-S^2} + \frac{\omega}{p})^2}$$

$$B_1 = \frac{-(\frac{\omega}{p} Z - S) \cos \phi + (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p}) \sin \phi}{(\frac{\omega}{p} Z - S)^2 + (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p})^2}$$

$$B_2 = \frac{-(\frac{\omega}{p} Z - S) \cos \phi - (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p}) \sin \phi}{(\frac{\omega}{p} Z - S)^2 + (\frac{m}{p} + \sqrt{1-S^2} - \frac{\omega}{p})^2}$$

$$B_3 = \frac{+(\frac{\omega}{p} Z - S) \cos \phi + (\frac{m}{p} - \sqrt{1-S^2} - \frac{\omega}{p}) \sin \phi}{(\frac{\omega}{p} Z - S)^2 + (\frac{m}{p} - \sqrt{1-S^2} - \frac{\omega}{p})^2}$$

$$B_4 = \frac{(\frac{\omega}{p}Z - S)\cos\phi - (\frac{m}{p} - \sqrt{1-S^2} + \frac{\omega}{p})\sin\phi}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} - \sqrt{1-S^2} + \frac{\omega}{p})^2}$$

$$C = \frac{\frac{\omega}{p}Z - S}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} - \sqrt{1-S^2} + \frac{\omega}{p})^2} - \frac{\frac{\omega}{p}Z - S}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p})^2}$$

$$D = \frac{\frac{\omega}{p}Z - S}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} - \sqrt{1-S^2} - \frac{\omega}{p})^2} - \frac{\frac{\omega}{p}Z - S}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} + \sqrt{1-S^2} - \frac{\omega}{p})^2}$$

$$F = \frac{\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p}}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} + \sqrt{1-S^2} + \frac{\omega}{p})^2} - \frac{\frac{m}{p} - \sqrt{1-S^2} + \frac{\omega}{p}}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} - \sqrt{1-S^2} + \frac{\omega}{p})^2}$$

$$G = \frac{\frac{m}{p} + \sqrt{1-S^2} - \frac{\omega}{p}}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} + \sqrt{1-S^2} - \frac{\omega}{p})} - \frac{\frac{m}{p} - \sqrt{1-S^2} - \frac{\omega}{p}}{(\frac{\omega}{p}Z - S)^2 + (\frac{m}{p} - \sqrt{1-S^2} - \frac{\omega}{p})^2}$$

Note that the first part of the equation represents the free vibration of the bridge, and the second term represents the forcing vibration.

Another solution, developed by A.N. Kryloff[3], considers that a vehicle of negligible mass, moving across a simple span at constant velocity. The solution is in following form:[8]

$$y = \frac{2PL^3}{EI\pi^4}\left\{\sum_{i=1}^{\infty} \frac{\sin\frac{i\pi x}{L}}{i^2\left[(i^2-\alpha^2)^2+(2Si\alpha)^2\right]}\sin(imt-\Psi_i) + \sum_{i=1}^{\infty}\frac{\sin\frac{i\pi x}{L}}{i^2\left[(i^2-\alpha^2)^2+(2Si\alpha)^2\right]^{\frac{1}{2}}}\frac{\alpha}{i}\frac{1}{\sqrt{1-S^2}}e^{-Sp_it}\sin\left(\sqrt{1-S^2}\,p_it-\Phi_i\right)\right\}$$

where y = deflection of beam at point x from the end of span

p = magnitude of constant force

W = weight per unit length of beam

L = Span

t = time in seconds

$\alpha = \frac{\pi v}{P_i L}$

S = damping coefficient of beam

$m = \frac{\pi v}{L}$ = forcing circular frequency

$P_i = \frac{i^2\pi}{L^2}\sqrt{\frac{EIg}{W}}$ = natural circular frequencies

$\Psi_i = \tan^{-1}\left[\frac{2S\alpha i}{i^2-\alpha^2}\right]$

$\Phi_i = \tan^{-1}\left[\frac{-\sin\Psi_i}{\frac{S}{\sqrt{1-S^2}}\sin\Psi_i - \frac{\alpha}{i_s}\cos\Psi_i}\right]$

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8. Biggs, J.M. and Suer, H.S., "Bridge Vibration", Progress Report No. 1, Cambridge, Mass., M.I.T., September, 1954.

The first series in the above equation represents the forced vibration and for practical velocities of the constant force the series reduces to an expression giving essentially the static deflection of the beam, that is, the deflection which would be produced by the load if it moved very slowly across the beam. The only dynamic effect then is given by the second series in the equation, which represents the free vibrations of the beam which are excited at the beginning of motion.

Theoretically then, the vibrations induced in a simple span bridge by an actual vehicle can be computed by dividing the forces applied by the vehicle into two parts - a constant force and an alternating force - solving each of the two equations given above and superimposing the results.

The foregoing investigations were all limited to simple-span bridges. Work has been carried on under Professor Biggs at the Massachusetts Institute of Technology to expand the solutions into multi-span bridges.

C. Objectives

The purpose of present investigation has been then to study experimentally the dynamic effects of a moving vehicle on two-span bridges. The specific objectives were as follows:

1. To design and construct a dynamically similar model to a typical two-span highway bridge.

2. To improve the damping system on the model vehicle, developed by R.M. Connell and R.J. Lamp[9] in their thesis work in partial fulfillment of the requirements for the degree of Master of Science.

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9. "Model Investigation of Effects of Vehicular Vibrations on Simple Span Bridge", Connell, R.M. and Lamp, R.J., Unpublished Thesis, Cambridge, Mass., M.I.T., 1955.

3. To ascertain the effect on the bridge vibrations of varying the natural frequency of the vehicle.

4. To ascertain the effect on the bridge vibrations of varying the velocity at which the vehicle crossed the span.

5. To ascertain the effect on the bridge vibrations of varying the magnitude of the alternating force.

6. To ascertain the effect on the bridge vibrations of varying the damping characteristics of the vehicle.

D. Scope

This study is an attempt to determine the effects of vehicle on bridge vibration that is objectionable to persons on the structure. Mid-span deflections and amplitudes of a dynamic nature due to a single-moving vehicle were taken. In keeping with the purpose, the dynamic stresses were not measured.

II. DESCRIPTION OF TEST APPARATUS

A. Prototype Bridge and Vehicle

The representative bridge selected was the Shirley Road Bridge on Concord-Westminster Highway, Massachusetts, a two-span, stringer type bridge with reinforced concrete pavement. The data are as follows:

Total Length (two at 73'-2")	146'-4"
Weight of Bridge	5768 lbs/ft.
Computed Composite EI	5.89×10^{12} lb.-in.^{2}

Use

$$f_i = (n_i)^2 \sqrt{\frac{EI\pi^2 g}{4L^4 W}} \quad [10]$$

10. "Natural Frequencies of Continuous Beams of Uniform Span Length", Ayre, R.S. and Jacobson, L.S., Journal of Applied Mechanics, Vol. 17, September, 1950.

where fi = Natural frequency of the ith mode in cps.

$n_1 = 1; \; n_2 = 1.250; \; n_3 = 2.000$

Substituting the computed composite EI in the equation, the natural frequencies of the bridge are found as follows:

Fundamental Mode	4.43 cps
Second Mode	6.91 cps
Third Mode	17.74 cps

The vehicle selected was a two-axle (six-wheel) Sterling-White 10 ton dump truck. It was tested by H.S. Suer in 1954.[11]

The data are as follows:

Wheel base of Vehicle	13 feet
Gross Load of Vehicle	43,165 lbs.
Load on Rear Axle	35,085 lbs.
Natural Frequency of Vehicle	3.1 cps

B. Dynamic Similarity

Models were constructed to operate dynamically similar to the given prototypes. The problem of providing this similarity was solved by dimensional analysis. The π-Theorem[12] was used in this case. The following dimensionless parameters were produced:

$$1 = \frac{p}{\omega}; \quad 2 = \frac{Lp}{v}; \quad 3 = \frac{p}{W}; \quad 4 = \frac{L}{l}; \quad 5 = \frac{L^3 W p^2}{EIg}$$ [13]

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11. "Dynamic Response of Simple Span Highway Bridges to Moving Vehicle Loads", H.S. Suer, Unpublished Thesis, Cambridge, Mass., M.I.T., 1955.
12. "Model Experiments and the Forms of Empirical Equations", Buckingham, Edgar, Transactions, A.S.M.E., Vol. 37, P. 263, 1915.
13. "Model Investigation of Effects of Vehicular Vibrations on Simple Span Bridges", Connell and Lamp, C.E. Thesis, M.I.T., Appendix, P. 77, 1955.

The model vehicle was first constructed. It had the following dimensions:

$l = 7.8$ inches

$P = 5.791$ lbs.

The parameters produced the following model bridge dimensions:

$L = 87.78$ inches

$W = 113.3$ lbs. or 15.49 lbs/ft.

Now, if the following ratio was selected for convenience:

$$\frac{\text{Frequency of Model Bridge}}{\text{Frequency of Prototype Bridge}} = Pr = 1$$

the parameters also fixed the following dimensions:

$$\left(\frac{W}{EI}\right)_r = 8000 \qquad \text{or} \qquad I = .00329 \text{ in.}^4$$

and these controlling ratios:

$$v_r = 1/20 \qquad\qquad \omega_r = 1$$

It should be noted that neither the velocity (v) nor the natural frequency of the vehicle (ω) could be fixed, since they would be varied during the tests.

C. Model Vehicle

The model was first designed and constructed by Richard M. Connell and Russell J. Lamp[13]. A complete description of the model is contained in their thesis[13] and hence only a brief description will be given herein.

The vehicle consisted of an aluminum chassis supported on two axles. The chassis in turn supported a cantilever beam of spring steel with a steel mass at its end. This arrangement was intended to simulate the springs and body of the prototype. The spring mass represented approximately 85% of the total weight. Originally, at the rear of the chassis a friction device was constructed to produce damping of the vehicular vibrations. The device consisted of a small wheel which was made to ride against the steel mass by means of a stretched coil spring. The amount of damping was controlled by changing the tension in the coil spring. This method was proved to be unsatisfactory during test runs. First, it did not provide a constant damping effect to simulate the condition of prototype. Second, the control of amount of damping by adjusting the tension in the coil spring was crude and hard to work with during tests. In this study, a new damping device was called for. See Figures 1 and 2.

1. Damping Device

A new damping device was designed and constructed to fulfill the need. It consisted of a dash-pot, filled with silicon oil. The dash-pot was placed in the rear of vehicle by means of a bracket attached to the vehicle chassis. A rubber tube was wrapped around the dash-pot to increase the hold.

A horizontal rod cantilevered out from the steel mass. Another rod was hung vertically from the horizontal rod. The rods were so fitted together that the vertical rod could rotate freely and remained vertical, when the steel mass was being sprung. At the end of vertical rod, a circular disc was attached, which submerged into the silicon oil, thereby furnishing a viscous damping of the vehicular vibrations. Silicon oil was used because it has the property that its viscosity remains constant under normal range of temperature. By changing the size of the disc and the oil with different viscosity, any desirable amount of damping could be achieved. See Figure 3.

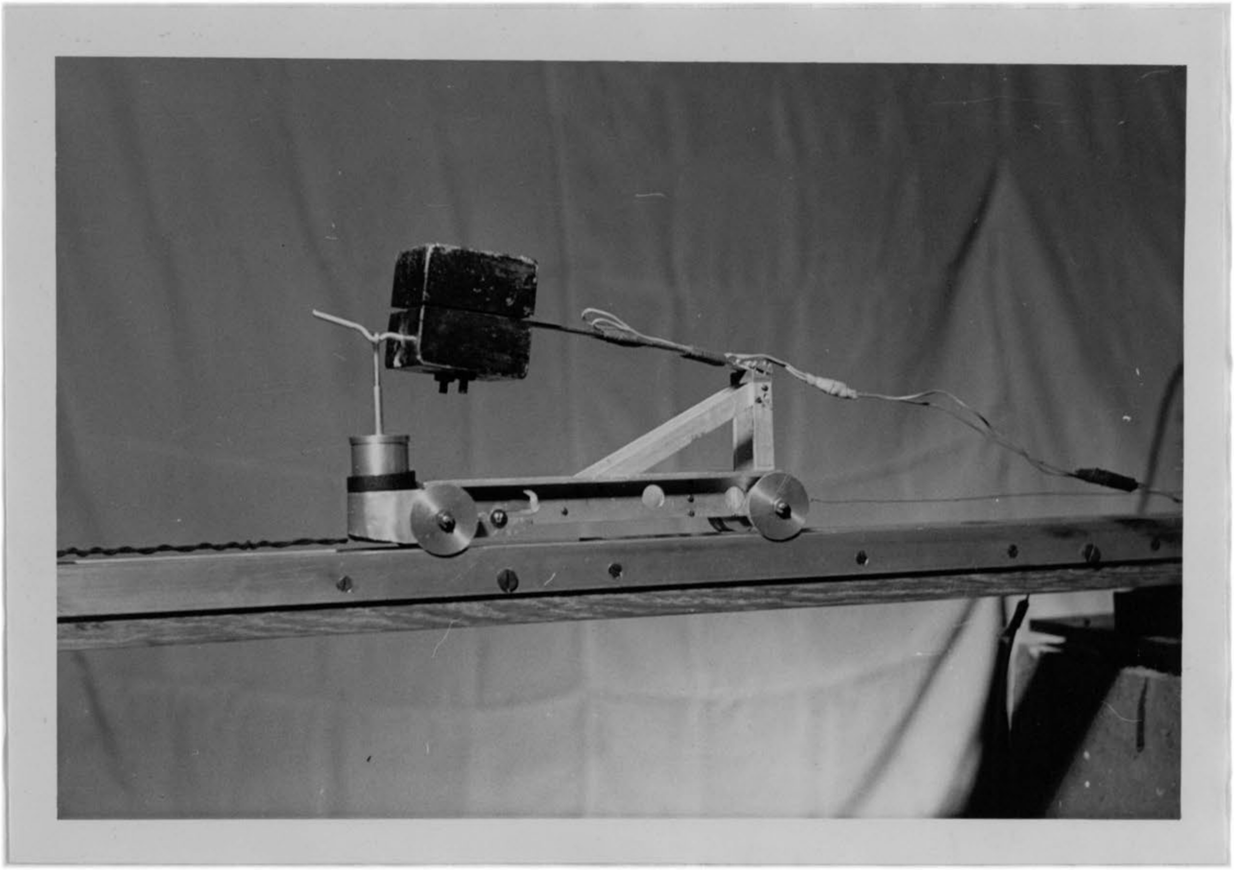
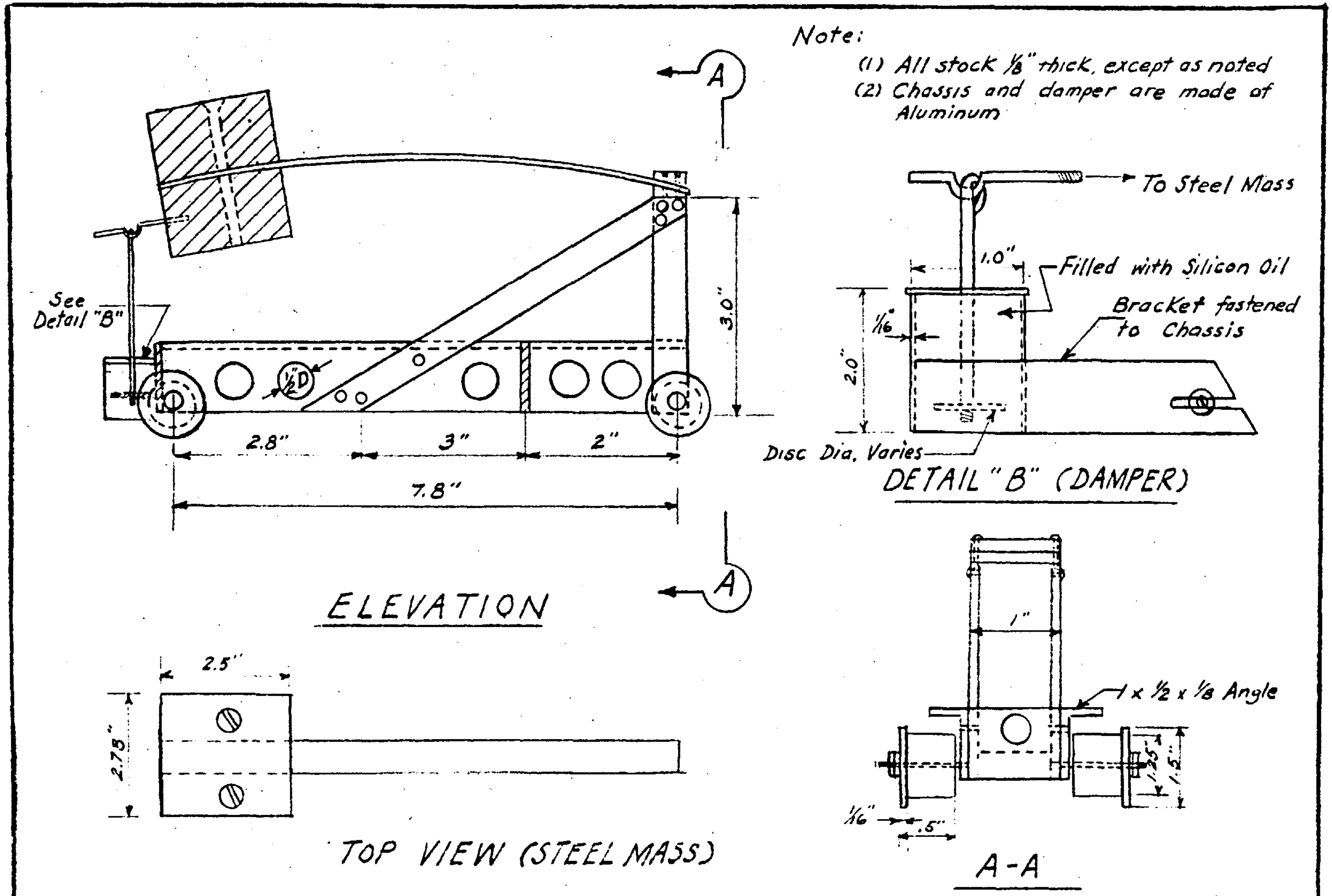


Figure 1 MODEL VEHICLE

Figure 2



PLAN OF MODEL VEHICLE (Not to Scale)

Figure 3 COMPONENTS OF DAMPING DEVICE

All parts in the damping device were made of aluminum stock. The total weight was very light compared with the total weight of the vehicle and, therefore, could be assumed negligible.

This method worked very satisfactorily during the tests. A nearly constant damping effect was achieved. Some of the typical damping curves are shown in Figure 4.

2. Springs

The natural frequency of fundamental mode of model bridge measured to be 4.54 cps and the frequency of second mode was 7.17 cps. In order to produce conditions close to resonance, one spring was to have a frequency of 4.54 cps and another to have a frequency of 7.17 cps. A third spring with frequency of 3.09 cps was used to simulate the frequency of the prototype vehicle.

Springs were all fabricated in the laboratory by trial and error, out of hacksaw blades.

D. Model Bridge

The model vehicle fixed the dimensions of the model bridge. Spacing between the wheels set the width at 2 1/2 inches. Other similitude considerations set the following dimensions:

$L = 87.78$ inches

$W = 113.3$ lbs. $\doteq 15.49$ lbs./ft.

$I = .00329$ in.^{4}

The bridge was composed of two stocks, one 2 1/2 in. x 3/16 in., and the other 1 1/4 in. x 3/32 in. They were fastened together by screws in such a manner as to introduce a camber equal to the total design load deflection. Thus, the bridge would remain level when it was turned over and loaded with the design load.

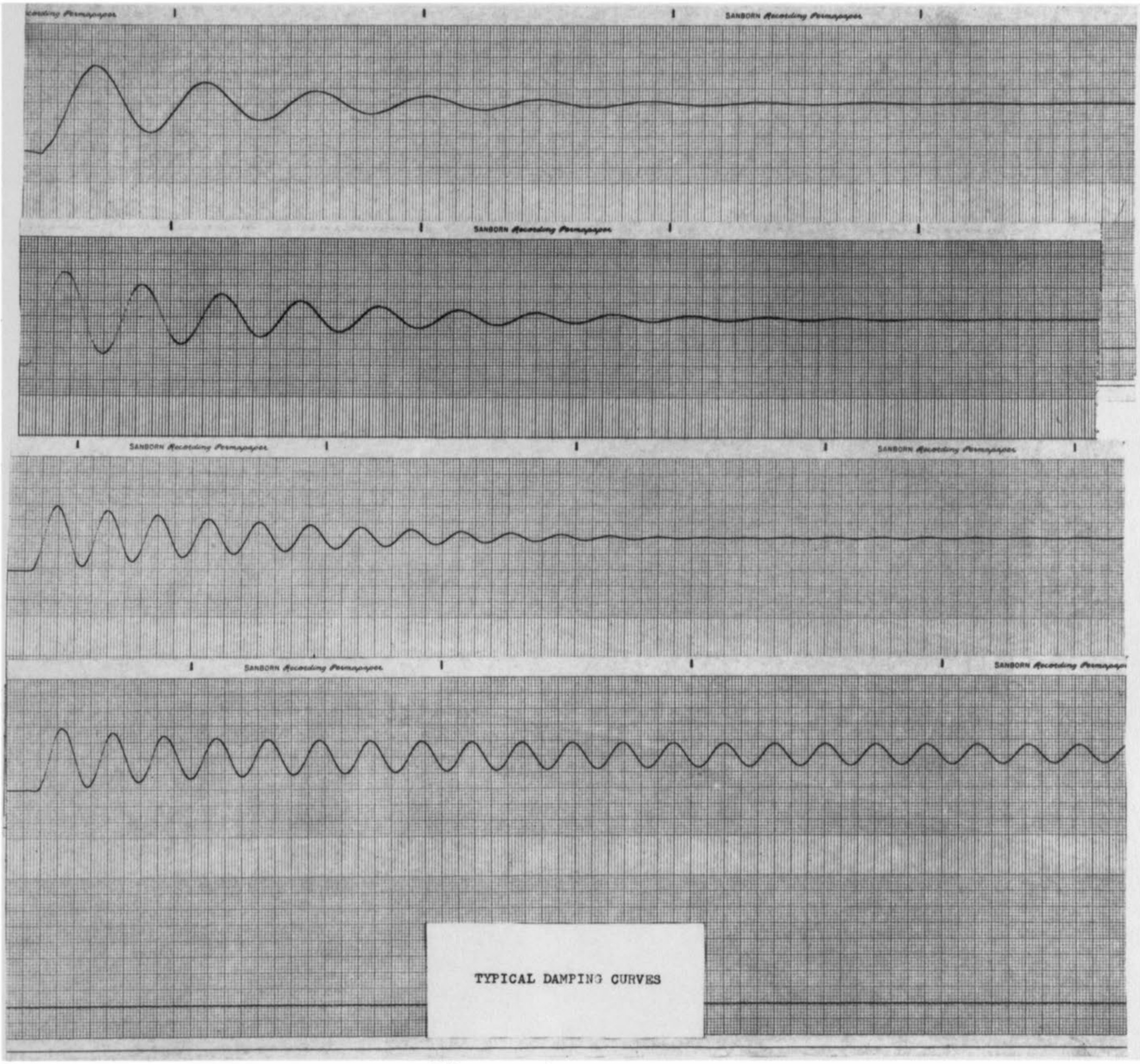


Figure 4

The cambering was accomplished by tapping holes in the 2 1/2 in. stock first for later fastening, as shown in Figure 5a. The pieces were placed on the supports, 1 1/4 in. stock lay on top of 2 1/2 in. stock and loaded with cambering load, as shown in Figure 5b. Since they were not fastened together, the effective moment of inertia of the cross-section was small, hence the deflection was much larger than it would result if the pieces were fastened together.

In this deflected position, the 1 1/4 in. stock was center punched (using the drilled holes in 2 1/2 in. stock for guides), the holes drilled, and the beams fastened together with screws. In so doing, the effective moment of inertia was greatly increased, hence when the load was released the beam rebound a lesser amount as shown in Figure 5c.

Now, the beam had a chamber which equals to the design load deflection, thereby when the beam was turned over and loaded, producing a loaded level beam.

Knowing the design load and moment of inertia of the cross-section, the cambering load can be computed as follows:

Let W_c = Cambering Load

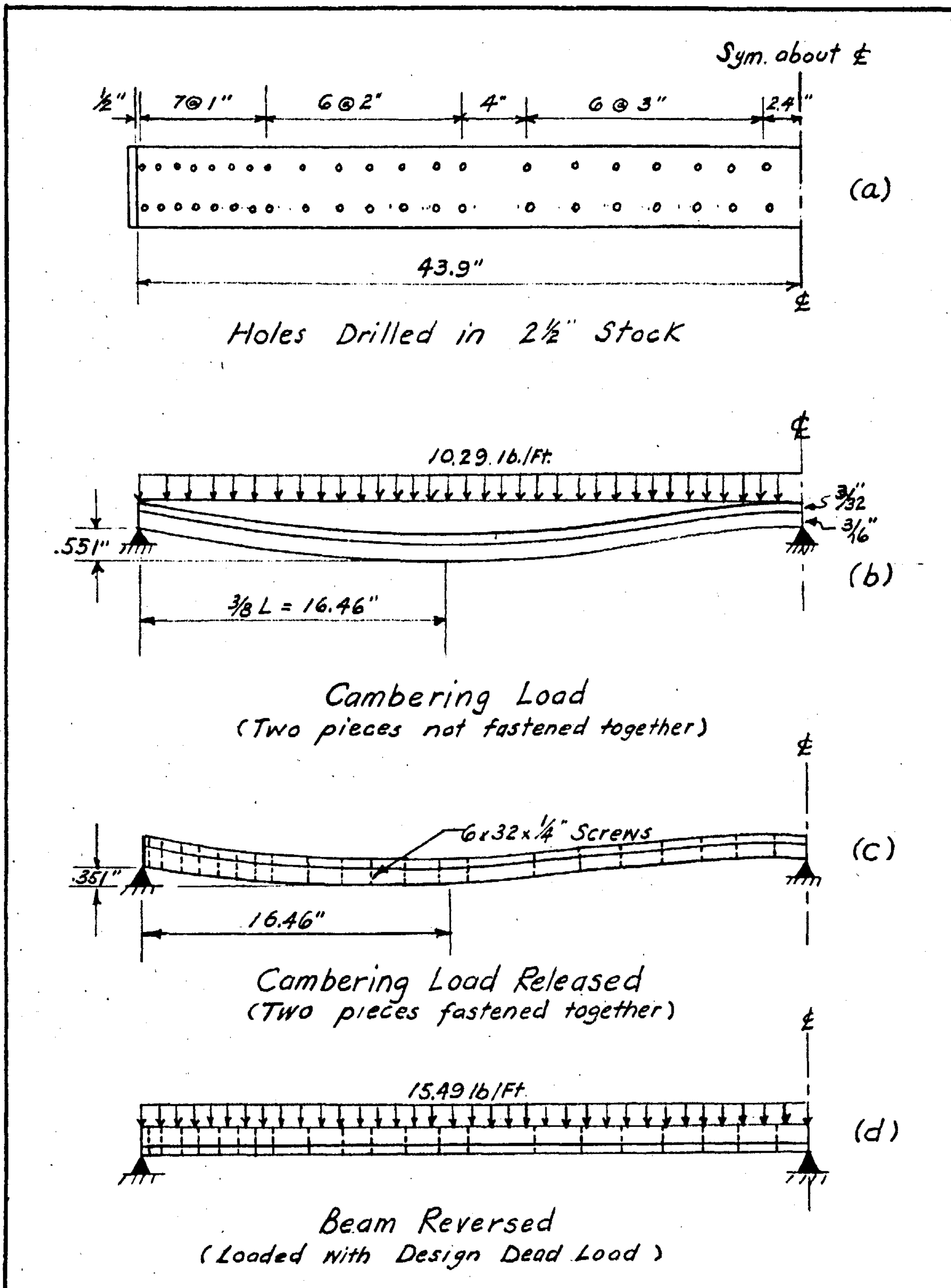
I_1 = Moment of Inertia of the cross-section when the two stocks are not fastened together.

Since the deflection is directly proportional to the load and inversely proportional to moment of inertia, the following must be true:

$$\frac{W_c}{I_1} - \frac{W_c}{I} = \frac{W}{I}$$

$$\therefore \quad W_c = \frac{W}{\left(\frac{I}{I_c} - 1\right)}$$

In the final stage, a small downward deflection existed. This was caused by difficulty in aligning the holes during the fastening operation. However, the dynamic similarity was achieved with good result. The natural



CAMBERING OF MODEL BRIDGE

Figure 5

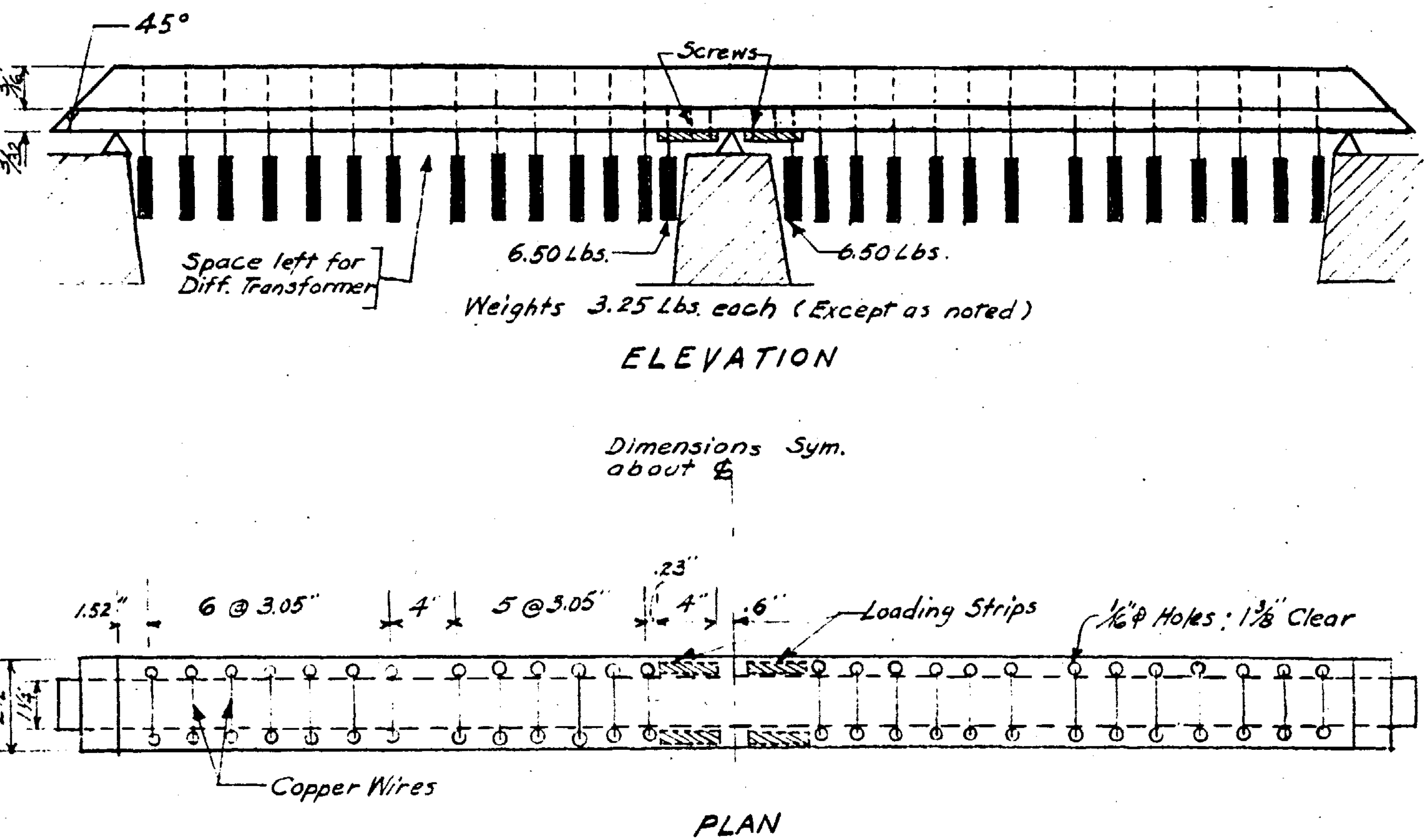
frequency was measured to be 4.54 cps at its fundamental mode and 7.17 cps at its second mode. The frequencies of the prototype for which the model bridge was designed were 4.43 cps and 6.91 cps. The percentage errors were 2.5 and 3.8 respectively. The damping of the bridge was found to be very small and could be neglected.

The dead load of the bridge consisted of the weight of the beam and thirty groups of cylindrical weights at 3.25 lbs. each. They were fastened to the bridge with copper wire. Holes, 1/16 inch in diameter were drilled across the span and the connecting wires were passed through these holes. Problem came up at the middle support where the concrete pedestal obstructed the hanging of the weight. This was solved by fastening four metal strips underneath the beam onto which weights were hung, thereby an uniform distirubution of the load was simulated along the strips. Space was also left out at the center of the spans for the instrumentation of differential transformer. See Figure 6.

The ends of the beam were cut in a 45° angle to ensure that the supports were directly underneath the road surface junction of the bridge with the approach and end sections. Figure 7 shows the complete model bridge.

E. Approach and End Tracks

The approach and end tracks provided the necessary roadway for accelerating and decelerating the vehicle. These sections consisted of two wooden beams with aluminum angles fastened along the top edge to provide a smooth roadway for the vehicle. Spacer bars were used between the aluminum angles to ensure parallel edges at 2 1/2 inches apart. The ends of the sections next to the bridge were cut on a 45° angle to provide a smooth junction at the beginning and end of the crossing.



MODEL BRIDGE LOADING
(Not To Scale)

Figure 6

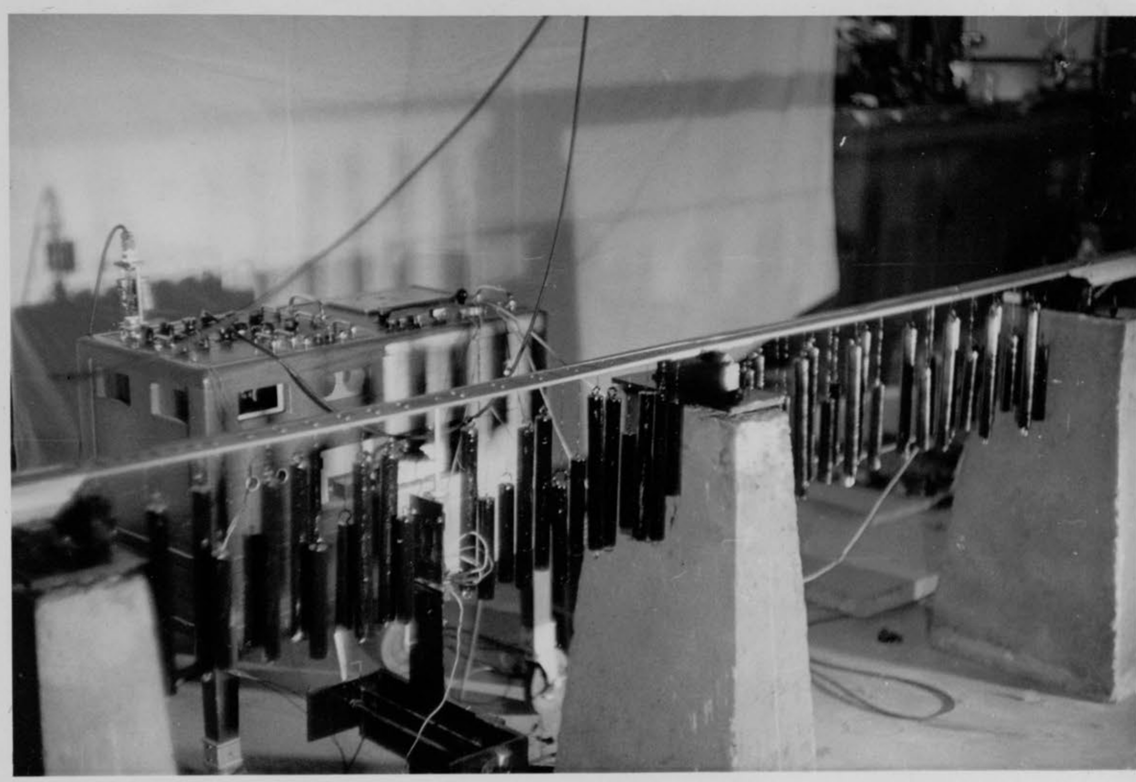


Figure 7 MODEL BRIDGE

F. <u>Set-Up</u>

The components were then assembled. (See Figure 8). The bridge was placed on three knife-edge supports resting on three concrete pedestals. The approach and end sections were volted to the pedestals at the ends of bridge to maintain alignment and level with the bridge. A gap was necessary to permit a free vibration of the bridge and at the same time to cause no discernible "bump" when the vehicle crossed the abutments. Trials showed that this was very hard to fulfill; a slight "bump" usually showed up in the tracing as an extraneous vibration of the vehicle. It was found that the trouble disappeared if the gap was filled with some bee wax. The bee wax met all three requirements; (1) The material was soft enough to ensure a free vibration; (2) It could be paved to smooth out any discontinuity in the junctions; (3) Wax does not conduct electricity to short circuit the remote time-marker.

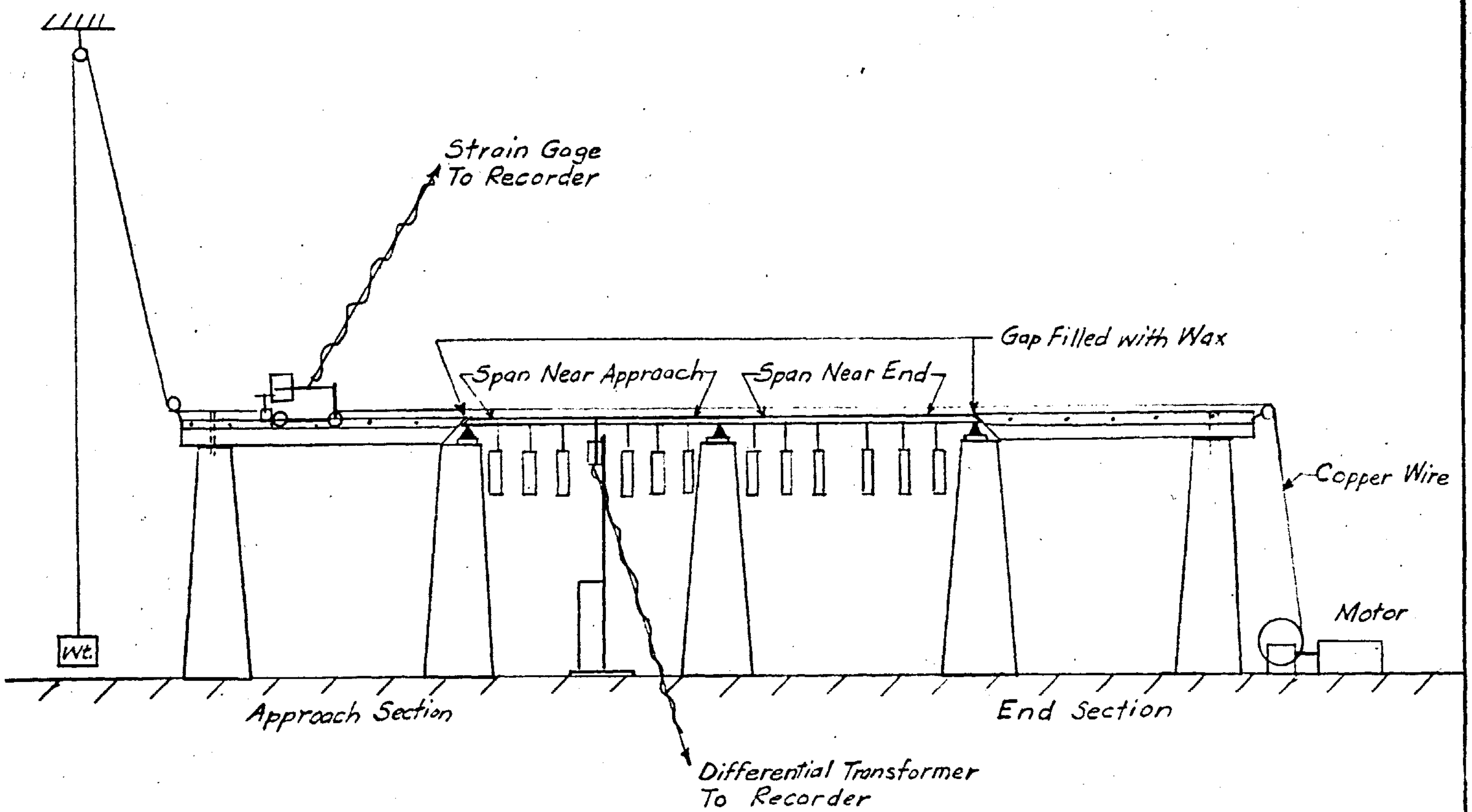
Copper wire was connected from a motor to the vehicle for pulling. Experience also showed that a line was necessary to be fastened to the rear of the vehicle and over a pulley arrangement to a weight, so that the vehicle could be kept from accelerating when going "down hill" as the bridge deflected.

G. <u>Control</u>

1. Velocity

To provide the normal range of velocities which might be expected from the prototype vehicle, three prototype velocities were chosen for testing: 20 MPH, 35 MPH, and 50 MPH. Model velocities should be: 1.0 MPH, 1.75 MPH, and 2.5 MPH respectively. A synchronous motor with a master speed control was used to obtain these velocities. Any velocity in the range from .459 MPH to 7.0 MPH could be maintained constant by

Figure 8



TEST APPARATUS SET-UP

(Not To Scale)

by the motor. A 7.5 to 1 gear reduction and a 8.5 inch pulley was attached to the output shaft to obtain the necessary direction of the pull.

2. Damping

The damping device in the vehicle permitted the change of damping effect by changing the size of the disc and silicon oil with different viscosity. Three different damping characteristics were chosen for this study: One was chosen to have as little damping as possible. The other two were having in the successive cycle approximately 80% and 50% of the amplitude of the preceeding cycle.

3. Alternation

Pieces of steel stock were cut down until two separate weights were obtained: 1.24 lbs., and 2.47 lbs., which represented respectively 25% and 50% of the sprung mass. When one of the weights was placed on top of the steel mass, it caused a certain deflection of the mass. An arm was used to hold down the mass in that position. The arm featured a notch in the lower end which fitted over the left rear axle and an angle which extended over the top of the mass. The angle was adjustable in height so that the mass might be held depressed in any position. Originally a release arm was used to extend into the path of the vehicle. As the vehicle crossed on to the span it knocked the hold-down arm free of the vehicle thus causing the vibration of the mass to begin. However, during several trial runs, it was found that the hold-down arm often fell on the junction of the approach section and bridge, thereby short-circuiting the time marker. This was improved by using a string instead of the release arm attached to the hold-down arm. One end of the string was fixed. As the vehicle was moving on to the bridge, the hold-down arm was pulled back by the string and free of the vehicle. The elasticity in the string was able to pull the arm far back off the bridge. The length of the string was so adjusted that the vehicle oscillation started just as the front,axle crossed on to the span.

4. Springs

Springs of frequency 3.09 cps, 4.54 cps, and 7.17 cps were used. All springs were fabricated out of hacksaw blades. Each spring was grinded and tested in the Sanborn recorder until it reach the right frequency.

H. Instrumentation

1. Sanborn Recorder

The experimental data was recorded by means of a Sanborn Strain Gage Amplifier (Model 64-500A) and Sanborn "Twin Viso" (Model 60) Recording system.

"The Sanborn Strain Gage Amplifier is designed to couple a bridge network containing strain sensitive elements to a Sanborn Direct Writing Recorder for the purpose of indicating the degree of unbalance of the bridge. The amplifier is of the conventional modulated carrier type in which the bridge is excited at 2500 cycles by a built-in oscillator which provides three volts of excitation voltage. The bridge output, which is proportional to the degree of unbalance, is amplified at 2500 cycles, rectified by a phase sensitive detector circuit, and the resulting DC is applied to a driver amplifier whose output is sufficient to operate the recorder."[14]

The "Twin-Viso" Model 60 Sanborn Recorder is basically a two channel direct-writing recording system with each channel operating independently of the other, but both registering phenomena simultaneously on the recording paper in true rectangular coordinates. Records are produced by a heated stylus in conjunction with heat sensitive paper. The recording paper is pulled over a sharp edge in the paper drive mechanism and the heated stylus wipes over this edge as it swings, thus producing records in rectangular coordinates.

A standard set of gears provides for a choice of paper speeds. For this test program a constant paper speed of 102 millimeters per second was used.

A separate writing arm operated electrically by remote control permits the signalling of start or finish of a test. The same writing arm is also used whenever desired to introduce one-a-second timing "pips" for a check on paper speed.

The Sanborn Recorder was used to record the magnitude and frequency of the bridge vibrations at mid-span, the magnitude and frequency of the alternating force, and the entrance and exit of the test vehicle on the bridge.

2. Strain Gage

A SR-4 strain gage, type A-11, mounted near the fixed end of the cantilever spring provided the recording of the alternating force. A dummy gage was used to complete the half bridge. The upper channel of

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14. "Sanborn Strain Gage Amplifiers", Bulletin 5a, Sanborn Company, Cambridge, Mass., January, 1954.

the recorder was used to obtain a trace of the vibration of the alternating force.

3. Differential Transformer

A Schaeritz Linear Variable Differential Transformer Type 250-SL was used to record mid-span deflections of the bridge model.

A Linear Variable Differential Transformer is a device which can provide a means of measuring mechanical motion electrically. Any physical quantity that can be converted into linear motion can thus be measured. The motion is picked up by displacement of a core in the magnetic field of a transformer.

The particular transformer used has a range of .250 in. either side of null point. The core, 1/4 in. in diameter and 3.5 in. long was fastened to a six inch length of aluminum rod which was screwed into a tapped hole at the center of the spans of model bridge. The transformer coils were supported from the floor by a rack and pinion type adjustable stand with leveling screws to obtain co-linearity of the core and coils. See Figure 9.

When using Differential Transformer with the Sanborn Strain Gage Amplifiers, it is important that the signal output from the transducer be in phase with the excitation voltage supplied by the built-in oscillator of the Strain Gage Amplifier. If not in phase, this has two effects on the system, namely: (1) sensitivity is reduced, (2) linearity is affected.

This difficulty was cured by the use of a phase correction network consisting of R and C. (See Figure 10).

The values of R and C can be found as follows:

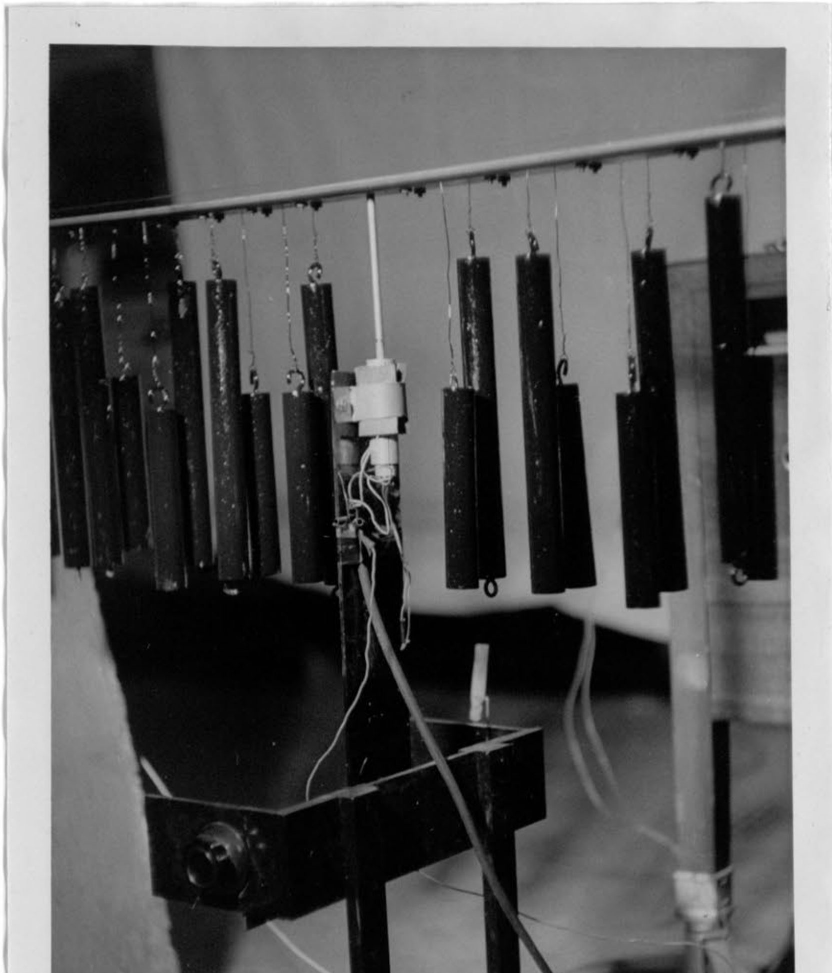

Figure 9 DIFFERENTIAL TRANSFORMER AND SUPPORT

1. Connect the transformer to the Strain Gage Amplifier. Connect the hot Y input terminal of an oscilloscope to B of the connector, the X input to D, and the scope ground to the amplifier chassis.

2. Adjust the capacity balance control on the amplifier to produce a straight line on the scope. This gets this control out of the way for the subsequent test. Also remove the plug-in resistor R3 on the amplifier panel.

3. Connect the scope Y input to pin A of the connector and observe the scope pattern as the core of the transformer is moved in and out. (Turn the amplifier attenuator to the OFF position.) The valves of R and C should be varied until the scope pattern is a horizontal straight line, and as the core is moved, the line should tilt but still be a line not an ellipse.

4. Once the phase shift network is set, replace plug-in resistor R3 and use the amplifier controls as usual.[15]

4. Remote Time Marker

A switching device, consisting of wires connected to the aluminum rails of both approach and end sections to one marker terminal, and the bridge to the other terminal, was used to signal the entrance and exit of the vehicle on the span.

As the front wheels of the vehicle crossed onto the span, the circuit was closed until the rear wheels passed onto the span. The process repeated as the vehicle moved off the span.

The entrance and exit of the vehicle were signified by the "pips" at the bottom of each trace.

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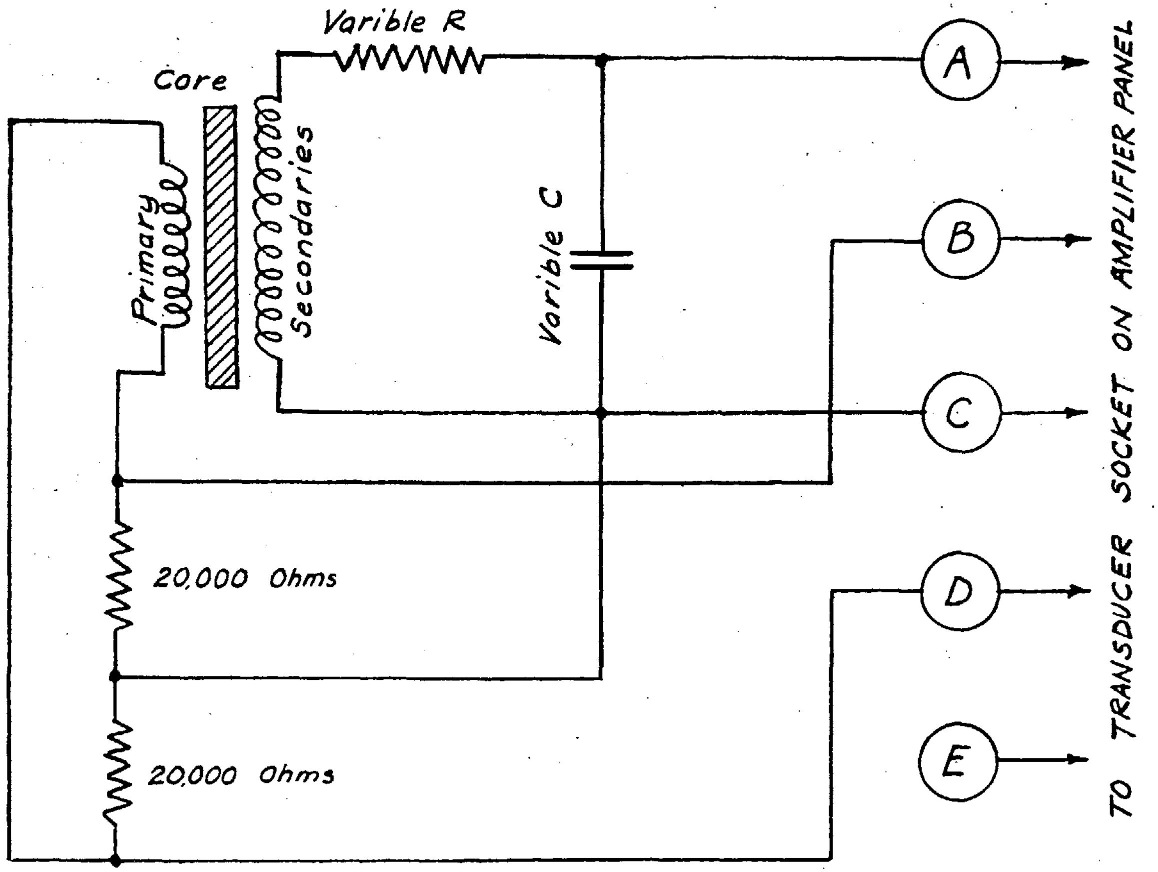
15. "The Right Angle", P. 5, Sanborn Company, Cambridge, Mass., August, 1953.

Figure 10



PHASE CORRECTION NETWORK OF DIFFERENTIAL TRANSFORMER

III. DESCRIPTION OF TESTS

A. Test Procedure

At the beginning of each day's testing, the Sanborn Recorder was "warmed" up for thirty minutes and the Resistance and Capacitance Controls were balanced. A determination of the natural frequency of the beam was made. The daily determinations were believed to be a good means to detect any changes that might have occurred in the Sanborn Recorder.

Following the determination, a short trace on the paper of the zero positions of the alternating force and the bridge was obtained. The bridge was then calibrated by the use of a weight, which when placed on a marked position on the bridge, would produce a deflection at mid-span of exactly 0.100 inch.

Next followed the calibration of the alternating force. One of the weights which represented 25% and 50% of the steel mass was placed on top of the block. The hold-down arm kept the mass in that deflected position. With the vehicle stationed at the approach track, a record trace was obtained by knocking off the hold-down arm by hand. This provided a calibration of the bridge deflection, of the alternating load, and permitted the later computation of damping coefficient. The alternating load hold-down arm was reset and the calibration weight was removed from the bridge. After closed all switches, the main trace was obtained by pulling the vehicle across the span.

A short trace of the zero positions of the alternating force and the bridge was again obtained after the vibrations had damped out. The copper wire was unwound from the pulley at the motor, the vehicle allowed to return to its starting position and was prepared for the next run.

B. <u>Description of Test Runs</u>

The span near the approach section was tested first. Three separate runs were made of the vehicle at the same velocity and spring frequency, using 0%, 25% and 50% alternation of the load. Keeping the same spring frequency, the entire procedure was repeated for the other two velocities. This made up a total of nine runs for one spring frequency, covering all variables in velocity and alternation.

The entire process was now repeated for the other two springs. This brought the total number of runs to 27, all with damping device removed (as little damping as possible). For the next six runs, the velocity was kept constant at 35 MPH (prototype) and alternation was kept constant at 25%. The vertical rod of the damping device was installed and for each of the three frequencies, runs were made with the following conditions of damping:

1. The amplitude of the successive cycle was about 50% of the preceeding one.

2. The amplitude of the successive cycle was about 80% of the preceeding one.

The differential transformer was now removed to the span near the end section. Nine runs were made under the condition that the velocity of 35 MPH (prototype) and alternation of 25% were kept constant. For each of the three frequencies, runs were made with the following conditions of damping:

1. Damping device removed (as little damping as possible).

2. The amplitude of the successive cycle was about 50% of the preceeding one.

3. The amplitude of the successive cycle was about 80% of the preceeding one.

One run was made at each span at a very slow velocity and with the spring mass blocked from oscillation. This provided curves which agree Fairly well with the static curves without any dynamic effect.

IV. RESULTS

A. Tracings

The tracings of the 44 runs are shown in Figures 11 through 25 inclusive. An index is included here for easy reference.

Fig. No.	Run No.	Span Tested	ω cps	% Alter- nation	v MPH	Damping Coeff.
11	1	Near Approach	3.09	25	20	.0027
	2	"	"	0	"	"
	3	"	"	50	"	"
12	4	"	"	"	35	"
	5	"	"	0	"	"
	6	"	"	25	"	"
13	7	"	"	"	50	"
	8	"	"	0	"	"
	9	"	"	50	"	"
14	10	"	4.54	"	"	.0056
	11	"	"	0	"	"
	12	"	"	25	"	"
15	13	"	"	"	35	"
	140	"	"	0	"	"
	15	"	"	50	"	"
16	16	"	"	"	20	"
	17	"	"	0	"	"
	18	"	"	25	"	"
17	19	"	7.17	"	35	.0065
	20	"	"	0	"	"
	21	"	"	50	"	"
18	22	"	"	"	20	"
	23	"	"	0	"	"
	24	"	"	25	"	"
19	25	"	"	"	50	"
	26	"	"	0	"	"
	27	"	"	50	"	"
20	28	"	4.54	25	35	.052
	29	"	"	"	"	.022
	30	"	3.09	"	"	.033

Fig. No.	Run No.	Span Tested	ω cps	% Alter- nation	v MPH	Damping Coeff.
21	31	Near Approach	3.09	25	35	.058
	32	"	7.17	"	"	.032
	33	"	"	"	"	.018
22	34	Near end	7.17	"	"	.0065
	35	"	"	"	"	.018
	36	"	"	"	"	.032
23	37	"	3.09	"	"	.0027
	38	"	"	"	"	.033
	39	"	"	"	"	.058
24	40	"	4.54	"	"	.052
	41	"	"	"	"	.022
	42	"	"	"	"	.0056
25	43	Static curve for span near approach section				
	44	" " " " " end "				

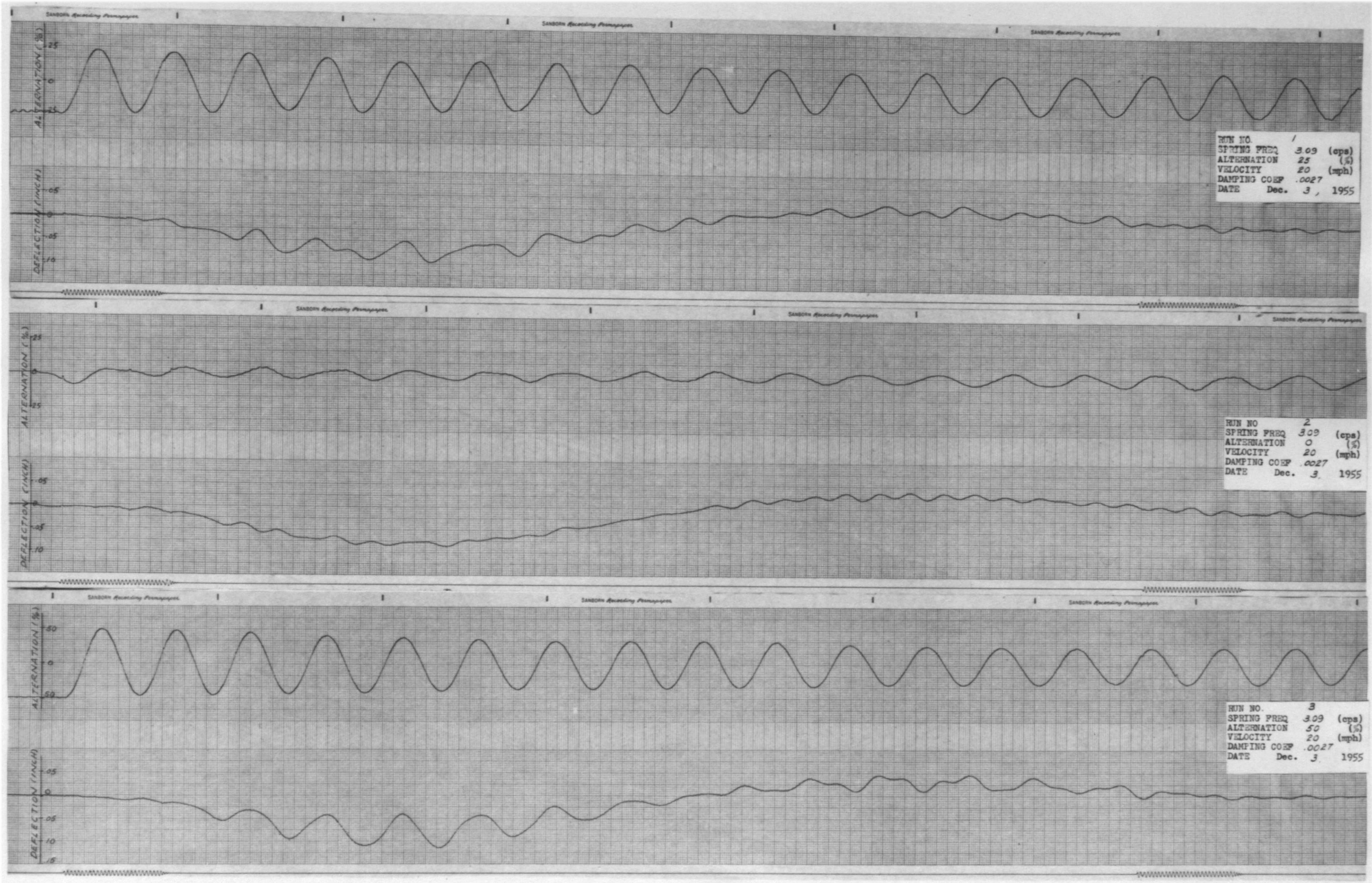

SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO. 1
SPRING FREQ 3.09 (cps)
ALTERNATION 25 (%)
VELOCITY 20 (mph)
DAMPING COEF .0027
DATE Dec. 3, 1955
RUN NO 2
SPRING FREQ 3.09 (cps)
ALTERNATION 0 (%)
VELOCITY 20 (mph)
DAMPING COEF .0027
DATE Dec. 3, 1955
RUN NO. 3
SPRING FREQ 3.09 (cps)
ALTERNATION 50 (%)
VELOCITY 20 (mph)
DAMPING COEF .0027
DATE Dec. 3 1955

Figure 11

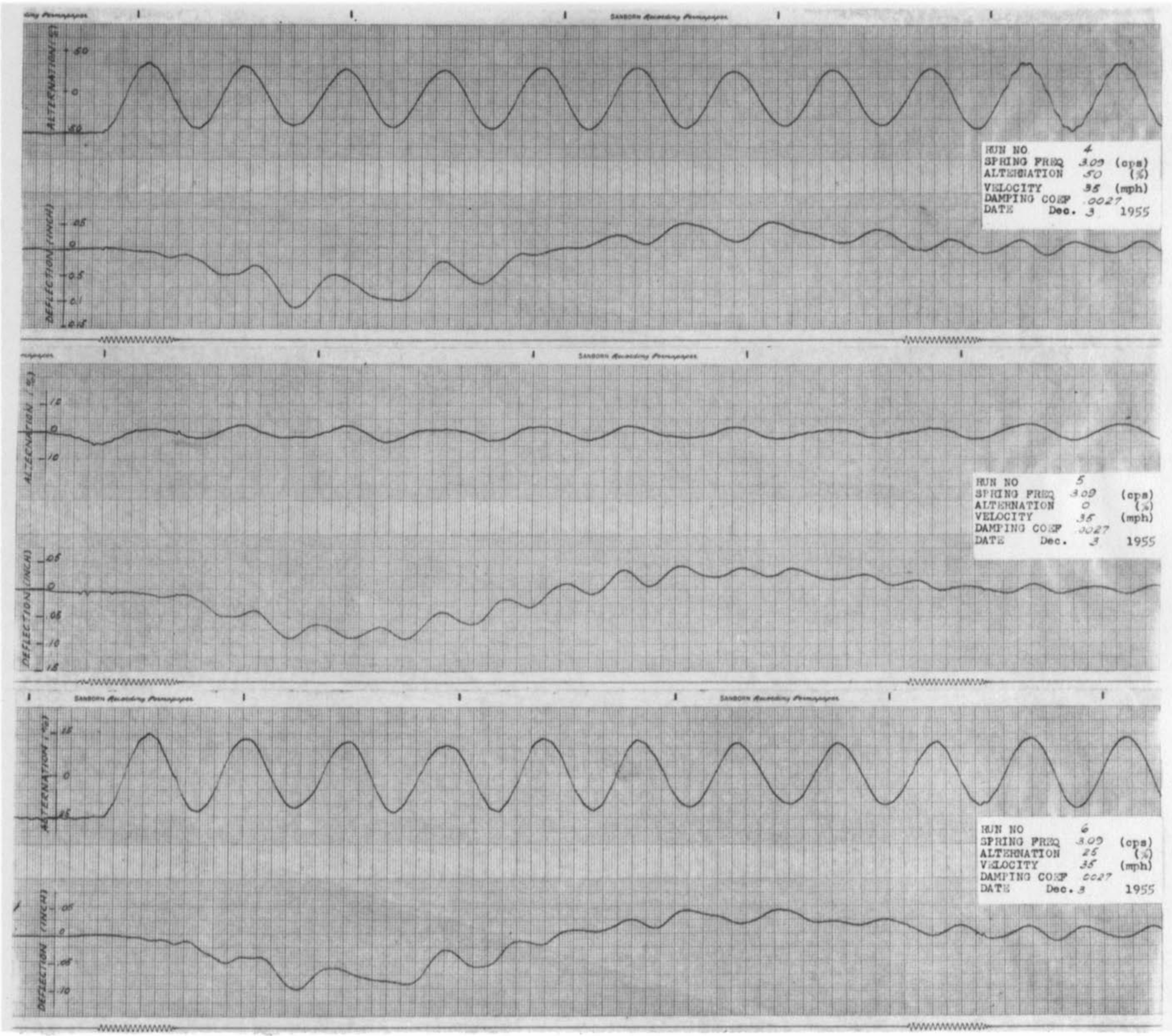

ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 4
SPRING FREQ 3.09 (cps)
ALTERNATION 50 (%)
VELOCITY 35 (mph)
DAMPING COEF .0027
DATE Dec. 3 1955
RUN NO 5
SPRING FREQ 3.09 (cps)
ALTERNATION 0 (%)
VELOCITY 35 (mph)
DAMPING COEF .0027
DATE Dec. 3 1955
RUN NO 6
SPRING FREQ 3.09 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .0027
DATE Dec. 3 1955

Figure 12

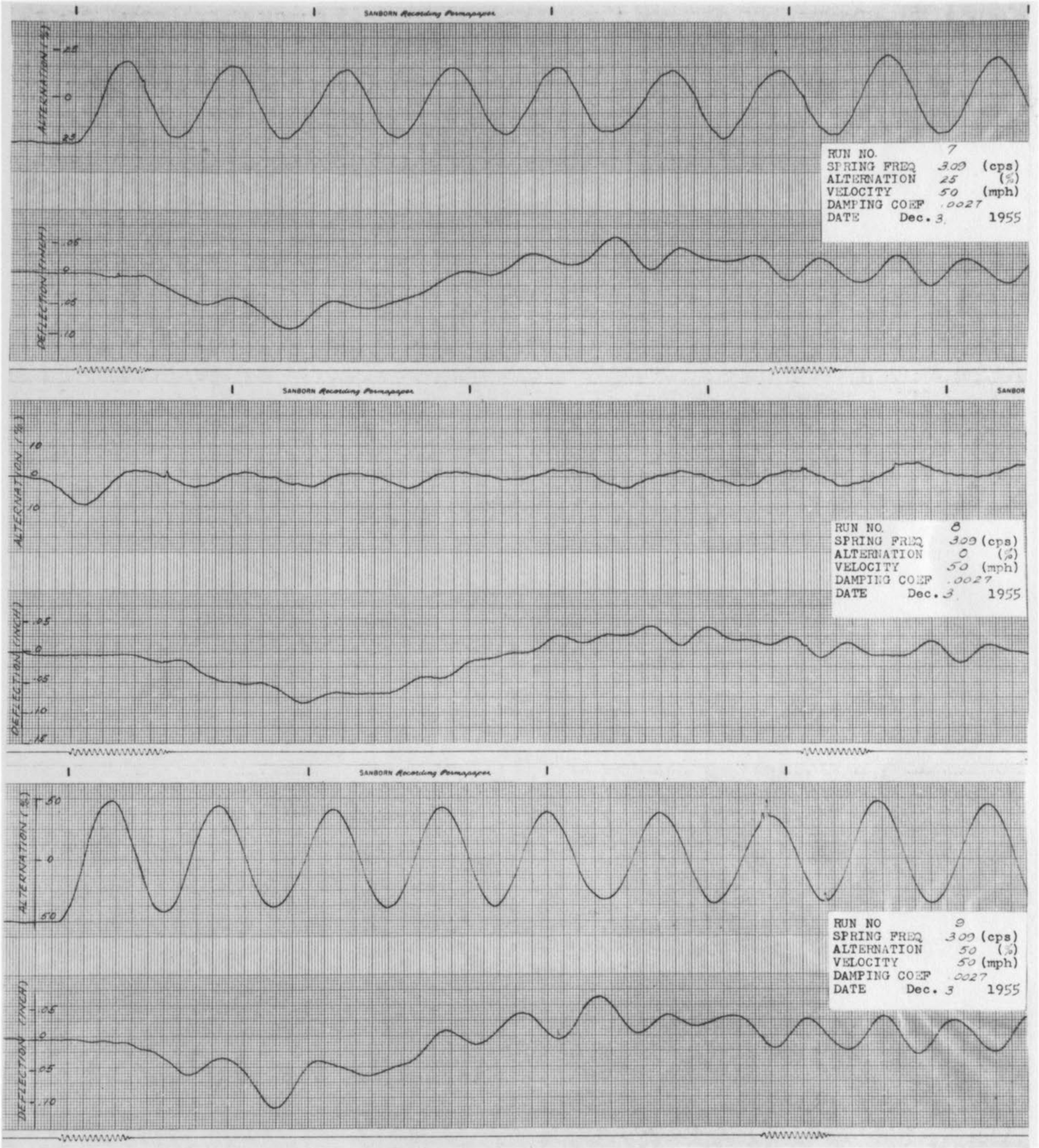

SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO. 7
SPRING FREQ 3.00 (cps)
ALTERNATION 25 (%)
VELOCITY 50 (mph)
DAMPING COEF .0027
DATE Dec. 3, 1955
SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO. 8
SPRING FREQ 3.00 (cps)
ALTERNATION 0 (%)
VELOCITY 50 (mph)
DAMPING COEF .0027
DATE Dec. 3 1955
SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 9
SPRING FREQ 3.00 (cps)
ALTERNATION 50 (%)
VELOCITY 50 (mph)
DAMPING COEF .0027
DATE Dec. 3 1955

Figure 13


SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 10
SPRING FREQ 4.54 (cps)
ALTERNATION 50 (%)
VELOCITY 50 (mph)
DAMPING COEF .0056
DATE Dec. 3 1955
RUN NO 11
SPRING FREQ 4.54 (cps)
ALTERNATION 0 (%)
VELOCITY 50 (mph)
DAMPING COEF .0056
DATE Dec. 3 1955
RUN NO 12
SPRING FREQ 4.54 (cps)
ALTERNATION 25 (%)
VELOCITY 50 (mph)
DAMPING COEF .0056
DATE Dec. 3 1955

Figure 14

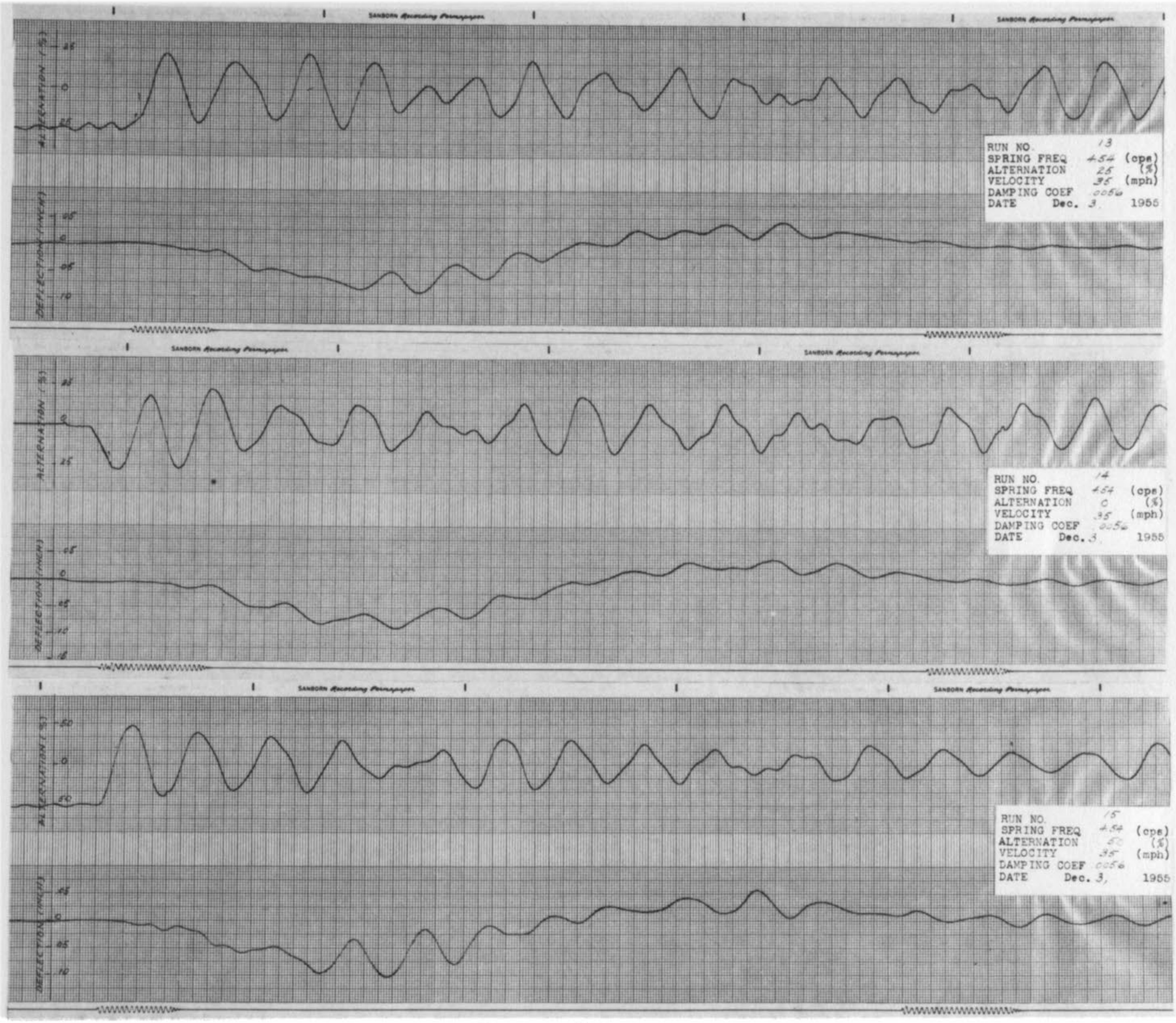

RUN NO. 13
SPRING FREQ 454 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .0056
DATE Dec. 3, 1955
RUN NO. 14
SPRING FREQ 454 (cps)
ALTERNATION 0 (%)
VELOCITY 35 (mph)
DAMPING COEF .0056
DATE Dec. 3, 1955
RUN NO. 15
SPRING FREQ 454 (cps)
ALTERNATION 50 (%)
VELOCITY 35 (mph)
DAMPING COEF .0056
DATE Dec. 3, 1955

Figure 15

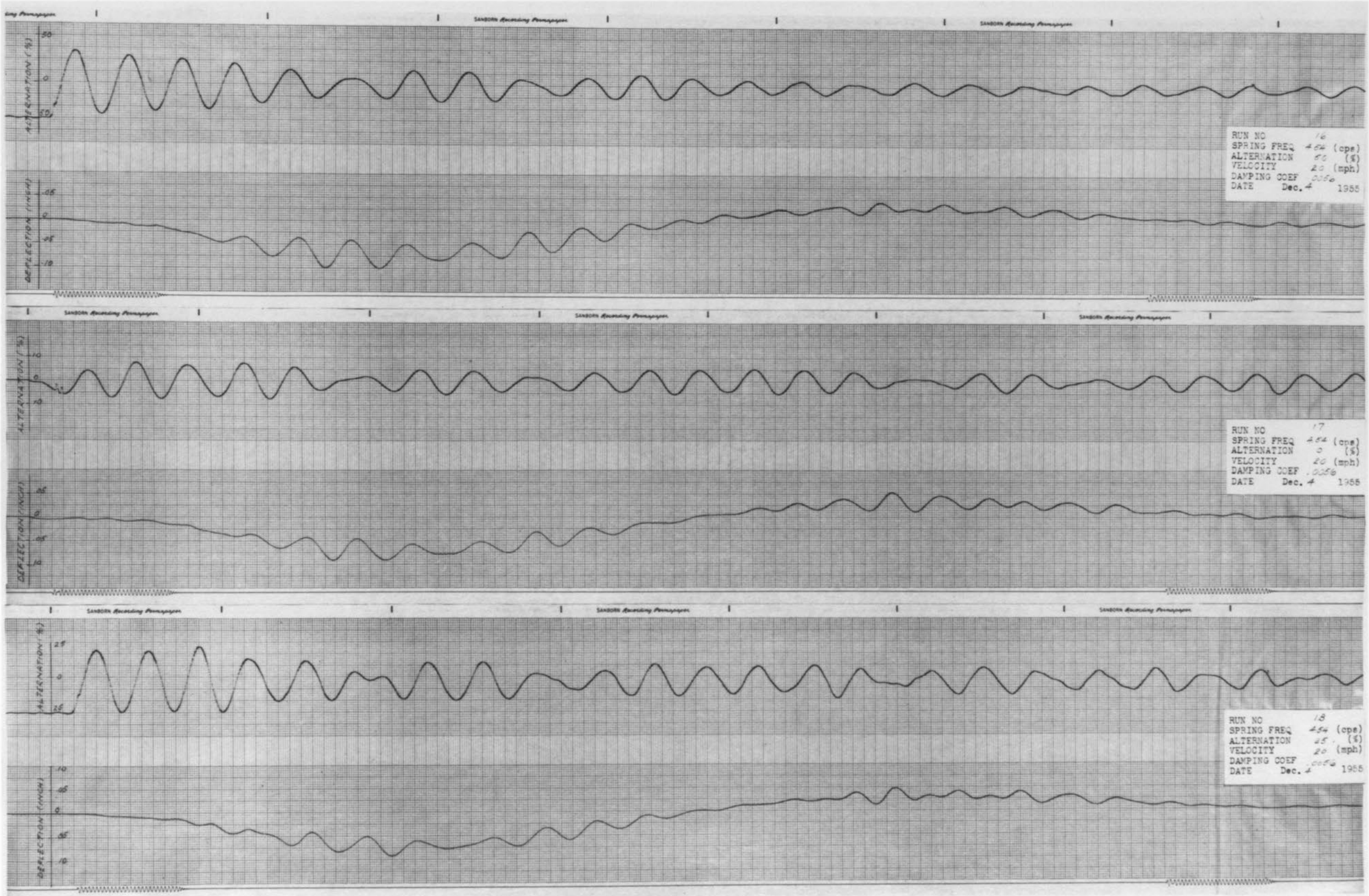
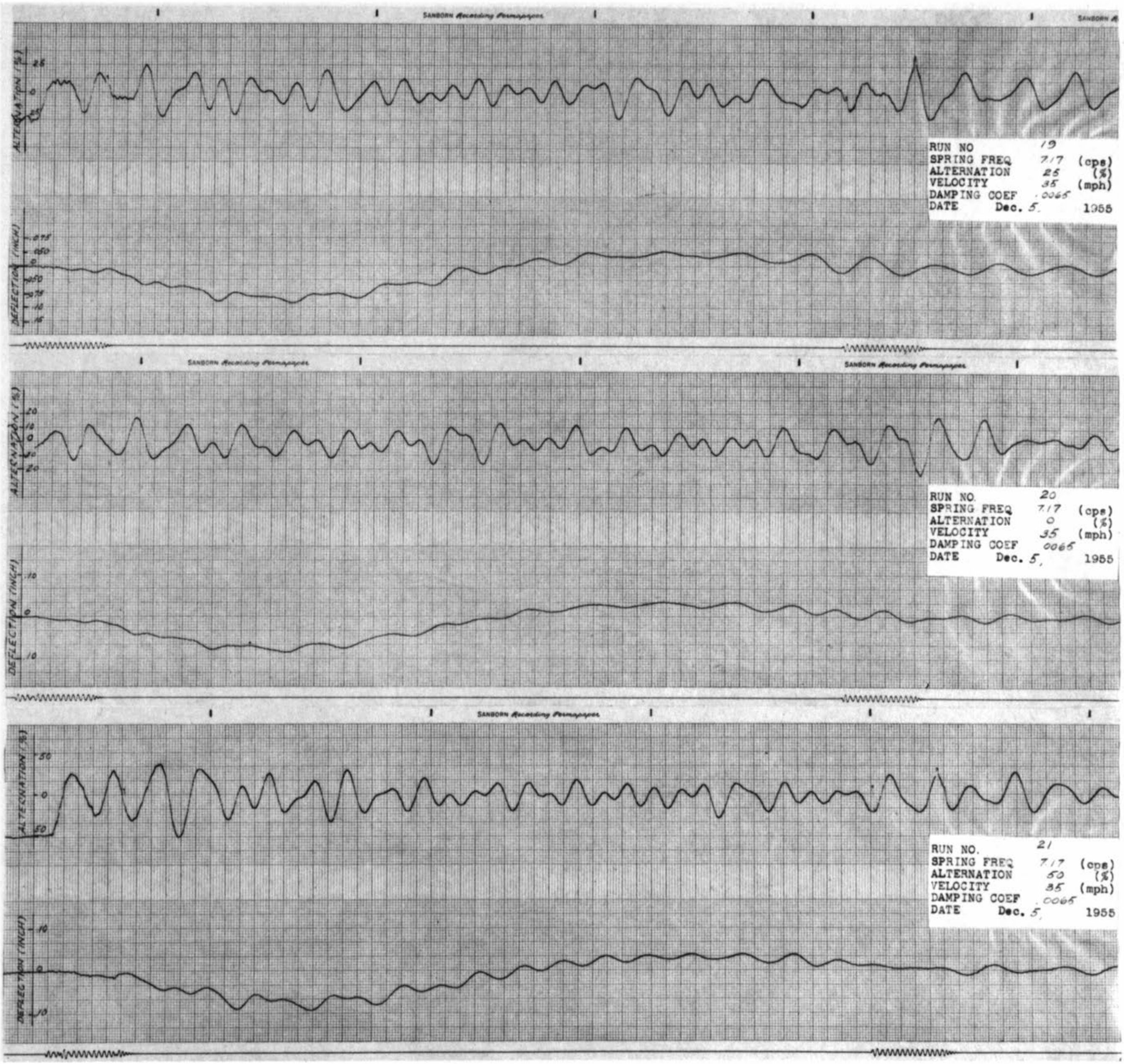

SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 16
SPRING FREQ 454 (cps)
ALTERNATION 50 (%)
VELOCITY 20 (mph)
DAMPING COEF .0056
DATE Dec. 4 1955
RUN NO 17
SPRING FREQ 454 (cps)
ALTERNATION 0 (%)
VELOCITY 20 (mph)
DAMPING COEF .0056
DATE Dec. 4 1955
RUN NO 18
SPRING FREQ 454 (cps)
ALTERNATION 25 (%)
VELOCITY 20 (mph)
DAMPING COEF .0056
DATE Dec. 4 1955

Figure 16



RUN NO 19
SPRING FREQ 7.17 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .0065
DATE Dec. 5, 1955
RUN NO. 20
SPRING FREQ 7.17 (cps)
ALTERNATION 0 (%)
VELOCITY 35 (mph)
DAMPING COEF .0065
DATE Dec. 5, 1955
RUN NO. 21
SPRING FREQ 7.17 (cps)
ALTERNATION 50 (%)
VELOCITY 35 (mph)
DAMPING COEF .0065
DATE Dec. 5, 1955

Figure 17

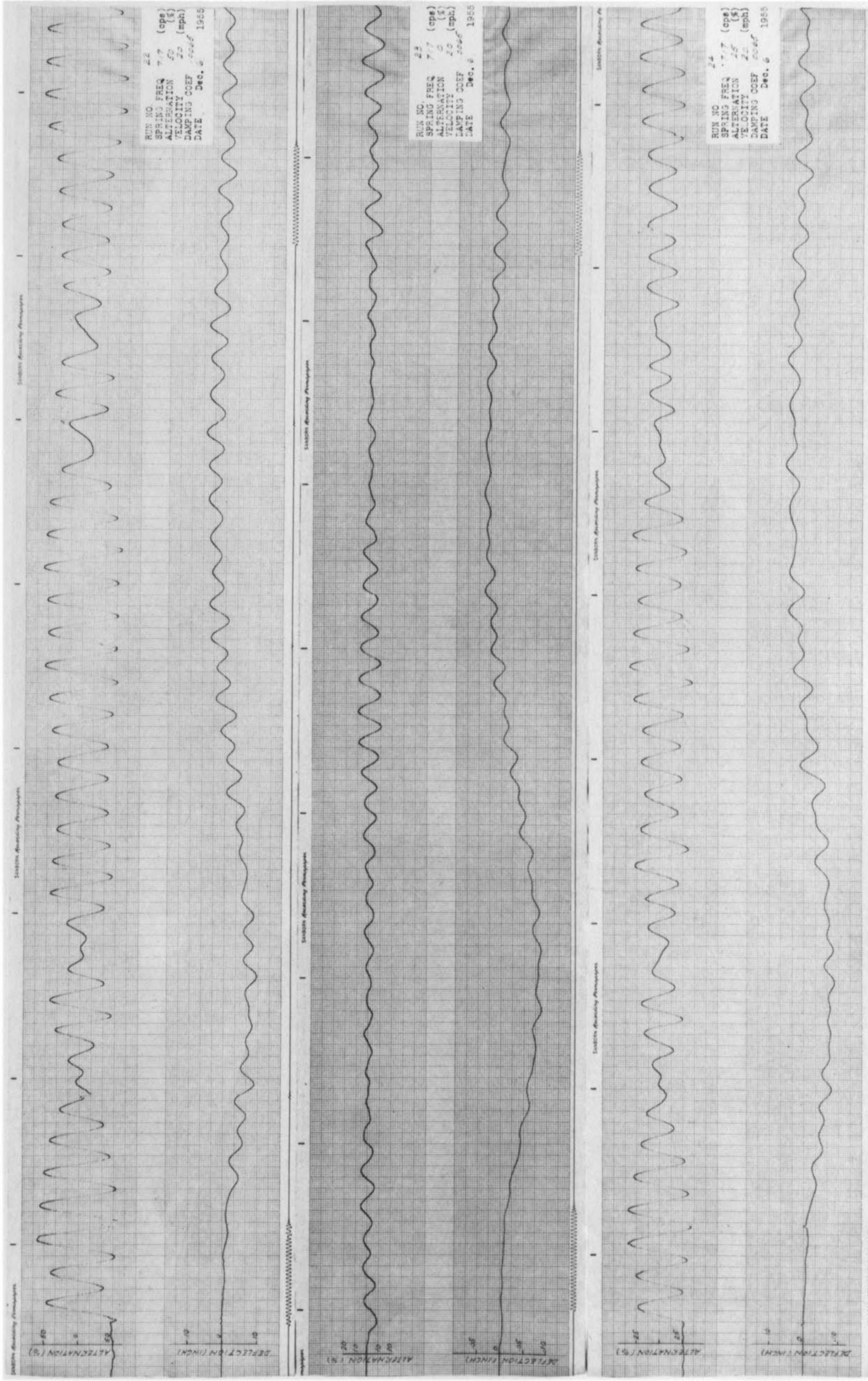

RUN NO. 22
SPRING FREQ 7.7 (cps)
ALTERNATION 50 (%)
VELOCITY 20 (mph)
DAMPING COEF .0065
DATE Dec. 6 1965
RUN NO. 23
SPRING FREQ 7.7 (cps)
ALTERNATION 0 (%)
VELOCITY 20 (mph)
DAMPING COEF .0065
DATE Dec. 6 1965
RUN NO 24
SPRING FREQ 7.7 (cps)
ALTERNATION 25 (%)
VELOCITY 20 (mph)
DAMPING COEF .0065
DATE Dec. 6 1965
ALTERNATION (%)
DEFLECTION (INCH)

Figure 18

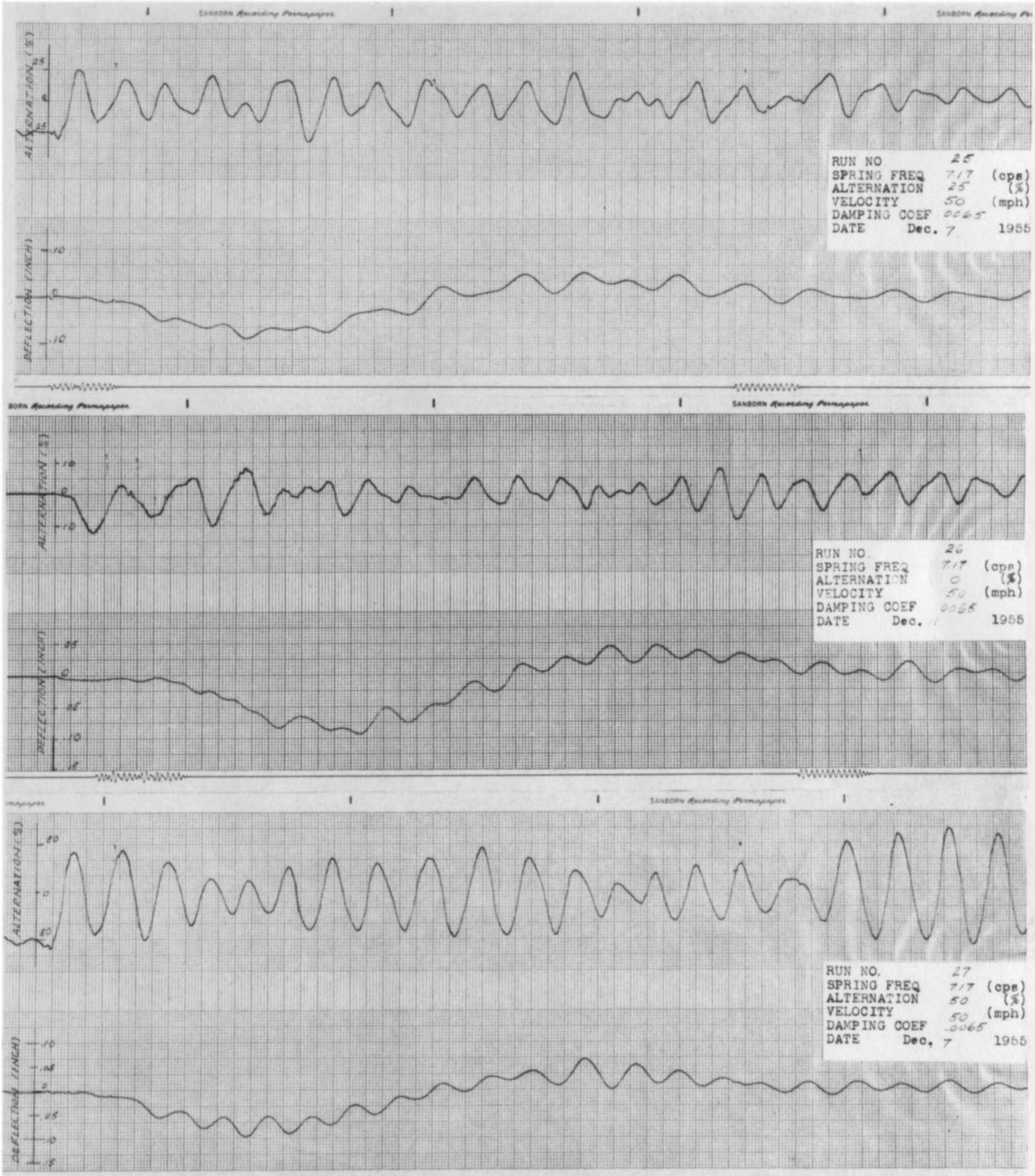

SANBORN Recording Permapaper
ALTERNATION (%)
RUN NO 25
SPRING FREQ 7.17 (cps)
ALTERNATION 25 (%)
VELOCITY 50 (mph)
DAMPING COEF .0065
DATE Dec. 7 1955
DEFLECTION (INCH)
RUN NO. 26
SPRING FREQ 7.17 (cps)
ALTERNATION 0 (%)
VELOCITY 50 (mph)
DAMPING COEF .0065
DATE Dec. 1955
RUN NO. 27
SPRING FREQ 7.17 (cps)
ALTERNATION 50 (%)
VELOCITY 50 (mph)
DAMPING COEF .0065
DATE Dec. 7 1955

Figure 19

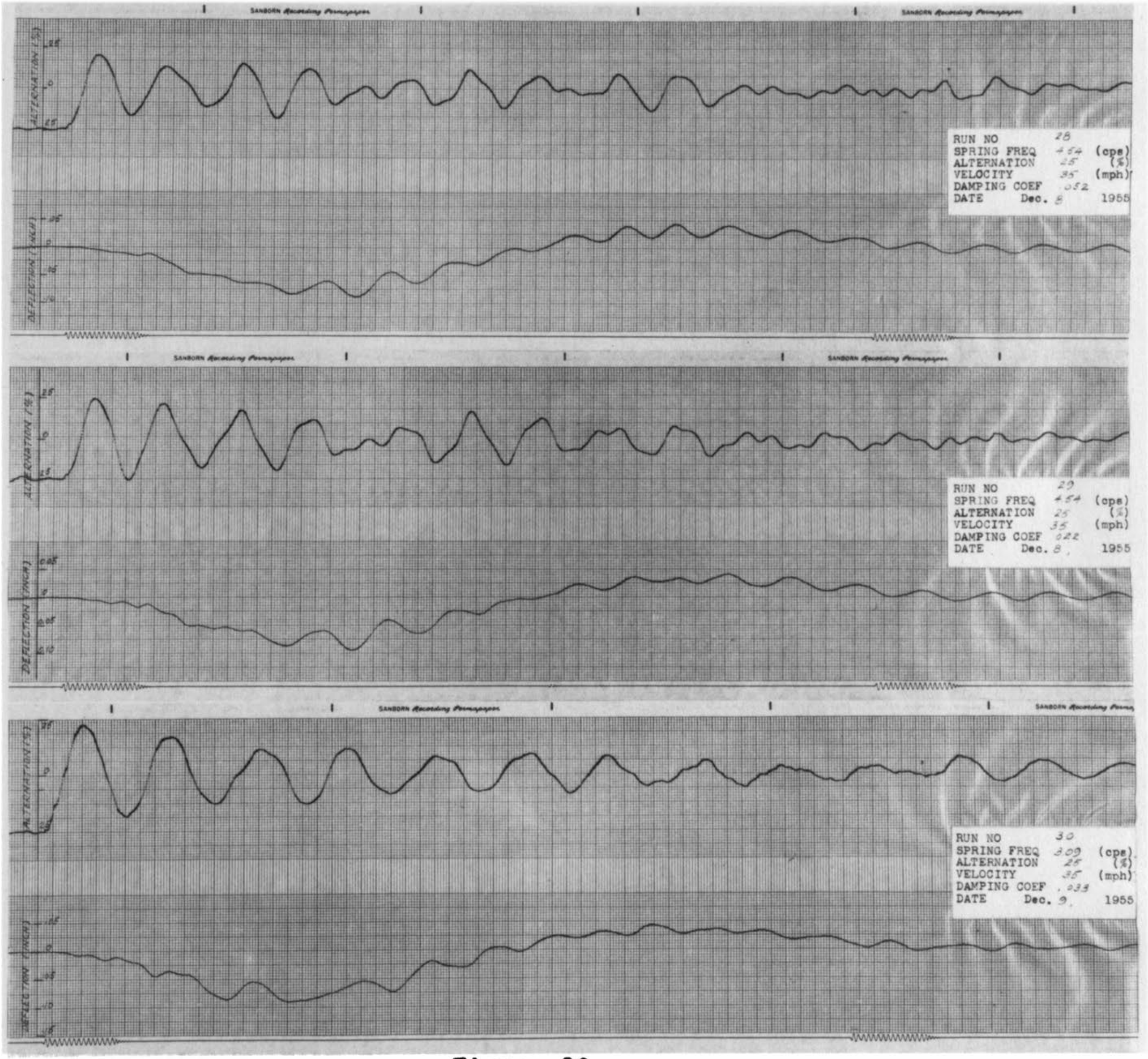

SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 28
SPRING FREQ 4.54 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .052
DATE Dec. 8 1955
RUN NO 29
SPRING FREQ 4.54 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .022
DATE Dec. 8, 1955
RUN NO 30
SPRING FREQ 3.09 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .033
DATE Dec. 9, 1955

Figure 20

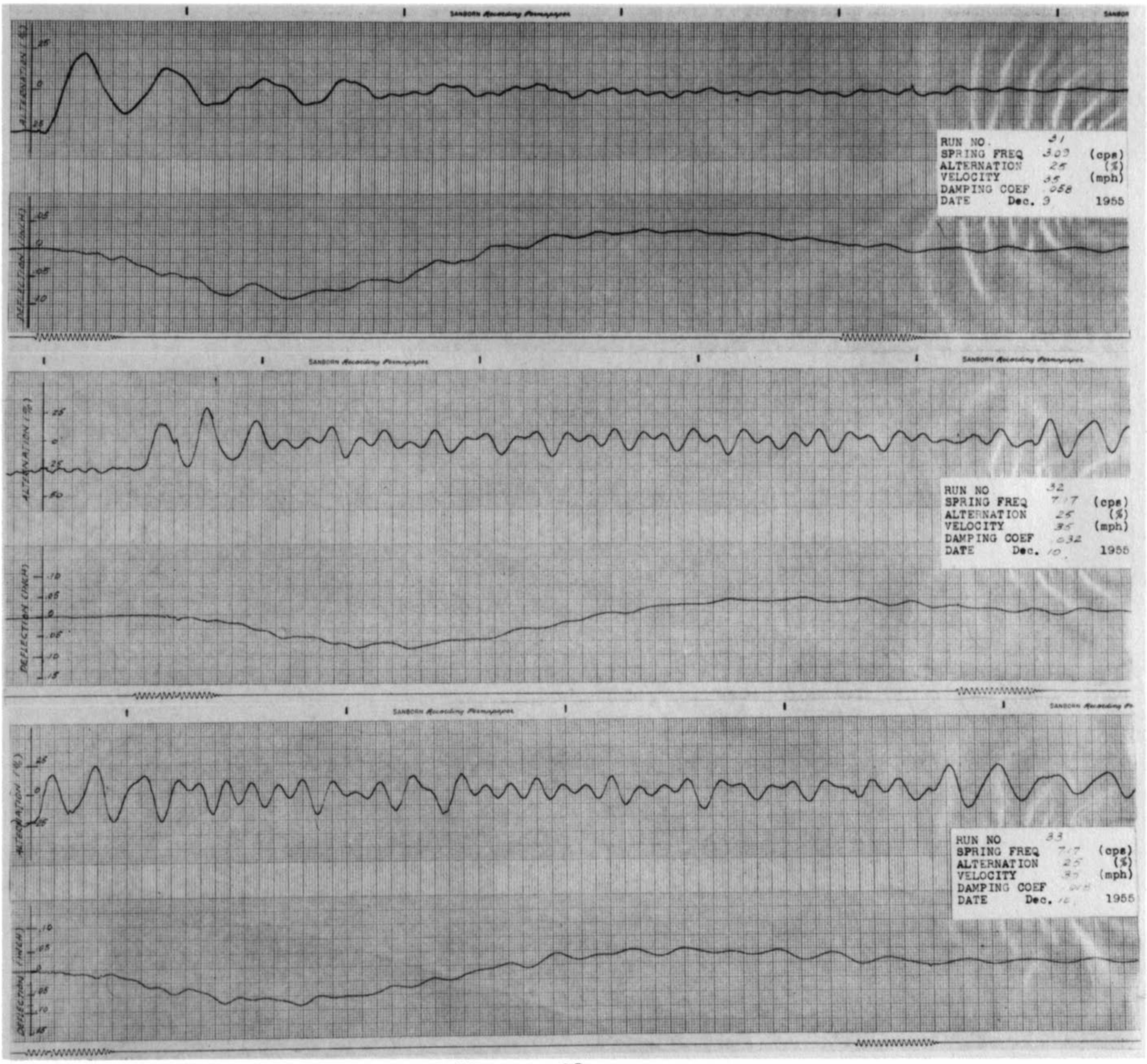

SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO. 31
SPRING FREQ 3.09 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .058
DATE Dec. 9 1955
RUN NO 32
SPRING FREQ (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .032
DATE Dec. 10 1955
RUN NO 33
SPRING FREQ (cps)
ALTERNATION 25 (%)
VELOCITY (mph)
DAMPING COEF
DATE Dec. 1955

Figure 21

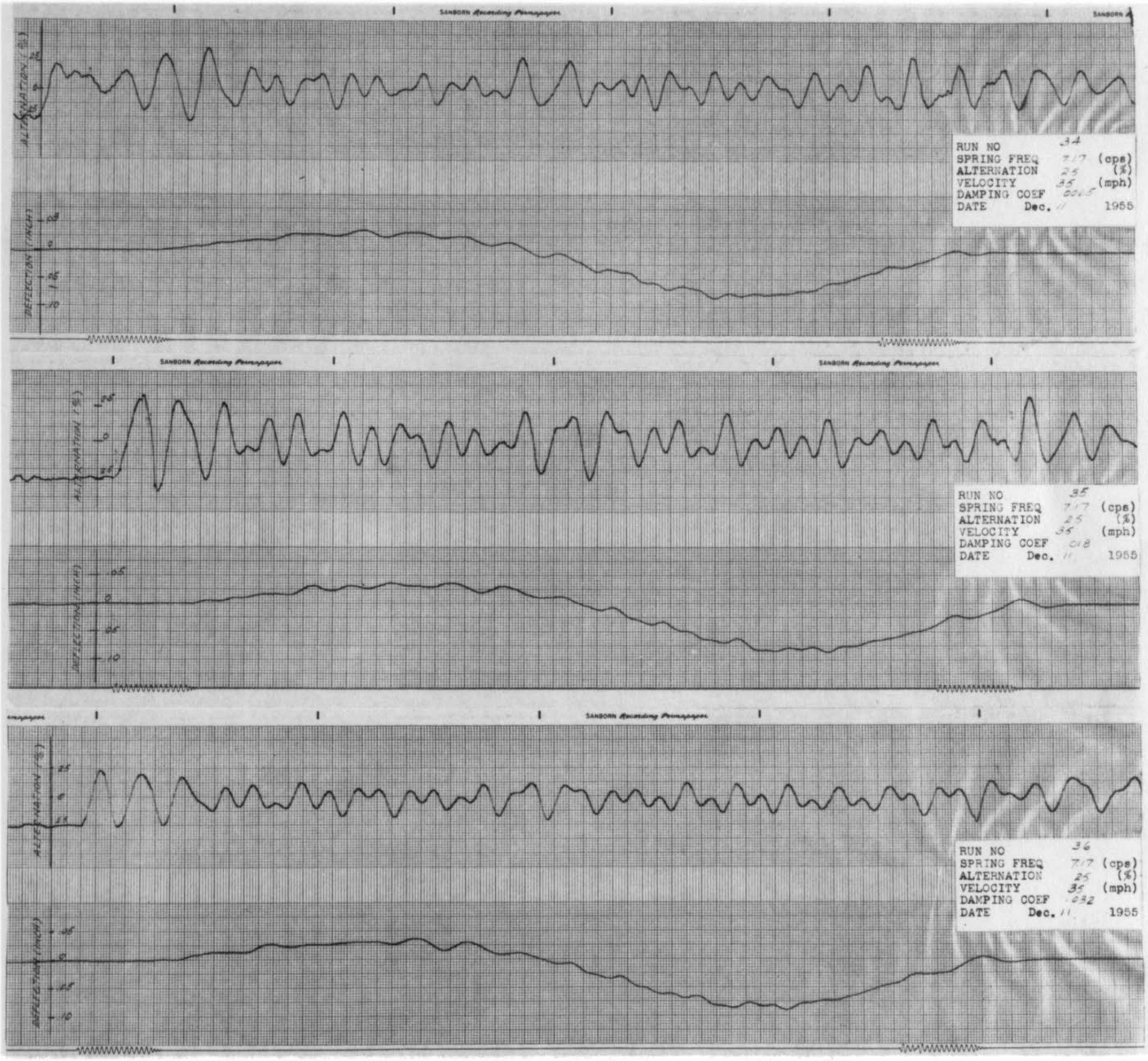

ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 34
SPRING FREQ 7.17 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .0065
DATE Dec. 11 1955
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 35
SPRING FREQ 7.17 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .018
DATE Dec. 11 1955
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 36
SPRING FREQ 7.17 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .032
DATE Dec. 11 1955

Figure 22

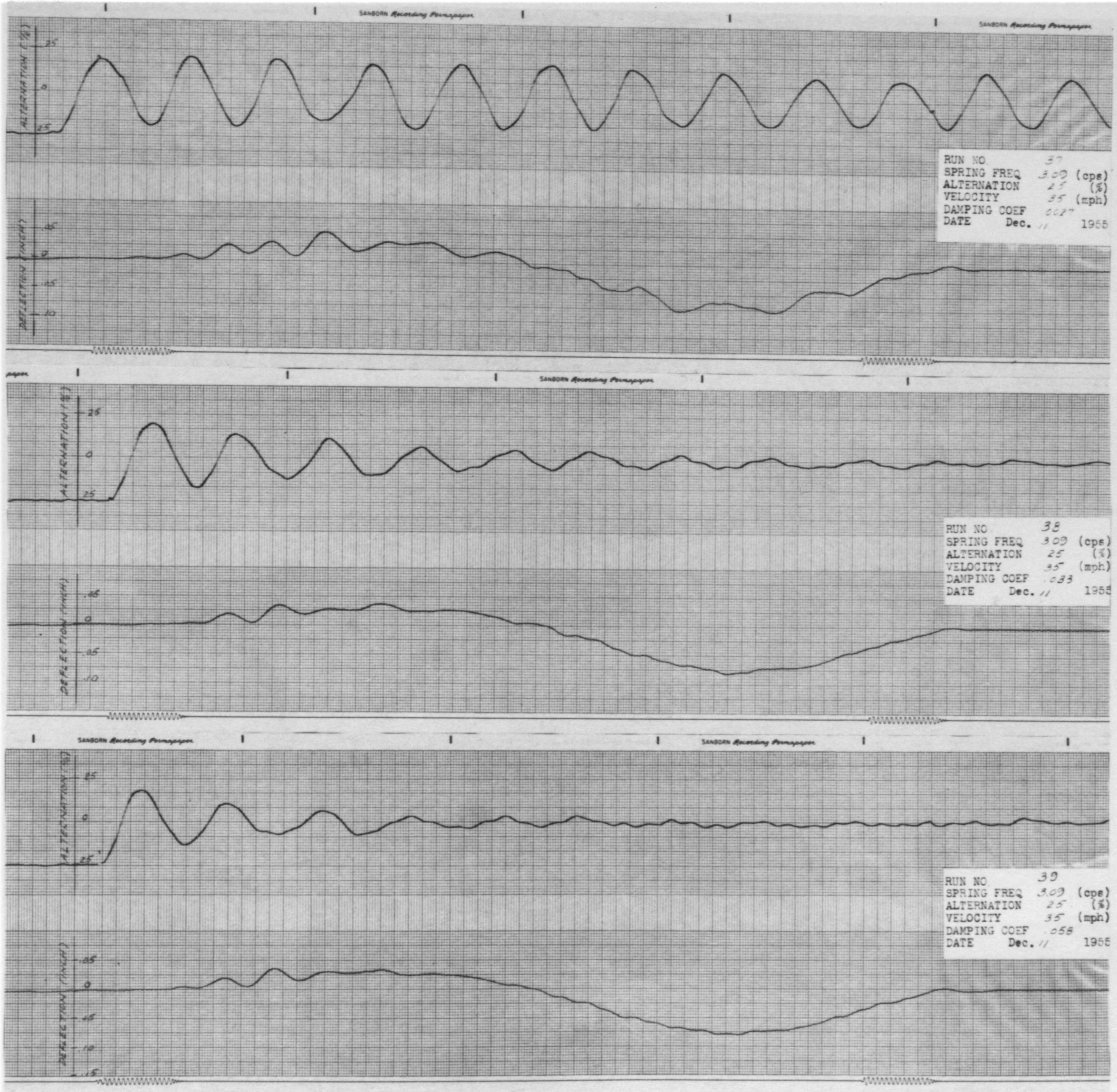
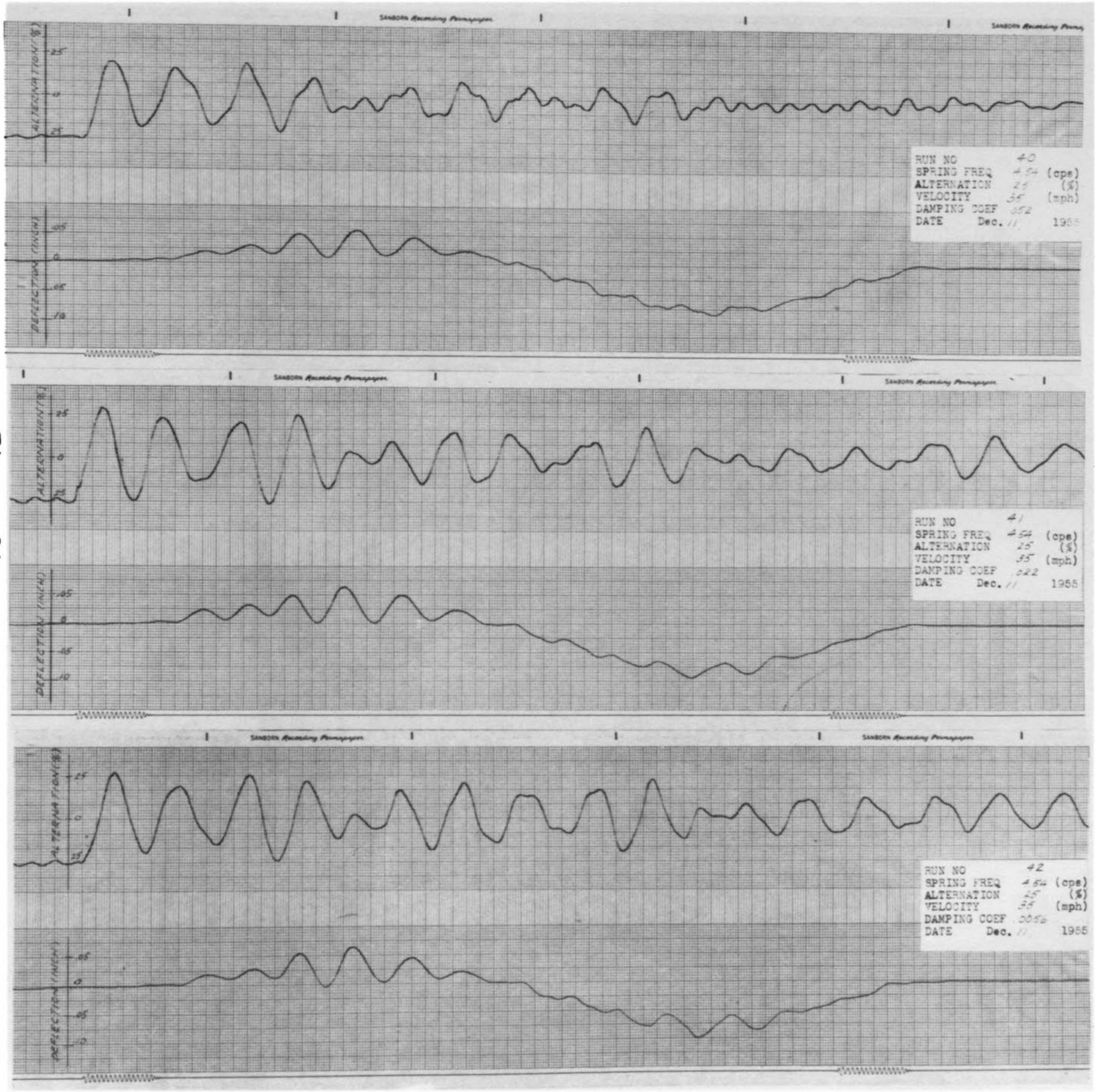
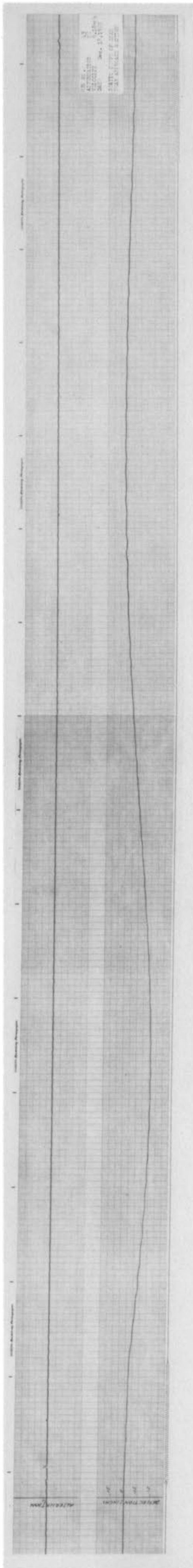

SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO. 37
SPRING FREQ 3.09 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .0027
DATE Dec. 11 1955
RUN NO. 38
SPRING FREQ 3.09 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .033
DATE Dec. 11 1955
RUN NO. 39
SPRING FREQ 3.09 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .058
DATE Dec. 11 1955

Figure 23



SANBORN Recording Permapaper
ALTERNATION (%)
DEFLECTION (INCH)
RUN NO 40
SPRING FREQ 4.54 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .052
DATE Dec. 11 1955
RUN NO 41
SPRING FREQ 4.54 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .022
DATE Dec. 11 1955
RUN NO 42
SPRING FREQ 4.54 (cps)
ALTERNATION 25 (%)
VELOCITY 35 (mph)
DAMPING COEF .0056
DATE Dec. 11 1955

Figure 24



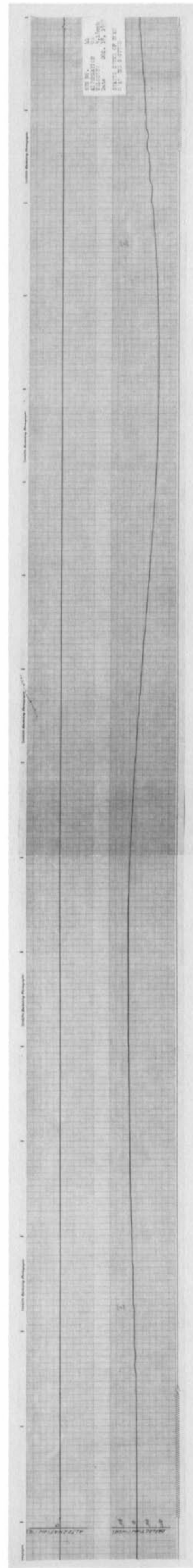


Figure 25

B. Summary of Test Result

The increased deflection of the bridge and the amplitude of the vibration seem to be the main objections of bridge vibration. They cause persons on the structure discomfort and a feeling of insecurity. Therefore, it was thought the best measure of the magnitude of the vibration lay in measuring the maximum deflection of the bridge, and also the largest amplitude of vibration which occurred as the vehicle passed over the bridge.

The maximum deflection ($\Delta max.$) was measured downward from the unloaded position. The maximum amplitude (Amax.) was measured as the largest sum of two succeeding positive and negative amplitudes. The maximum values were then divided by static deflection to give significance to the dynamic effect.

A summary of all the values is shown in Table 1. The nature of the frequency of the excited bridge vibrations is also included in the table.

V. DISCUSSION OF RESULTS

A. Effect of Spring Frequency

The ratios of deflection $\left(\frac{\Delta max}{\Delta st.}\right)$ and amplitude $\left(\frac{Amax}{\Delta st.}\right)$ were plotted against the vehicular frequency in Figure 26. In keeping the vehicular damping constant, only values of Run No. 1 to 27 inclusive were used. Different symbols were used to represent different velocities. In addition, distinction also made on each symbol to identity various alternations. Thus, a line connecting the identical symbols will show the effect of vehicular frequency only, other parameters being kept constant. The trend of the lines would then show a general relationship

Table 1

SUMMARY OF TEST RESULTS

Figure No.	Run No.	Span Tested	ω cps	% Alternation	v MPH	Damping Coeff. (Z)	Δmax. inch	Δmax./Δst.	Amax. inch	Amax./Δst.	Notes Re. Mode of Bridge Vibration
11	1	Near approach	3.09	25	20	.0027	.0965	1.097	.0500	.621	Combination of 1st and 2nd Modes.
	2	"	3.09	0	20	.0027	.0855*	1.060	.0205*	.265	Predominatly 2nd Mode.
	3	"	3.09	50	20	.0027	.1150#	1.428	.0740#	.920	Starting with ω gradually shifting to combination of 1st and 2nd Modes.
12	4	"	3.09	50	35	.0027	.1105#	1.371	.0785#	.975	Combination of ω and 1st Mode and some indistinct 2nd Modes.
	5	"	3.09	0	35	.0027	.0910*	1.129	.0495*	.615	Well-defined 2nd Mode.
	6	"	3.09	25	35	.0027	.0955#	1.185	.0610#	.758	Combination of ω and 1st Modes.
13	7	"	3.09	25	50	.0027	.0960#	1.190	.0505#	.623	Combination of ω and 1st Modes.
	8	"	3.09	0	50	.0027	.0850	1.054	.0325	.404	Combination of 1st and 2nd Modes.
	9	"	3.09	50	50	.0027	.1160	1.440	.0810	1.006	Combination of ω and 1st Modes.
14	10	"	4.54	50	50	.0056	.1095	1.360	.0890	1.107	1st Mode only.
	11	"	4.54	0	50	.0056	.0970	1.203	.0550	.684	Combination of 1st and 2nd Modes.
	12	"	4.54	25	50	.0056	.0975	1.210	.0655	.815	Combination of 1st and 2nd Modes.
15	13	"	4.54	25	35	.0056	.0920	1.141	.0550	.684	Combination of 1st and 2nd Modes.
	14	"	4.54	0	35	.0056	.0895	1.110	.0405	.503	Combination of 1st and 2nd Modes.
	15	"	4.54	50	35	.0056	.1035	1.284	.0875	1.087	Predominatly 1st Mode.
16	16	"	4.54	50	20	.0056	.1050	1.302	.0630	.784	Predominatly 1st Mode.
	17	"	4.54	0	20	.0056	.0910	1.129	.0500	.622	Predominatly 1st Mode.
	18	"	4.54	25	20	.0056	.0925	1.148	.0430	.535	Predominatly 1st Mode.
17	19	"	7.17	25	35	.0065	.0907*	1.127	.0600*	.745	Predominatly 2nd Mode.
	20	"	7.17	0	35	.0065	.0860*	1.068	.0333*	.414	Combination 1st and 2nd Modes.
	21	"	7.17	50	35	.0065	.0933	1.159	.0507	.630	Predominatly 2nd Mode.
18	22	"	7.17	50	20	.0065	.0953*	1.183	.0560*	.696	Well-defined 2nd Mode.
	23	"	7.17	0	20	.0065	.0875*	1.087	.0300*	.373	Combination of 1st and 2nd Modes.
	24	"	7.17	25	20	.0065	.0867*	1.077	.0720*	.895	Predominatly 2nd Mode.
19	25	"	7.17	25	50	.0065	.0907*	1.127	.0586*	.728	Predominatly 2nd Mode.
	26	"	7.17	0	50	.0065	.0900*	1.120	.0410*	.509	Predominatly 2nd Mode.
	27	"	7.17	50	50	.0065	.0987*	1.226	.0453*	.563	All 2nd Mode.
20	28	"	4.54	25	35	.052	.0895	1.110	.0400	.497	Predominatly 1st Mode.
	29	"	4.54	25	35	.022	.0950	1.180	.0540	.670	Predominatly 1st Mode.
	30	"	3.09	25	35	.033	.0910#	1.129	.0555	.690	Combination of ω, 1st and 2nd Modes.
21	31	"	3.09	25	35	.058	.0905	1.124	.0265	.330	Combination of 1st and 2nd Modes.
	32	"	7.17	25	35	.032	.0853*	1.060	.0190*	.237	Predominatly 2nd Mode.
	33	"	7.17	25	35	.018	.0893*	1.109	.0250*	.311	Predominatly 2nd Mode.
22	34	Near end	7.17	25	35	.0065	.0850*	1.054	.0230*	.286	Predominatly 2nd Mode.
	35	"	7.17	25	35	.018	.0860*	1.068	.0205*	.255	Predominatly 2nd Mode.
	36	"	7.17	25	35	.032	.0905*	1.124	.0200*	.249	Predominatly 2nd Mode.
23	37	"	3.09	25	35	.0027	.0840#	1.044	.0425	.528	Combination of ω, 1st and 2nd Modes.
	38	"	3.09	25	35	.033	.0825	1.025	.0085	.106	No distinctive Mode.
	39	"	3.09	25	35	.058	.0805	1.000	.0050	.062	No distinctive Mode.
24	40	"	4.54	25	35	.052	.0850*	1.054	.0150	.187	Combination of 1st and 2nd Modes.
	41	"	4.54	25	35	.022	.0950	1.180	.0300	.373	Combination of 1st and 2nd Modes.
	42	"	4.54	25	35	.0056	.1000	1.242	.0410	.510	Combination of 1st and 2nd Modes.
25	43	Near approach	--	--	9.18	---	.0805	1	0	--	
	44	Near end	--	--	9.18	---	.0805	1	0	--	

* Max. value occurs at Second Mode.
# Max. value occurs at frequency of the alternating force.

between the magnitude of the bridge vibration and vehicular frequency.

The plot shows that in the case of maximum deflection the change in frequency from 4.54 cps (corresponds to bridge natural frequency, fundamental mode) to 7.17 cps (Second mode) resulted in a decrease in the magnitude of the maximum deflection. However, there is no such clear pattern existing, when the frequency changes to 3.09 cps. Note that the resonant frequencies do not necessarily produce larger deflections.

In the case of maximum amplitude, only two-thirds of the cases show a decrease in the magnitude of the maximum amplitude, when the frequency changes from 4.54 cps to 7.17 cps. Again, no clear pattern exists when the vehicle frequency decreases to 3.09 cps. As was in the case of deflection, the resonant frequencies do not necessarily produce maximum amplitudes.

B. Effect of Velocity

Figure 27 shows the ratios of deflection and amplitude versus vehicular velocity. In plotting the graph same method as in Figure 26 was used. The lines in the graph show the effect of velocity only, other parameters, alternation and frequency being held constant.

The plot does not show any definite trend. The only possible conclusion is that the velocity does affect the magnitude of deflection and amplitude, but no simple relationship can be formulated.

C. Effect of Alternation

Figure 28 shows the ratio of deflection versus the percent alternation of the load.

This plot, unlike the two previous ones, shows a very definite pattern. The magnitude of the ratio of deflection increases with an increase of alternation, other parameters, velocity and spring frequency

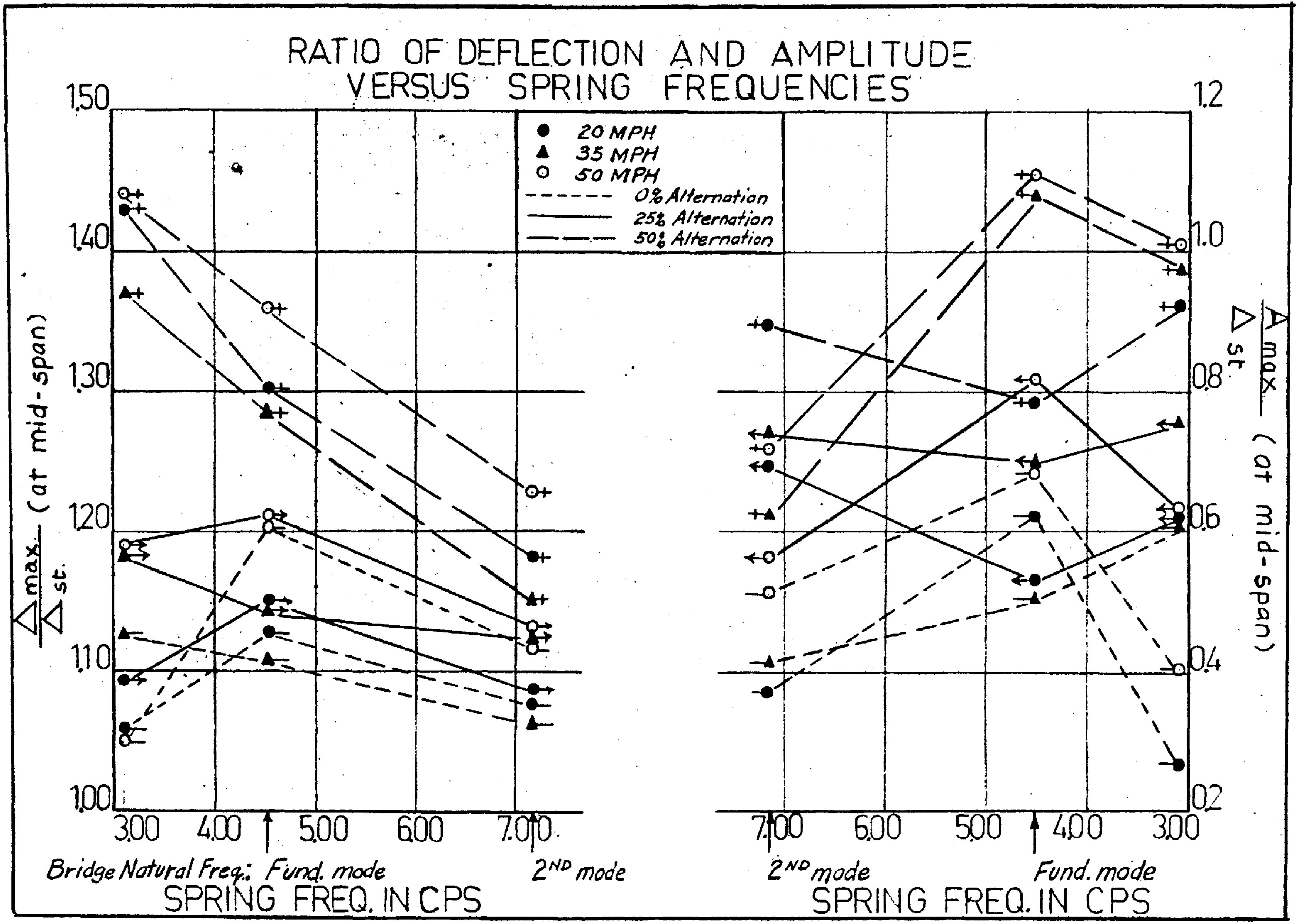
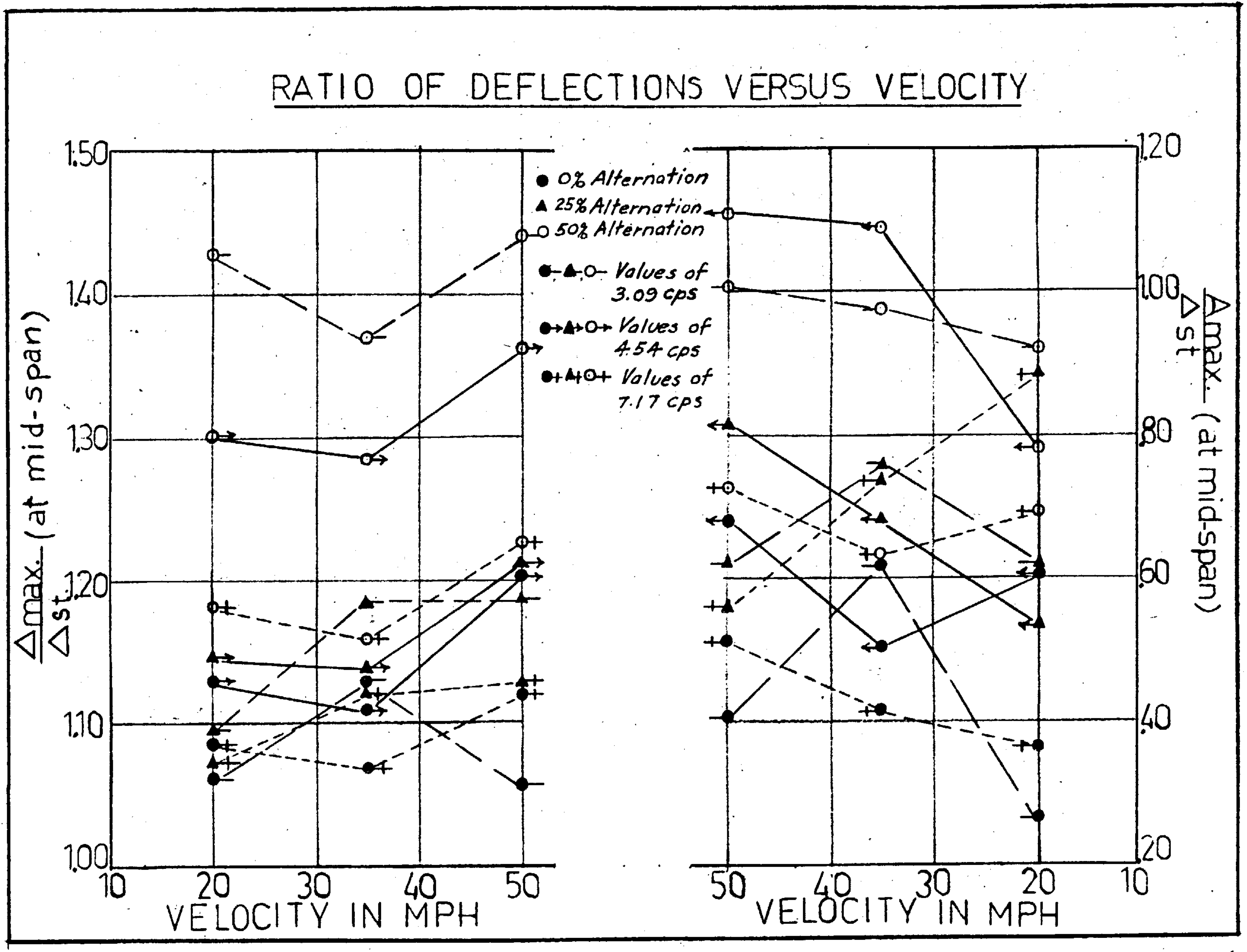

RATIO OF DEFLECTION AND AMPLITUDE
VERSUS SPRING FREQUENCIES
20 MPH
35 MPH
50 MPH
0% Alternation
25% Alternation
50% Alternation
1.50
1.40
1.30
1.20
1.10
1.00
1.2
1.0
0.8
0.6
0.4
0.2
$\frac{\Delta_{max}}{\Delta_{st.}}$ (at mid-span)
$\frac{\Delta_{max}}{\Delta_{st.}}$ (at mid-span)
3.00
4.00
5.00
6.00
7.00
7.00
6.00
5.00
4.00
3.00
Bridge Natural Freq.: Fund. mode
2ND mode
2ND mode
Fund. mode
SPRING FREQ. IN CPS
SPRING FREQ. IN CPS

Figure 26



RATIO OF DEFLECTIONS VERSUS VELOCITY
0% Alternation
25% Alternation
50% Alternation
Values of 3.09 cps
Values of 4.54 cps
Values of 7.17 cps
Δmax./Δst (at mid-span)
VELOCITY IN MPH
VELOCITY IN MPH
1.50
1.40
1.30
1.20
1.10
1.00
1.20
1.00
.80
.60
.40
.20
10
20
30
40
50
50
40
30
20
10

Figure 27

being held constant. A line is drawn to connect the main values. The slope of the line is greater between 25% and 50% alternation than that between 0% and 25%. It is observed that at zero alternation the mean maximum deflection is 9% greater than static, with variation from 5% to 21% greater. At 25% alternation, the main value is 13% greater than static, with variation from 8% to 22%. At 50% alternation, the main deflection is 30% greater with a range of 16% to 44% greater.

Figure 29 shows the ratio of amplitude versus the per cent alternation of the load.

This plot too shows a very definite trend. The magnitude of the amplitude increases almost linearly with an increase in the alternation force. The main values are 48%, 68%, and 88% of the static deflection at 0%, 25%, and 50% alternation respectively. The ranges of variation are: 27% to 68%, 52% to 88%, 58% to 111% greater at 0%, 25%, and 50% alternation respectively. It was observed that although maximum deflection and amplitude usually occur at same point on the vibration curve, but their magnitudes are not necessarily proportionate.

D. <u>Effect of Damping</u>

Runs 6, 13, 19, and 28 to 42 inclusive in which the vehicular velocity and alternation were constant at 35 MPH and 25% respectively, provided values in plotting Figure 30. It shows the ratio of deflection versus vehicular damping coefficients. The abscissa is plotted in log scale. Note there are two groups of values existed; one for the span near approach section, one for the span near end section. Different lines are also used to identify different spring frequencies.

The plot, although it is not very conclusive, shows a general trend for an increase in damping coefficient to cause a decrease in the magnitude of the ratio of deflection.

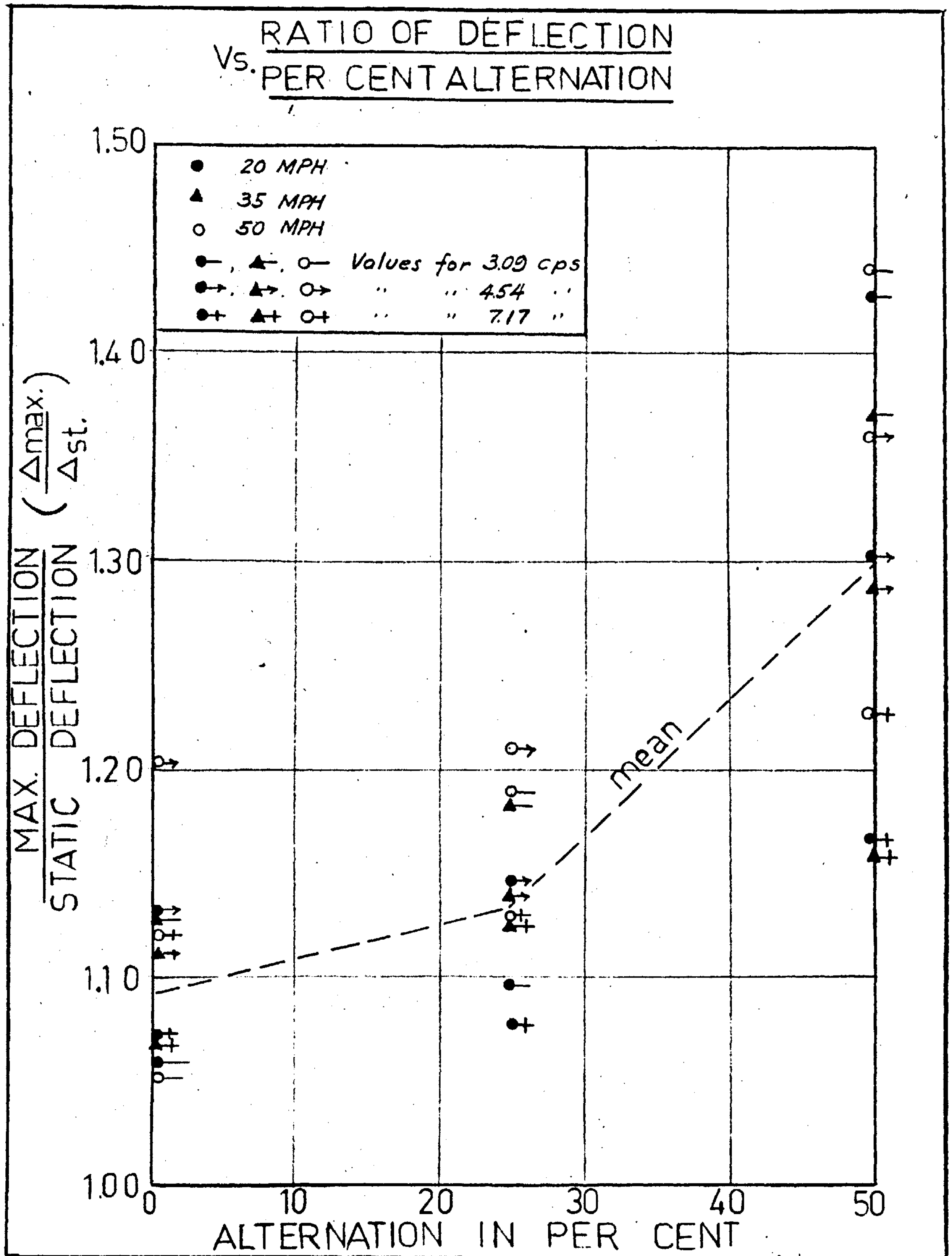

RATIO OF DEFLECTION
Vs. PER CENT ALTERNATION
20 MPH
35 MPH
50 MPH
Values for 3.09 cps
" " 4.54 "
" " 7.17 "
1.50
1.40
1.30
1.20
1.10
1.00
0
10
20
30
40
50
mean
MAX. DEFLECTION / STATIC DEFLECTION ($\frac{\Delta max.}{\Delta st.}$)
ALTERNATION IN PER CENT

Figure 28

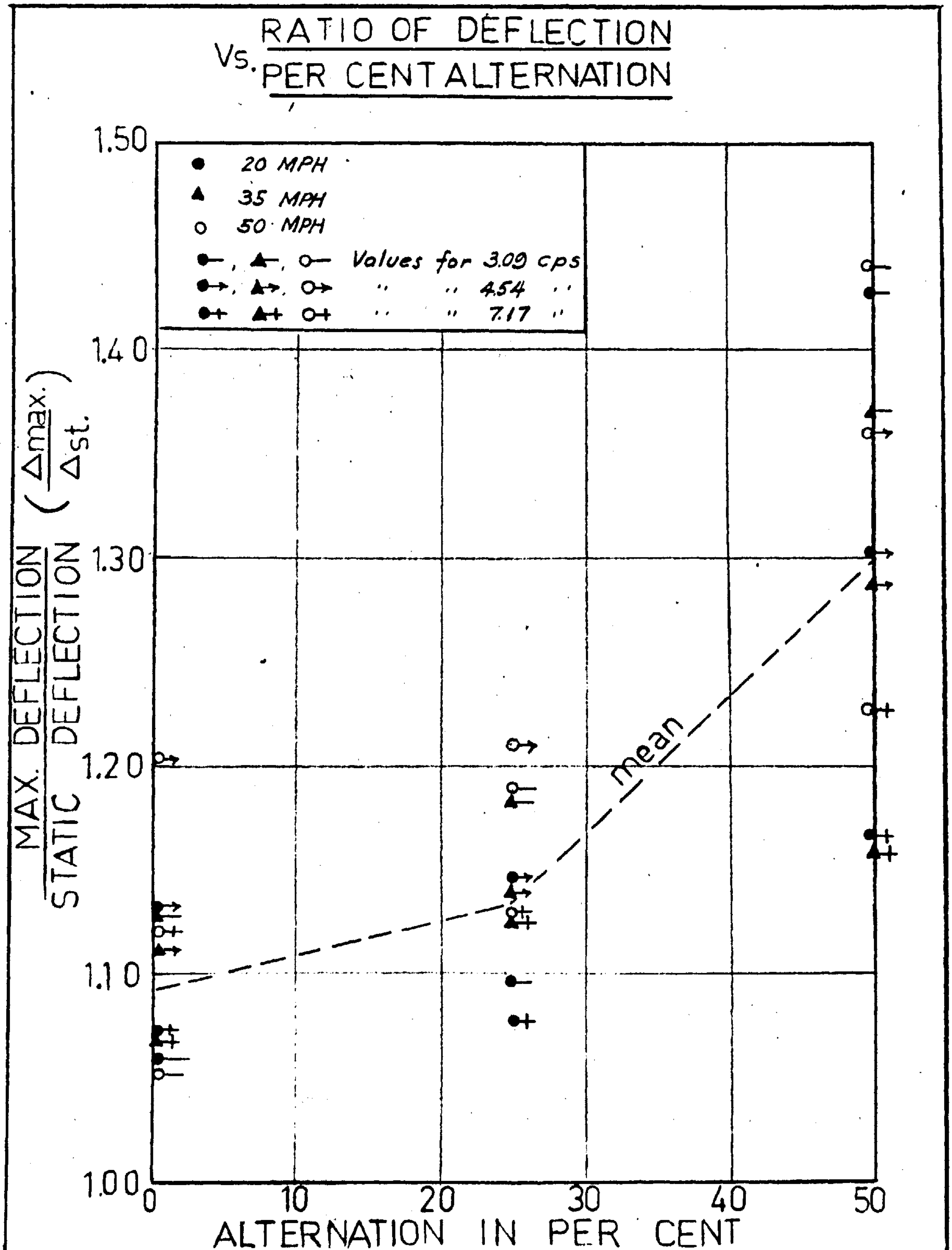

Vs. RATIO OF DEFLECTION
PER CENT ALTERNATION
20 MPH
35 MPH
50 MPH
Values for 3.09 cps
" " 4.54 "
" " 7.17 "
mean
MAX. DEFLECTION / STATIC DEFLECTION (Δmax. / Δst.)
1.50
1.40
1.30
1.20
1.10
1.00
0
10
20
30
40
50
ALTERNATION IN PER CENT

Figure 29

Same runs were used in plotting Figure 31, which shows the ratio of amplitude versus log damping coefficient.

The plot shows a much more clear pattern than that of Figure 30. A definite decrease of the amplitude is caused by an increase in the vehicular damping coefficient. The plot also follows what it was expected; that the amplitudes are lesser for the span near the end section than the span near the approach section. This is because of the damping-out of the alternation force when the vehicle crossed the span near the end, thus a lesser dynamic effect resulted.

E. Effect on Bridge Frequency

Figure 32 shows the per cent occurance of the bridge vibration against various vehicular quantities. The per cent occurance was obtained by dividing the number of total runs by the number of appearances of a certain mode. When a combination of modes occurred, only the approximate percentage part of that certain mode was counted.

The plots show that the frequency of the alternating force has an important effect upon the frequency of the bridge vibration. The spring frequency used, especially at the resonant frequency, either fundamental or second mode appears predominantly in the bridge vibration.

No such conclusion can be made in connection with the velocity and alternation of the vehicle.

VI. CONCLUSION

The important conclusions of this study are listed below. It should be kept in mind that they are based only on the data that was available.

1. There is an effect on the maximum deflection and amplitude of the bridge of varying the natural frequency of the vehicle. No definite conclusion can be made, however, to the exact relationship. The

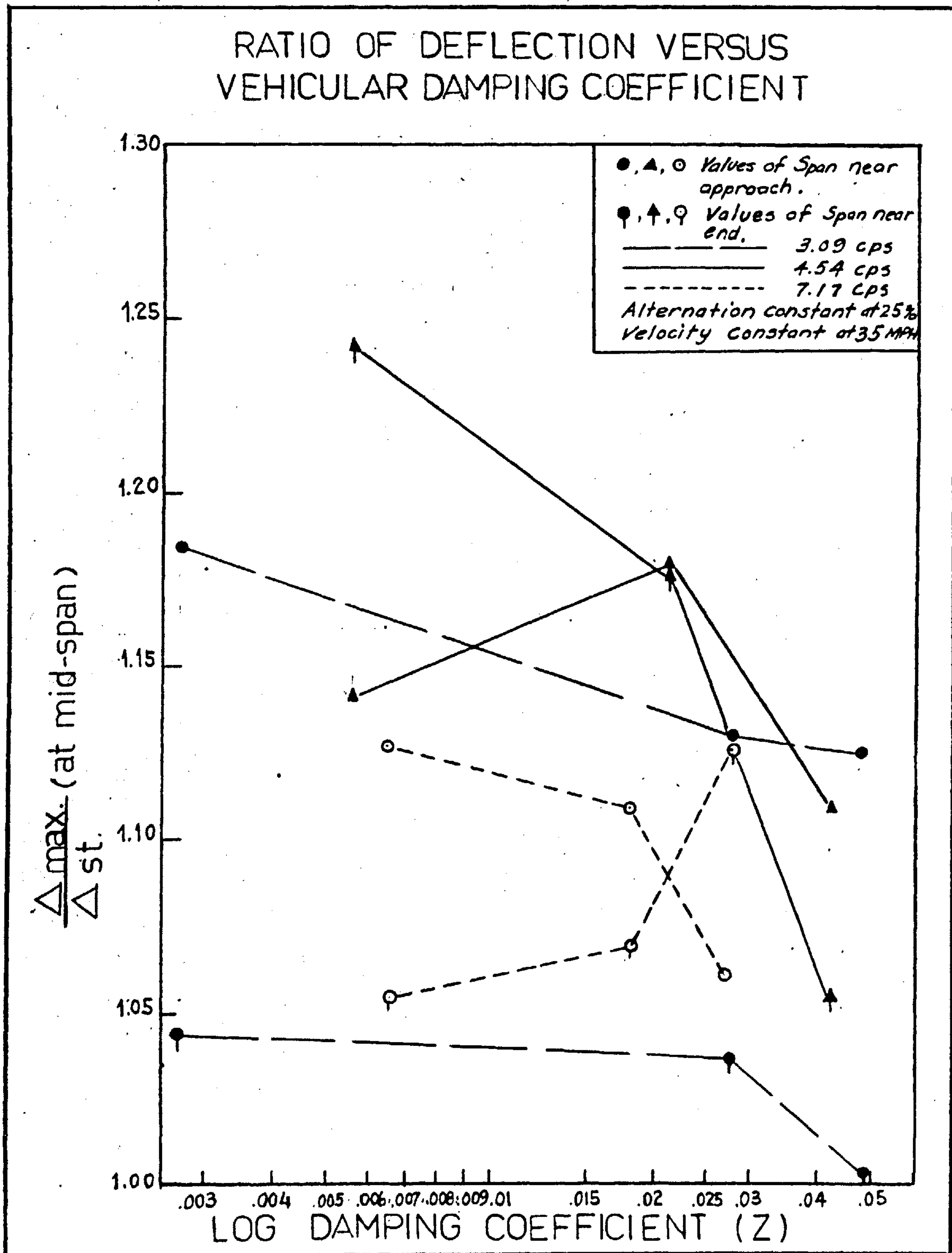

RATIO OF DEFLECTION VERSUS
VEHICULAR DAMPING COEFFICIENT
●, ▲, ⊙ Values of Span near approach.
Values of Span near end.
3.09 cps
4.54 cps
7.17 cps
Alternation constant at 25%
Velocity Constant at 35 MPH
Δ max. / Δ st. (at mid-span)
1.30
1.25
1.20
1.15
1.10
1.05
1.00
.003
.004
.005
.006
.007
.008
.009
.01
.015
.02
.025
.03
.04
.05
LOG DAMPING COEFFICIENT (Z)

Figure 30

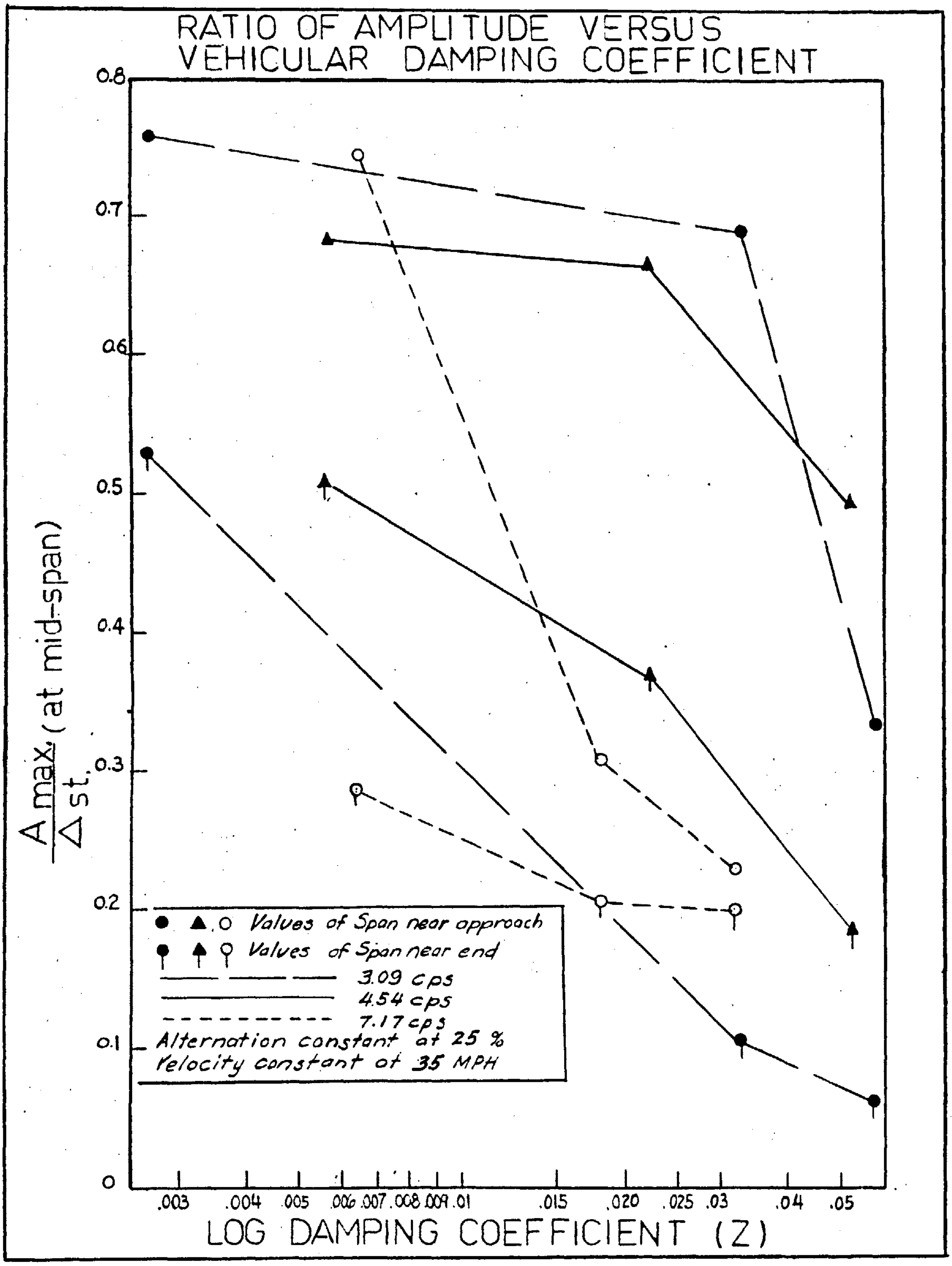

RATIO OF AMPLITUDE VERSUS
VEHICULAR DAMPING COEFFICIENT
A max (at mid-span) / Δ st.
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Values of Span near approach
Values of Span near end
3.09 cps
4.54 cps
7.17 cps
Alternation constant at 25 %
Velocity constant at 35 MPH
.003 .004 .005 .006 .007 .008 .009 .01 .015 .020 .025 .03 .04 .05
LOG DAMPING COEFFICIENT (Z)

Figure 31

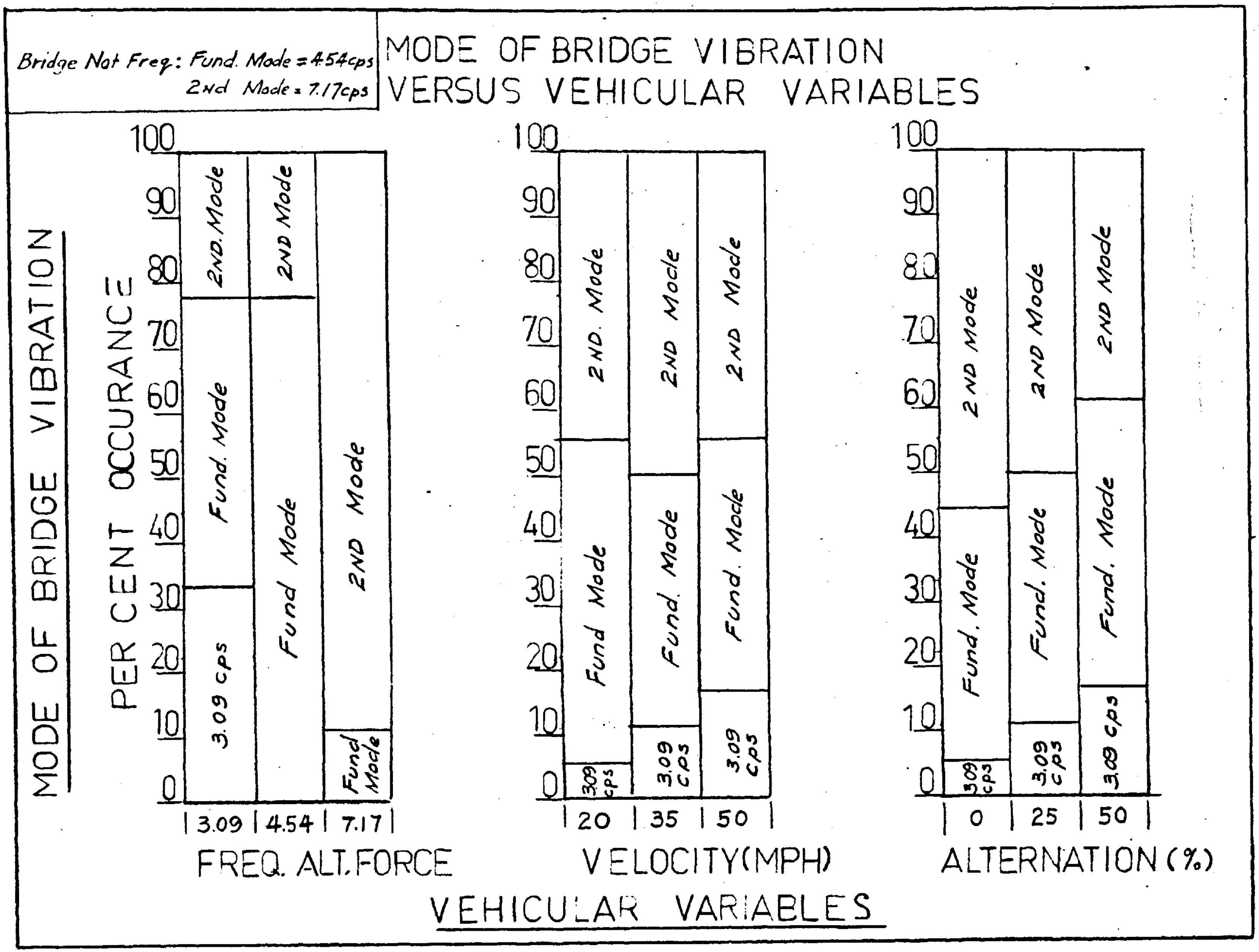

Bridge Nat Freq: Fund. Mode = 4.54 cps
2nd Mode = 7.17 cps
MODE OF BRIDGE VIBRATION
VERSUS VEHICULAR VARIABLES
MODE OF BRIDGE VIBRATION
PER CENT OCCURANCE
100
90
80
70
60
50
40
30
20
10
0
2ND. Mode
Fund. Mode
3.09 cps
2ND Mode
Fund Mode
2ND Mode
Fund Mode
3.09
4.54
7.17
FREQ. ALT. FORCE
2ND. Mode
Fund Mode
3.09 cps
2ND Mode
Fund. Mode
3.09 cps
2ND Mode
Fund. Mode
3.09 cps
20
35
50
VELOCITY (MPH)
2 ND Mode
Fund. Mode
3.09 cps
2ND Mode
Fund. Mode
3.09 cps
2ND Mode
Fund. Mode
3.09 cps
0
25
50
ALTERNATION (%)
VEHICULAR VARIABLES

Figure 32

resonant frequencies do not necessarily produce larger deflections and amplitudes of oscillation.

2. There is annoticeable effect on the maximum deflection and amplitude of the bridge of varying the velocity of the vehicle, but no conclusive relationship can be formulated.

3. An increase of the alternating force causes almost a linear increase of the maximum deflection and amplitude.

4. The damping characteristics of the vehicle has an inverse effect on the magnitude of the amplitude of the bridge. The larger the damping coefficient of the vehicle, the smaller the amplitude will be. The deflection is generally affected in the same manner.

5. A resonant vehicular frequency will cause the bridge to have predominantly the same frequency as the vehicle.

6. The damping device designed proved to be satisfactory and may be used in the future study. A constant damping effect can be achieved through the device.

VII. RECOMMENDATION FOR FURTHER STUDY

Further research is needed to expand the study of this thesis so that a complete solution is possible to define the effects of vehicular vibrations on highway bridges. First, a verification of test results with the theoretical solution is desirable. Device should also be designed to simulate the damping of the bridge so that its effect can be studied. The most serious vibrations have been observed in multi-span bridges, hence the model study should be continued to prototypes with a larger number of spans.

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