AN INVESTIGATION OF WEAKNESSES IN SELSYN DATA TRANSMISSION SYSTEMS

By

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Foreword

A fundamental knowledge of servo-mechanisms on the part of the reader has been assumed in this thesis in order to make it reasonably concise. It is therefore with the author's apologies that the reader with no background in servo-mechanisms is asked to read Professor H. L. Hazen's paper, "Theory of Servo-Mechanisms" (see bibliography), before reading this thesis. The reader will also find it helpful if he has some familiarity with selsyn eq uipment and this also may be obtained by perusing some of the references in the bibliography.

Acknowledgment

The author is greatly indebted to Professor G. S. Brown for his background in servo-mechanisms and for his opportunity to undertake the problem of this thesis. Credit for most of the basic ideas of this thesis should be given to Professor Brown and his associates, and of the latter particularly Mr. A. C. Hall.

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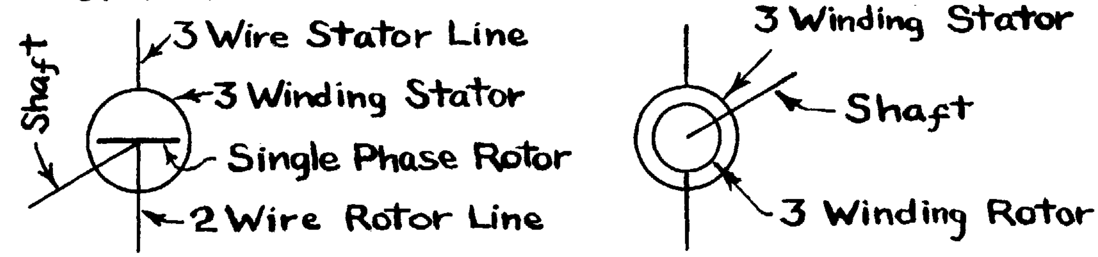
Non-Standard Symbols Used in this Thesis

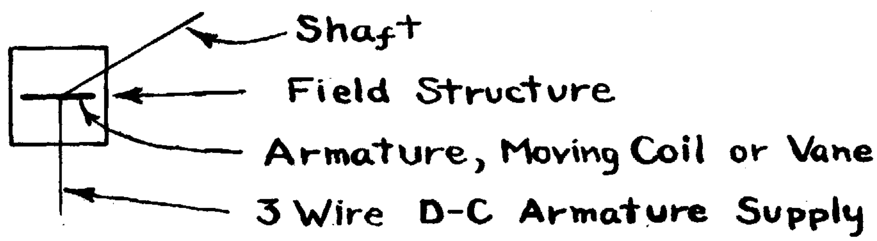
Selsyn

Transmitter, Master or Receiver

Differential

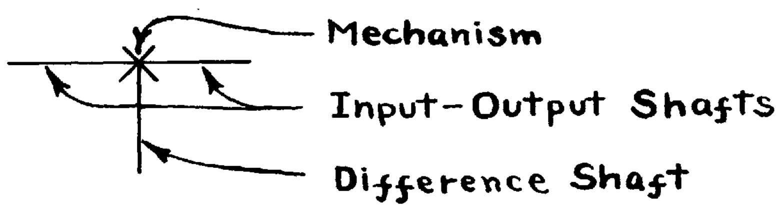


Torque Motor



Mechanical Differential

Ratio :- 1:1:1



Servo-Motor

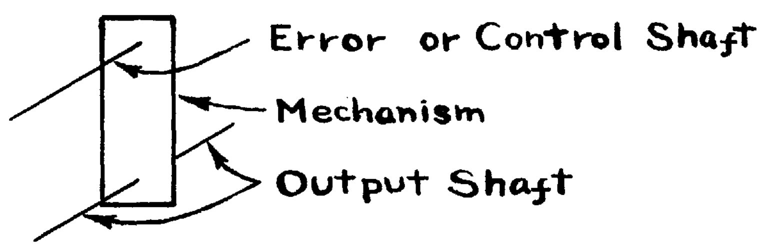


Fig. 1a

I Introduction - Nature of Problem

This thesis is concerned with the general problem of mechanically driving several "loads" in as near synchronism as possible with an input shaft which itself cannot be materially loaded and can have no mechanical connection to the driven "loads". Fortunately, this problem had a working solution at the commencement of this work and so, more specifically, the object of the thesis was to improve certain elements in the system used.

Basically, the system used consisted of a selsyn* data transmission system operating into servo-mechanisms at the receiving points. The selsyns are a-c machines which, in the case of transmitter (master) or receiver units, have 3-winding stators and single phase shuttle-wound rotors; and, in the case of differential units, 3-winding stators and 3-winding rotors. The operation of a master transmitter, differential, and receiver transmitter is analogous to a 1:1:1 mechanical differential with the exceptions that it has no backlash and is not as stiff.

Schematically, the overall system is shown in fig. 1 b. Each combination of a servo-motor and its associated differential and receiver selsyns are to be referred to as a receiver servo. The so-called "receiver" selsyn is usually similar in construction to the master transmitter selsyn and

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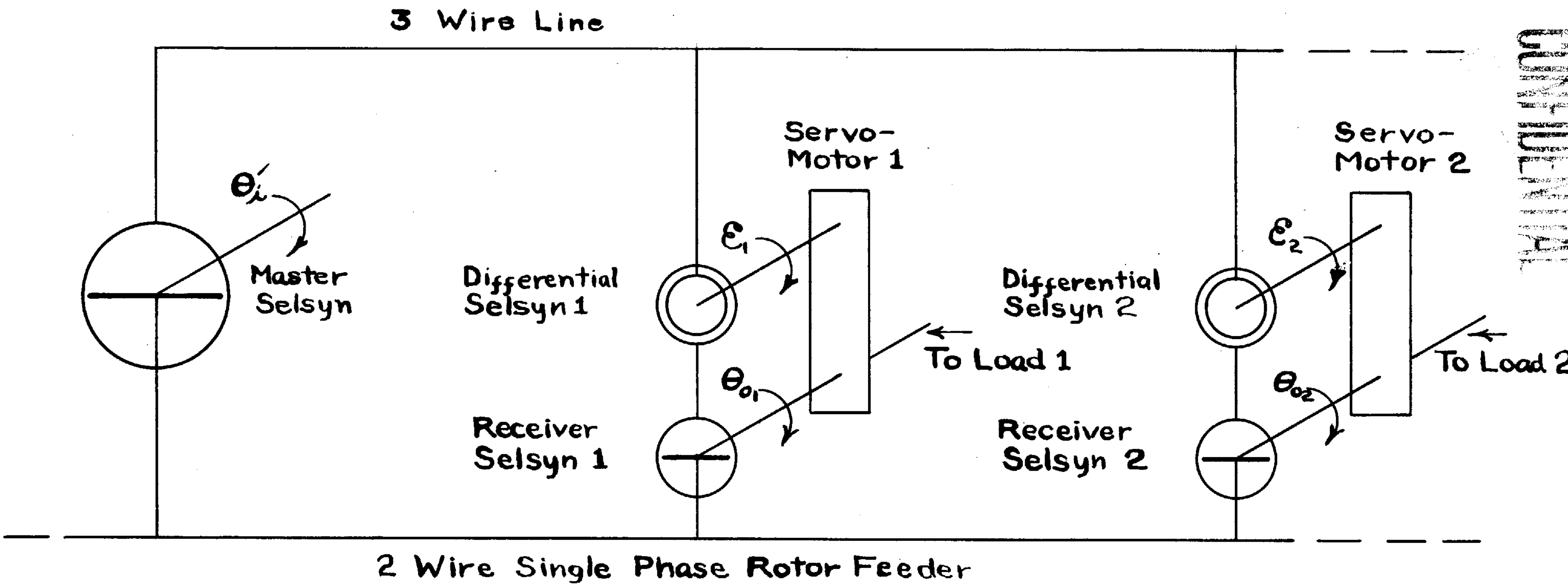
* "Selsyn", General Electric Company's trade name for self-synchronous induction motor systems, will be used exclusively in this thesis. Other words frequently used are "autosyn" (referring to system) and synchro-transmitter or receiver (referring to specific units in system).

Present System



Note: Selsyn Symbols are:
Circles — 3 Winding Stators or Rotors
Bars — 1 Winding (Single Phase) Rotors
All mechanical shafts are drawn 30° from horizontal

Figure 1b

may be regarded as a transmitter itself. However, since it and differential selsyn form a combination that may be considered to replace a receiver selsyn and mechanical differential, it is referred to as a receiver selsyn. Usually the receiver and differential selsyns are smaller than the master selsyn; the difference in size increases with increasing number of receiver servos operated from a master unit.

The operation of the system is as follows: Change the master selsyn angle θ_i'. The differential selsyns (one for each receiver servo) will tend to be deflected and displace the servo error shafts by angles $\mathcal{E}_1, \mathcal{E}_2$, etc. It is the property of a servo-motor that a displacement of its error shaft will cause its output shaft to rotate. Changes in positions of output shafts by angles θ_{o1}, θ_{o2}, etc., equal to change in θ_i' cause the differentials to return to their old equilibrium positions and, $\mathcal{E}_1, \mathcal{E}_2$, etc., again being zero, the servo-motors cease causing the output shafts to move. Thus equilibrium is restored after output shafts have "followed up" the input shaft.

The above is a qualitative explanation of how the system would function if one receiver servo could have no effect on any of the others in parallel with it. Actually, however, if the servo motor output shaft of any receiver servo becomes jammed so that it falls out of synchronism with the master selsyn, the differential and receiver selsyns in combination will act as a transmitter and inject into the system an extraneous signal. This extraneous signal will cause the other receiver servos to be in error and not "know" it, since they cannot distinguish between the extraneous signal and the signal of the master selsyn. The word coercion is used to describe both the phenomena in general and the extraneous signal or error therefrom. In any event the context should make the meaning of the term coercion clear.

The problem of this thesis is: first, to find a selsyn system or the equivalent which will not be subject to coercion; and second, to insure an increase in the stiffness to inertia ratio of any system adopted over that of commercial selsyn links. The reason for the latter objective will become clear later.

Throughout this thesis the selsyn system (or its equivalent) is looked upon as the remote control link which extends the input shafts of the individual servo-receivers and joins them together at the master unit or sending station. Thus, the thesis is a study of the remote controlling of servomechanisms (although admittedly as applying to a particular servo.)

II Servo-Mechanisms

Because this thesis is primarily concerned with the use of selsyns in conjunction with servo-mechanisms, a slight digression into the subject of automatic control is in order.

A servo-mechanism is a system, mechanical or electro-mechanical, consisting of a servo-motor, an input-output comparator (commonly in the form of differential in the case of a mechanical comparator), and a control network whose input is the output of the comparator (ordinarily referred to as the error or error function) and whose output is the physical quantity actuating the servo-motor. (The control network and servo-motor together are referred to as the controller.) If the system functions in such a way as automatically to maintain the input-output deviation as measured by the comparator within designed limits, then this system is a true servo-mechanism (servo for short.) Thus a servo is a "closed cycle" control system.

Servos may have one physical quantity as an input and another as an output providing the input-output comparison is made according to some arbitrary law. For purposes of analysis, it is assumed that the arbitrary law operates external to the servo and hence the input and output have the same dimensions.

The oldest and most common example of a servo-mechanism is the human being when engaged in some control operation such as steering a ship or automobile. Examples of electro-mechanical servos are, among others, those used on automatic die sinking machines in which the relative position of the feeler or contour detector and cutting head is used to actuate the latter in such a manner as to "follow" the master pattern contour.

It is probably apparent that the term "servo-mechanism" is very broad and in specific cases it may be possible to draw a boundary through some part of a "large" servo in such a way as to include a "little" servo within. For example, consider the overall steering mechanism of a large ship. The intricate machinery that connects the gyro-element of the compass and the rudder might be considered as one servo or it may be broken down into the device connecting the gyro-element to the master compass indicator (the compass "follow-up") and the automatic steering mechanism proper. Throughout this discussion, an attempt will be made to keep the servo-mechanism as small as possible by considering in addition to the servo-motor only such equipment as is absolutely necessary in order to meet the definition of a servo as previously stated.

Servo-mechanisms, like all physical systems may be subjected to dynamical analysis. The fundamental approach by analytical methods has been used by many investigators but unfortunately only for the purpose of solving their particular problems and it was left to Hazen, Brown (see bibliography) and possibly a few others to synthesize and extend the work of the earlier men. As a result of Hazen's and Brown's work a generalized approach to automatic control (and particularly servo problems) has emerged.

Without repeating the analytical work here and merely presenting results obtained by the Hazen-Brown approach, the general operational expression for servo deviation or error is:

$$\mathcal{E}(t) = \frac{p^{\ell}(a_m p^m + a_{m-1}p^{m-1} + \cdots + a_1 p + a_0)\,\theta_i(t) + p^{\ell}T_o(t)}{b_n p^n + b_{n-1}p^{n-1} \cdots\cdots + b_1 p + b_0}$$

where: $p = \frac{d}{dt}$

$\theta_i(t)$ = input time function

$\theta_o(t)$ = output time function

$\mathcal{E}(t) = \theta_i(t) - \theta_o(t)$ = error time function i.e. deviation of input and output as functions of time.

$T_o(t)$ = other positive forcing functions beside the controller's — generally simply the negative of the load.

This expression postulates a linear system. So far mechanical and electrical systems have usually been approximated as linear with sufficient accuracy for preliminary investigations of stability.

In many high performance mechanical servos, the load torques and other torques foreign to the controller are negligible in comparison with the inertia torques; hence a preliminary analysis of a proposed servo system often neglects the $T_o(t)$ term. The p^{ℓ} factor is present only in integral or special types of controllers (an integral type controller has a restoring torque or force proportional to the time integral of the error). Since the servos of this thesis do not use integral control, there is no need of retaining the p^{l} factor in the general expression for error. Furthermore, the a_0 coefficient is never encountered in practice (except possibly in case of certain integral controllers) and therefore can be deleted. (It signifies a steady-state error proportional to steady-state input.) Thus the most general expression for error needed in this thesis is:

$$\mathcal{E}(t) = \frac{(a_m p^m + a_{m-1} p^{m-1} + \cdots + a_1 p)\theta_i(t)}{b_n p^n + b_{n-1} p^{n-1} + \cdots + b_1 p + b_0}$$

The stability of a servo is determined by the denominator of the error expression. Routh's stability criteria may be applied in order to determine whether or not a given servo is stable; but only a solution of the equation will determine a servo's speed of response. The speed of response is primarily limited by the least damped exponential of the complimentary function of the general solution, i.e., by the smallest root (or real part thereof) of the denominator's auxiliary equation. (This statement is true for step function inputs because the numerator has no physical effect on account of infinite accelerations, and is a first approximation in other cases.)

The steady-state velocity error (error with constant input velocity after transients have died) is found by setting $p = 0$ in the error as a function of velocity expression and is equal to $\frac{a_1}{b_0}\dot{\theta}_{i\,s.s.}$. <u>This and the smallest denominator root form two very important bases of comparison of servo systems.</u>

III <u>Selsyn Systems for Remote Control, General</u>

The several references in the bibliography will be found to deal with the electrical properties of selsyns and their steady-state analyses along a-c machinery lines. In the work immediately to follow we are little concerned with the electro-magnetic analysis of selsyn systems. What is of concern is the dynamic behavior of such systems from the mechanical view point. The electrical properties are of interest only because they influence the mechanical operation of these systems.

It is found by experience that at low speeds (few percent of synchronous) selsyn systems act as masses connected by springs and dash-pots. For the present, the electro-magnetic damping is of little interest since it is usually augmented by some form of external damping. The damping is to be looked upon as something that can be adjusted after the springs and masses are given values.

In the analysis about to be attempted, the viewpoint will be as follows:

Given a complete servo-mechanism with a mechanical differential for a comparator. Its symbol will be a rectangle with output and input shafts representing the controller and an X representing the comparator. The symbols for input, output and error shaft angles will respectively be $\theta_i, \theta_o, \mathcal{E}$. The servo-mechanism's characteristic equation will for purposes of obtaining remote control expressions be:

$$\mathcal{E} = \frac{N(p)\,\theta_i}{D(p)}$$

where:

$N(p)$ = numerator as a function of p

$D(p)$ = denominator as a function of p

θ_i = input (as a function of time)

Now let the servo have its input shaft extended by a remote control system. This additional system will have an input θ_i', an output θ_i, and an error $\mathcal{E}_s$ equal to $\theta_i' - \theta_i$. The total error of the overall system therefore will be $\mathcal{E}_s + \mathcal{E} = \mathcal{E}'$.

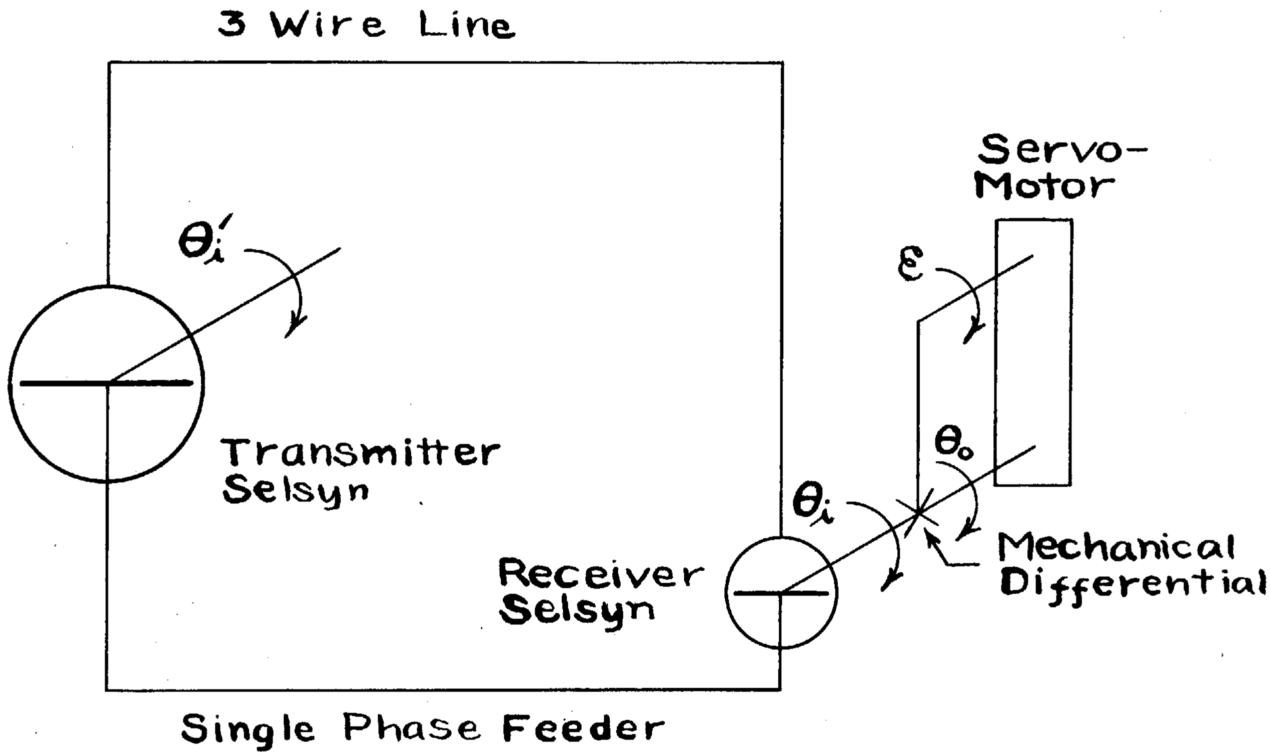
Four remote control systems will be considered.

System I: (fig. 2)

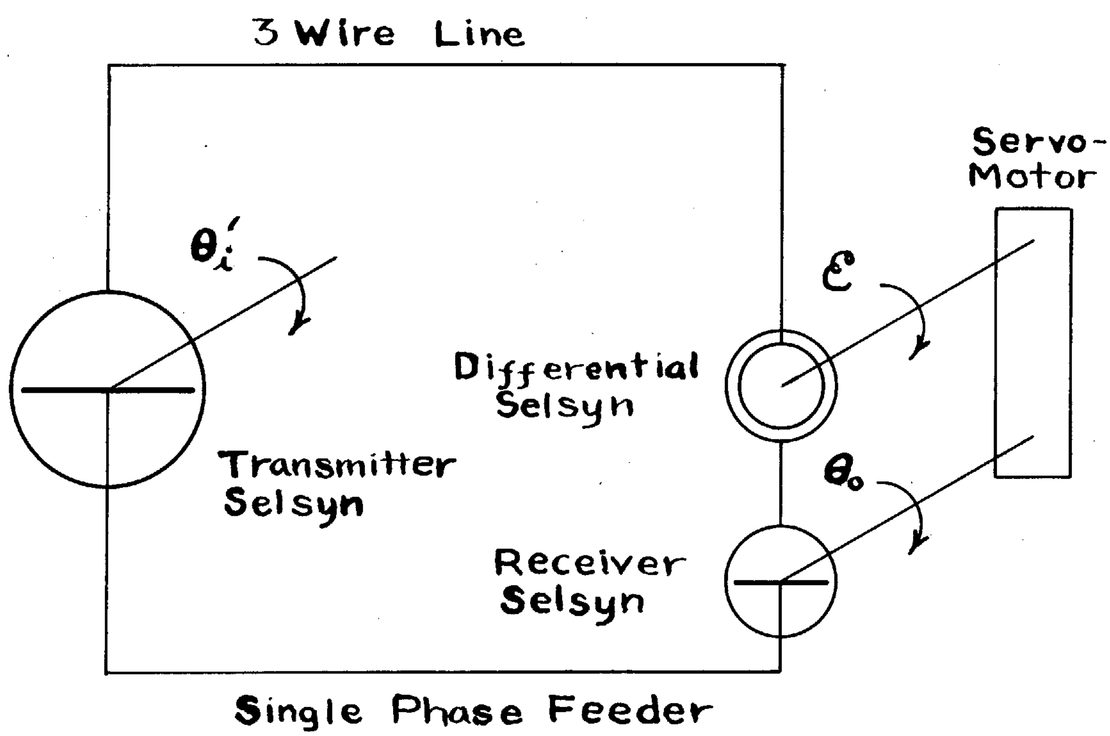
Simply a selsyn transmitter and receiver, with a transmitter driven by system input θ_i' and receiver driving a servo input θ_i. The symbols J, f, k_e will denote respectively the total receiver inertia (including load), total damping (internal plus external), the transmitter-receiver stiffness resulting from their electrical inter-connection.

System II: (fig 3)

Selsyn transmitter, differential, and receiver. Transmitter as above; receiver driven by output of servo θ_o; and differential driving servo's error shaft $\mathcal{E}$. J, f, k_e as above except referring to differential. Additional spring stiffness (added between differential shaft and its case or some fixed object) is denoted by k_s. This k_s is used to



System I
Fig. 2



System II
Fig. 3

increase the stiffness to inertia ratio of the selsyn system under certain conditions as will be seen later. In this and following systems θ_i has no physical reality and is merely a convenient artifice.

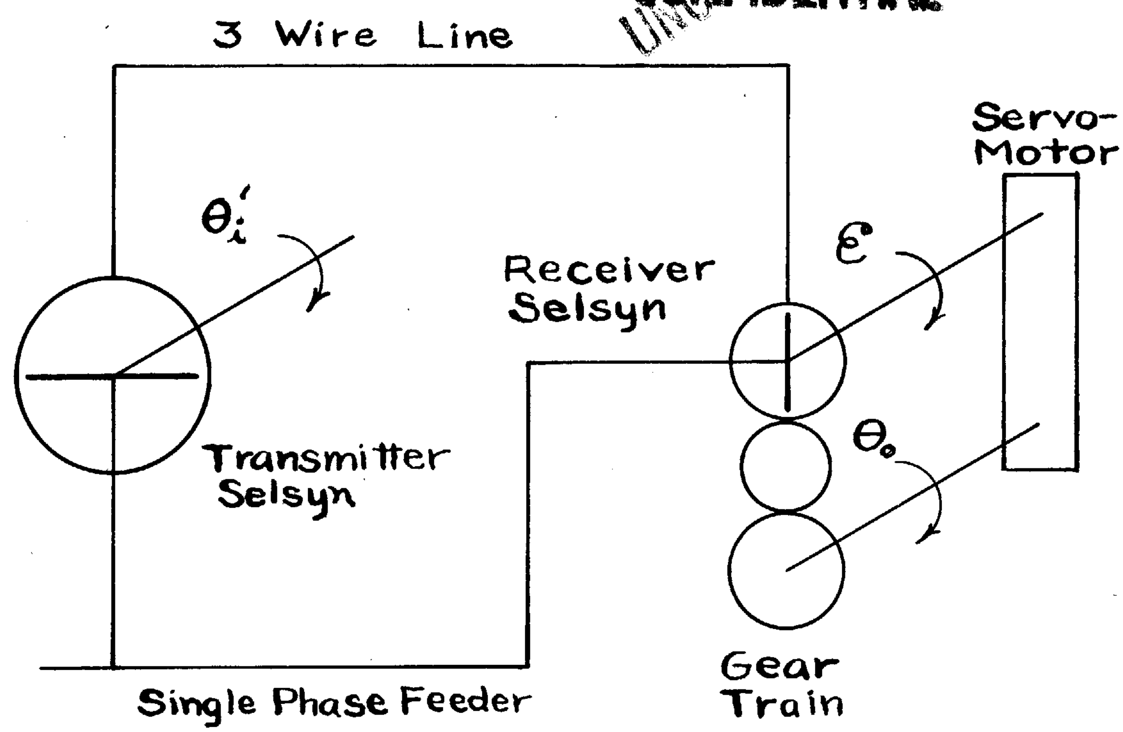
For completeness it might be mentioned that the addition of the receiver rotor's inertia to the output of the servo ordinarily is negligible compared to the load inertia although this is a point that must always be checked. The same applies to the receiver torques.

System III: (fig 4)

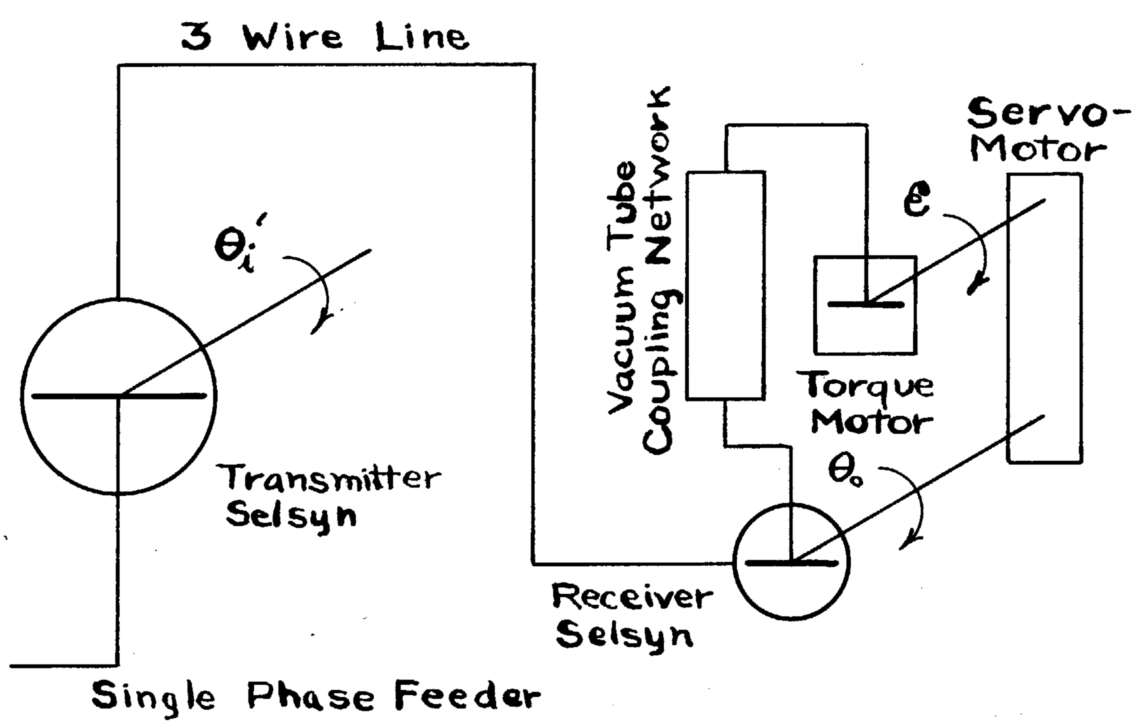
Selsyn transmitter, receiver rotated with servo output. Transmitter as before; receiver's stator driven by output of servo θ_0 ; and receiver's rotor driving error shaft $\mathcal{E}$. J, f, k_e, k_s refer to receiver's rotor. In the analysis of this system, windage torques and the like acting on the rotor of the receiver because of the relative motion of rotor and stator are neglected.

System IV: (fig. 5)

Transmitter, receiver, torque motor and coupling network. In this system, the transformer voltage of the receiver rotor is used to operate a vacuum tube amplifier which in turn operates a torque motor. The torque motor is simply a special design of d-c motor which turns through only a fraction of a revolution. Transmitter is as before; receiver as in system II except rotor winding is feed into coupling network; and torque motor replaces differential selsyn. The property of a receiver used in this manner is to give a voltage proportional to $\theta_i' - \theta_0$



System III
Fig. 4



System IV
Fig. 5

in magnitude and phase 0 or 180 degrees depending on sign of same. (This is true providing $\theta_i - \theta_o$ is small). J, f, k_e, and k_s refer to torque motor; k_e is the torque motor's electrical stiffness resulting from the overall operation of the remote control system.

IV Selsyn Systems for Remote Control, Analysis

System I (see fig. 2):

$$\theta_i = \frac{k_e \theta_i'}{Jp^2 + fp + k_e} \quad \text{(Selsyn operation)}$$

$$\varepsilon = \frac{N(p)\,\theta_i}{D(p)} \quad \text{(Servo operation)}$$

$$\varepsilon = \frac{N(p)}{D(p)} \frac{k_e \theta_i'}{Jp^2 + fp + k_e} \quad \text{(Substitution)}$$

$$\varepsilon_s = \theta_i' - \theta_i \quad \text{(Definition)}$$

$$\varepsilon_s = \frac{(Jp^2 + fp)\,\theta_i'}{Jp^2 + fp + k_e} \quad \text{(Substitution)}$$

$$\varepsilon' = \varepsilon + \varepsilon_s \quad \text{(Definition)}$$

$$\varepsilon' = \frac{[k_e N(p) + (Jp^2 + fp) D(p)]\,\theta_i'}{(Jp^2 + fp + k_e) D(p)}$$

System II (see fig. 3):

$$\varepsilon = \frac{k_e(\theta_i' - \theta_o)}{Jp^2 + fp + k_e + k_s} \quad \text{(Selsyn operation)}$$

$$\varepsilon = \theta_i - \theta_o \quad \text{(Definition)}$$

$$\theta_i - \theta_o = \frac{k_e(\theta_i' - \theta_o)}{Jp^2 + fp + k_e + k_s}$$

$$\text{Or} \quad \theta_i = \frac{k_e\theta_i' + (Jp^2 + fp + k_s)\theta_o}{Jp^2 + fp + k_e + k_s}$$

$$\mathcal{E}_s = \theta_i' - \theta_i \qquad \text{(Definition)}$$

$$\mathcal{E}_s = \frac{(Jp^2 + fp + k_s)(\theta_i' - \theta_o)}{Jp^2 + fp + k_e + k_s}$$

$$\mathcal{E} = \frac{N(p)}{D(p)}\theta_i \qquad \text{(Servo operation)}$$

$$\mathcal{E} = \frac{N(p)}{D(p)}\left[\frac{k_e\theta_i' + (Jp^2 + fp + k_s)\theta_o}{Jp^2 + fp + k_e + k_s}\right]$$

$$\mathcal{E}' = \mathcal{E} + \mathcal{E}_s \qquad \text{(Definition)}$$

$$\mathcal{E}' = \frac{\left\{\begin{matrix} N(p)\,k_e\theta_i' + N(p)(Jp^2 + fp + k_s)\theta_o \\ + D(p)(Jp^2 + fp + k_s)\theta_i' - D(p)(Jp^2 + fp + k_s)\theta_o \end{matrix}\right\}}{D(p)(Jp^2 + fp + k_e + k_s)}$$

Eliminating θ_o by $\theta_i' - \mathcal{E}' = \theta_o$

$$\mathcal{E}' = \frac{(Jp^2 + fp + k_e + k_s)\,N(p)\,\theta_i'}{(Jp^2 + fp + k_s)\,N(p) + k_e\,D(p)}$$

Eliminating θ_o from the expressions for $\mathcal{E}$ and $\mathcal{E}_s$:

$$\mathcal{E} = \frac{k_e\, N(p)\, \theta_i'}{(Jp^2 + fp + k_s)\, N(p) + k_e\, D(p)}$$

$$\mathcal{E}_s = \frac{(Jp^2 + fp + k_s)\, N(p)\, \theta_i'}{(Jp^2 + fp + k_s) N(p) + k_e\, D(p)}$$

System III (see fig. 4)

$$\mathcal{E} = \frac{k_e(\theta_i' - \theta_o)}{Jp^2 + fp + k_e + k_s} \quad \text{(Selsyn operation)}$$

This is identical to the corresponding expression for the selsyn differential of the preceeding analysis. Results for this system are therefore identical to those of the preceeding system.

System IV

$$\mathcal{E} = \frac{k_e(\theta_i' - \theta_o)}{Jp^2 + fp + k_s} \quad \text{(Selsyn operation)}$$

Notice that only k_s appears in the denominator because the electrical "stiffness" is one directional and can exercise no restoring torque.

Going through the same steps as in system II it is found that:

$$\mathcal{E}' = \frac{(Jp^2 + fp + k_s)\,N(p)\,\theta_i'}{(Jp^2 + fp + k_s - k_e)\,N(p) + k_e\,D(p)}$$

$$\mathcal{E} = \frac{k_e\,N(p)\,\theta_i'}{(Jp^2 + fp + k_s - k_e)\,N(p) + k_e\,D(p)}$$

$$\mathcal{E}_s = \frac{(Jp^2 + fp + k_s - k_e)\,N(p)\,\theta_i'}{(Jp^2 + fp + k_s - k_e)\,N(p) + k_e\,D(p)}$$

V Considerations Concerning the Remote Controlling of a Particular Servo

In order to illustrate the usefulness of the general equations derived in the preceeding section, a typical remote control problem is going to be solved. For illustrative purposes a high performance servo-motor is used. When this servo-motor has its output and error shafts rigidly coupled to the input shaft by means of a mechanical differential, its error equation is:

$$\mathcal{E} = \frac{p\,\theta_i}{p + 210}$$

This equation represents better than current practice; it says that any transients will be diminished by a factor of $\frac{1}{\epsilon}$ in $1/210$ seconds and that the steady-state velocity error is $\dot{\theta}_i/210$.

In applying a remote control system to such a servo it is desirable not to injure the performance by more than a factor of two or three because, if the addition of a remote control system reduces performance more than this, there is not much reason for using such a high performance servo in the first place. In other words, a chain is no better than its weakest link -- making a strong link stronger will not increase the strength of the chain.

The present purpose is to determine the conditions our particular servo imposes on the four remote control systems previously discussed in order that the overall performance in each case may remain high. The method will be to substitute the values of $N(p)$ and $D(p)$ for the servo on hand into the expressions for overall error, and from the conditions on steady-state velocity error and speed of response determine what the selsyn system coefficients J, f, k_e, k_s should be.

System I

$$\mathcal{E}' = \frac{[k_e N(p) + (Jp^2 + fp) D(p)]\,\theta_i'}{(Jp^2 + fp + k_e) D(p)}$$

$$\mathcal{E} = \frac{N(p)\,\theta_i}{D(p)} = \frac{p\,\theta_i}{p+210}$$

$$N(p) = p \;; \quad D(p) = p + 210$$

$$\mathcal{E}' = \frac{[k_e p + (Jp^2 + fp)(p+210)]\,\theta_i'}{(Jp^2 + fp + k_e)(p+210)}$$

Steady-state velocity error:

$$\mathcal{E}'_{ss} = \frac{k_e + 210 f \;\; \dot{\theta}_{i_{ss}}}{210\, k_e}$$

Let the ratio of new steady-state velocity error to old be C; then, since in the steady state $\dot{\theta}_i = \dot{\theta}_i'$:

$$\frac{C}{210} = \frac{k_e + 210 f}{210\, k_e}$$

This is one equation relating performance and selsyn system parameters. For others it is necessary to look at the denominator. First divide both numerator and denominator by J:

$$\mathcal{E}' = \frac{\left[\frac{k_e}{J}p + \left(p^2 + \frac{f}{J}p\right)(p+210)\right]\theta_i'}{\left(p^2 + \frac{f}{J}p + \frac{k_e}{J}\right)(p+210)}$$

Write the desired denominator factors as (A may be either real or imaginary):

$$(p+B+A)(p+B-A)(p+210)$$

$$(p^2 + 2Bp + B^2 - A^2)(p+210)$$

Equating coefficients:

$$\frac{f}{J} = 2B$$

$$\frac{k_e}{J} = B^2 - A^2$$

Going back to the steady-state velocity error condition:

$$\frac{k_e}{J}(C-1) = 210\frac{f}{J} = 420B$$

$$\frac{k_e}{J} = \frac{420B}{C-1}$$

Without further specification of desired performance it is impossible to weigh B against C in any rational manner, so, remembering our original objective of not cutting performance down by a factor of more than the order of 2 or 3, let us arbitrarily set $C = 3$ and $B = 100$ Then:

$$\frac{k_e}{J} = \frac{420 \times 100}{3-1} = 21,000/\text{second}^2$$

$$\frac{f}{J} = 2 \times 100 = 200/\text{second}$$

And the denominator factors are:

$$(p + 100 + j\,105)(p + 100 - j\,105)(p + 210)$$

Commercial selsyns have a $\frac{k_e}{J}$ of the order of 2300; until they are considerably improved, this system will not be capable of giving performance commensurate with that of the servo alone.

Systems II and III

$$\mathcal{E}' = \frac{(Jp^2 + fp + k_e + k_s)\,N(p)\,\theta_i'}{(Jp^2 + fp + k_s)\,N(p) + k_e\,D(p)}$$

$$N(p) = p \quad ; \quad D(p) = p + 210$$

$$\mathcal{E}' = \frac{[Jp^3 + fp^2 + (k_e + k_s)p]\,\theta_i'}{Jp^3 + fp^2 + (k_e + k_s)p + 210\,k_e}$$

Ratio of new to old steady-state velocity error is:

$$C = \frac{k_e + k_s}{k_e}$$

Dividing both numerator and denominator by J and substituting Ck_e for k_e+k_s:

$$\mathcal{E}' = \frac{\left(p^3 + \frac{f}{J}p^2 + \frac{Ck_e}{J}p\right)\theta_i'}{p^3 + \frac{f}{J}p^2 + \frac{Ck_e}{J}p + 210\frac{k_e}{J}}$$

Writing the desired denominator as:

$$(p + B)(p + B + A)(p + B - A)$$

Or:

$$p^3 + 3Bp + (3B^2 - A^2)p + B^3 - BA^2$$

Equating coefficients:

$$\frac{f}{J} = 3B$$

$$\frac{c\,k_e}{J} = 3B^2 - A^2$$

$$\frac{210\,k_e}{J} = B^3 - BA^2$$

It appears that there is some ratio of A to B which will make k_e/J a minimum. Eliminating k_e/J from the last two expressions and solving for A (substituting d for 210 to make result general):

$$c\,(B^3 - BA^2) = d\,(3B^2 - A^2)$$

$$-A^2 = \frac{3dB^2 - cB^3}{cB - d}$$

$$\frac{k_e}{J} = \frac{B^3 - BA^2}{d}$$

$$\frac{k_e}{J} = \frac{cB^4 - dB^3 + 3dB^3 - cB^4}{cdB - d^2} = \frac{2B^3}{cB - d}$$

Maximizing with respect to B:

$$(cB - d)\,6B^2 = 2cB^3$$

$$4cB^3 = 6dB^2$$

$$B = \frac{3d}{2C}$$

$$-A^2 = \frac{\frac{27}{4}\frac{d^3}{C^2} - \frac{27}{8}\frac{d^3}{C^2}}{\frac{3}{2}d - d} = \frac{27}{4}\frac{d^2}{C^2}$$

$$A = \pm j\sqrt{3}\ \frac{3}{2}\frac{d}{c}$$

$$\frac{A}{B} = \pm j\sqrt{3}$$

Again let $C = 3$:

$$B = 105$$

$$A = \pm j\,182$$

$$\frac{k_e}{J} = 22{,}000 / \text{second}^2$$

$$\frac{f}{J} = 315 / \text{second}$$

And the denominator factors are:

$$(p + 105)(p + 105 + j\,182)(p + 105 - j\,182)$$

It is interesting to compare this "optimum" denominator with an actual denominator of the servo using a commercial selsyn differential system. With the servo system as set up at the beginning of this thesis, the denominator was:

$$(p + 43.6)(p + 2.2 + j90)(p + 2.2 - j90)$$

System IV

$$\mathcal{E}' = \frac{(Jp^2 + fp + k_s)\, N(p)\, \theta_i'}{(Jp^2 + fp + k_s - k_e) N(p) + k_e D(p)}$$

$$N(p) = p \; ; \quad D(p) = p + 210$$

$$\mathcal{E}' = \frac{(Jp^3 + fp^2 + k_s p)\, \theta_i'}{Jp^3 + fp^2 + k_s p + 210 k_e}$$

Obviously with the particular servo at hand this case is identical to that of systems II and III with the single exception that the necessary k_s here is greater than the previous k_s by k_e.

VI Selsyn Coercion

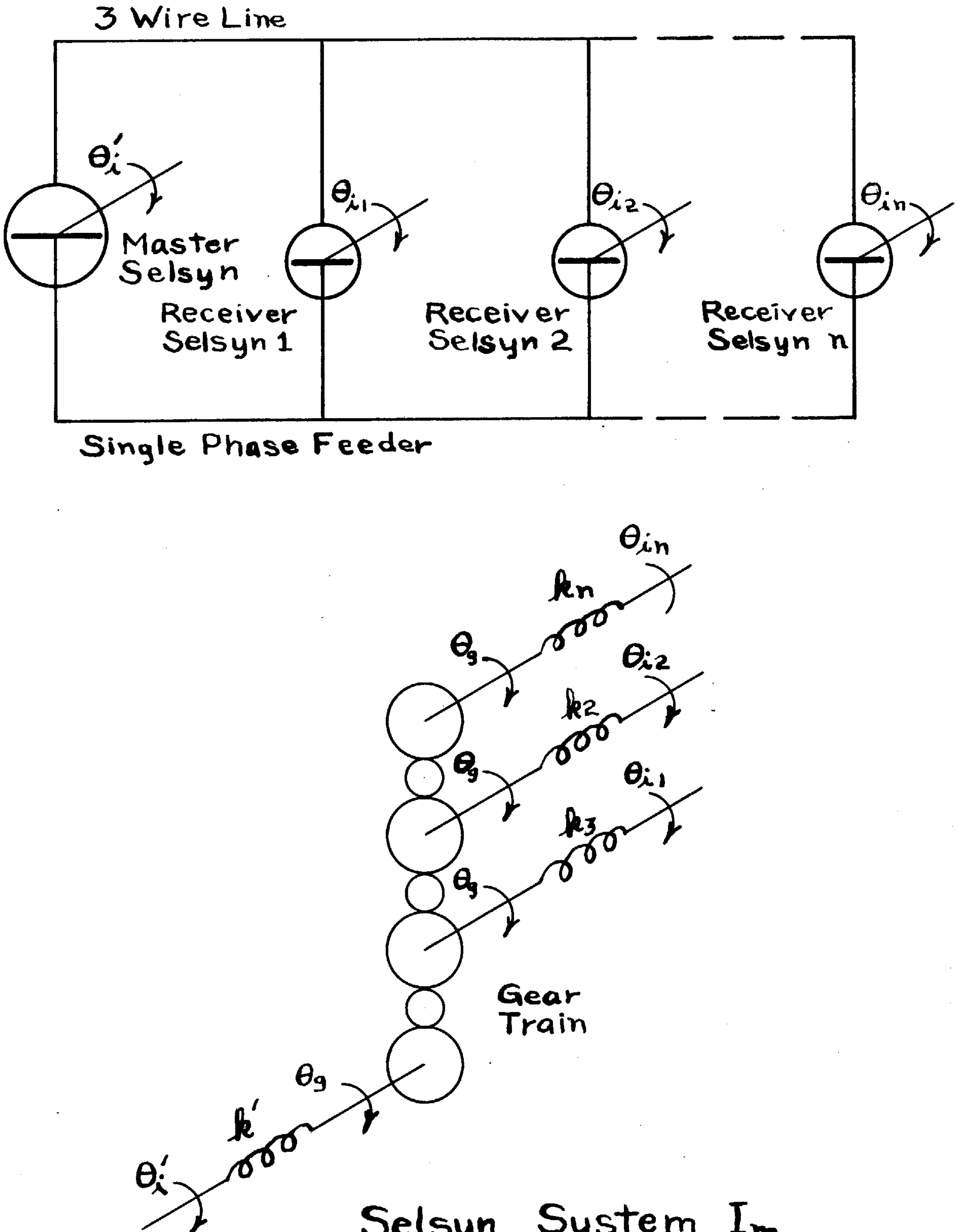
The preceeding discussion has been concerned with the requirements imposed on a selsyn remote control system by the servo-mechanism with which it is used. These are not the only requirements, however, in the case of multiple receiver systems.

The object of multiple receiver systems is to have a single input ultimately followed by several output members. Corresponding to the four selsyn systems previously analysed (systems I, II, III, IV), there are four multiple receiver systems which will be denoted as systems Im, IIm, IIIm, and IVm. The latter systems are identical with the former except several receiver servos are used.

No general electro-magnetic analyses of the selsyn components of multiple receiver systems is known to have been developed up to the present time, probably because of the complexity of the problem. Usually the selsyns used in servo work are single phase "instrument type" machines and, because of their small size, salient pole rotors are generally used except in differentials where of necessity cylindrical rotors are required.

Coercion simply is the term used to convey the general concept that the torque on a particular θ_i (or ε) shaft is a function of not only θ_i' (or $\theta_i' - \theta_o$) but also every other receiver's shaft angle; or that the open-circuit voltage of a receiver rotor in system IV is a function of all the other receiver θ_o S as well as its own $\theta_i' - \theta_o$. The former effect is the sum of the effects of salient poles, and electromagnetic characteristics which would be present even with cylindrical rotors; whereas, the latter effect is solely attributable to salient pole action and would be eliminated if cylindrical rotors were used. Coercion is inherently caused by the receivers acting as small scale transmitters and results from the extraneous signals so introduced.

Reflection and experiment on the part of the author seem to indicate coercion to be so intimately related to torques as to justify the conclusion that coercion is a function of torques only. It would require a thorough analysis and much experimental work to prove this but to justify it qualitatively, consider the selsyn system Im. (Similar notation to that of system I will be used; subscripts $1, 2, 3, \ldots n$ will be used to distinguish the various receivers.) By elementary experiments it is found that a certain k (stiffness factor) may be associated with each selsyn unit. This k is a function of the impedances of rotor and stator circuits, of the impressed voltage, and depends upon how the various units are connected. Within deflection limits of 20 to 30 degrees, k may be considered constant (k varies approximately as the cosine). The mechanical analogue of the selsyn system consists of a group of interconnected flywheels and springs, but limiting our attention to steady-state torques only the springs are important. (In fig. 6, only the springs are shown.) θ_g in fig. 6 is what might be called the angle of transmission and is the angle an unloaded selsyn would assume.



Selsyn System I_m
And Its Mechanical
Equivalent
Fig. 6

Letting:

$\tau' =$ master transmitter torque
$\tau_1, \tau_2 --- \tau_n =$ receiver torques

$\theta_i' =$ master angle

$\theta_{i1}, \theta_{i2}, --- \theta_{in} =$ receiver angles

$k' =$ master stiffness

$k_1, k_2, --- k_n =$ receiver stiffnesses

$\theta_g =$ transmission angle

$\varepsilon_{s1}, \varepsilon_{s2},$ etc. = selsyn errors $(\theta_i' - \theta_{i1}, \text{etc})$

These equations immediately follow:

$$\tau' = \tau_1 + \tau_2 ---- + \tau_n \quad \text{(no motor action)}$$

$$\tau' = (\theta_i' - \theta_g)k', \quad \tau_1 = (\theta_g - \theta_{i1})k_1 \text{ etc.}$$

Solving for ε_{s1}:

$$\varepsilon_{s1} = \theta_i' - \theta_{i1} = (\theta_i' - \theta_g) + (\theta_g - \theta_{i1}) = \frac{\tau'}{k'} + \frac{\tau_1}{k_1}$$

$$\varepsilon_{s1} = \frac{\tau_1 + \tau_2 ----- + \tau_n}{k'} + \frac{\tau_1}{k_1}$$

Defining coercion as the part ~~fraction~~ of ε_{s1} caused by influences other than involved in the coerced receiver, we have:

Coercion of Receiver 1 = $\Delta\varepsilon_{s1}$ (defining $\Delta\varepsilon_{s1}$)

$$\Delta\varepsilon_{s1} = \frac{\tau_2 ---- + \tau_n}{k'}$$

Notice that the coercion is in this case a function of the torques on all receivers except the one in question, and of the stiffness of the master transmitter. The stiffness of the master unit depends on its series impedance and the applied voltage, approximately varying directly as the square of the applied voltage and inversely as the square of the series impedance. This makes the stiffness a function of frequency -- decreasing with increasing frequency, voltage remaining constant. The expression for coercion would seem to hold as long as $\theta_i' - \theta_g$ is within 20 or 30 degrees regardless of the values of $\theta_g - \theta_i$ for the several receivers because only k' appears in the expression for coercion. (Only reason for the limited deflection in first place is the non-linearity of the k's which are approximately cosinusoidal functions of deflection.)

Extension of the above reasoning gives for system IIm and IIIm:

$$\text{Coercion} = \frac{\sum \text{torques of either differentials or receiver units causing coercion}}{\text{stiffness of master unit}}$$

The function of the selsyns in system IVm is slightly different from that in the other systems. Here the receiver selsyn's rotor is attached to the output shaft and its voltage (obtained by virtue of transformer action) is used to operate a torque motor through the intermediary of a vacuum tube coupling network. This voltage is a sinusoidal function of selsyn error and for small error may be regarded as a voltage whose magnitude is proportional to the error and whose phase is determined by the sign of the error (i.e., a 180 degree phase shift occurs when the error passes through zero.) At least this is the kind of operation that would exist if there were no coercion.

At first sight it might appear that a new attack on the coercion aspect of this case is needed. However, reasoning qualitatively, we can associate with θ_g an electrical angle representing the angle which a receiver would have to have for zero voltage. The amount that θ_i' differs from θ_g is still dependent upon the sum of the torques of the receiver rotors independent of whether those torques result from saliency or electrical loading of the rotors. Hence the same expression for coercion would seem to hold.

TO REDUCE COERCION IN SYSTEMS Im, IIm, IIIm, ABOUT ALL THAT CAN BE DONE IS TO MAKE k' LARGE RELATIVE TO THE RECEIVER TORQUES UNDER WORST CONDITIONS. THIS MEANS USING A LARGE MASTER UNIT COMPARED TO THE RECEIVER UNITS. ANY ATTEMPT TO REDUCE COERCION BY INSERTING IMPEDANCES IN SERIES WITH THE INDIVIDUAL RECEIVERS WILL REDUCE THEIR STIFFNESS TO INERTIA RATIOS (WHICH ARE ALREADY TOO LOW FOR OPTIMUM SERVO OPERATION) IN THE SAME PROPORTION.

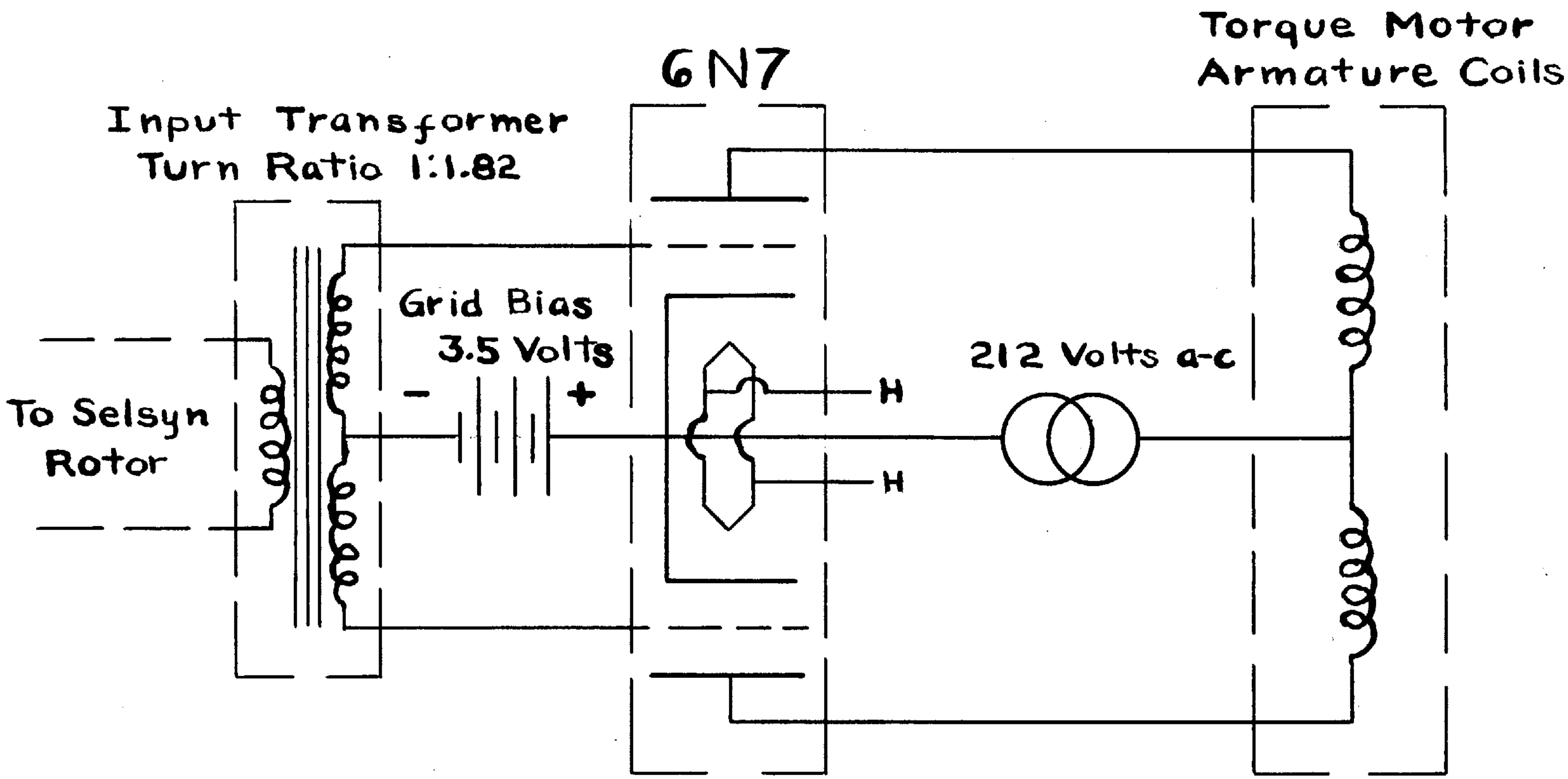
WITH SYSTEM IVm, ON THE OTHER HAND, USE OF SERIES IMPEDANCES WILL REDUCE THE VOLTAGE PER UNIT ANGLE OF ERROR ONLY APPROXIMATELY AS THE SQUARE ROOT OF THE REDUCTION IN COERCION. THIS IS BECAUSE THE RECEIVER TORQUES VARY AS THE SQUARE OF THE TOTAL SERIES IMPEDANCE, WHEREAS, THE VOLTAGES VARY LINEARLY.

VII Vacuum Tube Coupling Network for Systems IV and IVm.

From the start the reader should understand that any coupling network used had to incorporate a minimum of vacuum tubes. In fact the service conditions of the completed servo equipment are such as to make it appear to those laying down the specifications that the number of vacuum tubes per receiver should be limited to one only.

This limitation on number of vacuum tubes immediately narrowed the choice of tube down to twin triodes, pentodes, i.e., down to two valves in single envelope if balancing circuits were to be avoided. Balancing circuits depend upon tube and balancing circuit stability for maintaining their zero adjustment. If the zero adjustment is thrown out, the servo-mechanism will have a corresponding constant deviation of its output. Because this is a highly undesirable condition, balancing circuits were ruled out.

The simplest possible vacuum tube for this application is a triode. From the limited number of twin triodes available, the 6N7 was choosen for preliminary work (because of its high transconductance.) Using this tube in the basic circuit shown in fig. 7, it was found that a transconductance between a-c input voltage and difference in d-c output current of the order of 1800 micromhos was easily obtained. Under these conditions the input impedance was about 4200 ohms.



Basic Vacuum Tube Circuit

Schematic Diagram – Cathode Resistor is used in lieu of Bias Battery in practical Circuits

Fig. 7

The input impedance as well as the transconductance are functions of grid bias and input transformer characteristics. The particular values given above represent the best transconductance that could be obtained by varying the grid bias. The transformer used was the best available at the time but undoubtedly could be bettered. Therefore it is believed that the values of transconductance and input impedance just cited are conservative.

The relationship between transconductance and output impedance has not been determined but it is suspected that for output impedances up to 5 or 6 thousand ohms the decline in transconductance will be negligible (plate resistance of the 6N7 is about 11,000 ohms.)

VIII Torque Motor, General

The torque motor is the device which converts the output of the vacuum tube coupling circuit in systems IV and IVm into mechanical angle on the error shaft of the servo-motor. The angle through which its rotor turns is small -- the specification being plus or minus 10 degrees from neutral with linearity, plus or minus 15 degrees maximum. (By linearity is meant no torque variation with angle if current is kept constant.) Since the input to the torque motor is from the two plates of the vacuum tube, it must have the equivalent of a center tapped armature winding. Actually two separate coils have been used in all models up to the present time.

Since the torque motor is an approach that replaces a selsyn, it should not have greater external dimensions than the medium sized selsyns generally used. Damping in addition to inherent electro-magnetic damping is provided by disks moving in oil kept at constant temperature or some similar scheme.

Up to the present, only rotary type machines have been used, although rectilinear "torque motors" or force motors might fit into the application somewhat better providing the same equivalent characteristics could be obtained. The fundamentals of design are the same in both cases, however, so only rotary type machines will be considered here.

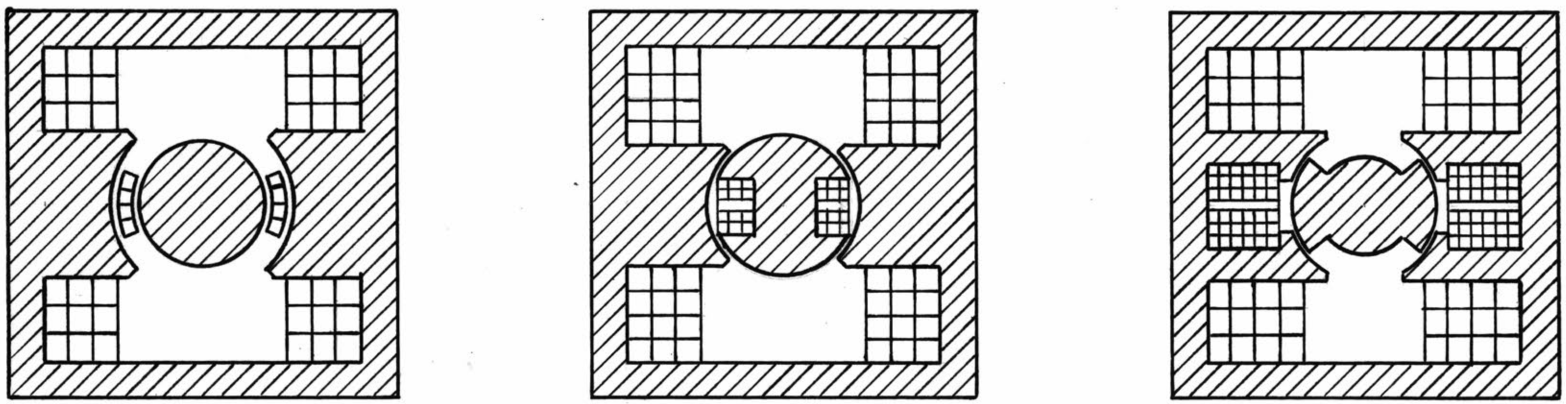
The three general types of torque motors to be investigated are: type 1, D'Arsonval or moving coil motor; type 2, shuttle armature motor (two pole d-c machine with two slot armature); type 3, iron vane motor.

Fig. 8, shows mid-section views of the three types looking along the axes of rotation.

Type 1 is merely a moving coil galvanometer on a large scale. It is only because the iron core stands still that this design has any possibility of competing with the moving iron machines because, although the torque is greatly diminished by the large air gap, the inertia is also reduced and it is simply a question of which is reduced the most.

Type 2 is simply a miniature d-c machine. No commutator or brushes are necessary because the motor only turns a fraction of a revolution, i.e. $\mathcal{E}$ and flexible pig-tails suffice to make armature connections.

Type 3 is a special design. The driving coils, which correspond to the armature coils in type 2, are placed in slots in the poles. These coils lie in a horizontal plane perpendicular to the paper of fig. 8; end turns are slightly bent to one side or the other in order to clear the rotor or vane shaft. The field coils, as in the other two types, are placed around the poles; they are in a vertical plane perpendicular to the paper of fig. 8. This design was conceived as a modification of type 2. In type 2, there is a torque on the poles equal and opposite to that on the armature; in type 3, the roles of armature and poles have been interchanged -- what was the armature is now the poles, what were the poles is now the armature. That this motor should work can be checked on



D'Arsonval
Type 1

Armature
Type 2

Vane
Type 3

Torque Motors
Fig. 8

the basis of conservation of energy. A change in vane angle changes the driving coil flux linkage, inducing a voltage which combined with the driving coil current results in work. Work out, means work in (by the geometry of the design there is no total flux change.)

At the time of the writing of this thesis no working model of a type 3 torque motor exists although one is under construction. Until this model is tried, this type is still in a hypothetical stage.

IX Torque Motor, Basic Design Of A Medium Size Unit

As a specific design problem the allowable dimensions on the torque motors are taken as those of a medium size selsyn and exclusive of shafts are 3 inches x 3 inches x 4 inches long. The last dimension limits the rotor and stator parts of the magnetic circuit to about 1½ inches of axial length -- the other 2½ inches going into bearings, springs for providing restoring torques, damping devices, and coil end turns.

Type 1 (D'Arsonval)

On the basis of previous experience, the flux density in the air gap of the type 1 machine will approximately vary inversely as the gap length and be independent of the radius of the stationary core providing that the radius is less than about 1¼ inch with 1/8 inch gap. This is based on the assumption that the magnetomotive force required for the iron is negligible in comparison with that required for the air gap, and that the available mmf is independent of the radius up to the limit cited. The latter is true because the winding space for the field coils is relatively independent of core radius and the permissible power dissipation is primarily fixed by the external surface area.

The electro-magnetic torque of the coil is:

$$\tau = NI\frac{\partial\Phi}{\partial\theta}$$

where:

τ = electromagnetic torque
NI = ampere turns of moving coil
Φ = coil flux linkage
θ = coil angle
$\frac{\partial\Phi}{\partial\theta} = \beta A$, β = air gap flux density, A = coil area.

In the preliminary "order of magnitude" calculation mechanical clearances and coil form thickness will be neglected. The angular width of the coil is limited by pole arc and desired angle of rotation; assuming 90 degree pole arc, allowing plus or minus 20 degrees for rotation and 5 degrees on each edge for fringing, the angular width of the coil should be about 40 degrees.

The flux density constant for this motor can be determined from the fact that on a similar frame size 10,000 gauss in a 0.01 inch air gap was obtained; i.e., $B = \frac{100}{l_g}$ gauss where l_g is the gap length in inches.

No. 44 awg is the smallest commercial size of magnet wire readily available. No. 50 is manufactured but is difficult to handle and procure. In winding a coil of this type an overall space factor of 2 or so is correct (space factor is the ratio of total winding area to Nd^2, N being number of turns and d, diameter of bare copper.) The winding area is $l_g r \frac{40}{57.3}$ where r is the mean radius of the coil. Since No. 44 wire is very nearly 0.002 inch in diameter, we have:

$$N = \frac{l_g r\, 40}{2\times(2\times10^{-3})^2\, 57.3} = 8.74\times10^4\, l_g r$$

$$\frac{\tau}{I} = \frac{8.74\times10^4\, l_g r\, 100}{10^4\, l_g}\times 2r\times1.5\times2.54^2$$

$$\frac{\tau}{I} = 1.69\times10^4\, r^2\ \frac{\text{dyne centimeter}}{\text{milliampere}}$$

Assume the average specific gravity of the coil to be one half that as if solid copper and neglect the inertia of the end connections and shaft by merely lumping it with the connected inertia of the servo motor error shaft. The latter has a magnitude of roughly 0.4 inch squared ounces. Thus the total inertia of moving coil plus load is:

$$J = 2 \times 1.5 \times l_g r \times \frac{40}{57.3} \times 2.54^3 \times \frac{8.82}{2} \times r^2 \times 2.54^2 + 0.4 \times 2.54^2 \times 28.35$$

$$J = 973\, l_g r^3 + 73.1 \text{ gram centimeters}^2$$

To maximize the torque/milliamp to inertia ratio the quantity $\cancel{\dfrac{r^2}{l_g r^3 + \frac{73.1}{973}}}$ must be a minimum. Obviously l_g should be as small as possible. Maximizing with respect to r, we find:

$$2\,(l_g r^3 + .075)\, r = r^2\, 3\, l_g r^2$$

$$l_g r^3 = 0.150$$

Let l_g be arbitrarily set equal to 0.3r, then $r^4 = \frac{.150}{.3}$, $r = 0.84$. Actually such a large r cannot be used in the frame size specified, hence the largest possible r which is about 5/8 inch is used. On this basis:

$$\frac{\tau}{I} = 0.66 \times 10^4 \ \frac{\text{dyne centimeter}}{\text{milliampere}}$$

$$J = 44.3 + 73.1 = 117.4 \ \frac{\text{dyne centimeter second}^2}{\text{radian}}$$

$$\frac{\tau}{IJ} = 56.2 \ \frac{\text{radian}}{\text{second}^2 \text{ milliampere}}$$

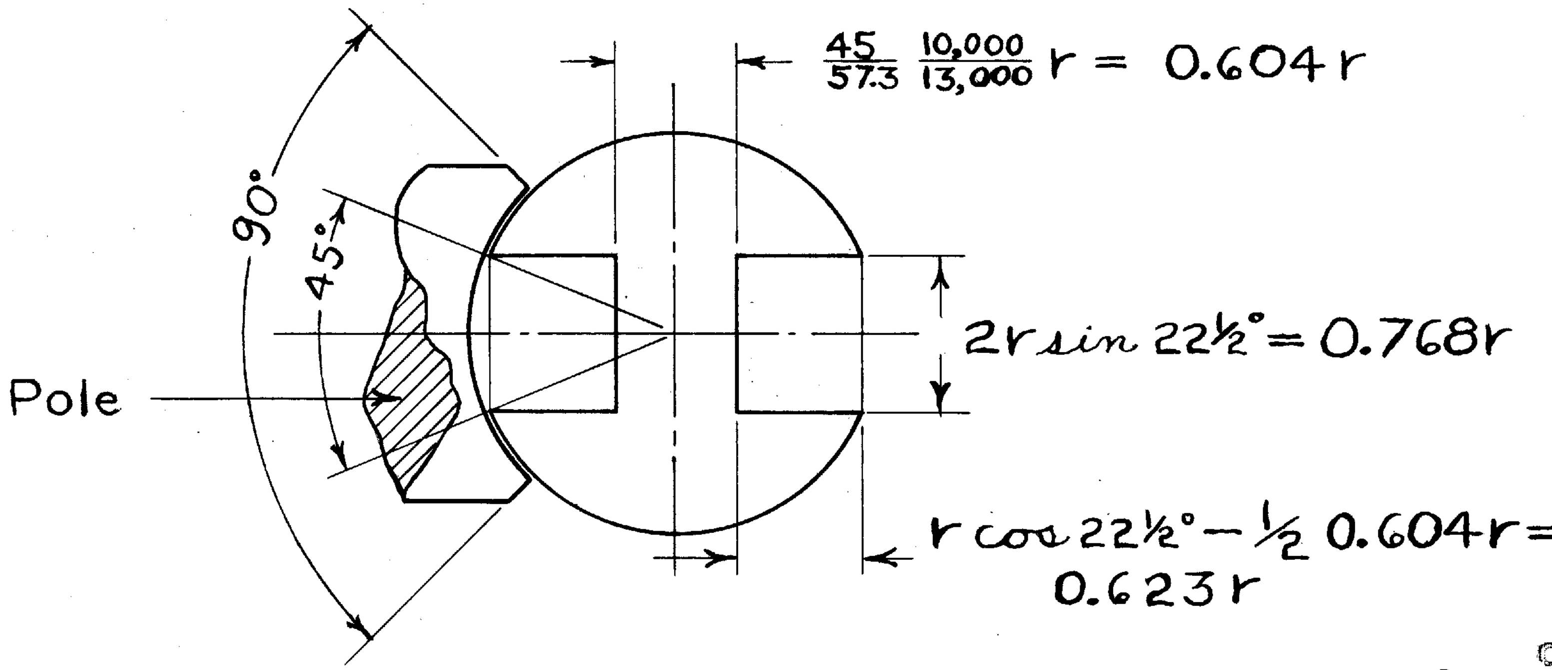
Since there are to be 2 coils in the winding area figured above, the torque/milliampere to inertia ratio will be one half of 56.2 if the calculation is to be made on a per coil basis.

Type 2 (Armature)

For both type 2 (armature) and type 3 (vane) the air gap flux density will be considered constant at 10,000 gauss since the gap length is small and the flux is limited by saturation of the iron.

In armature design, the area of winding space is limited by the angle of rotation in two ways: 1) by width of slot; 2) by depth of slot (the slot cannot be so deep as to reduce the center web cross section below that necessary to transmit the total flux per pole when the armature is turned to its extreme angle.) The first limitation is obvious; the second is related to the desired flatness of the torque-angle curve. By experience it is found that if the flux density in the web is kept below 13,000 gauss maximum, the torque-angle curve will be satisfactory. Usually the torque-angle curve will be practically flat for about two-thirds of the range of rotation. Reference to fig. 9, will clarify the calculation of winding area.

Making calculations corresponding to those in type 1, we find (using a space factor of 2½ to allow for extra slot insulation needed because the coils are surrounded on three sides by iron):



Winding Area $= .768r \times .623r = 0.478r^2$

Type 2 Torque Motor Armature

Fig. 9

$$N = \frac{0.478 r^2}{2\tfrac{1}{2} \times (2\times 10^{-3})^2} = 4.78 \times 10^4 r^2$$

$$\frac{\partial\phi}{\partial\theta} = 1.5 \times 2 \times r \times 6.45 \times 10^4 = 19.37 \times 10^4 r$$

$$\frac{\tau}{I} = \frac{\frac{\partial\phi}{\partial\theta} N}{10^4} = \frac{4.78\times 10^4 r^2 \times 19.37 \times 10^4 r}{10^4} = 92.5 \times 10^4 r^3$$

Consider the armature to be a solid iron cylinder of radius r and length $2\frac{1}{2}$". (The extra length is used because a "cradle type" of armature was adopted in order to facilitate winding. The weight of copper in the winding does not entirely make up for the removal of the iron in the slots but the approximation is good enough for order of magnitude purposes.) Thus the inertia is:

$$J = 2.54^5 \frac{2.5\,\pi\, r^4}{2} 7.85 + 73.1 = 3260 r^4 + 73.1$$

Maximizing $\dfrac{r^3}{r^4 + \frac{73.1}{3260}}$:

$$(r^4 + .0224) 3r^2 = 4r^3 r^3$$

$$r^4 = .0672 \quad r = 0.51''$$

$$\frac{\tau}{I} = 12.25 \times 10^4 \; \frac{\text{dyne centimeter}}{\text{milliampere}}$$

$$J = 219 + 73 = 292 \; \frac{\text{dyne centimeter second}^2}{\text{radian}}$$

$$\frac{\tau}{IJ} = 472 \; \frac{\text{radian}}{\text{second}^2 \text{ milliampere}}$$

This motor is better by a factor of 8 than the moving coil machine. In actual practice a machine of not exactly optimum dimensions gave 162 rad/sec^2-milliampere.

Type 3

In type 3 (vane) torque motors the armature windings or driving coils are placed in slots in the poles. Frame size limitations cause the winding space to decrease with increasing armature or vane radius. This is true because the pole width at points removed from the pole face is fixed by frame dimensions and field coil size, (the field coil is constant in size for a given air gap if the flux density is kept constant) but the slot in the pole must be narrow enough to leave sufficient iron for the transmission of the air gap flux (plus leakage.) As the radius increases, the air gap flux increases because of increasing area, and therefore the slot width must be decreased with increasing radius. Furthermore, the length of pole and hence length of slot must decrease with increasing radius. By making several scale drawings of this type of torque motor, the following function was empirically derived:

Winding area $= 1\frac{1}{4}(1-r)^2$ inches^2

where r is vane radius in inches

In the winding of this machine no. 40 wire having a bare diameter of 0.003145 inch is to be used instead of no. 44. The purpose of this is to compensate for the longer coil length required in this design and make the impedance of this machine comparable with that of type 2. Making calculations similar to the previous ones:

$$N = \frac{1\frac{1}{4}(1-r)^2}{2\frac{1}{2} \times (3.145 \times 10^{-3})^2} = 5.06 \times 10^4 (1-r)^2$$

$$\frac{\partial \phi}{\partial \theta} = 1.5 \times 2 \times r \times 2.54^2 \times 10^4 = 19.37 \times 10^4 r$$

$$\frac{\tau}{I} = \frac{19.37 \times 10^4 \times 5.06 \times 10^4 (1-r)^2 r}{10^4}$$

$$\frac{\tau}{I} = 98 \times 10^4 (1-r)^2 r \quad \frac{\text{dyne centimeter}}{\text{milliamp}}$$

The vane consists of a solid piece of steel which may be considered to be built up of a cylinder of radius 5/8 r, and two 80 degree segments of outside radius r and inside radius 5/8 r. (See fig. 8.) The total inertia is:

$$J = \frac{1.5\pi r^4 \left[\left(1-(5/8)^4\right)\frac{2 \times 80}{360} + 5/8^4\right]}{2} 7.85 \times 2.54^5 + 73.1$$

$$J = 1030 r^4 + 73.1 \text{ gram centimeter}^2$$

Maximizing $\dfrac{(1-r)^2 r}{r^4 + \frac{73.1}{1030}}$:

$$(r^4 + .071)\left[-2r(1-r) + (1-r)^2\right] = (1-r)^2 4r^4$$

$$-r^5 + 3r^4 + .213 r = .071$$

$r = .267$ inches for maximum τ/IJ

$$\frac{\tau}{I} = 14.0 \times 10^4$$

$$J = 5.2 + 73.1 = 78.3$$

$$\frac{\tau}{IJ} = 1790 \ \frac{\text{radian}}{\text{second}^2 \text{ milliampere}} \text{ for } r = 0.267$$

The construction of a machine with such a small r is difficult. The machine under construction actually has an r of ½ inches for which $\frac{\tau}{IJ} = 892 \frac{\text{radian}}{\text{second}^2 \text{ milliampere}}$ or

on a per coil basis:

$$\frac{\tau}{IJ} = 446 \frac{\text{radian}}{\text{second}^2 \text{ milliampere}}$$

This motor, even after making allowance for construction facilities, is twice as good as the type 2 (armature) motor.

X System IV_m Considered As A Whole

By experiment it has been found that the open circuit voltage output of the selsyns of system IV_m is of the order of 23 volts per radian of $\theta_i' - \theta_o$. This output is obtained with sufficient impedance in the receiver lines to make coercion negligibly small. Under these conditions the impedance of the receiver rotor is about 1000 ohms.

Using the vacuum tube coupling network previously described (with its input impedance of 4200 ohms and trans-conductance of 1800 micromhos) in conjunction with the above "coercionless" selsyn system, an input to the torque motor of about 33 milliamperes per radian is obtained.

If a type 3 torque motor with a half inch vane radius is used, it should be possible to obtain a stiffness to inertia ratio of the order of 15,000 per second squared (compare with 2300 for commercial selsyns subject to coercion.)

XI Conclusion

None of the systems of remote control at present is capable of meeting the requirements imposed by the particular servo considered. System IV_m comes nearest to doing so and offers more immediate promise than any of the other systems. Under the specifications laid down, system IV_m is capable of higher performance than any of the other systems regardless of whether coercion is considered or not.

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