

Characterizing the Performance of Footballs

by

Abdiel Mireles

Submitted to the
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ABSTRACT

This thesis aimed to characterize the performance of footballs by establishing a new metric based on the variation in flight paths in similar free kicks. The variation due to the aerodynamic properties could be described by a single metric and be used in accreditation testing as a means to gauge the performance of the football. Football players could also use this metric to their advantage on the field, as opposed to less tangible measures like the coefficient of lift. The plausibility for such a parameter was established by creating a mathematical model describing the flight path and simulating results when aerodynamic parameters vary.

Additionally, this study establishes the plausibility of creating a low cost transportable setup to test footballs as they are kicked. Currently, testing of footballs includes wind tunnel tests and expensive and bulky camera systems. Plausibility is established by proving that translational velocity can be measured with light, portable setups. This was accomplished by using commercial grade cameras in conjunction with computational tools.

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I. Introduction

Sports technology continues its advances in professional sports equipment to improve the performance of players. Improvement in football equipment is limited to the performance of players' boots and official match footballs. Advances in manufacturing and materials have allowed for the engineering of alternative designs in the surface and seams of footballs. Features in these designs alter the aerodynamic behavior of the footballs. FIFA, the accreditor of footballs for professional leagues has standards and conducts tests for seven parameters: circumference, roundness, rebound, water absorption, weight, pressure loss, shape and size retention, and balance.^[1] None of these parameters account for the flight behavior of the football. Other studies as those performed by Passmore et. al. have been able to experimentally determine flight characteristic parameters like coefficient of drag for stationary footballs^[2]. Rogers et. al. experimentally confirmed these measurements by using real kicks that allowed them to model the flight path and verify this with experimental kicks^[3]. The coefficient of drag, although beneficial in modeling the flight path and properties of the football, gives a player minimal information to use in a real kick.

Players are the most interested in the behavior of the football. By mastering the flight path of the football players can gain an advantage by creating a desired kick difficult to stop. On the other hand, a goalkeeper or defender can predict the trajectory of a kick and adapt accordingly. Learning the coefficient of drag does not aid players in making these predictions. Additionally, Passmore et. al. conducted their tests in a wind tunnel^[2] and Rogers et. al. conducted their experiments inside a laboratory with an experimental robot^[3]. Although these setups are very useful in the studies, they do not necessarily reflect real game conditions. The purpose of this study is to create a new metric that can characterize the flight path and allow for the assessment of acceptable flight behavior in real game conditions. In order to validate the model with experimental data, this thesis also attempted to establish the viability of creating a low cost, portable experimental setup to capture flight path characteristics. This was done by developing a mathematical football flight model of a free kick based on key initial characteristics of a flight path including velocity, direction, and spin rate. The final position of the football and its deviation from a simple kinematic flight path was used to establish a new parameter. The second part of this thesis was done by using commercial grade, low cost, and low transport components to measure the initial flight conditions of a football and establishing the accuracy of the measurements.

II. Mathematical Flight Path Model

We must understand the aerodynamics of the football flight path in order to establish a new metric to characterize it. One way to understand and compare the flight trajectory was to create mathematical model. Free kicks are the most studied kicks in football due to their aesthetics and high level of skill required to achieve. In addition, free kicks are the most controlled kick in football since the football is stationary, a barrier in its trajectory with known parameters exists, and the intended target is known. Thus, we chose to use the free kick in our model.

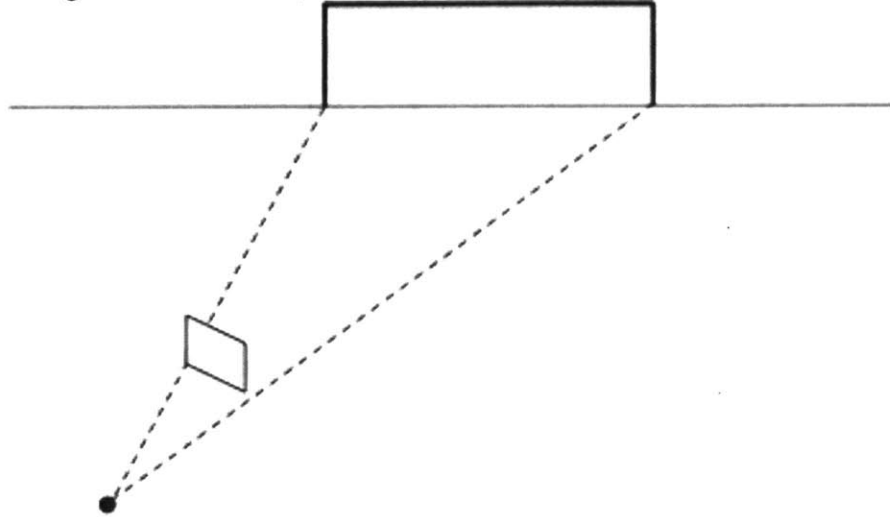


Figure II-1: A diagram depicting the basic setup for a free kick in football. A defensive wall composed of a varying number of field players obstructs the football's path towards the goal. The goalkeeper decides the position and size of the goal.

There are four major aspects to take into consideration when describing the flight path of a free kick: the initial conditions of the flight path i.e. the direction, velocity, and spin rate at which the football is kicked, the forces acting on the football during its flight, the defensive wall, and the goal itself. Figure II-1 shows the basic setup for a free kick including the defensive wall and the goal. A defensive wall is intended aid the goalie by obstructing the ball's path and is usually made up of two to five players depending on the position of the kick. In order to score, a player must kick the football in such a way that its flight path clears the defensive wall and hits the intended target within the goal.

II.1 Equations of motion

A football's flight path can be modeled by first order differential equations that determine the position of the football in time. These differential equations are dependent on the forces the football is subjected to and rely on the initial conditions of the football flight path to be solved. Initial conditions include velocity and direction of the kick, as well as the spin rate. Initial conditions are taken immediately after the football loses contact with the foot and are depicted in Figure II-2a. After the football loses contact with the foot, its motion is subject to different forces as shown in the free body diagram of the resulting forces on the football depicted in Figure II-2b.

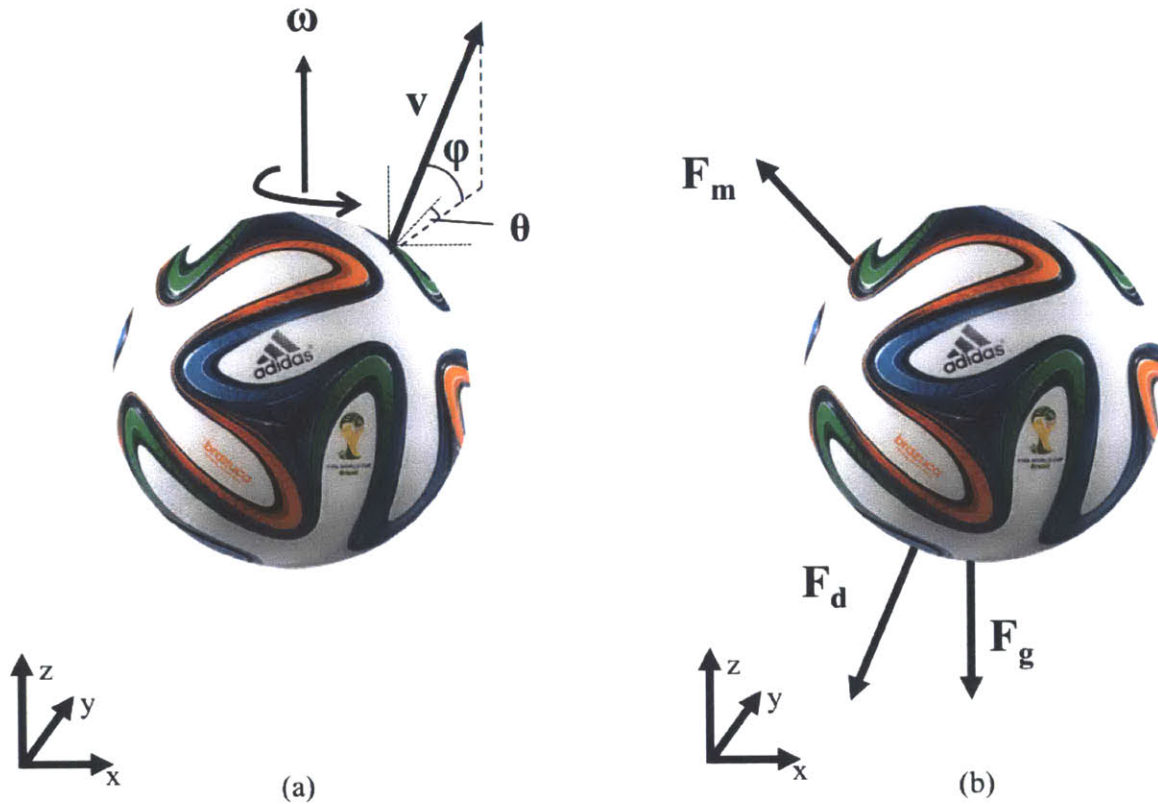


Figure II-2: After impact with the foot, the football's motion can be characterized at any point in time by its rotational velocity ω and its translational velocity v as shown in (a). When in flight, the football is subject to different forces as depicted in (b). Lift force, F_m , (also known as the Magnus effect in this case) is due to the football's rotational velocity and its direction perpendicular to both translational and rotational velocities. Drag force, F_d , is due to the air friction and its direction is opposite the translational velocity. Gravitational force, F_g , is due to the mass of the football and always points towards the ground.

If we take the origin of our coordinate plane to be the spot at which the free kick is taken, coordinates x, y, z describe the lateral position of the football, distance from the kick, and the football elevation, respectively, and ω describes the rotational speed of the football. Velocity vector v can be described as $v = v[\cos\phi \sin\theta, \cos\phi \cos\theta, \sin\phi]^T [x \ y \ z]^T$ where v is the magnitude of speed of the football, θ is the angle given by the direction of the kick relative to the y - z plane, and ϕ is the angle given by the direction of the kick relative to the y - x axis. The equations of motion of the football can be determined by using the free body diagram illustrated in Figure in Figure II-2b. If we model the football as a point mass, the sum of forces yields:

$$\mathbf{F} = \mathbf{F}_m + \mathbf{F}_d + \mathbf{F}_g \quad (1)$$

where F_m is the lift force due to the rotation of the football, F_d is the drag force due to air friction, and F_g is the force due to gravity.

Force components F_m and F_d are a result of the aerodynamic properties of the football. Drag force is a result of the friction between the air being displaced and the surface of the football. The direction of this force is the opposite of the direction of velocity. If the direction of the velocity is described by direction vector $\mathbf{u} = [\cos\varphi \sin\theta, \cos\varphi \cos\theta, \sin\varphi]^T [\mathbf{x} \ \mathbf{y} \ \mathbf{z}]^T$, the drag force can be described by the following equation:

$$\mathbf{F}_d = -\frac{1}{2} C_d \rho A v^2 \mathbf{u} \quad (2)$$

where ρ is the density of the air, A is the cross sectional area of the football, and C_d is the coefficient of drag.

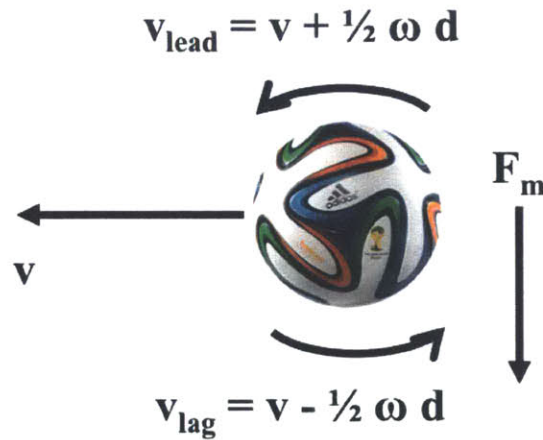


Figure II-3: Due to the rotation of the football, one side of the football is traveling at a faster velocity than the other side. This difference in velocities also creates a difference in pressure. This is known as the Magnus effect and results in a lateral acceleration. This acceleration is perpendicular to both the translational velocity and the rotational velocity. This direction can also be described as pointing from the leading side to the lagging side of the football.

Known as the Magnus effect, the lift component of force is a result of a difference in pressure on either side of the ball cause by the rotation of the football. A pressure differential creates a force between the two sides leading to a lateral displacement. Rotational speed makes the football travel faster on the leading side than on the trailing side. The difference is equal to the rotational speed times the diameter of the ball as illustrated in Figure II-3. For the purpose of this model, we will assume that rotational velocity has a constant magnitude and is spinning in the positive direction parallel to the y - z plane. That is to say, the model will assume a curled kick struck directly on its side. This effect is present in many other phenomena and has been studied extensively. It can be described by the following equation:

$$\mathbf{F}_m = \frac{1}{2} C_l \rho A v^2 (\mathbf{n} \times \mathbf{u}) \quad (3)$$

where ρ and A are, again, the air density and the cross sectional area of the football, respectively, C_l is the coefficient of lift, and \mathbf{n} is the unit vector describing the direction of rotational velocity. Due to our previous assumptions on the ball spin, rotational velocity only has a component in the z-direction and its unit vector is defined as $\mathbf{n} = [0, 0, 1]^T$.

Force component F_g is simply the force due to gravity acting on the mass of the football and can be expressed as

$$\mathbf{F}_g = -mg\mathbf{n}. \quad (4)$$

The sum of forces is related to the football's acceleration by the equation

$$\mathbf{F} = m\ddot{\mathbf{r}}. \quad (5)$$

where m is the mass of the football and $\ddot{\mathbf{r}}$ is the acceleration of the ball. Combining equations (1), (2), (3), and (4) we find that

$$\mathbf{F} = \frac{1}{2}C_l\rho Av^2 (\mathbf{n} \times \mathbf{u}) - \frac{1}{2}C_d\rho Av^2 \mathbf{u} - mg \mathbf{n} \quad (6)$$

Combining equations (5) and (6) and evaluating the direction vector expressions we find an expression for the acceleration with three components, one for each x, y, z direction. Each of the three components yields an equation of motion for each of the coordinate directions. The result is the following system of ODEs:

$$\ddot{x} = -\frac{1}{2}C_l\rho Av^2 \cos\theta \cos\varphi - \frac{1}{2}C_d\rho Av^2 \sin\theta \cos\varphi \quad (7)$$

$$\ddot{y} = \frac{1}{2}C_l\rho Av^2 \sin\theta \cos\varphi - \frac{1}{2}C_d\rho Av^2 \cos\theta \cos\varphi \quad (8)$$

$$\ddot{z} = -mg \quad (9)$$

These equations of motion can be solved numerically using computational tool MATLAB by specifying values for the constants and initial conditions. A script was created in MATLAB such that the constants could be altered depending on the game or testing conditions as well as the dimensions of the football being tested. For the purposes of this study, the script approximates the coefficients of drag and lift as those found in the study done by Passmore for one of the FIFA approved balls.^[2]

II.2 Defensive wall

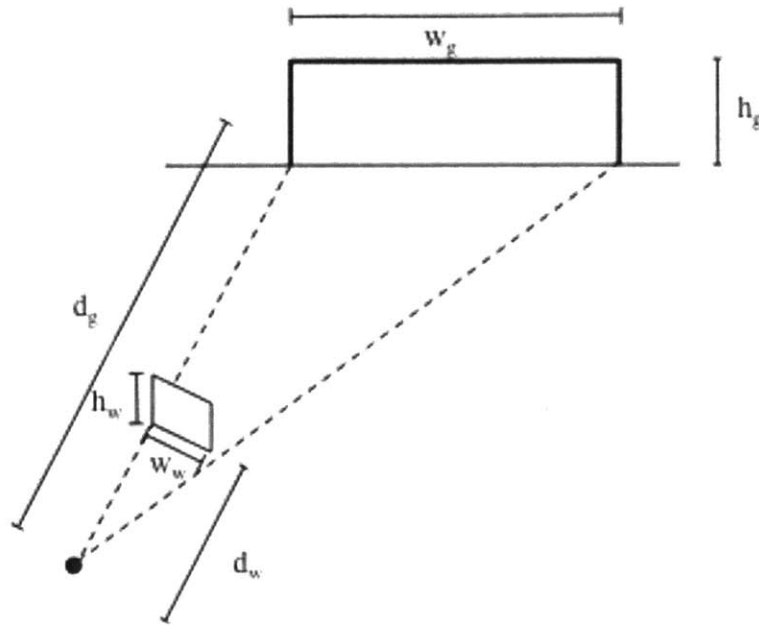


Figure II-4: A diagram depicting the basic setup for a free kick in football. The defensive wall in the script is perpendicular to the path from the ball to the nearest post. Distance from the ball to the wall, d_w , width of the goal w_g , and height of the goal h_g were chosen as those set by FIFA regulations. ^[1] The height of the wall was approximated as two meters and the width as $.25w_g$. Distance from the ball to the goal is an input to the script.

The defensive wall was determined by calculating a direct line from the position of the free kick to the nearest post. Although the position and width of the defensive wall changes at the goalkeeper's discretion and depending on the position of the free kick, we have chosen to cover the post nearest to the ball as one of the bounds and the width to be a quarter of the goal's width. An upper bound on wall height was approximated as two meters since players are allowed to jump to block the football. The orientation of the wall is perpendicular to the line connecting the position of the free kick to the nearest post as shown in Figure II-4. Another obstacle in real game conditions during a free kick is the goalkeeper's presence. Since the goalie does not affect the football flight path, the goalkeeper was left out of the mathematical model for simplification.

II.3 Simulation

A MATLAB script was created such that free kicks could be modeled. The script takes the position of the free kick as well as the initial conditions of the kick: \mathbf{v} , $\boldsymbol{\omega}$, $\boldsymbol{\theta}$, and $\boldsymbol{\varphi}$ as described in Figure II-2 (a) as well as the game or testing conditions like the football's dimensions and the air density. The script then calculates the position of the wall and the kick is simulated. A flight path is then plotted. Final position is also calculated at either the first bounce or the goal line, whichever one occurs first. This final position is given in x,y,z coordinates and are used to determine whether or not the football hits the intended target. Alternatively, the script can take the position of a free kick as the only input. It then determines different combinations of \mathbf{v} , $\boldsymbol{\omega}$, $\boldsymbol{\theta}$, and $\boldsymbol{\varphi}$ that result in a shot on target.

II.4 Results and recommendations

In this model assumes the coefficient of drag, the coefficient of lift, and rotational velocity remain constant; in an actual kick all of the aforementioned parameters change. It is in the variation of these parameters that opportunity for a new metric arises. The coefficients of drag and lift are affected by multiple conditions. The first is the velocity of the object: the coefficients change as velocity increases and fluid properties change. As velocity increases the air flowing around the football changes from laminar flow to turbulent flow. As seen in Passmore's study, the coefficients of lift and drag vary with increasing velocities and become unpredictable in the transition region. ^[2] The second condition is surface properties and its effect on the boundary layer that forms around an object when a fluid flows around it. For example, the Brazuca football used during the 2014 FIFA World Cup™ has small bumps on its surface altering the boundary layer. This in turn changes the flow directly around the football altering the interaction and forces between the surface of the ball and the air, thus causing variation in the coefficients. The third factor is the seams on the ball. Advanced manufacturing techniques allow for intricate designs with irregular patterns. Seams on the ball affect the airflow and the effect is very dependent on the shapes of the seams. Intricate designs cause the surface interacting with the air to be different depending on which part of the seam is in contact with the air, meaning the orientation of the ball when kicked will affect its aerodynamic properties.

Wind tunnel tests can determine the aforementioned coefficients of drag and lift; however, we propose a different approach and two new metrics to characterize the football's behavior and performance. The first metric proposed assesses the performance of the football and could be added to ball accreditation tests conducted by FIFA. As previously stated, many factors affect the coefficient of drag and lift. If a ball is kicked with the same initial conditions (i.e. direction, translational and rotational velocities), then the final position of the ball will vary from kick to kick. Over a large sample, the final position should regress to a normal distribution. Figure II-5 shows a simulation of a varying coefficient of lift with normal distribution. The standard deviation from the mean provides a useful metric to characterize the performance of the football in real game conditions. If the deviation is small, the football has a consistent predictable flight path. If the deviation is large, the football has an unpredictable flight path that can cause unwanted performance.

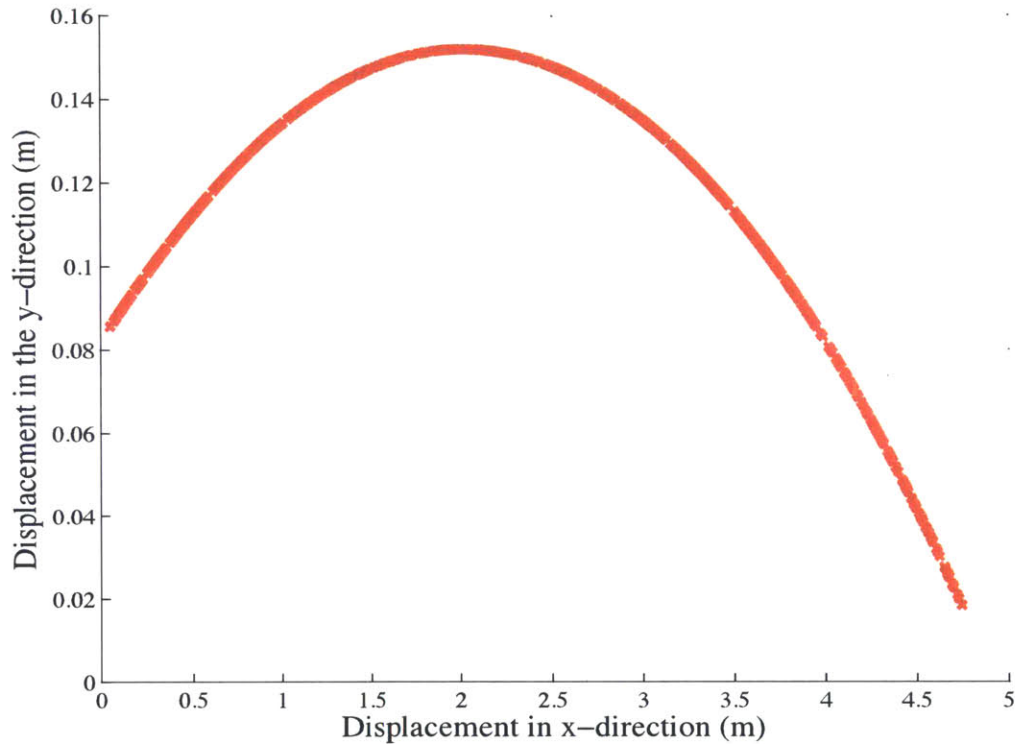


Figure II-5: A plot of ball landing locations of flight paths with varying coefficient of lift. MATLAB’s random generator with a normal distribution was used to create a random number of coefficients of lift between 0.14 and .18. A simulation of kicks with the same initial conditions was then run with each coefficient. The landing location deviates heavily in the x-direction implying that the force due to Magnus effect varies greatly as the coefficient of lift changes. The standard deviation in this direction is 1.33 meters.

During the 2010 FIFA World Cup™ a debate sparked over the behavior of the Jabulani football used for every match. Some argued that the flight of the Jabulani was unpredictable and lead to players committing mistakes albeit all the goals it led to. ^[4] Thus, a metric like the deviation from final position, tested under normal game conditions, could provide a comparison on the predictability of the ball across a range of velocities. The accrediting committee could create an upper bound on the deviation and thus ensure players are content with the performance of the ball.

The second metric that can be tested is in benefit of the player. A simplified flight path can be obtained when the lift and drag forces are ignored. Such an approximation results in simple kinematic equations of motion, which results in a two-dimensional parabolic flight path. When the lift force is taken into account, the additional force creates translational displacement. The two different models result in different final positions of the football. Lateral displacement is the difference between the final positions yielded by the two models. As rotational velocity varies, so does the lift force due to the Magnus effect. If the initial velocity is held constant, the lateral displacement will vary as a function of rotational velocity (due to its direct impact on the lift force equation, but also its effect on the coefficient of drag). Lateral displacement will vary

between types of football due their aerodynamic properties. Maximum lateral displacement would give professional players an idea of how the ball behaves under different spin conditions. Thus, players could make adjustments to take advantage of these properties since the behavior of the ball is explained to them in a language they can relate to.

III. Experimental Setup

In order to validate the mathematical football flight model and the claims made, experimental data was necessary. We desired a low cost, portable system capable of capturing all of the data necessary. Although their setup contained more components than our experiments required, Rogers et. al. used equipment that can be expensive and could not be easily moved. We sought a system that could be transported with minimal setup time to a field or indoors facility where a test subject could attempt the free kicks the mathematical model simulated.

III.1 Camera system

Like Rogers et.al. a system of cameras was selected to capture the football in the initial stages of its trajectory to determine the initial conditions of the trajectory. To satisfy the portability constraint, the rig that would hold all of the components in place had to be of a reasonable size. As a starting point, the size of the rig was set at one and a half meters high. Constraints came mostly from the materials available and a reasonable weight for a single person to carry. The size of the rig imposed limitations on the system that allowed for design considerations. Since the rig would be a meter and a half high, the field of view was expected to be around a meter by 0.3 meters. An upper bound of 30 meters per second was set on the velocity of the ball. Although kicks can reach higher speeds, they are fairly uncommon so this speed was chosen for design. A simple formula relating speed , s , and distance across the field of view, d_f would allow for a calculation of the necessary frame rate by its relation to time, t ,

$$t = \frac{d_f}{s} . \quad (10)$$

Cameras will capture the ball at different positions in time as it travels through the field of view. This allows recording of a plot of distance vs. time. To calculate velocity, MATLAB's least squares approximation is used to create a best-fit line of the data. As can be seen from equation (10), the slope of the best-fit line is the speed at which the football is traveling. We estimated that to obtain a good approximation for a velocity, around 10 different points were necessary. That is to say, that the football in flight had to be captured in at least 10 different frames. As such, the frame rate, f , was calculates as

$$f = \frac{1}{t} * 10 . \quad (11)$$

Thus, for our purposes we needed approximately 300 fps. A close, nominal frame rate was 240 fps. Such a frame rate would still allow for 8 different points to be obtained for a ball travelling at maximum speed. This amount of points would be sufficient to obtain the velocity. Once the frame rate was calculated, we looked for readily available commercial grade cameras that could provide the necessary frame rate. One such camera was that of the iPhone 6. In its slow motion settings, the iPhone 6 camera has a frame rate of 240 fps with a resolution of 720 pixels. Additionally, the iPhone 6 proved to be useful during acquisition and storage of the footage. Both cameras could be connected to the same computer. With a special application, the iPhone 6

cameras could be controlled simultaneously and the videos uploaded and filed as they were being captured.

The next step in the design was to decide how many cameras were necessary to obtain the data and where they would be placed. By obtaining the position of the ball in two perpendicular planes, the position of the ball could be mapped in three-dimensional space. For example, if the coordinates (x,y) of the ball were recorded on one plane and the coordinates (y,z) were recorded in a different plane, the three-dimensional coordinates (x,y,z) can be created. Furthermore, in order to obtain the direction in which the ball is traveling, the angles θ and ϕ need to be calculated. Taking all of this information into account we found that two cameras are sufficient to reconstruct the three-dimensional trajectory of the football. In order to calculate ϕ , one camera had to be position perpendicular to the ground, or parallel to the $x-z$ plane. In order to calculate θ , one camera had to be placed directly above the football parallel to the ground, or parallel to the $x-y$ plane. A diagram of the camera setup is shown in Figure III-1 to illustrate the setup.

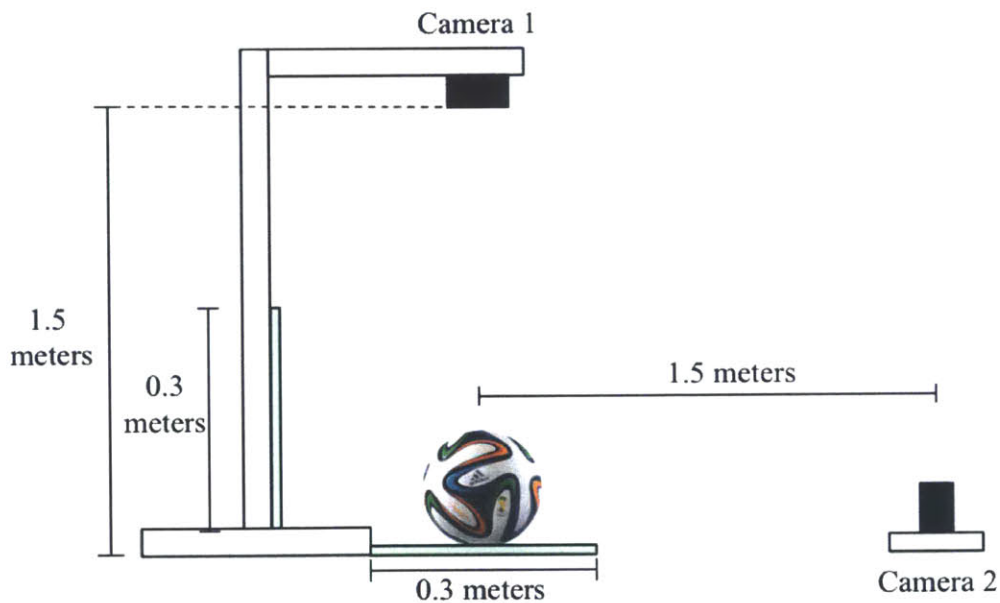


Figure III-1: A diagram depicting the overall camera setup. Clear boxes represent the 80-20 Aluminum bar structure to hold the cameras in place. Lightly shaded boxes served as backdrops that enabled the algorithm to work properly and carry out calculations by working as calibration images. Dark shaded boxes represent the cameras and their positions relative to the ball.

Designing the experimental setup had one critical goal: to enable the algorithm to calculate the position of the ball at different points in time and derive the velocity from those points. In order to enable the algorithm, a calibration was necessary. To ensure that known measurements appeared within the image, a checkered patterned backdrop made up of 10cm colored squares was used. The color of the squares was optimized experimentally for reasons that will be explored later. A backdrop was positioned so that both cameras would yield similar images. In order to calculate distances using the calibration image, however, additional calculations had to be made due to the nature of the camera.

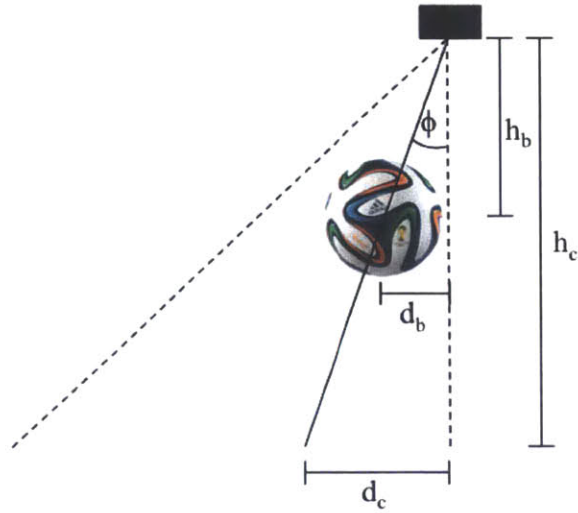


Figure III-2: A diagram depicting the conical nature of the camera’s field of view. Due to this, the distance travelled by the ball is related to the distance on the backdrop by the relationship $d_b = d_c * \frac{h_b}{h_c}$.

The field of view of the camera has a conical shape as illustrated in Figure III-2, therefore, the distances closer to the camera appear to be larger than they actually are. To correct for this discrepancy, a correction factor composed of the camera height, h_c , and ball height, h_b , was used. The distances in question are related by the following equation:

$$\sin\phi = \frac{d_b}{h_b} = \frac{d_c}{h_c} \quad (12)$$

Therefore, the distance travelled by the ball was calculated from the distances on the calibration image by the following equation:

$$d_b = d_c * \frac{h_b}{h_c}. \quad (13)$$

The last step in the experimental setup was to find an appropriate placement of the ball. While kicking, the foot and leg of the player can obstruct the images and make it difficult to process. Furthermore, while the foot is still in contact with the ball it is subject to forces that our model is not accounting for. Therefore it is ideal to start recording the flight path of the ball once contact with the foot is lost. This part of the setup was discovered by trial and error. A good position for the ball is .25 meters before the start of the field of view of the cameras. With the setup done and calibrated, an algorithm was developed to process the images captured by the cameras.

III.2 Image processing

An algorithm was developed in MATLAB to process the images and obtain the center positions of the ball in each frame. Velocity of the ball is then calculated using the information acquired by the script. Figure III-3 provides a detailed representation of the algorithm. Frames from each video recording of the football are processed as individual images. The MATLAB script then searches for an image that is significantly different than the first base image i.e. it searches until the ball comes into frame. Once the ball is in frame, it subtracts the base image from this current image. The result is an array of three values describing each pixel. A variable threshold for these values is set. When the values describing each pixel are below the set threshold, all three values describing the pixel are set to zero. When the values are above the threshold, all values are set to maximum value. The result is a black image with the football imprinted in white color as shown in Figure II-3 (c).

Each pixel on the image has a coordinate pair (i,j) representing its position within the image. If the pixel has been set to white, the coordinate pair is recorded. Once the image has been processed, the average of all the i coordinates and the sum of all the j coordinates are taken. The result is the weighted average of the coordinate. This coordinate pair is taken as the center of the football in the image. This process is represented by the expression below:

$$(i,j)_{center} = \left(\frac{\sum_1^k i}{k}, \frac{\sum_1^k j}{k} \right) \quad (14)$$

where k is the number of white pixels. An approximate diameter of the ball is also calculated. This approximate diameter and the calculated center of the ball is used to repeat the comparison process with a lower threshold, but limiting the search to the approximate radius of the ball. This allows for a clearer image but also eliminates any outliers that may introduce error. Figure III-3 (d) shows the resulting image.

This process is then repeated for the subsequent frames until the algorithm detects a frame in which the ball is no longer visible. The output is a series of images depicting what the algorithm extracts as the football in each frame as shown in Figure III-3 (e) and an array containing the position of the center of the ball in meters. The position of the football is converted to meters using the 10cm squares in the background image and the conversion factor from equation banana.

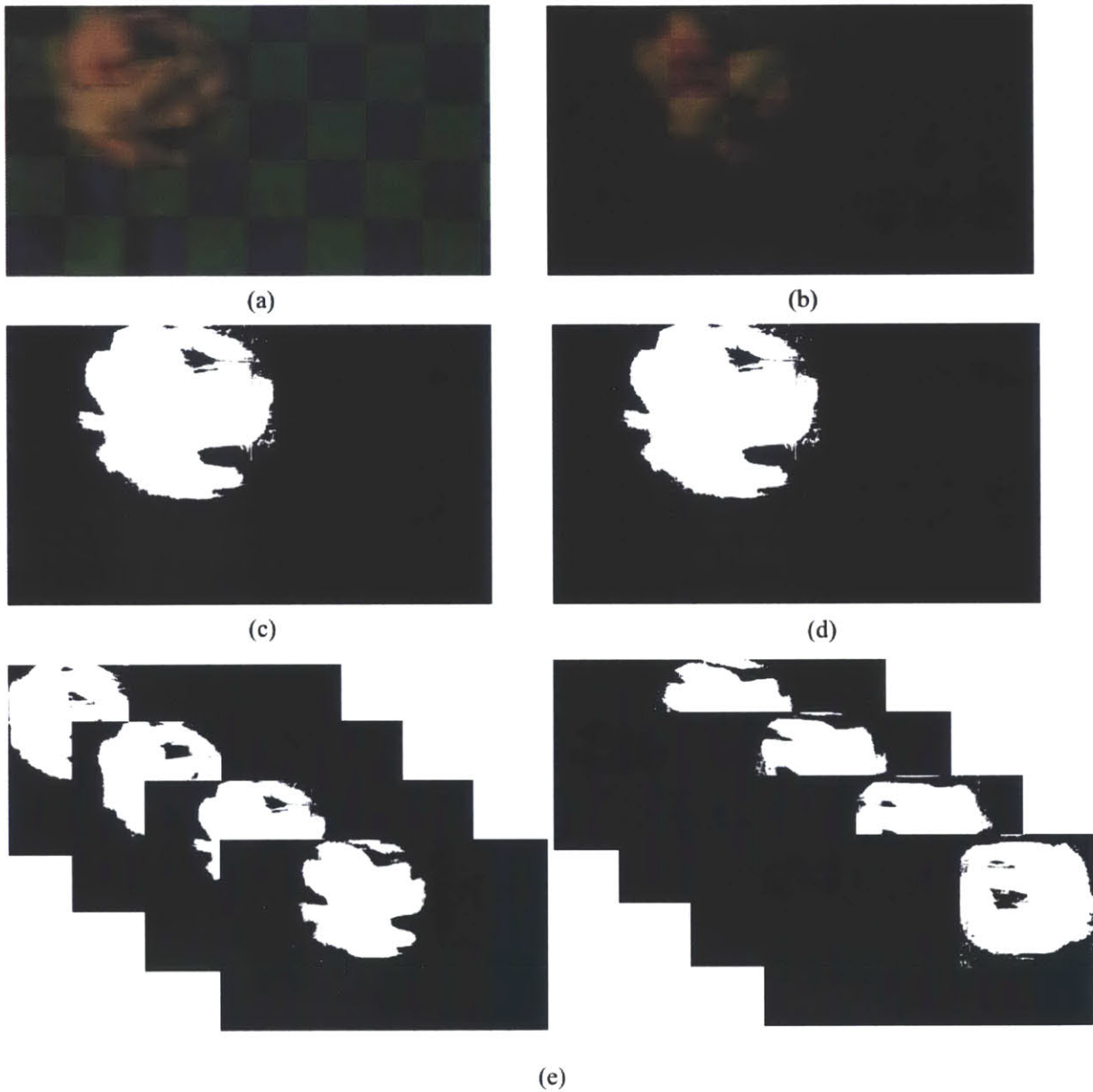


Figure III-3: Images describing the image-processing algorithm. Image (a) shows one of the frames acquired in the high-speed video. Image (b) shows the first step of the algorithm. A background image is subtracted from image (a). The result is a distorted partial image of the football. Images (c) and (d) are taken after the algorithm evaluated whether or not a pixel is part of the background by comparing the color values to a set threshold. Image (d) has a lower threshold than (c); outlier pixels in (c) are removed in (d). Image sequence (e) shows the process for the entire visible flight path.

III.3 Velocity approximation

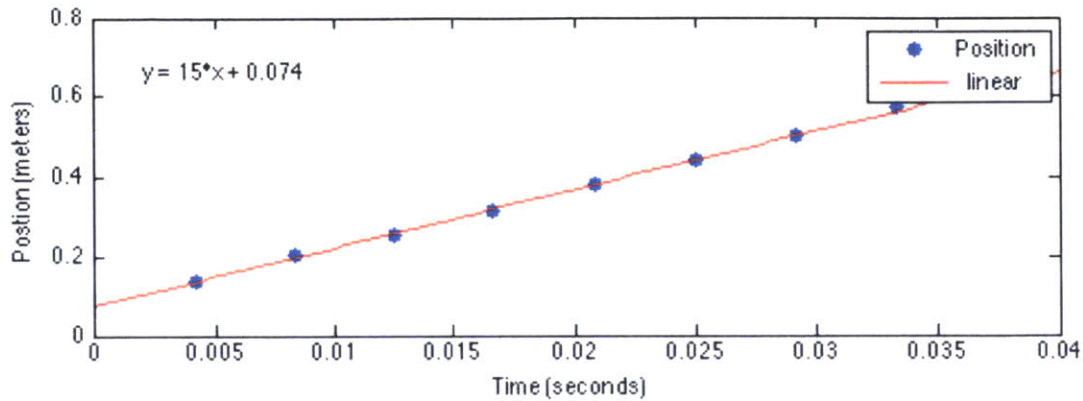


Figure III-4: The plot shows the y position as a function of time. Each point corresponds to a different frame in the captured video. MATLAB's least squares approximation was used to create a linear fit of the data. The slope of the data can be approximates as the speed of the football (in this case 15 meters per second), which is consistent with the kicking capabilities of the test subject.

Once the position of the ball is calculated as it crosses the field of view, the velocity can be approximated. MATLAB has different mathematical tools for approximations. We used the least squares to create a linear fit of the position vs. time plot. The slope of this fit can be approximated as the speed in this direction. Figure III-4 shows a plot of the position of the ball in the y direction as referred to throughout this paper. A linear fit yields a velocity of approximately 15 meters per second. This velocity is consistent with that of an average casual player. Our test subject falls under this category.

III.4 Error analysis

It is evident from Figure III-3 that the algorithm does not capture the ball in its entirety. Due the designs on the ball is difficult to capture the colorful areas with our current method. Depending on the orientation of the ball and the designs visible to the camera, different areas of the ball are missing as the image gets processed. This means that the calculate centers of the ball have a margin of error. An effort to quantify this error was done by placing the ball at a known location with a random orientation for 100 trials to understand this source of error. The standard deviation analysis yielded an error of 16.84 pixels, or approximately .76 cm. If the error were maximal from one frame to the other, the error would result in a difference of approximately 1.82 meters per second.

Other sources of unexplored error are the low quality images due too the high speed of the football. A higher frame rate would likely solve this issue but would require more expensive equipment.

IV. Summary and conclusions

In this thesis we showed that a mathematical model along with real game test kicks yields useful information. With a high number of kicks statistical analysis could create new ways of assessing a football's quality, mainly by quantifying its predictability. We also attempted to create a transportable and cost efficient way to measure a ball's rotational and translational velocity after a kick. Error due to the pattern on the ball is large but can be further diminished with a more advanced computational technique. As a result, we deem that obtaining the translational velocity of the football as it begins its flight path feasible with a low cost and transportable test setup. Computational power can overcome the limitations of the equipment in this case. Rotational velocity, however, we were unable to capture and unable to explore other solutions. Future work would have to evaluate whether or not rotational velocity can be obtained with inexpensive equipment. Furthermore, Rogers explains that for a period of time after the kick the football's motion is irregular ^[3]. We studied the football's motion directly after the kick suggesting our data may be susceptible to this irregular motion.

V. References

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^[4]Gosh, Pallab. "Engineers defend World Cup ball amid criticism." *BBC News*. 4 June 2010.