

Utilization-Aware Adaptive Back-Pressure Traffic Signal Control

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Abstract—Back-pressure control of traffic signal, which computes the control phase to apply based on the real-time queue lengths, has been proposed recently. Features of it include (i) provably maximum stability, (ii) low computational complexity, (iii) no requirement of prior knowledge in traffic demand, and (iv) requirement of only local information at each intersection. The latter three points enable it to be completely distributed over intersections. However, one major issue preventing back-pressure control from being used in practice is the utilization of the intersection, especially if the control phase period is fixed, as is considered in existing works. In this paper, we propose a utilization-aware adaptive algorithm of back-pressure traffic signal control, which makes the duration of the control phase adaptively dependent on the real-time queue lengths and strives for high utilization of the intersection. While advantages embedded in the back-pressure control are kept, we prove that this algorithm is work-conserving and achieves the maximum utilization. Simulation results on an isolated intersection show that the proposed adaptive algorithm has better control performance than the fixed-period back-pressure control presented in previous works.

I. INTRODUCTION

One major challenge confronting almost all metropolises in the world is traffic congestion. Besides the costly efforts made in increasing the traffic network capacity and restricting the traffic load, there have been extensive research works in traffic control algorithms design in order to improve the performance of traffic networks. The traffic signal at intersections assigns the rights of way, a set of which is called a control phase, to incoming vehicles and coordinates the conflicting traffic flows. The traffic control algorithm decides which feasible control phase to apply during a certain period of time. The most widely used adaptive traffic signal control algorithms [1]–[3] apply control phases according to a periodic pre-defined schedule, which can be updated based on the traffic measures in order to achieve good performance.

The back-pressure traffic signal control, which can be completely distributed over intersections, has been proposed recently [4]–[7]. Advantages of it are that (i) the stability, which is defined as bounded queue lengths, is proved to be the maximum; (ii) the computational complexity of the algorithm is $\mathcal{O}(1)$; (iii) the decision is based on the real-time queue lengths and requires no a priori information of the traffic demand; (iv) the control algorithm processes only local

information. In particular, an implementable back-pressure algorithm with a fixed control phase period is presented in the latest paper along this research direction [7].

Despite all the advantages that have been well discussed in the literature, one major challenge about the back-pressure traffic signal control is that the utilization of the intersection can be poor, especially when the control phase period is fixed. This adversely affects the control performance. The main contribution of this paper is that we propose a utilization-aware adaptive back-pressure traffic signal control algorithm, which keeps the mechanism in favor of the system stability. The period of the control phase adaptively depends on the real-time queue lengths. While existing works on back-pressure traffic signal control discuss or prove the work-conservation, none are able to achieve the maximum utilization of the intersection. Both these two properties of this algorithm are proved. The algorithm has the computational complexity of $\mathcal{O}(1)$, requires no information of the traffic demand, and only processes local information, i.e., the real-time queue lengths at the intersection. These ensure that the algorithm is completely distributable. Simulation results on an isolated intersection compare the control performance of this proposed adaptive algorithm with the one presented in [7].

Organization of this paper is as follows: Section II presents modeling of the intersection, arriving vehicles, and control phases. The proposed algorithm is elaborated in Section III. Properties of the work-conservation and maximum utilization are proved in Section IV. Simulations results are reported in Section V and Section VI makes concluding remarks.

II. SYSTEM MODELING

Modeling of a signalized intersection is necessary as the basis to develop back-pressure control algorithms. In this section, we describe the queueing network used to model the intersection. The junction works as a server consisting of a set of links and enabling traffic flow. Vehicles arrive exogenously according to the Poisson distribution from the outside of the network and endogenously from other intersections within the network. Each control phase opens a subset of all links, allowing transfers of vehicles in certain directions.

A. Queueing Network

We model the signalized intersection as a directed graph of nodes $N = \{N_i | i \in \mathcal{N}\}$. Each node represents a road with queueing vehicles. The set of incoming roads is denoted as $NI = \{N_i | i \in \mathcal{NI}\}$. The set of outgoing roads is $NO = \{N_i | i \in \mathcal{NO}\}$ and $N = NI \cup NO$. The junction is composed of links $L = \{L_i^{i'} | i \in \mathcal{NI}, i' \in \mathcal{NO}\}$. Opening

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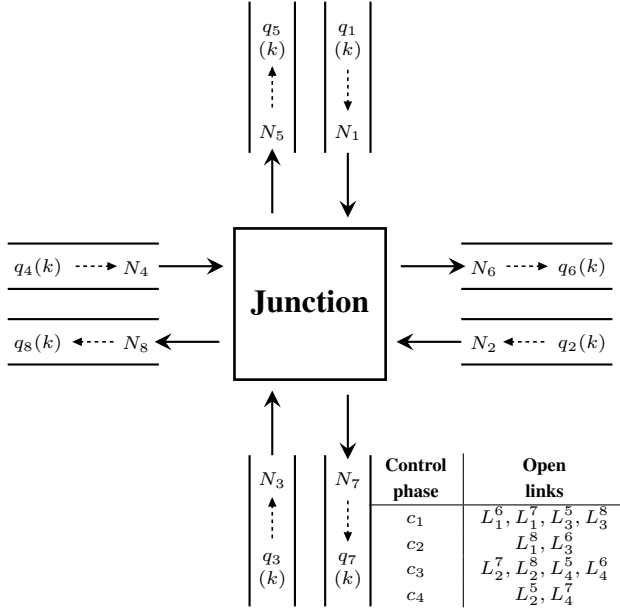


Fig. 1. An example intersection with four incoming nodes and four outgoing nodes. Open links corresponding to four control phases are tabularized.

a link $L_i^{i'}$ enables vehicles queueing at N_i to reach $N_{i'}$. As is common in queueing network control, the time is discretized at instants, which are denoted by t_k , where $k \in \mathbb{N}$. Assuming that the number of vehicles queueing at an incoming node N_i ($i \in \mathcal{NI}$) leaving for $N_{i'}$ ($i' \in \mathcal{NO}$) at t_k is $q_i^{i'}(k)$. The queue length at the node N_i at the time instant t_k is then

$$q_i(k) = \sum_{i' \in \mathcal{NO}} q_i^{i'}(k). \quad (1)$$

The capacity of N_i is denoted as W_i , which is the maximum number of vehicles that the road N_i can accommodate. When W_i is reached, no vehicles are able to enter N_i . As illustrated in Figure 1, the example intersection has eight nodes representing four incoming roads and four outgoing roads. There are twelve links in the junction. For instance, opening the link L_1^6 enables vehicles on the road N_1 to turn left and reach the road N_6 .

B. Arriving Vehicles

For an intersection on the border of a traffic network, vehicles arrive exogenously from the outside of the network and endogenously from other intersections of the network. An intersection only connected to intersections within the network does not have arriving vehicles from the outside. The exogenous process is modeled by a discrete random variable \mathcal{X} , which has a Poisson distribution with the parameter $\lambda > 0$. The probability mass function of X is given by

$$\text{Prob}(\mathcal{X} = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \quad (2)$$

which calculates the probability that a vehicle arrives at the time instant t_k , counting from the time 0. The expected value of \mathcal{X} is λ . We use the turning probabilities p_l and p_r to differentiate arriving vehicles that are going to different

nodes. When a vehicle arrives, the probability that it will turn left is p_l , the probability that it will turn right is p_r , and the probability that it will go straight is then $1 - p_l - p_r$. Taking the node N_3 in Figure 1 as an example,

$$q_3^8 = p_{3,l}q_3, \quad q_3^6 = p_{3,r}q_3, \quad q_3^5 = (1 - p_{3,l} - p_{3,r})q_3. \quad (3)$$

Denoting $A_i^{i'}(k)$ as the total number of vehicles that have both exogenously and endogenously arrived in the node N_i at t_k and are leaving for $N_{i'}$ since the time 0, the queueing dynamics is then

$$q_i^{i'}(k+1) = q_i^{i'}(k) + A_i^{i'}(k+1) - A_i^{i'}(k) - S_i^{i'}(k, k+1), \quad (4)$$

where $S_i^{i'}(k, k+1)$ is the number of vehicles leaving N_i and reaching $N_{i'}$ during the time between t_k and t_{k+1} .

C. Control Phases

The set of control phases at an intersection is denoted by $C = \{c_j | j \in J\}$, and $c_j \subset L$. In other words, corresponding to each phase c_j , a subset of L are opened. For the example shown in Figure 1, there are four control phases in total. For instance, when c_2 is applied, the links L_1^8 and L_3^6 are opened, allowing vehicles queueing at N_1 and N_3 to make a right turn. It is noted that the transition phase (i.e., the period when the yellow light is on to clear vehicles in the junction) is denoted as $c_0 = \emptyset$. That is, no links are open during the transition phase. The control decision to make is which phase to apply. Assuming that the full service rate for vehicles going from N_i to $N_{i'}$ is $\mu_i^{i'}$, the maximum number of vehicles that can be transferred from N_i to $N_{i'}$, when c_j is applied for a period of Δt , is $\mu_i^{i'} \Delta t$. There are three factors determining if this maximum number can be reached. First, the control phase has to open the link from N_i to $N_{i'}$, i.e., $L_i^{i'} \subset c_j$. Second, there have to be sufficient queueing vehicles at N_i leaving for $N_{i'}$. Third, the queue at $N_{i'}$ cannot exceed its capacity $W_{i'}$. Therefore, the number of vehicles transferred from N_i to $N_{i'}$ during the period from t_k to t_{k+1} is calculated in (5).

III. UTILIZATION-AWARE ADAPTIVE TRAFFIC CONTROL

The back-pressure control essentially implements a state-feedback control law, which decides the phase to apply at every time instant, based on the system state, i.e., the lengths of the queues at the intersection:

$$c(k) = \phi(Q(k)). \quad (6)$$

The control law is ϕ and $c(k) \in C$ is the selected control phase. The set of all queue lengths is $Q(k) = \{q_i | i \in \mathcal{N}\}$.

The function $b = f(q)$ is used to map the queue length to a pressure value. For each link $L_i^{i'}$, we can then construct the gain of a link as

$$g(L_i^{i'}, k) = (b_i^{i'}(k) - b_{i'}(k))\mu_i^{i'}, \quad (7)$$

which is the full service rate of the link $L_i^{i'}$ multiplied by the pressure difference between the queue at the incoming node N_i targeting $N_{i'}$ and the queue at the outgoing node $N_{i'}$. It is noted that the gain can be both positive and negative.

$$S_i^{i'}(k, k + \Delta k) = \begin{cases} \min(\mu_i^{i'}(t_{k+1} - t_k), q_i^{i'}(k) + A_i^{i'}(k+1) - A_i^{i'}(k), W_{i'} - q_{i'}(k)) & : \text{if } c_j \supset L_i^{i'}; \\ 0 & : \text{otherwise.} \end{cases} \quad (5)$$

$$g(L_i^{i'}, k) = \begin{cases} \beta G_{\min} & : \text{if } q_{i'}(k) = W_{i'}; \\ \alpha G_{\min} & : \text{if } q_{i'}(k) < W_{i'} \wedge q_i^{i'}(k) = 0; \\ (b_i^{i'}(k) - b_{i'}(k))\mu_i^{i'} & : \text{otherwise.} \end{cases} \quad (8)$$

In principle, the link with a higher gain, which indicates the urgency to get served (a larger imbalance between the queues of the incoming and outgoing nodes) and the better efficiency if served (a larger full service rate), should get the priority to be opened. There are two special scenarios to consider. First, when the outgoing node $N_{i'}$ reaches its capacity, i.e., $q_{i'} = W_{i'}$, no vehicles can enter $N_{i'}$ and thus $L_i^{i'}$ should not be opened. Second, when there are no vehicles queuing at the incoming node N_i going to $N_{i'}$, i.e., $q_i^{i'} = 0$, if $L_i^{i'}$ is opened, the effective service rate is smaller than $\mu_i^{i'}$, indicating low utilization of the junction, since only newly arrived vehicles will be served. Taking these two scenarios into account, we update (7) with (8), where G_{\min} is smaller than the minimum value that (7) can take. The parameters β and α are positive integers, and

$$\beta > \alpha > 1, \quad (9)$$

in order to differentiate the two special scenarios from the normal scenario that the link has the full service rate, and also from each other. In this work, the mapping function is

$$b = f(q) = q. \quad (10)$$

Therefore,

$$G_{\min} < \min_{i \in \mathcal{N}L, i' \in \mathcal{NO}} -W_{i'}\mu_i^{i'}, \quad (11)$$

is a negative number. The three scenarios in (8) are exclusive to one another and a given link gain corresponds to a unique scenario. For each control phase c_j ,

$$g(c_j, k) = \sum_{L_i^{i'} \in c_j} g(L_i^{i'}, k), \quad (12)$$

is the sum of all link gains and

$$g_{\max}(c_j, k) = \max_{L_i^{i'} \in c_j} g(L_i^{i'}, k), \quad (13)$$

is the maximum link gain.

The proposed utilization-aware adaptive back-pressure traffic signal control algorithm decides at each time instant whether the current control phase should continue (i.e., $c(k+1) = c(k)$) or the transition phase should start (i.e., $c(k+1) = c_0$), and in the latter case, which control phase should be applied after the transition phase. The algorithm is summarized in Algorithm 1 and elaborated as follows. Ties are randomly broken.

- In the input, $Q(k)$ is the set of queue lengths of all nodes at the current time instant t_k , P is the set of

Algorithm 1: The utilization-aware adaptive back-pressure traffic signal control algorithm

Input: $Q(k) = \{q_i | i \in \mathcal{N}\}, P = \{p_{i,l}, p_{i,r} | i \in \mathcal{N}\mathcal{I}\}, W = \{W_i | i \in \mathcal{N}\}, c(k), \Delta k$

Output: $c(k+1), c(k+\Delta k)$

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1 if  $g_{\max}(c(k), k) > 0$  then
2   |  $c(k+1) = c(k)$ ;
3 else
4   | if  $\max_{c_j \in C} g_{\max}(c_j, k) > \alpha G_{\min}$  then
5     |  $C' = \{c_j | g_{\max}(c_j, k) > \alpha G_{\min}\}$ ;
6     |  $c' = \arg \max_{c_j \in C'} g(c_j, k)$ ;
7   | else
8     |  $c' = \arg \max_{c_j \in C} g_{\max}(c_j, k)$ ;
9   | end
10  | if  $c' == c(k)$  then
11    |  $c(k+1) = c'$ ;
12  | else
13    |  $c(k+1) = c_0$ ;
14    | Executing Lines 4-9, replacing  $k$  with  $k + \Delta k$ ;
15    |  $c(k + \Delta k) = c'$ ;
16  | end
17 end
18 return  $c(k+1), c(k + \Delta k)$ ;

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turning probabilities of all incoming nodes, W is the set of road capacities, $c(k)$ is the current control phase, and Δk is the period of the transition phase. It can be seen that all the inputs are local to the intersection and that no traffic demand is required.

- In the output, $c(k+1)$ is the control phase of the next time instant to be decided. If the transition phase c_0 is returned to $c(k+1)$, the control phase after the transition $c(k + \Delta k)$ is also returned.
- When a certain control phase is applied, we would like to decrease the gains of all links belonging to this phase to 0. That is, efforts are made to achieve a balance between the queue lengths of the incoming and outgoing nodes, which is the essence of the back-pressure control to obtain high system stability. Therefore, as long as there is still a link with a gain larger than 0, the current control phase $c(k)$ remains. (Lines 1-2)
- Once the gains of all links belonging to the current control phase are equal to or smaller than 0, a possible change can be considered. In general, there are three

levels of priorities. The last priority goes to links with full outgoing nodes, since opening them is a total waste of the intersection. The second priority goes to links with no queues at the incoming nodes. They are the two special scenarios that have been discussed when deriving (8). The first priority goes to links that do not belong to the former two categories, and are able to achieve the full service rates (high utilization of the intersection). Among those control phases containing at least one link with the first priority, the one with the largest sum of link gains is selected. (Lines 4-6)

- If no control phases contain at least one link with the first priority, any one with at least one link with the second priority can be selected. (Line 8)
- If the selected control phase is the same as the current one, the current control phase $c(k)$ remains. (Line 11)
- If the selected control phase is different from the current one, the transition phase is applied, after which Lines 4-9 are executed again to compute the control phase to apply then. (Line 13-15)
- At the starting time of 0 when there is no current control phase, $c(0)$ is decided by executing Lines 4-9, setting $k = 0$.

Since the number of roads is fixed for an intersection, the computational complexity of Algorithm 1 is $\mathcal{O}(1)$.

IV. PROPERTIES OF THE ALGORITHM

In this section, we prove that Algorithm 1 is work-conserving and achieves the maximum utilization. Before the proofs, two lemmas are presented.

Lemma 1 When a control phase is computed by executing Lines 4-9 of Algorithm 1, if there exists a control phase with at least one link whose outgoing node does not reach the capacity, i.e., the link gain is larger than βG_{\min} , the selected control phase must have at least such a link.

Proof Lemma 1 is proved by contradiction. Assuming that the outgoing nodes of all links belonging to the selected control phase reach their capacities, the maximum link gain of the selected control phase is then βG_{\min} according to (8) and (13). Therefore, it is Line 8 that is executed to generate the selected control phase, indicating that the maximum link gains of all other control phases are also βG_{\min} . This contradicts that there is a control phase with at least one link whose outgoing node does not reach the capacity. \square

Lemma 2 When a control phase is computed by executing Lines 4-9 of Algorithm 1, if there exists a link with the full service rate, i.e., the link gain is larger than αG_{\min} , the selected control phase has a link with the full service rate.

Proof According to (8), a link with the full service rate means that the control phase this link belongs to has the maximum link gain larger than αG_{\min} . Therefore, Lines 5-6 are executed, indicating that the maximum link gain of the selected control phase is larger than αG_{\min} . It can thus be concluded that there is at least a link with the full service rate in the selected control phase. \square

Now we define the work-conservation and prove that Algorithm 1 is work-conserving.

Definition 1 In the context of traffic signal control, an algorithm is work-conserving if at the beginning of each time instant, which is not part of the transition phase, that an intersection is idle (i.e., no vehicles are served) is the sufficient condition of that the outgoing nodes of all links must be at their capacities.

Theorem 1: Algorithm 1 is work-conserving.

Proof We prove Theorem 1 by contradiction. It is assumed that there exists a link whose outgoing node does not reach the capacity. Since the intersection is idle, all the links of the current control phase have outgoing nodes reaching their capacities. Therefore, the maximum link gain of the current control phase is $\beta G_{\min} < 0$, which means that a decision was made after the last time instant to select the current control phase by executing Lines 4-9 of Algorithm 1, whether the control phase of the last time instant is the same as the current phase or the transition phase. According to Lemma 1, when a control phase is computed by executing Lines 4-9 of Algorithm 1, if there is a control phase with at least one link whose outgoing node does not reach the capacity, the selected control phase must have at least such a link. Referring to the assumption at the beginning of this proof, the current control phase must have at least a link that does not reach the capacity, which contradicts that the intersection is idle. We can then conclude that Theorem 1 is proved. \square

Now we define the maximum utilization and prove that if there is a control phase with the maximum utilization, the selected control phase has the maximum utilization.

Definition 2 The maximum utilization of a control phase is defined as that there is at least one link with the full service rate belonging to this control phase.

Theorem 2: If at the beginning of a time instant, which is not part of the transition phase, there is a link with the full service rate, the selected control phase has the maximum utilization.

Proof There are two situations to consider. First, the selected control phase is the continuation of the control phase of the last time instant by executing Line 2 of Algorithm 1. Since the maximum link gain of the selected control phase is larger than 0, there must be at least one link with the gain larger than 0. According to (8), this link has the full service rate. Second, the selected control phase is computed by executing Lines 4-9 of Algorithm 1. From Lemma 2, the selected control phase has a link with the full service rate. Summarizing these two situations, Theorem 2 is proved. \square

It can be seen that the proposed utilization-aware adaptive back-pressure traffic signal control algorithm is work-conserving and selects the control phase with the maximum utilization whenever possible.

V. SIMULATION RESULTS

An isolated intersection as shown in Figure 1 is simulated in this section. Turning probabilities of arriving vehicles for all four incoming nodes are shown in Table I. It is assumed that vehicles going to different nodes have different dedicated lanes. The transition phase has a fixed period of 4s for the yellow light. We simulate four representative traffic

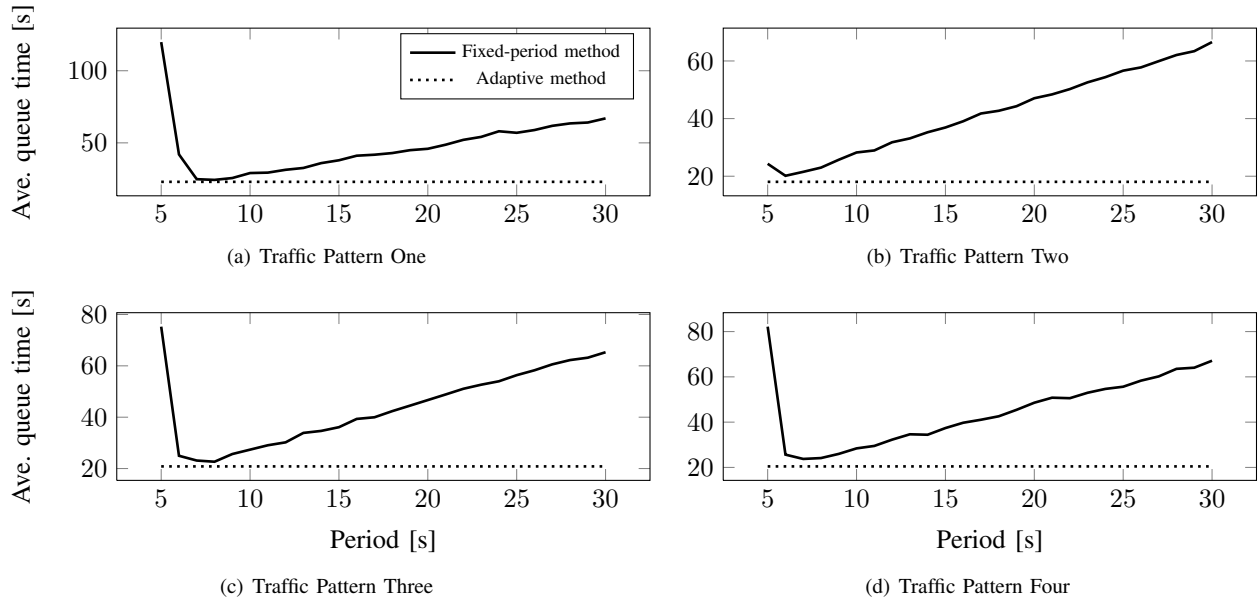


Fig. 2. Performance comparison of the conventional fixed-period control and the proposed utilization-aware adaptive control for four traffic patterns

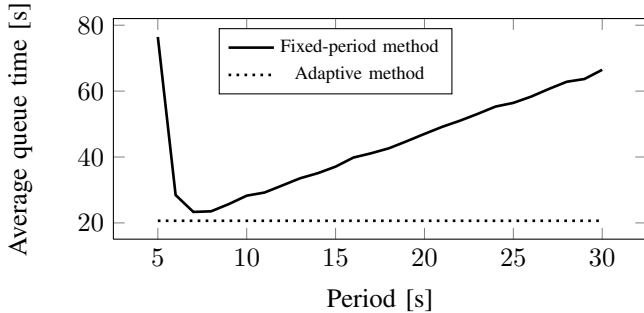


Fig. 3. Performance comparison of the conventional fixed-period control and the proposed utilization-aware adaptive control for the mixed pattern

patterns, whose parameters of vehicles arrival are presented in Table II. In the first pattern, two adjacent nodes N_1 and N_2 have more frequent arrival of vehicles. In the second, the incoming traffic for the four nodes is uniform. In the third, two nodes N_1 and N_3 that are opposite to each other have relatively heavy incoming traffic. In the fourth, vehicles arrive at N_1 more frequently than the other three nodes. Each pattern is simulated for half an hour. A mixed traffic pattern of 2 hours combining the above four individual patterns is also evaluated.

Results comparing the conventional back-pressure traffic signal control method as presented in [7] and our proposed utilization-aware adaptive algorithm are reported in Figure 2, Figure 3, and Table III. As is demonstrated and also intuitively expected, for a given traffic pattern, the system performance (i.e., the average waiting time of a vehicle) varies with the choice of the fixed period that a control phase is applied for. In order to deploy the optimal period, the traffic pattern must be known a priori, which contradicts one of the major advantages the back-pressure traffic signal control brings. Besides, it can be seen that for different

TABLE I

TURNING PROBABILITIES OF ARRIVING VEHICLES FOR ALL FOUR INCOMING NODES

Node	1	2	3	4
Right-turning probability p_r	0.4	0.3	0.4	0.3
Left-turning probability p_l	0.2	0.3	0.3	0.4

TABLE II

PARAMETERS OF VEHICLES ARRIVAL FOR FOUR TRAFFIC PATTERNS

Pattern	Description	λ_1 [s]	λ_2 [s]	λ_3 [s]	λ_4 [s]
I	adjacent heavy nodes	3	5	7	9
II	uniform nodes	6	6	6	6
III	opposite heavy nodes	3	7	5	9
IV	single heavy node	3	9	9	9

TABLE III

PERFORMANCE COMPARISON BETWEEN THE FIXED-PERIOD AND UTILIZATION-AWARE ADAPTIVE BACK-PRESSURE CONTROL ALGORITHMS FOR FIVE TRAFFIC PATTERNS

Pattern	Fixed-Period Control		Adaptive Control
	Optimal Period	Performance	Performance
I	8s	24.33s	22.98s
II	6s	20.15s	18.05s
III	8s	22.66s	20.84s
IV	7s	23.74s	20.45s
Mixed	7s	23.33s	20.64s

traffic patterns, this optimal period could be different, which makes tracking the dynamics of traffic patterns with the optimal periods challenging. The proposed utilization-aware adaptive back-pressure control method performs better than the conventional method with the optimal period. To better visualize the differences, taking the pattern I as an example, we compare the applied control phases in Figure 4 and Figure 5. The queue lengths at the node N_4 for both methods are presented in Figure 6.

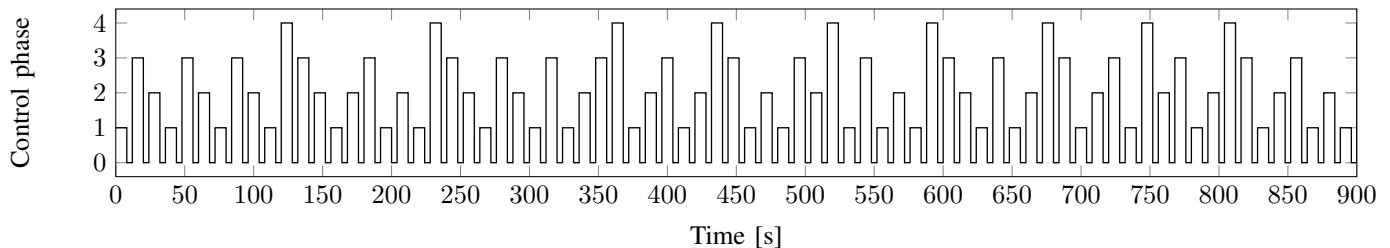


Fig. 4. Control phases of the fixed-period traffic signal control method for 15 minutes in the pattern I. The optimal period 8s is taken. The number j on the vertical axis represents the control phase c_j . 0 indicates that the yellow light is on.

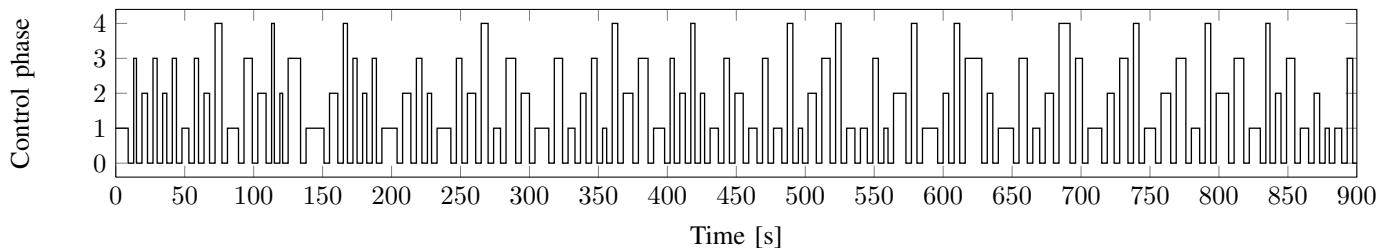


Fig. 5. Control phases of the utilization-aware adaptive traffic signal control method for 15 minutes in the pattern I.

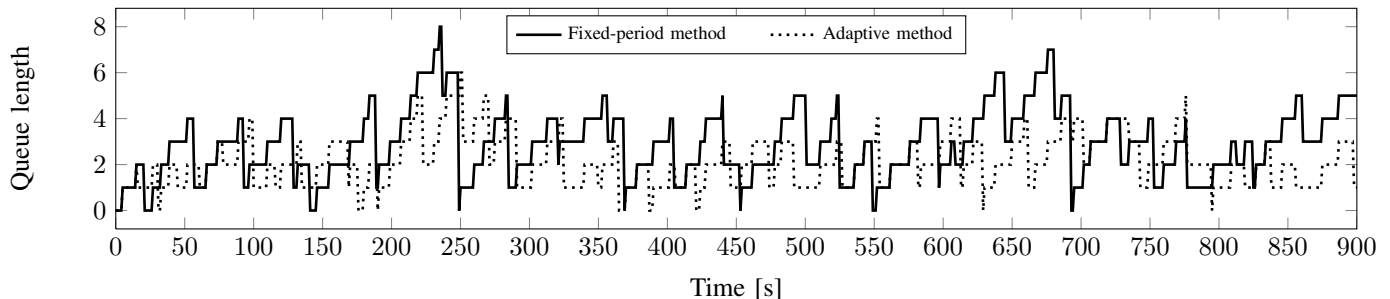


Fig. 6. Queue lengths of both fixed-period (8s) and adaptive methods for 15 minutes at the node N_4 in the pattern I

VI. CONCLUDING REMARKS

Back-pressure traffic signal control has attractive features to solve the congestion problem confronting metropolises. One weak point preventing the back-pressure control from being used in real-world intersections is the utilization. While existing algorithms discuss or prove the work-conservation, none are able to achieve the maximum utilization. Both properties are proved for our proposed utilization-aware adaptive back-pressure traffic signal control algorithm. There are two underlying reasons why the proposed algorithm has these two important properties and excellent control performance. First, unlike previous works on back-pressure traffic signal control, which solely focuses on the system stability, we try not to compromise the utilization while striving for system stability. Second, unlike the fixed control phase period, this algorithm is executed every time instant to make a decision on the control phase to apply and thus makes the duration of each control phase adaptive. This is possible due to the low computational complexity of the algorithm. As part of the future work, a real-world network of intersections can be simulated to further demonstrate the superior performance of our proposed algorithm. It is also worth investigating how

much performance improvement can be achieved by making the decision based on information not only from the local intersection but also neighbors.

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