Realization of the Harper Hamiltonian with Ultracold Atoms in Optical Lattices

Realization of the Harper Hamiltonian with Ultracold Atoms in Optical Lattices

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Abstract: We experimentally realized the Harper Hamiltonian with charge neutral, ultracold atoms in optical lattices using laser-assisted tunneling and a potential energy gradient. The energy spectrum of this Hamiltonian is the fractal Hofstadter butterfly.

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Systems of charged particles in magnetic fields have led to many discoveries in science such as the integer [1] and fractional quantum Hall effects [2] and have become important paradigms of quantum many-body physics. Generalizations have led to the study of topological insulators, initially in condensed matter [3] but also more recently in photonic systems [4, 5]. We have proposed and implemented a scheme which realizes the Harper Hamiltonian [6], a lattice model for charged particles in magnetic fields, whose energy spectrum is the fractal Hofstadter butterfly [7].

We experimentally realize this Hamiltonian with ultracold, charge neutral bosonic atoms of ^{87}Rb in a two-dimensional optical lattice by creating an artificial gauge field using laser-assisted tunneling and a potential energy gradient provided by gravity [8]. A schematic of our setup is shown in Fig. 1 (a),(b) and (c). The laser-assisted tunneling process is characterized by studying the expansion of the atoms in the lattice as shown in Fig. 1 (d).

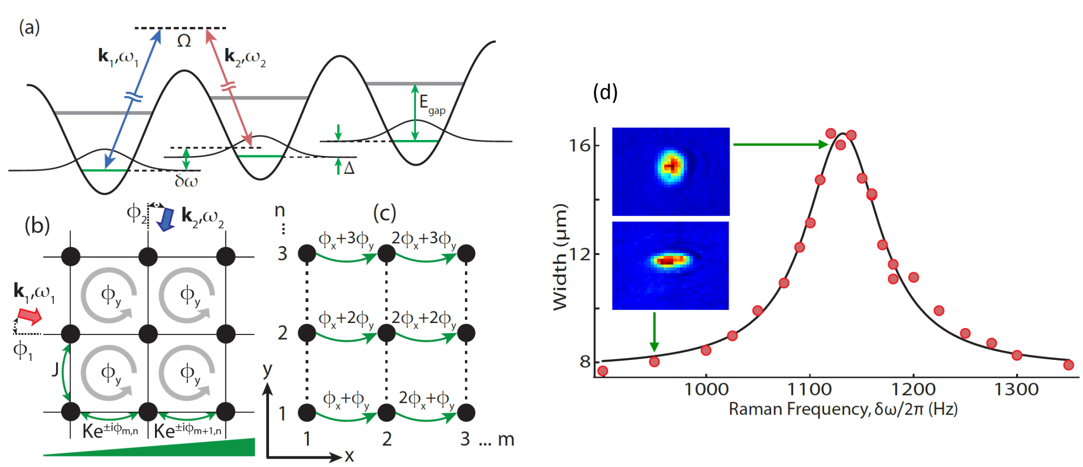

Fig. 1. (a) Laser-assisted tunneling in the lowest band of a tilted lattice with an energy offset Δ between neighboring sites and two-photon Rabi frequency Ω in energy units. (b) Experimental geometry to generate uniform magnetic fields using a pair of laser beams and a potential energy gradient. Tunneling along the x-direction with amplitude K imprints a spatially varying phase $\phi_{m,n}$ with site indices (m,n). (c) A schematic depicting the position-dependent phases of the tunneling process. (d) *In situ* cloud width as a function of Raman detuning $\delta\omega$ after an expansion of 500 ms. The line is a Lorentzian fit to the experimental data centered at 1133 Hz, consistent with the gravitational offset between sites. Pictures (of size 135×116 μm) show typical column densities on and off resonance.

In a uniform lattice, atoms are free to tunnel. However, in the presence of a uniform energy offset between neighboring lattice sites, tunneling along the offset direction is suppressed. Applying a pair of laser beams that are frequency detuned to the offset induces Raman transitions which re-establish tunneling and create the Harper Hamiltonian,

$$H = -\sum_{m,n} \left(K e^{-i\phi_{m,n}} \hat{a}^\dagger_{m+1,n} \hat{a}_{m,n} + J \hat{a}^\dagger_{m,n+1} \hat{a}_{m,n} + H.c. \right), \quad (1)$$

where $\hat{a}^\dagger_{m,n}$ ($\hat{a}_{m,n}$) is the creation (annihilation) operator of a particle at lattice site (m,n) and $\phi_{m,n} = \delta\mathbf{k} \cdot \mathbf{R}_{m,n} = m\phi_x + n\phi_y$ is a spatially varying phase, where the energy offset is in the x-direction. For our particular experimental setup, the magnitude of the tunneling amplitudes in terms of the bare tunneling amplitudes $J_{x,y}$, the energy offset Δ, and the two-photon Rabi frequency in energy units Ω can be written $K = J_x J_1(2\Omega/\Delta)$ and $J = J_y J_0(2\Omega/\Delta)$, where $J_n(x)$ are Bessel functions of the first kind of order n. The experimentally determined atomic cloud width qualitatively agrees with the Bessel function behavior as shown in Fig. 2 (a). We can also suppress nearest-neighbor tunneling while inducing next-nearest-neighbor tunneling with the appropriate laser detuning as shown in Fig. 2 (b).

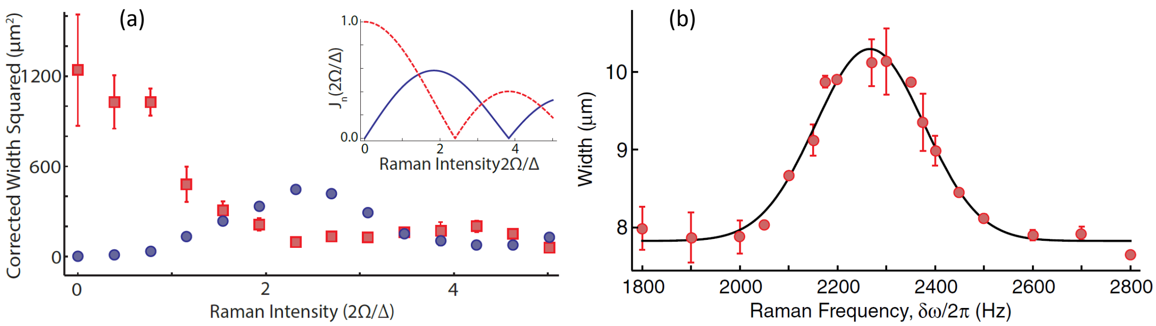

Fig. 2. (a) *In situ* cloud width expansion as a function of resonant Raman laser intensity shows the laser-assisted tunneling rate K along the tilt direction (blue circles) and the tunneling rate J along the transverse direction (red squares). Data taken at lattice depths of $9E_r$ and hold time of 1500 ms. Inset: Theoretical prediction for the tunneling rates K and J in terms of Bessel functions. (b) Next-nearest-neighbor tunneling induced and observed. The center is at 2Δ (compare to Fig. 1 (d)).

Furthermore, this scheme can be extended to realize spin-orbit coupling and the spin Hall effect for neutral atoms in optical lattices by modifying the motion of atoms in a spin-dependent way by laser recoil and Zeeman shifts due to magnetic field gradients [9]. One major advantage of our scheme is that it does not rely on near-resonant laser light to couple different spin states. Our work is a step towards studying novel topological phenomena with ultracold atoms.

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