Network Value Concept in Airline Revenue Management

by

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Abstract

A connecting passenger occupies a seat on each of the flight leg of his itinerary. Moreover, for a given fare class, the fare of a connecting passenger is lower than the sum of the fares of the local passengers on the traversed legs. If the demand is high, giving availability to a connecting passenger may displace local passengers and the airline would lose revenue. The objective of this thesis is to evaluate methods that airlines can use to better estimate the network revenue value of connecting passengers for the purpose of determining seat availability.

In this thesis we analyze and compare two different ways of estimating the network revenue value of the connecting passengers. The first approach consists of estimating the displacement cost of the connecting passenger on all the traversed legs by the shadow prices associated with the capacity constraints of a network linear program (LP). The second one is a prorated fare convergence technique developed in this thesis. The fares of the connecting passengers are prorated on each of the traversed legs using an estimation of the expected marginal revenue of the last seat on the legs. The existence and uniqueness of the limit for each prorated fare sequence are also proven.

We have compared the performance of different seat inventory control models that incorporate these two network revenue estimation techniques. The optimization/booking simulation uses demand forecasts from an airline's Yield Management historical database. The seat inventory control methods that use the network revenue value concepts perform up to 1.50% better than the existing fare class control approach at a high demand scenario (82% average load factor). Moreover, the prorated fare convergence technique performs better than the LP shadow price displacement cost approach especially if the demand is controlled by a bid price mechanism. Indeed, for a high demand scenario and a relatively high number of re-optimizations along the booking process, the prorated fare convergence method performs 0.12% better than the shadow price approach for a bid price control mechanism. Finally, the revenue difference between the two methods is both significant and robust with respect to demand variations.

Thesis Advisor: Professor Peter P. Belobaba
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NOTATION

σ_j	Standard deviation forecast.
$\Pi(1,..,j)$	Number of seats protected for the first j highest ranked ODF on a flight leg. (i.e., $\Pi_{1,2,3}$ is the joint protection level for the first three fare classes).
$\Pi(j)$	Incremental Protection level for ODF j.
BL(j)	Booking Limit of ODF j.
BP(n)	Bid price of OD n.
C_k	Number of seats available on flight leg k.
$Card(L_j)$	Cardinality of L_j (number of legs traversed by ODF j).
DC(j,k)	Displacement cost of ODF j on leg k.
D_j	Actual Demand for ODF j.
$\overline{D}_j$	Mean forecasted demand for ODF j.
Dadj	Demand adjustment parameter.
$EMSR_j(S)$	Expected Marginal Revenue of selling the S^{th} seat to ODF j.
EMSRc(k)	Critical EMSR value of leg k.
ENRV(k)	Expected Network Revenue Value of the last available seat on leg k.
f_j	Revenue Value of ODF j.
L_n	Set of legs traversed by OD n.
NRV(j,k)	Network Revenue Value of ODF j on leg k.
OD	Origin-destination itinerary (i.e., BOS-JFK-PAR).
ODF	Origin-destination and fare class.
p_j	Probability density distribution of the demand for ODF j .
$\overline{P}_j(S)$	Probability that the demand for ODF j is greater than S.
SF_k	Set of ODF traversing leg k.
SP(k)	Shadow Price of leg k.
W	Number of revision points of the booking process.
IN(n,k)	Set of ODF defining the critical EMSR value for iteration n of the convergence process.
Z	Z-factor, $\forall j \in \{1,..,J\}$, $\sigma_j = Z \times \sqrt{\overline{D}_j}$.
Z_T	Z-factor of the observed total demand.

Network Value Concept in Airline Revenue Management

1 Introduction

1.1 Thesis Objectives

In its 1987 annual report American Airlines described the objective of Revenue Management as to sell "the right seats to the right customers at the right price." This statement, as simple as it may seem, is not trivial to implement. Nonetheless, the airlines have a very effective way to manage the passenger requests by controlling the number of seats available to each fare product at any given time. This practice is called Seat Inventory Control. The main objective is, therefore, to generate the maximum revenue by controlling the airline seat inventory. In practice, this consists of designing decision-making models that help the airline to decide whether to accept or reject a customer request in order to optimize the revenue generated over the entire network along the entire booking process.

Most of the major U.S. airline networks are designed as a hub and spoke system. This system enables airlines to realize economy of scale because, for a given demand, the airlines need to operate fewer aircraft, reducing its operating costs. Nonetheless, this network design requires that a significant number of passengers connect at the hub airport. These passengers are called connecting passengers as opposed to the "local" passengers that fly only on one leg of the network. Hence, the origin-destination and fare (ODF) products that are offered to the potential passengers can be classified into two separate types, the connecting and the local passengers.

A connecting passenger occupies a seat on each flight leg he traverses. Therefore, if the demand is high, giving availability to a connecting passenger may displace several local passengers. The airline could lose revenue taking this decision, as, for a given fare class, a connecting fare is usually lower than the sum of the local fares from each segments of the connecting passenger itinerary. Consequently, one major issue of Revenue Management is to estimate the correct revenue contribution of the connecting passenger on each leg that he traverses. In this thesis, two different ways of estimating the revenue contribution of a fare product are analyzed.

The first approach consists of estimating the revenue loss, called displacement cost, from selling a set of seats to a connecting passenger instead of several local passengers. The second approach that is proposed in this thesis consists of prorating the total fare of the connecting ODF on each leg they traverse. The model uses a prorated fare convergence process to estimate the network revenue value of all the fare products on each leg.

The objective of this thesis is to propose and compare new models that estimate the network revenue value of the different ODF offered on each leg of an airline network. Both concepts, displacement cost estimate and prorated fare, are tested on a network of an operating international airline, using two seat inventory control mechanisms, booking limit and bid price, which are explained in Chapter 2.

1.2 Structure of the Thesis

Following the introductory chapter, the intention of the second chapter is to explain the seat inventory control techniques commonly used in the industry. The Booking Limit and Bid Price seat inventory control methodologies are described and compared. In the booking limit control section, we present the EMSRb mathematical model.

In the third chapter, the Network Revenue Value (NRV) concept is explained. We propose two different approaches to estimate the NRV of each ODF on a traversed leg. The first technique consists of estimating the displacement cost of a connecting ODF. We analyze two methods of estimating the displacement cost of a connecting ODF based on two different mathematical optimization models, a deterministic linear program and the stochastic EMSRb model. In Section 3, we introduce a new way of estimating the NRV by prorating the connecting ODF along the legs they traverse. The prorated fares are the limit values of a convergence process. We have analyzed the existence and the uniqueness of the limits of the prorated fare sequences on both a small and a general network.

In Chapter 4, the network value concepts are implemented with the booking limit and bid price control mechanisms introduced in Chapter 2, using actual data from a major airline. Robustness of the different seat inventory control models is analyzed using sensitivity analysis with respect to several parameters, such as the number of times the optimization model is revised.

Finally, the fifth chapter summarizes the contribution of this research and proposes potential research in Revenue Management.

2 Seat Inventory Control Models

2.1 Introduction

Seat inventory control is the process of determining the number of seats to allocate to each offered ODF. Practically, the objective is to protect a number of seats for the high fare business passengers who book close to departure time. If the airline decides to protect too many seats, some seats that could have been sold stay empty.

Seat inventory control models have two distinct entities: a mathematical optimization model and a control mechanism. In this chapter, two control mechanisms, Booking Limit and Bid Price, are presented. The booking limit control mechanism is explained using the Expected Marginal Seat Revenue (EMSRb) optimization model developed by Belobaba in 1992. Moreover, two important concepts of the booking limit control mechanism, nesting and virtual classes, are explored at a leg level. The Bid Price concept is then presented and analyzed. Each booking control mechanism has been illustrated with a small example. The last section of this chapter compares the two approaches in terms of revenue performance and implementation.

2.2 Booking Limit Control Mechanism

2.2.1 Objective

The objective is to "determine the booking limit on each fare that will maximize total revenues for a future scheduled flight departure."[1] This process is dynamic, as the number of available seats and the cumulative forecasted demand have to be updated along the booking process.

2.2.2 Motivation

In order to prevent high fare business passengers from buying the low fare products, the airlines have placed restrictions such as advance purchase or partial/non refundability that penalizes the customers who do not show up for the flight. These restrictions have induced that the discount fares book sooner than the full fare passengers do. Moreover, the business passengers usually do not know their time schedule long in advance and consequently tend to book closer to departure time compared to the leisure passengers. Therefore, taking into account both the product restrictions and the demand behavior, the objective for the airlines is to retain the correct number of seats for the potential customers who would be ready to pay high fares, by not selling these seats to the low fare classes who request a seat earlier in the booking process. In other words, the problem consists of finding the right number of seats to protect for potential high fare passengers who may request for a seat close to departure time.

2.2.3 The EMSRb Model

2.2.3.1 The Model

The EMSRb model is widely used by airlines all over the world. The technique consists of estimating the booking limit for each fare class on each leg of the network. The technique is therefore, leg based and the objective is to generate joint protection levels for higher fare types

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[1] "Application of a Probabilistic Decision Model to Airline Seat Inventory Control," P.P. Belobaba, Operations Research, Vol. 37, No.2, March-April 1989.

to lower fare types.

The expected marginal seat value of allocating S seats to fare type j is defined as,

$$EMSR_j(S) = f_j \times \int_S^{\infty} p_j(r)dr = f_j \times \overline{P}(S) \quad (2.1)$$

The Expected Marginal Revenue of selling S seats to fare class j is the probability that the demand for class j exceeds S, times the revenue value of a class j fare product. The EMSR value operator of a fare type is decreasing with respect to S, the number of seats allocated to the fare type. More generally, the EMSR curve obtained from the methodology explained below is decreasing with respect to the number of seats. Moreover, the demand forecast distribution is assumed to be gaussian in this thesis.

The EMSR model is an heuristic that consists of protecting enough seats for a high fare class until the expected value that the next seat is below the revenue value of selling the seat to the fare class that has the next lower revenue ranking. Then, the number of seats that are saved for a specific fare type, called the protection level, is obtained. The booking limit corresponding to a fare class is the number of seats that are not protected for the higher priced fare classes. The booking limit of a fare class is always greater than the booking limits of lower fare classes. This concept, called nesting, is discussed below but first, the EMSRb methodology is described on a leg traversed by J+1 fare types.

We assume that we have a criterion for ranking the fare type that traverses the leg in terms of desirability to the network. This network value concept is addressed in Chapter 3. We assume for an illustrative purpose that the first fare class is the one that has the highest ranking whereas the last one has the lowest ranking on the leg.

Methodology

We define the index j as the fare class ranked at the j^{th} position.

Step 1: First ranked fare class

The first step considers only one fare type corresponding to the first ranked fare class. Π_1, the protection level of to the first fare class is obtained by,

$$\text{Max}(\Pi_1) \qquad \Pi_1 \in N \tag{2.2}$$

Subject to: $f_1.\Pr ob(D_1 \geq \Pi_1) \geq f_2$

As seats are integer, the number of seats saved for the first fare class is found by iterating Π_1 until its expected marginal seat revenue, $f_1.\Pr ob(D_1 \geq \Pi_1)$, is lower than the next fare class value, f_2.

Step 2: First and second ranked fare classes are combined

The second step consists in combining the first and second ranked fare classes. The EMSR value corresponding to these two fare classes is defined as the weighted average fare value time the probability that the aggregated demand is greater than the number of seats that are protected for the set of fare classes,

$$\text{EMSR}\ (\Pi_{1,2}) = \frac{(f_1 . \overline{D}_1 + f_2 . \overline{D}_2)}{\overline{D}_1 + \overline{D}_2} \times \Pr ob(D_1 + D_2 \geq \Pi_{1,2}) \tag{2.3}$$

The joint protection level, $\Pi_{1,2}$, for the first and the second fare classes is found by,

$$\text{Max}(\Pi_{1,2})$$

subject to: $\text{EMSR}(\Pi_{1,2}) \geq f_3 \quad \text{with} \quad \Pi_{1,2} \in N$ (2.4)

Moreover,

$$\Pi_2 = \Pi_{1,2} - \Pi_1 \tag{2.5}$$

The protection level of the fare class 2 is determined incrementally by subtracting the protection level of the fare class 1, already computed in step 1, from the protection level of fare classes 1 and 2 combined.

The method is continued until the model performs J iterations.

Step J: The first J fare classes that traverse the leg are combined

This last step consists of finding the protection level, $\Pi_{1,\ldots,J}$, of the first J fare classes, which traverse the leg. As above, the EMSR of the combined fare classes is,

$$\text{EMSR}\,(\Pi_{1,\ldots,J}) = \frac{(\sum_{j=1}^{J} f_j \,.\, \overline{D}_j)}{\sum_{j=1}^{J} \overline{D}_j} \times \text{Prob}(\sum_{j=1}^{J} D_j \geq \Pi_{1,\ldots,J}) \tag{2.6}$$

And $\Pi_{1,\ldots,J}$ is found by solving

$$\text{Max}(\Pi_{1,\ldots,J})$$

subject to: $\text{EMSR}(\Pi_{1,\ldots,J}) \geq f_{J+1}$ with $\Pi_{1,\ldots,J} \in N$ (2.7)

The protection level of the second to last fare class, Π_J, is then obtained by,

$$\Pi_J = \Pi_{1,\ldots,J} - \sum_{i=1}^{J-1} \Pi_i \tag{2.8}$$

Therefore, the number of seats to be protected for the first J classes has been found. The booking limit for each fare class is then obtained by

$$\begin{cases} BL_1 = C \\ BL_j = Max(0\ ,\ C - \Pi_{1,..j-1}) \qquad \forall j \in \{1,..,J+1\} \end{cases} \tag{2.9}$$

The booking limit of the highest ranked fare class is set to be equal to the number of seats available on the leg. Furthermore, the booking limit of a specific fare class is the number of seats that are not protected for the higher-ranked fare classes provided that this number is positive.

2.2.3.2 Nesting Concept

The equation (2.9) encompasses a very effective concept called nesting. Its advantages are explained through a simplified example. An airline operates on Boston-Paris flight and proposes four fare classes to its customers. Moreover 70 seats are available for this flight. Information for each fare class, about forecasted mean demand, forecasted standard deviation and fare, is summarized in Table 2-1 below.

BOS-PAR

Fare Class	$\overline{D}_j$	σ_j	fare ($)
Y	10	5	1000
B	15	7	700
M	20	9	500
Q	30	13	350

Table 2-1: Demand Information for each Fare Class on BOS-PAR.

The booking limits of the four different fare classes are found using the EMSRb seat allocation model, explained above. Please see the Section A of the appendix for complete details about the booking limit calculation for each fare class.

Fare Class	Π_j	BL_j
Y	7	70
B	15	63
M	24	48
Q	N.A	24

Table 2-2: Booking Limit for each Fare Class using EMSRb Technique.

Suppose that a big advertising company decides to organize a meeting in Paris inviting a lot of business executives from Boston. The actual number of high-yield passenger requests, D_Y and D_M, is more than what was forecasted originally as can be observed in Table 2-3.

Fare Class	$\bar{D}_j$	D_j	Sold	Spilled
Y	10	13	13	0
B	15	20	20	0
M	20	13	13	0
Q	30	31	24	7
Total	75	77	70	7

Table 2-3: Forecasted Demand, Actual Demand, and Number of Seats Sold and Spilled.

Assuming that the lower fare class requests book first and that no revision of the booking limits is possible during the booking process, one can observe that the nested EMSRb model spills only 7 Q class requests and happens to be the optimal strategy to adopt. As opposed to the nesting approach, the partitioned approach consist of setting distinct seat allocations such that the sum of the booking limits of all the fare classes on a leg is equal to the number of seats available on the leg ($BL_Y+BL_M+BL_B+BL_Q=70$ in the example). If the fare class inventory structure were partitioned we would have expected some high fare potential customers to be spilled, as the actual number of requests for the high fare is well above what was originally forecasted.

Therefore, a nested fare class structure gives more flexibility with regard to the number of high fare passenger requests. In other words, "the impact of errors in the demand forecasts of higher valued class is reduced."[2] In practice, assuming a booking limit control, the nested models perform much better than the partitioned models.

2.2.3.3 Virtual Class Concept

Airlines forecast the demand for a particular flight all along the booking process. Nonetheless, the number of requests, for a given ODF, along the booking process is usually a very low integer number. For example, for a given airline, the number of potential Y class passengers on Boston-Paris connecting in New York JFK, 120 days prior to departure, is very small. Therefore, the forecast of passenger demand on an ODF by ODF basis carries a lot of uncertainty. To lower the risk of forecasting error, airlines have the possibility to aggregate the fare types into *virtual* classes.

The aggregation technique used in this thesis is explained below through an example. Let assume that the network consists of two legs, namely, BOS-ORD and ORD-LAX. Three types of OD traverse the network. Two local ODs with respect to each leg (i.e., BOS-ORD and ORD-LAX) and one connecting OD (BOS-ORD-LAX). For each OD, the airline offers three fare types (Y, B and Q) to their customers.

We assume that the network revenue value (NRV) for each ODF has been estimated by, for example, one of the techniques proposed in Chapter 3.

---

[2] E.L Williamson[14]

UA 593: BOS-ORD

Fare Class	$\bar{D}_j$	σ_j	NRV ($)
$Y_{BOS-LAX}$	13	4	900
$Y_{BOS-ORD}$	11	3	750
$B_{BOS-LAX}$	14	4	620
$B_{BOS-ORD}$	12	4	500
$M_{BOS-ORD}$	25	5	200
$M_{BOS-LAX}$	35	7	180

UA 189: ORD-LAX

Fare Class	$\bar{D}_j$	σ_j	NRV ($)
$Y_{BOS-LAX}$	13	4	850
$Y_{ORD-LAX}$	12	4	800
$B_{ORD-LAX}$	10	3	690
$M_{ORD-LAX}$	15	4	450
$B_{BOS-LAX}$	14	4	440
$M_{BOS-LAX}$	35	7	130

Table 2-4: Network Revenue Value and Forecasted Demand Information for each ODF.

As one can observe, the connecting ODF for fare type B (BOS-ORD-LAX itinerary) has different ranking on each leg as the marginal network value of seats on the two legs are different.

Let assume arbitrarily that the ODF are bucketed into four different virtual classes, VC_j, for $j \in \{1,2,3,4\}$. The virtual classes are constructed such that the mean demand forecast is, as much as possible, evenly distributed among the virtual classes.

UA 593: BOS-ORD

VC	Fare Type	$\bar{D}_{Vj}$	σ_{Vj}	NRV($)
V1	$Y_{BOS-LAX}$, $Y_{BOS-ORD}$	24	5	831
V2	$B_{BOS-ORD}$, $B_{BOS-LAX}$	26	5.7	565
V3	$M_{BOS-ORD}$	25	5	200
V4	$M_{BOS-LAX}$	35	7	180

UA 189: ORD-LAX

VC	Fare Class	$\bar{D}_{Vj}$	σ_{Vj}	NRV($)
V1	$Y_{BOS-LAX}$, $Y_{ORD-LAX}$	25	5.7	826
V2	$B_{ORD-LAX}$, $M_{ORD-LAX}$	25	5	546
V3	$B_{BOS-LAX}$	14	4	440
V4	$M_{BOS-LAX}$	35	7	130

Table 2-5: Network Revenue Value and Forecasted Demand Information for each Virtual Class.

The network value associated with each virtual class is the average of the network value of the ODFs belonging to the virtual bucket weighted by the corresponding mean demand forecasts. The network revenue value of V_j is therefore:

$$NRV(V_j) = \frac{\sum_{ODF \in V_j} [\overline{D}_{ODF} \times NRV(ODF)]}{\sum_{ODF \in V_j} \overline{D}_{ODF}} \qquad j \in \{1,2,3,4\} \qquad (2.10)$$

The forecasted mean demand and the forecasted standard deviation associated with each virtual class, V_j, is defined as:

$$\overline{D}_{Vj} = \sum_{ODF \in Vj} \overline{D}_{ODF} \qquad j \in \{1,2,3,4\} \qquad (2.11)$$

and,

$$\sigma_{Vj} = \sqrt{\sum_{ODF \in Vj} \sigma_{ODF}^2} \qquad j \in \{1,2,3,4\} \qquad (2.12)$$

The booking limit of each virtual class, for each leg, has been found using the EMSRb method explained above assuming that the number of available seats on BOS-ORD and ORD-LAX is, respectively, 90 seats and 105 seats. Furthermore, the booking limit of each ODF belonging to a virtual class has the same booking limit value on the leg. The booking limit of each virtual class and the ODFs are summarized in Table 2-6 below.

UA 593: BOS-ORD

Virtual Class	Fare Type	$BL(V_j)$
V1	$Y_{BOS\text{-}LAX}$, $Y_{BOS\text{-}ORD}$	90
V2	$B_{BOS\text{-}ORD}$, $B_{BOS\text{-}LAX}$	70
V3	$M_{BOS\text{-}ORD}$	36
V4	$M_{BOS\text{-}LAX}$	13

UA 189: ORD-LAX

Virtual Class	Fare Class	$BL(V_j)$
V1	$Y_{BOS\text{-}LAX}$, $Y_{ORD\text{-}LAX}$	105
V2	$B_{ORD\text{-}LAX}$, $M_{ORD\text{-}LAX}$	86
V3	$B_{BOS\text{-}LAX}$	58
V4	$M_{BOS\text{-}LAX}$	34

Table 2-6: Booking Limit of the Virtual Classes on both Legs.

The booking limit of the connecting ODFs is given by,

$$BL(ODF) = \min_{LEG}[BL(ODF, LEG)] \tag{2.13}$$

The booking limit of a connecting ODF is the smaller booking limit value over all the legs traversed. Thus, a given connecting ODF must have strictly positive booking limits over all the legs of its itinerary in order to be open to bookings.

Therefore, for the two-leg network the booking limits are summarized in Table 2-7.

Fare Class	Local BOS-ORD	Local ORD-LAX	Connecting BOS-ORD-LAX
Y	90	105	90
B	70	86	58
M	36	86	13

Table 2-7: ODF Booking Limit using EMSRb model.

Virtual bucketing brings an additional degree of freedom to the airline. The number of virtual classes to be used depends on how reliable the demand forecasts are. In the example analyzed above, it would have probably been better to use more virtual classes, especially to control the local requests on ORD-LAX as a booking limit of 86 seats seem to be loose for the local M class. The extreme case, where each fare value is a virtual class is called OD by OD booking limit control method. Nonetheless, forecast errors may be amplified by a large number of virtual classes. Therefore, the number of virtual classes to choose is a trade-off between forecasting error robustness and accuracy of the control mechanism.

Conclusion

Nesting and virtual class are two important concepts used in leg based booking limit control mechanisms. Nesting improves the robustness of the control mechanism with respect to demand variations and performs significantly better than the partitioning technique[3]. Virtual classes give the opportunity to design the inventory structure by grouping the ODFs into buckets and become more robust with regard to forecasting errors. In their thesis, Williamson[14] and Wei[13] have analyzed in great details the nesting and virtual class concepts.

2.3 Network Bid Price Control Mechanism

2.3.1 Definition

The network bid price value associated with and Origin-Destination (OD) itinerary is:

$$BP(n) = \sum_{m \in L_n} ENRV(m) \tag{2.14}$$

With ENRV(m), the Expected Network Revenue of the last available seat on leg m and L_n the set of legs traversed by OD n. Different techniques to estimate ENRV are proposed in Chapter 3. The objective of this section is to explain the bid price concept and to highlight its advantages and shortcomings. In this section we assume the ENRV value for each leg to be estimated using a mathematical technique explained in Chapter 3.

[3] E.L Williamson[14]

2.3.2 The Concept

The motivation behind the bid price concept is the following. The objective is to find an estimate of the expected potential revenue that can be generated by an itinerary at a given time of the booking process. As defined earlier the ENRV is the expected network revenue value of the last seat on a leg. Therefore, if a customer is ready to pay more for an itinerary (OD) than what the airline can expect to gain from the seats it will consume, then the request should be accepted. Therefore, the bid price associated with an OD is the sum of the ENRV over its itinerary. If an ODF has a fare greater than the bid price value corresponding to its itinerary then all the bookings from this particular ODF are accepted until the bid prices are revised. Conversely, if the fare is below the bid price value, then the requests are rejected. Therefore, the bid price concept is a binary (rejection/acceptance) decision making process.

2.3.3 Sub-Optimality Discussion

The objective of this section is to explain, through an example, the risk of taking a wrong decision by using a bid price control mechanism. The technique used to compute the bid price values is the Network Deterministic Bid Price (NDBP) in which bid price values are estimated using linear programming shadow prices. The technique is analyzed in Chapter 3.

Two fare classes, Y and B, are offered, on a two leg network, BOS-ORD and ORD-LAX, with, respectively, 50 and 60 available seats. Moreover, we assume that no revisions of the bid price values are possible until the departure time.

OD	Fare Class	$\overline{D}_j$	Fare ($)
BOS-ORD	Y	12	750
	B	23	290
ORD-LAX	Y	11	800
	B	25	340
BOS-ORD-LAX	Y	10	1000
	B	18	540

Table 2-8: Fares and Forecasted Demand for each ODF.

The bid price values are found using NDBP,

OD	BOS-ORD	ORD-LAX	BOS-ORD-LAX
BP ($)	290	260	550

Table 2-9: Bid Price value for each OD.

Of all the ODF, only the fare of the connecting B class ODF is below its corresponding bid price value. Thus, this ODF is the only one spilled and all the ODF are open except the B class connecting ODF. Let assume that the demand follows a Poisson process[4]. Therefore, the probability mass function is used below to compute the probability of occurrence of several events.

The probability that 5 seats stay empty on both legs is nearly 50% and the probability that 5 seats are left empty on leg 2 is greater than 90%. Moreover, the mean demand of denied B-Class connecting requests is 18. As the probability that the spill of B-Class connecting fare class is greater than five is very high it is very likely that allocating five seats to the B-Class connecting fare class would generate more revenue than by implementing the bid price control strategy. Moreover, this strategy would increase the load factor. It is clear in this example that, if not revised, the bid price control mechanism is sub-optimal. Nonetheless, by revising the bid prices more often along the booking process, the performance of the bid price control is likely to improve. In fact, the bid price control mechanism is very sensitive to revisions. The more often the model is revised the better it performs. In theory, the bid price values should be re-optimized after each booking.

[4] $\mathrm{Prob}(x=k) = \frac{\mu^k e^{-\mu}}{k!}$, with μ the mean, x the random variable and k an integer number.

2.4 Comparison Between Control Mechanisms

Booking limit and bid price are two mechanisms that control the acceptance or the rejection of customer's requests. This section attempts to highlight the difference between the two mechanisms. The bid price control tends to be easier to implement than the booking limit control mechanism because each bid price has to be computed for each OD whereas the booking limits have to be computed for each ODF. Therefore, bid price control mechanism tend to require less operations than the booking limit control mechanism. Nonetheless, the bid price control model is inclined to be more sensitive to revisions compared to the booking limit control model. If the bid price values are overestimated, too many connecting passengers are spilled resulting in revenue losses as explained above through an example. Moreover, bid price control is an acceptance/rejection technique. Therefore, it losses a degree of freedom compare to the booking limit control that can restrict the number of seats to be sold using an integer number instead of an open/close criterion. Finally, booking control tends to be more robust to forecasting errors. According to our simulation, the booking limit models perform slightly better than the bid price control models for the same number of revisions.

2.5 Conclusion

Two techniques to control the passenger requests have been highlighted, namely, network bid price and booking limit control mechanisms. As explained above, the booking limit mechanism, using EMSRb model, is leg based. If the published fares are considered as input to this booking control model, the connecting ODFs will have an advantage over the local ODFs. The objective of the next chapter is to propose some techniques to estimate the network revenue value of each ODF offered on a network. Two different optimization tools will be used, a deterministic mathematical programming approach and the stochastic EMSRb model analyzed in Chapter 2.

3 NETWORK REVENUE VALUE CONCEPT

3.1 Motivations

The EMSRb model is a leg based methodology and therefore fails to consider the set of legs traversed by the connecting ODF. The motivation behind the network revenue value concept is to take into account in the EMSRb model the fact that a connecting passenger occupies several seats on the network. As mentioned before, the fares of the multi-leg long haul ODF tend to be greater than the one of the local ODF, for the same fare class. Ranking the ODF with respect to their actual fare favors the connecting ODF against the local ODF in the booking control mechanism. Thus, connecting passengers may displace high fare local passengers, especially if the demand is high. As explained in this chapter, ranking the ODF according to their total fares would result in relatively high critical EMSR values. Therefore, If the number of passenger requests is controlled by a bid price method, many requests would be denied (especially the connecting ODF) and some seats are likely to stay empty although they could have been sold to the denied passenger requests.

The objective of the network revenue concept is to improve the performance of the seat inventory control models by estimating the network value of each ODF on the network. In this thesis, two different ways of estimating the network revenue value of an ODF have been considered. The first approach measures the cost of displacing a seat for a connecting ODF whereas the second method consists of prorating the connecting ODF fares over their itinerary.

3.2 Displacement Cost Estimation

The first way of analyzing the network revenue value consists of estimating the revenue loss, called displacement cost, from selling a set of seats to a connecting passenger instead of several other ODFs that would generate more revenue. The displacement cost can be seen as an estimate of the opportunity cost or the revenue that the airline would lose by taking the wrong decision to sell the seats to a connecting passenger. Formally, the displacement cost of a connecting ODF j with the itinerary L_j on a leg k is defined as,

$$DC(j,k) = \sum_{\substack{m \in L_j \\ m \neq k}} ENRV(m) \qquad (3.1)$$

With, ENRV(m), the Expected Network Revenue Value of the last available seat on leg m.

Therefore, the network revenue value of ODF j on leg k is

$$NRV(j,k) = Max\ (\ 0\ ,\ f_j - DC(j,k)\ ) \qquad (3.2)$$

And $NRV(j,k) = f_j$ for all j such that $Card(L_j)=1$.

$Card(L_j)$ corresponds to the number of legs traversed by ODF j. If the ODF traverses only one leg (i.e., $Card(L_j)=1$) then the NRV for a local ODF is equal to its total fare as the displacement cost of a local ODF is by definition null.

If the decision model overestimates the displacement cost, the connecting fare product has a lower availability because their network revenue values are lower. Moreover, if the actual demand happens to be low, this strategy will result in empty seats that could have been sold to the denied connecting passenger requests. Therefore, overestimating the displacement cost may entail a high revenue loss fcr the airline especially if the demand is low. Conversely, if the model estimates too low a displacement cost, then the long haul connecting ODF will have a higher ranking. If the demand is high, this strategy may displace some high fare local passengers as priority is given to the connecting ODF.

Airline seats are perishable goods because when the booking process is closed, the empty seats represent a revenue loss for the airline. Therefore, intuitively it seems to be wiser to underestimate the displacement cost as in the worse case this strategy would displace local high-fare passengers by giving priority to long haul connecting passengers. Moreover, if the number of customer requests varies significantly from one leg to another ("bottle neck" problem) then, underestimating the displacement cost is likely to work well as more seats are sold on the legs that have a low demand. Nonetheless, if the demand is high, this strategy is likely to deny high fare local requests and would result in a revenue loss for the airline. Therefore, a good estimation of the displacement cost is important for the airline in order to generate more revenue.

Two approaches to estimate the displacement cost of a connecting ODF are presented in this section. The first one is based on a deterministic mathematical programming technique whereas the second one is based on the EMSRb model.

3.2.1 Linear Programming Approach

In this section, we present a deterministic mathematical programming approach to estimate the expected network revenue value of the last seat on a leg, called ENRV. A discussion of the shadow price concept is conducted and several mathematical programming alternatives are pointed out.

Let consider the Linear Program

$$\text{Max } (c'x) \tag{3.3}$$

$$\text{Subject to } \begin{cases} Ax \le b \\ x \ge 0 \end{cases}$$

Where the coefficients c_j, b_k and $A_{j,k}$ are all positive (with $j \in \{1,..,J\}$ and $k \in \{1,..,K\}$).
According to the strong duality theorem[1],

$$c'\, x^* = p^{*\prime}\, b \tag{3.4}$$

Where x* is the optimal solution of the primal LP and p* is the optimal solution of the dual LP. The optimal solutions exist because the primal problem is feasible (the null vector is a feasible solution) and the objective function is bounded from above over the feasible space. An upper bound to the feasible objective values is:

$$\underset{j}{\text{MAX}}\left(f_j \times \underset{k}{\text{Min}}\left(\frac{b_k}{a_{j,k}} \text{ with} \cdot a_{j,k} \neq 0 \right)\right) \tag{3.5}$$

Because b, c and the matrix A are all positive.
According to (3.4), the revenue of the optimal solution increases by p_k^* if b_k is increased by one unit. The idea is, therefore, to estimate the displacement cost on a leg by the optimal dual value associated with the capacity constraint of the corresponding leg. The optimal dual value associated with a capacity constraint is also called the shadow price of the leg. The definition of the Linear Program (LP) is presented below.

[1] "Introduction to Linear Optimization", D. Bertsimas, J. N. Tsitsiklis, Athena Scientific, 1997.

3.2.1.1 *The Model*

The LP consists of optimizing the revenue subject to the capacity constraints and the demand constraints. In fact, the objective is to maximize the expected revenue as the right hand sides of the demand constraints are the forecasted mean demand for each ODF.

The LP is formally expressed for J ODF and K legs as,

$$\text{Max}\left(\sum_{j=1}^{J} f_j \cdot X_j\right) \tag{3.6}$$

Subject to

$$\begin{cases} \sum_{j \in ODF(k)} X_j \le C(k) & \forall\, k \in \{1,..,K\} & \textit{Capacity constraints} \\ X_j \le \overline{D}_j & \forall\, j \in \{1,..,J\} & \textit{Demand constraints} \\ X_j \ge 0 & \forall\, j \in \{1,..,J\} & \end{cases}$$

The shadow price for a leg k, SP(k), is defined as the optimal solution to the dual problem associated with the capacity constraint on leg k. As explained above, SP(k) corresponds to the additional revenue generated if the k^{th} capacity constraint is increased by one unit in the above mathematical problem. In theory, SP(k) is the optimal network marginal revenue of an additional seat on leg k provided that the actual demand is what has been forecasted originally. Nonetheless, the optimal solution to the primal LP, as explained by Wei[13], is very often degenerate on an airline network. Therefore, the dual problem is very likely to have many solutions. As a consequence, the optimization algorithm[4] gives one of the possible shadow prices that depends on the methodology used to find the optimal solution (i.e., Simplex, Interior point methods).

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[4] for the experimental part of this thesis, we have been using the Simplex model.

What we have been looking for in practice is the expected revenue of the existing last seat on a leg. Therefore, another option is to estimate the ENRV(k) by the difference of the optimal objective value of (3.6) and the LP where C(k) is replaced by C(k-1), provided that at least one seat is available on leg k. This technique is called "True SP(Cap-1)." This technique, although simple to understand, requires that k+1 LPs be solved for each calculation of the shadow prices and may be computationally expensive to implement for an airline.

Wei[13] has analyzed several options to estimate ENRV using a linear programming shadow price approach. She found in practice that estimating the ENRV by the shadow prices from (3.6) generates a revenue comparable to other LP estimation techniques (like the True SP(Cap-1)) if the demand is controlled by a booking limit mechanism.

In this thesis, the shadow prices are the optimal dual values of the LP defined in (3.6).

3.2.1.2 Demand Robustness Issues

A model is said to be robust if it performs well with relatively high demand variability. For example, the nested control mechanisms are more robust than the partitioned ones, as explained in Chapter 2. If the demands were known with certainty before the beginning of the booking process, the linear program formulated in (3.6) would provide the optimal seat allocation, which would be partitioned. However, actual demand is stochastic and the partitioned deterministic LP model, where the booking limits are simply the solution of (3.6), performs poorly as the demand variability increases (Williamson[14]). In other words, the model, although capturing the network structure, is not robust with respect to demand variations. Similar to Curry[7], a way to relax the demand constraint is to consider the following LP

$$\text{Max}\left(\sum_{j=1}^{J} \text{EMSR}_j(X_j) \times X_j\right) \tag{3.7}$$

Subject to

$$\begin{cases} \sum_{j \in ODF(k)} X_j \le C(k) & \forall k \in \{1,..,K\} \quad \textit{Capacity constraints} \\ X_j \ge 0 & \forall j \in \{1,..,J\} \end{cases}$$

X_j is the decision variable that represents the number of seats to allocate to fare class j. $EMSR_j(X_j)$ is the expected marginal revenue of allocating X_j seats to fare class j. Therefore, the objective value is the expected revenue if the allocation strategy $\{X_j\}_{j=1,...,J}$ is implemented. The mathematical programming representation (3.7) is more complicated to solve than (3.6) as $EMSR_j$ is a non-linear integral function of X_j. Solving such a problem would require some approximation of the EMSR integral function and, moreover, the solution would still be partitioned.

3.2.1.3 Conclusion

The shadow price associated with a leg on the deterministic LP, (3.6), has been considered as an estimate of the expected marginal revenue of the last seat available on the leg. As explained above, shadow prices are based on a deterministic model that fails to consider the stochastic nature of the demand. A more stochastic approach, based on the EMSRb model explained in Chapter 2, is proposed, next, to estimate the displacement cost of a connecting ODF on a leg.

3.2.2 EMSR Approach

As opposed to the LP model, the EMSRb model presented in Chapter 2 takes into consideration the probability of the booking event to occur. The calculation of the displacement cost using the EMSRb model is presented in this section. The method is then analyzed.

3.2.2.1 *Definition*

We define the Critical EMSR value, EMSRc, as the expected revenue value of the last seat available on a leg obtained from the EMSRb model. The calculation of a critical EMSR value on a leg is illustrated in Section A of the appendix of the thesis. The displacement cost of connecting ODF j on leg k is, formally,

$$DC(j,k) = EMSRc(k) \tag{3.8}$$

3.2.2.2 *Process of the Method*

The published fares of each ODF are used as input to compute the EMSR curve. Then, the value of the last seat on each leg, EMSRc, is read on the curve. The NRV for each ODF on each traversed leg is then computed according to (3.2). As explained in Chapter 2, the ODF traversing a leg can be aggregated into virtual buckets. A dashed line in the figure below features this option. The other technique that consists of taking the ODF individually is called OD by OD. Both techniques are developed in Chapter 4. Figure 3-1 below summarizes the process to obtain the EMSRc value on each leg.

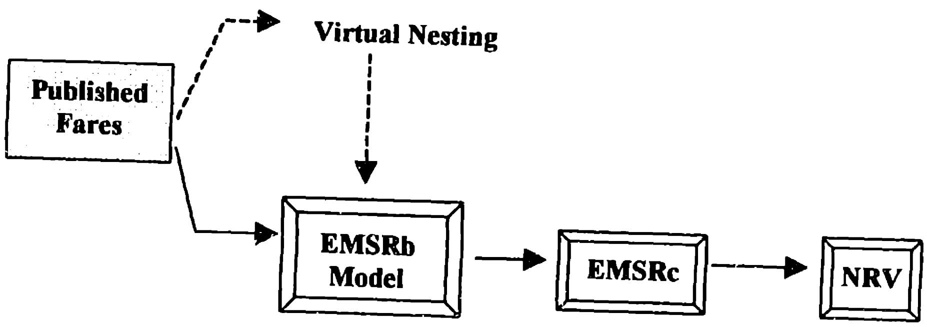

Figure 3-1: EMSRc Displacement Cost Estimation Process.

3.2.3 Shortcomings of the Method

In this section, we have proposed to estimate the ENRV by the critical EMSR value on the leg. Two arguments highlight the shortcomings of this approach. First, as mention in the introduction of Chapter 3, EMSRb is a leg-based methodology and therefore does not take into account the network path of each ODF. Moreover, the fares of the connecting ODF are counted several times if we use the total itinerary fares of the ODF traversing a leg as input to the EMSRb mathematical model. Assuming that a connecting ODF j traverses two legs, leg 1 and 2, the total fare f_j is taken into account in the computation of both EMSRc(1) and EMSRc(2). Therefore, f_j is considered in estimation the displacement cost on both legs 1 and 2. Thus, the EMSRc values tend to overestimate the displacement cost of the connecting ODF on a traversed leg and result in denying too many connecting ODF requests as their NRV are low.

In this thesis, we do not analyze the performances of the method that consists in estimating the displacement cost by the EMSRc value with the total fares as input to the EMSRb model. Wei[13] has conducted a detailed analysis of the performances of the method. In the experimental part of this thesis, we only consider the shadow price concept, presented in Section 2.1 of this chapter, to estimate the displacement cost of a connecting ODF on a leg. Furthermore, in Section 3.3, we propose a new technique that consists of prorating the connecting fare over the legs traversed by the corresponding ODF.

3.2.4 Conclusion

Two ways of estimating the displacement cost of a connecting ODF have been presented in this section. The first approach estimates the displacement cost using the shadow prices of a deterministic LP where the right-hand sides of the demand constraints are the mean demand forecast.

As opposed to the deterministic mathematical programming representation, the second approach encompasses the stochastic nature of the demand as it estimates the displacement cost by the EMSRc from the EMSRb heuristic model. The method considers the full fare of each connecting ODF on all the leg traversed and, therefore, tends to overestimate the displacement cost, which result in spilling too many connecting ODFs.

In this thesis, we propose a new methodology that consists in prorating the connecting ODF over their itinerary. This new technique is presented in the next section.

3.3 Prorated Fare Approach

3.3.1 Motivation

In this section, we propose a new technique that consists of prorating the fare of the connecting ODFs along the legs they traverse. The revenue generated by a potential request for a connecting ODF on a leg is the prorated fare of the ODF on the leg, as opposed to the EMSRc displacement cost estimate which take into account the total fares of the ODFs as input to the mathematical model. Moreover, the prorated concept takes into consideration the structure of the network and the more legs an ODF traverses the lower its average prorated fares tend to be. Furthermore, the new prorated fare methodology is a zero sum approach that is more robust to demand variation as analyzed in Chapter 4, where the seat inventory control models are tested on an actual airline network.

The objective is to find a way to prorate the fare of each connecting ODF over the legs they traverse. Williamson[14] proposes to prorate the connecting ODF according to their leg length. This technique does not take into consideration the demand on each traversed leg. For example, let us consider the ODF that traverses two legs such that the first segment is a short haul whereas the second is a long haul as happens frequently in a hub and spoke network structure. Moreover, suppose that the demand on the first short haul flight is high whereas the demand for the second long haul flight is low. The prorated fare of the connecting ODF would be higher on the long haul flight than on the short haul flight using the distance prorated technique. Therefore, the connecting ODF is likely to have a high priority on the second leg where the demand is low and a low priority on the first leg where the demand is high. Consequently, there is a high number of connecting ODF requests that would be denied as their priority on the high demand leg is low. Thus, a significant number of seats on the low demand leg, which could have been sold to the denied connecting ODF, stay unoccupied resulting in a revenue loss for the airline. Therefore, the distance-prorated approach does not perform well in such a case.

The proration technique has to encompass the stochastic nature of the demand. Coming back to the above example, the objective is to find a technique such that the prorated fare of the connecting passenger on the low demand long haul leg is lower than the prorated fare of the same ODF on the high demand short haul leg. Therefore, the connecting ODF would have a better protection level on the network. Assuming that the demand is low on all the traversed legs, the technique should gives more or less equal weight to all the legs in order to reduce the risk that some connecting ODF are spilled. In the next section we propose a new technique to prorate the fare of the connecting ODF.

3.3.2 Definition

3.3.2.1 Prorated Fare

In this thesis, we propose a new approach based on a prorated fare concept. The prorated fare of a connecting ODF j on leg k is defined as

- If $\sum_{m \in L_j} \text{EMSRc}(m) \neq 0$ then, $$\text{PRF}[j,k] = \frac{\text{EMSRc}(k)}{\sum_{m \in L_j} \text{EMSRc}(m)} \times f_j \qquad (3.9)$$

- If $\sum_{m \in L_j} \text{EMSRc}(m) = 0$ then, $$\text{PRF}[j,k] = \frac{f_j}{\text{card}(L_j)} \qquad (3.10)$$

The prorated fare of a connecting ODF on a leg is the ratio of the critical EMSR value on the leg and the sum of the critical EMSR values on all the traversed leg, times the total fare, provided that the sum of the EMSRc over the traversed legs is strictly positive. If the sum of the critical EMSR is null then the fare is evenly distributed over all the traversed legs, card(L_j) corresponding to the cardinality of the set L_j or the number of legs traversed by ODF j. As the critical EMSR values are by definition positive, this corresponds to the case where all the EMSRc are zeros for all the traversed legs. In other words, the demand forecast is very likely to be well below the capacity on all the legs belonging to L_j. Therefore, we decide to prorate evenly the fare of the connecting ODF j as we forecast that some seats will be available at the end of the booking process on all the legs belonging to L_j.

As in the displacement cost approach, the prorated fares of the local ODF are simply equal to their total fare. Therefore,

$$\text{If } L_j = \{k\}, \ \text{PRF}[j,k] = f_j \qquad (3.11)$$

3.3.2.2 Lagrangian Coefficients

The Lagrangian coefficient, θ, corresponding to the ODF j on leg k is

- If $\sum_{m\in L_j} EMSRc(m) \neq 0$ then, $\theta_{j,k} = \frac{PRF(j,k)}{f_j} = \frac{EMSRc(k)}{\sum_{m\in L_j} EMSRc(m)}$ (3.12)
- If $\sum_{m\in L_j} EMSRc(m) = 0$ then, $\theta_{j,k} = \frac{1}{card(L_j)}$ (3.13)

Note that, $\theta_{j,k}$ is no more than the percentage of the published fare of ODF j assigned to leg k and therefore $\theta_{j,k} \in [0;1]$. If ODF j is a local ODF on leg k, then, $\theta_{j,k} = 1$.

3.3.2.3 Properties

By definition of the prorated fares and the Lagrangian coefficients, for all j,

$$\sum_{k\in L_j} PRF(j,k) = f_j \qquad \forall j \in \{1,..,J\} \tag{3.14}$$

$$\sum_{k\in L_j} \theta_{j,k} = 1 \qquad \forall j \in \{1,..,J\} \tag{3.15}$$

Moreover, for all connecting ODF j, the ratio of the prorated fare on two legs is equals to the ratio of the critical EMSR values on these legs, provided that the EMSRc are not null.

$$\frac{PRF(j,m)}{PRF(j,k)} = \frac{EMSRc(m)}{EMSRc(k)} \qquad \forall (m,k) \in L_j. \tag{3.16}$$

3.3.3 Convergence Model

The revenue inputs that are used initially in the EMSRb model to compute the critical EMSR value on leg k are the total itinerary fares of the ODF traversing leg k. Therefore, the EMSR values are overestimated as the total itinerary fares of the connecting ODF are taken into account in the EMSRb model. A convergence model using the prorated fares as input to the EMSRb model has been developed to address this problem and is presented in the section below. First, the process of the method is explored and then the existence of the convergence limit is discussed.

3.3.3.1 Process of the Method

Figure (3.2) summarizes the process of the prorated fare convergence method.

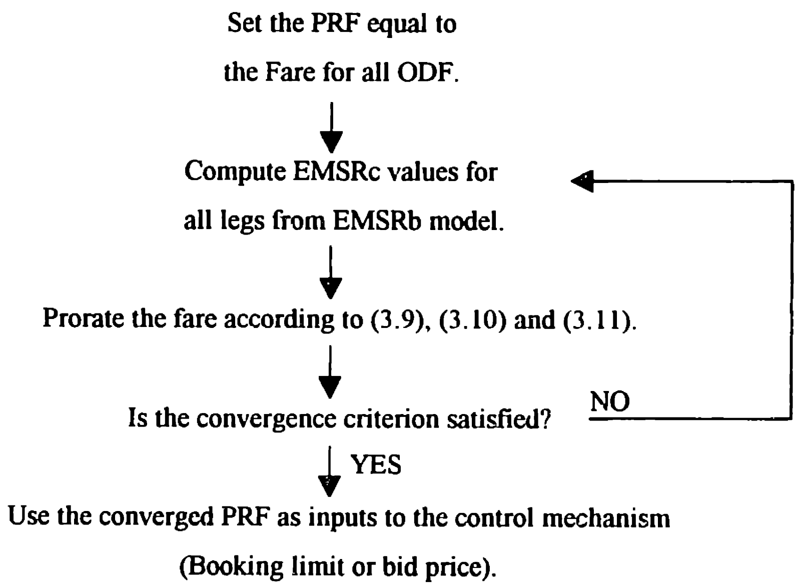

Figure 3-2: Prorated Fare Convergence Mechanism.

We have arbitrarily decided to set initially the prorated fares to their respective total fares for each ODF. They are next used to compute the critical EMSR value on each leg using the EMSRb mathematical model. The PRF for all ODF are then computed using (3.9), (3.10) and

(3.11). Then, the PRF are compared with the previous one (the initial fares for the first iteration). If the convergence criterion (explained in more details in the next section) is satisfied the last computed PRF are used as inputs to the control mechanism, analyzed in Chapter 2. Conversely, if the convergence criterion is not satisfied, the last computed PRF are used as inputs to the EMSRb model. The new PRF are computed and the convergence criterion is checked.

3.3.3.2 Convergence Criterion

An index, n, is added to the prorated fare definition in order to denote the iteration number of the convergence process. Therefore, PRF(j,k,n) is the prorated fare associated with ODF j on leg k after n iterations of the convergence process. The initial PRF, PRF(j,k,0), is arbitrarily set to f_j as explained in Figure 2 above. Similarly, the Lagrangian coefficient of the connecting ODF j on leg k for the n iterations of the convergence process is $\theta_{j,k,n}$. Moreover, $(\theta_{j,k,n})_{n \in N}$ is defined as the sequence of the Lagrangian coefficients for the connecting ODF j on leg k. Equation (3.15) still holds for all iterations n,

$$\sum_{k \in L_j} \theta_{j,k,n} = 1 \qquad \forall j \in \{1,..,J\} \text{ and } \forall n \in N \tag{3.17}$$

The convergence criterion that we have used in this thesis is

$$\underset{j,k}{\text{MAX}} \; | \text{PRF}[j,k,m] - \text{PRF}[j,k,m-1] | < \$5 \qquad m \in N^*. \tag{3.18}$$

The largest difference between two consecutive prorated fare iterations, for all ODF over all the legs, has to be less than $5 in order to satisfy the convergence criterion. The first set of PRF that satisfy the convergence criterion are used as inputs to the booking control mechanisms (i.e., booking limits or bid prices).

3.3.4 Convergence Analysis

The objective of this section is to analyze the existence of the convergence limit of the prorated fare sequence, or similarly the Lagrangian sequence, corresponding to the connecting ODF traversing a network. In the first place, the convergence is explored on a small two-leg network traversed by three ODF. Then, we discuss the convergence existence on a more general network.

3.3.4.1 Simple Case

<u>The Network</u>

The simple linear network, illustrated below, consists of three nodes A, B, C and two legs, A-B and B-C. Three ODFs 1, 2 and 3 traverse the network. One connecting (ODF 3) and two locals (ODF 1 and ODF 2). The connecting ODF traverses both legs whereas ODF 1 and 2 are offered respectively on only leg AB and BC.

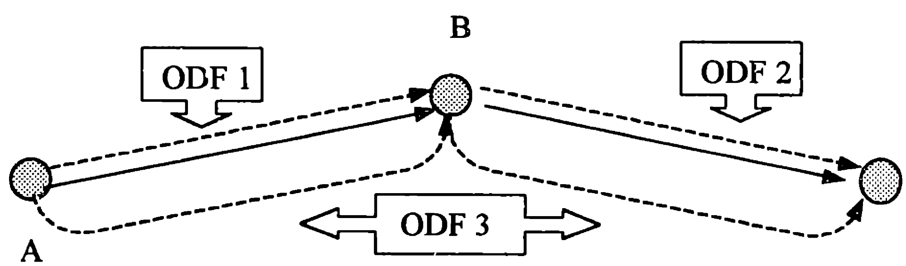

Figure 3-3: Two Legs, Three ODF Linear Network

The letters j, k and n denote respectively: the ODF, the leg number and the convergence iteration. In this small example, $j \in \{1,2,3\}$, $k \in \{1,2\}$, $n \in \{0,..,+\infty\}$.

Moreover, $\overline{D}_j$ is defined, as the mean demand forecast and f_j is the published fare corresponding to ODF j. The table below summarizes the mean demand forecast and the total fare of the ODF traversing both legs.

Leg A-B			Leg B-C		
j	$\overline{D}_j$	f_j	j	$\overline{D}_j$	f_j
1 3	$\overline{D}_1$ $\overline{D}_3$	f_1 f_3	2 3	$\overline{D}_2$ $\overline{D}_3$	f_2 f_3

Table 3-1: Forecasted Mean Demand for all ODF traversing both legs

Definition

- Probability

We define α, β, and δ as:

$$\alpha = \text{Prob } (D_1+D_3 > \text{Avail}[1])$$

$$\beta = \text{Prob } (D_2+D_3 > \text{Avail}[2])$$

$$\delta = \text{Prob } (D_3 > \text{Avail}[1])$$

With Avail[k] ($k \in \{1,2\}$), the number of seats available on leg k. Note that α, β, and δ are constant all along the convergence process as the number of seats available and the demand distribution are given. Moreover, no assumption has been made concerning the demand probability density function. The demand can well be gaussian, gamma or anything else.

- Prorated Fare and Lagrangian Coefficients

The prorated fare of a local ODF is equal to its corresponding published fare. Therefore, f_1

and f_2 are the prorated fares of the local ODF on leg A-B and B-C respectively.

PRF(3,k,n) and $\theta_{3,k,n}$ are defined, respectively, as the prorated fare and the Lagrangian coefficient of the connecting ODF (i.e., ODF 3) on leg k for the n iteration of the convergence process. As explained in (3.12), $\theta_{3,k,n}$ is the ratio of the prorated fare and the total fare of the connecting ODF (i.e., $\theta_{3,k,n} = PRF(3,k,n) / F_3$).

According to (3.17), the prorated fares and the Lagrangian coefficients satisfy the conditions

$$\begin{cases} PRF(3,1,n) + PRF(3,2,n) = 1 & & (3.19) \\ \theta_{3,1,n} + \theta_{3,2,n} = 1 & \forall\, n \in \{0,..,+\infty\} & (3.20) \end{cases}$$

- Demand coefficients

$\gamma_{1,1}$ and $\gamma_{3,1}$ are defined as the expected percentage of, respectively, local and connecting requests on the first leg. Similarly, $\gamma_{2,2}$ and $\gamma_{3,2}$ are, respectively, the percentage of local and connecting forecasted requests on the second leg.

$$\gamma_{1,1} = \frac{\overline{D}_1}{\overline{D}_1 + \overline{D}_3} \qquad \gamma_{2,2} = \frac{\overline{D}_2}{\overline{D}_2 + \overline{D}_3} \qquad \gamma_{3,1} = \frac{\overline{D}_3}{\overline{D}_1 + \overline{D}_3} \qquad \gamma_{3,2} = \frac{\overline{D}_3}{\overline{D}_2 + \overline{D}_3} \qquad (3.21)$$

Convergence Analysis

We are only interested in studying the convergence of the sequence $(\theta_{3,1,n})_{n \in N}$ because the convergence of this sequence determines, mutatis mutandi, the convergence of the other sequence $(\theta_{3,2,n})_{n \in N}$ as $\theta_{3,1,n} + \theta_{3,2,n} = 1 \; \forall n \in N$.

On the simple network define above, two cases have to be considered. First, when the EMSRc value on a leg is, for all the iterations of the convergence process, on the piece of curve generated by the compound of both the connecting (i.e., ODF 3) and the local ODF (i.e., ODF 1 or 2 depending on the leg). The second case corresponds to the situation when, for some iterations of the convergence process, the EMSRc is obtained from the piece of curve generated by the connecting ODF only.

We define IN(k,n), the set of ODF that defines the critical EMSR value of leg k for the n^{th} iteration. In other words, IN(k,n) is the set of ODFs that generate the point on the EMSR curve that corresponds to the EMSRc value.

In the first case, explained above, for all iterations n, IN(1,n) = {ODF 1, ODF 3}. Therefore, for all the iterations of the convergence process, the critical EMSR values are of Type 1 illustrated in Figure 3-4 below. Whereas in the second case, it exists an iteration n such that, IN(1,n) = {ODF3}. For several iterations, the EMSRc is defined by ODF 3 only, corresponding to Type 2 in the illustration below.

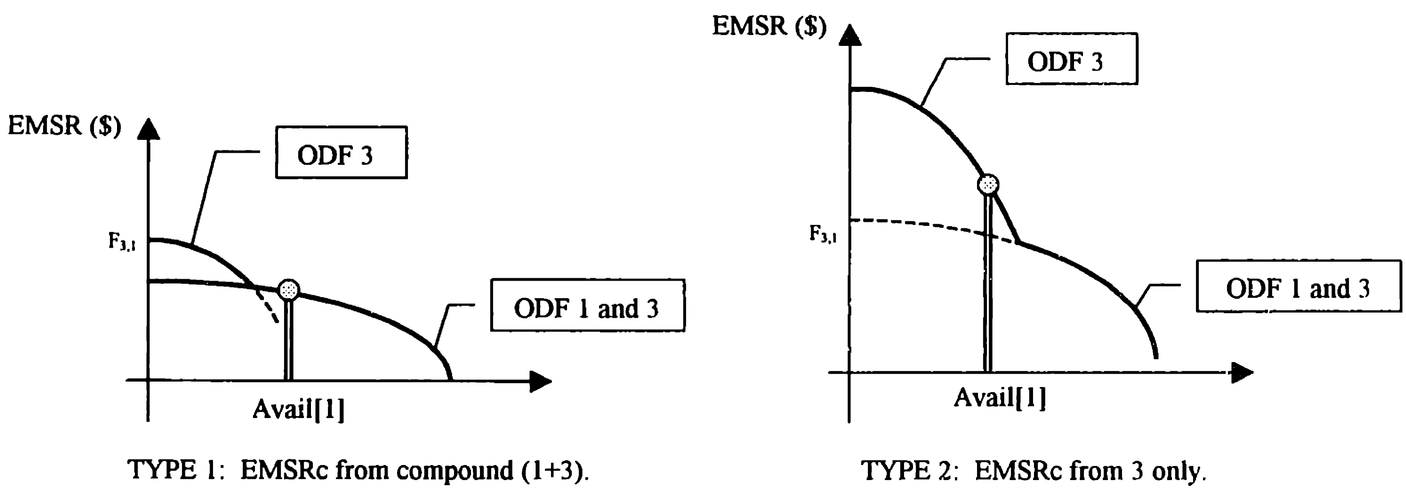

Figure 3-4: Two Types of EMSRc values.

<u>CASE 1</u>: IN(1,n) = {ODF 1, ODF 3} and IN(2,n) = {ODF 2, ODF 3} $\forall n \in N$.

For all the iterations of the convergence process, the set of ODFs that generates the point on the EMSR curve corresponding to EMSRc, IN(1,n) and IN(2,n), is the compound of ODF 3 with, respectively, ODF 1 and 2. In other words, the critical EMSR value on both legs is always of Type 1, as illustrated in Figure 3-4. Therefore, the EMSRc operator keeps the same definition all along the convergence process on both legs.

The critical EMSR on both legs for the n iterations are:

$$\text{EMSRc}\,[1,n] = \alpha\ [\gamma_{1,1}\, f_1 + \gamma_{3,1}\, \text{PRF}(3,1,n\text{-}1)] = \alpha\, f_3\, [\gamma_{1,1}\, f_1/f_3 + \gamma_{3,1}\, \theta_{3,1,n\text{-}1}] \quad (3.22)$$

$$\text{EMSRc}\,[2,n] = \beta\ [\gamma_{2,2}\, f_2 + \gamma_{3,2}\, \text{PRF}(3,2,n\text{-}1)] = \beta\, f_3\, [\gamma_{2,2}\, f_2/f_3 + \gamma_{3,2}\, \theta_{3,2,n\text{-}1}] \quad (3.23)$$

By definition of the prorated fare, we have:

$$\theta_{3,1,n} = \frac{\text{EMSRc}[1,n]}{(\text{EMSRc}[1,n] + \text{EMSRc}[2,n])} \quad (3.24)$$

As explained in (3.13), if the sum, over both legs, of the critical EMSR values for the same iteration n (i.e., EMSRc[1,n] + EMSRc[2,n]) is null then we have decided arbitrarily to prorate equally the total itinerary fares of the connecting ODF over the traversed legs. In other words, $\theta_{3,1,n} = \theta_{3,2,n} = ½$. The intuitive reason is that if both critical EMSR values are equal to zero, the demand is expected to be well below the capacity. In such case the objective is to give the maximum chance for a connecting request to be accepted by equally prorating the connecting fare on all the traversed legs.

After some simple algebra, and using equation (3.20), (3.22), (3.23) and (3.24), we find that

$$\theta_{3,1,n} = \frac{a + b \times \theta_{3,1,n-1}}{c + d \times \theta_{3,1,n-1}} \tag{3.25}$$

with

$$\begin{cases} a = \alpha\, \gamma_{1,1}\, F_1/F_3 \\ b = \alpha\ \ \gamma_{3,1} \\ c = \alpha\, \gamma_{1,1}\, F_1/F_3 + \beta\, \gamma_{2,2}\, F_2/F_3 + \beta\, \gamma_{3,2} \\ d = \alpha\ \ \gamma_{3,1} - \beta\, \gamma_{3,2} \end{cases}$$

Similarly for leg 2, we would have obtained the parameters a, b, c, d by replacing 1 by 2 and α by β in the above formulas. But, as we mention above, we are only interested in the convergence of the prorated fare of the connecting ODF on leg 1 as this would induce the convergence of the sequence of the prorated fares on the other leg (i.e., leg 2).

We next, prove that the sequence of the Lagrangian coefficients corresponding to the connecting ODF on leg 1 converges.

Theorem: The sequence $(\theta_{3,1,n})_{n\in N}$ converges.

Proof:

According to (3.35), all the elements of the sequence $(\theta_{3,1,n})_{n\in N}$ are on the curve: $Y = F(X)$ with

$$Y = F(X) = \frac{a + b \times X}{c + d \times X} \qquad X \in [0;1] \tag{3.26}$$

The sequence $(\theta_{3,1,n})_{j\in N^*}$ is said to be homographic.

The first derivative of the function F is:

$$F'(X) = \frac{bc - ad}{(c + dX)^2} \tag{3.27}$$

with

$bc\text{-}ad = \alpha\ \gamma_{3,1}\ (\alpha\ \gamma_{1,1}\ F_1/F_3 + \beta\ \gamma_{2,2}\ F_2/F_3 + \beta\ \gamma_{3,2}) - \alpha\ \gamma_{1,1}\ F_1/F_3\ (\alpha\ \gamma_{3,1} - \beta\ \gamma_{3,2})$

$\Leftrightarrow$ $bc\text{-}ad = \alpha\ \beta\ \gamma_{3,1}\ \gamma_{3,2} + \alpha\ \beta\ \gamma_{3,1}\ \gamma_{2,2}\ F_2/F_3 + \alpha\ \beta\ \gamma_{1,1}\ \gamma_{3,2}\ F_1/F_3$

$\Leftrightarrow$ $bc\text{-}ad \geq 0$ as all the coefficients of the sum are positive.

Therefore, $\forall X \in [0;1]$, $F'(X) \geq 0$ and the function F is increasing.

Moreover, the second derivative is:

$$F''(X) = \frac{-2d(bc - ad)}{(c + dX)^3} \tag{3.28}$$

Let define X^* the value, such that $F(X^*)=X^*$

Lemma 1: The first derivative of F in X* is less than one: $F'(X^*) < 1$

Lemma 1 is proved for both cases: F is convex or concave.

- $d \leq 0$ (i.e., $\alpha \gamma_{3,1} \leq \beta \gamma_{3,2}$)

If $d \leq 0$ according to (4) the function F is convex as $F''(X) \geq 0$. Moreover, as we have proved before, F is increasing. The function F is illustrated on the figure below.

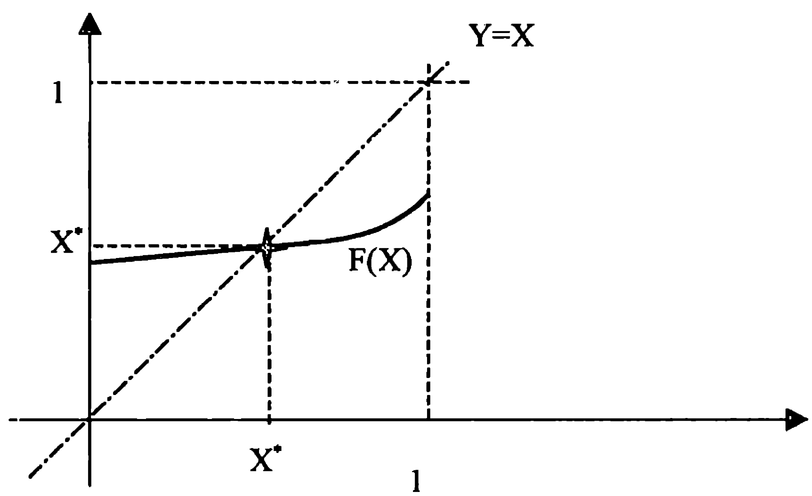

Figure 3-5: F Curve Supporting the Lagrangian Coefficient Sequence

By looking at the definition of a, b, c, d we have:

$$0 \leq F(0) = \frac{a}{c} < 1 \tag{3.29}$$

$$0 \leq F(1) = \frac{a+b}{c+d} < 1 \tag{3.30}$$

As F is convex we have

$$\forall\, t \in [0;1],\quad F'(X^*)\,(t - X^*) + F(X^*) \leq F(t)$$

The tangent line to the curve of F in X^* is below the curve F.

If $F'(X^*)$ were greater than one we would have:

$$\forall\, t\in[0;1],\quad t - X^* + F(X^*) \leq F(t)$$

Therefore, for t=1, $1 - X^* + F(X^*) \leq F(1)$

As $F(X^*) = X^*$, we would have that $F(1) \geq 1$. But, this result is in contradiction with (3.30) and therefore $F'(X^*) < 1$.

- $d > 0$ (i.e., $\alpha\, \gamma_{3,1} > \beta\, \gamma_{3,2}$)

If $d > 0$ the function F is <u>concave</u> as $F''(X) < 0$. Therefore,

$$\forall\, t\in[0;1],\quad F'(X^*)\,(t-X^*) + F(X^*) > F(t)$$

The tangent line to the curve of F in X^* is above the curve of F.

For $t = 0$, $F(0) < F'(X^*)\,(-X^*) + F(X^*)$

If $F'(X^*) \geq 1$ we would have

$$F(0) < F(X^*) - X^*$$

But by definition of the fixed point, $F(X^*) = X^*$ and therefore, according to the above result $F(0) < 0$. But, by definition of the prorated coefficients, $F(0) \geq 0$. Therefore, we have found a contradiction and have proved that when $d > 0$, $F'(X^*) < 1$.

Consequently, independently of the sign of d, the derivative of the function F in X* is strictly less than one ($F'(X^*) < 1$).

The sequence $(\theta_{3,1,n})_{n\in N}$ is bounded as all its elements belong to [0;1]. Moreover, the function F is increasing implying that the sequence is increasing if the first element of the sequence is below X* and decreasing if the first element is above X*. Therefore, after N iterations all the elements of the sequence belong to the vicinity S of X*.

$\exists N / \forall n > N, \ | F(\theta_{3,1,n}) - F(X^*) | \ \leq \ F'(X^*) \times | \theta_{3,1,n} - X^* |$

For all integer q we have , $| F(\theta_{3,1,N+q}) - X^* | \ \leq \ F'(X^*)^q \times | \theta_{3,1,N} - X^* |$

As q goes to infinity the sequence on the right hand side converges to zero because it is a geometric sequence with $r = F'(X^*) < 1$. Therefore, the sequence $(\theta_{3,1,n})_{n \in N}$ converges to the fixed point X^*.

Therefore, the sequence $(\theta_{3,1,n})_{j \in N}$ converges. We also know that $\theta_{3,1,n} + \theta_{3,2,n} = 1$ and conclude that the sequence $(\theta_{3,2,n})_{j \in N}$ also converges.

Uniqueness of the limit

Suppose that there are two different real X and W such that the sequence $(\theta_{3,1,n})_{j \in N}$ converges to both X and W. We have $|X-W| \leq |X - \theta_{3,1,n} + \theta_{3,1,n} - W| \leq |X - \theta_{3,1,n}| + |W - \theta_{3,1,n}|$
But, by definition of the convergence of a sequence the right hand side term converges to zero and therefore, W and X are equal. Thus, the limit of the sequence is unique.

Conclusion

We have proven that the sequence $[F(3,1,n)]_{n \in N}$ converges to a unique limit on a simple network, under the assumption that for all the iterations of the convergence process, the set of ODF that generates the point on the EMSR curve for both legs is the compound of the connecting ODF and the local ODF.

CASE II: The IN sets are not always IN(1,n)={ODF1,ODF3} and IN(2,n)={ODF2,ODF3} along the convergence process.

We define the sets S and T such that

S = {n ∈ N such that: IN(1,n)={ODF1,ODF3} and IN(2,n)={ODF2,ODF3}} and

T = {n ∈ N such that: IN(1,n)≠{ODF1,ODF3} and/or IN(2,n)≠{ODF2,ODF3}}.

If T is finite and therefore S is infinite (as $S \cup T = N$), it exists an integer M such that: for all n greater than M, the critical EMSR value is on the piece of curve generated by the compound of both the connecting and the local ODF. In other words, when n is greater than an integer value M, we know that we meet the assumption of case I. Therefore, the sequence, $(\theta_{3,1,n})_{n \in N}$, converges.

Nonetheless, the sequence may not converge when the sets, S and T, are both infinite as we may have a cycling problem. The sub-sequences $(U_n)_{n \in N}$ and $(V_n)_{n \in N}$ defined respectively as $(\theta_{3,1,n})_{n \in S}$ and $(\theta_{3,1,n})_{n \in T}$, do not convergence to the same fixed points.

$$\left.\begin{array}{l} U_n = \theta_{3,1,n\,(n \in S)} \xrightarrow[\infty]{} u \\ \\ V_n = \theta_{3,1,n\,(n \in T)} \xrightarrow[\infty]{} v \end{array}\right\} \quad \text{where} \quad u \neq v \tag{3.31}$$

In other words, the critical EMSR value is computed using two different sets of ODF and the model never stops switching between the two sets of ODF. If the two sub-sequences, U_n and V_n, converge to two different fixed points, the model ends up cycling between these two different values, u and v, and does not converge.

Moreover, if after a certain number of iterations, only the connecting ODF (or the local ODF) are considered in the computation of the EMSR critical values, the convergence can be proved the same way as in case I. In this case, the function F has the form $\frac{aX}{b + cX}$ and we would show similarly the convergence of the sequence.

Conclusion:

As long as the EMSRc operator is defined by the same set of ODF for an infinite consecutive number of iterations, the sequence $(\theta_{3,1,n})_{n\in N}$ converges. The sequence may not converge if the EMSR operator is defined by several different sets of ODF.

On leg 1, for example, we define the two sets N_1 = {n such that IN(1,n) = {ODF1, ODF3}} and N_2 = {n such that IN(1,n) = {ODF3}}. If N_1 and N_2 have both an infinite cardinality then the prorated fare of ODF 3 on leg 1 may not converge to a unique limit value as the two sub-sequences $(\theta_{3,1,n})_{n\in N1}$ and $(\theta_{3,1,n})_{n\in N2}$ may have different limits.

3.3.4.2 General Case

We have analyzed the simple case of a network with two flight legs and three ODF. In fact we can generalize the results that we have proven on a simple network for any network as long as a connecting OD traverses no more than two flight legs. If this is the case, the sequence $\theta_{j,k,n}$ can be expressed as a continuous fraction of the form $\frac{a + b\times\theta_{j,k,n-1}}{c + d\times\theta_{j,k,n-1}}$ with a, b, c, and d strictly positive real, specific to the connecting OD j on leg k. The prorated sequence, $(\theta_{j,k,n})_{n\in N}$, is homographic and we would prove the convergence of the sequence similarly as we did above for the simple case. Therefore, if the connecting passengers traverse no more than two flight legs, the prorated fare sequence converges.

When some connecting passengers traverse more than two legs, the problem becomes much more complicated as the sequence $(\theta_{j,k,n})_{n\in N}$ are dependent on each other. In this section we prove the existence and the uniqueness of the convergence limit of each sequence $(\theta_{j,k,n})_{n\in N}$.

Definitions:

Let assume that a connecting OD, arbitrarily called c, traverses the first 3 legs of the network (i.e., leg 1, 2 and 3). We define CO(k) and LO(k) as respectively, the set of connecting OD and local OD traversing leg k. TO(k) is defined as the set of OD traversing leg k and therefore, $TO(k) = LO(k) \cup CO(k)$.

Moreover, we define the set L_j as the set of legs traversed by OD j and therefore, $L_c = \{1,2,3\}$. $\theta_{c,k,n}$ is defined as the n^{th} iteration of the prorated coefficient corresponding to the connecting OD, c, on leg k.

Expression of the Lagrangian vector corresponding to OD c:

The vector of Lagrangian coefficients corresponding to OD c for iteration n is defined by the equations below.

$$\theta_{c,1,n} = \frac{a_1 + \sum_{i \in CO(1)} b_{i,1} \times \theta_{i,1,n-1}}{\sum_{k \in L_c} a_k + \sum_{k \in L_c} \sum_{i \in CO(k)} b_{i,k} \times \theta_{i,k,n-1}} \tag{3.32}$$

$$\theta_{c,2,n} = \frac{a_2 + \sum_{i \in CO(2)} b_{i,2} \times \theta_{i,2,n-1}}{\sum_{k \in L_c} a_k + \sum_{k \in L_c} \sum_{i \in CO(k)} b_{i,k} \times \theta_{i,k,n-1}}$$

$$\theta_{c,3,n} = \frac{a_3 + \sum_{i \in CO(3)} b_{i,3} \times \theta_{i,3,n-1}}{\sum_{k \in L_c} a_k + \sum_{k \in L_c} \sum_{i \in CO(k)} b_{i,k} \times \theta_{i,k,n-1}}$$

$$\text{And } \sum_{k \in L_c} \theta_{c,k,n} = 1 \quad \forall n \in N \tag{3.33}$$

by definition of the Lagrangian coefficients.

Where we define

$$a_k = \left[\sum_{i \in LO(k)} \left(\overline{D}_i \times F_i \right) \right] \times \frac{\text{Prob}\left(\sum_{i \in TO(k)} D_i \geq C_k \right)}{\sum_{i \in TO(k)} \overline{D}_i} \tag{3.34}$$

and

$$b_{j,k} = \overline{D}_j \times F_j \times \frac{\text{Prob}\left(\sum_{i \in TO(k)} D_i \geq C_k \right)}{\sum_{i \in TO(k)} \overline{D}_i} \tag{3.35}$$

- a_k corresponds to the sum of the local fares weighted by the mean demand forecast, times the probability that the total demand on leg k is greater than the number of seats available on the leg divided by the sum of all the mean demand forecast of all the ODF traversing leg k. Thus, a_k is a constant as the fare of the local ODF are not affected by the prorated fare technique.
- $b_{j,k}$ corresponds to the ratio of the mean demand forecast of connecting ODF j and the sum over all the ODF traversing leg k times the total fare of the connecting ODF j and the probability that the total demand on leg k is greater than the number of seats available on the leg.

For all the Lagrangian coefficients corresponding to a connecting ODF, the denominator is the same as one can observe in (3.32). It corresponds to the sum of the critical EMSR values for the $(n-1)^{th}$ iteration of the convergence process. But this value is dependent on all the PRF of the connecting OD traversing one of the leg belonging to L_c. Therefore, the Lagrangian coefficient of ODF c on leg k is dependent on the Lagrangian coefficients of all the ODF that traverse one of the legs traversed by c (i.e., one of the leg belonging to L_c).

The convergence of the sequence seems not trivial to prove, as the prorated sequences are dependent on each other. We first prove the existence and then the uniqueness of the limit of each sequence $(\theta_{j,k,n})_{n \in N}$ of Lagrangian coefficients.

Existence of the limits:

Bolzano-Weierstrass theorem states that every bounded sequence has a limit point. But for all iterations n, $\theta_{j,k,n} \in [0;1]$. Therefore, the sequences $(\theta_{j,k,n})_{n\in N}$ are bounded for all OD j and all legs traversed k as all the iterations belong to the interval [0;1]. Therefore, there exists at least one limit point for each sequence. Nonetheless, it is hard to prove that the limit point for each sequence $(\theta_{j,k,n})_{n\in N}$ is unique. The theorem of Bolzano-Weierstrass proves the existence but not the uniqueness of the limit point.

Uniqueness of the limits:

Let consider the operator G that transforms the matrix $\left[x_{j,k}\right]_{j=1,..J}^{k=1,..,K}$, where j is the OD index and k the leg index, into $\left[g(x_{j,k})\right]_{j=1,..J}^{k=1,..,K}$ with $g_{j,k}$ the function:

$$g_{j,k}: \quad [0;1] \longrightarrow [0;1]$$

$$x_{j,k} \longrightarrow \frac{a_k + \sum\limits_{i\in CO(k)} b_{i,k} \times x_{i,k}}{\sum\limits_{m\in L_j} a_m + \sum\limits_{m\in L_j} \sum\limits_{i\in CO(m)} b_{i,m} \times x_{i,m}} \qquad (3.36)$$

G is the function that transform the (n-1)th into the nth iteration of the convergence process. Therefore, the limit matrix X^* is solution to the system of (J×K) equations:

$$X = G(X) \qquad (3.37)$$

If we prove that G is a contraction operator, for which the distance between images of any two distinct points is less than the distance between the points, then the Banach theorem proves the existence and the uniqueness of a fixed point and this point can be obtained by the method of successive approximation for any initial point.

Lemma 2: $\max_{c,k} \left\| \nabla g_{c,k}(X^*) \right\| < 1$.

The proof of this lemma is developed in Appendix Section C.

We can use the Banach theorem to prove the uniqueness of the limit points.

Banach[2] theorem: If the contraction operator U maps a complete metric space X onto itself, then we have a unique fixed point.

Conclusion: The operator G is a contraction and therefore the sequence $(\theta_{j,k,n})_{n \in N}$ converge toward a unique fixed point.

3.3.4.3 Virtual Class Versus OD by OD Convergence

The prorated fare convergence method distributes the fare of the connecting ODF over the legs they traverse. Two different ways of computing the EMSRc values have been tested in the experimental analysis of this thesis. As explained in Chapter 2, the airline has the possibility to aggregate the ODF into virtual buckets with regard to the NRV of the ODF traversing the leg. The fare associated with the virtual class is the sum of the fares of the ODF belonging to the virtual class weighted by the mean demand forecast as explained in (2.10). Then, the EMSRc values are computed using the virtual nesting EMSRb model. The prorated fare convergence model that is based on the virtual class EMSRb model is called VC-convergence model. If the EMSRb model considers all the ODF traversing a leg as individual entities the convergence model is called OD by OD convergence model. Both methods are tested and analyzed in Chapter 4.

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[2] S. Banach (1892-1945) one of the founder of functional analysis.

3.3.4.4 Conclusion

As opposed to the displacement cost concept, the prorated convergence approach constrains the sum of the NRV of a connecting ODF over all the traversed legs to be equal to the fare of the connecting ODF. The NRV are the limit values of a prorated fare convergence process. As discussed above, the existence of the limit depends on the stability of the EMSRc operator. If the operator is defined by the same set of ODF after a certain number of iterations of the convergence process, the model converges. Nonetheless, some cycling problems may occur when the EMSRc operator is computed using different sets of ODF in successive iterations. The quality of the convergence using actual data is discussed in Chapter 4.

3.4 Summary

A connecting ODF, which traverses several legs, is not taken in account into the leg based EMSRb model. The objective of the network revenue value concept is to estimate the revenue contribution of the connecting ODF on each leg they traverse. Two different techniques to estimate the network revenue value (NRV) have been highlighted in this chapter.

The first concept, called displacement cost, consists of estimating the opportunity cost of selling a seat to a connecting ODF as it may displace several ODF that would have generated more revenue. Two approaches have been considered. The first approach is deterministic and uses mathematical programming concepts. The displacement cost on a leg is estimated by the shadow price associated with the corresponding leg. As opposed to the previous technique, the second approach is stochastic and estimates the displacement cost of a leg by the critical EMSR value on this leg. The latter technique has not been tested in this thesis as we have highlighted several shortcomings. The most important problem is that the technique overestimates the displacement cost on a leg as it considers the total fare of a connecting ODF on each traversed leg, resulting in spilling too many connecting passengers. The displacement cost concept is not a zero sum technique as the sum of the NRV of each ODF over the traversed legs is most of the times different from the total fare.

In this thesis, we propose a new way to estimate the NRV of a connecting ODF. The method is based on a prorated fare convergence model. As opposed to the first technique, the sum of the NRV of a connecting ODF over the traversed legs equals the total itinerary fare of the ODF. The new technique takes into account both the structure and the demand pattern of the network. If the demand forecast is high on a leg the prorated fare of the connecting ODF will be high. Conversely, if the demand is low, less weight is assigned to the leg as the probability that some seats are available on the flight is high. Therefore, the prorated fare model optimizes the availability of a connecting ODF given that the sum of the prorated fare is the total fare. We have proven the existence and the uniqueness of the limit on a small network and highlight the potential cycling problems in the application using the EMSRb model. Moreover, we have proven the existence and the uniqueness of the limit point for a general network under the assumption that the EMSRc operator keeps a stable definition.

The NRV are then used as input to the control mechanism that uses the EMSRb model. Figure 3-6 below summarizes all the concepts described in Chapter 2 and 3.

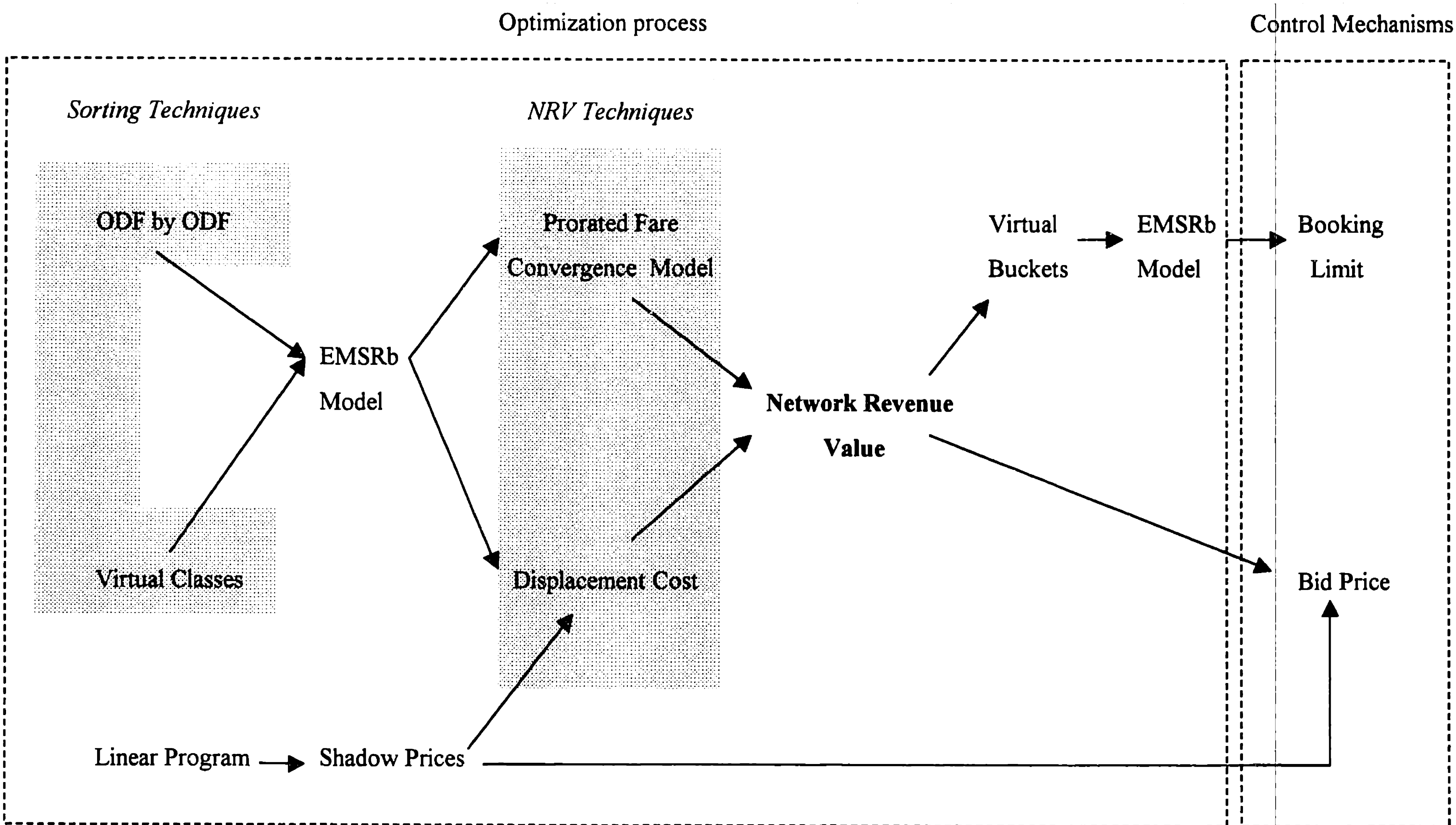

Figure 3-6: Summary of Network Value Seat Inventory Control Models.

4 Case studies

The objective of Chapter 4 is to measure the performance of some seat inventory control methods, using the network revenue value concept on a major airline network. The booking process simulator is first described and the assumptions that it makes with regard to the passenger behavior modeling are highlighted in Section 1. Next, we present the database that has been used as input to the booking process simulation in our experimental work.

In Section 3 of this chapter, we present the different methods that combine the NRV estimation techniques, described in Chapter 3, with the booking control mechanisms introduced in Chapter 2. We define the base case model for all the performance analysis conducted in Chapter 4. The performance of each model is analyzed when the seat inventory control model is optimized only once (static optimization) and then, when it is optimized at the beginning of each booking period (dynamic optimization).

In Section 4, we have analyzed the robustness of the different seat inventory control methods developed in this thesis with respect to several parameters, like the number of booking periods and the number of revision points. The performance of the seat inventory control methods are then compared to the optimal strategy where the revenue is maximized by assuming that the number of requests for each ODF was known before the beginning of the booking process. Finally, an analysis of the convergence speed of the prorated fare convergence method is conducted and we propose an algorithm to decrease the number of iterations before satisfying the convergence criterion.

4.1 Simulation Model

In "real life", the chance to observe the same passenger requests at the same time of the booking process for each ODF over all the network under different seat inventory control methods is non-existent. Simulation is a practical and powerful tool to compare the performance of different methods under the exact same arrival process. Therefore, the seat inventory control methods analyzed in this thesis have been tested on an integrated seat inventory control simulator developed in the Flight Transportation Laboratory at MIT. In the section, the structure of the simulation process is explained and the inputs of the methods are highlighted.

4.1.1 Model Structure

The model, developed by Williamson[14], simulates the booking process for the same number of departures on all the flights offered by the airline on a subset of its network. Moreover, the simulation is a multi-stage process as airlines adjust their booking limits many times along the booking process. The simulator model revises the booking limits corresponding to each ODF at the beginning of each booking period, called revision point (RP). Figure 4-1 below summarizes the time line of the booking process with W defined as the number of revision points or booking periods during the booking process.

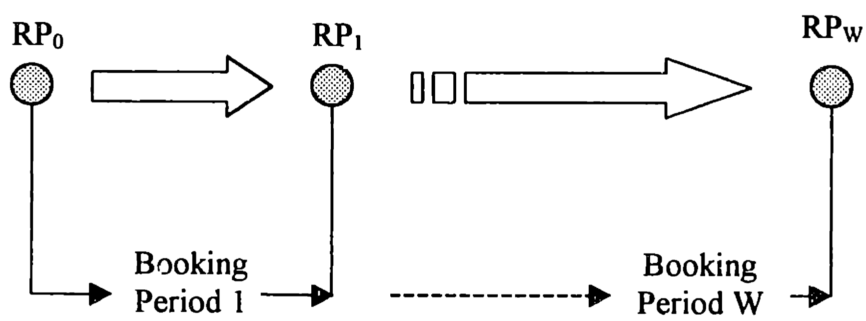

Figure 4-1: Time Line of the Booking Process

In Section 3 of this chapter, we present the performance of the seat inventory control methods for 18 booking periods and in Section 4 we conduct a sensitivity analysis of several seat inventory control methods with respect to W.

The simulated demand follows a Poisson process where the arrival rate of the simulated demand for each ODF equals the mean demand forecast of the corresponding ODF, obtained from an airline's Yield Management historical database. Therefore, the random demand generator only considers the ODF demand forecast to generate the number of requests per booking period for each ODF. Nonetheless, we can estimate the standard deviation of each ODF with respect to their mean demand forecast and then use them as input to the EMSRb mathematical model. Therefore, we can control the variance of the demand forecasts in the mathematical model using the Z-factor parameter, as we explain in Section 3 of this chapter. These remarks are important and have to be kept in mind throughout the performance analysis of the different seat inventory control methods.

In the data set used in the simulations, the low fare requests tend to arrive before the high fare requests because of the fare restrictions imposed by the airline. This booking pattern characteristic is observed on leg 10.

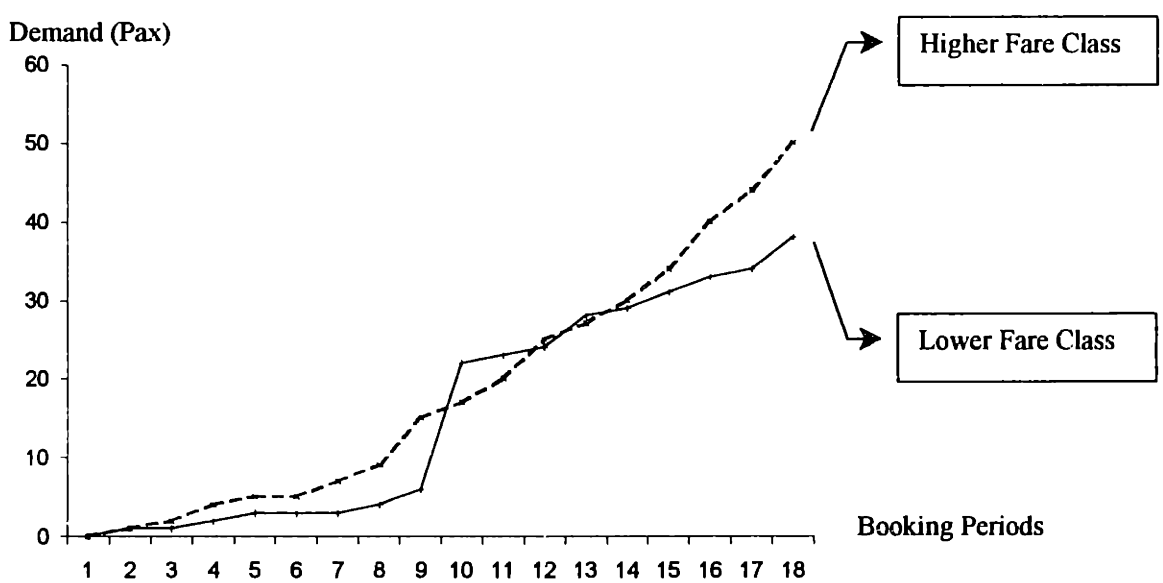

Figure 4-2: Accumulated Demand Curve for High and Low Fare Classes on Leg 1.

4.1.2 Model Assumptions

The objective of this research is to compare seat inventory control methods assuming that the passenger requests follow a Poisson process with a rate equal to the mean demand forecast for each offered ODF. As Rohrs pointed out, the goal is to "find the right model for the situation, big enough to capture needed behavior, small enough to allow answers and understanding."[1] Introducing new assumptions would increase the complexity of the analysis. Nonetheless, it is worth highlighting the assumptions of the simulation process.

Some assumptions have been made in the simulation model. First we assume that the fare for each ODF and the total capacity for each flight stay the same all along the booking process. Nonetheless, if these inputs were not constant, they could easily be incorporated into the simulation software. Moreover, the simulation does not consider competitor behaviors, as a change in a competitor's seat allocations may affect the demand of the given airline in the real world.

Some assumptions relative to the passenger demand have been made. First, the simulation model assumes that the demand arrivals follow a Poisson process, as it is a natural model to represent the customer arrival process. Alstrup et al. claim that airline ODF requests have a Poisson distribution[2]. Therefore, we assume that the arrival process has IID[3] and exponentially distributed inter-arrival intervals. Furthermore, the simulated demand is unbiased with regard to the demand forecasts as the rate of the Poisson process is the mean demand forecast, for each ODF. In other words, over a large number of departures, the

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[1] C. Rohrs, "Discrete Stochastic Processes," MIT course notes, unpublished, 1998.

[2] Alstrup J., S. Boas, O. Madsen and R. Vidal, "Booking Policy for Flight With Two Types of Passengers," European Journal Opnl. Res. 27, 274-288, 1986.

[3] Independent and Identically Distributed.

average of the simulated demand for each ODF on each booking period has a mean equal to the forecasted demand. In the next section, one can observe in Table 4-1 that for 20 simulated departures the total simulated demand is very close to the total demand forecast.

If a request is rejected the simulation does not allow recapture, as sell up (the possibility that a denied passenger decides to buy a higher fare) is not considered in the simulation. Therefore, a denied request is a revenue loss for the airline. No cancellation of bookings is considered and thus, no overbooking strategy has been taken into account in the experimental work presented in this thesis.

4.2 Network Characteristics

In this section, we present quantitatively the supply and demand characteristics of an actual large airline. We will use the presented network for all the experimental work addressed in this thesis.

4.2.1 Supply

The network consists of 102 legs and 34 nodes. Three nodes generate more than four flight legs and can be considered as hubs. Moreover, the airline offers to its customers 1066 OD and 7 fare classes that are defined by fare types. Thus, 7,462 ODF are offered on the network. On average, the highest fare is slightly more than four times the lowest fare of the corresponding OD. Finally, each OD traverses at most 4 legs of the network. In this thesis, we average the performance of each model over 20 simulated departures.

4.2.2 Demand

As expected, the mean of the total simulated demand over the 20 departures is very close to the mean demand forecast as one can observe in Table 4-1. Three different demand scenarios have been considered: a low, a moderate and a high demand scenario. For each scenario the arrival rates have been multiplied by a demand factor of 0.80, 1.00, 1.20, for respectively, the low, the moderate and the high demand scenarios. Please note that the moderate scenario corresponds to the simulation based on the actual data supplied by the airline. As far as local/connecting demand mix is concerned, the percentage of local requests is about 68% of the simulated traffic.

Demand Scenario	**Low**	**Moderate**	**High**
Demand Adjustment	**0.80**	**1.00**	**1.20**
Local Demand	3,712	4,621	5,544
Connecting Demand	1,694	2,130	2,546
Total Demand[4]	5,406	6,751	8,089
Total Demand Forecast	5,395	6,644	8,092
Average Load Factor[5]	63.07%	75.55%	82.48%

Table 4-1: Simulated, Forecasted Total Demand and Average Load Factor.

[4] For 18 booking periods

[5] The average load factor is obtained using the Base Case model, EMSRb Leg Based Fare Class, presented in section 4.3.3.

In the next section we analyze the performance of the inventory control methods for 18 booking periods.

4.3 Performance Analysis

4.3.1 Introduction

The objective of this section is to analyze the performance of several seat inventory control methods that take into account the network revenue values of each ODF. Under a booking limit control mechanism, we have tested two mathematical methods to estimate the NRV of the connecting ODF:

- Displacement cost of connecting ODF using the shadow price concept. For a connecting ODF, the displacement cost on a leg is the sum of the shadow prices over the other legs traversed. The NRV is the difference between the fare and the displacement cost if the fare is greater than the displacement cost. Otherwise, the NRV is set to zero.

- Convergence of the connecting ODF prorated fare. The NRV is the sum of the EMSRc over the leg traversed.

Both methods have been described in detail in Chapter 3. When the passenger requests are controlled by a bid price mechanism, we have tested two mathematical techniques to compute the expected network revenue value of the last available seat on a leg (ENRV):

- Shadow price concept from deterministic LP.

- Critical EMSR values obtained from the prorated fare convergence method described in Chapter 3. Two convergence techniques have been tested: OD by OD and virtual classes (VC). (We remind our readers that the ODF are either considered independently or aggregated into virtual classes (VC) in the EMSRb model (OD) for the purpose of calculating the critical EMSR values).

The network bid price value is either the sum of the critical EMSR values, resulting from the prorated fare convergence algorithm, or the shadow prices corresponding to the capacity constraints in the deterministic LP.

The booking limit control methods have been tested running the network optimization model only once (static optimization) and at the beginning of each booking period (dynamic optimization). In all cases the booking limits are re-calculated at the beginning of each booking period. First, we define an important parameter, called Z-factor, and then we present the base case seat control model for all the performance analysis in this section.

4.3.2 EMSR Inputs

As analyzed in Chapter 2, the EMSRb mathematical model assumes that the demand is a Gaussian random variable with its mean equals to the demand forecast. We define the forecasted standard deviation as a function of the mean demand forecast:

$$\sigma_j = Z \times \sqrt{\overline{D}_j} \qquad (4.1)$$

With Z defined as the Z-factor parameter and σ_j and $\overline{D}_j$ the standard deviation and the mean of the demand forecast of ODF j. It is worth mentioning that the standard deviation in our simulation is important only as an input to the EMSR calculations. In all cases, the "actual," or simulated, demand variance is always equal to the mean as the passenger demand is simulated using a Poisson process. Therefore, the Z parameter affects the EMSR model but has no impact on the simulated demand. We have conducted sensitivity analysis with respect to this parameter in Section 4 of this chapter.

The Z-factor can be interpreted as a risk factor. As shown in the practical analysis of the EMSRb mathematical model in Section A of the appendix, if we increase the Z-factor, fewer seats will be protected for the high ranked passengers (i.e., the ODF with high NRV).

In practice, the airlines are conservative in their seat protection because if they protect too many seats for the high ranked ODF they risk denying too many requests. As a consequence, some seats that could have been sold would stay empty. Even if the demand follows a Poisson process in our simulation we use data from an actual demand Yield Management database. Therefore, in accordance with the airline practice we have first considered a Z-factor of 2 in the performance analysis of the different seat inventory control methods proposed in this thesis. In summary, in this section all EMSR calculations have been performed with a Z-factor of 2.

In Section 4 we analyze the sensitivity of the different seat inventory control methods if the Z-factor is changed. Next, we present the seat inventory control model used as a base case for performance comparisons.

4.3.3 Base Case

In the experimental work presented in this thesis, we use the EMSRb Leg Based Fare Class (LBFC) model with Z equals two as a base case for comparison. First, we describe the model and then, we present the network performance of the model in the simulation context.

LBFC consists of grouping the ODF into booking classes according to their fare type. As the airline offered 7 fare types, the ODF are grouped into 7 booking classes. Then, the booking limits according to each class are found using the EMSRb mathematical model presented in Chapter 2. The booking limit of each ODF is then derived using (2.12). The performance of LBFC is presented in Table 4-2.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3,074,627	3,739,074	4,206,056
Local Passengers Spilled	4	103	443
Connecting Passengers Spilled	10	137	448
Avg. Leg Load Factor	63.08 %	75.67 %	82.77 %
Avg. Rev. per Pax ($/Pax)	570.29	574.35	584.29
Avg. Rev. per Avail. Seat ($/Seat)	279.16	339.48	381.88

Table 4-2: Performance of LBFC Model with Z=2, Base Case Model.

The LBFC control model has served as base case to compare the performance of the methods that we propose in this section. Bear in mind that the base case model does not use any NRV concept but takes the full fare of each ODF into account on a leg independent basis.

4.3.4 Simulated Methods

In this section, we describe explicitly the eight seat inventory control methods tested. But first, we repeat that two types of convergence methods are considered in this thesis. The first model is an OD by OD convergence model that considers each ODF as a separate entity in the EMSRb mathematical model whereas in the second model, VC convergence model, the ODF are grouped into virtual classes. The aggregation technique is explained in Section 2.3.3 of Chapter 2. In the performance analysis, the ODF are aggregated into at most 16 virtual classes. The reason it is "at most" is because if less than 16 ODF on a leg have non-zero demand forecast than the number of virtual classes will be less than 16. Therefore, the idea is to compute the critical EMSR values with different entities (ODF and virtual classes) as inputs to the EMSRb mathematical model. We present below the seat inventory control methods simulated in the thesis.

<u>**LPODBL**</u> LP Shadow Price Displacement Cost Estimation and OD by OD Booking Limit Control.

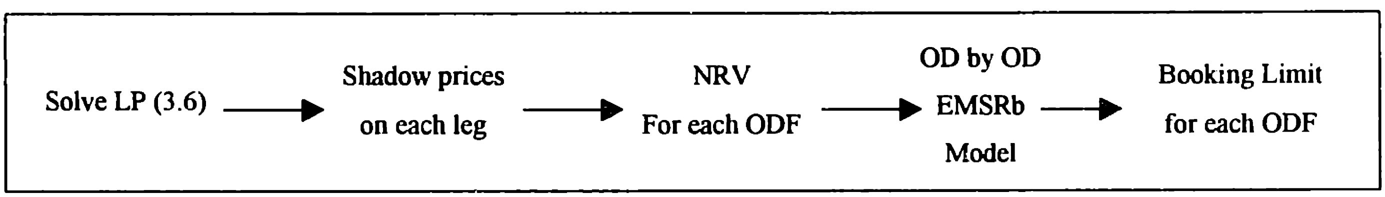

Figure 4-3: Process of LPODBL.

<u>**LP16BL**</u> LP Shadow Price Displacement Cost Estimation and VC Booking Limit Control.

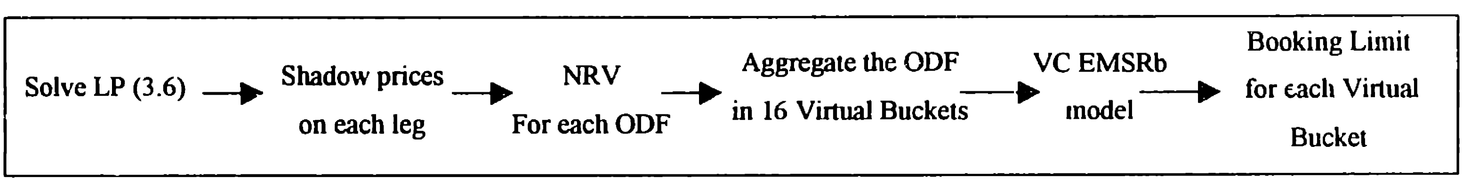

Figure 4-4: Process of LP16BL.

ODCODBL OD by OD Prorated Fare Convergence and OD by OD Booking Limit Control.

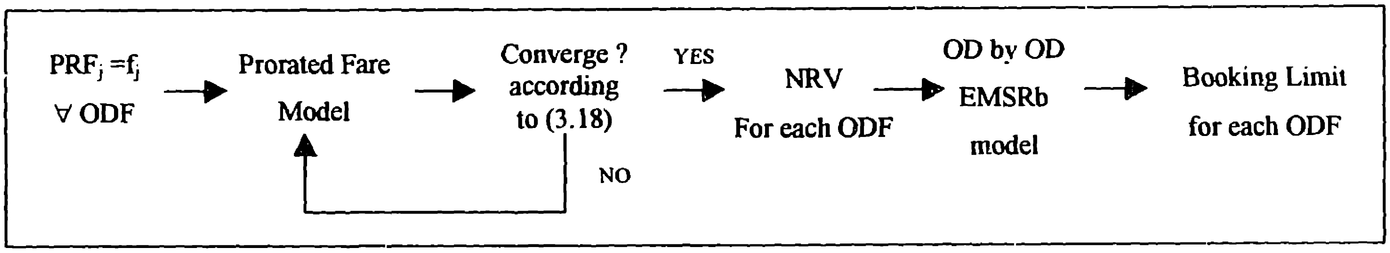

Figure 4-5: Process of ODCODBL.

ODC16BL OD by OD Prorated Fare Convergence and VC Booking Limit Control.

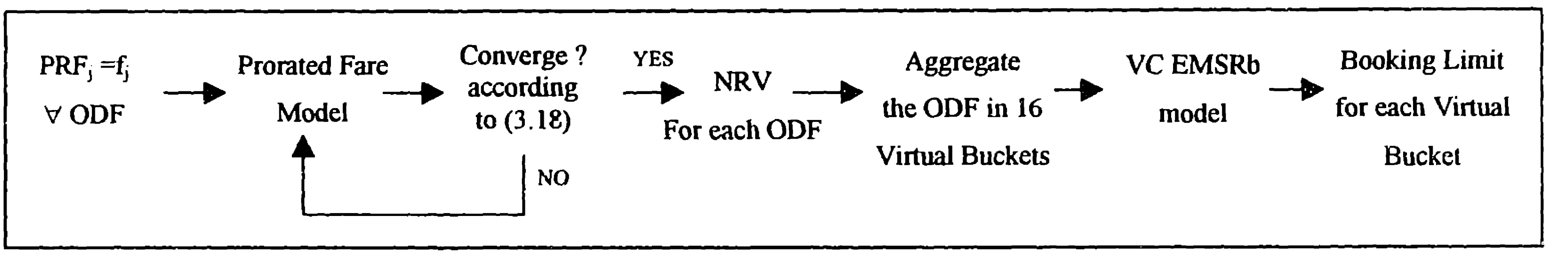

Figure 4-6: Process of ODC16BL

VCC16BL VC Prorated Fare Convergence and VC Booking Limit Control.

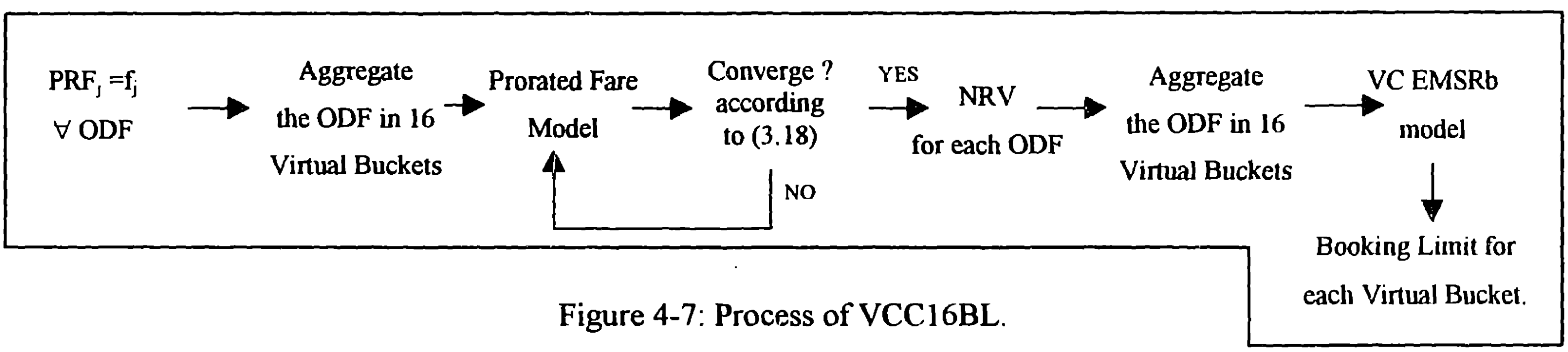

Figure 4-7: Process of VCC16BL.

LPBP LP Shadow Price Displacement Cost Estimation and Bid Price Control.

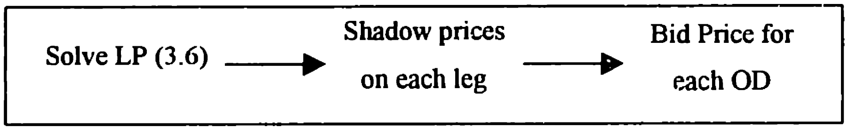

Figure 4-8: Process of LPBP.

ODCBP OD by OD Prorated Fare Convergence and Bid Price Control.

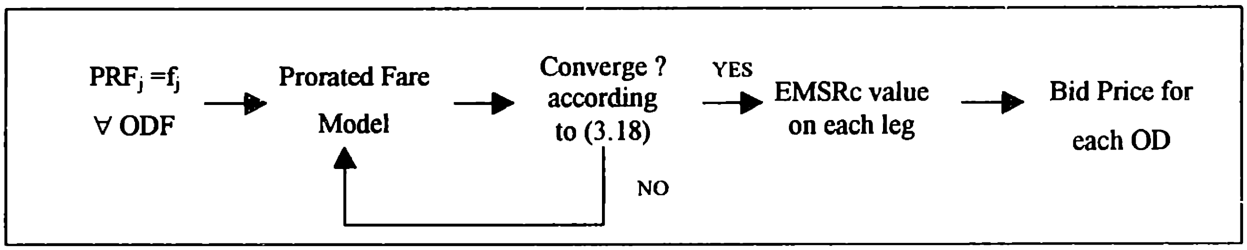

Figure 4-9: Process of ODCBP.

VCCBP VC Prorated Fare Convergence and Bid Price Control.

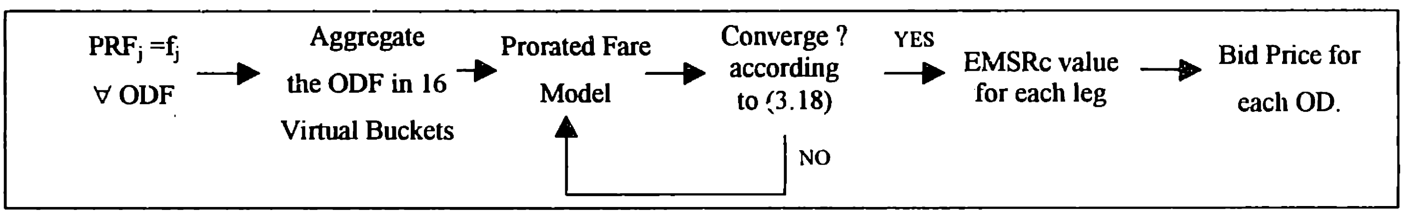

Figure 4-10: Process of VCCBP.

Summary Table

#	Name	Methodology	Optimization Model	Control Mechanism
1	LPODBL	Displacement Cost	LP-Simplex	OD Booking Limit
2	LP16BL	Displacement Cost	LP-Simplex	VC Booking Limit
3	ODCODBL	Prorated Fare	OD-CONV	OD Booking Limit
4	ODC16BL	Prorated Fare	OD-CONV	VC Booking Limit
5	VCC16BL	Prorated Fare	VC-CONV	VC Booking Limit
6	LPBP	Shadow Prices	LP-Simplex	Bid Price
7	ODCBP	EMSRc	OD-CONV	Bid Price
8	VCCBP	EMSRc	VC-CONV	Bid Price

4.3.5 Static Optimization

In this section, we analyze the static cases that consist of determining the NRV of all the ODF only once at the beginning of the booking process. We present the revenue performance of the methods that control the demand using a booking limit mechanism and then we highlight some information concerning the different booking limit methods introduced in Section 4.3.4, such as average load factor and number of refused requests along the booking process.

4.3.5.1 Expected Revenue

As far as expected revenue is concerned, all the methods using the network revenue value concept perform significantly better than the base case for the moderate demand scenario (demand generated using the actual demand forecasts) and for the high demand scenario (demand adjusted by a 1.20 factor). Moreover, the prorated fare convergence methods (OD by OD and VC) perform slightly better than the LP shadow price displacement cost approach as one can observe in Table 4-3 below where the percentage revenue gain of the seat inventory control methods with respect to the base case are summarized.

MODEL / Dadj	0.80	1.00	1.20
BASE CASE ($)	3,074,627	3,739,074	4,206,056
LPODBL	0.03%	0.27%	1.13%
LP16BL	0.03%	0.31%	1.17%
ODCODBL	0.02%	0.53%	1.20%
ODC16BL	0.03%	0.54%	1.21%
VCC16BL	0.03%	0.51%	1.26%

Table 4-3: Revenue Gain with respect to the Base Case, Static Optimization and Booking Limit Control.

We have conducted a statistical analysis (paired sample t-test[6]) in order to determine if the methods using the prorated fare convergence model perform statistically better than the one

[6] See DeGroot[8] p485 for a technical description of the t-test.

using the LP displacement cost approach. We have found that for the same control mechanism (OD by OD and 16 VC booking limit), the prorated fare convergence approach performs statistically better (over 20 simulated departures) than the LP displacement cost approach for a demand adjustment of 1.00 (moderate demand scenario). For low and high demand scenarios (demand adjustments of respectively 0.80 and 1.20) nothing can be said about the relative revenue performance of the two approaches as the paired sample t-test statistics are lower than the critical values (1.76) for 90% of confidence. Table 4-4 summarizes the paired sample t-test values.

Book. Control/Dadj	1.00	1.20
ODBL	5.08	1.33
VCBL	5.00	1.51

Table 4-4: Paired Sample T-test Statistics, Static Optimization and Booking Limit Control.

4.3.5.2 Average Load Factor and Number of Passengers Spilled

As far as average load factor is concerned, the load factors observed for the seat inventory control methods using NRV are lower than the load factors observed for the base case for all demand scenarios. Among these methods, the prorated fare convergence methods generate a slightly higher average load factor than the LP displacement cost methods as one can observed in the table below.

MODEL / Dadj	0.80	1.00	1.20
BASE CASE	63.08%	75.67%	82.77%
LPODBL	62.81%	74.39%	80.79%
LP16BL	62.80%	74.41%	80.91%
ODCODBL	62.84%	74.63%	81.32%
ODC16BL	62.85%	74.71%	81.35%
VCC16BL	62.85%	74.71%	81.37%

Table 4-5: Average Load Factor, Static Optimization and Booking Limit Control.

Two remarks are worth mentioning looking in more detail at the number of passengers spilled using the proposed methods. First, we find out that the base case model spills much fewer connecting passengers than the methods that use the NRV concept. Moreover, at a high demand level, the base case spills more local passengers compared to the NRV methods.

The first result makes sense as the seat inventory control methods using the NRV concept give lower availability to the connecting passengers compared to the base case as the NRV are by definition lower than the initial fares for the connecting ODF. The second observation can be explained by the following argument. When the network demand is high (i.e., demand adjusted by 1.20) less seats are protected for the local passengers if the demand is controlled by the base case model because many seats have been already sold to the connecting ODF. In other words if the airline gives high seat availability to the connecting passengers a higher number of local passengers will be spilled when the demand is high. As one can observe below in Table 4-6, for a high demand scenario, the base case spilled much more local passengers compared to the NRV methods. Moreover, among the methods using the NRV concept, the prorated fare convergence methods tend to spill less local passengers as one can observe in Table 4-6 below.

LOCAL PASSENGERS			
MODEL / Dadj	0.80	1.00	1.20
BASE CASE	4	103	443
LPODBL	25	112	350
LP16BL	25	113	340
ODCODBL	15	90	301
ODC16BL	14	89	294
VCC16BL	14	91	300

CONNECTING PASSENGERS			
MODEL/ Dadj	0.80	1.00	1.20
BASE CASE	10	137	448
LPODBL	8	182	570
LP16BL	8	180	569
ODCODBL	13	180	571
ODC16BL	13	176	572
VCC16BL	13	177	566

Table 4-6: Average Spilled Passengers over the 20 Departures, Static Optimization and Booking Limit Control.

4.3.5.3 Average Revenue per Seat and per Passenger

In essence, the NRV methods protect fewer seats for the connecting passengers than the base case method, which uses the full fare for each ODF. The NRV methods give lower rankings to the connecting ODF and therefore more availability is assigned to the local ODF. Therefore, the NRV methods tend to generate higher average revenues per available seat because a connecting ODF generates less revenue than the sum of the local ODF on its itinerary for the same fare class. Therefore, the NRV methods result in higher average revenues per available seat than the base case model, as one can observe in Table 4-7 below. Moreover, the NRV methods perform higher revenue per passenger than the base case as the requests of the low fare connecting ODF are more likely to be denied if the network demand is high.

Dadj	0.80		1.00		1.20	
1 or 2	1	2	1	2	1	2
BASE CASE	279.16	570.29	339.48	574.35	381.88	584.29
LPODBL	279.25	572.46	340.41	580.65	386.20	593.35
LP16BL	279.25	572.51	340.53	580.71	386.36	592.64
ODCODBL	279.22	571.90	341.27	579.98	386.46	589.73
ODC16BL	279.23	571.77	341.32	579.61	386.50	589.28
VCC16BL	279.24	571.77	341.22	579.70	386.68	589.60

Table 4-7: Ave. Revenue per Available Seat (1) and Ave. Revenue per Passenger (2), Static Optimization and Booking Limit Control.

4.3.5.4 Conclusion

The NRV methods give more availability to the local passengers and especially to the high fare local ODF when the network demand is high. This remark explains the performance characteristics of the NRV methods. First, the revenue per passenger is higher if a seat allocation model uses the NRV concept to control the demand. Moreover, the average load factor tends to be lower for the NRV methods because the connecting ODF have lower availability, compared to the base case, resulting in a higher spill of connecting ODF. As connecting passengers occupy many seats on the network, the average load factor tends to be lower for the NRV methods.

According to the paired sample t-test, the NRV methods generate revenues that are higher than the base case model for the moderate and the high demand scenarios (demand levels of respectively 1.00 and 1.20). The paired sample t-test is not significant for the low demand scenario (0.80 demand level) and therefore, nothing can be said.

If we compare the NRV methods among each other, using the same statistical test, the prorated fare convergence approach performs better in terms of expected generated revenue than the LP displacement cost approach for the moderate demand scenario. Nonetheless, the revenue difference between the methods using the NRV concept (prorated fare convergence versus LP displacement cost) is not statistically significant for the low and high demand scenarios and therefore, the methods can be assumed to perform similarly at these demand levels. Finally, the methods using LP displacement cost approach spill more local requests and more or less the same number of connecting requests than the one using the prorated fare convergence methods, resulting in lower average load factors. In the next section we analyze the seat inventory control methods that revise the NRV of each ODF dynamically along the booking process.

4.3.6 Dynamic Re-Optimization

In this section, we present the seat inventory control methods that re-optimize the NRV corresponding to each ODF at the beginning of each of the 18 booking periods. The NRV are computed using the number of seats available and the remaining ODF demand forecast corresponding to the booking periods that are still to come before departure. First, we compare the performance of the seat inventory control methods where the requests are controlled using booking limits. Then, we analyze the performance of the methods with a bid price control mechanism.

4.3.6.1 Booking Limit Control

The seat inventory control methods using the NRV concept perform significantly better than the base case. As one can observe looking at Table 4-3 and 4-8, the revenue difference with respect to the base case is greater if the seat inventory control methods are re-optimized at the beginning of each booking period. The improvement is more significant for the prorated fare convergence methods (+0.25% on average) than for the LP displacement cost methods (+0.12% on average). As one can observe in Table 4-8, the prorated fare convergence methods generate more revenue than the LP displacement cost methods for a booking control mechanism.

MODEL / Dadj	0.80	1.00	1.20
BASE CASE	3,074,627	3,739,074	4,206,056
LPODBL	0.04%	0.51%	1.30%
LP16BL	0.03%	0.51%	1.24%
ODCODBL	0.04%	0.62%	1.49%
ODC16BL	0.05%	0.61%	1.44%
VCC16BL	0.05%	0.64%	1.48%

Table 4-8: Revenue Performance Compared to the Base Case, Dynamic Optimization and Booking Limit Control.

We have conducted a paired-sample t-test in order to assert whether or not the revenue difference between the prorated fare convergence methods and the LP displacement cost methods is statistically significant.

Methods / Dadj	1.00	1.20
ODCODBL vs. LPODBL	3.56	4.67
VCC16BL vs. LP16BL	3.17	4.65

Table 4-9: Paired Sample T-test Statistics, Dynamic Optimization and Booking Limit Control.

As one can observe in Table 4-9, if the booking limits are found OD by OD, the paired-sample t-test statistics is well above the critical value (1.76) for 90% of confidence and 19 degrees of freedom. It means that the revenue generated by ODCODBL model for 18 booking periods is statistically larger than the revenue generated by the LPODBL model for a demand adjustment of 1.00 and 1.20. Similarly, if the demand is controlled using virtual classes, the paired-sample t-test statistics are still well above the critical value. Therefore, for the moderate and the high demand scenarios, the prorated fare concept performs statistically better than the LP displacement cost approach if the demand is controlled by a booking limit mechanism.

We have conducted an analysis of the revenue generated over the 20 simulated departures by the ODCODBL and LPODBL methods.

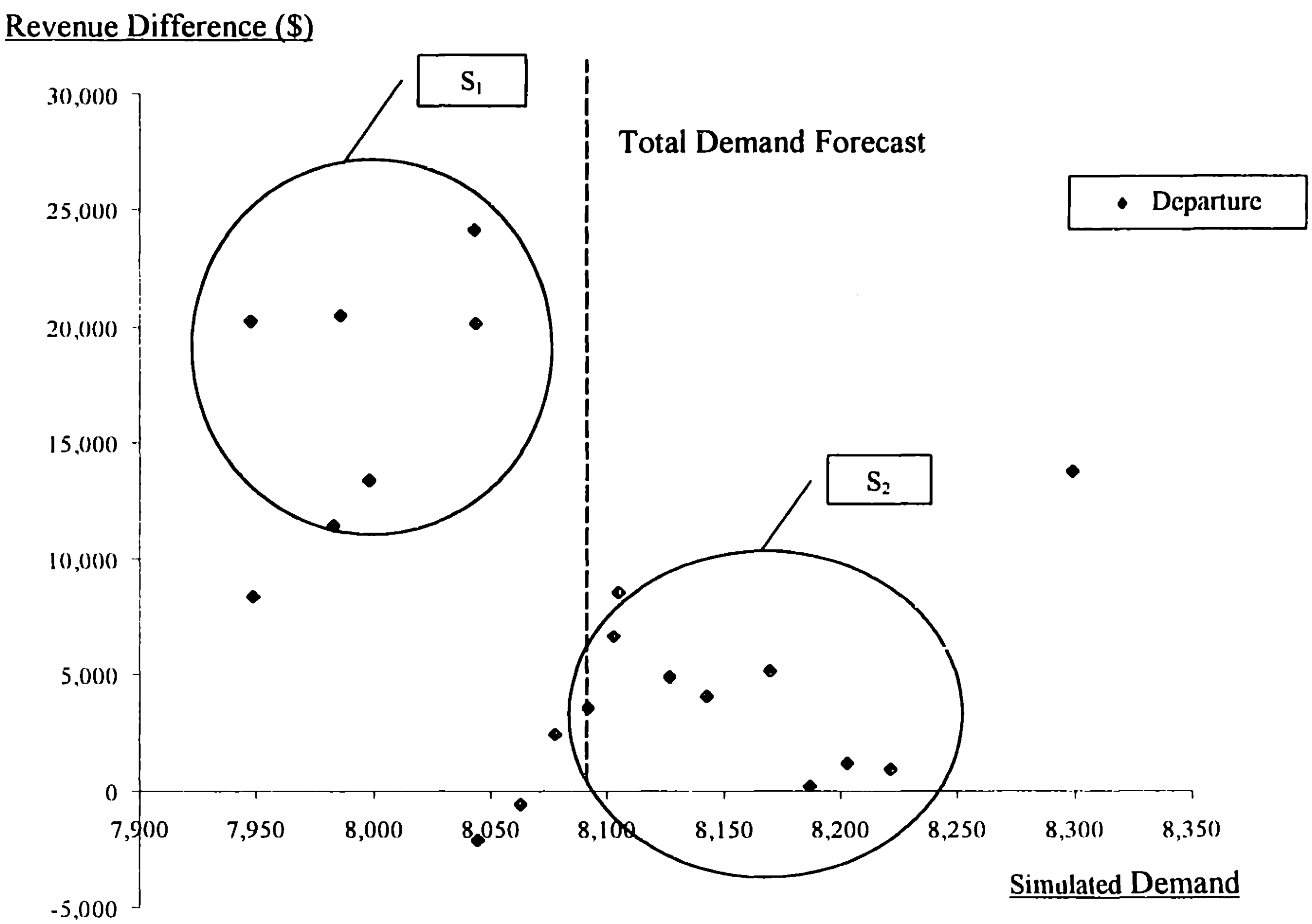

Figure 4-11: Revenue difference between ODCODBL and LPODBL with respect to the simulated demand for each departure.

As one can observe looking at the Figure 4-11 above, ODCODBL generates significantly more revenue than LPODBL when the simulated demand is lower than the total mean demand forecast (S_1) and tend to generate similar revenues if the demand is greater than the total demand forecast (S_2). Therefore, when the demand is lower than expected the OD by OD convergence model performs much better than the LP displacement cost model with a booking limit control mechanism. We have computed the k-factor (i.e., the ratio of the estimated standard deviation and the mean of the generated revenue over the 20 simulated departures). For ODCODBL, the k-factor is 1.05% whereas it is 1.16% for the LPODBL. Therefore, the prorated fare convergence model is somewhat more robust to demand variations than the LP displacement cost model if the demand is controlled by a booking limit mechanism.

The explanation of these interesting observations follows. The displacement cost concept, analyzed in Section 2, is not a "zero sum approach" (as the sum of the NRV of a connecting ODF over the traversed legs is in general not equal to its published fare). If the actual demand happens to be lower than the initial forecast for most of the connecting ODF then, the shadow prices are likely to be higher than what they should have been if the demand were known beforehand. Therefore, the connecting ODF will have a lower availability. This strategy is likely to result in spilling too many connecting passengers and, therefore, a relatively high number of seats which could have been sold to the denied connecting passengers stay empty. In summary, the displacement cost concept can lead to underestimating the NRV of the connecting passengers on all the traversed legs if the network demand happens to be low resulting in a revenue loss for the airline.

As opposed to the displacement cost concept, the prorated fare technique is a zero sum concept. If the demand happens to be lower than what has been forecasted initially, the technique does not discriminate against the connecting ODF because their ranking is not directly dependent on the forecasted demand, as it is for the displacement cost ([illegible]), but on the ratio of the forecasted demand. In other words, the ranking of the connecting ODF is dependent on the ratio of the total demand forecast on the traversed legs rather than the total demand considered for each leg individually. Therefore, the seat inventory control methods based on the prorated fare convergence technique tend to spill less connecting passengers and more local passengers compare to the methods using LP displacement cost concept. Conversely, when the demand is higher than expected the prorated fare convergence model tends to spill less local and more connecting passengers. This argument is sustained by the following analysis.

The difference between ODCODBL and LPODBL in terms of spilled passengers for the two departures is summarized in Table 4-10. Departure 8 corresponds to the departure with the lowest simulated demand and departure 14 corresponds to the one with the highest demand among the 20 simulated departures.

Departure #	Total Demand (Pax)	Δ Local Spill (Pax)	Δ Connecting Spill (Pax)
8	7948	16	-4
14	8299	-4	11

Table 4-10: Difference between ODCODBL and LPODBL in Terms of Number of Passengers Spilled for Specific Departures.

In summary, when the demand is much lower then expected the prorated fare convergence technique spills more local and less connecting requests then the LP displacement cost concept. Conversely, when the demand is higher then expected, the prorated fare convergence technique tends to spill less local and more connecting requests then the LP displacement cost concept.

When the demand is low the airline should favor the connecting passengers as it is very unlikely that they will displace local passengers on all the traversed legs. The prorated fare convergence technique seems to implement the strategy stated above better than the LP displacement cost technique. Therefore, this argument explains why ODCODBL performs much better then LPODBL for low demand departures as observed in Figure 4-11. In other words, the prorated fare convergence method seems to be more robust to demand variations then the LP displacement cost approach.

Average Number of Passenger Spilled

Averaging over the 20 departures, we find out that the NRV methods spill more local passengers and less connecting passengers when the methods are optimized dynamically (Table 4-11 below) than when they are optimized only once at the beginning of the booking process (Table 4-6). Moreover, as one can observe in Table 4-11 below, the prorated fare convergence methods spill on average less local and less connecting passengers than the LP displacement cost methods for the moderate and the high demand scenarios, when the NRV are optimized dynamically.

LOCAL PASSENGERS			
MODEL	0.80	1.00	1.20
BASE CASE	4	103	443
LPODBL	24	119	356
LP16BL	24	116	342
ODCODBL	18	106	332
ODC16BL	17	103	326
VCC16BL	11	97	332

CONNECTING PASSENGERS			
MODEL	0.80	1.00	1.20
BASE CASE	10	137	448
LPODBL	8	165	548
LP16BL	10	168	555
ODCODBL	10	163	525
ODC16BL	10	162	525
VCC16BL	9	159	522

Table 4-11: Average Passenger Spill over the 20 Departures, Dynamic Optimization and Booking Limit Control.

Average Load Factor

If the demand is controlled by a booking limit mechanism, the average load factors tend to be higher if the NRV are revised at the beginning of each booking period than if they are computed only once at the beginning of the booking process. This finding has been derived by comparing Table 4-5 and Table 4-12, which is presented below. Moreover, the prorated fare convergence methods generate higher load factors than the LP displacement cost methods but are lower than the base case load factors.

Methods / Dadj	0.80	1.00	1.20
BASE CASE	63.08%	75.67%	82.77%
LPODBL	62.81%	74.62%	81.12%
LP16BL	62.78%	74.58%	81.12%
ODCODBL	62.84%	74.78%	81.85%
ODC16BL	62.87%	74.83%	81.91%
VCC16BL	62.99%	74.99%	81.93%

Table 4-12: Average Load Factor, Dynamic Optimization and Booking Limit Control.

Average Revenue per Available Seat and Average Revenue per Passenger

The average revenue per available seat achieved by the prorated fare convergence method tends to be higher if the NRV are optimized dynamically along the booking process, looking at Table 4-6 and 4-13. The LP displacement cost methods tend to have higher revenue per passenger then the prorated fare convergence methods as they give more seat availability to the connecting passengers who, for a given fare class, have higher fares. Nonetheless, the prorated fare convergence methods have higher average revenue per available seat than the LP displacement cost methods as they tend to favor more high fare local passengers.

Dadj	0.80		1.00		1.20	
1 or 2	1	2	1	2	1	2
BASE CASE	279.16	570.29	339.48	574.35	381.88	584.29
LPODBL	279.25	572.44	341.22	581.12	386.83	592.90
LP16BL	279.24	572.52	341.20	581.06	386.62	592.06
ODCODBL	279.27	572.11	341.60	580.49	387.58	590.23
ODC16BL	279.29	571.97	341.57	580.05	387.39	589.48
VCC16BL	279.30	571.15	341.66	579.43	387.52	589.84

Table 4-13: Av. Revenue per Available Seat (1) and Av. Revenue per Passenger (2), Dynamic Optimization and Booking Limit Control.

4.3.6.2 Network Bid Price Control

In this section, we analyze the performance of bid price control methods. The bid price value associated with each OD is the sum of the ENRV values over all the traversed legs. The bid price value for an OD at a given time corresponds to the revenue that the airline expects to gain from the last seat on each of the traversed legs. We remind the reader that ENRV of a leg corresponds to the expected network revenue value of the last seat on the leg. For the LPBP model, the ENRV value is the shadow price on a leg whereas it is the EMSRc value for the ODCBP and the VCCBP methods. Moreover, we have used a Z-factor of two for both prorated fare convergence methods (OD and VC).

As far as generated revenue is concerned, the prorated fare convergence methods perform slightly better than the LPBP. Nonetheless, with 90% of confidence the paired-sample t-test tells us that estimating the ENRV by the critical EMSR values obtained with the converge prorated fare performs similarly as using the LP shadow prices at all demand levels.

MODEL / Dadj	0.80	1.00	1.20
BASE CASE	3,074,627	3,739,074	4,206,056
LPBP	0.03%	0.48%	1.05%
ODCBP	0.03%	0.48%	1.06%
VCCBP	0.04%	0.49%	1.08%

Table 4-14: Revenue gain with respect to the Base Case, 18 Booking Periods, Bid Price Control Mechanism.

If the demand is controlled by a bid price mechanism, estimating the ENRV of each ODF on a leg using the critical EMSR value from the prorated fare convergence model or the LP shadow price seem to generate similar revenues for a Z-factor equal to two for 18 booking periods and re-optimizations. We expect both methods to improve their performance for 36 re-optimizations. Nonetheless, as we will analyze in the next section, the performance of the prorated fare convergence is improved if the Z-factor is chosen more carefully.

4.3.7 Summary

The revenue performance of the different seat inventory control methods for demand adjustments of 1.00 and 1.20 are summarized in Figure 4-12 below.

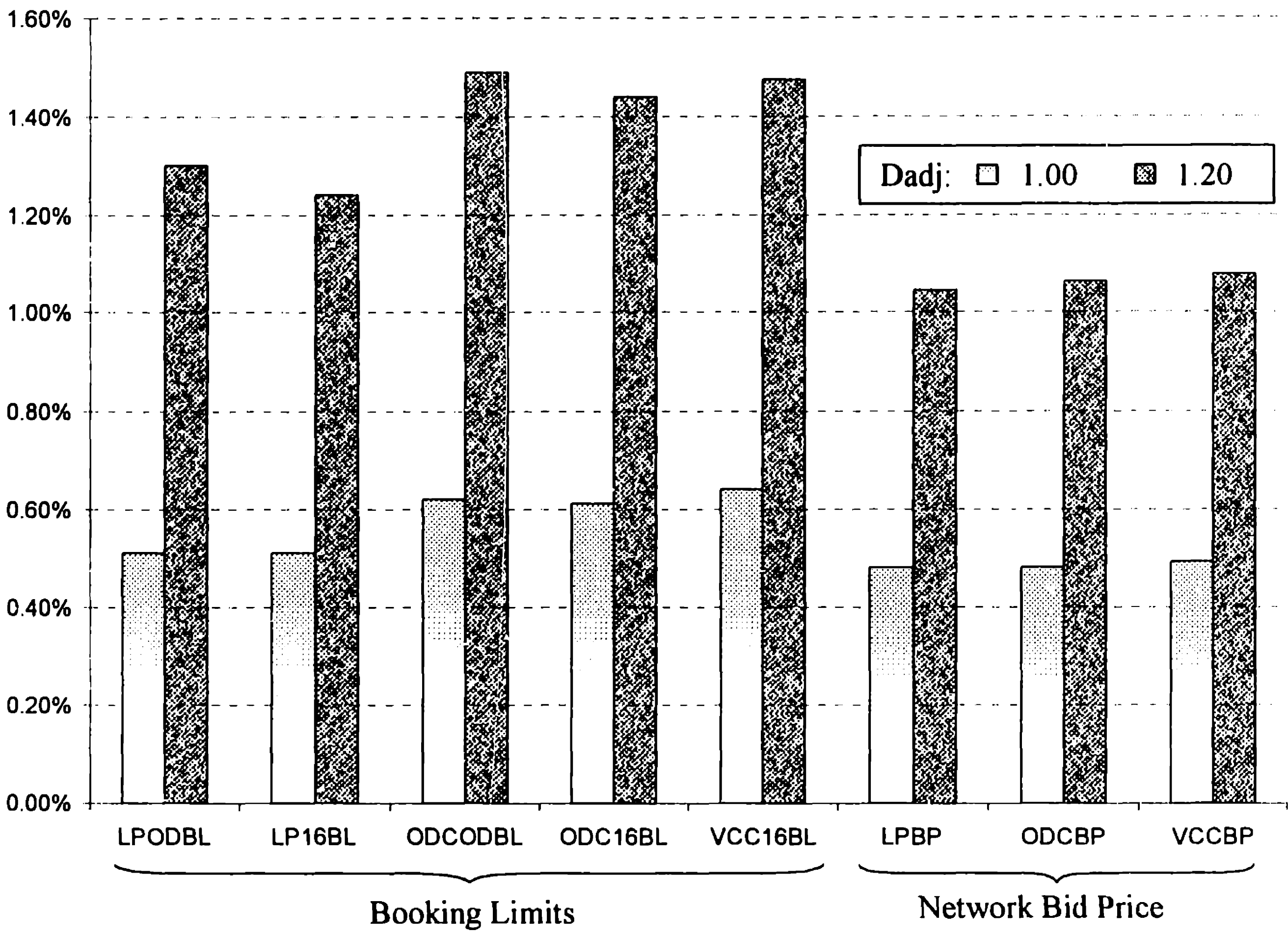

Figure 4-12: Revenue Gain of the Seat Inventory Control Methods, for 18 Booking Periods, Dynamic Optimization.

As one can observe, the booking limit control mechanisms perform better than the bid price control methods for 18 booking periods. The bid price control technique is an open/closed mechanism as opposed to the booking limits that set the maximum number of seats to be sold to a specific ODF. In addition, the NRV are re-optimized only 18 times. As we will highlight in the next section, the bid price control mechanism is very sensitive to the number of revisions (in between two revision points, the longer the time interval, the more likely the bid prices are to be incorrect after a certain number of booking requests).

The seat inventory control method generates more revenue if the NRV of each connecting ODF is estimated using a prorated fare convergence technique than with an LP displacement cost technique. Nonetheless, the revenue difference is significant only for the booking limit control mechanism and for the moderate and the high demand scenarios. For the network studied in this thesis the prorated fare concept performs approximately 0.20% better, for the high demand scenario, than the LP displacement cost approach if the demand is controlled with booking limits and is statistically the same if the demand is controlled using bid prices. However, if we choose the Z-factor more carefully the performance of the prorated fare method with a bid price control mechanism is improved significantly as emphasized in the next section.

Moreover, if the demand is controlled by a booking limit mechanism the prorated fare method tends to spill less connecting passengers than the LP displacement cost method when the demand is lower than expected. Therefore, the prorated fare convergence method seems to be more robust to demand variations than the LP displacement cost approach.

In the next section we analyze the robustness of the different seat inventory control methods by conducting sensitivity studies of several parameters.

4.4 Robustness Analysis

We have made several assumptions in the performance analysis conducted in the previous section. First, we have assumed a Z-factor of 2 used as input to the EMSRb calculations for both the bid price and the booking limit control mechanisms. Moreover, we have assumed 18 periods along the booking process. We will in this section analyze how the methods perform if we change the Z-factor and if we use 36 booking periods with 36 revision points.

Moreover, we have assumed that the simulated demand follows a Poisson process. Although this discrete stochastic process is widely used to model the time at which arrivals enter a system, it assumes that the variance of the simulated demand is equal to its estimated mean. We may be interested to measure the relative performance of the different seat inventory control methods when the variance of the simulated demand is greater than the mean. Therefore, we have constructed an arrival process for which the variance of the simulated demand is greater than its mean but where the simulated ODF demand is unbiased. In other words, the average simulated demand over the 20 simulated departures is approximately equal to the demand forecast for each ODF.

We start our analysis by looking at the sensitivity of the seat inventory control methods with respect to the Z-factor used as input.

4.4.1 Z-Factor

We remind our reader that the Z-factor does not affect either the simulated demand or the LP shadow prices but only affects the critical EMSR values obtained from the EMSRb mathematical method. Therefore, the NRV that are estimated using LP shadow prices are independent of the Z-factor. Moreover, the Z-factor analysis has been conducted for 18 booking periods.

4.4.1.1 Booking Limit Control Mechanism

We first study the sensitivity of the performance of ODCODBL (OD by OD prorated fare convergence method with OD by OD booking limit control mechanism) and LPODBL (LP displacement cost with OD by OD booking limit control mechanism) with respect to the Z-factor. The NRV computed in the LPODBL method are independent of the Z-factor but the booking limits that are calculated with the EMSRb method depend on the Z-factor values. Four values of the Z-factor, used as input to the EMSR calculations, have been considered, namely: 1.30, 1.50, 2.00 (Case analyzed in Section 2) and 2.50. The greater the Z-factor, the flatter the EMSR curve as one can figure out by looking at the construction of the EMSR curve in the Appendix (Section A). Therefore, the greater the Z-factor, the lower the number of seats protected for the high ranked ODF on the leg. As one can observe below in Table 4-15, the ODCODBL method is much less sensitive to changes in the Z-factor parameter than the LPODBL.

METHOD	Z-Factor	1.30	1.50	2.00	2.50
ODCODCBL	Revenue ($)	4,269,687	4,269,826	4,268,857	4,266,698
	Local Pax Spill	333	336	332	331
	Connecting Pax Spill	525	522	525	526
	Ave. Load Factor	81.85%	81.88%	81.85%	81.85%
LPODCBL	Revenue ($)	4,265,858	4,263,779	4,260,525	4,255,599
	Local Pax Spill	347	348	356	363
	Connecting Pax Spill	530	537	548	562
	Ave. Load Factor	81.61%	81.45%	81.12%	80.76%
DIFFERENCE (ODCODCBL - LPODCBL)	Δ Revenue ($)	3,829	6,047	8,332	11,099
	Δ Local Pax	-14	-12	-24	-33
	Δ Connecting Pax	-5	-15	-23	-36
	Δ Ave. Load Factor	0.24%	0.43%	0.73%	1.09%

Table 4-15: Sensitivity Study of ODCODBL and LPODBL with respect to the Z-factor at 1.20 Demand Adjustment.

As the Z-factor increases, the revenue difference between ODCODBL and LPODBL increases. According to the paired sample T-test statistics, the ODCODBL method performs statistically better, with 90% of confidence, than the LPCODBL for all analyzed Z-factor levels. Moreover, the ODCODBL method spills more or less the same number of local and connecting passengers for all the considered Z-factor values whereas, the LPODBL method spills more and more local and connecting passengers as the Z-factor increases (resulting in a decreasing average load factor). In summary, the ODCODBL method seems to be more robust than the LPODBL method to variations of the Z-factor parameter. ODCODBL considers the Z-factor value in the NRV estimation whereas the SP calculation in LPODBL is independent of the Z-factor. Nonetheless, for both methods the booking limits are obtained using the EMSRb mathematical method, which is dependent on the Z-factor. Therefore, as the Z-factor gets larger ODCODBL adjusts the NRV estimates according to the Z-factor whereas the SPs are not affected.

4.4.1.2 Bid Price Control Mechanism

We have analyzed the performance of ODCBP (bid price control method where the ENRV are estimated by the critical EMSR values obtained from the prorated fare convergence method) and of LPBP (ENRV are estimated by the LP shadow prices). As one can observe in Table 4-16, ODCBP generates the best revenue for a Z-factor of 1.00.

ODCBP	Z-factor	0.70	1.00	1.50	2.00	2.50
	Revenue ($)	4,255,003	4,255,970	4,254,978	4,250,808	4,247,020
	Local Pax Spill	335	333	335	335	335
	Connecting Pax Spill	485	489	489	493	496
DIFFERENCE (ODCBP -- LPBP)	Δ Revenue ($)	4,940	5,907	4,915	745	-3,043
	Δ Local Pax	5	4	6	6	5
	Δ Connecting Pax	10	13	13	18	20
	Δ Ave. Load Factor	-0.33%	-0.36%	-0.38%	-0.46%	-0.49%
	Δ Ave. Rev. per Pax	1.89	2.19	2.21	2.00	1.65
	Δ Ave. Rev. per Avail. Seat	0.45	0.53	0.44	0.07	-0.28

Table 4-16: Z-factor Sensitivity Study of ODCBP and LPBP for a Demand Level of 1.20.

Therefore, the expected revenue is maximized if the relationship between the variance and the mean in the EMSRb method is the same as the one corresponding to the demand generator (Poisson arrival process). This observation is important but not surprising given that the actual simulated demand comes from a Poisson process and, therefore, the variance of all ODF demand is equal to its expectation (i.e., the forecast).

In Section 4.4 we conduct a performance comparison of ODCBP and LPBP when the variance of the simulated demand is greater than the mean.

According to the paired sample T-test statistics, the ODCBP method performs statistically better than the LPBP, for a Z-factor of one, for the high demand scenario with 90% of confidence. Before presenting in more detail the performance of the ODCBP method for a Z-factor of one, we highlight some observations concerning the Expected Network Revenue Value (ENRV).

4.4.1.3 ENRV Analysis

We have collected all the SP and the EMSRc corresponding to each leg for the first simulated departure at a 1.20 demand level. Figure 4-13 below illustrates the average ENRV over all the legs on the network for the two estimation techniques (SP from LP and EMSRc prorated fare convergence).

As one can observe, at the beginning of the booking process the average of the SP is lower than the average of the EMSRc obtained from the prorated fare convergence method. But the average EMSRc becomes lower than the SP from booking period 16 until the end of the process.

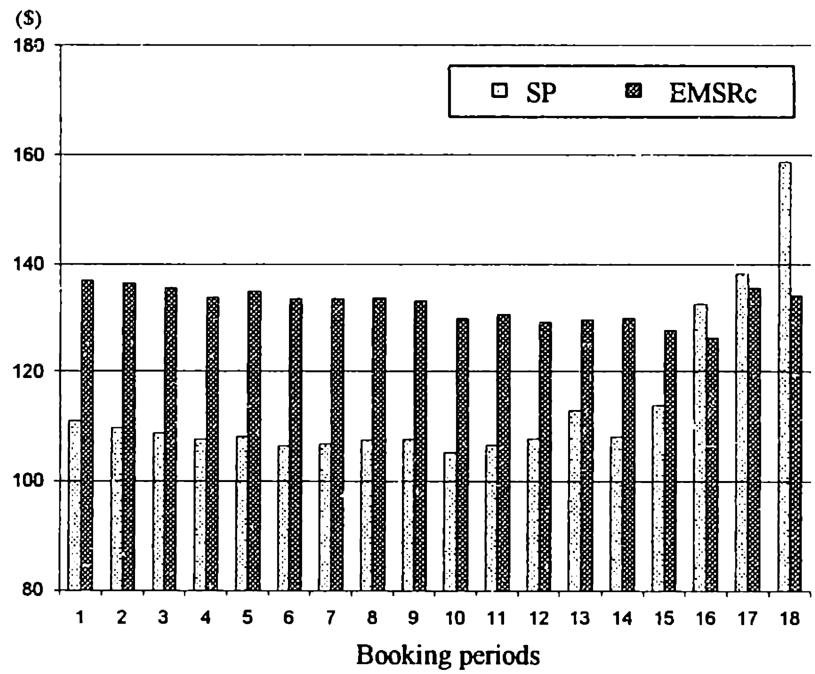

Figure 4-13: Average ENRV Value over the Legs of the Network for Dadj=1.20, Bid Price Control Mechanism.

As the booking process goes on, the forecasted ODF requests still to come have higher fares. If few seats are left open to passenger requests on leg k and the demand forecast for the ODF traversing leg k is high, then the shadow price corresponding to leg k will be high. We observe that at the last revision point (beginning of the 18^{th} booking period) the non-zero shadow prices (corresponding to the binding constraints) are always higher than the EMSRc values obtained with the prorated fare convergence method. Therefore, close to departure time the LP approach is likely to deny more ODF requests than the convergence approach.

We have also compared, for each leg on the network, the average over the 18 booking periods of the EMSRc from the OD by OD prorated fare convergence method and the LP shadow prices (SP) using a network bid price control mechanism. As one can observe looking at Section B of the appendix, at a high demand level (1.20 demand adjustment) when the load factor on a given leg is relatively low (below 85%), the average EMSRc values tend to be higher than the average SP. This observation can be explained by the fact that when the load factor on a leg is low, the corresponding capacity constraint is likely to be not binded and therefore, the SP is zero. The EMSRc value is less often equal to zero as it includes the probability that the demand will be greater than the capacity. Therefore, when the ENRV are averaged over the booking periods, the SP tend to be lower than the EMSRc for legs with a relatively low demand.

4.4.1.4 Performance Analysis for Z=1

For 18 booking periods, the ODCBP method performs the best for a Z-factor of one. We present, in this section, a comparative analysis of the performances of ODCBP and LPBP.

As far as generated revenue is concerned, the prorated fare convergence methods perform better than the LPBP at all demand levels. Nonetheless, the paired-sample t-test tells us that ODCBP performs better, with 90% of confidence, than LPBP at only 1.20 demand level. At lower demand adjustments (i.e., 0.80 and 1.00) all the methods perform statistically the same according to the paired-sample t-test.

METHOD / Dadj	0.80	1.00	1.20
BASE CASE	3,074,627	3,739,074	4,206,056
LPBP	0.03%	0.48%	1.05%
ODCBP	0.05%	0.55%	1.19%
VCCBP	0.04%	0.53%	1.21%

Table 4-17: Revenue gain with respect to the Base Case, Z=1, Bid Price Control.

The EMSRc prorated fare convergence methods tend to spill more local and connecting ODF requests than the LP bid price method. As we have observed previously, at 1.20 demand adjustment, the average over all the legs of the EMSRc values tend to be higher than the SP for most of the booking periods (15 out of 18). Although this result has been observed for the first departure only, this argument may explain why the ODCBP method spills more connecting passengers than the LPBP method as one can observe in Table 4-18.

LOCAL PASSENGERS			
METHOD	0.80	1.00	1.20
LPBP	7	93	330
ODCBP	9	102	333
VCCBP	11	114	357

CONNECTING PASSENGERS			
METHOD	0.80	1.00	1.20
LPBP	8	133	475
ODCBP	8	136	489
VCCBP	7	132	480

Table 4-18: Average Passenger Spill over 20 Departures, Z=1, Bid Price Control.

Moreover, the VCCBP spills significantly more local passengers than the two other methods at a high demand level.

At a high demand level, the average load factor is higher for the LPBP method than for the prorated fare convergence methods as the LPBP spills both less connecting and local ODF. The table below quantitatively illustrates the previous statement.

METHOD / Dadj	0.80	1.00	1.20
BASE CASE	63.08%	75.67%	82.77%
LPBP	63.08%	75.65%	82.89%
ODCBP	63.06%	75.43%	82.53%
VCCBP	63.04%	75.42%	82.52%

Table 4-19: Average Load Factor, Z=1, Bid Price Control.

The average revenue per passenger and per available seat performed by the LPBP method tend to the lower than the one performed by the convergence methods. Although LPBP spills less connecting and local passengers than the two other methods, it has a lower revenue per available seat than the prorated fare convergence methods at all demand levels as one can observe in Table 4-20 below.

Dadj	0.80		1.00		1.20	
1 or 2	1	2	1	2	1	2
LPBP	279.25	570.52	341.11	575.76	385.88	583.45
ODCBP	279.28	570.78	341.33	577.24	386.41	585.64
VCCBP	279.28	570.94	341.29	577.89	386.51	587.01

Table 4-20: Av. Revenue per Available Seat (1) and Av. Revenue per Passenger (2), Z=1, Bid Price Control.

4.4.1.5 Conclusion

If the demand is controlled by a booking limit mechanism, we have observed that the ODCODBL method is more robust to Z-factor variations than the LPODCBL method (LP displacement cost approach). For all the Z-factors considered, ODCODBL method generates statistically higher revenues than LPODCBL for the high demand scenario. Moreover, ODCODBL spills more or less the same number of connecting and local requests regardless of the Z-factor whereas the more we increase the Z-factor the more ODF requests, the LPODCBL method spills. As far as the bid price control mechanism is concerned, the ODCBP method generates, as expected, the most revenue for a Z-factor of one and generates statistically a greater revenue than the LPBP bid price method for a 1.20 demand adjustment. In the next section, we study the effect of re-optimizing more often the ENRV by increasing the number of booking periods.

4.4.2 Number of Booking Periods

We have analyzed the performance of the different methods for 36 booking periods. The objective is to compare the relative performances of the NRV methods if the number of booking periods is doubled. Is it worth it for an airline to forecast its demand more often? Which seat inventory control method works the best if the method is re-optimized more often considering a Poisson arrival process? First, we explain how we went from 18 to 36 booking periods and then we highlight the performance of the base case for 36 periods. We then, present the performance of the different seat inventory control method for 36 booking periods.

As we have pointed out in Section 4-1, the arrival rate of each ODF for 18 booking periods is the demand forecast. In order to generate the simulated demand for 36 booking periods, the ODF arrival rate is divided by two and the demand is generated based on these new arrival rates. Formally, the arrival rates for each ODF becomes λ':

$$\lambda'_{2\times r-1} = \frac{\lambda_r}{2} \quad r \in \{1,..,18\} \tag{4.2}$$

$$\lambda'_{2\times r} = \frac{\lambda_r}{2} \quad r \in \{1,..,18\} \tag{4.3}$$

The simulated demand depends on the number of booking periods as it determines the arrival rate vector. Therefore, one has to bear in mind that we can analyze the performance of the seat inventory control methods only for the same number of booking periods. In Table 4-21 we present the simulated demand matrix for 36 booking periods.

Demand Adjustment	0.80	1.00	1.20
Local Demand	3,691	4,646	5,536
Connecting Demand	1,705	2,127	2,530
Total Simulated Demand	5,395	6,773	8,067

Table 4-21: Simulated Demand for 36 Booking Periods.

4.4.2.1 *Base Case*

As explained above, the performance of the base case method (LBFC) is different for 18 (defined in 4.3.3) and 36 booking periods. For a high demand scenario, the base case method generates a lower revenue for 36 booking periods than for 18 booking periods because the simulated demand is slightly lower as one can observe comparing Table 4-2 for 18 booking periods and Table 4-22 for 36 booking periods. Table 4-22 summarizes the performance of the base case method for 36 booking periods.

Demand Adjustment	0.80	1.00	1.20
Total Revenue	3,084,473	3,758,991	4,204,280
Local Passengers Spilled	2	110	436
Connecting Passengers Spilled	6	127	436
Avg. Leg Load Factor	63.04%	75.98%	82.69%
Avg. Rev. per Pax ($/Pax)	572.59	575.16	584.31
Avg. Rev. per Avail. Seat ($/Seat)	280.05	341.29	381.72

Table 4-22: Performances of LBFC Method (Z=2), 36 Booking Periods, Base Case Method.

4.4.2.2 *Performance Analysis*

We first present in Table 4-23, the revenue performance of the different seat inventory control methods where the demand is controlled using booking limits with a Z-factor of two. The Z-factor, input to the EMSRb mathematical model, is equal to two for the reasons given in Section 4.3.2 for 18 booking periods.

METHOD / Dadj	0.80	1.00	1.20
BASE CASE	3,084,473	3,758,991	4,204,280
LPODBL	0.03%	0.56%	1.51%
LP16BL	0.01%	0.48%	1.29%
ODCODBL	0.02%	0.55%	1.53%
ODC16BL	0.02%	0.55%	1.46%
VCC16BL	0.03%	0.55%	1.46%

Table 4-23: Revenue Gain for 36 Booking Periods, Booking Limit Control Method.

As one can notice, the revenue performance of the seat inventory control methods have improved as we re-optimize more often the NRV.

The performance of the bid price control methods is summarized in the table 4-24.

METHOD / Dadj	Z-factor	0.80	1.00	1.20
BASE CASE	2	3,084,473	3,758,991	4,204,280
LPBP	N.A	0.03%	0.47%	1.38%
ODCBP	1	0.04%	0.57%	1.50%
VCCBP	1	0.03%	0.55%	1.46%
ODCBP	2	0.04%	0.54%	1.36%
VCCBP	2	0.01%	0.51%	1.35%

Table 4-24: Revenue Gain for 36 Booking Periods, Bid Price Control Method.

If the demand is controlled using the bid price mechanism, ODCBP and VCCBP perform better for a Z-factor of one. Therefore, the prorated fare convergence method with network bid price control performs better for both 18 and 36 booking periods if the variance of the demand forecast is estimated by the mean demand forecast in the EMSRb mathematical method. As explained in Section 4.4.1.2 this result is not surprising as the demand follows a Poisson process.

Moreover, according to Table 4-23 and 4-24, controlling the demand using booking limit or bid price mechanisms generates very similar expected revenues if the prorated fare convergence technique is used to estimate the ENRV. Thus, the prorated fare bid price method is very sensitive to revisions. The more the method is revised, the more accurate the bid price values and the greater the generated revenue. For 36 booking periods, ODCBP (Z=1) performs as well as LPODBL. Similarly, VCCBP (Z=1) performs as well as VCC16BL at all demand levels. Therefore, no matter what sorting method is used (i.e., OD by OD or VC) in the prorated fare convergence method, the expected revenue obtained by controlling the demand with a bid price mechanism is more or less the same as the one generated with a booking limit control mechanism for 36 re-optimizations of the ENRV.

The LP bid price method and the LP displacement cost with VC booking limit method do not perform as well as the other methods. According to the paired sample t-test, the LPBP and LP16BL perform statistically lower (with 90% of confidence) than all the booking limit control prorated fare convergence methods at a 1.20 demand level. Moreover, as far as bid price methods are concerned, ODCBP performs statistically better than LPBP at both 1.00 and 1.20 demand levels.

4.4.2.3 Conclusion

For 36 booking periods ODCBP and VCCBP perform better if the Z-factor is one. This result confirms what we have found for 18 booking periods. Moreover, for 36 booking periods, the bid price control mechanism performs as well as the booking limit control mechanism if the NRV are estimated by the critical EMSR values obtained from the prorated fare convergence method with a Z-factor of one. Furthermore, as far as bid price control is concerned, the ODCBP method generates statistically more revenue than the LPBP method at both 1.00 and 1.20 demand levels. In the next section, we compare the expected revenue generated by some NRV methods to the upper bound method that corresponds to the generated revenue if the demand were deterministically known at the beginning of the booking process.

4.4.3 Upper Bound

4.4.3.1 Upper Bound Method

How fare are we from the optimal strategy? The optimal strategy would be easy to implement if we knew, beforehand, the demand at the end of the booking process for each of the ODF on the network. In other words, if we knew deterministically the exact number of passenger for each ODF just before departure time, we would be able to optimize the revenue according to the number of seats available on the network. Therefore, the Upper Bound method is constructed as explained below in Figure 4-14.

Simulate ODF demand for
The entire booking process

↓

Compute the total simulated demand for each
leg over the entire booking process

↓

Solve LP (3.6) with actual cumulated ODF simulated
demand as inputs

↓

Optimal objective value is the Upper Bound
revenue for the departure.

Figure 4-14: Upper Bound Method.

Note that we are able to relax the integrality constraints in LP (3.6) because the optimal solution to the linear program is integer independently of the integrality constraints (the optimal solution is at an extreme point of the feasible space).

4.4.3.2 Performance Analysis

As one can observe in Table 4-25, all the seat inventory control methods get closer to the optimal strategy as the number of re-optimizations is increased from 18 to 36. Furthermore, the best seat inventory control method, ODCODBL, is 1.39% from the optimal strategy for 36 booking periods.

Methods / Book. Periods	18	36
Upper Bound ($)	4,342,357	4,328,726
LBFC (Base Case)	-3.14%	-2.87%
LPODBL	-1.89%	-1.41%
ODCODBL	-1.69%	-1.39%
LPBP	-2.13%	-1.53%
ODCBP	-1.99%	-1.42%

Table 4-25: Revenue Performance Compared to the Upper Bound Strategy
for 18 and 36 Booking Periods.

4.4.4 Variance of the Simulated Demand

4.4.4.1 Motivations

In this section, we analyze the robustness of the seat inventory control methods with respect to demand variations. For simulation purposes, the Poisson arrival process is widely used because of its attractive characteristics (memoryless property, independent increment property). However, the Poisson process assumes that the variance of the counting process is equal to its expected value. In the real world, this assumption is far from being always true. The objective of this section is to test the seat allocation methods on a new arrival process where the variance of the simulated demand is greater than the mean. The simulated demand should be close enough to the initial demand forecast (unbiased property) as we expect the forecast to be accurate over a large number of trials of the same departure. Therefore, we have designed a demand generation process where the expected value is close to the initial demand forecast but where the estimation of the variance can be increased.

4.4.4.2 Performance Analysis

We define Z_T the Z-factor for the observed simulated demand by:

$$Z_T = \sqrt{\frac{\hat{\sigma}_T^{\,2}}{\hat{D}_T}} \qquad (4.4)$$

with $\hat{D}_T = \frac{1}{20}\sum_{i=1}^{20} D_i$ (D_i :observed demand of departure i). (4.5)

and $\hat{\sigma}_T^{\,2} = \frac{1}{19}\sum_{i=1}^{20}(D_i - \hat{D}_T)^2$ (4.6)

Note the difference between Z_T and Z defined in (4.1). The former corresponds to the Z-factor assumed to be the variance of the demand input for each ODF in the EMSRb mathematical model whereas the later is a measure of the simulated demand variation over the 20 departures. In other words, Z_T is a measure of the variation of the observed demand whereas Z is a parameter that affects the variance inputs of the ODF in the EMSRb mathematical method. Therefore, Z affects the strategy of the EMSRb seat inventory control method whereas Z_T is just a measurement of the observed demand variations. In the following analysis we assume that Z is always 1.0 as input to the EMSRb method.

Z_T corresponds to the ratio of the unbiased estimation of the standard deviation and the total mean demand over the 20 simulated departures. Note that for the Poisson process, Z_T should be close to one because the variance equals the mean for the Poisson probability distribution. We observe that Z_T = 1.05 for the Poisson simulation which confirms the theoretical result. The figure below summarizes the performance for LPBP and ODCBP methods at a 1.20 demand level and for 18 booking periods.

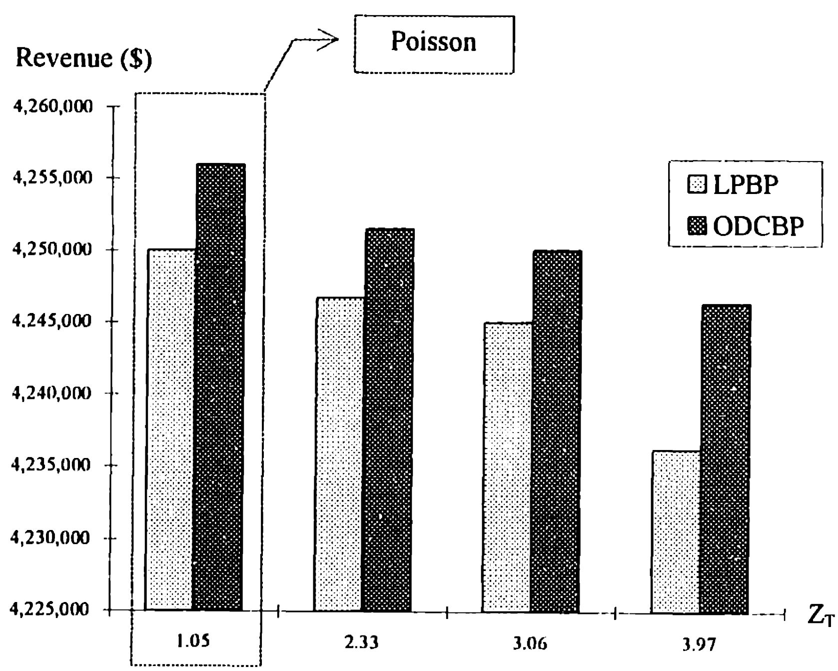

Figure 4-15: Revenue Performance Sensitivity of LPBP and ODCBP with Respect to Z_T for the High Demand Scenario and 18 Booking Periods.

For all Z_T, the ODCBP method performs better than the LPBP method. Moreover, for a Z_T of nearly 4.0, the revenue difference between ODCBP and LPBP is twice as much as the revenue difference observed for a Z_T of around 1.0 (Poisson process).

The LPBP method, which only considers the mean demand forecast (see equation (3-6)), provides estimations of the ENRV that become less reliable as the demand variability increases. The LPBP method fails to consider the stochastic nature of the passenger arrival process. Conversely, the ODCBP method incorporates the variance of the demand forecasts in the EMSRb optimization method. In other words, the ODCBP method takes into consideration the probability distribution of the demand forecast. In summary, the difference between ODCBP and LPBP methods seems to be robust to demand variability. In the next section we analyze the convergence speed of the prorated fare convergence method.

4.4.5 Convergence Speed

In this section, we have analyzed the convergence speed of the prorated fare convergence methods on the network used for the performance analysis. We remind our reader that two convergence methods have been proposed in this thesis: A virtual class (VC) and an OD by OD prorated fare convergence method.

Performance Analysis

In Section 3 of Chapter 3 we have proved the convergence of the prorated fare sequences. In this section, we give some quantitative results concerning the number of iterations required by the prorated fare convergence method before satisfying the convergence criterion[1].

The VC convergence method does not converge as well as the OD by OD convergence methods. It is not unusual to reach 100 iterations, without meeting the $5 convergence

[1] See (3-18) for an explicit formulation of the convergence criterion.

criterion, under VC convergence. We believe that this problem is due to the fact that for the VC convergence method, a fixed number of 16 virtual classes were considered (if an ODF has a relatively high demand, it may encompass several virtual classes) whereas, for the OD by OD convergence method, each ODF is, in a sense, taken as a virtual class, providing many more points on the EMSR curve. Therefore, the VC-convergence method converges slower and cycling is more likely as fewer points are considered in the EMSR curve computation. In summary, the OD by OD convergence method converges much more quickly than the VC-convergence method. We present below some quantitative information concerning the OD by OD convergence method.

In practice, on a network of 102 flight legs and 1066 ODF, cycling of the OD by OD convergence method is very rarely observed. We have run an OD by OD convergence method where the requests are controlled using an OD by OD EMSRb bid price method with a Z-factor of 2. Statistics about the number of iterations before meeting the convergence criterion for 20 simulated departures and 18 booking periods, for each departure, are summarized in Table 4-26 below.

	Number of iterations
Mean	10
Standard Deviation	6
Maximum	48
Minimum	2
% above 20	8.9%
% less than 10	60.8%

Table 4-26: Statistics about the Number of Iterations of the OD by OD Prorated Fare Convergence Model.

Only 8.9% of the time, the number of convergence iterations is greater than 20 whereas, 60.8% of the time, the number of iterations is less than 10. The convergence is relatively monotonic. (The highest difference between two iterations decreases nearly monotonically). In Chapter 5, we propose a method to speed up the convergence of the prorated fare, which is based on the successive approximation technique.

4.5 Summary

In this chapter, we have evaluated the performance of the different seat inventory control methods that estimate the network revenue value of the connecting ODF. The six different methods tested are based on two techniques to estimate the NRV: Displacement cost using shadow prices from an LP and the prorated fare convergence model. Both concepts are explained in Chapter 3. Two results obtained using the MIT integrated optimization/booking simulator are worth highlighting.

First, if the demand is controlled by a bid price mechanism, the prorated fare convergence method with a Z-factor of one performs statistically better than the LP shadow price method for the high demand scenario. Figure 4-16 summarizes the performance of the bid price methods.

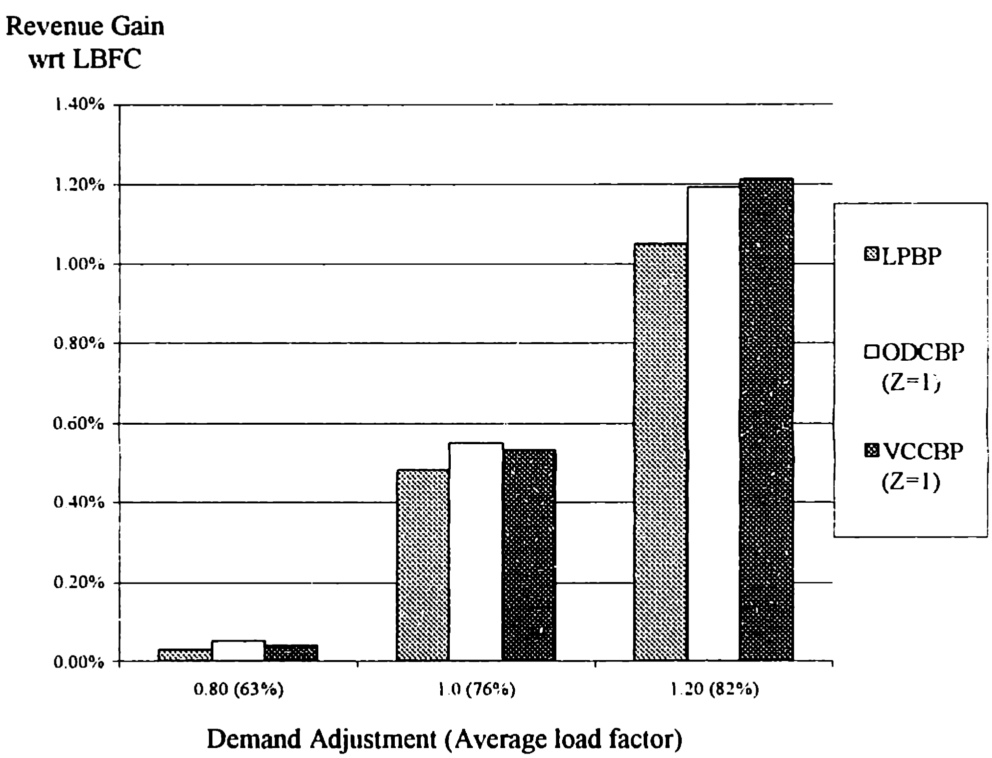

Figure 4-16: Revenue Gain wrt the Base Case (LBFC) for Bid Price Controlled Methods, 18 Booking Periods.

The second important result is that under each of the network seat inventory control mechanisms tested in this thesis, the prorated fare convergence approach has always generated higher network revenues than a deterministic LP approach. For example with 36 booking periods (and re-optimizations), if the demand is controlled using booking limits and the OD by OD nesting strategy, the prorated fare convergence approach performs better than the LP displacement cost approach. Using the same booking control mechanism but with a virtual class nesting strategy, the prorated fare convergence method performs 0.17% better than the LP displacement cost approach. If the demand is controlled using bid prices, the OD by OD prorated fare convergence approach performs 0.12% better than the LP shadow price approach. Using the same booking control mechanism but with a virtual class nesting strategy as input to the EMSRb model, the prorated fare convergence method performs also better than the LP shadow price approach. In Figure 4-17 below we present the revenue performance of the seat inventory control methods for 36 booking periods and for the high demand scenario (82% average leg load factor).

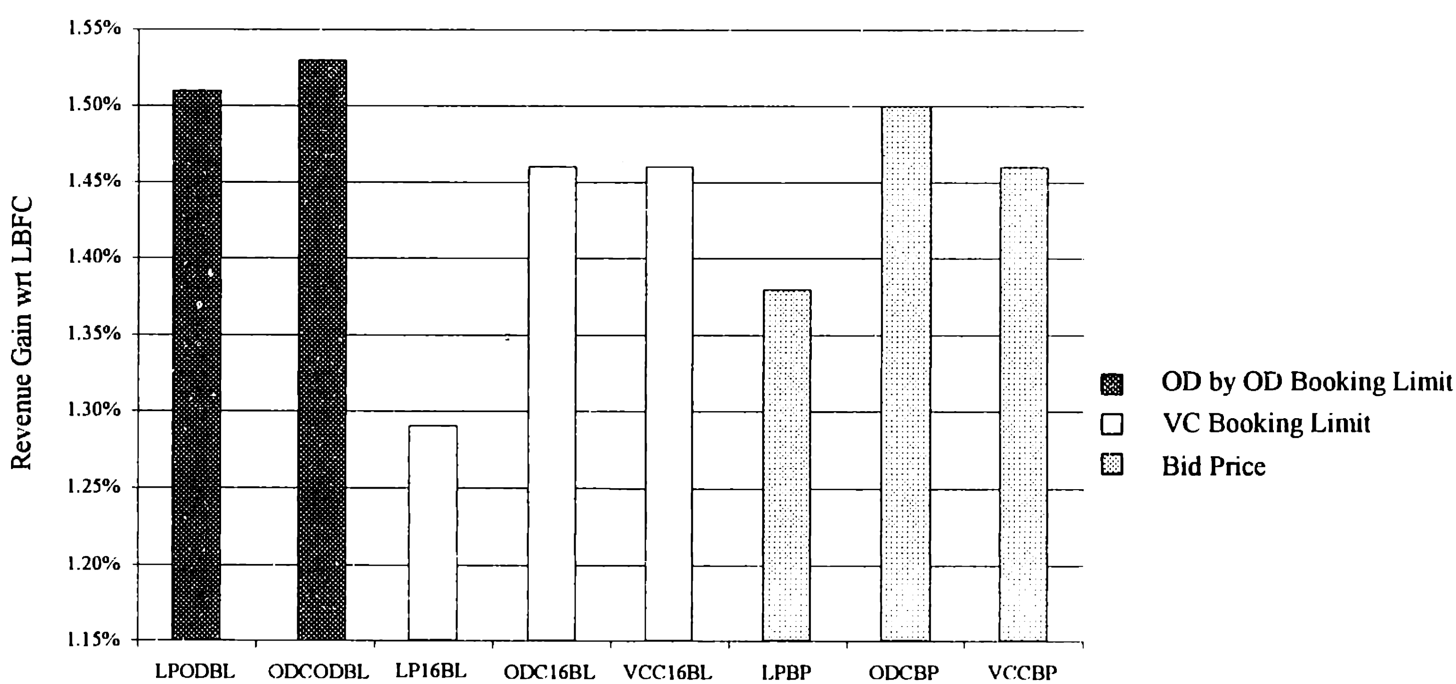

Figure 4-17: Revenue Gain wrt the Base Case (LBFC) for 36 Booking Periods.

In summary, when the NRV are re-optimized 18 or 36 times along the booking process, the prorated fare convergence techniques perform always better than the LP shadow price methods. Indeed, if the demand is controlled by network bid price mechanism, the prorated fare convergence method performs up to 0.16% better than the LP shadow price approach for 18 booking periods and up to 0.12% for 36 booking periods. Moreover, as far as booking limit control mechanism is concerned, estimating the ENRV by the EMSRc from the prorated fare convergence method generates up to 0.24% additional revenue compare to the LP shadow price approach for 18 booking periods and up to 0.17% for 36 booking periods.

5 Conclusion

In this chapter, we first summarize the research contributions by highlighting the main results found in this thesis and then, we propose several directions for future research.

5.4 Research Contribution

The objective of this thesis is to develop and test different techniques to estimate the network revenue values (NRV) of the origin destination and fare itineraries offered by an airline, on each traversed leg. The major contribution of this thesis is to propose techniques to estimate the NRV that consider the stochastic nature of the passenger demand and that satisfy the seat inventory nesting property. This technique prorates the fares of the connecting ODF according to the critical EMSRc operator on the legs traversed. The method is a convergence method as the prorated fares are used as inputs to the EMSRb mathematical model until the convergence criterion is satisfied. We have proved the existence and the uniqueness of the convergence limit of the prorated fare sequence under the assumption that the operator is obtained by the same set of ODF over a large consecutive number of iterations. In practice, the convergence on a network of an actual airline is reasonably quick (on average no more than ten iterations are necessary).

In order to evaluate the performance of the new method, we have conducted an empirical analysis on an actual airline network where the passenger arrivals constitute a Poisson process. We have found that the method performs always better than the LP shadow price approach for both the booking limit and the bid price control mechanisms. Moreover, if the number of booking periods is relatively high (36 revision points along the booking process) the NRV methods perform approximately 1.50% better than the base case method (Leg Base Fare Class, commonly used in the industry) for a high but realistic demand scenario (the average leg load factor is 82%),.

Furthermore, the prorated fare convergence method tends to be more robust to demand variations than the LP simplex approach. The former method encompasses the stochastic information of the demand by incorporating the variance whereas the latter considers only the mean demand forecast. Therefore, if the demand is controlled using booking limits, the NRV obtained from the displacement cost approach tend to spill too many connecting passengers when the observed demand is lower than expected (underestimated the demand forecasts) as it tends to overestimate the displacement costs (underestimate the NRV of the connecting ODF). Conversely, the prorated fare convergence method estimates the NRV of each connecting ODF such that the sum of the NRV over the traversed legs is equal to the total fare. In summary, prorating the connecting fare reduces the risk to underestimate systematically the NRV of the ODF.

5.5 Further Research Directions

5.5.1 Tests on Different Airline Networks

In Chapter 4 of this thesis, the different seat inventory control methods have been tested on an actual airline network. This computational experiment gives us some information about the performance of the methods. Nonetheless, it would be interesting to compare the performance of the proposed seat inventory control methods on different airline network. We would be able to test the robustness of the results with respect to the airline environments (network type and demand pattern). In other words, do the seat inventory control methods using the prorated fare convergence algorithms developed in this thesis perform better than the methods using the LP displacement cost approach on different networks? Is the performance of the prorated fare convergence algorithm correlated with some features of the network offered by the airline, like the percentage of connecting ODF?

5.5.2 Optimality Analysis

The prorated fare convergence method with an OD by OD sorting strategy performs approximately 1.40% lower than the upper bound (the actual demand is known with certainty at the start of the booking process) in terms of expected revenue. This result has been obtained from the experimental analysis conducted on a given network and with a Poisson process. No theoretical result has been derived about the optimality of the prorated fare technique. In other words, is the critical EMSR operator the optimal measure to prorate the fare of the connecting ODF along their itineraries?

5.5.3 Convergence Speed Improvement

A sub gradient technique can be implemented to speed up the convergence. The method estimates dynamically the derivative of the curve g (see general case analysis). If the absolute value of the derivative is less then one, we consider the new function G such that:

$G(x) = \dfrac{g(x) - g'(x) \times x}{1 - g'(x)}$. We expect the number of iterations of the convergence method to decrease if the above technique is implemented.

5.5.4 Stochastic Programming

The main drawback of the deterministic LP approach is that it does not encompass the stochastic nature of the passenger demand. The objective is therefore, to consider several demand scenarios in the estimation of ENRV for each leg of the network. The Average Plan method[1] can be used to approximate the shadow prices associated with the capacity

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[1] F. Jauffred, "Stochastic Optimization for Robust Planning in Transportation," MIT, 1997.

constraints. The technique considers different demand scenarios with an associated probability of occurrence. A large area of research is still to be explored in stochastic programming applied to the seat allocation problem. Nonetheless, the mathematical programming techniques fail to incorporate the nesting properties that are most desirable for airline seat inventory control. A question that remains unanswered and open to further research is whether a non-nested stochastic programming technique can perform better and be more robust to demand variations than the nested prorated fare convergence method developed in this thesis?

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APPENDIX

SECTION A: THE EMSRb CURVE AND THE BOOKING LIMITS

The objective of this part is to illustrate, through a simple example, the construction of the EMSRb curve and the calculation of the booking limits associated with each ODF using the EMSRb method.

We assume that the demand for each fare is normally distributed ($D_j \approx N(\overline{D}_j, \sigma_j)$) and that the number of seats available on the flight, at the time of the booking limit calculation, is 70 seats. The forecasting information and the fare of each ODF are summarized in the table below.

BOS-PAR

Fare Class	$\overline{D}_j$	σ_j	fare ($)
Y	10	5	1000
B	15	7	700
M	20	9	500
Q	30	13	350

Table A-1: Forecasted Demand and Standard Deviation, Fare for each Fare Class.

1 Protection for Y fare class

According to the first step of the EMSRb algorithm, formally explained in the second section of chapter 2, the number of seat to protect for the Y fare class is found solving,

$$\text{Max } \Pi_Y$$

$$\text{Subject to} \quad f_Y \times \overline{P}(\Pi_Y) \geq f_B \tag{A1}$$

This corresponds to find the largest integer value, Π_1^*, such that:

$$1000 \times \overline{P}(\Pi_Y) \geq 700 \Rightarrow \overline{P}(\Pi_Y) \geq \frac{7}{10} \tag{A2}$$

But,

$$\overline{P}(\Pi_Y) = P(D_Y \geq \Pi_Y) = P(\frac{D_Y - \overline{D}_Y}{\sigma_Y} \geq \frac{\Pi_Y - \overline{D}_Y}{\sigma_Y}) = P(\frac{D_Y - 10}{5} \geq \frac{\Pi_Y - 10}{5}) \qquad \text{(A3)}$$

The random variable Z, equals to $\frac{D_Y - 10}{5}$, is normally distributed with a 0 mean and unit variance. Therefore, using the table of the Standard Normal Distribution Function, it comes that,

$P(Z \geq \frac{7-10}{5}) = 0.72$ for Π_Y=7, and $P(Z \geq \frac{8-10}{5}) = 0.66$ for Π_Y=8. Therefore, the largest number Π_Y that meets constraint (A1) is 7. Consequently, 7 seats have to be protected for Y class, according to the EMSRb model.

2 *Protection for Y and B fare classes*

The fare, the forecasted demand and the forecasted standard deviation of the Y and B classes combined are:

$$f_{Y,B} = \frac{\overline{D}_Y \times f_Y + \overline{D}_B \times f_B}{\overline{D}_Y + \overline{D}_B} = \frac{10 \times 1000 + 15 \times 700}{10 + 15} = \$820 \qquad \text{(A4)}$$

$$\overline{D}_{Y,B} = \overline{D}_Y + \overline{D}_B = 10 + 15 = 25 \qquad \text{(A5)}$$

$$\sigma_{Y,B} = \sqrt{\sigma^2{}_Y + \sigma^2{}_B} = \sqrt{25 + 49} = 8.60 \qquad \text{(A6)}$$

$\Pi_{Y,B}$, the number of seats to protect for the compound Y and B classes is found by solving

$$\text{Max } \Pi_{Y,B}$$

$$\text{Subject to} \quad f_{Y,B} \times \overline{P}(\Pi_{Y,B}) \geq f_M \qquad \text{(A7)}$$

Therefore,

$$\bar{P}(\Pi_{Y,B}) \geq \frac{f_M}{f_{Y,B}} \Rightarrow \bar{P}(\Pi_{Y,B}) \geq \frac{500}{820} \Rightarrow P(Z \geq \frac{\Pi_{Y,B} - \bar{D}_{Y,B}}{\sigma_{Y,B}}) \geq 0.61$$

$$\Rightarrow P(\frac{\Pi_{Y,B} - 25}{8.6}) \geq 0.61$$

Using the same method as in the first step, the number of seats to protect for the combined Y and B class is 22 seats.

3 Protection for Y, B and M fare classes

The fare, the forecasted demand and the forecasted standard deviation of Y, B, and M classes combined are:

$$f_{Y,B,M} = \frac{\bar{D}_Y \times f_Y + \bar{D}_B \times f_B + \bar{D}_M \times f_M}{\bar{D}_Y + \bar{D}_B + \bar{D}_M} = \frac{10 \times 1000 + 15 \times 700 + 20 \times 500}{10 + 15 + 20} = \$678 \quad \text{(A8)}$$

$$\bar{D}_{Y,B,M} = \bar{D}_Y + \bar{D}_B = 10 + 15 + 20 = 45 \quad \text{(A9)}$$

$$\sigma_{Y,B,M} = \sqrt{\sigma^2{}_Y + \sigma^2{}_B + \sigma^2{}_M} = \sqrt{25 + 49 + 81} = 12.45 \quad \text{(A10)}$$

$\Pi_{Y,B,M}$, is by definition the number of seats to protect for the combined Y, B, and M classes. $\Pi_{Y,B,M}$ is found by solving

$$\text{Max } \Pi_{Y,B,M}$$

$$\text{Subject to} \quad f_{Y,B,M} \times \bar{P}(\Pi_{Y,B,M}) \geq f_Q \quad \text{(A11)}$$

Therefore,

$$\bar{P}(\Pi_{Y,B,M}) \geq \frac{f_Q}{f_{Y,B,M}} \Rightarrow \bar{P}(\Pi_{Y,B,M}) \geq \frac{300}{678} \Rightarrow P(Z \geq \frac{\Pi_{Y,B,M} - \bar{D}_{Y,B,M}}{\sigma_{Y,B,M}}) \geq 0.44$$

$$\Rightarrow P(\frac{\Pi_{Y,B,M} - 45}{12.45}) \geq 0.44$$

According to the standard normal distribution table, we find that the number of seats to protect for the combined Y B and M class is 46 seats.

4 Booking Limits

The following protection levels have been calculated:

Π_Y = 7 seats

$\Pi_{Y,B}$ = 22 seats

$\Pi_{Y,B,M}$ = 46 seats

The booking limit of each fare class is therefore:

BL(Y) = C = 70 seats

BL(B) = C - Π_Y = 70 – 7 = 63 seats

BL(M) = C - $\Pi_{Y,B}$ = 70 – 22 = 48 seats

BL(Q) = C - $\Pi_{Y,B,M}$ = 70 – 46 = 24 seats

The booking limits of the different ODF proposed on BOS-PAR are summarized in the figure below.

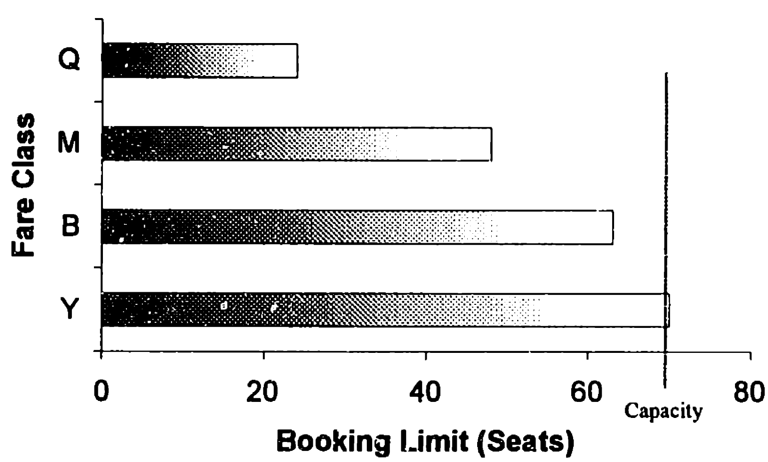

Figure A-1: Booking Limit for each Fare Class on BOS-PAR.

5 EMSR curve

The objective of this section is to explain how the EMSR curve is drawn based on the EMSRb algorithm. According to the optimal booking limit strategy explained by Curry[1], the Expected Marginal Seat Revenue curve is decreasing and convex with respect to the seat allocation. The objective is to force the EMSR curve from the EMSRb heuristic, to be both decreasing with respect to the seat allocation.

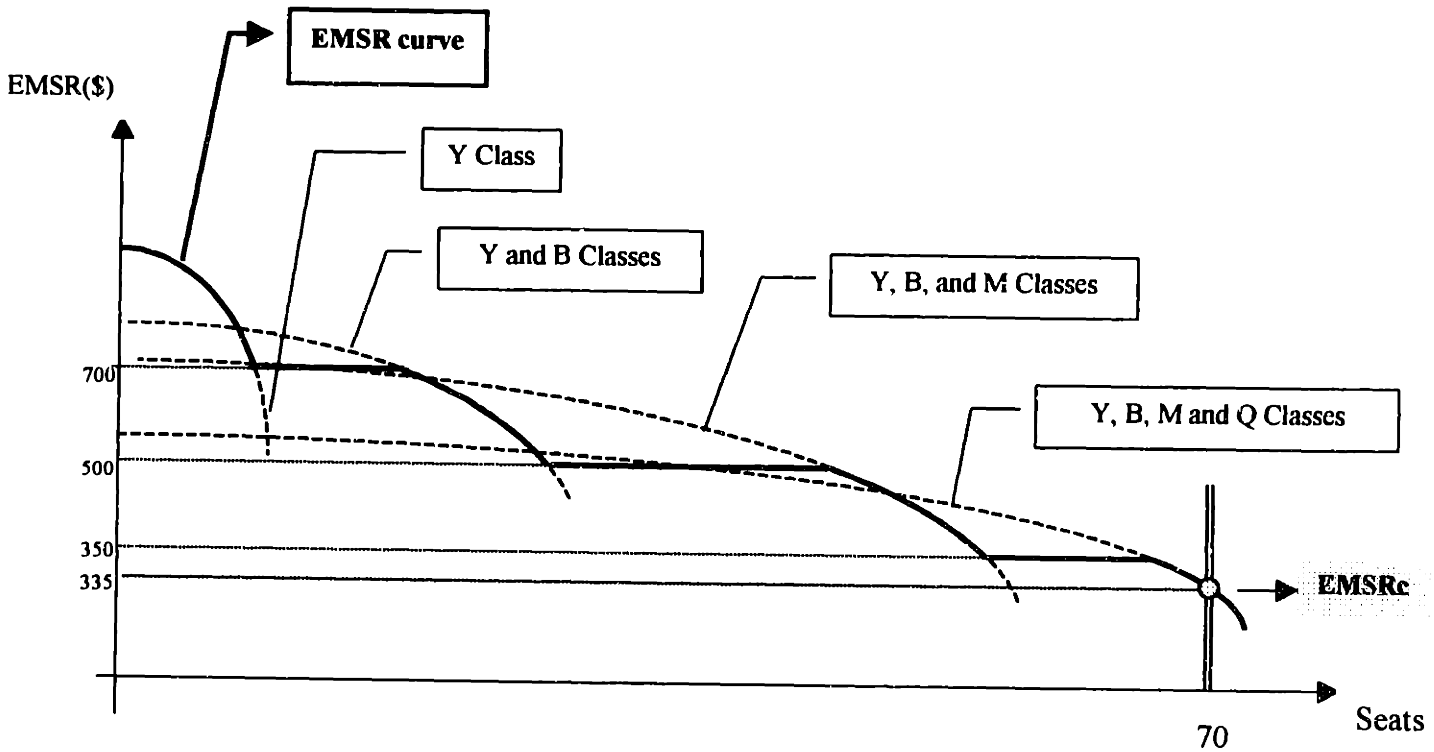

Figure A-2: EMSR curve using EMSRb model for BOS-PAR.

The EMSR curve is cut by the lowest fare of the active combined fare class set. Therefore, as one can observe in figure 2, above, the EMSR curve is cut by the fares corresponding to B and M classes. Therefore, using this heuristic, the EMSR curve is

[1] Curry[6].

decreasing with respect to the allocated seats. Nonetheless, the EMSR curve is not convex with respect to the allocated seats.

6 *Critical EMSR Value*

We define the critical EMSR value, EMSRc, as the EMSR value of the number of seats available on the leg. In our example, 70 seats are available on BOS-PAR flight. As one can observe in Figure 2, above, the compound of all the ODF generates the EMSR piece of curve that defines the critical EMSR value.

Therefore, on BOS-PAR, the critical EMSR value is

$$\text{EMSRc(BOS-PAR)} = f_{Y,B,M,Q} \times \text{Prob}(D_{Y,B,M,Q} \geq 70) \tag{A12}$$

With

$$f_{Y,B,M} = \frac{\overline{D}_Y \times f_Y + \overline{D}_B \times f_B + \overline{D}_M \times f_M + \overline{D}_Q \times f_Q}{\overline{D}_Y + \overline{D}_B + \overline{D}_M + \overline{D}_Q}$$

$$= \frac{10 \times 1000 + 15 \times 700 + 20 \times 500 + 30 \times 350}{10 + 15 + 20 + 30} = \$546.7$$

$$\overline{D}_{Y,B,M} = \overline{D}_Y + \overline{D}_B = 10 + 15 + 20 + 30 = 75$$

$$\sigma_{Y,B,M} = \sqrt{\sigma^2{}_Y + \sigma^2{}_B + \sigma^2{}_M + \sigma^2{}_Q} = \sqrt{25 + 49 + 81 + 169} = 18$$

Using (A12), it comes that

<u>EMSRc(BOS-PAR) = $333.5</u>

The EMSRc value for the flight is represented in Figure 2 above.

SECTION B: OD-CONV EMSRc AND LP SHADOW PRICES (MEAN OVER THE 18 BOOKING PERIODS OF THE FIRST DEPARTURE, DADJ=1.20)

Leg #	SP	EMSRc	Av. L.F	Leg #	SP	EMSRc	Av.L.F
1	0.00	0.00	52.0%	31	0.00	11.06	77.5%
2	0.00	20.75	89.5%	32	0.00	20.62	78.5%
3	661.72	654.95	96.5%	33	0.00	13.36	75.5%
4	24.11	248.40	94.0%	34	2.89	59.83	81.5%
5	615.00	720.62	96.0%	35	72.28	91.14	90.5%
6	0.00	80.15	80.0%	36	0.00	0.24	58.5%
7	704.11	532.15	100.0%	37	0.00	0.12	56.5%
8	479.50	463.62	97.0%	38	0.00	2.58	62.5%
9	434.28	425.32	98.5%	39	143.94	135.15	90.0%
10	219.72	244.37	95.5%	40	37.89	72.12	88.0%
11	0.00	149.97	92.0%	41	0.00	0.04	50.5%
12	0.22	153.65	93.5%	42	0.00	7.95	74.5%
13	349.67	343.68	97.0%	43	47.06	76.18	85.0%
14	0.00	0.29	62.0%	44	0.00	0.20	56.5%
15	0.00	8.21	77.5%	45	40.06	72.79	94.0%
16	0.00	0.00	47.0%	46	0.00	27.62	87.0%
17	0.00	53.85	82.0%	47	0.00	31.32	88.0%
18	0.00	1.40	63.0%	48	3.56	27.21	92.5%
19	0.00	0.00	39.0%	49	121.78	128.85	90.0%
20	0.00	95.10	93.0%	50	0.00	11.25	90.0%
21	0.00	0.00	22.5%	51	144.78	149.31	92.5%
22	157.39	150.53	97.0%	52	0.00	0.00	40.0%
23	0.00	1.64	65.0%	53	0.00	0.27	63.5%
24	0.00	0.01	58.5%	54	31.94	46.69	87.5%
25	118.56	117.22	95.0%	55	0.00	0.00	28.0%
26	88.22	86.08	99.0%	56	156.72	143.80	95.0%
27	0.00	0.00	34.0%	57	225.61	139.09	92.5%
28	0.00	0.08	31.0%	58	149.67	103.65	94.5%
29	9.56	106.85	92.5%	59	41.39	142.62	97.5%
30	163.44	192.24	93.0%				

60	49.11	66.93	93.5%
61	6.67	51.83	93.5%
62	15.61	149.06	93.0%
63	0.00	8.95	87.5%
64	337.44	300.18	98.0%
65	0.00	5.45	72.0%
66	0.00	7.67	69.5%
67	0.00	35.76	82.0%
68	0.00	72.10	87.0%
69	0.00	65.06	93.0%
70	33.44	79.25	96.5%
71	201.94	177.89	95.5%
72	0.00	0.01	48.5%
73	0.00	4.39	75.5%
74	190.61	158.59	97.5%
75	0.00	2.79	66.0%
76	67.33	270.75	92.0%
77	405.00	386.79	98.5%
78	207.33	207.03	97.0%
79	0.00	17.24	77.5%
80	0.00	63.28	90.0%
81	38.50	114.36	83.0%
82	0.00	43.59	79.0%
83	172.61	258.08	97.0%
84	203.28	168.10	95.5%
85	0.00	3.28	67.0%
86	491.89	452.52	96.5%
87	2.56	19.35	86.5%
88	424.00	397.53	99.0%
89	1097.94	1037.43	96.5%
90	258.50	289.84	97.0%
91	600.28	557.43	97.5%
92	713.50	609.89	96.5%
93	168.72	139.23	96.5%
94	141.67	165.62	97.5%
95	0.00	53.26	70.5%
96	0.00	2.02	71.0%
97	418.94	405.86	98.0%
98	0.00	38.58	76.0%
99	0.00	0.03	51.0%
100	0.00	47.60	90.0%
101	0.00	43.11	80.0%
102	170.50	167.10	98.0%

SECTION C: PROOF OF LEMMA 2

Lemma 2: $\underset{c,k}{MAX}\left\|\nabla g_{c,k}(X^*)\right\| < 1$ with distance of vector V defines as $\|V\| = \underset{i}{MAX}(|v_i|)$.

For a given OD c on a traversed leg k we have

$$\nabla g_{c,k}(X^*) = \sum_{l\in L_c}\sum_{i\in CO(l)} \frac{dg_{c,k}}{dx_{i,l}}$$

two cases have to be considered:

- $l = k$

In that case, $\left|\frac{dg_{c,k}}{dx_{i,k}}\right| = \left|\frac{b_{i,k}\times D - b_{i,k}\times N}{D^2}\right|$

$$\text{With } \frac{a_k + \sum_{i\in CO(k)} b_{i,k}\times x_{i,k}}{\sum_{m\in L_j} a_m + \sum_{m\in L_j}\sum_{i\in CO(m)} b_{i,m}\times x_{i,m}} = \frac{N}{D}$$

Therefore,

$$\left|\frac{dg_{c,k}}{dx_{i,k}}\right| = \left|\frac{b_{i,k}\times D\times(1-N/D)}{D^2}\right|$$

But, $\frac{N}{D} = g_{c,k}(X^*) = x^*_{c,k}$

Thus,

$$\left|\frac{dg_{c,k}}{dx_{i,k}}\right| = \left|\frac{b_{i,k}\times D\times(1-x^*_{c,k})}{D^2}\right| \le \frac{b_{i,k}}{D} < 1$$

- $l \neq k$

We show similarly that

$$\left|\frac{dg_{c,k}}{dx_{i,l}}\right| = \left|\frac{b_{i,l} \times x^{*}{}_{c,k}}{D}\right| < 1 \qquad \text{as } x^{*}{}_{c,k} < 1$$

Conclusion: c and k, $\left\|\nabla g_{c,k}(X^{*})\right\| < 1$.
