Guastavino Structural Calculations

by

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ABSTRACT

The Guastavino Company designed and constructed thousands of incredible thin shell masonry domes, arches, and staircases in America from the 1880s to the 1960s. This thesis traces the design process of the Guastavino Company from the late 1890s to the mid-1900s through their original structural drawings held at the Avery Library Archives at Columbia University.

The drawings are analyzed to reveal the Guastavino Company's innovation of graphic statics techniques, advanced calculations, and strategic designs. No one has ever studied the original drawings. The design process for Guastavino Jr.'s arches and domes is formed through the examination of the barrel vault calculation drawing for the St. Francis de Sales Church (Philadelphia, PA – 1908) and the dome calculation drawing for the Cathedral of Saint John the Divine (NY, NY – 1909). The research presents the ingenuity of the Guastavino Company in their structural calculations and in particular, Rafael Guastavino Jr.'s outstanding contributions.

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This thesis would not have been possible without the dedicated support and resources from many individuals. I would like to thank everyone along the way that pointed me in the right direction and introduced me to the Guastavinos.

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There are multiple people in Asheville, North Carolina that introduced me to the personal life and estate of Guastavino Sr. Peter Austin kindly shared his time and research on the Guastavino Archives to help me understand the endless resources that are still in the archives. Michael Murphy and Helen Johnson from Christmount graciously toured us around the former Rhododendron estate at Black Mountain and shared their knowledge of the family. From the St. Lawrence Basilica, John Toms introduced me to Diane Wright who maneuvered through the snow to show us the hidden secrets of the church. Thank you all for sharing your Guastavino knowledge and for making my Asheville trip very memorable.

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1 Introduction

1.1 Motivation

Advances in engineering transform thick and heavy ancient masonry constructions into thin and sleek form-found structures that are molded around their forces. From the 1880s to the 1960s, the Guastavino Company created thousands of masonry vaults, arches, staircases, and domes across America. Their structures embody the transition from the old world design rules of thumb to precise calculations through graphic statics.

It is vital to learn the design intent and assumptions behind Guastavino structures to praise their advances but also to understand their techniques. The life span of their constructions is dependent on proper maintenance. Rehabilitation work that honors the integrity of structures stems from a deep understanding of the original design. Guastavino structures have only been tested for around one hundred years, and there have been no failures. The Guastavinos are admired for introducing thin shell masonry structures to America. The history of the Guastavinos and some of their projects has been closely studied to learn from their success. However, their design process and analysis techniques have never been analyzed.

Graphic statics is an analytical and design tool used by Guastavino Jr. to calculate forces on arches and domes and to shape more efficient structures. The method has been used by world renowned designers such as Gustave Eiffel (1832-1923), Antoni Gaudi (1852-1926), Robert Maillart (1872-1940), and Guastavino Jr. (1872-1950) to design symbolic structures like the Eiffel Tower, Parque Güell, the Salginatobel Bridge, and the Cathedral of St. John the Divine. This thesis investigates the calculations of the Guastavino Company original drawings to learn from their design advancements and to present their design process for practitioners rehabilitating them.

1.2 The Guastavinos

1.2.1 Rafael Guastavino Sr.

Rafael Guastavino Sr. (1842-1908) studied at the *Escola Especial de Mestres d'Obres* (Special School for Masters of Works) from 1861 to 1872 to earn his *mestre d'obres*, or master builder title. In Spain, Guastavino Sr. is most recognized for the Batlló Factory (Barcelona, 1875), which displays the applicability of his thin shell structural vaulted system to a large scale and quick construction. Guastavino Sr. continued to construct structures in Spain until 1881, when he decided to move to America with his youngest son, Rafael Guastavino Jr. (1872-1950).

Guastavino Sr.'s American fame began in 1889 with the Boston Public Library, led by McKim, Mead, and White. After experiencing difficulty landing full building projects, Guastavino Sr. rebranded himself as a fireproof construction company and focused on his structural vaulting system. He introduced the vaulting system in place of large iron beam floor systems as an efficient, fireproof, and rapid construction method. Guastavino Sr. is best known for his work on the City Hall Subway Station, Queensborough Bridgemarket, and the St. Lawrence Basilica. In 1908, Guastavino Sr. passed away. His final resting place is in his final construction and masterpiece, the St. Lawrence Basilica in Asheville, North Carolina [Ochsendorf, 2010].

1.2.2 Rafael Guastavino Jr.

Guastavino Jr. took over the Guastavino Company towards the end of Guastavino Sr.'s life when he retired to North Carolina. Guastavino Jr. only attended school for six years. He learned the majority of his skills as an apprentice for his father from the age of fifteen.

In 1909, Guastavino Jr. elevated the popularity of his father's company by constructing one of the largest free-standing masonry domes in the world, the Cathedral of St. John the Divine. He is also widely known for his work on the Grand Central Station Oyster Bar, the Nebraska State Capital, and the Smithsonian Institution. Apart from his design and construction inventions, Guastavino Jr. experimented with acoustic and glazed tiles. Guastavino Jr. retired in 1943 and passed away in 1950. The company was permanently closed in 1962 [Ochsendorf, 2010].

1.2.3 Conclusions

Guastavino structures are incredibly thin masonry shells that can support an impressive amount of weight through their form and geometry. There are over four-hundred Guastavino structures in New York alone and hundreds more in the rest of America. There might not be a single architect or builder that can say they have that many structures of their own in New York City, one of the greatest architecture capitals of the world.

1.3 Literature Review

1.3.1 Guastavino Literature

The popularity of and curiosity in the function of Guastavino structures in the early 1900s passed by the end of the Company in 1962. Professor George R. Collins (1917-1993), the late art historian at Columbia University, was captivated by Guastavinos' St. Paul's Chapel in 1961 and reintroduced the world to the Guastavinos. Collins rescued Guastavino Company documents that were almost destroyed and began the Guastavino Archives at Columbia's Avery Library. In 1968, Collins published "The Transfer of Thin Masonry Vaulting from Spain to America" to present the hundreds of seemingly forgotten Guastavino structures that populate major cities like New York, Boston, and Philadelphia. Since his passing in 1993, Janet Parks has taken over the archive collection. Parks greatly assisted the research process in the Avery Archives for this thesis and shared information on various sources. In 1996, Parks and Alan G. Neumann wrote "The Old World builds the New: The Guastavino Company and the technology of Catalan vault, 1885-1962" to share the contents and knowledge of the archived collection. More recently, Parks wrote "Rafael Guastavino and Cass Gilbert: A Match made in Minnesota" (2012) to present Guastavino Sr.'s correspondence with Cass Gilbert, the architect, on the Minnesota State Capital in St. Paul. The letters disclose some information on Guastavino Sr.'s little known design techniques.

The Spanish book, "Las Bóvedas De Guastavino En América" (Huerta, 2002), encloses essays on Guastavino constructions and has an extensive collection of drawings from the Avery Library Collection. Huerta's essay, "The Mechanics of Timbrel Vaults: A Historical Outline" (2003) discusses the history of timbrel vaults and Guastavino Sr. and Jr.'s designs theories.

The recent book "Guastavino Vaulting" (Ochsendorf, 2010) and public exhibition *Palaces for the People* (2012) have spiked excitement and curiosity in Guastavino structures again. The book serves to present the massive achievements of the Guastavino Company through Guastavino Sr. and Jr.'s life history, projects, and, most importantly, their designs. However, the book does not go into detail on the methods of calculation.

Guastavino structure scholars, such as Collins, Huerta, and Ochsendorf have inspired many students to conduct research on the Guastavino's. The following theses provide historical and technical information on specific Guastavino projects. Katherine Milkovich (M.S. Department of Historic Preservation, University of Pennsylvania) recounts the history and presents conservation issues of Guastavino constructions in "Guastavino Tile Construction: An Analysis of a Modern Cohesive Construction Technique" (1992). Lisa Mroszczyk (B.S. Department of Architecture, MIT) discusses the history of "Rafael Guastavino and the Boston Public Library" (2004). Hussam Dugum (M. Eng. Department of Civil and Environmental Engineering, MIT) investigates the "Structural Assessment of the Guastavino Masonry Dome of the Cathedral of Saint John the Divine" (2013).

1.3.2 Graphic Statics Literature

Graphic statics is a powerful design and analysis tool that was used by the Guastavino Company starting in the early 1900s to create many of their structures. The method relies on the geometry and weight of a structure to determine the forces in arches and domes graphically. There are many resources available to learn graphic statics. Ochsendorf and Mueller created the *Structural Design Lab* website at

MIT, which provides publications and interactive tools on graphic statics. Allen and Zalewski published "Shaping Structures" (1998) and "Forms and Forces" (2009), which explain graphic statics methods and powerful applications.

Fivet and Zastavni published a journal article for the International Association for Shell and Spatial Structures (IASS) on "Robert Maillart's Key Methods from the Salginatobel Bridge Design Process (1928)" (Fivet and Zastavni, 2012). They investigate Maillart's original drawings to understand the design process behind the iconic structure and the power of form-finding with graphic statics. Fivet's analysis of the Maillart graphic statics drawings is the inspiration for the methodology of this thesis. Fivet's expertise in the history and application of graphic statics greatly influenced the progress and findings in this thesis.

Several theses researched the properties, designs, and collapse mechanisms of masonry domes. Wanda Lau (M.S. Building Technology, Department of Architecture, MIT) wrote the "Equilibrium Analysis of Masonry Domes" (2006), where she developed the modified thrust line analysis for masonry domes. Lau clearly details the methodology behind Eddy and Wolfe's graphic statics methods and creates an interactive program available online. Jennifer Zessin (Ph.D Building Technology, Department of Architecture, MIT) wrote the journal article, "Collapse Analysis of Unreinforced Masonry Domes and Curving Walls" (2012). Zessin investigates and defines the masonry collapse mechanisms for domes and curving walls.

1.3.3 Open Research Areas

As mentioned, plenty of resources detail the life history, projects, and achievements of the Guastavinos. However, little information exists on their structural calculations. Huerta summarizes the analysis techniques of the Guastavinos and Dugum checks the structural safety of the Cathedral of St. John the Divine in New York. However, no research has directly analyzed the graphic statics diagrams found in the Avery Library Guastavino Archives. This thesis applies an investigative analysis method for the Guastavino calculation drawings, as executed by Fivet and Zastavni in the Maillart calculation drawings. The diagrams contain information that can reveal the design process implemented on the thousands of Guastavino structures found around America.

Some research has portrayed strong opinions on the Guastavino calculation methods. However, there are many questions left unanswered. In many instances, researchers combine the design accomplishments of Guastavino Sr. and Guastavino Jr. instead of separately praising them. The designer behind many of the structures is still unknown. Allen claims,

> Graphic statics permitted Guastavino to give a funicular shape to each of his vaults. This minimized bending stresses while producing shapes that were generally parabolic or catenary in section rather than circular. [Allen, 2004]

Based on the available records, Guastavino Sr. did not appear to use graphic statics in any of his structures. Guastavino Jr. used graphic statics; however, little is known about the design and calculation process of the vault. A generalized shape for the structures has also not been investigated through the original drawings. Collins alludes that the Guastavino vaulting structures were not calculated and questions if they could be calculated at all.

> There is, as we shall see, not only something spectacular and gravity-defying about these vaults – but also an air of mystery as to precisely how and why they function as they do, and whether a precise theory can be evolved to explain and/or calculate them structurally. [Collins, 1968]

The calculations for the structures are hidden in the original drawings. This thesis investigates these documents to determine the mysterious function of the extremely thin and long spanning masonry constructions.

The Cathedral of St. John the Divine in New York, NY has hundreds of original drawings in the Guastavino Archives at Avery Library in Columbia University. The Cathedral dome has an impressive span of 132 feet and an incredible thinness of only 4 inches for a significant portion of the dome. The dome is the second longest masonry dome in the world, behind the Duomo in Florence, Italy that spans 148 feet. The calculation and designs behind the Cathedral dome are undetermined. The documents at the archives have never been completely searched to find the calculations for the dome. There is one known calculation drawing for the dome, but it has never been closely analyzed.

The Guastavino Company designed many structures very quickly. It is questionable whether each Guastavino structure design was original or if they were repeating designs based on ratios, experience, and builders' intuition. The design process behind Guastavino structures is unknown, yet there are thousands of them around America. Ochsendorf learned in his research, "In 1910 alone ... they were building 100 buildings at once... it's almost unfathomable today that a construction company would be working on a hundred buildings at once" [Stamberg, 2013]. The original design capacity and intention behind the structures is important for practitioners maintaining and rehabilitating them to understand.

The drawings behind many Guastavino structures do not exist; however, the documents that survive can be used to form a data base to understand the general design process. No one has ever traced the calculation drawings in the Avery Archives to learn the design and validate their methods. The information the Guastavino calculations reveal should be spread for people to learn from and advance further.

1.4 Problem Statement

This thesis investigates the original calculation drawings of the Guastavino Company to understand their design methods on masonry arches, barrel vaults, and domes. The majority of Guastavino constructions do not have calculation drawings and little is known about their design.

The research seeks to answer:

- What are the calculation documents that exist in the Avery Library Guastavino Archives at Columbia University and what generalities can be made about them?

- What was the design process behind a Guastavino Company arch, barrel vault, and dome?
    - How were the forces calculated?
    - How did they choose a shape for a structure?
    - How did they decide on the layers of brick?
    - How was the metal reinforcing designed?

- Who was conducting the graphic statics analysis?

- How did the Guastavino Company designs, calculations, and structures change over time?

None of these questions have been answered in previous literature. This thesis provides a hypothesis on the Guastavino design process and orders the sequence of the Guastavino's calculation methods. A table useful for practitioners rehabilitating Guastavino structures is created. The results are based on the analysis of the original graphic statics drawings and on available texts on the Guastavino Company and graphic statics.

1.5 Thesis Outline

The introduction section presents the history and primary literature on the Guastavino Company, family, and calculation methods. The open areas of research concerning the original design drawings are identified.

Chapter 2 lists the process implemented in this thesis.

Chapter 3 introduces the calculation methods of Guastavino Sr. and Jr. It also details the graphic statics knowledge available to the Guastavinos in the early 1900s.

Chapter 4 gives a broad overview of the graphic statics calculation drawings found in the Avery Library Guastavino Archives. The graphic statics methods and dates of the drawings are compared.

Chapter 5 focuses on the original barrel vault drawing for the St. Francis de Sales Church in Philadelphia, PA.

Chapter 6 analyzes the rubric calculation drawings and compares the figures to graphic statics literature available at the time.

Chapter 7 traces the design of the dome of the Cathedral of St. John the Divine. A theoretical design process behind the dome is presented.

The final chapters summarize the results and findings from this thesis and identify future research areas.

2 Methodology

2.1 Introduction

This thesis would not be possible without the preservation work by George R. Collins and the archival work led by Janet Parks at Avery Library. The research for this thesis assumes that the drawings in the Avery Library Archives are representative of the entire works of the Guastavino Company.

A thorough understanding of graphic statics and the Guastavino Company history is necessary to decipher the drawings and design methods. After the drawings are retrieved and photographed at the archive, they are categorized and prepared for analysis. The lines on three drawings are analyzed to develop the theoretical design process for barrel vaults or arches and domes. 15 graphic statics drawings are found in total for arches, barrel vaults, and domes. Two tables are assembled to compare various parameters for arches and domes separately.

2.2 Analysis of Existing Drawings

2.2.1 Guastavino Archives

The Guastavino Archives at Avery Library, Columbia University contain thousands of documents categorized in six series: Architectural Drawings, Administrative and Technical Records, Project Files, Factory Orders, Slides, and Sample Products and Fragments. The online Finding Aid for the Archive is very useful to locate drawings by city, project, date, and content. This thesis focuses on the calculation drawings available in the Architectural Drawings series. The drawings that show calculations are usually titled "Stress Diagrams," the Guastavino Company title for graphic statics applications.

All of the Architectural Drawings and Project File Records available are searched for "Stress Diagrams" and only 26 drawings with graphic statics were found. The Project Files series are not examined as closely as the Architectural Drawings series, but they may include letters that describe the use of stress diagrams on projects. The 26 drawings detail 14 separate projects. Some of the 26 drawings contain duplicates of the same analysis. From all of the drawings, 15 different calculations are analyzed in this thesis.

The existence of more stress diagrams is unknown. It is unlikely that these are the only stress diagrams that were ever created. More drawings could have been produced that were lost or destroyed. The research for this thesis is based on the surviving materials in the Avery Library Archives.

A search is also run in the online Finding Aid for domes in general. Dozens of projects are looked at to see if graphic statics is used on the dome and just not mentioned in the title of the drawing. Some stress diagrams are found that did not have stress diagrams in the title. The online Finding Aid is updated to include "stress diagrams" in the title.

In total, 37 projects are recorded; the amount of drawings for each project ranges. Over 100 drawings are photographed. Every graphic statics drawing found as well as other drawings available for each project that detailed floor plans and general information are documented. The photographs of each drawing are saved and organized under the project file name given at the Finding Aid Columbia website.

2.2.2 Document Preparation

The drawings are photographed instead of scanned since scanning degrades the document. The Avery Library has a service to professionally photograph the drawings to attain the best quality. Since over a hundred drawings are searched for this thesis, the quality of personal photographs is sufficient.

Once photographed, each drawing is prepared for analysis. The photo is transformed in Adobe Photoshop to straighten the drawing and adjust coloring. The drawing is then scaled in AutoCAD based on information provided. If there is no dimension or scale written on the drawing, the drawing is scaled based on researching the structure.

There may be some inconsistency with the dimensions recorded. Some of the drawings examined are over a hundred years old. The paper is very fragile, thin, and in some cases torn. Differences in air and humidity may have changed the size of the paper as well, resulting in inaccurate dimensions when the drawing is measured. The documents are often bended, creating skewed lines. It is difficult to straighten skewed lines on the photographs and best approximations are made. The presented dimensions for structures are collected from the drawings and verified from other available resources.

The graphic statics calculation method requires various projections of geometry and different views of the structure on one drawing. One drawing is used to design an entire structure. For example, the Cathedral of St. John the Divine has approximately five different perspective views on one drawing to design a 132 feet spanning dome. This structure would require hundreds of pages of construction drawings today. In 1909, there was only one sheet of paper used to calculate everything.

In order to present the drawings clearly, several color-coded drawings are made to identify the different components and views of the structure.

2.2.3 Detailed Analysis of Original Calculation Drawings

Three drawings are closely analyzed:

- Barrel Vault: St. Francis de Sales Church- Philadelphia, PA 1908
- Dome: Rubric Diagrams- Date Assumed 1906-1907
- Dome: Cathedral of St. John the Divine - Manhattan, NY 1909

The main objective for the analysis is to answer:

- What line came first and why?
- What are the employed assumptions?

Each drawing is studied to develop a design methodology for an arch and dome. The general process is to trace lines on the drawing and form connections. To decide which lines to trace, graphic statics knowledge to design an arch and dome is used. The graphic statics procedure is implemented on the actual drawing.

Two models are created for each arch and dome. The first model is based on tracing lines on the actual drawing to determine the dimensions. The accuracy of the original drawings is based on the tools used: a pencil, ruler, protractor, and compass. The lines are often skewed and slightly off. Another level of error is added since the lines measured are from a photograph of the drawing, and not the original drawing. The second model is created using the annotated dimensions from the original drawing. Even though the dimensions were written correctly on the drawing, the original lines do not always reflect these values. The drawing is recreated using precise computer accuracy. The rough dimensions in the actual drawing are compared to precise computer model dimensions to see if precision yields different structural results.

After the models are created and verified, several parameters are checked for to understand the design assumptions in the drawing. The following areas are examined to develop a design process:

- Previously Defined Parameters
- Shape
- Segments
- Lune (Only applicable for Domes)
- Funicular Shape to find Center of Gravity of Geometry (Only applicable for Vaults)
- Load Line
- Forces
- Brick Layers
- Metal Placement
- Construction

Once a design process is assumed, it is implemented and compared to the original graphic statics drawing. Different assumptions are tested by recreating numerous graphic statics calculations until a design process yields similar results to the Guastavino Company drawings.

For the Cathedral of St. John the Divine, Dugum conducted a structural assessment of the dome using graphic analysis and membrane analysis techniques. Dugum's analysis is compared to the Guastavino analysis.

2.2.4 Broad Analysis of Existing Calculation Drawings

Graphic statics drawings of eight domes and seven arches are analyzed and compared to develop a drawing record table, located in Appendix A. The following parameters are considered for each drawing:

- Architect
- Date
- Location
- Span at Base
- Height
- Radius for Structure Curvature
- Amount of Radii for Construction
- Brick Thickness
- Loading Considered
- Drawing Scale
- Load Scale
- Guastavino Stamp
- Lune Dimension Considered (Only applicable for Domes)
- Load Table (Only applicable for Domes)

General conclusions on Guastavino construction methods are assumed and presented.

Only drawings that contain graphic statics were used to collect data for the drawing record tables. There are hundreds of construction drawings in the Avery Library Archives that detail dimensions of different Guastavino structures. The calculations behind the majority of the structures were not found. The purpose of the drawing record tables is to compare the parameters for projects that were definitely designed using graphic statics. The design behind the other structures without calculation drawings cannot be absolutely proven.

The chronology of graphic statics drawings for barrel vaults, arches, and domes is determined. The range of graphic statics techniques and metal reinforcement calculation methods are investigated to conclude if the methods change. The sequence of calculation techniques used on Guastavino domes is recorded and compared to the graphic statics knowledge published at the time. The different metal reinforcement calculation methods are summarized and the validity of the calculation is questioned. The decision for metal placement in columns for certain projects is also considered.

The authorship of the Guastavino Company drawings is questionable since they are rarely signed. There is no literature that definitively claims an author to the design drawings. A basic handwriting analysis is applied to the graphic statics drawings to determine the engineer behind the calculations. Calculation drawings and other construction drawings are used to make conclusions. The date, chronology, calculation techniques, and handwritings on drawings are closely compared to make new contributions to the authorship.

2.3 Conclusions

The following assumptions are made on the Guastavino Company materials in this thesis:

- The materials that exist in the Avery Library Archives represent all of the available calculation drawings for the Guastavino Company.
- There is a level of inaccuracy in the information presented due to the condition of the original drawings and the error introduced in reproducing photographs of the drawings.

The materials can be used to make the following assessments:

- The theoretical design process for Guastavino arches and domes can be determined.
- The comparison of graphic statics calculations for arches and domes can be used to develop a chronology to the Company's calculation knowledge and applications.
- The theoretical design process for Guastavino Company structures can be applicable to determine the conditions of other Guastavino constructions that do not have calculation drawings.

3 Guastavino Design History

3.1 Introduction

The history of graphic statics is presented in this section to present the tools that were available to the Guastavinos in the early 1900s. The design strategies of Guastavino Sr. and Jr. reflect the engineering knowledge at the time. Documents written by Guastavino Sr. and drawings by Guastavino Jr. reveal the Guastavino company construction mentalities, design strategies, and metal reinforcement usage in their structures.

3.2 Graphic Statics Background for Arches and Domes

3.2.1 Introduction

Graphic statics uses geometry and forces for a set of assumptions to determine the ideal shape of a structure. It was first published in 1858 by W. J. Rankine (1820-1872) [Kurrer, 2008]. For masonry arches and domes, graphic statics is a tool used to find the thrust line, the imaginary line of forces in the structure proving equilibrium. The development of graphic statics for domes was ongoing in the early 1900s. The origin of dome graphic statics analysis methods can be found in the work of Johann Schwedler (1823-1894), Henry Turner Eddy (1844-1921), August Föppl (1852-1924), William Dunn (1904), and William S. Wolfe (1921) [Kurrer, 2008].

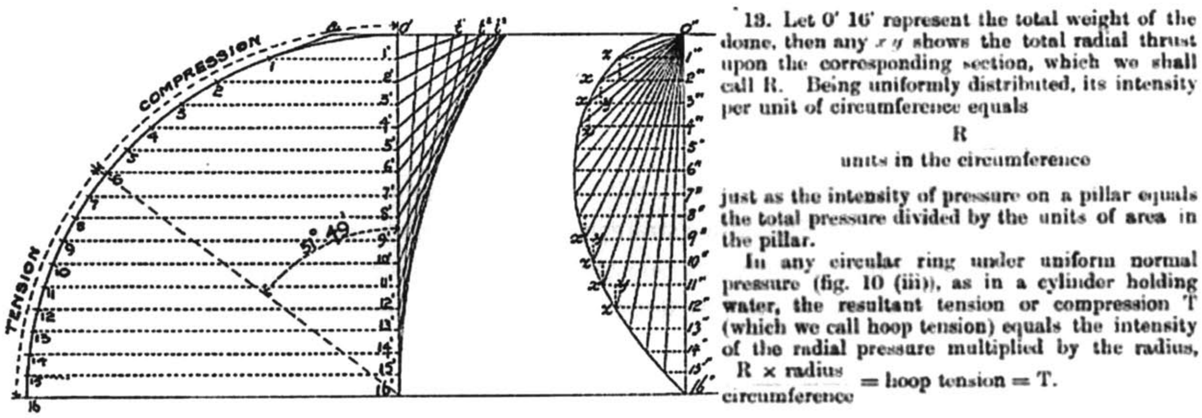
The application of graphic statics is referenced in Lau and Zessin's theses. Lau provides a step by step explanation of Eddy's and Wolfe's methods in her thesis and on an interactive MIT website [Lau, 2006].

3.2.2 Timeline of Graphic Statics for Arches and Domes

In 1866, Schwedler first introduces membrane theory for spatial frameworks. He proved that membrane forces act on the meridians and latitudes of a dome [Kurrer, 2008]. As Lau points out, by limiting the thrust line to the middle of the dome, Schwedler underestimates the ability of the dome to find a stable solution.

In 1878, Eddy publishes a graphical method to calculate the tensile and compressive hoop forces in a dome. He identifies the transition of compressive to tensile hoop forces at 51° 49'. Eddy models the dome as a series of arches and constrains the thrust line to the middle-third of a dome to ensure only compressive solutions. With this constraint, he doesn't explore the possibility of the dome to develop tensile hoop forces to add to the structural stability of the dome. Eddy limits the greatest horizontal thrust to the compressive region in the dome. This method is conservative because it models the top of a dome in the compressive hoop force region as a cap supported by separate arches [Lau, 2006].

In 1881, Föppl applies Eddy's theory to masonry domes. In 1904, Dunn reintroduces Eddy's graphic procedure in North America for domes in a paper, "Notes on the Stresses in Framed Spires and Domes" [Kurrer, 2008]. To find the hoop tension, Dunn states to take the radial thrust for each section of the dome, multiply it by the radius and divide it by the circumference, as shown in Figure 3.1 below.



19. Let 0′ 16′ represent the total weight of the dome, then any xy shows the total radial thrust upon the corresponding section, which we shall call R. Being uniformly distributed, its intensity per unit of circumference equals

$$\frac{R}{\text{units in the circumference}}$$

just as the intensity of pressure on a pillar equals the total pressure divided by the units of area in the pillar.

In any circular ring under uniform normal pressure (fig. 10 (iii)), as in a cylinder holding water, the resultant tension or compression T (which we call hoop tension) equals the intensity of the radial pressure multiplied by the radius,

$$\frac{R \times \text{radius}}{\text{circumference}} = \text{hoop tension} = T.$$

Figure 3.1: Dome Stresses and Hoop Tension Calculation [Dunn, 1904]

In 1921, Wolfe introduced a new method based on the membrane theory of domes, developed from Schwedler's theory. The major difference between Eddy's method and Wolfe's method is that Wolfe proves that tensile hoop forces add to the stability of masonry domes. Wolfe models the dome as a series of lunes, instead of a series of arches like in Eddy's model. He allows the tensile region of the dome to contribute to the maximum horizontal thrust. As Lau (2006) states, "What was significant about Wolfe's approach was its development of a zero-hoop force thrust line path that deviated from the median surface thrust line when tensile strength in the masonry is required." Wolfe's graphic method provides a calculation for the tensile hoop forces in the dome.

3.2.3 Conclusions

In the early 1900s when Guastavino Jr.'s Company was designing the domes, the available literature in English was Eddy's method, which Dunn republished specifically for masonry in 1904. Guastavino Jr.'s Company designed domes using the principles of Eddy's method and developed their own design methodology that was similar to Wolfe's method. Guastavino Jr.'s calculation drawings prove that he founded his own analysis method in 1909 before Wolfe's method was published in 1921. The theoretical Guastavino design process is presented in Chapter 7.

3.3 Guastavino Sr. Design History

3.3.1 Education

Guastavino Sr.'s design techniques likely came from his education (1861-1872) at the *Escola Especial de Mestres d'Obres* in Barcelona where he was taught by the best professors of the time, Juan Torras (1827-1910) and Elias Rogent (1821-1897) [Ochsendorf, 2010]. Antoni Gaudi (1852-1926), known for his graphic statics use in Barcelona, also graduated from this school in 1878 and was taught by the same professors. Even though Gaudi used graphic statics, it was not in the curriculum when Gaudi attended school [Nonell, 2001]. Since Guastavino Sr. attended the school before Gaudi, he would likely not have been exposed to the analysis method.

3.3.2 Methods

Once in America, Guastavino Sr. introduced thin masonry vaults on structures like the Boston Public Library and the City Hall Subway Station in New York. Guastavino Sr. describes his design methods in his book [Guastavino, 1893]. Guastavino Sr. presents formulas for calculating the general form of his arches. He wrote that to calculate the thickness of the arch, he uses a formula given by Dejardin. Dejardin wrote a book in 1845 based on the La Hire (1695) equilibrium analysis [Huerta, 2003]. Dejardin formulas were used as a rule of thumb to design arches and arch bridges in the late 19th century [Ceraldi, 2010]. Guastavino Sr. follows Dejardin formulas to determine the thickness of an arch under a distributed load and self-weight. For domes, Huerta (2003) explains that Guastavino Sr. made many inaccurate approximations. He approximated the area of a sphere as a half cylinder with the same radius and then assumed that since the weight of the dome is half of that of a barrel vault, the thrust would be half. Guastavino Sr. was aware that his own design strategies were great approximations that may be inaccurate. He writes,

> We must repeat here that we do not pretend to have an absolutely mathematical formula, but one practical enough to give sufficient security for safe construction. We are here also considering the dome as not one of voussoirs, but as a simple cast dome working as a single piece. [Guastavino Sr., 1893]

3.3.3 Application

Guastavino Sr. applied this design method for the Boston Public Library. On one drawing, Figure 3.2 and Figure 3.3, he explains his computations for a vault and claims that he has four times the amount of material necessary to handle the load.

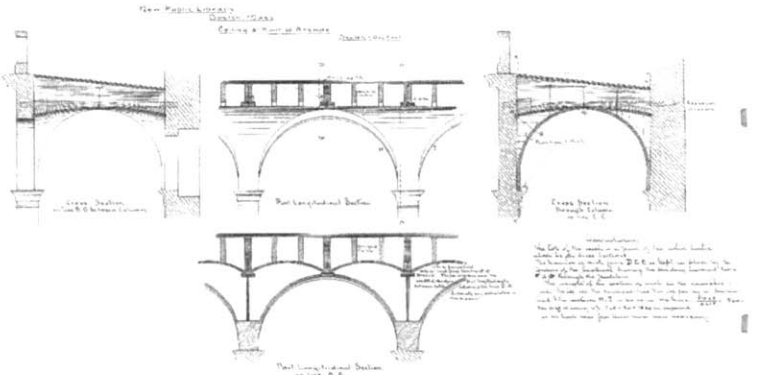
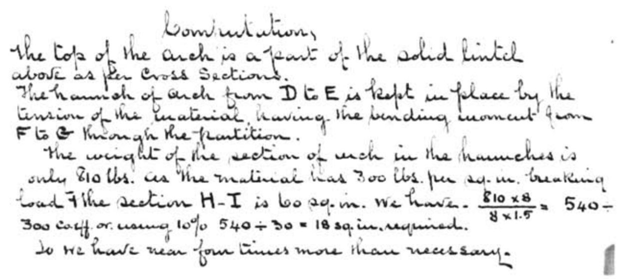


Figure 3.2: Guastavino Sr. Boston Public Library Drawing, Boston, MA, 1889 (Appendix B) [Avery Library]



Computations: The top of the arch is a part of the solid lintel above as per cross sections. The haunch of arch from D to E is kept in place by the tension of the material having the bending moment from F to G through the partition. The weight of the section of arch in the haunches is only 810 lbs, as the material has 300 lbs. per sq. in. breaking load the section H-I is 60 sq. in. We have $\frac{810x8}{8*1.5} = 540 \div 300$ clf or using 10% 540÷30 = 18 sq. in. required. So we have near four times more than necessary.

Figure 3.3: Guastavino Sr. Boston Public Library Calculation

A series of letters from 1895 to 1904 between Rafael Guastavino Sr. and Cass Gilbert on the Minnesota State Capital Project reveal Guastavino Sr.'s design mentality and his view of other dome constructions.

> … masonry construction requires perfect repose and rigidity and even settlements to avoid complications [,] and to enterprise a dome of masonry as you intent [sic] there is never enough precaution. Please bear in mind that nearly all the domes built in the world are more or less in constant repairation [sic] and I am against the use of St. Peter dome imitations on that account and please excuse my observation. [Parks, 2012]

The letters expose some of the calculations that Guastavino Sr. was making on the project. "He [Guastavino] assumes the lantern will weigh 120-130 tons and the domes below it are calculated to carry 172 tons "allowing for wind pressure"" [Parks, 2012]. Guastavino Sr. was somehow calculating the wind pressure on his structure in 1896. This is the only mention of wind pressure on a Guastavino design until 1934 for the Museum of Natural History when Guastavino Jr. is leading the company [Avery Library]. Letters from 1900 prove that Guastavino Sr. is calculating the amount of iron he needed at the base of a dome using the formulas and principles detailed in his book. He claims that the "Thrust of main dome at base or level of iron ring [is] 290,000 lbs" [Parks, 2012]. Not many records exist for Guastavino Sr. calculations. The letters prove that some structural calculations were performed. However, the person behind the calculations is questionable whether it was Guastavino Sr. or Jr. since they were both involved in the correspondence.

Guastavino Sr. is very critical of the construction techniques of the domes in Europe.

> "He argued that engineers' calculations, being based on old formulae, were not reliable, as these formulae had been found defective in practice, and that the domes in Europe, upon which they were based, were always defective in construction. He constantly urged the necessity of careful and accurate workmanship." [Parks, 2012]

Guastavino Sr. relies heavily on his personal builder's knowledge to understand how the dome worked. He states that the builder himself and proper brick laying is vital to the performance of the dome.

Guastavino Sr. understood that there were tensile forces in domes. "The material of a dome is not only working by compression, but in consequence of its form it is also working by tension, because the thrust depends upon the form and not on the material" [Guastavino, 1893]. Even though he just stated that there are tensile forces in the dome, a few pages later he writes "... if we build the ceilings in the form of domes, and if they are well applied and properly built, we have, practically, no thrust whatever" [Guastavino, 1893]. Guastavino Sr. inaccurately states that if the domes are built with the right material and workmanship, the dome will not experience thrust. Regardless, Guastavino Sr. still placed a significant amount of metal in his structures. As Huerta (2003) writes, "... there is a clear contradiction between both manners of thinking and the ensuing 'schizophrenia' is manifested in Guastavino [Sr.]'s writing and speaking, but not in the constructed work, which is the best proof of Guastavino [Sr.]'s mastery."

It is uncertain if Guastavino Sr. placed metal arbitrarily or based on the transition of compressive forces to tensile forces on a dome. From the photographs of a kiln on the Guastavino Sr. property at Black Mountain, NC, Figure 3.4, Guastavino Sr. most likely did not place the metal based on tensile thrusts. It is placed at the top of the dome in a ring and then at the base before the opening to the dome. There is no more information on the dimensions of the dome or metal to determine the exact location of the metal. The Grace Universalist Church dome in Lowell, MA proves that the Company understood the location of tensile hoop forces, as explained in Chapter 4.

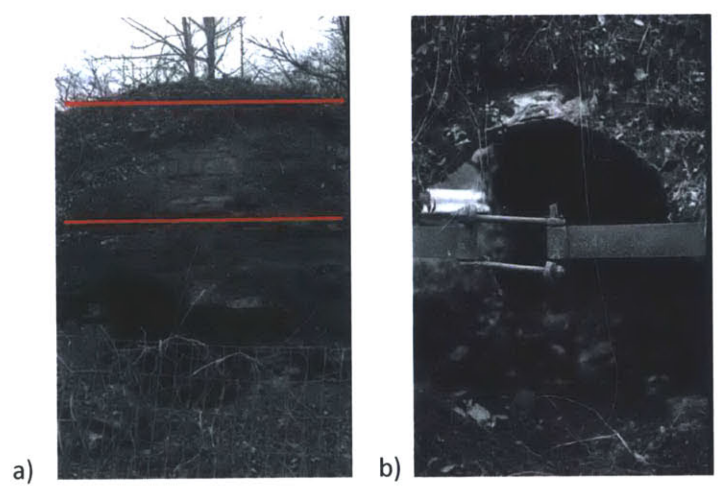


Figure 3.4: Guastavino Sr.'s. Black Mountain Estate – a) Kiln Dome (red lines represent metal placement) b) Enlarged Image of Metal Connection [Johnson, 1970's]

3.3.4 Conclusion

Guastavino Sr. follows his own calculation methods to determine horizontal thrusts and metal quantities. He was critical of other engineers that used "old formulae". It is unknown how Guastavino Sr. was estimating wind pressure on the Minnesota State Capital or how accurate the approximations for the main dome thrusts were. It would be interesting to study the Minnesota State Capital Dome and determine the thrust using a Graphical Analysis.

Guastavino Jr. does not seem to follow Guastavino Sr.'s design methods in the majority of his projects. The Dejardin formulas Guastavino Sr. is using only show up once in the drawings found in the Guastavino Archives at Avery Library. This drawing is discussed further in the following section.

3.4 Guastavino Jr. Design History

3.4.1 Introduction

Guastavino Jr. did not have any formal engineering training. He started to work as an apprentice for his father at the age of fifteen in 1887 [Ochsendorf, 2010]. Guastavino Sr. most likely did not know graphic statics, as explained in the previous section. There are three main questions behind the use of graphic statics after Guastavino Sr. passed away:

- How did Guastavino Jr. learn graphic statics?
- Was Guastavino Jr. producing the diagrams and calculations himself?
- What graphic statics methods are used?

Since Guastavino Jr. did not go through a formal education, he learned on the job and from other engineers and architects he interacted with. The earliest Guastavino graphic statics drawing that exists is from February 28th, 1906 for the St. Columbus R. C. Church Vault (Appendix B) in Philadelphia PA with architect Henry Dagit. There is a R. Guastavino Company stamp on the drawing, however it is not signed. The next drawing is from December 4th, 1906 for the St. Paul's Chapel Dome in Columbia University (Appendix B) with Howells and Stokes. This is the only project where the files in the Avery Archives were under Howells and Stokes instead of with the other Guastavino drawings. Under the title of the drawing, it says Nelson Goodyear Consulting Engineer. This is also the only known stress diagram associated with Guastavino constructions that is signed.

3.4.2 Author Speculation

Nelson Goodyear (1872-1917) was an architecture student and chemist from a family of inventors. His cousin, William Henry Goodyear (1846-1923), was an architect and Professor at The Cooper Union for the Advancement of Science and Art in New York and the University of Chicago. William looked after Nelson and took him on trips to Italy to educate him on architecture. Since William was a professor, it is reasonable that William could have taught Nelson graphic statics. There is not much information available on Nelson Goodyear, however, he is cited by the Architectural League of New York as a draughtsman for Howells and Stokes [Architectural League of New York, 1899]. In his obituary in *The Architectural Record*, John Mead Howells (1868-1959), of Howells and Stokes, writes, "Goodyear was also a chemist, and when he gave up architecture he became probably the best known acetylene engineer in this country and the inventor of much of the apparatus for the application of this and other gases" [Howells, 1917].

Guastavino Jr. most likely learned graphic statics from Goodyear while working on the St. Paul Chapel project. Ochsendorf (2010) writes that Howells and Stokes hired Goodyear to determine the safety of adding a lantern to the top of the dome. Ochsendorf questions, if Guastavino Jr. knew graphic statics at the time, he would have completed the analysis himself.

The next Guastavino graphic statics drawings for a dome in the Avery Archives are the Girard Trust Building dome in 1907 and the St. Francis de Sales dome in 1908 (Appendix B). The drawings are not signed. However, after a handwriting analysis detailed in the Chapter 4, the authorship is potentially linked to Guastavino Jr.

3.4.3 Graphic Statics Knowledge in the Early 1900s

Graphic statics knowledge for arches and domes was still being developed in the 1900s. An explanation of the methods is given in Section 3.2 The graphic analysis that Goodyear conducts in 1906 follows Eddy's methodology. He assumes that the greatest horizontal thrust from the dome is at the end of the compressive region. With this limitation, Goodyear does not explore the possibility for tensile hoop forces to contribute to the stability of the structure. He follows Eddy's method and limits the thrust line to a compressive-only solution in the middle one third of the dome. The St. Paul's Chapel dome is two domes that are buttressed together. This system presents more thickness to enclose the thrust line.

In 1904, Dunn reintroduces Eddy's method in a publication in *The Architectural Journal of The Royal Institute of British Architects* [Dunn, 1904]. Since Guastavino Jr. did not have any formal education, it is unlikely that he would just learn from this article. If he was interested in graphic statics for domes before interacting with Goodyear, he potentially could have learned even before 1904 from previous literature published on Eddy's Method. After learning the basics from Goodyear, Guastavino Jr. most likely learned more about graphic statics from Dunn's 1904 publication, as mentioned by Huerta [2003].

Guastavino Jr. invented his own graphic analysis for domes stemming from Eddy's method and Goodyear's analysis. Guastavino Jr. considers the horizontal thrust from the entire dome. Eddy's more conservative method does not account for tensile hoop forces contributing to the domes stability. Calculation drawings, referred to as Rubric Diagrams (Chapter 6) in this thesis, were found in the Guastavino Archives. There is no date on these drawings, however, by observing the chronology of graphic statics methods in Guastavino drawings, the date is set between 1906 and 1907 (Chapter 4). The Rubric drawings, Figure 3.5, show Guastavino Jr. mimicking Eddy's method as Dunn republished it in 1904, Figure 3.6.

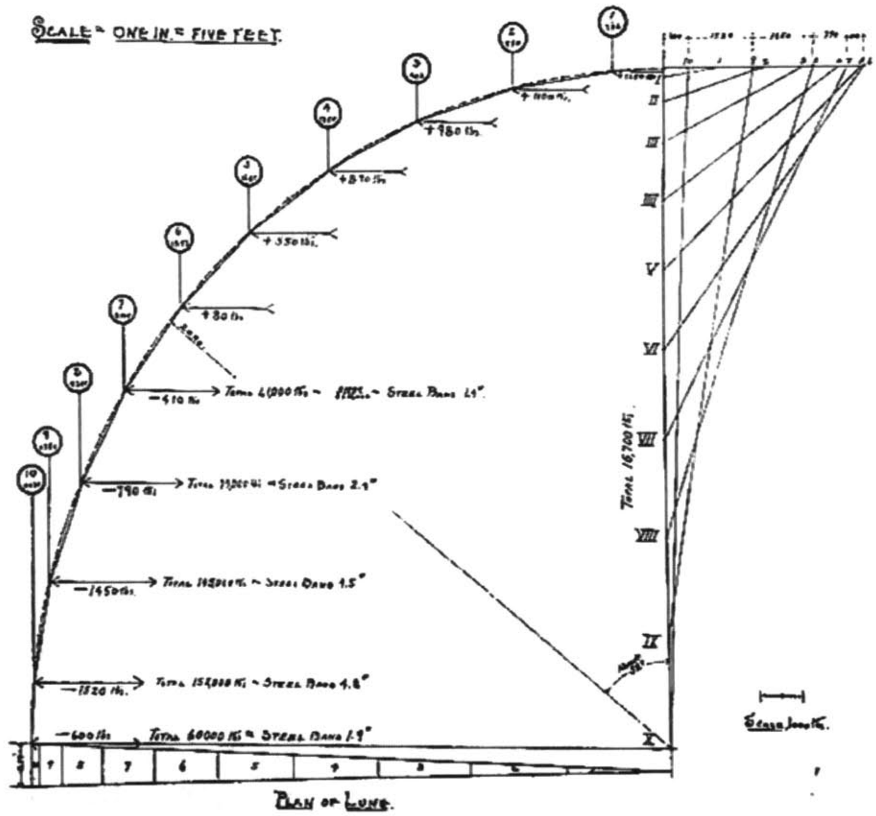


Figure 3.5: Rubric Drawing 1, 1906-1907? (Appendix B) [Avery Library]

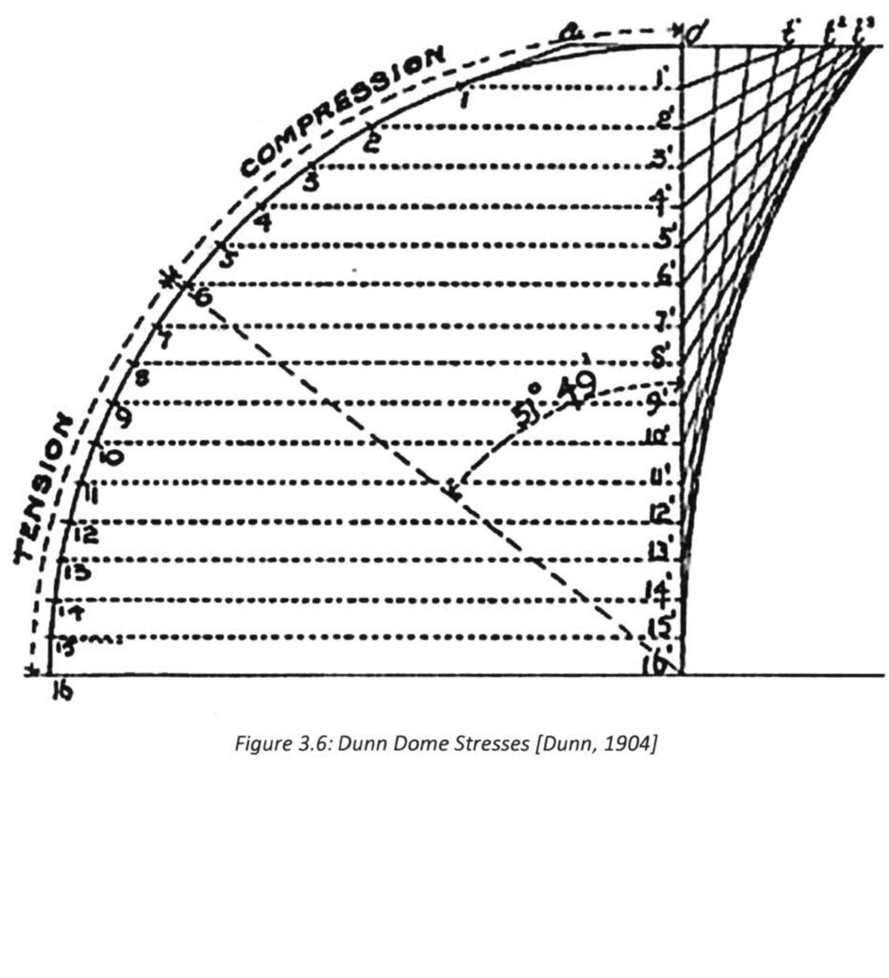


Figure 3.6: Dunn Dome Stresses [Dunn, 1904]

The following Rubric drawing, Figure 3.7, shows Guastavino Jr.'s innovation.

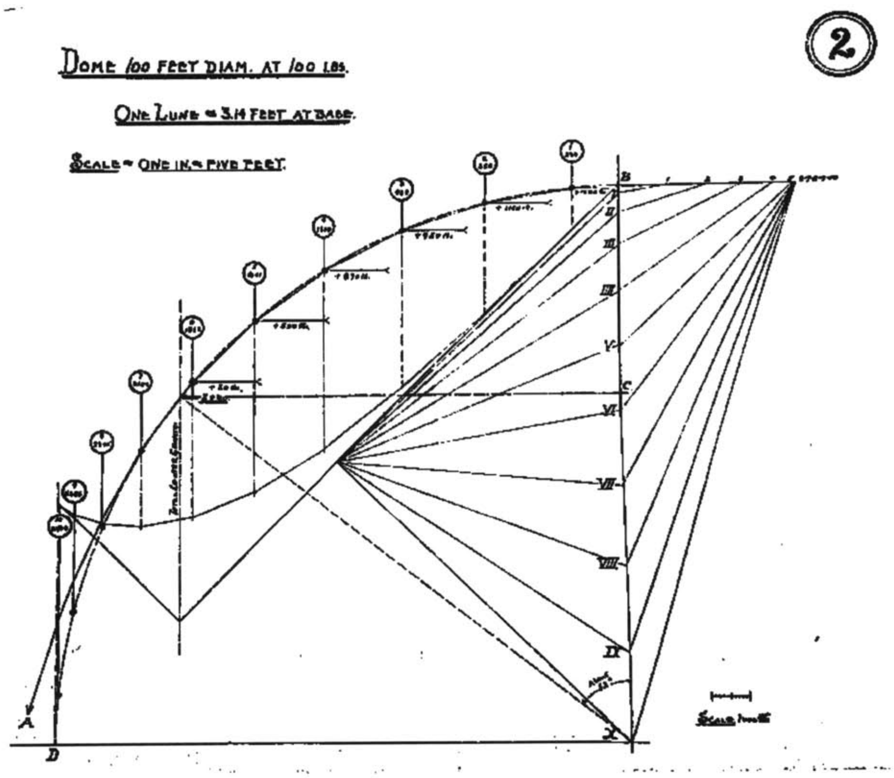


Figure 3.7: Rubric Drawing 3, 1906-1907? (Appendix B) [Avery Library]

In Figure 3.7, Guastavino Jr. assumes that the masonry cannot take any tension and he restricts the maximum horizontal thrust to the compressive region to find the amount the dome thrusts outwards. He finds the thrust line through the dome assuming the masonry cannot handle tension and determines the location in relation to the original curve. Figure 3.5 shows calculations for the amount of metal reinforcement in the tensile region of the dome. Guastavino Jr. is exploring the behavior of masonry domes in Figure 3.5 and Figure 3.7. These drawings are further analyzed in Chapter 6.

The Girard Trust Building (1907) and St. Francis de Sales (1908) (Appendix B) domes show a different design approach from Goodyear's analysis of St. Paul's Chapel. Both of the analyses do not limit the greatest horizontal thrust to the compressive region of the dome. They calculate the tensile thrust to size and strategically place metal reinforcement in the dome. The Girard Trust dome begins a buttress system at the tensile hoop force level. The St. Francis de Sales dome is the first example of a Guastavino dome to place metal starting at the level where tensile hoop forces begin. Chapter 4 expands on the designs of these domes. Goodyear may have introduced Guastavino Jr. to graphic statics, however Guastavino Jr. quickly learned from the method and transformed it. Tensile hoop force calculations are first seen on the drawing for the dome of the Cathedral of St. John the Divine, detailed in Chapter 7.

3.4.4 Conclusion

Guastavino Jr.'s design method assumes that the tensile hoop forces contribute to the stability of the dome. It is uncertain if other designers at the time were using the same method as Guastavino Jr. However, this idea and practice was not published until Wolfe's method in 1921. The Guastavino Company under Guastavino Jr. created some extremely complicated drawings and analyses.

3.5 Conclusions on Guastavino Design History

The application of graphic statics for domes was still being developed in the early 1900s when the Guastavinos were constructing masonry domes. The following conclusions are made on the Guastavino design techniques from analyzing original Guastavino Company letters, drawings, and documents:

- Dunn widely introduced graphic statics method for masonry domes in 1904 from Eddy's method. Wolfe's method for domes was not published until 1921.

- Guastavino Sr. uses "rule of thumb" design theories. He claims that his cohesive constructions do not need metal to restrain tensile thrusts, but he places a significant amount of metal in his structures anyway.

- Guastavino Jr. most likely learned graphic statics from Goodyear on the St. Paul's Chapel (1906) project and then probably learned more from Dunn's 1904 article.

- Graphic statics analysis methods are seen in Guastavino Company drawings after Guastavino Sr.'s death in 1908.

- The calculations in 1909 consider tensile hoop forces in the dome and size metal using these forces. This method is not published until Wolfe 1921.

4 Comparison of Existing Drawings

4.1 Objective

A wide scope of drawings ranging from 1895 to 1947 from the Avery Library Archives are considered. The drawings with calculations are used to compile a table comparing the date, architect, span, thickness, and design parameters of Guastavino Company structures.

The drawings are used to answer the following questions:

- How did the Guastavino Company calculations range for the graphic statics drawings available?
- What were the calculations for metal bands and how did they change over the years?
- Who was performing the calculations?
- Were design decisions made to simplify construction?

4.2 Chronology of Calculations

4.2.1 Sequence of Graphic Statics Applications

The Guastavino Company drawings from the early 1900s reveal their graphic statics methods. The first graphic statics drawing found in the Guastavino Archive collection is the St. Columbus R.C. Church arch (1906) (Appendix B). The first graphic statics drawing for a dome is the St. Paul's Chapel dome (1906) (Appendix B). This analysis was executed by Nelson Goodyear, a consulting engineer, who most likely introduced graphic statics dome analysis to the Guastavino Company.

In less than five years, the Guastavino Company experimented with a few different design theories for domes, chronicled in Table 4.1. The dates given are from the drawings and represent when the drawing was finalized, not necessarily when it was designed.

The drawings are expanded upon in the Section 4.2.2. Each dome drawing lends more insight to understand the Guastavino Company's knowledge of graphic statics analyses for domes. The reasoning behind the author of the drawings is validated in Section 4.3. The author is referred to as the Guastavino Company if the author is still uncertain after the handwriting analysis.

Table 4.1: Chronology of Guastavino Company Graphic Statics Drawings

Calculation Method	Date	Title	Type	Author
1	2.28.1906	St. Columbus R.C. Church Stress Diagram	Arch	Guastavino Jr. (Presumed)
2	12.04.1906	St. Paul's Chapel	Dome	Nelson Goodyear
3	1906-1907?	Rubric Drawings	Dome	Guastavino Jr. (Presumed)
4	9.5.1907	Girard Trust Building	Dome	Guastavino Jr. (Presumed)
5	4.16.1908	St. Francis de Sales Church	Dome and Barrel Vault	Guastavino Jr. (Presumed)
6	1.18.1909	Cathedral of St. John the Divine	Dome and Arch	Guastavino Jr. (Presumed)

The method seen in the Cathedral of St. John the Divine shows the final establishment of Guastavino Jr.'s design process.

4.2.2 Graphic Statics Methods in Arch and Barrel Vault Drawings

Six original calculation drawings are compared to determine trends in the graphic statics methods for arches and barrel vaults. A barrel vault and arch analysis are grouped together since a section of a barrel vault is just an arc that continues for a long span. The following drawings presented are some of the graphic statics drawings found for the arches and barrel vaults in the Avery Archives. Their general geometry and thicknesses are noted, if possible.

St. Columbus Roman Catholic Church, Philadelphia, PA – February 28, 1906

This analysis for an arch is the first graphic statics drawing in the Avery Library Guastavino Archives, Figure 4.1. The question mark and three lines were found on the drawing. It seems that someone wanted to determine the curvature of the arch and found these three circles as approximate radii to construct the interior arch face. The three radii shown are not great approximations of the curvature. As seen in Chapter 5 for the St. Francis de Sales barrel vault, the radii specified are formed by tangential circles. Therefore the center of the circles would not be at the same point. The drawing does not detail any dimensions so it cannot be properly scaled.

The weight of the buttresses are incorporated in the thrust line analysis to find the thrust in the buttresses. The buttresses have metal detailed in them, which could be based on the thrust line calculations. The correlation between the thrust values and the metal placement were not studied in this thesis.

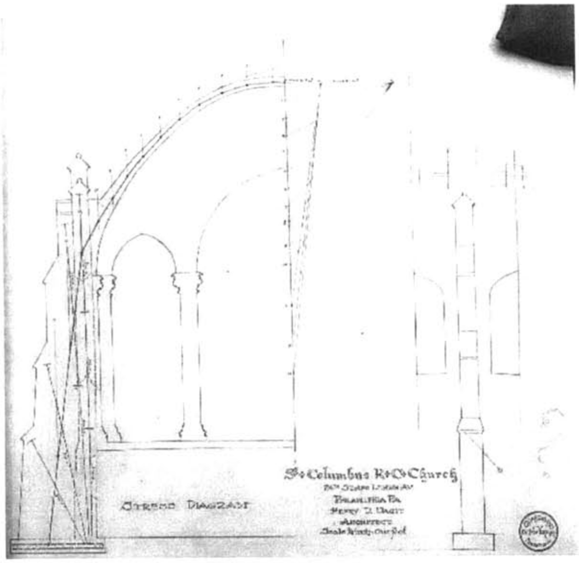


Figure 4.1: St. Columbus R.C. Church Arch, Philadelphia, PA 1906 (Appendix B) [Avery Library]

St. Francis de Sales Barrel Vault, Philadelphia, PA – April 18, 1908

The Guastavino Archives have two drawings for the barrel vault, a working drawing (Figure 4.2) and a final drawing (Figure 4.3). The working drawing shows the process behind the design. The correlation between the working and final drawings is presented in depth in Chapter 5. The span of the vault is 56 feet, the height is 22 feet, and the thickness is 5 inches. The vault is based on a circle of radius 32 feet. The working drawing shows the bottom of the vault defined by a single circle. In the final drawing, it is dimensioned using three tangential radii. The columns show metal in the final drawing, but there were no further calculations found.

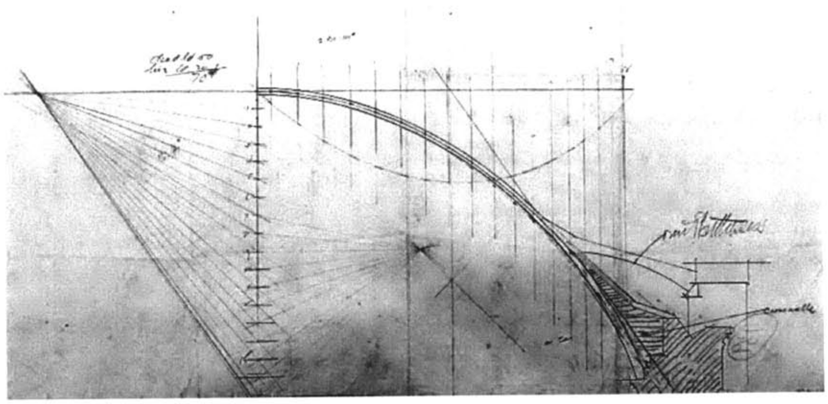


Figure 4.2: St. Francis de Sales Church Barrel Vault, Working Drawing, Philadelphia, PA 1908 (Appendix B) [Avery Library]

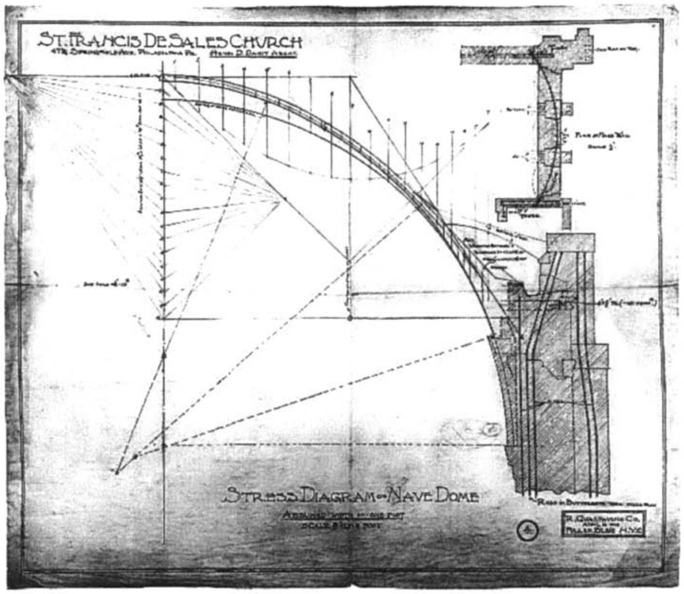


Figure 4.3: St. Francis de Sales Church Barrel Vault, Final Drawing (Appendix B) [Avery Library]

<u>Cathedral of St. John the Divine Nave Vault, NY, NY – 1909</u>

The Nave vault for the Cathedral drawing does not show any dimensions, as seen in Figure 4.4. A single circle that fits the arc precisely cannot be found. Three collinear circles are approximated to fit the curve. Two separate graphic statics analyses are shown on the drawing. The first one considers just dead load. The second analysis considers an asymmetric loading of live load and dead load only on the right half of the vault. This is the first drawing that shows asymmetric loading conditions. The nave vault was not constructed in the end. The nave was covered with metallic trusses instead [Huerta, 2003].

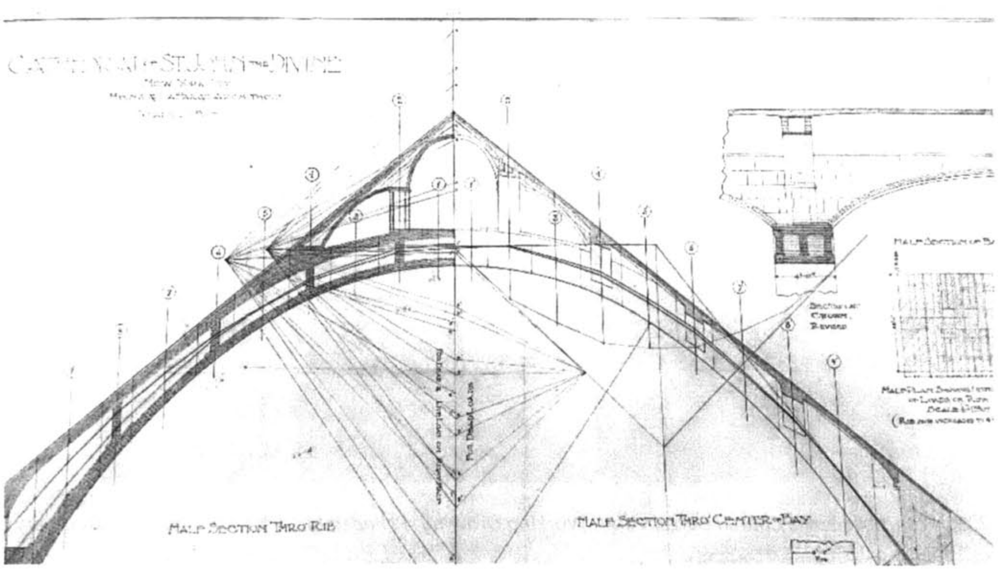


Figure 4.4: Cathedral of St. John the Divine Nave Vault, NY, NY 1909 (Appendix B) [Avery Library]

St. Patrick's Church, Philadelphia, PA – June 20, 1910

The church arch, Figure 4.5, has a span of 50 feet, a height of 10 feet, and an average thickness of 5 inches. The inside curvature is based on a single 50 foot radius circle.

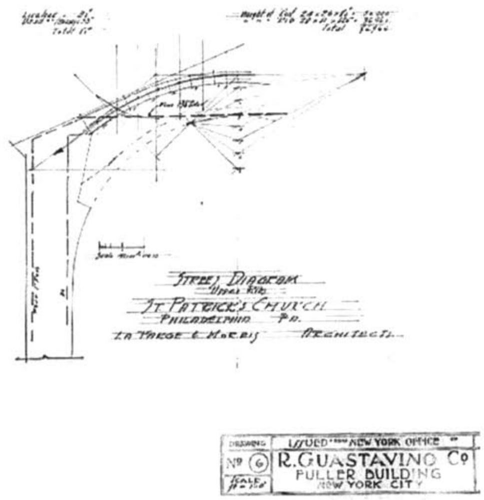


Figure 4.5: St. Patrick's Church Arch, Philadelphia, PA 1910 (Appendix B) [Avery Library]

Trinity College Chapel, Washington D.C. – April 14, 1921

The chapel arch, Figure 4.6, spans 38 feet, has a height of 17.5 feet, and a thickness of 12 inches. The radius is based on a single 20 foot circle. The arch is twice as thick as other arches due to a heavy roof on top of it. The thrust is extended through the column. The column was designed based on the thrust from the arch.

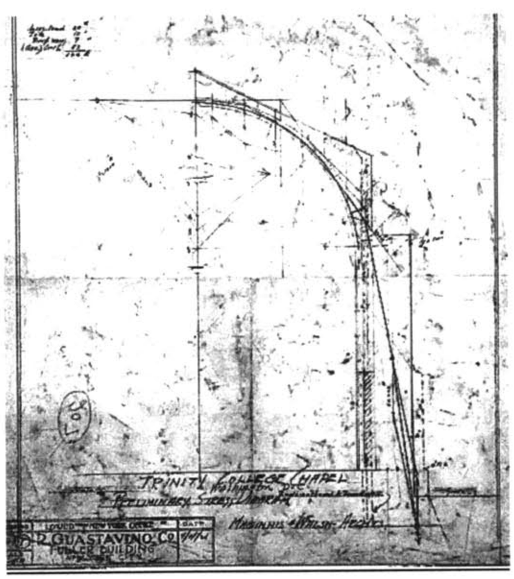


Figure 4.6: Trinity College Chapel Arch, Washington, D.C. 1921 (Appendix B) [Avery Library]

St. John's R.C. Church, Jersey City, NJ – July 14, 1931

The barrel vault, Figure 4.7, spans 38 feet, has a height of 11 feet and a thickness of approximately 6 inches. The vault is based on one circle with a radius of 19 feet.

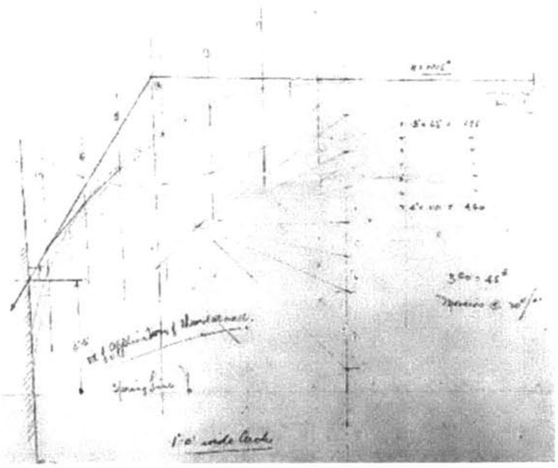


Figure 4.7: St. John's R.C. Church Barrel Vault, Jersey, City, NY 1931 (Appendix B) [Avery Library]

4.2.2.1 Barrel Vaults and Arches Summary

The barrel vaults and arches found in the Guastavino Avery archives are all designed with the same graphic statics techniques. They all begin with boundary conditions, a circular base curve, and an assumed brick thickness to find the funicular shape and center of gravity of the arch. Then the thrust is directed towards the abutments and a thrust line is found within the extents of the arch. The thrust location and brick thickness is iteratively altered until a solution is found. This design process is detailed for the St. Francis de Sales barrel vault in Chapter 5.

The main differences seen on the arch drawings are the detail in the buttress designs. In some drawings, the rebar is shown in the columns but not calculated on the drawing itself. There may have been other drawings showing that calculation that were destroyed, but this is not verified.

4.2.3 Graphic Statics Methods in Dome Drawings

Eleven original drawings for domes from the Avery Archives are compared. The graphic statics methods vary for domes. The engineer behind every drawing is uncertain. However, assumptions based on the handwritings are presented in Section 4.3. The following domes presented show the Guastavino Company development of graphic statics methods in the early 1900s.

Grace Universalist Church, Lowell, MA - 1895

This is the first and only drawing found that Guastavino Jr. signs his name (Figure 4.8). Guastavino Jr. designed and led the construction for this dome when he was only 23 [Ochsendorf, 2010]. The curvature of this dome follows a radius of 35 feet perfectly. The thickness varies from 6 inches at the base to 4 inches at the top. This dome shows Guastavino Jr.'s understanding of tensile hoop forces forming at around 52° since he ends the dome at this level and introduces a barrel vault to restrain the dome. He also increases the thickness of the dome at this level.

Guastavino Jr. demonstrates his understanding of the tensile and compressive stress boundary in a dome in 1895, well before Dunn publishes Eddy's method in 1904. Guastavino Jr. either found this relation in the dome on his own or he had previous knowledge from Eddy's method or other literature on domes.

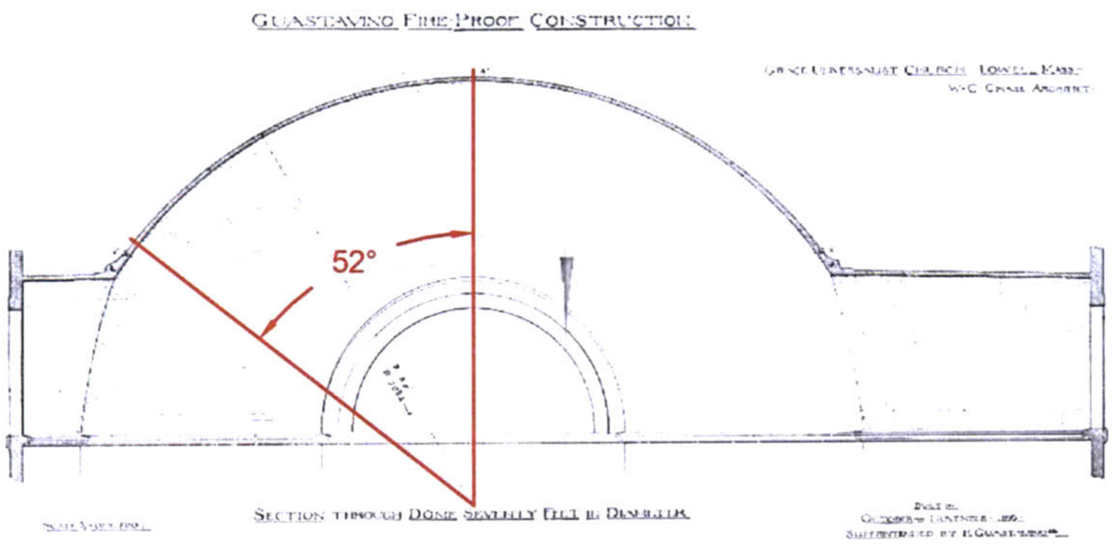


Figure 4.8: Grace Universalist Church Dome, Lowell, MA 1895 – Red Lines added by Author to indicate Compression to Tension Boundary on Dome (Appendix B) [Avery Library]

Bank of Montreal, Canada - December, 1903

The span of the dome (Figure 4.9) is 73 feet and the thickness of the outer dome is approximately 6 inches. There is no graphic statics analysis shown for this dome. However, there is a metal band at the base of the dome (Figure 4.10). This is the earliest drawings found that has a dome with a metal band detailed at the base.

The dome consists of an inner and outer shell. Both domes are based on circles and follow their curvature perfectly, as shown in Figure 4.9. There are stepping bricks built up along the side of the dome, starting at exactly 52° from the radius of the outer dome. The placement of these bricks emphasizes Guastavino Jr.'s understanding of tensile stress in a dome, before he was exposed to Goodyear's graphic statics analysis in 1906. Guastavino Jr. most likely created this drawing. The handwriting on the Bank of Montreal drawing matches the signed Grace Universalist Church drawing (Section 4.3). It is interesting that Guastavino Jr. writes a note at the base of the drawing in Spanish. Unfortunately, the text is blurred and could not be deciphered.

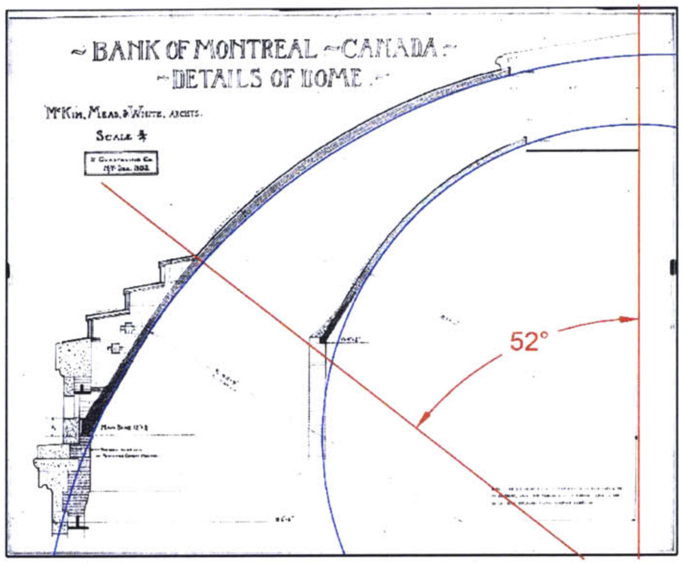


Figure 4.9: Bank of Montreal Dome, Canada 1903 – Red Lines added by Author to indicate Compression to Tension Boundary on Dome (Origin of lines is center of outside dome shell) (Appendix B) [Avery Library]

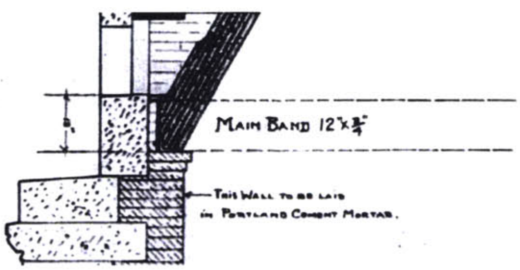


Figure 4.10: Bank of Montreal Dome, Canada 1903 (Appendix B) [Avery Library]

St. Paul's Chapel, Columbia University, NY, NY – December 4, 1906

Consulting Engineer: Nelson Goodyear

This drawing possibly introduced the Guastavino Company to graphic statics for domes. The method used in the analysis assumed the maximum horizontal thrust is from the compressive region. There don't seem to be any details of metal on the graphic statics drawing, as seen in Figure 4.11. The dome is actually a double dome connected with buttresses. The double dome is effective to contain the thrust. Wolfe's graphic method for domes (1921) to calculate hoop forces did not exist yet. The method shown in this drawing is based on Eddy's method (1877) to calculate the thrusts of the dome. Goodyear calculates the thrust line for the inner shell and outer shell of the dome. He also finds the funicular shape of the dome to locate the center of gravity of each shell. Goodyear does not appear to calculate any hoop forces in the analysis.

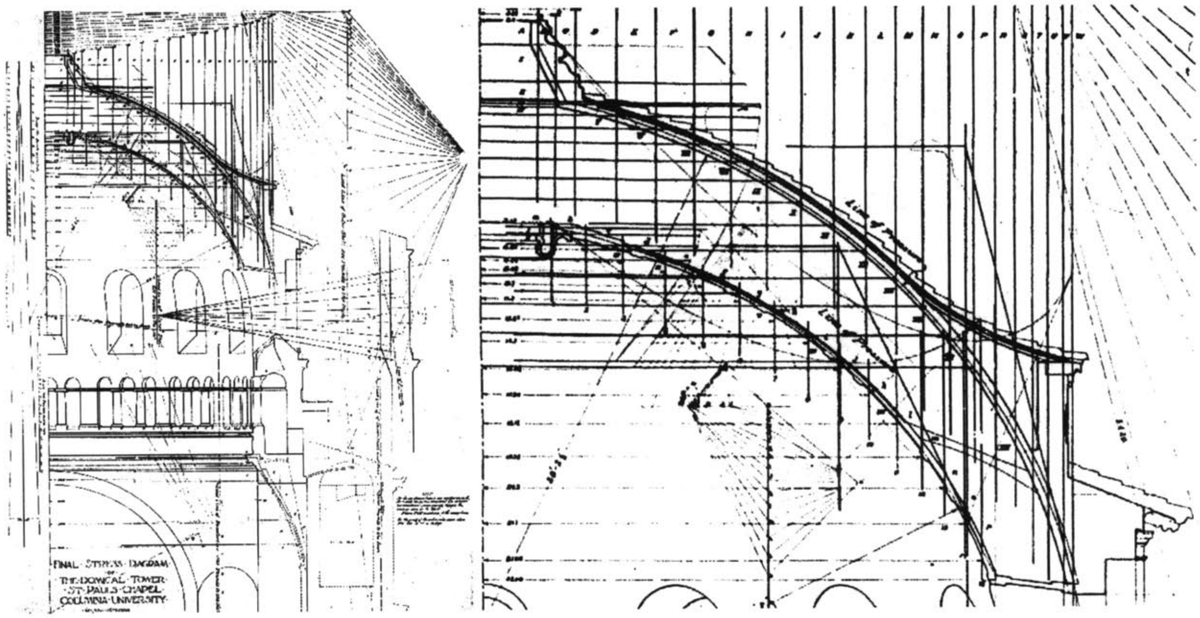


Figure 4.11: St. Paul's Chapel, Columbia University, NY, NY 1906 (Appendix B) [Avery Library]

<u>Rubric Drawing- Between 1906 and 1907</u>

Two drawings, referred to as rubric drawings in this thesis, show Guastavino Jr. exploring the change of compressive to tensile stresses in a dome, as seen in Figure 4.12. Rubric Drawing 3 shows the amount a 100 feet span dome thrusts outwards. The analysis finds the dome thrusts 2.5 feet at the base. With this analysis, Guastavino Jr. determines the amount a dome thrusts outwards if the dome cannot handle tension. From this finding, Guastavino Jr. strategically limits the thrust at the base of domes in future projects, detailed in Chapter 7.

Instead of using a double shell dome like the St. Paul's Chapel project, Guastavino Jr. contains the thrust by introducing metal reinforcement in a single dome instead.

Rubric Drawing 1 shows that the metal bands are calculated based on the horizontal thrust calculated from the force polygon. That thrust value is then multiplied by the total amount of lunes to get the full thrust for the entire dome. The accuracy of this metal calculation method and more analysis of these drawings are presented in the Chapter 6.

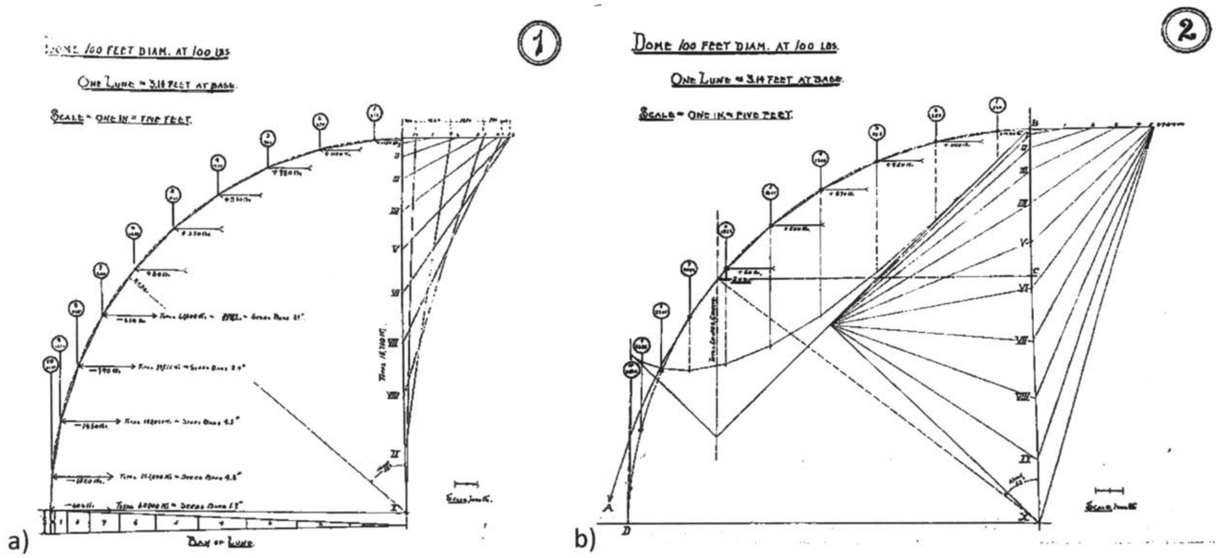


Figure 4.12: Guastavino Rubric Drawings a) Drawing 1 and b) Drawing 3, 1906-1907? (Appendix B) [Avery Library]

<u>Girard Trust Building Dome, Philadelphia, PA – September 5, 1907</u>

The span of the dome is 98 feet. The curvature of the dome is based on a circle with radius 49 feet. Guastavino Jr. limits the tensile thrust in the dome by ending the dome before the middle of the base circle. This drawing applies concepts from the rubric drawings analyzed in Chapter 6.

The force diagram shows the tensile forces in the dome were considered and calculated, unlike the St. Paul's Chapel analysis (Figure 4.11) where only a compressive solution was calculated. The force diagram also shows the measurement of tensile thrust at the base of the dome, 1000 lbs, between segments 9 and 10. Following the rubric drawings section analysis (Figure 4.12), that force is multiplied by the amount of lunes in a full dome, 72 in this case, and then used to size the tension band at the base of the dome. Using the same tensile stress for metal as the rubric drawing, 29,000 psi, the result is 2.5 inches. This calculation does not explain the thickness of the 6 inch metal band. As mentioned in Chapter 6, this calculation is blurry in the rubric drawings, so the value and origin of 29,000 is uncertain.

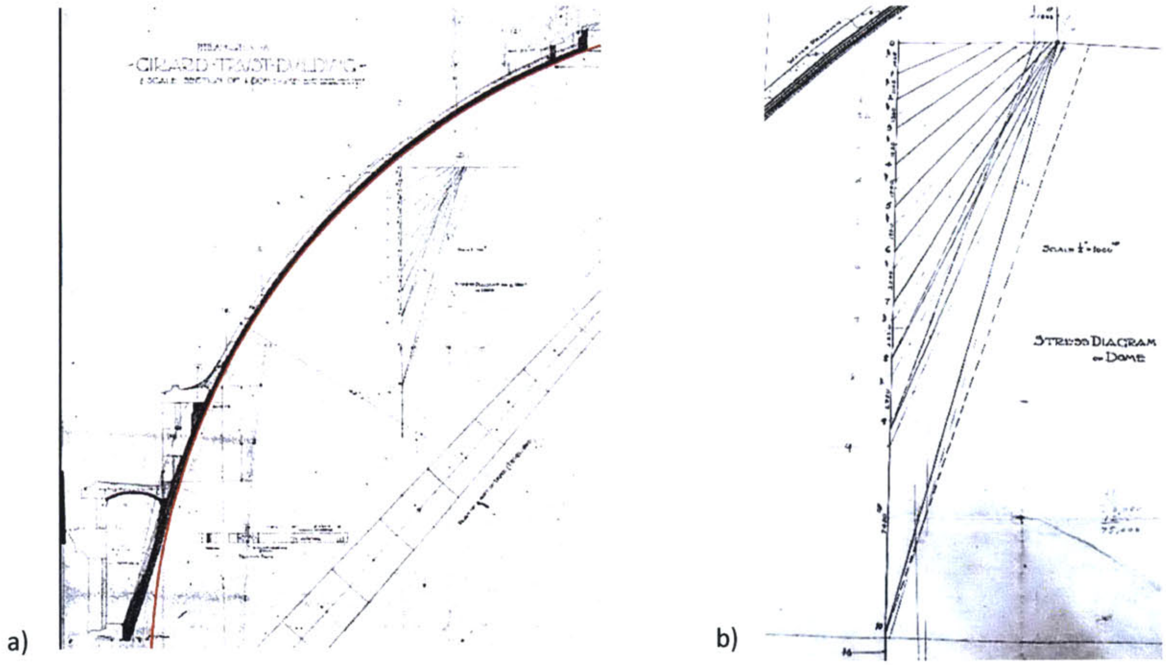


Figure 4.13: Girard Trust Dome – a) Constant 49' Radius Curvature – Red Lines added by Author for Circle Placement b) Stress Diagram Enlargement (Appendix B) [Avery Library]

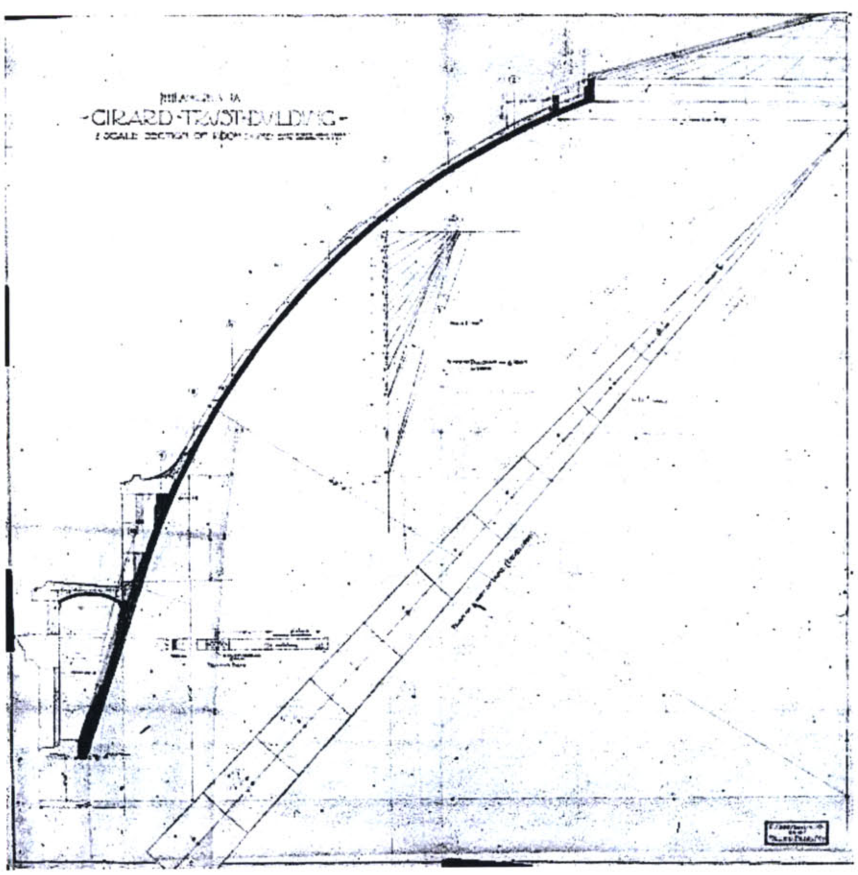


Figure 4.14: Girard Trust Building Dome, Philadelphia, PA 1907 (Appendix B) [Avery Library]

St. Francis de Sales Dome, Philadelphia, PA - April 16, 1908

This is the first Guastavino Company drawing that shows metal placed throughout the dome starting at 52°, and not just at the base as a ring (Figure 4.15). There is no information on the drawing to explain the calculation behind the metal rods in the dome. However, the drawing is cut off at the force diagram. If the lines are extended, it is apparent that Guastavino Jr. is calculating tensile forces in the dome. He is not limiting the maximum thrust to the compressive region of the dome. It is possible that he calculated the metal placement similarly to the Cathedral of St. John the Divine analysis with hoop forces. However, this cannot be verified with the materials available in the Avery Library Archives.

There is a calculation based on Guastavino Sr. Dejardin calculation techniques to show the total thrust from the dome. The metal placed in the dome and the calculation for the dome thrust is detailed in Section 4.3.4.

The curvature of the dome is precisely a circle, approximately 34'-0". The height of the dome is equal to the radius of the dome at the base, each approximately 31'-0". The curvature of the dome is similar to the thrust in the barrel vault for St. Francis de Sales. The relationship is detailed in Chapter 5. Tensile forces were not taken into account to alter the dome shape since it is a perfect circle curve. The Guastavino Company method to shape the dome based on tensile forces is expanded upon in Chapter 7. This drawing shows the transition from Guastavino Sr. design methods to the use of graphic statics in designs to place metal more accurately in domes.

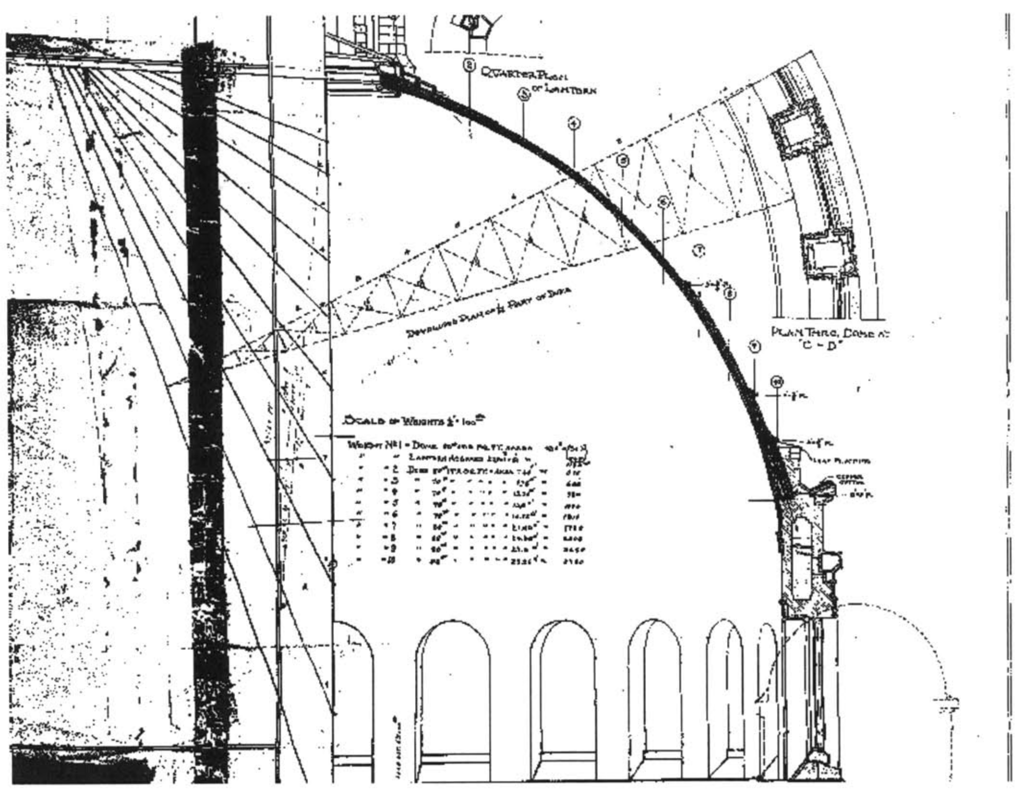


Figure 4.15: St. Francis de Sales Church Dome, Philadelphia, PA 1908 (Appendix B) [Avery Library]

Cathedral of St. John the Divine, NY, NY – January 18, 1909

The analysis for this dome spanning 132 feet is the most complicated and intricate drawing found in the Avery Archives, Figure 4.16. The structure and the drawing itself are a masterpieces. Hoop forces and meridional forces are calculated for the first time in this drawing. The dome is almost a perfect circle that deviates slightly at the base to enclose tensile hoop forces. This project shows the transition of the Guastavino Company from approximate calculations in previous domes to a completely accurate model. The handwriting in this drawing is traced to Guastavino Jr., as explained in Section 4.3. An extensive analysis of the drawing is presented in Chapter 7.

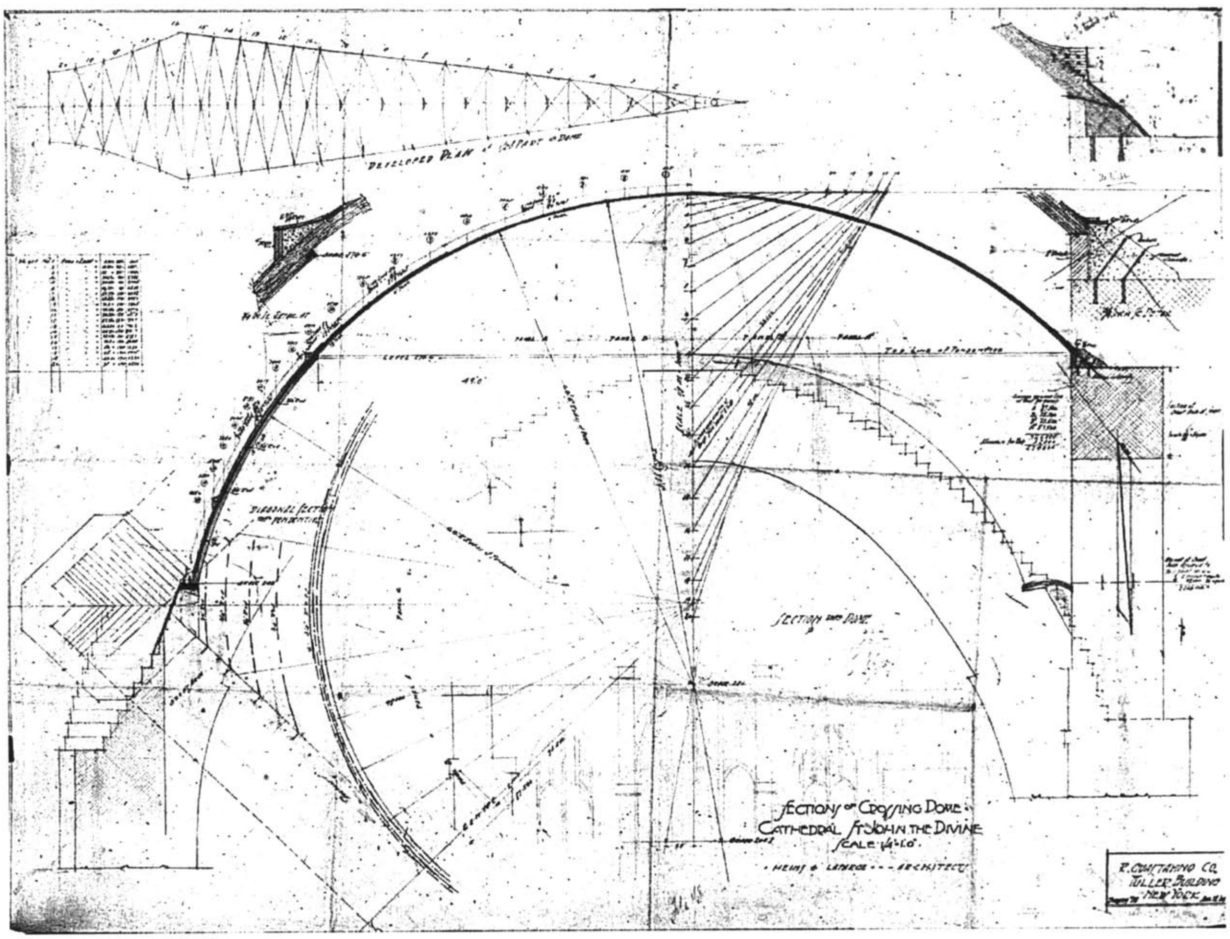


Figure 4.16: Cathedral of St. John the Divine Dome, NY, NY 1909 (Appendix B) [Avery Library]

After the Cathedral of St. John the Divine project, hoop force calculations are seen in all of the Guastavino Company structures.

Trinity Chapel Dome, Washington, D.C. – September 14, 1922

This dome, Figure 4.17, is based on a circle with a 20 foot radius, and the span is 36 feet and the height is 12 feet. There are proper tensile hoop force calculations shown to find the amount of metal necessary throughout the dome. The arch for this project, Section 4.2.2, uses the same base circle as the dome.

The tensile hoop forces begin much earlier than 52°. The weight from the addition of the lantern causes the dome to burst outwards. Eddy describes how to alter the tensile and compressive behavior in domes by adding a lantern or creating an opening to subtract weight from the top [Eddy, 1904].

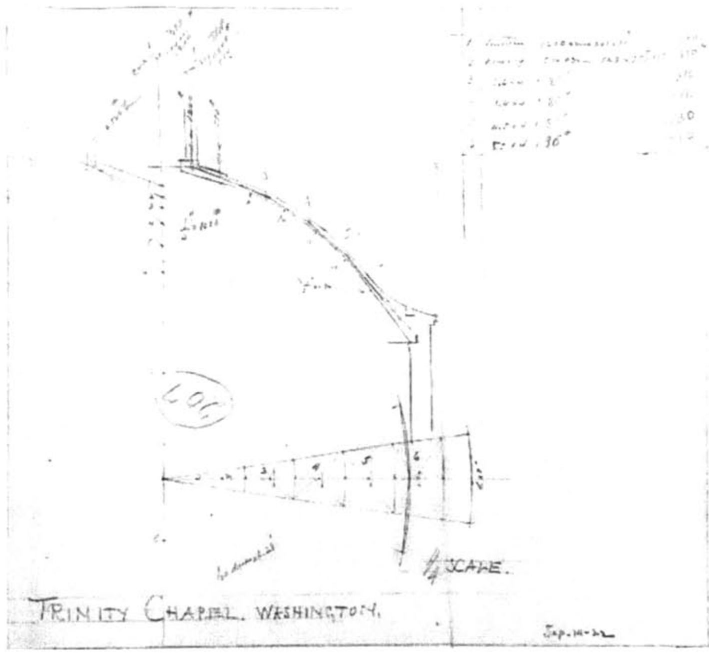


Figure 4.17: Trinity Church Chapel Dome, Washington D.C., 1922 (Appendix B) [Avery Library]

St. John's R.C. Church, Jersey City, NY – July 14, 1931

The dome dimensions seen in Figure 4.18 are similar to the Trinity Chapel Dome, Figure 4.17. It is based on a circle with a 19 foot radius. The span is 28 feet and the height is 11 feet. The barrel vault for this project uses the same base curvature as the dome.

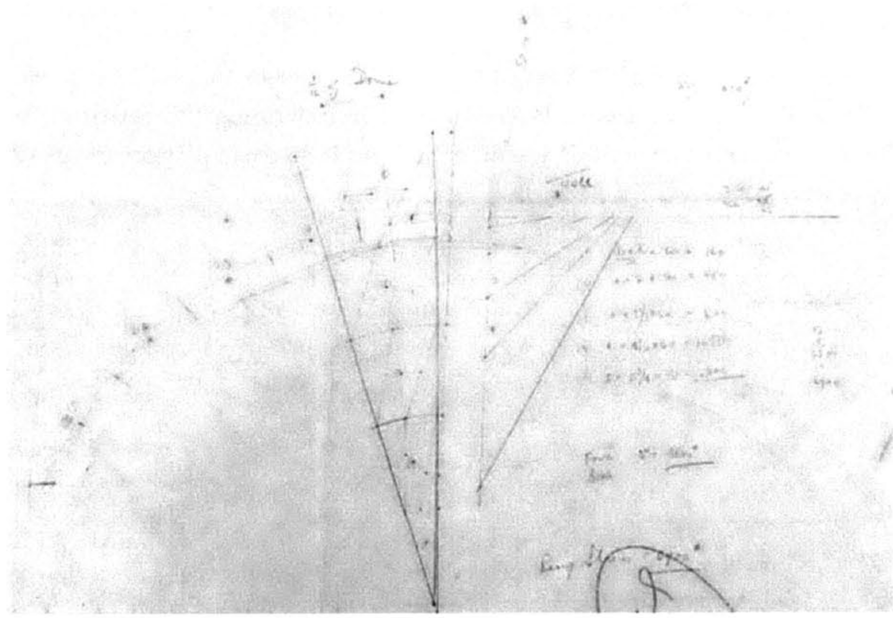


Figure 4.18: St. John's R.C. Church, Jersey City, NY, 1931 (Appendix B) [Avery Library]

Planetarium American Museum of National History, NY, NY – January 29, 1934

This drawing, Figure 4.19, shows the advancement of the Guastavino Company. The appearance of the drawing is more professional than the other drawings in the archives and all of the design equations are summarized on the drawing. The analysis considers dead load, roof load for the entire dome and for half the dome, and wind load for the near side and far side. There are numerous equations detailed for ring stress, meridian stress, wind stresses, deformations, materials, and more. The author of this drawing is unknown. The text is not a free-hand handwriting like the other Guastavino Company Drawings.

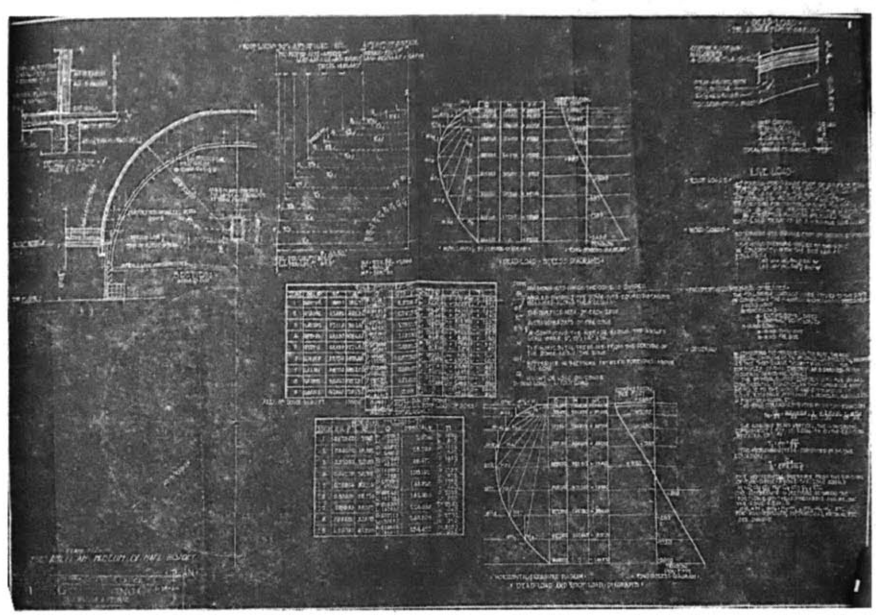


Figure 4.19: Planetarium American Museum of National History, NY, NY 1934 (Appendix B) [Avery Library]

Second Church Christ Scientist, Cleveland, Ohio - 1946

The drawing seen in Figure 4.20 shows calculations for tensile thrusts. The original dome was constructed in 1917 and there are drawings located in the Avery Library Archives that were not looked at in this thesis. The graphic statics analysis on the drawing from 1946 could be from the 1917 drawing.

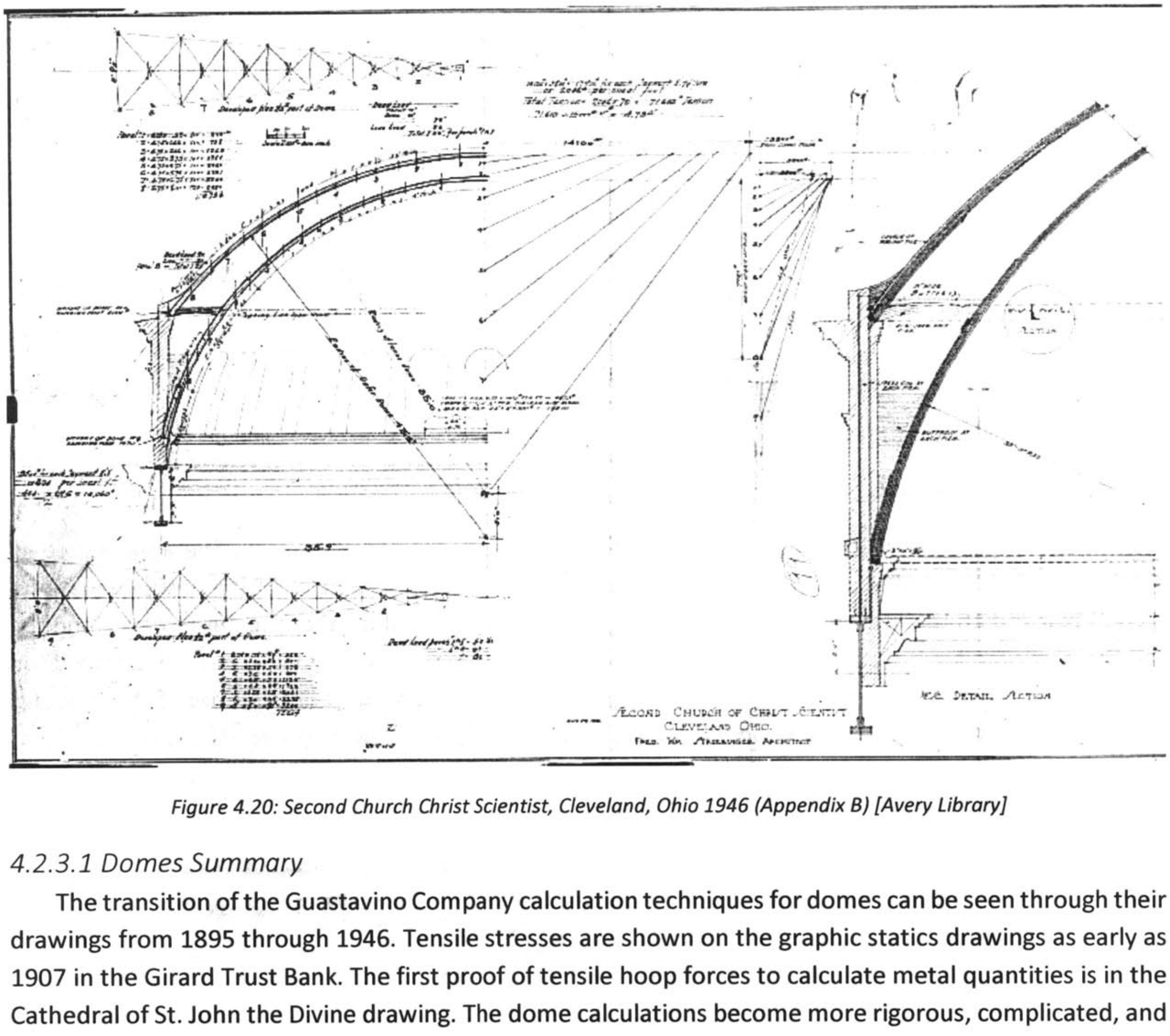


Figure 4.20: Second Church Christ Scientist, Cleveland, Ohio 1946 (Appendix B) [Avery Library]

4.2.3.1 Domes Summary

The transition of the Guastavino Company calculation techniques for domes can be seen through their drawings from 1895 through 1946. Tensile stresses are shown on the graphic statics drawings as early as 1907 in the Girard Trust Bank. The first proof of tensile hoop forces to calculate metal quantities is in the Cathedral of St. John the Divine drawing. The dome calculations become more rigorous, complicated, and detailed towards the 1930s.

4.2.4 Metal Calculations

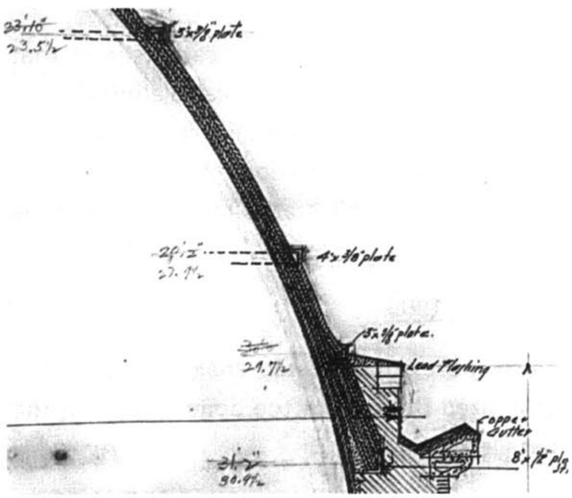
The Guastavino Company introduced a range of calculations for the steel or iron in their domes. The following examples summarize the most interesting projects that led to their final design methodology. This section introduces similarities for metal calculations on the Guastavino Company drawings. The calculations are not analyzed for each drawing.

4.2.4.1 Metal Placed Throughout Dome

St. Francis de Sales Dome, Philadelphia, PA - April 16, 1908

There is metal placed in the dome at around 53°, as seen in Figure 4.21 and detailed in Table 4.2. Since the drawings are photographed and then scaled, it is assumed that in actuality this metal is at 52°. This is the first Guastavino Company drawing that details metal placed at four levels in the dome, instead of just a base metal ring.

a)



b)

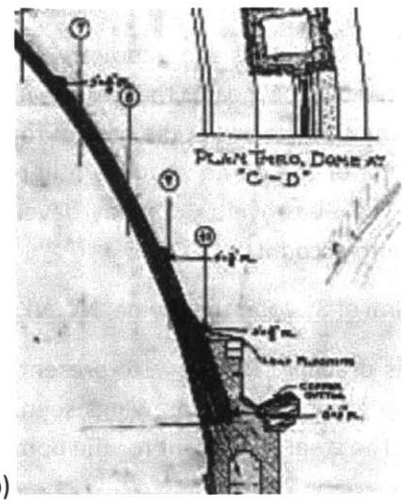


Figure 4.21: St. Francis de Sales Dome, 1908 - Metal Placement a) Working Drawing b) Final Drawing (Appendix B) [Avery Library]

Table 4.2: St. Francis de Sales Dome, 1908 - Metal Placement

Segment	**Location**	**Amount**
6 and 7	53.2°	3" x 3/8" Plate
8 and 9	66.7°	4" x 3/8" Plate
9 and 10	72.5°	3" x 3/8" Plate
Base	78.3°	8" x ½" Plate

The metal is placed based on the location of tensile forces in the dome. There is no reinforcement placed between segments 7 and 8. By reconstructing the polygon, Guastavino Jr. may have found that there is no tensile thrust in that region and does not place any metal there. There are no hoop force calculations shown on this drawing, however, it's very possible he followed the same methodology as on the Cathedral of St. John the Divine to design the dome.

There is a note on the drawing, Figure 4.22, which leads us to believe that Guastavino Jr. used Guastavino Sr. Dejardin techniques to determine the thrust on beams. This drawing was completed in April 1908 and Guastavino Sr. passed away in February 1908. It is possible that Guastavino Sr. influenced the design of this dome. Guastavino Sr. did not personally write this since this handwriting is seen on many drawings past 1908.

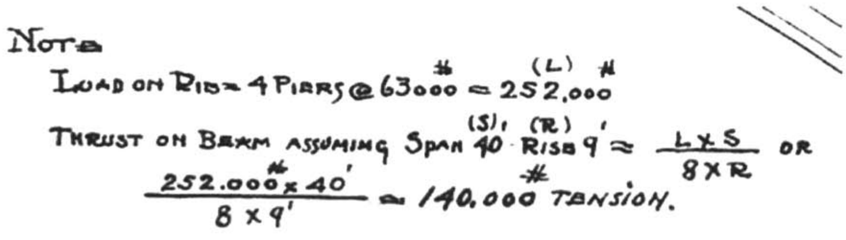


Figure 4.22: St. Francis de Sales Church, 1908 - Thrust Calculation (Appendix B) [Avery Library]

Milkovich details the geometry of the dome in her thesis. Her research shows that the dome is supported by steel beams that rest on four arches [Milkovich, 1992]. The calculation above would be used to size the steel beams at the base of the dome. The use of steel beams in a dome construction to contain the thrust of the dome is unlike other Guastavino dome constructions. This drawing was planned out before the Guastavino Company developed their new graphic statics method that takes tensile hoop forces into account.

Cathedral of St. John the Divine, NY, NY – January 18, 1909

This drawing, Figure 4.16, presents a much more sophisticated analysis compared to any of the previous graphic statics drawings seen. Steel is sized throughout the dome based on the tensile hoop forces. The steel placement for the dome is detailed in Chapter 7.

Trinity Chapel Dome, Washington, D.C. – September 14, 1922

The calculations for this project are sloppy in comparison to previous analyses on drawings, as seen in Figure 4.23. The author of this drawing is most likely not Guastavino Jr. since the handwriting is very different. Hoop forces are used to calculate the metal reinforcement. The tensile hoop force is 2,500 lbs. A tensile metal strength of 52,000 lbs, a value seen in other Guastavino constructions for metal reinforcing, yields a circular metal rod with diameter 0.25 inches. The rods placed have a diameter of 0.5 inches, applying a safety factor of 2. He only finds the hoop forces for one of the segments. The 1/2" rods are a conservative placement for the dome.

The metal rods are placed very close to the top of the dome, far before 52°. The graphic statics analysis shows that there are tensile hoop forces forming at that level. As explained in the dome section above, the addition of the lantern creates these forces.

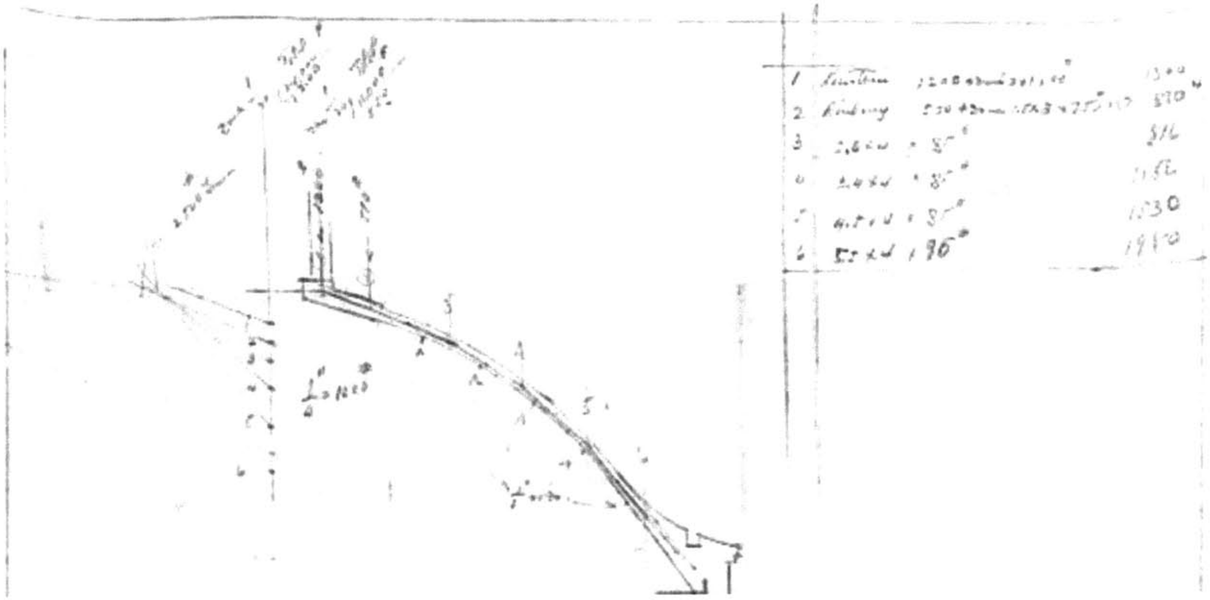


Figure 4.23: Trinity Chapel Dome, Washington, D.C., 1922- Metal Placement (Appendix B) [Avery Library]

St. Boniface R.C. Church, Pittsburgh, PA – June, 3 1926

Metal rods are dimensioned throughout the base of the dome, as seen in Figure 4.24.

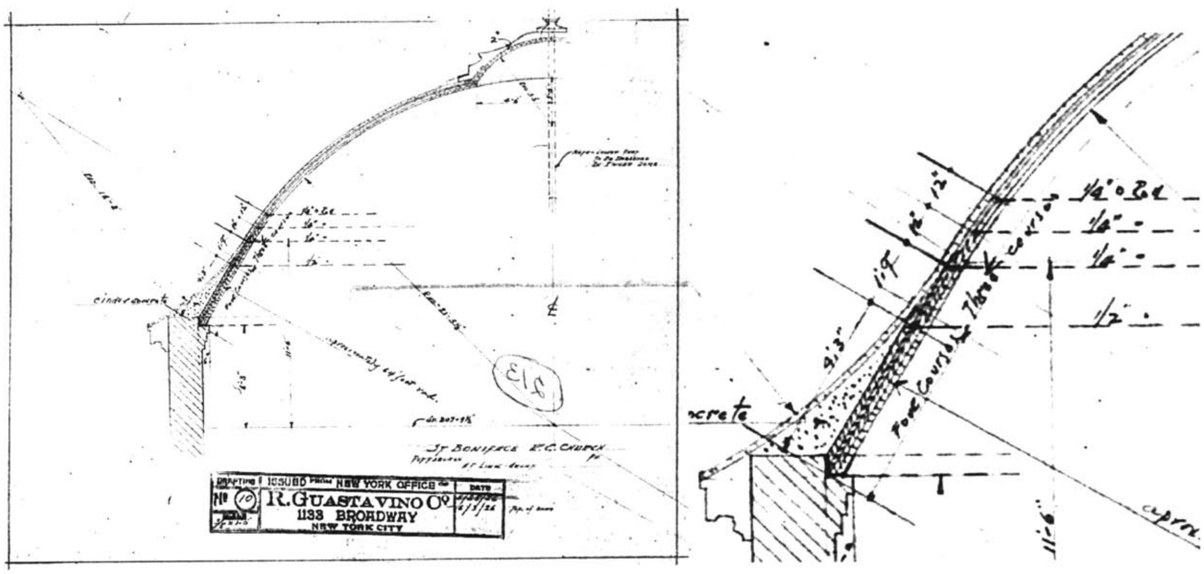


Figure 4.24: St. Boniface R.C. Church, Pittsburgh, PA, 1926 - Metal Placement (Appendix B) [Avery Library]

4.2.4.2 Metal Ring Tension Bands

The first projects with a metal band are the Bank of Montreal in 1903 (Figure 4.25) and the Girard Trust Building (Figure 4.26) in 1907. After this the placement of metal throughout the dome is more popular in Guastavino Company domes. Then in the 30s and 40s, there are a few projects that show ring tension calculations. In the later projects, the ring tension is calculated as a hoop force from the beginning of the dome to the base of the dome. This ring tension is most likely used to size the tension band. All of these drawings have a single tension band at the base of the dome.

Bank of Montreal, Canada - December, 1903

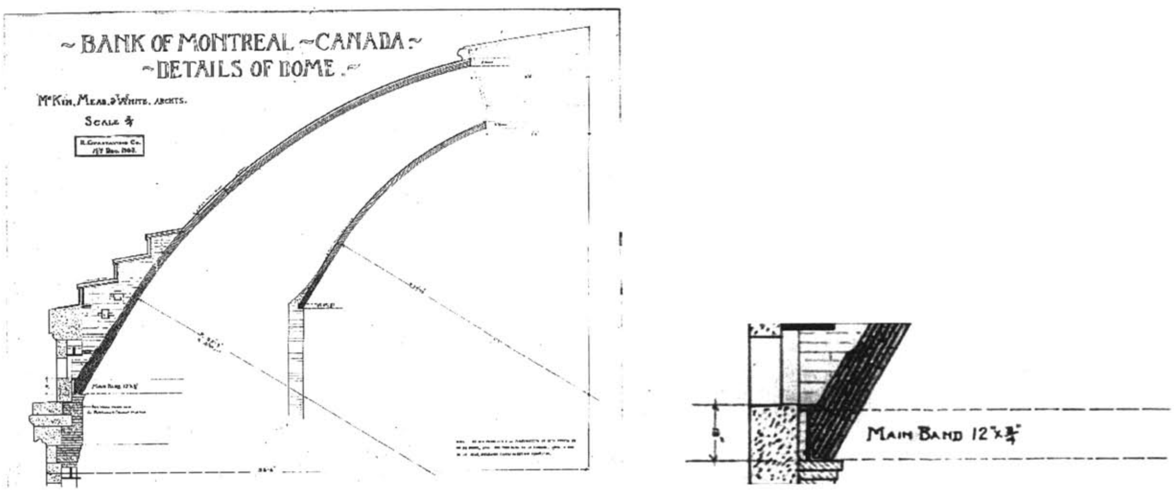


Figure 4.25: Bank of Montreal, 1903 - Metal Main Band (Appendix B) [Avery Library]

Girard Trust Building Dome, Philadelphia, PA – September 5, 1907

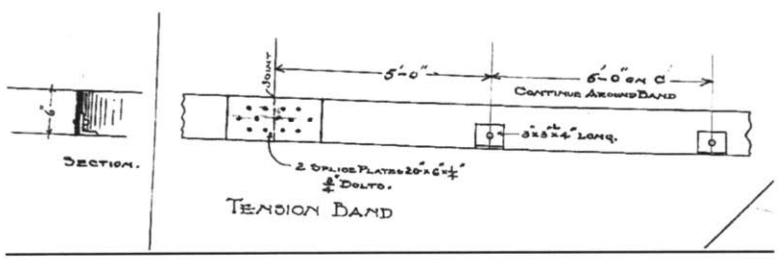


Figure 4.26: Girard Trust Building Dome, 1907 - Metal Main Band (Appendix B) [Avery Library]

Dime Savings Bank, Brooklyn, NY – April 1, 1931

The drawing in Figure 4.27 states “$Max\ Ring\ Stress\ = \frac{88k}{16} = 5.5"\ steel$” The origin of these values is not further analyzed in this thesis.

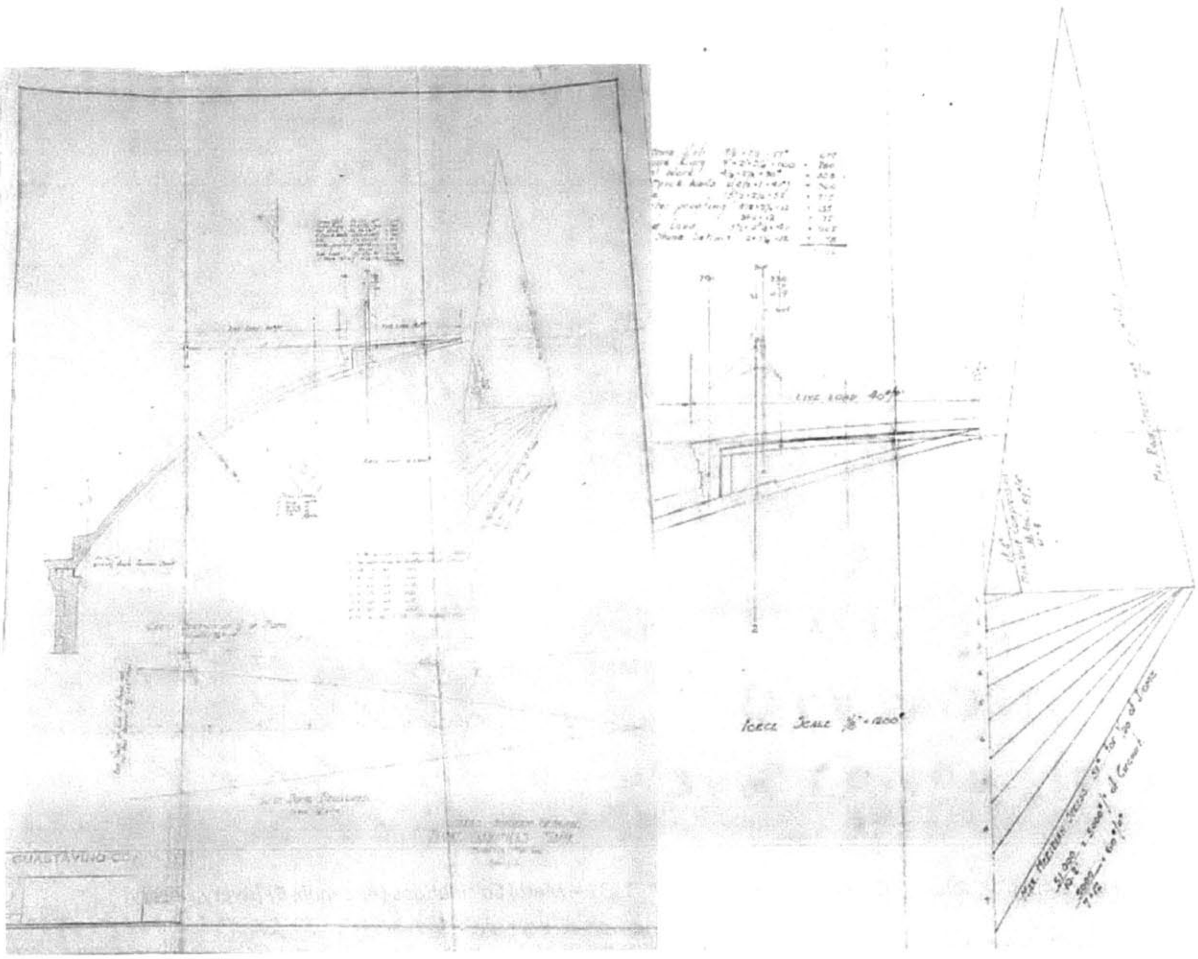


Figure 4.27: Dime Savings Bank, 1931 - Main Metal Band (Appendix B) [Avery Library]

St. John's R.C. Church, Jersey City, NY – July 14, 1931

There are hoop force calculations shown in Figure 4.28. The ring stress is 6900 lbs. Figure 4.29 shows that 6-5/8" bars were calculated and the values are related to the hoop forces on the dome.

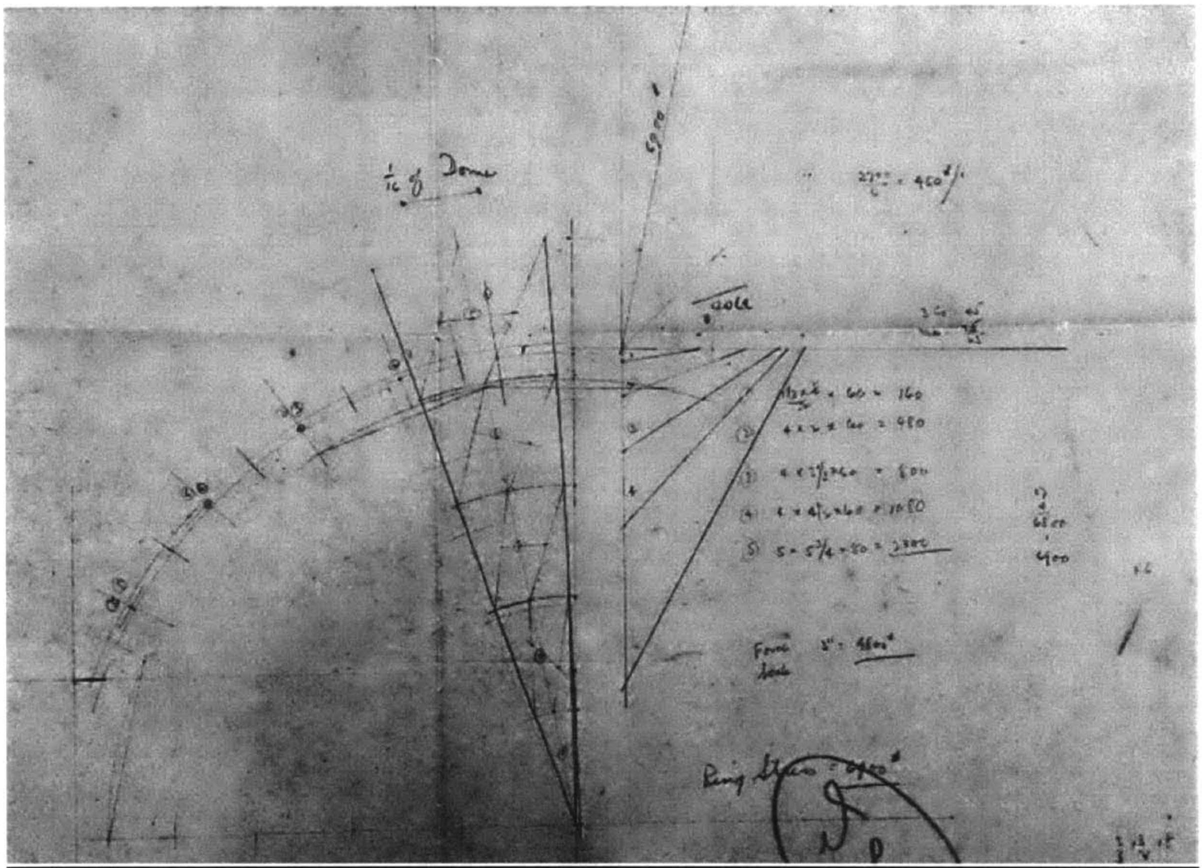


Figure 4.28: St. John's R.C. Church, Jersey City, NY - 1931 – Metal Calculations (Appendix B) [Avery Library]

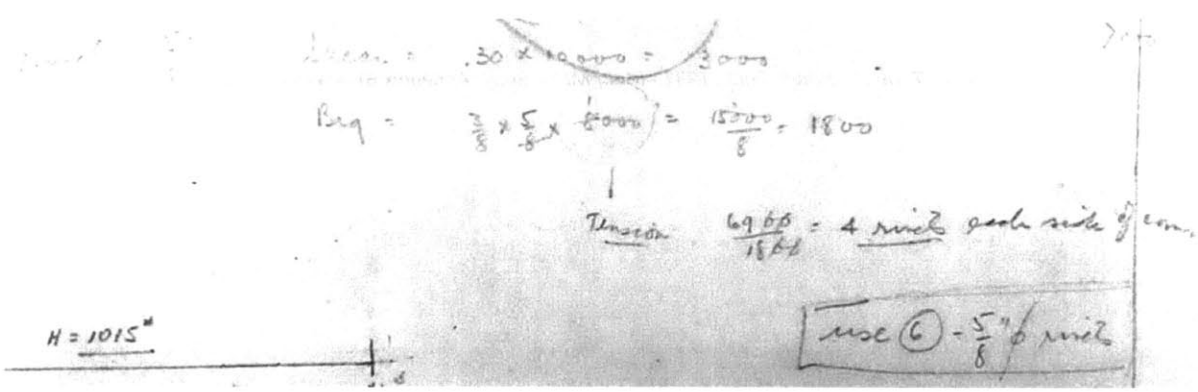


Figure 4.29: St. John's R.C. Church, Jersey City, NY - 1931 - Hoop Force Calculations (Appendix B) [Avery Library]

Supreme Court Building, Tallahassee, Florida – 1947

The drawing, Figure 4.30 and Figure 4.31, shows a ring tension of 24,200 lbs.

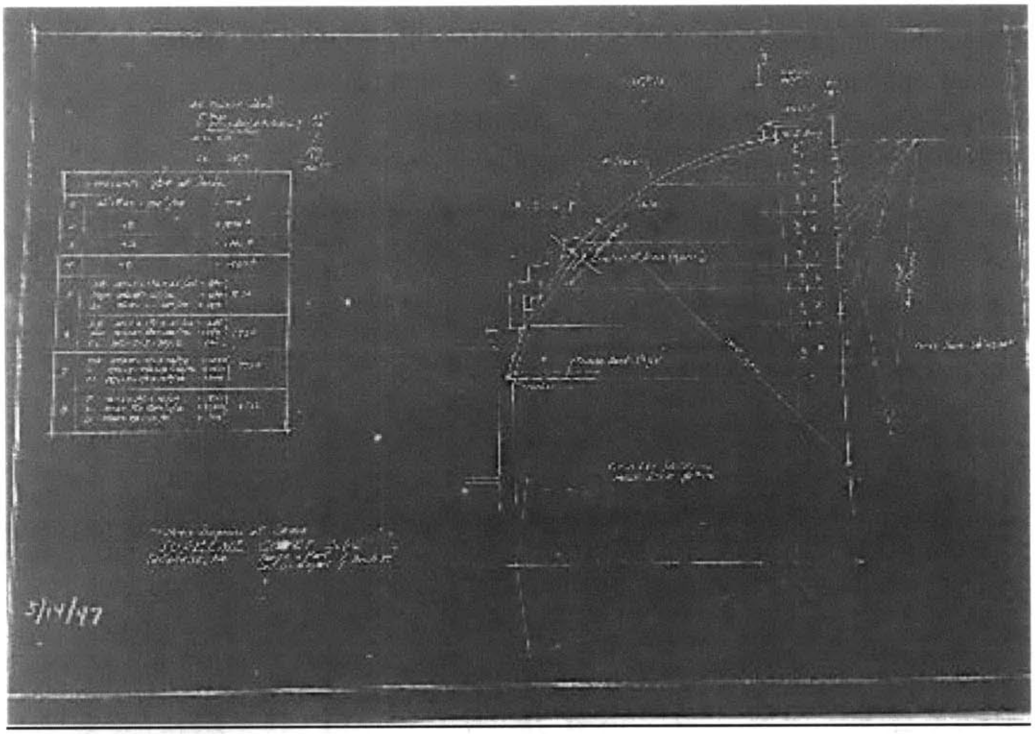


Figure 4.30: Supreme Court Building, Tallahassee, Florida - 1947, Metal Reinforcement Calculations

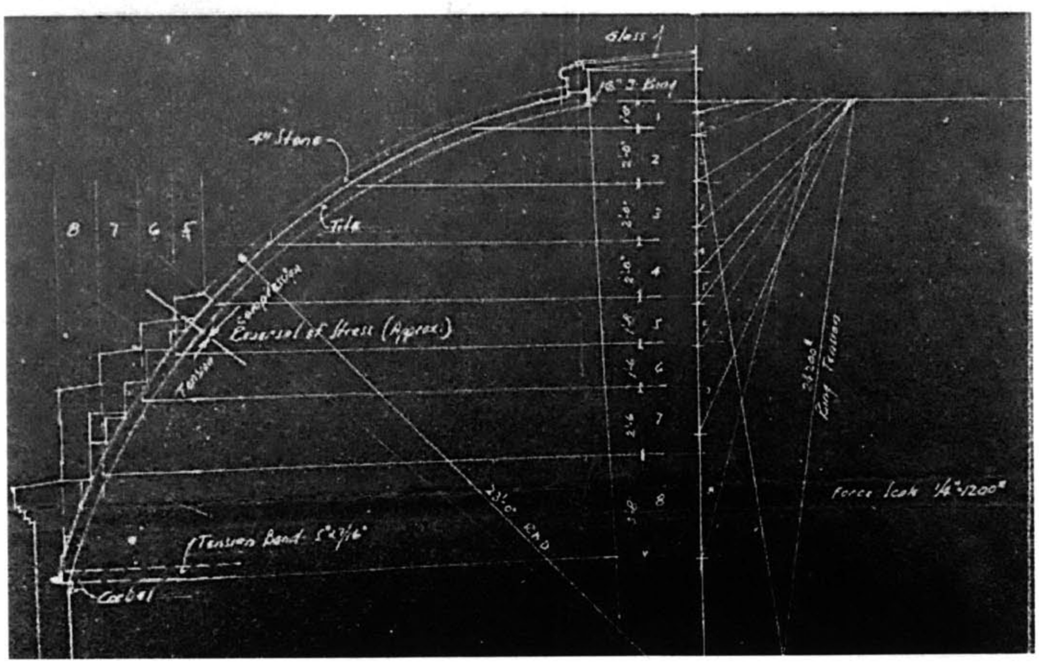


Figure 4.31: Supreme Court Building, Tallahassee, Florida - 1947, Ring Tension Calculation

4.2.4.3 Metal Reinforcement in Columns

The following drawings, Figure 4.32, show placement or calculations for metal in columns. It is uncertain if the metal is steel or iron.

- St. Columbus Roman Catholic Church, Philadelphia, PA – February 28, 1906
- St. Francis de Sales Barrel Vault, Philadelphia, PA – April 18, 1908
- Trinity College Chapel, Washington D.C. – April 14, 1921

a) b) c)

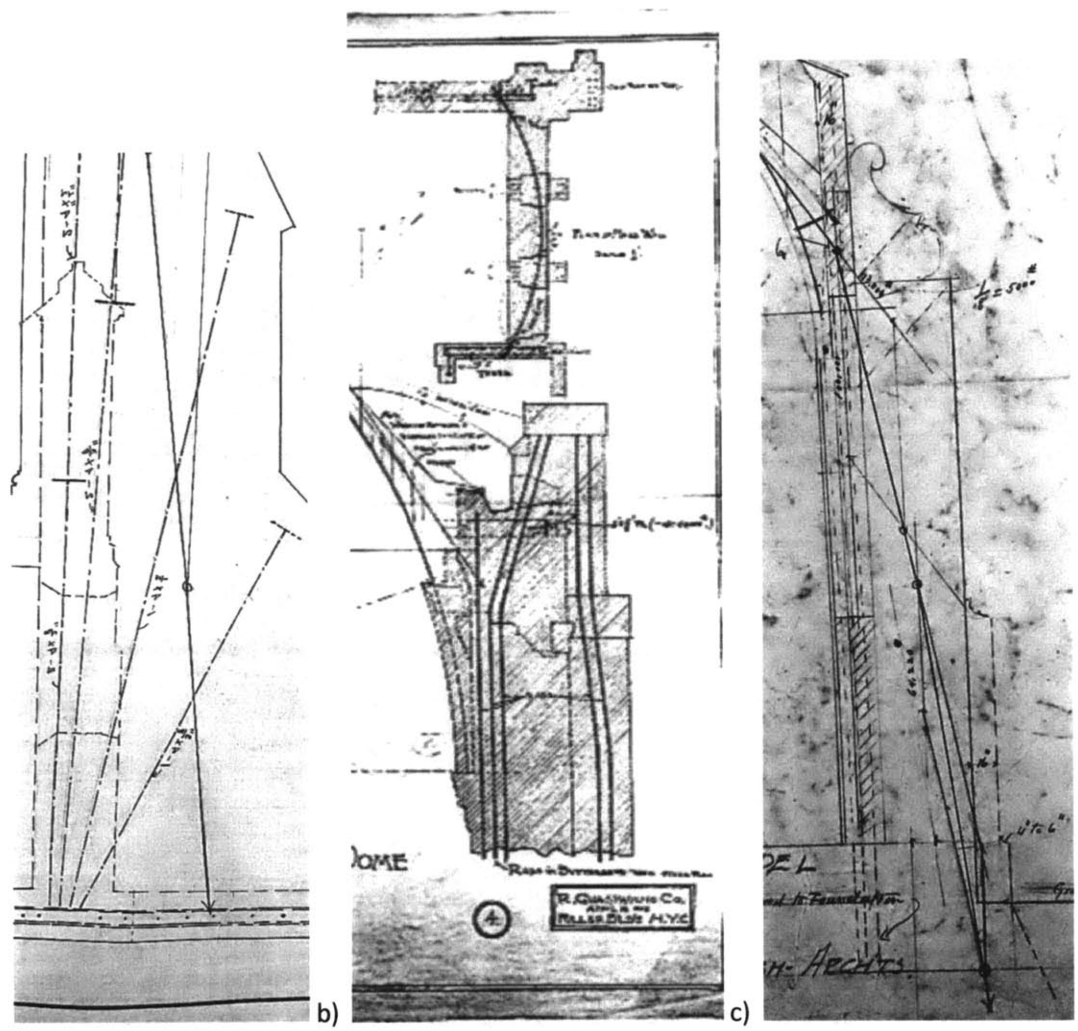


Figure 4.32: a) St. Columbus R.C. Church, b) St. Francis de Sales Church, c) Trinity College Chapel - Metal Reinforcement in Columns (Appendix B) [Avery Library]

4.3 Comparison of Handwritings

The handwritings across the Guastavino Company graphic statics drawings from the Avery Library archives are compared to determine the authorship. Other projects are also referenced for more information. The architects were ruled out as the designers of the drawings since the same handwriting is seen in multiple instances when the architects are different. The only drawing found that is signed by Guastavino Jr. is the Grace Universalist Church in Lowell, Massachusetts. This drawing is the source that many drawings are connected to. The process and information outlined in this section is a possibility. There is no definite way to define the author of the drawings. Letters from the Avery Archives can be further explored to find more information.

<u>Grace Universalist Church (1895) → Tennis Shelter Prospect Park (1906)</u>

The letter "S" between the two projects is identical, as seen in Figure 4.33 and Figure 4.34. Guastavino Jr. is trying to be very neat in the first drawing so it is less of a free-hand script. However, there resemblance is still recognizable, especially in the world "scale."

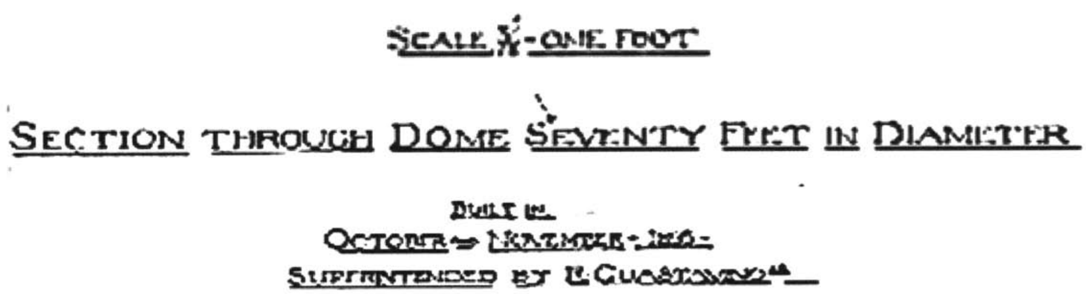


Figure 4.33: Grace Universalist Church (1895) Handwriting (Appendix B) [Avery Library]

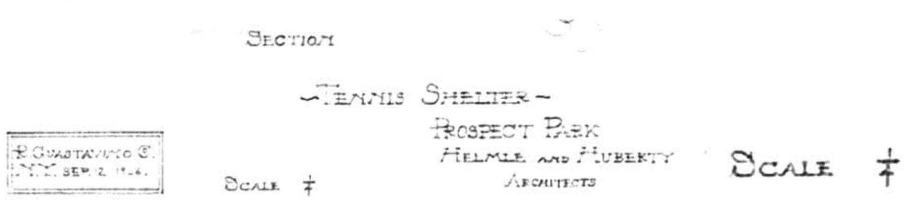


Figure 4.34: Tennis Shelter Prospect Park (1906) Handwriting (Appendix B) [Avery Library]

Grace Universalist Church (1895) → Bank of Montreal (1903)

The "radius" call out on the drawings is the same, as seen in Figure 4.35.

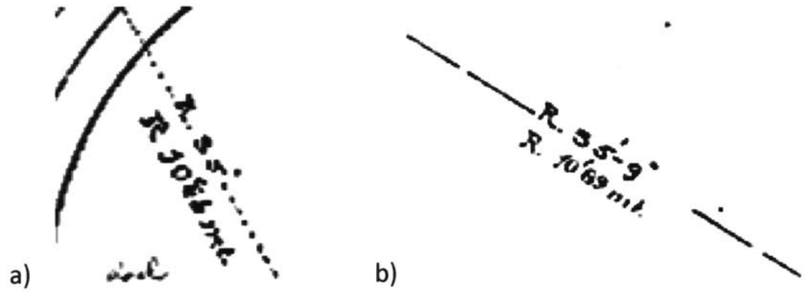


Figure 4.35: a) Grace Universalist Church (1895) vs. b) Bank of Montreal (1903) Handwriting (Appendix B) [Avery Library]

Bank of Montreal (1903) → Girard Trust Building (1907)

The word "band" is relatively the same for the two drawings (Figure 4.36). The letter "S" is also the same one as observed in the Grace Church and Tennis Shelter drawings.

Figure 4.36: a) Bank of Montreal (1903) vs. b) Girard Trust Building (1907) Handwriting (Appendix B) [Avery Library]

St. Columbus R.C. (1906) → Girard Trust Building (1907) → St. Francis de Sales Final Drawing (1908)

The phrase "Stress Diagram" is identical in these series of drawings see in Figure 4.37.

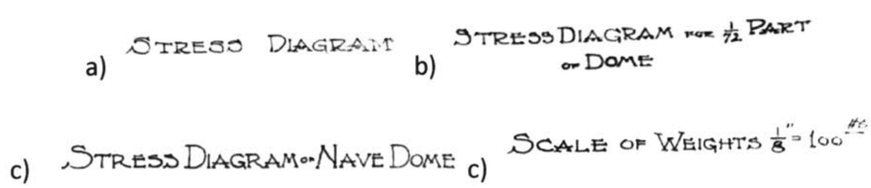


Figure 4.37: a) St. Columbus R.C. (1906) vs. b) Girard Trust Building (1907) vs. c) St. Francis de Sales (1908) (Appendix B) [Avery Library]

Girard Trust Building (1907) → St. Francis de Sales Working and Final Drawings (1908)

The handwritings of the notes match, especially the letters "St," seen in Figure 4.38.

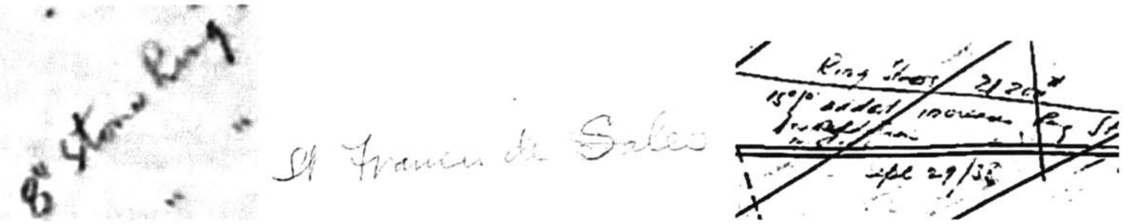


Figure 4.38: Girard Trust Building (1907) vs. St. Francis de Sales Drawings (1908) (Appendix B) [Avery Library]

St. Francis de Sales Working Drawing (1908) → Cathedral of St. John the Divine Panel Planning (1909)

The style of the free-hand handwritings seen in Figure 4.39 may be related.

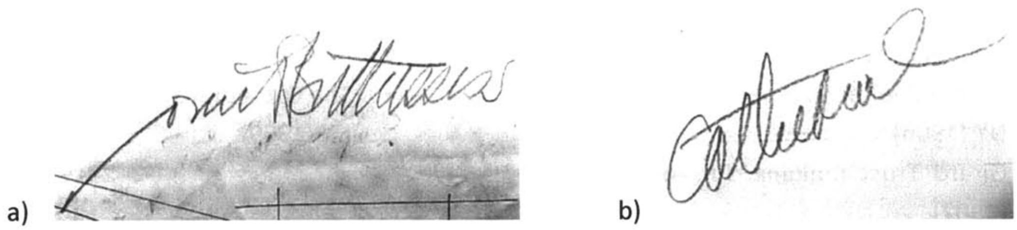


Figure 4.39: a) St. Francis de Sales Working Drawing (1908) vs. b) Cathedral of St. John the Divine Panel Drawing (1909) (Appendix B) [Avery Library]

Cathedral of St. John the Divine Panel Planning (1909) → Cathedral of St. John the Divine Final (1909)

The calculations and text on the drawings in Figure 4.40 are identical.

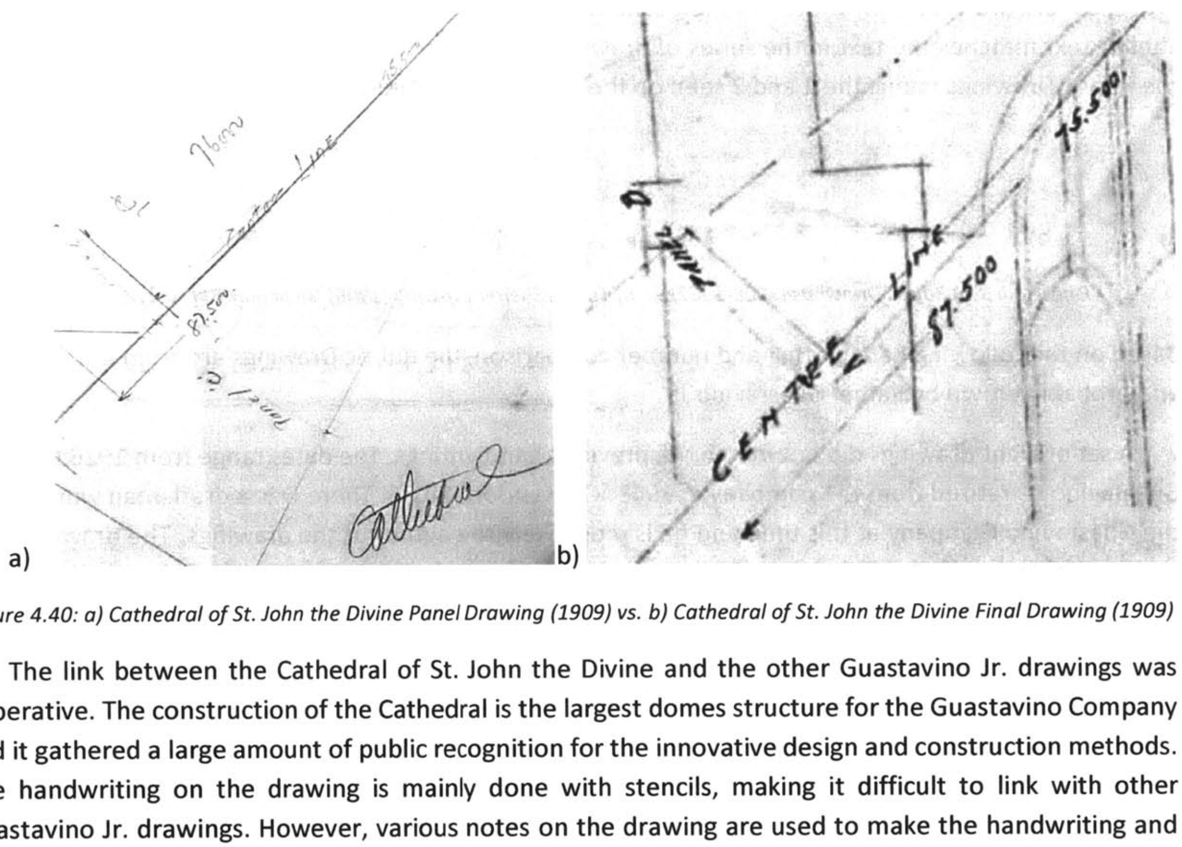


Figure 4.40: a) Cathedral of St. John the Divine Panel Drawing (1909) vs. b) Cathedral of St. John the Divine Final Drawing (1909)

The link between the Cathedral of St. John the Divine and the other Guastavino Jr. drawings was imperative. The construction of the Cathedral is the largest domes structure for the Guastavino Company and it gathered a large amount of public recognition for the innovative design and construction methods. The handwriting on the drawing is mainly done with stencils, making it difficult to link with other Guastavino Jr. drawings. However, various notes on the drawing are used to make the handwriting and design connection.

The main drawings that are analyzed in this thesis were focused on in the handwriting analysis. Once these drawings were connected, the following drawings from the early 1900s all had similar handwritings. The main link for the drawings was the letter "S." They can all be found the Appendix for further comparison. These drawings include:

- Grace Universalist Church, Lowell, MA (1895)
- Bank of Montreal, Canada (1903)
- St. Columbus R.C. Church, Philadelphia, PA (1906)
- Tennis Shelter Prospect Park, Brooklyn, NY (1906)
- Girard Trust Building, Philadelphia, PA (1907)
- Williamsburg Bridge, Brooklyn, NY (1907)
- St. Francis de Sales Church, Philadelphia, PA (1908)
- Elephant House, Bronx, NY (1908)
- The Cathedral of St. John the Divine, NY, NY (1909)

All of these drawings found in the Avery Library Archive are very possibly drawn and analyzed by Guastavino Jr. However, there is no definitive text to verify this assumption.

The author of the Rubric Diagrams needed further investigation. The text in the Tennis Shelter in Prospect Park drawing from 1906 was important to link the Rubric drawings to Guastavino Jr. The Tennis Shelter text matches the text in the series of drawings mentioned above. The 1 and 2 numerals seen on the Rubric Drawings mimic the 1 and 2 seen on the Prospect Park Drawings, as seen in Figure 4.41.

Figure 4.41: a) Rubric Drawings (1906-1907) vs. b) Tennis Shelter Drawing (1906) (Appendix B) [Avery Library]

Based on the following handwriting and number comparison, the Rubric Drawings are from around 1906 and probably drawn by Rafael Guastavino Jr.

A set of eight drawings did not match the previous handwritings. The dates range from 1926 to 1947. Guastavino Jr. retired from the company in 1943 [Ochsendorf, 2010]. There was a draftsman working for the Guastavino Company at this time and he is most likely the author of the drawings. The drawings can be found in Appendix B.

These drawings are as follows:

- St. Barbara's Church, Brooklyn, NY (1926)
- St. Boniface R.C. Church, Pittsburgh, PA (1926)
- Dime Savings Bank, Brooklyn, NY (1931)
- Planetarium Metropolitan Museum of National History, NY, NY (1934)
- St. Louis Art Museum, St. Louis, MO (1937)
- Buhl Planetarium, Pittsburgh, PA (1938)
- Second Church of Christ Scientist, Cleveland, OH (1946)
- Supreme Court Building, Tallahassee, FL (1947)

4.4 Conclusions

The comparison of drawings available in the Avery Library Archives revealed valuable information about the graphic statics methods, the metal usage on projects, and the author of the drawings.

- The barrel vault and arch graphic statics calculation drawings are consistent throughout the entire Guastavino Company collection of drawings from Avery Library.

- The comparison of the dome drawings from 1895 to 1946 reveal the emergence and changes in the graphic statics calculation of the Guastavino Company. The drawings show a strong understanding of tensile stresses in a dome.

- Steel or iron metal calculations on various drawings shows the Guastavino Company used graphic statics, and at a certain stage tensile hoop forces, to design the necessary amount of metal in their structures.

- The handwriting analysis presents a potential connection from the Grace Universalist Church drawing which Guastavino Jr. signs, to many of the dome projects in the early 1900s, including the Cathedral of St. John the Divine dome drawing.

- The structures are form-found using the funicular shape of the forces and geometry. Generally, the design begins with a circle that defines the inside curvature of the structure.

- More careful analysis is required to understand the designs behind the structures presented in this Chapter.

This chapter introduced many of the Guastavino Company structures that pertain graphic statics calculations. A few of the intricate drawings deserve a more thorough analysis.

The following drawings are further analyzed in the subsequent chapters.

- St. Francis de Sales Church Barrel Vault – Working Drawing (Chapter 5)
- St. Francis de Sales Church Barrel Vault – Final Drawing (Chapter 5)

- Rubric Drawing 1 (Chapter 6)
- Rubric Drawing 2 (Chapter 6)
- Rubric Drawing 3 (Chapter 6)

- Cathedral of St. John the Divine Dome Drawing (Chapter 7)
- Cathedral of St. John the Divine Dome Panel Drawing (Chapter 7)

5 Analysis of St. Francis de Sales Church - Barrel Vault Drawings

5.1 St. Francis de Sales Background

St. Francis de Sales Church is located in Philadelphia, PA on the corner of 47th and Springfield Streets. The parish had two different and less extravagant locations before Rev. Michael J. Crane pushed for the construction of the church in 1907 [Milkovich, 1992]. He hired Henry Dagit, a Philadelphia architect that Guastavino Jr. worked with frequently. Two projects they collaborated on also contain graphic statics records in the Avery Archives: St. Columbus Roman Catholic Church (Philadelphia, 1906) and St. Ann's Church (Washington D.C., 1945). Rev. Crane specifically hired the Guastavino Company for portions of the construction. Rev. Crane stated,

> "all Dome work, Nave vault work, choir gallery and Sanctuary vaults and Four Tower domes together with all the necessary steel work as may be required by the Department of Building Inspection of Philadelphia and in accordance with the Architect's directions, who will supervise the work..." [Milkovich, 1992]

Rev. Crane specifically states the amount of steel used had to be approved by the Department of Building. Guastavino Jr. has the graphic statics calculations proving the necessary amount of steel. Guastavino Jr.'s domes and vaults are incredibly thin and his analysis methods were not common knowledge. Even though he calculated the amount of steel required, it would be difficult to convince the Department that more steel was superfluous.

Guastavino Jr. and Dagit collaborated from the beginning of the St. Francis de Sales project. Guastavino Jr. was involved in five domes, one main dome and four smaller tower domes, and several barrel vaults. The following analysis will only go into detail on the Nave Dome, which is the main barrel vault.

5.2 Objective

There are three graphic statics drawings for St. Francis de Sales Church in the Avery Library Archives. Two of the drawings detail the barrel vault, and the third calculates the main dome. There is a working drawing and a final drawing for the vault. In the archives, most of the Guastavino drawings are final drawings used for construction. It is difficult to learn about the design process of the structures from a final drawing. A working drawing has more notes and erased lines that can lend insight into the design process. The author of this drawing is Guastavino Jr., as proven in the *Handwriting Analysis* section.

The objective of the following analysis is to form a design process for Guastavino Jr. barrel vaults. The analysis can be replicated for other Guastavino Jr. barrel vaults to understand their design as well.

5.3 Explaining the Graphic Statics Drawing

The working drawing for St. Francis de Sales is not the first stress diagram for the project. Guastavino Jr. uses pen on linen which cannot be erased and would not be used while calculating the arrangement of a vault. For this drawing, Guastavino Jr. already decided on several parameters of the vault: the thickness of the brick, the dead and live loads, and the vault section considered. These are decisions that would be chosen after an initial graphic statics analysis based on the boundary of the vault and an assumed thickness.

The approximate thickness of the brick for the vault is 5 inches. He is using a dead load of 50 psf for the bricks and a live load of 20 psf. Guastavino Jr. considers a 1′ wide vault section with 2′ wide segments. He assumes the distributed weight of the vault is 70 psf. The weight per each segment is 70 psf acting over 1′ of the vault and a 2′ wide segment, equaling 140 lbs. Based on these parameters, he chooses a load scale of ½″ = 140 lbs.

It is very helpful to have both the working and final stress diagram drawings to see if the design changed. The load scales vary for each drawing, as explained in the *Load Line* section below. The following analysis compares the drawings and traces the lines to understand the design process. The drawings were recreated based on the original drawing dimensions. There may be some discrepancies between the assumed model and the original drawings, as described in the Chapter 2.

The drawings in Figure 5.1 and Figure 5.2 use the following color scheme:

- Vault extents: Blue (Thicker)
- Load Line: Red
- Funicular Shape to find Center of Gravity of Geometry Lines: Orange (Dashed)
- Force Lines: Green
- Buttress and Abutment: Black

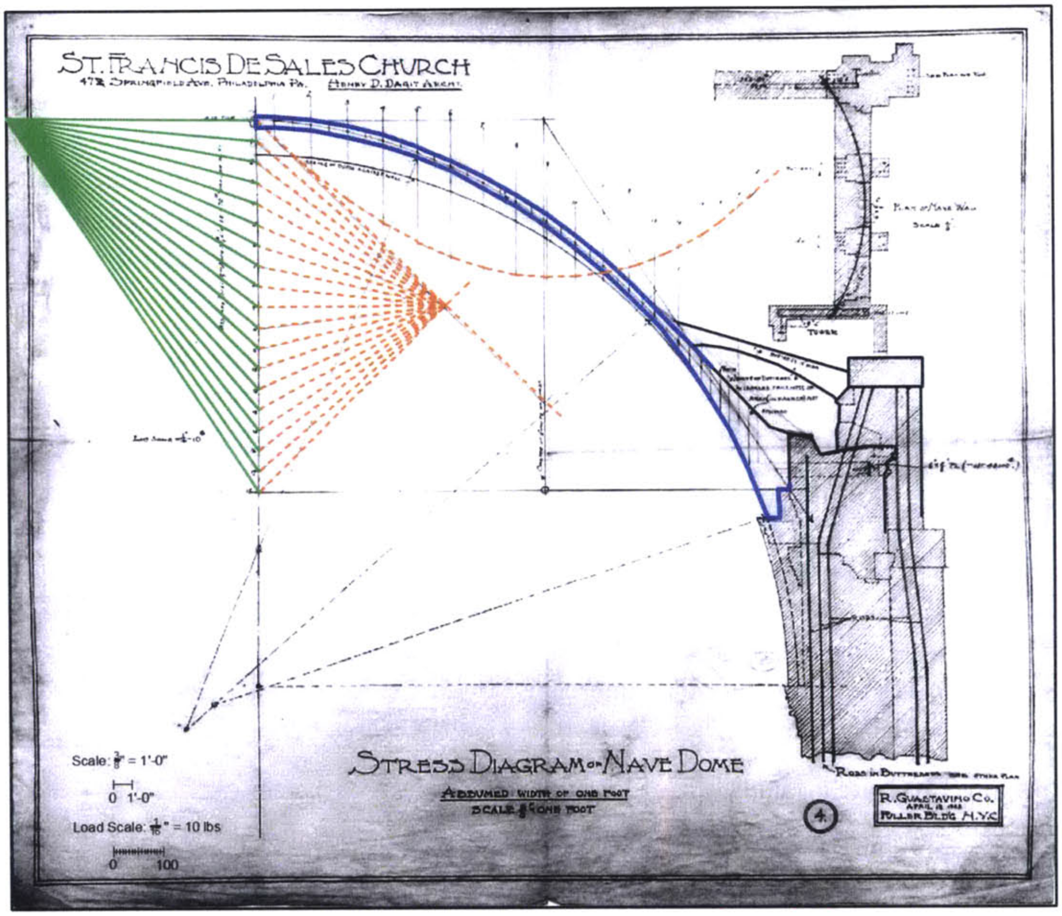


Figure 5.1: St. Francis de Sales Church Final Drawing - Color Scheme drawn by Author (Appendix B) [Avery Library]

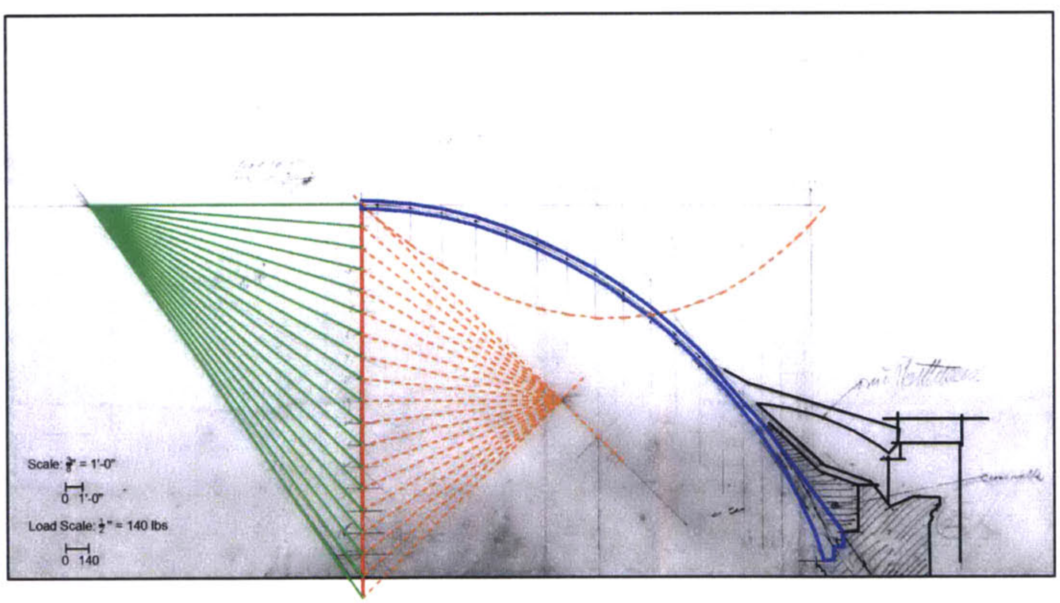


Figure 5.2: St. Francis de Sales Church Working Drawing - Color Scheme drawn by Author (Appendix B) [Avery Library]

5.4 Questions

❖ Previously Defined Parameters

The general dimensions of the space to enclose are predetermined. The span of the vault is defined by the church geometry. It is uncertain whether the height of the vault is specifically designed to lower forces or if it is simply a workable geometry. The original working drawing is cut off at the bottom. The following figures of the working drawing extend the load line as it would be drawn.

❖ Shape

The general shape of the vault is based on a circle. The radius at the base of the vault is approximately 28 feet and the height is 22 feet. A circle with a radius of approximately 32 feet, that intersects both the radius and the height of the vault, defines the inside curvature for the project. A full quarter section of the circle is not used. To apply graphic statics, a dead and live load must be defined. The first thickness of the vault is chosen based on Guastavino Jr.'s past experience. After he has a general idea of the thrust line, he can refine the thickness of the vault. The funicular shape and thrust line for the specified geometry and loading are found to determine the thickness of the vault. The *Brick Layers Section* expands upon the thickness design of the vault.

The main difference between the working and final drawings is that a lower distributed load is considered on the vault. Instead of using 140 lbs on each segment, only 122.5 lbs is applied. Even though the loading changes, the thickness of the vault does not change from the working drawing to the final drawing. Guastavino Jr. does not redesign the vault for the new loads. The main difference in the drawings is the inner curvature of the vault. Guastavino Jr. refines the curvature and defines it with three different radii, instead of just one radius as in the working drawing. The circles of the three radii are all tangent circles that intersect along the vault curvature to assure a smooth curve. Since the inside curvature is defined by radii, the construction could be completed without extensive scaffolding, as explained in the *Construction* section below.

❖ Segments

To analyze the vault, a 1 foot section is considered. This section is cut in half to form an arc, as seen in Figure 5.3. Guastavino Jr. uses the 32 feet radii circle (green) that defines the inside face of the vault to divide the arc into segments. This allows a constant load for every segment in the force diagram. The circumference of the arc is approximately 36 feet. The 36 feet is divided into 18 segments, each 2 feet. A vertical line (red) is drawn at the intersection of the segment and the circle. The vertical lines are used to construct the funicular shape of the vault.

Generally with graphic statics, the arc curve would be divided into segments using a constant angle, instead of the circumference of the circle. The arc length value is used for further arch calculations while the angle value is not used. Even though the arc length and the angle of the segment are directly related, Guastavino Jr. wants to minimize the error in the arc length value since the analysis is done by hand.

The following drawings are based on the assumed dimensions of the drawing. There may be some discrepancies as described above in the *Explaining the Graphic Statics Drawing* section.

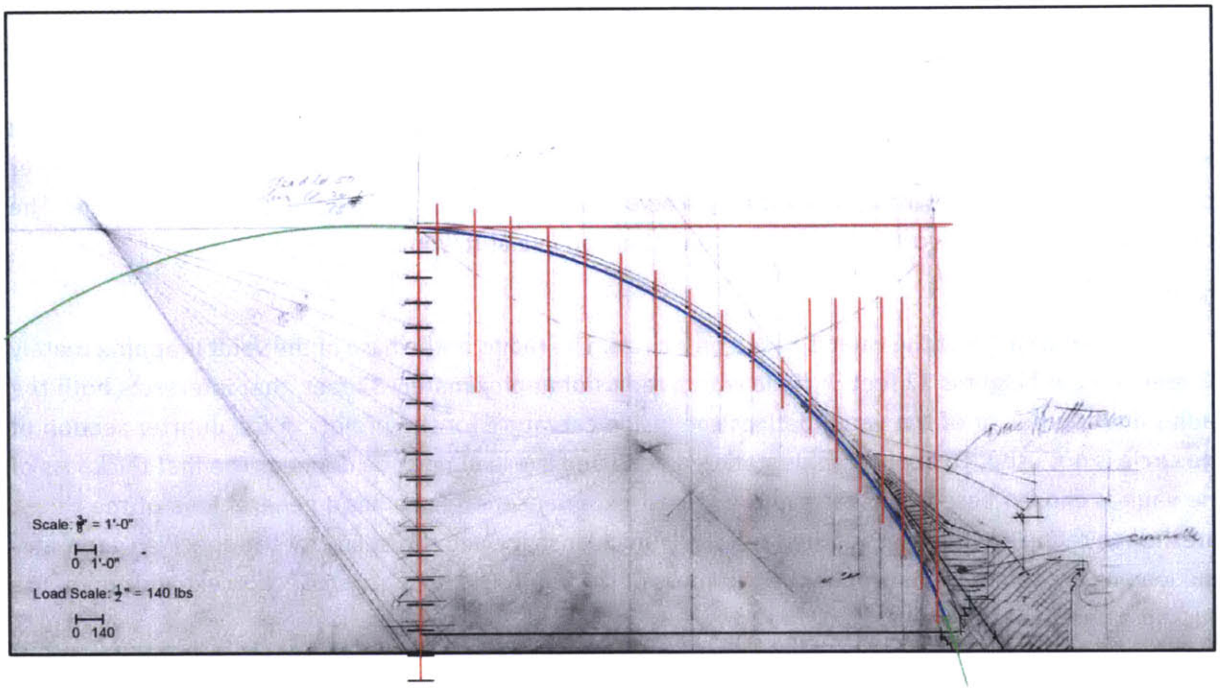


Figure 5.3: St. Francis de Sales Working Drawing Segments

❖ Load Line

Guastavino Jr. uses a dead load of 50 psf and a live load of 20 psf. From experience and iterations, he assumes the thickness of the vault will be a constant 5 inches, approximately 10 psf per inch of the vault. Every segment on the load line is the same dimension, minimizing the analysis error. He chose the scale of the loads as ½" = 140 lbs. At the base of the final drawing, Guastavino Jr. writes that he "assumed width of one foot." Therefore, all of the load calculations are for a barrel vault width of one foot. The weight for the 70 psf, is multiplied by 1 foot for the width of the vault, and 2 feet for each segment, amounting to 140 lbs per segment, as seen on the drawing. Every segment of the load line for the working drawing is ½", corresponding to 140 pounds.

❖ Funicular Shape to find Center of Gravity of Geometry Lines

The load line is used to determine the funicular shape and the center of gravity of the entire arc. A line is drawn at 45 degrees from the top and bottom of the load line. Then, a line is drawn from each load line segment to the intersection of the 45 degree lines. These lines are used to draw the funicular shape of the vault. The first funicular line (orange) is placed at the intersection of the circle (blue) and vertical line (red). It is cut when it meets the second vertical line. The second funicular line is placed at the intersection of the first funicular line and the second vertical line. The process continues until the full curve is constructed. Then, the first and last funicular lines are extended until they intersect. Their intersection represents the axis of the resultant of all the loads applied on the arc. A vertical line is drawn at this point until it intersects a horizontal line (red) extended from the top of the dome. A line is then extended from this point to the lower region of the abutment, shown in Figure 5.4 in green. Guastavino Jr. has a general idea of where he wants to direct the thrust in the abutments so he chooses the location.

For the thesis analysis, the original drawing has an erased line where the green line is placed. On the curvature of the vault, there is also a bottom curve erased. Therefore as a first step, this thrust location in the abutment is chosen and investigated.

In terms of the load line, the thrust occurs at Segment 18, so the thrust line is transferred to the base of the load line at 18. When this line is extended to the horizontal line at the top of the dome, the horizontal distance of the intersection is the maximum horizontal thrust of the vault. All of this thrust must be taken up by the abutments.

The calculation of the abutment is not shown in any of the drawings found the in the Avery Archives. Guastavino Jr. most likely first assumed a weight of the abutment and determined the necessary thickness of the abutment to withstand the vault thrust.

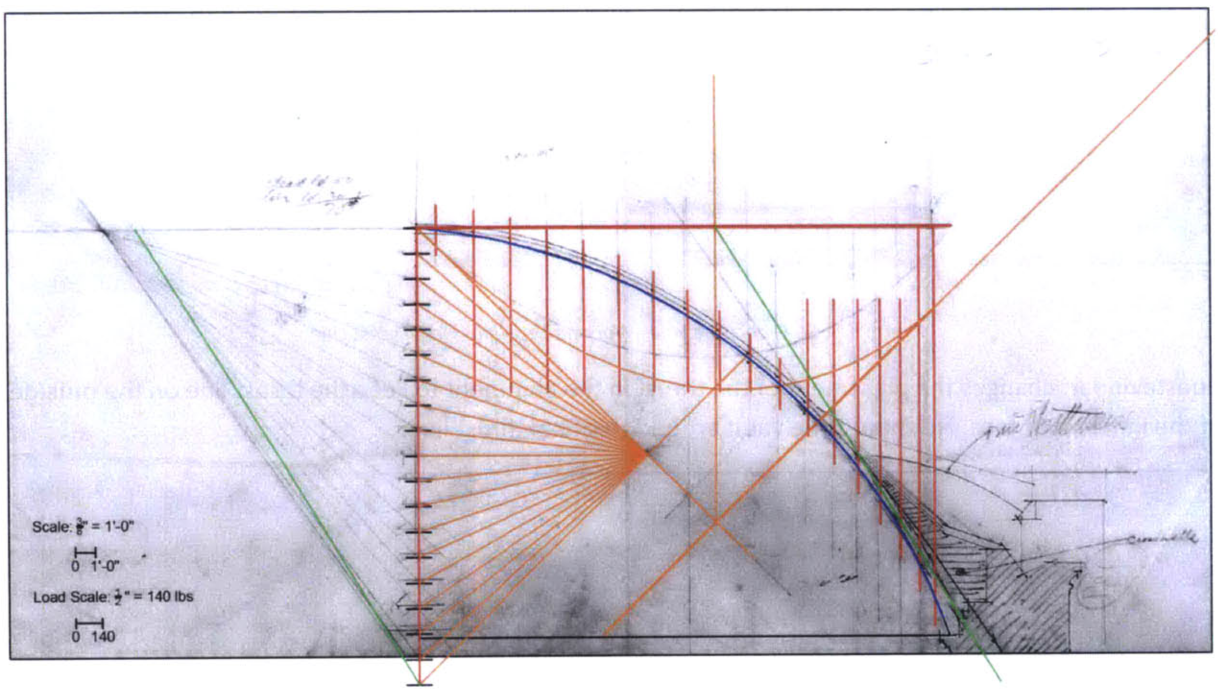


Figure 5.4: St. Francis de Sales Working Drawing - Funicular Shape to find Center of Gravity of Arch

The discrepancies between the original drawing lines and the traced lines are from the poor quality of the drawing. There are bents in the original drawing paper that could not be transformed in Adobe Photoshop. For the most part, the results are fairly accurate and the thrust line is in the proper location on the drawing. The funicular line in orange is the most distorted. The curvature shown on the drawing for the funicular curve does not follow the funicular lines used to create the curve. The origin of the funicular curve shown is undetermined.

❖ Forces

Now that the maximum horizontal thrust is determined, a line is drawn from each load line segment to the maximum horizontal thrust point. These lines replicate the process with the funicular shape. After each line is transferred, the thrust line of the dome is found.

On the working drawing, two iterations can be identified. First, Guastavino Jr. directs the thrust line to the lower third of the abutment. Guastavino Jr. has defined the inside of the vault using the circle and he does not want to change this curvature for construction purposes. He wants the thrust line to stay on the outside of the inside curvature. The thrust line (green) for this arrangement crosses the inside vault boundary (blue line) in the middle area of the vault, as can be seen in Figure 5.5.

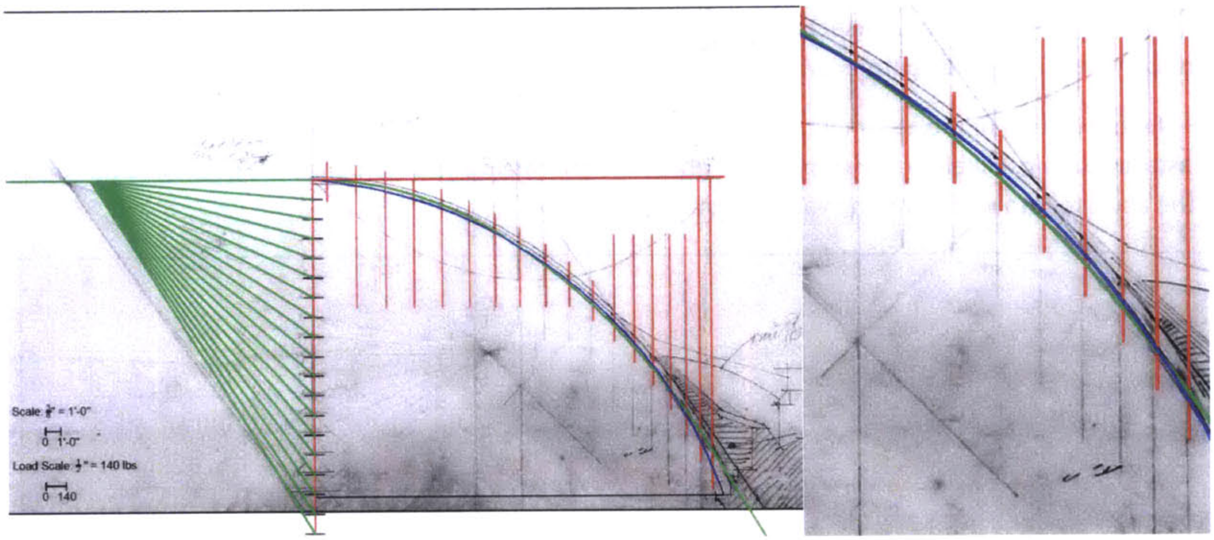


Figure 5.5: St. Francis de Sales Working Drawing- First Thrust Line

Guastavino Jr. changes the placement of the thrust in the abutment to keep the thrust line on the outside of the inner vault face. He shapes the vault around this final thrust line.

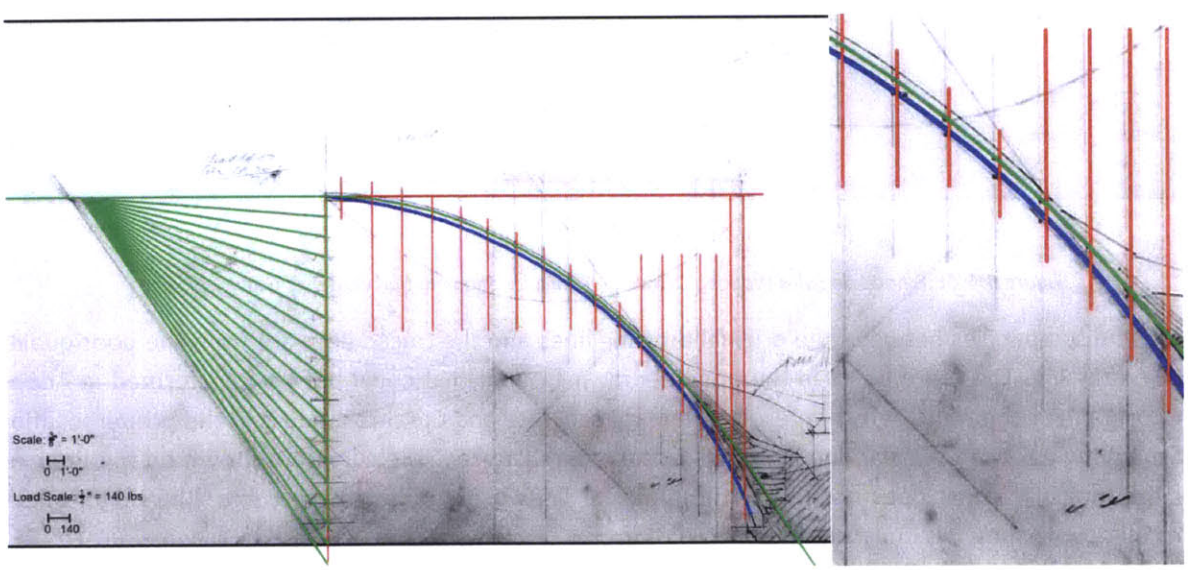


Figure 5.6: St. Francis de Sales Working Drawing - Second Thrust Line

The placement of this first thrust line is based on faint lines that can be seen on Guastavino Jr.'s drawing. The thrust line positions perfectly with points that Guastavino Jr. later erased. Both of the thrust lines are depicted on Figure 5.7. The first thrust line is dashed in Purple and the final thrust line is solid in green.

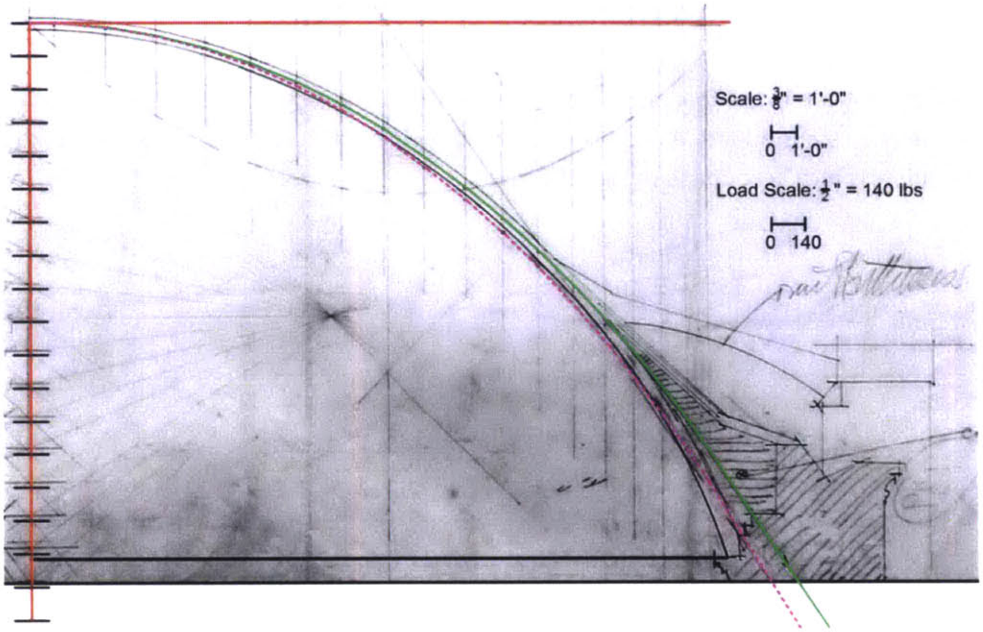


Figure 5.7: St. Francis de Sales Working Drawing - Thrust Line Comparison

❖ Brick Layers

After the final thrust line (green) of the dome is found, the difference (purple) between the inside of the vault and the thrust line is determined, as seen in Figure 5.8. Guastavino Jr. wants to enclose the thrust line in the middle of the vault. He mirrors the distance between the inside vault and the thrust line over the thrust line to find the thickness of the vault (light blue).

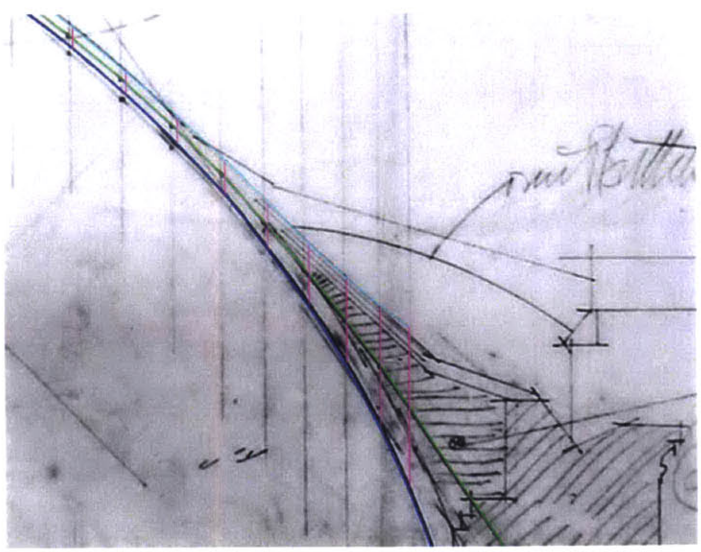


Figure 5.8: St. Francis de Sales Working Drawing - Brick Layer Thickness

Guastavino Jr. introduces the buttress around segment 12 where the difference between the thrust line and the interior vault rapidly increases. The top of the vault, the first 11 segments, is standardized to 5 inches. The difference between the thrust line and the interior vault is assumed as two-thirds of the vault thickness. The average of the distance for the first 11 segments is taken. The final third is added on the other side of the thrust line to get the thickness of the vault of 5 inches.

Guastavino Jr. only alters the thickness at the base of the dome where the vault is thrusting outwards. This is where the buttress is added. As he notes on his drawing in Figure 5.9, he does not take the added weight of the buttress into account to design the vault.

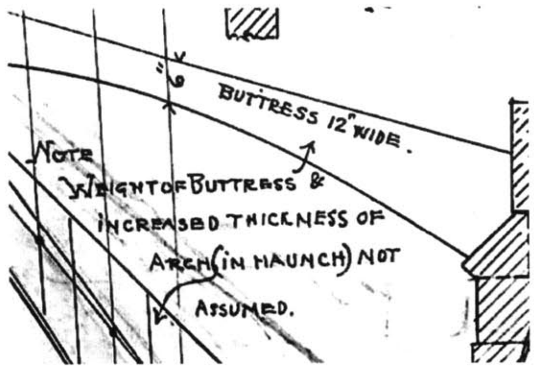


Figure 5.9: St. Francis de Sales Working Drawing - Buttress Detail

❖ Metal Placement

There is no metal placed in the vault itself. There is only metal in the abutments, as seen in Figure 5.10. The calculation for the metal amount in the abutments is not found. It is possible, as mentioned earlier, that the Department of Buildings in Philadelphia required more metal than necessary in the columns to contain the thrust from the vault. In other Guastavino Company barrel vaults and arches, as seen in Chapter 4, the thrust line is extended into the abutment and the weight of the abutment is used to determine the necessary thickness of the wall.

a) b)

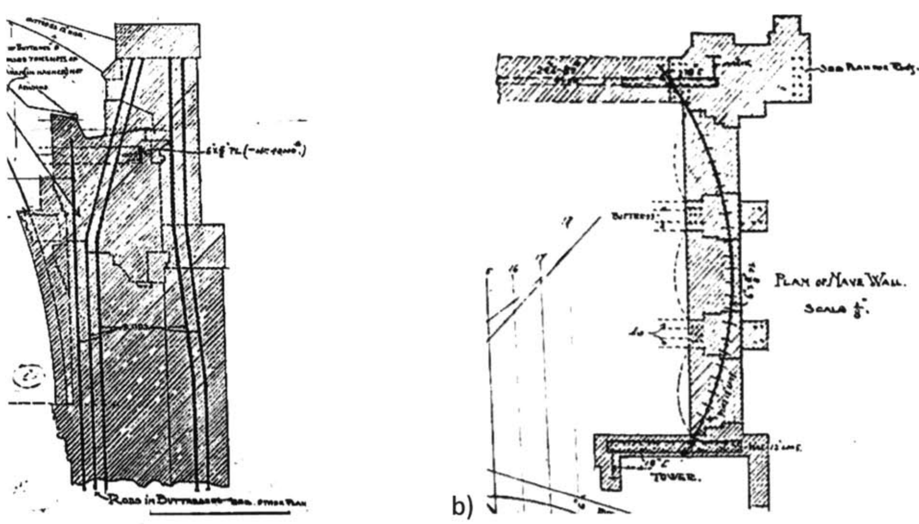


Figure 5.10: Metal Placement in a) Buttress Cross Section and b) Nave Wall Plan View

In the top right of the final drawing, there is a Plan of the Nave Wall. The length of the barrel vault is approximately 34 feet. There is a 6" x 5/8" plate placed in the wall. Based on the arc of the wall, it is assumed that Guastavino Jr. applies a form of pretension to the wall. As the barrel vault is thrusting outwards, the wall wants to move outwards. There is a constant thrusting along the wall, like a distributed load on a beam. There is no calculation found for the area of the metal. Figure 5.11 shows the elevation of the church. The front view elevation on the right shows the barrel vault. The side elevation on the left shows the plan of nave wall, boxed in red.

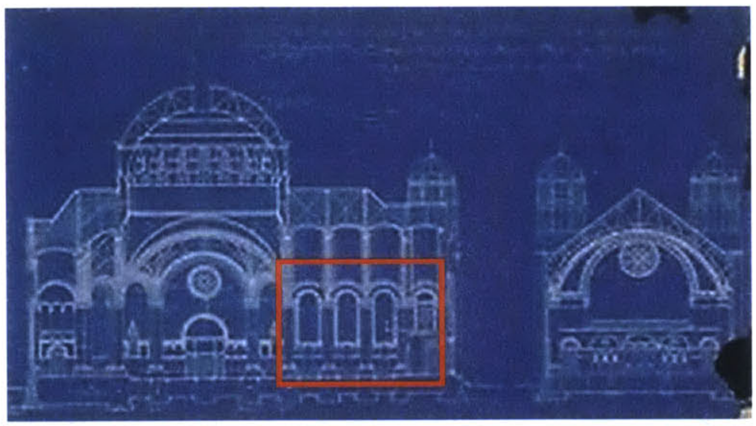


Figure 5.11: St. Francis de Sales Elevation Blueprint – Red Box drawn by Author to Identify Plan of Nave Wall [Dagit (1907)]

❖ Construction

In the final drawing, Guastavino Jr. specifies three radii for the inner curve of the vault, as seen in Figure 5.12. During his design process, he does not change the interior of the vault; he wants to keep a constant radius. For the construction of the Cathedral of St. John the Divine, Guastavino Jr. uses a suspended cable and follows the cable around the curvature of the dome to place bricks. This process is detailed further in Chapter 7. It is possible that Guastavino used the same principle to construct this vault. On the final drawing, Guastavino specified three locations along the vault. Each of the locations specify a different radius. The circles are tangent to each other along the curvature of the vault. During construction, Guastavino Jr. would know when to change the radius for the bricks to continue following the curvature of the vault. The tangent radii are related so Guastavino Jr. would know how to change the centering for the suspension cables.

There are no known documents to support this assumption. There are some detailed letters on the St. Francis de Sales project in the Avery Library Archives that were not reviewed in this thesis. Potentially, there is information on the construction of the vault in those letters.

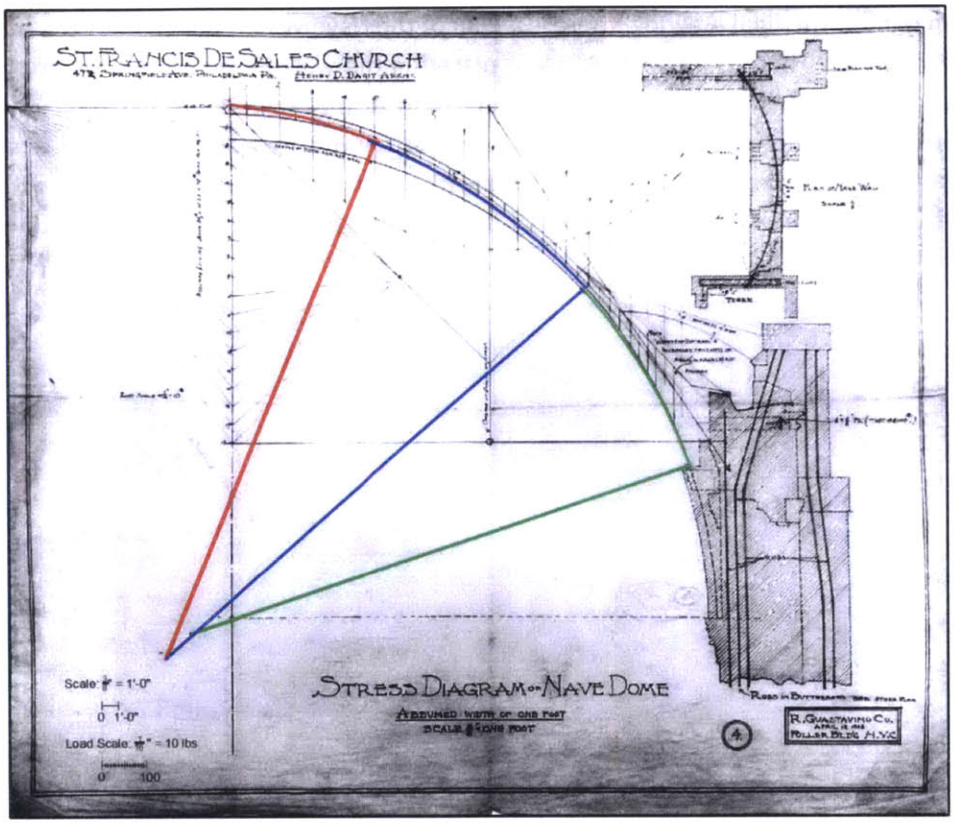


Figure 5.12: Triple Circle Radius Interior Face Definition

❖ Comparison of the Drawings

The three stress diagrams for St. Francis de Sales Church are two Nave Vault Drawings and one Main Dome Drawing. The curvature of the three drawings is connected.

The load scale is different for the working drawing (1/2" = 140 lbs) and the final drawing (1/16" = 10 lbs). The full drawing scales (3/8" = 1'-0") are the same. To compare the load lines, the load scale must be converted from one drawing to the next. The load for each segment in the working drawing is 140 lbs, as explained above. The final drawing considers a different total load. Each load line is 7/16", translating to 70 lbs. However, when this value is converted to the working drawing load scale, it is 122.5 lbs.

It is uncertain why Guastavino Jr. lowered the loading on the vault. The length of each segment is still 2 feet and a 1 foot width of the arch is still considered. Guastavino Jr. draws a line at 80 lbs on the final drawing. The 80 lbs translates to 140 lbs at the working drawing scale. Guastavino Jr. draws the line for the previous loading for an inexplicable reason. The curvature and thicknesses of the vaults are the same. Figure 5.13 and Figure 5.14 compare the final thrust lines for the vault under the 140 lbs (dashed red) and 122.5 lbs (green) loading.

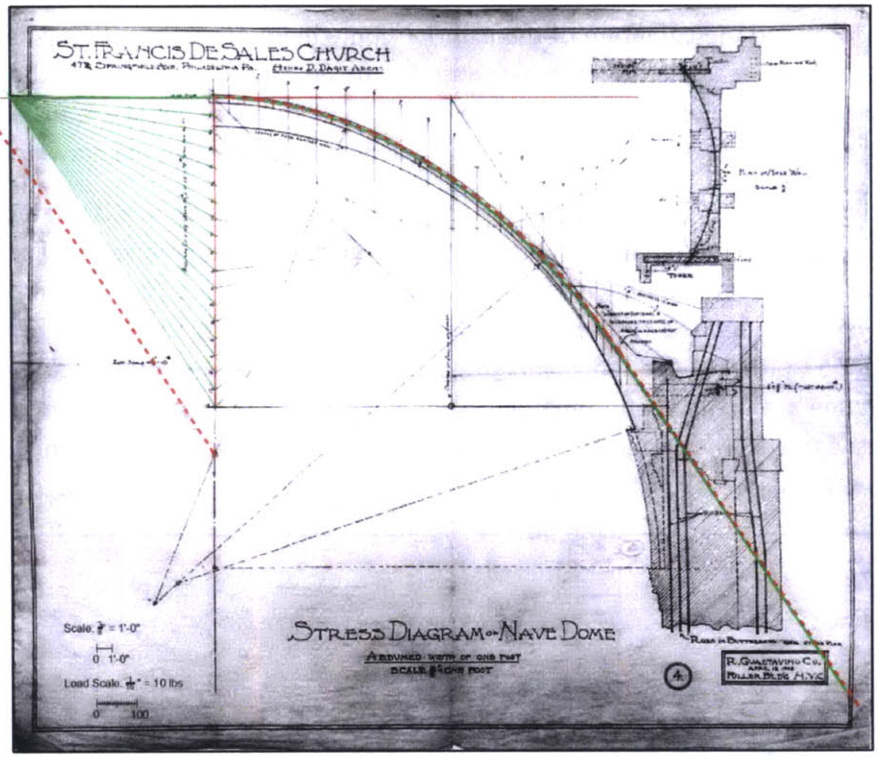


Figure 5.13: St. Francis de Sales Final Drawing – Dashed Red Line -Working Drawing Force Load Scale Value and Thrust line Shown, Solid Green Line – Final Drawing Load Scale Value and Thrust line

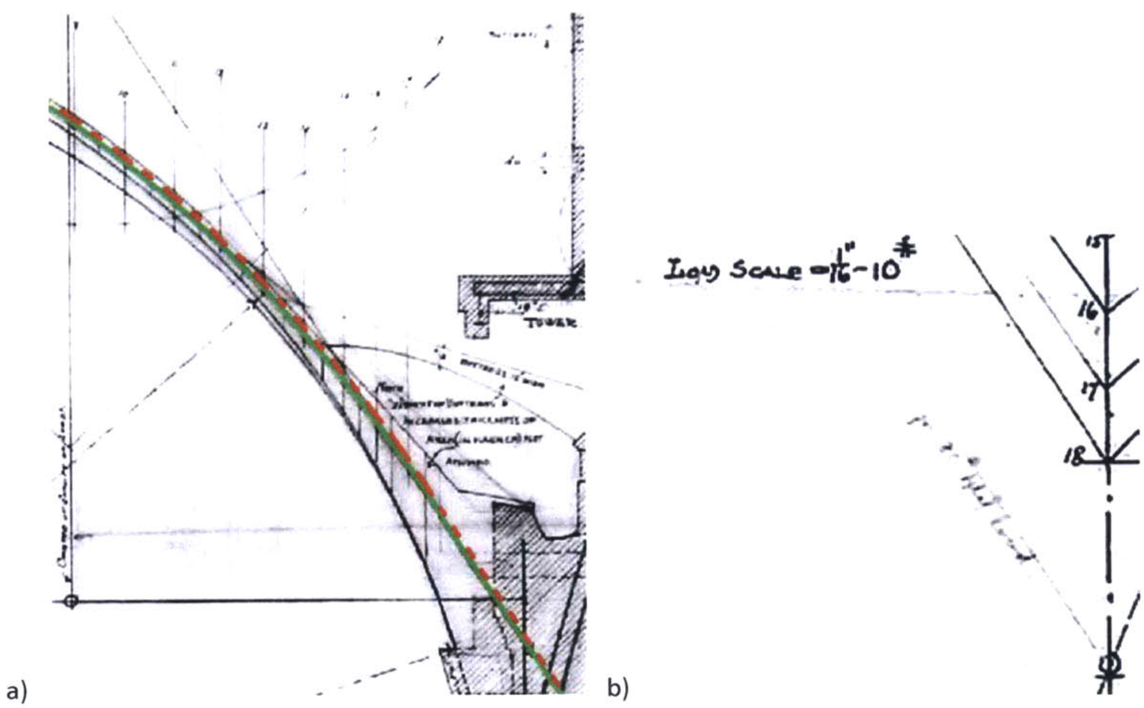


Figure 5.14: St. Francis de Sales Final Drawing – a) 70 lbs Line Load in Green, 80 lbs Line Load in Dashed Red, b) 80 lbs Line Total Load Close Up from Final Drawing

Even though the load taken into consideration from one drawing to the next changes, it is not a design iteration to form-find the curvature of the exterior dome more precisely as the general curvature does not change. With the scale that Guastavino chose for the final drawing, 1/16" = 10 lbs, the bottom of the load line lines up perfectly with the base of the dome. By making the lines line at the same level, it seems like they are related and adds a level of obscurity and aesthetic appeal to the drawing.

The curvature of the vault and the dome for St. Francis de Sales Church is related, as seen in Figure 5.15. The base diameter of the main dome is 62 feet. The curvature of the vault is approximately a circle with a radius of 32 feet. Guastavino based the height and curvature of the vault to match with the dimensions of the dome. The top half of the dome follows the thrust line (green) of the barrel vault, as seen in Figure 5.6. The red line represents the inside circle that defines the curvature of the dome. The green thrust line deviates at approximately 54°, the location where compressive hoop forces in a dome become tensile forces. The drawing is bent towards the center of the dome, explaining the discrepancy in the 54° value. The dome is further analyzed in Chapter 4.

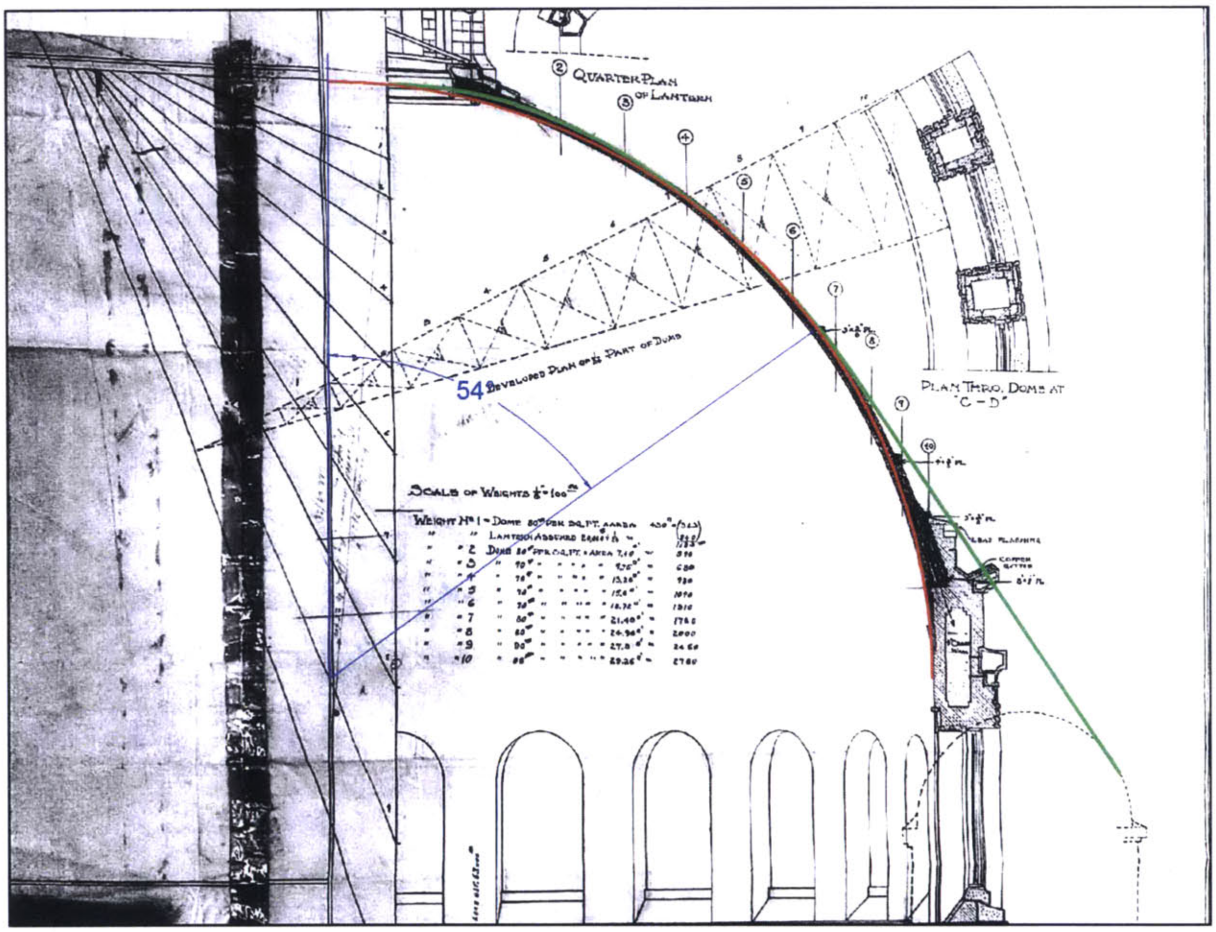


Figure 5.15: St. Francis de Sales Dome - Barrel Vault Thrust Line

5.5 Conclusions

The rigorous analysis of the barrel vault for the St. Francis de Sales Church represents the typical design process for a Guastavino Company barrel vault or arch. Since the working and final drawing for the project exists, the process behind the design can be seen. The following conclusions are found:

- Guastavino Jr. personalizes graphic statics parameters, such as the segment quantity, load scale, and vault width, based on the vault geometry to make the analysis as simple as possible.

- Guastavino Jr. begins the analysis with a circle for the interior face of the vault. The interior circle allows for construction without extensive scaffolding that defines the curvature of the vault.

- He uses graphic statics to find the thickness of the vault. Once the thrust line is found, the distance between the original circle and thrust line is found. That distance is then mirrored over the thrust line to find the extents of the arch.

- The interior of the arch is sometimes altered from the base circle into two or three tangential circles to find a thinner interior arch.

6 Analysis of the Rubric Drawings

6.1 Drawing Background

Three drawings for a 100 foot span dome with the height equal to the radius of the dome, 50 feet, show Guastavino Jr. learning and expanding Dunn's graphic statics methods. This ratio is famously exhibited on the Pantheon in Rome. There is no definitive date or authorship written on the drawings. Chapter 4 argues that the drawings were created between 1906 and 1907 by Guastavino Jr. The drawings are a rubric for a generic dome that is a perfect circle and spans 100 feet. The dimensions and design parameters on the drawing are chosen to easily replicate for other dome sizes.

6.2 Objective

The objective of the following analysis is to determine:

- What is the difference between Guastavino Jr.'s and Dunn's analysis?
- What was the purpose of these drawings?
- Was Guastavino Jr. learning graphic statics through these drawings?
- Are these drawings used to design any dome span by just scaling the dimensions?

6.3 Explaining the Graphic Statics Drawings

Each drawing shows a dome with 100 feet diameter, 50 feet height, and a 100 psf loading. The lune is 3.14 feet at the base. The dimensions chosen for the analysis are easily scaled.

The first drawing, Figure 6.1, shows the horizontal thrusts in each segment of the dome assuming tensile forces develop. The red zigzag lines relate to the compressive thrusts and the thin solid blue lines relate to tensile thrusts. The lines along the arc for each segment relate to the thrust values.

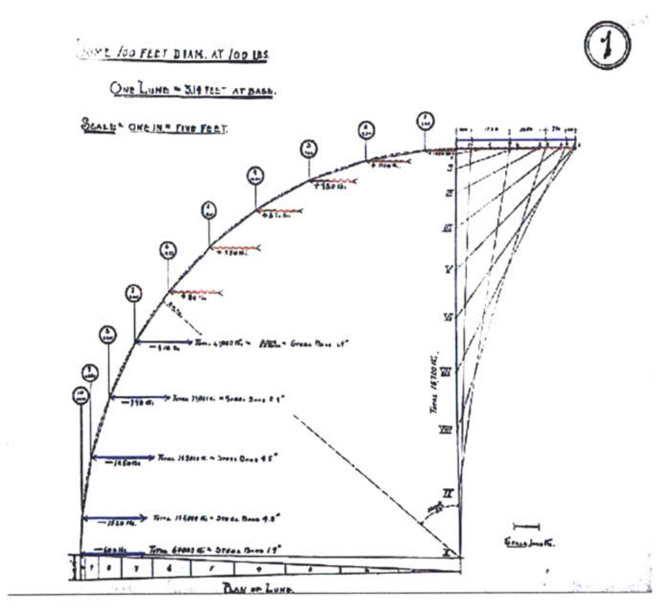


Figure 6.1: Rubric Drawing 1 (Appendix B) [Avery Library]

Table 6.1: Rubric Drawing 1 - Horizontal Thrust, Full Tensile Thrust, and Metal Band Values

Segment	**Horizontal Thrust (lbs)**	**Full Tensile Thrust (lbs)**	**Metal Band (inches)**
1	1250		
2	1100		
3	980		
4	870		
5	550		
6	80		
7	-410	41,000	1.4
8	-790	79,000	2.4
9	-1450	145,000	4.5
10	-1520	152,000	4.8
End	-600	60,000	1.9

The second drawing, Figure 6.2, takes apart the force diagram and details the forces on each segment of the dome. It clearly identifies which parts are in tension or compression. By taking apart each segment of the force polygon, Guastavino Jr. can visualize the forces that are occurring in that segment of the dome.

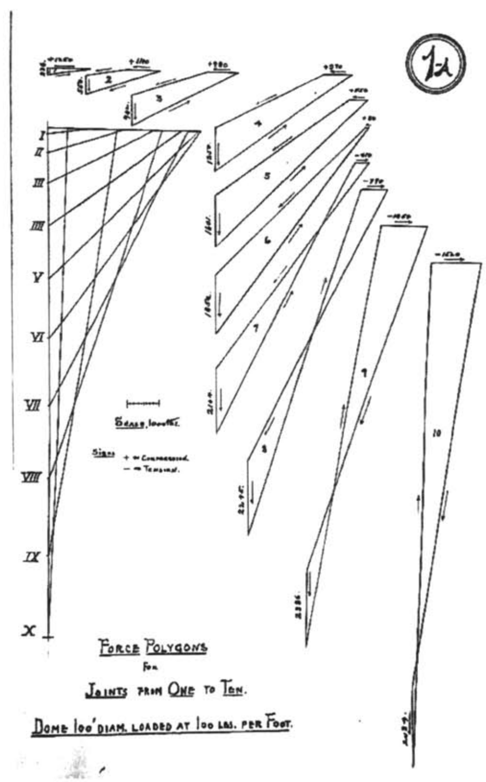


Figure 6.2: Rubric Drawing 2 - Force Polygons (Appendix B) [Avery Library]

The third rubric drawing, Figure 6.3, is the most interesting component of the rubric drawings. The limit state analysis of masonry dictates that masonry cannot handle tensile forces and instead the base of the dome splays outwards. Guastavino Jr. assumes that the masonry cannot take any tension and he restricts the maximum horizontal thrust to the compressive region, like Eddy's method. Then, he draws the rest of the meridional lines to the maximum compressive horizontal thrust value and transfers the meridional lines from the tensile region to the arc segment of the dome. Guastavino Jr. finds the amount the dome thrusts outwards, approximately 2.5 feet. The graphic analysis in the Rubric drawings introduces Guastavino Jr.'s design innovation for domes.

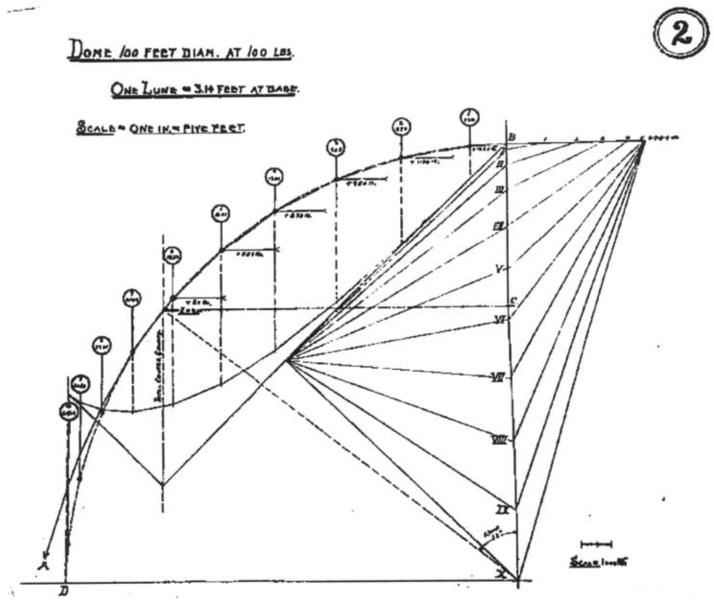


Figure 6.3: Rubric Drawing 3 (Appendix B) [Avery Library]

6.4 Questions

- Shape

The shape of the dome is a circle that has a radius of 50 feet.

- Segments

The arc portion is divided into 10 equal segments. The length of each segment is 7.85 feet. The arc length of a quarter circle: $c = \frac{\pi d}{4}$. To divide the quarter circumference into 10 even segments, each arc length is: $L = \frac{\pi d}{40} = 7.85\ feet.$

- Lune

The lune is shown at the base of drawing one. The description of Drawing 1 states: "One lune is equal to 3.14 feet at the base." A lune would normally be chosen as a fraction, e.g. 1/20th, of the total dome. By choosing the lune to equal 3.14 feet, or PI, at the base, the fraction of the lune is actually $\frac{1}{diameter}$, in this case, 1/100th of the dome. With this lune section, the values can easily be scaled to a lune of any diameter.

- Lune representing 1/20th Dome:

$$Section\ Angle = \frac{1}{20} * 360° = 18°$$

$$L = \frac{c}{20} = \frac{\pi d}{20}$$

- Lune with base arc length of 3.14:

$$L = 3.14 \qquad c = \pi d$$

$$L = 3.14 = \frac{\pi d}{d} = \pi$$

$$Section\ Angle = \frac{1}{d} * 360° = 3.6°$$

$$L = r\theta = 50' * \frac{3.6° * \pi}{180} = 3.14$$

❖ Load Line

The area of each segment is calculated form the lune. That area is then multiplied by 100 psi to calculate the load line.

❖ Forces

As seen in drawing one along the dome arc, Figure 6.4, after the tensile thrusts are determined for each section, the values are multiplied by 100. The lune segment represents $\frac{1}{diameter}$ or $\frac{1}{100}$ section of the dome. To get the total tensile thrust, Guastavino Jr. multiplied the tensile thrusts by 100.

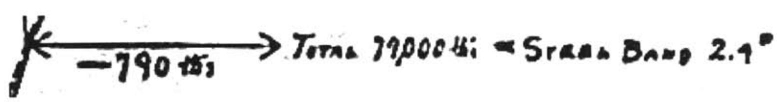


Figure 6.4: Rubric Drawing 1 - Tensile Thrust and Metal Band Example (Appendix B) [Avery Library]

Guastavino Jr. finds that the tensile thrust for one lune section is 790 lbs. By multiplying the tensile thrust by 100, Guastavino Jr. is attempting to model the total amount every lune section creating the full dome thrusts outwards. This method is not accurate to determine the tensile thrust of the dome. It does not project the tensile thrust of the dome and eliminates the necessary three-dimensional aspect of the forces in a dome. The tensile stresses can be determined by calculating hoop forces as detailed in Wolfe's Method.

❖ Metal Calculations

The engineer performing this analysis is aware that compressive thrust forces in a dome turn compressive at 51.8°, as drawn.

The first drawing shows metal calculations based on the horizontal thrusts in the tensile section, as seen in Figure 6.5. However, the method shown is questionable to size metal in a dome. The tensile hoop forces need to be found to size metal. Tensile hoop force calculations are not seen in Guastavino Company drawings until the Cathedral of St. John the Divine dome seen in Chapter 7.

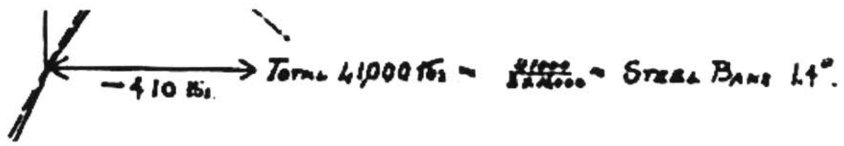
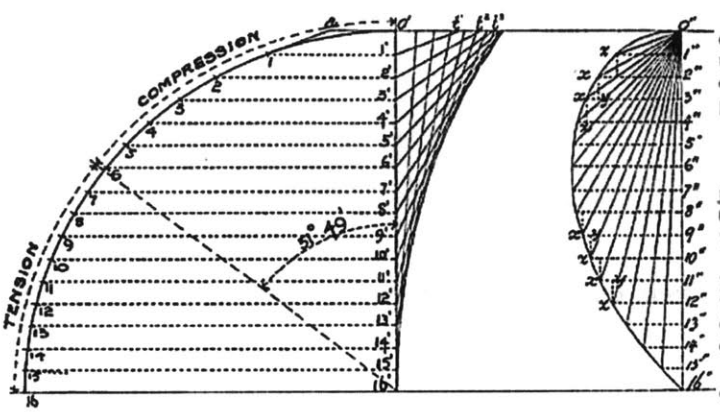


Figure 6.5: Rubric Drawing 1 - Tensile Thrust and Metal Band Calculation (Appendix B) [Avery Library]

Since Guastavino Jr. is influenced by Dunn's publication, the equation Dunn presents for hoop tension is interesting, detailed in Figure 6.6.



13. Let 0' 16' represent the total weight of the dome, then any *x y* shows the total radial thrust upon the corresponding section, which we shall call R. Being uniformly distributed, its intensity per unit of circumference equals

$$\frac{R}{\text{units in the circumference}}$$

just as the intensity of pressure on a pillar equals the total pressure divided by the units of area in the pillar.

In any circular ring under uniform normal pressure (fig. 10 (iii)), as in a cylinder holding water, the resultant tension or compression T (which we call hoop tension) equals the intensity of the radial pressure multiplied by the radius,

$$\frac{R \times \text{radius}}{\text{circumference}} = \text{hoop tension} = T.$$

Figure 6.6: Dunn - Hoop Tension Calculation [Dunn, 1904]

In the text, Dunn states: $T = hoop\ tension = \frac{radial\ pressure * radius}{circumference}$. This equation can be simplified to: $T = \frac{radial\ pressure}{2\pi}$. The value the force is divided by in Guastavino Jr.'s drawing, Figure 6.5, is unclear. If Guastavino Jr. was following Dunn's method, he would not multiply the radial force by 100.

This unclear value would represent the strength of the Metal used times the arc length of each segment, if the metal band value is in inches. If the metal band value is in inches squared, then the value is just the strength of the metal used. Working backwards, the value in the denominator is around 29,000 psi.

Using Wolfe's method and finding the tensile hoop forces at this location in the dome, the total force is only approximately 6,700 lbs, as compared to 41,000 lbs. Applying the same metal strength, the metal band only needs to be 0.23 inches. The analysis shown in the Rubric Diagrams is most likely inaccurate. However, it applies a safety factor of over 6.

Since this drawing was not used to create any dome, there is no cause for concern that a dome is reinforced incorrectly. However, if this method was used, then the metal reinforcement surpasses the amount necessary by over 6 and is very conservatively designed. Chapter 4 carefully considers the methods used to calculate tensile thrusts in various dome projects.

6.5 Guastavino Jr. versus Dunn's Analysis

The analysis in the drawings presented in Figure 6.7 are extremely similar.

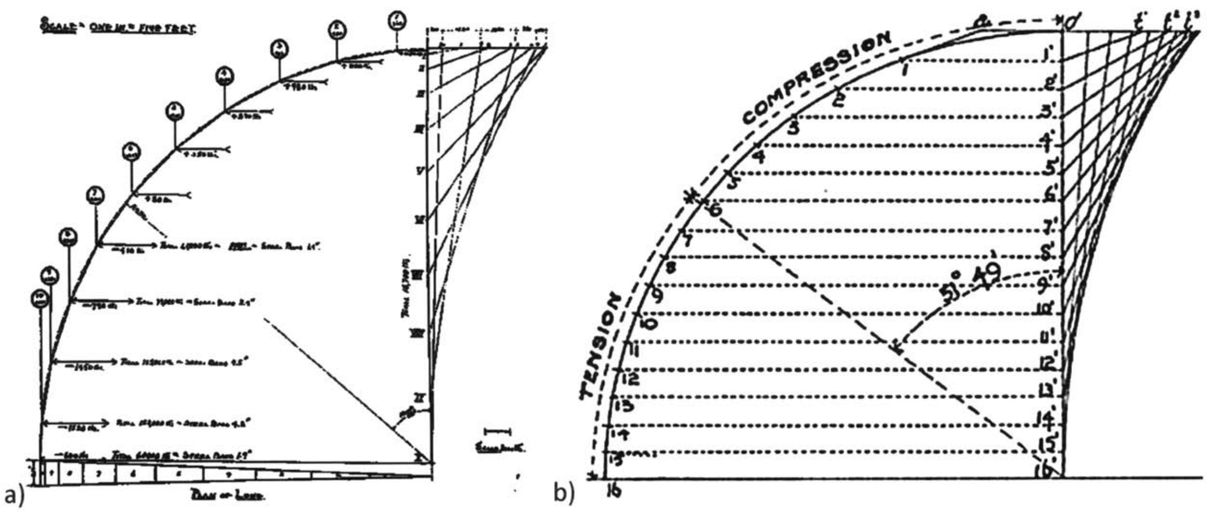


Figure 6.7: a) Rubric Drawing 1 [Avery Library] vs. b) Dunn 1904 Dome Stress Calculation Comparison [Dunn, 1904]

Guastavino Jr. points out the same tension and compression regions. The overall set up of the drawings is different. Guastavino Jr. creates segments based on the circumference of the circle. He divides the quarter circle into 10 segments with equal arc length. Dunn makes the weight of each segment the same but the arc length is different. For graphic statics calculations, it is more important for the arc length value to be accurate. To calculate hoop stresses, the forces are divided by the length of the segment times the thickness of the brick. Since the arc length of each segment is uniform, there is less error in calculating the stresses.

In the third rubric drawing, Figure 6.3, Guastavino Jr. adapts principles from Eddy's method to understand the amount his dome is thrusting outwards. This principle is clearly applied to the design of the Cathedral of St. John the Divine, as detailed in Chapter 7. After Guastavino Jr. understands the amount the thrust line leaves the extents of the dome, he adjusts the arrangement and thickness of bricks. Once the thrust line is not deviating a significant amount, he can still adjust the base curvature of the dome or add more thickness at this location to accommodate the thrust line.

Another incredibly innovative principle Guastavino Jr. learns from this analysis is how to choose the base circle for his projects. As seen in other Guastavino drawings, detailed in Chapter 4, Guastavino Jr. never builds a dome that is a perfect semicircle. He always begins the dome for a project with a perfect circle. However, if the dome is based on a circle that has a radius of 66 feet, the total height of the dome will only extend to 52 feet. By limiting the height of the dome, the maximum thrust at the base of the dome is averted. These rubric drawings taught Guastavino Jr. how to manipulate the design of a dome.

6.6 Conclusions

The rubric drawings present the design mentality of Guastavino Jr. and his process towards developing his design methodology for domes.

- The original principles are based on Eddy's method, republished by Dunn in 1904.
- It appears that Guastavino Jr. is learning how to analyze masonry domes.
- Guastavino Jr. calculates the amount a full dome, assuming it has no tensile capacity, splays at the base and develops new methods to design domes to avoid and cope with tensile forces.
- The metal quantities calculated in Rubric Drawing 1, Figure 6.1, do not appear to be accurate, but they are highly overestimated.

7 Analysis of the Cathedral of St. John the Divine - Dome Drawings

7.1 Cathedral of St. John the Divine Background

Guastavino Jr.'s involvement in the Cathedral of St. John the Divine, NY, NY (Figure 7.1) was highly publicized for the immensity of the dome and the innovative construction system. The Cathedral was in the process of construction around 1909. The middle of the church was planned to be enclosed with a tall spire, but high construction costs and time constraints halted the project. Guastavino Jr. proposed to enclose the approximately 98 feet square (132 feet diagonal) with a thin masonry dome instead. He was able to convince the church that his design would be faster and cheaper than any alternative. Guastavino assured a low price by claiming he does not need any scaffolding for the dome [Dugum, 2013]. Shockingly, the dome was meant to be a temporary construction to last only 10 years. At this point, it has lasted over 100 years [Ramazzotti, 2001].

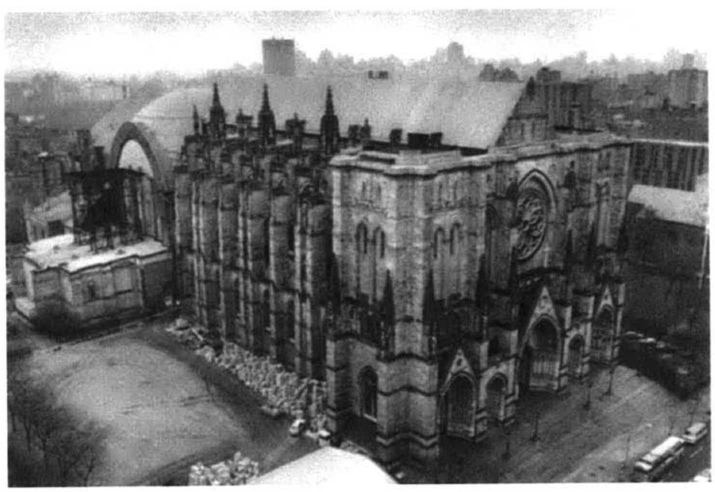


Figure 7.1: Cathedral of St. John the Divine Aerial View [Cathedral Images (1909)]

Guastavino Jr. was very proud of his achievements and publically praised the thinness of his dome in comparison to the largest domes in the world: the Duomo in Florence (139 feet), Pantheon, Rome (142 feet), St. Paul's, London (3 feet less), and the St. Sophia's mosque, Constantinople (115 feet), as seen in the same order in Figure 7.2 [Ochsendorf, 2010].

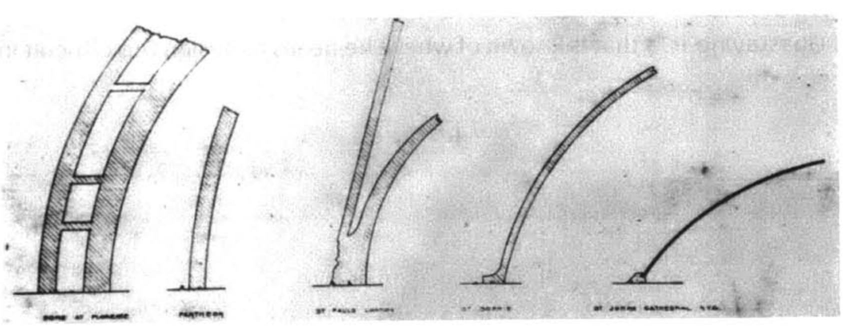


Figure 7.2: Dome Thickness Comparison Drawing (Appendix B) [Avery Library]

This dome is the first time the Company constructs with steel bars between tiles [Ochsendorf, 2010]. However, this is not the first time that Guastavino Jr. adds reinforcement along the curvature of the dome. He adds steel plates at various levels of the St. Francis de Sales dome in 1908, as presented in Chapter 4. In 1910, Guastavino Jr. patents the placement of steel within tile layers along a dome with an oculus, as seen in Figure 7.3. The patent shows steel at the top of the dome near the oculus and at four layers at the base of the dome. The layers are closer together at the base of the dome to handle the higher tensile hoop forces.

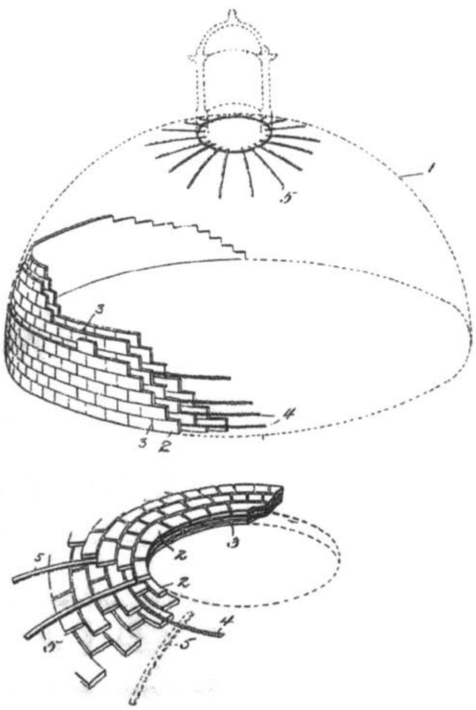


Figure 7.3: Guastavino Jr. Patent - Steel Placement in Dome (Appendix B) [Avery Library]

When Guastavino Jr. became involved with the project, the arches were already constructed in preparation for the spire, as seen in Figure 7.4 [Dugum, 2013]. The existing arches are the foundation of the design decisions for the geometry of the dome. The interface between the arches and the dome is difficult to design accurately without 3-dimensional modeling tools. Guastavino Jr. projects different views of the arches and the dome to understand the exact angles between the two structures. This is the first and only dome of Guastavino Jr.'s that is known of where he needs to design this difficult interface section.

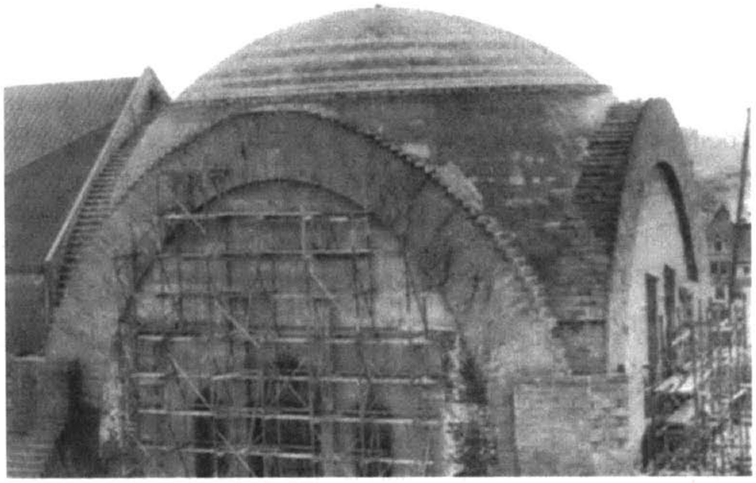


Figure 7.4: Cathedral of St. John the Divine Dome Elevation Photograph [Cathedral Images (1909)]

The Cathedral of St. John the Divine is the largest Guastavino Company dome. The dome is only 4 inches thick at the crown and it spans 132 feet diagonally. This extreme thinness and long span would be very difficult to achieve without proper calculations and a deep understanding of the forces. The two graphic statics calculation drawings behind the dome are an accomplishment for the Guastavino Company. The main drawing, Figure 7.6, shows the entire analysis of the dome. The second drawing, Figure 7.11, only shows the calculation of the dome thrust onto the arches. The first drawing is the most complicated graphic statics drawing found in the Guastavino Archives. The complexity stems from the preexisting arches. The drawing shows a new level of understanding tensile hoop forces in a dome. The construction technique was also publicized as a great new advancement in construction.

7.2 Explaining the Graphic Statics Drawing

The main drawing depicts many different views of the dome. The second drawing of the dome is a draft for the top view intersection of the arches with the dome. All of the views are necessary to visualize and design the dome and the interface of the dome with the arches. Figure 7.5 shows the 3-dimensional elevation of the dome. Figure 7.6 to Figure 7.11 illustrate the various views of the dome.

Each figure is outlined with a different color. Each color represents a view of the dome. The color of the overlaid lines correspond to the figure with the same colored outline. In each figure, there are multiple section cut lines. The color of each section line relates to the view of the figure with the same colored outline.

Figure 7.6 shows the diagonal top view of the arches and their intersection with the dome in purple. The lines extending from the middle of the drawing form the panels of the dome. The orange section cut is the front arch elevation view. The green section cut is the elevation view of the diagonal dome segment. The light blue section cut shows the elevation view of the straight segment of the dome.

Figure 7.7 represents the diagonal elevation view in green. It shows the diagonal segment of the dome, the meridional stresses, and the base arch viewed on a diagonal in green. The blue section cut is the projected lune of the dome. The purple section cut is the top view of the dome.

Figure 7.8 shows the straight elevation view of the dome in light blue. It shows the straight segment and the panels of the dome and the straight arch elevation. The blue section view is the projected lune. The orange section cut is the front arch elevation view. The purple section is the top view of the dome.

Figure 7.9 shows the front elevation view of the arch in orange. The light blue section cut shows the cross section of the arch. The purple section cut represents the top view of the arch.

Figure 7.10 is the projected lune segment of the dome in blue. The diagonal lune segment cut is shown in green and the straight lune segment section is shown in light blue.

Figure 7.11 is the second graphic statics drawing for the Cathedral. The section shown is the same as Figure 7.6, the top diagonal view of the dome in purple. The drawing shows the panel segments of the dome, each $1/20^{th}$ of the full dome. The light blue lines show the side elevation view of the arches, the same arch view as Figure 7.8 in light blue. The green section cut shows the diagonal segment of the dome. The light blue section cut shows the straight segment of the dome. The orange section cut is the front elevation view of the arches.

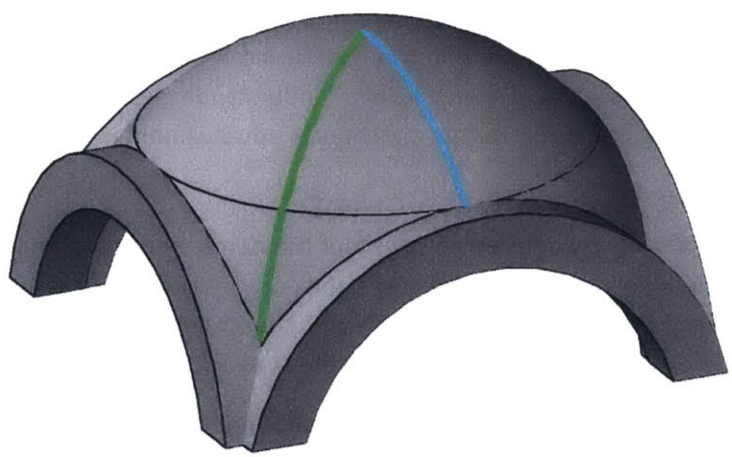


Figure 7.5: 3-Dimensional Elevation View of Cathedral Dome: Straight Segment (Blue), Diagonal Segment (Green)

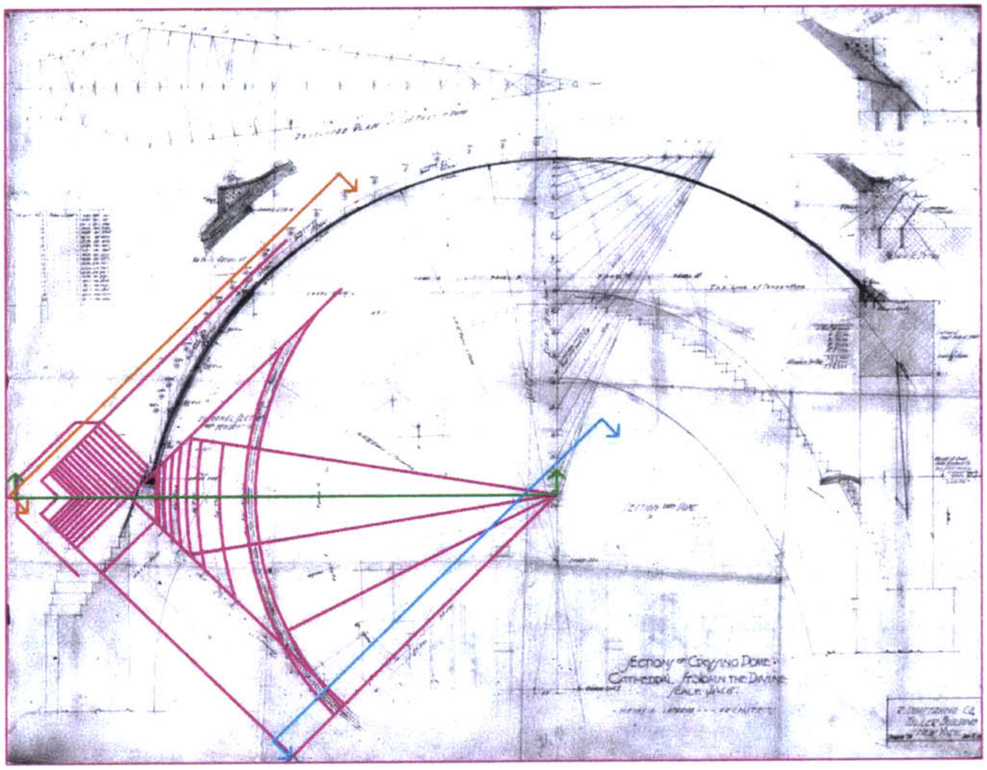


Figure 7.6: Cathedral Drawing - Top Diagonal View [Avery Library]

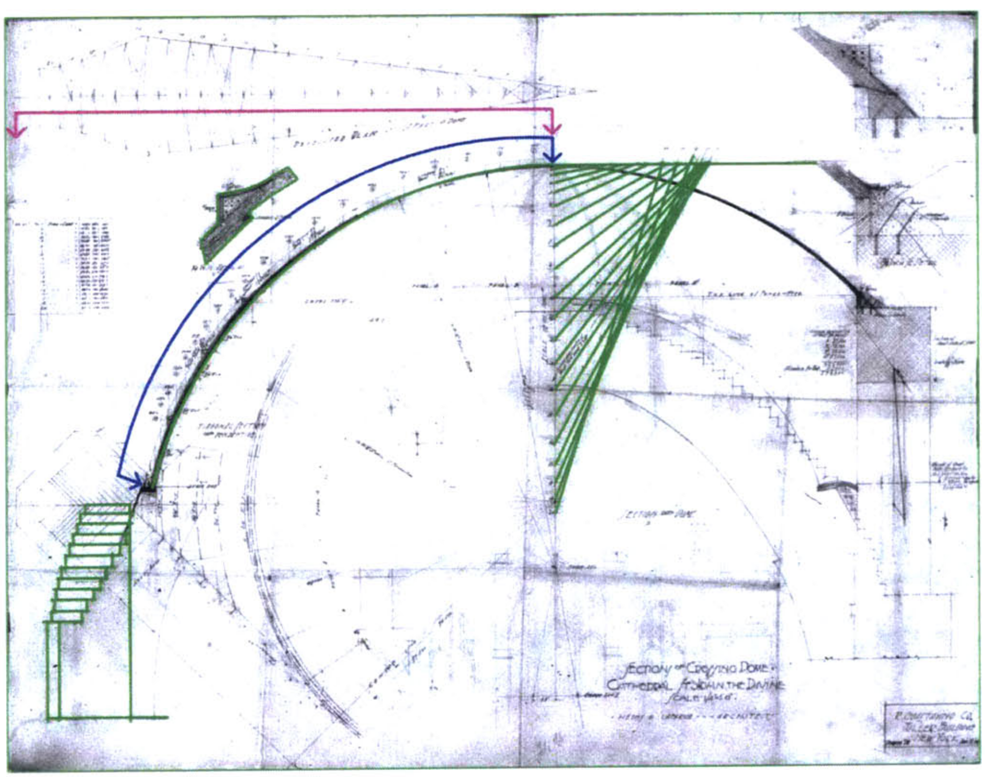


Figure 7.7: Cathedral Drawing - Diagonal Dome Elevation View [Avery Library]

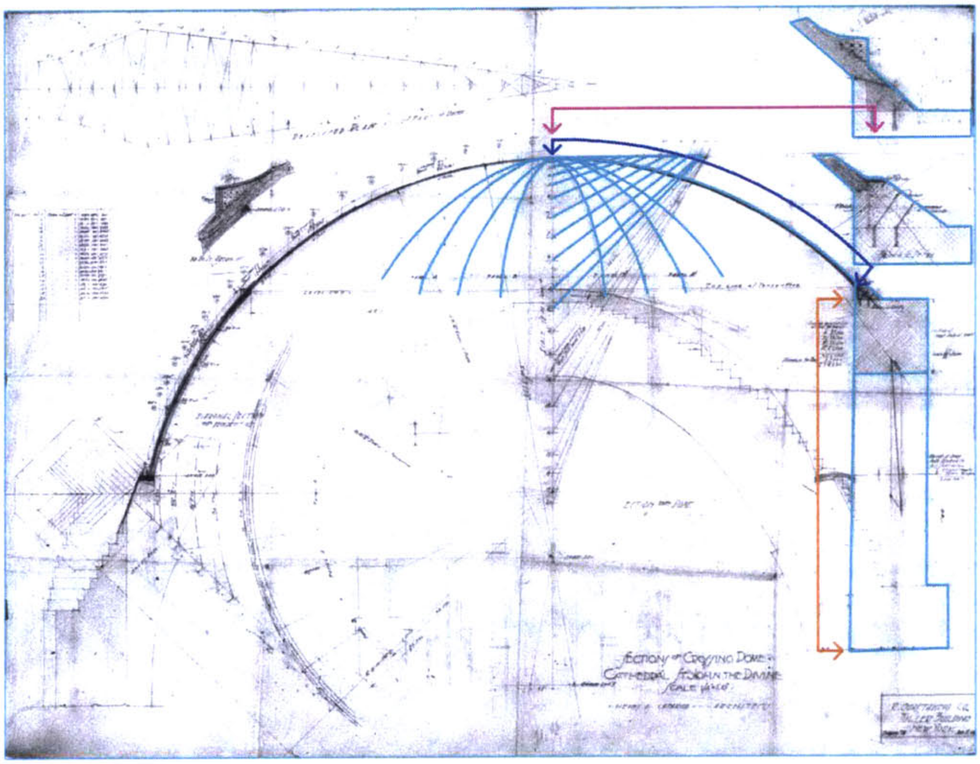


Figure 7.8: Cathedral Drawing - Straight Dome Elevation View [Avery Library]

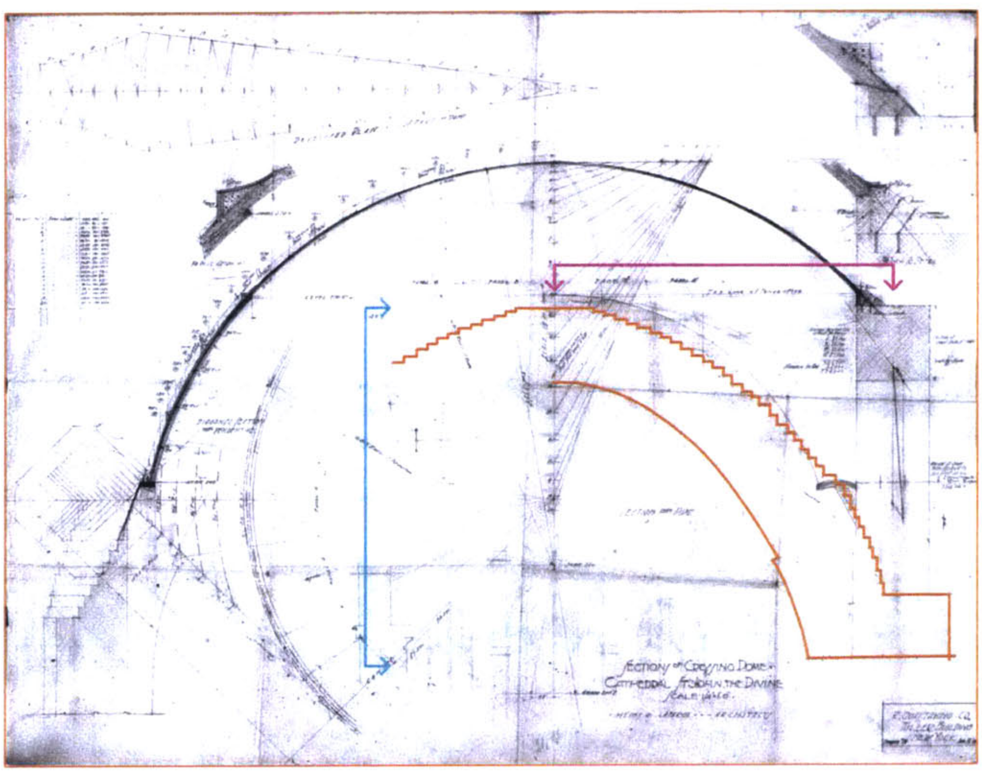


Figure 7.9: Cathedral Drawing - Front Arch Elevation View [Avery Library]

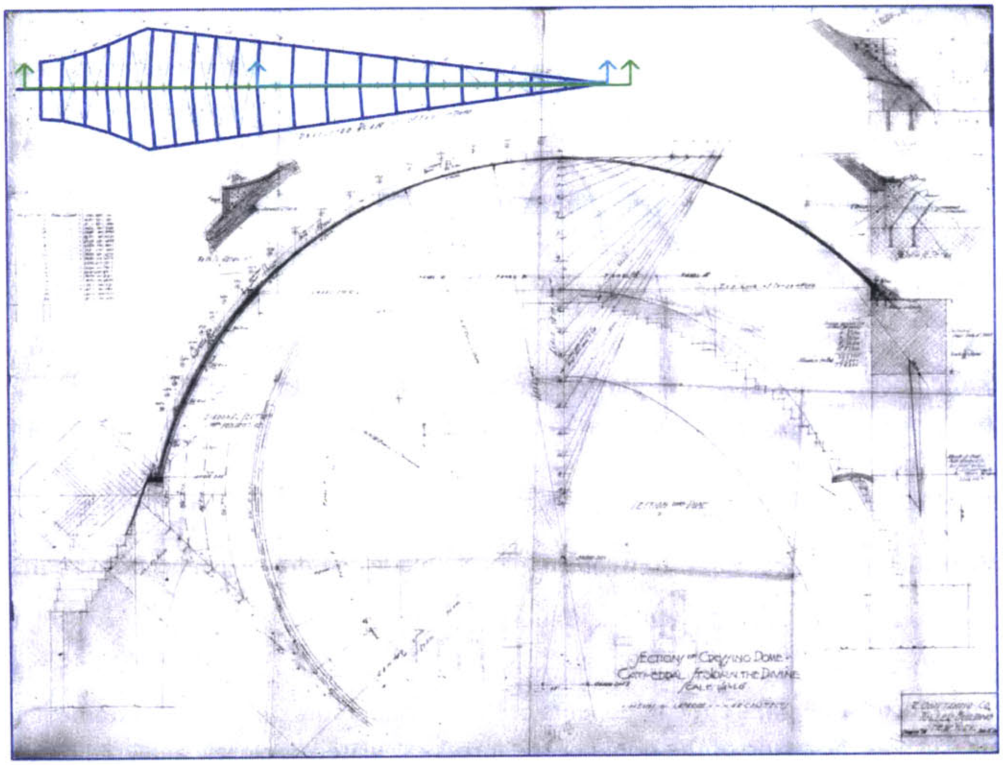


Figure 7.10: Cathedral Drawing - Projected Lune Section [Avery Library]

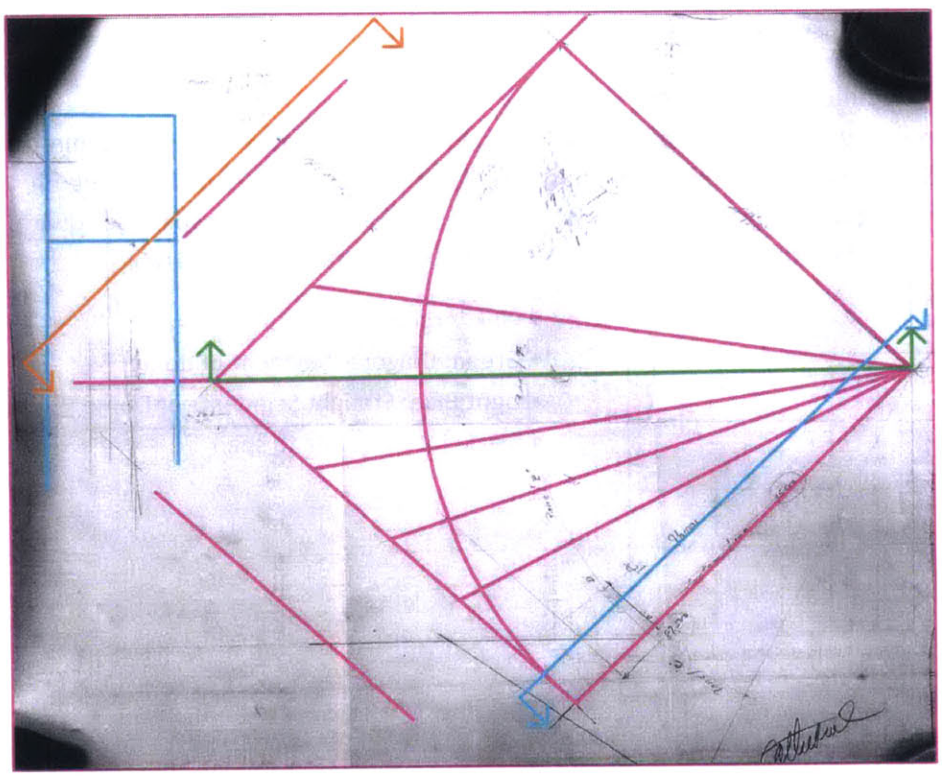


Figure 7.11: Cathedral Drawing 2 - Top View Dome Panel Division [Avery Library]

7.3 Objective

A significant amount of research has been done on the Cathedral history, geometry, and stability. However, no research has ever looked into the exact calculation techniques Guastavino Jr. used to design the dome. This thesis examines the following questions on the Cathedral of St. John the Divine:

- What are the assumptions behind the Guastavino Jr. analysis?
- What forces were calculated to design the Cathedral of St. John the Divine?
- What calculation methods did Guastavino Jr. use?

The analysis is based on the drawings attained from the Avery Library Archives. There are hundreds of drawings and documents archived on the Cathedral of St. John the Divine, but only three of them are stress diagrams, and only two of them pertain to the dome. The stress diagram archived is the final draft for the project. There must have been other working drawings to arrive at this final dome solution. The second drawing is on tracing paper and calculates the thrust from the dome on the arches.

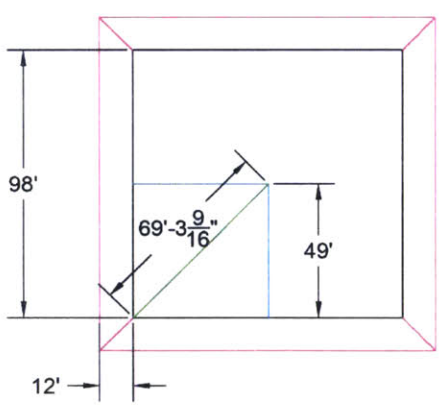
The design theory is established from information found in the drawings and from sources on the Cathedral.

7.4 Questions

This section focuses on different components of the graphic statics drawing for a dome. Each aspect of the design, such as the shape of the dome or the forces calculated, is closely analyzed on the original drawing. Many of the explanations cannot be verified on the drawing or with other sources. The design process presented is one possibility for the design of the dome. It is uncertain if this process was absolutely performed by the Guastavino Company.

❖ Previously Defined Parameters

Guastavino Jr. was not involved in the initial planning of the Cathedral. He was hired to enclose a square area, 98 feet each, defined by four massive pillars that were each 21 by 21 ft [Dugum, 2013]. There were already four grand arches (12 feet thick, 56 feet high) constructed that span the square. Guastavino Jr.'s task was to enclose the space, Figure 7.12, in the cheapest, fastest way possible.



- Pink: Arches
- Green: Diagonal Segment of dome
- Light Blue: Straight Segments of Dome
- Black: Interior Area to span

Figure 7.12: Cathedral Dome Layout Dimensions

❖ Panels

The dome is divided into 20 panels, each 18°. Three of those panels are unique. As seen in Figure 7.13, Panel C represents the diagonal segment of the dome, and Panel B represents the straight segment of the dome. These are the two panels that are designed for in the drawing. The dashed blue lines shown represent the location where the dome meets the arches. The dashed red lines represent the different segment divisions of the dome.

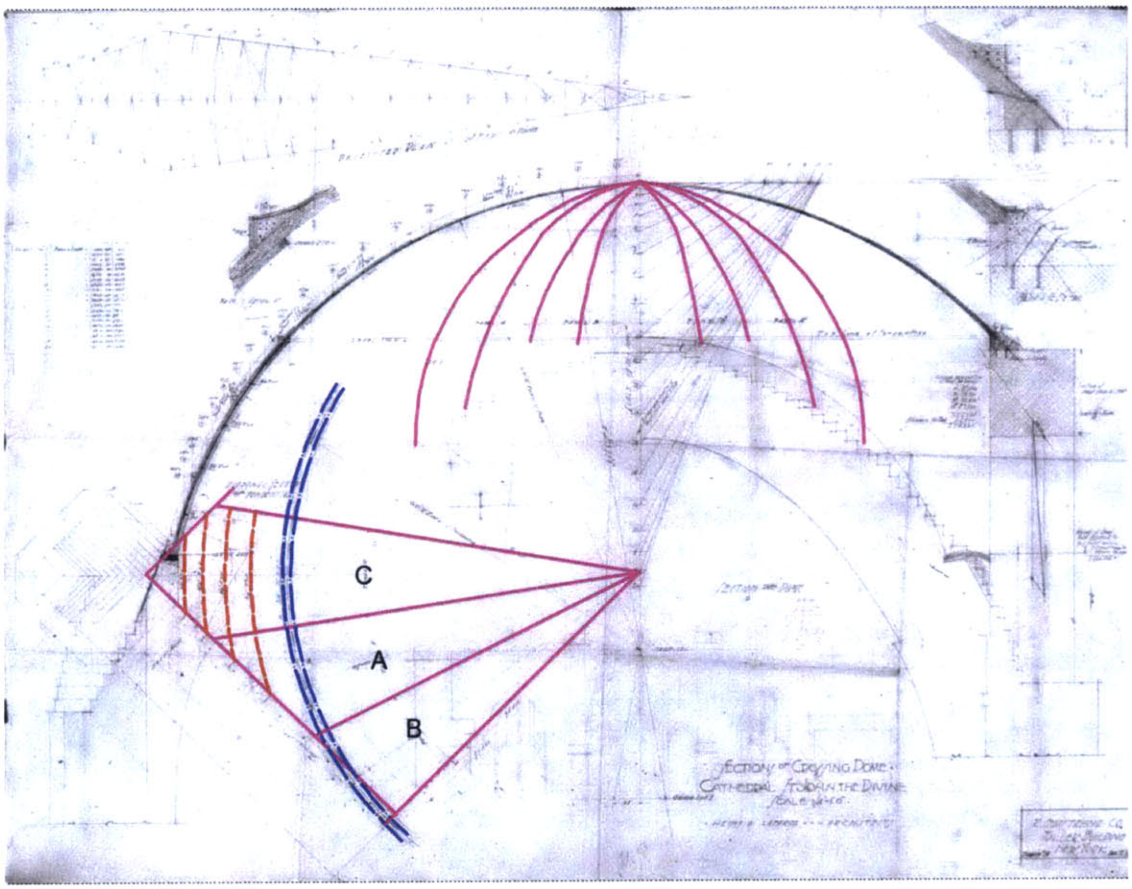


Figure 7.13: Cathedral: Panel (Purple lines) Layout (Dashed Blue Lines Represent the Top of Arches in Top View) [Avery Library]

❖ Shape

Guastavino Jr. went through several iterations to find the shape of this dome. Each step of the analysis is uncertain. However, a process theory is formed. The shape begins with a general circular curve. The thrust line is found for the dome assuming the horizontal thrust is limited to the compressive region of the dome. The shape and the thickness of brick layers is changed until the thrust lies within the thickness of the bricks. Then the thrust line is found assuming the horizontal thrust extends to the tensile region of the dome. The hoop forces can be found in the tensile region of the dome to determine the necessary amount of steel to constrain the thrust. The exact process of altering the radius for the dome is attempted in this thesis to arrive at the same dome layout that Guastavino Jr. found.

Unlike certain other Guastavino Company dome projects, the project began with the arches in place. The arches constrain the dimensions of the dome and define the location to direct the dome thrust.

The following process is one possibility. The first step was to find a dome that fit with the base arches. The dome is formed with two different spheres. The radius of the dome is such that the 51.8° latitude happens below the intersection of the dome with the arches. Guastavino Jr. wants to limit the tensile stresses in this region so he alters the curvature of the dome below the arches. The top sphere is based on a circle with radius 69'-3 9/16", as shown in Figure 7.12. The second sphere has a radius of approximately 66 feet. Each of these curvature follow the inside face of the dome well.

The transition in the spheres occurs at the top of the arches. The arches force the transition of the spheres. As seen in Figure 7.14, the sphere must fill and end on the inside face of the arches. If the radius of the first sphere (69' – 3 9/16") is kept constant, the base of the dome lands on the outer edge of the arches. By changing the curvature to a smaller circle, Guastavino Jr. redirects and limits the thrust to a better location on the arches. The thrust line for a dome with a top radius of 69'-3 9/16" and a base radius of 66'-0" is calculated. With this dome, the base thrust is over 2 feet. (See Figure C.4 in Appendix C) Therefore, Guastavino Jr. continues to alter the dome shape that has a more continuous curvature.

Figure 7.14: Cathedral Dome Curvature Photograph [Cathedral Images (1909)]

It seems that he chooses a central location and draws two circles: one that fits the curve of the previous top dome well and transitions smoothly into the base 66 feet circle. The radius for the top dome is 66'-6", as seen on the final drawing in Figure 7.15. The base dome becomes 66'-2" in the final drawing.

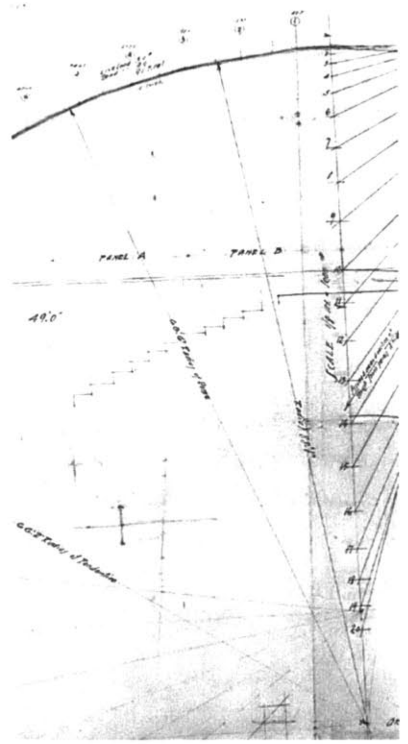


Figure 7.15: Cathedral - Original Drawing Final Radii [Avery Library]

The shape of the curve is difficult to determine. The following scenarios are tested to find the same meridional curves from the original drawing:

- Constant 69' - 3 9/16"
- Constant 66'-0"
- Top at 69' - 3 9/16", Bottom 66'-0"

None of the curves has the exact profile and meridional lines that Guastavino Jr. determined. The following analysis in this thesis transferred the meridional lines from Guastavino Jr.'s drawing to the curve of the dome. The lines are exactly the thrust line through the dome.

The tensile forces that form in the final dome are lower than the other scenarios examined. The *Forces* section compares the forces and thrusts from the dome scenarios. Guastavino Jr. skillfully manipulated the curvature and thickness of the dome to limit the tensile forces at the base of the dome.

Guastavino Jr. strategically decided on the height of the dome. The dome had to span 132 feet diagonally and fill in the space between the arches. The curvature of the second dome is based on a 66 foot sphere, but the actual height of the full dome is only 52 feet. Guastavino Jr. deliberately chose a shorter height to limit the thrust in the dome and to assure the dome intersects the top of the arches. Based on the Rubric Drawings in Chapter 5, Guastavino Jr. learned that the thrust at the base of the dome is the greatest. By limiting the height of the dome, he does not construct the very base of the 66 foot dome and therefore limits the thrust of the dome.

The tension region of the dome does not begin until the part of the dome between the arches. Guastavino Jr. chose this height to limit the tension to this portion of the dome spanning the arches. This is a conservative and smart design decision. The dome is not continuous at the base; it spans between the arches in segments. The total tensile thrusts from the dome do not develop in the same manner as a continuous dome.

The overall span of the dome at the base diagonal is 132 feet, approximately half of the second sphere radius (66 feet). Guastavino Jr. investigates the behavior of a spherical dome in the Rubric diagrams, detailed Chapter 6. The Pantheon in Rome also exhibits a spherical design and possibly influenced Guastavino Jr.'s design.

- Segments

One of the first decisions in constructing the force diagram is to determine the amount of segments for the arc of the dome. The circumference of the full diagonal arc is 90 feet. The arc length of the straight arc is 55 feet and the diagonal arc is 35 feet. Guastavino Jr. divides the dome into 20 segments based on the arc length of each segment, as seen in Figure 7.16. He decides to use the arc length instead of a common angle to divide the dome since the arc length dimension is later needed to find the hoop forces.

The straight arc has 10 segments, each 5'-6". The diagonal arc also has 10 segments, each 3'-6". The center point of the segments is the middle of the 66 foot base circle, not the base of the dome at 52 feet. The change in segments occurs at 48 degrees from the vertical, using the center of the sphere. Membrane theory states that the compression in a dome will change to tension at around 52°. Guastavino Jr. wants more details on the location of tensile stresses. He makes the second set of segments smaller. With smaller segments, Guastavino can place the steel more accurately in the dome. The center of gravity of each segment is found.

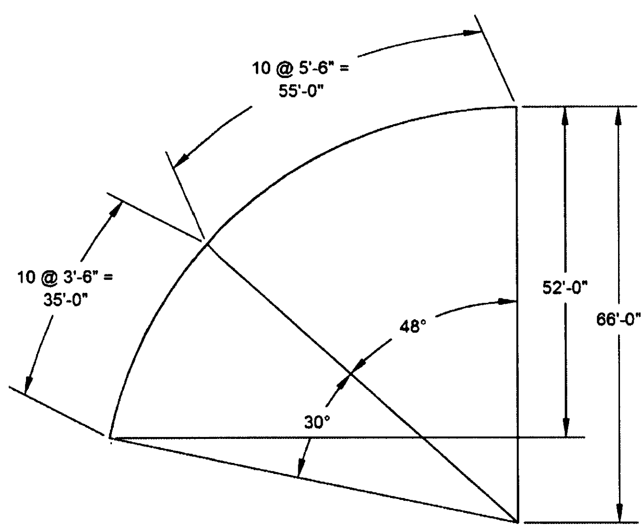


Figure 7.16: Cathedral Dome - Arc Segment Division (Drawing Based on Constant 66'-0" Circle)

The dimension of the segments 5'-6" and 3'-6" match the lines on the drawing very well. However, there is a note on the drawing stating that segment one has an arc length of 5'-9", instead of 5'-6". To arrive at the same total arc length, this value would make the second set of segments 3'-3" instead of 3'-6". The segments with 5'-9" and 3'-3" do not match the segment lines on the drawing as well as the previous set. This analysis uses the first set of segments since the segment lines match up accurately on the drawing.

❖ Lune

A lune is a projected top view of a dome slice that flattens the three-dimensional shape. It is used to determine the area of each section of the dome. There are 20 segments, 10 at 5'-6" and 10 at 3'-6". The lune is 1/20th of the full dome, or 18 degrees. Each lune segment is divided into a triangle to calculate the area and to locate the center of gravity of the lune. Guastavino Jr. recorded the areas he used for the analysis on his drawing, as seen in Figure 7.22.

Guastavino Jr. uses the same lune for the straight (light blue) and diagonal (green) portions of the dome, as seen on Figure 7.17. The straight portion ends at segment 10. The diagonal lune is difficult to construct without three-dimensional software. The intersection of the dome with the arches dictates the dimensions of the domes and areas for the lune segment. The geometry of the lune determines the areas accounted for in the graphic statics analysis.

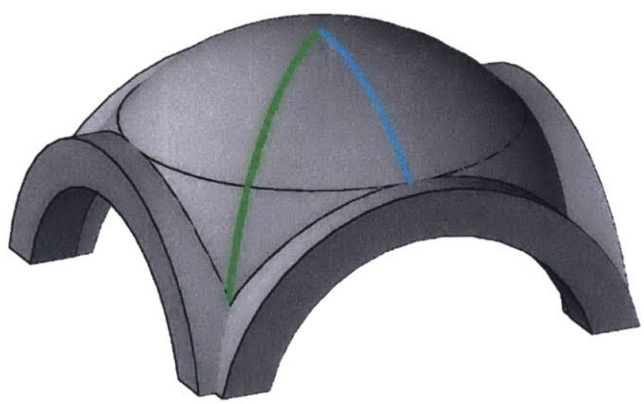


Figure 7.17: Cathedral 3-D Dome Model- Diagonal Arc Blue, Straight Arc Red

The lune that Guastavino Jr. uses on his drawing, shown in Figure 7.18, to get the areas of the segments is slightly smaller than 18°, most likely due to the inaccuracy of hand tools. The top view projection of the lune on the drawing is accurate, as seen in Figure 7.20. The lune in Figure 7.19 has the proper angle and the light blue lines to represent the dome projected onto the arches is the proper length as well. The strange jump in the lune is undetermined. For the rest of this analysis, the areas from the lune that Guastavino Jr. found are used to replicate his results.

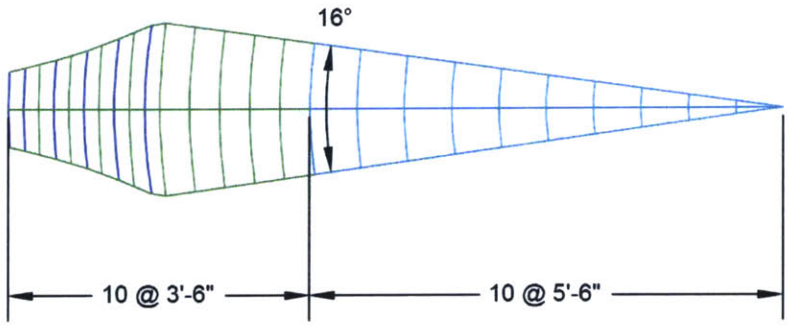


Figure 7.18: Cathedral - Projected Lune from Guastavino Original Drawing

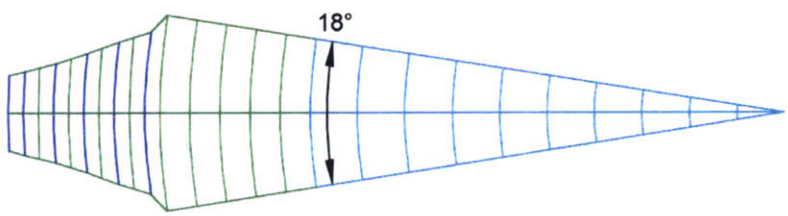


Figure 7.19: Cathedral - Projected Lune Precise Dimensions Compared to Original Drawing Lune

Guastavino Jr. projects the geometry of the arches onto the diagonal panel of the dome. Using the center of gravity of each segment, he finds the distance the dome spans between arches for each segment of the dome. These segments are then translated to the lune, as seen in Figure 7.20.

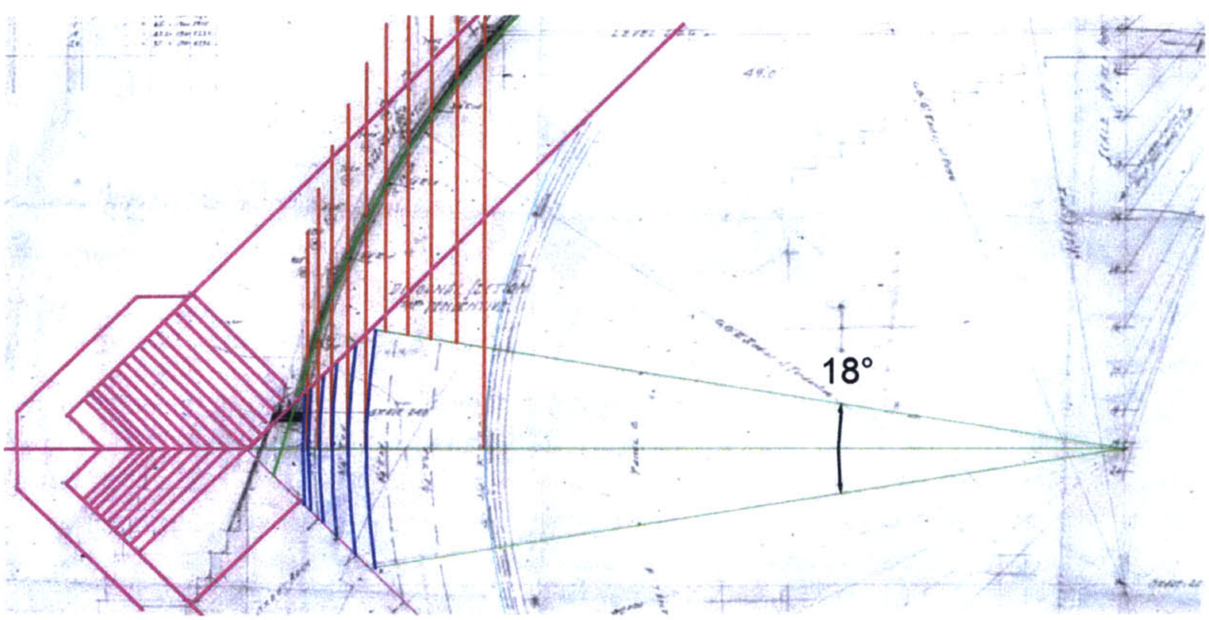


Figure 7.20: Cathedral - Original Drawing Lune Segment Calculation

❖ Load Line

The lune determined the areas of each segment. The load line represents the weight of each segment represented as vectors. The scale chosen for the load line is 1/8" = 1000 lbs. The weights are determined by multiplying the area of the segment times the distributed dead and live loads. The live and dead loads vary with the layers of bricks in that segment. Table 7.1 below summaries the loadings. The color of each segment title corresponds to the segments on Figure 7.21. In general, the live load considered was 50 psf. The dead load was the thickness of the brick times 10, for example a 4 in. thick segment had a dead load of 40 psf. Figure 7.21 shows the thickness distribution of the bricks on the arc segment of the dome.

Table 7.1: Cathedral - Dome Dead and Live Loads per Segment Division

Segment	Thickness (inches)	Dead Load (psf)	Live Load (psf)	Total Load (psf)
1 to 6	4	40	50	90
7 to 9	6	60	50	110
10 to 11	7 ½	70	50	120
12 to 20	12	100	30	130

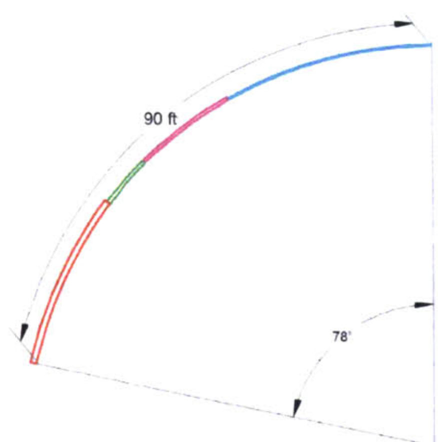


Figure 7.21: Cathedral - Dome Brick Thickness Division

The final loads for each segment are calculated using the total distributed loads and weights per segment. Table C.1 in Appendix C summarizes the areas, loads, and load line distances Guastavino used for the design. The loads from the original drawing are shown in Figure 7.22.

Weight No.	1.	Area x Load	-	4.50 x 90 =	405.
" "	2.	" "	-	13.75 x 90 =	1237.
	3.	" "	-	22. x 90 =	1980.
	4.	" "	-	30.25 x 90 =	2722.
	5.	" "	-	38.05 x 90 =	3465.
	6.	" "	-	46.75 x 90 =	4207.
	7.	" "	-	57.75 x 110 =	6352.
	8.	" "	-	63.25 x 110 =	6957.
	9.	" "	-	71.5 x 110 =	7865.
	10.	" "	-	79.75 x 120 =	9570.
	11.	" "	-	54.25 x 120 =	6510.
	12.	" "	-	57.75 x 130 =	7507.
	13	" "	-	61.25 x 130 =	7962.
	14	" "	-	64.75 x 130 =	8417.
	15	" "	-	67.55 x 130 =	8781.
	16	" "	-	66.5 x 130 =	8645.
	17	" "	-	56. x 130 =	7280.
	18	" "	-	45.5 x 130 =	5915.
	19	" "	-	40.2 x 130 =	5220.
	20.	" "	-	35. x 130 =	4550.

Figure 7.22: Cathedral - Original Drawing Dome Segment Area Calculations

❖ Forces

Guastavino Jr. designs the dome by considering the amount the dome thrusts outwards at the base. Eddy's method assumes that the maximum horizontal thrust in a dome is limited to the compressive region of the dome. Using this principle, Guastavino Jr. finds the amount the dome thrusts outwards. He alters the layers of bricks to redistribute the weight of the dome until the dome thrust at the base is contained in the bounds of the dome. Then he recalculates the meridional and hoop forces in the dome assuming the maximum horizontal thrust extends into the tensile region of the dome. He uses these hoop forces to calculate the steel reinforcement necessary to handle the thrust in the dome. This analysis is only shown for the final brick arrangement of the dome in Figure 7.25. The lines replicate the dome Guastavino Jr. has in the final drawing.

Meridional and hoop forces are calculated using the load line, lune section, and the geometry of the dome. To calculate the meridional forces, each segment of the arc is redrawn on its' corresponding location on the load line and extended to the top horizontal line. For example in Figure 7.23, to calculate the meridional force of segment 5 (blue), the average length of meridional lines 4 (blue) and 5 (purple) are taken. Then, they are divided by the average length of the respective lines of the lune (average of W5 and W4). The meridional stresses are recorded as pounds per linear foot.

To calculate the hoop forces, a line is drawn perpendicular to each segment of the lune. Each segment has two perpendicular lines. These lines are then transferred to the top horizontal line of the force diagram. For example, to calculate the hoop force of segment 5 (purple), the line perpendicular to the lune at segment 5 is transferred to the intersection of M4 (blue) and the horizontal thrust line. The other perpendicular line is transferred to the intersection of M5 (purple) and the thrust line. The length of the line where the two hoop force lines (purple H5) intersect is the hoop force for segment 5. The hoop force for segment 4 is shown as well (blue).

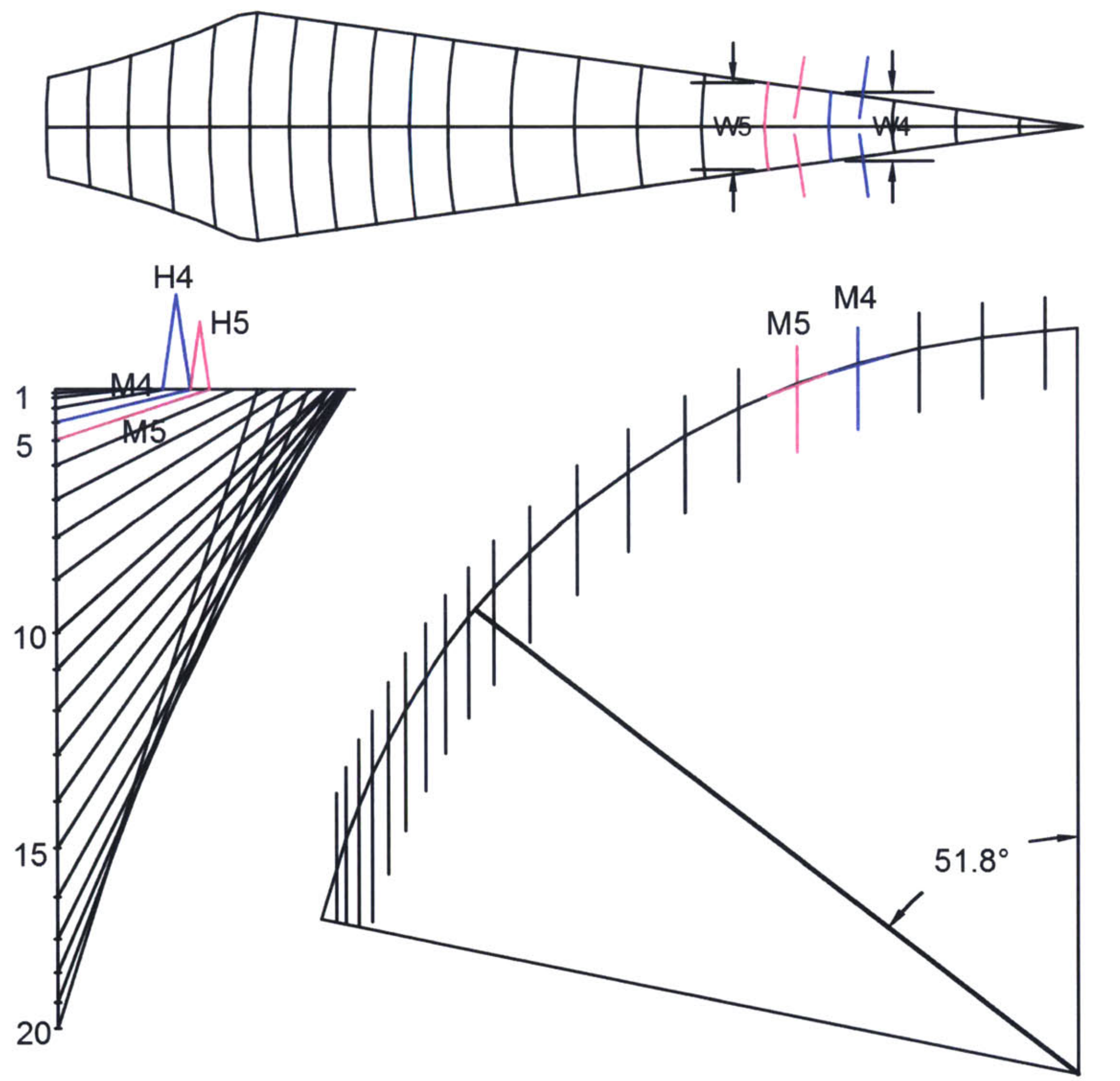


Figure 7.23: Cathedral - Meridional and Hoop Force Graphic Statics Calculations for Segment 5

The meridional and hoop forces and stresses for Guastavino Jr.'s dome are recorded in Tables C.2, C.3, and C.4 in Appendix C.

Limit state analysis assumes masonry has no tensile capacity and as a result, the base of the dome splays outwards [Heyman, 1995]. According to membrane theory, tensile forces begin in a dome at 51.8°. Guastavino Jr. first finds the amount the dome thrusts outwards limiting the maximum horizontal thrust of the dome to the compressive region. The following load diagram determines the horizontal thrust. It can be seen that the modified thrust line leaves the dome by 1'-7". The thickness of the dome at this point is 1'-0", which does not contain the thrust line. However, the dome meets the piers at this point which act as abutments and will keep the dome from thrusting outwards. As seen on Figure 7.24, the base of the dome follows this thrust line into the abutment.

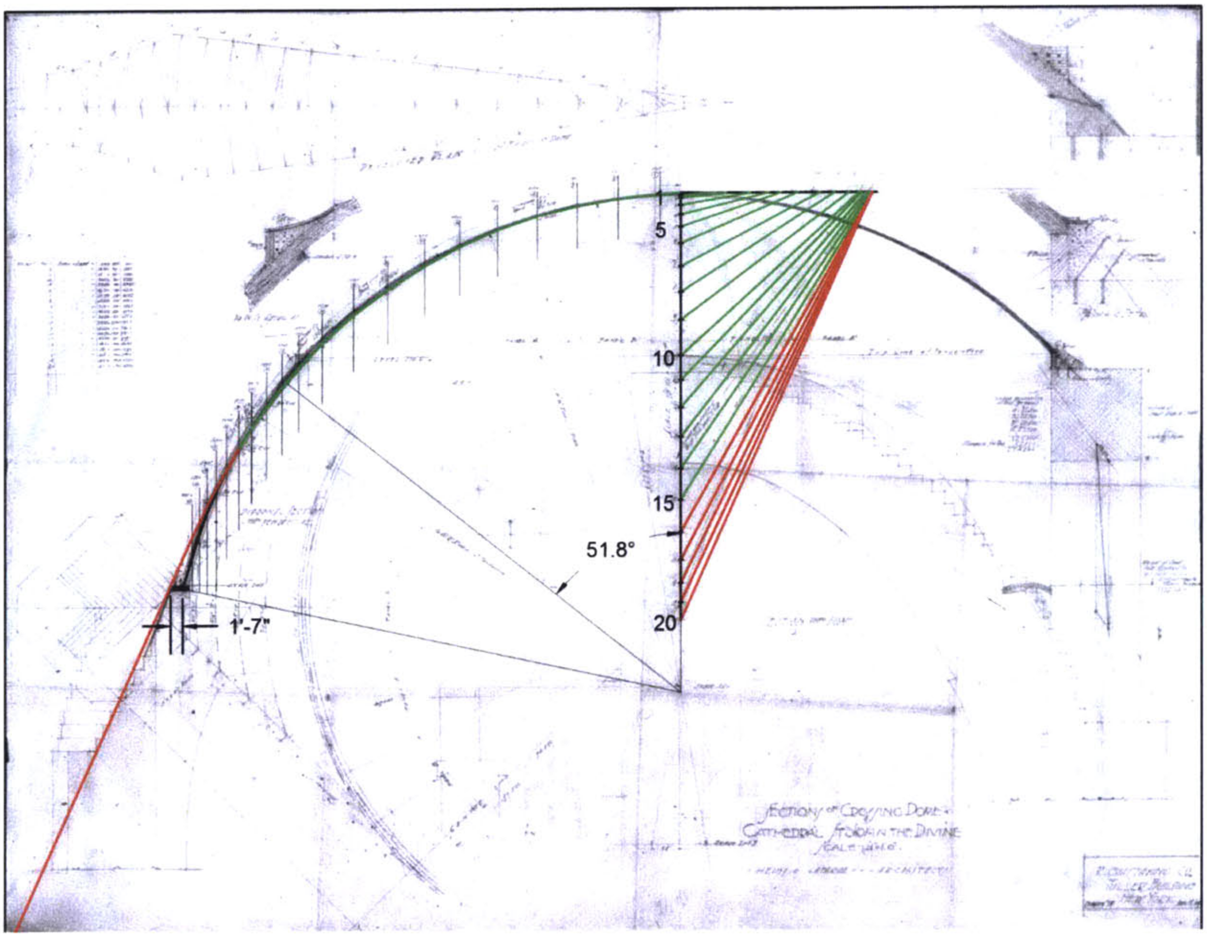


Figure 7.24: Cathedral - Dome Original Drawing Thrust Line Calculation, Limiting Max. Horizontal Thrust to Compressive Region

Once the final brick arrangement is chosen, Guastavino Jr. reiterates the design allowing tensile forces to contribute to the horizontal thrust of the dome. Tensile hoop forces first form in segment 11 at 51.8°, as seen in the force diagram where the purple line crosses the meridional lines of the previous segment in Figure 7.25. Hoop force calculations are used to determine the amount of steel necessary to resist the tensile forces. Steel requirements are summarized in the following section.

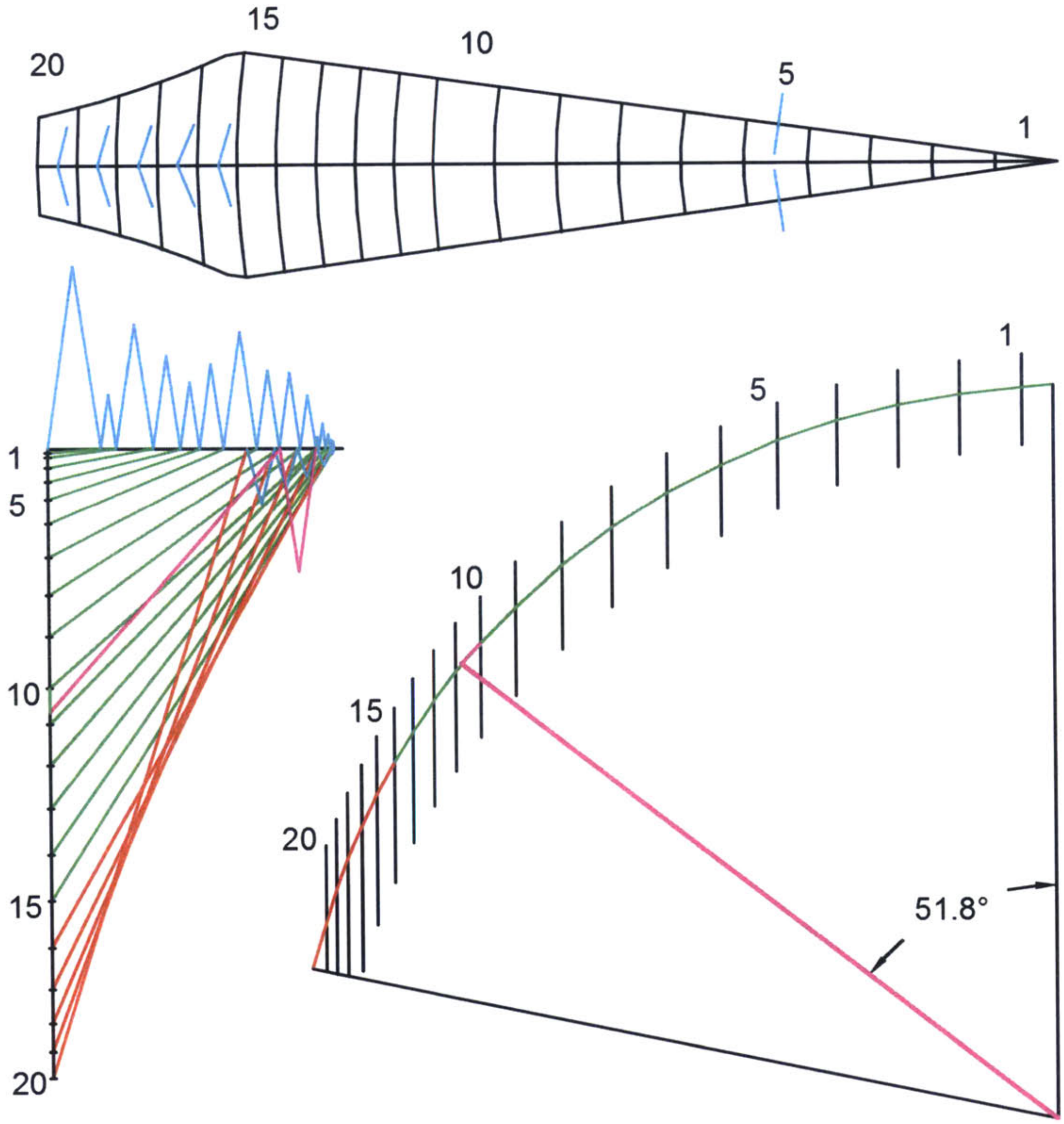


Figure 7.25: Cathedral - Dome Meridional and Hoop Force Calculations with Full Horizontal Thrust

❖ Comparison to Drawing Values:

The original drawing has several annotations that proves Guastavino Jr. completed a meridional and hoop force analysis.

In Figure 7.26, Guastavino found the "Average Meridional Stress between 10" to be 33 psi, with a 44,760 lbs meridional force acting over a lune width of 15' and a 7.5" brick layer. This analysis found the stress to be 46 psi, with a 60,000 lbs meridional force acting over a lune width of 14.6'. The discrepancies lie in the precision of Guastavino Jr.'s tools. This analysis used AutoCAD to draw and measure.

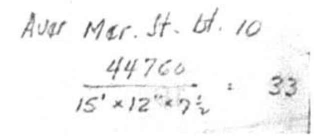


Figure 7.26: Cathedral Drawing - Meridional Stress Calculation Comparison [Avery Library]

In Figure 7.27, Guastavino found the "Average Unit Compressive Stress [for] Segment #1" to be 89 psi, with a 24,500 lbs hoop force acting over a dome segment of 5'-9" and a 4" brick layer. This analysis found the compressive stress in segment 1 to be 126 psi, with a 33,000 lbs hoop force acting over a dome segment of 5'-6" and a 4" brick layer. This note on the drawing was significant to determine the segment division for the dome. The 5'-9" value does not accurately correspond to every line on Guastavino Jr.'s drawing. With the 5'-6" value used in this analysis, the lines are much closer to the drawings'. Again, the discrepancies lie in the low precision of the tools.

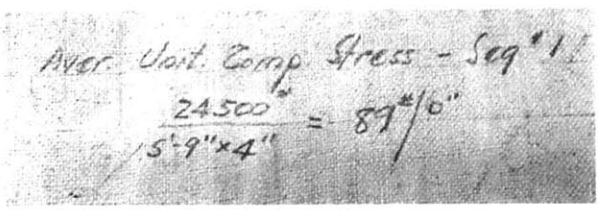


Figure 7.27: Cathedral Drawing - Hoop Stress Calculation Comparison [Avery Library]

The hoop forces can be clearly seen in Figure 7.28. Guastavino finds the hoop force for segment 8 to be 15 kips, this analysis finds 14.2 kips.

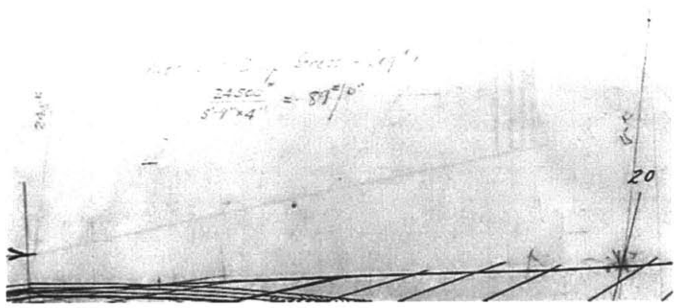


Figure 7.28: Cathedral Drawing - Hoop Stress Calculation Comparison 2 [Avery Library]

Guastavino clearly used the hoop forces to calculate the amount of steel necessary to restrain the dome from thrusting outwards, as seen in Figure 7.29. At segment 17, Guastavino notes a force of 3000 lbs. This analysis finds the force around segment 17 to be 3350 lbs. At segment 19, Guastavino notes a force of 8000 lbs. This analysis find the force around segment 19 to be 7400 lbs. At segments 13 and 15, Guastavino wrote a 0, but placed a ¾" rod at this location anyway. This analysis does not find any tension at segments 13 or 15 either. Guastavino placed steel at this location of the dome since it is after 51.8°. He knew that tensile forces could form at this location, even if his calculations didn't precisely reflect it.

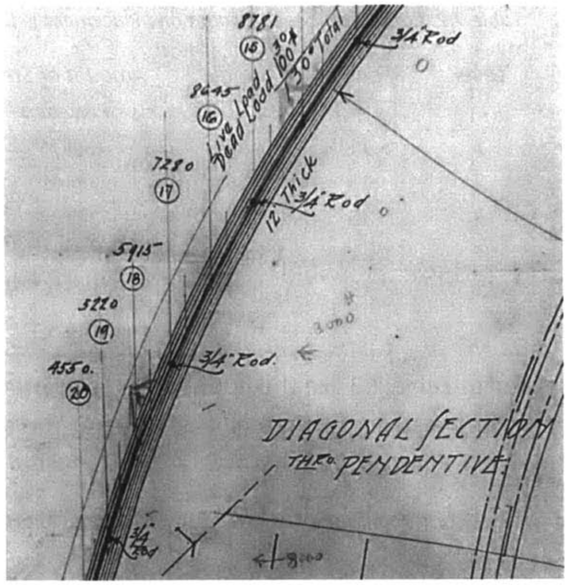


Figure 7.29: Cathedral Drawing - Hoop Stress Values at Dome Base [Avery Library]

The values found on the drawing verify the type of analysis Guastavino Jr. performed. The lines that are traced are very faint and difficult to trace properly at times. This is also a source of error in comparing the calculations.

❖ Steel Placement

Steel is placed at five levels in the dome. The amount of steel is calculated from the hoop forces. As mentioned in the Forces Section, tensile forces begin to form in the dome at 51.8°. Guastavino placed five layers of ¾" rods at the base of the dome, as illustrated in Figure 7.30 and detailed in Table 7.2.

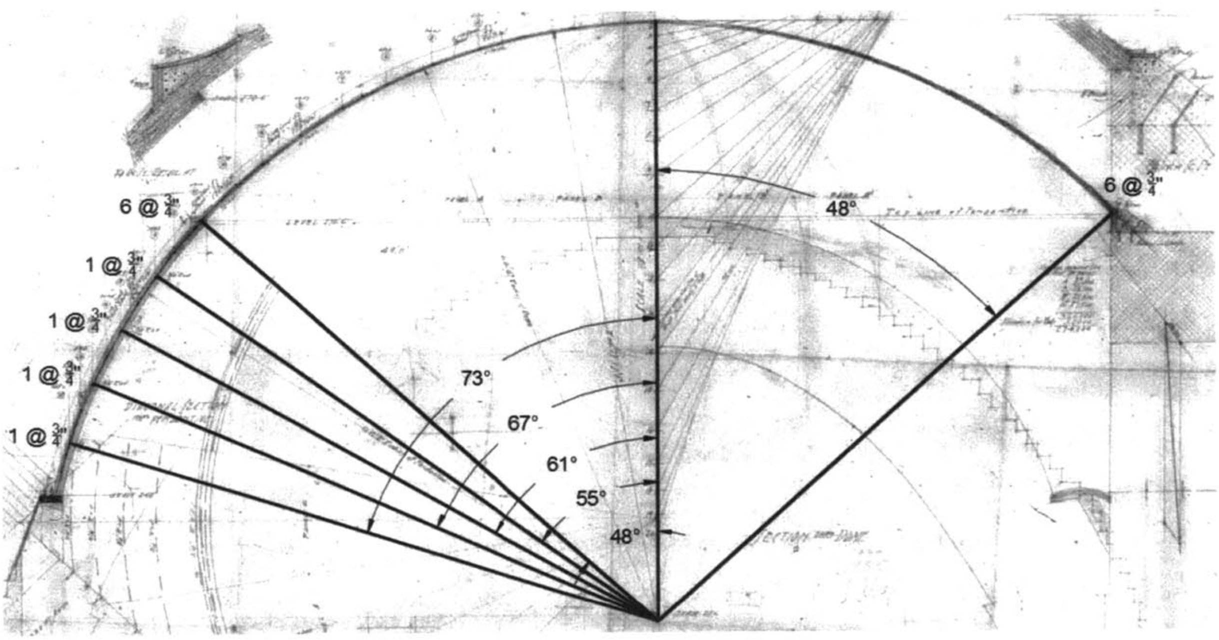


Figure 7.30: Cathedral Drawing - Steel Amount and Placement in Dome [Avery Library]

Table 7.2: Cathedral - Steel Amount and Placement in Dome

Level	*Segment*	*Angle*	*Amount of Steel Provided*
1	11	48°	6 @ ¾"
2	13	55°	1@ ¾"
3	15	61°	1@ ¾"
4	17	67°	1@ ¾"
5	19	73°	1@ ¾"

The tensile forces provided in Table 7.3 are the hoop forces calculated in the *Forces* Section. The area of steel needed assumes the tensile strength of steel is 52,000 psi. The tensile strength of steel is based on a note on the original drawing, Figure 7.31.

Table 7.3: Cathedral - Hoop Stress Values and Steel Necessity Compared to Existing Steel at Base of Dome

Segment	*Angle*	*Force (lbs)*	*Tensile Stress (psi)*	*Diameter of Steel Necessary (in)*	*Diameter of Steel Provided*	*Safety Factor*
11	48°	2100	0	0	4.5	6
T/C	51.8°	-23,000	113	0.75	-	
13	55°	2600	0	0	-	
15	61°	1370	0	0	0.75	4.6
16	64°	-1100	2.2	0.16	-	
17	67°	-3500	6.6	0.29	0.75	1.2
18	70°	-5500	11	0.37	-	
19	73°	-7400	15	0.43	0.75	0.8
20	76°	-12,200	20.3	0.50	-	

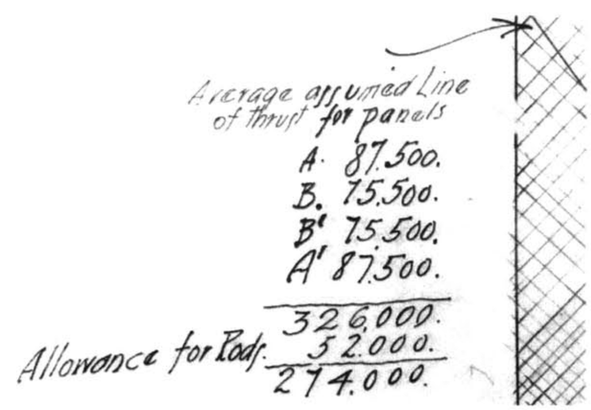


Figure 7.31: Cathedral Drawing - Steel Strength for Rods Example [Avery Library]

The radius of steel necessary calculated in Table 7.3 assumes the steel rods are circular. The safety factor calculated assumes the ¾" rod of steel takes the stress from two consecutive segments. Based on these calculations, we find that Guastavino Jr. possibly did not apply much of a safety factor to the amount of steel needed towards the base of the dome. The maximum tensile force is at the base of the dome. It can be assumed that Guastavino did not provide more steel at the base since the dome was constrained at this point by the piers and the arches. Also, this lower portion of the dome is not a full dome. These are the pendentive sections. The hoop forces would not be as large since there is no continuous hoop. The dome is restrained at these levels by massive arches that take the thrust from the pendentives. As seen in Figure 7.32, Guastavino assumes a tensile force of 5000 lbs at the base level, while this analysis finds 12,200 lbs. It is not certain what this value is based on.

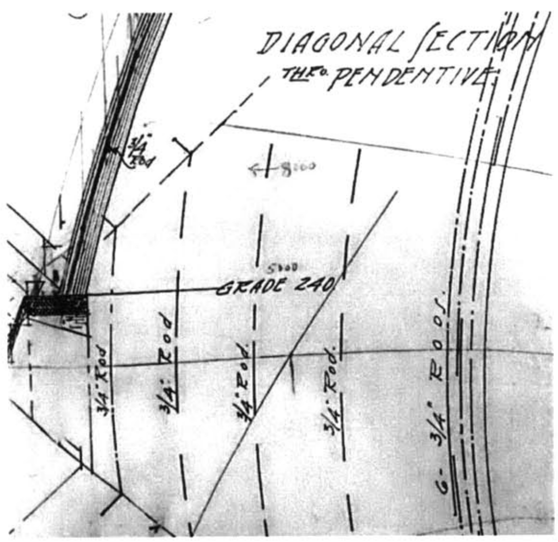


Figure 7.32: Cathedral Drawing - Hoop Force Value at Base Level [Avery Library]

Another possibility for not placing more steel towards the base is that the dome was supposed to be a temporary construction. The strength of steel taken into account for this analysis is somewhat arbitrary; it is uncertain the strength of steel he was using. The results from this analysis may be inaccurate since the strength of steel he was using is not absolutely verified.

There is a large amount of steel, 6 rods at ¾" each, at 48°. This is the location where the top dome joins the arches. The steel is located in the compressive portion of the dome. However, the transition to tensile forces begins at 51.8° shortly after. The tensile force at this level is very significant, 23,000 lbs. This amount of tensile force is almost three times higher than most of the tensile forces at other locations in the dome.

Another possibility for the amount of steel is to act as a steel band for the top part of the dome. Even though Guastavino Jr. analyses the diagonal segment of the dome as somewhat of a full dome, the system changes entirely at the top of the arches. The base of the dome is restrained between arches. The hoop forces are not continuous as in the top of the dome. If the top dome is treated as its own system, the large amount of steel at the base of the dome is a steel band. A calculation for ring stress is seen in other Guastavino Company projects, as detailed in Chapter 4. As seen in Figure 7.33, only the straight portion of the dome, or top part of the dome, is analyzed. The force for the steel band is 170,000 lbs, the large hoop force seen. The Guastavino Company process behind calculation ring stresses is undetermined. The St. John's R.C. Church dome drawing in Chapter 4 is a good example to further analyze to determine the ring stress calculation. The steel area required to contain this force, using 52,000 psi for the strength of steel, is 3.26 in^2 of steel, or a solid 2 inch diameter steel rod. The drawing shows 4.5 inches of solid steel placed. This is possibly the reasoning behind the steel placement, with a safety factor of over 2.

Dugum presents a theory in his thesis on the steel placement at this level. Dugum states that it is possible that during construction, construction workers needed to be supported by the dome before it was completed [Dugum, 2013]. It is possible that this is an added benefit of the steel, however this large amount of steel would not be needed to support a construction worker on the dome. The steel was likely placed to serve a greater structural purpose.

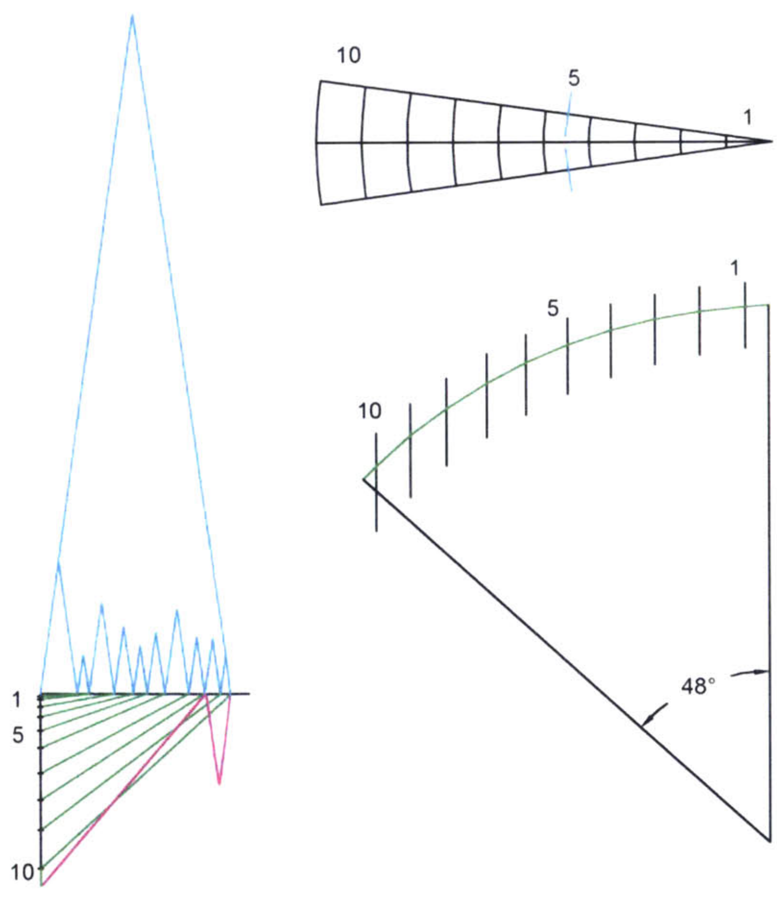


Figure 7.33: Cathedral - Meridional, Hoop Force, and Ring Stress Calculations for Straight Dome Segment

❖ Brick Layers

The drawing presented is the only stress diagram found for the Cathedral of St. John the Divine. It is likely that there were more working drawings before this final drawing that show Guastavino Jr.'s design iterations to achieve this final shape and brick layout. To determine a process behind Guastavino Jr.'s brick thickness, two different brick arrangements were analyzed. The loads, load line, meridional forces, hoop forces, force line, and steel placement must be recalculated with each iteration of dome thickness. It is a very tedious process, especially calculating the loads by hand.

The theoretical design process to choose a brick thickness is as follows:

- Choose a dome geometry and a constant brick thickness
- Calculate loads, areas, and completely a hoop and meridional stress analysis
- Increase the brick thickness where the tensile forces begin and decrease the brick thickness in the compressive region at the top
- Recalculate forces and stresses
- Compare results
- Continue iterations until:
    - The forces are reasonably distributed
    - The meridional stress thrust line, assuming the maximum horizontal thrust is in the compressive region, stays within the thickness of the dome and abutments.

For this procedure, a constant 66'-0" radius dome is used for simplicity. The shape is not exactly the same as the dome on Guastavino Jr.'s final drawing. The segment spacing is also slightly different, at 5'-9" and 3'-3". Therefore, the final arrangement design stresses are different from the design stresses presented earlier in this section. The stresses presented in this section serve to show that the stresses vary greatly based on the brick layout and brick thicknesses.

The first brick thickness is a constant 6 inches. The next iteration uses 6 inches and 12 inches for the lower portion of the dome.

For the constant 6 inch brick layout, the compressive stress in the first segment is 96.8 psi and in the last segment, 140.5 psi. For an evenly split 6 inch and 12 inch brick layout, the first compressive stress is still 96.8 psi, but the stress in the last segment becomes 77.6 psi. Adding thickness to the base of the dome greatly reduces the meridional stresses. The final dome layout achieves 118.8 psi and 76.6 psi, respectively. The first compressive stress increases, however since this region is in compression the bricks can easily handle it. The goal is to develop a brick arrangement to lower the stresses in the tensile region at the base of the dome. Guastavino Jr. achieved this by reducing the thickness in the highly compressive region, and increasing the thickness at the base of the dome to contain the thrust.

The hoop stresses show the same distribution of forces as the meridional stresses. The minimum and maximum stresses for hoop and meridional stresses are presented in Table 7.4.

Table 7.4: Cathedral - Meridional and Hoop Stress Calculations for Varying Brick Thickness (6"; 6" and 12"; Final Layout)

Segment	*Meridional (6") (psi)*	*Meridional (6", 12") (psi)*	*Meridional (Final Layout) (psi)*	*Hoop (6") (psi)*	*Hoop (6", 12") (psi)*	*Hoop (Final Layout) (psi)*
1	**96.8**	**96.8**	**118.8**	**92.5**	**92.5**	113.5
20	140.5	77.6	76.6	-87.4	-47.4	-24.3

Tables C.5 and C.6 in Appendix C present the meridional and hoop stresses for each segment of the dome.

The hoop stresses are reduced at the base of the dome with each iteration. The constant 6 inch thickness iteration is the only iteration that shows tensile forces forming in Segment 11. Tensile hoop forces exist at Segment 11 for every dome in this analysis since 51.8° occurs in this segment. However, since the segment break is not exactly at 51.8°, the tensile hoop force is not always seen. Since the forces are higher in the thin dome, the tensile stresses govern that region. The amount of steel needed to reduce the thrust in the dome varies with the thicknesses of the bricks as well. Guastavino minimizes the amount of steel necessary with the iterations. Since the hoop stresses are decreasing the dome, the amount of steel needed decreases as well. In the final dome layout needs the least amount of steel. The results can be found in Table C.7 in Appendix C.

The amount the thrust line deviates at the base of the dome, assuming the maximum horizontal thrust is constrained to the compressive region of the dome, decreases with each brick layout iteration, as seen in Figure 7.34. The red lines represent the tensile forces. The green lines represent the compressive forces. The perpendicular line through the dome represents the tensile to compressive force boundary (51.8°) on the dome. The final design achieves the smallest thrust displacement.

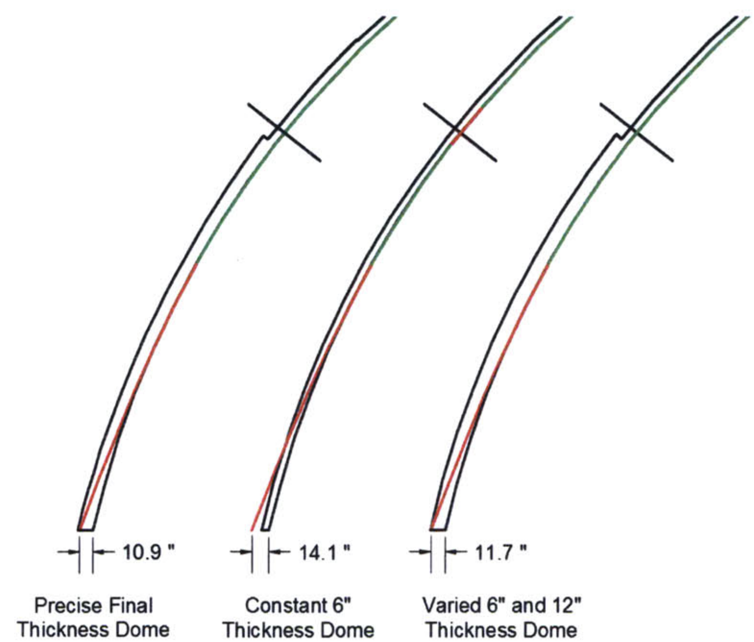


Figure 7.34: Cathedral - Thrust Line Deviation at Base of Dome for Various Brick Thicknesses (Final Layout; 6"; 6" and 12")

The results in this section are for a constant 66'-0" diameter circle for the dome and the segment sizes are 5'-9" and 3'-3". This is not the exact arrangement of the final dome as seen in Guastavino Jr.'s final drawing. The purpose of this section is to illustrate that the stresses vary with the thickness and arrangement of the bricks.

❖ Dome Shape Sphere Comparisons

As mentioned in the *Shape* section, the base sphere for the dome shape is undetermined. Four different spheres were considered. The force diagram and the deviation of the thrust line from the base of the dome are compared for each arrangement. The results can be seen in Figures C.1, C.2, C.3, and C.4 in Appendix C.

- Shape 1 (S1): Constant 69'-3 9/16"
- Shape 2 (S2): Constant 66'-0"
- Shape 3 (S3): Top at 69'-3 9/16", Bottom 66'-0"
- Shape 4 (S4): Trace of Final Drawing Meridional Lines

The comparison shows that the constant 69'-3 9/16" (S1) dome radius thrusts approximately 1'-1". Tensile forces begin to form at segment 15.

When the base of the dome is changed to a 66'-0" radius (S3), the thrust becomes 2'-1'. Since the dome is less continuous, the thrust greatly increases. The entire base of the dome is in tension, beginning at segment 11.

The constant 66'-0" dome (S2) only has a base thrust of 10". However, the thrust line enters the inside face of the dome, creating an unstable dome.

The final drawing arrangement (S4) thrusts 1'-7". Even though the base thrust of S1 is less, this dome does not fit within the arch constraints, as mentioned in the *Shape* section. Therefore, through several meticulous iterations, Guastavino Jr. arrives at a dome shape that has the least tensile forces forming throughout the dome.

❖ Construction

The construction of the dome occurs incredibly fast. It began on May 1st, 1909 and ended just three months and sixteen days later on August 16th, 1909 [Ramazzotti, 2001]. The main reason the construction can take place so quickly is that scaffolding is not necessary for the placement of each brick on the dome. Instead, Guastavino Jr. came up with an ingenious construction system that assures the accurate placement of each brick on the dome.

The stability and performance of the dome is based on the calculated curvature. The precise placement of each brick is important to avoid irregular in the shape that could decrease strength of the dome. As Ramazzotti writes,

> The issue [curvature irregularities] was resolved in the following way: Four steel cables at 6 mm, anchored at the end of the granite arches, tightened by threads and welded to a metal plate that materialized the geometric center of the spherical dome sleeves. In the center of the plate, 20.3 cm square, a metallic wire is fixed by a bolt with

a cable. The cable extends to the ground where it is anchored to a counterweight with a turnbuckle, in order to control the tension... Another set of wires are attached to the plate and the other end is held by workers on scaffolding, allowing them to plot the correct radial geometry and lay bricks on the dome. (Translated from Spanish)

Figure 7.35 represents this construction device.

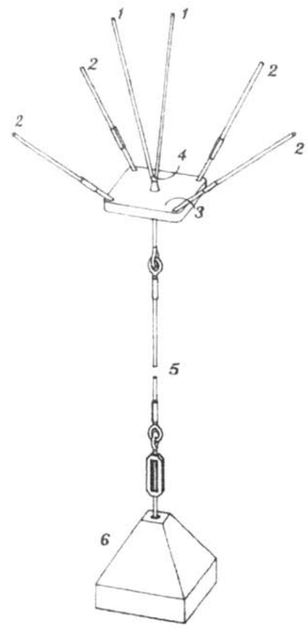


Figure 7.35: Cathedral - Construction Centering Device [Ramazotti, 2001]

The device only had two wires the construction workers could work with, which greatly limited all of the masons to work quickly. The second part of the device is what allowed the rapid dome construction. Since the curvature of the dome was defined, a wooden framework could be constructed on the outside of the dome along the curvature. The wood was purely used for geometric reasons and not structural support. After a segment of the dome was constructed, the location for the next arc was dictated by the wooden framework. Part of the framework was secured with just a pin so the wood could rotate to the exact angle of the dome, as seen in Figure 7.36 [Ramazotti, 2001].

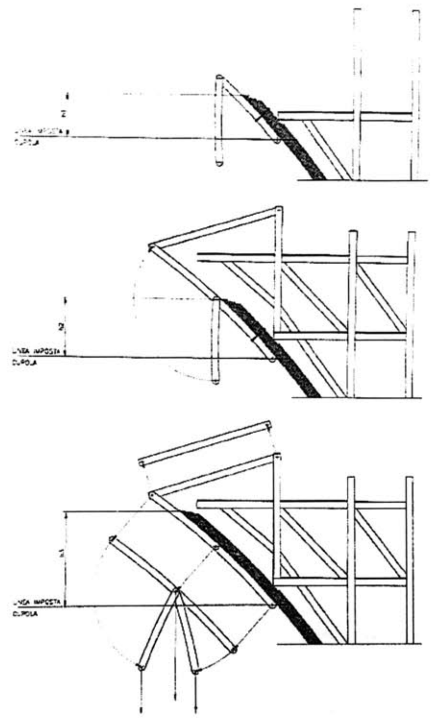


Figure 7.36: Cathedral - Geometry Scaffolding Guide for Dome Construction [Ramazotti, 2001]

It is uncertain if this is the first time the Guastavino Company used this construction process. As seen in Chapter 4, the inside face of their constructions is always a circle. The base circle design choice seen in many Guastavino structures could be driven by constructability purposes.

7.5 Dome Thrust Transfer to Arches

Guastavino Jr. calculates the thrust from the dome on to the arches. The exact process to determine the thrust is not verified, however the following procedure is a possibility. Based on the Panel divisions seen in Figure 7.13, he assumed the thrust from the three panels onto the arches is around the center of Panel A. As seen in, Figure 7.37, he assumes that the average thrust from panel A is between segments 13 and 14, blue line. It is not at the middle of panel A, orange line, since there is more weight on the left hand side of the panel.

The extension of the center of gravity of segments 13 and 14 on the dome are shown in red. From these lines, an arc, in green, is drawn to show the segment on the panel. The blue line drawn extends from the middle of segments 13 and 14 to the beginning of the panel. On the force diagram, a line, blue, is drawn between segments 13 and 14. Along the line is written, "Assumed mean direction of thrust from Panels A to A'." Panels A to A' encompass four panels A, B, B' and A', as shown on the drawing in light blue lines. The angle of this line is the direction of the thrust from the panels. When this line is mirrored over a vertical line, the angle is shown into the arches. This line is exactly the same orientation as the line on the drawing. The weight from the arches is drawn as a force vector, light blue at the right. The resultant of these two forces, red, is found within the enclosure of the arches, assuring the thrust from the dome is constrained by the arches.

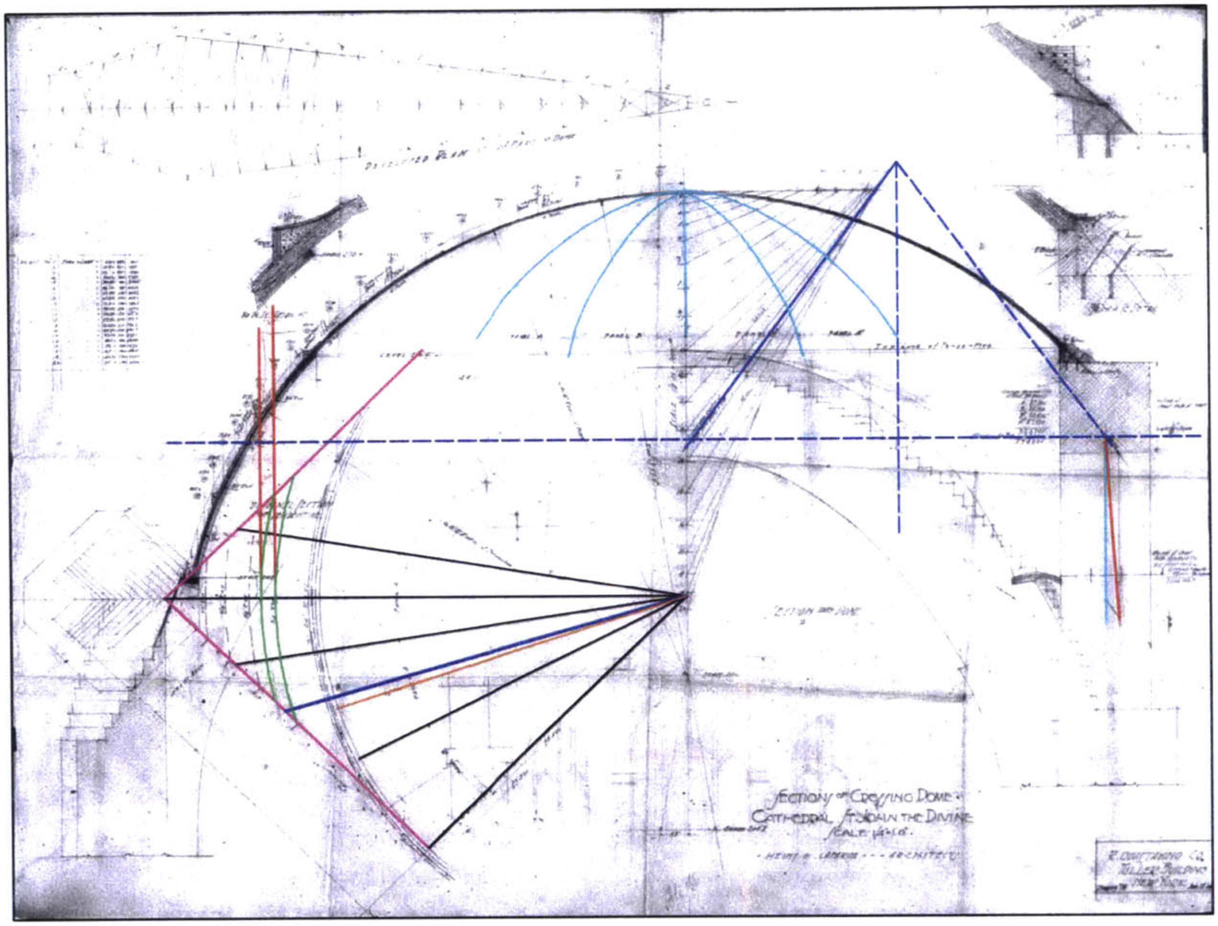


Figure 7.37: Cathedral - Thrust from Dome on to Arches

The process to determine the value of the thrust is not completely understood. It is possibly from the meridional lines in the force diagram. Panel A extends to segment 14. The thrust value for this meridional line is 88,000. The average meridional line for Panel B is 11. The thrust value for this meridional line is 70,000. The values Guastavino Jr. uses are 87,500 and 75,500, as seen in Figure 7.38.

These values are determined on the panel top view, as seen in Figure 7.38. Strangely, the exact length of the yellow line is 98,000 (value written on the drawing), the light blue line is 75,500, and the orange line is 87,500. The dark blue lines seem to be the projection of the forces onto the end of Panel B. The end of Panel B represents the middle arc of the dome, of the dome, or the straight segment of the dome. The exact determination of these forces is uncertain. It seems like there is a relation to the length of the meridional forces that correlate to the middle of each panel.

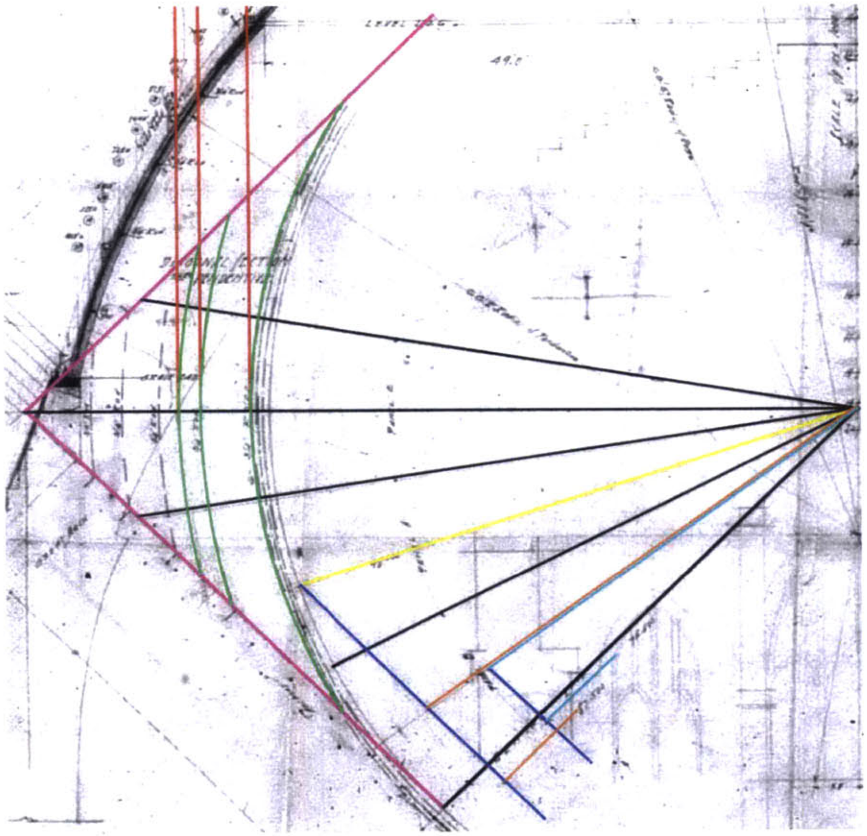


Figure 7.38: Cathedral - Panel Top View Thrust Values

The thrusts are added to a total thrust of 326,000 lbs, assuming four panels, A, B, B', A', contribute to the thrust at the middle of the arch, as seen in Figure 7.39. The reason for the 52,000 lbs value subtracted for allowance for rods is undetermined as well. The total weight of the arch is written as 2,246,000 lbs. These values are used to find the thrust line through he arches, as seen in Figure 7.40. These values and calculations can also be seen on the second Cathedral drawing, Figure 7.41.

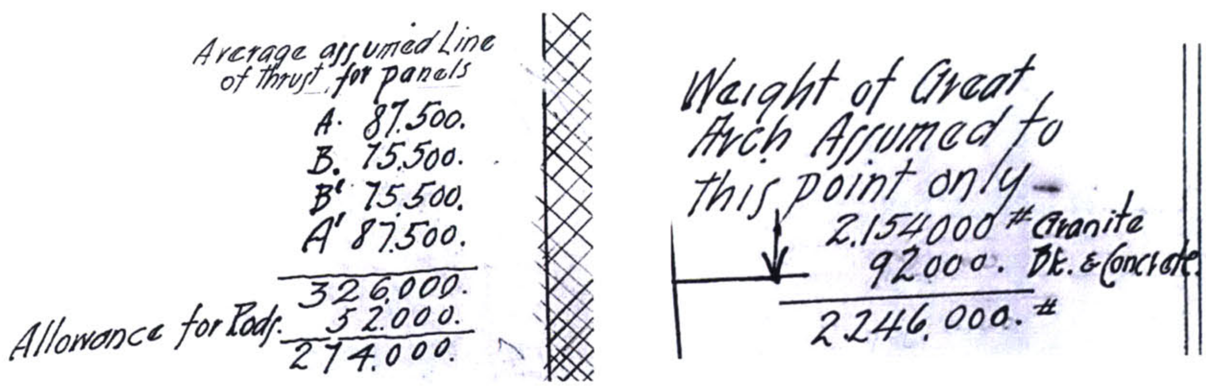


Figure 7.39: Cathedral - Values for Thrust Lines and Weight of Great Arch

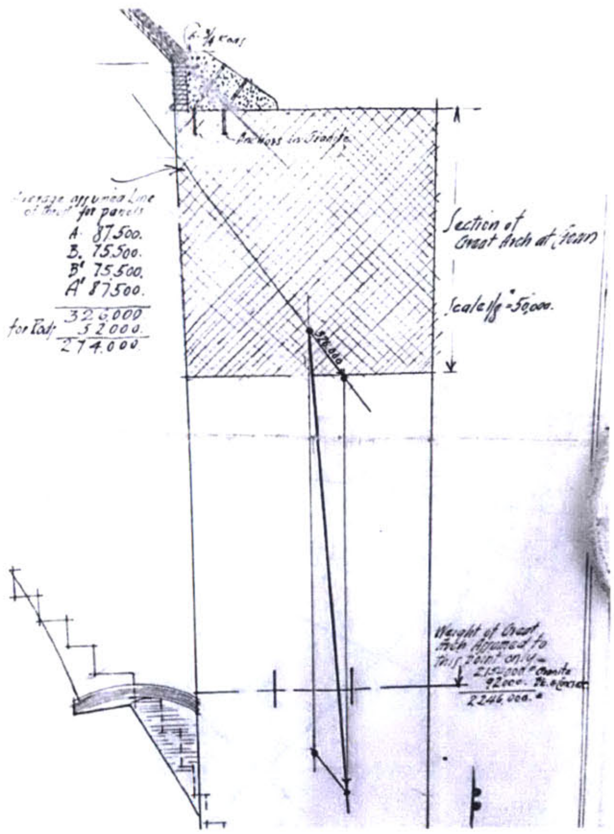


Figure 7.40: Cathedral - Thrust of Dome onto Arches

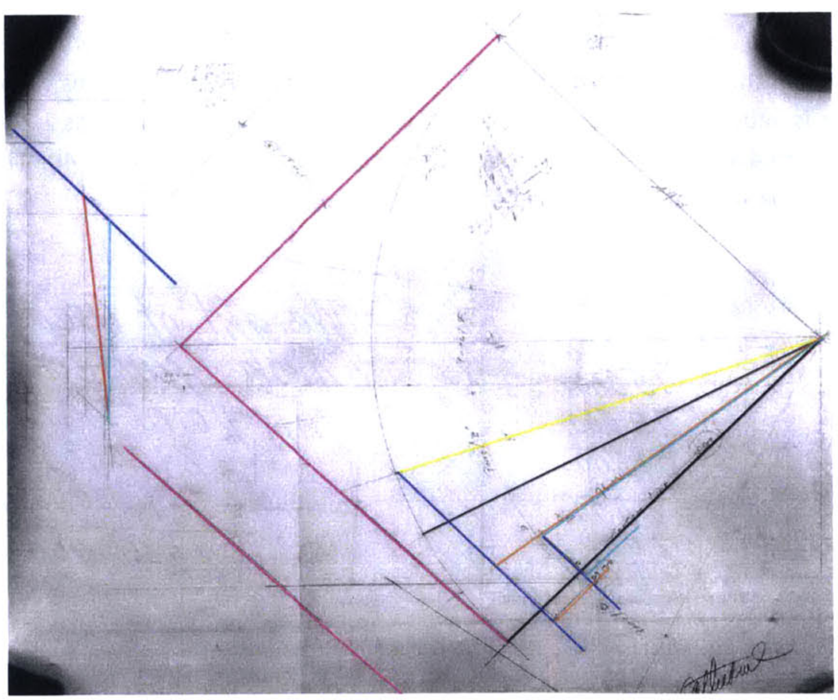


Figure 7.41: Cathedral 2 - Thrust of Dome onto Arches Calculation

7.6 Discrepancies in Force Values

Guastavino was using a pencil, ruler, and compass. He did not have more than two decimal points of precision in his analysis. The dome was reanalyzed using AutoCAD Software in two ways. The first estimates values to achieve results as close as possible to Guastavino's. The second is a precise analysis without approximations. The results were compared to determine if graphic statics analysis by hand is accurate enough.

In the lune drawing, Guastavino Jr. is off by one degree on each side. When the full area is accounted for, as shown in Figure 7.19, the total load difference is approximately 9000 pounds. The load scale for the drawing is 1/8" = 1000 lbs, amounting to 1.2" short. When the load lines are compared for the two models, the differences are negligible. The maximum difference in the length of a segment line is 0.15 inches, and the average difference is 0.06 inches. Therefore, a separate precise model is unnecessary.

7.7 Comparison to Dugum Thesis

Dugum's thesis assesses the structural stability of the Cathedral of St. John the Divine. He performs a membrane and graphical analysis and compares the results of the two methods. There are some discrepancies in Dugum's model versus Guastavino Jr.'s model. Dugum analyzes the dome only under its' self-weight while the original model also includes a distributed live load on the dome. The lune section Dugum considers is different as well, as seen in Figure 7.42, which leads to inconsistencies in segment areas and loads.

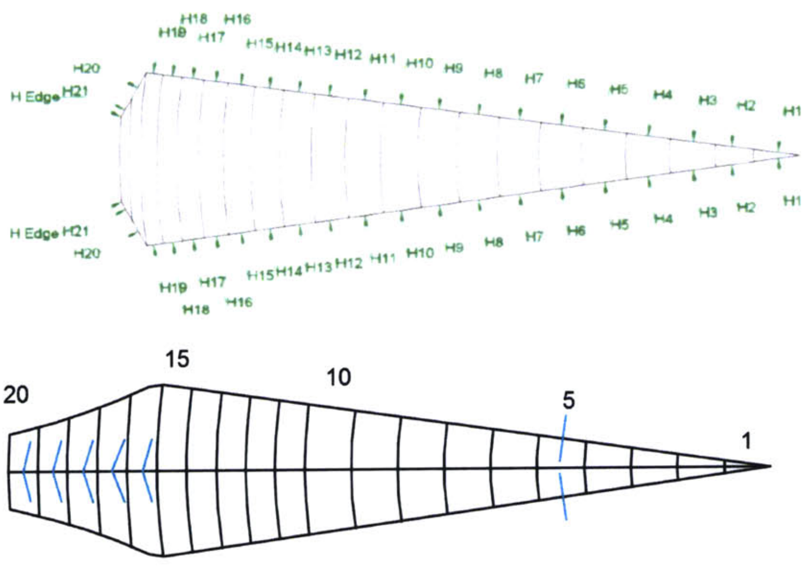


Figure 7.42: Cathedral - Dugum Lune vs. Original Drawing Lune [Dugum, 2013]

The final hoop and meridional stresses are compared, as seen in Table C.8 and Figure C.5 in Appendix C. The meridional forces accounted for in Dugum's analysis are almost half the amount throughout the dome. Therefore, the stresses found are also lower. For example, at approximately 71°, Dugum's stresses and forces versus this analysis' are 4363 psf versus 8121 psf, and 71,795 lbs versus 112,100 lbs, respectively. A major factor for this discrepancy is the lune section Dugum uses. The meridional force is divided by the width of the corresponding lune segment. Dugum's lune dimensions vary since his model does not account for the curvature of the interface between the arches and the dome. The forces are also significantly different. This variance is possibly due to segment division Dugum uses on the arc section. The hoop forces are more comparable. The forces only vary at most by 200 psf. At approximately 76.5°, Dugum's stresses are 2715 psf and this analysis finds 2927 psf.

The reason for the placement of a large amount of steel in the dome at the top of the arches is debatable as well. Dugum assumes the steel is placed to ensure the stability of the dome under construction to help support the workers. This thesis finds the amount of steel may act as a ring stress band, as seen in many Guastavino constructions discussed Chapter 4.

7.8 Conclusions

The thinness of the dome of the Cathedral of St. John the Divine is only possible due to the lengthy, iterative, and meticulous calculation process exhibited in the original Guastavino Company drawing. The drawing is a masterpiece that details the entire calculation of the structure.

The following findings are the main contributions to the design process behind the extraordinary structure:

- Five different views of the dome and arches are projected onto one drawing.
- Calculations are performed for the entire dome and its thrust onto the arches on one drawing.
- The existing arches dictate the initial curvature of the dome.
- Graphic statics is used to calculate the meridional and hoop forces in the dome.
- Steel rods are sized based on the calculated hoop forces.
- Two circles that deviate at the top of the arches define the shape of the dome.
- Multiple graphic statics calculations are implemented to achieve the final dome geometry.
- The curvature of the dome and the brick thickness layout is manipulated to limit the tensile forces at the base of the dome.
- The constructability of the dome is a primary design consideration.

Guastavino Jr. designed the entire structure on one drawing. He did not model the very specific details of the dome, like a full finite element model would today; he only focused on the critical elements of the design.

The portions of the dome that he designed are:

- Diagonal Lune
- Transversal Lune (Straight Segment)
- Buttresses (Only a stability check since the arches were already built)

In these sections, he calculated the thrust and hoop forces to assure the stability and the stresses occurring in the dome. These calculations were sufficient to design the entire dome on one drawing.

8 General Conclusions

The original Guastavino Company drawings contain a wealth of information on the Company's structural design analysis methods. This thesis assumes that the Guastavino Archives at Avery Library in Columbia University hold the entire Guastavino drawing collection available.

The drawings are analyzed to learn their development in design tools and strategies. The calculation drawings form a timeline for the different design methods of the Guastavino Company. The graphic statics drawings for the barrel vault of the St. Francis de Sales church and the dome of the Cathedral of St. John the Divine are thoroughly analyzed to develop the Company's design methodology behind these structures.

The following findings are presented in this thesis:

- The design strategy for structures transitions in the Company around the time of Guastavino Sr.'s death in 1908. Coincidentally, around this time in 1904, Dunn publishes new literature on graphic statics design techniques for masonry domes.

- Most likely, Guastavino Jr. was the first to use graphic statics on Guastavino Company structures, beginning around 1906.

- Guastavino Jr. is generating geometries and determining the thickness of structures using graphic statics only. The analyzed working-drawings comprise all the calculations required to assess the stability of his domes. No additional calculations were needed.

- Tensile hoop force calculations are used to size steel reinforcement in Guastavino Company domes after the Cathedral of St. John the Divine project in 1909. Before 1909, the steel bands provided are found to be generally over-sized.

- The hoop force calculations Guastavino Jr.'s drawings exhibit in 1909 were not published until 1921 by Wolfe. Guastavino Jr. made innovative advancements not only with his structures, but also in his calculation methods.

- Guastavino Jr. most likely creates the Rubric Drawings to learn and understand the behavior of masonry domes by comparing the thrust lines for a dome with and without the maximum compressive thrust extending into the tensile region of the dome.

- The analysis of the Cathedral of St. John the Divine dome reveals that the curvature of the dome and the thickness and layout of the bricks is manipulated iteratively to limit the tensile forces at the base of the dome.

- Guastavino Jr. form-finds the curvature of domes using the funicular shape of the loads and geometry. He often predefines the inside face of the dome as a circular shape to facilitate construction.

- The Guastavino Company domes with graphic statics drawings analyzed in this thesis have a circular inside shape at the crown of the dome. The base sometimes deviates from the circle to match the funicular shape of the force polygon.

9 Future Work

This thesis provides a framework to further analyze more Guastavino graphic statics original drawings. There are plenty of areas to research on the Guastavino calculations and the archival collection at Avery Library.

Potential areas of additional research include:

- Determine if the design for steel and brick layers given on the Guastavino original drawings match the as-built conditions by investigating existing structures.

- How were the vast majority of Guastavino Company Structures designed if no calculation drawings exist for them? Were the drawings ever created, lost, or still waiting to be discovered?

- What information can be found on graphic statics calculations in the correspondence letters, mainly held in the Avery Library Guastavino Archives, between the Guastavino Company and architects on a project? Can the design process be recreated from the letters and drawings available?

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11 Appendices

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Appendix A: Drawing Records

Table A.1: Information for Graphic Static Arch and Barrel Vault Drawings

Project	*Date*	*Location*	*Architect*	*Arch/ Barrel Vault*	*Span*	*Height*	*Brick Dim.*	*Radius of Inside Circle*	*Amt of Rad*	*Loads Considered (psf)*			*Drawing Scale*	*Load Scale*	*G.C. Stamp*
										Total	**DL**	**LL**			
St. Columbus R.C. Church	1906	Phil., PA	Henry D. Dagit	Arch	-	-	-	-	3	-	-	-	¼" = 1'-0"	1/8" = 1000 lbs	Yes
St. Francis de Sales	1908	Phil., PA	Henry D. Dagit	Barrel Vault	56'	22'	5"	32'-0"	3	70	50	20	3/8" = 1'-0"	1/16" = 10 lbs	Yes
Cathedral of St. John the Divine	1908	NY, NY	Heinz and LaFarge	Arch	-	-	-	-	-	-	-	30	½" = 1'-0"	1/8" = 1000 lbs	Yes
St. Patrick's Church	1910	Phil., PA	La Farge and Morris	Arch	50'	10'	~5"	50'-0"	1	80	50	30	1/8" = 1'-0"	1" = 40,000 lbs	Yes
Philadelphia Electric Company	1915	Phil., PA	John T. Windrim Architect	Barrel Vault	-	-	-	-	-	70	40	30	¾" = 1'-0"	½" = 165 lbs	Yes
Trinity College Chapel	1921	Wash., D.C.	Maginnis and Walsh	Barrel Vault	38'	17'-6"	12"	20'-0"	1	100	70	30	¼" = 1'-0"	1/8" = 1000 lbs	Yes
St. Johns R.C. Church	1931	Jersey City, NJ	J.T. Ronland Jr.	Arch	38'	11'	~6"	19'-0"	1	65-100	-	-	-	3" = 1200 lbs	No

Table A.2: Information for Graphic Static Dome Drawings

Project	*Date*	*Location*	*Architect*	*Tension*	*Hoop*	*Span*	*Height*	*Brick Dim.*	*Rad of Int. Circ*	*Amt. of Rad*	*Loads Considered (psf)*			*Dwg. Scale*	*Load Table*	*Load Scale*	*Lune*	*G.C. Stamp*
											Total	**DL**	**LL**					
St. Paul's Chapel, Columbia	1906	NY, NY	Howells and Stokes	No	No	46'	15'	4"-12"	~ 23'	1	-	-	-	½" = 1'-0"	Yes	Varies	1/16	No
Girard Trust Building	1906 /7	Phil., PA	Allen Evans, McKim, Mead, &White Architects NYC	Yes	No	98'	-	-	-	-	-	-	30	¾" = 1'-0"	No	½" = 1000 lbs	1/72	Yes
St. Francis de Sales	1908	Phil., PA	Henry D. Dagit	Yes	No	62'	31'	6" - 12"	34'	-	80-95			3/8" = 1'-0"	Yes	1/8" = 100 lbs	1/24	Yes
Cathedral of St. John the Divine	1908	NY, NY	Heinz and LaFarge	Yes	Yes	132'	52'	4" , 6", 7 1/2", 12"	66'	3	90 - 130	40 - 100	30-50	¼" = 1'-0"	Yes	1/8" = 1000 lbs	1/20	Yes
Trinity College Chapel	1921	Wash., D.C.	Maginnis and Walsh	Yes	Yes	36'	12'	6" - 9"	20'	1	85-90	70	30	¼" = 1'-0"	Yes	¼" = 1000 lbs	1/20	Yes
Dime Savings Bank	1931	Brooklyn, NY	Halsey, McCormack & Helmer	Yes	Yes	66'	20'	4" - 9"	-	1	-	134 *	30-40	3/8" = 1'-0"	Yes	1/8" = 1200 lbs	1/20	Yes
St. Johns R.C. Church	1931	Jersey City, NJ	J.T. Ronland Jr.	Yes	Yes	38'	11'	4" - 5"	19'	1	60-80	-	-	-	Yes	3" = 4800 lbs	1/16	No
Supreme Court Building	1947	Tallahassee, FL	Yonge and Hart	Yes	Yes	43'	16'-6"	5.5"	23'	1	160	130 *	30-40	¼" = 1'-0"	Yes	¼" = 1200 lbs	1/24	No

Notes: *Radius of Inside Circle* – the radius of the circle that the inside curvature of the arch is based on

Amt. of Rad – the amount of radii that are used to represent the inside curvature of the arch

Tension – states if the graphic statics design allows the maximum horizontal thrust to extend into the tensile region of the dome

Hoop – states if hoop forces were calculated on the drawing to size steel quantities

Lune – dimensions the size of the lune section designed for on the drawing

G.C. Stamp – states if the drawing has a Guastavino Company stamp or title block on it

Appendix B: Drawing Index

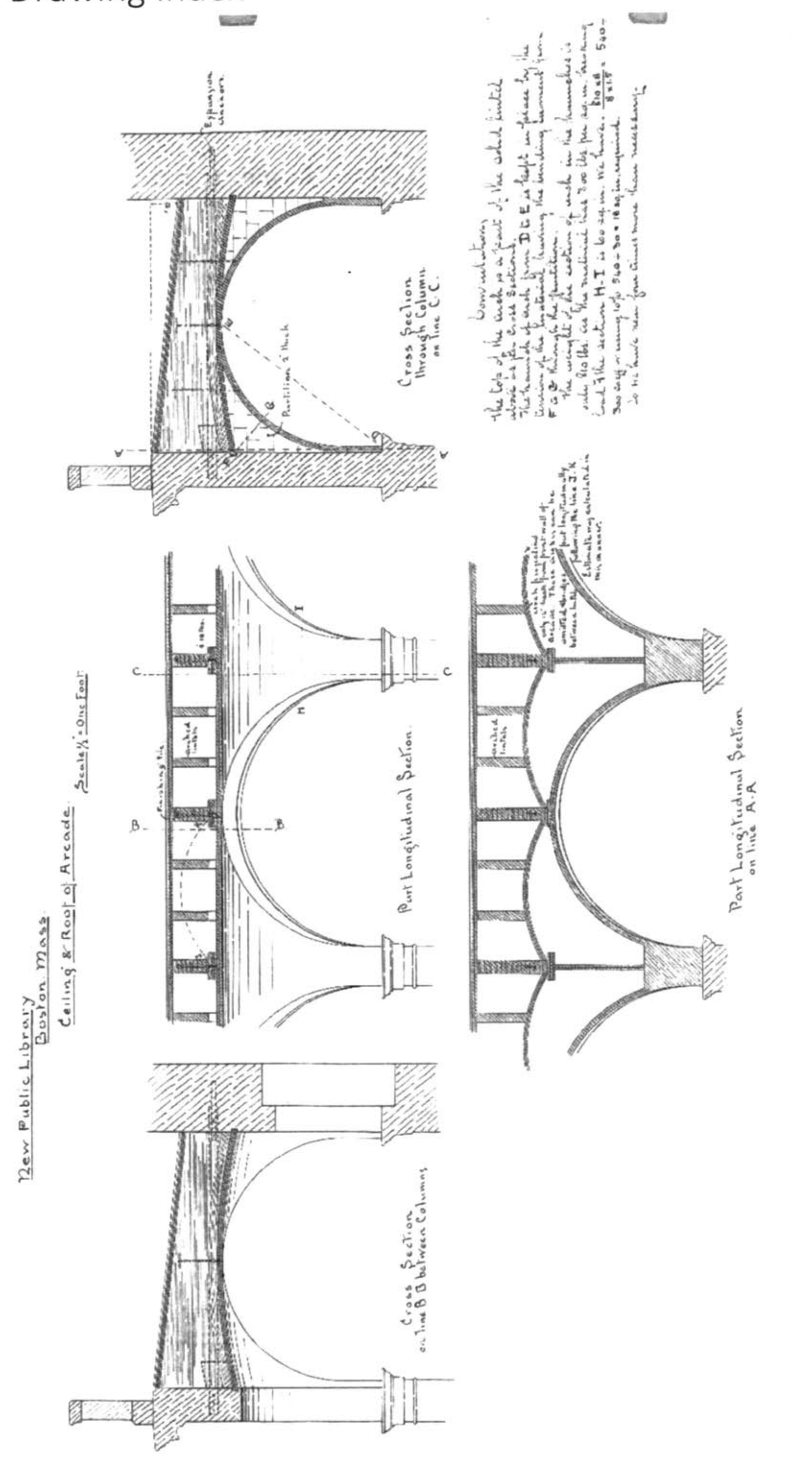

Figure B.1: Boston Public Library, Boston, MA 1889 [Avery Library]

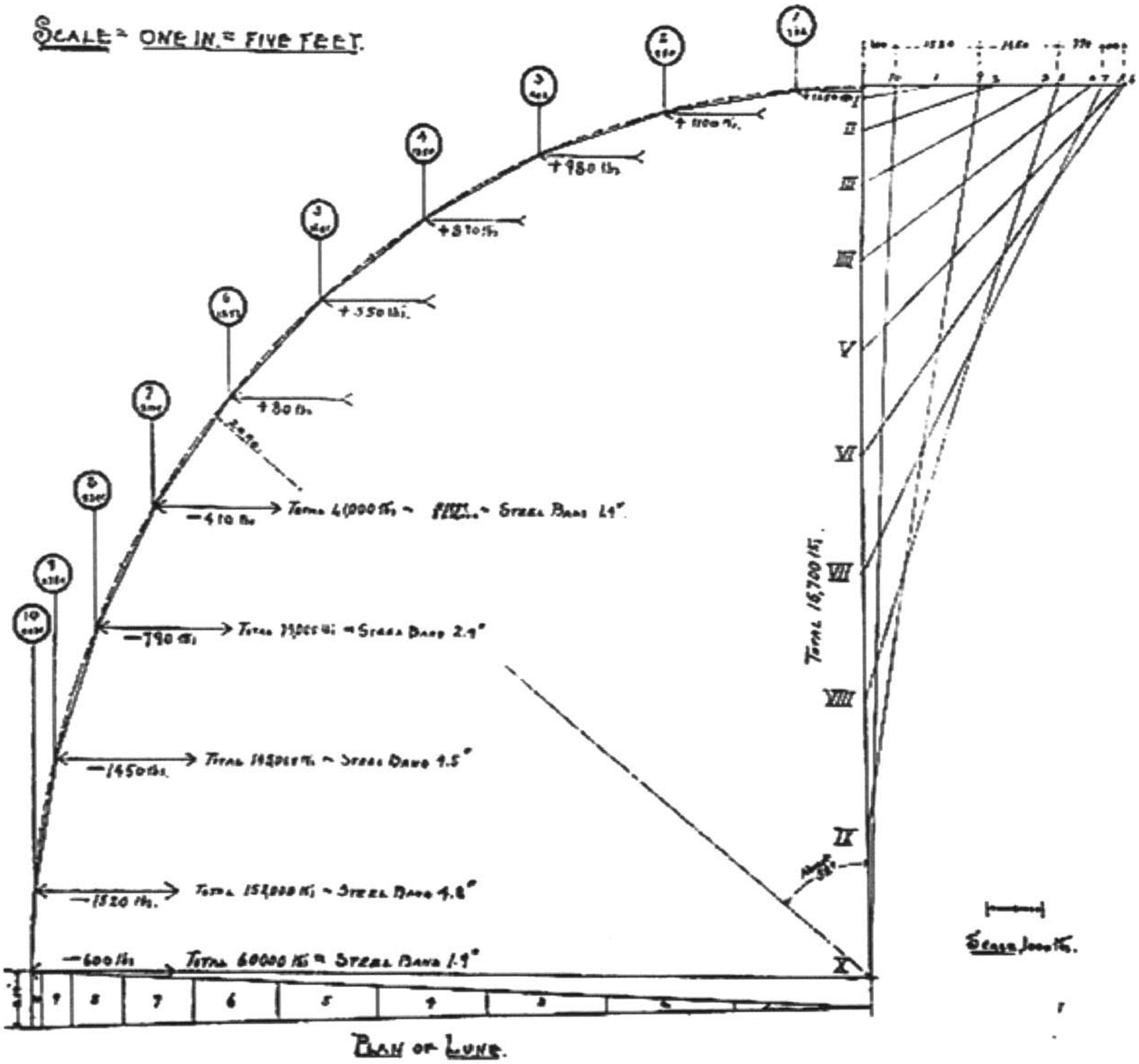

Figure B.2: Rubric Drawing 1, 1906-1907? [Avery Library]

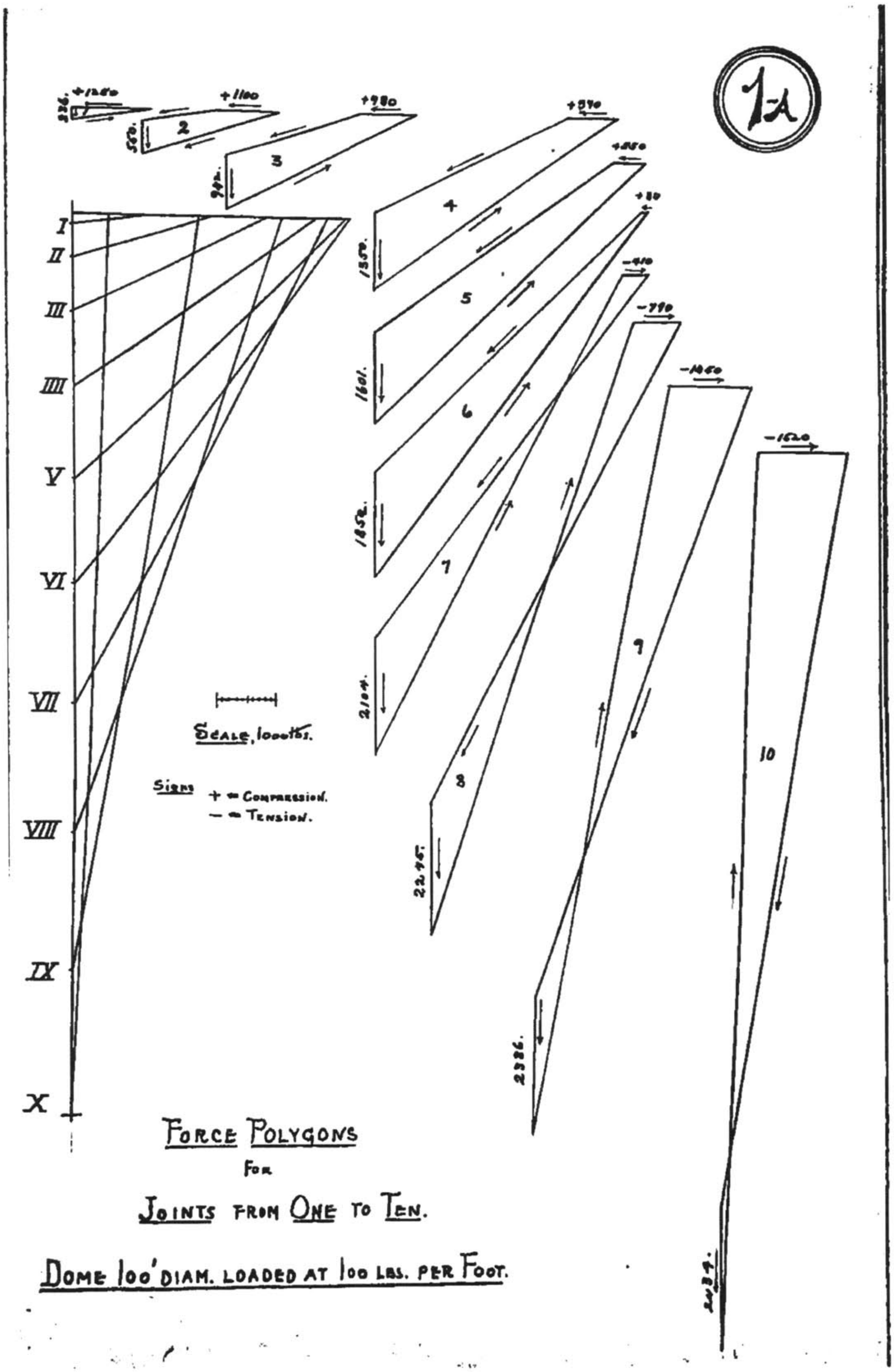

Figure B.3: Rubric Drawing 2, 1906-1907? [Avery Library]

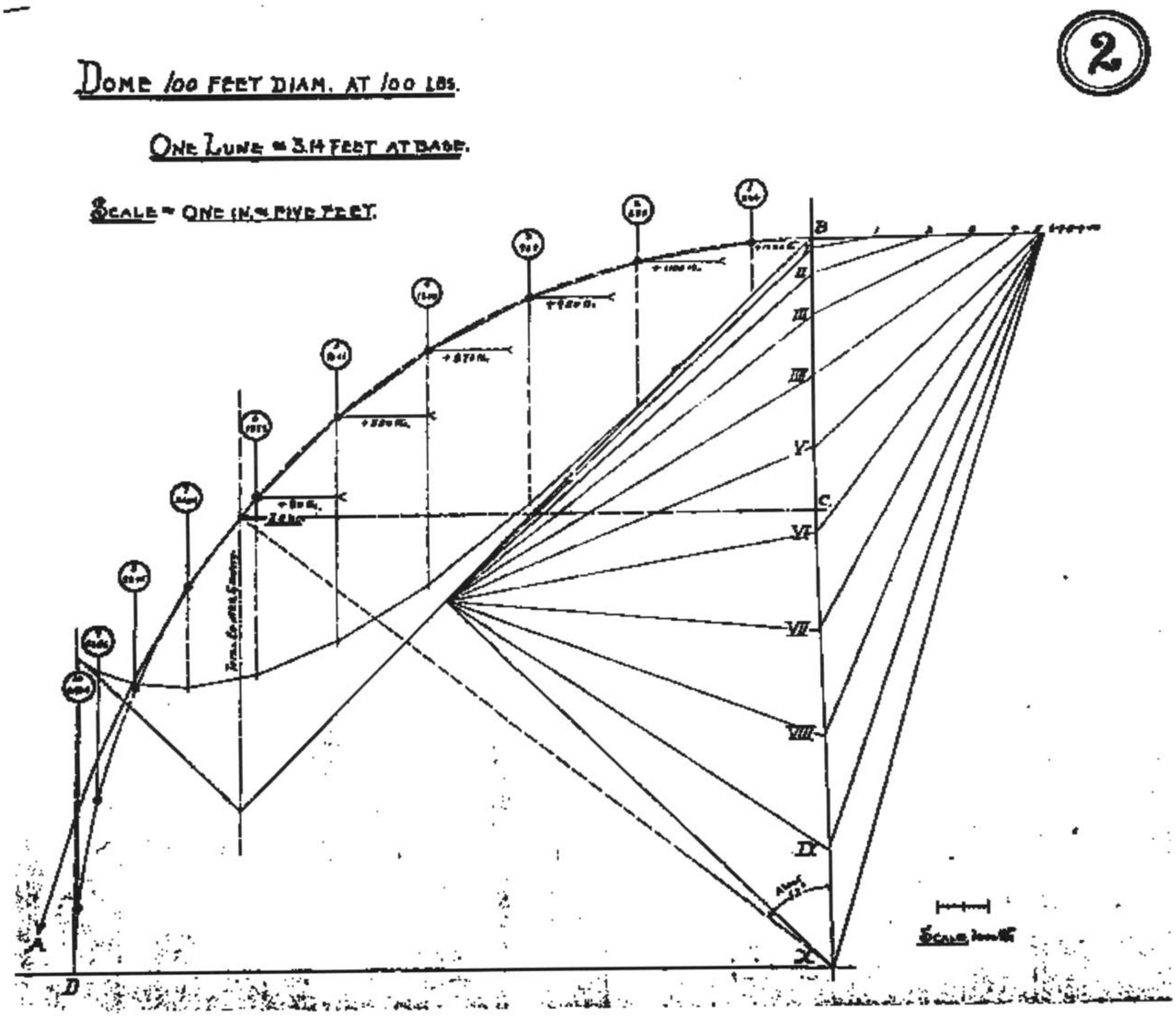

Figure B.4: Rubric Drawing 3, 1906-1907? [Avery Library]

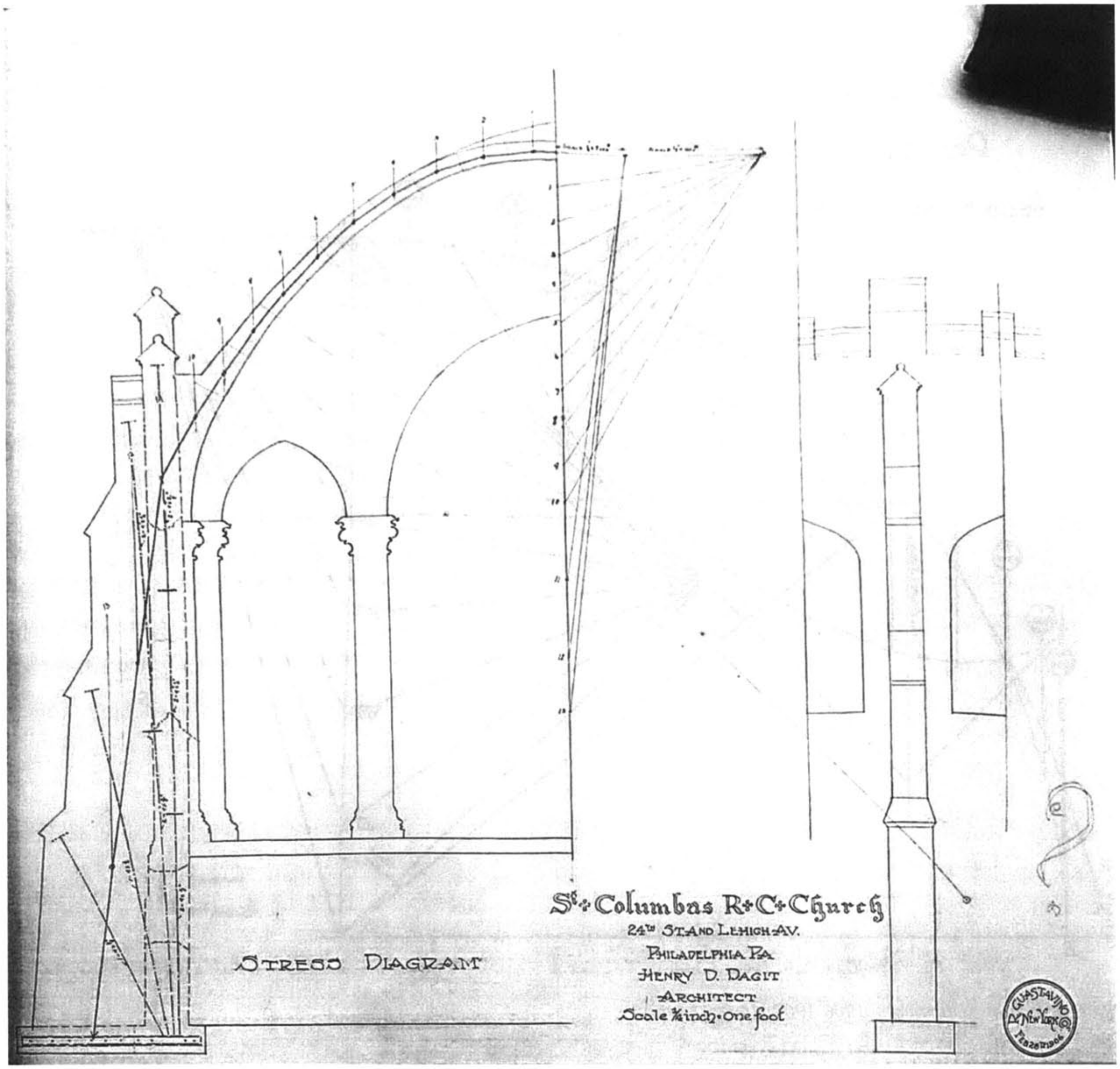

Figure B.5: St. Columbus. R.C. Church Arch, Philadelphia, PA 1906 [Avery Library]

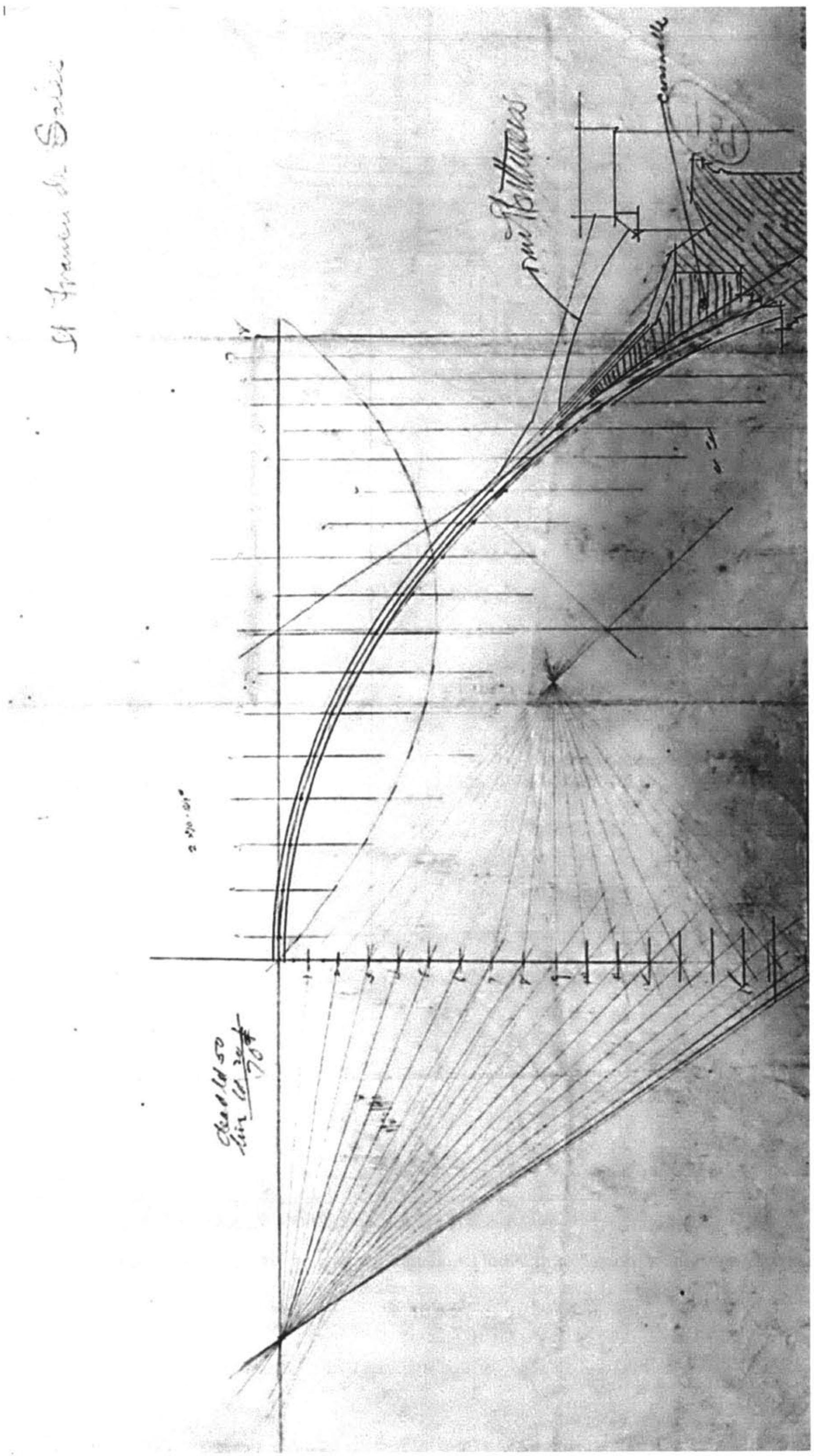

Figure B.6: St. Francis de Sales Church Barrel Vault, Working Drawing, Philadelphia, PA 1908 [Avery Library]

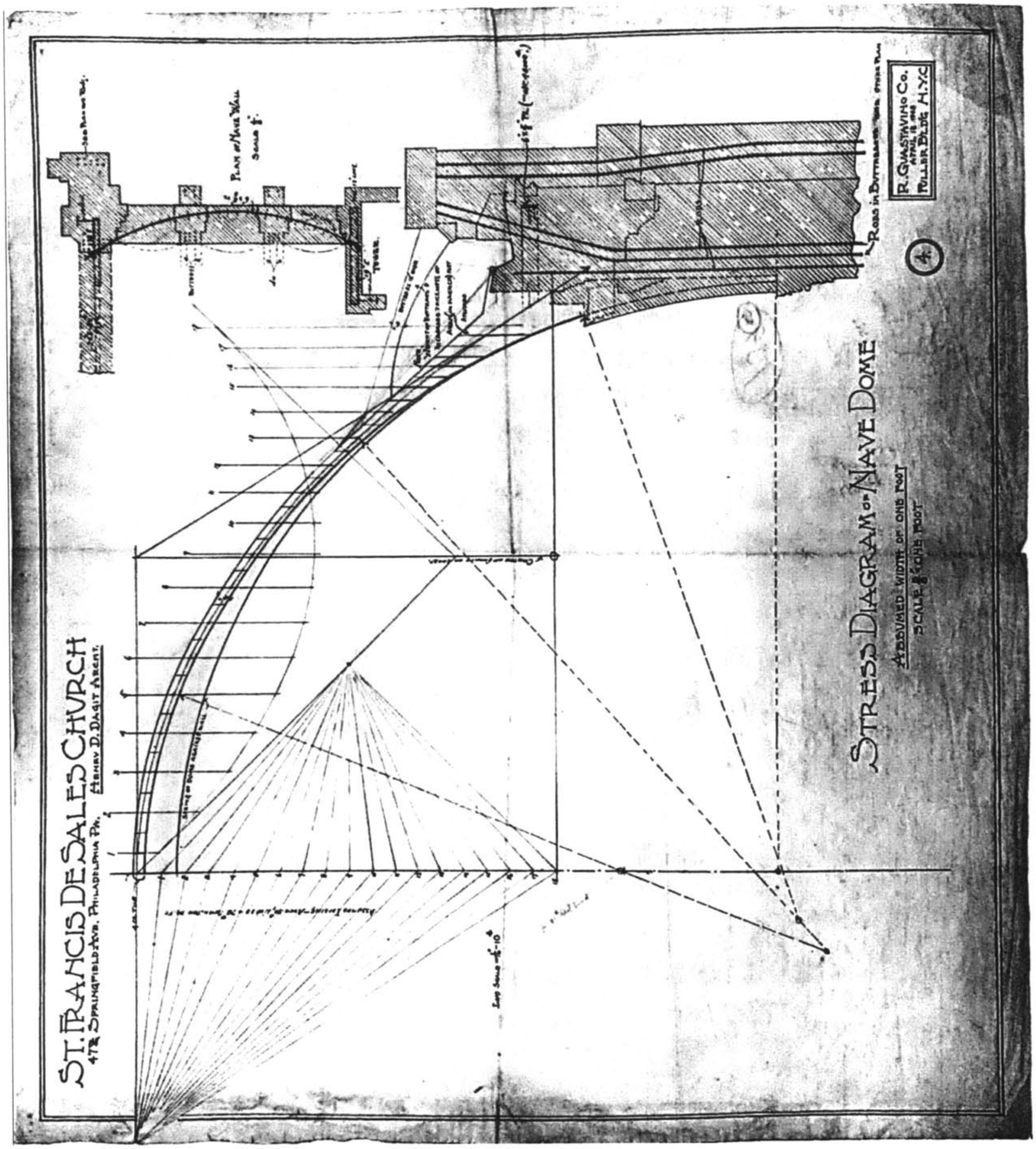

Figure B.7: St. Francis de Sales Church Barrel Vault, Final Drawing, Philadelphia, PA 1908 [Avery Library]

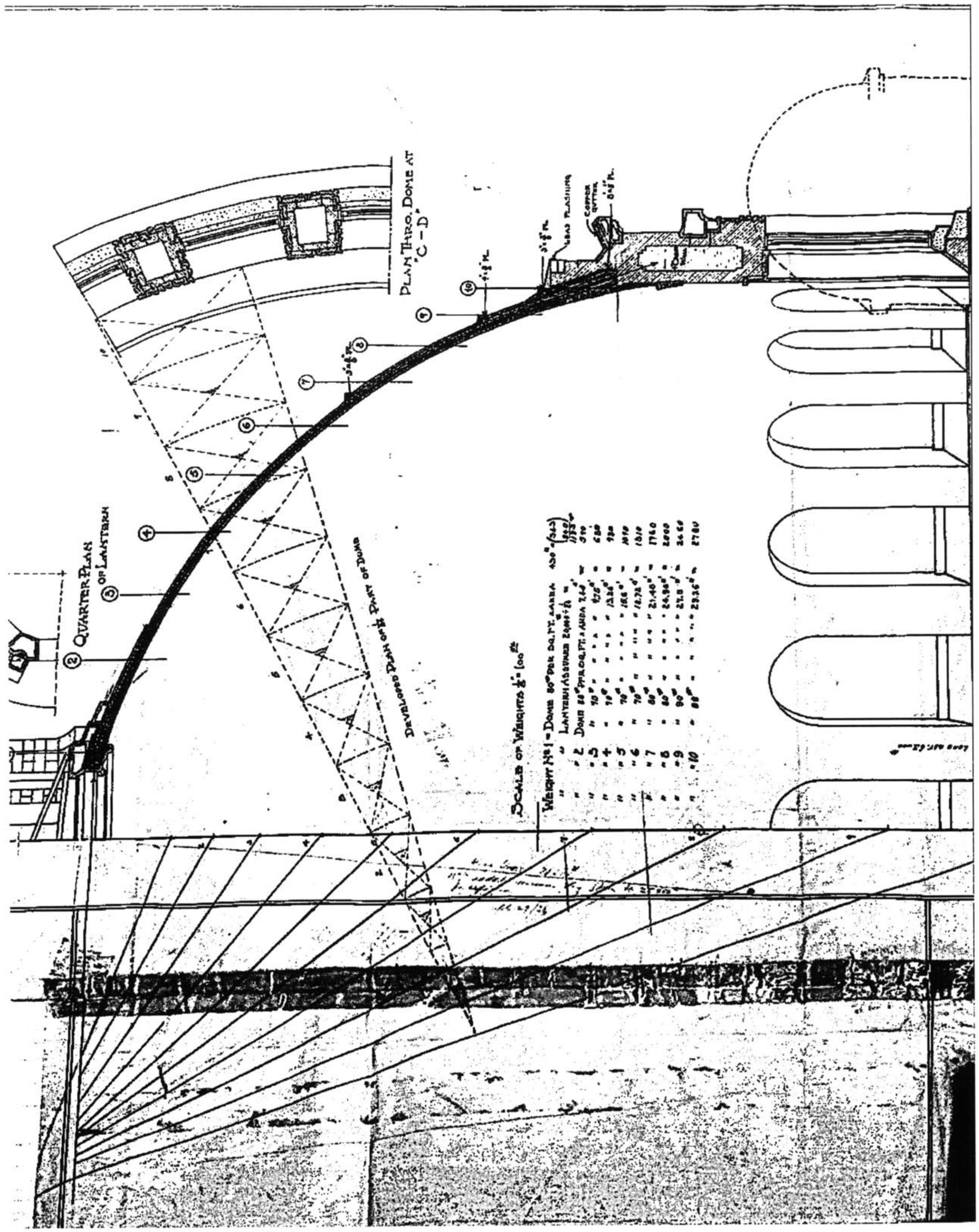

Figure B.8: St. Francis de Sales Church Dome, Philadelphia, PA 1908 [Avery Library]

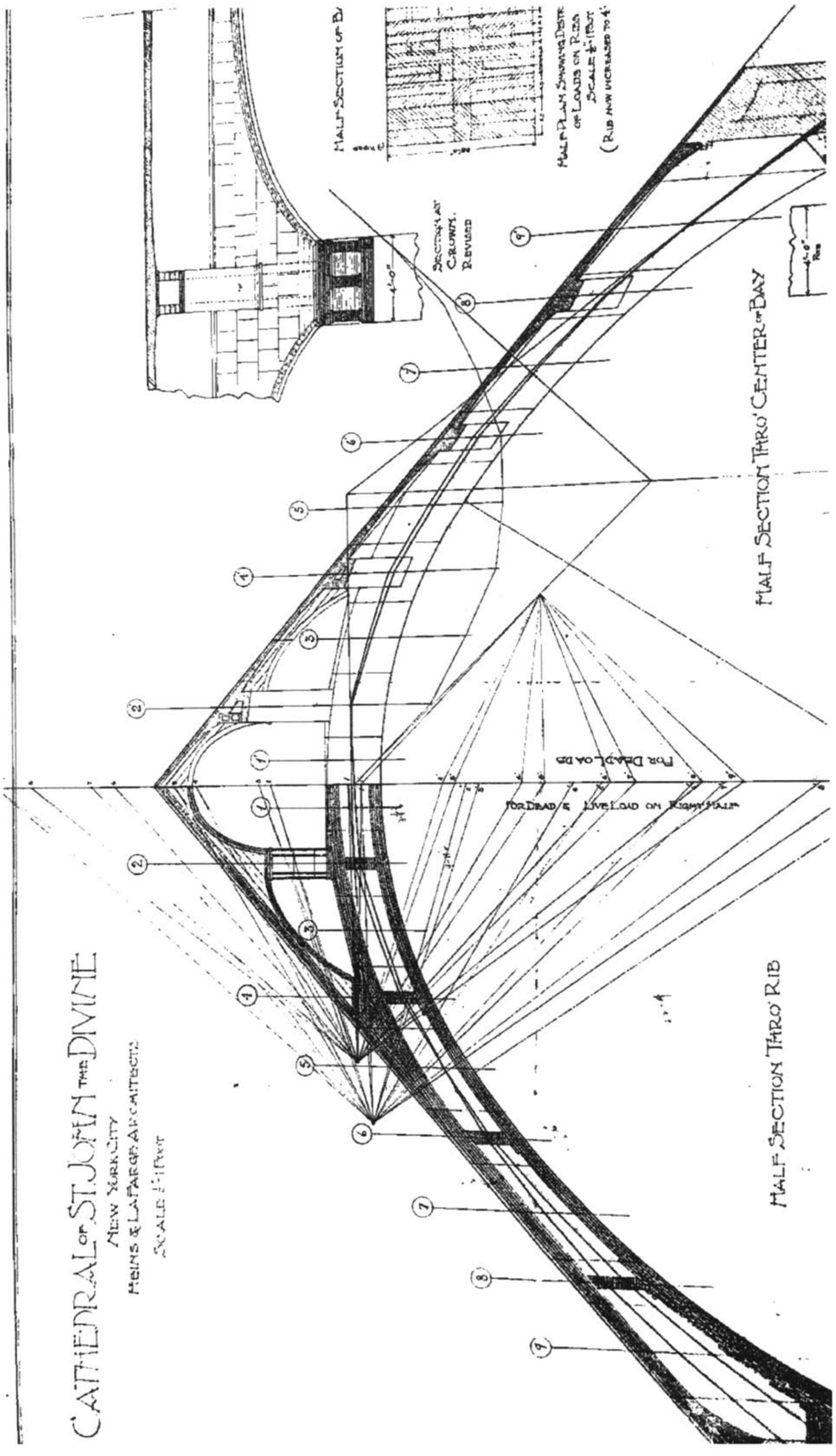

Figure B.9: Cathedral of St. John the Divine Nave Vault, NY, NY 1909 [Avery Library]

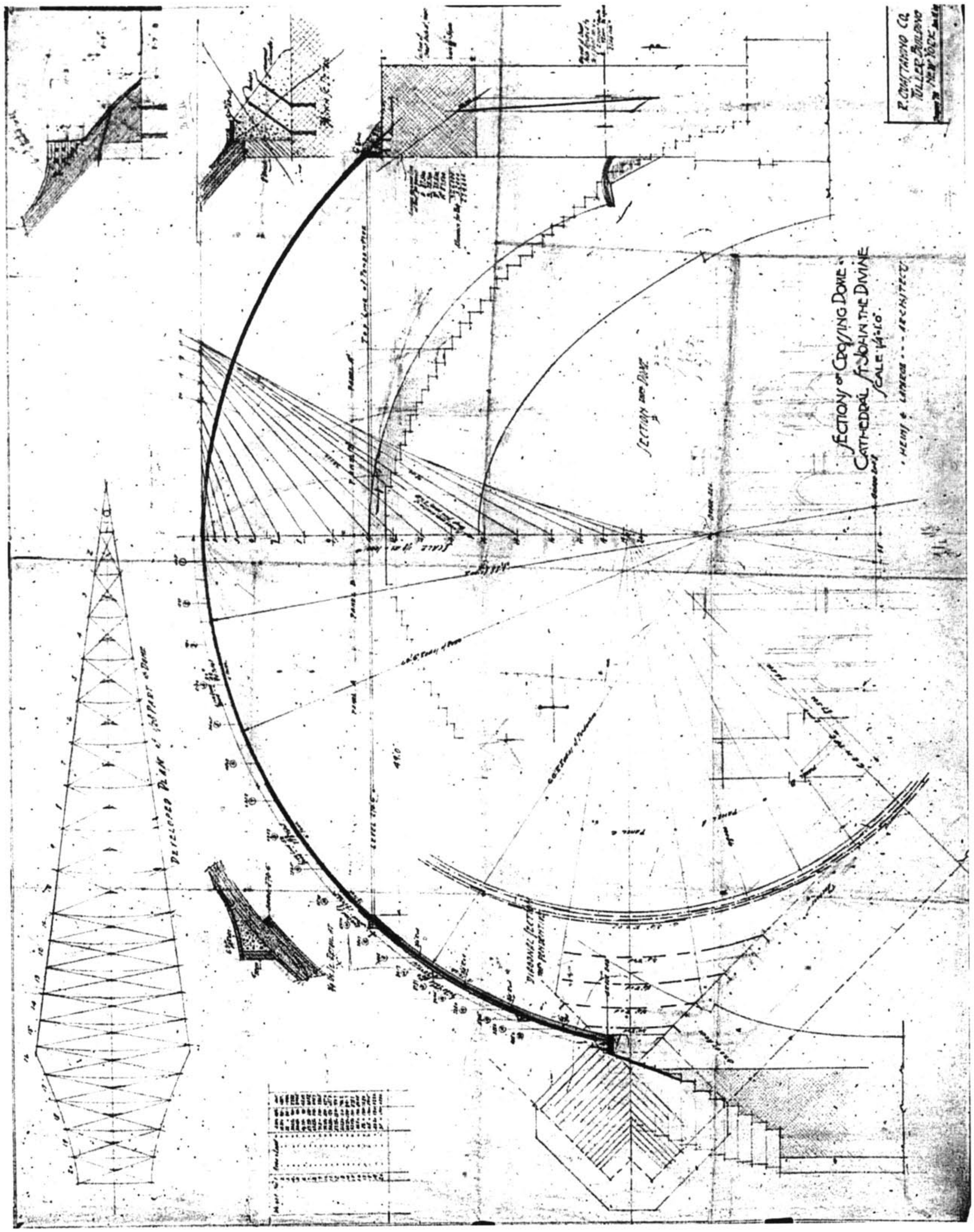

Figure B.10: Cathedral of St. John the Divine Dome, NY, NY 1909 [Avery Library]

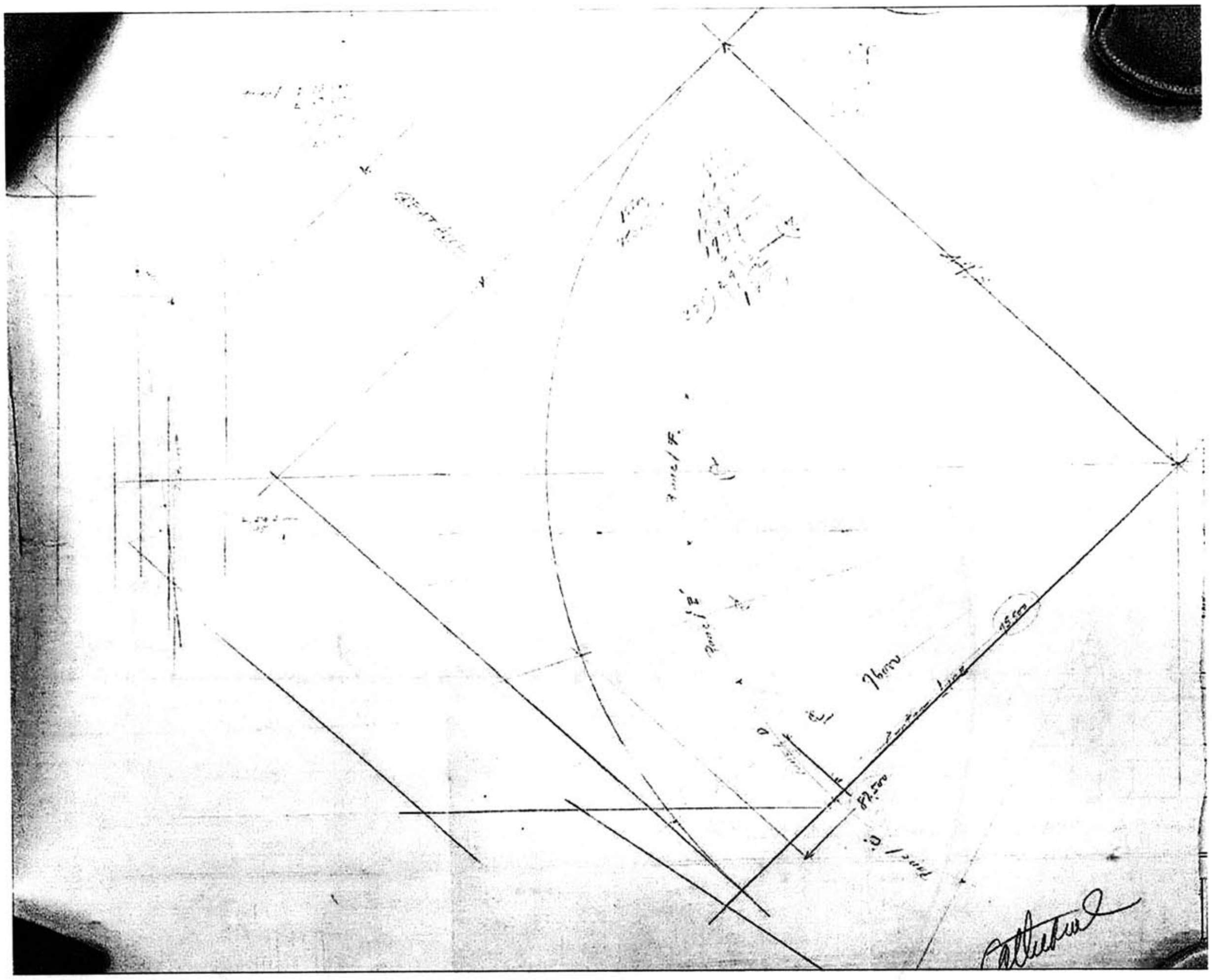

Figure B.11: Cathedral of St. John the Divine, Planning Drawing, NY, NY 1909 [Avery Library]

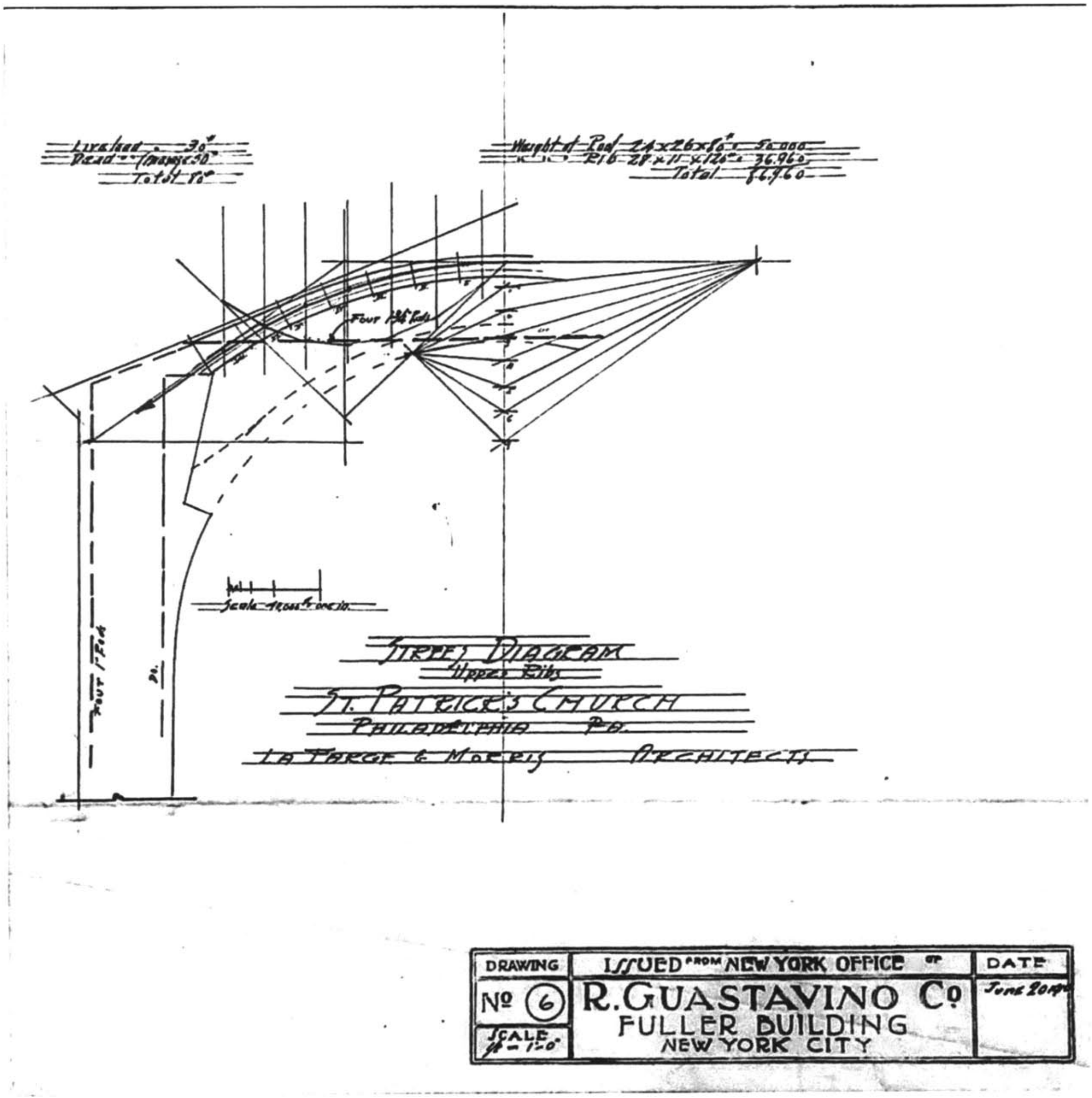

Figure B.12: St. Patrick's Church Arch, Philadelphia, PA 1910 [Avery Library]

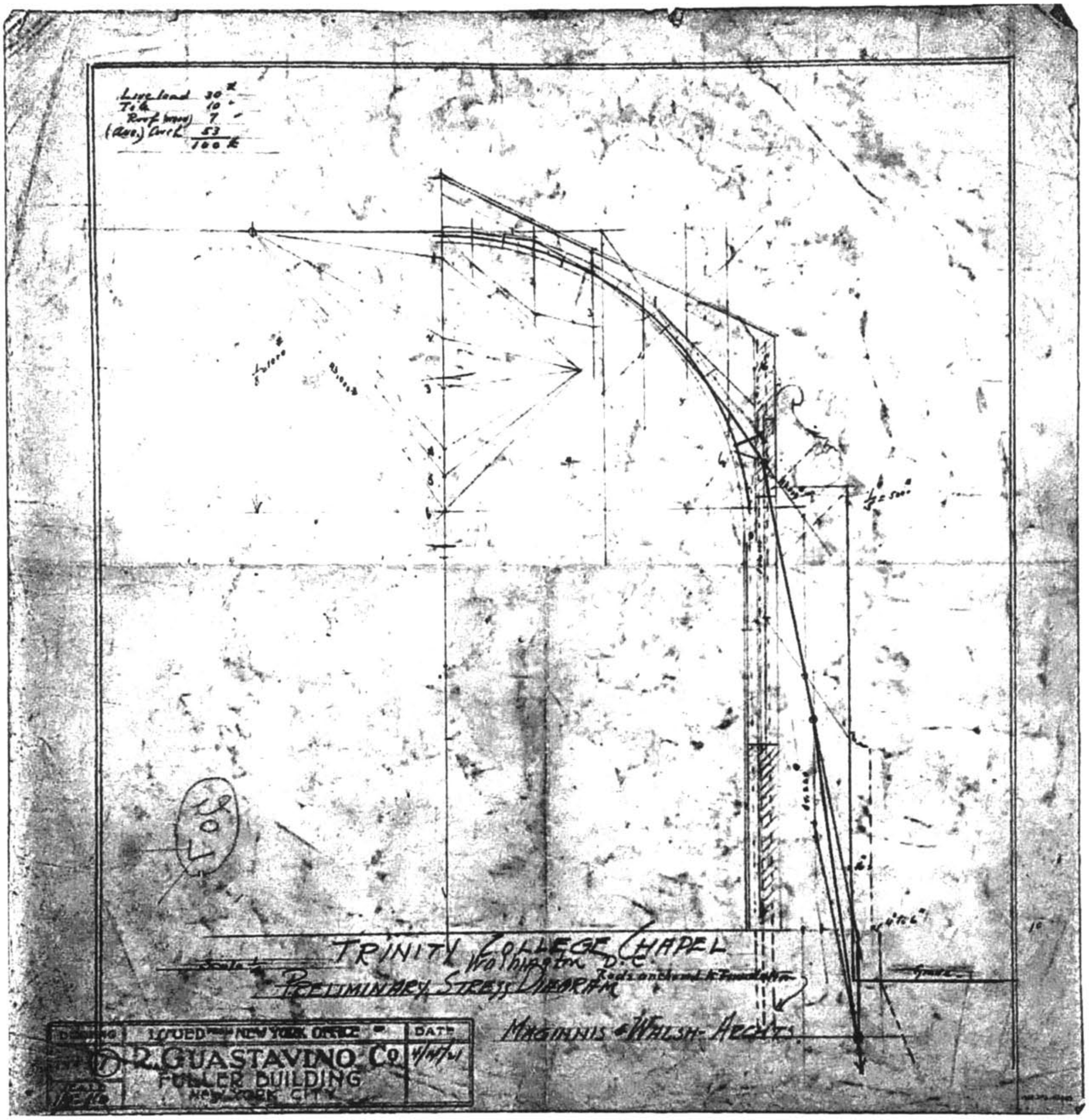

Figure B.13: Trinity College Chapel Arch, Washington D.C. 1921 [Avery Library]

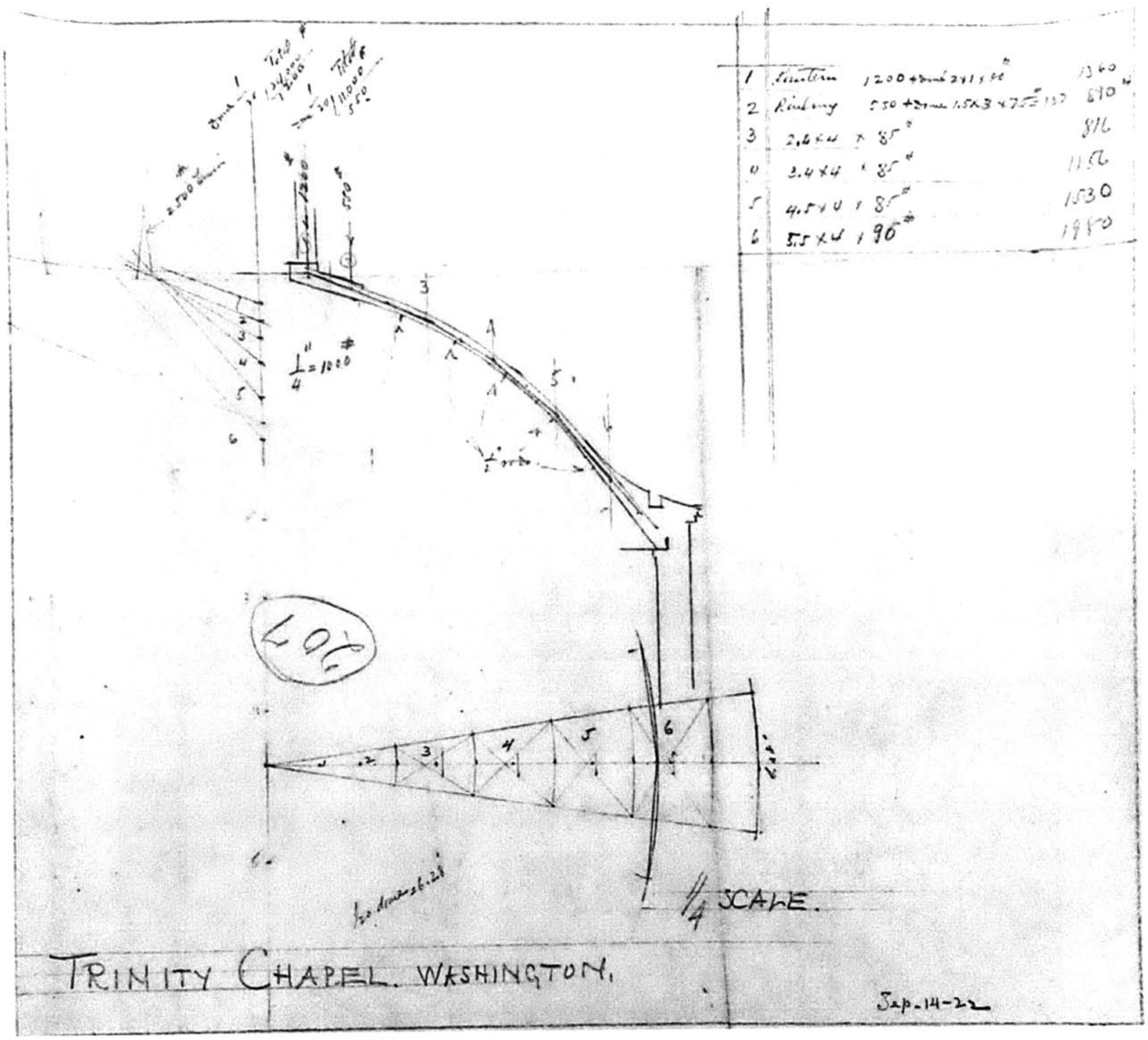

Figure B.14: Trinity College Chapel Dome, Washington, D.C. 1922 [Avery Library]

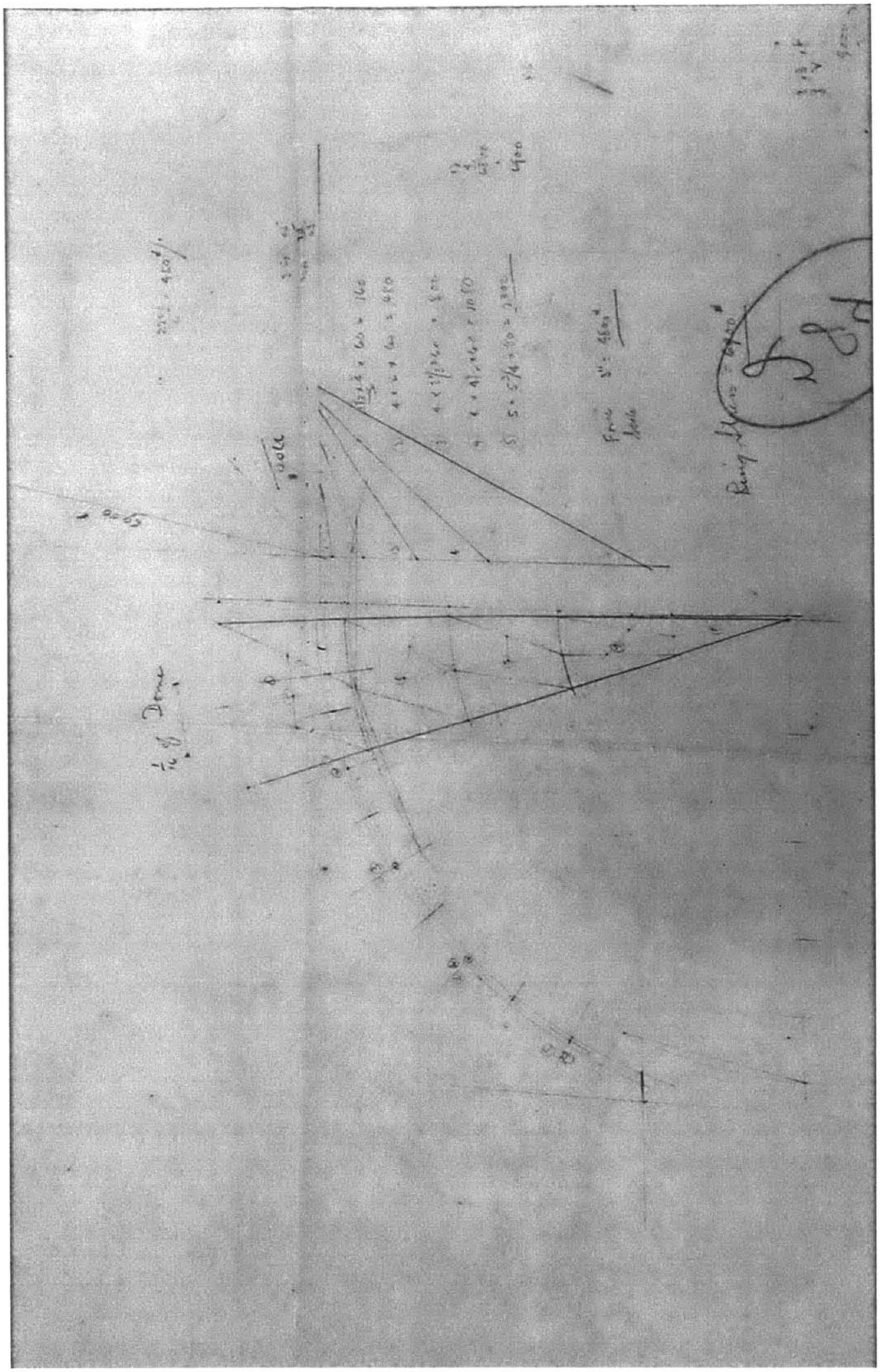

Figure B.15: St. John's R.C. Church Dome, Jersey City, NY 1931 [Avery Library]

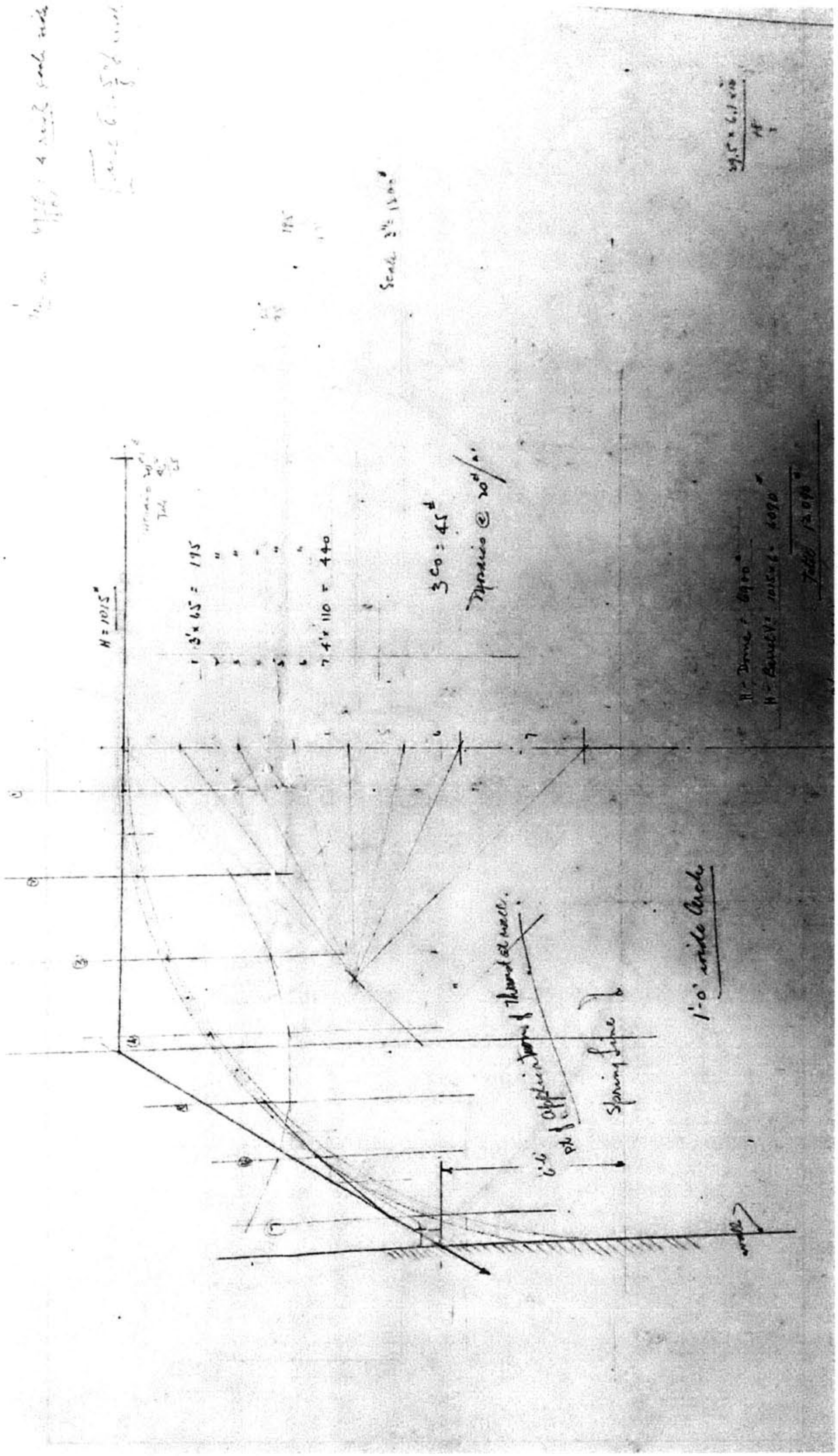

Figure B.16: St. John's R.C. Church Barrel Vault, Jersey City, NY 1931 [Avery Library]

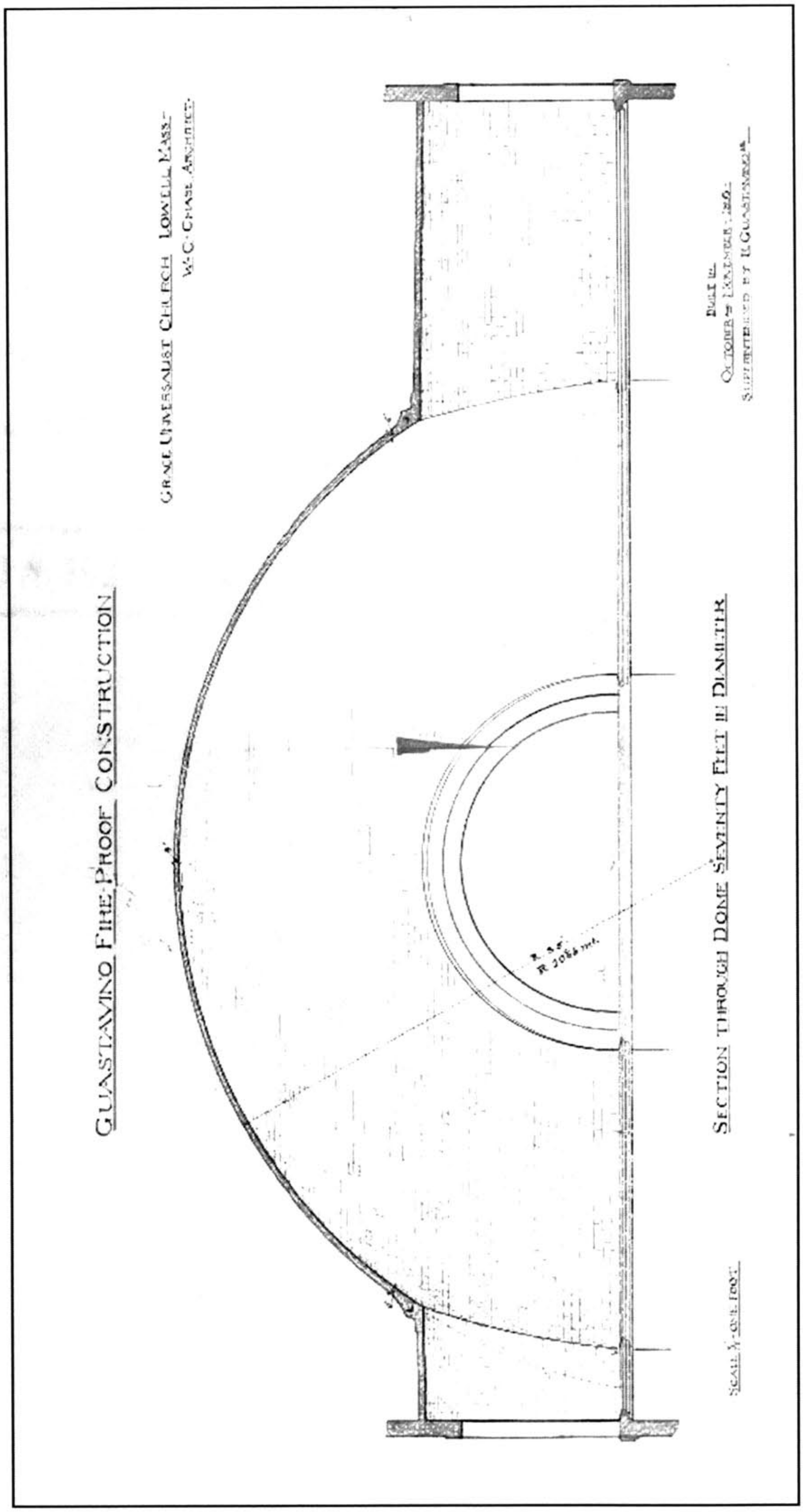

Figure B.17: Grace Universalist Church Dome, Lowell, MA 1895 [Avery Library]

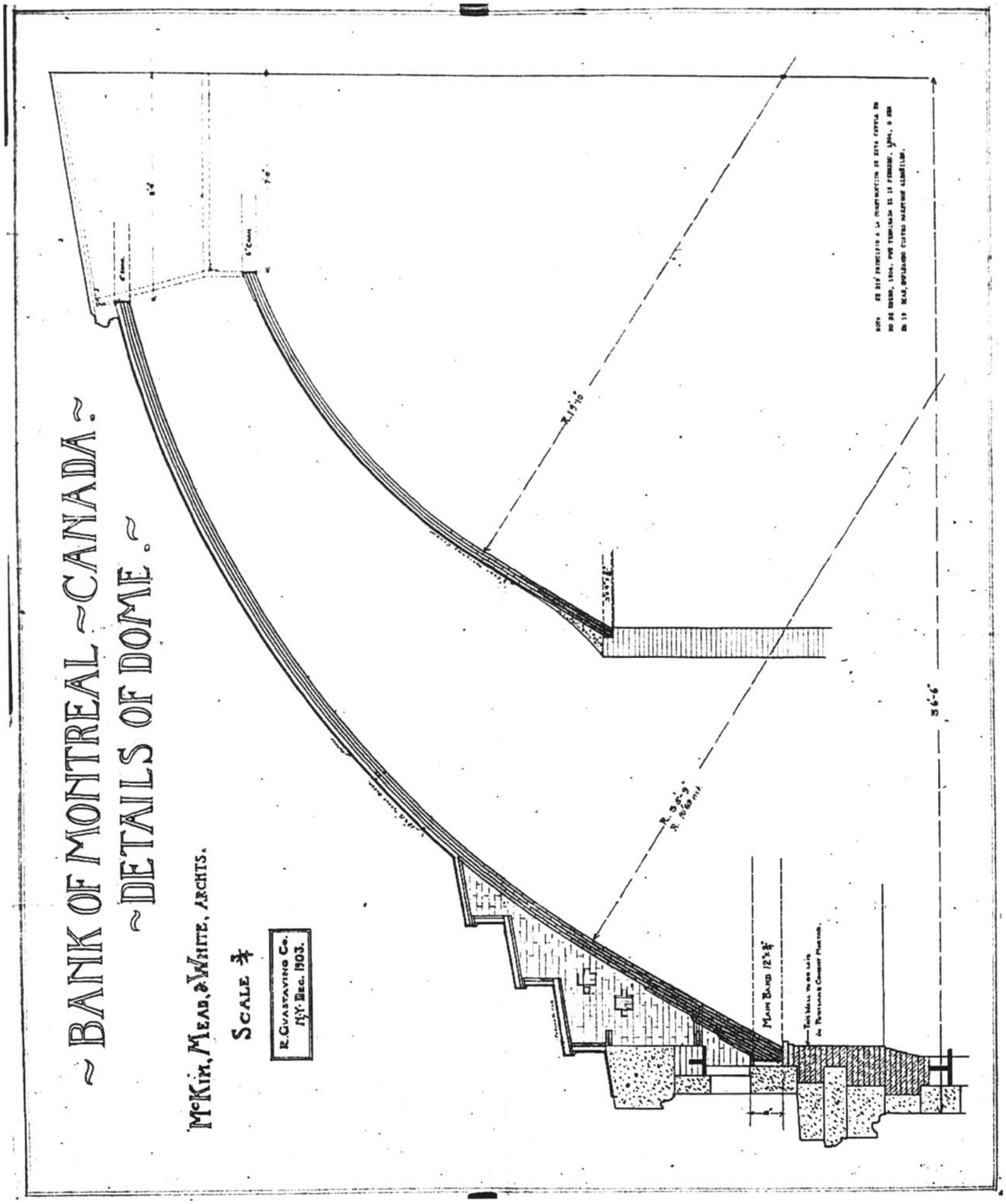

Figure B.18: Bank of Montreal Dome, Canada 1903 [Avery Library]

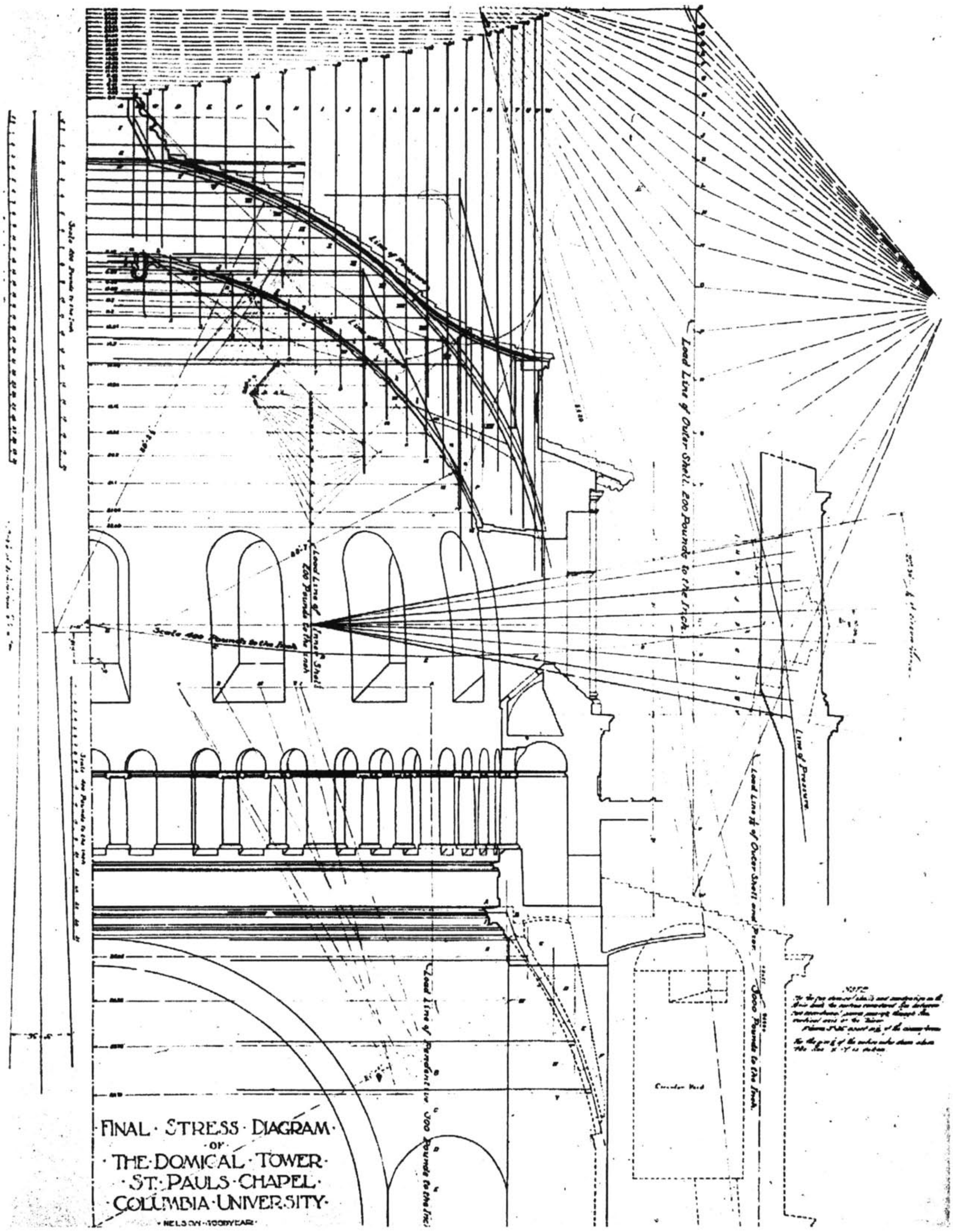

Figure B.19: St. Paul's Chapel, Columbia University, NY, NY 1906 [Avery Library]

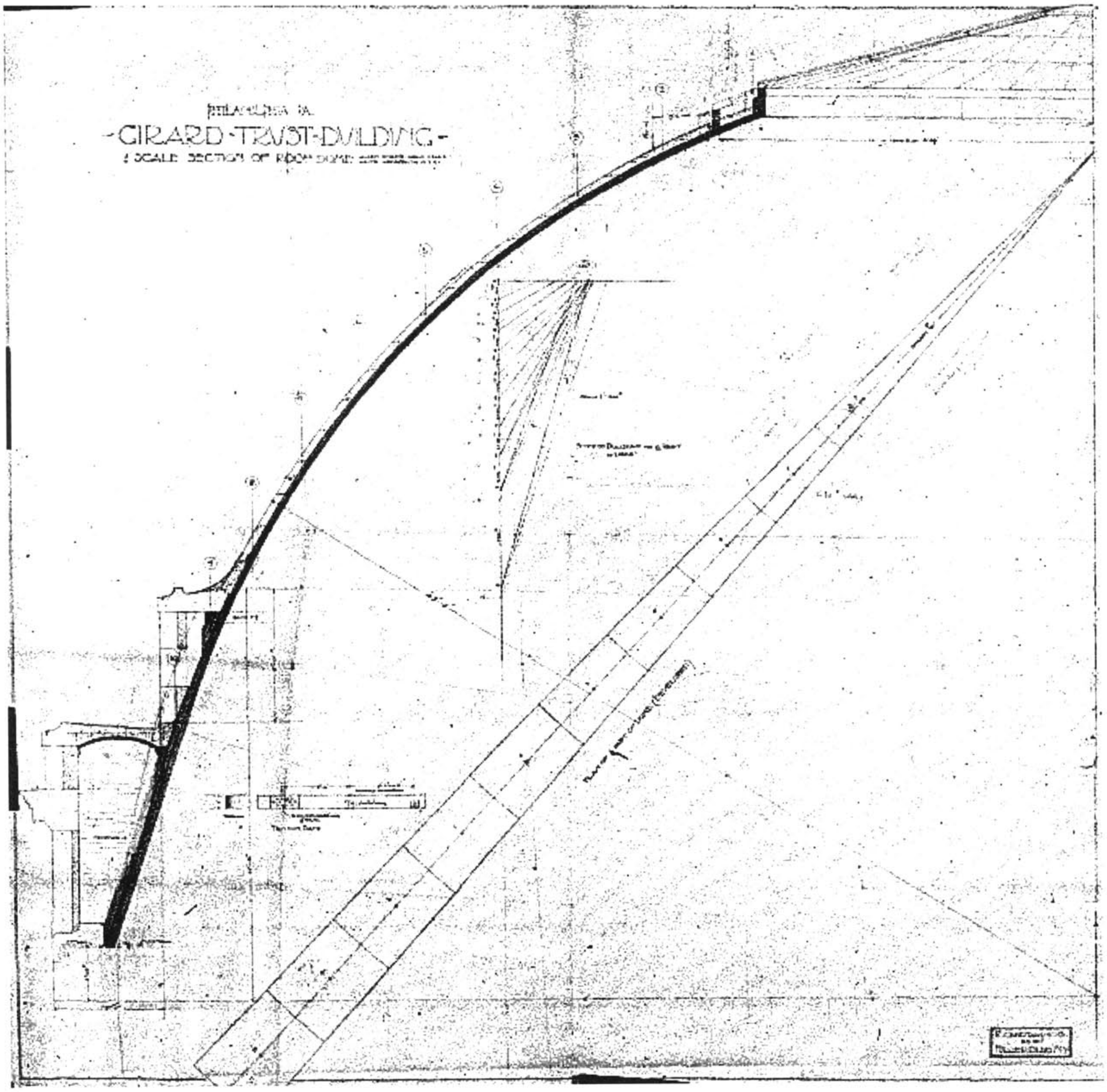

Figure B.20: Girard Trust Building Dome, Philadelphia, PA 1907 [Avery Library]

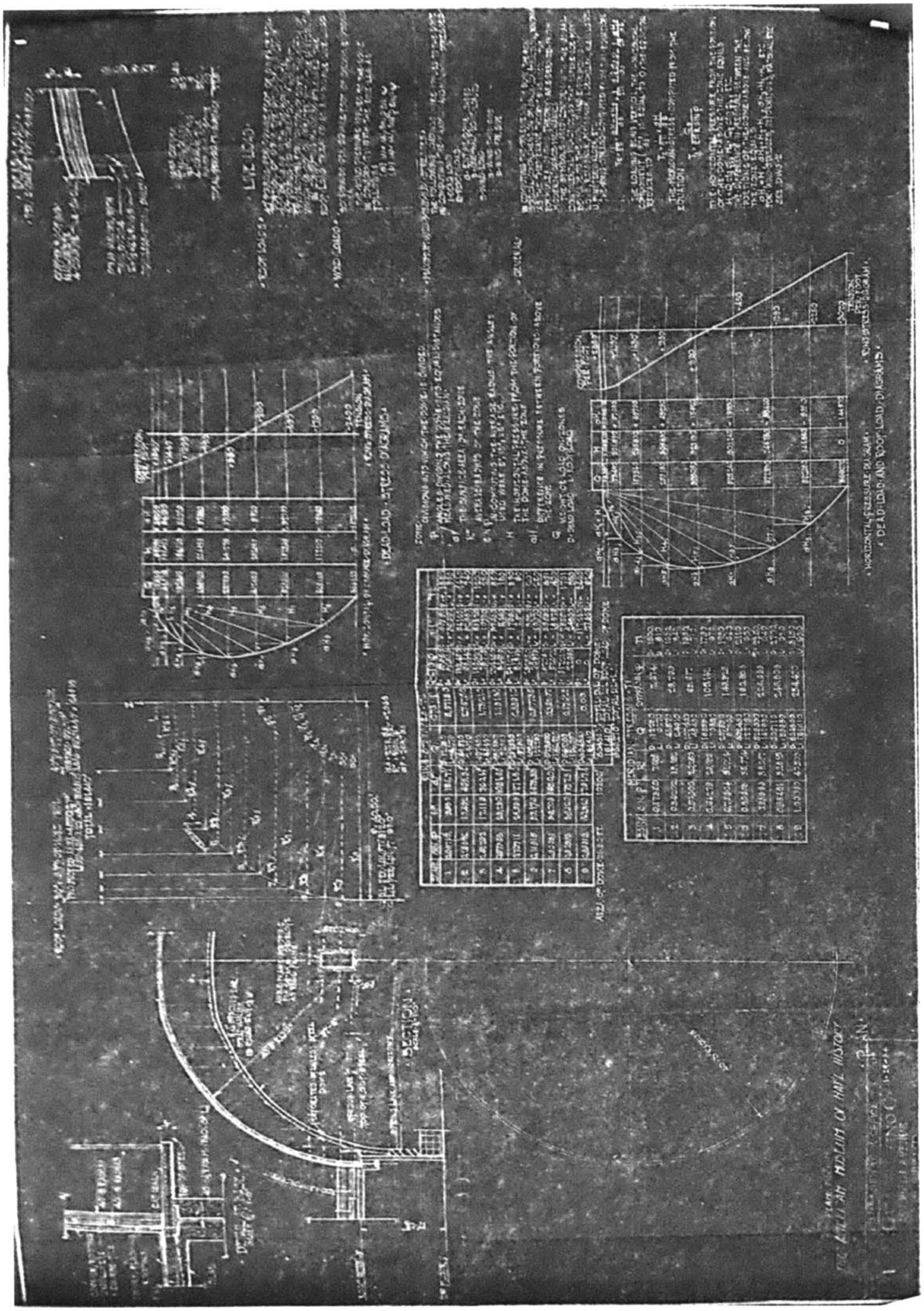

Figure B.21: Planetarium American Museum of National History, NY, NY 1934 [Avery Library]

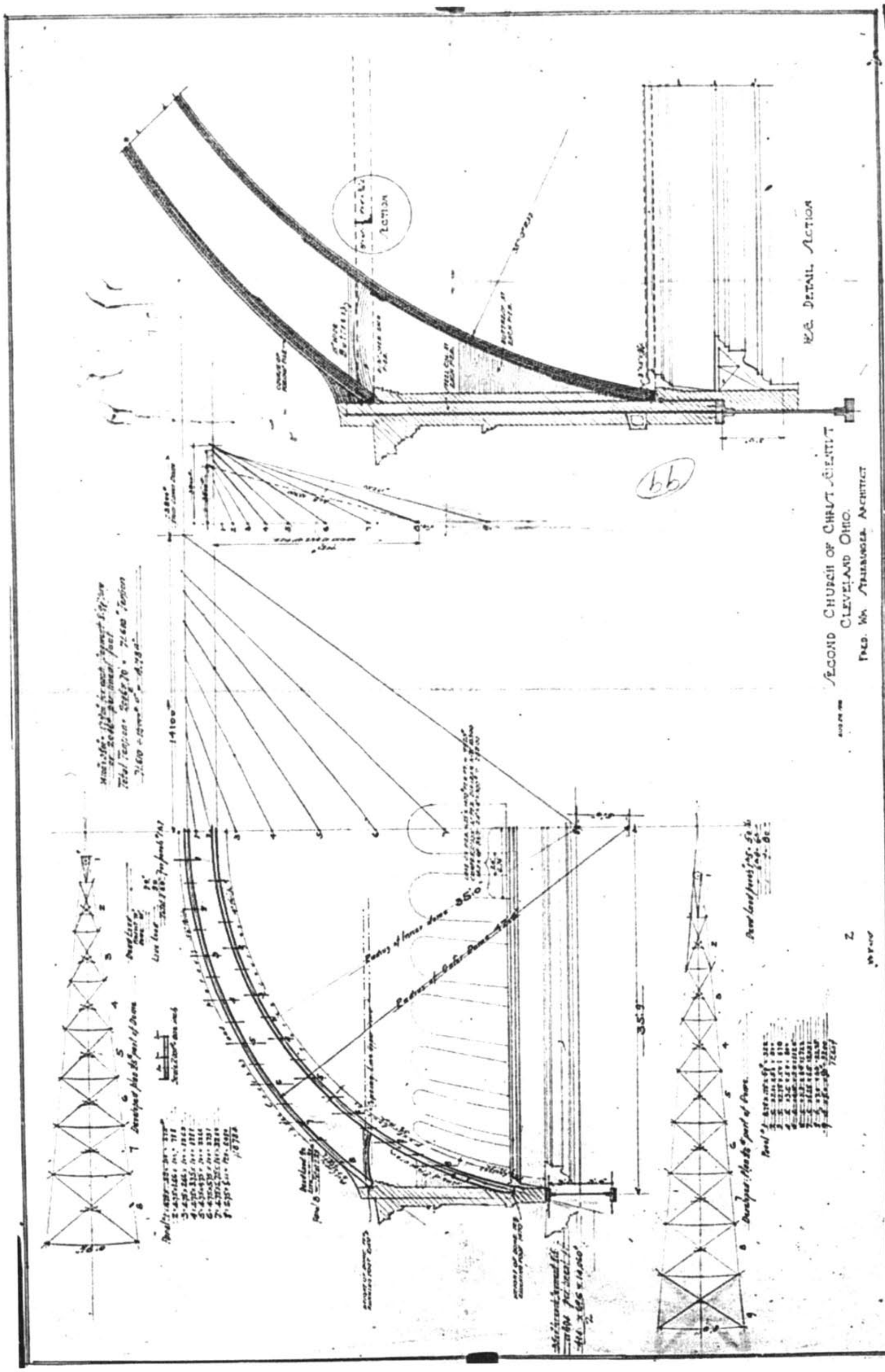

Figure B.22: Second Church Christ Scientist, Cleveland, Ohio 1946 [Avery Library]

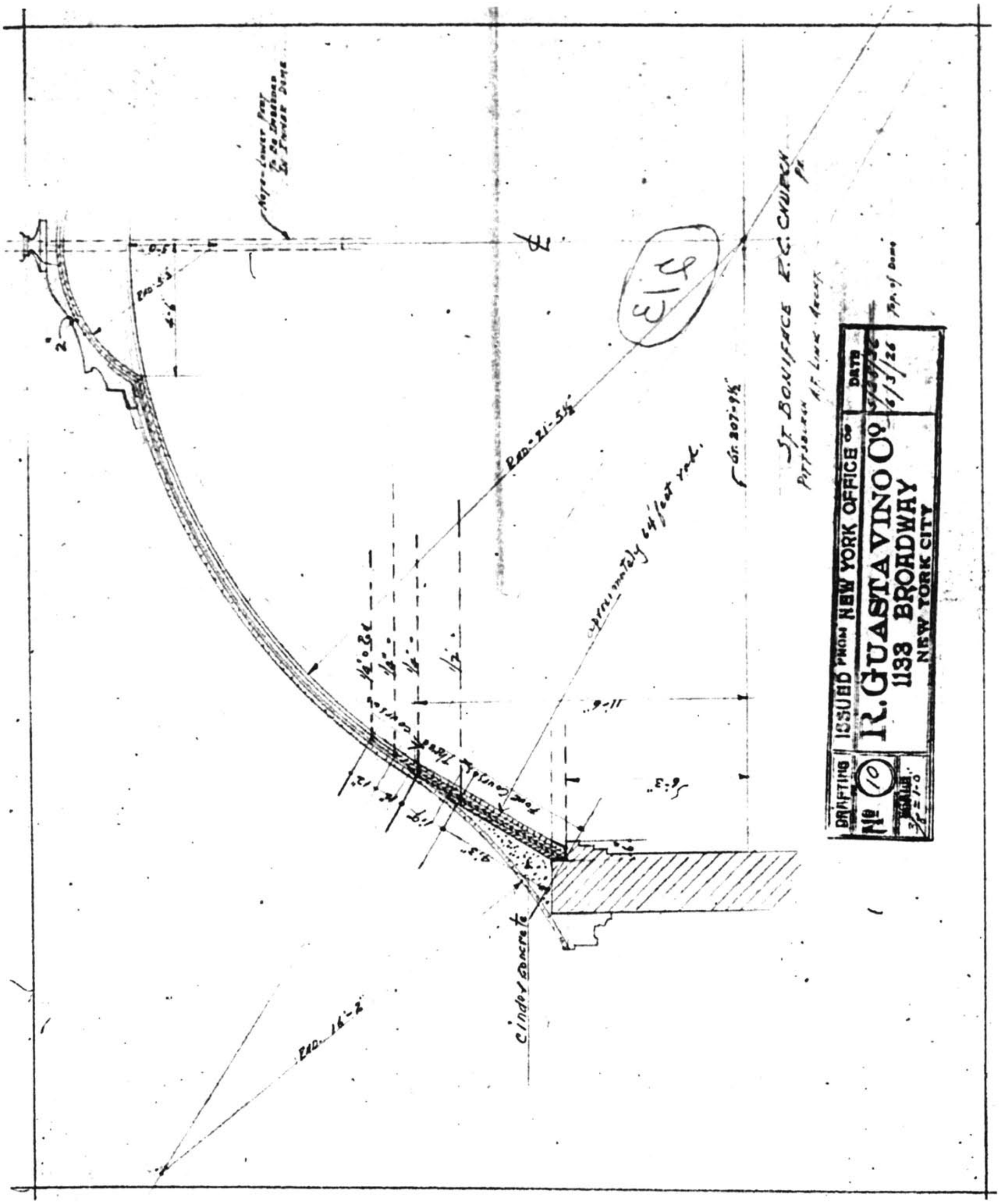

Figure B.23: St. Boniface R.C. Church, Pittsburgh, PA 1926 [Avery Library]

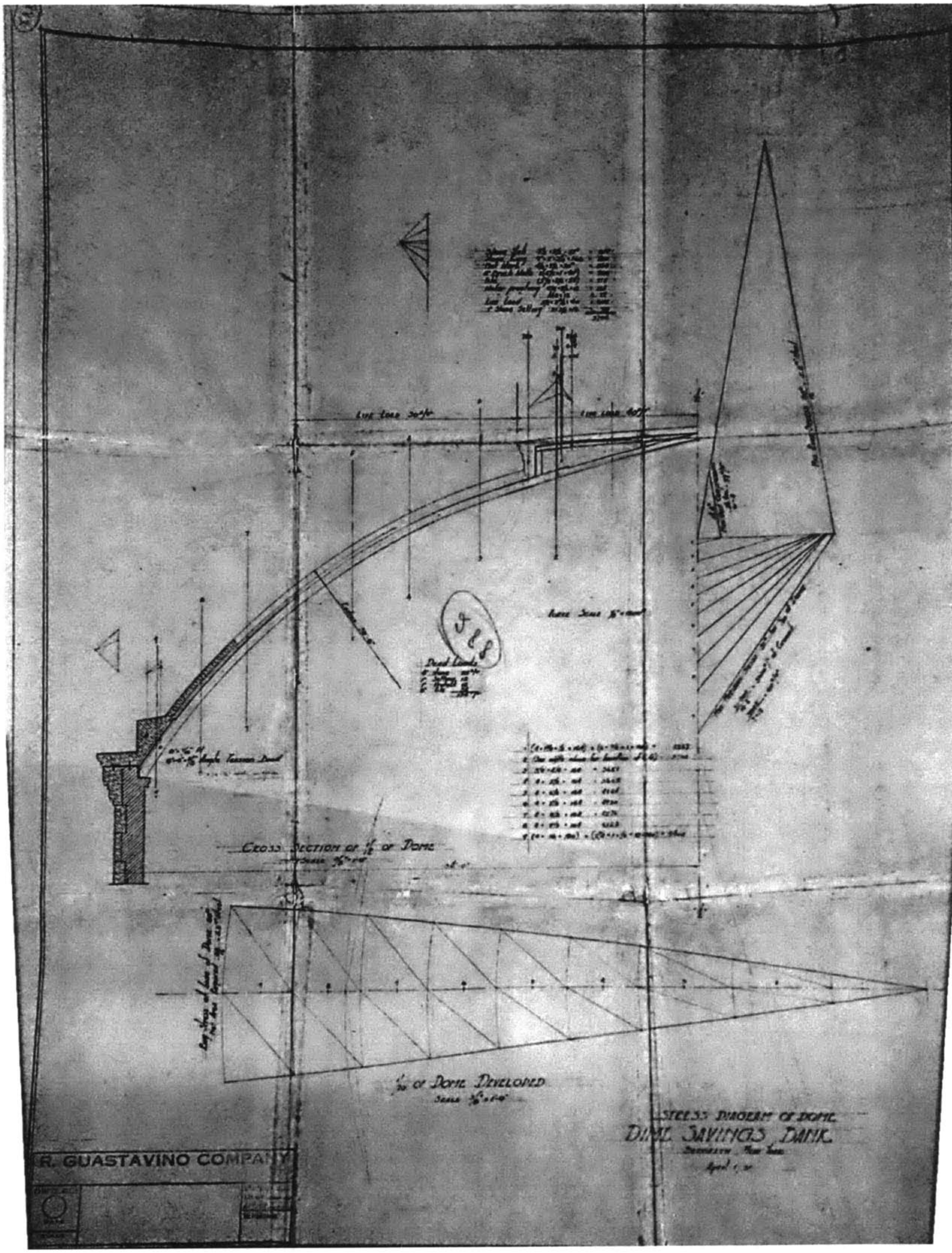

Figure B.24: Dime Savings Bank Dome, Brooklyn, NY 1931 [Avery Library]

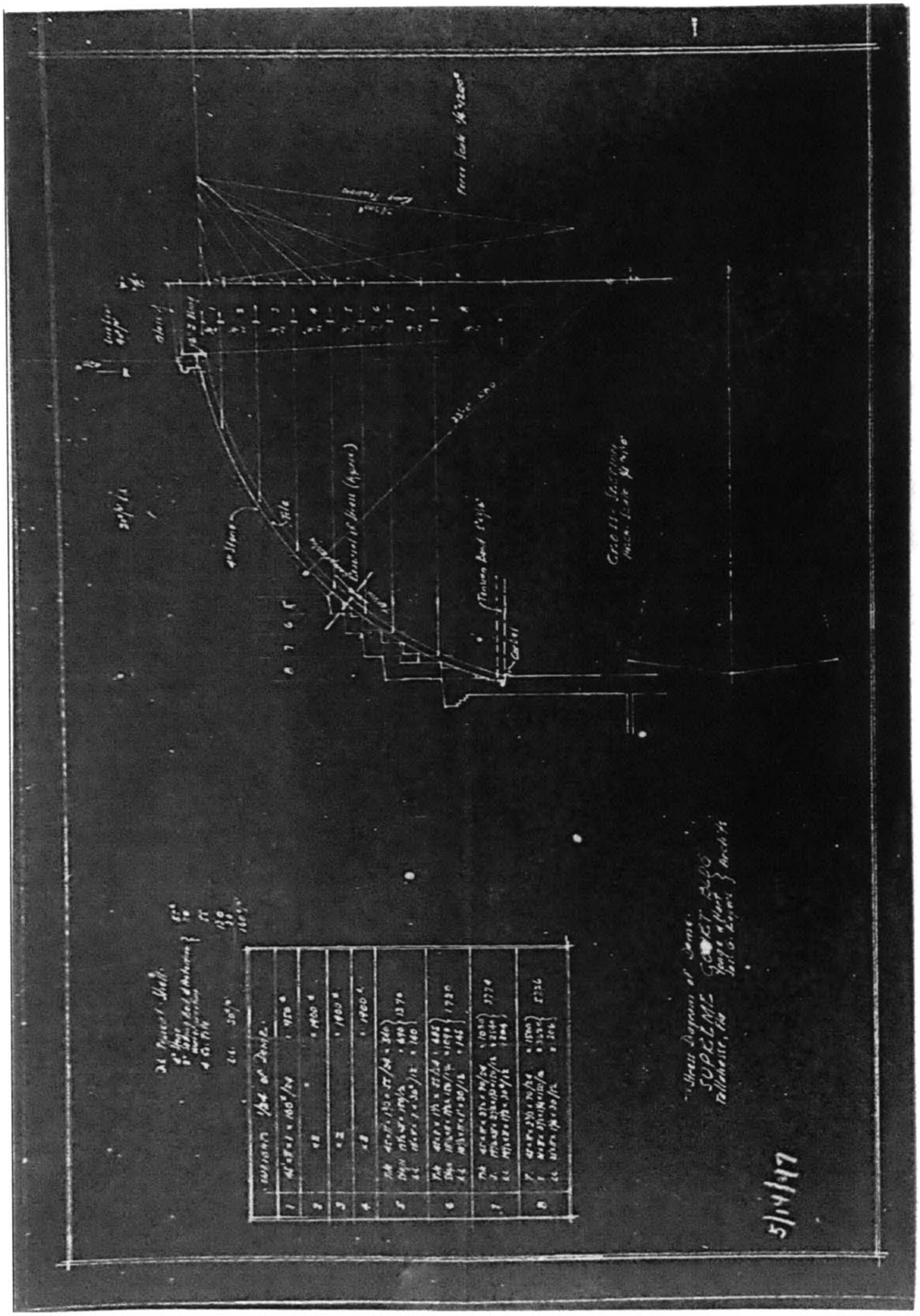

Figure B.25: Supreme Court Building, Tallahassee, Florida 1947 [Avery Library]

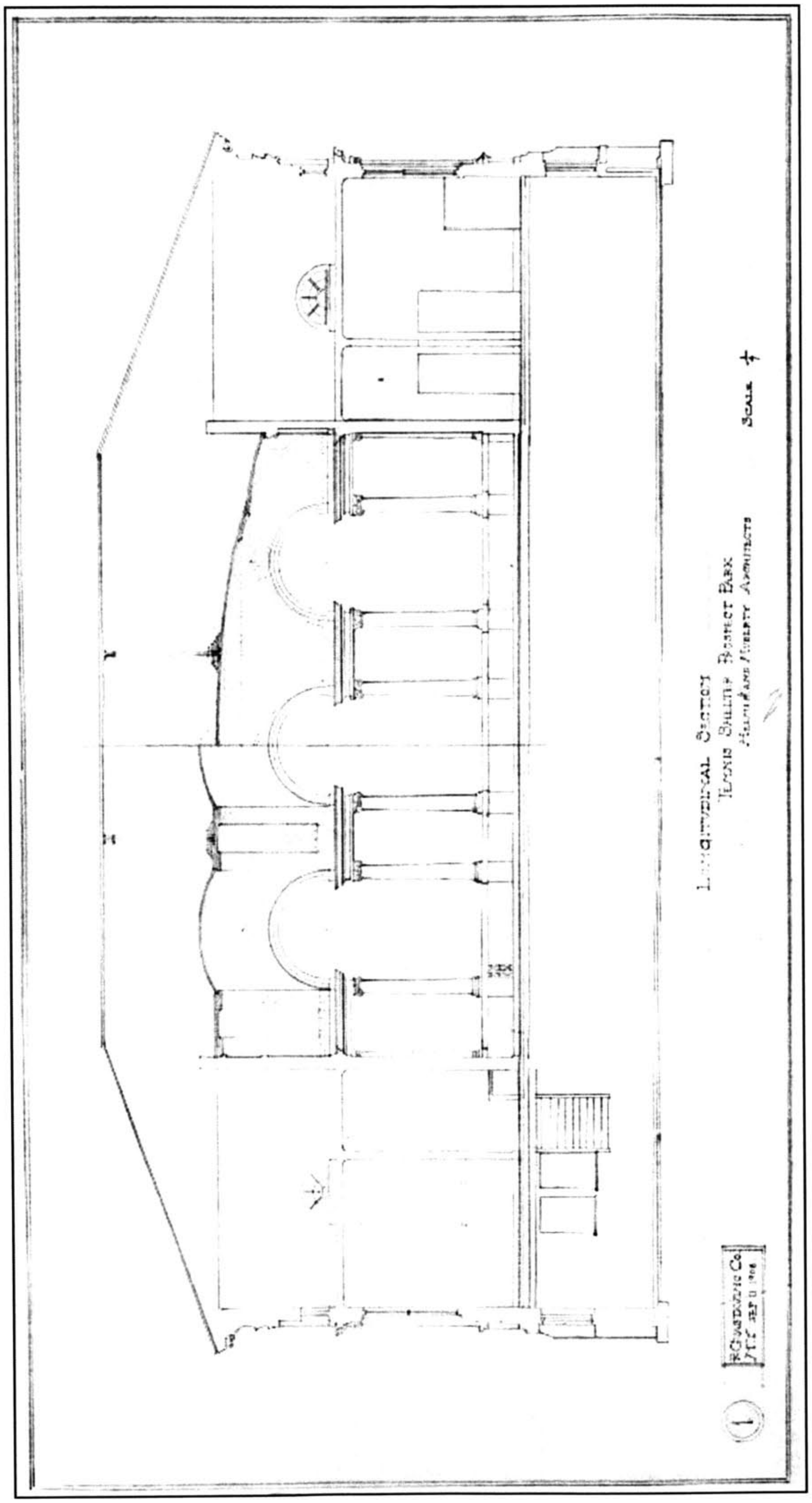

Figure B.26: Tennis Shelter Longitudinal Section, Prospect Park, Brooklyn, NY 1906 [Avery Library]

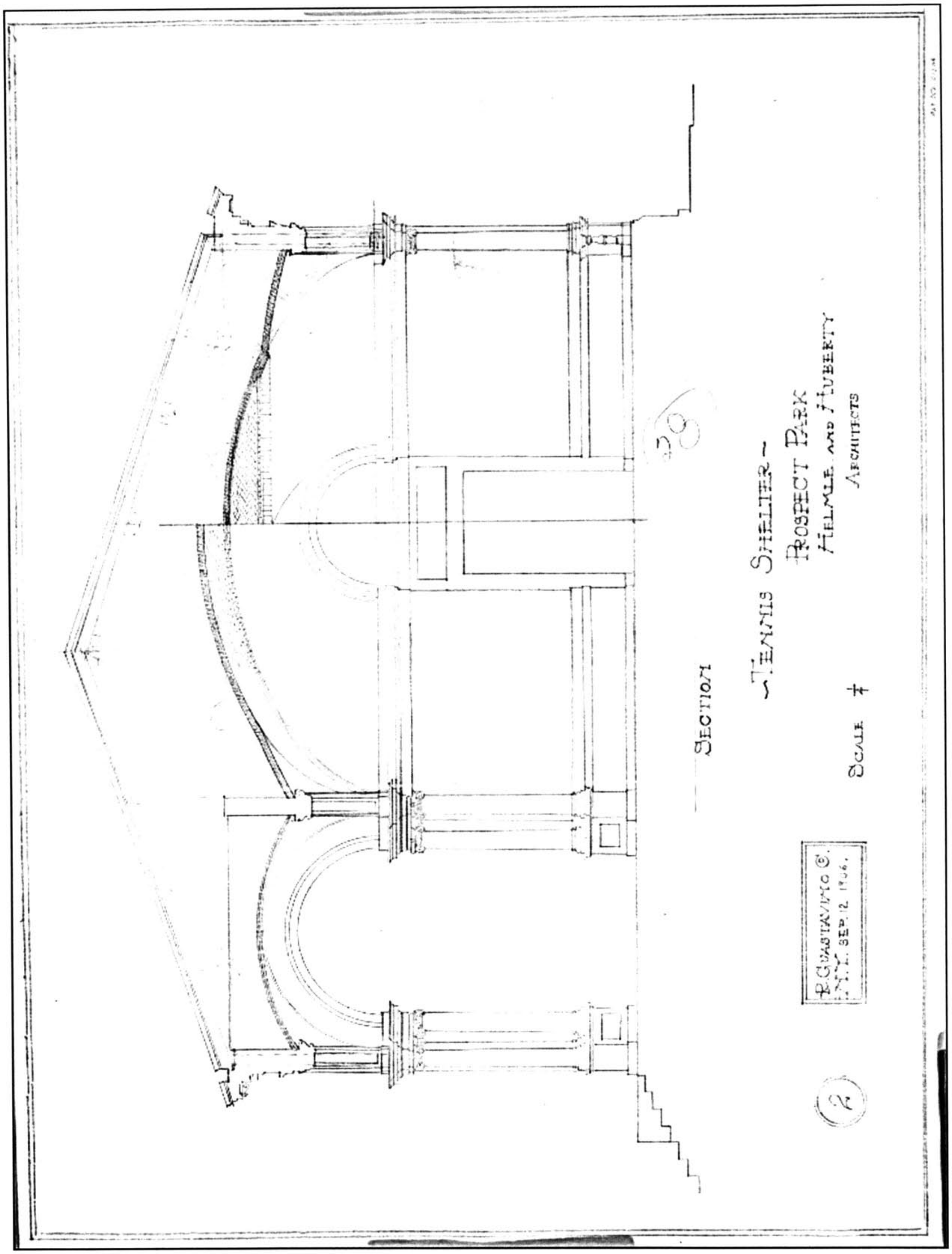

Figure B.27: Tennis Shelter Section View, Prospect Park, Brooklyn, NY 1906 [Avery Library]

Appendix C: Cathedral of St. John the Divine Calculations and Drawings

Table C.1: Detailed Dome Segment Area Calculations

Segment	*Area (ft^2)*	*Total Distributed Load Considered (psf)*	*Load Applied (lbs)*	*Load Converted to Distance (inches)*
1	4.50	90	405.0	0.05
2	13.75	90	1237.5	0.15
3	22.00	90	1980.0	0.25
4	30.25	90	2722.5	0.34
5	38.05	90	3424.5	0.43
6	46.75	90	4207.5	0.53
7	57.75	110	6352.5	0.79
8	63.25	110	6957.5	0.87
9	71.50	110	7865.0	0.98
10	79.75	120	9570.0	1.20
11	54.25	120	6510.0	0.81
12	57.75	130	7507.5	0.94
13	61.25	130	7962.5	1.00
14	64.75	130	8417.5	1.05
15	67.55	130	8781.5	1.10
16	66.50	130	8645.0	1.08
17	56.00	130	7280.0	0.91
18	45.00	130	5850.0	0.73
19	40.20	130	5226.0	0.65
20	35.00	130	4550.0	0.57

Table C.2: Meridional Forces and Stresses With and Without Tension Considered

Segment	*Meridional Forces (lbs)*	*Lune Width (ft)*	*Meridional Stresses (plf)*	*Brick Thickness (in)*	*Meridional Stresses (Tension) (psi)*	*Meridional Stresses (Without Tension) (psi)*
1	4,648	0.77	6,072	4	126.5	126.5
2	10,705	2.30	4,662	4	97.1	97.1
3	15,414	3.83	4,027	4	83.9	83.9
4	21,279	5.36	3,971	4	82.7	82.7
5	25,912	6.89	3,761	4	78.4	78.4
6	30,770	8.42	3,654	4	76.1	76.1
7	37,552	9.95	3,774	6	52.4	52.4
8	44,949	11.48	3,915	6	54.4	54.4
9	52,061	13.01	4,001	6	55.6	55.6
10	59,811	14.54	4,113	7.5	45.7	45.7
11	66,309	15.80	4,198	7.5	46.6	46.6
12	71,898	16.77	4,287	12	29.8	29.8
13	78,249	17.74	4,410	12	30.6	30.6
14	84,961	18.72	4,539	12	31.5	31.5
15	92,021	19.69	4,673	12	32.4	32.4
16	99,086	19.06	5,199	12	36.1	36.2
17	105,327	16.49	6,389	12	44.4	44.7
18	110,030	13.80	7,972	12	55.4	56.4
19	113,645	11.55	9,835	12	68.3	70.4
20	116,717	9.67	12,070	12	83.8	87.3

Table C.3: Hoop Forces and Stresses

Segment	Hoop Forces (lbs)	Unit Length (ft)	Hoop Stresses (plf)	Brick Thickness (in)	Hoop Stresses (psi)
1	33310	5.5	504.7	4	126.2
2	9923	5.5	150.4	4	37.6
3	22828	5.5	345.9	4	86.5
4	16954	5.5	256.9	4	64.2
5	12124	5.5	183.7	4	45.9
6	15480	5.5	234.5	4	58.6
7	21234	5.5	321.7	6	53.6
8	14213	5.5	215.3	6	35.9
9	13836	5.5	209.6	6	34.9
10	9769	5.5	148	7.5	19.7
11	2100	3.5	50	7.5	6.7
C/T	-22597	2.23	-846.1	7.5	-112.8
12	4511	3.5	107.4	12	9.0
13	2572	3.5	61.24	12	5.1
14	1490	3.5	35.47	12	3.0
15	1371	3.5	32.65	12	2.7
16	-1088	3.5	-25.9	12	-2.2
17	-3347	3.5	-79.7	12	-6.6
18	-5483	3.5	-130.6	12	-10.9
19	-7421	3.5	-176.7	12	-14.7
20	-10246	3.5	-243.9	12	-20.3

Table C.4: Meridional and Hoop Force and Stress Quantities for Cathedral Drawing

Segment	Meridional Forces (lbs)	Meridional Stresses (psf)	Hoop Forces (lbs)	Hoop Stresses (psf)
1	4,648	126.5	33310	126.2
2	10,705	97.1	9923	37.6
3	15,414	83.9	22828	86.5
4	21,279	82.7	16954	64.2
5	25,912	78.4	12124	45.9
6	30,770	76.1	15480	58.6
7	37,552	52.4	21234	53.6
8	44,949	54.4	14213	35.9
9	52,061	55.6	13836	34.9
10	59,811	45.7	9769	19.7
11	66,309	46.6	2100	6.7
C/T	--	--	-22597	-112.8
12	71,898	29.8	4511	9.0
13	78,249	30.6	2572	5.1
14	84,961	31.5	1490	3.0
15	92,021	32.4	1371	2.7
16	99,086	36.1	-1088	-2.2
17	105,327	44.4	-3347	-6.6
18	110,030	55.4	-5483	-10.9
19	113,645	68.3	-7421	-14.7
20	116,717	83.8	-10246	-20.3

Table C.5: Meridional Stresses for 6", 6" and 12", and the Final Layout Brick Thicknesses

Segment	Final Layout (psi)	Brick Layout (in)	6" Brick Thickness (psi)	Brick Layout (in)	6" and 12" Brick Thickness (psi)	Brick Layout (in)
1	118.8	4	96.8	6	96.8	6
2	92.6	4	75.5	6	75.5	6
3	75.0	4	61.1	6	61.1	6
4	70.5	4	57.4	6	57.4	6
5	68.6	4	55.9	6	55.9	6
6	67.8	4	55.3	6	55.3	6
7	46.6	6	55.2	6	55.2	6
8	49.0	6	55.5	6	55.5	6
9	51.0	6	56.1	6	56.1	6
10	42.7	7.5	56.9	6	56.9	6
11	44.2	7.5	57.6	6	29.1	12
12	28.5	12	58.2	6	30.0	12
13	29.7	12	59.4	6	31.1	12
14	31.0	12	60.7	6	32.2	12
15	32.1	12	62.0	6	33.2	12
16	35.0	12	67.8	6	36.7	12
17	42.0	12	81.3	6	44.3	12
18	52.7	12	100.5	6	55.0	12
19	65.3	12	122.1	6	67.2	12
20	76.6	12	140.5	6	77.6	12

Table C.6: Hoop Stresses for 6", 6" and 12", and the Final Layout Brick Thicknesses

Segment	*Final Layout (psi)*	*Brick Layout (in)*	*6" Brick Thickness (psi)*	*Brick Layout (in)*	*6" and 12" Brick Thickness (psi)*	*Brick Layout (in)*
1	113.5	4	92.5	6	92.5	6
2	37.3	4	30.4	6	30.4	6
3	51.1	4	41.6	6	41.6	6
4	51.5	4	41.9	6	41.9	6
5	47.9	4	39.0	6	39.0	6
6	43.9	4	35.7	6	35.7	6
7	41.0	6	31.6	6	31.6	6
8	35.1	6	28.2	6	28.2	6
9	24.7	6	18.9	6	18.9	6
10	20.6	7.5	14.7	6	14.7	6
11	3.6	7.5	-6.4	6	3.7	12
T/C	-21.0	7.5	-12.4	6	-9.6	12
12	11.6	12	7.8	6	9.8	12
13	7.7	12	2.2	6	6.1	12
14	6.1	12	0.9	6	4.7	12
15	-2.0	12	-13.0	6	-3.4	12
16	-5.5	12	-15.4	6	-6.7	12
17	-7.1	12	-16.9	6	-8.3	12
18	-9.2	12	-19.3	6	-9.9	12
19	-15.7	12	-34.9	6	-18.5	12
20	-24.3	12	-87.4	6	-47.4	12

Table C.7: Amount of Steel Necessary for 6", 6" and 12", and the Final Layout Brick Thicknesses

Segment	Final Layout (in)	Brick Layout (in)	6" Brick Thickness (in)	Brick Layout (in)	6" and 12" Brick Thickness (in)	Brick Layout (in)
11	-	7.5	0.096	6	-	12
T/C	0.194	7.5	0.192	6	0.166	12
12	-	12	-	6	-	12
13	-	12	-	6	-	12
14	-	12	-	6	-	12
15	0.075	12	0.137	6	0.098	12
16	0.125	12	0.148	6	0.139	12
17	0.143	12	0.156	6	0.155	12
18	0.162	12	0.166	6	0.168	12
19	0.212	12	0.223	6	0.230	12
20	0.264	12	0.354	6	0.369	12
Sum of Steel Needed	1.174		1.471		1.325	

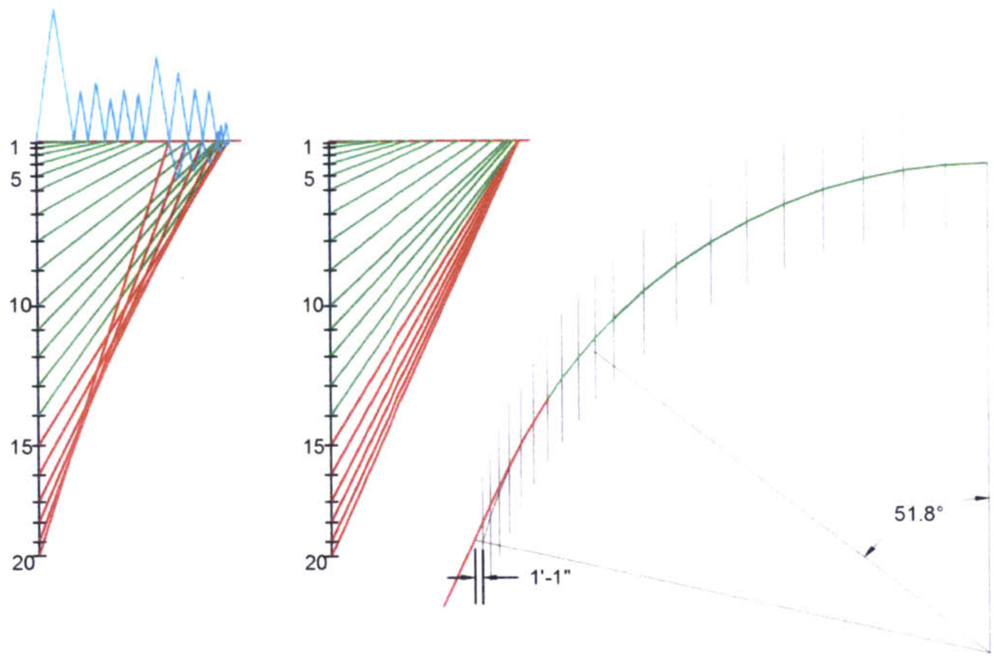

Figure C.1: Initial Sphere Shape 1 (S1): Constant 69'-3 9/16"

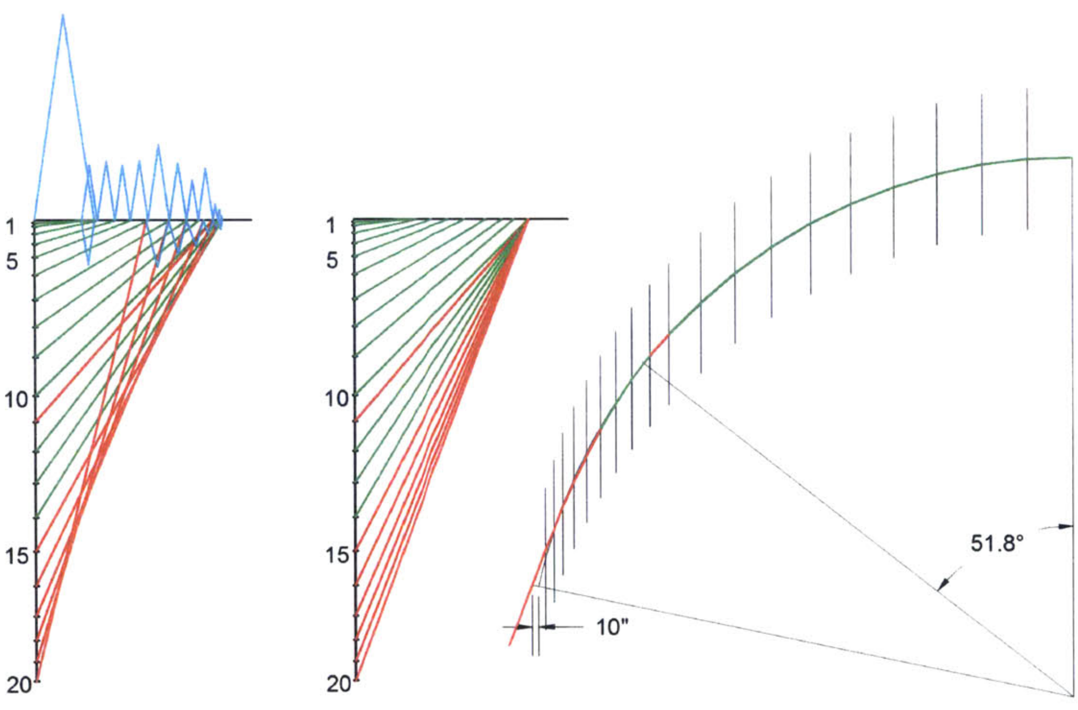

Figure C.2: Initial Sphere Shape 2 (S2): Constant 66'-0"

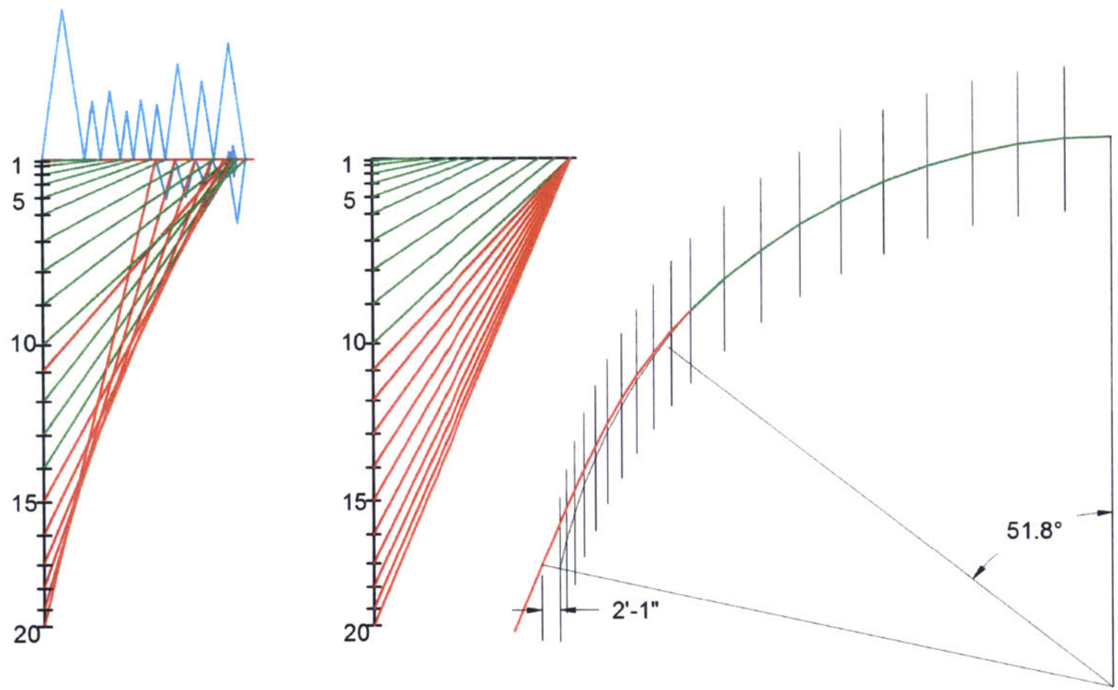

Figure C.3: Initial Sphere Shape 3 (S3): Top 69'-3 9/16", Bottom 66'-0"

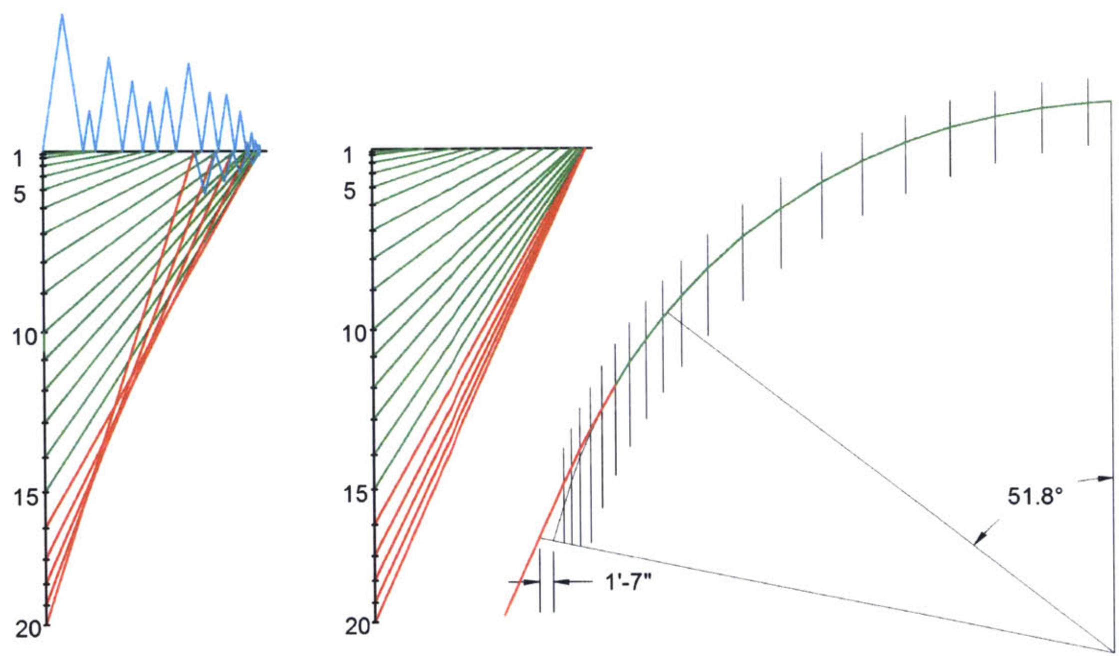

Figure C.4: Initial Sphere Shape 4 (S4): Trace of Final Drawing Meridional Lines

Table C.8: Values of Guastavino Dome Meridional and Hoop Forces and Stresses to Compare to Dugum Values

Segment	Angle (Degrees)	Meridional Stresses (psf)	Meridional Forces (lbs)	Hoop Stresses (psf)	Hoop Forces (lbs)
1	2.5	18216	4,648	18169	33310
2	7.5	13985	10,705	5413	9923
3	12.5	12082	15,414	12452	22828
4	17.5	11914	21,279	9247	16954
5	22.5	11284	25,912	6613	12124
6	27.5	10963	30,770	8444	15480
7	32.5	7547	37,552	7722	21234
8	37.5	7830	44,949	5168	14213
9	42.5	8002	52,061	5031	13836
10	48	6580	59,811	2842	9769
11	51.2	6716	66,309	960	2100
C/T	51.8	--	--	-16245	-22597
12	54	4287	71,898	1289	4511
13	56.8	4410	78,249	735	2572
14	59.5	4539	84,961	426	1490
15	62.5	4673	92,021	392	1371
16	65.1	5207	99,242	-311	-1088
17	68	6440	106,172	-956	-3347
18	70.8	8121	112,094	-1567	-5483
19	73.5	10131	117,065	-2120	-7421
20	76.5	12567	121,523	-2927	-10246

Diagonal Lune: Meridional			lbs	Diagonal Lune: Hoop			
Top H. Thrust	0.0	psf	-1380			psf	lbs
M1	3.8	-3541	-1383	H1	1.9	-3417	-4869
M2	7.5	-4030	-2867	H2	5.6	-3537	-5039
M3	11.3	-4027	-4371	H3	9.3	-3543	-5048
M4	15.0	-3982	-5791	H4	13.0	-3210	-4574
M5	18.8	-3972	-7217	H5	16.8	-2865	-4082
M6	22.5	-3983	-8654	H6	20.5	-3052	-4348
M7	26.3	-4013	-10109	H7	24.2	-2649	-3775
M8	30.0	-3753	-13398	H8	27.9	-5252	-9359
M9	33.8	-3283	-15662	H9	31.7	-2149	-4598
M10	37.5	-3502	-18336	H10	35.4	-2534	-5422
M11	41.3	-3792	-22398	H11	39.1	-3991	-8880
M12	45.0	-3522	-26850	H12	42.8	-3633	-9715
M13	48.8	-2932	-32358	H13	46.6	-1564	-5690
M14	52.5	-2838	-38934	H14	50.3	-2007	-8586
M15	56.2	-3157	-45434	H15	54.0	-1343	-5744
M16	59.9	-3460	-51920	H16	57.7	-682	-2917
M17	63.6	-3757	-58436	H17	61.5	0	0
M18	67.3	-4055	-65041	H18	65.2	681	2916
M19	71.0	-4363	-71795	H19	68.9	1342	5742
M20	74.2	-6573	-77181	H20	72.6	2392	10235
M Support	78.0	-9546	-76414	H21	76.4	2715	11615
M Horizontal			-15889	H Edge			62079

Figure C.5: Dugum Values for Meridional and Hoop Forces and Stresses for Cathedral Dome