

**Spatial Distribution of Dynamic Amplification in Progressive  
Collapse Analysis of Building Frames**

by

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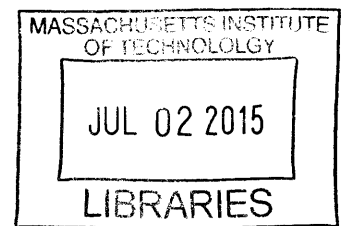
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Submitted to the Department of Civil & Environmental Engineering on May 21, 2015  
in partial fulfillment of the requirement for the Degree of Master of  
Engineering in Civil & Environmental Engineering

## **Abstract**

The purpose to use Dynamic Application factor is to design and analysis the building and structures more efficient under the progressive collapse where one can only use only the Linear Static Analysis and times the results by the DAF instead of using more complex Linear Dynamic Analysis. It is suggested in the code that Dynamic Amplification Factor should be 2 to account for the all the dynamic effect, however, the simulation from this paper suggest otherwise. Throughout the four case analysis in this paper, none of them has more than half of the design members with qualified Dynamic Amplification Factor which suggests that the current code has failed to predict a efficient loading condition for one to analysis the building under the progressive collapse condition.

**Thesis Supervisor: Pierre Ghisbain**

**Title: Lecturer in Civil & Environmental Engineering**





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# 1.Introduction

Progressive collapse is a phenomenon where a local failure in a structure causes a partial or full-scale collapse of the whole structure. ASCE 7 defines this phenomenon as "the spread of an initial local failure from elements to element, eventually resulting in the collapse of the entire structure of a disproportionately large part of it" (ASCE7-10,2010). The phenomenon is often seen when a tragic event or accident happens, resulting in loss of hundred of lives.

## 1.1 Examples

The following sections are some examples of life losses and building damages caused by progressive collapse:

### 1.1.1 Alfred P.Murrah Federal Building (Oklahoma City Bombing)

The Alfred P.Murrah Federal Building was a nine-stores building of RC slab/columns. Measuring 61.5m\*21.5m. The frame had 10.7m\*6.2m typical bays. Four of the north-face columns, spaced at 12.3m and unsupported for two stories, formed the atrium at the street level. On April of 1995, a car bomb estimated to contain about 1.8 tons of high explosives destroyed three of the four front columns and a center column. The 200mm thick slabs separated from the columns located in the center, resulting in 8 of the 10 bays collapsing in the northern half of the building. This tragic event caused 168 fatalities and \$50 million in damage.



**Figure 1: Alfred P. Murrah Federal Building**

### **1.1.2 World Trade Center**

On September 11, 2001, two commercial airliners were hijacked and each flew into one of the two 110-story towers. The structure damage sustained by each tower from the impact which, combined with the ensuing fires, resulted in the total collapse of both buildings. Following the event, the Federal Emergency Management Agency (FEMA) and Structural Engineering Institute of the American Society of Civil Engineers and several other Federal agencies and professional organizations deployed a team of civil structural and fire protection engineers to study the performance of buildings at World Trade Center.

### **1.1.3 Ronan Point**

In May 1968, the Ronan Point Apartment building collapsed in Newham, England. A gas explosion on the 18<sup>th</sup> floor took out a precast concrete column,

resulting in a chain reaction that caused partial collapse. The upper floor collapsing caused an impact load on the lower floors causing them collapsing as well. This event killed 4 people and injured 7 others, fostering interest in studying the behavior of progressive collapse in building code (Pearson and Delatte 2005).



**Figure 2: Ronan Point**

## **1.2 Types of Collapse**

For purpose of clear explanation, Starossek have categorized all the types of the progressive collapse scenarios into 5 groups:

### **1.2.1 Domino Collapse**

Domino Collapse occurs when there is overturning in one element where the fail of the element causes a lateral impact on the adjacent elements, which experience a horizontal force that is transmitted by a static and dynamic impact, which causes that element to overturn as well. The failures propagate in an overturning direction.

### **1.2.2 Instability-type Collapse**

Instability collapse occurs when there is failure of a bracing element that stabilizes load bearing elements. The load-bearing elements is no longer stable so that a small perturbation causes immediate collapse.

### **1.2.3 Pancake Collapse**

Pancake Collapse when there is an initial failure of vertical load bearing element. The separation of structure components leads to an impact and failure in the floor below. This failure propagates vertically through the building causing a pancake type collapse.

### **1.2.4 Section Collapse**

Section Collapse occurs when a member under bending moment or tension tie is cut, the internal force redistributed into the remaining cross section, corresponding to an increase in stress that would rupture further along the member.

### **1.2.5 Zipper Collapse**

Zipper Collapse occurs when rupture of one cable cause the redistribution of forces and impulse due to sudden failure, resulting in adjacent cable to rupture in transverse direction as well.

### **1.3 General Analysis Types**

There are four general analyses available for engineers in practice; however, in practice one usually chooses to use linear static and linear dynamic analysis for simplicity and efficiency reasons.

#### **1.3.1 Linear Static**

Linear Static analysis procedure is the simplest analysis to run. Without accounting both dynamic and nonlinearity, one should always evaluate the structure based on the demand to capacity ratio (DCR). The DCR is the ratio for the maximum moment determined by the computer analysis to the maximum plastic moment inherent to the beam. If the DCR exceeds 1, the structure could experience the progressive collapse and should be redesigned.

#### **1.3.2 Nonlinear Static**

Nonlinear Static analysis procedure is a static analysis, which takes account for the nonlinearity problem. Plastic hinges forms in the structure to account for the nonlinearity properties, however, it doesn't takes into account the dynamic effect. The computation time would be greater than normal linear static analysis due to the convergence issues.

#### **1.3.3 Linear Dynamic**

Dynamic analyses are considered more accurate comparing with just static analysis because they incorporate dynamic effects such as inertial and damping. The dynamic load combination is applied to the structure and DCR's are calculated to determine the likelihood of structure for progressive collapse. The disadvantage of this analysis is that it's oblivious to nonlinearity in structural design.



#### **1.3.4 Nonlinear Dynamic**

Nonlinear Dynamic analysis is most complete and reliable procedure for general analysis. It takes into account both nonlinearity and dynamic factors. It models the dynamic removal of the column and analyzes the time history of the structural response and allows the structure to enter the inelastic range. It gives more accurate information regarding the larger deformation, energy dissipation through material yielding. It is also time consuming.

#### **1.4 Dynamic Amplification factor**

The code require one to use a Dynamic Amplification factor to account for the dynamic effect when one designs for based on static analysis results. The current code chooses to use 2 as the Dynamic Amplification factor when one designs for both linear and nonlinear static analyses. Linear Static analysis is amplified to account for both dynamic and nonlinear effect and it is amplified by same factor of 2.

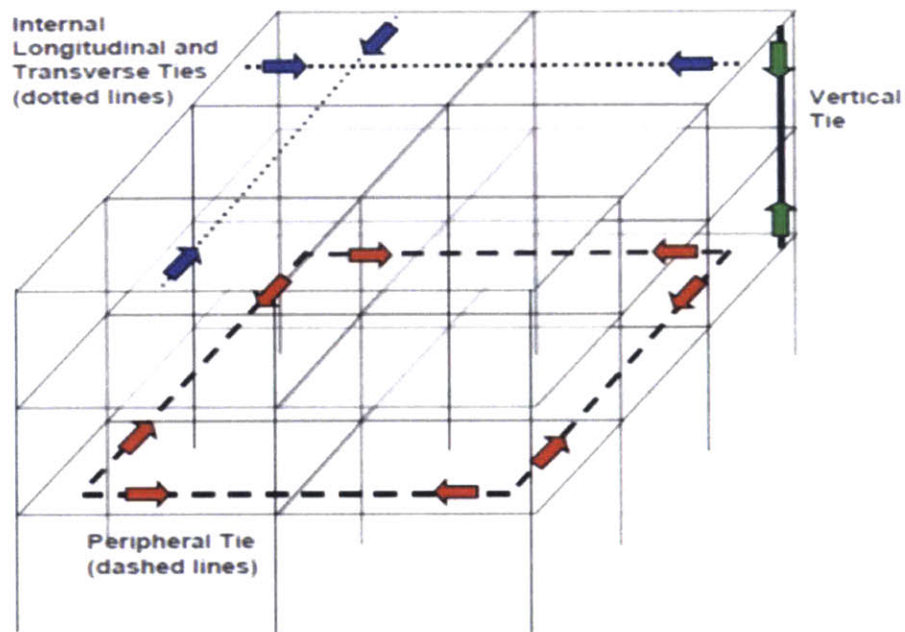
For general design, engineers usually chose to use static analysis for computational efficiency and then multiply the results with the Dynamic Amplification factor to account for the dynamic effect, after which more conservative sizing is chosen to account for safety reason. Since the linear static method is advised to be used only for small structures (maximum 10 story building) where the nonlinear and dynamic effects are easily predicted, it's reasonable to use a dynamic amplification factor less than 2(Ruth et al,2006). Using the code-required Dynamic Amplification factor of 2 would make the structure overdesigned, and it is desirable for one to find more accurate results from the following research.

#### **1.5 Progressive Collapse Design Method**

The following sections include three design methods that has been developed and used in engineering for the purpose of design against progressive collapse.

### 1.5.1 Tie Forces

The tie force method uses mechanical tie between structural members to integrate the structure in case of the local failure. The method simplified an indeterminate structure to a locally determined structure using a tie to approximate the effect. There are four types of structural ties based on the functionality of the structural members: longitudinal, transverse, peripheral and vertical.



**Figure 3: Tie Force Method**

This method utilizes the design strength and continuity to redistribute loadings caused by local failure.

### 1.5.2 Alternate Path Method

The principal of using the alternate path method is to analyze the structure force distribution in the event of local failure using various analysis method. Once the damage is present such as after the removal of load bearing elements, while analyzing the redistribution of the force, the method requires to find the primary

contribution from the remaining structures component and to design each component based on the new distribution. The component capacities are based on either the deformation or force controlled, therefore if the analysis procedure shows certain component or connections do not meet the strength criteria, the elements need to be redesigned to account for the progressive collapse requirement.

### **1.5.3 Enhanced Local Resistance**

The enhanced local resistance method is used in conjunction with other progressive collapse methods, but it is not a standalone method to design for progressive collapse. The principle of this design is to utilize ductile failure mechanism when the column is loaded to failure. This method requires to use a over strength factor to ensure the design strength meets the flexural and shear demands of loading conditions, based on occupancy category of the building. The implementation of the Enhanced Local Resistance method allows for the development of resistance for critical elements and provides more sufficient strength and alternate load paths (Krauthammer, 2002). It is more applicable to high-rise buildings.

### **1.6 Objectives**

The goal of this thesis is to analyze one building structure and how each component behaves under progressive collapse condition, with different columns removed in different locations. For the purpose of design, linear static analysis is usually preferred for its simplicity and by using the Dynamic Amplification Factor to account for dynamic aspect of the design, however, it's proven by many that this method could overdesign the structure, therefore we need to explore more detailed and accurate analysis to evaluate and design more efficient structure. The essence is to understand the building's behavior in a progressive collapse loading condition and determine whether the structure is over designed with Dynamic Amplification factor of 2 or it is under designed. Analyzing various cases with the columns of

different floor being removed would help one to explore the possibility how and when to utilize the Dynamic Amplification Factors the will be analyze using linear static and dynamic static analysis. The sequence of how to remove the column and the exact number of the column being removed will be discussed in details in the next section.

## 2. Building Modeling

### 2.1 Building Parameters

A reliable analytical model should be built before further testing. Sap2000 is chosen to model a 2D building with 10 stories and 4 bays. The floor distance is 12 foot apart while the bays expand 24 foot. The 3D span of the building (into the page direction) is 25 feet. For the simplicity of the model, all the connections are models as moment resisting connection and all of bottom columns are pinned with only moment release. The loading parameter for design are listed as following:

- DL: Dead load -6" concrete slab
  - DL=4.5kPa
- LL: Live Load of a mixed residential and office building
  - LL=3.83KPa
- WL: Wind load of 15Psf as the parameter to adjust for the purpose of the mode shape reality.

One thing needs to be noted here is that the wind load is not fixed at 15psf, because there is no bracing or damper are put here for resisting the lateral loads. The model needs to be developed to withstand the mode shape and frequency requirements, which indicates a less than 1 second period according with ASCE7, therefore, stiffer member needs to be developed with even higher lateral wind load to achieve the design goal. So the joint load applied to this model is chose to be:

$$P_{wind} = 15psf * 25ft * 24ft * factor 10 = 45kips$$

The design of the section member is realized using the sizing tool from SAP2000. One needs to set up the loading cases to let SAP2000 do a static analysis of the structure and therefore the tool would choose from a pool of section that one chosen to provide optimal section sizes for the loading conditions. The optimal section provided here is based on the strength of a member to take 90% of loading requirement. For the general purpose, AISC W24 is the selection pool for the beams and AISCW18 is the selection pool for the columns. Another thing need to pay

attention here is that the reason for us to chose these member sizes is purely for design purpose, because these are common section members used in practice. Many iterations are needed here for the purpose of optimal solution and one should check if there was any change after each run or simply run 20 times. The loading combination are listed as below based on ASCE:

- $1.4(D + F)$
- $1.2(D + F + T) + 1.6(L + H) + 0.5(Lr \text{ or } S \text{ or } R)$
- $1.2D + 1.6(Lr \text{ or } S \text{ or } R) + (L \text{ or } 0.8W)$
- $1.2D + 1.6W + L + 0.5(Lr \text{ or } S \text{ or } R)$
- $1.2D + 1.0E + L + 0.2S$
- $0.9D + 1.6W + 1.6H$
- $0.9D + 1.0E + 1.6H$

The mass of the each element is assigned to each element member to account for the self weight of the beams. After the optimal selection by SAP, the model needs to be manually switched to symmetric selections because the wind load only applies to one side. The resulting selection is provided in the picture next page. It's a realistic design with stiffer bottoms. The first mode shape has a period of only 0.94303 second, which meets mode shape requirement for a 10-story building. As mentioned above, one could use different wind load to adjust the mode shape to have a desired period if 1 second, in this case, however, the final solution is same to have period of 0.94303 because the beam and column selection pool is not large enough. The same color indicates that they are the same selection for sizing.



The following is the capacity graph of the building after the section has been selected based on 90% strength.

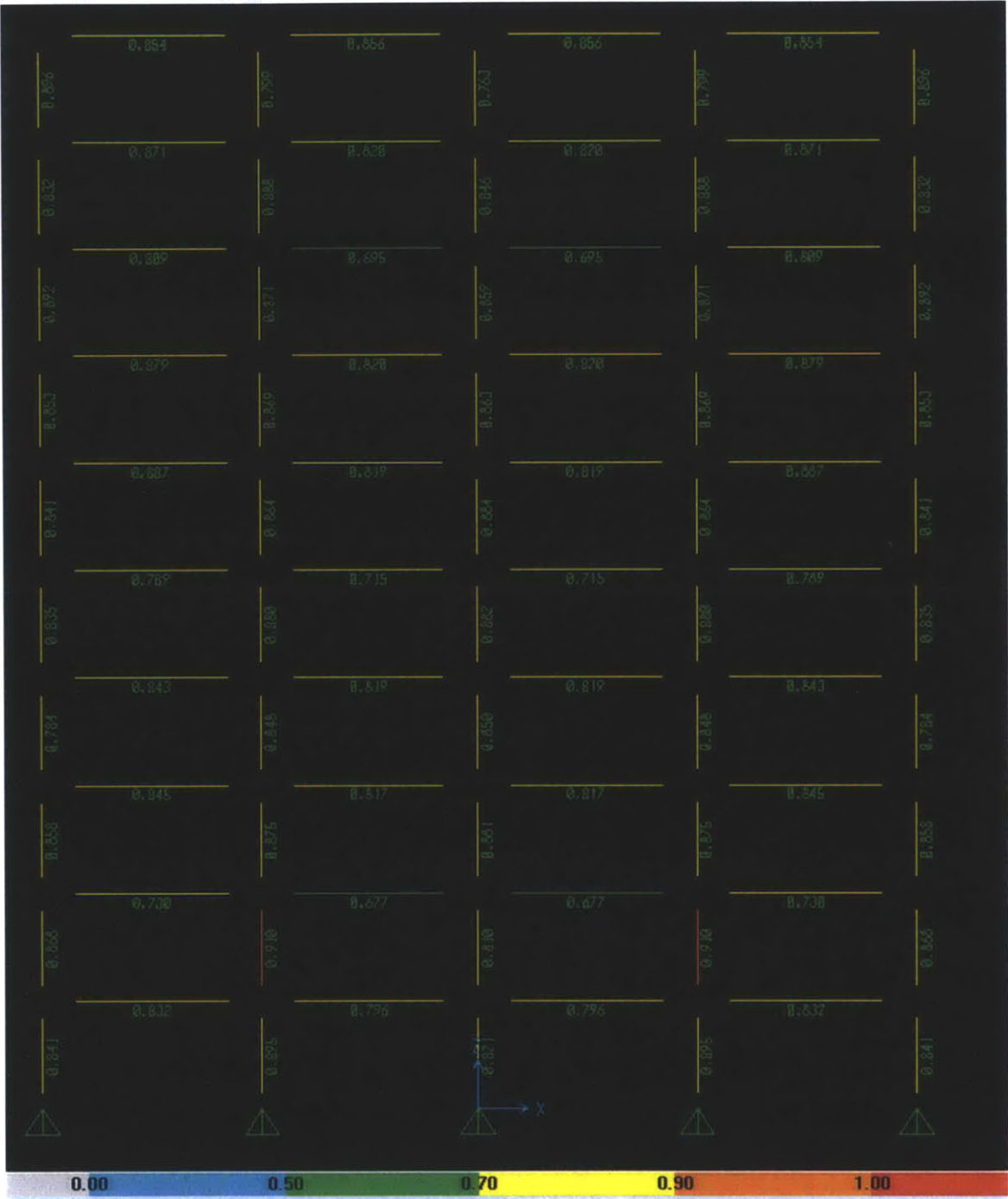


Figure 5: Capacity for each member under the design load



## 2.2 Analysis Procedure

The U.S. General Services Administration (GSA) has issued a general guideline for evaluating the progressive collapse potential of a building. This document provides valuable suggestion of the analysis procedure and how to evaluate the results. According to GSA, the alternate path method is the preferred method.

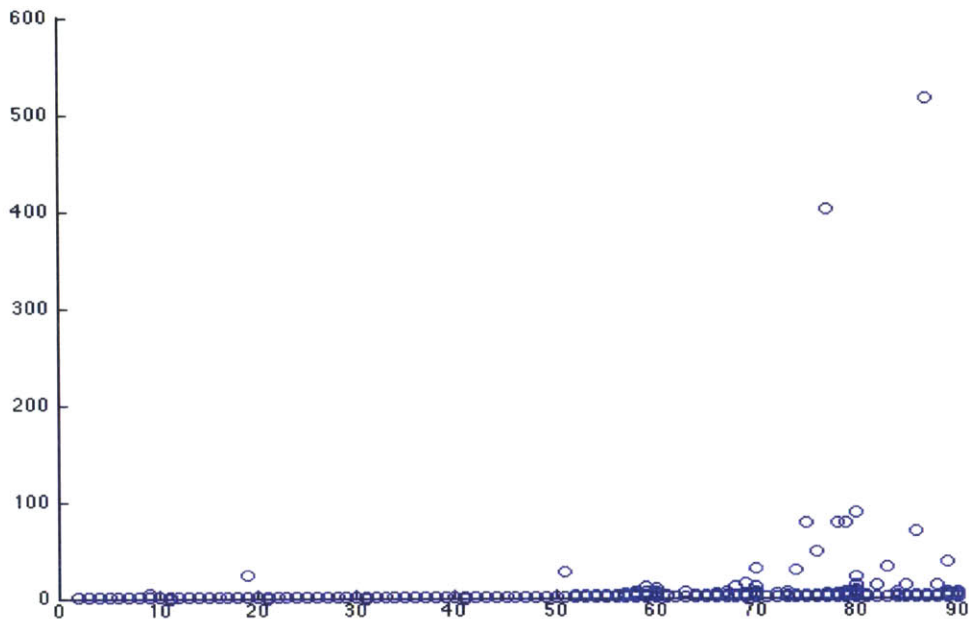
The guidelines also provides the loading combination for analyzing progressive collapse:

- Static Analysis:  $P=2*(DL+0.25LL)$
- Dynamic Analysis:  $P=DL+0.25LL$

P represents the loading under progressive collapse condition. DL means dead load and LL represent live load, which is consistent with the loading provided in section 2.1. The factor of the live load is always smaller because it is assumed that at the extreme event the full live load will not be acting in the structure. The factor of 2 is the dynamic amplification factor introduced above.

### 2.2.1 Modeling Assumption

The damping ratio of this model is set to be 2% for the dynamic analysis and 0.01 second is used to capture all the dynamic effect with 5000 steps, which gives the total time history function to go to 50 seconds. When one sets the model from the dynamic analysis and static analysis under the same loading condition, neglecting the DAF assumption, at the end of the dynamic effect, one should expect to get the same result as the static analysis. However, the following picture indicates otherwise.



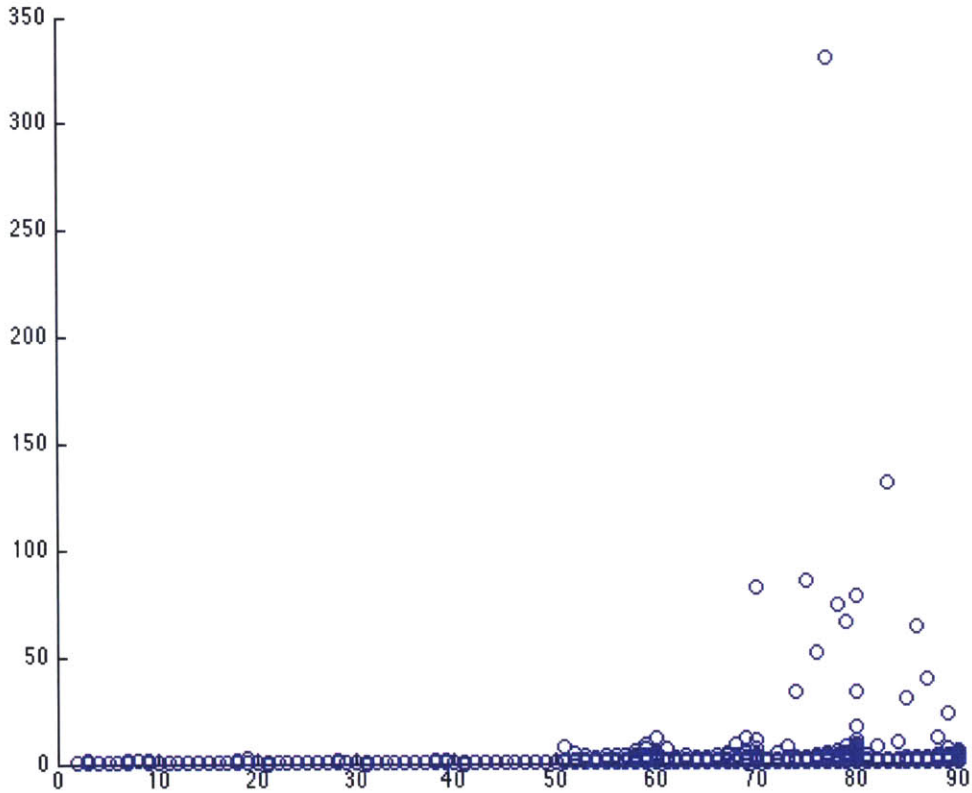
**Figure 6: Dynamic to Static Ratio under 1% Damper**

Sap2000 provide the moment analysis from the static analysis with different location of the members, for the column with size of 12 feet, moments are given at 0 feet, 6 feet and 12 feet; for the beam with 24 feet span, moments are given every two foot.

The graph indicates the ratio between dynamic analysis at the last step and the static analysis under the damping ratio of 1%, one can interpret easily from the graph that the ratio goes up to more than 600, which has two reason to be discussed here:

1. The moment at the in middle of an element approaches 0 so when it is used to divide, the ratio goes up significantly;
2. The top of the building is not as stiff as the bottom so one can easily see the top columns are the ones has largest ratio because the damping is not efficient enough to assimilate the effect under 50 seconds

With the above reasons listed, damping ratio is adjust to 2% and it is very conceived in the following picture:



**Figure 7: Dynamic to Static Ratio under 2% Damper**

The difference is significantly improved from 550 to 350, which verifies the reason using 2% damping ratio is much more efficient and applicable, which is also a good estimate of the actual amount of inherent damping in steel frames.

For this study, one needs to extract the moment values from the information sheets provide by SAP2000, for which a Matlab code is created and provided in the Appendix section for reference.

### **2.2.2 Procedure for Linear Static Analysis**

- Build the model with the desired column removed based on the structure model provided
- Set the load case under progressive collapse condition without the dynamic amplification factor of 2, as same as the Dynamic Analysis loading condition
- Perform the linear static analysis of the model
- Evaluate the results

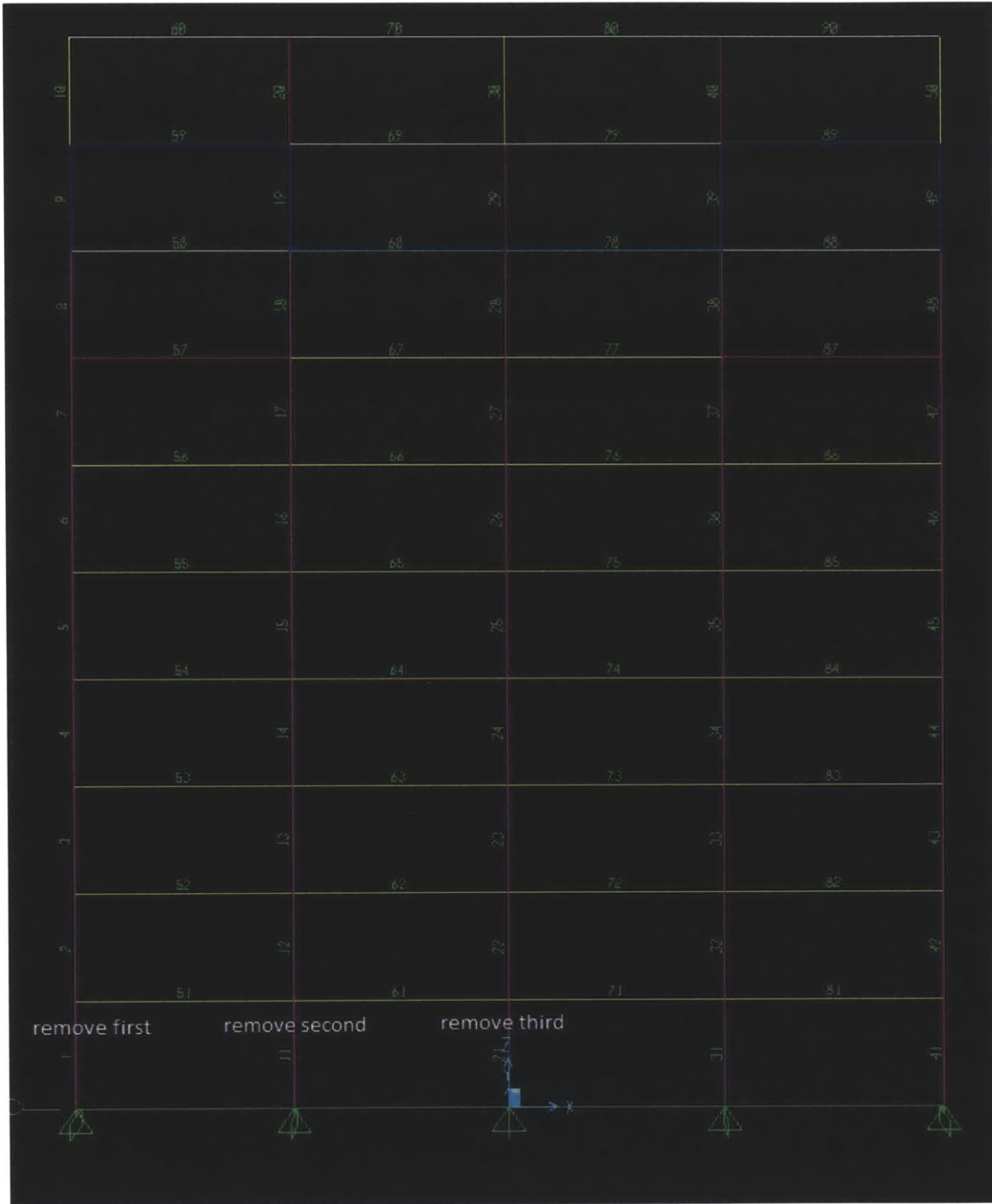
### **2.2.3 Procedure for Linear Dynamic Analysis**

- Build the model with the desired column removed based on the structure model provided
- Set the load case under progressive collapse condition with progressive collapse loading for dynamic analysis
- Perform the linear dynamic analysis with modal analysis use time history function
- Evaluate the results

### **2.2.4 Sequence of Column Removal**

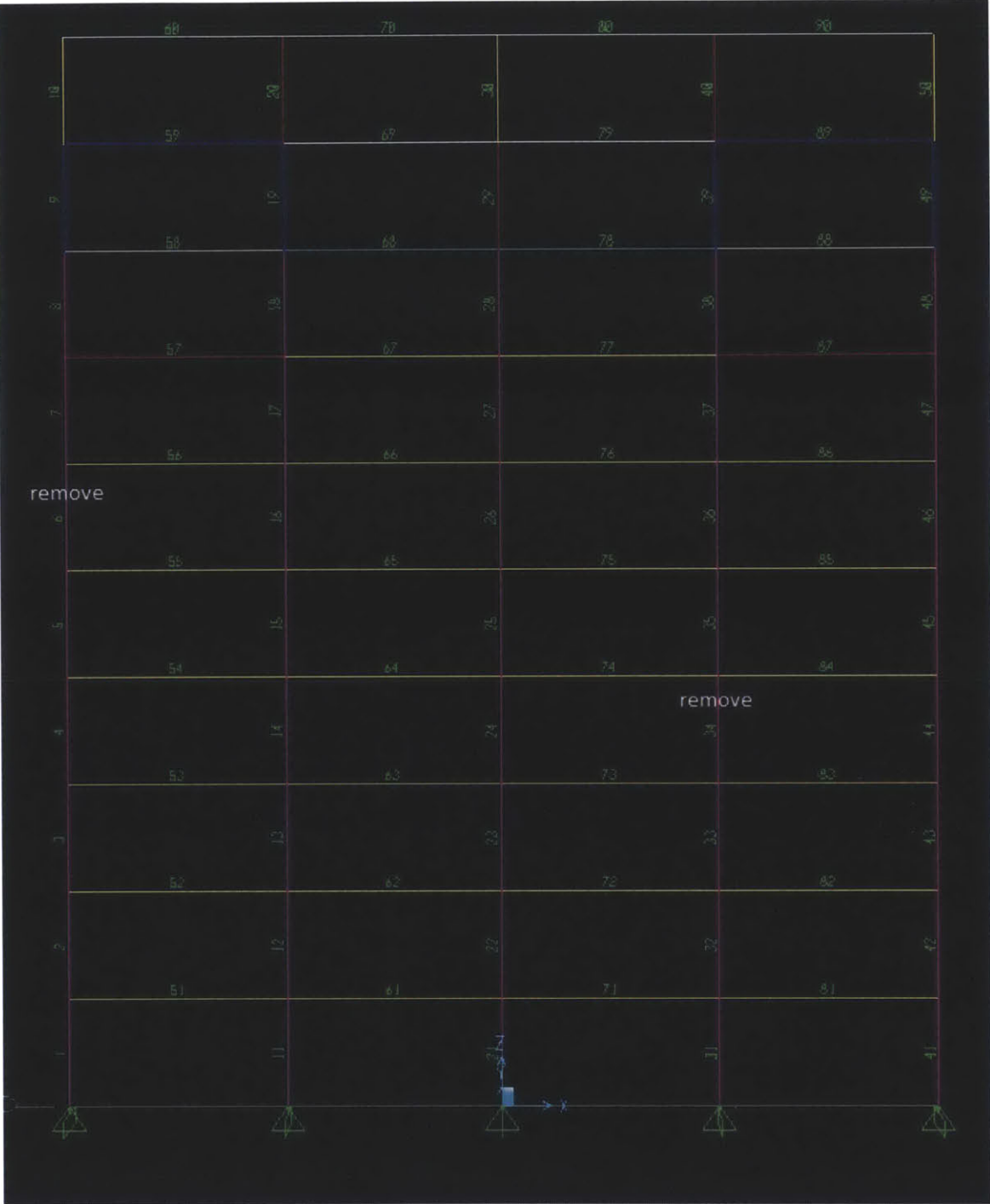
The following picture is the numbered picture of the building structure, where all the members have been property numbered for further discussion.

The following picture is the removing sequence of this simulation:



**Figure 8: Column Removing Sequence**

And for further discussion, one should put one combination case with two columns removed under the progressive loading.



**Figure 9: Combination Remove Case for the Building**

### 3. 2D Frame Analysis Results

#### 3.1 Introduction

Analysis on the 10-story building with 4 bays has been performed for different columns removal scenarios. All cases have been analyzed with both linear static and linear dynamic analysis methods. Both analyses were used to determine the ratio of moments from dynamic analysis and static analysis. For this paper, all the analyses are done under the assumption that the columns are removed immediately and the results are all recorded after the removal. For dynamic analysis, the time frame is set to be 50 seconds to capture the dynamic effects. Modes superposition method All the modeling case are using the same loading factors:  $P=DL+0.25LL$ .

#### 3.2 Definition of Results

Some of the ratios are listed as the equation here for easier indications later:

1. Dynamic Amplification Factor (DAF): ratio between the maximum moment from the dynamic analysis and the moment from the static analysis:

$$DAF = \frac{M_d}{M_s}$$

$M_d$  = Max Moment from the Dynamic Analysis

$M_s$  = Moment from the Static Analysis

2. End Moment Factor (EMF): ratio between the moment from the dynamic analysis at the last second and moment from the static analysis

$$EMF = \frac{M_{dlast}}{M_s}$$

$M_{dlast}$  = Max Moment from the Dynamic Analysis at the last second

$M_s$  = Moment from the Static Analysis

3. End Horizontal Displacement Factor (EHDF): ratio between the horizontal displacement from the dynamic analysis at the last second and horizontal displacement from the static analysis.

$$EHDF = \frac{DH_{dlast}}{DH_s}$$

$DH_{dlast}$  = Horizontal Displacement from the Dynamic Analysis at the last  
second

$DH_s$  = Horizontal Displacement from the Static Analysis

4. End Vertical Displacement Factor (EVDF): ratio between the maximum vertical nodal displacement from the dynamic analysis and displacement from the static analysis.

$$EVDF = \frac{DV_{dlast}}{DV_s}$$

$DV_{dlast}$  = Vertical Displacement from the Dynamic Analysis at the Last Second

$DV_s$  = Vertical Displacement from the Static Analysis

5. Horizontal Displacement Factor (HDF): ratio between the maximum horizontal displacement from the dynamic analysis and horizontal displacement from the static analysis.

$$HDF = \frac{DH_d}{DH_s}$$

$DH_d$  = Max Horizontal Displacement from the Dynamic Analysis

$DH_s$  = Horizontal Displacement from the Static Analysis

6. Vertical Displacement Factor (VDF): ratio between the maximum vertical nodal displacement from the dynamic analysis and displacement from the static analysis.



$$VDF = \frac{DV_d}{DV_s}$$

$DV_d$  = Max Vertical Displacement from the Dynamic Analysis

$DV_s$  = Vertical Displacement from the Static Analysis

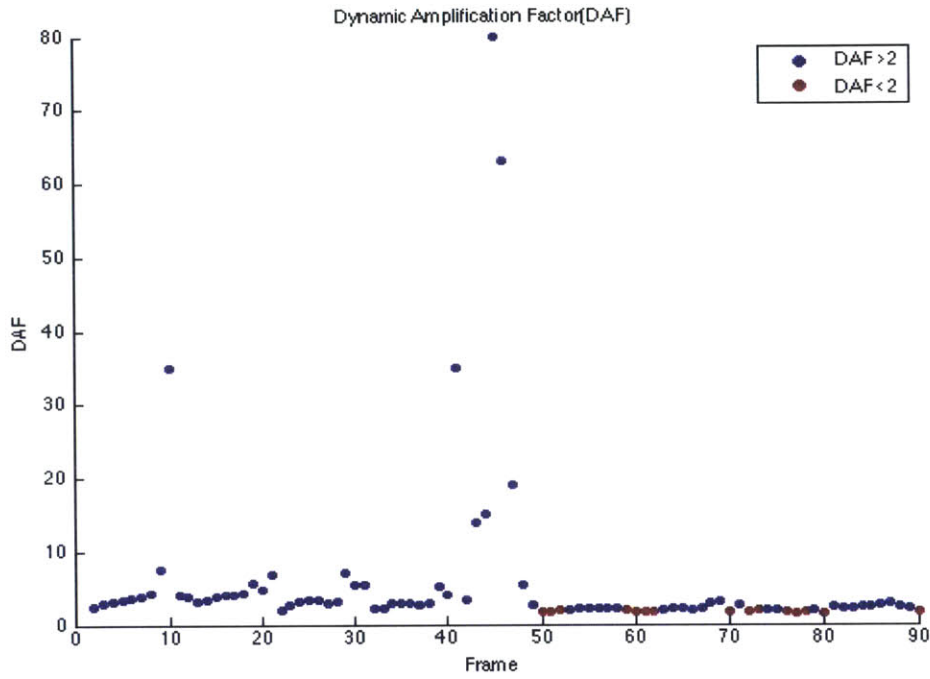
The main focus of this section is to interpret the DAF of the building with the column removed. EMF, HDF and VDF are the measured primarily to verify that the models have behaved appropriately.

### **3.3 Data Collection**

All of the data on DAF, EMF, EVDF, EHDF, VDF and HDF has been produced for all four cases with both Linear Static Analysis and Linear Dynamic Analysis. These data has been organized in case by case format.

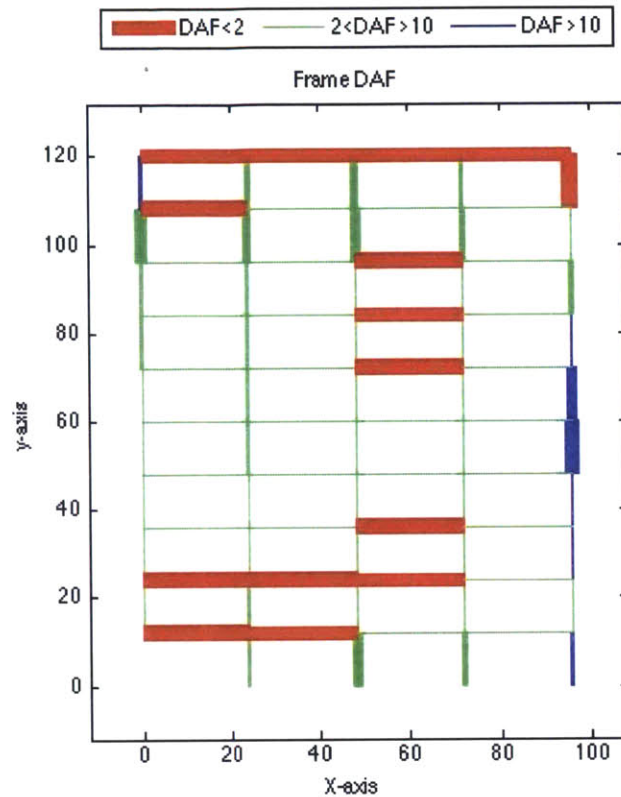
#### **3.3.1 Case 1 (exterior column removal)**

The results from the first case on DAF are captured in the following graphs:



**Figure 10 DAF for Case 1**

This is the result one obtained from the Matlab code, which is developed for better presentation of the data; further discussion has been included in the next section. Another script was developed to put the value of the DAF onto the frame structure for better presentation:

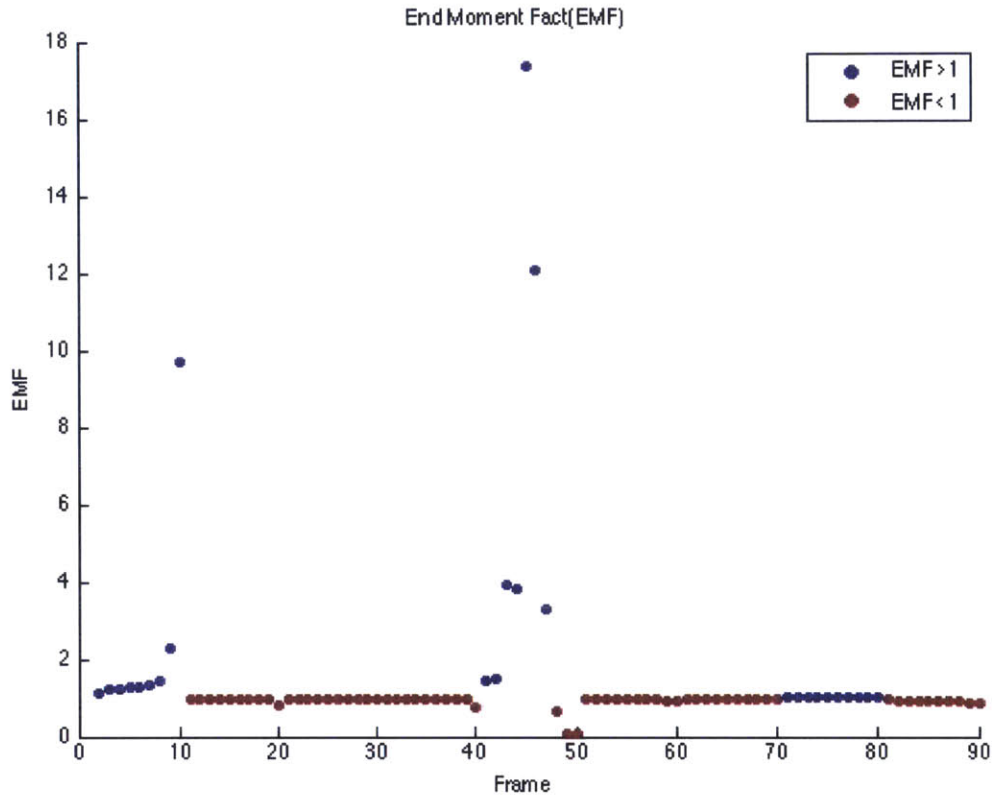


**Figure 11: DAF in Case 1**

The thickness of each member represents its relative ratio to its assign DAF range. For example, a thick red member means the it's closer to 2 than a think red member and a thin blue member has a DAF close to 10 than a thick member. Most of the members has a dynamic amplification bigger than 2 and less than 10. The reason for the DAF exceed 10 is because the moment under Linear Static analysis is very small because it is under only the vertical load condition, while for dynamic analysis, the horizontal effect is considered as well. Due to the small magnitude state of the moment under the linear static Analysis, the DAF got magnified disproportionately, which causes the blue section.

The green members should be paid close attention to because their DAF exceeds 2 and all of them are under the right loading cases and they are magnified the DAF.

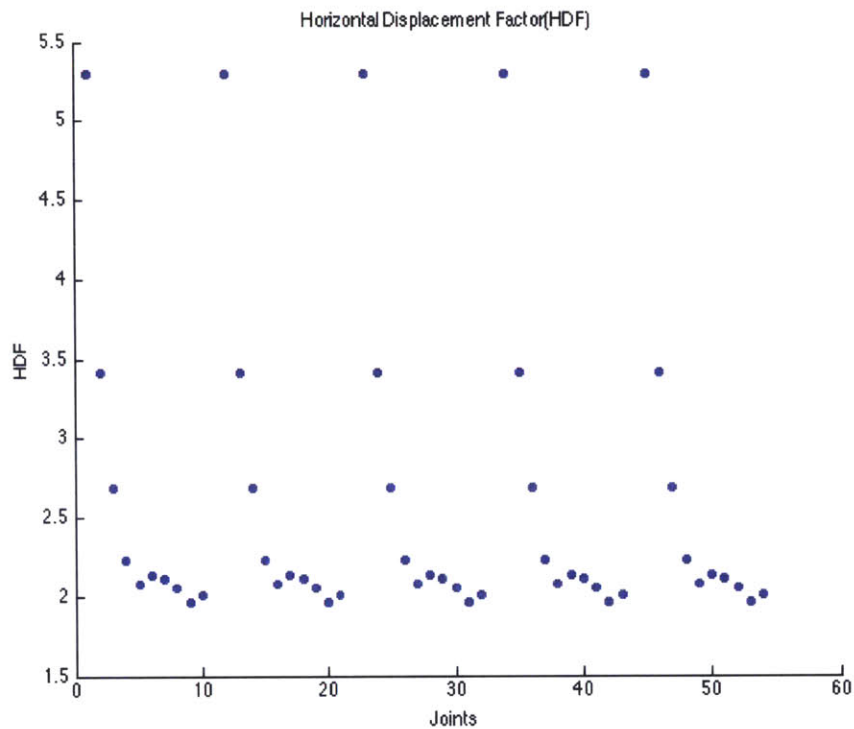
There is also a moment measurement at the end of the Linear Dynamic Analysis and it is compared with the Linear Static Analysis as listed:



**Figure 12: EMF for Case 1**

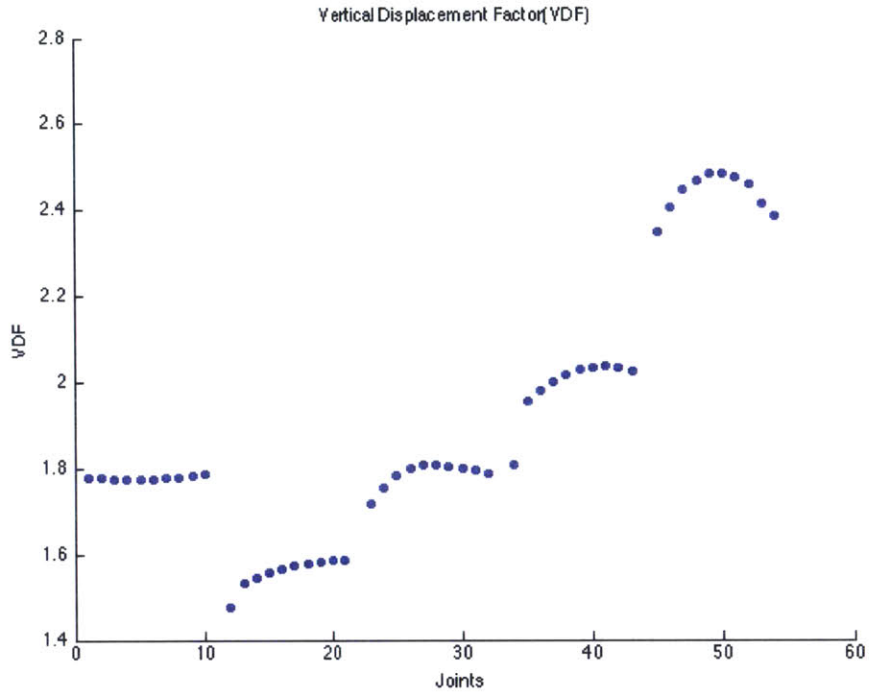
Most of the member show a ratio close to 1 as they should, and the outliers are due to the small value of some members under Linear Static Analysis. The magnitude problem resurfaces again for the end comparison.

To confirm the accuracy of the model, the SAP2000 has produced the displacement results from the analysis case and one has organized to put the data into ratio forms to present more clear results.



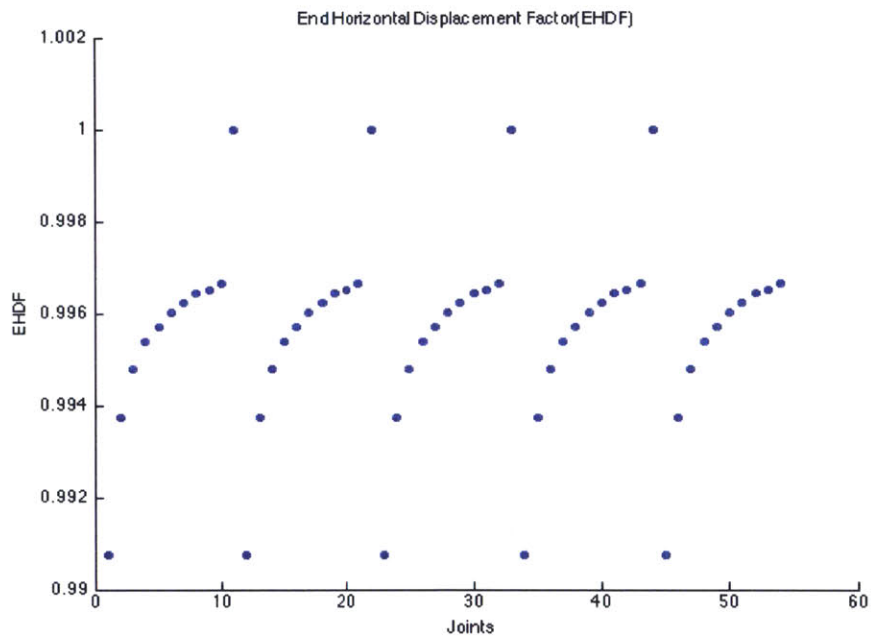
**Figure 13: HDF for Case 1**

The expected HDF should be around 2 in theory, however some of them exceed 5 and they are located on the top of the structure with the height of the building of 120 ft. The height does magnify the horizontal displacement under the model shape that moves in the horizontal direction.

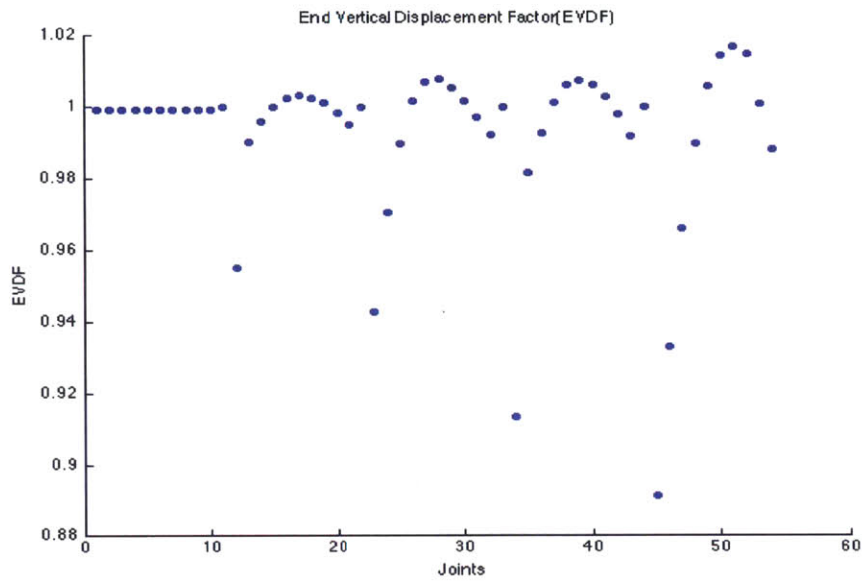


**Figure 14: VDF for Case 1**

The VDF shows around 2, which is reasonable because the vertical results should not be affected much by the dynamic analysis due to the weight of the building.



**Figure 15: EHDF for Case 1**



**Figure 16: EVDF for Case 1**

The EVDF and EHDF has further proved that the model behaves correctly since the end moment under Linear Dynamic Analysis should match the Linear Static analysis since they are under the same loading condition.

### 3.3.2 Case 2 (interior column removal)

The DAF result has been produced in the second case as follows:

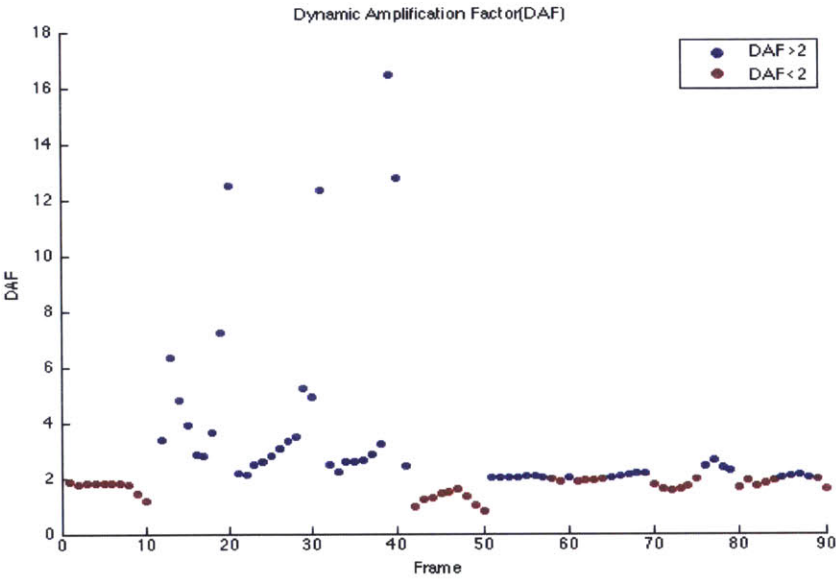
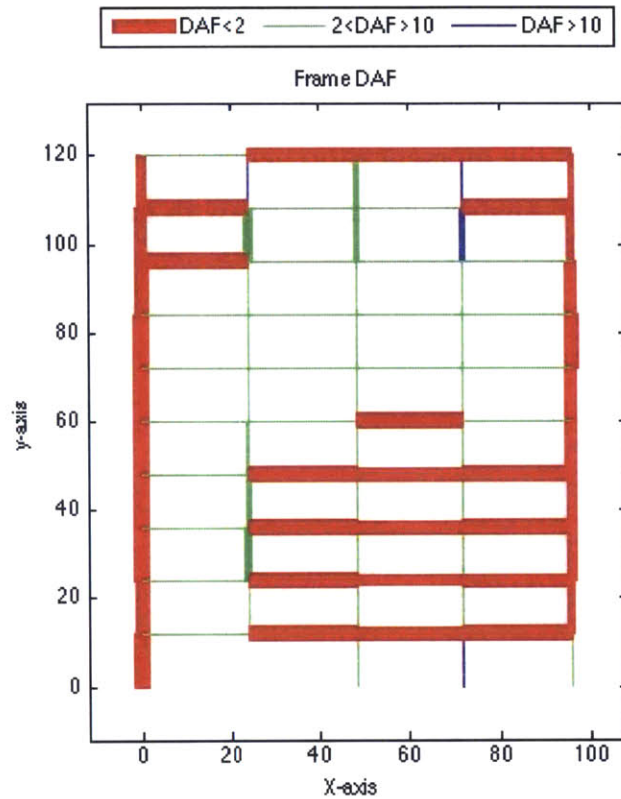


Figure 17: DAF for Case 2

And the frame presentation has been produced as follows:





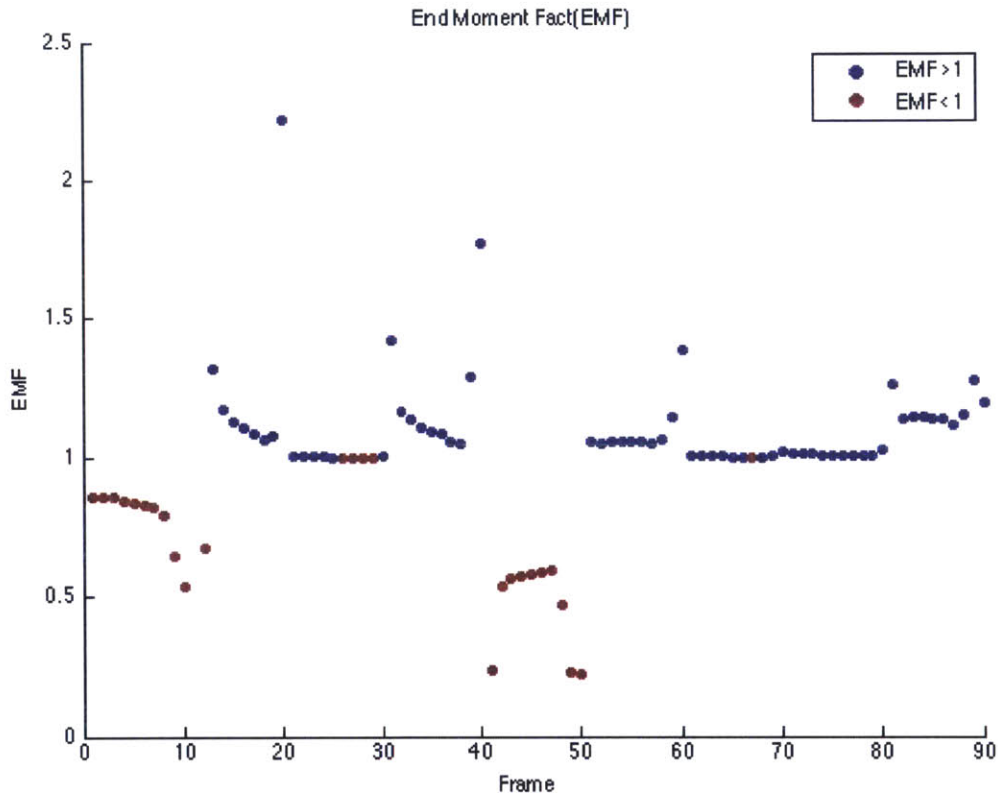
**Figure 18: DAF for Case 2 on Model**

A similar argument could be made here as the previous case because the geometry of this building is similar so they share a lot of similar traits. Likewise, a lot of the members still has a dynamic amplification bigger than 2 and less than 10. The reason for the DAF to exceed 10 is because the moment under Linear Static analysis is very small because it is under only the vertical load condition, while for dynamic analysis, the horizontal effect is considered as well. Due to the small magnitude state of the moment under the linear static Analysis, the DAF got magnified disproportionately, which causes the blue section.

The green members are mostly around the removed column because they tend to take most of the load after the underneath column is removed. The removed column

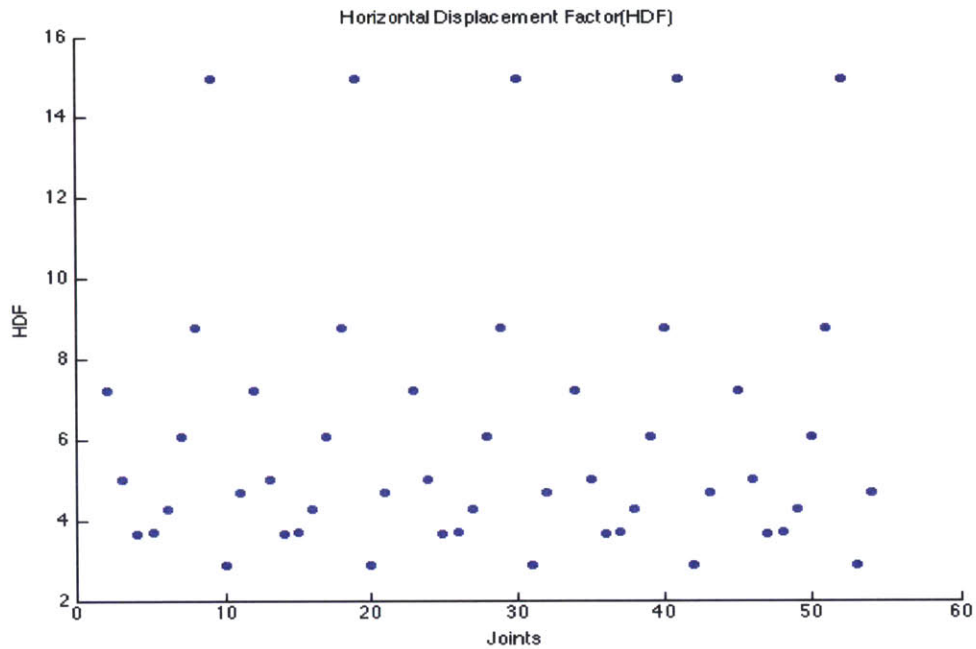
is closer to the center than the Case 1, resulting less green members because the force is distributed better than the previous case.

And the confirmation of the moment ratio between moment measurement at the end of the Linear Dynamic Analysis and compare with the Linear Static Analysis as listed:



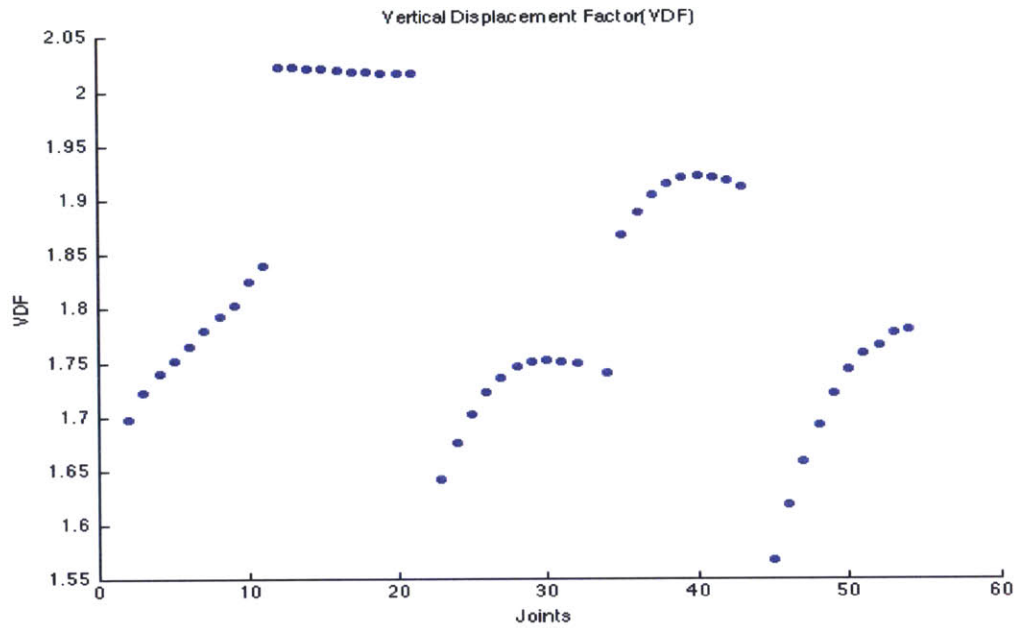
**Figure 19: EMF for Case 2**

To confirm the accuracy of the model, the SAP2000 has produced the displacement results from the analysis case and one has organized to put the data into ratio forms for clearer presentation.



**Figure 20: HDF for Case 2**

The HDF is surprisingly large, as 2 was expecting at the beginning, but if one put close attention to the geometry, it is clear that most of the building members would be hard to move because the removed column is closer to the center of the building. As the Dynamic Analysis include the modes with horizontal deformation, the HDF showed its effect.



**Figure 21: VDF for Case 2**

The VDF shows in the reasonable range between 1.55 and 2.05, which is expected from the theory. The vertical displacement between the two analyses should be around 2 if the provided is accurate.

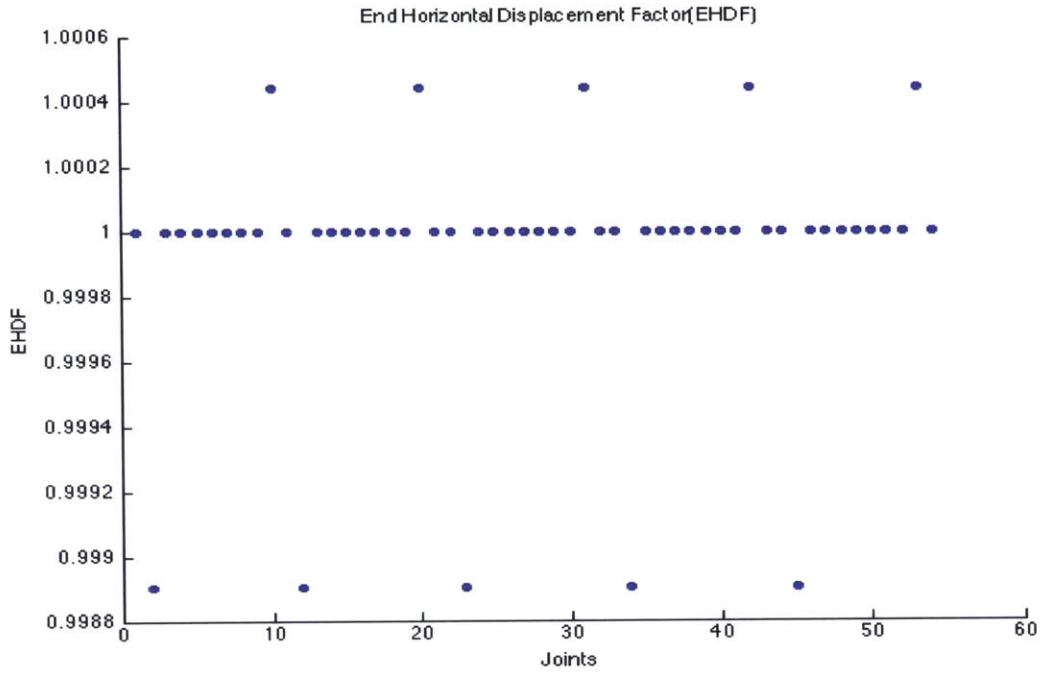


Figure 22 EHDF for Case 2

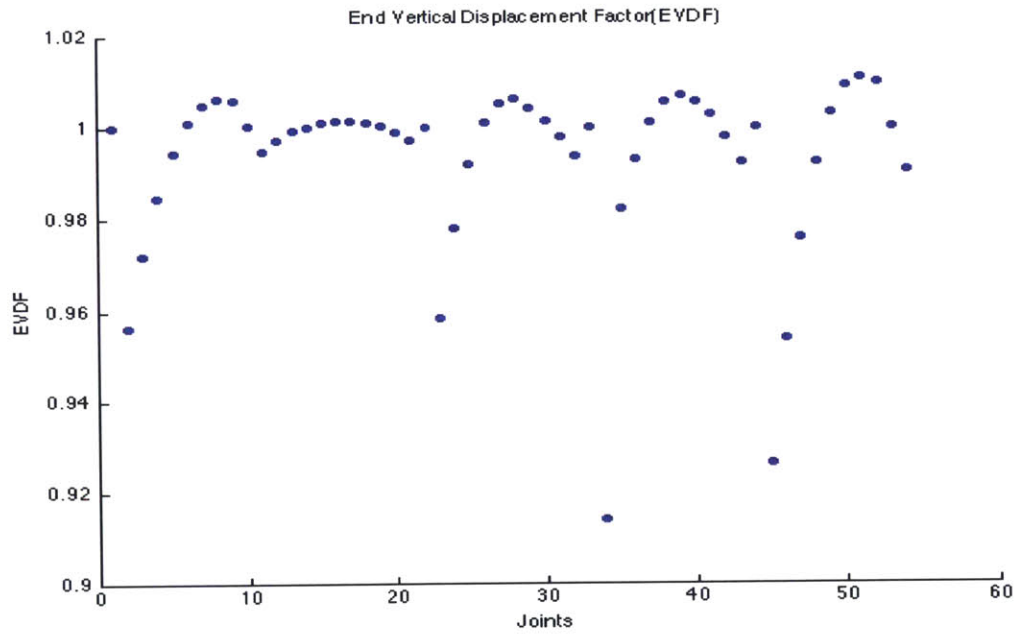


Figure 23: EVDF for Case 2

The EHDF and EVDF show that there is not much discrepancy between the Linear Dynamic and Linear Static analysis.

### 3.3.3 Case 3 (middle column removal)

The DAF result have been produced in the third case:

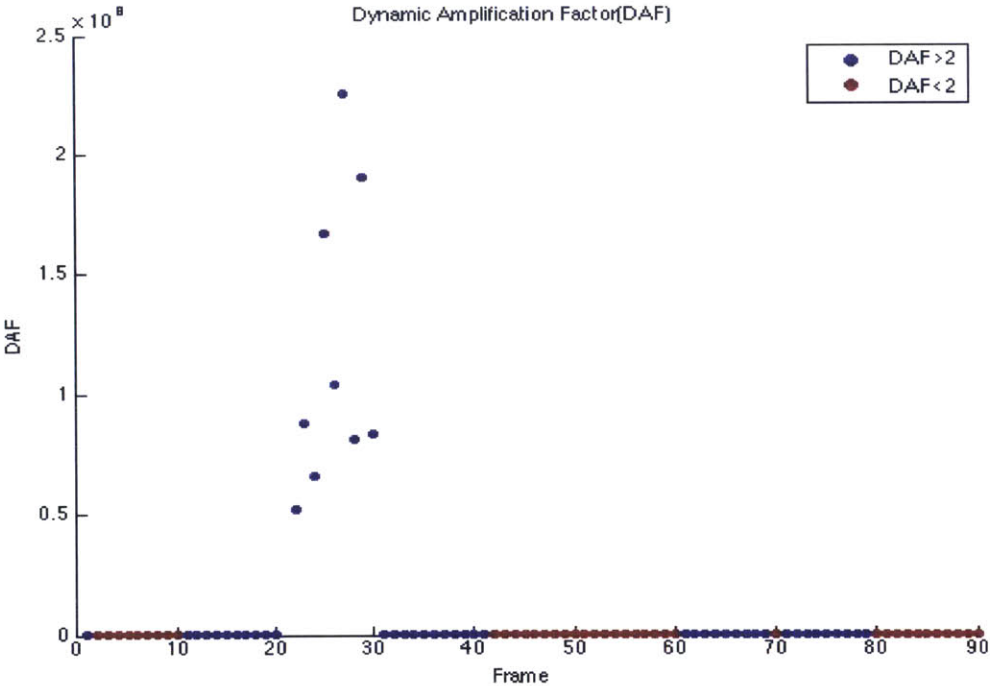
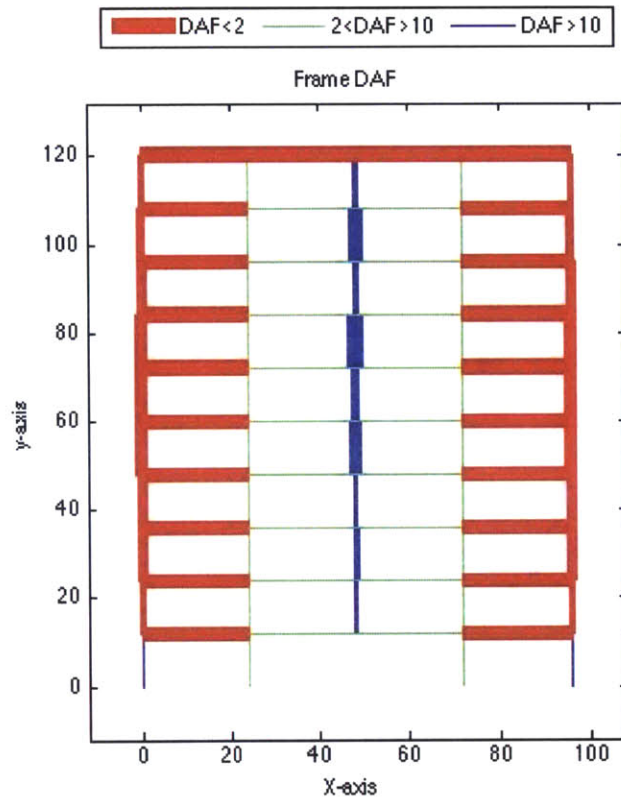


Figure 24: DAF for Case 3

And the frame presentation has been produced as follows:

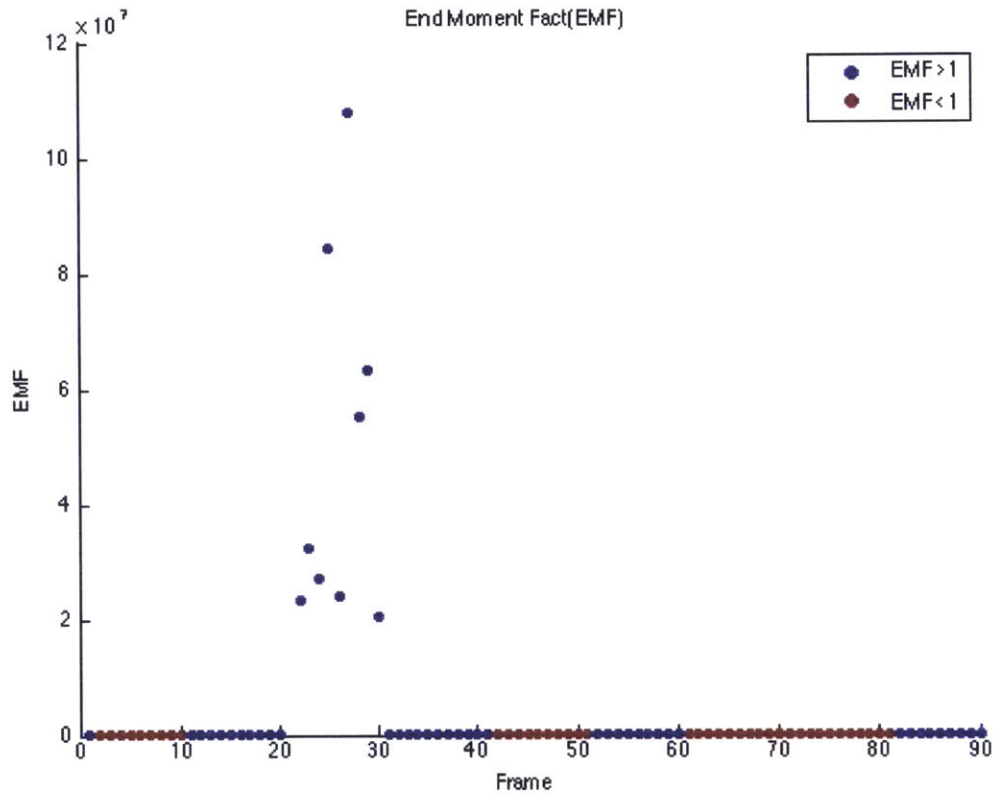


**Figure 25: DAF for Case 3 on Model**

The geometry of the building is symmetry so that the vertical member barely takes any moment because it only takes the axial forces resulting from the floor bending. So when Dynamic Analysis includes the horizontal displacement it radically magnifies the ratio.

However, the middle section in green causes a problem for evaluation, as they exceeds the limit because the moment under the Dynamic Analysis is simply larger than the Static Analysis by more than a factor of 2.

And the confirmation of the moment ratio between moment measurement at the end of the Linear Dynamic Analysis and compare with the Linear Static Analysis is listed:

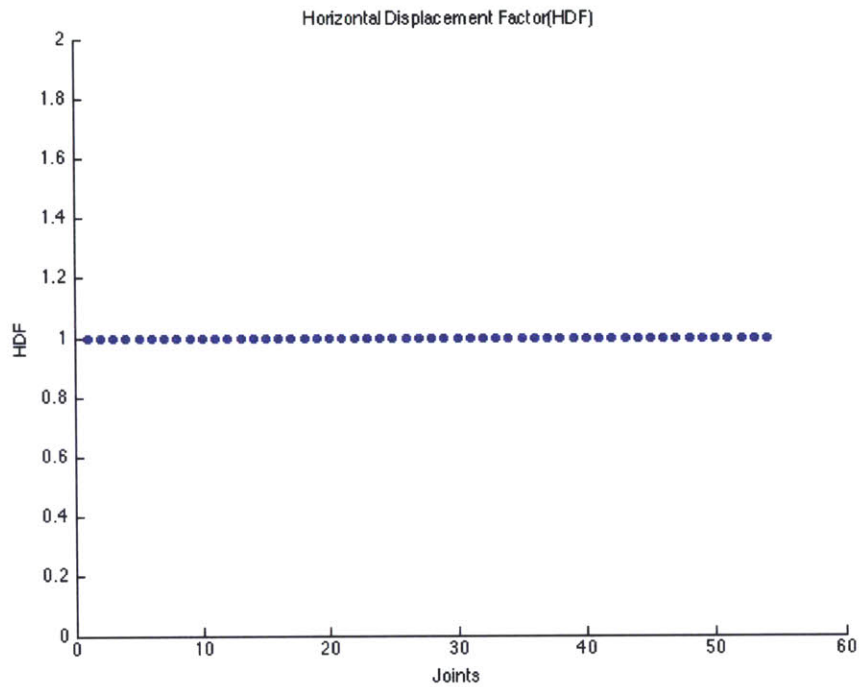


**Figure 26: EMF for Case 3**

The EMF almost identically matches the results from the DAF because the symmetry geometry of the case only the center columns shows such significant jump from the rest of the columns.

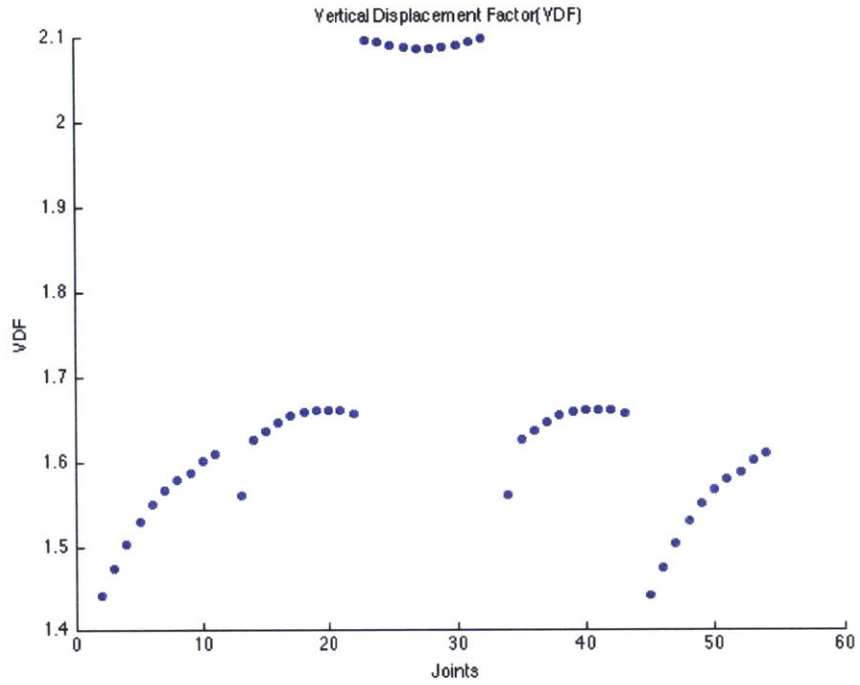
To confirm the accuracy of the model, the SAP2000 has produced the displacement results from the analysis case and one has organized to put the data into ratio forms to present clearer results:





**Figure 27: HDF for Case 3**

Due to the symmetry of the building, the horizontal displacement is 0. One thing needs to be paid attention to is that the displacement under both Analyses is close to 0 but not 0, however it's  $10^{-18}$  magnitude made it to 0 for the purpose of calculation.



**Figure 28: VDF for Case 3**

The vertical displacement showed a reasonable range between 1.4 and 2.1. It matches the theoretical value of the amplification factor. And the only slightly large value comes from the middle section where the largest discrepancy is expected and it is still in a reasonable range.

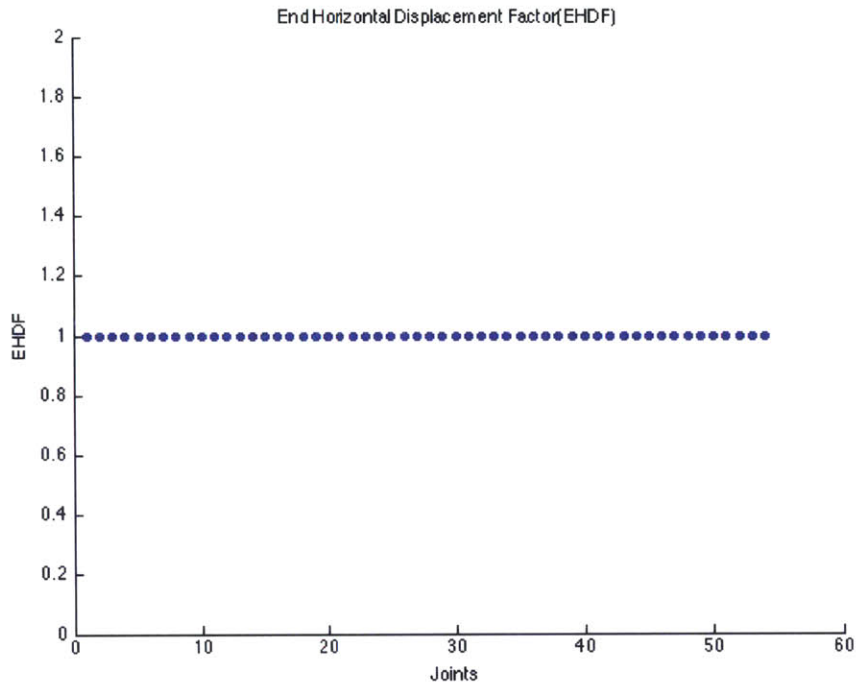


Figure 29: EHDF for Case 3

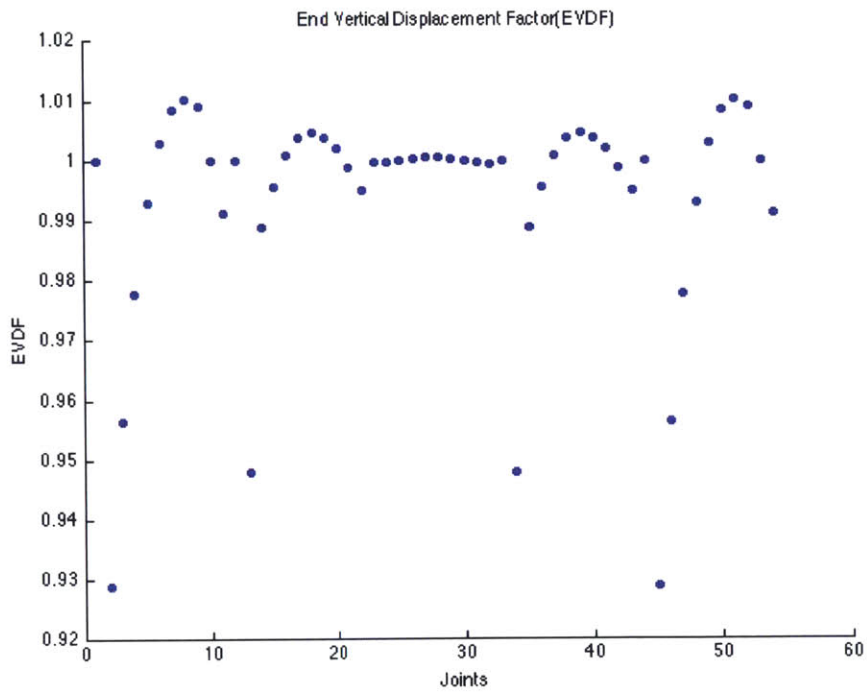


Figure 30: EVDF for Case 3

A similar argument can be made here about the EHDF here as the HDF. The EVDF of the building shows a better confirmation about the geometry of the building structure.

### 3.3.4 Case 4(combination columns removal)

The results from the Case 4 on DAF is captured in the following graph:

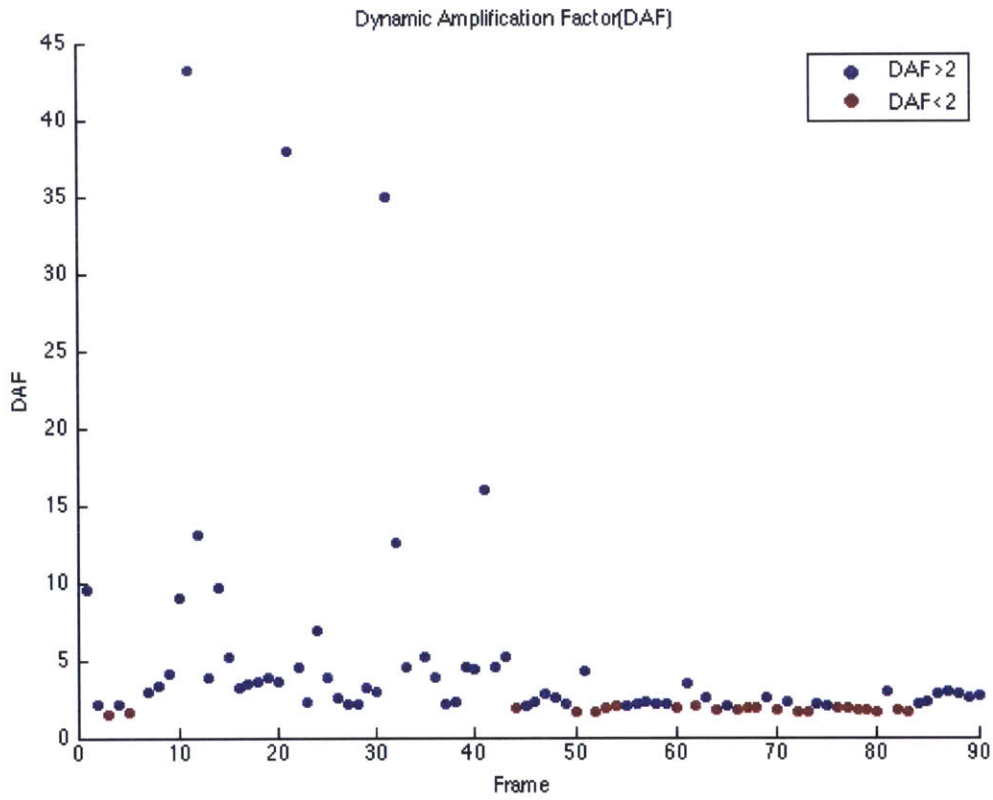
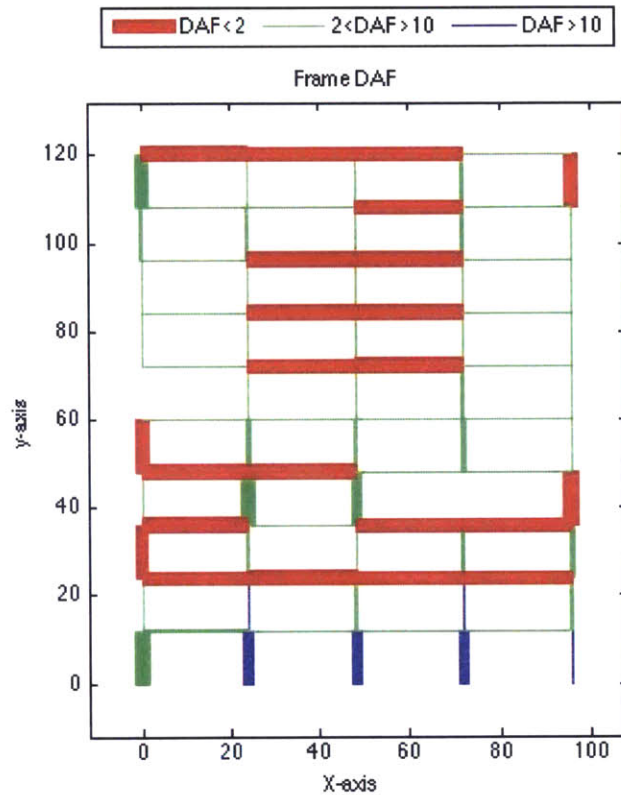


Figure 31: DAF for Case 4

And the frame presentation has been produced as follows:

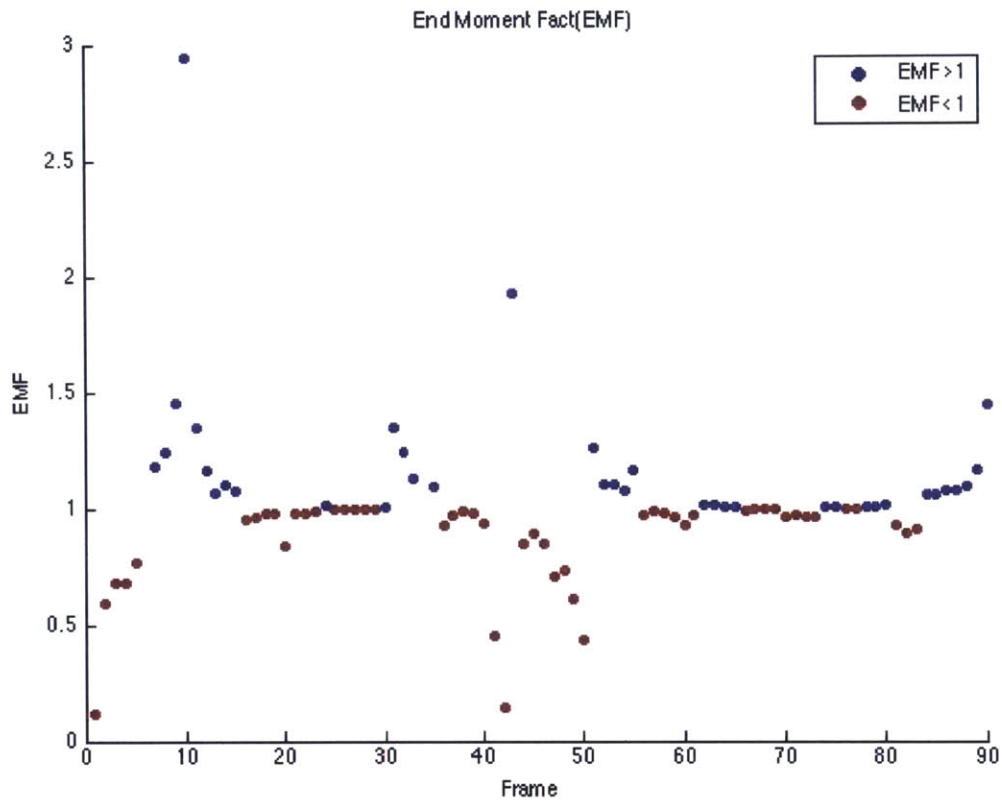


**Figure 32: DAF for Case 4 on the Model**

The case 4 is a combination case of them both case 1 and 2 with slight change in the election of the removed columns. The results are fairly interesting: the asymmetry feature of the building is made the bottom right column (member 41) different in blue than the bottom left column (member 1), however, one removed column on the left and one removed on the right has also made the middle three column (member 11,21,31) experienced somewhat symmetrical loading under the static loading case, but the Dynamic Analysis has magnified the result to the blue section.

It is hard to find a pattern about the green section once there are more than one column has been removed. The results of the simulation needs for further detailed study.

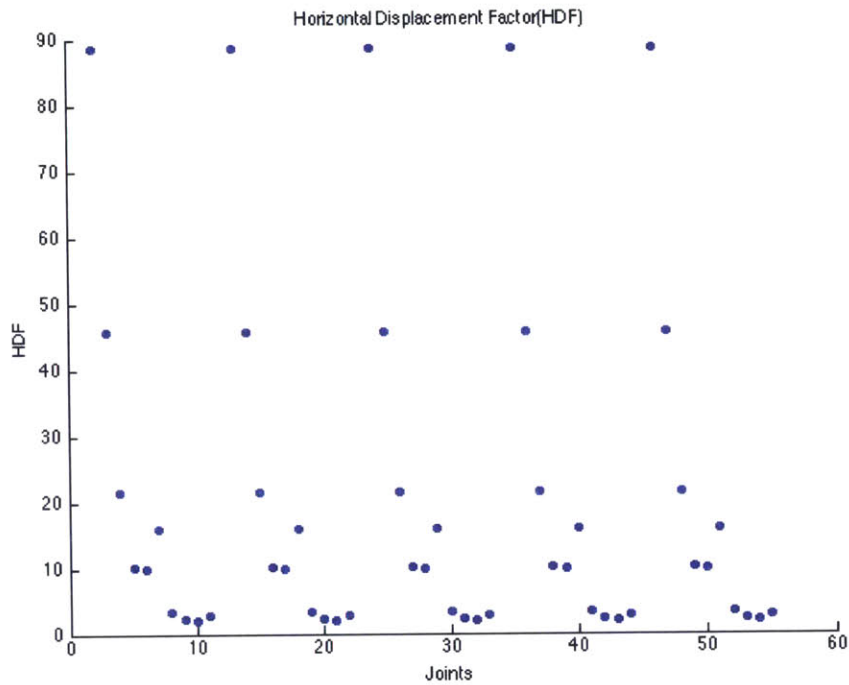
And the confirmation of the moment ratio between moment measurement at the end of the Linear Dynamic Analysis and compare with the Linear Static Analysis as listed



**Figure 33: EMF for Case 4**

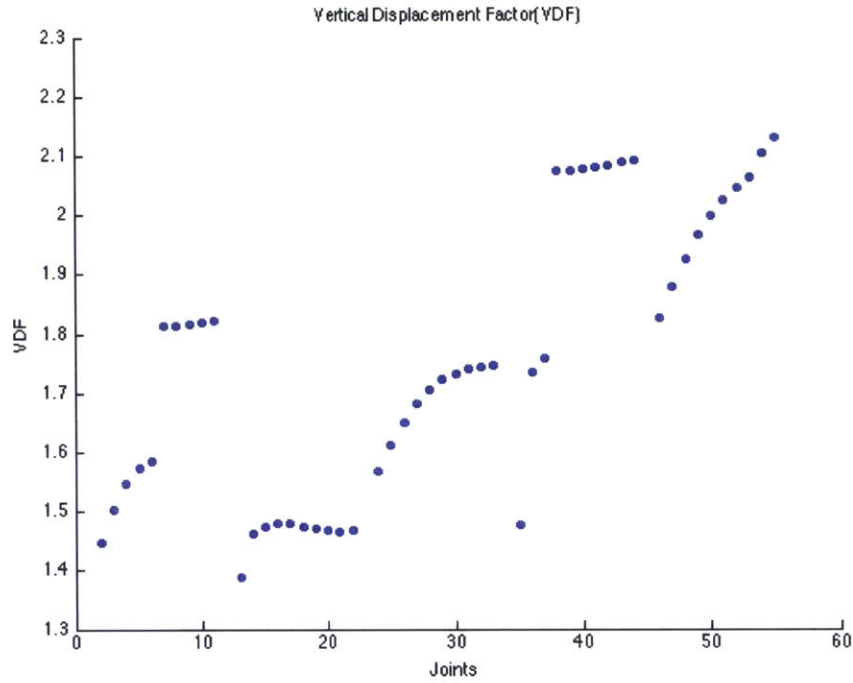
The EMF is mostly under or equal 1 which is a good sign for confirming our model, the three outliers are from the middle three members that have similar traits as symmetrical components of the building.

To confirm the accuracy of the model, the SAP2000 has produced the displacement results from the analysis case and one has organized to put the data into ratio forms to present more clear results.



**Figure 34: HDF for Case 4**

The bottom three layer of columns has showed less desirable results but because the location of them are attached to the ground so it is not easy for them to show the horizontal displacement under the regular loading case, while the mode shape displacement has the edge to show more reflection in the horizontal direction.



**Figure 35: VDF for Case 4**

The vertical displacement showed a reasonable range between 1.3 and 2.2. It matches the theoretical value of the amplification factor. And the only slightly large value comes from the top of the member 34, which has been removed, therefore largest discrepancy is expected and it is still in a reasonable range.



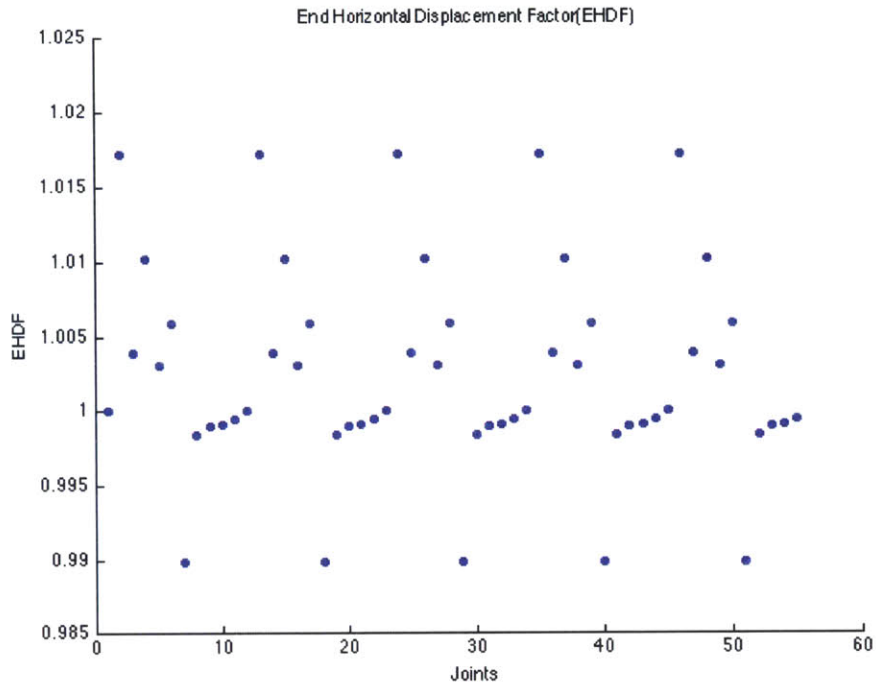


Figure 36: EHDF for Case 4

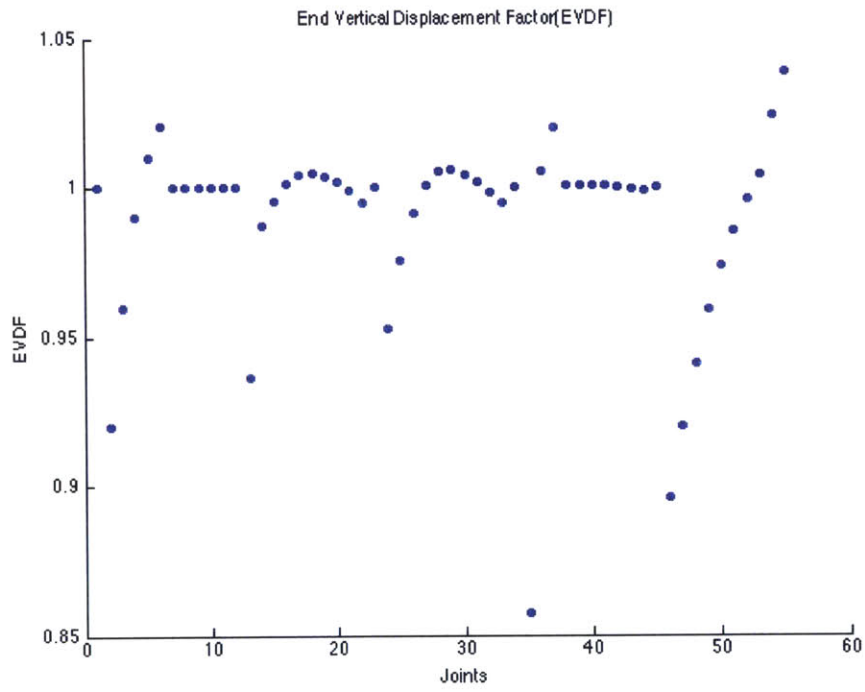


Figure 37: EVDF for Case4

Both of the EHDF and EVDF have shown remarkably close to the theoretical value of 1, furthermore proved the credibility of the models.

## 4. Summary

The general results from the previous section have been generated from the raw data and the following table is a the summary of the data.

Member	case 1	case2	case3	case4
1	REMOVED	1.88	47.69	9.52
2	2.52	1.78	0.77	2.11
3	2.82	1.80	1.20	1.53
4	3.14	1.82	1.29	2.19
5	3.47	1.82	1.36	1.62
6	3.73	1.82	1.41	REMOVED
7	3.86	1.81	1.43	2.94
8	4.22	1.74	1.30	3.36
9	7.50	1.47	1.03	4.15
10	34.91	1.19	0.85	8.98
11	4.11	REMOVED	2.20	43.17
12	3.91	3.39	2.21	13.06
13	3.04	6.35	2.28	3.82
14	3.46	4.84	2.37	9.68
15	3.88	3.93	2.39	5.22
16	4.14	2.84	2.44	3.21
17	4.19	2.81	2.48	3.49
18	4.32	3.67	2.53	3.62
19	5.67	7.22	2.68	3.88
20	4.87	12.51	3.03	3.61
21	6.99	2.17	REMOVED	37.99
22	2.01	2.13	52495738.98	4.48
23	2.64	2.50	88165680.47	2.24

Member	case 1	case2	case3	case4
24	3.26	2.61	66422912.86	6.89
25	3.45	2.83	166965973.53	3.86
26	3.33	3.08	103869209.81	2.54
27	2.96	3.34	225252525.25	2.13
28	3.23	3.51	81772575.25	2.13
29	7.08	5.23	190084550.35	3.28
30	5.59	4.92	83619222.21	2.98
31	5.48	12.33	2.20	34.87
32	2.13	2.49	2.21	12.50
33	2.29	2.22	2.28	REMOVED
34	2.92	2.60	2.37	4.48
35	2.93	2.62	2.39	5.25
36	2.85	2.67	2.44	3.84
37	2.59	2.88	2.48	2.21
38	2.83	3.22	2.53	2.27
39	5.26	16.47	2.68	4.52
40	4.10	12.78	3.03	4.44
41	35.04	2.45	47.69	16.00
42	3.48	0.98	0.77	4.59
43	13.82	1.22	1.20	5.22
44	15.01	1.31	1.29	1.93
45	79.87	1.46	1.36	2.01
46	62.99	1.51	1.41	2.27
47	18.95	1.58	1.43	2.78
48	5.39	1.34	1.30	2.58
49	2.77	1.03	1.03	2.14
50	1.86	0.80	0.85	1.62



Member	case 1	case2	case3	case4
51	1.84	2.03	1.74	4.24
52	1.91	2.00	1.58	1.64
53	2.04	2.02	1.69	1.90
54	2.14	2.04	1.73	1.98
55	2.24	2.05	1.77	2.02
56	2.29	2.06	1.80	2.16
57	2.29	2.03	1.79	2.25
58	2.14	1.95	1.73	2.23
59	1.92	1.88	1.71	2.11
60	1.67	2.04	1.96	1.86
61	1.65	1.89	2.13	3.45
62	1.79	1.91	2.26	1.99
63	2.10	1.95	2.23	2.52
64	2.28	1.98	2.30	1.79
65	2.26	2.03	2.37	2.01
66	2.07	2.09	2.45	1.73
67	2.32	2.14	2.52	1.86
68	2.90	2.20	2.45	1.94
69	3.16	2.17	2.27	2.50
70	1.73	1.77	1.79	1.76
71	2.66	1.63	2.13	2.32
72	1.70	1.55	2.26	1.60
73	1.93	1.61	2.23	1.61
74	2.06	1.70	2.30	2.15
75	2.00	2.00	2.37	2.05
76	1.83	2.46	2.45	1.93
77	1.59	2.66	2.52	1.94

Member	case 1	case2	case3	case4
78	1.84	2.41	2.45	1.79
79	2.06	2.30	2.27	1.74
80	1.63	1.66	1.79	1.60
81	2.52	1.91	1.74	2.99
82	2.16	1.70	1.58	1.76
83	2.24	1.84	1.69	1.66
84	2.36	1.94	1.73	2.19
85	2.49	2.02	1.77	2.36
86	2.67	2.10	1.80	2.76
87	2.89	2.13	1.79	3.02
88	2.52	2.04	1.73	2.78
89	2.21	1.97	1.71	2.56
90	1.64	1.61	1.96	2.64

<b>REMOVED</b>	The member has been removed
1.64	2<DAF<=5
7.50	5<DAF<20
34.91	20<DAF

Only the clear cells listed above have a Dynamic Amplification Factor less than 2, and the rest of the cell are not passing the requirement. Bar charts have been created to present the results of the study, the blue section represent the qualified member for which Dynamic Amplification Factor is less than two:

## Percentage of DAF for Case 1

■ DAF<2 ■ 2<DAF<=5 ■ 5<DAF<20 ■ 20<DAF

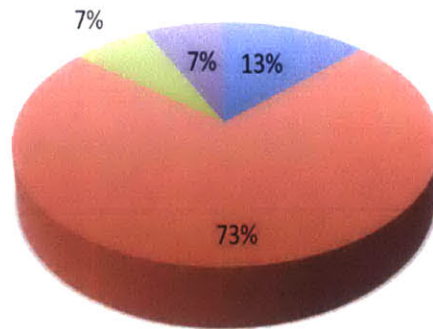


Figure 38: Percentage of DAF for Case 1

## Percentage of DAF for Case 2

■ DAF<2 ■ 2<DAF<=5 ■ 5<DAF<20 ■ 20<DAF

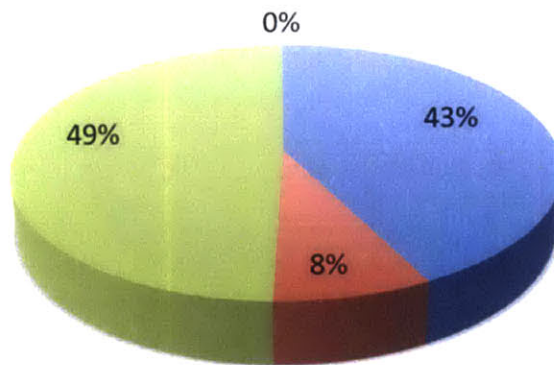


Figure 39: Percentage of DAF for Case 2

### Percentage of DAF for Case 3

■ DAF<2 ■ 2<DAF<=5 ■ 5<DAF<20 ■ 20<DAF

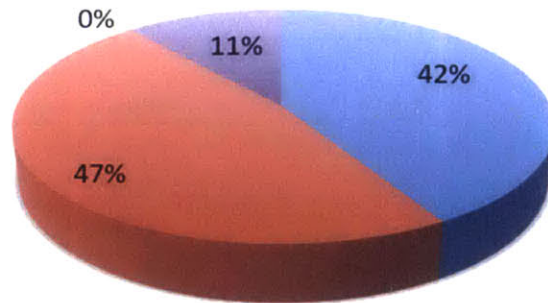


Figure 40: Percentage of DAF for Case 3

### Percentage of DAF for Case 4

■ DAF<2 ■ 2<DAF<=5 ■ 5<DAF<20 ■ 20<DAF

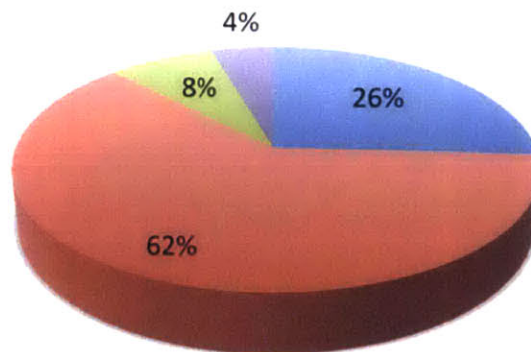


Figure 41: Percentage of DAF for Case 4



## 5. Conclusion

The purpose to use Dynamic Application factor is to design and analyze the building and structures more efficiently under progressive collapse where one wishes to use only Linear Static Analysis instead of using more complex Linear Dynamic Analysis. It is suggested in the code that the Dynamic Amplification Factor should be 2 to account for the all the dynamic effect, however, the simulations from this paper suggest otherwise. In conclusion, throughout the four case analysis in this paper, none of them has more than half of the design members to have the Dynamic Amplification Factor qualified, which suggests that the current code has failed to predict an efficient loading condition for one to analyze the building in progressive collapse condition.

The model is only a primary 2D structure; therefore, a more sophisticated and complete model needs to be developed for further study. This paper has not yet discover the pattern of where and how the member has DAF exceed 2 would occur on the structure or the exact method to evaluate the structure more systematically that one can actually make it more applicable to practice. A practical model should be developed to test the theoretical results for confirmation.

More work and effort needs to be put into this topic for further and more complete study, so that it is more efficient and meaningful for the civil engineers to design structures for progressive collapse.

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## 7. Appendix

The section is to show the Matlab code that has been developed for this paper, people could use later on to fortify the results of this research. Moment Reader is developed to automatically search and find the maximum moment for each member and Building Structure section would help to draw each section as indicated in the case. Draw Results section would help one to develop all the graphical results for this research.

### ***Moment reader:***

```
clear all;
filename1 = 'l4ds.xlsx';
filename2= 'l4ls.xlsx';
filename3 = 'l4dsl.xlsx';

sheet=3;
A = xlsread(filename1,sheet);
C = xlsread(filename2,sheet);
D = xlsread(filename3,sheet);
n=size(A,1);
B=zeros(n/2,2);
BM=zeros(88,4);
ds_ls=zeros(88,1);
dsl_ls=ds_ls;
B(:,2)=C(:,2);
i=1;
k=1;
while i<=n

    if A(i+3,1)==A(i,1)
        counter=3;
        b_start=(i+1)/2;
        for l=1:counter
            B(b_start+l-1,1)=max(A(i+1-1,2),A(i+1-1+counter,2));
        end
        BM(k,2)=max(B(b_start:(b_start+2),1));
        i=i+6;
        k=k+1;
    else
        b_start=(i+1)/2;
        counter=13;
        for l=1:counter
            B(b_start+l-1,1)=max(A(i+1-1,2),A(i+1-1+counter,2));
        end
        BM(k,2)=max(B(b_start:(b_start+12),1));
```

```

        i=i+26;
        k=k+1;
    end

end
i=1;
k=1;
while i<=n/2

    if D(i+3,1)==D(i,1)
        counter=13;

        BM(k,1)=max(D(i:i+counter-1,1));
        BM(k,3)=max(D(i:i+counter-1,2));
        i=i+counter;
        k=k+1;
    else

        counter=3;
        BM(k,1)=max(D(i:(i+counter-1),1));
        BM(k,3)=max(D(i:(i+counter-1),2));
        i=i+counter;
        k=k+1;
    end

end

i=1;
k=1;
while i<=n/2

    if C(i+3,1)==C(i,1)
        counter=13;

        BM(k,4)=max(C(i:i+counter-1,2));
        i=i+counter;
        k=k+1;
    else

        counter=3;

        BM(k,4)=max(C(i:i+counter-1,2));
        i=i+counter;
        k=k+1;
    end

end

for i=1:88
    ds_ls(i)=BM(i,2)/BM(i,4);
    dsl_ls(i)=BM(i,3)/BM(i,4);

end
figure(1);
for i=1:88;

```

```

if ds_ls(i)>2
h1=scatter(BM(i,1),ds_ls(i),25,2,'filled');
hold on
else
    h2=scatter(BM(i,1),ds_ls(i),25,8,'filled');
    hold on
end
end
title('Dynamic Amplification Factor(DAF)');
xlabel('Frame') % x-axis label
ylabel('DAF') % y-axis label
legend([h1,h2], 'DAF>2', 'DAF<2')

h4=1;
figure(2);
for i=1:88;
if dsl_ls(i)>=1
h3=scatter(BM(i,1),dsl_ls(i),25,6,'filled');
hold on
else
    h4=scatter(BM(i,1),dsl_ls(i),25,10,'filled');
    hold on
end
end
hold off
title('End Moment Fact(EMF) ');
xlabel('Frame') % x-axis label
ylabel('EMF') % y-axis label

legend([h3,h4], 'EMF>1', 'EMF<1')

```

### **Building structure**

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%building structre%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
bay=4;
floor=10;
bay_width=24;
floor_height=12;
member=floor*(bay*2+1);
t=zeros(member,2);
n=zeros((floor+1)*(bay+1),2);
for i=1:floor+1
    for j=1:bay+1
        n(i+(j-1)*(floor+1),1)=(j-1)*bay_width;
        n(i+(j-1)*(floor+1),2)=(i-1)*floor_height;
    end
end

```

```

        end

    end
    k=1;
    for j=1:bay+1;
    for i=1:floor;

        t(k,1)=i+(j-1)*(floor+1);
        t(k,2)=i+(j-1)*(floor+1)+1;
        k=k+1;
    end
end

```

```

for i=1:bay
    for j=2:floor+1
        t(k,1)=(i-1)*(floor+1)+j;
        t(k,2)=i*(floor+1)+j;
        k=k+1;
    end
end

```

```

end

```

```

nElements = size(t,1)-1;
remove1=6;
remove2=33;
t(remove1,:)=[];
t(remove2,:)=[];
figure(3)

```

```

drawFrame(n, t, ds_ls,1, 10)

```

```

maximum displacement reader

```

```

clc;
clear all;
filename1 = 'llds_disp.xlsx';
filename2 = 'l1_disp.xlsx';
sheet=3;
A = xlsread(filename1,sheet);
B = xlsread(filename2,sheet);

n=size(A,1);
C=zeros(n/2,2);
D=zeros(n/2,4);
D(:,1:2)=B(:,5:6);
for i=1:(n/2)
    C(i,1)=max(A(i*2-1,1),A(i*2,1));
    C(i,2)=max(A(i*2-1,2),A(i*2,2));
end

```



```

        D(i,3)=C(i,1)/abs(B(i,3));
        D(i,4)=C(i,2)/abs(B(i,4));

end
% figure(1)
% scatter(1:n/2,D(:,1));
% figure(2)
% scatter(1:n/2,D(:,2));
figure(3)
scatter(1:n/2,D(:,3));
figure(4)
scatter(1:n/2,D(:,4));

```

### ***Draw the Result***

```

% Function to draw analysis results

function [] = drawFrame(N, T, F, thkMin, thkMax)

% Input
%
% N = node coordinates
% (number of nodes)-by-2 matrix with N(n,1) and N(n,2) the X and
% Y coordinates of node n
%
% T = truss topology
% (number of elements)-by-2 matrix with T(e,1) and T(e,2) the indices
of
% the starting and ending nodes of element e
%
% F = dsl/lr
% (number of elements)-by-1 matrix with F(e,1) the force in element e.
% May be empty.
%
% thkMin = line thickness used to draw elements with zero moment.
% May be empty.
%
% thkMax = line thickness used to draw elements with maximum moment.
% May be empty.
%

nNodes = size(N,1);
nElements = size(T,1);

% Verify and complete line thickness input

if isempty(thkMin) == 0
    sizeOfThk = size(thkMin);
    if sizeOfThk(1,1) ~= 1 || sizeOfThk(1,2) ~= 1

```

```

        error('Line thickness input must be empty or a number')
    end
end

if isempty(thkMax) == 0
    sizeOfThk = size(thkMax);
    if sizeOfThk(1,1) ~= 1 || sizeOfThk(1,2) ~= 1
        error('Line thickness input must be empty or a number')
    end
end

if isempty(thkMin) && isempty(thkMax)
    thkMin = 1;
    thkMax = 1;
elseif isempty(thkMax)
    thkMax = thkMin;
elseif isempty(thkMin)
    thkMin = thkMax;
elseif thkMin > thkMax
    temp = thkMin;
    thkMin = thkMax;
    thkMax = temp;
end

% Determine line thicknesses and colors

thicknesses = zeros(1,nElements);
colors = zeros(nElements,3);

if isempty(F)
    thk = 0.5*(thkMin+thkMax);
    for e=1:1:nElements
        thicknesses(1,e) = thk;
        colors(e,:) = [0,0,0]; % black
    end
else
    sizeOfF = size(F);
    if sizeOfF(1,1) ~= nElements || sizeOfF(1,2) ~= 1
        error('Invalid input')
    end

    thkFact = (thkMax-thkMin)/max(max(F), -min(F));
    for e=1:1:nElements
        thicknesses(1,e) = thkMin + thkFact*abs(F(e,1));

        if (F(e,1) < 2)
            colors(e,:) = [1,0,0]; % red
            thicknesses(1,e) = (thkMax-thkMin)*(F(e,1)/2);
            h1=plot([N(T(e,1),1), N(T(e,2),1)], [N(T(e,1),2),
N(T(e,2),2)], 'linewidth', thicknesses(1,e), 'Color', colors(e,:));

            hold on;
            elseif (F(e,1) < 10)

```

```

        colors(e,:) = [0,1,0]; % green
        thicknesses(1,e) = (thkMax-thkMin)*((F(e,1)-2)/8);
        h2=plot([N(T(e,1),1), N(T(e,2),1)], [N(T(e,1),2),
N(T(e,2),2)], 'linewidth', thicknesses(1,e), 'Color', colors(e,:));
        hold on;
        else
            colors(e,:) = [0,0,1]; % blue
            thicknesses(1,e) = (thkMax-thkMin)*((F(e,1)-10)/max(F));
            h3=plot([N(T(e,1),1), N(T(e,2),1)], [N(T(e,1),2),
N(T(e,2),2)], 'linewidth', thicknesses(1,e), 'Color', colors(e,:));
            hold on;
        end
    end
end

```

```

% Set axes

```

```

minX = min(N(:,1));
maxX = max(N(:,1));
minY = min(N(:,2));
maxY = max(N(:,2));
gap = 0.1*max(maxX-minX, maxY-minY);

```

```

axis equal
axis([minX-gap, maxX+gap, minY-gap, maxY+gap])
hold off
title('Frame DAF ');
xlabel('X-axis') % x-axis label
ylabel('y-axis') % y-axis label

```

```

hleg1=legend([h1,h2,h3], 'DAF<2', '2<DAF>10', 'DAF>10');
set(hleg1, 'Location', 'Northoutside');
set(hleg1, 'Orientation', 'horizontal')

```

```

end

```

### ***Check for displacement***

```

clc;
clear all;
filename1 = 'l1ds_disp.xlsx';
filename2 = 'l1_disp.xlsx';
sheet=3;
A = xlsread(filename1,sheet);

```

```

B = xlsread(filename2, sheet);

n=size(A,1);
C=zeros(n/2,2);
D=zeros(n/2,4);
D(:,1:2)=B(:,5:6);%dsl/ls
for i=1:(n/2)
    C(i,1)=max(A(i*2-1,1),A(i*2,1));
    C(i,2)=max(A(i*2-1,2),A(i*2,2));
    D(i,3)=C(i,1)/abs(B(i,3));%ds/ls
    D(i,4)=C(i,2)/abs(B(i,4));%ds/ls
end
figure(1)
scatter(1:n/2,D(:,1),25,'filled');
title('End Horizontal Displacement Factor(EHDF)');
xlabel('Joints') % x-axis label
ylabel('EHDF') % y-axis label

figure(2)
scatter(1:n/2,D(:,2),25,'filled');
title('End Vertical Displacement Factor(EVDF)');
xlabel('Joints') % x-axis label
ylabel('EVDF') % y-axis label

figure(3)
scatter(1:n/2,D(:,3),25,'filled');
title('Horizontal Displacement Factor(HDF)');
xlabel('Joints') % x-axis label
ylabel('HDF') % y-axis label

figure(4)
scatter(1:n/2,D(:,4),25,'filled');

title('Vertical Displacement Factor(VDF)');
xlabel('Joints') % x-axis label
ylabel('VDF') % y-axis label

```