Dynamic Response of 3D Printed Beams with Damping Layers

by

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BEng Civil Engineering
Université Laval, 2013

SUBMITTED TO THE DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF ENGINEERING IN CIVIL AND ENVIRONMENTAL ENGINEERING
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
JUNE 2015

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Submitted to the Department of Civil and Environmental Engineering on May 19, 2015 in Partial Fulfillment of the Requirements for the Degree of Master of Engineering in Civil and Environmental Engineering

ABSTRACT

3D printers are a relatively new technology and they could be used in the future to 3D print structural components in buildings or bridges. The main advantages of using 3D printing would be the optimization of the structures. Effectively, with 3D printers, it is possible to generate polymers with different strengths and stiffnesses in the same structure. It is also possible to print very complex shapes and forms.

This thesis will focuses on the dynamic response of 3D printed beams with damping layers. More precisely, natural frequency and damping ratio will be analysed in order to find the optimal location of the damping layers. For this experiment, three methods are used, one with an accelerometer, one with a high speed camera and one with a piezoelectric actuator. Characterization of the 3D printed material has been made to predict results.

For the results, it is possible to conclude that using softer material as damping layer reduces the beam frequencies but increases the damping ratio. Also, in order to get the most efficient beam in terms of damping properties, the damping layers need to be close to the top and bottom surfaces as strains are larger. Finally, it can be say that a high speed camera is the best device to investigate the dynamic response of 3D printed materials.

Thesis Supervisor: Markus J. Buehler
Title: Professor of Civil and Environmental Engineering
Acknowledgements

I would personally like to thank Professor Markus Buehler, Professor Jerome Connor and Dr. Zhao Qin for their guidance and support for this thesis. I would also like to thank the Fonds de recherche du Québec – Nature et technologies for their generous funding.

As some might know, the MEng program is a very intensive program and I would like to thank my family for their constant support and help during the past nine months. This amazing opportunity would not have been possible without them.

I would like to express my gratitude to the MEng group of 2015 for their help and friendship, especially Isabelle Su. This year would not have been the same without you.
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Chapter 1 - Introduction

1.1 Motivation

The phenomenon of resonance happens when the frequency of an external force is equal to the natural frequency of a structure. Under resonance, structures experience large amplitude oscillations as they store kinetic energy. This phenomenon needs to be avoided as it can compromise people's comfort and lead to the collapse of civil engineering infrastructures. Thus, engineers need to be careful when designing structures; high-rise buildings and bridges need to be damped to avoid resonance under lateral loadings, such as wind or earthquake.

Nowadays, concrete and steel are the most common materials used to build civil engineering structures. However, as engineers and architects want to build more efficient and creative structures, other materials are making their way into construction fields. For example, fiber-reinforced plastic can be used as rebar in concrete and aluminum can be seen on pedestrian bridges. Other materials that could be seen in the future could be polymers. Actually, with 3D printers, polymers with different strengths and stiffnesses can be generated in the same structure. It is also possible to create an infinite number of forms and shapes with this technology. In brief, almost any architectural design would be possible with polymers. It could even be possible to remove space-consuming damping devices in high-rise buildings by including damping material layers directly in the structures.

1.2 Scope of Work

As 3D printers and polymers are still far from being found on construction sites, this thesis will primarily investigate the impact of adding damping material layers in 3D printed beams on the dynamic response of the beams. For this project, two layers of damping material are added inside a beam made of a stiffer material. Furthermore, different damping materials and layer locations are experimented with in order to find the most optimal configuration.

To get satisfying results, many steps need to be followed. First of all, a literature review on structural dynamic and 3D printing is necessary to have a better understanding of the experimental project. Also, materials available to perform 3D printing need to be selected
carefully and tested for their properties. It is important that relatively stiff materials are chosen for damping materials as the beams need to remain rigid. After the initial experimental tests, simple configurations of 3D printed beams with and without damping layers are produced and tested in the laboratory to determine their damping ratio and dynamic response under external loading. For this test, beams are clamped to a table to simulate cantilever beams. They are then struck and different devices are used to investigate the dynamic response. The dynamic response of 3D printed beams will be analyzed with an accelerometer, a high speed camera and a piezoelectric actuator.
Chapter 2 - Literature Review

2.1 Structural Dynamics

2.1.1 Dynamic Response for a Single Degree of Freedom System

The differential equation governing the response of a viscous damped system is given by:

\[ m\ddot{u} + c\dot{u} + k = p_0 \]  

(1)

where \( m, c, k, u, p_0 \) are respectively the mass, damping parameter, relative displacement to ground and external excitation. Depending on the situation, the external excitation can be arbitrary, step, periodic or pulse.

The natural frequency and the damping ratio of a structure are the following:

\[ \omega = \sqrt{\frac{k}{m}} \]  

(2)

\[ \xi = \frac{c}{2m} \]  

(3)

Three different situations can be observed. When \( \xi = 1 \), the structure experiences critical damping and returns to its equilibrium state without any oscillations. When \( \xi > 1 \), the structure is overdamped and does not oscillate. Finally, when \( 0 < \xi < 1 \), the structure oscillates until it reaches its equilibrium position. The last case is the most common case in structural engineering and it is called an underdamped system. Figure 1 represents free vibrations of the three cases presented (Chopra, 1995). Note that when \( \xi = 0 \), the structure is not damped and should oscillate forever.

![Figure 1: Free Vibration](image)
Other important parameters related to the dynamic response of a structure are the period (T), the duration of one full cycle of oscillation, and frequency (f). Both parameters are presented in the following equations:

\[ T = \frac{2\pi}{\omega} \]  
\[ f = \frac{1}{T} \]

### 2.1.3 Damping in Structures

In civil engineering, common structures have a damping ratio between 5% and 20%. Almost no structures have a damping ratio over 20% as it becomes too expensive. The following table, from N.M. Newmark and W.J. Hall, presents the desired damping ratio for different types of structures depending on the stress level.

<table>
<thead>
<tr>
<th>Stress Level</th>
<th>Type and Condition of Structure</th>
<th>Damping Ratio Recommended (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working stress, no more than</td>
<td>Welded steel, prestressed concrete, well reinforced concrete (only slight cracking)</td>
<td>2-3</td>
</tr>
<tr>
<td>about 0.5 yield point</td>
<td>Reinforced Concrete with considerable cracking</td>
<td>3-5</td>
</tr>
<tr>
<td></td>
<td>Bolted and/or riveted steel, wood structures with nailed or bolted joints</td>
<td>5-7</td>
</tr>
<tr>
<td>At or just below yield point</td>
<td>Welded steel, prestressed concrete (without complete loss in prestress)</td>
<td>5-7</td>
</tr>
<tr>
<td></td>
<td>Prestressed concrete with no prestress left</td>
<td>7-10</td>
</tr>
<tr>
<td></td>
<td>Reinforced concrete</td>
<td>7-10</td>
</tr>
<tr>
<td></td>
<td>Bolted and/or riveted steel, wood structures with bolted joints</td>
<td>10-15</td>
</tr>
<tr>
<td></td>
<td>Wood structures with nailed joints</td>
<td>15-20</td>
</tr>
</tbody>
</table>
2.1.4 Existing Damping Systems

This section presents different damping devices that are currently used in high-rise buildings.

2.1.4.1 Tuned Mass Damper and Tuned Liquid Damper

The tuned mass damper was invented in the early 1900s. It is composed of a mass on a roller, a spring and a damper that is attached to the main structure. It absorbs vibration and controls displacements of the building. Tuned mass dampers are primarily used to mitigate motion, due to wind load, to avoid uncomfortable motion for occupancy. As tuned mass damper can only be tuned for one of the building’s natural frequencies, it is tuned for the frequency that is the most likely to happen. This frequency is often the first fundamental frequency of the building as it is the lowest one. Thus, when the external excitation is close to the natural frequency, the response amplitude is considerably reduced. However, this system does not work when the external frequencies get further from the frequency tuned. An alternative damping system needs to be added when a building is likely to experience two resonance frequencies. Another disadvantage of tuned mass dampers is that they are space consuming as the mass needs to move (Connor & Laflamme, 2014). Figure 2 shows the tuned mass damper system that is used for the Taipei 101 building.

Figure 2: Tuned Mass Damper for the Taipei 101 Building in Taiwan
Tuned liquid dampers are more recent. They were invented in the late 1990s. The most popular type is the U-shape tuned liquid column damper. It uses water flow from one column to the other to dissipate vibration energy. This system plays exactly the same role as a tuned mass damper. However, it requires less maintenance and uses less valuable space. Effectively, tuned liquid dampers can use pre-existing tanks in buildings.

2.1.4.2 Viscous Dampers and Friction Dampers
Viscous dampers can be used in buildings and bridges. To dissipate vibration energy, this system uses a resisting force that is created when a fluid is forced through the piston head orifices. The resisting force is controlled by the velocity of the rod.

2.1.5 How to Evaluate Damping
Different ways can be used to estimate the damping ratio of structures or materials in the laboratory. The half-power bandwidth method and the logarithmic decrement method can both be used to determinate the damping ratio of beams. However, another method has to be used to determine the damping properties of viscoelastic materials.

2.1.5.1 Half-Power Bandwidth Methods
The half-power bandwidth method uses the first natural frequency to determine the damping ratio of the structure. In order to determine the damping ratio in the laboratory, the beam needs to be clamped to a table to simulate a cantilever beam. An external source of excitation is attached to the free end of the beam forcing the beam to experience different known frequencies. Also, an accelerometer is used to measure and record the free end displacement of the beam (Sawant, 2014). A peak in amplitude is observed for each natural frequency as the beam goes into resonance. The first natural frequency corresponds to the largest amplitude oscillation and is the lowest resonance frequency. Figure 3 shows an example of the oscillation amplitude under resonance. The damping ratio can be found with the following formula:
where \( w_n \) is the natural frequency.

\[
\xi = \frac{1}{2} \left( \frac{\omega_2 - \omega_1}{\omega_n} \right)
\]  \hspace{1cm} (6)

**Figure 3:** Determination of Equivalent Viscous Damping

### 2.1.5.2 Logarithmic Decrement Method

The logarithmic decrement method uses almost the same laboratory experiment. However, instead of forcing the beam to experience different frequencies, the beam is struck and goes into free vibration. A function, like the one presented in Figure 4, is recorded. In this Figure, \( u_x \) is the maximal oscillation displacement (Chopra, 1995). In order to find the damping ratio of the structure, the following relation between successive peaks can be used:

\[
\frac{u_i}{u_{i+1}} = \exp \left( \frac{2\pi \xi}{\sqrt{1 + \xi^2}} \right)
\]  \hspace{1cm} (7)

Since \( \xi \) is very small and \( \sqrt{1 + \xi^2} \approx 1 \),

\[
\ln \left( \frac{u_i}{u_{i+1}} \right) = 2\pi \xi
\]  \hspace{1cm} (8)
2.1.5.3 Viscoelastic Damper

For viscoelastic materials another method is used to calculate their damping properties as those materials are too soft to make a beam. To do the experiment, a viscoelastic damper is fabricated by bonding thin sheets of damping material to steel plates. Figure 5 shows a setting that can be made in the laboratory. The grey shaded areas represent the damping material and the black shaded areas are steel plates. Also, an external force, F, is applied on the system and the displacement, u, is measured (Connor & Laflamme, 2014).

In this system, all motion is due to the shear deformation of the viscoelastic material as the shear modulus of the damping material is remarkably smaller than the elastic modulus of steel. Damping properties of the material can be estimated as the loss factor ($\eta$), a measurement of intrinsic damping. $\delta$ is the phase shift between stress and strain and its value varies from 0, for an elastic material, to $\pi/2$, for a viscous material. The loss factor can also be calculated by the ratio of the storage modulus ($G_s$) over the loss modulus ($G_l$).

$$\eta = \frac{G_l}{G_s} = \tan(\delta) \quad (9)$$
2.2 3D Printing

2.2.1 Background and Process
3D printing is a new technology and was first introduced in the late 1980s. One of the earliest developments was made at the Massachusetts Institute of Technology by 3D Systems. The most recent 3D printer at MIT uses the triple-jetting technology, the most recent technology. 3D printers using the triple-jetting technology jet layers of curable liquid photopolymer onto a build tray (Stratasys, 2015). It has many advantages over the previous technology, inkjet printing. It produces more complex shapes, smoother surfaces and more intricate details. It is also faster, for tooling and 3D print, is more precise and permits the inclusion of different material’s properties or colors in the same model in order to create composite structures.

2.2.2 Materials Available at MIT
Three main materials are available at MIT. VeroMagenta and VeroCyan are rigid opaque materials and TangoBlackPlus is a rubber-like material. Their material properties are shown in Table 2 and are provided by Stratasys, the manufacturer.
### Table 2: Material Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>ASTM</th>
<th>Units</th>
<th>Metric</th>
<th>Units</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>VeroGray RGD850, VeroBlackPlus RGD875, VeroWhitePlus RGD835, VeroYellow RGD836, VeroCyan RGD841, VeroMagenta RGD851</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile strength</td>
<td>D-638-03</td>
<td>MPa</td>
<td>50-65</td>
<td>psi</td>
<td>7250-9450</td>
</tr>
<tr>
<td>Elongation at break</td>
<td>D-638-05</td>
<td>%</td>
<td>10-25</td>
<td>%</td>
<td>10-25</td>
</tr>
<tr>
<td>Modulus of elasticity</td>
<td>D-638-04</td>
<td>MPa</td>
<td>2000-3000</td>
<td>psi</td>
<td>290,000-435,000</td>
</tr>
<tr>
<td>Flexural Strength</td>
<td>D-790-03</td>
<td>MPa</td>
<td>75-110</td>
<td>psi</td>
<td>11000-16000</td>
</tr>
<tr>
<td>Flexural Modulus</td>
<td>D-790-04</td>
<td>MPa</td>
<td>2200-3200</td>
<td>psi</td>
<td>320,000-465,000</td>
</tr>
<tr>
<td>HDT, °C @ 0.45MPa</td>
<td>D-648-06</td>
<td>°C</td>
<td>45-50</td>
<td>°F</td>
<td>113-122</td>
</tr>
<tr>
<td>HDT, °C @ 1.82MPa</td>
<td>D-648-07</td>
<td>°C</td>
<td>45-50</td>
<td>°F</td>
<td>113-122</td>
</tr>
<tr>
<td>Izod Notched Impact</td>
<td>D-256-06</td>
<td>J/m</td>
<td>20-30</td>
<td>ft lb/inch</td>
<td>0.375-0.562</td>
</tr>
<tr>
<td>Water Absorption</td>
<td>D-570-98</td>
<td>24hr</td>
<td>1.1-1.5</td>
<td>%</td>
<td>1.1-1.5</td>
</tr>
<tr>
<td>Tg</td>
<td>DMA, E'</td>
<td>°C</td>
<td>52-54</td>
<td>°F</td>
<td>126-129</td>
</tr>
<tr>
<td>Shore Hardness (D)</td>
<td>Scale D</td>
<td></td>
<td>83-86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rockwell Hardness</td>
<td>Scale M</td>
<td></td>
<td>73-76</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polymerized density</td>
<td>ASTM D792</td>
<td>g/cm³</td>
<td>1.17-1.18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ash content VeroGray, VeroWhitePlus</td>
<td>USP281</td>
<td>%</td>
<td>0.23-0.26</td>
<td>%</td>
<td>0.23-0.26</td>
</tr>
<tr>
<td>Ash content VeroBlackPlus</td>
<td>USP281</td>
<td>%</td>
<td>0.01-0.02</td>
<td>%</td>
<td>0.01-0.02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Property</th>
<th>ASTM</th>
<th>Units</th>
<th>Metric</th>
<th>Units</th>
<th>Imperial</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>TangoBlackPlus FLX980 and TangoPlus FLX930</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tensile strength</td>
<td>D-412</td>
<td>MPa</td>
<td>0.8-1.5</td>
<td>psi</td>
<td>115-220</td>
</tr>
<tr>
<td>Elongation at break</td>
<td>D-412</td>
<td>%</td>
<td>170-220</td>
<td>%</td>
<td>170-220</td>
</tr>
<tr>
<td>Compressive set</td>
<td>D-395</td>
<td>%</td>
<td>4-5</td>
<td>%</td>
<td>4-5</td>
</tr>
<tr>
<td>Shore Hardness (A)</td>
<td>D-2240</td>
<td>Scale A</td>
<td>26-28</td>
<td>Scale A</td>
<td>26-28</td>
</tr>
<tr>
<td>Tensile Tear resistance</td>
<td>D-624</td>
<td>Kg/cm</td>
<td>2-4</td>
<td>Lb/in</td>
<td>18-22</td>
</tr>
<tr>
<td>Polymerized density</td>
<td>ASTM D792</td>
<td>g/cm³</td>
<td>1.12-1.13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 3 – Methodology

3.1 Characterization of 3D Printed Materials

3.1.1 Young’s Modulus

Characterization tests need to be performed on the available 3D printed materials in order to determine the Young’s modulus, as this modulus is needed to calculate the natural frequencies of the beams. Thus, a tensile test was performed on every mixture of VeroMagenta and TangoBlackPlus. Table 3 shows every material tested with their name according to the manufacturer and shore hardness. For example, FLX-MK-S95 is composed of a greater ratio of stiff over rubber-like material than FLX-MK-S85.

Table 3: 3D Printed Materials

<table>
<thead>
<tr>
<th>Name</th>
<th>Shore Hardness</th>
</tr>
</thead>
<tbody>
<tr>
<td>VeroMagenta</td>
<td>100</td>
</tr>
<tr>
<td>FLX-MK-S95</td>
<td>95</td>
</tr>
<tr>
<td>FLX-MK-S85</td>
<td>85</td>
</tr>
<tr>
<td>FLX-MK-S70</td>
<td>70</td>
</tr>
<tr>
<td>FLX-MK-S60</td>
<td>60</td>
</tr>
<tr>
<td>FLX-MK-S50</td>
<td>50</td>
</tr>
<tr>
<td>FLX-MK-S40</td>
<td>40</td>
</tr>
<tr>
<td>TangoBlackPlus</td>
<td>27</td>
</tr>
</tbody>
</table>

Tensile tests were performed at the civil engineering laboratory. Figure 6 shows the required dimensions for a dog bone of category IV according to the ASTM D638-10. This category has been chosen as it can be used for rigid, semi-rigid and non-rigid plastics. It is also the specimen to use to compare rigidity of materials. A gage of 12.5 mm was used and a displacement rate of 5mm/min was chosen. However, the gage had to be removed for FLX-MK-S60 and softer materials. The gage produced too much initial displacement on specimens as the material has more rubber-like properties. Furthermore, only one specimen of each type of material has been tested instead of the five prescribed by the ASTM D638-10 for time constraint. Figure 7
represents the experimental setup for the tensile test and Table 4 presents the dimensions of each specimen narrow area.

Figure 6: Specimen Dimensions - ASTM

Figure 7: Tensile Test of Plastics
<table>
<thead>
<tr>
<th>Specimens</th>
<th>Thickness (mm)</th>
<th>End 1</th>
<th>Middle</th>
<th>End 2</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>VeroMagenta</td>
<td>3.99</td>
<td>4.00</td>
<td>3.99</td>
<td>3.99</td>
<td></td>
</tr>
<tr>
<td>Width (mm)</td>
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<td>6.02</td>
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<tr>
<td>Width (mm)</td>
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<td>5.83</td>
<td>5.88</td>
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<td>TangoBlackPlus</td>
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<td>5.68</td>
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<td>Area (mm²)</td>
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<td></td>
<td>22.34</td>
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</tbody>
</table>
3.1.2 Viscoelastic Properties

To calculate the viscoelastic properties of the 3D printed materials, a creep test was performed on them. A creep test consists of applying a constant load on a beam and measuring its deflexion over time. The creep function and viscosity can be determined by the following equations:

\[
J_c(t) = \frac{3E_1\delta(t)}{P_0L^3} \quad \text{For a cantilever beam}
\]

\[
J_c = J_c(t = 0^+) = \frac{1}{(E_1 + E_2)}
\]

\[
J_c = J_c(t = \infty) = \frac{1}{(E_1)}
\]

\[
J_c(t) = J_c - (J_c - J_{ce}) \cdot \exp\left(-\frac{t}{\tau_c}\right)
\]

\[
\eta = \frac{\tau_c E_1 E_2}{(E_1 + E_2)}
\]

where \(J_c\), \(J_{cg}\), \(J_{ce}\), \(\tau_c\) and \(\eta\) are respectively the creep function, the glassy compliance, the equilibrium compliance, the creep retardation time and the viscosity.

Table 5 presents information on the beams used and Figure 8 shows the experimental setup of the creep test. Note that no test was performed for FLX-MK-S70 or softer materials as the beams would be not rigid enough for a creep test.

Table 5: Creep Test – Load, Length, Cross Section

<table>
<thead>
<tr>
<th>Specimens</th>
<th>P (N)</th>
<th>L (mm)</th>
<th>Cross Section (mm x mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VeroMagenta</td>
<td>0.0981</td>
<td>51</td>
<td>20 x 3</td>
</tr>
<tr>
<td>FLX-MK-S95</td>
<td>0.0981</td>
<td>71</td>
<td>20 x 3</td>
</tr>
<tr>
<td>FLX-MK-S85</td>
<td>0.0981</td>
<td>200</td>
<td>20 x 3</td>
</tr>
</tbody>
</table>
3.2 Determination of the Frequency and Damping Ratio of a 3D Printed Beam

All beams have a rectangular cross section to simplify the experiment. The beam lengths are 210 mm in order to get a free span of 200 mm as a cantilever beam. This dimension was chosen to fit the maximal build size of the 3D printer, the Object260 Connex3. The beam widths are 20 mm as typical beam widths are 1/10 of the span. In addition, the depth of the beam is chosen in order to get a precise frequency, as natural frequencies of a beam depend on their cross sections. For the first set of beams, each beam has exactly the same cross section but is using different materials as the damping layer. This set is composed of the following beams, Pink (VeroMagenta only), Pink95 (VeroMagenta/ FLX-MK-S95) and Pink85 (VeroMagenta/ FLX-MK-S85). For the second set of beams, the location of the damping layers is variable but the ratio of stiff over damping material remains constant. This set is composed of Pink85V1 and Pink85V2 and the reference beam is Pink85. An extra specimen has been built to investigate the dynamic response with a piezoelectric actuator. This specimen is the same as Pink95 but it is 300 mm
long and 45 mm wide to be able to fit the actuator on the beam. Figure 9 presents the different beam cross sections in millimeters.

Figure 9: Beam Configurations
3.2.1 Prediction by hand calculation

3.2.1.1 Beam Theory

The natural frequency, $f$, of a cantilever beam is given by the following equation, where $Kn$ is 3.52 for the first natural frequency:

$$f = \frac{Kn}{2\pi} \sqrt{\frac{EI}{mL^4}}$$  \hspace{2cm} (15)

Consequently, an equivalent stiffness can be found by fixing the natural frequency. The total height of the beam can be calculated using equations (16) to (19). Every parameter is represented in Figure 10. For this experiment, the total height of the beam was fixed to 2 mm in order to get frequencies around 10 Hz. Note that the mass of the 3D printed material is 1g/cm$^3$.

$$E_e = E_s \ (I_1 + I_2) + E_s \ (I_2)$$  \hspace{2cm} (16)

$$I_1 = 2 \left[ \frac{b(h_1)^3}{12} + bh_1 \left( \left( H_t - \frac{h_1}{2} \right) - \frac{H_t}{2} \right)^2 \right]$$  \hspace{2cm} (17)

$$I_2 = 2 \left[ \frac{b(h_2)^3}{12} + bh_2 \left( \left( H_t - h_1 - \frac{h_2}{2} \right) - \frac{H_t}{2} \right)^2 \right]$$  \hspace{2cm} (18)

$$I_3 = \frac{[b(h_3)^3]}{12}$$  \hspace{2cm} (19)

![Figure 10: Beam Dimensions](image)

Figure 10: Beam Dimensions
3.2.1.2 Correction Factor for Shear

As the ratio of $E_{\text{stiff}}$ and $E_{\text{soft}}$ is relatively high, hand calculations might be more accurate if the effect of shear is taken into account. The following equation can be used to calculate a correction factor to the frequency of a cantilever beam:

$$f = f_B \tau_n \ast \left[ 1 - \frac{K \ast r^2}{2L^2} \ast \left( 1 + \frac{E}{k'G} \right) \right]$$

(20)

where $r$ is the radius of gyration of the section and $k'$ can be approximated to 0.5.

However as the ratio $r^2$ over $L^2$ is very small, the correction factor is ≈1. Thus, shear should not have an effect on the frequency of the beam.

3.2.2 Experimental study

3.2.2.1 Accelerometer

The first set of beams (Pink, Pink95 and Pink85) has been used for this experiment. Figure 11 shows the experimental setup to investigate the dynamic response. Note that the logarithmic decrement method was chosen for this project. The accelerometer and software used are called “Slam Stick.”

Figure 11: Experimental Setup – Accelerometer
3.2.2.2 High Speed Camera

Both sets of beams are used for this experiment. The setup and MatLab code to analyse the beam were made by Justin Chen, a PhD candidate at MIT. The setup is shown in Figure 12. Beams are clamped to simulate a cantilever column and are excited by a small pressure close to the support. While beams go into free vibration, the high speed camera records their movement. Data are then analysed with a MatLab code that was created to generate the displacement of multiple points along the column and calculate frequencies.

![Figure 12: Experimental Setup - High Speed Camera](image)

3.2.2.3 Piezoelectric Actuator

A piezoelectric actuator can be used to determine the frequency of a beam. The ceramic actuators used are those represented in Figure 13. Those types of actuators contract and expand as a response to a small imposed strain. When a direct current is applied, the cantilever beam will deflect and the deflection will depend on the intensity of the current. When an
alternative current is applied, the cantilever beam will go into vibration. The natural frequency of the beam is found when the amplitude is maximal. For this experiment the actuators are bond with epoxy on each side of the beam at the support. Figure 14 shows the experimental setup and the wide beam is used.

Figure 13: Piezoelectric Actuator

Figure 14: Experimental Setup - Piezoelectric Actuator
Chapter 4 – Results and Discussions

4.1 Characterization of 3D printed materials

4.1.1 Young’s Modulus

In order to determine the Young’s modulus, the stress-strain curve of each material has been generated, as presented in Figure 15 and Figure 16. VeroMagenta is on a separate graph as its maximal stress value is more than 10 times the maximal stress value of other mixtures.

![Figure 15: VeroMagenta Stress-Strain Curve](image)

![Figure 16: Stress-Strain Curve for Other Mixtures](image)
As one can observe, the maximal stress decreases significantly between VeroMagenta and FLX-MK-S75. Note that failure was reached only for the four stiffer materials. However, the maximal stress and stress at failure are not required in order to calculate the Young’s modulus. The initial slope of the stress-strain curve represents the Young’s Modulus and those values are presented in Table 6.

Table 6: Young’s Modulus for 3D Printed Materials

<table>
<thead>
<tr>
<th>Specimens</th>
<th>E (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VeroMagenta</td>
<td>2469</td>
</tr>
<tr>
<td>FLX-MK-S95</td>
<td>228</td>
</tr>
<tr>
<td>FLX-MK-S85</td>
<td>47</td>
</tr>
<tr>
<td>FLX-MK-S75</td>
<td>18</td>
</tr>
<tr>
<td>FLX-MK-S60</td>
<td>15</td>
</tr>
<tr>
<td>FLX-MK-S50</td>
<td>7</td>
</tr>
<tr>
<td>FLX-MK-S40</td>
<td>2</td>
</tr>
<tr>
<td>TangoBlackPlus</td>
<td>1</td>
</tr>
</tbody>
</table>

4.1.2 Viscoelastic Properties

The following table presents the viscous properties obtained from the creep test. It is possible to see that the viscous properties decrease when a softer material is used. To verify the current results, a similar creep test has been performed on simply supported beams made of the same materials and equivalent results were obtained. The creep functions are detailed in Appendix A.

Table 7: Viscoelastic Properties

<table>
<thead>
<tr>
<th>Specimens</th>
<th>η (GPa*s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>VeroMagenta</td>
<td>202.6</td>
</tr>
<tr>
<td>FLX-MK-S95</td>
<td>6.7</td>
</tr>
<tr>
<td>FLX-MK-S85</td>
<td>2.1</td>
</tr>
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</table>
4.2 Determination of the frequency and damping ratio of a 3D printed beam

4.2.1 Prediction by hand calculation

The following table presents the predicted natural frequencies of the beams.

Table 8: First Natural Frequency - Hand Calculations

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Frequency (Hz)</th>
<th>Specimens</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pink</td>
<td>11.80</td>
<td>Pink85V1</td>
<td>11.13</td>
</tr>
<tr>
<td>Pink95</td>
<td>10.69</td>
<td>Pink85</td>
<td>10.58</td>
</tr>
<tr>
<td>Pink85</td>
<td>10.58</td>
<td>Pink85V2</td>
<td>8.92</td>
</tr>
</tbody>
</table>

As one can observe, frequencies become lower when a softer material is used, as $E_2$ decreases. Also, higher frequencies are observed when the top (external) layers are thicker as it increases the equivalent stiffness of the beam.

4.2.2 Experimental study

4.2.2.1 Accelerometer

Four tests were performed on each beam. The accelerometer was placed at the free end of the beam (200 mm for the support) for the first two tests and the accelerometer was placed at 150 mm from the support for the last two tests. Detailed results of the behavior of the beams can be found in Appendix B. Figure 17 shows the first natural frequency and Table 9 shows the damping ratio measured.
Figure 17: First Natural Frequency - Accelerometer
Table 9: Average Damping Ratio - Accelerometer

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pink - 200 mm</td>
<td>2.0%</td>
</tr>
<tr>
<td>Pink - 150 mm</td>
<td>2.4%</td>
</tr>
<tr>
<td>Pink95 - 200 mm</td>
<td>3.4%</td>
</tr>
<tr>
<td>Pink95 - 150 mm</td>
<td>3.3%</td>
</tr>
<tr>
<td>Pink85 - 200 mm</td>
<td>4.4%</td>
</tr>
<tr>
<td>Pink85 - 150 mm</td>
<td>4.7%</td>
</tr>
</tbody>
</table>

As one can see, the damping ratio increases when a softer material is used as the damping layer. However, the obtained frequencies do not match with the hand calculations, even for the homogeneous material. This difference, for Pink when the accelerometer is placed at the free end, is 6 Hz and gives a relative error over 100%. Another inaccuracy is that the frequency measured changes depending on the location of the accelerometer. When the accelerometer is closer to the support, the frequency is higher. This difference is significant; the relative error between Pink-200 mm and Pink-150 mm is 47%. However, the frequency should be the same as it does not depend on the location of the accelerometer (see equation (15)). Thus, the mass of the accelerometer is too heavy compared to the mass of the beams. Effectively, with this experiment, the frequency obtained is more similar to a SDOF system instead of a continuous cantilever beam. Another method is then required to verify the previous conclusion.

4.2.2.2 High Speed Camera

4.2.2.2.1 First Set

Table 10 shows the measured frequency and damping ratio obtained with the high speed camera.
Table 10: Experimental Results - High Speed Camera - First Set

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pink</td>
<td>11.95</td>
<td>2.3%</td>
</tr>
<tr>
<td>Pink95</td>
<td>11.30</td>
<td>4.0%</td>
</tr>
<tr>
<td>Pink85</td>
<td>11.14</td>
<td>6.1%</td>
</tr>
</tbody>
</table>

As it can be seen, the measured frequencies are very similar to the predicated frequencies. The largest relative error is for Pink95 with 6%. The relative error for Pink and Pink85 are 1% and 6% respectively. Thus, the weight of the accelerometer played an important role on the previous experiment. Also, as seen with the accelerometer, the damping ratio increases when a softer material is used as the damping layer. An increase of 4% can be seen between the reference specimen and Pink85. Additional data are provided in Appendix C.

4.2.2.2.1 Second Set

Table 11 shows the measured frequencies and damping ratio obtained for the second set of beams.

Table 11: Experimental Results - High Speed Camera - Second Set

<table>
<thead>
<tr>
<th>Specimens</th>
<th>Frequency (Hz)</th>
<th>Damping Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>P85V1</td>
<td>10.26</td>
<td>6.0%</td>
</tr>
<tr>
<td>Pink85</td>
<td>11.14</td>
<td>6.1%</td>
</tr>
<tr>
<td>P85V2</td>
<td>9.00</td>
<td>6.4%</td>
</tr>
</tbody>
</table>

As it can be seen, the optimal position for the damping layers is close to the top and bottom surfaces as displacements are maximal. The increase is 0.36% between P85V1 and P85V2. However, it is preferable that damping layers are covered by stiff material layers to protect them. For the frequencies, obtained frequencies and expected frequencies are similar. The
relative errors are 8%, 5% and 2% for P85V1, Pink85 and P85V2 respectively. More details are available in Appendix D.

4.2.2.3 Piezoelectric Actuator

Results were compared with hand calculations. Table 12 shows the results obtained for the wide beam.

Table 12: First Natural Frequency – Actuator and Hand Calculations

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Actuator</th>
<th>Hand Cal.</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>7.90</td>
<td>10.10</td>
</tr>
<tr>
<td>250</td>
<td>17.00</td>
<td>15.79</td>
</tr>
</tbody>
</table>

As one can see, the relative error between hand calculations and the piezoelectric actuator is around 28% and 7% for the first and second test respectively. No equipment was added at the free end of the beam to measure its displacement because the mass of the device would have influenced the results. Therefore, it was harder to get natural frequency as the maximal amplitude of the beam had to be estimated.
Chapter 5 - Conclusion

Many conclusions can be drawn from this thesis. First, it was possible to characterize 3D printed materials. The Young’s modulus of the stiffer 3D printed material is 2 500 MPa and it decreases rapidly when more rubber-like material are added. In addition, the viscosity of the stiffer material is higher and this property also decreases when the stiffness decreases. As expected according to hand calculations and theory, using softer material as damping layer reduces the beam frequencies but increases the damping ratio. Also, in order to get the most efficient beam in terms of damping properties, the damping layers need to be close to the top and bottom surfaces. Damping layers are more efficient where strains are higher. However, using softer materials for the damping layers or place the damping layers closer to the surface decreases the natural frequencies of the beams. Another conclusion is that the best way to analyse the dynamic response of 3D printed material is the high speed camera, as no equipment has to be installed on the beams. 3D printed beams are so light that the weight of a simple accelerometer has a big influence on the results.

To push forward this study, finite element models should be made to determine even more efficient beams in terms of damping. It would be also interesting to use finite element to not only look at damping and frequencies but to look also into tensile and compressive strengths and deflection.

In summary, the use of polymer could be an interesting avenue for the future. As this thesis showed, it was possible to create different beams with different strengths and stiffnesses using mixtures of two polymers. It was also possible to vary natural frequencies and damping ratio while keeping dimensions and stiff over soft material ratio constant. Thus, structural components could be optimized with the use of 3D printed materials. However, for now, 3D printed materials are not strong enough. Their highest modulus of elasticity is 2500 MPa, which is 80 times lower than steel, and their maximal tensile strength is about 55 MPa compare to 400 MPa for steel. Therefore, major improvements have to be made before seeing 3D printed materials as structural components in buildings.
References


Appendix A

Creep functions for the creep tests:

1. VeroMagenta
2. FLX-MK-S95
3. FLX-MK-S85
Appendix B

Data for the experiment with the accelerometer (Pink):
Data for the experiment with the accelerometer (Pink95):

![Graph for Test 1 - 200 mm](image)

![Graph for Test 2 - 200 mm](image)
Data for the experiment with the accelerometer (Pink85):

![Graph of Test 1 - 200 mm](image1)

![Graph of Test 2 - 200 mm](image2)
Appendix C

First natural frequency:

Displacement:
Amplitude vs. Time (s)

Graph 1: Pink95

Graph 2: Pink85

Amplitude

Time (s)
Appendix D

First natural frequency:

Displacement: