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Simulation of the effect of stress-induced anisotropy on borehole compressional wave propagation

Xinding Fang¹, Michael C. Fehler¹, and Arthur Cheng²

ABSTRACT

Formation elastic properties near a borehole may be altered from their original state due to the stress concentration around the borehole. This can lead to an incorrect estimation of formation elastic properties measured from sonic logs. Previous work has focused on estimating the elastic properties of the formation surrounding a borehole under anisotropic stress loading. We studied the effect of borehole stress concentration on sonic logging in a moderately consolidated Berea sandstone using a two-step approach. First, we used an iterative approach, which combines a rock-physics model and a finite-element method, to calculate the stress-dependent elastic properties of the rock around a borehole subjected to an anisotropic stress loading. Second, we used the anisotropic elastic model obtained from the first step and a finite-difference method to simulate the acoustic response of the borehole. Although we neglected the effects of rock failure and stress-induced crack opening, our modeling results provided important insights into the characteristics of borehole P-wave propagation when anisotropic in situ stresses are present. Our simulation results were consistent with the published laboratory measurements, which indicate that azimuthal variation of the P-wave velocity around a borehole subjected to uniaxial loading is not a simple cosine function. However, on field scale, the azimuthal variation in P-wave velocity might not be apparent at conventional logging frequencies. We found that the low-velocity region along the wellbore acts as an acoustic focusing zone that substantially enhances the P-wave amplitude, whereas the high-velocity region caused by the stress concentration near the borehole results in a significantly reduced P-wave amplitude. This results in strong azimuthal variation of P-wave amplitude, which may be used to infer the in situ stress state.

INTRODUCTION

Borehole acoustic-logging data provide important information about formation elasticity (Mao, 1987; Sinha and Kostek, 1995). Monopole and cossidipole measurements are widely used for determining the formation of P-wave velocity and S-wave anisotropy (Sinha and Kostek, 1995, 1996; Winkler et al., 1998; Tang et al., 1999, 2002). Most conventional unfractured reservoir rocks, such as sands, sandstones, and carbonates, show very little intrinsic anisotropy in an unstressed state (Wang, 2002). However, stress-induced anisotropy caused by the opening or closing of the compliant and crack-like parts of the pore space due to tectonic stresses can significantly affect the elastic properties of rocks. Drilling a borehole in a formation strongly alters the local stress distribution. When the in situ stresses are anisotropic, drilling causes the closure of opening of cracks in the formation around a borehole and leads to an additional stress-induced anisotropy. Winkler (1996) experimentally measures the azimuthal variation of the P-wave velocity in a direction parallel to a borehole that was subjected to a uniaxial stress loading and showed that the borehole stress concentration has a strong impact on the velocity measurements. To fully understand the effect of borehole stress concentration on borehole sonic logging, a thorough analysis of the propagation of waves in a 3D borehole embedded in a medium with stress-dependent elastic properties needs to be conducted.

The stiffness tensor of the formation around a borehole is governed by the constitutive relation between the stress field applied around the borehole and the elasticity of the rock with microcracks embedded in the matrix. Several approaches (Sinha and Kostek, 1996; Winkler et al., 1998; Tang et al., 1999; Brown and Cheng, 2002).
Sinha (2000, 2003) study the influence of borehole stress concentration on the rock when it is subjected to anisotropic stress loading. Liu and Sinha (2000, 2003) study the response of the elastic properties of the rock around a borehole. Then, we use a finite-difference method to simulate the wave propagation along the borehole and study the effect of stresses on borehole P-wave propagation.

All approaches (Sinha and Kostek, 1996; Winkler et al., 1998; Tang et al., 1999; Brown and Cheng, 2007; Fang et al., 2013) that have been used for calculating the stress-induced formation stiffness changes around a borehole are based on the data measured from compression experiments to determine the mechanical behavior of a rock under stress. The effect of tensile stress on rock stiffness is either neglected or determined by extrapolating the data from the compressive to the tensile regime. This extrapolation has no physical basis (Fang et al., 2013). Although crack opening under tension can be studied through uniaxial or triaxial experiments (Stanichits et al., 2006), further research is needed to study how to quantitatively determine the effect of crack opening from laboratory data in the calculation of borehole stress-induced anisotropy. Thus, the effect of stress-induced crack opening is not considered in this study. Moreover, the inelastic effect due to irreversible mechanical damage is also neglected because the rock-physics model of Mavko et al. (1995), which is used by Fang et al. (2013), is purely elastic. Instead of focusing on rock-physics modeling, our objective here is to discuss the importance of wave propagation simulation in the study of the effect of stress on borehole sonic logging.

We first give a brief review of the method of Fang et al. (2013) for calculating the borehole stress-induced anisotropy, and then compare the numerical simulation results with the laboratory measurements of Winkler (1996) for a moderately consolidated Berea sandstone. We then simulate the effect of stress on sonic logs on the field scale. The work presented in this paper gives us a new understanding of the characteristics of sonic-wave propagation in a borehole.

**BRIEF REVIEW OF THE METHOD FOR BOREHOLE STRESS-INDUCED ANISOTROPY CALCULATION**

In the approach of Fang et al. (2013), the stress-induced anisotropy around a borehole is obtained through an iterative process (Figure 1) that combines the method of Mavko et al. (1995) and an FEM. First, we measure the P- and S-wave velocities versus hydrostatic pressure for a given rock sample. These data are used to calculate the stress dependent crack compliance of the rock. Second, we apply the workflow shown in Figure 1 to calculate the stiffness tensor of the rock around a borehole subjected to a given stress loading. In the workflow, we first use Mavko’s model to calculate the stiffness of an intact rock under a given stress loading and use it as an initial model. Second, we insert a borehole into the initial model and use an FEM to calculate the stress field around the borehole. We then iteratively use Mavko’s model to calculate the stiffness tensor of each element in the model based on the local stress tensor and replace the old stiffness tensor with the updated one. After the first iteration, the model becomes heterogeneous due to the spatially varying stress field. We use the finite element to calculate the stress distribution in the updated model and iterate over those steps inside the loop shown in Figure 1, until the model stiffness converges to a stable value. The output from the iteration is the stiffness tensor of the model as a function of space and applied stress. Validation of the finite-element program is presented in Appendix A.

**FINITE-DIFFERENCE MODELING**

We use the P- and S-wave velocities versus hydrostatic pressure data (shown in Figure 4 of Fang et al., 2013) measured from a moderately consolidated Berea sandstone sample to construct a model for our wave propagation simulation. Table 1 lists the properties of the rock sample (i.e., sample 1). To validate the applicability of our modeling to simulate stress effect on sonic logs, we first compare our numerical simulations with the laboratory measurements of Winkler (1996), in which the P-wave velocity versus azimuth

<table>
<thead>
<tr>
<th>Sample</th>
<th>$V_p$ (km/s)</th>
<th>$V_s$ (km/s)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>Porosity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample 1 (Fang et al., 2013)</td>
<td>2.83</td>
<td>1.75</td>
<td>2198</td>
<td>17.7</td>
</tr>
<tr>
<td>Sample 2 (Winkler, 1996)</td>
<td>2.54</td>
<td>N/A</td>
<td>N/A</td>
<td>22</td>
</tr>
</tbody>
</table>

Figure 1. Workflow for computation of stress-induced anisotropy around a borehole (from Fang et al., 2013). FEM and M represent finite-element method and the method of Mavko et al. (1995), respectively. See text for explanation.

Table 1. Parameters of the Berea sandstone samples in an unstressed state. Samples 1 and 2 are, respectively, the rocks used in our simulation and in the experiment of Winkler (1996).
around a borehole in sandstone samples with and without applied uniaxial stress was measured. After the comparison with Winkler’s high-frequency experiment, we upscale our model to the field scale and study the simulation results at frequencies suitable for field sonic logging.

Comparison with Winkler’s laboratory measurements

Winkler (1996) conducts acoustic experiments on a Berea sandstone sample and a Hanson sandstone sample to measure the P-wave velocity versus azimuth around a borehole. We cannot directly simulate the laboratory experiments because the P- and S-wave velocities of the rock samples versus hydrostatic pressure, which are the necessary input to determine the stress dependent crack compliance in our model construction, are not available. Instead, we use the measurements made by Fang et al. (2013) and scale the results to those of Winkler (1996). Our comparison is limited to the Berea sandstone because we only have velocity versus pressure data for a Berea sandstone sample (sample 1 in Table 1). As shown in Table 1, the Berea sandstone sample used in Winkler’s experiment has similar properties to the sample used in our simulation, so we can expect that they have similar responses under stress. In Winkler’s experiment, the Berea sandstone sample having dimensions of $15 \times 15 \times 13$ cm and with a 2.86 cm (1.125 in) diameter borehole parallel to the short dimension was placed in a water tank for conducting acoustic measurements. The P-wave velocity at each azimuth was measured parallel to the borehole axis using a directional transducer and two receivers, which were 7 and 10 cm, respectively, away from the transducer. The average P-wave velocity before stress was applied to the sample was approximately $2.54$ km/s and there was little variation with azimuth. When Winkler’s model is scaled to a 20 cm (8 in) borehole, the corresponding frequency of the received acoustic signals is $30$ kHz.

We built a borehole model with the same geometry of the experiment configuration of Winkler (1996), so that the numerical results are comparable to the laboratory measurements. Figure 2 shows the geometry of our borehole model. The formation is Berea sandstone (sample 1 in Table 1) and the borehole is water saturated. A 2.86 cm (1.125 in) borehole is at the center of the model along the $z$-direction. A uniaxial stress is applied normal to the borehole in the $x$-direction. The direction of the applied uniaxial stress is defined as $0^\circ$. A 0.64 cm (1/4 in) diameter piston source, which mimics the 1/4 in diameter directional transducer in Winkler’s experiment, is used in the simulation. A schematic of the piston source is shown in Figure 3. The source amplitude is tapered from the center to the edges using a Hanning window that is shown as the dashed curve in Figure 3. Source time function is a Ricker wavelet with a 213 kHz center frequency, the corresponding frequency is $30$ kHz in a 20 cm (8 in) borehole. Figure 4 shows a snapshot at 0.011 ms of the pressure field in the borehole excited by a piston source pointing at $30^\circ$. We can see that the wavefield excited by the piston source has good directionality and is antisymmetric with respect to the source plane. Receivers, which are shown as the blue circles in Figure 2, are placed in water and are 0.7 cm away from the borehole axis along the source direction. Perfectly match layer is used at all model boundaries to avoid boundary reflection. The boundary effect is not considered in the comparison because Winkler (1996) only measures the time of the first arriving P-wave and the distance between the model boundary and the wellbore is almost six times the P-wave wavelength.

The elastic model obtained from the method of Fang et al. (2013) contains 21 independent elastic constants, which are functions of the applied stress and position. For 10 MPa stress loading, we calculate the average value of the stiffness tensor of the formation around the borehole and plot it in Figure 5. Color intensity of each box in Figure 5 represents the average value of the corresponding component in the stiffness tensor ($6 \times 6$ matrix notation). The nine

Figure 2. Borehole model geometry. A piston source (red circle) is located at the borehole center. Receivers (blue circles) are 0.7 cm away from the borehole center along the source direction that is indicated by the red arrow. A uniaxial stress is applied in the x-direction. Borehole diameter is 2.86 cm.

Figure 3. Schematic showing the 1/4 in diameter piston source used in the simulation. Arrows indicate source excitation direction. Dashed curve represents a Hanning window that is used to taper the source amplitude from the center to the edges. Source orientation angle $i$ is measured from the positive x-direction.
components inside the dashed green lines of Figure 5 are approximately two orders of magnitude larger than the others. This indicates that only nine elastic constants (i.e., $C_{11}$, $C_{12}$, $C_{13}$, $C_{22}$, $C_{23}$, $C_{33}$, $C_{44}$, $C_{45}$, and $C_{66}$) in the stiffness tensor are important. We call these nine elastic constants as the dominant components. In the following, we will show that the nine dominant components determine the characteristics of wave propagation in a borehole and the remaining 12 components in the stiffness tensor have negligible effect. Figure 6 shows the variations of the nine dominant elastic constants around the borehole in the $x$–$y$ plane for a 10 MPa uniaxial stress applied along the $x$-direction. Properties of the model are invariant in the $z$-direction because of model symmetry. As seen in Figure 6, the rock around the borehole becomes inhomogeneous and anisotropic under a stress loading. The diagonal components in the stiffness tensor (i.e., $C_{ii}$, $i = 1, 2, \ldots, 6$) are significantly different from each other. Due to the stress concentration at $\pm 90^\circ$, the stiffness of the rock increases from the stress loading direction ($\theta$) to the direction normal to the loading stress ($\pm 90^\circ$).

We use a 3D staggered grid finite-difference method (Cheng et al., 2000) for P- and S-waves in the $x$–$y$ plane at zero stress state is illustrated by the solid and dashed curves, respectively, in Figure 7. Numerical error in the $z$-direction is of the same order of magnitude of that in the $x$–$y$ plane because grid spacing is uniform in all three directions. From Figure 7, we can see that the difference between the true velocity (i.e., formation velocity) and the actual velocity (i.e., grid velocity) of wave propagation in the simulation is less than 0.01% for P- and S-waves in all directions within the source frequency range. The azimuthal variation of P-wave velocity in the borehole caused by 10 MPa uniaxial stress is approximately 10% (Winkler, 1996), which is several orders of magnitude larger than that caused by numerical error. Thus, the numerical error has negligible effect on the results. Figure 8 shows a comparison of the seismograms simulated from two models, respectively, containing 21 (dashed red) and nine dominant (solid black) elastic constants for a piston source at $30^\circ$ and 10 MPa uniaxial stress loading. As shown in Figure 8b, the difference (multiplied by $10^3$) between the wavefields recorded in these two models is negligible. This indicates that the wave propagation in the borehole is reliably simulated when using only the nine dominant elastic constants and the rest of the stiffness tensor can be neglected. Therefore, we only use the nine dominant components and assume the other components in the stiffness tensor are equal to zero in the simulations below. However, this is the only case for a borehole that is aligned with the symmetry plane of an anisotropic formation. When the borehole is oblique to the symmetry plane, a general anisotropic stiffness tensor with 21 elastic constants might need to be used in the simulation.

Figure 9 shows the pressure profiles recorded in the borehole for sources at 10 different orientations when the model is subjected to 10 MPa uniaxial stress. The $0^\circ$ and $90^\circ$ are along the $x$- and $y$-directions, respectively. The refracted P-waves, which have almost linear moveouts, are marked by the dashed red lines. To show the weak refracted P-waves, we saturate the wavefields for plotting. In Figure 9f–9j, we can see that the refracted P-waves vanish when the offset is larger than approximately 12 cm. We will discuss the cause of this in the next section. The repeating hyperbolic events arriving after approximately 0.07 ms are the multiple reverberations inside the borehole.

Figure 10 shows the waveforms at two receivers located at $z = 7$ and 10 cm (source at $z = 0$ cm), which are the positions of the near and far receivers in Winkler’s experiment, for sources at 10 different orientations. The first arriving P-waves at $z = 7$ and 10 cm should be the refracted P-wave because the refracted P-wave starts to appear at approximately $z = 3 \sim 4$ cm and is still present at $z = 10$ cm, as shown in Figure 9. At each source orientation, we divide the distance between the two receivers by the delay between the refracted P-wave arrival times, which are indicated by the red circles in Figure 10, to get the P-wave velocity. Figure 11 shows the azimuthal variation of the normalized P-wave velocity obtained from our numerical simulations together with the data measured
by Winkler (1996) for 10 MPa uniaxial stress. The modeling data (squares) and the measured data (circles) are normalized separately by the corresponding P-wave velocity of the rock sample at zero stress state. We simulate 10 sources with orientation varying from 0° to 90° in steps of 10°. Using symmetry, we replicate the data from 0° to 90° to the other three quadrants for plotting. As shown in Figure 11, our numerical results agree well with the laboratory measurements of Winkler (1996). Figure 12 shows the comparison of our numerical results (squares) and Winkler’s laboratory measurements (solid curves) for 5, 10, and 15 MPa uniaxial stresses. For 5 and 15 MPa uniaxial stresses, Winkler (1996) does not show the original measured data points but only the best fits. Winkler (1996) measures the velocities during loading and unloading cycles. We only compare the data measured during the loading process because the velocity versus pressure data used in our model construction are measured in an increasing stress process. The impact of stress-strain hysteresis is not considered in our study. To understand the propagation of sonic waves in the 3D inhomogeneous borehole environment, we compare our 3D results with 2D simulation results, which are shown as dashed curves in Figure 12. In the 2D finite-difference simulations (Wang and Tang, 2003), we take 10 2D profiles of the model on radial planes at azimuths from 0° to 90° in steps of 10° and then simulate the wave propagation at each azimuth separately in 2D cylindrical coordinates (r–z coordinates) by assuming that model properties are azimuthally invariant in each 2D model. Validation of the 2D finite-difference program is presented in Appendix B. The velocity variation range of the results from 3D simulations (squares) is smaller than that of the 2D results (dashed curves) because the properties of the 3D model vary azimuthally and the waves are sensitive to some average of the properties of the formation in all directions.

As shown in Figure 12, Winkler (1996) finds that the measured P-wave velocities have broad maxima and cusped minima and can be better fit using an exponential function instead of a cosine function, which is that expected from the cosine dependence of stress near a borehole (Jaeger et al., 2007). Our numerical results (squares) obtained from 3D finite difference simulations have very similar azimuthal variation as the measured data (solid curves), whereas the results given by the 2D simulations (dashed curves) show a variation trend close to a cosine behavior. If the propagation of the refracted P-wave follows a straight wave path along the wellbore in the source excitation direction, then the P-wave velocity versus source direction should show a cosine function variation, which has been predicted by the theoretical calculations of Sinha and Kostek (1996) and Fang et al. (2013) and is also confirmed by the 2D simulation results shown in Figure 12. The broad maxima and cusped minima shown in the measured data and the 3D numerical results in Figure 12 suggest that the propagation of the refracted P-wave does not follow a straight wave path along the wellbore. The first arriving P-wave finds the fastest path through a higher velocity zone to reach a receiver.

The overall variation of our numerical results with azimuth is a little bit smaller than that of the measured data because the rock sample used in the experiment of Winkler (1996) is more compliant than our rock sample, as the porosity of our sample is lower and the velocity before applying stress is higher. Another difference between the numerical results and the measured data occurs at 0° and 180°, in which the laboratory measured velocities for 10 and 15 MPa uniaxial stresses are smaller than that for 5 MPa uniaxial stress. This may be caused by the opening of micro cracks induced...
Figure 8. (a) Comparison of the seismograms simulated from two models, respectively, containing 21 (dashed red) and nine dominant (solid black) elastic constants for a piston source at 30° and 10 MPa stress loading and (b) difference (multiplied by $10^5$) between the seismograms shown in (a).

Figure 10. (a and b) Seismograms recorded at $z = 7$ and 10 cm, respectively, when the model is subjected to 10 MPa stress loading. Source is at $z = 0$ cm. Red circles indicate the arrival times of the refracted P-wave. The 0° and 90° are along the x- and y-axis directions, respectively.

Figure 9. Pressure profiles recorded in the borehole for sources at 10 different directions when a 10 MPa uniaxial stress is applied in the x-direction (i.e., 0°). The number above each panel indicates the source direction. The 0° and 90° are along the x- and y-directions, respectively. Wavefields are saturated for plotting to show the weak refracted P-waves, which are marked by the dashed red lines.
by tensile stresses, whose effect increases with increasing loading stress. However, crack opening caused by tensile stress is neglected in our model, so the normalized velocities in the numerical results increase with the increase of loading stress at 0° and 180°.

The good agreement between the laboratory measurements and numerical results in Figure 12 suggests that the effect of crack opening is relatively small for a moderately consolidated Berea sandstone under relatively low uniaxial loading stresses. Winkler (1996) finds that the azimuthal variation of P-wave velocity is closer to a cosine behavior and the P-wave velocity measured along the loading stress direction (i.e., 0°) is substantially smaller than that at zero stress loading for the Hanson sandstone sample used in his experiment. This indicates that crack opening caused by tensile stress could be an important factor affecting borehole sonic logs for certain types of rock. However, this is beyond the scope of this paper and will be a future research topic.

Field scale simulations

In the previous section, we have shown that the numerical simulation results match the laboratory measurements of Winkler (1996) very well. This demonstrates the applicability of the approach of Fang et al. (2013) for estimating the azimuthal variation of anisotropic elastic properties around a borehole. However, the source frequency (i.e., 30 kHz for a 20 cm borehole) used in the above simulations is much higher than the conventional logging frequency used in the field. In this section, we conduct a simulation by upscaling the borehole model constructed previously to a 20 cm borehole and replacing the piston source with a monopole source, which is commonly used in measuring formation P-wave velocity from borehole sonic logging (Tang and Cheng, 2004). The scaled model may not represent a realistic borehole, around which the rocks show multiscaled heterogeneity in the field (Sato et al., 2012). However, this simplified model provides us a means to understand the basic physical nature of sonic wave propagation in a borehole.

The center frequency of the monopole source used in wireline sonic logging is usually approximately 10 kHz. To investigate the acoustic response in a borehole as a function of frequency, we will simulate monopole sources of 10, 15, and 20 kHz center frequencies. A 20 kHz source can be achieved in a logging-while-drilling tool. Our source wavelet is a Ricker wavelet. The dominant P-wave wavelength changes approximately from 9 to 28 cm when the source frequency drops from 30 (laboratory scale) to 10 kHz.
to 10 kHz (field scale). We can expect that the waves excited from a 10 kHz monopole source penetrate deeper into the formation where properties are less affected by the stress concentration and thus the azimuthal variation of their velocity will be smaller than that in the laboratory measurements of Winkler (1996). To make scales consistent with typical field scales, we place receivers with offsets from 1 to 5 m at every 10° from 0° to 90° relative to the applied stress direction (0°). All receivers are 5 cm away from the borehole center.

Figure 13 shows the seismograms recorded at five different offsets and at 10 different azimuths for 10 kHz monopole source when the model is subjected to 10 MPa loading stress. The P-waves, which arrive at times between the two dashed red lines in each panel, are dispersive and extend over a longer time window as offset (i.e., source-to-receiver distance) increases. At a given offset, the difference of the P-wave arrival times at different azimuths is too small to be directly measured from the trace data in Figure 13.

From our previous comparison with laboratory measurements, we know that the refracted P-wave vanishes beyond a certain offset, which is approximately 12 cm for the 2.86 cm diameter laboratory scale borehole, at azimuths close to the direction normal to the loading stress, as shown in Figure 9. When we upscale the model to a 20 cm borehole, this critical offset changes to approximately 84 cm, which is less than the minimum offset of the receivers in a sonic logging tool. This suggests that the recorded first arriving P-wave may not be the refracted P-wave in a field borehole measurement. However, Figure 13 shows that P-waves appear at all azimuths for a 10 kHz monopole source. The P-wave amplitude shows significant azimuthal variation that is also observed in Figure 10. We should keep in mind that the borehole size and source frequency in this section are different from those used for comparison with the laboratory measurements. To further investigate the physical nature of the P-wave, we conducted 2D simulations in a manner similar to the 2D simulations used to construct Figure 12. We took 10 2D radial profiles of the model at azimuths from 0° to 90° and simulated the wave propagation at each azimuth separately in 2D cylindrical coordinates. The 2D simulation results shown in Figure 14 indicate that the P-wave amplitude decreases with the increase of azimuth and the P-wave disappears starting from 50° at offsets larger than 1 m, whereas the P-wave is present at all azimuths in Figure 13. The disappearance of P-wave in Figure 14 at azimuths close to 90° is caused by the negative radial stiffness gradient. As shown in Figure 6, the radial gradient of model stiffness changes from positive (radially increasing) to negative (radially decreasing) as angle changes from 0° to 90°. The positive stiffness gradient along the loading stress direction favors guided waves and produces strong P-wave amplitude along the wellbore. In contrast, the negative stiffness gradient near 90° inhibits guided waves and dissipates the P-wave amplitude and makes the refracted P-wave disappear after a certain offset. This is confirmed by the 2D simulation results shown in Figure 14. In Figure 13, the recorded P-waves at azimuths near 90° are the P-wave leaking from the neighboring negative stiffness gradient region at smaller azimuths because the positive stiffness gradient cannot support the existence of refracted waves as shown in Figure 14. Although we show only the 2D simulation results for a 10 kHz monopole source, the 2D simulation

Figure 14. Seismograms obtained from 2D simulations at 10 azimuths for 10 kHz monopole source and 10 MPa loading stress. Source is at z = 0 m. Seismograms at each azimuth are simulated independently using the model on the radial plane along the corresponding azimuth. The refracted P-waves occur between the two dashed red lines in each panel.

Figure 15. (a-d) Variations of the refracted P-wave amplitude versus azimuth for data recorded at z = 2, 3, 4, and 5 m, respectively. The 10 kHz monopole source is located at z = 0 m. Black, red, and blue curves show the results for 5, 10, and 15 MPa loading stresses, respectively. The amplitudes of all data are normalized by the same value.
results for 15 and 20 kHz monopole sources have similar features. This indicates that P-wave is more influenced by the properties of the formation around the borehole along the loading stress direction.

The azimuthally changing stiffness gradient has a big impact on the P-wave amplitude. Figures 15–17 show the variations of the P-wave amplitude versus azimuth for monopole sources of 10, 15, and 20 kHz center frequencies, respectively, at four different offsets. The value of amplitude used in plotting is the normalized mean absolute value of the amplitude of the P-wave in the selected time window, which is shown as the dashed red window in Figure 13. In Figures 15–17, black, red, and blue curves show the variations of the P-wave amplitude for 5, 10, and 15 MPa loading stresses, respectively. For each source frequency, all amplitudes are normalized by the same value. We can see that the P-wave amplitude decreases with increasing offset and increases with increasing loading stress. The P-wave amplitude shows maxima at 0° and 180°, whereas P-wave amplitude has minima at 90° and 270°. For 10 and 15 kHz monopole sources, the amplitude versus azimuth variations shown in Figures 15 and 16 are similar to a cosine function. However, the amplitude shows broad minima and cusped maxima for the 20 kHz source, as shown in Figure 17. This is similar to the P-wave velocity variation shown in Figure 12 except that the roles of maxima and minima reverse. The amplitude variation shown in Figure 17 deviates from those shown in Figures 15 and 16 because the source with higher frequency generates waves of shorter wavelength that probe a region within a narrower azimuth. Winkler (1997) finds that the focusing of acoustic waves propagating through the low-velocity channels in a borehole, which were created either by tensile stress concentrations or by mechanical damage, can cause high-amplitude bright spots in the acoustic data measured in borehole experiments. We show here that this borehole acoustic focusing effect is present even in the absence of the effects of tensile stress and mechanical damage. Compared with the azimuthal variation of velocity, the amplitude is much more sensitive to the change of formation properties around the borehole. It varies by several times between the loading stress direction and the direction normal to the loading stress. This suggests that the P-wave amplitude may be used to study the in situ stress state.

For comparison, Figure 18 shows the azimuthal variations of the Stoneley wave dispersion for three different loading stresses. We treat receivers at each azimuth as an individual array and process the data separately for each azimuth. We show only the Stoneley dispersion greater than 10 kHz because they do not show any azimuthal variation below that frequency. The overall slowness of the Stoneley wave decreases with increasing loading stress. For all three loading stresses, we can see that the Stoneley wave slowness increases from 90° to 0° when the frequency is greater than 12 kHz. The percentage change of the Stoneley wave slowness from 90° to 0° at 20 kHz increases from approximately 4% to...
for 5 MPa loading stress to approximately 5% for 15 MPa loading stress. Compared with the P-wave amplitude, Stoneley wave slowness is much less sensitive to the formation property changes around the borehole and its azimuthal variation might be too small to measure in the field.

CONCLUSIONS

We studied the borehole azimuthal acoustic response caused by borehole stress concentration using a finite-difference method and an iterative method combining rock physics and finite-element modeling. We compared our numerical results with the published laboratory measurements for the Berea sandstone sample. The consistency of the azimuthal variation of the normalized P-wave velocity between the numerical and the experimental data measured during a loading cycle suggests that the constitutive relation between an applied stress field and stiffness of the rock around a borehole can be accounted for correctly in the rock-physics modeling. Due to the preference of the wavefield to propagate through a higher stiffness around the borehole axis, the stiffness matrix (equation A-5) reduces to

\[
\begin{bmatrix}
  s_{xx} & s_{xy} & s_{xz} & s_{yy} & s_{yz} & s_{zz}
\end{bmatrix}
\]

where \( s_{ij} \) are the components of the stiffness tensor in a strongly azimuthal variation in P-wave amplitude, which has maxima and minima in directions parallel and normal to the loading stress, respectively. The high sensitivity of the P-wave amplitude to the azimuthal variation of formation properties may suggest that the P-wave amplitude versus azimuth variation, which is easy to measure, can be used to study the in situ stress state around a borehole. We also find that the Stoneley slowness has stress-dependent variation greater than approximately 12 kHz. However, the variations with stress may be too small to measure in field data.

The radial changes of stiffness around the borehole shown in our model are also consistent with the established idea that there is a crossover of the velocities as one goes away from the borehole. We will examine the quantitative nature of that in a separate paper.

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APPENDIX A

VALIDATION OF THE FINITE-ELEMENT PROGRAM

We use COMSOL to do the finite-element calculation in our modeling. The accuracy of the finite-element program is validated through comparison with the analytical solution (AS) of the stress field around a borehole when it is subjected to the compression of a uniaxial stress in the direction normal to the borehole axis (i.e., plane strain situation).

The elastic deformation of a medium under static loading is governed by the equation of equilibrium:

\[
\begin{aligned}
&\frac{\partial s_{xx}}{\partial x} + \frac{\partial s_{yx}}{\partial y} + \frac{\partial s_{zx}}{\partial z} = 0, \\
&\frac{\partial s_{xy}}{\partial x} + \frac{\partial s_{yy}}{\partial y} + \frac{\partial s_{zy}}{\partial z} = 0, \\
&\frac{\partial s_{xz}}{\partial x} + \frac{\partial s_{yz}}{\partial y} + \frac{\partial s_{zz}}{\partial z} = 0,
\end{aligned}
\]

where the body force is assumed to be zero, and the constitutive relation

\[
\sigma = C \varepsilon,
\]

with

\[
\sigma = \begin{bmatrix} s_{xx} & s_{xy} & s_{xz} & s_{yy} & s_{yz} & s_{zz} \end{bmatrix}^T,
\]

\[
\varepsilon = \begin{bmatrix} e_{xx} & e_{xy} & e_{xz} & e_{yy} & e_{yz} & e_{zz} \end{bmatrix}^T,
\]

\[
C = \begin{bmatrix}
  c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
  c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
  c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\
  c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\
  c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\
  c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66}
\end{bmatrix},
\]

where \( s_{ij} \) and \( e_{ij} \) are the components of stress and strain and \( C \) is the stiffness matrix. The boundary conditions are stresses at model boundaries equal to the external loading stresses. Borehole boundary is assumed to be stress free.

When the formation elasticity exhibits orthorhombic symmetry and one of the elastic symmetry planes is perpendicular to the borehole axis, the stiffness matrix (equation A-5) reduces to

\[
\begin{bmatrix}
  c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
  c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
  c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\
  c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\
  c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\
  c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66}
\end{bmatrix}
\]
For this special case and under plane strain assumption, the stresses around a circular borehole that is subjected to triaxial compression can be analytically given as (Amadei, 1983)

\[
s_{xx} = T_x + \text{Re}[T_x(i\gamma_1\mu_1^2 - i\gamma_2\mu_2^2) - T_y\mu_1\mu_2(\gamma_1\mu_1 - \gamma_2\mu_2)],
\]
(A-7)

\[
s_{yy} = T_y + \text{Re}[T_x(i\gamma_1 - i\gamma_2) - T_y(\gamma_1\mu_2 - \gamma_2\mu_1)],
\]
(A-8)

\[
s_{xy} = \text{Re}[T_x(-i\gamma_1\mu_1 + i\gamma_2\mu_2) + T_y\mu_1\mu_2(\gamma_1 - \gamma_2)],
\]
(A-9)

\[
s_{zz} = T_z - (a_{31}s_{xx} + a_{32}s_{yy})/a_{33},
\]
(A-10)

with

\[
y_k = \frac{i\mu_k - 1}{(\mu_2 - \mu_1)(\frac{x + \mu_1}{a})^2 - \mu_k^2 - 1 + \frac{x + \mu_1}{a}\sqrt{\left(\frac{x + \mu_1}{a}\right)^2 - \mu_k^2 - 1}}
\]
(k = 1, 2),
(A-11)

\[
\mu_1 = i\sqrt{\frac{2\beta_1 + \beta_{66} + \sqrt{(2\beta_{12} + \beta_{66})^2 - 4\beta_{11}\beta_{22}}}{2\beta_{11}}},
\]
(A-12)

\[
\mu_2 = i\sqrt{\frac{2\beta_1 + \beta_{66} - \sqrt{(2\beta_{12} + \beta_{66})^2 - 4\beta_{11}\beta_{22}}}{2\beta_{11}}},
\]
(A-13)

\[
\beta_{ij} = a_{ij} - \frac{a_{13}a_{3j}}{a_{33}},
\]
(A-14)

where \(x\) and \(y\) are the coordinates on the borehole cross section, \(a\) is borehole radius, \(a_{ij}\) is the \(ij\)-component of the compliance matrix, which is the inverse of the stiffness matrix in equation A-6, \(i = \sqrt{-1}\) is the imaginary unit, \(\text{Re}[\ast]\) is the notation for the real part of the complex expression in the brackets, \(T_x, T_y,\) and \(T_z\) are the loading stresses in the \(x, y,\) and \(z\)-directions, respectively, and borehole axis is in the \(z\)-direction.

An orthorhombic anisotropic model is used for validating the finite-element program. The nine nonzero elastic constants of the formation are taken to be \(c_{11} = 17.4,\ c_{22} = 15.4,\ c_{33} = 11.7,\ c_{12} = 7.1,\ c_{13} = 6.3,\ c_{23} = 6.5,\ c_{44} = 3.1,\ c_{55} = 3.5,\) and \(c_{66} = 3.8\) GPa. Figure A-1 shows the comparison of the numerical result (first column), which is computed using the finite-element program, with the AS (second column), which is given by equations A-7–A-10. The \(s_{xx}, s_{yy}, s_{zz},\) and \(s_{xy}\) are the four nonzero stress components in the \(x-y\) plane. The uniaxial stress applied in the \(x\)-direction is 10 MPa. Borehole radius is 0.1 m. The stress field computed from the finite-element program, as shown in the first column of Figure A-1, is almost identical to the AS shown in the second column. Their difference (third column) is approximately two orders of magnitude smaller. This proves the validity and accuracy of the finite-element program.

**APPENDIX B**

**VALIDATION OF THE FINITE-DIFFERENCE PROGRAMS**

To validate the 3D and 2D finite-difference simulation programs, we compare the results for wave propagation along a fluid-filled borehole calculated using the two finite-difference methods (Cheng et al., 1995; Wang and Tang, 2003) with the solutions obtained from the discrete wavenumber method (Cheng and Toksöz, 1981).

The 3D finite-difference program solves the elastic wave equation in Cartesian coordinates:

- \(s_{xx}\) (FEM)
- \(s_{xx}\) (AS)
- Difference (x10)

- \(s_{yy}\) (FEM)
- \(s_{yy}\) (AS)
- Difference (x10)

- \(s_{zz}\) (FEM)
- \(s_{zz}\) (AS)
- Difference (x10)

- \(s_{xy}\) (FEM)
- \(s_{xy}\) (AS)
- Difference (x10)

Figure A-1. Comparison of the stress field around a borehole obtained from the FEM with the AS for 10 MPa uniaxial stress loading in the \(x\)-direction. The third column shows the difference (multiplied by 10) between the FEM result (1st column) and the AS (2nd column). Positive and negative signs denote compressive and tensile stresses, respectively.
Figure B-1. (a and b) Comparisons of the results obtained from finite-difference methods (dashed blue and dashed red) and discrete-wavenumber method (solid black) for 10 kHz monopole and 3 kHz dipole sources, respectively. Dashed red and blue traces are, respectively, obtained from 3D finite-difference modeling in Cartesian coordinates and 2D finite-difference modeling in cylindrical coordinates.

\[
\frac{\partial^2 u_x}{\partial t^2} = \frac{\partial}{\partial x} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right),
\]

\[
\frac{\partial^2 u_y}{\partial t^2} = \frac{\partial}{\partial y} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right),
\]

\[
\frac{\partial^2 u_z}{\partial t^2} = \frac{\partial}{\partial z} \left( \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} \right),
\]

where \(u_x, u_y, \) and \(u_z\) are, respectively, displacements in the \(x, y,\) and \(z\)-directions; \(t\) is time; and \(\rho\) is density. The constitutive relation is given by equation A-2.

The 2D finite-difference program solves the elastic wave equation in cylindrical coordinates (\(r-z\) coordinates) by assuming that the model properties are azimuthally invariant.

Figure B-1 shows the comparison of the results obtained by the finite-difference methods (dashed blue and dashed red) and the discrete-wavenumber method (solid black) for monopole (Figure B-1a) and dipole (Figure B-1b) sources. The center frequencies are 10 and 3 kHz for monopole and dipole sources, respectively. The formation is a homogeneous vertically transversely isotropic medium. The vertical P- and S-wave velocities are 2830 and 1750 m/s, respectively, anisotropy parameters are \(\varepsilon = \gamma = 0.1\) and \(\delta = 0\), and the density is 2198 kg/m\(^3\). Borehole fluid is water and the borehole radius is 10 cm. Receiver array records the pressure along the borehole axis direction at positions 5 cm away from the borehole center. For the dipole simulation, receivers are in the source inline direction. The finite-difference grid size is 0.2 cm (i.e., 1/100 borehole diameter). From Figure B-1, we can see that the finite-difference results (dashed traces) agree well with the discrete wavenumber solutions (solid traces). This demonstrates the accuracy of our finite-difference programs.

REFERENCES

This article has been cited by:

1. Xinding Fang, Arthur Cheng, Michael C. Fehler. 2015. Investigation of borehole cross-dipole flexural dispersion crossover through numerical modeling. GEOPHYSICS 80:1, D75-D88. [Abstract] [Full Text] [PDF] [PDF w/Links]