THE STOCHASTIC BEHAVIOR OF CONSUMPTION AND SAVINGS

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TABLE OF CONTENTS

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ABSTRACT

This thesis studies the behavior of consumption and savings in the presence of labor income, horizon, and preferences uncertainty. Special emphasis is put in understanding the potential role of these elements in explaining the rejections of the Life Cycle/Permanent Income Hypothesis (henceforth LCH/PIH) found in the postwar US data.

Chapter I obtains closed form solutions for the problem solved by a representative consumer whose labor income is uncertain and uninsurable (at least partially), and possesses a utility function with convex marginal utility. For tractability, the latter is specialized to a negative exponential -or constant absolute risk aversion- utility function. The income process, on the other hand, has a general ARIMA representation. One of the contribution of this chapter is to provide a new and very simple solution technique for this class of dynamic-rational expectation models.

When marginal utility is convex, consumers facing uninsurable labor income uncertainty tend to consume less early in life in order to build up savings to face the possibility of bad realizations of future income. The importance of this precautionary savings motive depends on the higher moments of the innovation of the income process, on the persistence of these shocks, on the consumer's horizon, and on the degree of convexity of marginal utility. It is shown that precautionary savings can potentially account for most of the puzzles found in the empirical literature on consumption of non-durables and services (e.g. excess smoothness of consumption to income innovations, excess sensitivity of consumption to anticipated changes in income, excessively high rate of consumption growth, and extremely low estimates of the degree of intertemporal substitution).

Chapter II extends the model derived in chapter I, in order to study the implications of precautionary savings for aggregate wealth accumulation. Particular emphasis is put on the finiteness and the

randomness of individual's horizons. After solving the problem of the individual consumer, aggregation is studied and bequests are made endogenous. The main conclusion is that for reasonable parameter values, precautionary savings can generate sizable amounts of wealth accumulation. A second conclusion is that for almost any reasonable set of parameters, reducing labor income uncertainty -and therefore wealth accumulation- is Pareto improving, even when generations are completely selfish.

Stochastic preferences are introduced in chapter III. Changes in preferences, or taste shocks, do not affect the intertemporal budget constraint. As such, they cannot have wealth effect, they can only have substitution effect. This observation is used to show that taste shocks have not had an important role in explaining the fluctuations of the postwar US aggregate consumption of non-durables. This result is obtained even in a model in which the interest rate is assumed to be constant. Given that the substitution effect of interest rate changes has the same properties of a taste shock process, this shows that there is very little intertemporal substitution in aggregate consumption.

Chapter IV extends the model of chapter III to study the expenditure on durable goods. The chapter shows that the Life Cycle/Permanent Income hypothesis is not a useful way to think about the short run behavior of aggregate expenditure on durable goods. Complementing the standard representative consumer model with a very general taste shock process, it was possible to check whether the most commonly blamed auxiliary assumptions could be held responsible for the resounding rejections of the LCH/PIH model found in previous studies. It was shown that the rejections are robust to the relaxation of these auxiliary assumptions.

Chapter V does not attempt an explanation for the short run behavior of durables expenditures. But it studies whether lower frequency data (annual) show some signal of the implications of the LCH/PIH being satisfied. It is shown that in fact, a slow adjustment model makes the data consistent with the LCH/PIH model. The durables puzzle is changed from plain irrationality to some degree of slowness, certainly a less dangerous result for economic theory.

Roughly speaking, the results of this thesis suggest that one can think of aggregate consumption as decisions taken by a representative agent who (i) is concerned with the higher moments of the labor income process, (ii) saves to reduce the consumption-risk of future bad realizations of this income, (iii) has stable preferences, (iv) responds primarily to wealth shocks but not to incentives to reallocate consumption across time (small substitution effects), and (v) adjusts slowly, specially so for decisions that concern durables purchases.

Thesis Supervisors: Olivier J. Blanchard
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INTRODUCTION

This thesis studies the behavior of consumption and savings in the presence of different sources of uncertainty, like labor income, horizon, and preferences. The consumption function literature was one of the areas of empirical work most severely affected by the Lucas(1976) critique. He noticed that in the framework of rational-dynamic models, if the process generating the forcing variables (e.g. income and wealth in the case of consumption) is not stable, neither will be the function itself. Hall(1978) responded by noticing that the life cycle/permanent income hypothesis (henceforth LCH/PIH) could be tested without the need of obtaining an explicit closed form solution for consumption. In particular, Hall pointed out that a test was possible by just studying the Euler equation of the optimization problem solved by a consumer; thereby circumventing the need to make explicit assumptions about the stochastic processes of the forcing variables. This insight spurred a long series of papers, most of them showing that in its purest form, the LCH/PIH tends to be rejected.

It did not take long, however, to realize that if any structural interpretation was to be given to these rejections, the Euler equation would not be sufficient. Flavin(1981) went back to the consumption function, although now derived from first principles[1]. She solved the problem of a representative consumer with quadratic utility function. The only source of uncertainty in her model was labor income, and the latter was represented by a stable ARMA process around a deterministic trend. She showed that adding the budget constraint (to the Euler equation) did not

[1] Sargent(1978) also derived a rational expectations consumption function.

alter Hall's test, but permitted a structural interpretation of the findings that income Granger-caused consumption. She coined the expression "excess sensitivity", referring to the fact that income seems to have a role in consumption revisions, that goes beyond its role as signaling of changes in permanent income.

Flavin's paper is a cornerstone in the new consumption literature, however the expression "excess sensitivity" has been somewhat misleading. Deaton(1986), and specially Campbell and Deaton(1987), show that if income has a unit root (as it seems to do), consumption responds too <u>little</u> - contrary to the interpretation given by researchers to the "excess sensitivity" expression- to income innovations. They called this discovery the "excess smoothness" of consumption to income innovations. They also showed that appropriately interpreted, Flavin's result show excess sensitivity of consumption to anticipated changes in income. Summarizing, consumption seems to react too little to <u>unanticipated</u> changes in income, but too much to <u>anticipated</u> changes in income (therefore the Granger causality result).

Although now a structural interpretation of the rejections was possible, it was not clear what to do next. The previous work used certainty equivalence assumptions, excluding higher moments of the distribution of the forcing variables from playing a role in the explanations. However, dynamic programming techniques do not seem to provide simple solutions for the case of more realistic utility functions than the quadratic, and general income processes. Perhaps one of the most important contributions of this thesis is to provide a simple solution technique that permits -under still very restrictive assumptions- this step. Using an exponential instead of a quadratic utility function, Flavin's and Campbell and Deaton's models are enriched by the presence of higher moments of income in the consumption function. The role of these higher moments are shown not only to be potential explanations for the excess smoothness/excess sensitivity puzzle, but also for the positive growth of consumption even in periods in which the real interest rate has been negative (Deaton 1986), and for the extremely low estimates (sometimes negative) of the degree of intertemporal substitution (Hall 1988).

Chapter I introduces the solution technique and shows the effect of human wealth uncertainty on the consumption function and path, when precautionary savings motives are present. This is a multiperiod extension of a result first derived by Leland(1968): when the utility function is such that marginal utility is convex, consumers facing uninsurable labor income uncertainty tend to consume less early in life, in order to build up savings to face the possibility of bad realizations of future income.

The importance of the precautionary savings motive depends on the higher moments of the innovation of the income process, on the persistence of these shocks, on the consumer's horizon, and on the degree of convexity of marginal utility. As mentioned before, it is shown that precautionary savings are potentially able to account for most of the puzzles found in the empirical consumption literature that use non-durables and services data[2]. Additionally, this chapter shows that contrary to what was initially thought, Hall's procedure is not immune to the Lucas critique, since the slope of the consumption path depends on the stochastic process of income.

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[2] In this thesis I do not examine the rejections of the Consumption CAPM model.

Competing -or perhaps complementary- explanations for some of these puzzles are liquidity constraints (e.g. Hayashi 1982 and 1988, Caballero 1986 and 1987, Campbell and Mankiw 1987), and general equilibrium considerations (e.g. Christiano 1987), however neither of them seems to give as complete an explanation as precautionary savings. Liquidity constraints can deal with excess smoothness and excess sensitivity, but not with the "abnormal" growth of consumption nor with the sometimes negative intertemporal substitution parameter estimates. General equilibrium considerations of the type suggested by Christiano, on the other hand, can explain the excess smoothness result, but not the excess sensitivity result, since the latter extends to procedures with flexible interest rates and proper instrumental variables.

The concern for savings has been overshadowed -in the macroeconomic literature- by the vast literature on consumption. Not surprisingly, however, savings are the other side of the consumption decisions, so the problems found in the latter are directly reflected in the former. This was pointed out by Campbell(1988), who showed that savings move too little to be consistent with the LCH/PIH, confirming the excess smoothness result mentioned before. Given that this is a symptom of the same problem found in consumption, the cures are also the same. Therefore, precautionary savings can also explain Campbell's findings.

A completely different literature on savings, developed more in the public finance rather than in the macroeconomics literature, refers to the source of savings accumulation. The second chapter of this thesis extends the model derived in chapter I, in order to study the implications of precautionary savings for aggregate wealth accumulation. Special emphasis is put on the finiteness and the randomness of individuals' horizons. Perhaps the most important result of this chapter is that for sensible parameter values, precautionary savings can generate sizable amounts of wealth accumulation. Therefore models in which the only source of uncertainty is the date of death -typically used in the public finance literature- are seriously misleading, since they omit one of the most important motives for wealth accumulation, the fear of facing a stream of bad income (health) realizations in the future.

In addition, this essay shows that: (i) labor income uncertainty reduces the response of savings to changes in the interest rate, (ii) contrary to the findings in social security models with no endogenous bequests and no labor income uncertainty, (ii.1) increases in the probability of dying early can increase the aggregate stock of wealth, even when the maximum length of life is kept unchanged, and (ii.2) when bequests are not received early in life, aggregate wealth may be reduced after an increase in the probability of dying early, even when the expected lifetime is kept constant. And (iii) using a very simple general equilibrium model, it is shown that in most of the cases income stabilization policies benefit not only current but also future generations, even when individuals are completely selfish.

Chapters I and II are mainly theoretical and are specially devoted to understanding the effects of precautionary savings on consumption and savings behavior. The second part of this thesis, chapters III to V, is primarily empirical. Chapter III uses the solution technique developed in chapter I, and adds stochastic preferences. By obtaining a closed form solution it is possible to disentangle and assess the relative importance of wealth and taste innovations in consumption fluctuations. The main insight is simple; taste shocks do not enter the budget constraint, therefore they cannot affect the present value of expenditure, they can only affect consumption through substitution effects. This has very distinctive implications for the time series of consumption, allowing for the identification of the taste shocks process. Contrary to the results in Hall(1984) and Fair(1986), this procedure shows no evidence of an important role for taste shocks in the postwar US fluctuations in consumption.

This result has implications far deeper than just the unimportance of taste shocks for consumption fluctuations. In fact, any substitution effect that has not been taken into account in the model (e.g. the substitution effect of interest rates changes when the model assumes a constant interest rate), can be represented in the form of a taste shock (unexplained change in the slope of the consumption path). Considering that the no-taste shocks result holds even in the model with constant expected interest rate, it seems that the degree of intertemporal substitution implicit in decisions with respect to non-durables consumption, is extremely low. This confirms and at the same time generalizes the result found by Hall(1988).

The remaining two chapters concentrate on durables expenditures, an area of the consumption literature that has generated the strongest rejections for the representative agent version of the LCH/PIH. Surprisingly, research in this area is very thin.

The seminal paper on the new durables literature is due to Mankiw(1982). He summarized Hall(1978)'s model as implying an AR(1) process for non-durables and services, to then extend this framework to the case of durable goods. Under the same assumptions used by Hall -time separable utility function, quadratic instantaneous utility function and constant interest rates- he showed that the same argument used for non-durables applies to the services provided by the stock of durables. In addition, at any point in time the stock of durables is formed by what is left from the previous period and the expenditure on durable goods in the current period. Combining this accumulation equation with the Euler equation of the optimization problem solved by the consumer, he showed that the AR(1) process for the stock implies an ARMA(1,1) process for the expenditure on durable goods. The MA coefficient is negative (with absolute value equal to one minus the rate at which the stock of durable depreciates) reflecting the fact that once a durable good is bought, it lasts for more than one period, thus reducing the expenditure required in the future to keep the stock of durables unchanged at the new level. He found that the U.S. post-war data reject this hypothesis in favor of a simple AR(1) process; except for a greater volatility, the time series behavior of durable goods looked very similar to that of non-durable goods.

Chapter IV of this thesis uses the insights of chapter III in order to study whether the stringent auxiliary assumptions used in Mankiw's model could be blamed for the rejection. Under the assumption of the validity of a representative agent model, almost any shock can be decomposed into a wealth and a substitution effect, and the latter can be seen as a taste shock. Most of economists understanding of the LCH/PIH is related to wealth effects. Moreover, Hall's testing procedure is designed precisely to capture the response of consumption to wealth innovations. This is the essence of the concept of permanent income. Substitution effects, on the other hand, depend heavily on the auxiliary assumptions. Different auxiliary assumptions imply different substitution effects. There are at least two alternatives to test whether some auxiliary assumptions are responsible for the rejections; the first one (structural) -the most obvious but at the same time most difficult- is to relax each of the auxiliary assumptions and find out whether Mankiw's puzzle disappears. A second alternative -suggested in this thesis- is to keep the simple Hall-Mankiw model, but add a very general taste shock process to it; remembering that almost any substitution effect can be represented by taste shocks, it is possible to see whether any particular auxiliary assumption is responsible for the rejections. This is done by comparing the taste shock process estimated, with the substitution effects (approximately) implied by a model in which the particular auxiliary assumption under study is relaxed.

This procedure has two mayor advantages over the former (structural): first, it permits to analyze a broad set of alternatives in one step, and second, it does not require to solve the far more complex model in which the auxiliary assumptions are relaxed (in fact these auxiliary assumptions are often made in order to be able to find closed form solutions). The disadvantages are directly related to the advantages: first, it does not provide uniformly most powerful tests against any single alternative, worse, in many cases it is not even possible to define a formal test, and second, given the lack of structure, there are fewer theoretical insights to be learned.

Applying this procedure to the quarterly US data on expenditure on durable goods show very negative results: the homogeneous representative agent version of the LCH/PIH, so useful in other areas of macroeconomics, does not seem to render the same insights for studying the short run behavior of aggregate durable goods. Worse yet, it does not even seem to be a good approximation.

Summarizing, this chapter's contributions are first, to provide a testing procedure that permits to analyze a broad set of alternatives. And second, to show that the set of alternatives that have a chance to explain the durables puzzle is very small. Slow adjustment seems to be the only type of explanation that has a chance to match the correlation (negative) between taste and wealth innovations found. However, high frequency data (i.e. quarterly) does not allow to make more precise statements about the speed of adjustment. The next chapter explores this alternative using annual data.

Chapter V is a sequel of chapter IV, and it changes the tone of the durables puzzle. This essay shows that contrary to what is found when quarterly data are used, annual data show that the behavior of durables is significantly different from the behavior of non-durables. Moreover, the differences point in the direction suggested by the LCH/PIH. The change in data frequency is not sufficient, however adding a very simple slow adjustment model allows for an extremely good characterization of the data. The adjustment seems to take approximately three years, with expenditure on durables adjusting more slowly than expenditure on non-durables and services. The puzzle is changed from plain irrationality to some degree of slowness, certainly a less dangerous result for economic theory. At the same time, this paper suggests that looking at low frequency data may be a useful practice when studying the speed of adjustment of a process.

Nonetheless, it is still the case that the conclusions of chapter IV hold. In the sense that, (i) the LCH/PIH model does not seem to be a very useful way to think about within the year responses of consumption to wealth innovations. And (ii) the apparent slow adjustment is not the result of mispecification (different from slow adjustment) in the short run model.

Roughly speaking, the results of this thesis suggest that one can think of aggregate consumption as decisions taken by a representative agent who (i) is concerned with the higher moments of the labor income process, (ii) saves to reduce the consumption-risk of future bad realizations of this income, (iii) has stable preferences, (iv) responds primarily to wealth but not to substitution effects and, (v) adjusts slowly, specially so for decisions that concern durables purchases.

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CHAPTER I

CONSUMPTION AND PRECAUTIONARY SAVINGS: EMPIRICAL IMPLICATIONS

INTRODUCTION

Most of the empirical research on the Life Cycle/Permanent Income hypothesis (henceforth LCH/PIH) which deals with the relation between income and consumption disturbances uses certainty equivalence assumptions. One possible explanation for the repeated use of this specification, is the degree of difficulty involved in obtaining closed-form solutions in the multiperiod optimization problem of a consumer who faces a random sequence of -uninsurable- labor income and whose utility function is not quadratic. The exceptions are few and in general they involve complex stochastic dynamic programming procedures (e.g. Merton 1971, Sibley 1975, Schechtman and Escudero 1977, Levhari, Mirman and Zilcha 1980).

Unfortunately, by using a quadratic utility function important aspects of the problem are ignored. Theoretical studies have shown that whenever the utility function is separable and has a positive third derivative ($U'''>0$) -a property of most of the utility functions used in theoretical macroeconomics- an increase in labor income uncertainty when insurance markets are not complete will reduce current consumption and alter the slope of the consumption path (Leland 1968, Sandmo 1970, Dreze and Modigliani 1972, Miller 1976). These results are confirmed, for the case of a CRRA (constant relative risk aversion) utility function and i.i.d. (independently identically distributed) labor income by the numerical simulations performed in Zeldes(1984).

This chapter studies the implications of precautionary savings on the time series behavior of consumption. Using a very solution technique it is possible to find a closed form solution for the consumption function for a special utility function with positive third derivative, the exponential. The main feature of this solution technique is that it is extremely flexible to the specification of the stochastic process followed by income. Hence, it nicely accommodates the typical needs of the macroeconometrician.

It is shown that besides its theoretical implications, precautionary savings can explain four results found in tests based on intertemporal optimization: (i) excess sensitivity of consumption to anticipated changes in income (e.g. Flavin 1981, Hayashi 1982), (ii) excess smoothness of consumption to unanticipated changes in income (e.g. Deaton 1986, Campbell and Deaton 1987), (iii) extremely low -sometimes negative- intertemporal substitution effects (e.g. Hall 1985, Runkle 1986), and (iv) persistent growth of consumption, even when the real interest rate has been negative (e.g. Deaton 1986).

This chapter is divided in five sections. Section I sets up the general problem and discusses some of the assumptions which are maintained in the chapter.

Section II first introduces a simple example with income following a random walk and with the interest and discount rates equal to zero. With this example the solution technique is explained and some preliminary results on the relation between risk aversion, the degree of uncertainty and consumption behavior are established. The results are then generalized by allowing for a general ARMA process (with possibly a unit root) for labor income and non-zero interest and discount rates. Particular emphasis is put in studying the relation between the consumption stochastic process, the consumption function, the degree of persistence of income shocks and the length of the consumer's horizon.

Section III explains in more detail the behavior of precautionary savings and their implications for the slope of the consumption path. It is shown, for example, that if the horizon is finite and income does not follow a random walk, then the slope of the consumption path changes through time, even when the interest and discount rates are constant.

Section IV presents three applications. The first shows that the effect of changes in the interest rate on precautionary savings can distort the measurement of the degree of intertemporal substitution. The second application shows that once the certainty equivalence assumption is dropped, tests of the LCH/PIH à la Hall(1978) are not longer immune to the Lucas(1976) critique. Finally, the third example shows that provided labor income can be described as a log-linear process the precautionary savings motive can reconcile the excess sensitivity and excess smoothness findings with the LCH/PIH. Section V concludes.

I. THE PROBLEM SET-UP AND MAIN ASSUMPTIONS

The problem to be solved is that of a representative consumer who has an horizon that extends to a known time T (that may be far enough to be approximated by infinity). This consumer takes decisions in discrete time and leaves no bequests. His utility function is time separable, and he works a fixed number of hours that remain constant throughout his life. For his work he receives "labor income". The latter is the only source of uncertainty in the model. He is also allowed to buy and sell a riskless bond subject to some solvency constraints, but there is no insurance market for labor income.

Formally the problem can be stated as:

$$\max_{\{c_{t+i}\}} E_t\left[\sum_{i=0}^{T-t} D^i U(c_{t+i})\right]$$

$$\text{s.t} \quad c_{t+i} = y_{t+i} + R_{t+i}\, s_{t+i-1} - s_{t+i} \qquad 0 \le i < T-t$$

$$c_T = y_T + R_T\, s_{T-1} \qquad \text{(P1)}$$

$$s_{t-1} \text{ given}$$

$$0 \le c_{t+i} \le y_{t+i} + R_{t+i}\, s_{t+i-1} + \sum_{j=1}^{T-t-i} y_{min,t+i+j} \prod_{k=1}^{j} R_{t+i+k}^{-1}$$

$D \equiv (1+\delta)^{-1}$, $R \equiv (1+r)$

with E_t : conditional (on information available at t) expectations operator.

δ : discount rate.

U : instantaneous utility function.

c : consumption.

y : labor income.

s : non-human wealth.

r : riskless return on the bond.

y_{min} : lower bound of y's support (in general it changes through time).

The last condition on program (P1) has been called the "Solvency Constraint"[1], and it is equivalent to impose the condition $Prob\{c_0 \geq 0, \ldots\ldots, c_T \geq 0\}=1$[2]. The first inequality in the solvency constraint is self explanatory, however it would not be satisfied almost surely, unless the second inequality is added. The latter puts an upper bound to consumption in each period. This upper bound guarantees that in each period total wealth (financial plus human) is positive, therefore allowing the first inequality to be satisfied in every period.

Throughout most of this chapter it will be assumed that (a) the solvency constraint is not binding, (b) the instantaneous utility function is exponential and (c) there is no retirement period. These assumptions allow to isolate the precautionary savings motive as the difference between the results in this paper and the results in standard certainty equivalence empirical models[3].

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[1] Sibley(1975).

[2] See appendix A in Sibley op.cit.

[3] Dreze and Modigliani(1972) showed that the effect of an increase in uncertainty on current consumption can be decomposed into an income and a substitution effect. The income effect is the change in consumption due to the new expected utility level resulting from a change in the degree of uncertainty. This effect is always negative in the presence of risk aversion. The substitution effect, on the other hand, is the change in consumption due to the change in the desired optimal wealth at the time of receiving the uncertain income. When the utility function is exponential, absolute risk aversion does not depend on the level of wealth hence the substitution effect is zero. This has important implications for the sensitivity of current consumption to income shocks. On the other hand, if the utility function exhibits decreasing absolute risk aversion (e.g. CRRA) then the substitution effect is negative since people care less about future uncertainty if they have more wealth at that time. Whenever uncertainty rises, it is optimal to shift to the future more resources than those indicated by the income effect. The opposite happens when the utility function exhibits increasing absolute risk aversion (e.g. quadratic utility).
The substitution effect of the CRRA has been used to explain excess

II. THE RELATION BETWEEN THE CONSUMPTION FUNCTION AND THE PROCESS FOR LABOR INCOME

This section presents two propositions which state the basic results of the paper. The discussion of these propositions stresses the characteristics of the innovation residual of the stochastic process of consumption. A discussion in more depth of the precautionary savings term is left for section III.

Although stochastic dynamic programming is the most used tool to solve the kind of problems shown in section I, here a different solution technique is developed which, under the restrictive assumptions made, facilitates the understanding of the propositions in this chapter. This is especially true in those cases where there is no closed form solution for the consumption function, since the procedure still generates some useful insights[4].

The general principle of the solution technique is very simple; (i) given the set-up of the problem "guess" the stochastic process for consumption[5] in such a way that the Euler equation is satisfied, and (ii) use the (ex-post) intertemporal budget constraint to infer, first the characteristics of the stochastic component of the consumption process and then the consumption function itself.[6]

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sensitivity of consumption to income news (Zeldes 1984). However, if the income process has a unit root this is counterfactual. See Campbell and Deaton(1987) and section IV.3 in this essay.

[4]See section IV.

[5]This is the main difference with (informal) dynamic programming techniques, where the guess is on the value function or on the consumption function itself.

[6]This procedure is closely related, in spirit, to the Martingale approach (Cox and Huang 1985). It could also be considered as an stochastic version of the procedures used under certainty by Modigliani and Brumberg(1954).

PROPOSITION 1:

If (a) the instantaneous utility function is exponential: $-(1/\theta)\exp(-\theta c_t)$ with θ: coefficient of risk aversion, (b) $r_t=\delta=0$, and (c) income follows a random walk: $y_{t+1} = y_t + e_{t+1}$, e_{t+1} i.i.d$(0,\sigma^2)$ $\varepsilon[e_{min},e_{max}]$, then,

(i) consumption follows a random walk with a positive drift,

(ii) the drift is increasing in the degree of risk aversion, and of riskiness in the Rothschild-Stiglitz(1970) sense,

(iii) the disturbance in the stochastic process of consumption is equal to the disturbance in the stochastic process of income, and

(iv) the consumption function can be decomposed, additively, in a term analogous to consumption under certainty equivalence and a precautionary savings term.

Proof:

Start by guessing the following stochastic process for consumption:

$$c_{t+1} = \Gamma + \phi c_t + v_{t+1} \qquad (1)$$

with v serially independent and $E_t(v_{t+1})=0$. The other moments of v as well as the parameters Γ and ϕ to be determined later.

The standard Euler equation arising from program (P1) takes the form[7]:

$$\exp(-\theta c_t) = E_t[\exp(-\theta c_{t+1})] \qquad (2)$$

Plugging (1) in (2) and solving for c_t, we get

$$\exp(\theta(\phi-1)c_t) = E_t[\exp(-\theta(\Gamma+v_{t+1}))] \qquad (3)$$

[7]Notice that at this stage the density of v is not known, however, under some regularity conditions, it is still possible to use the linear properties of the expectations operator.

It is clear that, unless the higher moments of v_{t+1} have very special characteristics (later it is shown that they do not), ϕ must be equal to 1. Otherwise, consumption would be determined by (3) regardless of the budget constraint! Hence, if (1) is a potential solution (i.e. able to satisfy all the first order conditions) it has to take the following form:

$$c_{t+1} = \Gamma + c_t + v_{t+1} \tag{1'}$$

and (3) can be written as

$$1 = E_t[\exp(-\theta(\Gamma+v_{t+1}))] \tag{3'}$$

therefore,

$$\Gamma = (1/\theta)\log E_t[\exp(-\theta v_{t+1})] \tag{3''}$$

Jensen's inequality tells us that

$$\log E_t[\exp(-\theta(\Gamma+v_{t+1}))] \geq E_t[-\theta(\Gamma+v_{t+1})] \tag{4}$$

By taking logs on both sides of (3') and using condition (4) obtains,

$$0 \geq -\theta\Gamma \quad (>0 \text{ for } \theta>0) \tag{5}$$

or

$$\Gamma > 0 \quad (\text{for } \theta>0) \tag{5'}$$

Further, the exponential utility function exhibits no satiation hence the realized intertemporal budget constraint at time t is[8]:

$$\sum_{i=0}^{T-t} (c_{t+i} - y_{t+i}) = s_{t-1} \tag{6}$$

Substituting, using the income and consumption processes (c) and (1') respectively, we obtain,

$$(c_t - y_t) + \sum_{i=1}^{T-t} \{c_t + \sum_{k=1}^{i} (\Gamma+v_{t+k}) - y_t - \sum_{k=1}^{i} e_{t+k}\} = s_{t-1} \tag{6'}$$

Solving (6') for c_t,

$$c_t = y_t + s_{t-1}/(T-t+1) - \Gamma(T-t)/2 - \{\sum_{i=1}^{T-t} \sum_{k=1}^{i} (v_{t+k}-e_{t+k})\}/(T-t+1) \tag{7}$$

[8]Notice that total repayment of debts is implicitly assumed, otherwise (6) does not need to be satisfied. This is the case, for example, in Yaari(1974), where he showed, using a weak law of large numbers, that under the assumption of an i.i.d. income process, when T goes to ∞, c goes to E(y). However, that $\Sigma e/T$ goes to 0 does not imply that Σe goes to zero.

Since at time t c_t is known, it should be apparent that the last expression in (7) must vanish (or at least become non-random). This restriction serves to identify the $\{v_{t+k}\}$s.

At time T-1 (7) becomes

$$c_{T-1} = y_{T-1} + s_{T-2}/2 - \Gamma/2 - (v_T - e_T)/2 \qquad (7')$$

but $(v_T - e_T)$ cannot be random, furthermore, $E_{T-1}[v_T]=E_{T-1}[e_T]=0$ by construction, hence $v_T=e_T$. Working backwards shows that $v_{t+k}=e_{t+k}$ for all k, therefore (7) simplifies to

$$c_t = y_t + s_{t-1}/(T-t+1) - \Gamma(T-t)/2 \qquad (8)$$

The first two terms in the consumption function (8) correspond to the certainty equivalent consumption function. The third term is what has been called "precautionary savings" in the literature.[9]

The fact that e is i.i.d. and $v_{t+k}=e_{t+k}$ for all k implies, by (3"), that Γ is constant, hence (1') satisfies all the FOC's of the problem and therefore it is the unique solution[10].

Furthermore, Definition II in Rothschild-Stiglitz(1970)[11] implies that Γ is increasing on the degree of riskiness since[12] $E[U']=-\theta E[U]$. Also, the size of the increase in expected marginal utility after an increase in uncertainty depends, positively, on the convexity of marginal utility, and the latter rises with θ. Hence, an increase on the degree of risk aversion raises Γ.

Q.E.D.

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[9]See Leland(1968), Miller(1974), Sibley(1975), Zeldes(1984), Barsky(1986) and Kuelhwein(1986) among others.

[10]The exponential utility is strictly concave on each period consumption, and each period consumption is strictly monotonic respect to current consumption (for $\theta>0$), hence the solution is unique.

[11]"..., a risk averter is defined as a person with a concave utility function. If x and y have the same mean, but every risk averter prefers x to y, i.e., if $E[U(x)] \geq E[U(y)]$ for all concave U then surely it is reasonable to say that x is less risky than y." (Rothschild-Stiglitz 1970 pp.226).

[12]Notice that given c_t, Γ is increasing in the expected marginal utility of c_{t+1}.

This first proposition is mainly used to explain the optimization procedure and introduce, in an organized way, the results arising from the combination of a utility function with $U'''>0$ and stochastic labor income.

The next proposition generalizes the results of proposition 1, providing a useful tool for macroeconometric studies,

PROPOSITION 2:

If (a) the instantaneous utility function is exponential as in proposition 1, and (b) income follows any ARMA process (with possibly a unit root),

then

(i) the stochastic process of consumption is a martingale with drift,

(ii) the drift is increasing in the degree of risk aversion, riskiness in the Rothschild-Stiglitz(1970) sense, and in the persistence of labor income shocks,

(iii) when the horizon is finite the drift is, in general, not constant; even when the interest and discount rates are constant,

(iv) the disturbance of the stochastic process of consumption is equal to the annuity value of the contemporaneous innovation in income, and

(v) when $r_t=\delta_t$, the consumption function can still be decomposed, additively, in a term analogous to that of the certainty equivalence case and a precautionary savings term.[13] [14]

Proof:

See Appendix II.

Even though the general proof is given in the appendix, most of the qualitative results of proposition 2 can be seen in the set-up where income (in either levels or first differences) follows an AR(1) process and the interest rate is positive and constant.

Suppose first that income is a stationary AR(1) process. Replace assumption (c) in proposition 1 (labor income follows a random walk) by (c') $y_{t+1} = b + \Phi y_t + e_{t+1}$ with $0 \leq \Phi < 1$, $b \geq 0$, and e_{t+1} having the same properties as before. Also define $\alpha \equiv R^{-1}$.

Using the analogue to equation (7) it is possible to find, after some algebra, that the residual in the stochastic process of consumption is equal to:

$$v_t = e_t[(1-(\alpha\Phi)^{T-t+1})/(1-\alpha\Phi)][(1-\alpha)/(1-\alpha^{T-t+1})] \qquad (10)$$

The term that postmultiplies e_t is called the annuity value and corresponds to the fraction of today's labor income shock effect on human wealth, that is consumed in the current period.

The time-dependence of the annuity value observed in (10) disappears when t is very far from T. In this case (10) converges to (10'),

$$v_t = e_t(1-\alpha)/(1-\alpha\Phi) \qquad (10')$$

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[13] If r is different from δ it is still possible to decompose the consumption function in a certainty equivalent term and a second term. However, the latter no longer has the interpretation of a pure precautionary savings term.

[14] Danny Quah has suggested, correctly, that the solution procedure is able to deal with income processes that do not have an ARMA representation.

The next section studies in detail the implications of (10) and (10') on precautionary savings.

On the other hand, when the process of income is stationary in first differences, (c) is replaced by (c") $y_{t+1}-y_t = \Phi^d(y_t-y_{t-1}) + e_{t+1}$ with $0<\Phi^d<1$.[15]

In this case the equivalent to equations (10) and (10') are (10") and (10'"), respectively;

$$v_t = e_t[(1-(\alpha\Phi^d)^{T-t+1})/(1-\alpha\Phi^d)] \qquad (10")$$

$$v_t = e_t/(1-\alpha\Phi^d) \qquad (10'")$$

with the same interpretation as before.

The next step is to write down the consumption function for each of the cases. Equations (11) and (11') correspond to the cases of income stationarity in levels and in first differences, respectively;

$$c_t = b/(1-\Phi) + s_{t-1}(1-\alpha)/\alpha(1-\alpha^{T-t+1}) - [\sum_{i=1}^{T-t}\alpha^i \sum_{k=1}^{i}\Gamma_{t+k-1}](1-\alpha)/(1-\alpha^{T-t+1}) + v_t/(1-\Phi L) \qquad (11)$$

$$c_t = y_t + (y_t-y_{t-1})\alpha\Phi^d(1-(\alpha\Phi^d)^{T-t+1})/(1-\alpha\Phi^d) + s_{t-1}(1-\alpha)/\alpha(1-\alpha^{T-t+1}) - [\sum_{i=1}^{T-t}\alpha^i \sum_{k=1}^{i}\Gamma_{t+k-1}](1-\alpha)/(1-\alpha^{T-t+1}) \qquad (11')$$

Once more, the consumption function can be decomposed in a certainty equivalence and a precautionary savings term. The latter looks more complex here than in proposition 1 because Γ_t is, in general, not constant.[16]

[15]This section specializes in positive autocorrelation coefficients in order to be consistent with the empirical findings (e.g. Campbell and Mankiw 1986, Campbell and Deaton 1987, and West 1986).
[16]See section III.

Furthermore, as in the certainty equivalent case, the sensitivity of consumption to changes in income is equal to the annuity value of the change:

$$dc_t/de_t = (dc_t/dv_t)(dv_t/de_t)$$

$$= [(1-\alpha)/(1-\alpha^{T-t+1})][(1-(\alpha\Phi)^{T-t+1})/(1-\alpha\Phi)] \qquad (12)$$

$$dc_t/de_t = [(1-(\alpha\Phi^d)^{T-t+1})/(1-\alpha\Phi^d)] \qquad (12')$$

with (12) and (12') corresponding to the cases of levels and first differences-stationary income processes, respectively.

The combination of the assumptions on the form of the utility function[17], the absence of liquidity constraints, the linearity of the stochastic process of income and the i.i.d. nature of income's innovations, ensures that there is a complete dichotomy between the effect of the precautionary savings motives and the sensitivity of consumption to unexpected changes in income[18].

In this framework, the effects of the interaction of the positive third derivative of the utility function, with risk aversion and uncertainty, are entirely reflected on the trend component of consumption. Any change, in either the uncertainty level or the degree of risk aversion, affects the slope of the expected path of consumption, but not the responsiveness of consumption to news about permanent income embodied in current income changes. The next section studies the trend component. Later, section IV, relaxes some of the assumptions of this section in order to allow for interaction between sensitivity of consumption to income changes and precautionary savings.

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[17] With its absence of Dreze-Modigliani substitution effect and possibility of negative consumption.
[18] In section IV.3 we relax some of these assumptions.

III. PRECAUTIONARY SAVINGS

After establishing the fundamental propositions of this chapter, section II stressed those results that are shared by the exponential and the certainty equivalence cases. Here, on the other hand, some of the differences between the two cases are highlighted. This is done by studying the behavior of the Γs, the main distinctive characteristic of the consumption function and process in the presence of precautionary savings motives.

III.1 Consumption Growth

Assume that $r=\delta$, so we can isolate the impact of the precautionary savings motive on the slope of the consumption path can be isolated. The expected path of consumption under certainty equivalence would be absolutely flat.

Propositions 1 and 2 show that in the case of an exponential utility function the drift term in the consumption process is positive. Its size depends on the higher moments of v and on the degree of risk aversion. Table 1 shows the drift term for different sets of parameters and distributions of e when income follows a random walk (the results are normalized to represent % of average annual consumption).

TABLE 1

SOME EXAMPLES OF Γ

Distrib.		Γ expression	values of Γ % $(\sigma/y, \theta c)$ (0.1,2)	(0.2,2)	(0.1,10)
(i)	N	$(\theta/2)\sigma^2$	1.000	4.000	5.000
(ii)	Be	$\log[(\exp(\theta\sigma)+\exp(-\theta\sigma))/2]/\theta$	0.990	3.900	4.340
(iii)	Be^b	$\log[0.1\exp(3\theta\sigma)+0.9\exp(-\theta\sigma/3)]/\theta$	1.190	5.650	9.760
(iv)	Be^g	$\log[0.9\exp(\theta\sigma/3)+0.1\exp(-3\theta\sigma)]/\theta$	0.840	2.840	2.320
(v)	U	$\{\log[\exp(\theta\sigma\sqrt{3})-\exp(-\theta\sigma\sqrt{3})]-\log(2\theta\sigma\sqrt{3})\}/\theta$	1.000	3.940	4.580

With

(i) $N(0,\sigma^2)$ a Normal distribution[19].

(ii) $Be(0,\sigma^2)$ a Bernoulli that takes values: $\begin{cases} \sigma & \text{w.p. } 0.5 \\ -\sigma & \text{w.p. } 0.5 \end{cases}$

(iii) $Be^b(0,\sigma^2)$ a Bernoulli that takes values: $\begin{cases} \sigma/3 & \text{w.p. } 0.9 \\ -3\sigma & \text{w.p. } 0.1 \end{cases}$

(iv) $Be^g(0,\sigma^2)$ a Bernoulli that takes values: $\begin{cases} 3\sigma & \text{w.p. } 0.1 \\ -\sigma/3 & \text{w.p. } 0.9 \end{cases}$

(v) $U(0,\sigma^2)$ a Uniform with support $[-\sigma\sqrt{3},\sigma\sqrt{3}]$.

[19]This distribution is used only as an approximation to more complex symmetric distributions. Distributions with unbounded (from below) support are meaningless for our analysis.

As noted earlier, these results are intimately related to the definition of riskiness in Rothschild and Stiglitz(1970). The drift in the random walk process followed by consumption rises with θ, σ, the worsening of the "bad state" (see the Bernoulli examples) and the enlargement of the support of e.

Whenever $U'''>0$ and uninsurable labor income is the only source of uncertainty, the expected path of consumption will be upward sloped. Given the expected present value of lifetime consumption, a positive Γ implies less consumption than under certainty equivalence during the early years and more during the late years. If the assumption of flat expected income path is added, precautionary savings produce a "humped savings" model (Harrod 1948) in the same way as retirement does in the Modigliani and Brumberg(1954) model.

III.2 Annuity Value and Precautionary Savings

Section II spends some time developing simple AR(1) cases to understand some of the statements of proposition 2. Those examples were used to study the role of the annuity value in the sensitivity of consumption to income's innovations.

Except for this sensitivity issue, the annuity value does not play any additional (theoretical) role under certainty equivalence. In the exponential utility case, however, this is no longer true. For a given distribution of the e_s, a change in the annuity value changes the second moment of the distribution of the v_s. Propositions 1 and 2 showed that this alters Γ, i.e. the slope of the consumption path, and hence precautionary savings behavior.

Equations (10) and (10") illustrate this issue. There it is possible to see that if $\Phi<1$ ($\Phi^d>0$) the variance of v_t is monotonically increasing (decreasing) over time. In fact, it is smaller (larger) than the variance of e_t except for t=T. When the utility function is such that $U'''>0$, the heteroskedasticity problem becomes more important than a simple correction of the standard errors because the drift term depends on the moments of v_t. If the moments of v_t are not constant across time neither is Γ_t, even when r and δ are constants. Here, Γ_t would increase (decrease) as t gets closer to T.[20] These effects are studied in more detail in chapter II.

Notice also, that as the annuity value of a shock decreases (Φ, Φ^d and/or α decreases), the variance of v, for a given variance of e, decreases, and the drift term becomes less important[21].

The behavior of precautionary savings in response to these changes in the annuity value, as well as changes in the distribution of income's innovations, have been disregarded in the literature. The next section shows how to reinterpret some well known results when consumers have a precautionary savings motive.

[20]Certainly, in an infinite horizon set-up this would not be an issue since the distance from the horizon does not change (significantly) as time passes.
[21]See table A2 in appendix II.

IV. APPLICATIONS

IV.1 Intertemporal Substitution Measurement

The next two propositions reflect the implications of precautionary savings on the measurement of the intertemporal substitution parameter. They are derived under the AR(1) -in levels and first differences- assumption in order to simplify the proofs. However, their implications extend to the more general cases. The AR(1) in levels represent those processes that are stationary, whereas the AR(1) in first differences represents those processes that have a unit root with long run effect larger than the immediate effect.[22]

PROPOSITION 3a:

If the assumptions of proposition 2 hold and income follows an AR(1) process with $0 \leq \Phi \leq 1$,

then,

(i) as compared with the standard result, a change in the interest rate induces more substitution between current and future consumption although,

(ii) this effect disappears when $\Phi=1$.

Proof:

First, it is useful to summarize the standard result in the following way: (a) the effect of a change in the interest rate, dr, on the difference between expected consumption at time t+1 and consumption today is $(1/\theta)dr$, and (b) the total (substitution) effect on current consumption is $\{\alpha/(1-\alpha)\}(1/\theta)dr$.

[22] The analysis is restricted to positive AR coefficients.

Now assume, for convenience, that $T=\infty$, then equation (11) becomes (11')[23];

$$c_t = b/(1-\Phi) + s_{t-1}(1-\alpha)/\alpha - \{\alpha/(1-\alpha)\}\Gamma + v_t/(1-\Phi L) \qquad (11')$$

with

$$\Gamma = (r-\delta)/\theta + (1/\theta)\log E_t[\exp(-\theta v_{t+1})] \qquad (13)$$

Define $z \equiv -(\alpha/(1-\alpha))[\Gamma-(1/\theta)(r_t-\delta)]$, so the new effects can be isolated, then

$$z = [-\alpha/(1-\alpha)](1/\theta)\log E_t[\exp(-\theta v_{t+1})] \qquad (14)$$

and

$$dz/dr = [\alpha^2/(1-\alpha)^2](1/\theta)\log E_t[\exp(-\theta v_{t+1})] - [\alpha/(1-\alpha)](1/\theta)[d\log E_t[\exp(-\theta v_{t+1})]/dVar(v_t)]\, dVar(v_t)/dr \qquad (15)$$

The first term in equation (15) corresponds to a wealth effect and it is always positive.

The second term is a substitution effect and proves part (i) of the proposition since,

$(1/\theta)d\log E_t[\exp(-\theta v_{t+1})]/dVar(v_t) > 0$ by proposition 1, and

$$dVar(v_t)/dr = \sigma^2 2\alpha^2(1-\alpha)(1-\Phi)/(1-\alpha\Phi)^3 \geq 0 \qquad (16)$$

Thus, disregarding the wealth effect, when precautionary savings are taken into account, an increase in the interest rate reduces current consumption by more than in the standard case.

The proof of part (ii) follows from evaluating (16) at $\Phi=1$.

Q.E.D.

PROPOSITION 3b:

If the assumptions of proposition 3a hold but

[23]The proof for finite horizons is qualitatively identical, although the algebra is slightly more complex.

$0<\Phi^d\leq 1$, with Φ^d the AR coefficient of the first difference of labor income, then compared to the standard result, a change in the interest rate induces less substitution between current and future consumption.

Proof:

It is possible to construct a term z^d in the same way as in proposition 3a. In this case, however, $Var(v_t)=\sigma^2/(1-\alpha\Phi^d)^2$, hence

$$dVar(v_t)/dr = -\sigma^2 2\alpha^2\Phi^d/(1-\alpha\Phi^d)^3 \leq 0 \qquad (16')$$

Q.E.D.

Propositions 3a and 3b have very important implications for the purpose of measuring the elasticity of intertemporal substitution by computing the effect of a change in r on the slope of the consumption path. At the very least, the interpretation of the results from such a procedure must be made with caution. If one really cares about the standard intertemporal substitution parameter, then the precautionary savings and intertemporal substitution effects must be disentangled. In particular, the conditions in proposition 3b would bias downward the estimator of $(1/\theta)$.[24] If, on the other hand, one cares about the total substitution effect, then the Lucas(1976)' critique becomes important; in the sense that this effect is not invariant to changes in the stochastic process of income.[25]

Tables A3 to A5 in appendix III show examples of these effects. From there it is possible to see that the biases are large only when the

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[24]See Hall(1985), Runkle(1986) for examples of small, and even negative, estimates of the intertemporal substitution parameter. They estimate it in a CRRA framework, but the logic of the problem is similar.

[25]This critique is concerned with procedures that explicitly solve the consumption path, and to maximum likelihood procedures with constant higher moments. It does not apply to procedures that estimate the Euler equation directly.

intertemporal substitution parameter ($1/\theta$) is already small. This means that proposition 3b does not offer an explanation for low intertemporal substitution estimates, but only for biases once it is already small. However, it could certainly explain negative estimates of the intertemporal substitution parameter (e.g. Hall 1985).

IV.2 The Lucas Critique

Changes in the stochastic process of income will also alter the rate of growth of consumption itself. This leads to the next proposition.

PROPOSITION 4:

Under the assumptions of proposition 2, tests of the LCH-PIH a la Hall(1978) are not immune to the Lucas(1976)' critique. Any policy measure that changes the stochastic process of income will also change the slope of the consumption path.

Proof:

Again, for simplicity, assume that $T=\infty$. Differentiating equation (13) we obtain,

$$d\Gamma_t/d\Phi = [d\Gamma_t/dVar(v_t)][dVar(v_t)/d\Phi] \qquad (17)$$ [26]

the first term is positive by proposition 1. In the levels case

$$dVar(v_t)/d\Phi = 2\alpha(1-\alpha)^2\sigma^2/(1-\alpha\Phi)^3 > 0 \qquad (18)$$

hence $d\Gamma_t/d\Phi > 0$.

In the first difference case,

$$dVar(v_t)/d\Phi^d = 2\alpha\sigma^2/(1-\alpha\Phi^d)^3 > 0 \qquad (18)$$

[26]Disregarding the effect of a change on Φ on the support of v. This would only make the result stronger.

hence $d\Gamma_t/d\Phi^d > 0$.

Q.E.D.[27]

Proposition 4 says that even when r=δ and both are constant, then if the stochastic process of the forcing variable is not stable and the utility function exhibits U'">0, the stochastic process of consumption will not have stable parameters. The original work by Hall(1978) dealt mainly with the certainty equivalence case. Unfortunately, the immunity of his procedure to the Lucas critique does not extend to cases where precautionary savings are present.

IV.3 Excess sensitivity/Excess smoothness

This final application shows the potential role of precautionary savings in explaining the excess sensitivity (Flavin 1981, Hayashi 1982) and the recent excess smoothness (Deaton 1986, West 1986, and Campbell and Deaton 1987) results.

Campbell and Deaton(1987) made clear that the apparently contradictory findings[28] of excess sensitivity and excess smoothness are in fact consistent. Excess sensitivity refers to the reaction of consumption to anticipated changes in income, whereas excess smoothness refers to the response of consumption to unanticipated changes in income. Moreover, if savings Granger cause income (they do!) then both, excess sensitivity and smoothness, reflect the violation of the same orthogonality condition.

Campbell and Deaton's result is reviewed and explained within the framework of this paper. The same is done for the better known excess sensitivity result.

[27]Extending this result to finite horizons is trivial.
[28]Both under certainty equivalence conditions.

The derivation of the variance conditions, in Campbell and Deaton(1987), is in the spirit of this paper. They start by stating the permanent income theory in a certainty equivalence framework. As a result they obtain the standard expectations "revision" formula[29];

$$c_{t+1}-c_t = r\sum_{i=0}^{\infty}(1+r)^{-i}(E_{t+1}-E_t)y_{t+i} \quad (19)$$

Afterwards they specify the stochastic process of income so that (19) simplifies to:

$$c_{t+1}-c_t = \beta e_{t+1} \quad (20)$$

with β the annuity value of an income shock, and e_{t+1} an i.i.d. residual with the same characteristics described before.

Following standard empirical results they argue that a log specification for income seems appropriate. They derive what they call "a logarithmic version of the permanent income model"[30]; in our notation

$$(c_{t+1}-c_t)/y_t \approx \beta^L e^L_{t+1} \quad (21)$$

with L standing for logarithmic version.

Their preferred specification for income is:

$$\log(y_{t+1}/y_t) = 0.00263 + 0.443\log(y_t/y_{t-1}) + e^L_{t+1} \quad (22)$$

The parameter estimates of equation (22) imply a β^L approximately equal to 1.8. They also retrieve the variance of e^L to then test the null hypothesis by comparing the standard deviation of the right and left hand side of (21). Their conclusion is that the two terms are far from being equal, as (23) shows,

$$[Var_t(c_{t+1}/y_t)/Var_t(\beta^L e^L_{t+1})]^{1/2} = 0.58 \quad (23)$$

[29]See for example Hall(1978), Flavin(1981) and Hayashi(1982).
[30]See appendix in Campbell and Deaton (1987).

hence consumption is too smooth to be consistent with this version of the life cycle-permanent income hypothesis.

They, as well as West(1986), suggest the need for research in the direction of models that exhibit smoother consumption than what is implied by this version of the LCH/PIH.

A log-linear process with i.i.d. residuals implies that in levels income will be conditionally heteroskedastic. The next proposition shows that once precautionary savings are taken into account, this heteroskedasticity may be responsible for the excess smoothness and excess sensitivity results.

PROPOSITION 5:

If the utility function is exponential and the stochastic process followed by income is such that the variance of its residual depends (positively) on the level of lagged income, then

(i) the sensitivity of consumption to income shocks is smaller than under certainty equivalence conditions, and

(ii) if in addition Γ is assumed constant then the change in consumption will be correlated with lagged variables, specially with lagged income.

Proof:

To simplify assume that $r=\delta$ and constant, $T=\infty$, and that the process for labor income can be described by $y_{t+1}= y_t(1+e'_{t+1})$ with e'_{t+1} i.i.d.$(0,\sigma'^2)$ $\varepsilon(-1,\infty)$.

Propositions 1 and 2 showed that the solution for the process and the level of consumption are of the form;

$$c_{t+1} = \Gamma_t + c_t + v_{t+1} \tag{24}$$

and

$$c_t = y_t + s_{t-1}(1-\alpha)/\alpha - E_t[\sum_{i=1}^{\infty} \alpha^i \sum_{k=1}^{i} \Gamma_{t+k-1}](1-\alpha) \tag{25}$$

Finding v in this problem is more complex than in propositions 1 and 2 since Γ_{t+1} is now a random variable that depends on y_{t+1}. The solution for v must take into account the effects of income shocks on the revision of expectations about future Γs. Suppose this solution is written as $v_{t+1}=a_{t+1-1}e'_{t+1}y_{t+1-1}$. Notice that $e'_{t+1}y_{t+1-1}$ is equivalent to e_{t+1} in propositions 1 and 2. Thus a_{t+1-1} equal to one means to go back to the type of solutions found before and there is no excess smoothness problem. However, part (i) of the proof consists in showing that once precautionary savings are taken into account $a_{t+1-1}<1$. The coefficient a_{t+1-1} can be defined as the expected sensitivity of consumption to income innovations, or as an index of excess smoothness.

To simplify the evaluation of (25) it is convenient to assume that the income innovation is normal[31]. In this case the sequence of Γs can be approximated using a Taylor expansion so:

$$E_t[\Gamma_{t+i}] \approx \Gamma_t(1+\sigma'^2)^i \tag{26}$$

hence

$$c_t = y_t + s_{t-1}(1-\alpha)/\alpha - \Gamma_t h \tag{27}$$

or

$$c_t = y_t + s_{t-1}(1-\alpha)/\alpha - \theta\sigma'^2 y_t^2 a_t^2 h/2 \tag{27'}$$

with $h \equiv [\alpha/(1-\alpha(1+\sigma'^2))]$, $(1+\sigma'^2)\alpha<1$.

[31]See table 1 for differences with other distributions.

Substituting the process for income, subtracting c_{t-1} and dividing by y_{t-1}, obtains

$$(c_t - c_{t-1})/y_{t-1} = -c_{t-1}/y_{t-1} + (1+e'_t) + s_{t-1}(1-\alpha)/\alpha y_{t-1}$$
$$- \theta\sigma'^2 y_{t-1}(1+e'_t)^2 a_t^2 h/2 \qquad (28)$$

Differentiating expression (28) with respect to e'_t yields,

$$dx_t/de'_t = 1 - \theta\sigma'^2 y_{t-1}(1+e'_t) a_t^2 h \qquad (29)$$

and

$$E_{t-1}[dx_t/de'_t] = 1 - \theta\sigma'^2 y_{t-1} a_t^2 h < 1 = E_{t-1}[dx_t^{ce}/de_t'] \quad (29')$$

with $x_t \equiv (c_t - c_{t-1})/y_{t-1}$ and $x_t^{ce} \equiv (c_t^{ce} - c_{t-1}^{ce})/y_{t-1}$. The superscript ^{ce} refers to certainty equivalence.

But $E_{t-1}[dx_t/de'_t]$ is the definition of a_t, hence $a_t < 1$, proving part (i) of the proposition.

On the other hand, part (ii) of the proposition is proved by noticing that tests for excess sensitivity use an expression of the form,

$$c_{t+1} - c_t = \Gamma + v_{t+1} + w_t \qquad (30)$$

with $w_t \equiv (\Gamma(y_t) - \Gamma)$.

In this set-up v_{t+1} is orthogonal to all information available at t, but w_t is <u>not</u>, let alone to lagged income (y_t).

Q.E.D.

The idea behind proposition 5 is simple. An unexpected change in income not only raises human wealth but also the variance of the <u>level</u> of future income, and as a result, precautionary savings. The latter partially offsets the wealth effect, reducing the responsiveness of current consumption to permanent changes in income.

Campbell and Deaton's result can be thought of in terms of proposition 5, since the log specification (with an i.i.d. residual) implies conditional heteroskedasticity on the process for the level of income. Table 2 shows possible values of a_t; it is possible to see that for very reasonable values of the coefficient of relative risk aversion and of the (quarterly) variance of individual labor income, a_t is substantially less than one. The value found by Campbell and Deaton(1987), 0.58, is perfectly attainable within the LCH/PIH context.[32]

TABLE 2

RELATIVE SENSITIVITY OF CONSUMPTION TO INCOME SHOCKS

(EXPONENTIAL UTILITY/QUADRATIC UTILITY)

(r=2%)[33]

	$\theta y=1$	$\theta y=2$	$\theta y=4$	$\theta y=10$
$\sigma'=0.025$	0.8250	0.7276	0.6132	0.4588
$\sigma'=0.035$	0.7076	0.5914	0.4743	0.3369
$\sigma'=0.045$	0.5832	0.4666	0.3611	0.2478

Note: σ' is the percentual standard deviation of labor income (quarterly).

Part (ii) of proposition 5, on the other hand, shows the effect of income following a log-process on the interpretation of the tests reported by Hall(1978), Flavin(1981) and Hayashi(1982), among others. A change in income implies a change in the expected rate of growth of consumption. Violation of the orthogonality condition may be caused, not by a failure of

[32] See appendix IV for details on the calculation of a_t.
[33] See appendix IV for tables with r=1% and 4%.

the LCH/PIH but by the conditional heteroskedasticity of the income process and its effect on the drift term.

Put it in other words, if the true process of income is an ARIMA in logs instead of in levels, then, unless researchers are willing to strongly maintain the quadratic utility function specification, much of the evidence most typically used to reject the LCH/PIH can be accounted for by the behavior of precautionary savings.

V. CONCLUSION

The aim of this chapter has been to study, analytically, the empirical implications of abandoning the certainty equivalence assumption, in order to take into consideration precautionary savings. Among other things, the presence of new "wealth" and substitution effects was noticed. This complicates and casts some doubts on previous results on measuring intertemporal substitution by looking at the changes on consumption growth after a change in interest rates.

It was also shown that if income follows an ARIMA process in logs instead of in levels, the excess sensitivity and excess smoothness results can alternatively be explained as a rejection of the certainty equivalence assumption in favor of a utility function with positive third derivative. Once this is done, the LCH-PIH becomes perfectly consistent with the data.

In order to obtain the previous results, this essay develops a simple solution technique that should be of independent value.

APPENDIX I

This appendix shows the effect of a change in Φ and/or Φ^d on Γ. The results are approximated to the case of a normal distribution. The numbers correspond to an annual model, and as a reference, average consumption is around 100.

TABLE A2

VALUE OF Γ FOR DIFFERENT Φs AND Φ^ds

		values of Γ ($\sigma/y,\theta c$)	
Autocorrelation	(0.1,2)	(0.2,2)	(0.1,10)
$\Phi = 0.9$	0.030	0.120	0.150
$\Phi = 1.0,\ \Phi^d = 0$	1.000	4.000	5.000
$\Phi^d = 0.1$	1.230	4.920	6.150

This table can be used as an example of some of the implications of propositions 2 and 4.

APPENDIX II

The steps of the proof of proposition 2 are identical to those of proposition 1. Only those elements that differ from the latter are stressed.

Assume first that the process followed by income is a stationary ARMA(p,q):

$$y_t = \sum_{k=1}^{p} a_k y_{t-k} + e_t + \sum_{m=1}^{q} b_m e_{t-m} \qquad \text{(aII.1)}$$

As before the guess for the consumption process is:

$$c_{t+1} = \Gamma_t + c_t + v_{t+1} \quad \text{(aII.2)}$$

Following exactly the same steps as in proposition 1 yields,

$$v_{t+k} = [(1+ \sum_{i=1}^{T-t} h_i \prod_{j=1}^{i} \alpha_{T-t+j})/(1+ \sum_{i=1}^{T-t} \prod_{j=1}^{i} \alpha_{T-t+j})]e_{t+k} \quad \text{(aII.3)}$$

with $h_i = b_i + \sum_{j=1}^{i} a_j h_{i-j}$,

the parameters of the MA representation of (aII.1).

As before,

$$\Gamma_t = (r_t-\delta)/\theta + (1/\theta)\log E_t[\exp(-\theta v_{t+1})] \quad \text{(aII.4)}$$

and c_t can be written as:

$$c_t = \{y_t + E_t[\sum_{i=1}^{T-t} y_{t+i} \prod_{j=1}^{i} \alpha_{t+j}]\}/\{1 + \sum_{i=1}^{T-t} \prod_{j=1}^{i} \alpha_{t+j}\}$$
$$+ \; s_{t-1}/\alpha \sum_{i=1}^{T-t} (1+ \prod_{j=1}^{i} \alpha_{t+j}) \quad \text{(aII.5)}$$
$$- [\sum_{i=1}^{T-t} \prod_{j=1}^{i} \alpha_{t+j} \sum_{k=1}^{i} \Gamma_{t+k-1}]/ \sum_{i=1}^{T-t} (1+ \prod_{j=1}^{i} \alpha_{t+j})$$

In the case of α_j constant and $T=\infty$, Γ becomes a constant and,

$$E_t[\sum_{i=1}^{T-t} y_{t+i}\alpha^i] = \alpha\{(h(L)-h(\alpha))/(L-\alpha)\}h^{-1}(L)y_t \quad \text{(aII.6)}$$

with L the lag operator and h(.) a polinomial.

If there is a unit root, on the other hand, the modification is straight forward. Suppose that the changes in income follow a stationary ARMA process:

$$(1-L)y_t = \sum_{k=1}^{p} a_k^*(1-L)y_{t-k} + e^*_t + \sum_{m=1}^{q} b_m^* e^*_{t-m} \quad \text{(aII.7)}$$

If α is assumed to be constant, then

$$v_{t+k} = (1+ \sum_{i=1}^{T-t} h_i^*\alpha^i)e^*_{t+k} \quad \text{(aII.8)}$$

and the consumption function is:

$$c_t = \{y_t + E_t[\sum_{i=1}^{T-t}(1-L)y_{t+i}\alpha^i]\} + s_{t-1}(1-\alpha)/\alpha(1-\alpha^{T-t+1})$$
$$\text{(aII.9)}$$
$$- [\sum_{i=1}^{T-t} \alpha^i \sum_{k=1}^{i} \Gamma_{t+k-1}](1-\alpha)/(1-\alpha^{T-t+1})$$

Also notice that in both cases if $r=\delta$, the decomposition of the consumption function into certainty equivalence and precautionary savings terms holds as before, hence the proof is complete.

Q.E.D.

APPENDIX III

This appendix presents some examples of the implications of proposition 3b (assuming $\Phi^d=0.48$)[34]. Tables A3 and A4 show the effect of a change on r (equal to 0.01), net of the traditional substitution effect, on Γ and the intertemporal substitution parameter. Table A5 shows the effect of this new substitution effect on current consumption.

TABLE A3

"NON-TRADITIONAL" INTEREST RATE EFFECT ON Γ

(r increases by 0.01)

	$\theta c=2$	$\theta c=4$	$\theta c=6$	$\theta c=10$
$\sigma/y=0.025$	-0.0033	-0.0066	-0.0099	-0.0165
$\sigma/y=0.035$	-0.0065	-0.0130	-0.0195	-0.0324
$\sigma/y=0.045$	-0.0107	-0.0214	-0.0322	-0.0536
"traditional"	0.5000	0.2500	0.1667	0.1000

[34]This is the value found by Campbell and Deaton(1987) for the model in first differences of the levels.

TABLE A4

INTERTEMPORAL SUBSTITUTION PARAMETER DOWNWARD BIAS

(%)

	$\theta c=2$	$\theta c=4$	$\theta c=6$	$\theta c=10$
$\sigma/y=0.025$	0.6600	2.6400	5.9400	16.5000
$\sigma/y=0.035$	1.3000	5.2000	11.7000	32.4000
$\sigma/y=0.045$	2.1400	8.5600	19.3200	53.6000

TABLE A5

"NON-TRADITIONAL" SUBSTITUTION EFFECT ON CURRENT CONSUMPTION

(increase on r = 0.01)

(%)

	$\theta c=2$	$\theta c=4$	$\theta c=6$	$\theta c=10$
$\sigma/y=0.025$	0.6600	1.3200	1.9800	3.3000
$\sigma/y=0.035$	1.3000	2.6000	3.9000	6.5000
$\sigma/y=0.045$	2.1400	4.2800	6.4200	10.7000

APPENDIX IV

Finding a_t amounts to find the positive root in equation (29'),

$$a_t = 1 - \theta\sigma'^2 y_{t-1} a_t^2 h \qquad \text{(aIV.1)}$$

with $h \equiv [\alpha/(1-\alpha(1+\sigma'^2))]$, $(1+\sigma'^2)\alpha<1$.

Call $g \equiv \theta\sigma'^2 y_{t-1} h$, then (aIV.1) can be re-written as;

$$a_t = 1 - g a_t^2 \qquad \text{(aIV.2)}$$

If the persistence is 1.8 (Campbell and Deaton 1987), then a good approximation is to replace g by $g' \equiv 1.8g$. Table 1's values correspond to the positive root of;

$$a_t = 1 - g' a_t^2 \qquad \text{(aIV.3)}$$

Section IV.3 presented the result for r=0.02. The postwar US data suggest an even smaller real interest rate. The results in table 2 are very sensitive to changes on the interest rate, hence similar tables where r=0.01 and 0.04 respectively are reported.

TABLE A6

RELATIVE SENSITIVITY OF CONSUMPTION TO INCOME SHOCKS

(EXPONENTIAL UTILITY/QUADRATIC UTILITY)

(r=1%)

	$\theta y=1$	$\theta y=2$	$\theta y=4$	$\theta y=10$
$\sigma'=0.025$	0.7033	0.5868	0.4700	0.4167
$\sigma'=0.035$	0.5244	0.4122	0.3147	0.2133
$\sigma'=0.045$	0.3017	0.2248	0.1649	0.1078

TABLE A7

RELATIVE SENSITIVITY OF CONSUMPTION TO INCOME SHOCKS

(EXPONENTIAL UTILITY/QUADRATIC UTILITY)

(r=4%)

	$\theta y=1$	$\theta y=2$	$\theta y=4$	$\theta y=10$
$\sigma'=0.025$	0.9023	0.8333	0.7383	0.5867
$\sigma'=0.035$	0.8277	0.7312	0.6170	0.4624
$\sigma'=0.045$	0.7458	0.6334	0.5150	0.3710

APPENDIX V

The purpose of this appendix is to show that under the maintained hypothesis of an exponential utility function and a log-income process, a log-consumption process may be a better description of consumption than the linear process shown in (30) which is typically used in sensitivity tests.

The log process will not solve the FOCs exactly, however it may provide a better approximation to the true consumption process which exhibits a flexible drift.

To see this assume that $r=\delta$, $T=\infty$ and that income follows a random walk in logs:

$$\log(y_{t+1}/y_t) = e_{t+1} \qquad \text{(aV.1)}$$

The solution for the consumption process can be approximated by;

$$\log(c_{t+1}/c_t) = \Gamma^L + v_{t+1} \qquad \text{(aV.2)}$$

with $\Gamma^L>0$, e and v i.i.d innovation residuals[35].

Section IV.3 showed that Γ, the precautionary savings term (in levels), is increasing in income (hence in consumption). Here, by letting the drift term to be constant in the log specification, this positive correlation (in levels) is achieved[36].

As suggested by Campbell and Deaton(1987), if part of the excess sensitivity can be explained by one argument, then also part of the excess smoothness should be accountable by the same argument. Substituting (aV.1) and (aV.2) in the intertemporal budget constraint and solving in the same way as in propositions 1 and 2, yields a consumption function of the form:

[35]Assume that their means are $(-1/2)\sigma_e^2$ and $(-1/2)\sigma_v^2$ respectively.
[36]Although it only depends linearly, as opposed to quadratically, on income. See equation (27'). This could explain why there is still some correlation left in the log-specification (see Nelson 1987).

$$c_t = y_t(1-\alpha\exp(\Gamma^L))/(1-\alpha) + s_{t-1}(1-\alpha\exp(\Gamma^L))/\alpha \qquad \text{(aV.3)}$$

with $\alpha\exp(\Gamma^T)<1$. But $\exp(\Gamma^T)>1$, hence the sensitivity of consumption to income shocks is less than one (the certainty equivalence response). As an example, if r=2% (annual) is assumed then $\Gamma^T=0.002$ implies that the sensitivity of consumption is cut by 40% (i.e. an expected 0.8% (annual) rate of growth of consumption due to precautionary savings would be enough to account for the excess smoothness result).

It is important to notice that the process (aV.2) could be also thought to be the solution of the CRRA case. In fact it is easy to show, using the same steps followed in proposition 1, that a log-consumption process satisfies the Euler equation in the CRRA case. However, it is no longer true that Γ^T is a constant. It depends on the mean and variances of total wealth, and both are changed after an income shock. This new dependence results from the negative Dreze-Modigliani substitution effect and the implicit non-negativity constraint of the CRRA.

In any case, whether the true utility function is exponential or CES is immaterial for the main purpose of the application IV.3. In both cases precautionary savings make the excess smoothness/excess sensitivity result consistent with the LCH/PIH hypothesis[37].

[37] In the CRRA precautionary savings show up in the condition $\Gamma^T>0$. In fact, if $\Gamma^T=0$ we are back on the certainty equivalence case.

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CHAPTER II

A NON LIFE CYCLE MODEL OF WEALTH ACCUMULATION

I. INTRODUCTION

This chapter addresses several issues related to individual and aggregate savings behavior in the presence of labor income and length of life uncertainty.

Usually models with uncertain horizons are designed to study the welfare implications of a funded social security system when annuity markets are incomplete. Here, on the other hand, the main concern is the interaction between horizon uncertainty and the absence of markets for labor income insurance. Even when no retirement existsS this model leads to a hump savings model (Harrod 1948) with implications very similar to those of the social security models.

It is also shown that for reasonable values of the coefficient of risk aversion, the degree of labor income uncertainty and the rate of survival, this model can generate steady state levels of wealth and bequests that far exceed those of the US economy.

Furthermore, it is shown that when labor income and horizon uncertainties interact, an increase in the probability of dying early can increase the aggregate stock of wealth even when the maximum length of life is kept unchanged. On the other hand, when bequests are not received early in life, aggregate wealth may be reduced after an increase in the probability of dying, even when expected lifetime is kept constant and the coefficient of relative risk aversion is large (greater than one). Both findings are contradictory with the results derived in models with exogenous bequests.

Here the coefficient of risk aversion plays a role much more important than in models in which only horizon uncertainty exists. Roughly speaking, aggregate steady state wealth moves one for one with changes in the degree of risk aversion[38]. Moreover, the degree of risk aversion is very important in assessing the welfare effect of policies that tend to stabilize labor income fluctuations.

This introduction is followed by five sections. Section II studies the interaction between the stochastic process of labor income and the precautionary savings of an individual. It is shown that in the presence of this type of savings the interest elasticity of savings is generally reduced. It is also shown that the disentangling of savings sources performed in Kotlikoff and Summers(1981 and 1986) (henceforth KS) is biased against hump-shaped savings models due to the omission of a source of dissavings: the income profile.

In section III individual savings are aggregated in order to determine the aggregate capital stock due to precautionary savings. Most of the section concentrates on studying the role of changes in labor income uncertainty and degree of risk aversion of consumers, on aggregate capital accumulation. Both the dynamics and steady states are studied.

Uncertain horizons are introduced in section IV. The emphasis is on the effects of changes in the probability of dying early when bequests are endogenous. It is shown that the long-run effects of an increase in labor income uncertainty and degree of risk aversion are increased in the presence of bequests. It is also shown that many of the implications

[38]This occurs because in the case of the exponential utility function, the specification adopted here, an increase in the coefficient of absolute risk aversion raises the convexity of marginal utility. This also applies to the isoelastic utility case. See proof of proposition 1 in chapter I.

derived in cases in which bequests are taken as exogenous are reversed once the endogeneity of intergenerational transfers is considered.

Section V presents some intertemporal welfare implications of policies that affect labor income riskiness. It is shown that for reasonable values of the coefficient of risk aversion, not only current but also future generations benefit from a reduction in the level of labor income uncertainty. Section VI concludes.

II. The Relation Between Savings and the Stochastic Process of Labor Income: Individual behavior

This section discusses some implications of precautionary savings due to labor income uncertainty for individual savings decisions. Special emphasis is put on the interaction between the stochastic process followed by labor income and the amount of savings generated by precautionary motives. The effect of the latter on the response of savings to permanent changes in the interest rate is also studied. These results are a direct implication of those found in chapter I for consumption behavior. The details of the optimization procedure can be seen there.

The underlying problem is that of a consumer-saver who has an horizon that extends to a known time T (that may be far enough to be approximated by infinity). This consumer takes decisions in discrete time and leaves no bequests. His utility function is a time separable constant absolute risk aversion, and he works a fixed number of hours that remain constant throughout his life. For his work he receives labor income. The latter is the only source of uncertainty (horizon uncertainty is introduced later).

He is also allowed to buy and sell a riskless bond subject to some solvency constraint, but there is no insurance market for labor income.

Most of the issues to be stressed in this paper can be dealt with through simple cases. It will be assumed that labor income follows an AR(1) process (with a positive autoregressive coefficient) either in levels or in first differences. It will be also assumed that the interest rate is constant[39].

If income follows an AR(1) process in levels:

(1) $y_{t+1} = b + \Phi y_t + e_{t+1} \quad 0 \leq \Phi < 1,\ b \geq 0$

with y: labor income

b,Φ: parameters

e: an independently, identically distributed residual,

then chapter I shows that the consumption function is:

(2) $$c_t = b/(1-\Phi) + W_{t-1}(1-\alpha)/\alpha(1-\alpha^{T-t+1})$$

$$-[\sum_{i=1}^{T-t} \alpha^i \sum_{k=1}^{i} \Gamma_{t+k-1}](1-\alpha)/(1-\alpha^{T-t+1}) + v_t/(1-\Phi L)$$

the innovation, v, is:

(3) $v_t = e_t[(1-(\alpha\Phi)^{T-t+1})/(1-\alpha\Phi)][(1-\alpha)/(1-\alpha^{T-t+1})]$

and the slope of the consumption path (i.e. the expected change in the level of consumption between t and t+1) is:

(4) $\Gamma = (r-\delta)/\theta + (1/\theta)\log E_t[\exp(-\theta v_{t+1})]$ (> 0 for $r \geq \delta$)

with α: 1/(1+r), r the riskless interest rate,

W: non-human wealth,

[39]Later, however, the effect of a permanent change in the interest rate is analyzed. To be consistent with the assumptions made, this experiment must be thought of as the comparison of the savings path under two alternative interest rates.

δ: discount rate,

θ: coefficient of absolute risk aversion.

The term that involves the Γs in (2) is what has been called income-precautionary savings in the literature (the word income has been added in order to distinguish from precautionary savings arising from uncertainty respect to the horizon; the latter will be preceded by the word horizon). The Γs are time varying because the variance of v is increasing over time, and Γ is increasing in the variance of v.[40] The latter rises with time because when income shocks are stationary the annuity value (the term after e_t in (3)) increases as time goes by.

If, on the other hand, income follows an AR(1) process in first differences:

(5) $y_{t+1}-y_t = \Phi^d(y_t-y_{t-1}) + e_{t+1}$ with $0<\Phi^d<1$

then the equivalents of (2) and (3) are:

$$(6)\ c_t = y_t + (y_t-y_{t-1})\alpha\Phi^d(1-(\alpha\Phi^d)^{T-t+1})/(1-\alpha\Phi^d) + W_{t-1}(1-\alpha)/\alpha(1-\alpha^{T-t+1}) - [\sum_{i=1}^{T-t}\alpha^i\sum_{k=1}^{i}\Gamma_{t+k-1}](1-\alpha)/(1-\alpha^{T-t+1})$$

and

$$(7)\ v_t = e_t[(1-(\alpha\Phi^d)^{T-t+1})/(1-\alpha\Phi^d)]$$

respectively.

In this case the annuity value decreases with time, hence Γ also decreases. Figures 1 and 2 show the path of Γ for different values of $\theta\sigma^2_e$, Φ and Φ^d (figures 1 and 2 differ only in that in the former $\theta\sigma^2_e=4$ whereas in the latter $\theta\sigma^2_e=8$).[41] All the numbers are normalized so they represent

[40] See chapter I.

[41] Γ is approximated by using the expression corresponding to a normal density of e ($\Gamma=\theta\sigma_v^2/2$). This is certainly a good approximation for the value of Γ in the case of bounded symmetric densities. See chapter I. The interest rate is assumed to be constant and equal to 4% a year.

percentage of average annual consumption. Three things are apparent. First, as noticed in chapter I, Γ is increasing in the degree of risk aversion, riskiness of labor income (for both of these results compare the values of the Γs in figure 1 with the value of the Γs in figure 2), and persistence of labor income shocks. Second, the finiteness of the horizon does not have much influence in the path of Γ except for the very final periods. This is especially true when Γ is large. And third, if the process of labor income is stationary in levels with a small autoregressive coefficient, the slope of the consumption path is not significantly affected by precautionary savings (see, for example, the path of Γ when $\Phi=0$).

After obtaining the consumption function, the savings function can be easily computed since $s_t=y_t+w_{t-1}[(1-\alpha)/\alpha]-c_t$.

Figures 3 to 6 show the expected path of savings (flow) and wealth through life when no initial wealth is inherited[42]. It is apparent that the model here described leads to a hump-shaped savings (stock) model (Harrod 1948). Income-precautionary savings have a role very similar to that of retirement in Modigliani-Brumberg(1954)'s life cycle model.

As noted earlier, it is also clear that in order for income-precautionary savings to be an important source of savings, income must exhibit strong persistence[43]. This certainly seems to be the case in real life (e.g. Campbell and Deaton 1987).

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[42]Through all these experiments $\theta\sigma^2_e=4$ and r=4%.

[43]Notice that if initial wealth is zero, the case in which $\Phi^d=0.4$ leads to negative consumption when young unless extremely good realizations of labor income occur. Therefore, the very large wealth accumulation shown in this is likely to be an upper bound (when the income realizations are such that the non-negativity constraints are never binding).

Now that, for the purposes of this section, the role of the distance from the final date has been assessed, assume that $T \to \infty$ so the formulae simplify. In particular:

(2') $c_t = b/(1-\Phi) + W_{t-1}(1-\alpha)/\alpha - \{\alpha/(1-\alpha)\}\Gamma + v_t/(1-\Phi L)$

(3') $v_t = e_t[(1-\alpha)/(1-\alpha\Phi)]$

(6') $c_t = y_t + (y_t - y_{t-1})\alpha\Phi^d/(1-\alpha\Phi^d) + W_{t-1}(1-\alpha)/\alpha - \{\alpha/(1-\alpha)\}\Gamma$

and

(7') $v_t = e_t/(1-\alpha\Phi^d)$

With this formulae at hand it is easy to analyze the implications of income-precautionary savings for interest rate elasticity of total savings. The response of savings to permanent changes in the real after tax rates of return is studied[44]. Summers(1981) shows that in the traditional two periods formulation the interest rate elasticity is downward biased. This happens because the two periods model does not take into account the effect of interest rates on human wealth.

Chapter I shows that once income-precautionary savings are taken into account, there are new wealth and substitution effects (in addition to those in Summers(1981))[45]. The new wealth effect can be seen directly in (2'). When the interest rate rises, the term $\alpha/(1-\alpha)$ goes down, i.e. the effective horizon (defined as 1/r) is reduced. Given Γ, this raises current consumption since there are "less" periods to save for. Therefore the new wealth effect reduces the response of savings to interest rate changes.

The substitution effect, on the other hand, comes through the effect of the effective horizon on the variance of v and hence on Γ. If income is stationary in levels, a shortening of the effective horizon raises the

[44] Summers (1984) argues that this is the question of primary concern.
[45] See propositions 3a and 3b in chapter I.

variance of v. The latter raises Γ and therefore increases the response of savings to interest rates changes. Clearly the opposite happens if income is stationary in first differences. And finally, if income follows a random walk there is no substitution effect.

Given the great persistence observed in empirical studies of labor income, it seems reasonable to believe that, overall, precautionary savings reduce the interest rate elasticity of savings.

Another issue to deal with is the relative importance of non-intergenerational transfers explanation of wealth accumulation. Kotlikoff and Summers(1981 and 1986) argue that the life cycle model is not the primary explanation of the US wealth formation. They claim that eliminating intergenerational transfers would reduce US wealth by at least 50%. They take this as the main proof to postulate that intergenerational transfers are the main source of wealth accumulation.

KS arrive to this conclusion using what they call "the direct calculation of the life cycle wealth". For this they take as null hypothesis the zero bequest implication of the certain horizon life cycle model. Afterwards, they compute the difference between consumption and (non-bequest) income profiles, and conclude that this difference cannot explain the bulk of wealth accumulation.

Here there is no retirement but there is also a zero final wealth condition, hence it is also affected by the KS criticism. In fact the KS critique generalizes to all the models that do not have intergenerational transfers as the main motive for wealth accumulation. From the point of view of this paper, KS have been successful in proving existence of intergenerational transfers (that need not be voluntary) but they are far from proving their relative importance. In fact, by measuring bequest-motive wealth as a residual they are biasing the conclusion towards intergenerational transfers.

The basic argument of KS for low importance of non-intergenerational transfer models of wealth accumulation is that consumption and income profiles look very similar hence their difference cannot generate much savings. However, this is misleading, as can be seen by making an analogy with project evaluation techniques, whenever one evaluates a project the comparison must be made between the situations with and without the project. In the context of this paper, with and without income-precautionary savings, taking as given the path of income. The slope of the latter has nothing to do with the income-precautionary savings motive, even though it will affect wealth accumulation.

For example, the previous section concluded that consumption is expected to grow by Γ units per period. If initial income is equal to c_0 and increases by Γ units per period, then there will be no wealth accumulation at all. However, given the interest rate, if income-precautionary savings did not exist there would be negative wealth accumulation since each consumer would dissave in the early years and pay back late in life. It is clear that given average income, a tilt in the deterministic slope of income will affect total wealth accumulation, but the amount of income-precautionary savings will remain unchanged. What KS attribute to non-intergenerational savings is in fact, in the setup of this paper, the sum of income-precautionary savings and a big negative savings due to the income profile. In their paper, KS ask what would happen if the intergenerational transfers did not exist, it seems fair to ask what would happen if precautionary savings did not exist. Considering that the ratio between private wealth and consumption in the US is around five, the numbers shown in tables 1, 2, 3, 6 and 7 below suggest effects considerably larger than 50% of total wealth accumulation[46].

Given this defense of non-intergenerational savings models, it seems rewarding to develop in more detail the income-precautionary savings model. The next section deals with the steady state and dynamic behavior of aggregate wealth resulting from income-precautionary savings.

III. Aggregation: The Certain Horizon Case

In order to highlight the precautionary savings effect, it is convenient from now on to assume that the rate of output and population growth, as well as the interest and discount rates, are all equal to zero.

It is also convenient to assume that labor income follows a random walk. This assumption is very useful since first, Γ becomes constant, and second, the marginal propensity to consume out of labor income is one throughout life, hence savings are isolated from the stochastic fluctuations of income.

In these circumstances the consumption function for an individual i whose final date is T_i, is:

(8) $c_{it} = y_{it} + W_{it-1}/(T_i-t+1) - \Gamma(T_i-t)/2$

[46]An even more extreme example of how misleading the KS approach can be, is to assume that there are bequests but these are received in mid or late life. If this is the case, non-liquidity constrained life cycle agents will borrow early in life against the bequest. As a result the KS procedure would reflect less life cycle savings although clearly the source of dissaving is precisely the intergenerational transfer.
For more details about bequests see section IV.

and savings (flow) can be defined as:

(9) $s_{it} \equiv y_{it} - c_{it} = \Gamma(T_i-t)/2 - W_{it-1}/(T_i-t+1)$

If no initial wealth was inherited, W_{it-1} corresponds only to accumulation due to income-precautionary savings, hence s_{it} can be written as follows:

(10) $s_{it} = \Gamma(T_i+1-2t)/2$

Summing over time,the total wealth accumulated at time t by the individual i is obtained:

(11) $W_{it} \equiv \sum_{j=1}^{t} s_{ij} = \Gamma(T_i-t)t/2$

Clearly the condition $W_{i0}=W_{iT}=0$ is satisfied by (11).

It is a well known result that in stationary economies with no bequest motives aggregate savings (flow) are zero in the steady state. The aggregate wealth, however, is certainly different from zero:

(12) $W \equiv \sum_{i=1}^{T} W_{it} = \Gamma T(T^2-1)/12$

with T the length of life.

W is the aggregate wealth due to income-precautionary savings, and is independent of the income profile and its realizations.

The residual of the income process should have bounded support, however it is not very misleading, and clearly simplifies the formulae, to assume that e is distributed normal $(0,\sigma^2)$. In this case:

(13) $\Gamma = (\theta/2)\sigma^2$

By doubling the degree of risk aversion or the variance of labor income, the steady state stock of wealth (due to precautionary savings) is also doubled[47]. Certainly an impressive relation. Table 1 shows how

[47]This one to one relation certainly depends on the constant disposition towards risk given by the utility function chosen here.

reasonable values of the coefficient of relative risk aversion (θc) and the standard deviation of labor income (in percentage terms σ/y), can generate sizable wealth accumulation. In fact, according to the Board of Governors of the Federal Reserve System(1981) the private wealth/private consumption ratio (W/C) in the US is around five. The income-precautionary saving by itself can generate values of W/C much larger than five.

Suppose now that there is an unexpected change in the degree of risk aversion and/or the level of labor income riskiness, so Γ changes. Equation (12) describes what happens in the new steady state. When the change occurs, however, there are individuals that had already lived some periods of their life under the old conditions. These individuals have path of savings different from those for people beginning their lives under the new regime. Appendix 1 shows the derivation of equation (14). The latter describes aggregate wealth h periods after the new conditions were set:

$$(14)\quad W^h = \Gamma^+ T(T^2-1)/12 - \{(\Gamma^+ - \Gamma^-)/2\} \sum_{k=h}^{T} (k-h)(T-k)$$

with Γ^+: new Γ, and

Γ^-: old Γ.

The transition lasts for T periods since by then all the individuals that were surprised by the new conditions are dead. Figure 7 plots W^h over average steady state consumption for a change in Γ/c from 2% to 4% a year. T is assumed to be equal to 100.

The speed of adjustment is faster at the beginning since the people that were already old when the change in conditions occurred are replaced by new, fully informed (respect to σ and θ), people. As time passes, however, those that are replaced by the latter are people that did have some time to partially adjust to the new conditions, hence the differences with the path of the new generations, and therefore the speed of adjustment, are reduced.

The chief contribution of this section has been to show how income-precautionary savings can generate sizable wealth stocks. It has also shown the transition towards a new steady state after a change in the coefficient of risk aversion and/or the degree of riskiness of labor income. In the next section uncertainty in the length of life is introduced. Special emphasis is put in studying the generation and implications of involuntary bequests. This will set up the framework to study, in section V, the welfare implications of income stabilization policies.

IV. Uncertain Horizons

The formal treatment of the role of uncertain horizons on the consumption-saving profiles starts with the work of Yaari(1965). He shows that if no fair annuity markets exist, the consumption path is flatter than under certainty since future periods are discounted more heavily.

Levhari and Mirman(1977) study the effect of changes in the degree of life-horizon uncertainty on the level of current consumption. Their main contribution is to disentangle the effects of a change in the probability of dying before T on current consumption. They show that an increase in the probability of dying before T has two effects: first, taking the expected life as given, savings tend to increase because of the risk of living longer than expected. And second, for a given T, savings are reduced because expected lifetime is shortened. Once the latter is compensated for, current consumption is almost always reduced by an increase in the probability of dying before T. Davies(1981) performs a series of simulations and concludes that the slow dissaving of the elderly may be explained by the horizon uncertainty.

All these previous studies refer to the behavior of a single representative agent, given initial wealth. If, on the other hand, an overlapping generation model is considered, then changes in the saving behavior of one generation affects the bequests received by the oncoming generations. Abel(1983) uses a two periods overlapping generation model to study these interrelations. He shows that changes in the economic environment can have effects on aggregate behavior which differ dramatically from the effects on individual behavior because of the endogenous adjustment of bequests. Using this setup he shows that introducing a fully funded social security system reduces savings.

Kotlikoff, Shoven and Spivak(1983) construct a four periods model with uncertainty only in the final period. They compare the involuntary bequest model with the perfect and the family insurance cases. Their conclusion is that any form of insurance has substantial effect on aggregate wealth accumulation.

Hubbard(1984) uses a very simple life-cycle model to study the general equilibrium effect of the introduction of a funded social security system. He shows that consumption of future generations is reduced due to the effect of the reduction in accidental bequests on the aggregate capital stock and factor prices[48].

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[48] Some other papers relevant for the topic here surveyed are: Fischer(1973), Sheshinsky and Weiss(1981), Abel(1985) and, Eckstein, Eichenbaum and Peled(1985).

This chapter goes back to the non-insurance partial equilibrium model but extends it in two directions. First, a multiperiod problem with strictly positive probability of dying in any period is solved. Second, and more importantly, this paper concentrates on the study of the interaction of this type of uncertainty with labor income uncertainty in generating wealth accumulation and involuntary bequests. In the next section some welfare considerations are made by adding a very simple general equilibrium structure.

It should be remembered that this model does not have retirement hence the standard idea of horizon-precautionary savings arising to cover the possibility of a longer than expected period after retirement does not apply. However, a very similar concept applies here as long as the consumption path is stepper than the income path. In fact, the assumptions here made are such that if income-precautionary savings do not exist, there is no wealth accumulation at all[49]. Individuals expect to receive today's income for as long as they live, hence living some extra periods per-se does not generate horizon-precautionary savings.

So, let the same assumptions of the previous section prevail. The only modification is to admit a strictly positive probability of dying before T_i. In addition assume that there are no markets for annuities. Then, as shown in appendix 2:

$$(15)\quad s_{it} = \sum_{j=1}^{T_i-t} \sum_{k=1}^{j} \Gamma_{t+k}^{*i}/(T_i-t+1) - W_{it-1}/(T_i-t+1)$$

$$(16)\quad \Gamma_{t+k}^{*i} \equiv \Gamma - (1-p_{t+k}^{i})/\theta$$

with p_{t+k}^{i} : probability (for individual i) of being alive in

[49]Moreover, when there is a strictly positive probability of dying before T, average wealth is negative.

period t+k+1, conditional on being alive in period t+k.

Replacing (16) in (15):

$$(15')\quad s_{it} = \Gamma(T_i-t)/2 - [W_{it-1} + \sum_{j=1}^{T_i-t} \sum_{k=1}^{j} (1-p_{t+k}^{i})/\theta]/(T_i-t+1)$$

To simplify things even further, assume that $p_{t+h}^{i}=p$ a constant for periods 1 to T_i-1 ($p_{T_i}\equiv 0$). Then (15') reduces to:

$$(15'')\quad s_{it} = \Gamma^*(T_i-t)/2 - W_{it-1}/(T_i-t+1)$$

This expression is similar to equation (9) in the previous section. The main difference is that now there are bequests (involuntary), hence W_{it-1} is formed not only by income-precautionary savings accumulation but also by bequests. If bequests are received at the beginning of life, then:

$$(18)\quad s_{it} = \Gamma^*(T_i+1-2t)/2 - W_{i0}/(T_i-t+1)$$

and summing over time,

$$(19)\quad W_{it} = \Gamma^*(T_i-t)t/2 + W_{i0}(1-t/T_i)$$

The aggregate wealth is defined as $W\equiv\Sigma_i p^i W_{it}$.

In equilibrium, bequests (W_{i0}) must be equal to:

$$(20)\quad W_{i0} = \sum_{t=0}^{T^*} p^t(1-p)W_t = (1-p)W$$

The steady state aggregate wealth is obtained by replacing (20) in (19) and aggregating over individuals:

$$(21)\quad W = \Gamma^* T[(T-1)(1-p^{T+1})-p(T+1)(1-p^{T-1})]/2[(1-p^T)(1-p)^2]$$

Table 2 presents the average steady state wealth to consumption ratio. Table 3 is the analog of table 2 but a correction is made on T so expected lifetimes are comparable with those of table 1.[50]

[50] T^* and T are related by $T^*=[\log(p-T(1-p)/\log p -1]$. For a given T, an increase in p raises T^* in order to keep the expected lifetime constant.

In the individual agent model with exogenous initial wealth, an increase in the probability of dying without altering T reduces, unambiguously, wealth accumulation[51]. This results from the increase in the discounting of future periods consumption and the shortening of lifetime. Table 2 shows that once the endogenous behavior of bequests is considered, the previous result is no longer unambiguous. For a low income-precautionary savings motive this is still true. However, if the income-precautionary savings motive is strong, the increase in bequests dominates the shortening of expected lifetime effect, and savings actually rise after an increase in the probability of dying before T.

Table 3 confirms the results found in Levhari and Mirman(1977) and Davies(1981). Once the expected lifetime is kept constant, an increase in the probability of dying raises wealth accumulation (horizon-precautionary savings) for reasonable parameter values. As will be seen later, this no longer holds when bequests are received later in life.

It is also apparent from comparing tables 1, 2 and 3, that the existence of horizon uncertainty makes stronger the effects of changes in the degree of riskiness of labor income or degree of risk aversion on steady state wealth.

Kotlikoff et al. stress the fact that in their model wealth accumulation is very sensitive to the income profile but not to the degree of risk aversion. In the model of this paper, on the other hand, by taking explicitly into account the income-precautionary savings effect it is possible to see that the coefficient of risk aversion has very important implications for wealth accumulation. The coefficient of risk aversion

[51] See Yaari(1965) and Levhari and Mirman(1977).

significantly affects the slope of the consumption path, hence it has an effect analogous to a change in the income profile in the Kotlikoff et al. model.

Finally, the wealth due to bequests, B, is:

$$(22)\ B = W_{i0}\left[\sum_{t=0}^{T} p^t - (1/T)\sum_{t=0}^{T} tp^t\right]$$

or

$$(22')\ B = W\{1-p(1-p^T)/T(1-p)\}$$

Then the ratio of wealth due to bequests to total wealth is:

$$(23)\ B/W = 1 - p(1-p^T)/T(1-p)$$

Tables 4 and 5 show the ratio of bequest to total wealth accumulation under different horizons and conditional probability of surviving one extra period (p). As stressed by many other authors, even though there is no bequest motive at all, it is still possible to generate sizable amounts of accidental bequests related wealth.

If bequests are not received at birth but at time t* later in life, and people can borrow against these bequests, or alternatively income precautionary savings are enough to finance the consumption against future bequests, then both the stock of wealth and the proportion of bequest originated wealth are reduced.

Equation (24) below is the generalization of equation (19) and describes wealth accumulation by individual i at time t when bequests are received at time t*:

$$(24)\ W_{it} = \Gamma^*(T_i-t)t/2 - W_i^*(t/T_i) \qquad \text{for } t<t^*$$

$$= \Gamma^*(T_i-t)t/2 + W_i^*(1-t/T_i) \qquad \text{for } t\geq t^*$$

Put in other words, for a given transfer, the path of consumption is the same as before (when t* was equal to t_0). Bequests, however, are received later in life, hence average wealth accumulation is smaller. As a result the aggregate steady state stock of wealth is also smaller. In fact, if the aggregate wealth when bequests are received at time t* is called W*, it is easy to show that (see appendix III):

(25) $W^* = p^{t^*}W$

Tables 6 and 7 show the equivalents to table 3 (W*/C) for t* equal to T*/2 and T* respectively. It is possible to see that if the income-precautionary savings motive is not very strong, wealth accumulation can be reduced by an increase in the probability of dying early, even when expected life is kept constant. This is certainly contradictory with the results found in models with exogenous determination of bequests.

Additionally, the importance of bequests as a source of wealth accumulation is reduced due to the early dissaving they originate. This effect is reflected in an extra negative term in equation (26) (respect to equation (23)):[52]

(26) $B^*/W^* = 1 - p(1-p^T)/T(1-p) - p^{T-t^*+1}(1-p^{t^*})$

Tables 8 and 9 show the equivalents of table 5 when t* is equal to T*/2 and T* respectively. The last example is extreme, however it is useful to realize that observed intergenerational transfers can be a very misleading measure of the importance of bequests in the aggregate stock of wealth.

In the next section some of these results are used to study the welfare implications of income stabilization policies.

[52]Note that (B*/W*) is clearly decreasing on t*:
$d(B^*/W^*)/dt^* = p^{T-t^*+1}\log(p) < 0$

V. Welfare Considerations

The public finance-macroeconomics literature has used the uncertain horizon framework to study the welfare implications of funded social security systems. Here, on the other hand, the model constructed has no retirement, hence it isolates the interaction between labor income uncertainty, capital accumulation and lifetime uncertainty.

The standard mechanism (e.g. Abel 1983, Kotlikoff et al. 1983, and Hubbard 1984) is to have a model in which the interaction between retirement and uncertainty in the length of life generates precautionary savings in addition to those generated by the retirement effect itself. The individual does not know how many periods after retirement he will live. Under reasonable assumptions on the functional form of preferences, the individual not only saves for the years he expects to live after retirement, but also for the eventuality of living beyond the expected lifetime. The latter is what has been called (horizon) precautionary savings[53].

[53]In the model of this chapter, on the other hand, there is no retirement, however the presence of income-precautionary savings produce an effect similar to it. If income-precautionary savings motives are strong enough (to offset the discounting effect of the probability of dying before T) then the expected consumption path is upward sloped. Given the flat expected income path assumed, if the individual lives longer than expected, consumption will be substantially larger than income, therefore financial wealth will be required to satisfy the gap. The last interaction is what gives origin to the horizon-precautionary savings.
Here all the wealth accumulation is ultimately due to income-precautionary savings hence the role of the degree of risk aversion is, contrary to the case of the "social security" models, crucial. In some sense, besides the standard roles of the coefficient of risk aversion, it accomplishes the same as the slope of the income path in the "social security" models.

In the absence of annuity markets these models lead to accidental bequests. The introduction of annuity markets is welfare improving for current generations, however the implied reduction in accidental bequests has a negative impact on the capital stock and as a result reduces welfare of future generations. As is standard in selfish overlapping generations models, the opening of a new market is not necessarily welfare improving for future generations. This framework is particularly suitable to study funded social security systems since the latter have the role of compulsory annuities.

In this essay, on the other hand, the main concern is not a social security system but policies that reduce labor income uncertainty. Any policy that reduces the degree of riskiness of labor income will clearly benefit current generations because of two reasons: first, they are risk averse, hence any reduction in uncertainty improves their welfare even if the consumption-savings profile does not change at all, and second, the consumption-saving profile does change in the direction of reducing savings. The latter reduces the average accidental bequest left, raising average consumption.

Future generations will add to these gains a third and negative effect since the reduction in bequests left by previous generations imply less initial wealth for them.

In the model presented up to here factor prices are not endogenous, and the interest rate (international) is assumed to be equal to zero, hence average consumption cannot change in the new steady state. This implies that the second and third effects cancel each other; the decrease in the accidental bequest left is identical to the decrease in the bequest received. However, the first effect remains, therefore the new steady state is associated with a higher level of welfare for every generation.

If factor prices are endogenized, future generations are affected by the reduction in the (now productive) capital stock. Which effect dominates depends on the preference and technology parameters. Figures 8 and 9 show some simulations of welfare gains for future generations after reducing the variance of labor income by twenty five per-cent starting from different uncertainty levels[54].

It is assumed that one unit of reduction in the capital stock represents 1/4 unit less of steady state consumption (output). Once more the degree of risk aversion is crucial. When θ is large the effect of the reduction in the stock of capital is significantly surpassed by the direct welfare effect of the reduction in uncertainty[55]. It is only for very small values of the coefficient of risk aversion that future generations are worse off. In general, if individual labor income uncertainty can be partially removed or insured, doing it is Pareto improving.

VI. CONCLUSION

This chapter has studied the role of labor income and horizon uncertainties in explaining wealth accumulation and intergenerational transfers. Among other things, the following results were derived: (i) labor income uncertainty reduces the response of savings to changes in the

[54]The only difference between figures 8 and 9 is that in the former the average level of consumption is kept constant across θ, whereas in the latter the output/capital ratio is kept constant.
[55]Certainly intermediate generations will also be better off since they enjoy a larger capital stock than the new steady-state generations.

real interest rate, (ii) precautionary motives can generate steady state levels of wealth far beyond those of the US economy, (iii) the coefficient of risk aversion and the timing of bequests play a crucial role, not only on the level of wealth and importance of intergenerational transfers, but also on the response of savings to changes in the probability of dying before the "last period", and (iv) unless the degree of risk aversion of consumers is very low, income stabilization policies benefit not only current but also future generations, even when individuals are completely selfish.

To conclude, it seems important to stress that according to these results it is reasonable to believe that precautionary savings are a major source of wealth accumulation. However the motives for these savings are also a major source of reduction in welfare and in the response of savings to interest rate changes.

APPENDIX I

Derivation of Equation (14)

It is easy to modify equation (10) in order to allow for initial endowment:

(A.1) $s_{it} = \Gamma(T_i+1-2t)/2 - W_{io}/T_i \quad t=1,\dots.T_i$

Aggregating over time we obtain:

(A.2) $W_{it} = \Gamma t(T_i-t)/2 + W_{io}(1-t/T)$

To solve for the dynamics after a surprise in Γ the problem is re-started, given wealth accumulated up to that time. Therefore, if i is the age of individual i at the time of the surprise, then individual wealth accumulation h periods after the shock is:

(A.3) $W_i{}^h = \Gamma^+h(T_i-h-i)/2 + W_{ii}(1-h/(T_i-i))$

with W_{ii}: wealth accumulated by individual i at the time of the surprise.

But $W_{ii}=\Gamma^- i(T_i-i)/2$. Replacing this in (A.3) we obtain:

(A.4) $W_i{}^h = W_{ih}{}^+ - i(T_i-h)(\Gamma^+-\Gamma^-)/2$

with $W_{ih}{}^+= \Gamma^+(i+h)(T_i-i-h)/2$.

Equation (14), here called (A.5), is obtained by summing (A.4) across individuals:

(A.5) $W^h = \Gamma^+T(T^2-1)/12 - \{(\Gamma^+-\Gamma^-)/2\} \sum_{k=h}^{T} (k-h)(T-k)$

APPENDIX II

Solution of the Uncertain Horizon Problem

The reader is referred to chapter I to see the details of the optimization technique. Here the solution is only sketched.

The problem of the representative consumer can be written as follows:

$$\text{Max } E_{T,y,t}\left[\sum_{i=0}^{T-t} \beta^i U(c_{t+i}) \right. \Leftrightarrow \text{Max } \sum_{i=0}^{T-t} \left(\prod_{j=0}^{i} p_{j-1} \right) \beta^i E_t[U(c_{t+i})]$$

s.t.

$$\sum_{i=0}^{T-t} \alpha^i (c_{t+i} - y_{t+i}) \leq \alpha^{-1} W_{t-1}$$

If the instantaneous utility function is exponential, then the stochastic process of consumption is:

$$\text{(A.6)} \quad c_{t+i} = \Gamma^*_{t+i-1} + c_{t+i-1} + v_{t+i}$$

with $\Gamma^*_t = (r - \delta + \log(p_t))/\theta + (1/\theta)\log E_t[\exp(-\theta v_{t+1})]$

The consumption function is obtained by substituting (A.6) and the process of income in the intertemporal budget constraint, and using the condition that if individual i lives to the final date then the intertemporal budget constraint must be satisfied with an equality:

$$\text{(A.7)} \quad c_t = y_t + W_{t-1}/(T-t+1) - \left[\sum_{i=1}^{T-t} \sum_{k=1}^{i} \Gamma^*_{t+k-1} \right]/(T-t+1)$$

$$- \left[\sum_{i=1}^{T-t} \sum_{k=1}^{i} (v_{t+k} - e_{t+k}) \right]/(T-t+1)$$

But consumption is known at time t hence the last term must cancel out. Furthermore, both v and e have expected value equal to zero. The unique solution then is $v_{t+k} = e_{t+k}$ for all k. Equation (15) follows from the definition $s \equiv y - c$.

APPENDIX III

Model with bequests received at time t*

W* is obtained by aggregating equation (24) across individuals we obtain:

$$\text{(A.8)}\quad W^* = \sum_{i=0}^{T} p^i W_i = W - W^*_1 \sum_{i=0}^{t^*-1} p^i$$

In equilibrium W^*_1 must be equal to the wealth left by those who die in each period, divided by the size of the cohort that receives the bequest:

$$\text{(A.9)}\quad W^*_1 = [(1-p)/p^{t^*}]W^*$$

Aggregate wealth is simply obtained by replacing (A.9) in (A.8).

It is also easy to derive the ratio of wealth generated by bequests, B*, to total wealth, W*:

$$\text{(A.10)}\quad B^* = W^*_1 \left(\sum_{i=t^*}^{T} p^i - (1/T) \sum_{i=0}^{T} ip^i \right)$$

then

$$\text{(A.11)}\quad (B^*/W^*) = 1 - p(1-p^T)/T(1-p) - p^{T-t^*+1}(1-p^{t^*})$$

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TABLE 1

WEALTH TO CONSUMPTION RATIO (W/C) [STEADY STATE]

CERTAIN HORIZON

T	($\theta c, \sigma/y$)		
	(2,0.1)	(2,0.2)	(10,0.1)
40	1.33	5.33	6.66
50	2.08	8.33	10.41
60	3.00	12.00	15.00
100	8.33	33.33	41.66

TABLE 2

WEALTH TO CONSUMPTION RATIO (W/C) [STEADY STATE]

UNCERTAIN HORIZON

p	T	$(\theta c, \sigma/y)$		
		(2,0.1)	(2,0.2)	(10,0.1)
	40	1.10	5.51	7.77
0.995	50	1.76	8.82	11.64
	60	2.60	13.00	17.16
	100	7.91	39.56	52.23
	40	0.89	5.65	7.91
0.990	50	1.45	9.23	12.93
	60	2.18	13.89	19.45
	100	7.15	45.47	63.66
	40	0.44	5.74	8.56
0.985	50	0.74	9.55	14.26
	60	1.13	14.63	21.83
	100	3.89	50.62	75.54

TABLE 3

WEALTH TO CONSUMPTION RATIO (W/C) [STEADY STATE]

UNCERTAIN HORIZON

p	T* (T)	(θc, σ/y)		
		(2,0.1)	(2,0.2)	(10,0.1)
	45(40)	1.41	7.06	9.32
0.995	58(50)	2.42	12.09	15.96
	73(60)	3.97	19.84	26.19
	139(100)	16.63	83.16	109.77
	52(40)	1.44	10.08	14.11
0.990	70(50)	2.82	19.74	27.63
	93(60)	5.47	38.27	53.58
	62(40)	1.22	15.81	23.59
0.985	95(50)	3.43	44.54	66.47
	162(60)	13.37	173.82	259.39

TABLE 4

RATIO OF BEQUEST ORIGINATED WEALTH TO TOTAL WEALTH (B/W)

p	T			
	40	50	60	100
0.995	0.096	0.118	0.139	0.215
0.990	0.181	0.218	0.253	0.372
0.985	0.255	0.304	0.347	0.488

TABLE 5

RATIO OF BEQUEST ORIGINATED WEALTH TO TOTAL WEALTH (B/W)

p				
T*	45	58	73	139
0.995	0.107	0.134	0.165	0.282
T*	52	70	93	
0.990	0.225	0.286	0.354	
T*	62	95	162	
0.985	0.356	0.473	0.630	

TABLE 6

WEALTH TO CONSUMPTION RATIO (W/C) [STEADY STATE]

UNCERTAIN HORIZON (t*=T*/2)

p	T* (T)	(θc,σ/y) (2,0.1)	(2,0.2)	(10,0.1)
	45(40)	1.26	6.31	8.33
0.995	58(50)	2.09	10.45	13.80
	73(60)	3.31	16.52	21.81
	139(100)	11.74	58.70	77.48
	52(40)	1.11	7.76	10.87
0.990	70(50)	1.98	13.89	19.44
	93(60)	3.43	23.98	33.58
	62(40)	0.76	9.90	14.77
0.985	95(50)	1.67	21.73	32.42
	162(60)	3.93	51.10	76.26

TABLE 7

WEALTH TO CONSUMPTION RATIO (W/C) [STEADY STATE]

UNCERTAIN HORIZON (t*=T*)

p	T* (T)	(θc,σ/y)		
		(2,0.1)	(2,0.2)	(10,0.1)
	45(40)	1.13	5.63	7.44
0.995	58(50)	1.81	9.04	11.93
	73(60)	2.75	13.76	18.16
	139(100)	8.29	41.43	54.69
	52(40)	0.85	5.98	8.56
0.990	70(50)	1.40	9.77	13.67
	93(60)	2.15	15.03	21.04
	62(40)	0.48	6.19	9.24
0.985	95(50)	0.82	10.60	15.81
	162(60)	1.16	15.02	22.42

TABLE 8

RATIO OF BEQUEST ORIGINATED WEALTH TO TOTAL WEALTH (B*/W*)

(t*=T*/2)

p				
T*	45	58	73	139
0.995	0.012	0.018	0.026	0.075
T*	52	70	93	
0.990	0.050	0.079	0.122	
T*	62	95	162	
0.985	0.125	0.227	0.426	

TABLE 9

RATIO OF BEQUEST ORIGINATED WEALTH TO TOTAL WEALTH (B*/W*)

(t*=T*)

p				
T*	45	58	73	139
0.995	-0.094	-0.117	-0.140	-0.217
T*	52	70	93	
0.990	-0.184	-0.214	-0.247	
T*	62	95	162	
0.985	-0.243	-0.247	-0.270	

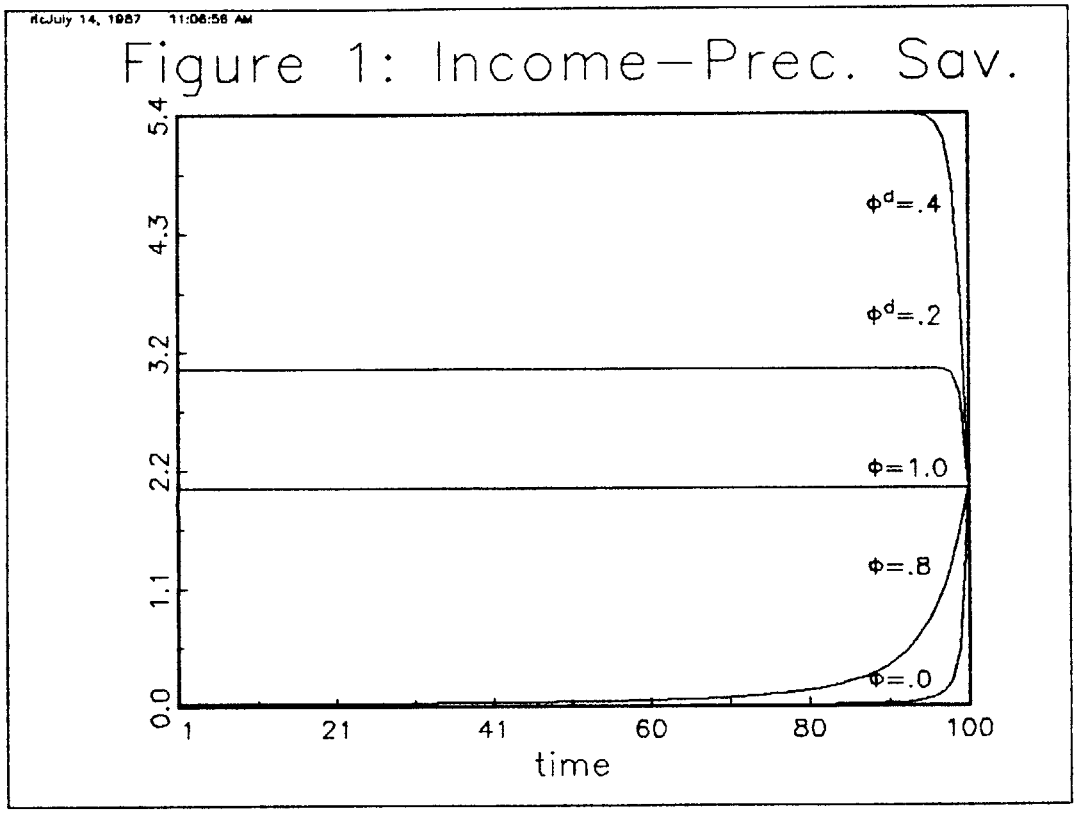
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Figure 1: Income–Prec. Sav.
5.4
4.3
3.2
2.2
1.1
0.0
1
21
41
60
80
100
time
φd=.4
φd=.2
φ=1.0
φ=.8
φ=.0

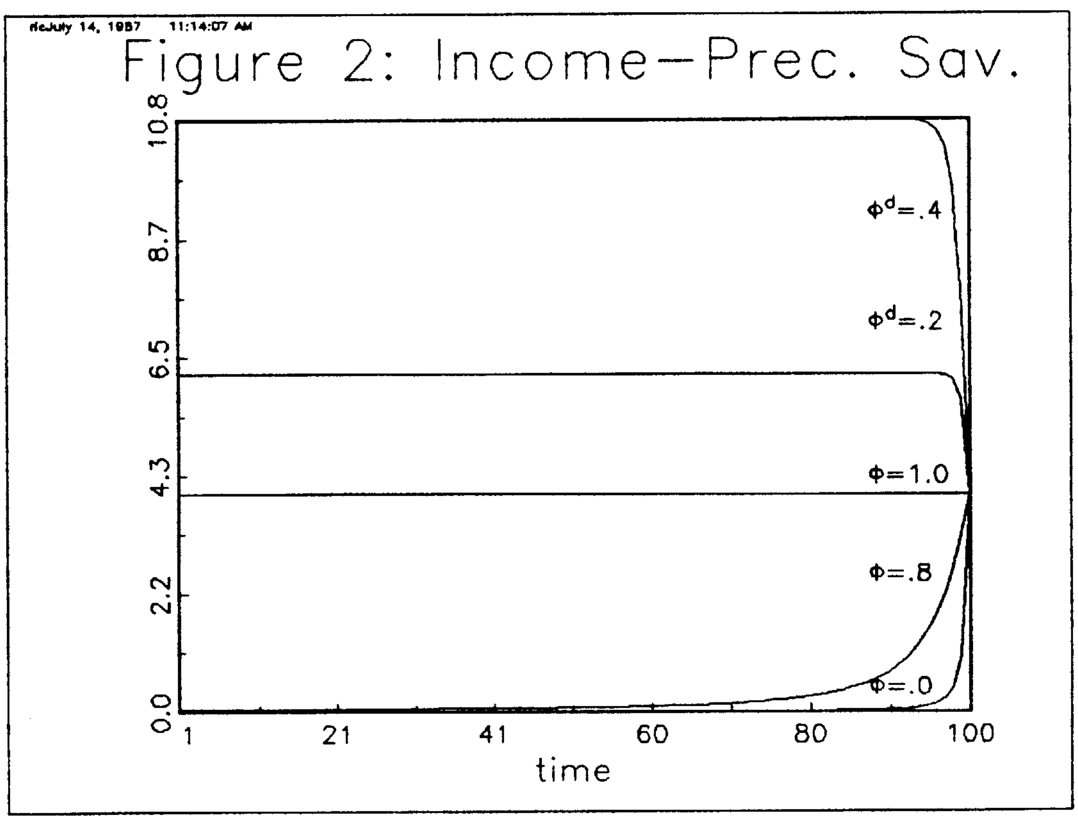
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Figure 2: Income–Prec. Sav.
10.8
8.7
6.5
4.3
2.2
0.0
1
21
41
60
80
100
time
φd=.4
φd=.2
φ=1.0
φ=.8
φ=.0

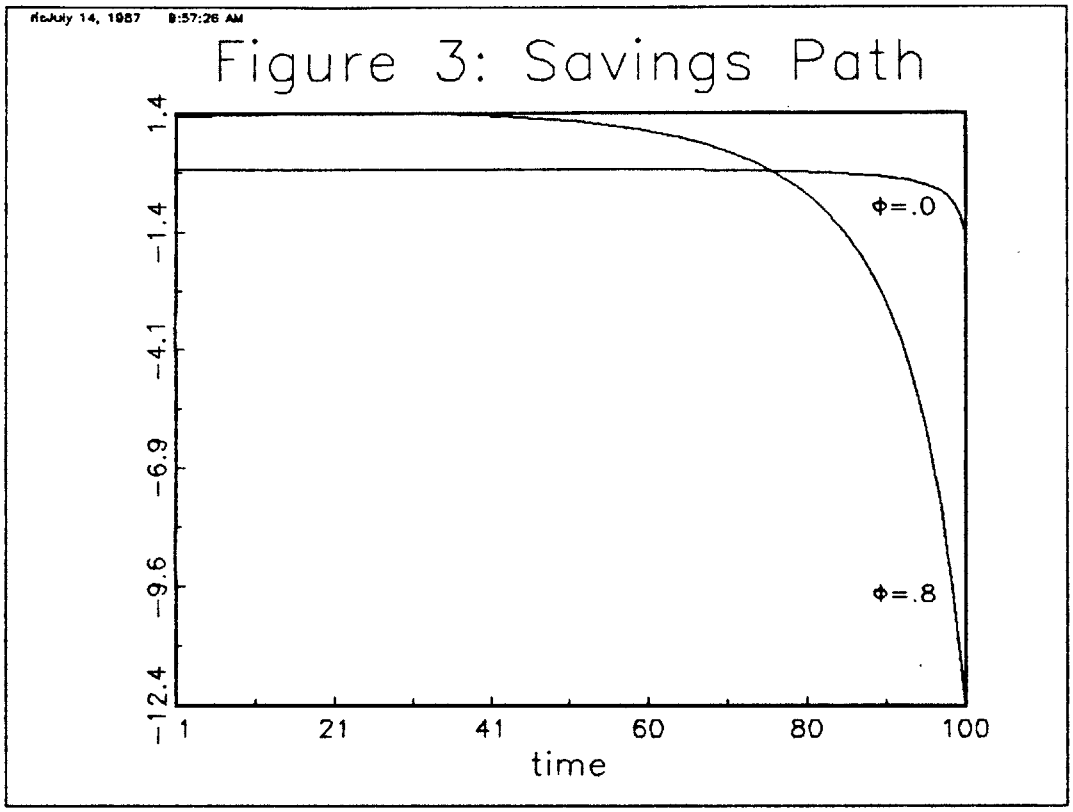
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Figure 3: Savings Path
1.4
-1.4
-4.1
-6.9
-9.6
-12.4
Φ=.0
Φ=.8
1
21
41
60
80
100
time

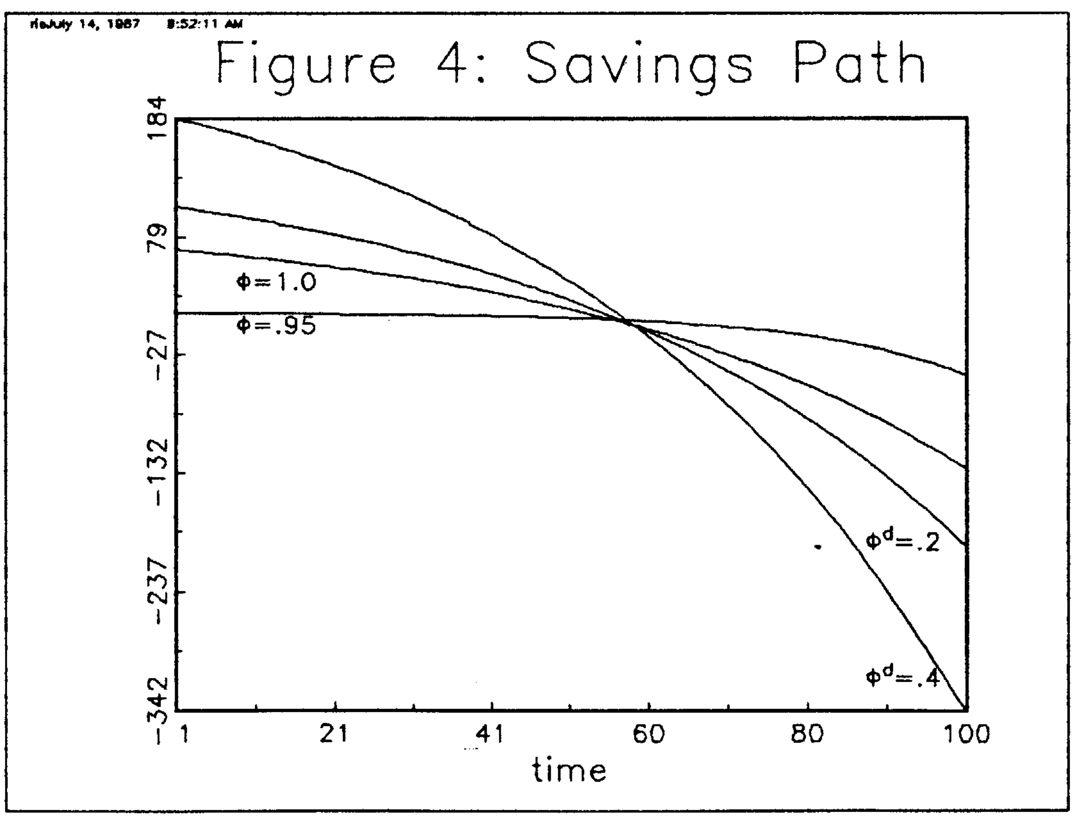
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Figure 4: Savings Path
184
79
-27
-132
-237
-342
Φ=1.0
Φ=.95
Φ^d=.2
Φ^d=.4
1
21
41
60
80
100
time

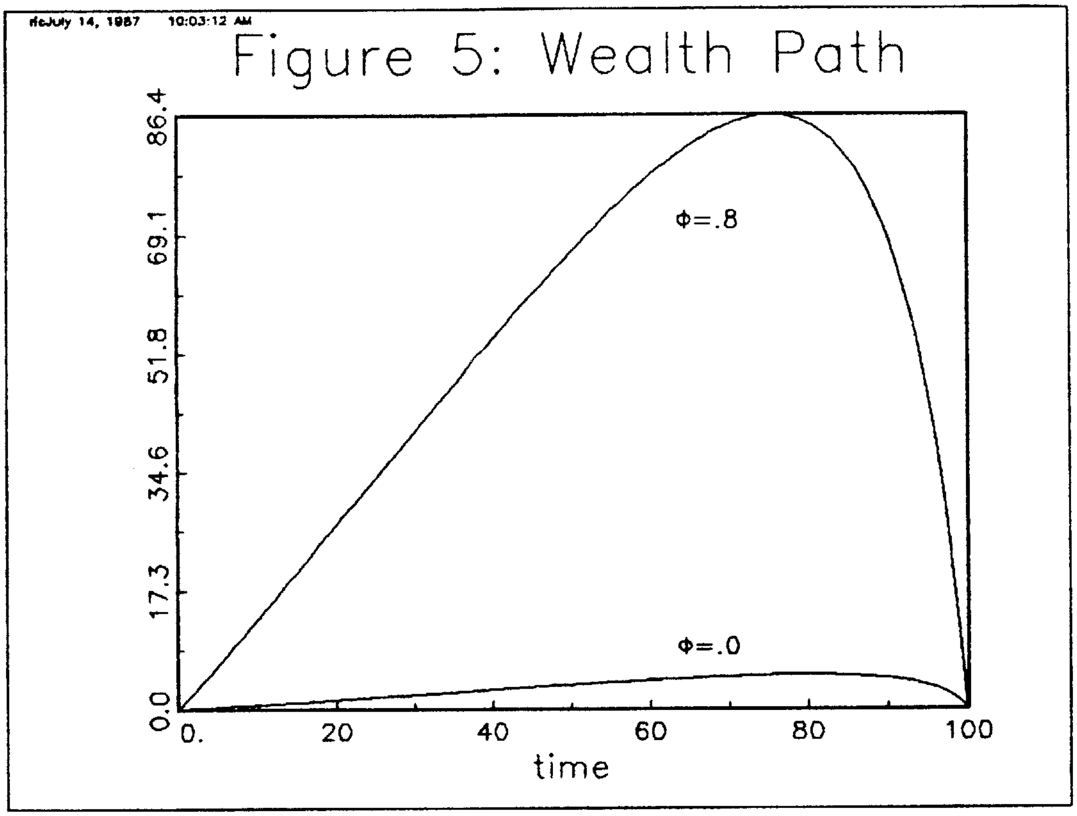

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Figure 5: Wealth Path
86.4
69.1
51.8
34.6
17.3
0.0
0.
20
40
60
80
100
time
Φ=.8
Φ=.0

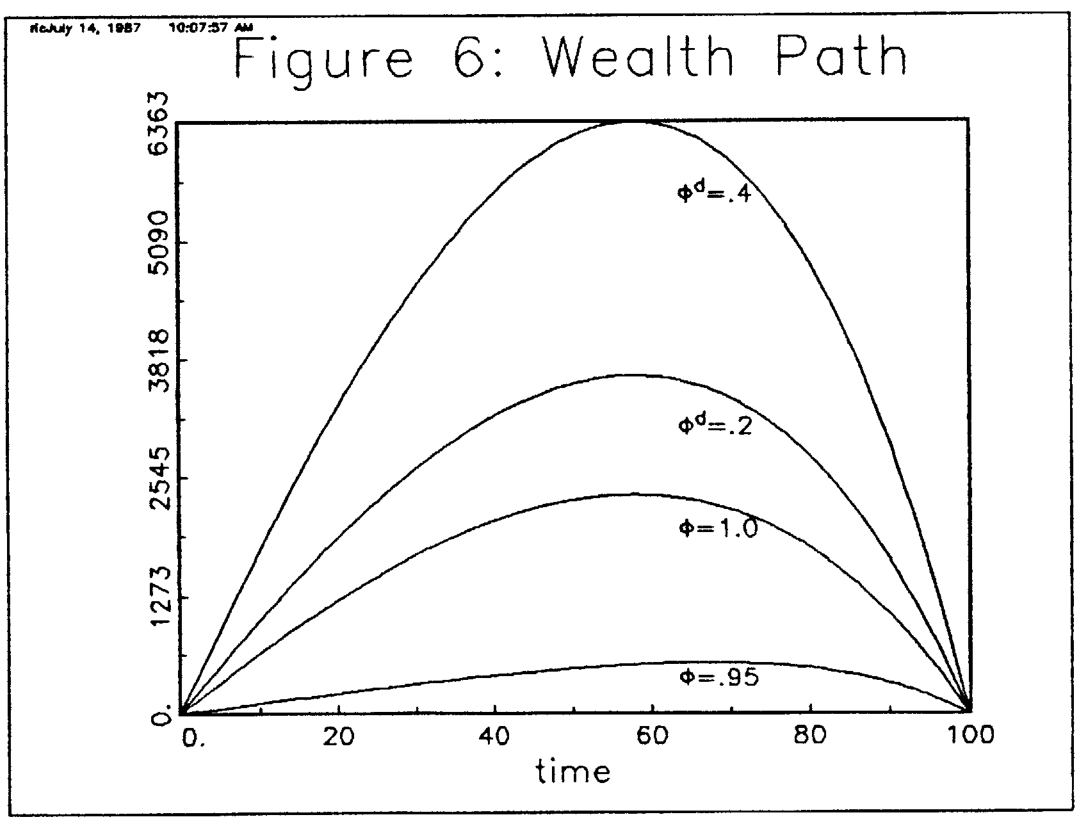

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Figure 6: Wealth Path
6363
5090
3818
2545
1273
0.
0.
20
40
60
80
100
time
Φd=.4
Φd=.2
Φ=1.0
Φ=.95

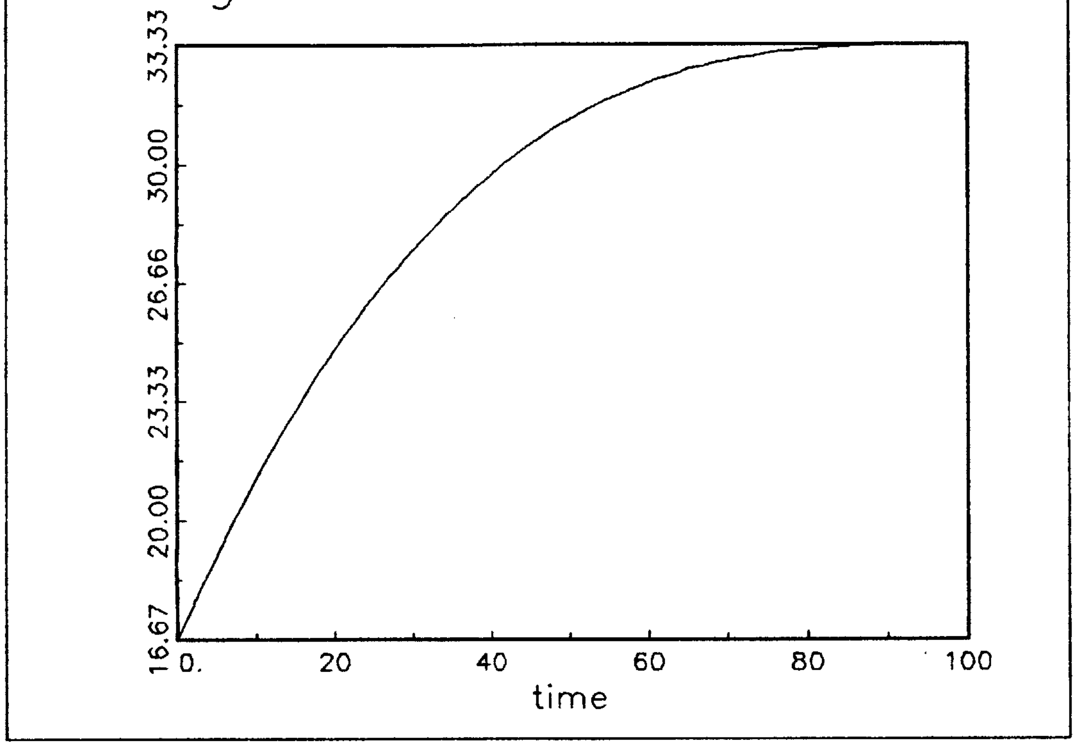

Figure 7: Transition Wealth

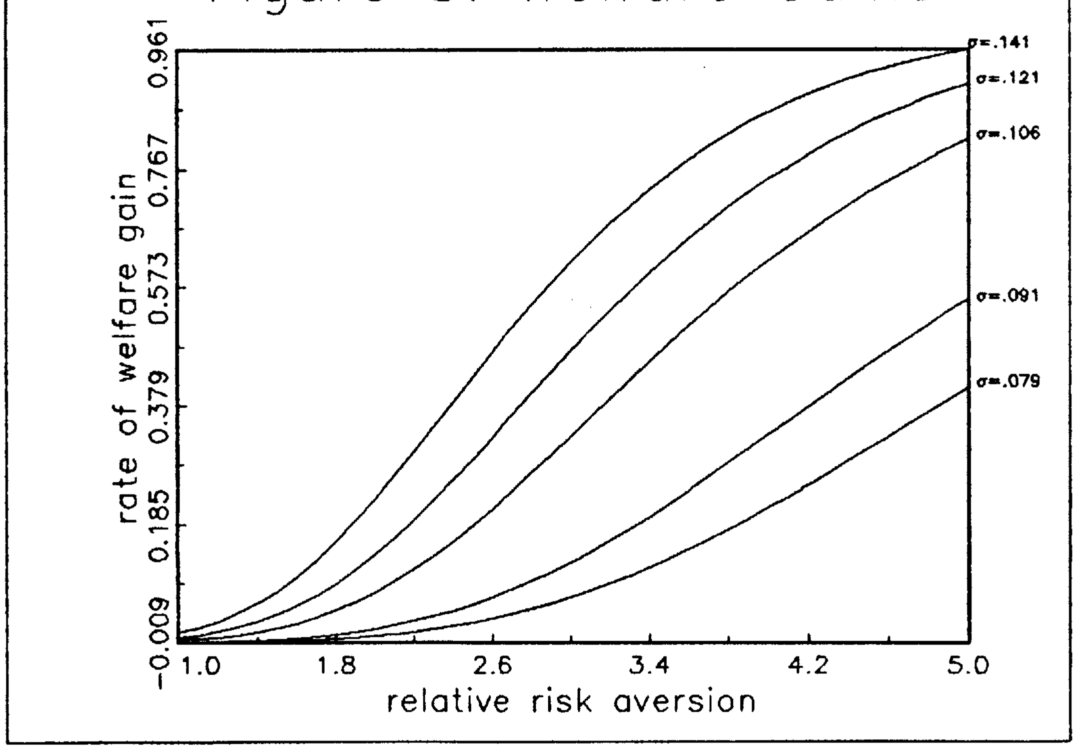

Figure 8: Welfare Gains

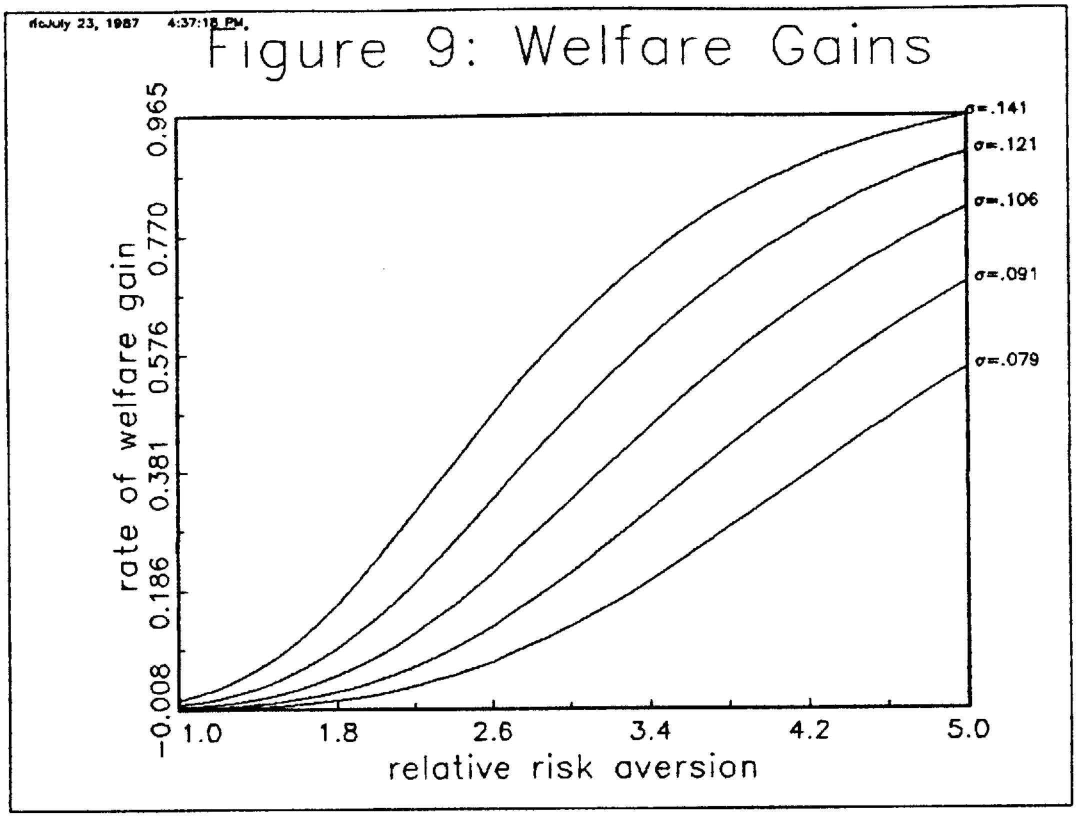

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Figure 9: Welfare Gains
rate of welfare gain
-0.008
0.186
0.381
0.576
0.770
0.965
1.0
1.8
2.6
3.4
4.2
5.0
relative risk aversion
σ=.141
σ=.121
σ=.106
σ=.091
σ=.079

CHAPTER III

THE ROLE OF TASTE SHOCKS IN CONSUMPTION FLUCTUATIONS

I. INTRODUCTION

The study of the role of consumption in the business cycle has not been an issue of primary concern in the postwar economic literature. This negligence has been rationalized by the small fluctuations in consumption as compared to fluctuations in other aggregate demand components. Studies of aggregate demand behavior have concentrated in understanding inventories and capital investment fluctuations (e.g. Blinder and Fischer 1981, Kydland and Prescott 1982, Abel and Blanchard 1986 and Bernanke 1983).

Exceptions are the works by Temin(1976) and Hall(1984). They both studied the unexplained component of consumption. The former concluded that the Great Depression had been caused by an unexplained shift in autonomous consumption. Hall studied the postwar period and concluded that shifts in consumption have been an important source of overall fluctuations in the aggregate economy. Recently, Fair(1986) performed simulation experiments with his macroeconometric model of the US (Fair 1984) and arrived at the same conclusion.

As these papers demonstrate the fact that one aggregate demand variable (e.g. consumption) fluctuates less than another (e.g. investment) does not say anything about the relative magnitude of their unexplained components, let alone about their relative importance in total aggregate demand fluctuations. Volatility is usually measured by the conditional variance of reduced form residuals. According to this metric, however, there will always be a combination of structural parameters and covariances that could imply that shocks to consumption are a major business cycle driving force, even though consumption itself does not fluctuate much. It is not in the scope of this paper to study the "transmission mechanisms", but their existence is crucial to make meaningful, for understanding the business cycle, the study of consumption's residuals.

Shocks to consumption can basically be divided in two types[1]: (i) wealth shocks[2] and, (ii) taste shocks[3]. For the purposes of this chapter a wealth shock will be defined as a perturbation that affects the perceived intertemporal budget constraint but not the utility function. Taste shocks will be defined as the complement, i.e. shocks that affect the utility function but not the budget constraint.

Taste shocks have a long history in the literature on consumption behavior. The most typical example is the idea of transitory consumption. However, to my knowledge, their relative importance in consumption fluctuations has not been assessed. This paper tries to do this.

The behavior of consumption in the presence of both wealth and taste shocks is derived, and the differences between the two in terms of the implications for the stochastic process followed by consumption are established. The main conclusion is that taste shocks have not been an important source (as compared to wealth shocks) of aggregate (non-durables) consumption fluctuations after World War II.

II. THEORY

Consider the intertemporal optimization problem faced, at time t, by a

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[1]Given that the tests will be applied to aggregate data, a third type of shocks should be allowed: aggregation shocks. This type of shocks is disregarded more for my ignorance of how to model them within this framework, than for any other "deep" reason.
[2]Wealth innovations can also be divided in shocks to (i) the interest rate and initial wealth (e.g. Samuelson 1969) and (ii) to labor income (e.g. Leland 1968). Shocks to the horizon (T) are a mix of the two types of shocks (e.g. Blanchard 1985).
[3]Actually, only taste shocks are truly consumption originated shocks.

representative consumer who lives for T periods, earns no labor income[4], has a given initial endowment and accumulates wealth in the form of a risky asset with a stochastic return r_t. He faces no liquidity constraints and suffers from unexpected changes in preferences. The utility function is assumed to be time separable, and the instantaneous utility belongs to the class of CRRA (constant relative risk aversion).

Formally, the problem is:

$$\max_{\{c_{t+i}\}} E_t\left[\sum_{i=0}^{T-t} D^i z_{t+i} c_{t+i}^{1-\tau}/(1-\tau)\right] \qquad \tau \geq 0$$

$$\text{s.t.} \quad c_{t+i} = R_{t+i}\, s_{t+i-1} - s_{t+i} \qquad 0 \leq i \leq T-t$$

(1) $$c_T = R_T\, s_{T-1}$$

$$s_{t-1} \text{ given}$$

$$D \equiv (1+\delta)^{-1}, \; R_t \equiv (1+r_t)$$

with E_t : the conditional -on information available at time t- expectation operator.

δ : the discount rate.

c : consumption.

y : labor income.

s : wealth.

r : risky rate of return.

z : taste shock.

τ : coefficient of relative risk aversion.

Additionally, assume that,

(a) $R_t = Re_t$ $\qquad$ e_t i.i.d. (independent, identically

[4]Alternatively, the labor income stream may be sold at time t, i.e. there is complete insurance.

distributed) with $e_{min}>0$, and

(b) $z_{t+1} = z_t^{\Phi} v_{t+1}$ v_{t+1} i.i.d. with $v_{min}>0$, and $-1 \leq \Phi \leq 1$.

Here taste shocks appear multiplicatively. This has the role of enhancing consumption in those periods with large realizations of z. The consumption decision will depend on the relation between today's and expected future realizations of z. The autocorrelation parameter (Φ) measures this relation. If instead of this multiplicative-type, shocks are assumed to affect the discount factor, the conclusions and "spirit" of the arguments, with respect to the time series behavior of consumption, remain unaltered[5] [6].

The type of problem shown in (1) is typically solved by stochastic dynamic programming. Here, however, the procedure developed in chapter I is used. This procedure significantly facilitates the solution, and makes clearer the form of the stochastic process of consumption. Even though using only the Euler equation is enough to discuss the identification of the intertemporal substitution parameter[7], finding a solution for the consumption level, and as a consequence disentangling the innovation in current consumption, is a <u>must</u> if the purpose is to assess the relative importance of taste shocks in consumption fluctuations.

The first step in this optimization procedure is to guess the form of the stochastic process followed by consumption. Under these circumstances the guess is:

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[5]Although the time series properties of the multiplicative and discount rate shocks under which they have maximum effect on consumption fluctuations are reversed (i.e. white noise in the case of multiplicative shocks, and random walk in the discount rate shocks case).
[6]It should be noted that with either type of shock -multiplicative or to the discount rate- the expected marginal rates of substitution depend on time distance and not on calendar time, hence the program is still time consistent.
[7]See Garber and King(1984).

(2) $c_{t+1} = \beta_t c_t u_{t+1}$

with β_t a slope term. The u_{t+1} is an independently distributed residual orthogonal to the subspace spanned by variables that belong to the information set at time t, and has other properties to be found later.

The Euler equation arising from program (1) is:

(3) $z_t c_t^{-\tau} = D\, E_t[R_{t+1} z_{t+1} c_{t+1}^{-\tau}]$

Using assumptions (a) and (b) and substituting the guess (2) into (3), the feasibility of the "guess" is confirmed and the slope of the consumption path is obtained:

(4) $\beta_t = (DR)^{(1/\tau)} z_t^{(\Phi-1)/\tau} E_t[e_{t+1} v_{t+1} (u_{t+1})^{-\tau}]^{(1/\tau)}$

or

(4') $\beta_t = \theta_t \; z_t^{(\Phi-1)/\tau}$

with $\theta_t = (DR)^{(1/\tau)} E_t[e_{t+1} v_{t+1} u_{t+1}^{-\tau}]^{(1/\tau)}$. The parameter θ_t may change across time due to possible changes in the higher moments of u. In what follows it will be assumed that $\theta_t = \theta$, a constant. Later it is shown that this assumption is correct when taste shocks are either white noise or a random walk, since in these cases the higher moments of u are constant. However, this is only an approximation for cases with $0<\Phi<1$.[8]

The next step is to write down the realized intertemporal budget constraint:

(5) $c_t + \sum_{i=1}^{T-t} c_{t+i} \prod_{j=1}^{i} R_{t+j}^{-1} = R_t s_{t-1}$

but

(6) $c_{t+i} = c_t \theta^i \prod_{j=1}^{i} u_{t+j} z_{t+j-1}^{(\Phi-1)/\tau}$

then

(7) $c_t \left(1 + \sum_{i=1}^{T-t} \theta^i \prod_{j=1}^{i} R_{t+j}^{-1} u_{t+j} z_{t+j-1}^{(\Phi-1)/\tau}\right) = R_t s_{t-1}$

[8] See chapter I for further details.

and replacing assumption (a) in (7)

$$(8)\quad c_t\left(1 + \sum_{i=1}^{T-t} (\theta/R)^i \prod_{j=1}^{i} (u_{t+j}/e_{t+j})\, z_{t+j-1}^{(\Phi-1)/\tau}\right) = R_t s_{t-1}$$

Given that u and e correspond to innovation residuals, the solutions for c_t and u_t are[9] [10];

$$(9)\quad c_t = s_{t-1}R_t / \left(\sum_{i=0}^{T-t} (\theta/R)^i z_t^{(\Phi^i-1)/\tau}\right)$$

$$(10)\quad u_t = e_t \left(\sum_{i=0}^{T-t} (\theta/R)^i z_{t-1}^{\Phi(\Phi^i-1)/\tau}\right) \Big/ \left(\sum_{i=0}^{T-t} (\theta/R)^i z_{t-1}^{\Phi(\Phi^i-1)/\tau} v_t^{(\Phi^{i-1}-1)/\tau}\right)$$

In fact, it is apparent from (10), (4'), (9) and (2) that if Φ=1 taste shocks are completely irrelevant as an explanation for consumption <u>fluctuations</u> [Note: Hall(1987) makes the following comment on the random walk taste shock case: "The easiest assumption [to regain identification in Euler equation procedures], though very special, is that shifts in preferences occur as random walk, so that the corresponding stochastic component in the first difference of consumption is unpredictable. Then the Euler equation has an extra stochastic term that satisfies the assumptions already made about the term that comes from the innovation in income or wealth [orthogonal to lagged variables]" [pp. 28]. Here, on the other hand, by using not only the Euler equation but also the intertemporal budget constraint, it is possible to go one step further. Equation (10') shows that when taste shocks follow a random walk their innovations are not only

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[9] See chapter I.
[10] The means of the residuals are adjusted so to avoid nuisance constants.

unpredictable but also they do not enter at all in the consumption innovation. It is precisely this disentangling of the consumption innovation which allows the assessing of the relative importance of taste shocks in consumption fluctuations.]. In this case,

(10') $u_t = e_t$

(11) $\beta_t = \theta = (DR)^{1/\tau}E_t[e_{t+1}^{1-\tau}v_{t+1}]^{1/\tau}$

(12) $c_t = [\{(\theta/R)-1\}/\{(\theta/R)^{T-t+1}-1\}]R_t s_{t-1}$

and

(13) $c_{t+1} = \theta c_t e_{t+1}$

hence the presence of permanent taste shocks affects θ, the slope of the consumption path, but not consumption fluctuations. Put in other words, when taste shocks follow a random walk it is the existence, not the realizations, of taste shocks which affect the consumption path.

This reveals the essential feature of taste shocks; since they do not affect the perceived budget constraint[11] (i.e. there is no income nor wealth effects), consumers know that the present value of consumption cannot change. Any increase in consumption today will come at the cost of sacrificing consumption tomorrow[12]. Therefore, after a taste shock has occurred there is no change in the permanent level of consumption, but only a decision to be taken with respect to the slope of the consumption path, and this depends only on the expected marginal rates of substitution.

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[11]It is assumed that consumers do not take into account the potential aggregate demand effect of an increase in their consumption.
[12]Certainly this will always be true, but in the case of a wealth shock (positive), increasing current consumption does not necessarily mean a reduction, respect to the before-shock situation, of future consumption, as it happens in the taste shock case.

Long lived changes in marginal utilities have a <u>smaller</u> effect on rates of substitution, therefore they are less likely to tilt the consumption path. In the extreme case of taste shocks following a random walk, the consumption path is completely unaffected by the realizations of these shocks, although the level of utility associated with the same consumption sequence changes.

By the same type of argument, a taste shock has its maximum effect on current consumption when it is a white noise ($\Phi=0$). A shock today affects the marginal rate of substitution between today and all future dates, leaving the expected marginal rate of substitution between future dates unaltered. A large change in current consumption can be compensated by an evenly spread (small) reduction in future consumption.

When $\Phi=0$ equation (10) becomes,

$$(10'')\quad u_t = e_t v_t^{(1/\tau)}\{(\theta/R)^{T-t+1}-1\}/ [\{(\theta/R)-1\}v_t^{(1/\tau)}+\{(\theta/R)^{T-t+1}-1\}]$$

Two results are apparent. First, as τ increases, the importance of taste shocks -relative to wealth shocks- decreases; when consumers are very risk averse, small changes in consumption have a large effect on marginal rates of substitution, hence the change in the consumption profile required to compensate for the changes in the marginal rates of substitution induced by a taste shock is small[13]. Second, if the parameters (τ, β, R and variances) are such that $(\theta/R)>1$, and v_t has a bounded support, then as T goes to infinity, u_t goes to $e_t v_t^{(1/\tau)}$ almost surely[14]. In this case the

[13]Certaily this depends on the taste shock specification. It is easy to see that if the instantaneous utility function is written as follows: $(zc)^{1-\tau}/(1-\tau)$, then this first result disappear. However, this specification is less appealing since the effect of any given shock on the marginal rate of substitution is τ-dependent, making the interpretation of the shock less clear.

[14]This has been done only for expository simplicity. None of the qualitative conclusions change if $(\theta/R)<1$ is assumed. On the other hand, the assumption $(\theta/R)>1$ has the inconvenient implication that unless s_{t-1} is $O_P((\theta/R)^T)$, consumption goes to zero as T goes to infinity.

stochastic process followed by consumption is:

(14) $c_{t+1} = \theta c_t e_{t+1} v_{t+1}^{(1/\tau)} v_t^{-(1/\tau)}$

taking logs in both sides of (14)

(15) $(1-L)\ln c_{t+1} = \ln \theta + \ln e_{t+1} + (1/\tau)(\ln v_{t+1} - \ln v_t)$

or

(15') $(1-L)\ln c_{t+1} = \ln \theta + \ln u_{t+1}^*$

with $\ln u_{t+1}^* \equiv \ln e_{t+1} + (1/\tau)(\ln v_{t+1} - \ln v_t)$ and L the lag operator. In the more general case, $\ln u_{t+1}^*$ is clearly more complicated (see (10)), however it is still true that the residual must exhibit serial correlation and the taste components must eventually exhibit negative serial correlation.

The test is now simple; the null hypothesis is that taste shocks are not an important source of consumption fluctuations. The alternative is that they are. If the alternative is true the demeaned rate of change in consumption cannot be white noise.

The next section shows that taste shocks do not seem to have had an important role in the fluctuations of consumption of non-durable goods.

III. EMPIRICAL EVIDENCE

III.1 Data

Which consumption series to use in Euler equation tests is an unsettled issue. There is, however, a generalized agreement on the properness of non-durables consumption. Assuming that the utility function is separable in the major categories of consumption, the model is estimated using only consumption of non-durables (45% of total consumption expenditure). The period extends from the first quarter of 1947 to the first quarter of 1986.

The real interest rate corresponds to the realized real return after taxes from the 3-month US Treasury Bills. Inflation is calculated using the rate of change in the non-durables deflator. Taxes correspond to the average tax rate given in Barro and Sahasakul(1983). All the data, but taxes, are from the Citibank Data Base.

III.2 Results

The null hypothesis is that taste shocks are not an important source of consumption fluctuations[15]. In this case the residuals in equation (15') should follow a white noise process. Most of the effort in this section is devoted to test the whiteness of these residuals[16]. The following equation is a generalization of (15') and will be the basis for the subsequent tests:

(16) $(1-L)\ln c_{t+1} = \text{const.} + (1/\tau)(E_t[R_{t+1}]-1) + \ln u_{t+1}^*$

This equation was estimated by ordinary least squares (OLS)[17] and instrumental variables (IV) using different instrument sets[18]. It should be

[15] Remember, from section I, that this does not mean that taste shocks do not exist. It only means that they do not alter the expected marginal rate of substitution between current and future consumption sufficiently, so as to significantly tilt the consumption path.

[16] The underlying CRRA utility function is a maintained hypothesis. In general this assumption should not harm the power of the test. Although its size would be wrong since specification errors are almost always reflected in some non-whiteness of the residual.
A potentially more important problem may result from time aggregation. Working(1960) showed that this kind of problem may induce a MA(1) (with positive coefficient) structure in the residual. This certainly would work against the power of the test.

[17] This is only justified if R_{t+1} is known at time t.

[18] For estimation purposes the realization of the interest rate instead of its conditional expected value is used. This implies that there is an extra term, equal to $R_{t+1}-E_t[R_{t+1}]$, in the residual. This problem is corrected by assuming that expectations can be written as linear projections on past interest rates.

clear that some of these estimates need not be consistent under the alternative hypothesis. In fact, if taste shocks exist and they do not follow a random walk, it is likely that none of the estimation procedures used will lead to consistent estimates, however this does not affect the fact that if taste shocks exist the residual of equation (16) must exhibit serial correlation[19]. The regression results can be seen in table 1.

As usual the intertemporal substitution parameter is small[20]. For the purposes of this paper, however, the most important column is the last one. The Q(d) test correspond to a Lagrange multiplier (henceforth LM) test for a high order ARMA(p,q) (here max(p,q)=36) as an alternative. It is distributed as a Chi-Square with d (here equal to 36) degrees of freedom. The critical values for 5% and 10% significance levels are 50.998 and 47.212 respectively. The white noise assumption cannot be rejected at these significance levels.

The Q statistic is often criticized for its lack of power, specially against alternatives representable by low order ARMA processes. Table 2 reports the LM tests for lower order ARMA processes[21]. This shows the result of tests against (the alternative) ARMA(p,q) processes with max(p,q) going from 2 to 8.[22] The results are similar to those obtained in the last column of table 1; there is no evidence against the white noise assumption

[19]Although the power of the test may be affected.
[20]See Hall(1985). Chapter I in this thesis shows that once precautionary savings are taken into account, negative intertemporal substitution effects are still consistent with the LCH/PIH hypothesis.
[21]These tests were performed using the heteroskedasticity-robust method suggested in Wooldridge(1987).
[22]Here the first coefficient was set to zero. This was only done after performing LM tests against ARMA(p,q) processes with max(p,q) going from 1 to 8, with no success in rejecting the null. The same was done with the case of max(p,q) going from 3 to 8. The results reported are those that were most "unfavorable" for the null.

at the 5% significance level. However, when the significance level is raised to 10%, equation (16) estimated by OLS shows marginal evidence in favor of a low order ARMA process.

This single rejection is not very worrisome since it is expected that the direct inclusion of R_{t+1} in the right hand side of equation (16) leads to specification error when estimated by OLS.

The Q statistic corresponding to the residuals from the non-heteroskedasticity-robust regression form of the LM tests are also reported. All of them are well within the critical values of the Chi-square at 5% and 10% significance levels.

Finally, Durbin's cumulative periodogram test is reported. This is a frequency domain alternative and consists of comparing the observed spectral distribution of the series, with the theoretical spectral distribution of a white noise. The critical values are given by the Kolmogorov-Smirnov goodness of fit test tables. Table 3 shows that, once again, there is no evidence against the white noise hypothesis.

IV. CONCLUSION

This essay does not attempt to explain what a "taste shock" means. However it proposes a working definition to distinguish between wealth and taste shocks.

Using this distinction it was possible to derive, from first principles, a consumption function and process in the presence of both types of shocks. Once the implications of each type of shocks were obtained, it was possible to construct a simple test to assess the relative importance of taste shocks in consumption fluctuations.

When this test was used to analyze the time series behavior of the postwar U.S. consumption expenditure on non-durables, no evidence of any significant role for taste shocks in aggregate consumption fluctuations was found.

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TABLE 1

EQUATION (EST. METH.)	CONSTANT	$1/\tau$	Q(36)
(15')	0.0060	-	40.82
(OLS)	(0.0007)		(0.267)
(16)	0.0055	0.252	39.83
(OLS)	(0.0007)	(0.066)	(0.304)
(16)	0.0054	0.288	40.37
(IV1)	(0.0007)	(0.104)	(0.283)
(16)	0.0063	-0.091	40.23
(IV2)	(0.0008)	(0.157)	(0.288)

Notes: (i) standard deviations in parenthesis, except for the last column where the marginal significance level is reported, (ii) Nobs=150, (iii) IV1: instr. are a constant and R_{t-1} (iv) IV2: instr. are a constant and R_{t-2}

TABLE 2

LM statistic, Ha: ARMA(p,q). First coeff.=0

Max(p,q)

	2	3	4	5	6	7	8	Q(36)
Chi-sq								
α=5%	3.84	5.99	7.82	9.49	11.1	12.6	14.1	51.00
α=10%	2.71	4.61	6.25	7.78	9.49	11.1	12.0	47.21
EQUATION								
(15)OLS	0.42	2.05	2.47	3.45	4.75	5.11	9.10	33.27
(16)IV1	0.02	1.77	1.84	2.54	2.85	4.34	7.50	34.11
(16)IV2	0.65	2.27	2.83	3.57	4.04	4.63	9.21	35.47
(16)OLS	2.18	5.07	5.79	6.09	6.09	7.51	9.67	33.24

Note: The first two rows correspond to the critical value of the Chi-square distribution.

TABLE 3

Durbin's Cumulated Periodogram Test

	(15)OLS	(16)IV1	(16)IV2	(16)OLS
DCP	0.046	0.035	0.055	0.049

Note: Kolmogorov-Smirnov critical values= 0.1124 and 0.0888, for 5% and 20% significance level, respectively.

CHAPTER IV

THE BEHAVIOR OF EXPENDITURE ON DURABLE GOODS: DISENTANGLING ITS DISTURBANCE

I. INTRODUCTION

Since Hall(1978)'s seminal paper, most of the rational expectations literature concerning the time series behavior of consumption has concentrated on non-durable goods and services. The basic hypothesis (and finding) is that under the life cycle/permanent income hypothesis (LCH/PIH) non-durables consumption should follow an AR(1) process: given that in any period consumers use all the information available to them to choose their desired level of consumption in the same period, the latter should be a sufficient statistic to predict the level of consumption in the following period.

Mankiw(1982) extended this framework to the case of durable goods. Under the same assumptions used by Hall -time separable utility function, quadratic instantaneous utility function and constant interest rates- he showed that the same argument used for non-durables applies to the services provided by the stock of durables. In addition, at any point in time the stock of durables is formed by what is left from the previous period and the expenditure on durable goods in the current period. Combining this accumulation equation with the Euler equation of the optimization problem solved by the consumer, he showed that the AR(1) process for the stock implies an ARMA(1,1) process for the expenditure on durable goods. The MA coefficient is negative (with absolute value equal to one minus the rate at which the stock of durable depreciates) reflecting the fact that once a durable good is bought, it lasts for more than one period, thus reducing the expenditure required in the future to keep the stock of durables unchanged at the new level[23]. He found that the U.S. post-war data reject

[23]Mankiw assumed that the time unit was a quarter. Appendix II in this chapter shows that the ARMA(1,1) model holds for quarterly data even if decisions are taken more frequently.

this hypothesis in favor of a simple AR(1) process; except for a greater volatility, the time series behavior of durable goods looked very similar to that of non-durable goods.

Startz(1986) discarded one of the possible sources of rejection in the previous models by dropping the assumption of separability between the consumption of non-durables and durables' services. He concluded that "..the behavior of durable goods purchases is found to be consistent with the life cycle/permanent income hypothesis..". However, he did not emphasize that the disturbance in his estimated equation for durable goods expenditure did not satisfy the MA(1) structure implied by the theory. Therefore Startz's result can be seen as consistent with Mankiw's finding[24].

These studies neither propose a metric to assess, in economic terms, the relative importance of these rejections nor do they show how far it is necessary to go in relaxing auxiliary assumptions to explain the departures from the LCH/PIH. This chapter tries to improve the understanding of these "disturbance based" rejections. In other words, it studies whether by relaxing enough auxiliary assumptions it is still possible to use, at least as an approximation, the simple, and therefore appealing, homogeneous-representative agent model.

[24] Williams(1972) also noticed the MA structure of the disturbance in a stock adjustment model. However he disregarded this restriction when estimating the model using quarterly data for cars and domestic electrical goods in the United Kingdom (1948-1967). As he recognizes, had this restriction be imposed it would have been rejected in favor of no MA terms. Stone and Rowe(1960) did not model the MA structure but also found a very large implicit depreciation rate in the interwar and postwar British data. They concluded that the behavior of durables looked very similar to that of non-durables.

Within the restrictions imposed by the homogeneous-representative agent model, there are different types of explanations for these rejections. Among them the most widely cited appear to be: (i) people do not behave according to the LCH/PIH, (ii) there are measurement errors and/or time aggregation problems (i.e. data problems in general), and (iii) there are taste shocks, changes in relative prices and interest rates, shocks to the higher moments of the wealth distribution function, misspecification of the functional form of utility, adjustment costs, and many other problems associated with the auxiliary assumptions required to obtain a simple time series representation of the consumption path.

The alternative explanations, (i), (ii) and (iii) differ substantially in their implications. If the LCH/PIH is not the basic criterion for the allocation of wealth across lifetime consumption, a large portion of economic theory runs into troubles, to say the least. On the other hand, if (ii) and/or (iii) are responsible for the rejections, then the implications are less dramatic. They only call for a better understanding of the data construction and of the validity of the auxiliary assumptions.

This chapter relaxes many of the restrictions imposed by (iii). This is achieved by relying on the idea that the LCH/PIH is a hypothesis about people's allocation of wealth across lifetime consumption. Its validity should therefore be judged according to the response of consumption (and savings) to wealth innovations. Moreover, this is precisely the implication that is meant to be tested in procedures like Hall's and Mankiw's. On the other hand, taste shocks, for example, do not enter the realized budget constraint, therefore they do not affect wealth. They can only affect the slope of the consumption path but not the expected present value of expenditures[25]. As a result they can lead to a rejection of Hall's and Mankiw's tests, even if the deep implications of the LCH/PIH model are satisfied. It turns out that within the representative agent model, the effect of almost every type of shock can be decomposed into a wealth and a substitution effect, and the latter will behave in the same (qualitative) way as the effect of taste shocks (and therefore can lead to "unwarranted"[26] rejections of the LCH/PIH). One way to avoid these "unwarranted" rejections is by relaxing some of the assumptions of type (iii). However, most of these assumptions are often necessary to derive empirically (and analytically) tractable models. The novelty of this paper is to notice that if only the substitution effects (as opposed to wealth effects) of (iii) are responsible for the "unwarranted" rejections, then it is not necessary to solve the complex problem itself. Instead, it is possible to solve a closely related (but much simpler) problem in which a very general taste shock process is added. By doing this, very general substitution effects can be taken into account.

The main contribution of the empirical section of this chapter is the characterization of the slope (taste) shock process[27]. Once this is done, tentative explanations of the behavior of durable goods stand on a much more solid ground; the slope shock process can be compared with the substitution effects that relaxing each of the auxiliary assumptions would imply. The conclusion is that if the LCH/PIH is taken as a maintained

[25]Accordingly, these shocks will be called slope (substitution) shocks.
[26]Unwarranted in the sense that they do not reflect a rejection of the deep implications of the LCH/PIH.
[27]Notice that once a general taste shocks process is allowed, the model becomes untestable in the traditional sense. However it is still the case that the implicit (the one that makes the data consistent with the LCH/PIH) taste shocks process can be estimated. Once this is done, it is possible to see whether the taste shocks process has any economic sense.

hypothesis -even after the auxiliary assumptions are relaxed to allow for such elements as taste shocks, precautionary savings, flexible interest rates, adjustment costs or habit formation, to name a few- the homogeneous-representative consumer model does not seem to be a good approximation for explaining the short run behavior of expenditure on durable goods. Furthermore, it is possible to show that adjustment costs can also be seen within the taste shocks framework. However, if a parsimony criterion is used to choose the taste shocks process, the implicit speed of adjustment is unreasonably low. The next chapter shows that this conclusion is severely affected by the parsimony criterion.

The basic claims of the paper are presented in the form of answers to four questions (Q1 to Q4) that are distributed among five sections. Section II explains the spirit of the approach and extends Mankiw's framework to allow for taste shocks. Although the particular case of taste shocks is considered, as noted before, most of the implications are shared by other kinds of slope shocks.

Section III presents preliminary evidence on the actual behavior of the disturbance in the stochastic process of expenditure on durable goods for the U.S.. It is shown that even though the almost non-existence of an MA(1) term is still the dominant characteristic of the series, there seems to be marginal evidence of higher order MA terms. Using the Granger and Morris(1976) rules for sums of series, it is possible to restrict the class of slope shocks processes that have a chance of fitting the data.

In section IV the evidence provided in section III is used to pose the problem in terms of a state space model, which is estimated by a maximum likelihood-Kalman filtering technique. The results provide not only an estimate of the underlying wealth and taste stochastic processes, but also minimum mean square error estimates of the time series path of wealth and slope shocks. Among other things, it is shown that (i) slope shocks play a role almost as important as wealth shocks in explaining the unconditional variance of changes in durable goods expenditure and, (ii) "standard" across-the-board (across all goods) taste shocks are not a plausible explanation for the time series behavior of expenditure on durable goods.

Section V contains the concluding remarks and analyzes several alternatives using the characterization of slope shocks given in section IV. Non-homogeneous taste shocks, precautionary savings, the effect of interest rates on the user cost of durables, liquidity constraints, adjustment costs, habit formation, leisure in the utility function and time aggregation do not seem to be sufficient explanations for the departures of the quarterly U.S. data on durable goods from the implications of the LCH/PIH.

II. GENERAL FRAMEWORK AND THEORY

II.1 General Framework

Before going into the details of the model it is important to devote a few lines to explain the basic methodology of this chapter. It starts from a tautological statement: it is always possible to write a residual as the sum of two disturbances (not necessarily independent)[28]. Therefore the change in expenditure on durable goods, (1-L)c, can always be written as follows:

[28]This statement refers to an identity, it has nothing to do with the realizability conditions shown in Granger and Morris(1976).

(0) $(1-L)c_{t+1} = tw_{t+1} + tv_{t+1}$

where tw and tv are the wealth and slope components of the disturbance, respectively.

The aim of the model presented below is to impose enough restrictions so that both disturbances can be distinguished. It will be shown that shocks that do not have wealth effects have very distinctive implications for the time series behavior of durables. This characteristic will be used to study the substitution component of very different type of effects.

The main difference with previous studies of the durables behavior, and consumption in general, is the presence of the disturbance tv. In Mankiw(1982)'s paper, for example, the main concern was to study whether the model originally used by Hall(1978) could explain the behavior of durables. Put in other words, whether the disturbance tw, assuming that tv≡0, was consistent with the LCH/PIH -i.e. followed an MA(1) with a negative MA coefficient equal (in absolute value) to one minus the depreciation rate of the stock of durables- or not. Here, on the other hand, the problem is approached from a different angle. Among the questions to be answered are: Assuming that the behavior of tw is consistent with the LCH/PIH implications, what does the behavior of tv look like? Is it possible for a pure substitution effect to account for the behavior of tv? or, assuming that certain specific auxiliary assumption is true, is tw consistent with the LCH/PIH?[29]

In answering this kind of questions, not only is a broad set of explanations for the durables puzzle assessed, but also the process for tv is characterized, providing a useful starting point for future research.

---

[29]Notice that this question includes Mankiw's hypothesis.

II.2 Theory

Consider adding a taste shock to Mankiw(1982)'s model of expenditure on durable goods. The problem to be solved by the consumer becomes:

$$(1) \quad \max_{\{c_{t+i}\}} E_t\left[\sum_{i=0}^{T-t} \beta^i U(k_{t+i}, z_{t+i})\right]$$

s.t

$$c_{t+i} = R_{t+i} S_{t+i-1} + y_{t+i} - S_{t+i}$$

$$k_{t+i} = (1-\delta) k_{t+i-1} + c_{t+i}$$

$$c_T = R_T S_{T-1} + y_T + (1-\delta) k_T / R_{T+1}$$

S_{t-1}, k_{t-1} given

with

E_t: expectation operator, conditional on all information available at time t,

β: subjective discount factor,

R: one plus the riskless real interest rate (henceforth assumed constant),

U(): instantaneous utility function,

k: stock of durable goods providing services to the consumer,

δ: depreciation rate of durable goods,

c: expenditure on durable goods,

S: non-human wealth,

y: labor income (only source of wealth uncertainty), and

z: stochastic taste parameter.[30]

[30]As long as taste shocks are homogeneous across all goods it is possible to concentrate out non-durables and services. The case of non-homogeneous taste shocks is discussed in section V.

U(.,.) is further specialized to a constant absolute risk aversion (CARA) utility function with multiplicative taste shocks[31],[32]:

(2) $U(k,z) = -(1/\theta)\exp(-\theta k)\exp(z)$

The effect of the multiplicative taste shock is to alter marginal utility ($U_k(k,z)=\exp(-\theta k)\exp(z)$): for a given k, a positive taste shock raises marginal utility. What happens to the (expected) marginal rates of substitution after a shock, and therefore to the allocation of expenditure on durables, depends on the stochastic process of the taste shock.

Taste shocks can follow a very general process. Assume that:

(3) $D(L)z_{t+1} = F(L)v^*_{t+1}$

$D(L)=1-D_1L-D_2L^2-\ldots-D_pL^{p'}$

$F(L)=1+F_1L+F_2L^2+\ldots+F_qL^{q'}$

where v^*_{t+1} is an independent and identically distributed disturbance with mean zero and variance σ_{v*}^2 [i.i.d.$(0,\sigma_{v*}^2)$] and L is the lag operator (i.e. $L^1x_t=x_{t-1}$).

Assume, for now, that D(L) is invertible, then:

(3') $z_{t+1}=G(L)v^*_{t+1}$ with $G(L)=D(L)^{-1}F(L)$

Labor income is assumed to be characterized by[33]:

(4) $A(L)(1-L)y_{t+1} = B(L)w^*_{t+1}$

$A(L)=1-A_1L-A_2L^2-\ldots-A_pL^p$

---

[31]Although the solution to this optimization problem is certainly influenced by the utility function chosen, the general implications are not. This is further discussed in section IV of this chapter.

[32]Diewert(1974) shares some of the questionable assumptions of this paper, e.g. no vintage effects, the services of durables are proportional to the stocks, depreciation is constant and exponential, and the relative prices are constant. However, in this paper the last two of these restrictions are relaxed later.

[33]Adding a drift term to the income process does not affect the nature of the solution. It only changes the permanent level of consumption.

$B(L)=1+B_1L+B_2L^2+\ldots+B_qL^q$

with w^*_{t+1} i.i.d.$(0,\sigma_{w*}^2)$.

The disturbances w^* and v^* can be contemporaneously correlated (σ_{w*v*} may be different from zero).

The Euler equation for this problem is:

(5) $\exp(z_t)\exp(-\theta k_t) = \beta RE_t[\exp(z_{t+1})\exp(-\theta k_{t+1})]$

Appendix I shows that the corresponding stochastic process for k is of the form:

(6) $k_{t+1} = \Gamma_t + k_t + e_{t+1}$

with e_{t+1} an innovation residual and Γ_t the "slope" of the consumption path.

Replacing (3') and (6) in (5) an expression for Γ_t is obtained:

(7) $\Gamma_t= H(L)v^*_t/\theta + (1/\theta)\log(\beta R) + (1/\theta)\log E_t[\exp(v^*_{t+1}-\theta e_{t+1})]$

with $H(L)=((G(L)/L)_+ - G(L))$. The $_+$ denotes the positive powers of L.

The first term in (7) is the effect of taste shock <u>realizations</u> on the slope between today's and tomorrow's consumption[34]. The second one is the standard slope term in the certainty case. Finally, the last one is the precautionary savings term[35]: when the utility function has a positive third derivative, people save in order to have an insurance against future labor income risk. This results in a stepper consumption path. If the horizon is long enough and both w^* and v^* are i.i.d. (as assumed), then the precautionary savings term is constant and (7) can be written as follows:

(7') $\Gamma_t = H(L)v^*_t/\theta + a_0$

[34]Notice that another taste disturbance appears in the third term, v_{t+1}, however in this case it is the existence, not the realizations of taste shocks that matters. If taste innovations are i.i.d. (as assumed) this term does not play any role in consumption fluctuations.
[35]See chapter I.

with

(8) $a_0 \equiv (1/\theta)\log(\beta R) + (1/\theta)\log E_t[\exp(v^*_{t+1}-\theta e_{t+1})]$

The next step is to identify e_{t+1}. The appendix shows that:

(9) $e_{t+1}= \phi w^*_{t+1}/(1+\alpha(\delta-1)) - (\alpha \sum_{i=0}^{\infty} \alpha^i H_i)v^*_{t+1}/\theta$

with ϕ the annuity value of an income shock and $\alpha\equiv 1/R$.

Equations (7) and (9) show that taste shocks affect the slope and the innovation of the stochastic process followed by the stock of durable goods. An example may clarify the issues. Assume that taste shocks follow an AR(1) process with autoregressive coefficient Φ, $0\leq\Phi\leq 1$. In this case $H_j=(\Phi-1)\Phi^j$, therefore:

(10) $\Gamma_t=(\Phi-1)z_t/\theta + a_0$

and

(11) $e_{t+1}= \phi w^*_{t+1}/(1+\alpha(\delta-1)) + [(1-\Phi)\alpha/(1-\alpha\Phi)]v^*_{t+1}/\theta$

Equation (10) shows the <u>substitution effect</u> of taste shocks. A positive taste shock, when $\Phi<1$, enhances consumption in this period relative to the following periods, therefore the expected change in consumption (Γ_t) is reduced. Equation (11), on the other hand, shows the restriction imposed by the fact that taste shocks do not enter the budget constraint hence they cannot affect the present value of lifetime expenditures. Taste shocks can only have substitution effects, but the latter depend only on the expected marginal rate of substitution. If the shocks are expected to persist for a long time, the marginal rates of substitution are only slightly affected, hence the incidence of taste shocks in today's consumption cannot be large. In the limit, when $\Phi=1$, there is no substitution effect at all, therefore consumption is not affected by the realizations of taste shocks[36].

[36]See chapter III.

At this stage it is possible to answer the first question:

Q1: Taking the implications of the LCH/PIH as satisfied, is there a taste shock process, with innovations that are independent of wealth innovations, that can explain Mankiw's stylized fact?

The answer is no[37]. In order to see this, use the equation of accumulation of durable goods ($k_{t+1}=(1-\delta)k_t+c_{t+1}$)[38] and equation (6) to obtain the process for expenditure on durable goods:

(12) $(1-L)c_{t+1} = \delta a_0 + (1-(1-\delta)L)H(L)v^*_t/\theta + e_{t+1} - (1-\delta)e_t$

Using a compact notation and replacing (11) in (12) leads to:

(12') $(1-L)c_{t+1} = \delta a_0 + w_{t+1} - (1-\delta)w_t + tv_{t+1}$

with $w_{t+1}\equiv\phi w^*_{t+1}/(1+\alpha(\delta-1))$

$tv_{t+1}\equiv\Psi(1-(1-\delta)L)H(L)v_t + v_{t+1} - (1-\delta)v_t$

$v_{t+1}\equiv\Psi^{-1}v^*_{t+1}/\theta$

$\Psi^{-1}\equiv-(\alpha\sum_{i=0}^{\infty}\alpha^i H_i)$

As mentioned before, by setting $tv\equiv0$ the model becomes identical to Mankiw's model[39]. Here, on the other hand, the question is whether there is a process for tv, independent of the w process, such that the disturbance

[37]Notice that this does not mean that people do not satisfy the intertemporal budget constraint. In fact this is a maintained assumption. The answer to Q2 shows that under this maintained assumption, the answer to Q! is negative due to the condition of independence between the taste and the wealth innovations.

[38]Wykoff(1970) criticizes the assumption of a constant and exponential rate of depreciation. He studied the price evolution of ten car models for the period 1950-69. He found that the price fell twice as much in the first year of use than in subsequent years. This modification can be easily accommodated in the framework of this essay: $k_{t+1}=(1-\delta_2)^2k_{t-1}+(1-\delta_1)c_t+c_{t+1}$, $\delta_1\approx2\delta_2$, then, disregarding the taste shock component, $(1-L)c_{t+1}=\delta a_0+(1-\delta_1)(1-L)c_t+w_{t+1}-(1-\delta_2)^2w_{t-1}$. The white noise result rejects this model with the same strength as the model with $\delta_1=\delta_2$. Additionally, imperfections in secondary markets suggest that prices may not be a good estimate of the utility-value of the car.

[39]Except for the additional restriction on the coefficient of lagged consumption (=1).

in (12'), call it u_{t+1} ($u_{t+1} \equiv w_{t+1} - (1-\delta)w_t + tv_{t+1}$), is white noise. One way to answer this question is by looking at the conditions that the covariogram must satisfy (or the spectrum, in a frequency domain explanation).

If u is white noise, then except for the variance, all the autocovariance terms are zero. Considering that substitution effects must introduce serial correlation in (1-L)k and (1-L)c, the zero-autocovariances restriction implies that in order to have any chance of satisfying the white noise process, tv_t must follow an MA(1) process ($tv_{t+1} = (1+bL)v_{t+1}$). If tv_t follows any higher order (or autoregressive) process then substitution and wealth effects cannot cancel each other in such a way that the white noise result is achieved[40].

The next step is to check whether there exists a coefficient b such that the following one lag-autocovariance zero restriction is satisfied[41]:

(13) $0 = -(1-\delta)\sigma_w^2 + b\sigma_v^2$

and at the same time $\Psi(1-(1-\delta)L)H(L)$ is not a function of L (hence tv_t follows an MA(1) process). As said before, the answer is <u>no</u>. By looking at the definition of H(L) below equation (7) it is clear that the condition on $\Psi(1-(1-\delta)L)H(L)$ can only be satisfied when taste shocks follow an AR(1) process with autoregressive coefficient equal to $(1-\delta)$; in this case $H(L) = \delta(1-(1-\delta)L)^{-1}$ so $tv_{t+1} = v_{t+1} - v_t(1-\delta(1-\Psi))$. Replacing the expression for Ψ is possible to see that, in this case, $b = -1/\alpha$, so (13) cannot be satisfied for any strictly positive σ_w and/or σ_v.

[40]It is easy to show that except for the case in which the real interest rate is equal to $-\delta$, b is different from $-(1-\delta)$.
[41]Longer lag autocovariances are zero since both tw and tv follow MA(1) processes.

A direct implication of this result is that transitory consumption, in the sense of Friedman(1957), cannot be rationalized in terms of taste shocks.

The next question relaxes the independence assumption:

Q2: Is there any taste shock process that could make condition (13) hold?

The answer is yes, although the characteristics of this process are very peculiar.

From the answer to Q1, the taste shock process must be an AR(1) with an autocorrelation coefficient equal to $(1-\delta)$.

The variance and first autocovariance of u (μ_0, μ_1) impose the following restrictions:

(14) $\mu_0 = (1+(1-\delta)^2)\sigma_w^2 + (1+\alpha^{-2})\sigma_v^2 + 2(1+(1-\delta)\alpha^{-1})\sigma_{vw}$

(15) $\mu_1 = 0 = -(1-\delta)\sigma_w^2 - \alpha^{-1}\sigma_v^2 - (1-\delta+\alpha^{-1})\sigma_{vw}$

therefore

(15') $\sigma_{vw} = -((1-\delta)\sigma_w^2 + \alpha^{-1}\sigma_v^2)/(1-\delta+\alpha^{-1})$

It is clear from the previous question that the covariance term has a crucial role in the explanation. This covariance term is bounded by the Cauchy-Schwartz inequality:

(16) $-1 \leq \sigma_{vw}/\sigma_w\sigma_v \leq 1$

Restriction (16) guarantees that the correlation coefficient (Φ_{vw}) belongs to [-1,1].

Replacing the definition of Φ_{vw} in (15") implies:

(17) $\Phi_{vw} = -[f(\sigma_w/\sigma_v)+(1-f)(\sigma_w/\sigma_v)^{-1}]$

with $f \equiv (1-\delta)/(1-\delta+\alpha^{-1})$.

Equation (17) describes a parabolic function. This is shown in figure 1. First of all, it is clear that Φ_{vw} is negative since $0 \leq f \leq 1$. Second, it is easy to see that if f=0.5 the parabolic curve "sits" in $\Phi_{vw}=-1$. When f=0.485 (this corresponds to $\delta \approx 5\%$ and a quarterly real interest rate around 1%) the maximum value of Φ_{vw} rises only to -0.9996.

But the Cauchy-Schwartz inequality restricts the solution to values of Φ_{vw} within the [-1,1] interval, therefore all the potential solutions must convey values of Φ_{vw} within the interval [-1.0, -0.9996]. Also, equation (17) implies that σ_w is only slightly larger than σ_v (e.g. if $\Phi_{vw}=-0.9996$, $\sigma_w=1.03\sigma_v$). If Φ_{vw} is less than -0.9996, there are two solutions for each Φ_{vw} however all of them are very close to each other[42].

Figure 2 determines the <u>level</u> of the standard deviations. The dashed line represents those points in which equations (14) and (15) are satisfied, whereas the area in between the two solid lines represent those points that satisfy equation (15) and the Cauchy-Schwartz inequality. This implies that there is a continuum of solutions, however they are all very close. This raises an issue of near identification that will be discussed in more detail in the empirical section.

As said before, the characteristics of this solution are very peculiar. Whenever there is a positive wealth innovation there is an offsetting "taste" shock that significantly curtails the increase in expenditure on durable goods in the first period. In the second period, people like the good more but expenditure only remains constant since there is still a portion $(1-\delta)$ of the good that is left from the previous period.

[42]As a curiosity, when $\Phi_{vw}=-1.0$, the solutions are either $\sigma_w=\sigma_v$ or $\sigma_w=1.05\sigma_v$. In the first case the white noise result is obtained by completely eliminating the response of durables to wealth innovations in the first period they occur.

This pattern is expected to remain forever, generating the white noise (of the changes in expenditure on durable goods) result.

As long as consumer's perception of their wealth is positively correlated with aggregate demand, the very strong negative correlation ($\Phi_{vw} \approx -1$) required suggests that slope shocks cannot correspond to across-the-board taste shocks. Moreover, it is very difficult to think of any slope effect that can lead to this result. Precautionary savings, substitution effects of interest rate changes and non-homogeneous taste shocks are more likely candidates, but they are far from flawless, as it is shown later in the paper. A more promising explanation comes through slow adjustment; people adjusting slowly is like if they were substituting current (future) for future (current) consumption after a positive (negative) wealth shock. Therefore the correlation between "taste" and wealth innovations appears to be negative. This alternative is further discussed later, with the conclusion that there does not seem to be a systematic short run adjustment pattern. Chapter V, on the other hand, shows that when the time unit analyzed is years instead of quarters, there seems to exist a well defined long run adjustment mechanism.

Besides the strong negative correlation, it was also shown that the variance of the slope innovations is almost as important as the variance of the wealth innovations. Therefore not only is the puzzle much more robust than what was initially thought, but it is also quantitatively important (in economic terms) in the explanation of the behavior of expenditure on durable goods.

In practice, changes in expenditure on durable goods are not exactly a white noise, although the MA(1) coefficient is very close to zero. However, some higher order MA coefficients are marginally different from zero when the significance level is around 10%. Most of what remains of this chapter studies how important are these slight departures for both, the negative correlation and the relative importance results.

The next section is devoted to study the time series of expenditure on durable goods. Section IV uses this evidence to disentangle the disturbance and to assess the relative importance of wealth and slope shocks.

III. SERIAL CORRELATION

The previous section showed that if the demeaned changes in expenditure on durable goods behaves as a pure white noise, both innovations (slope and wealth) must have very similar variances and almost perfect negative correlation. In the sample, however, there are some slight departures from the pure white noise behavior. It is important to determine the size of these departures to then, in section IV, measure their incidence on the conclusions obtained in section II for the pure white noise case.

This evidence is reported in table 1. This table shows the estimates of the coefficients under different assumptions about the time series process of u in equation (18) below:

$$(18)\quad (1-L)c_{t+1} = \delta a_0 + u_{t+1}$$

The rows in the table state the assumption on the process of u. The data correspond to deseasonalized quarterly expenditure on durable goods as reported by NIPA[43], detrended by the deterministic rate of growth of GNP[44].

[43]The data correspond to the period 1947:2-1987:1.
[44]Similar tests and procedures were performed in per-capita terms. The main conclusions did not change in any important way.

The first two rows correspond to the cases already analyzed in section II, however it is interesting to see what happens in row (2). In this case the MA coefficient is not strictly zero; this implies that (15') above has to be re-written as follows:

(15") $\sigma_{vw} = -\{((1-\delta)\sigma_w^2 + \alpha^{-1}\sigma_v^2)/(1-\delta+\alpha^{-1})\} - \mu_1/(1-\delta+\alpha^{-1})$ with $\mu_1 < 0$.

In terms of figure 1, this shifts the parabolic function up increasing the range of possible solutions. However, given that the MA coefficient is so small (remember that with no slope shocks it should approximately be -0.95) this still leaves the problem as one of near-identification[45]. The equality constraints are only two, (14) and (15"), for three unknowns, σ_w, σ_v and σ_{vw}; however the Cauchy-Schwartz inequality guarantees that all the possible combinations of the three parameters are "close" to each other. Table 1 shows that in all the models which are not overparametrized, the small and negative MA_1 coefficient remains as one of the most important characteristics.

The following three rows show that there is marginal evidence of an MA(2) and especially of an MA(3) term. The rules for sums of series[46] suggest that if these coefficients are different from zero the component of the disturbance due to slope shocks must follow an MA(3) process. This can be obtained in two ways: (a) the "taste" shock process follows an MA(1) or (b) it follows an ARMA(1,2) with the autoregressive coefficient equal to 1-δ. These two possibilities arise because in the ARMA(1,2) case there is a coincidental reduction[47].

[45]See Fisher(1966).
[46]See Granger and Morris(1976) or Engel(1984).
[47]See Granger and Morris(1976).

If taste shocks follow an MA(1) process, i.e.:

(19) $z_{t+1} = v^*_{t+1}(1+\tau_1 L)$

the change in expenditure on durables goods follows an MA(3):

(20) $(1-L)c_{t+1} = \delta a_0 + (1-(1-\delta)L)w_{t+1} + [1-(1-\tau_1)¥L-\tau_1¥L^2]$

$(1-(1-\delta)L)v_{t+1}$

with $¥\equiv[\alpha(1-\tau_1)+\alpha^2\tau_1]^{-1}$

On the other hand, when the taste shock follows an ARMA(1,2) with an autoregressive coefficient equal to $(1-\delta)$, then:

(21) $(1-(1-\delta)L)z_{t+1} = v^*_{t+1}(1+\tau_1 L+\tau_2 L^2)$

so that the change in expenditure on durable goods also follows an MA(3):

(22) $(1-L)c_{t+1} = \delta a_0 + (1-(1-\delta)L)w_{t+1}\ [1-(\delta-\tau_1)¥L-(\delta(1-\delta+\tau_1)-\tau_2)¥L^2]$

$(1-(1-\delta)L)v_{t+1} - v_{t-2}\delta¥(\tau_2+(1-\delta)(1-\delta+\tau_1))$

with $¥\equiv[\alpha(\delta-\tau_1)+\alpha^2(\delta(1+\tau_1-\delta)-\tau_2)/(1-\alpha(1-\delta))]^{-1}$.

Summarizing, in both cases the equation for the change in expenditure on durable goods can be written as follows:

(23) $(1-L)c_{t+1} = \delta a_0 + w_{t+1}(1-(1-\delta)L) + v_{t+1}[1+a_1L+a_2L^2+a_3L^3]$

although the definition of the coefficients a_i changes according to the specification chosen.

The next section uses these restrictions to attempt to disentangle the wealth and slope processes more precisely.

IV. THE UNDERLYING WEALTH AND TASTE PROCESSES

IV.1 The State Space Model

The purpose of this section is twofold: first, to estimate the parameters governing the underlying wealth and slope processes; and second, to construct estimates of the slope and wealth components (tw and tv) in order to assess their relative importance in the fluctuations of expenditure on durable goods.

Fortunately, the theoretical section provides enough restrictions to make the use of state space models, together with filtering and smoothing techniques, particularly appropriate for these purposes.

The basic state space model can be written as follows[48]:

(24) $x_{t+1} = A_{t+1}x_t + b_{t+1} + B_{t+1}u_{t+1}$

(25) $m_t = Z_t x_t + S_t o_t$

with x_t: a (Kx1) vector of state variables,

A_t, B_t: (KxK) fixed matrices (not to be confused with the polynomials in

section II),

b_t: a (Kx1) non-stochastic vector,

u_t: an (Mx1) white noise process,

m_t: an (Nx1) measurement process,

Z_t: an (NxK) fixed matrix,

S_t: an (NxP) fixed matrix,

o_t: a (Px1) white noise process.

The white noise vector processes are assumed to be Gaussian and jointly independent. Each of the noise processes are allowed to be contemporaneously correlated.

Equation (24) is called the transition equation and describes the dynamics of a vector of states, x. At least one of these state variables is

[48]For details, generalizations and explanations, see Anderson and Moore(1979). Other sources are: Chow(1975,1984), Judge et al.(1985), and Meinhold and Singpurwalla(1983).

unobservable. Equation (25) is called the measurement equation and serves to extract information about the state, x, from the (observable) measurement process m.

A maximum likelihood-Kalman filter approach is used to estimate this system. Besides obtaining the parameter estimates, the filter provides estimates of the state vector. These estimates are used to assess the relative importance of wealth and taste shocks in explaining changes in expenditure on durable goods. An alternative set of estimates of the state vector generated by an optimal fixed interval smoother is also reported[49].

The basic experiment in this section consists of estimating equation (23) by means of a state space model like the one described in (24) and (25). The measurement vector m_t has a single element, $(1-L)c_t$. The state vector x_t has five elements:

(26) $x_t=[(1-L)c_t \;\; w_t \;\; v_t \;\; v_{t-1} \;\; v_{t-2}]'$

The matrices A_t, B_t, and the vector b_t are assumed to be constant:

(27) $$A=\begin{bmatrix} 0 & -(1-\delta) & a_1 & a_2 & a_3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(28) $b=[\delta a_0 \;\; 0 \;\; 0 \;\; 0 \;\; 0]'$

(29) $$B=\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

[49]The main difference between filtering and smoothing is that the latter allows for a delay. This delay means that more information is used in estimating the states. In particular, here the estimates are obtained using an "optimal fixed interval smoother". This uses all the information available in the sample to predict each state. It is apparent that the tradeoff with filtering is the substantial increase in computer programming complexity as well as memory needs.

The white noise vector u contains the wealth and taste innovations:

(30) $u_t=[w_t \;\; v_t]'$

The variance covariance matrix Σ_u and the vector Z, are also assumed to be invariant across time:

(31) $$\Sigma_u=\begin{bmatrix}\sigma_{ww} & \sigma_{wv}\\ \sigma_{wv} & \sigma_{vv}\end{bmatrix}$$

(32) $Z_t=[1 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$

It is assumed that there is no measurement error hence $o_t \equiv 0$ for all t.

The main results of this procedure are reported in the answers to questions Q3 and Q4.

Q3: Given the order of the slope shock process, and assuming that the implications of the LCH/PIH are correct (i.e. the wealth disturbance follows an MA(1) with coefficient $-(1-\delta)$ and δ is around 5%), what does the slope process look like? Can these shocks be "plausibly" described as taste shocks?

Q4: Assuming that the slope and wealth innovations are independent, what does the LCH/PIH term (or the wealth term) look like?

The answers to these questions are given in tables 2 and 3. Table 2 presents the results under the assumption of slope shocks following an ARMA process with autoregressive coefficient equal to $(1-\delta)$ and $\delta \approx 5\%$. Except for the new MA terms, the conclusions are very similar to those derived in section II. The values of the likelihood function in the bottom row of the table show that the dominant characteristic is the very strong negative correlation between wealth and slope innovations. Also, as expected, their variances are very similar.

Table 3, on the other hand, presents the results under the assumption of slope shocks following a pure MA process. Remember, however, that given that in the ARMA case there is a coincidental reduction, both specifications, the MA and the ARMA, are potentially able to generate a process for changes in the expenditure on durable goods of exactly the same order. In particular, an MA(3) process (the order of the process for changes in durable's expenditure) can be generated by the sum of the MA(1) wealth disturbance and a slope shocks process following either an ARMA(1,2) (with autoregressive coefficient equal to $(1-\delta)$) or an MA(1) process. The advantage of the latter is that there is one parameter less to estimate, therefore either δ or Φ_{vw} can be directly estimated. The cost of this, however, is that it becomes less likely for the sum of slope and wealth shocks to satisfy the realizability conditions (i.e. their ability to, once summed, generate the MA process followed by the change in expenditure on durable goods).

Columns (1) and (3) in table 3 show the results under the assumption of the LCH/PIH implications being satisfied (i.e. δ=0.05). Once more the implied correlation for wealth and slope innovations is very close to minus one. It is also clear (the likelihood values are significantly smaller than for the ARMA(1,2) case with $\Phi_{vw}\approx-1$) that the realizability conditions are in fact not fully satisfied.

Columns (2) and (4) in the same table answer question Q4: when the innovations are assumed to be independent, the implicit rate of depreciation of durable goods goes to values above 0.9 (per quarter), therefore rejecting any reasonable value for the durability of these goods.

The first two columns in table 4 show the correlation between the total (as compared with the innovations) wealth and slope effects and the change in the expenditure on durable goods. As expected, the former component exhibits a very high correlation with the latter. Columns (3) and (4) in the same table present the contribution of both sources to the fluctuation of the durables' expenditure. It is possible to see that not only their innovation but also their contribution to the unconditional variance of changes in expenditure on durable goods is about the same.

Summarizing, under the maintained assumption of the LCH/PIH implications being satisfied, the slope process seems to follow an ARMA(1,2) with autoregressive coefficients equal to 0.95, and moving average coefficients changing in their relative importance according to the exact value of the correlation between slope and wealth innovations. An alternative formulation, the MA(1), does not satisfy all the realizability conditions but it has the advantage of providing a single solution. In this case, the MA coefficient is around 0.5. The most striking characteristic, however, is the still very strong negative correlation between wealth and slope innovations. If one believes in the positive correlation between aggregate demand and output, this clearly rules out any sensible across-the-board taste shock explanation. Any increase in the desire for current consumption should raise aggregate demand, output, and possibly interest rates. If the output effect is stronger than the interest rate effect, taste and wealth shocks should exhibit positive correlation.

A more plausible explanation points towards changes in the higher moments of wealth. Chapter I proposes a mechanism in which the interaction between the changes in the higher moments of wealth and precautionary motives produces a slope effect that partially offsets wealth shocks. The problem with this explanation, however, is that it implies a similar behavior for non-durables. Chapter III shows that "taste" shocks do not seem to be an important source of non-durables' fluctuations. In that paper the model was in logarithms, however relaxing the log specification can at most uncover a precautionary savings effect on non-durables of about half the wealth effect size (see appendix V in chapter I). Moreover, buying a durable good today guarantees some consumption of the services of the goods tomorrow, therefore the effects of precautionary savings are less important for expenditure on durable than in non-durable goods[50].

This suggests that if the homogeneous-representative consumer model is thought to be a close approximation for the durables case, an important part of the explanation must involve factors that do not affect all goods homogeneously and hopefully affect more durables than non-durables: individual taste shocks, substitution effects of interest rates (Mankiw 1985, 1986), adjustment costs, and the like. These explanations are discussed in more detail in section V.

Finally, figures 3a and 3b show the path of the wealth and the slope components (tw,tv), respectively, when the parameters of the model correspond to those presented in column (4) of table 3. Within the models that have some explanatory chance, this one provides a lower bound for the volatility of wealth and slope components, however it is still the case that the fluctuations would be much wider if the slope component did not exist. Alternatively, the implausible large estimates of the wealth

[50]This difference between durables and non-durables is attenuated as the degree of imperfection in the secondary market for durable goods becomes more important.

effects, can be taken as another symptom of the failure of the representative agent-LCH/PIH model to describe the short run behavior of durables expenditures.

One way to assess the magnitude of the estimated wealth effect is to compute the consumption volatility implied by one of the components of wealth. With this purpose in mind, the univariate representation of labor income is computed, using as proxy for the latter national income, compensation of employees and wages and salaries[51]. All these series are well described by an ARI(1,1) model. The autoregression coefficients are 0.47, 0.54 and 0.53 respectively. And the standard deviation of their innovations are 0.011, 0.0084 and 0.0088, respectively. These values imply standard deviations of human wealth annuities equal to 0.021, 0.018 and 0.019 respectively. Equation (9) shows that when goods are durables the annuity value of wealth appears multiplied by $1/(1-\alpha(1-\delta))$, leading to a maximum value of 0.35 for the national income measure. This is certainly un upper boundary since not all goods are durables, however it is still the case that the implied wealth effect is half the implicit estimate found in the last column of table 2.

V. ANALYSIS OF ALTERNATIVES AND CONCLUSIONS

The novelty of this chapter is to provide a theoretical basis for disentangling the residual of the stochastic process of consumption, permitting the analysis of a broad set of alternatives in a unified way. The main insight of the theoretical section is to show the crucial role played by the intertemporal budget constraint in distinguishing between the

[51] All the series were detrended and normalized so their averages are equal to average expenditure.

implications of shocks that have a wealth effect and shocks that have only a substitution effect.

After deriving the theoretical results, filtering and smoothing techniques were used to estimate the underlying stochastic processes and to generate estimates of the realizations of wealth and slope shocks. The results show that in the framework of the homogeneous-representative agent model, slope shocks have a role almost as important as wealth shocks in explaining the observed fluctuations of expenditure on durable goods.

Once the importance of these slope shocks is shown, the most bothersome issue is their strong negative correlation with wealth shocks. This definitely rules out any sensible explanation in terms of across-the-board taste shocks.

One possible explanation for such negative correlation is the presence of precautionary savings: if the income process is conditionally heteroskedastic and people have precautionary saving motives, then an increase in income not only raises current consumption but also raises the slope of the consumption path. This change in the slope is a pure substitution effect and therefore can be described (approximately) by a taste shock process. However this implies that the same slope shock process (with even more strength) should be observed in non-durables, and that is not the case. This suggests that a large part of the explanation must rely on something that does not homogeneously affect durable and non-durable goods. The easiest explanation would certainly be to have true taste shocks that only affect durables. This generates an additional distribution effect (from one type of good to the other) that behaves exactly like a wealth shock for a single series, therefore it can downward bias the measurement of the relative importance of taste shocks. More importantly, as long as the distribution effect dominates the true wealth effects and exhibits negative correlation with the intertemporal substitution component of the shock, it is also potentially able to account for the negative correlation between wealth and slope innovations. However, in this case changes in expenditure on durables and non-durables should be negatively correlated, which is not true for U.S. quarterly data.

Another possible explanation comes through the substitution effect missed by assuming a constant interest rate. Mankiw(1986) shows that due to the effect of a change in the interest rate on the user cost of durables, the latter ought to be much more responsive to real interest rate changes than non-durables[52]. However, if interest rate changes are permanent (as he assumed) this should behave exactly as a (pure) distribution shock: after an increase in the interest rate the ratio of non-durables consumption to the stock of durables should be permanently reduced, therefore they would not be registered as slope shocks by using only the durables series. In reality, changes in interest rates are less persistent so in addition to the distribution effect (and to the traditional income, wealth and substitution effects) there is a slope effect as the one studied in this paper (as the interest rate changes the optimal allocation of expenditure between durables and non-durables changes). But the real interest rate seem to follow a stationary AR(1) process, therefore the slope and distribution effects would be positively instead of negatively correlated.

Adjustment costs, of the kind most often used, are not likely explanations either. Quadratic adjustment costs could explain the puzzle;

[52] This point was also noticed by Hamburger(1967), among others.

slow adjustment matches the concept of a taste shock that is negatively correlated with wealth shocks: whenever people perceive an increase in wealth they increase their desired stock of durables, however costs of adjusting too quickly induce them to adjust the stock gradually or, in the words of this chapter, to substitute away from today's consumption in favor of tomorrow's consumption. Furthermore it is possible to show that there is a one to one relation between the stochastic process of the taste shock and the partial adjustment mechanism. However, the implied speed of adjustment is too low; the average adjustment period would be five years and only 19% of the adjustment would be completed within one year. Chapter V, however, shows that when the speed of adjustment is low, looking at high frequency data may be misleading, therefore this conclusion must be taken with caution. Moreover, the annual model presented in the next chapter, suggests a much faster adjustment. The one sided (S,s) model (Bar-Ilan and Blinder 1987) departs from the representative agent specification and is conceptually much more appealing than the quadratic model, however it does not solve the puzzle; on the contrary, it deepens it. Changes in the distribution of the population after a wealth shock should lead to even smaller estimated depreciation rates.[53]

Habit formation can -in the same way quadratic adjustment costs can- account for the behavior of aggregate durable goods: whenever people increase their stock of durables they raise marginal utility of future consumption (respect to today's marginal utility), precisely the effect of a negatively correlated taste shock. However the degree of "addiction" seems to be to high: once the effect of today's increase in the stock of

[53] There would also be additional low frequency oscillations.

durables on tomorrow's utility is taken into account, the net result is that people would not perceive almost any benefit from the stocks but only from the expenses on durable goods.

As long as separability between consumption and leisure is assumed, the exclusion of leisure from the utility function cannot explain the findings: If taste shocks are homogeneous across all goods and leisure, then there would be a negative correlation between wealth (income) and taste innovations, but labor and consumption would not exhibit the right business cycle comovement[54]. On the other hand, if taste shocks are of the type that change the marginal rate of substitution between consumption and leisure, the covariance between consumption and labor becomes consistent with business cycle comovements but inconsistent with the negative correlation between taste and wealth shocks found in this essay.

Another possibility is that there are supply elements that determine the process for durable expenditures. In fact most of the analysis in this paper has implicitly assumed a very elastic short run supply of durables. If this is not the case, however, the analysis still goes through (the demand condition still has to be satisfied), although now relative prices and/or real interest rates should respond to changes in expenditure. The extremely low correlation between the slope innovation and any measure of relative price changes (e.g. the correlation between relative prices (durables/non-durables) innovations and the slope innovations is around -0.05), suggest that these are not elements of primary relevance in explaining the puzzle[55].

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[54]This is just an extension of Barro and King(1984) propositions on the implications of a separable utility function for business cycle comovements.

[55]See Bils(1987) for a theoretical model of cyclical rigidities of prices of durable goods.

Different kinds of explanations are the measurement errors and time aggregation problems. However if measurement errors are to be responsible for the rejections, they have to be at least as important as wealth shocks. If that is the case, then there is not much sense in using these series at all. If, on the other hand, time aggregation is responsible, the problems should disappear with the use of higher frequency data. However, repeating the procedures of this paper on monthly data yields an implicit depreciation rate of about 75% (monthly!), so the problem is far from disappearing. Moreover, the results shown in chapter V suggest that the puzzle is much less important when annual data are used, therefore, at least for the LCH/PIH component, the decision period seems to be longer, not shorter, than one quarter.

Finally, liquidity constraints have no hope of explaining the puzzle within the context of the homogeneous-representative agent model. Caballero(1987) shows that in fact liquidity constraints can produce excess smoothness[56], but the serial correlation would not appear until the time when the liquidity constraint is actually binding. Moreover, given that in the procedure utilized the wealth component is extracted from the expenditure series itself, the discontinuities in the periods immediately after the constraints are binding are interpreted as independent wealth innovations and therefore they cannot explain the slope shock process found. The homogeneous-representative agent model, so useful in other areas of macroeconomics, does not seem to render the same insights for the study of the short run behavior of aggregate durable goods. Worse yet, it does not even seem to be a close approximation.

[56]Excess smoothness refers to the lack of response of consumption to income (non-stationary) innovations. See Deaton(1986), West(1986) and Campbell and Deaton(1987). This could also be seen as a negative correlation between wealth and taste innovations.

Summarizing, this chapter raises more questions than provides answers, however its main contributions seem to be useful: first, it provides a metric to assess the relative importance of the rejection results, second, it proposes a procedure that permits studying a broad set of alternatives, third, it shows that contrary to what was commonly thought, taste shocks are subject to well defined constraints that, in this case, permit to rule them out as an explanation for the durables puzzle. And fourth, it establishes a clear characterization of the object to be explained in future research.

APPENDIX I

This appendix solves the optimization problem using the technique developed in chapter I. The reader is referred to that chapter for details.

First make a guess on the process for the stock of durables:

(I.1) $k_{t+1} = \Gamma_t + a_t k_t + e_{t+1}$

The next step is to use the Euler equation and the budget constraint to find Γ_t, a_t and the innovation e_{t+1}, and to check whether (I.1) is a feasible solution.

Replacing (I.1) in the Euler equation (5), leads to the following:

(I.2) $\exp(z_t)\exp(-\theta k_t(1-a_t)) = \beta R\exp(-\theta\Gamma_t)E[\exp(z_{t+1})\exp(-\theta e_{t+1})]$

Without loss of generality it is possible to assume that Γ_t is not a linear function of k_t (this is an identification assumption). This implies that a_t must be equal to one, otherwise k_t would be determined by the Euler equation, regardless of the budget constraint! Given that the exponential utility function exhibits no satiation, this would violate the first order conditions almost surely. Using this result plus the fact that $z_{t+1}-z_t=H(L)v_t+v_{t+1}$, it is possible to find Γ as a function of an expectation that involves e_{t+1} (equation (7)):

(I.3) $\Gamma_t= H(L)v^*_t/\theta + (1/\theta)\log(\beta R) + (1/\theta)\log E_t[\exp(v^*_{t+1}-\theta e_{t+1})]$

The next step is to write down the budget constraint:

(I.4) $\sum_{i=0}^{T-t} \alpha^i(c_{t+i}-y_{t+i}) = \alpha^{-1}S_{t-1}$

but

(I.5) $c_{t+i} = k_{t+i} - (1-\delta)k_{t+i-1}$

replacing (I.1) in (I.5):

(I.5') $c_{t+i} = c_{t+i-1} + (1-(1-\delta)L)(\Gamma_{t+i-1}+e_{t+i})$

therefore

(I.5") $c_{t+i} = c_t + \{\Gamma_{t+i-1}-(1-\delta)\Gamma_{t-1}\} + \delta \sum_{j=1}^{i-1} \Gamma_{t+j-1} + e_{t+i}-(1-\delta)e_t + \delta \sum_{j=1}^{i-1} e_{t+j}$

The process for income can be very general, however given the structure of the problem there is no interaction between the income process and the dependence of e on v* and δ. The more general case can be seen in chapter I, here this process is simplified to a random walk so that the algebra is easier:

$$(I.6) \quad y_{t+i} = y_t + \sum_{j=1}^{i} w^*_{t+j}$$

Replacing (I.5) and (I.6) in (I.4):

$$(I.7) \quad \sum_{j=1}^{T-t} \alpha^i \{(c_t - y_t - (1-\delta)\Gamma_{t-1} - (1-\delta)e_t) + (\Gamma_{t+i-1} + e_{t+i}) \quad {}^{i-1}$$
$$+ \delta \sum_{j=1}^{i-1} (\Gamma_{t+j-1} + e_{t+j}) - \sum_{j=1}^{i} w^*_{t+j}\} = \alpha^{-1} S_{t-1}$$

Substituting (I.3) in (I.7) and using the information structure (i.e. the fact that at time t all the variables indexed by t-h ($h \geq 0$) are known), it is possible to show that the sequence of e's are identified by solving the following T-t' equations:

$$(I.8) \quad \sum_{i=1}^{T-t} \alpha^i \{ \sum_{j=1}^{i} e_{t+j} + 1[i>1]\delta \sum_{j=1}^{i-1} e_{t+j} + 1[i>1] \sum_{j=0}^{i-2} H_j v^*_{t+i-j-1}$$
$$+ 1[i>2]\delta \sum_{j=2}^{i-1} \sum_{k=0}^{j-2} H_k v^*_{t+j-k-1} - \sum_{j=1}^{i} w^*_{t+j}\} = 0 \quad \text{for } t=t' \text{ to } T-1.$$

The problem is solved by induction. At time T-1, $(1-(1-\delta)\alpha)e_T = w^*_T$. And the recursion is:

$$(I.9) \quad e_{T-j}\{1-(\alpha(1-\delta))^{T-j+1}\}\{1+\alpha\delta(1-\alpha^{T-j})/(1-\alpha)\} = w^*_{T-j}(1-\alpha^{T-j})/(1-\alpha)$$
$$-\{(1-\alpha)/(1+\alpha\delta(1-\alpha^{T-j}))\}\, \alpha(\sum_{i=0}^{T-j} \alpha^i H_i) v^*_{T-j}/\theta$$

Taking the limit when T goes to infinity, equation (9) in the paper is obtained[57].

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[57]Notice that the only difference is the annuity value in front of w*. However this happens because in the random walk case the annuity value is one.

APPENDIX II

TIME AGGREGATION BIAS

It is apparent that in a frictionless world with continuous information arrival, there is no reason for decisions to be taken on a quarterly basis. The fact that people take a quarter as the time unit corresponds to data availability not to theoretical restrictions. The purpose of this appendix is to show that contrary to what happens in the case of non-durables, time aggregation does not introduce serious biases on the ARMA representation of the expenditure on durables process.

The intuition is the following: Working(1960) showed that time aggregation of a process that follows a random walk in continuous time introduces an artificial MA coefficient converging to 0.25. On the other hand, it is obvious that if instead of a random walk, the original (almost) continuous time process is white noise, then time aggregation does not have any affect, the (time) aggregate is still white noise. The theoretical process for non-durables is a random walk, therefore Working's limit applies. However expenditure on durables, specially as the time interval shrinks, follows a process very close to a white noise, therefore the size of the upward bias in the MA(1) coefficient is small. Furthermore, as table A.1 below shows, the small bias works in the right direction since the depreciation rate becomes higher as the time unit enlarges, so the required absolute value of the MA coefficient is reduced (remember that the absolute value of the MA coefficient is equal to one minus the depreciation rate).

If the quarterly and the quarter/n depreciation rates are denoted by δ and δ_n respectively, then

(A.1) $(1-\delta_n)^n = (1-\delta) \quad \Rightarrow \quad \delta_n = 1 - (1-\delta)^{1/n}$

Also define $y \equiv (1-L)c_q$ and $x \equiv (1-L)c_{qn}$ with L the lag operator, c_q and c_{qn} the expenditure on durables in a quarter and in a quarter/n respectively. A simple calculation shows that y and x are related as follows:

$$\text{(A.2)}\quad y_t = \sum_{i=0}^{n-1} x_{tn-i} + \sum_{i=0}^{n-1} x_{tn-i-1} + \ldots\ldots + \sum_{i=0}^{n-1} x_{tn-i-n+1}$$

$$= \sum_{j=0}^{2(n-1)} a_j x_{tn-j} \qquad \text{with } a = 1+j \quad \text{for } j \le n-1$$
$$= (2n-1)-j \quad \text{for } j \ge n$$

<u>Proposition</u>

If n is greater than one, and x follows an MA(1) then y also follows an MA(1).

Proof:

It follows directly from Proposition 4 in Engel(1984). If x follows an MA(q) process then y follows an MA(q') process with $q' \le [(q+2(n-1))/n]$. Where [s] denotes the integer part of s. Here q is equal to one, then $q' \le [2-1/n] < 2$.

Q.E.D.

Furthermore, if $r_y(k)$ and $r_x(k)$ denote the k-autocovariances of the quarterly and quarterly/n data respectively, then

$$\text{(A.3)}\quad r_y(k) = \sum_{i=0}^{2(n-1)} \sum_{j=0}^{2(n-1)} a_i a_j r_x(kn+j-i)$$

The statement in the proposition can also be seen in A.3; if $r_x(h)=0$ for $h \ge 2$, then it is apparent that $r_y(h)=0$ for $h \ge 2$.

Replacing the zero autocovariance restrictions in A.3 yields,

$$\text{(A.4)}\quad r_y(0) = r_x(0) \sum_{i=0}^{2(n-1)} a_i^2 + 2r_x(1) \sum_{i=1}^{2(n-1)} a_i a_{i-1}$$

and

$$r_y(1) = r_x(0) \sum_{i=0}^{n-2} a_i a_{i+n} + r_x(1)\{ \sum_{i=0}^{n-1} a_i a_{i+n-1} + \sum_{i=0}^{n-3} a_i a_{i+n+1}\}$$

or

(A.5) $\rho_y(1) \equiv r_y(1)/r_y(0) = (z_1 + \rho_x(1)z_2)/(z_3 + 2\rho_x(1)z_4)$

with $z_1 \equiv \sum_{i=0}^{n-2} a_i a_{i+n}$

$$z_2 \equiv \sum_{i=0}^{n-1} a_i a_{i+n-1} + \sum_{i=0}^{n-3} a_i a_{i+n+1}$$

$$z_3 \equiv \sum_{i=0}^{2(n-1)} a_i^2$$

$$z_4 \equiv \sum_{i=0}^{2(n-1)} a_i a_{i-1}$$

Replacing the expressions for a_i and a_j above yields,

$z_1 = (n-1)^2 + (n-2)^2(n-1)/2 - (n-2)(n-1)(2n-3)/6$

$z_2 = n^2 + (n-1)^2 n/2 - (n-1)n(2n-1)/6 + (n-2)^2 + (n-3)^2(n-2)/2$

$- (n-3)(n-2)(2n-5)/6$

$z_3 = n^2 + (n-1)n(2n-1)/6 + (2n-1)^2(n-1) - (2n-1)(n-1)(3n-2)$

$+ (n-1)(2n-1)(7n-6)/6$

$z_4 = (n-1)n/2 + 2n(2n-1)(n-1) - (4n-1)\{2(n-1)(2n-1)-(n-1)n\}/2$

$+ (n-1)(2n-1)(4n-3)/3$

The next step uses the relation between the correlogram and the MA coefficients:

(A.6) $\rho_x(1) = -(1-\delta)^{1/n}/\{1+(1-\delta)^{2/n}\}$

and

(A.7) $\rho_y(1) = ma_y(1)/(1+ma_y(1)^2)$

The one lag autocorrelation $\rho_y(1)$ is obtained by solving A.5 given $\rho_x(1)$ (or given δ), the $ma_y(1)$ coefficient is obtained from A.7. The time aggregation bias can be measured by comparing this $ma_y(1)$ coefficient with the value of $-(1-\delta)$. Table A.1 below shows that, unless the depreciation rate is extremely high, this bias is not of practical importance.

TABLE A.1

QUARTERLY MA(1) COEFFICIENT

	n				
	2	5	10	100	10^6
-(1-δ)					
-0.95	-0.9499	-0.9499	-0.9499	-0.9499	-0.9353
-0.80	-0.7997	-0.7996	-0.7996	-0.7996	-0.7955
-0.10	-0.0406	-0.0241	-0.0218	-0.0210	-0.0210
-0.00	0.1716	0.2500	0.2633	0.2679	0.2680

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TABLE 1
THE SERIAL CORRELATION

	Coefficients							
	MA_1	MA_2	MA_3	MA_4	AR_1	LLF	Q(18)	Q(36)
(1) WN	-	-	-	-	-	279.5	21.8	36.1
(2) MA(1)	-0.073	-	-	-	-	280.0	19.9	32.5
	(0.080)							
(3) MA(2)	-0.067	0.102	-	-	-	280.7	18.2	30.3
	(0.080)	(0.082)						
(4) MA(3)	-0.063	0.117	-0.154	-	-	282.6	14.9	25.6
	(0.080)	(0.082)	(0.082)					
(5) MA(4)	-0.062	0.116	-0.153	0.036	-	282.6	14.7	25.8
	(0.080)	(0.082)	(0.082)	(0.085)				
(6) ARMA(1,1)	0.552	-	-	-	-0.631	281.1	17.5	29.6
	(0.743)				(0.858)			
(7) ARMA(1,2)	0.605	0.068	-	-	-0.672	281.7	16.2	28.3
	(0.533)	(0.090)			(0.526)			
(8) ARMA(1,3)	0.114	0.105	-0.133		-0.117	282.6	14.8	25.9
	(0.547)	(0.091)	(0.103)		(0.540)			
(9) ARMA(1,4)	-0.075	0.117	-0.154	0.040	0.013	282.6	14.7	25.8
	(2.173)	(0.157)	(0.269)	(0.343)	(2.172)			

Notes: -Standard errors in parenthesis.
-LLF: value of the log-likelihood

TABLE 2
STATE SPACE MODEL
(Slope Process:ARMA(1,2), $\Phi=1-\delta$)

Φ_{vw}	-0.990	-0.995	-1.000	-0.990	-0.995	-1.000	-0.990	-0.995	-1.000
COEFF.									
τ_1	-	-	-	0.072	0.036	0.010	-0.152	0.089	0.008
				(0.021)	(0.017)	(0.003)	(0.050)	(0.014)	(0.004)
τ_2	-	-	-	-	-	-	-0.173	0.075	-0.003
							(0.035)	(0.018)	(0.004)
σ_w	0.299	0.355	0.550	0.241	0.340	0.682	0.155	0.217	0.629
	(0.017)	(0.020)	(0.050)	(0.032)	(0.033)	(0.078)	(0.018)	(0.020)	(0.090)
σ_v	0.285	0.340	0.509	0.253	0.341	0.642	0.123	0.236	0.589
	(0.018)	(0.020)	(0.048)	(0.027)	(0.027)	(0.077)	(0.020)	(0.027)	(0.089)
LLF	386.3	401.1	421.7	390.3	402.9	425.6	402.3	414.9	425.9

Note: -Standard errors in parenthesis.

TABLE 3

STATE SPACE MODEL

(Slope process: MA)

	WHITE NOISE		MA(1)	
τ_1	0.000	0.000	0.466 (0.032)	0.468 (0.056)
σ_w	0.090 (0.006)	0.039 (0.006)	0.106 (0.008)	0.041 (0.008)
σ_v	0.041 (0.005)	0.008 (0.015)	0.067 (0.006)	0.000 (0.008)
Φ_{vw}	-0.976 (0.004)	0.000	-0.989 (0.007)	0.000
δ	0.050	0.962 (0.141)	0.050	0.927 (0.063)
LLF	371.6	422.7	404.2	422.7

Notes: -Constants are not reported.
-When no standard error is reported, it means that the coefficient was fixed at that value.

TABLE 4

RELATIVE IMPORTANCE IN EXPENDITURE FLUCTUATIONS

	Q1		Q2		Q3
(Table.col)	WEALTH	TASTE	WEALTH	TASTE	
*2.7	0.822	-0.658	3.023	2.267	1.333
*2.9	0.744	-0.722	21.19	20.46	1.036
*3.3	0.723	-0.453	2.655	2.046	1.298
3.4	0.994	0.159	1.000	0.000	-
S3.3	0.613	-0.235	2.350	1.903	1.235

Notes: Q1=Cov(x,y)/(sdv(x)sdv(y)) with x=(1-L)c y=tw,tv.
Q2=sdv(y)/sdv(x).
Q3=Q2(tw)/Q2(tv)
*: models in which the LCH/PIH implications are satisfied.
S: smoothing.

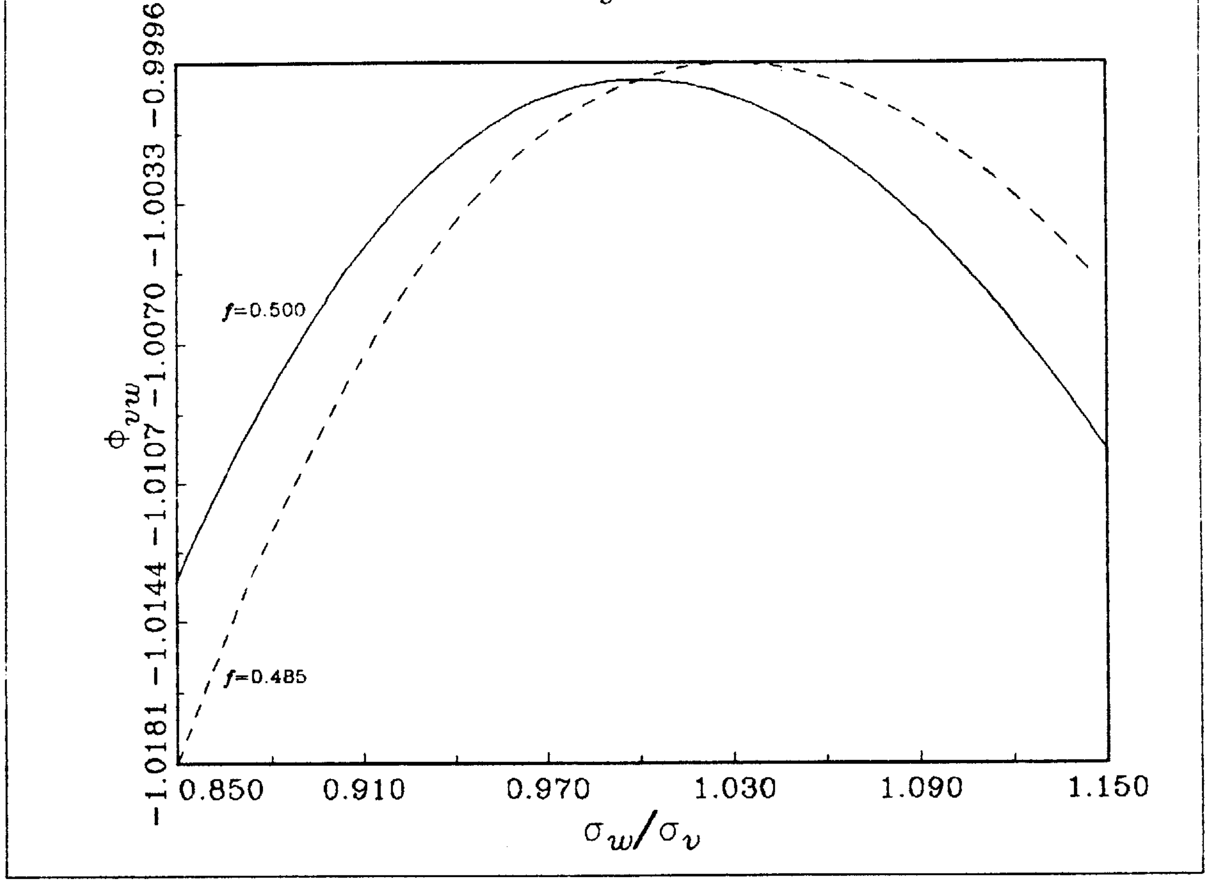


Figure 1

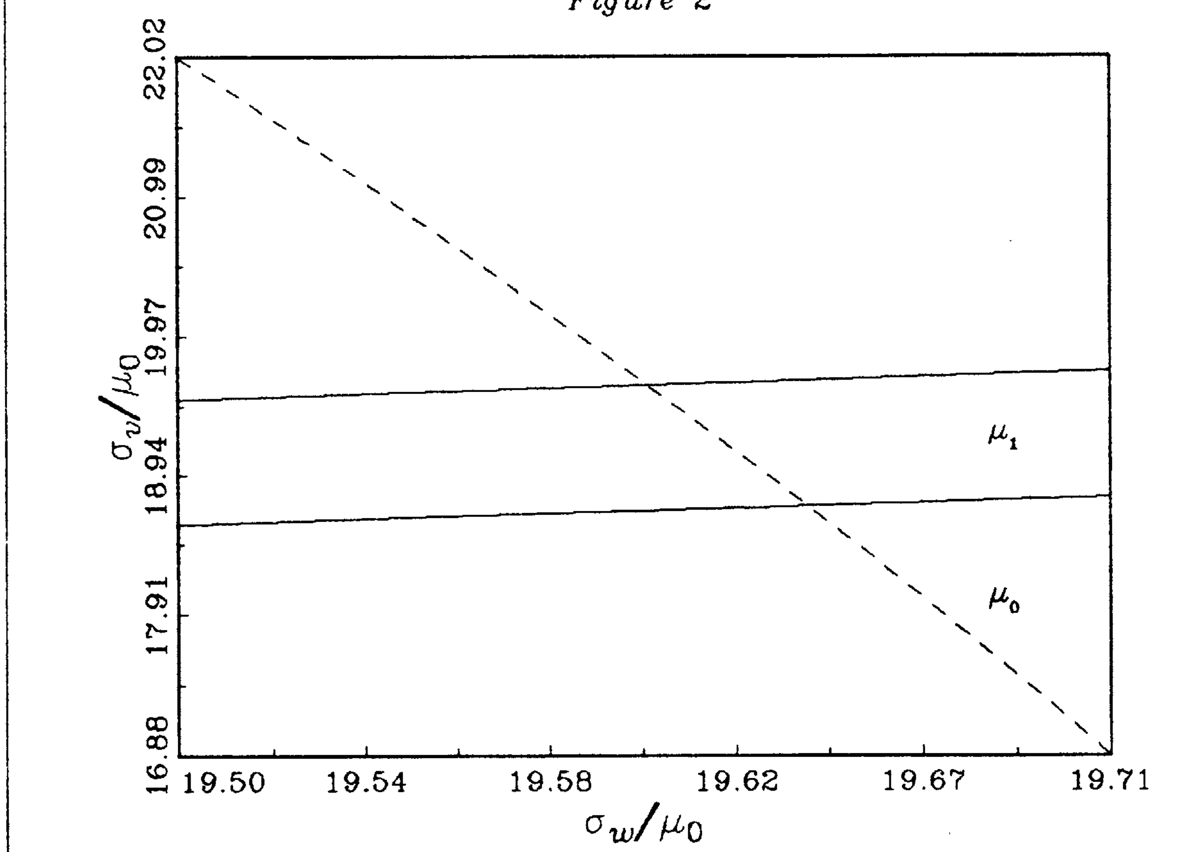
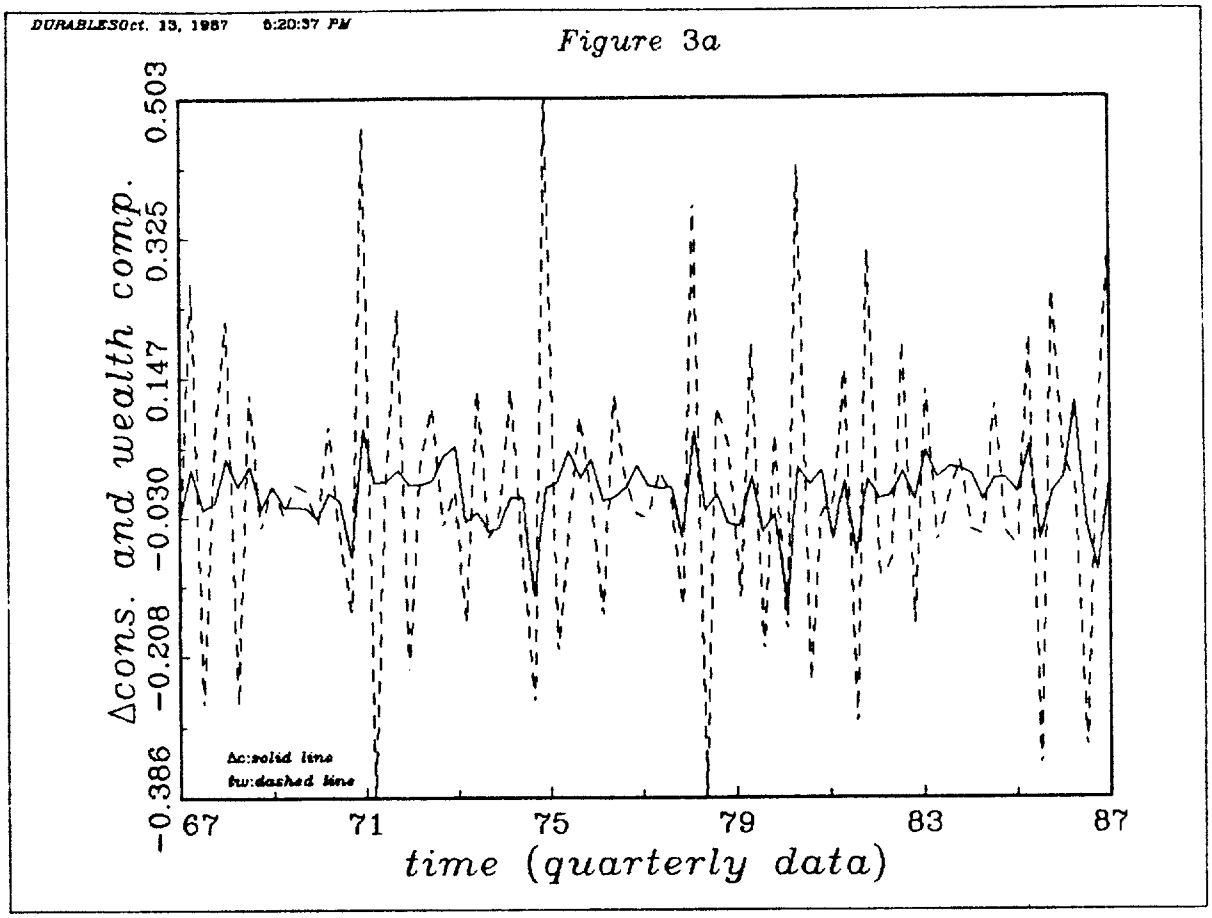
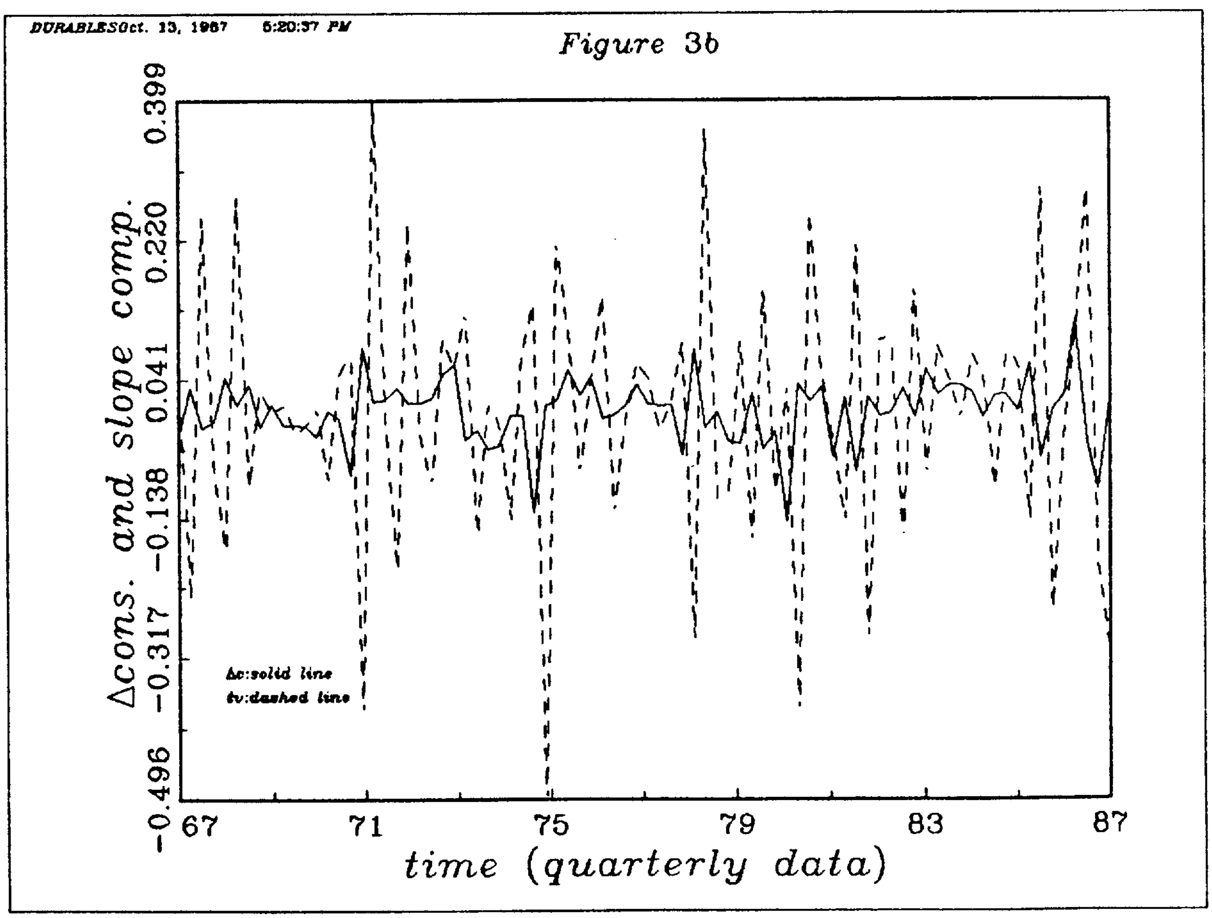


Figure 2


DURABLESOct. 13, 1987 5:20:37 PM
Figure 3a
Δcons. and wealth comp.
0.503
0.325
0.147
-0.030
-0.208
-0.386
Δc:solid line
tw:dashed line
67
71
75
79
83
87
time (quarterly data)


DURABLESOct. 13, 1987 5:20:37 PM
Figure 3b
Δcons. and slope comp.
0.399
0.220
0.041
-0.138
-0.317
-0.496
Δc:solid line
tv:dashed line
67
71
75
79
83
87
time (quarterly data)

CHAPTER V

CONSUMPTION EXPENDITURES: A CASE FOR SLOW ADJUSTMENT

I. INTRODUCTION

Few mainstream economists would strongly object to the implications of the life cycle-permanent income hypothesis (henceforth LCH/PIH)[1]; denying this hypothesis is tantamount to denying many of the basic principles used by economists in their modeling efforts. It is not surprising, then, that the seminal works of Modigliani and Brumberg(1954) and Friedman(1957) were followed by innumerable attempts to test the validity of the LCH/PIH theory.

The advent of rational expectations spurred a whole new literature starting with the Sargent(1978) and Hall(1978) papers that tested the joint LCH/PIH-rational expectations hypothesis.

Hall's paper has been specially important for the consumption literature in the last decade. He noticed that the rational expectations hypothesis implies that consumers should use all the information available to them at each moment in time to take their consumption decision. The LCH/PIH, on the other hand, implies that expected marginal utilities of consumption should be equalized across time. The interaction of these two implications -plus some standard assumptions on the specification of preferences and the sources of uncertainty- makes today's consumption a sufficient statistic to forecast tomorrow's consumption; this is the now famous random walk hypothesis.

Mankiw(1982) noticed that when Hall's insight is applied to durables, the disturbance in a regression of current expenditure on lagged expenditure should exhibit a first order moving average (MA(1)) structure (as opposed to the non-durables case in which this disturbance should be

[1]For the purposes of this chapter I will not stress the differences between the LCH and the PIH. Moreover, I will take them as one.

white noise). Furthermore, the MA coefficient should be negative and its absolute value equal to one minus the rate at which the stock of durables depreciates. The empirical implementation of this model has brought strong rejections of the implications of the LCH/PIH model. Mankiw used quarterly U.S. postwar data and found that contrary to the theory, the disturbance in the equation for durables expenditures behaved as a white noise. In other words, the time series behavior of durables' expenditures exhibited the same type of behavior as expenditures on non-durables. Chapter IV in this thesis showed that this unpleasant result holds even when the model is expanded to allow for the possibility of very general substitution effects, so that only the wealth component of the shocks -the component most clearly related to the LCH/PIH- is subjected to the MA(1) structure.

There is nothing in the theory, however, suggesting the frequency at which decisions are made; researchers often use the data with the highest available frequency. This paper does not intend to deal with the problem of the optimal frequency for decision making[2], but it shows that when annual instead of quarterly data are used, there is a clear difference between the time series behavior of durables and non-durables. Furthermore, this difference cannot be explained in terms of time aggregation problems and points in the direction suggested by the LCH/PIH.

An alternative way to interpret this result is by claiming that when the speed of adjustment is very slow, using lower frequency data may be more revealing. The trade off between time aggregation problems and precision of the adjustment pattern estimates, seem to favor the latter.

[2] Some preliminary work by the author suggests that when there is a cost associated with decision-taking (it is difficult to solve the optimization problem!), the optimal decision rules are a mix of state and time dependent rules. In very stable periods the time rule tends to dominate the state component of the rule.

In addition to the change in the data frequency, this chapter generalizes Hall's and Mankiw's models adding preference uncertainty (taste shocks) and slow adjustment. It is shown that non-durable goods adjust faster than durable goods expenditures. The of adjustment of durables to wealth and taste shocks seems to be around 55% in the same year of the shock, and 90% and 100% one and two years later, respectively.

Another interesting fact is that most of the fluctuations in both goods seem to come from wealth shocks, therefore the LCH/PIH shows to be a useful way to think about annual fluctuations in aggregate expenditures. The primary importance of the LCH/PIH component can also be seen through the high correlation between the innovations of durables and non-durables expenditures; in sharp contrast to the quarterly data evidence[3] -in which this correlation is below 0.4- the annual data show a coefficient of correlation around 0.7.

This introduction is followed by five sections. Section II presents the model. It shows the optimization problem solved by an agent who consumes a durable and a non-durable good, and faces wealth and taste uncertainty. Then it presents the aggregate behavior under the assumption that people are heterogeneous in terms of the time they take to adjust to these shocks.

Section III presents evidence on the time series processes of durables and non-durables. Particular emphasis is placed on showing the differences in the MA structure of both processes and the consistency of the results with the model proposed in section II.

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[3] See Startz(1986).

Section IV shows that durables and non-durables are not cointegrated, therefore taste shocks (or substitution effects in general) that affect the distribution of expenditure between durables and non-durables have permanent effects. This simplifies the subsequent state space representation of the model.

The speed of adjustment, the implicit depreciation rate of durables, as well as the tests on the cross equation restrictions imposed by the model, are all presented in section V. A maximum likelihood-Kalman filter approach is used to estimate the model and the results are very promising. Finally, a fixed interval smoother is used to construct estimates of the underlying source of uncertainty and assess the effect of slowness on expenditure fluctuations.

Section VI presents the conclusions and discusses rejection results found in previous papers (e.g. Mankiw 1982 and Caballero 1987d), in the light of the findings of this paper.

II. THE MODEL

II.1 The Frictionless Model

This subsection presents the standard intertemporal optimization model used in macroeconomic studies of consumption, but adds two less common features; first, durables and non-durables (and services) are jointly modeled[4], and second, there is a taste shock that affects the marginal rate of substitution across time and goods. Given the assumptions necessary to obtain linear models, the latter is the simplest way to introduce a second source of uncertainty in order to avoid singularities in the joint representation of durables and non-durables expenditures.

[4] Bernanke(1985) and Startz(1986) also modeled the joint process of durables and non-durables.

The problem to be solved by each consumer is:

$$(1)\quad \max_{\{cn_{t+i},cd_{t+i}\}} E_t\left[\sum_{i=0}^{T-t} \beta^i U(cn_{t+i},k_{t+i},z_{t+i})\right]$$

s.t

$$cn_{t+i} + cd_{t+i} = R_{t+i}S_{t+i-1} + y_{t+i} - S_{t+i}$$
$$k_{t+i} = (1-\delta)k_{t+i-1} + c_{t+i}$$
$$cn_T + cd_T = R_T S_{T-1} + y_T + (1-\delta)k_T/R_{T+1}$$
$$S_{t-1},\ k_{t-1} \text{ given}$$

with

E_t: expectations operator, conditional on all information available at time t,

β: subjective discount factor,

R: one plus the riskless real interest rate (assumed constant),

U(): instantaneous utility function,

k: stock of durable goods providing services to the consumer,

δ: depreciation rate of durable goods,

cn: expenditure on non-durable goods,

cd: expenditure on durable goods,

S: non-human wealth,

y: labor income (only source of wealth uncertainty), and

z: stochastic taste parameter.

The relative prices have been assumed constant, however this does not represent a strong restriction since it is always possible to replicate the effects of changes in relative prices by a taste shock (substitution effect) and a wealth shock.

U(.,.,.) is further specialized to a separable constant absolute risk aversion (CARA) utility function with multiplicative taste shocks on the durable good[5]:

[5]In order to reduce the number of constants both goods have been assumed to be equally important in the utility function. This does not have any important implication for the results presented in the paper.

(2) $U(cn,k,z)= -(1/\tau)[\exp(-\tau cn)+\exp(-\tau k)\exp(z)]$

with τ the parameter of absolute risk aversion.

More details about the distinctive characteristics of taste shocks can be seen in chapters III and IV of this thesis[6]. As said before, here the distribution-taste shock is introduced mainly to avoid the theoretical singularity in the variance covariance matrix of the innovations of durables and non-durables that would arise if only one source of uncertainty existed. Furthermore, if z is non-stationary cn and cd will not be cointegrated, simplifying the estimation of their joint process even further[7].

Taste shocks can follow a very general process. Assume that:

(3) $D(L)z_{t+1} = F(L)v^*_{t+1}$

$D(L)=1-D_1L-D_2L^2-...-D_pL^{p'}$

$F(L)=1+F_1L+F_2L^2+...+F_qL^{q'}$

where v^* is an independent and identically distributed disturbance with mean zero and variance σ_{v*}^2 [i.i.d.$(0,\sigma_{v*}^2)$].

To simplify the exposition assume, for now, that D(L) is invertible, then:

(3') $z_{t+1}=G(L)v^*_{t+1}$ with $G(L)=D(L)^{-1}F(L)$

Labor income can also follow a general process[8]:

(4) $A(L)(1-L)y_{t+1} = B(L)w^*_{t+1}$

$A(L)=1-A_1L-A_2L^2-...-A_pL^p$

[6] Chapters III shows that taste shocks, of the type that generate serial correlation in the changes of expenditure, are not an important source of fluctuations for non-durable goods. The specification in (2) imposes this result as a restriction.
[7] See section IV.
[8] Adding a drift term to the income process does not affect the nature of the solution. It only changes the permanent level of consumption.

$B(L)=1+B_1L+B_2L^2+...+B_qL^q$

where w^*_{t+1} is i.i.d.$(0,\sigma_{w*}^2)$. The disturbances w^* and v^* can be contemporaneously correlated (σ_{w*v*} may be different from zero).

Apart from the intertemporal budget constraint, the first order conditions are:

(5a) $\exp(-\tau cn_t) = \beta RE_t[\exp(-\tau cn_{t+1})]$

(5b) $\exp(z_t)\exp(-\tau k_t) = \beta RE_t[\exp(z_{t+1})\exp(-\tau k_{t+1})]$

(5c) $(1-\alpha(1-\delta))\exp(-\tau cn_t) = \exp(z_t)\exp(-\tau k_t)$

Appendix I presents the intermediate steps in deriving the stochastic process of non-durables and durables shown below:

(6a) $(1-L)cn_{t+1} = \Gamma_t + \lambda_1 w^*_{t+1} - \lambda_2 v^*_{t+1}$

(6b) $(1-L)cd_{t+1} = \{\varepsilon_t + \lambda_1 w^*_{t+1} + \lambda_3 v^*_{t+1}\}(1-(1-\delta)L)$

with $\Gamma_t = \log(\beta R) + E_t[\exp(-\tau(\lambda_1 w^*_{t+1}-\lambda_2 v^*_{t+1}))]$

$\varepsilon_t = \log(\beta R) + E_t[\exp(-\tau(\lambda_1 w^*_{t+1}-\lambda_3 v^*_{t+1}))] + H(L)v^*_t/\tau$

$H(L) = [(G(L)/L)_+ - G(L)]$ with $_+$ denoting the positive powers of L.

λ_1 = annuity value of an income shock$/(2-\alpha(1-\delta))$

$\lambda_2 = [(1+\epsilon)-\alpha(1-\delta)]/[\tau(2-\alpha(1-\delta))]$

$$\epsilon = (2-\alpha(1-\delta))\alpha \sum_{i=0}^{\infty} \alpha^i H_i$$

$\lambda_3 = (1+\epsilon)/\{\tau(2-\alpha(1-\delta))\}$

Taste shocks do not enter the budget constraint therefore they cannot affect the present value of expenditure. In the context described here taste shocks have two effects: First, they affect the marginal rate of substitution between consumption of durables and non-durables, changing the allocation of expenditure between these two goods. From the point of view of each series this is equivalent to a wealth effect in the sense that it does not change the slope of the consumption path; the main difference, however, is that it leads to a negative comovement between durables and nondurables expenditures. And second, they alter the marginal rate of substitution between consumption of durables today and in the future, changing the intertemporal allocation of expenditure in this good. This adds serial correlation to the time series of expenditure on durable goods (see the term H(L) in ε) and it is explained in more detail in Caballero(1987c and 1987d). For the purposes of this chapter this second effect will be assumed to be negligible. This is done by assuming that the distribution-taste shocks follow a random walk[9]. In this case the second effect disappears since $H_i \equiv 0$ for all i: a taste shock only reallocates expenditure from one good to the other. The presence of this type of shocks contributes to explain why the correlation between durables and non-durables innovations, as traditionally measured, is not as high as was expected (e.g. Startz(1986) computed a correlation of 0.38). It is shown later, however, that when all the corrections suggested here are made, this correlation raises to approximately 0.7. It is also interesting to notice that as long as the goods that are affected in opposite directions by a taste shock have different durability, aggregate expenditure is also affected by these distribution-taste shocks.

Additionally, under the assumption of random walk taste shocks, both series -durables and non-durables- are not be cointegrated, thus facilitating the estimation of their processes. This persistence property will be tested in section IV. The lack of more complex dynamics in the

[9]The persistence component of this assumption is tested -and not rejected- in section IV below.

shocks is needed as an identification assumption: it will be assumed that most of the serial correlation of the durables process, in addition to the MA coefficient implied by the LCH/PIH, will be due to slow adjustment. Therefore, taste shocks will be left only with the role of breaking the perfect correlation between the innovations of the durables and non-durables processes, and given the constant interest rate and relative price assumptions, to justify the non-cointegration result found in section IV.

If taste shocks are assumed to follow a random walk, then (6a) and (6b) simplify to:

(7a) $(1-L)cn_{t+1} = a_0 + u^n_{t+1}$

(7b) $(1-L)cd_{t+1} = a_1 + u^d_{t+1} - (1-\delta)u^d_t$

with $u^n_{t+1} \equiv \lambda_1 w^*_{t+1} - \lambda_2 v^*_{t+1}$

$u^d_{t+1} \equiv \lambda_1 w^*_{t+1} + \lambda_3 v^*_{t+1}$

a_0 and a_1 are constants.

II.2 Slowness

This essay does not pursue an explanation of why some people do not react immediately. This is certainly a fascinating topic, however the main purpose here is to provide a new stylized fact. With this in mind, slowness is introduced by assuming that everybody bears the same wealth and taste shocks but they react with different lags. In this case aggregate expenditure on durable and non-durables, CD and CN respectively, can be described as follows:

(8a) $$(1-L)CN_{t+1} = a_2 + \sum_{i=0}^{npn} \theta^n_i u^n_{t+1-i}$$

(8b) $$(1-L)CD_{t+1} = a_3 + \theta^d_0 u^d_{t+1} + 1[npd>0] \sum_{i=1}^{npd} \{\theta^d_i - \theta^d_{i-1}(1-\delta)\} u^d_{t+1-i} - \theta^d_{npd}(1-\delta)u^d_{t-npd}$$

with npn: number of periods (after the shock) in which the adjustment of non-durables is completed,

npd: number of periods in which the adjustment of durables is completed,

$\theta^d{}_i, \theta^n{}_i \geq 0$ and $\theta^n{}_0 = \theta^d{}_0 = 1$.

It is apparent from equations (8a) and (8b) that the MA structure of the aggregate expenditure on both goods differs substantially as long as $\delta << 1$. For example, if $\theta^n{}_i = \theta^d{}_i$ for all i, the sum of the difference of the MA coefficients of each process (durables minus non-durables) is equal to $-(1-\delta)\Sigma_i \theta_i$ (see equations (10a) and (10b) below). This difference in the MA structure, due to the difference in the durability of the goods, is the main characteristic to be tested in the empirical section.

The proportion of people who adjust their consumption of good j in each period is $\theta^j{}_i / \Sigma_i \theta^j{}_i$. An alternative interpretation is to assume that everybody adjusts at the same speed but slowly ($\theta^j{}_i / \Sigma_i \theta^j{}_i$ per period).

Before concluding this section it is convenient to point out a technical issue. If $\theta^d{}_i$ differs from $\theta^n{}_i$ for some i, there are people with different adjusting times for different goods. If this is the case, it is likely that there are short run cross-equation (goods) effects. According to Bernanke(1985)'s finding of separability between durables and non-durables consumption, and the numerical simulations presented in Lam(1986), this only suggests the presence of a parameter of excess sensitivity in the good with shorter adjustment period. Since this parameter has no role whatsoever in the results presented in sections III and IV, this is only introduced later in section V.

The next section presents preliminary evidence showing that (1-L)CN and (1-L)CD have very different time series behavior. Moreover, the differences point in the direction implied by the LCH/PIH.

III. DO DURABLE GOODS BEHAVE DIFFERENTLY THAN NON-DURABLE GOODS?

This section has two objectives: first, it shows that the MA structures of durables and non-durables differ in the direction suggested by the model of the previous section, and second, it tries to determine the length of the adjustment period of both goods (npn and npd).

The data correspond to annual private expenditure on durables and non-durables and services as reported by NIPA for the period 1947-86. All the series were detrended by using the deterministic trend of GNP, as indicated by an overlapping generations model with population growth and technological progress. In order to check whether the detrending procedure is responsible for the results, all the tests were re-run using per-capita data. Also the restriction on the lagged expenditure coefficient (equal to one) was relaxed. In this case the results showed an even stronger difference in the MA structure of both goods. Furthermore, none of the main conclusions was altered[10].

Table 1 reports the estimates of the MA coefficients of the disturbance of equation (9a) and (9b) below:

(9a) $(1-L)CN_{t+1} = a_0 + tu^n_{t+1}$

(9b) $(1-L)CD_{t+1} = a_1 + tu^d_{t+1}$

with tu^n and tu^d disturbances, and a_0 and a_1 constants.

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[10]These results are available from the author upon request.

Each row represents a different assumption on the dynamic processes of tu^n and tu^d.

The results on durables suggest the presence of large and negative MA coefficients after the first lag. It is clear that the frictionless model will not by itself be able to explain the data (this would require an MA_1 coefficient large and negative), but the sign and size of the MA_2 and MA_3 coefficients are specially encouraging for the "slow" version of the model. A parsimony criterion suggests that an MA(3) model is appropriate to describe the behavior of durables, this implies that npd=2, i.e. people take two periods, after the period when the shock takes place, to complete the adjustment.

As said before, the pattern for non-durables and services is very different from that of durables; only the first MA coefficient is significantly different from zero, and it is large and positive. This can be taken as evidence of a faster adjustment since the implied lag is one (npn=1). However, it could also be taken as evidence of a much higher frequency in the decision making for non-durables expenditures. In this case the positive MA coefficient would be reflecting time aggregation problems as suggested by Working(1960). Which one is the right interpretation is not very important for the main result of this chapter; the fact that both goods behave very differently, and that this difference in behavior is consistent with the implications of the LCH/PIH, is independent of the interpretation of the MA coefficient in the non-durables series.

Summing the MA coefficients (MA_i) leads to the following expressions:

$$\text{(10a)} \quad \sum_{i=1}^{npn} MA_i^n = \sum_{i=1}^{npn} \theta_i^n$$

$$\text{(10b)} \quad \sum_{i=1}^{npd} MA_i^d = -(1-\delta) + \delta \sum_{i=1}^{npd} \theta_i^d$$

Therefore unless the fraction of the people who adjust their expenditure on durables during the period of the shock is very small, the sum of the MA coefficients on durables should be negative and large in absolute value. The sum of the MA coefficients of the non-durables series, on the other hand, should be positive. These sums are shown in the sixth column of table 1. The evidence is very suggestive and makes the differences between the two series even more apparent. It seems safe to conclude that there is strong evidence in favor of the LCH/PIH-slow adjustment model presented in the previous section. Section V will make these statements much more precise. The next section establishes some preliminary results which are necessary for the procedures used later in the chapter.

IV. ARE TASTE SHOCKS PERMANENT?

The model presented in section II implies that both series, CN and CD, are integrated of order one, I(1). The addition of taste shocks and slow adjustment does not destroy the unit root (due to the LCH/PIH model) result.

Nonetheless, the error correction and later the co-integration literature (e.g. Davidson et al.(1978), Granger(1981), Engle and Granger(1987)) have stressed the idea that two (or more) series may each be integrated and therefore have infinite unconditional variance, but there may exist one or more linear combinations of them that are stationary. If this is the case, the series are called cointegrated (of order one in this loose explanation). In other words, this is a steady state concept: the level of the variables can wander everywhere but some linear combination of the variables is mean reverting.

This framework is very appropriate for the model this paper deals with; if the only source of uncertainty was wealth, then the innovations of the process CN and CD would be perfectly collinear. In other words, both series would be I(1) but there would be no sense in estimating both equations together since the variance-covariance matrix of their innovations would be singular. By adding a taste shock, a second source of uncertainty (with different effects on each good) was introduced, and therefore the singularity problem was avoided. However, it is still the case that if taste shocks are stationary then there is only one source with long run effects (wealth), therefore estimating the system (8a)-(8b) would lead to inconsistent estimates (Engle and Granger (1987)) since it would omit an error correction term in the durables equation[11]. On the other hand, if taste shocks are non-stationary, CD and CN need not be cointegrated. In this case a taste shock can permanently alter their relation, so there would be no steady state concept.

Testing for cointegration, therefore, is important to determine whether taste shocks are permanent or not, but more relevant for the purposes of this chapter, it determines whether it is possible to give a meaningful interpretation to the parameters obtained from estimating the system (8a)-(8b) without the addition of an error correction term.

[11] In this framework of homothetic preferences, if wealth was the only source of long run changes, then the cointegrating vector would be such that the error correction term would correspond to the (pseudo) difference of expenditure in both goods. In other words, the (pseudo) difference of expenditure in both goods would be stationary.

This paper uses three of the statistics proposed in Engle and Granger(1987), all of them based on either of the following cointegrating regressions[12]:

(11a) $CN_t = a_5 + a_6 CD_t + \omega_{1t}$

(11b) $CD_t = a_7 + a_8 CN_t + \omega_{2t}$

The first statistic is the CRDW, i.e. the Durbin-Watson statistic (DW) of each regression. If ω_{1t} and ω_{2t} are non-stationary, i.e. CN and CD are not cointegrated, the DW approaches zero. Therefore a large DW can be taken as evidence of cointegration.

The second statistic is the DF, i.e. the t-statistic (absolute value) on a Dickey-Fuller auxiliary regression. After retrieving the estimates of ω_{1t} and ω_{2t} the first difference of each of these residuals is constructed. The auxiliary regression consists of regressing this difference on the lagged level of the estimated disturbances. The DF statistic is the t-statistic on the coefficient of the lagged level disturbance.

The third statistic used here is the ADF (augmented DF). This statistic is identical to the previous one but the auxiliary regression includes lagged changes in the disturbances in the right hand side.

Engle and Granger(1987) performed Monte-Carlo experiments to generate estimates of the critical values and power of these statistics. None of the models they used to generate the data correspond to equations (8a) and (8b), however the DF and ADF statistics do not seem to be very sensitive to the changes in the assumptions about the stationary component of the shocks (the source of differences between their models and (8a)-(8b)). More

[12]The statistics may differ between the two equations, because of the well known result that the inverse of the slope estimate is different from the slope estimate of the reverse regression.

problematic is the sample size, they used 100 observations, significantly more than the 39 observations used in this paper. Fortunately, Engle and Yoo(1986) present tables for different sample sizes, including the case of 50 observations which is used as a reference here.

The drift terms are another issue. All the previous statistics were derived for the zero drift case. The drifts in (8a) and (8b) are not significantly different from zero therefore this should not represent major problems. In any event, the same set of statistics is reported for the case in which the data is further detrended so the drift terms are numerically equal to zero in the sample. In both cases, however, the critical values should be taken with caution. Stock and Watson(1986) show, for their cointegration-statistic, that when the drift term is estimated the critical values rise (in absolute value).

The columns of table 2 show the values of these statistics in the cases explained above. Specially considering that the sample is smaller than 50 observations and that the drift terms are estimated, the results seem to be clear evidence on the non-cointegration of CD and CN. As said before, this suggests that taste shocks (or substitution effects in general) have permanent effects. Furthermore, (8a) and (8b) can be estimated as a system without the need of an error correction term. This is done in the next section.

V. DEEP PARAMETERS AND AGGREGATE BEHAVIOR

This section is divided in two parts: the first part estimates and tests the parameters and restrictions imposed by the system formed by equations (8a) and (8b); the second part shows the relation between wealth and taste shocks and the path of expenditure on durables and non-durables. Also some simulation experiments are performed to show the effect of slowness on the dynamics of both goods.

V.1 Deep Parameters

This subsection starts with the assumption -based on the evidence in section III- that the adjustment is completed in two or less periods after the shock.

The system formed by equations (8a) and (8b) is estimated by maximum likelihood. The prediction errors to construct the likelihood are formed using a Kalman filter. The details can be seen in appendix II.

Table 3 presents the results. Column (1) corresponds to equation by equation estimates of equations (8a) and (8b)[13]. The precision of the estimates is substantially improved when the system is jointly estimated. The results of the joint estimation that are equivalent to those of column (1) are shown in column (2). It is possible to see that the value of the likelihood function is substantially larger. Hereon the analysis concentrates on the joint estimation results.

Column (2) shows the results with no cross equation restrictions. The first very promising result is the estimate of the annual rate of depreciation of durables, 0.35. This is still high, however it is much more reasonable than the rates of depreciation above 0.95 per quarter obtained in previous studies (e.g. Mankiw (1982) and chapter IV in this thesis). The second important result is that the correlation between the innovations of

[13]These results do not exactly match (although they are not significantly different) the coefficients implied by the MA estimates obtained in section III. This is not surprising. The problem is very non-linear and they were estimated using different software packages; the estimates in section III were computed using MicroTSP version 5.1, whereas the estimates in this section were computed using GAUSS version 1.49b.

durables and non-durables and services is 0.7, considerably higher than what was found before (Startz (1986) found a correlation coefficient of the innovations of his model equal to 0.38), and much closer to what one would expect if the LCH/PIH is the driving force of consumption decisions. The estimates of $\theta^d{}_1$ and $\theta^n{}_1$ are large and significant. The estimates of $\theta^d{}_2$ and $\theta^n{}_2$, on the other hand, do not appear very significant, however the results in column (5) show that the restriction $\theta^d{}_2=\theta^n{}_2=0$ can be rejected at the 5% significance level. It is also encouraging to see that all the estimates of the θs are positive as suggested by the slow adjustment model.

Column (3) imposes the constraint of equal speed of adjustment in both goods; the likelihood ratio statistic (LR) for this hypothesis is 4.4 and therefore cannot (marginally) be rejected at the 10% significance level, however the implicit rate of depreciation rises and it becomes less reasonable. In fact when the rate of depreciation is kept fixed at 0.35 (see column (4)) these equality restrictions ($\theta^d{}_1=\theta^n{}_1$ and $\theta^d{}_2=\theta^n{}_2$) can be rejected at the 2% significance level.

Table 5 presents the speed of adjustment implied by the set of models shown in table 3. Overall, the results are sensible: people seem to adjust their stock of durables more slowly than their level of consumption of non-durables and services. For durable goods the adjustment takes three periods. Approximately 55% of the adjustment is completed in the year of the shock, 35% one year later, and 10% after two years. Non-durables and services, on the other hand, show numbers around 70%, 25% and 5% for the same periods of adjustment.

V.2 Slowness and the Impact of Wealth and Taste Shocks

This subsection shows the dynamic responses generated by wealth and taste shocks. Special emphasis is put on the effect of slowness on these dynamics.

The basic model corresponds to equations (8a) and (8b), and the parameter estimates correspond to the model with no cross equation restrictions on the θs. For the purpose of studying the effect of wealth and taste shocks it is convenient to re-estimate the model replacing the disturbances of each equation for their equivalent expressions in terms of wealth and taste innovations. This is done by substituting u^n and u^d by their definitions (shown below equation (7b)) and assuming that wealth and taste shocks are independent. The only difference with those definitions is a parameter γ premultiplying the wealth innovations in the equation for non-durables and services. This parameter is introduced to take into account the difference in size of the expenditures on these goods and the expenditure on durables, and the cross effects due to different speeds of adjustment. In addition, all the coefficients λ are normalized so $\lambda_1=\lambda_3=1$.

As expected, the results are identical to those in column (2) of table 3, with the exception of the new decomposition of disturbances. The estimates are reported in table 4. The parameter γ shows that the relative response of durables to wealth innovations is much larger than that of non-durables and services. In fact the latter represent approximately 80% of consumption expenditures but γ is only 1.5. It is also interesting to notice that the standard deviation of wealth innovations is approximately 40% larger than the standard deviation of taste innovations.

Figures 1 to 6 show the effect of a wealth and a taste innovation on both goods under a "slow" (corresponding to the parameters estimated in this paper) and a "fast" ($\theta_1=\theta_2=0$) adjustment regime. Figures 1 and 2 correspond to the effects of taste and wealth shocks under the slow regime, figures 3 and 4 show the effects of equivalent taste and wealth shocks under the fast regime. Figures 5 and 6 show the effects of taste and wealth shocks on total consumption expenditure. The main lesson of these experiments is that under the slow regime a single shock can produce paths of expenditure close to those observed in the business cycle.

Figures 7 to 11 use the estimates of the state vector (underlying disturbances) obtained with a fixed interval smoother. Figures 7 and 8 show the path of (1-L)CD and (1-L)CN respectively, and the wealth and taste components (remember that "component" here refers to the weighted sum of the current and lagged innovations). It is apparent that the wealth component is the main driving force of both processes, and specially so for the non-durables and services process.

Figures 9, 10 and 11 show the path of the sum of wealth and taste components of (1-L)CD, (1-L)CN and (1-L)CT (total expenditure), and compare these paths with the paths these variables would have followed had the same innovations occur under the fast regime. Clearly, the latter would have led to much wider and less persistent fluctuations of consumption expenditure.

Summarizing, the taste shock-slow model not only fits the data well but also has implications for aggregate fluctuations that are very much in accord with the concept of the business cycle that many economists have in mind.

VI. CONCLUSION AND GENERAL DISCUSSION

This paper presents an augmented LCH/PIH model of durable and non-durable expenditures, in which people are heterogeneous in the speed at which they react to wealth and taste shocks. The annual postwar US data provide strong support to the model. This contrasts sharply with the evidence found when quarterly data are used.

Mankiw(1982) used durables-quarterly postwar U.S. data and found no evidence of an MA term; he concluded that the stochastic process followed by expenditure on durables is very similar to the process followed by non-durables and services, therefore rejecting the LCH/PIH implications. Most of the papers on durables that followed Mankiw's paper eluded the "lack of an MA term" problem, and therefore left an important failure of the theory unexplained. Chapter IV in this thesis also used quarterly data and extended Mankiw's model to allow for taste shocks and other omitted potential sources of rejections. The results were very negative: the durables-quarterly data puzzle is robust to all these generalizations, and worse, whatever is causing the rejection has at least as much explanatory power as the LCH/PIH. The conclusion of that chapter was that the representative agent version of the LCH/PIH did not seem to have much chance of explaining the facts.

After this strong rejection, one of the first models that comes to mind is the (S,s) model. Its microfoundations are very sensible for the case of durables and the aggregation has been worked out in Bar-Ilan and Blinder(1987). Unfortunately, it does not take long to realize that aggregation, at least of the form implied by this model, far from solving the quarterly data puzzle exacerbates it. If the wealth innovation is small, so the steady state distribution of consumers between the bands remains unchanged (uniform), the MA coefficient should be the same as the one obtained in the frictionless representative-consumer model. If, on the other hand, the shock is large so that the distribution of consumers after the shock skews towards one of the barriers[14], the MA coefficient should be even closer to minus one than what is suggested by the representative agent model, since immediately after the shock most of the people will be far from the one barrier that leads to replacement purchases.

The magnitude, as well as the robustness of the puzzle found for the case of durable goods using quarterly data, may lead one to believe in an irrational behavior by consumers. This essay has shown, however, that this need not be the case. In fact the slow adjustment version of the LCH/PIH model was supported by the U.S. annual data. The puzzle to be explained now is why the results differ so much when quarterly or annual data are used[15]. One simple example of how this could be done is the following: suppose that people take decisions once a year (and at the same time) but distribute their shopping almost uniformly along the four quarters (0.22,0.24,0.26,0.28). Then the quarterly changes in expenditure on durable goods would not be covariance stationary, however table 6 shows -for the case in which the data are generated by the annual model[16]- that they would look very close to a white noise with some higher order MA terms; the type of result usually found in the postwar U.S. data (see bottom rows in table 6). An alternative explanation is that people actually take decisions on a

[14]Actually this intuition, as well as Bar-Ilan and Blinder's model, is much better suited for the one sided (S,s) model.

[15]Bernanke(1984) stresses another dimension of this difference. He notices that studies using panel data seem to show more consistency with the implications of the LCH/PIH than aggregate data studies do. It is interesting to notice, however, that most of the panel data studies use annual data, so part of their better success could be due to the frequency problem noticed in this paper.

[16]The annual data were generated by equation (8b) using the parameters estimated in the paper. The disturbances were drawn from a normal distribution. The quarterly data were then generated by dividing the expenses according to the weights given in the example.

quarterly basis but they adjust very slowly and irregularly. If this is the case, the MA estimates in the quarterly model will be very small and unprecise. Therefore, estimating the model with annual data is just a way to combine the autocovariances[17] of the quarterly data in such a way that is more revealing. The bottom rows of table 6 show the value of the first eight MA coefficients when quarterly data are used; interestingly, the sum of the MA terms is negative and equal to -0.541, reflecting the dynamics implied by the combination of consumption smoothing and durability of goods[18]. The research now needs to orient towards understanding the source of slowness, the short run dynamics and the length of the decision period, but this is far less important, for the fundamentals of economics, than plain irrationality.

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[17]Call xa_t the annual change in expenditure on durable goods between periods t-1 and t, and xq_{tq} the quarterly change in expenditure on durable goods between periods tq-1 and tq, then

$$xa_t = \sum_{j=0}^{6} a_j xq_{tq-j} \quad \text{with } a_0=a_6=1,\ a_1=a_5=2,\ a_2=a_4=3 \text{ and } a_3=4$$

so using proposition 4 in Engel(1984) it is possible to show the relation between the annual autocovariances, $r_{xa}(k)$, and the quarterly autocovariances, $r_{xq}(k)$:

$$r_{xa}(k) = \sum_{j=0}^{6} \sum_{i=0}^{6} a_i a_j r_{xq}(4k+j-i)$$

[18]Adding MA terms to 12 (computational problems require to set the first two MA coefficients equal to zero) lowers the sum of MA coefficients to -0.664.

APPENDIX I

This appendix solves the optimization problem using the technique developed in chapter I. The reader is referred there for details.

First make a guess on the processes of the expenditure on non-durable goods and of the stock of durables:

(I.1) $cn_{t+1} = \Gamma_t + a_{1t}cn_t + u^n_{t+1}$

(I.2) $k_{t+1} = \zeta_t + a_{2t}k_t + u^d_{t+1}$

The next step is to use the first order conditions (including the budget constraint) to find Γ_t, ζ_t, a_{1t}, a_{2t}, and the innovations u^n_{t+1} and u^d_{t+1}, and to check whether (I.1) and (I.2) are feasible solutions.

Replacing (I.1) and (I.2) in the Euler equations (5a) and (5b) respectively, leads to the following:

(I.3) $\exp(-\tau cn_t(1-a_{1t})) = \beta R\exp(-\tau\Gamma_t)E_t[\exp(-\tau u^n_{t+1})]$

(I.3') $\exp(z_t)\exp(-\tau k_t(1-a_{2t})) = \beta R\exp(-\tau\zeta_t)E_t[\exp(z_{t+1})\exp(-\tau u^d_{t+1})]$

Without loss of generality it is possible to assume that Γ_t and ζ_t are not linear function of cn_t and k_t respectively (this is an identification assumption). This implies that a_{1t} and a_{2t} must be equal to one[19], otherwise cn_t and k_t would be determined by the Euler equation, regardless of the budget constraint! Given that the exponential utility function exhibits no satiation, this would violate the first order conditions almost surely. Using this result plus the fact that $z_{t+1}-z_t=H(L)v_t+v_{t+1}$, it is possible to find Γ_t and ζ_t as functions of an expectation that involves u^n_{t+1} and u^d_{t+1}:

(I.4) $\Gamma_t = (1/\tau)\log(\beta R) + (1/\tau)\log E_t[\exp(-\tau u^n_{t+1})]$

$= \Gamma$ under the i.i.d. assumptions on v^* and w^*.

[19]Notice that the first order condition (5c) implies that $a_1=a_2$.

(I.4') $\zeta_t = H(L)v^*_t/\tau + (1/\tau)\log(\beta R) + (1/\tau)\log E_t[\exp(v^*_{t+i} - \tau u^d_{t+i})]$

The next step is to write down the budget constraint:

$$\text{(I.5)} \quad \sum_{i=0}^{T-t} \alpha^i (cn_{t+i} + cd_{t+i} - y_{t+i}) = \alpha^{-1} S_{t-1}$$

but

$$\text{(I.6)} \quad cn_{t+i} = cn_t + i\Gamma + \sum_{j=1}^{i} u^n_{t+j}$$

and

$$\text{(I.6')} \quad cd_{t+i} = k_{t+i} - (1-\delta)k_{t+i-1}$$

replacing (I.2) in (I.6') yields:

$$\text{(I.6")} \quad cd_{t+i} = cd_{t+i-1} + (1-(1-\delta)L)(\zeta_{t+i-1} + u^d_{t+i})$$

therefore

$$\text{(I.6''')} \quad cd_{t+i} = cd_t + \{\zeta_{t+i-1} - (1-\delta)\zeta_{t-1}\} + \delta \sum_{j=1}^{i-1} \zeta_{t+j-1} + u^d_{t+i} - (1-\delta)u^d_t + \delta \sum_{j=1}^{i-1} u^d_{t+j}$$

The process for income can be very general, however given the structure of the problem there is no interaction between the income process and the dependence of u^n and u^d on v^*. The more general case can be seen in chapter I, here this process is simplified to a random walk so that the algebra is easier:

$$\text{(I.7)} \quad y_{t+i} = y_t + \sum_{j=1}^{i} w^*_{t+j}$$

Replacing (I.6), (I.6''') and (I.7) in (I.5) yields:

$$\text{(I.8)} \quad \sum_{i=0}^{T-t} \alpha^i \{(cn_t + cd_t - y_t - (1-\delta)\zeta_{t-1} - (1-\delta)u^d_t) + (\zeta_{t+i-1} + u^d_{t+i}) + i\Gamma + 1[i>1]\delta \sum_{j=1}^{i-1} (\Gamma_{t+j-1} + u^d_{t+j}) + \sum_{j=1}^{i} (u^n_{t+j} - w^*_{t+j})\} = \alpha^{-1} S_{t-1}$$

Substituting (I.4) and (I.4') in (I.8), and using the information structure (i.e. the fact that at time t all the variables indexed by t-h ($h \geq 0$) are known), it is possible to show that a linear combination of the sequences $\{u^d\}$ and $\{u^n\}$ is identified by solving the following T-t' equations:

$$(I.9)\quad \sum_{i=1}^{T-t} \alpha^i \{ \sum_{j=1}^{i} u^d_{t+j} + 1[i>1]\delta \sum_{j=1}^{i-1} u^d_{t+j} + 1[i>1] \sum_{j=0}^{i-2} H_j v^*_{t+i-j-i} + 1[i>2]\delta \sum_{j=2}^{i-1} \sum_{k=0}^{j-2} H_k v^*_{t+j-k-1} + \sum_{j=1}^{i} (u^n_{t+j} - w^*_{t+j}) \} = 0$$

for t=t' to T-1.

The problem is solved by induction. At time T-1, $u^n_T+(1-(1-\delta)\alpha)u^d_T=w^*_T$. And the recursion is:

$$(I.10)\quad u^d_{T-j}\{1-((1-\delta)\alpha)^{T-j+1}\}\{1+\alpha\delta(1-\alpha^{T-j})/(1-\alpha)\} = (w^*_{T-j}+u^n_{T-j})(1-\alpha^{T-j})/(1-\alpha) - \{(1-\alpha)/(1+\alpha\delta(1-\alpha^{T-j}))\}\ \alpha(\sum_{i=0}^{T-j} \alpha^i H_i)v^*_{T-j}/\theta$$

Taking the limit when T goes to infinity and re-arranging the terms yields:

$$(I.11)\quad u^n_t + u^d_t(1-\alpha(1-\delta)) = w^*_t - [\alpha(1-\alpha(1-\delta)) \sum_{i=0}^{\infty} \alpha^i H_i]v^*_i/\tau$$

but when taste shocks follow a random walk, $H_i \equiv 0$ for all i, then

$$(I.12)\quad u^n_t + u^d_t(1-\alpha(1-\delta)) = w^*_t$$

On the other hand, the first order condition (5c) implies:

$$(I.13)\quad u^d_t - u^n_t = v^*_t/\tau$$

Combining (I.12) and (I.13) it is possible to obtain the solutions:

$$(I.14a)\quad u^d_t = [w^*_t+v^*_t/\tau]/[2-\alpha(1-\delta)]$$

$$(I.14b)\quad u^n_t = [w^*_t-(1-\alpha(1-\delta))v^*_t/\tau]/[2-\alpha(1-\delta)]$$

APPENDIX II

This appendix shows the state space representation of the model.

The basic state space model can be written as follows[20]:

$$(II.1)\quad x_{t+1}= A_{t+1}x_t + b_{t+1} + B_{t+1}u_{t+1}$$

[20]For details, generalizations and explanations, see Anderson and Moore(1979). Other sources are: Chow(1975,1984), Judge et al.(1985), and Meinhold and Singpurwalla(1983).

(II.2) $m_t = Z_t x_t + S_t o_t$

with x_t: a (Kx1) vector of state variables,

A_t, B_t: (KxK) fixed matrices,

b_t: a (Kx1) non-stochastic vector,

u_t: an (Mx1) white noise process,

m_t: an (Nx1) measurement process,

Z_t: an (NxK) fixed matrix,

S_t: an (NxP) fixed matrix,

o_t: a (Px1) white noise process.

The white noise vector processes are assumed to be Gaussian and jointly independent. Each of the noise processes are allowed to be contemporaneously correlated.

Equation (II.1) is called the transition equation and describes the dynamics of a vector of states, x. At least one of these state variables is unobservable. Equation (II.2) is called the measurement equation and serves to extract information about the state, x, from the (observable) measurement process m.

A maximum likelihood-Kalman filter approach is used to estimate this system. The estimates of the state vector used to construct figure 7 to 11 are obtained by means of an optimal fixed interval smoother.

The system formed by equations (8a) and (8b) can be written on these terms. The measurement vector m_t has two elements, $(1-L)CN_t$ and $(1-L)CD_t$. The state vector x_t has seven elements:

(II.3) $x_t = [(1-L)CN_t \;\; u^n_t \;\; u^n_{t-1} \;\; (1-L)CD_t \;\; u^d_t \;\; u^d_{t-1} \;\; u^d_{t-2}]'$

The matrices A_t, B_t, and the vector b_t are constant:

$$(II.4)\quad A=\begin{bmatrix} 0 & \theta^n{}_1 & \theta^n{}_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (1-\theta^d{}_1(1-\delta)) & (\theta^d{}_1-\theta^d{}_2(1-\delta)) & -\theta^d{}_2(1-\delta) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$(II.5)\quad b=[a_2\ 0\ 0\ a_3\ 0\ 0\ 0]'$$

$$(II.6)\quad B=\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The white noise vector u contains the innovations of each equation:

$$(II.7)\quad u_t=[u^n{}_t\ u^d{}_t]'$$

The variance covariance matrix Σ_u and the vector Z, are also assumed to be invariant across time:

$$(II.8)\quad \Sigma_u=\begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix}$$

$$(II.9)\quad Z_t=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

It is assumed that there is no measurement error hence $o_t \equiv 0$ for all t. With this all the elements necessary to estimate the model in section V.1 are given. The model used in section V.2 is similar to this one. The main difference is that the residuals are decomposed into wealth and taste shocks.

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TABLE 1
THE MA STRUCTURE

	Coefficients							
	MA_1	MA_2	MA_3	MA_4	MA_5	ΣMA	Q(8)	LLF
I.DURABLES								
(1) WN	-	-	-	-	-	-	5.69	-119.3
(2) MA(1)	0.124	-	-	-	-	0.124	4.54	-119.0
	(0.165)					(0.165)		
(3) MA(2)	0.073	-0.217	-	-	-	-0.144	4.11	-118.3
	(0.169)	(0.171)				(0.244)		
(4) MA(3)	-0.064	-0.275	-0.375	-	-	-0.714	2.45	-115.8
	(0.173)	(0.174)	(0.177)			(0.338)		
(5) MA(4)	-0.028	-0.290	-0.332	-0.336	-	-0.913	1.59	-114.3
	(0.177)	(0.178)	(0.181)	(0.182)		(0.417)		
(6) MA(5)	-0.048	-0.330	-0.354	-0.295	0.076	-0.951	0.90	-114.3
	(0.179)	(0.181)	(0.184)	(0.185)	(0.184)	(0.460)		
II. NON-DURABLES AND SERVICES								
(1) WN	-	-	-	-	-	-	4.38	-130.2
(2) MA(1)	0.393	-	-	-	-	0.393	0.58	-127.4
	(0.164)					(0.164)		
(3) MA(2)	0.391	-0.069	-	-	-	0.322	0.49	-127.3
	(0.167)	(0.168)				(0.236)		
(4) MA(3)	0.391	-0.070	-0.044	-	-	0.277	0.42	-127.3
	(0.169)	(0.170)	(0.170)			(0.293)		
(5) MA(4)	0.390	-0.071	-0.038	-0.031	-	0.250	0.45	-127.3
	(0.172)	(0.172)	(0.173)	(0.174)		(0.343)		
(6) MA(5)	0.387	-0.064	-0.045	-0.017	-0.065	0.196	0.35	-127.3
	(0.174)	(0.175)	(0.175)	(0.176)	(0.180)	(0.388)		

Notes: -Columns 1 to 5: moving average coefficients.
-Column 6: sum of moving average coefficients.
-Column 7: Portmanteau statistic. (8 degrees of freedom).
-Rows: estimated process.
-Standard errors in parenthesis.
-LLF: value of the log-likelihood

TABLE 2

TESTS FOR COINTEGRATION

	DW	DF	ADF(1)	ADF(2)	ADF(7)
EQUATION					
(10a)	0.158	0.521	1.367	0.955	1.318
(10b)	0.150	0.361	0.311	0.319	0.643
NO DRIFT					
(10a)	0.147	1.182	1.801	1.713	1.819
(10b)	0.526	2.429	2.916	2.563	2.047
SIGNIFICANCE		CRITICAL VALUES			
1%	1.00/1.49	4.32	4.32	4.32	4.32
5%	0.78/1.03	3.67	3.67	3.67	3.67
10%	0.69/0.83	3.28	3.28	3.28	3.28

Notes: -DW: Durbin-Watson statistic.
-DF: Dickey-Fuller statistic.
-ADF(1): Augmented DF statistic (one lag of the LHS variable).
-ADF(2): Augmented DF statistic (two lags of the LHS variable).
-ADF(7): Augmented DF statistic (seven lags of the LHS variable).

TABLE 3
STATE SPACE MODEL
(Durables/Non-durables shocks decomposition)

	(1)	(2)	(3)	(4)	(5)
θ^d_1	0.701	0.682	0.439	0.507	0.077
	(0.204)	(0.148)	(0.130)	(0.118)	(0.228)
θ^d_2	0.319	0.178	0.060	0.124	0.000
	(0.260)	(0.183)	(0.139)	(0.125)	
θ^n_1	0.363	0.338	θ^d_1	θ^d_1	0.362
	(0.164)	(0.133)			(0.114)
θ^n_2	-0.074	0.068	θ^d_2	θ^d_2	0.000
	(0.182)	(0.134)			
σ_{ud}	5.022	5.075	5.048	5.162	5.122
	(0.582)	(0.594)	(0.582)	(0.607)	(0.580)
σ_{un}	6.311	6.439	6.398	6.431	6.343
	(0.719)	(0.756)	(0.733)	(0.741)	(0.719)
$\rho_{ud,un}$	-	0.700	0.647	0.634	0.619
		(0.089)	(0.097)	(0.100)	(0.099)
δ	0.350	0.348	0.513	0.348	1.077
	(0.245)	(0.131)	(0.227)		(0.228)
LLF	-173.4	-162.0	-164.2	-165.9	-165.5

Notes: -Columns (1) to (5): estimates of the state space model. The assumptions can be seen directly in the table.
-Standard errors in parenthesis.

TABLE 4
STATE SPACE MODEL
(Wealth and taste shocks decomposition)

θ^d_1	θ^d_2	θ^n_1	θ^n_2	σ_w	σ_v	γ	δ	LLF
0.682	0.180	0.338	0.068	4.124	2.950	1.533	0.346	162.0
(0.148)	(0.183)	(0.133)	(0.134)	(0.718)	(0.392)	(0.243)	(0.130)	

Note: -Standard errors in parenthesis.

TABLE 5

SPEED OF ADJUSTMENT

PERIODS AFTER THE SHOCK	(1)	(2)	(3)	(4)	(5)
			DURABLES		
(0)	0.51	0.54	0.67	0.61	0.93
(1)	0.33	0.37	0.29	0.31	0.07
(2)	0.16	0.09	0.04	0.08	0.00
			NON-DURABLES AND SERVICES		
(0)	0.73	0.71	0.67	0.61	0.73
(1)	0.27	0.24	0.29	0.31	0.27
(2)	0.00	0.05	0.04	0.08	0.00

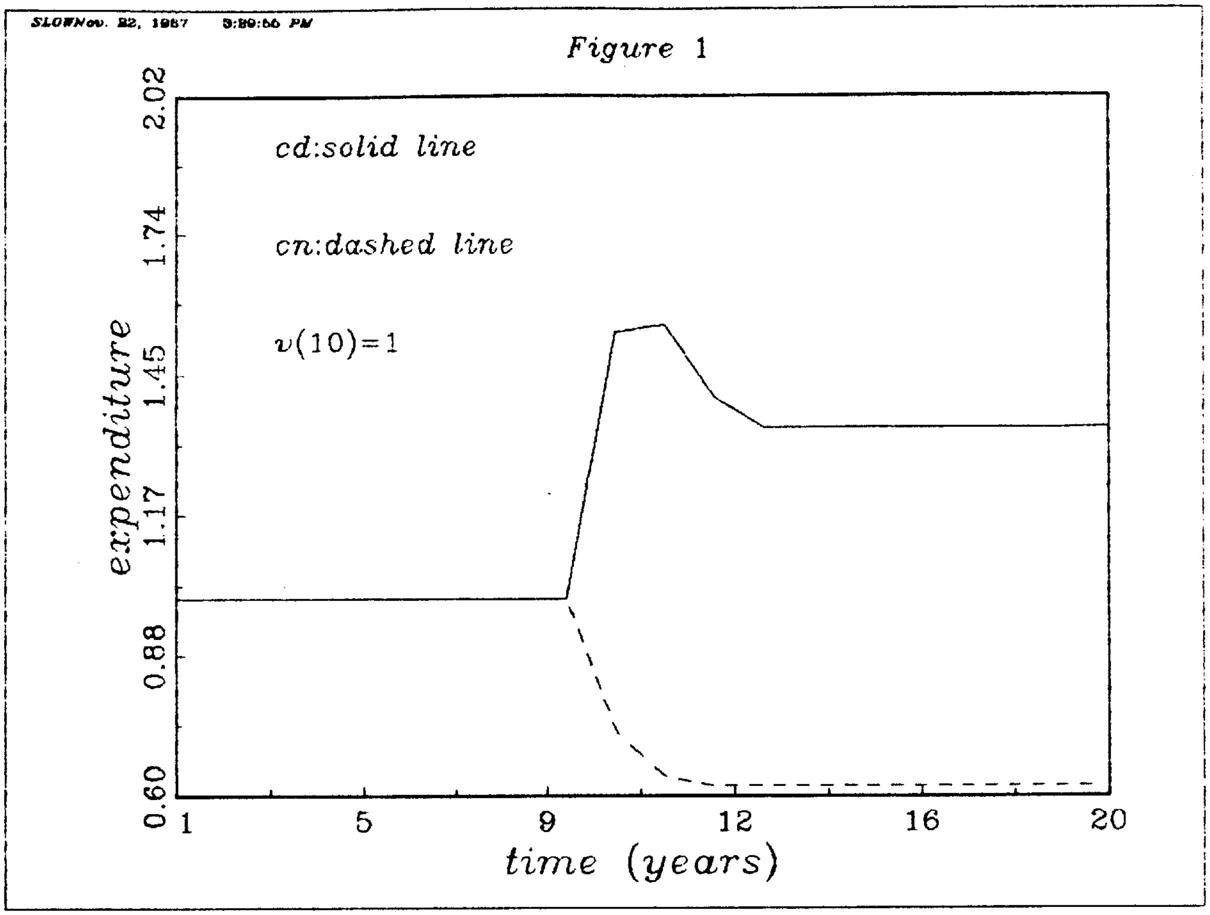
Note: the columns correspond to the columns of table 3.

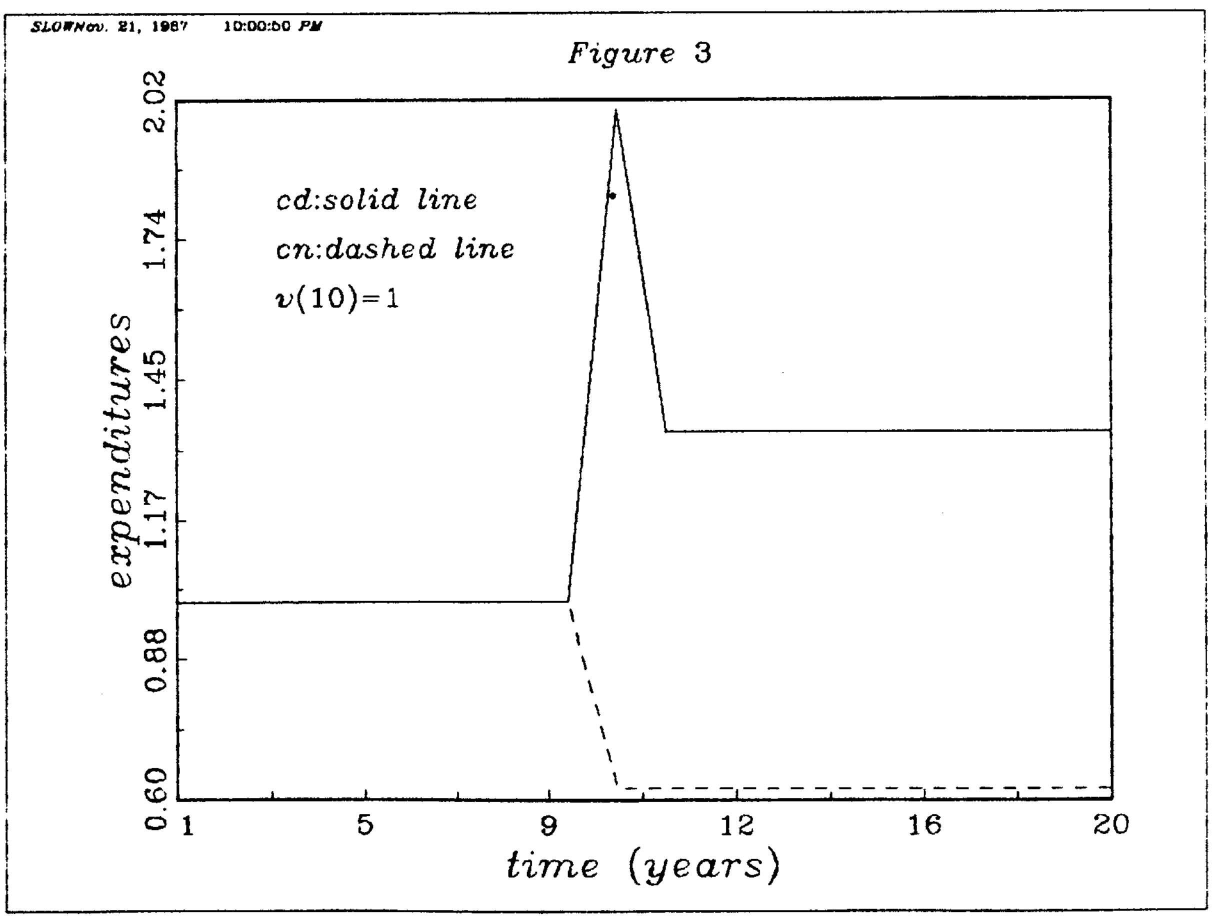
TABLE 6

EXAMPLE: ANNUAL DECISIONS-RANDOM QUARTERLY DISTRIBUTION OF EXPENSES

	Coefficients							
	MA_1	MA_2	MA_3	MA_4	MA_5	MA_6	MA_7	MA_8
(1) MA(1)	-0.020	-	-	-	-	-	-	-
	(0.071)							
(2) MA(2)	-0.021	-0.020	-	-	-	-	-	-
	(0.071)	(0.072)						
(3) MA(3)	-0.021	-0.024	-0.028	-	-	-	-	-
	(0.072)	(0.072)	(0.072)					
(4) MA(4)	-0.020	-0.024	-0.027	0.010	-	-	-	-
	(0.072)	(0.072)	(0.072)	(0.072)				
(5) MA(8)	-0.043	-0.046	-0.049	-0.074	-0.036	-0.037	-0.038	-0.184
	(0.073)	(0.073)	(0.073)	(0.073)	(0.073)	(0.073)	(0.073)	(0.073)
Σ -0.507								
(0.202)								
(6) MA(8)T	-0.092	0.123	-0.170	0.008	0.019	-0.082	-0.068	-0.279
	(0.081)	(0.083)	(0.083)	(0.086)	(0.086)	(0.086)	(0.086)	(0.087)
Σ -0.541								
(0.234)								

Notes: -The columns correspond to the moving average coefficients.
-The rows correspond to the estimated process.
-T: actual estimates using US quarterly data.
-Σ denotes the sum of the MA coefficients in the previous row.


SLOWNov. 22, 1987 9:89:56 PM
Figure 1
cd:solid line
cn:dashed line
v(10)=1
expenditure
2.02
1.74
1.45
1.17
0.88
0.60
1
5
9
12
16
20
time (years)


SLOWNov. 21, 1987 10:00:50 PM
Figure 3
cd:solid line
cn:dashed line
v(10)=1
expenditures
2.02
1.74
1.45
1.17
0.88
0.60
1
5
9
12
16
20
time (years)

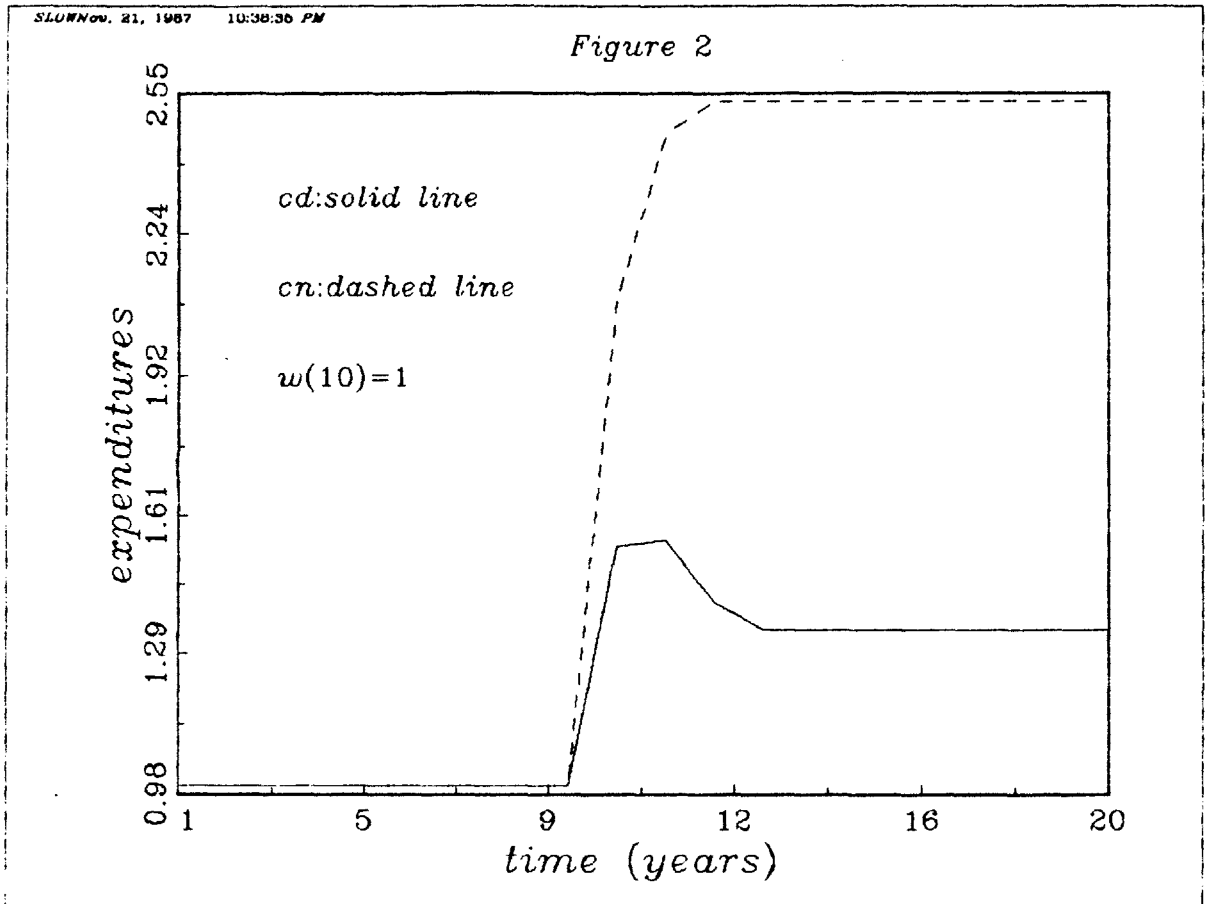

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Figure 2
2.55
2.24
1.92
1.61
1.29
0.98
expenditures
cd:solid line
cn:dashed line
w(10)=1
1
5
9
12
16
20
time (years)

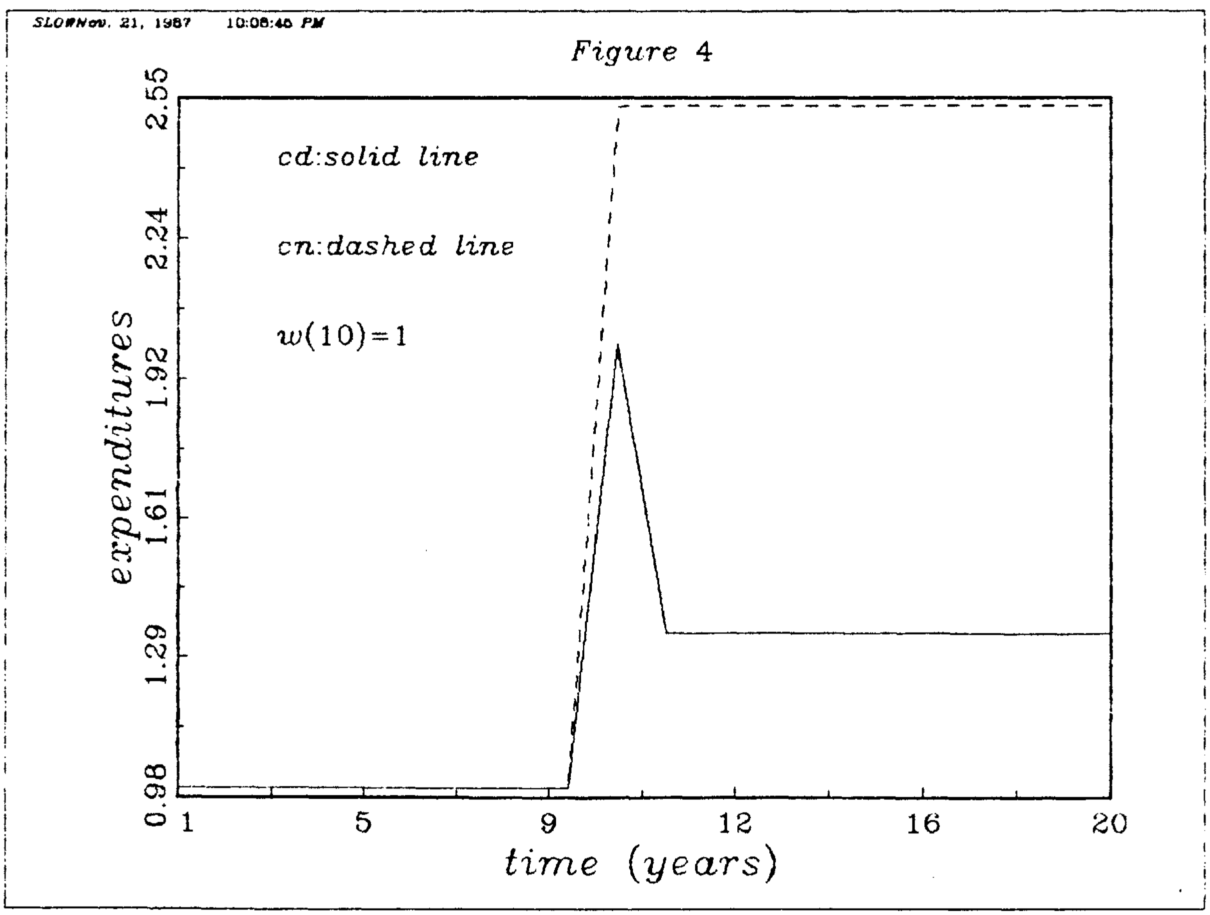
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Figure 4
2.55
2.24
1.92
1.61
1.29
0.98
expenditures
cd:solid line
cn:dashed line
w(10)=1
1
5
9
12
16
20
time (years)

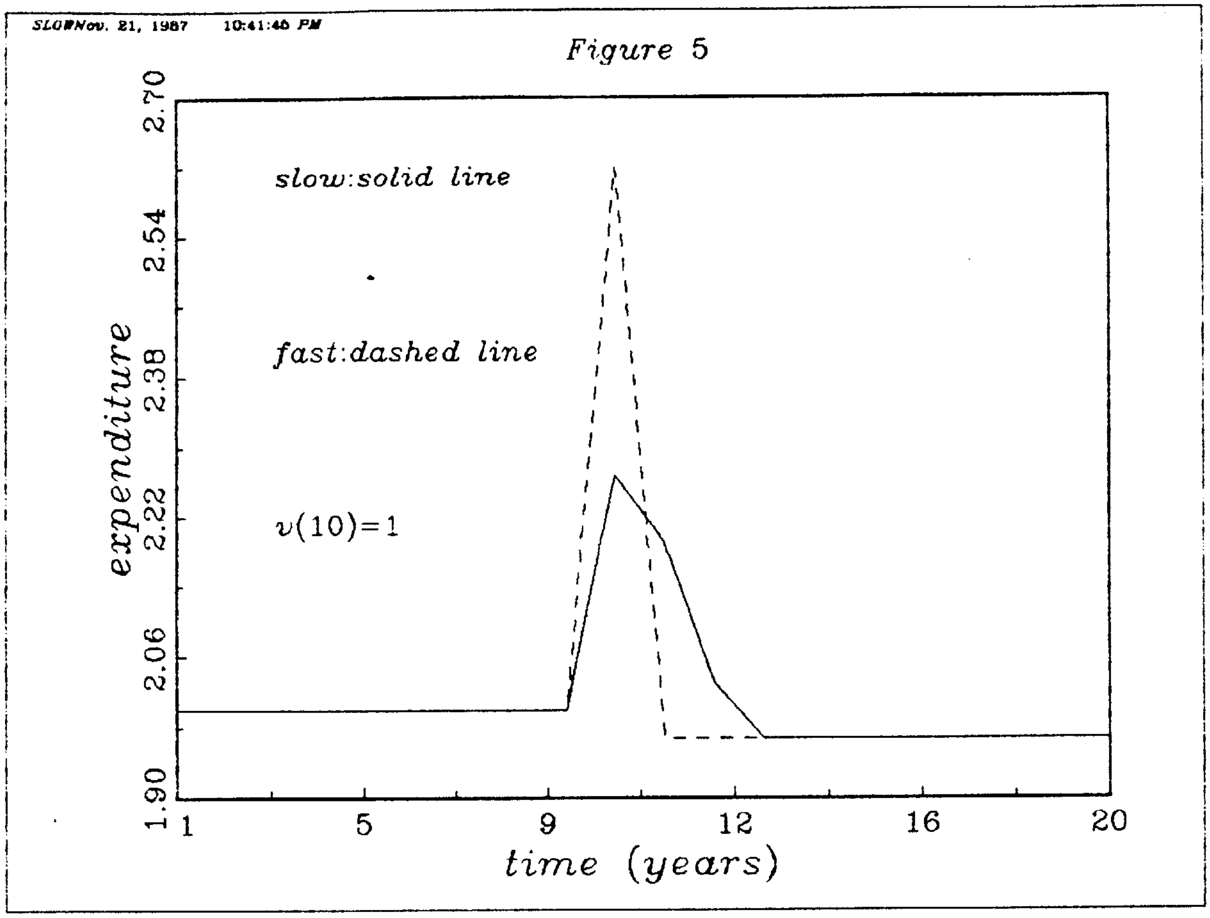
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Figure 5
slow:solid line
fast:dashed line
v(10)=1
expenditure
2.70
2.54
2.38
2.22
2.06
1.90
1
5
9
12
16
20
time (years)

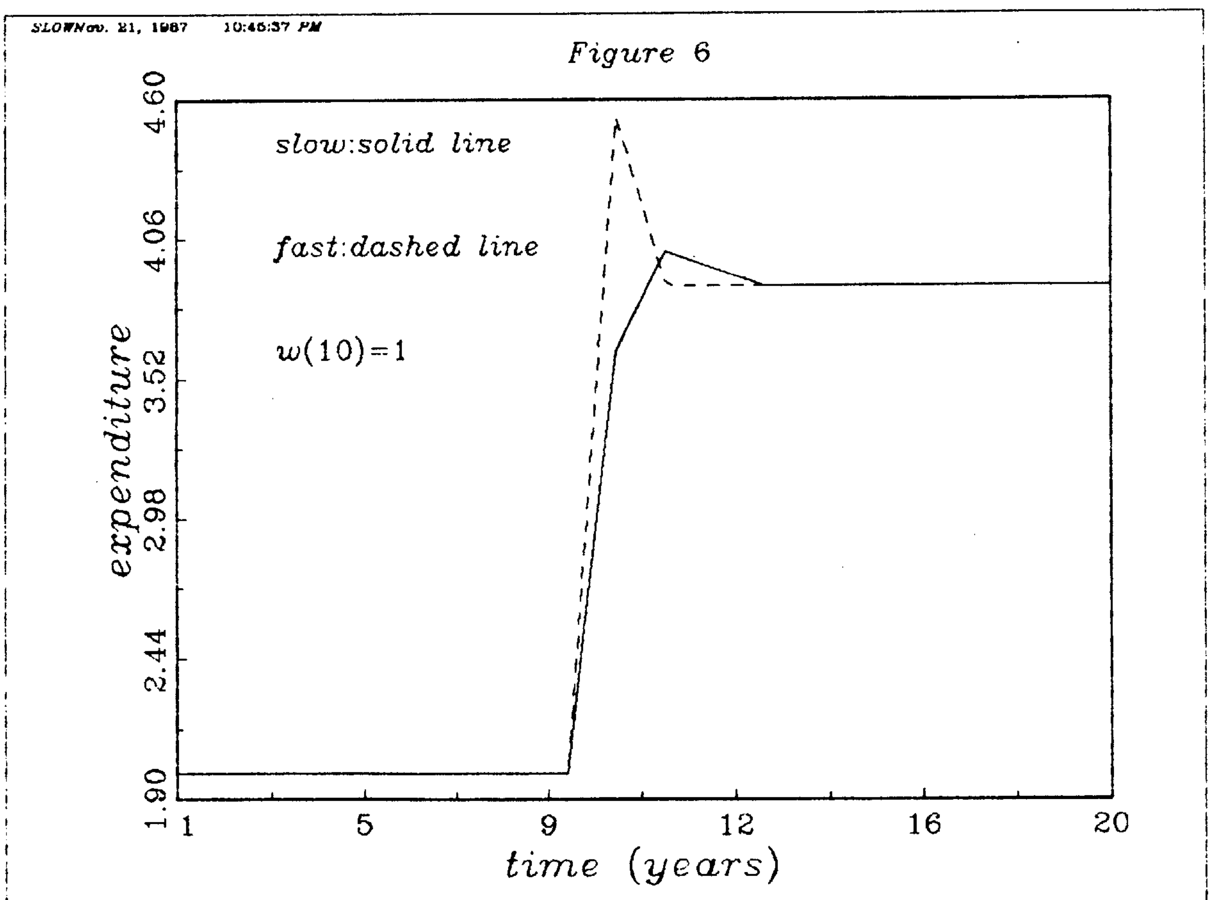
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Figure 6
slow:solid line
fast:dashed line
w(10)=1
expenditure
4.60
4.06
3.52
2.98
2.44
1.90
1
5
9
12
16
20
time (years)

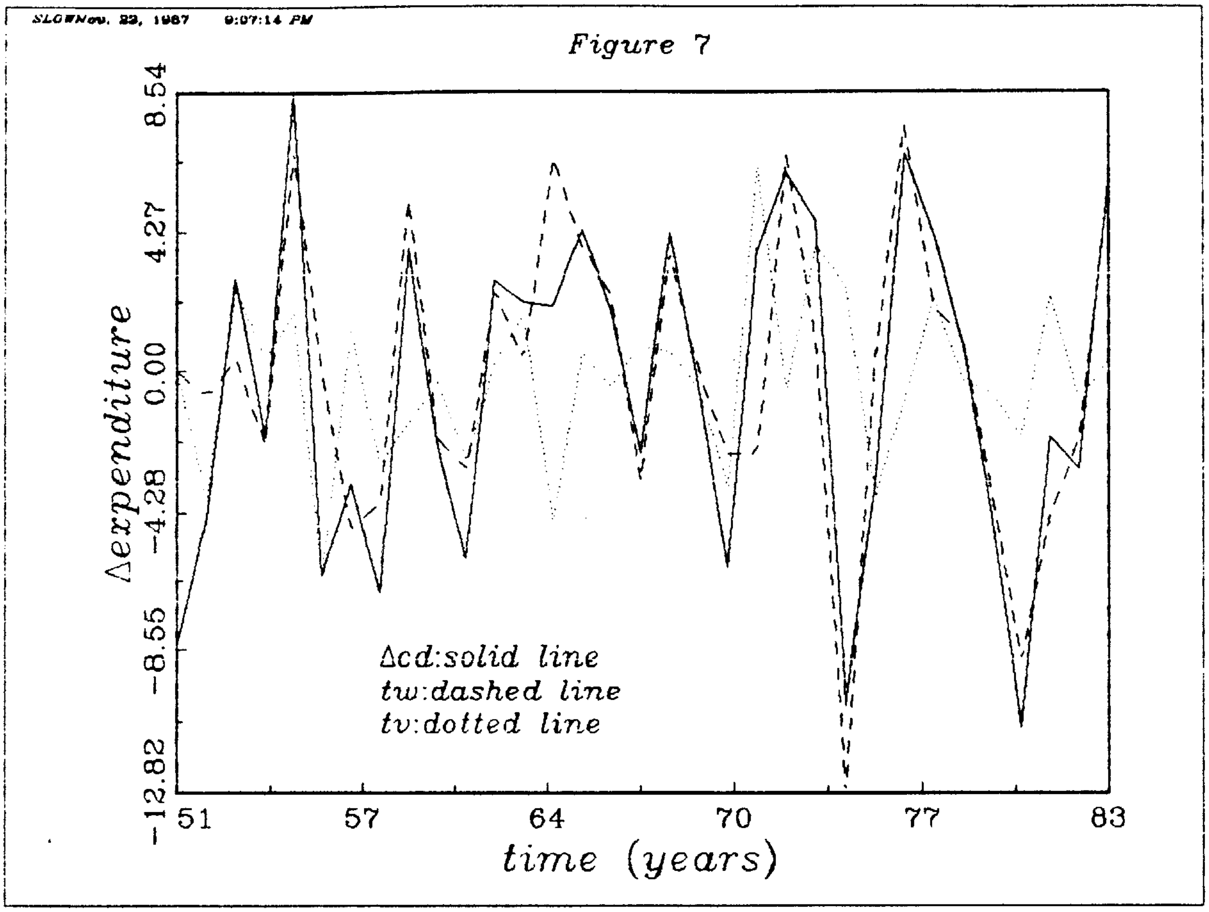
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Figure 7
8.54
4.27
0.00
-4.28
-8.55
-12.82
Δexpenditure
Δcd:solid line
tw:dashed line
tv:dotted line
51
57
64
70
77
83
time (years)

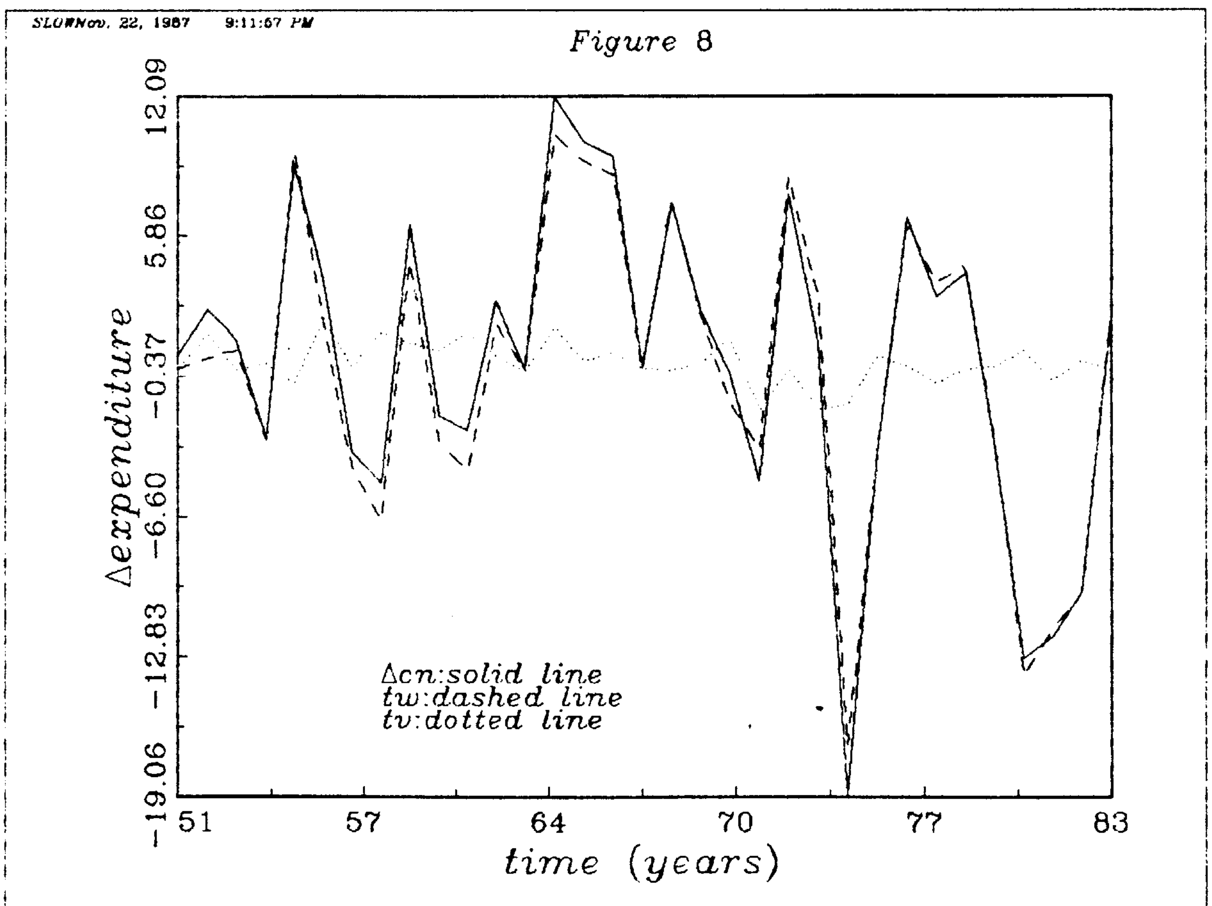
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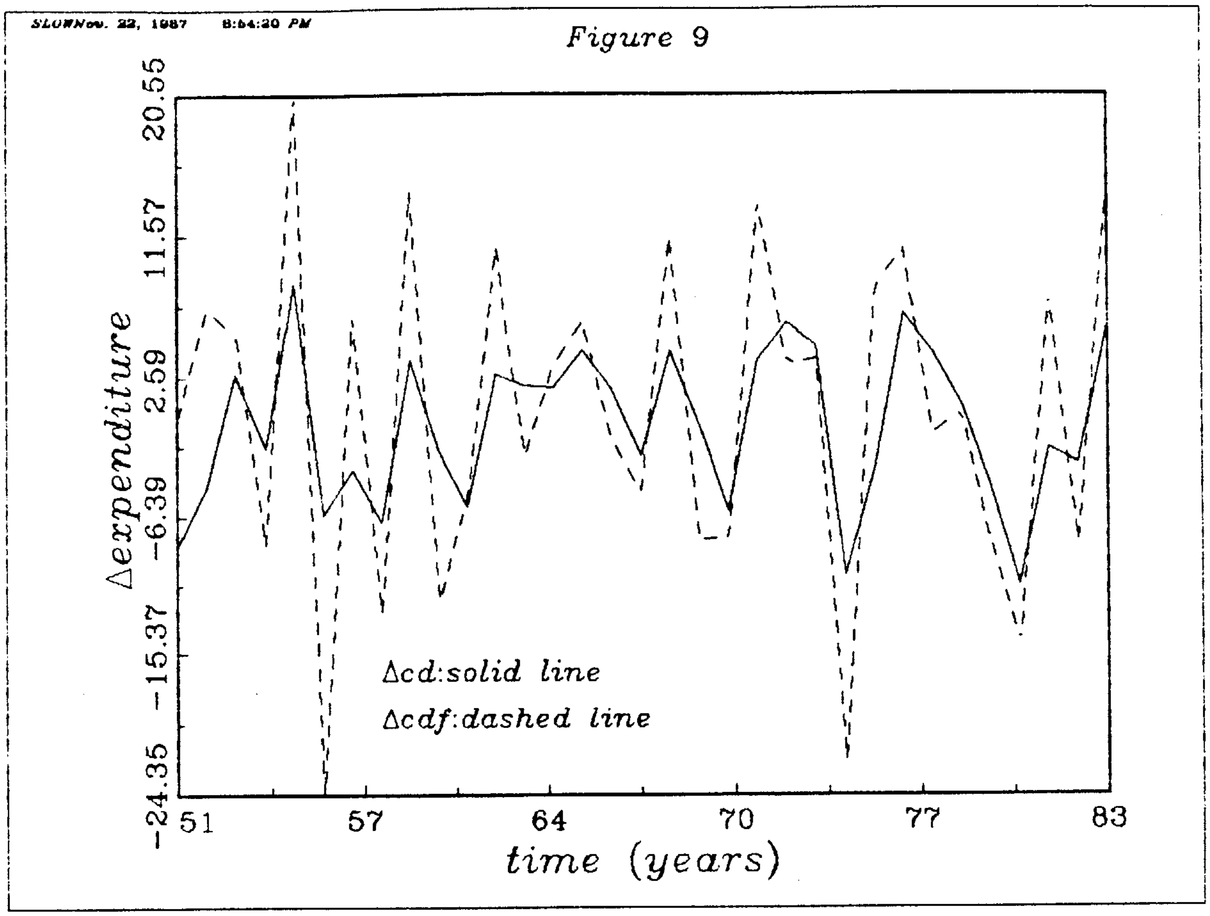
Figure 8
12.09
5.86
-0.37
-6.60
-12.83
-19.06
Δexpenditure
Δcn:solid line
tw:dashed line
tv:dotted line
51
57
64
70
77
83
time (years)


SLOWNov. 22, 1987 8:54:20 PM
Figure 9
20.55
11.57
2.59
-6.39
-15.37
-24.35
Δexpenditure
Δcd:solid line
Δcdf:dashed line
51
57
64
70
77
83
time (years)

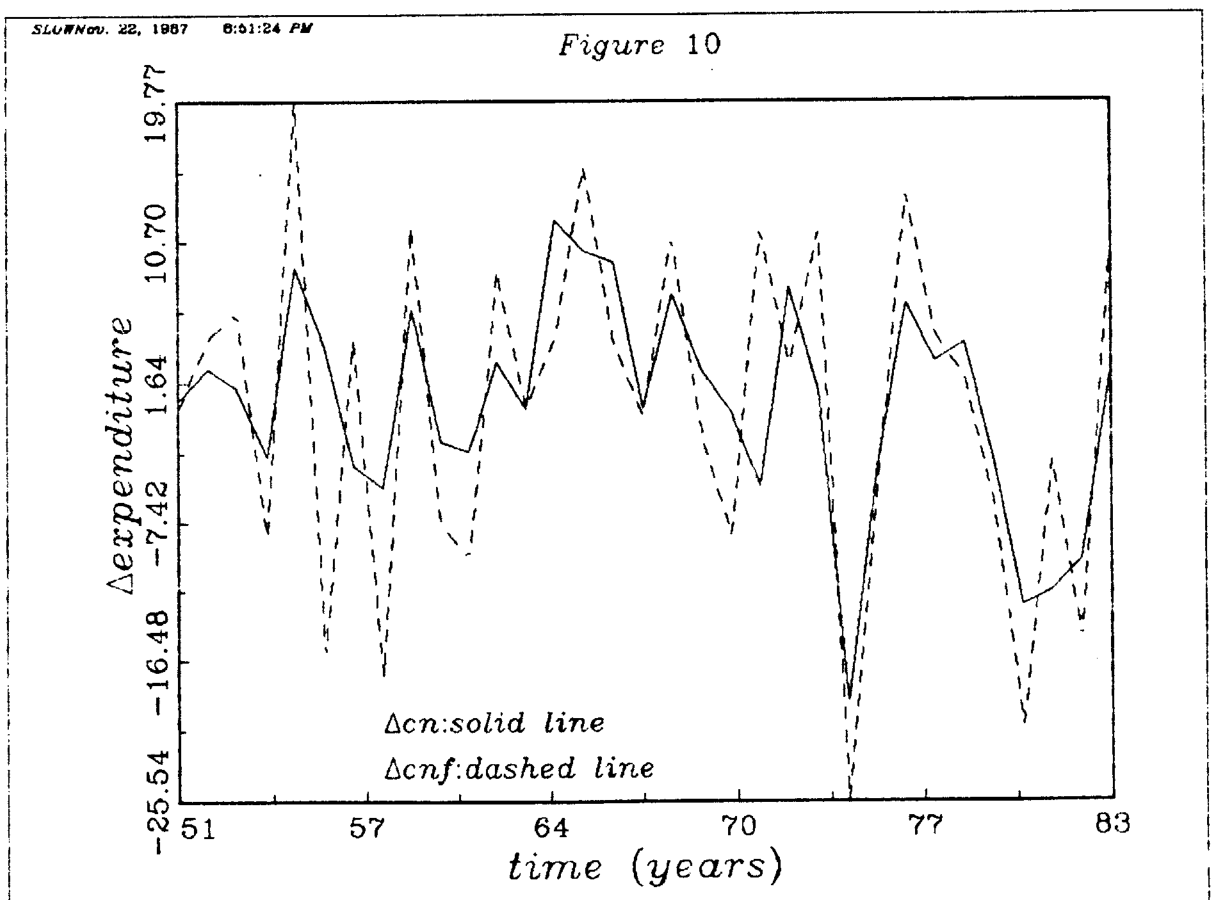

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Figure 10
19.77
10.70
1.64
-7.42
-16.48
-25.54
Δexpenditure
Δcn:solid line
Δcnf:dashed line
51
57
64
70
77
83
time (years)

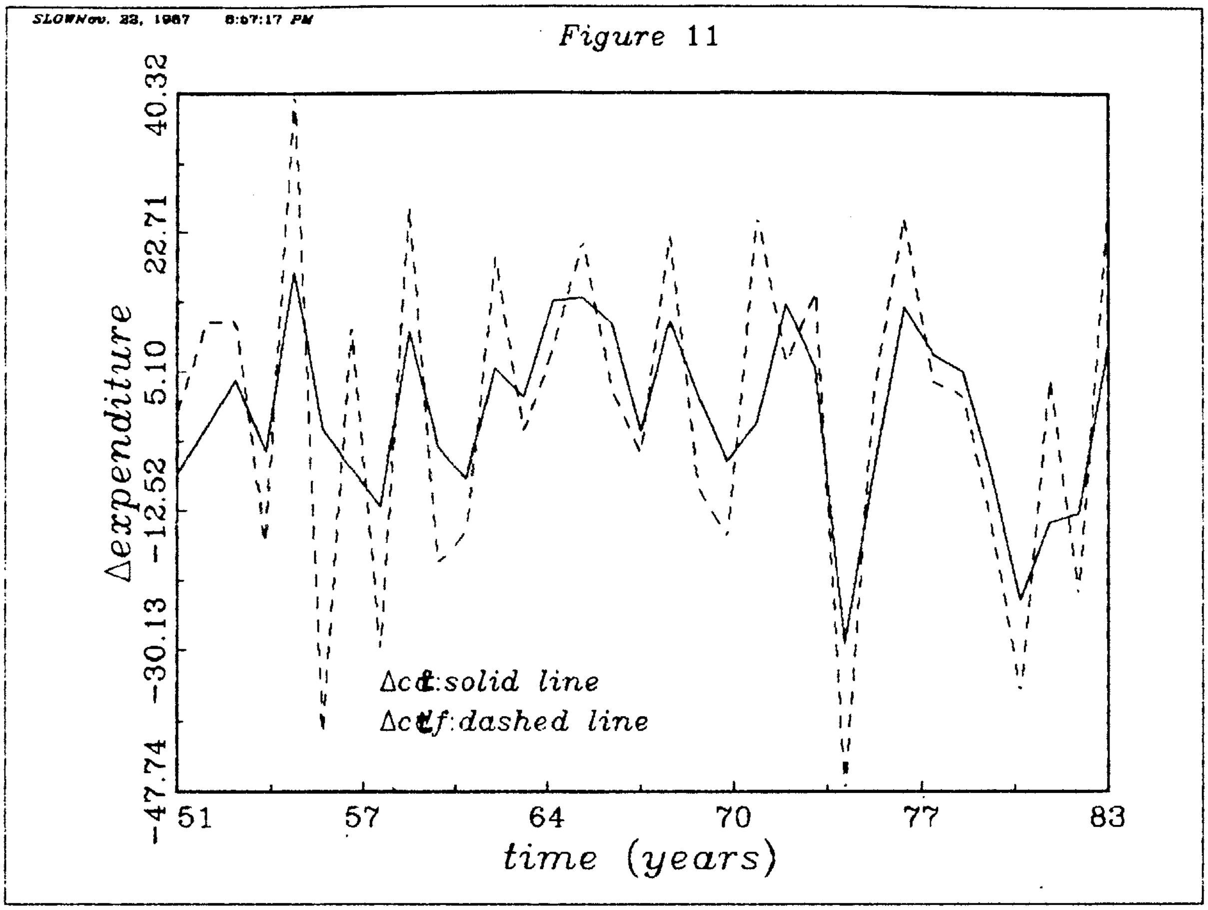
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Figure 11
Δexpenditure
40.32
22.71
5.10
-12.52
-30.13
-47.74
Δct:solid line
Δctf:dashed line
51
57
64
70
77
83
time (years)