INVESTIGATION ON REGULATORS IN QUANTUM ELECTRODYNAMICS.

by

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ABSTRACT

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Submitted to the Department of Physics on July 5, 1958, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Several attempts have been made in order to connect an eventual suppression of the divergences inherent to the present formulation of Quantum Electrodynamics, with the high energy behaviour of some known processes.

Whereas the theoretical situation is found to be fairly unclear, some crucial experiments are required in order to shed some light as to which of the three somewhat contradictory models we have constructed is closer to reality.

Thus, the following phenomena should be investigated:

- large angle pair creation.
- large angle bremstrahlung.
- large angle Moller scattering.

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ABSTRACT

We present in this work three models which are able to suppress the divergences of approximate versions of Quantum Electrodynamics. It is indeed argued that, in view of the smallness of the fine structure constant, not only the first terms of a perturbation expansion, or of an expansion according to the number of particles involved in intermediate states, gives a fair approximation, but furthermore, that it is in these terms that a breakdown of electrodynamics should be sought.

Our goal is to connect the high energy behaviour of relevant physical processes with the suppression of the divergences.

The first model assumes the existence of a photon cut off, whose observable consequences are clearly stated, and of a fermion cut off which, although unable to give a satisfactory description of phenomena involving virtual fermions, points to the conclusion that these have no obvious connection with the creation of virtual pairs in vacuo.

The second model is based on formal analytic relationships exhibited by the lowest order perturbation theory and connects the divergences with the high behaviour of the phenomena which, while they are well described here were left out by the first model.

The third model, aiming at a better understanding of the preceding one, contradicts its theoretical basis without necessarily invalidating the observable consequences that were drawn from it.

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TABLE OF CONTENTS

INTRODUCTION

Although Quantum Electrodynamics, at the present stage of its development, has been able to give a very accurate description of the experimental situation, it is generally believed that the theory is yet incomplete. The present scheme, formulated through a Lagrangian formalism, involves two phenomenological constants, the so-called bare mass m and bare charge e of the charged particle, in terms of which one should be able to calculate all physical quantities. In particular, one should be able to compute in terms of these parameters, the observed mass $m=m_0+\delta M$ and the observed charge $e_0=Z_3^{1/2}e$ of the charged particle. Conversely, all physical quantities should be expressible in terms of e_0 and m_0.

Unfortunately, it is well known that, if one tries to compute m_0 and e_0 in terms of m and e, in the framework of perturbation theory, one gets infinite answers. A third "infinite" quantity occurs in perturbation theory, namely the probability Z_2 for a "bare" fermion to be in a physical fermion state. The occurrence of these infinities is generally attributed to the sharp localizability of fields in local interaction, which allow states of arbitrarily high momenta to be reached in virtual processes.

Unless otherwise specified, we shall consider the system of a charged fermion field interacting with the Maxwell field.

Whereas perturbation theory yields infinite results for the renormalization constants Z_2, Z_3, δm, the use of renormalized equations of motion , independently of a perturbation expansion, shows that, under the assumption that no abnormal feature appears in the theory, (such as the occurrence of states with negative normalizations or energies), either Z_2 or Z_3 is equal to zero (1). This not physically surprising result can be interpreted by saying that, owing to the possible occurrence of an infinite number of states, the probability for each one of them to be realized in a bare particle state vanishes.

Since the perturbation method does not seem to be responsible for the difficultids encountered, and, as one feels that the smallness of the physical charge $(e_0^2/4\pi\hbar c = \frac{1}{137})$ responsible for the coupling should justify using a series expansion in powers of the fine structure constant, it is legitimate to try and modify the present scheme as approximated by its lowest order terms in this expansion.

When using the perturbation expansion, one has to recognize infinite integrals as being terms contributing to the implicitely defined renormalization constants. This problem has been solved by means of so-called regularization methods, (2), (3), (4), which essentially consist of an alteration of the theory at short space-time intervals (or, what is the same, for large virtual four-momenta), preserving the invariance built in the theory.

Such procedure allow for the unambiguous recognition and separation of standard integrals which become infinite when the theory is restated into its initial form, and are identified with terms of the perturbation expansion of the renormalization constants. The remaining expressions, which involve rather low virtual four-momenta, have finite limits, which are presumably independent of the cut off procedure.

One may argue that it is not meaningful to formulate the theory in terms of bare parameters, because the bare mass and charge may not be actually measurable. However, one should at least require the mathematical formalism to be free from inconsistencies; if furthermore one can show that it should be possible, at least in principle, to measure the renormalization constants, then one must require that they come out finite from the theory.

Beside these questions of principle, one would like to be able to compute such quantities as mass differences between neutral and charged, otherwise similar particles (e.g. neutrons and protons, charged and neutral pions, hyperons, etc.).

It has been suggested by a number of authors, (5), (6), that such mass effects may be described in terms of a modification of electrodynamics at high energies of the type previously mentioned; to be more specific, a simple alteration of the high energy behavior of the Maxwell field has been found to give a satisfactory account of the experimental data.

More recently, (7), (8), a possible alteration of electrodynamics at high energies was invoked in trying to explain the somewhat puzzling state of affairs concerning the difference between the charge distribution radii of the neutron and the proton.

Whereas it may be hoped that these pnenomena will be understood , provided electrodynamics is not considered as a closed theory, cut offs may still be used to give a phenomenological description of electrodynamic processes in an energy range where either various couplings come into play, or the concept of local fields becomes dubious.

As most of the work done along these lines has been concerned with the study of observable effects due to an alteration of the Maxwell laws,it seems useful to investigate also those phenomena which would be affected by a modification of the Dirac law.

The aim of this investigation is to review in a simple minded way the known electrodynamic processes in so far as they can exhibit high energy deviations from their description in terms of the conventional theory. Many simple features can be understood in the light of the usual covariant perturbation theory, as, for instance, the classification of electrodynamic processes into two classes: those involving virtual photons, and those involving virtual fermions.

Whereas it is quite natural to take perturbation theory as a starting point, since it is the only version of electrodynamics usable for practical calculations,

the global method has the advantage that it fully exploits the fundamental properties implied by the usual form of the interaction (1). As we intend to describe phenomenologically physical effects which fall outside the scope of a theory whose language we shall keep using, we shall be forced to introduce some formal inconsistencies which, we believe, is a way to hide our ignorance of the "true" theory. We shall therefore try, as much as possible, to give physical arguments in order to justify such a situation. Since many extrapolations of a given theory can be found, which mainly depend upon the form of the theory one starts from, and the properties one decide to keep, it is insructive to vary both of these determining factors.

We shall propose in the body of this work three models, and compare their predictions; it is encouraging to see that, in spite of large differences, they have in common a substantial number of features.

The first two models are based on properties exhibited by the covariant perturbation theory. The philosophy we adopt is the following: since most of the radiative corrections become usually sizable at energies where electrodynamics may get mixed up with the dynamics due to other couplings, it is reasonable to look for a breakdown of pure electrodynamics in the effects predicted by the lowest order perturbation theory. Whereas the divergent processes depend on the very high energy tail of the theory, we should require that the contribution

they get from "pure" electrodynamics (as described by the lowest order terms of the perturbation expansion) should be finite.

We shall now proceed to give a brief description of our three models, each of them being improved or explained by the next one.

Model I is studied in the first three chapters:
- In Chapter I, we discuss what is thought to be a minimal regularization in the framework of the covariant perturbation theory, that is to say, a high energy modification which suppresses divergences in this order. In spite of the well known conceptual inconsistencies which arise in this model, it is argued that it may give a more accurate description of phenomena than the usual theory does, at least in a suitable energy range.
- In Chapter II, we shall study those electrodynamic processes which are affected by a modification of the Maxwell laws, and emphasize the similarities and differences between such a breakdown and the existence of particle spreads as may be produced by interactions. This will be more or less a review of previous works.
- In Chapter III, we shall show that the model proposed in Chapter I is not adequate to describehigh energy modifications of processes involving virtual fermions, as a consequence of the requirement of local gauge invariance. If however drastically simplifying assump-

-tions are made, it is found that there is no obvious connection between the regularization of vacuum polarization and high energy deviations of the Compton and related effects from what they are thought to be according to the conventional scheme.

One salient feature of this model is that the use of a detailed local description of phenomena(implied in particular in the requirement of local gauge invariance) forces us to argue extensively in terms of virtual particles, so that physical interpretations are made very difficult.

Model II has been constructed in order to remedy this defect , and is closer in spirit to the philosophy we have previously defined. It is based on the observation that the occurrence of divergences in the lowest order of perturbation theory can be blamed on the too slow decrease of the forward Compton amplitude. It makes fairly definite predictions about the behaviour of processes involving virtual fermions, which can easily be tested experimentally.(Chapter IV).

In Chapter V, we study general properties of the S-matrix in electrodynamics, in order to see whether the formal connection between the divergent processes and the Compton process, on which Model II is based, is a consequence of the "$j_\mu A_\mu$"coupling, or of more fundamental properties. The first alternative seems

to be the correct one. Whereas the approximation method used here is not a perturbation expansion, which makes it hard to establish a connection with Models I and II, and requires a slight change in our philosophy, it seems more correct to connect the regularization of vacuum polarization in the "lowest" approximation with a finite extension of the fermion current, a damping of the Compton amplitude only being required in higher orders.

Whichever model is closer to reality, there emerges from this investigation the necessity of performing a number of experiments in order to test electrodynamics:

- Moller scattering of weakly coupled particles at high energies. Although this is not experimentally possible at present in view of magnitude of the energies required, this would ascertain the existence of particle radii or of a photon cut off, if any.
- Pair creation and Bremstrahlung at large angles and high energies.
- A more accurate measurement of magnetic moments.
- Also, a better experimental and theoretical knowledge of the strong interactions will allow to separate out electrodynamic effects in phenomena in which the role played by these interactions is understood, but not quantitavely known at present.

CHAPTER I. FORMULATION OF A REGULATOR THEORY.

For our present purpose, we may consider Quantum Electrodynamics as an S-matrix theory, the S-matrix being expandable in powers of the coupling constant, and a functional of local free field operators(in-fields). We recall the main characteristics:

-Invariance under the full Lorentz group.

-Invariance under charge conjugation as implied by the T.C.P. theorem.

-Gauge invariance.

-Unitarity of the S-matrix.

-Existence of a complete set of in-states, with positive energies and normalizations.

-Causality.

The S-matrix one conventionally starts from is well known to be:

$$(I,1) \qquad S = 1 + \sum \frac{(-i)^n}{n!} \int dx_1 \cdots dx_n \, P\left[j_{\mu_1}(x_1) \cdots j_{\mu_n}(x_n)\right] \mathcal{P}\left[A_{\mu_1}(x_1) \cdots A_{\mu_n}(x_n)\right]$$

Whereas it has formally all the properties we have listed, these have to be used at each step, when one states the mathematical prescriptions which allow to extract meaningful answers from an otherwise ill-defined formalism. Thus, the stability properties which one would like the S-matrix

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From now on, we shall follow the notations of reference(9).

to have,(namely, it should leave invariant the vacuum and one-particle states), are introduced through the infinite invariant mass, charge, and wave function renormalizations. The usual regularization procedures,(2),(3),(4), which realize this program, are based on the observation that, when the S-matrix is rewritten in normal ordered form, it involves the following basic expressions:

-the bare vertex operator: $\gamma_\mu \delta(x-\xi)\delta(\xi'-\xi)$

-the bare photon propagator: $-i D^c_{\mu\nu}(\xi-\xi') = T\langle 0|A^{in}_\mu(\xi)A^{in}_\nu(\xi')|0\rangle$

-the bare fermion propagator: $i S^c(x-x') = T\langle 0|\psi^{in}(x)\bar{\psi}^{in}(x')|0\rangle$

It is well known that, if one limits oneself to the lowest non vanishing order of the perturbation expansion, it is enough to regularize the photon propagator and the fermion loop.

As we shall limit ourselves to a study of these lowest order terms, and since we want to correlate the effects of such regularizations on various phenomena, we shall propose a realization of these regulators which allows such connections.

We assume that the photon and fermion fields interact with abnormal fields, in such a way that the bare propagators and vertex are modified in a relativistic and gauge invariant way. In other words, we assume that they are replaced by the corresponding renormalized quantities emerging from these interactions. We therefore write,

in momentum space:

$$(I,2) \qquad D_c(k) \longrightarrow D_{cR}(k) = \int_0^{\infty} \frac{\pi_R(a^2)\,da^2}{k^2+a^2-i\epsilon}$$

$$(I,3) \qquad S_c(p) \longrightarrow S_{cR}(p) = \int_{-\infty}^{+\infty} \rho_R(a)\,da\, \frac{ip-a}{p^2+a^2-i\epsilon}$$

These forms can be deduced from very general principles, (10),(11),(12), and it can be shown that, if such propagators emerge from interactions between local fields, they cannot give rise to a regularization, (i.e. $\rho_R(a) > 0, \pi_R(a^2) > 0$) unless abnormal features appear, such as the occurrence of states with negative probabilities. The situation in which interacting vector fields are involved is somewhat peculiar; however, the non-positiveness of the weight functions does not seem to save the situation in this case either.

As we shall see presently, regularization requirements imply not only the non-positivenes of the weight functions $\pi_R(a^2)$, $\rho_R(a)$, but also a physically inacceptable renormalization condition on the interactions introduced. Thus, if one does not allow the regulator fields to be vector fields, so that the weight functions ought to be positive definite, one has to admit that the theory involves negatively normalized states. It seems therefore little attractive to replace the present inconsistencies of electrodynamics by those of a wider scheme which may have no good physical interpretation. One may however take the

viewpoint that, if these new inconsistencies which are supposed to replace the old ones occur at high enough energies, this model may be more accurate than the old scheme, in the medium energy range where the cut off effects are only size dependent, whereas it completely breaks down at higher energies where the inconsistencies appear, and where these effects start to depend on the shape of the cut off functions. More specifically, let

$$\pi_R(a^2) = \delta(a^2) + \chi(a^2);\ \chi(a^2) \neq 0 \text{ for } a^2 > M_{ph}^2 \qquad (I,4)$$

$$\rho_R(a) = \delta(a - m_0) + \sigma(a);\ \sigma(a) \neq 0 \text{ for } |a| > M_F \qquad (I,5)$$

where $\chi(a^2)$ and $\sigma(a)$ are non positive definite functions. This model should cover the energy range $(0, M_{ph})$ or $(0, M_F)$, whichever the narrower. One point to be cleared is whether these energies are absolute ,or whether they are measured in units of the charged fermion rest mass; we shall have to come back to this on several occasions, although, if these cut offs are connected at all with the idea that there exists a fundamental natural length, one should think that the first viewpoint is the correct one.

Our model has to be completed by an appropriate alteration of the vertex operator, in order to preserve local gauge invariance:

(I,6) $$\gamma_\mu \rightarrow \Gamma_{\mu R}(p',p)$$

where $\Gamma_{\mu R}(p',p)$ must fulfill the generalized Ward identity (13), (14), (see Appendix B):

(I,7) $$i(p'^\mu - p^\mu)\,\Gamma_{\mu R}(p',p) = S_{CR}^{-1}(p') - S_{CR}^{-1}(p)$$

Let us remark that, without violating gauge invariance, one may multiply $\Gamma_\mu(p',p)$ by a form factor $F(p-p')^2$ which goes to unity for zero momentum transfer, and describes a spread of the particle current. Since each vertex is accompanied either by an external photon line ($k^2=0$, $F(0)=1$), or by a photon propagator, such a form factor will have effects similar to those produced by an alteration of the type indicated in Eq.I,2; however, these two kinds of modifications can, in principle, be distinguished, as we shall see later. Removing the form factor into the photon propagator, one can write Eq.I,7 under the following form:

(I,8) $$\Gamma_{\mu R}(p',p) = S_{CR}^{-1}(p')\int \rho_R(a)\,da\,\frac{ip'-a}{p'^2+a^2-i\epsilon}\,\gamma_\mu\,\frac{ip-a}{p^2+a^2-i\epsilon}\,S_{CR}^{-1}(p) + \Gamma_\mu^{(1)}(p',p)$$

where

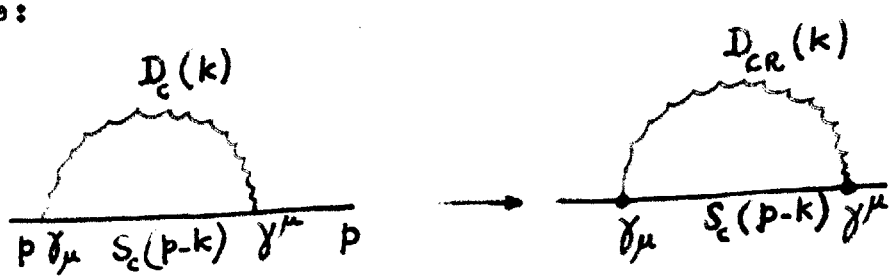
(I,9) $$i(p'^\mu - p^\mu)\,\Gamma_\mu^{(1)}(p',p) = 0$$

About the first term in Eq.I,8, one should remark that it does not contribute to anything else than the ordinary current, when it is evaluated for free particles. In particular, all the magnetic moment effects are contained in $\Gamma_\mu^{(1)}(p,p')$. With the help of these modified propagators and

vertex, one may now investigate under what conditions the divergences are suppressed in the lowest order of perturbation theory.

A- Fermion self energy and electrodynamic vertex.

We shall only study here the modification of the self energy due to the regularization of the photon line:



In the case of ordinary electrodynamics, the initial linear divergence reduces to a logarithmic one, so that the minimal regularization condition is:

$$\int_0^\infty \pi_R(a^2)\,da^2 = 0 \qquad \text{(I,10)}$$

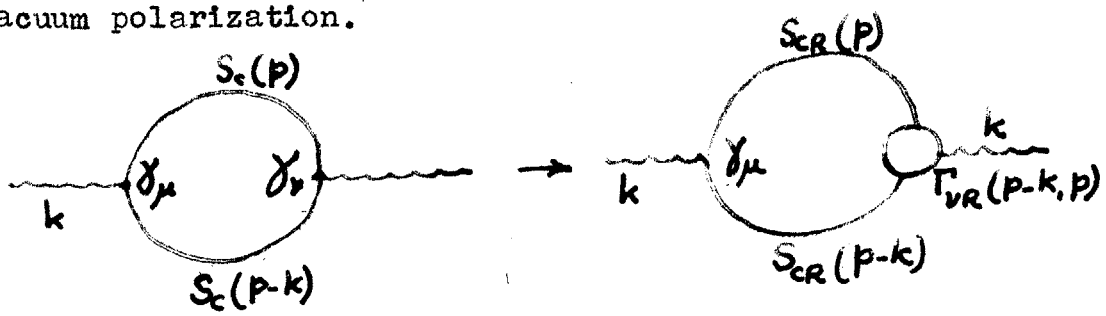
which means from the point of view of the abnormal interactions introduced that $Z_{3\,abn.} = \infty$; this is of course completely inconsistent with the interpretation of Z_3 as a probability. The use of a stronger regularization:

$$\int_0^\infty \pi_R(a^2)\,da^2 = \int_0^\infty a^2\,\pi_R(a^2)\,da^2 = 0 \qquad \text{(I,11)}$$

in the phenomenological calculation of the neutron-proton mass difference should not be retained as a fundamental condition in electrodynamics, since the spread of the

nucleon current produced by mesonic effects makes it superfluous. We shall put the study of the fermion cut off effects aside ; this will be done in Chapter III. Of course all that we said about the self energy stays valid for the vertex.

B- Vacuum polarization.



We first remark that the vacuum polarization diagram contains one vertex and one regularized vertex since a particle-antiparticle pair has to be created with the usual interaction before the abnormal couplings come into play. Thus, one must write:

$$\text{(I,12)}\qquad \Pi^R_{\mu\nu}(k) = \frac{ie^2}{(2\pi)^4}\,\mathrm{Tr}\int d^4p\;\gamma_\mu\, S_{cR}(p-k)\,\Gamma^R_\nu(p-k,p)\,S_{cR}(p)$$

$$= \frac{ie^2}{(2\pi)^4}\,\mathrm{Tr}\int d^4p \int \rho_R(a)\,da\;\gamma_\mu\,\frac{i(\not p-\not k)-a}{(p-k)^2+a^2}\,\gamma_\nu\,\frac{i\not p-a}{p^2+a^2} + \Pi^{(1)}_{\mu\nu}(k)$$

$$\approx \int \rho_R(a)\,da\;\Pi_{\mu\nu}(k,a) + \Pi^{(1)}_{\mu\nu}(k)$$

where $\Pi^{(1)}_{\mu\nu}(k)$ is the part of $\Pi^R_{\mu\nu}(k)$ containing $\Gamma^{(1)}_\mu(p-k,p)$ and the sign $\approx$ was written because one should not actually interchange the order of the -k and -a integrations; $\Pi_{\mu\nu}(k,a)$ is the conventional expression for the second order vacuum polarization due to a fermion of mass a. As is well known, this expression, as it stands, is not gauge invariant and exhibits a quadratically divergent photon mass. If one

takes the view that this divergence is accidental and due to unallowed manipulations of a theory which was initially gauge invariant, then a weak regularization condition

$$\int \rho_R(a)\,da = 0 \tag{I,13}$$

suffices to suppress the logarithmic divergence of the charge renormalization constant, provided that $\Gamma_\mu^{(1)}(p',p)$ is sufficiently damped at high momenta.

Thus, we have achieved both the Feynman photon cut off and the gauge invariant Pauli-Villars fermion loop regularization, at the price of well known inconsistencies,(15),(16), but in such a way that we may hope to express the high energy modifications of other processes in terms of the cut off parameters.

To summarize, the minimum regularization conditions required in order to suppress the serious divergences of the conventional electrodynamics in the lowest order of perturbation theory,are:

$$\int \pi_R(a^2)\,da^2 = 0;\quad \int \rho_R(a)\,da = 0 \qquad \begin{matrix} |p^2| \\ |p'^2| \end{matrix} \Gamma_\mu^{(1)}(p',p) \to 0 \quad |p^2|, |p'^2| \to \infty \tag{I,14}$$

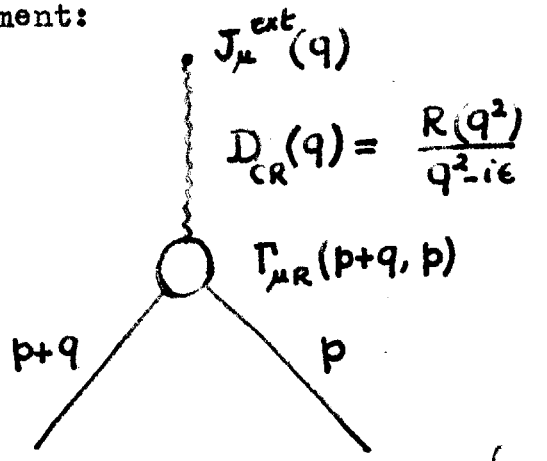
In the following chapters, we shall be concerned with a study of some observable consequences of this simple minded model. However, it will be seen in Chapter III that not enough has been said about gauge invariance, and, in particular, that one cannot describe processes involving virtual fermions without making extra assumptions.

CHAPTER II. PHOTON CUT OFF CORRECTIONS.

We shall investigate here those processes which are affected by an alteration of the Maxwell laws, namely, those which involve the exchange of virtual photons. We may expect that the deviations will be all the more important that these photons are more virtual-that is to say further off their mass shell-, which occurs when their source suffers a large momentum transfer.

A- Physical interpretation of the photon cut off.

In all processes involving an external current, one finds as a portion of the relevant diagram the following element:



The cut off function $R(q^2) = \int d\alpha^2 \pi_R(\alpha^2) \frac{q^2}{q^2+\alpha^2-i\epsilon}$ vanishes in view of the regularization condition I,14, when $q^2 \to \infty$. Furthermore, from Eq.I,4, one deduces that $R(0)=1$.

The effect of the photon cut off is to produce an apparent spread of the external current, when the result is interpreted in terms of conventional electrodynamics.

In other words, if one says that one sees an external current $J_\mu^{ext.}(q)$ creating a field according to Maxwell's laws, one may as well say that the true current distribution is $J_\mu^{true}(q)$, but that electrodynamics is modified according to Eqs;I,2;I,14:

$$(II,1) \qquad J_\mu^{apparent}(q) = J_\mu^{true}(q)\, R(q^2)$$

For instance, a static charge distribution $\rho^{true}(\vec{r})$ characterized by the mean square radius

$$(II,2) \qquad \langle r^2 \rangle^{true} = \int r^2 \rho^{true}(\vec{r})\, d^3r \; ; \quad \left(\int \rho(\vec{r}) d^3\vec{r} = 1 \right)$$

looks like

$$(II,3) \qquad \rho^{app.}(\vec{r}) = \frac{\rho^{true}(\vec{r})}{R(-\nabla^2)}$$

with a mean square radius

$$(II,4) \quad \langle r^2 \rangle^{app.} = \int r^2 \frac{\rho^{true}(\vec{r})}{R(-\nabla^2)}\, d^3\vec{r} = \int \left(\frac{1}{R(-\nabla^2)}\, r^2 \right) \rho^{true}(\vec{r}) d^3r = \langle r^2 \rangle^{true} + 6R'(0)$$

(R being a dimensionless function, $R'(0)$ is a squared length which we now call $1/M_{ph.}^2$).

This argument also applies to magnetization distributions. Of course, higher momenta of a current distribution will depend on higher derivatives of the cut off function, which is to say, will be more sensitive to its shape. As was previously pointed out, the introduction of a form factor in the vertex plays a similar role and produces an apparent charge spread similar to that produced by the photon cut off.

However, if the model is to be taken seriously, one should admit that each particle is characterized by a vertex form factor which describes its intrinsic charge distribution, whereas the photon acts with the same cut off on all particles; we must nevertheless keep in mind that the cut off mass may be a universal function of the masses of the particles between which the photon is exchanged. As such a behaviour would imply a strong acausality, we shall work with the first hypothesis.

Thus, one should in principle be able to determine the photon cut off and the charge distribution of each type of particle: if one studies Moller for two kinds of particles, A and B, (A-A, B-B, A-B), at low enough energies that one does not investigate the structure, but only the size of these particles, one can measure the three combinations $\langle r_A^2\rangle+\langle r_A^2\rangle+\frac{6}{M_{ph}^2}$, $\langle r_B^2\rangle+\langle r_B^2\rangle+\frac{6}{M_{ph}^2}$, $\langle r_A^2\rangle+\langle r_B^2\rangle+\frac{6}{M_{ph}^2}$. But, as we shall see in the next section, this is a matter of principle for the time being, owing to the magnitude of energies which can be produced at present in laboratories. Let us finally remark that virtual photons are necessary in order to measure a current distribution, since for real photons $k^2=0$, so that only the total charge is seen. (In particular, Compton scattering is not suitable and rather measures the deformability or polarizability of the distribution).

Of course, what we said about charged particle

scattering also applies to the bound state problem, although it is clear that the energies involved are usually very low; however, in view of the great accuracy of the experimental methods applied in this field, it may prove worthwhile to investigate cut off corrections to energy levels of bound systems.

B- Order of magnitude of the photon cut off.

BI. The proton radius.

From the proton mean square charge radius measured at Stanford,(7),one deduces:

$$(II,5)\qquad \frac{6}{M_{ph.}^2} = \langle r_p^2\rangle^{\text{Stanford}} - \langle r_p^2\rangle^{\text{mtr.}} - \langle r_e^2\rangle^{\text{intr.}} < \left(.8 \pm .05\ 10^{-13}\ \text{cm}\right)^2$$

where $\langle r_p^2\rangle^{\text{intr.}}$ is the intrinsic mean square spread of the proton which is produced by mesonic effects, rather than by an electrodynamic breakdown, and $\langle r_e^2\rangle^{\text{intr.}}$ the mean square radius of the electron, if any. Thus,

$$(II,6)\qquad M_{ph.} < .64\ M_p \qquad (M_p = \text{proton mass})$$

One may speculate about the eventual possibility of a more accurate determination: whereas calculations of the neutron and proton charge radii in the fixed source theory (17), are consistent with the value measured at Stanford for the proton, and therefore come out much too large in the neutron case, relativistic corrections seem to subsyantially reduce this value, (18). Calculations of the magnetic moment distribution also seem to follow a similar

trend,(18). It is not impossible that, when the treatment of mesonic effects is improved, one shall be able to understand the neutrality of the neutron as due to a cancellation of the mesonic cloud by the neutron core, spread over a region with a radius of the order of the nucleon Compton wavelength,(8). Then, although there are effects which may yield a mesonic cloud larger for the proton than for the neutron,(e.g. strange particle effects,(20), inequality of the pionic coupling constants,(21)),it seems hard to get a proton radius larger than $\langle r_p^2 \rangle^{\frac{1}{2}} \simeq .5\, 10^{-13}$ (obtained from Tanaka's result with $f^2 = .09$). If one takes this tentative value for the proton radius, one may interpret the Stanford experiment either with $\langle r_e^2 \rangle = 0$, $M_{ph.} \sim .7 M_p$ or with $M_{ph.} \simeq \infty$, $\langle r_e^2 \rangle^{\frac{1}{2}} \sim .63\, 10^{-13}$ cm. or some intermediate values.

BII. The neutron proton mass difference.

The cut off parameter obtained from the proton radius can be compared to that resulting from the fitting of the neutron proton mass difference,(5),(6). If one computes this mass difference, assuming that the nucleons are punctual Dirac particles with anomalous magnetic moments, one finds that the experimental value can be fitted with an effective photon cut off $\lambda = \frac{4}{3} M_p$. One may have doubts about the validity of this calculation if the photon cut off is high enough to allow for photons that are hard enough to investigate details of the nucleon

structure, in which case it would be a bad approximation to consider the nucleons as rigid punctual current distributions, (see K. Huang's argument in (6).).The situation is however not so bad as it seems at first sight; indeed, if one treats the nucleons as extended distributions, one sees that the effective photon cut off is given by:

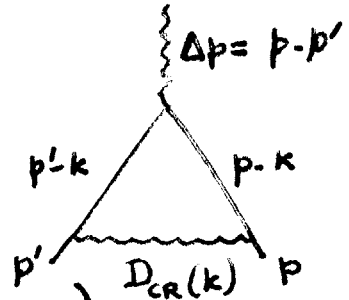
$$(II,7) \qquad \frac{6}{\lambda^2} = \frac{6}{M_{ph}^2} + 2\langle r_p^2\rangle^{int.} = 2\langle r_p^2\rangle^{Stanford} - \frac{6}{M_{ph}^2} - 2\langle r_e^2\rangle^{int.}$$

provided that the intrinsic electric and magnetic sizes are the same. If one assumes $M_{ph}=\infty$, one deduces $r_c = .7\ 10^{-13}$ cm. If, on the contrary, one assumes $\langle r_e^2\rangle = 0$, one gets $M_{ph} \sim .5\, M_p$. Thus, under either hypothesis, one has an order of magnitude agreement with the previous estimate. If the electron is assumed to be punctual, the photon cut off is low enough and the nucleonic sizes small enough, that photons of the cut off energy can only see the nucleon size, but not its structure. If, on the other hand, the electron is not punctual, and the photon cut off high enough, (it may be infinite), these conclusions are not valid, and the approach followed by Wick, (22), is more reliable, as it allows in principle to take into account the deformability of the nucleons; the experimental situation is however much too poor to take advantage of this latter formulation.
At any rate, it is clear why the result found by Wick has the wrong sign: since he used the Stanford radius, his calculation is roughly equivalent to a calculation

with such a small photon cut off, ($\lambda = \sqrt{\frac{6}{32}}\, M_n \simeq .45 M_n$), that the contribution of pair states is negligible, and that it is known to yield a negative mass difference, (6). To conclude this section, we should like to remark that the evaluation of the neutron proton mass difference, which in some respects may be thought of as replacing a Moller scattering experiment, will be a very sensitive test of the existence of a photon cut off, once the nuclear matrix elements are better known; this is so because of the fast variation of this quantity as a function of the cut off value.

C. Photon cut off correction to the Schwinger magnetic moment.

Selecting in the lowest order radiative correction to the electrodynamic vertex the term proportional to $\sigma_{\mu\nu}\Delta p^{\nu}$ yields: (23)

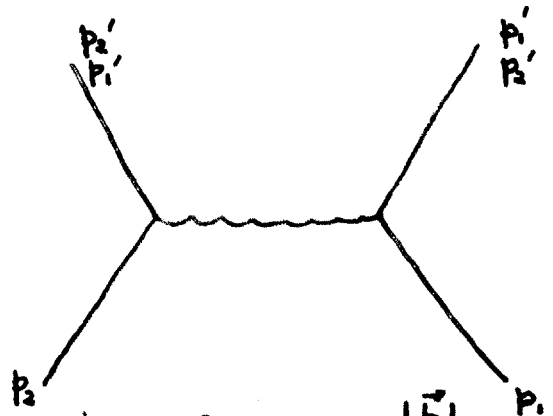


$$(II,8)\qquad \mu \simeq \frac{e}{2m_e}\left(1 + \frac{\alpha}{2\pi}\left(1 - \frac{2}{3}\frac{m_0^2}{M^2_{ph}}\right) + \cdots\right)$$

Whereas this cut off correction is very small for the electron, it is for the μ-meson of the same order of magnitude as the fourth order radiative correction, ($.7\,\frac{\alpha^2}{\pi^2}$), (24), with the opposite sign, and, for the time being, two orders of magnitude remote from the obtainable accuracy, (25).

D. Moller scattering.

As was previously mentioned, the cut off corrections become important when the momentum transfer is of the order of the cut off mass squared:



$$\Delta p^2 = (p_1 - p_1')^2 = 2\epsilon_1\epsilon_1'(1-\beta_1\beta_1'\cos\theta_1) \simeq M_{ph}^2 , \quad \beta = \frac{|\vec{p}|}{\epsilon}$$

that is to say, at high energies and large angles, when the ordinary Moller scattering is small. One finds indeed, in the center of mass frame of reference, (see the derivation in Appendix A) :

$$(II,9)\quad \frac{\Delta d\sigma_{Moll}}{d\sigma_{Moll}} \simeq -\frac{4(\epsilon^2 - m_0^2)}{M_{ph}^2}\sin^2\theta\left[\frac{12\gamma^4 - 16\gamma^2 + 5 - (3\gamma^2+1)(\gamma^2-1)\sin^2\theta}{\frac{(\gamma^2-1)^2}{4}\sin^4\theta - (2\gamma^4 - \gamma^2 + \frac{1}{4})\sin^2\theta + (2\gamma^2-1)^2}\right]$$

where $\gamma = \frac{\epsilon}{m}$.

$$\left.\frac{\Delta d\sigma}{d\sigma}\right|_{\theta=0,\pi} = 0 \quad , \text{ as expected.}$$

$$\left.\frac{\Delta d\sigma}{d\sigma}\right|_{\frac{\pi}{2}} = -\frac{4\epsilon^2}{M_{ph}^2}\,\frac{9\gamma^4 - 14\gamma^2 + 6}{9\gamma^4 - 14\gamma^2 + 4} \simeq -\frac{4\epsilon^2}{M_{ph}^2}$$

In order to observe a 10% correction, one needs a center of mass energy $\epsilon \gtrsim \frac{M_{ph}}{2}\frac{1}{3} \sim 100$ Mev, which corresponds, in the laboratory frame, to an incident electron energy of the order of 40 Bev.

In the case of μ-e scattering, there is no exchange scattering, and one finds, in the center of mass frame:

(II,10) $$\frac{\Delta d\sigma_{\mu e}}{d\sigma_{\mu e}} \simeq -\frac{4\epsilon^2\beta_\mu^2}{M_{ph}^2}\sin^4\frac{\theta}{2}$$

which is maximum for backward scattering. In order to observe a 10% deviation, one must require a laboratory μ-meson energy, (the electron being taken at rest), of the order of 20 Bev.

These experiments are thus at present outside the range of experimental possibilities.

E. Bound state problems.

In the case of the hydrogen atom, the energy shift due to the finite proton size is, (26):

(II,11) $$\Delta E = \frac{1}{6}\langle r_p^2\rangle \psi^2(0)e^2$$

where $\psi(0)$ is the electron wave function evaluated at the position of the proton. In the ground state, it amounts to .11 Mcs, and, as was pointed out before, does not indicate anything more about the photon cut off than the Stanford experiments do.

In conclusion, there does not seem to be at the moment any practicable new experiment which might decide for or against the existence of a photon cut off. Whereas electron-electron scattering requires too high energies, our knowledge of nuclear phenomena does not allow us to squeeze any more information out of the values of the nucleon radii or of their mass difference.

CHAPTER III. FERMION CUT OFF CORRECTIONS.

We have seen that, in spite of the inconsistencies inherent to the model, the theoretical situation concerning the photon cut off is fairly clear, that is to say, one can calculate photon cut off corrections, and, in principle, test the result experimentally. In the medium energy range where the model is assumed to be valid, all the results are expressible in terms of the size of the cut off, and are insensitive to the shape of the cut off function.

In the case of the fermion cut off, the problem is much more undetermined. The first difficulty which comes up is that, in view of the non-measurability of the fermion field, no obvious interpretation proposes itself. Secondly, we shall see that the requirement of local gauge invariance implies that the fermion cut off function does not suffice to describe the high energy behaviour of physically observable effects. Deviations from the usual formulas will of course manifest themselves in processes where virtual fermions are involved far off their mass shell, i.e; when the corresponding matrix element of the conventional theory is small, due to the occurrence of a large retardation denominator . We have seen furthermore that gauge invariance requires, together with an

alteration of the propagator, a modification of the vertex, which is to say, essentially, of the current. One easily interpretable effect due to this alteration would be, for free particles, to produce an additional magnetic moment. However, since we are concerned here with alterations connected with the regularization of vacuum polarization, we shall not study such an effect, and thus , we shall definitely have to deal with modificatfions which affect virtual particles.

The elementary processes involving virtual fermions are the following:

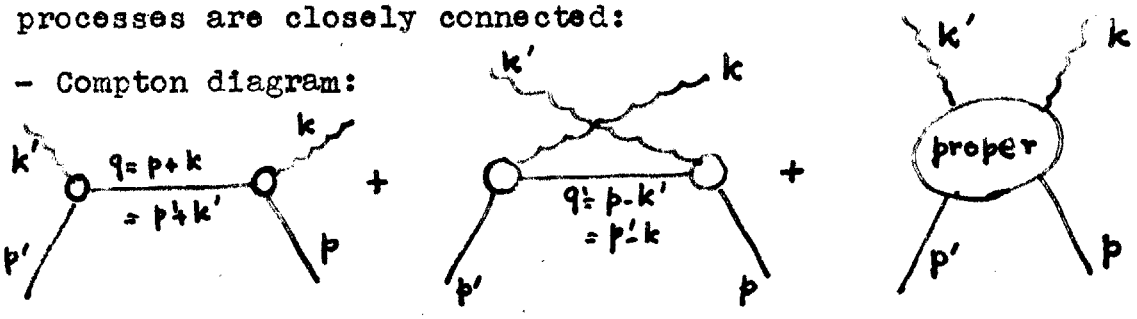
-Fermion self energy.

-Radiative corrections to the scattering in an external field;(in particular, the Schwinger magnetic moment.).

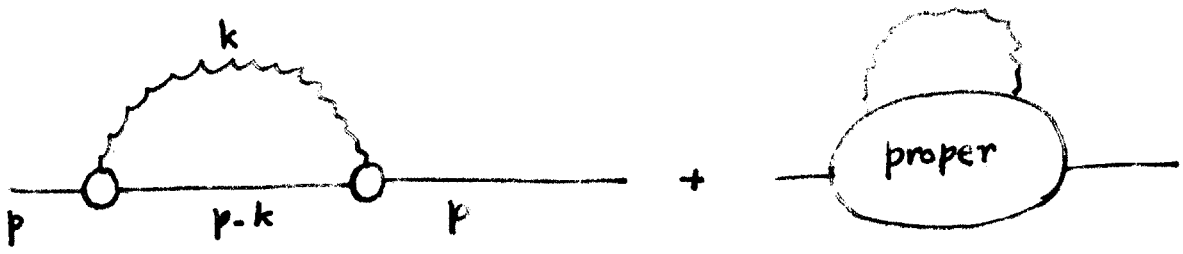
-Processes of the type "Compton scattering",(Compton scattering, pair creation, bremstrahlung, in an external field).

In the usual perturbation theory, these processes are closely connected:

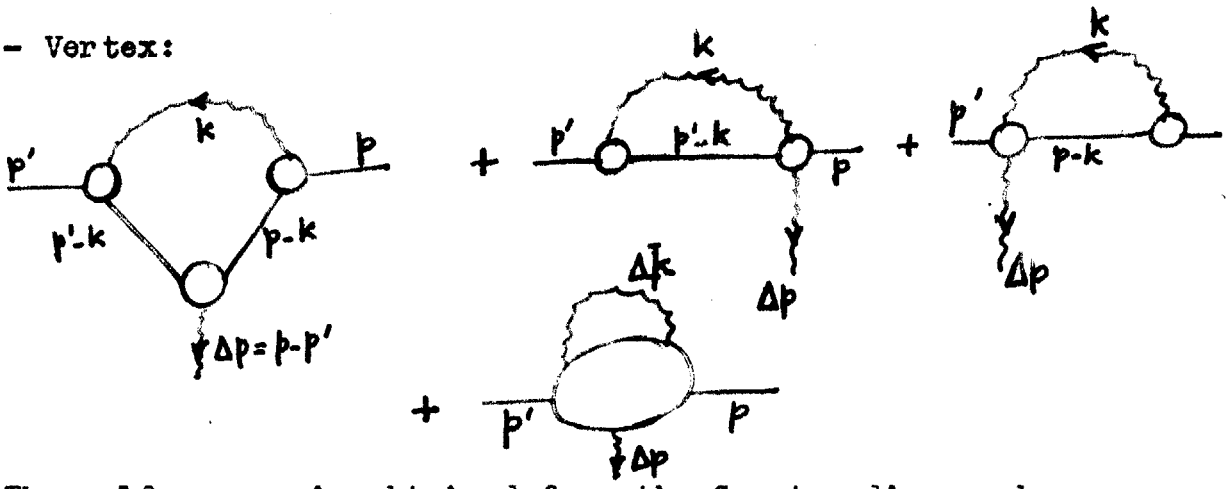
- Compton diagram:



- Self energy diagram:



- Vertex:



The self energy is obtained from the Compton diagram by closing up the photon lines into a single virtual photon line, whereas the vertex is obtained by inserting a photon in all possible ways in the self energy diagram. Since the Compton diagram plays a prominent role, we shall first make a few remarks concerning the gauge invariance requirements it is subjected to.

A. Gauge invariance problems related to the Compton process;.

It is shown in appendix B that the gauge invariance condition relating the vertex operator to the fermion propagator is not sufficient to insure the gauge invariance of the Compton matrix element. In other words, the two "improper" diagrams which only involve the modified vertex and propagator, do not combine into a gauge invariant contribution, the lack of gauge invariance that they exhibit being compensated by that of the proper Compton part.

Let us call $C_{\mu\nu}(p';k';p,k)$ the operator whose

matrix element between free fermion states $\bar{u}(p'), u(p)$, yields essentially the amplitude for Compton scattering of a photon of momentum k, polarization ν, from an initial fermion of momentum p, into a photon of momentum k', polarization μ, and a fermion of momentum p'. Then we define the proper amplitude $\Gamma_{\mu\nu}(p',k';p,k)$ through:

$$C_{\mu\nu}(p',k';p,k) = \Gamma_{\mu R}(p', p'+k')\, S_{cR}(p'+k')\, \Gamma_{\nu R}(p'+k', p) + \Gamma_{\nu R}(p', p'-k)\, S_{cR}(p'-k)\, \Gamma_{\nu R}(p'-k, p) + \Gamma_{\mu\nu}(p',k';p,k) \tag{IV,1}$$

there being understood that the four-momenta are linked by momentum conservation:

$$p'+k' = p+k$$

The gauge invariance conditions are, (see Appendix B):

$$i k'_\mu\, C_{\mu\nu}(p',k';p,k) = -\Gamma_\nu(p',q')\, S_{cR}(q')\, S_{cR}^{-1}(p) + S_{cR}^{-1}(p')\, S_{cR}(q)\, \Gamma_\nu(q,p) \tag{IV,2}$$

$$C_{\mu\nu}(p',k';p,k)\, i k_\nu = \Gamma_\mu(p',q)\, S_{cR}(q)\, S_{cR}^{-1}(p) - S_{cR}^{-1}(p')\, S_{cR}(q')\, \Gamma_\mu(q',p)$$

where q and q' are the intermediate fermion momenta occurring in the uncrossed and crossed diagrams, respectively. The corresponding conditions on the proper part $\Gamma_{\mu\nu}(p',k';p,k)$ are:

$$i k'_\mu\, \Gamma_{\mu\nu}(p',k';p,k) = \Gamma_\nu(q,p) - \Gamma_\nu(p',q') \tag{IV,3}$$

$$\Gamma_{\mu\nu}(p',k';p,k)\, i k_\nu = \Gamma_\mu(p',q) - \Gamma_\mu(q',p)$$

(notice that the matrix elements of the right hand sides

of Eq.IV,2, between free particle states vanish, which is consistent with the gauge invariance condition usually quoted).

If one makes the gauge invariant approximation $\Gamma^{(1)}_{\mu}(p',p)=0$, one can write:

$$C_{\mu\nu}(p',k';p,k) = S^{-1}_{cR}(p')\int \rho_R(a)da\, \frac{i\not{p}'-a}{p'^2+a^2}\gamma_\mu \frac{i\not{q}-a}{q^2+a^2}\gamma_\nu \frac{i\not{p}-a}{p^2+a^2} S^{-1}_{cR}(p)$$

(IV,4)

$$+ S^{-1}_{cR}(p')\int \rho_R(a)da\, \frac{i\not{p}'-a}{p'^2+a^2}\gamma_\nu \frac{i\not{q}'-a}{q^2+a^2}\gamma_\mu \frac{i\not{p}-a}{p^2+a^2} S^{-1}_{cR}(p)$$

$$+ C^{(1)}_{\mu\nu}(p',k';p,k)$$

where $k' C^{(1)}_{\mu\nu}(p',k';p,k) = C^{(1)}_{\mu\nu}(p',k';p,k)\, k_\nu = 0$

Thus, whereas one would have expected that the cut off corrections would have set in through the improper part, one sees that these corrections are cancelled by terms of the proper part in such a way that gauge invariance is preserved. The matrix element of the first part of $C_{\mu\nu}(p',k';p,k)$ appearing in Eq.IV,4; (excepting $C^{(1)}_{\mu\nu}(p',k';p,k)$), is exactly identical to the usual expression, because the inverse propagators $S^{-1}_{cR}(p')$, $S^{-1}_{cR}(p)$, select under the integral those terms which have a pole for $i\not{p}+M=0$, $i\not{p}'+M=0$.

We must take this result as a proof that, under the assumption of local gauge invariance, the cut off corrections to the Compton process, if they exist, are not determined by the cut off function responsible for the regularization of vacuum polarization. We shall reach a similar conclusion in a somewhat more transparent analysis to be found in Chapter V, whereas the contradictory point of view assumed in the next Chapter will lead

to this new analysis in a natural way, whereas the consequences of the contradictory model will serve as a basis for the description of the high energy behaviour of the Compton process and related phenomena, and will be justified in a different way.

B. Self energy and vertex.

The description of these two quantities is just as ambiguous as that of processes of theCompton type. We shall not investigate any more the self energy since in the physically interesting case of the neutron proton mass difference, the strong couplings play such a prominent role.

If one wants to make a statement about the cut off correction to the radiative magnetic moment, one has first to hope that the proper diagram is small compared to the others;(whereas this is impossible to do in the case of Compton scattering where the proper part is known to have a contribution of the same order of magnitude as that of the improper part, as a consequence of gauge invariance,one can do it here because the photon is virtual). Such a simplification would be physically acceptable if the vertices on which the photon is attached, are more strongly damped towards high momenta in the proper part than in the others. Then one has the gauge invariant approximation:

$$(IV,5)\quad \Lambda^{I}_{\mu}(p',p) = \frac{ie^2}{(2\pi)^4}\int d^4k'\, D_{CR}(k')\Big[\Gamma_\nu(p',p'-k')S_{CR}(p'-k')\,\Gamma_\mu(p'-k',p-k')S_{CR}(p-k')\,\Gamma^\nu(p-k',p)$$
$$+\Gamma_\nu(p',p'-k')S_{CR}(p'-k')\,\Gamma_{\mu\nu}(p'-k',\Delta p;\,p,-k')$$
$$+\Gamma_{\nu\mu}(p',-k';\,p-k',-\Delta p)\,S_{CR}(p-k')\,\Gamma^\nu(p-k;\,p)$$

which fulfills the Ward identity: $i(p'_\mu - p_\mu)\Lambda^{I}_{\mu}(p',p) = \Sigma^{I}(p') - \Sigma^{I}(p)$

(The upper index I means that the proper diagram has been neglected both in the vertex and in the self energy)

Thus, for free particles, one can rewrite:

$$(IV,6)\quad \Lambda^{I}_{\mu}(p',p)\Big|_{\substack{ip=-m\\ ip'=-m}} = \frac{ie^2}{(2\pi)^4}\int d^4k'\, D_{CR}(k')\Big[\Gamma_\nu(p',p'-k')S_{CR}(p'-k')C_{\mu\nu}(p'-k',\Delta p,p-k')$$
$$+C_{\nu\mu}(p'-k';\,p-k',-\Delta p)\,S_{CR}(p-k')\,\Gamma^\nu(p-k',p) - \Gamma_\nu(p',p'-k')S_{CR}(p'-k')\Gamma_\nu(p'-k',p')S_{CR}(p')\Gamma_\mu(p',p)$$
$$-\Gamma_\nu(p',p'-k')S_{CR}(p'-k')\Gamma_\mu(p'-k',p-k')\,S_{CR}(p-k')\,\Gamma^\nu(p-k',p)$$
$$-\Gamma_\mu(p',p)\,S_{CR}(p)\,\Gamma_\nu(p,p-k')\,S_{CR}(p-k')\,\Gamma^\nu(p-k',p)$$
$$= \Sigma^{I}(p')\,S^{\circ}(p')\,\left[1-S^{-1}_{CR}(p')S^{\circ}(p')\right]\gamma_\mu$$
$$+\gamma_\mu\left[1-S^{\circ}(p)\,S^{-1}_{CR}(p)\right]S^{\circ}(p)\,\Sigma^{I}(p)$$
$$+\frac{ie^2}{(2\pi)^4}\int d^4k'\, D_{CR}(k')\Big[\gamma_\nu\,S^{\circ}_{c}(p'-k')\,S^{-1}_{CR}(p'-k')\,S^{\circ}(p'-k')\,\gamma_\mu\,S^{\circ}_{c}(p'-k')\,\gamma^\nu$$
$$+\gamma_\nu\,S^{\circ}_{c}(p-k')\,\gamma_\mu\,S^{\circ}(p-k')\,S^{-1}(p-k')\,S^{\circ}(p-k')\,\gamma^\nu$$
$$-\Gamma_\nu(p',p'-k')S_{CR}(p'-k')\,\Gamma_\mu(p'-k',p-k')S_{CR}(p-k')\,\Gamma^\nu(p-k',p)\Big]$$

The first two terms correspond to charge renormalization whereas the magnetic moment is contained in the integral. In view of the unreliability of the result we would get, we shall not pursue in this direction any further.

Our conclusion on all this is that the description of processes involving virtual fermions mainly depends on the structure of the Compton matrix element, rather than on that of the charged particle propagator.

This is specifically a consequence of local gauge invariance. In the next chapter, we shall try to somewhat change the point of view we have taken here: recognizing the important role played by the Compton process, we shall try to take it as the fundamental entity describing the physics of virtual fermions, rather than the more abstract propagators.

CHAPTER IV. THE "COMPTON CUT OFF"

We have seen that an attempt to formulate a gauge invariant fermion cut off in terms of the elementary operators which characterize the covariant perturbation theory, (propagators and vertex), has failed to provide an unambiguous description of the elementary electrodynamic processes which involve virtual fermions. The scheme we previously proposed had furthermore the unpleasant feature that it was difficult to interpret physically, because its formulation forced us to argue in terms of virtual particles; this is due to the very detailed interpretation of the Feynman theory, one drawback of which is the separation of transition amplitudes into several parts, each of which has no meaning, except in the framework of this interpretation. Thus, we imposed gauge invariance in a much more artificial and detailed way than if we had just required that the transition amplitudes themselves should be gauge invariant, as this latter demand makes only restrictions on the dependence of physical phenomena upon the states of free incoming particles.

We however gained the feeling that processes of the same type as Compton scattering deserve a special attention, because they cannot be fully expressed in terms of the regularized operators of perturbation theory. We should like here to reverse the situation, and, stressing

the important role played by the Compton amplitude in the structure of the lowest order approximation in perturbation theory, take it as a basis for an alternative cut off procedure, susceptible of a more direct physical interpretation. More precisely, instead of first trying to make finite the fermion self energy and vacuum polarization, we should like to modify the Compton amplitude at high energy in such a way that the basic infinities are removed.

A. Connection between the Compton amplitude and the divergent diagrams.

We shall rewrite here the results of second order perturbation theory in a more suggestive, (may be misleading) way than has been done before. We recall that the Compton matrix element is given by:

$$(IV,1)\quad \langle p',k',\epsilon' | C | p,k,\epsilon \rangle = (2\pi)^4 ie^2\, \bar{u}(p')\left[\not\epsilon' \frac{i(\not p+\not k)-m}{(p+k)^2+m^2}\not\epsilon + \not\epsilon \frac{i(\not p-\not k')-m}{(p-k')^2+m^2}\not\epsilon'\right] u(p)$$

where p', k', (p,k), are the final, (initial), fermion and photon momenta, ϵ', (ϵ), representing the final, (initial) photon polarization. (We have omitted the trivial normalization factors $\frac{1}{(2\pi)^3}\frac{m}{E E'}\frac{1}{(2\pi)^3}\frac{1}{\sqrt{2\omega 2\omega'}}$). The matrix elements for pair creation and bremstrahlung are obtained from Eq. IV,1. by the substitution law, (cf. (9), p.162), as a consequence of the local character of the interaction, (i.e. the positive and negative frequency parts of the free field operators

are symmetrically involved in the S-matrix), and of the fact that the S-matrix is the same functional of the external electromagnetic field as it is of the quantized Maxwell field. The self energy operator, on the other hand, is given by:

$$\Sigma(p) = \frac{ie^2}{(2\pi)^4} \int \frac{d^4k}{k^2} \gamma_\mu \frac{i(\not{p}-\not{k})-m}{(p-k)^2+m^2} \gamma_\mu \tag{IV,2}$$

whereas the vacuum polarization kernel is usually written as:

$$\Pi_{\mu\nu}(k) = \frac{ie^2}{(2\pi)^4} \int \frac{d^4p}{p^2+m^2} \operatorname{tr} \gamma_\mu \frac{i(\not{p}-\not{k})-m}{(p-k)^2+m^2} \gamma_\nu (i\not{p}-m) \tag{IV,3}$$

which only fulfills the gauge invariance requirement within a shift of origin in k-space, which, forbidden in a quadratically divergent integral, (cf. (9), p.457), produces the well known spurious photon mass, unless a strong regularization of the fermion loop is imposed; thus, it is just as correct to write it in the symmetrized form:

$$\Pi_{\mu\nu}(k) = \frac{ie^2}{(2\pi)^4} \int \frac{d^4p}{p^2+m^2} \frac{1}{2} \operatorname{tr} \left[\gamma_\mu \frac{i(\not{p}-\not{k})-m}{(p-k)^2+m^2} \gamma_\nu + \gamma_\nu \frac{i(\not{p}+\not{k})-m}{(p+k)^2+m^2} \gamma_\mu \right] (i\not{p}-m) \tag{IV,4}$$

We now want to express the two divergent operators in terms of the forward Compton effect; the forward Compton matrix is:

$$C^f(p,k,\epsilon',\epsilon) = \not{\epsilon}' \frac{i(\not{p}+\not{k})-m}{(p+k)^2+m^2} \not{\epsilon} + \not{\epsilon} \frac{i(\not{p}-\not{k})-m}{(p-k)^2+m^2} \not{\epsilon}' \tag{IV,5}$$

or

$$(IV,5') \quad C^{f}_{\mu\nu}(p,k) = \gamma_\mu \frac{i(\not{p}+\not{k})-m}{(p+k)^2+m^2}\gamma_\nu + \gamma_\nu \frac{i(\not{p}-\not{k})-m}{(p-k)^2+m^2}\gamma_\mu$$

Now, we can express the self energy in terms of the Compton matrix:

$$(IV,6) \quad \Sigma(p) = \frac{1}{2(2\pi)^4}\int C^{f}_{\mu}{}^{\mu}(p,k)\,\frac{d^4k}{k^2-i\epsilon}$$

The electromagnetic mass itself is given by:

$$(IV,6') \quad \delta m = \bar{u}(p)\Sigma(p)u(p) = -\frac{1}{4m}\,\mathrm{tr}\,\Sigma(p)(i\not{p}-m)$$
$$= -\frac{1}{4m}\,\frac{1}{2(2\pi)^4}\int \mathrm{tr}\; C^{f}_{\mu}{}^{\mu}(p,k)(i\not{p}-m)\,\frac{d^4k}{k^2-i\epsilon}$$

it is a weighted integral of the Compton matrix element over a virtual photon spectrum.

The form used in Eq.IV,4 to describe vacuum polarization is however not very suitable because, although it is formally gauge invariant, actual computation exhibits the well known lack of gauge invariance which is in particular associated with the appearance of a quadratically divergent photon mass. This formal trouble is associated with the fact that expressions of the type $T\langle 0|j_\mu(x), j_\nu(x')|0\rangle$ are not well defined, since $[j_\mu(x), j_\nu(x')] \neq 0$ for $x \doteq x^0$; as is explained in Chapter V; on the contrary, $T\langle 0|j_\mu(x).j_\mu(x')|0\rangle$ can be assigned an unambiguous meaning, since $[j_\mu(x).j_\mu(x')] = 0$ for $x \doteq x^0$. Thus, we shall express $\Pi_{\mu\nu}(k)$ in terms of the better defined $\Pi_\mu{}^\mu(k)$.

Now, if one writes:

$$(IV,7) \quad \Pi_{\mu\nu}(k) = C(k^2)k_\mu k_\nu + D(k^2)g_{\mu\nu}$$
$$= (k_\mu k_\nu - k^2 g_{\mu\nu})C(k^2)$$

whereby we assume that $\Pi_\mu{}^\mu(k) = -3k^2C(k^2) = +3D(k^2)$

one has to require:

(IV,8) $\Pi_\mu^\mu(0) = 0$, C(0) finite.

Thus, eliminating off hand the spurious photon mass, we shall replace Eq.IV,4 by:

(IV,9) $$\Pi_\mu^\mu(k) = \frac{ie^2}{(2\pi)^4}\int \mathrm{tr}\left[C_\mu^{F\mu}(p,k) - C_\mu^{F\mu}(p,0)\right](i\not{p}-m)\frac{d^4p}{p^2+m^2-i\epsilon}$$

Thus, $$C(0) = -\frac{1}{3k^2}\frac{d}{dk^2}\Pi_\mu^\mu(k^2)\Big|_{k^2=0}$$

B) The Compton cut off.

In view of the formal connections we just established, we may want to blame the occurrence of divergences on the fact that the forward Compton matrix does not decrease fast enough at high energies. We may put:

(IV,10) $$C_{\mu\nu}(p';k';p,k) = g(p\cdot p', k\cdot k')\,C^0_{\mu\nu}(p';k';p,k)$$

where $C^0_{\mu\nu}(p';k';p,k)$ is the usual Compton matrix, and where the cut off function $g(p\cdot p';k\cdot k')$ exhibits charge conjugation invariance and crossing symmetry. The g-function must have the following properties:

a) $g(p\cdot p';k\cdot k')\to 0$ when $p\cdot p'\to\infty$ or $k\cdot k'\to\infty$, in order to make the wave function renormalization constants finite.

b) $g(-m^2;0) = 1$, in order to preserve the Thompson limit;

$g(p^2;k^2) \neq 1$: $p^2 \sim M_{1c}^2$, $k^2 \sim M_{2c}^2$

c) $g(p\cdot p', k\cdot k')$ does not have any singularities in the first and third quadrant of the p.p' and k.k' complex planes so that condition a) makes it to be a cut off function.

d) If this function were required to suppress the divergent photon mass, one should impose the condition:

but we do not believe that this condition is physically meaningful.

With such prescriptions, the dependence of the cut off function on p.p' provides the finiteness of vacuum polarization, whereas its dependence on k.k' provides a substitute for a photonncut off, which makes the self energy finite.

In what follows, we shall study observable consequences of these two cut offs.

C) Physical consequences.

-The neutron proton mass difference.

The cut off factor to be used here is

which fixes the value of M_{2c} at about one nucleon mass or more, if one believes that an appreciable part of the mass difference is provided by this Compton cut off.

-Compton scattering.

As one always has $k.k' < 0$, $p.p' < 0$, one comes to the conclusion that, although our cut off was initially based on the formal connection between the Compton matrix and the divergent processes, the cut off function is not involved in these two classes of processes for the same values of its arguments. If, however, the same analytic form is used for positive and negative values of the arguments, one sees that both cut off effects are important for large angle scattering:

$$|p.p'| \sim |k.k'| \sim M_c^2 \gg m^2 \; ; \; k.k' = \omega\omega'(1-\cos\theta) = \omega^2 \frac{1-\cos\theta}{1+\frac{\omega}{m}(1-\cos\theta)}$$

the characteristic energy parameter is $\frac{\omega}{M_c}\frac{m_c}{M_c}$, so that very large energies would be required in order to exhibit deviations from the usual behaviour; this is forbidding in view of the smallness of the cross section at high energy.

-Pair creation.

The two cut off effects are quite different in this case: the first one can be observed if $p_+ . p'_- \sim M_{ic}^2$, that is to say, when both members of the produced pair come out with large energies and at large angle from each other, whereas $k.k' = k.(p_+ + p_-)$ can be large if either of the produced particle comes out with a large energy, at large angle from the photon, even though the other one tends to go forward (k.p small).

-Bremstrahlung.

The first cut off can be observed when the incident energetic particle suffers a large deflection associated with a small energy loss, whereas the second cut off appears when $k.k' \sim k'.(p - p')$, that is to say, when a hard photon is radiated at large angle from the incident particle, (then, p' is small).

D) Concluding remarks.

-Experimental.

As the second cut off needs not be invoked in order to suppress the divergences, one would be tempted to think that the crucial experiments to be performed are the following:

-energetic pair creation of two particles at large angle from each other.

-observation of a Bremstrahlung process in which the radiating particle suffers a small energy loss, but a large deviation.

On the other hand, the existence of the second Compton cut off can be tested by studying:

- observation of one energetic member of a pair coming out at a large angle from the producing photon.

-High energy, large angle Bremstrahlung.

-Theoretical.

With the photon cut off studied in Chapter III, and the Compton cut offs which we have just investigated, one can describe high energy deviations of the main electrodynamic processes from their behaviour as predicted by the conventional lowest order perturbation theory. Such deviations are consistent with the finiteness of the renormalization constants. Both of these cut offs have simple physical interpretations.
As their construction was essentially based on the analytic forms of the amplitudes predicted by second order perturbation theory, we should like to study in the next and last chapter what basic properties of the conventional scheme imply such analytic structures.

CHAPTER V. REMARKS ON THE STRUCTURE OF THE S-MATRIX.

A. The S-matrix constructed from the Lagrangian formalism.

Although we have already given a brief summary of the properties of the conventional S-matrix at the beginning of Chapter I, we should like to insist on a few specific points. We have pointed out that the introduction of renormalization constants was made necessary by the stability requirement:

(V,1) $$S|0\rangle = |0\rangle \quad ; \quad S\,|1\,\text{particle}\rangle = |1\ \text{particle}\rangle$$

We shall presently see, however, that even if these conditions were fulfilled , the local and causal structure implied by the presence of time ordered products would leave some ambiguity. Indeed, it is well known that the time ordered product of two operators -which has only a meaning if they"commute" outside the light cone- is not well defined when the two points it refers to coincide. If these two operators commute at the same space-time point, their time ordered product can be smoothly defined when the two points coincide, but if they do not, it is only defined within a quasi-local operator,(27). Thus, whereas the S-matrix is, from this point of view, well defined, its matrix elements, obtained by the application of Wick's theorem, are not; thus, besides the inconsistency implied by the divergence of the conventional renormali-

zation constants, there is still left the possibility to introduce new constants. The attitude one usually takes is to use as few constants as possible, besides those w which come out infinite, that is to say, to define the time ordering function, each time this is possible, as:

(V,2) $\epsilon(x) = +1$, $x \in$ future light cone ; $\epsilon(x) = -1$, $x \in$ past light cone

$$\epsilon(0) = 0$$

In the next section, we shall try to see how far one can go in constructing a local electrodynamics, starting from an S-matrix with reasonable properties, since the Lagrangian formalism, which implies the "existence" of a continuous set of evolution operators, and forces the coupled field operators to satisfy canonical commutation rules, has proved difficult to modify.

B. Unitarity and causality of a "local" S-matrix.

We start from an S-matrix, a functional of free in-field operators which describe assemblies of uncoupled physical particles, (27), (28), (29). The S-matrix can be written in normal ordered form:

$$(V,3)\quad S = 1 + \sum_{n,n',m} \int dx_1 \cdots dx_n\, dx'_1 \cdots dx'_{n'}\, dy^1 \cdots dy^m\, F^{(x_1 \cdots x_n, x'_1 \cdots x'_{n'}, y_1 \cdots y_m)}_{\alpha_1 \cdots \alpha_n, \beta_1 \cdots \beta_{n'}, \mu_1 \cdots \mu_m} \times$$

$$\times : \psi_{\alpha_1}(x_1) \cdots \psi_{\alpha_n}(x_n)\, \bar{\psi}_{\beta_1}(x'_1) \cdots \bar{\psi}_{\beta_{n'}}(x'_{n'})\, A_{\mu_1}(y_1) \cdots A_{\mu_m}(y_m) :$$

As it is written, it only involves Fourier transforms of the F-functions for momenta on the mass shell; however, it will soon prove necessary to extend their definition for virtual momenta in order to specify the local properties of the theory.

The stability conditions:

(V,4) $$S|0\rangle = |0\rangle \quad ; \quad S|1\,\text{particle}\rangle = |1\,\text{particle}\rangle$$

imply the vanishing of some of these Fourier transforms for particular values of their arguments, which will be used in several connections.

The S-matrix is assumed to be unitary:

(V,5) $$S S^{\dagger} = S^{\dagger} S = 1$$

As to the condition of microscopic causality, it has been formulated by Bogoliubov and coworkers under the following form: (27), (28)

(V,6) $$\frac{\delta}{\delta A(x)}\left(-i S^{\dagger}\frac{\delta S}{\delta B(y)}\right) = 0 \qquad x \gtrsim y$$

and proved under the assumption that there exist unitary evolution operators- i.e. in the framework of a Lagrangian formalism-; here, $A(x)$ and $B(y)$ are any two functions or operators entering the functional definition of the S-matrix; the functional differenciation is defined in the same way as it is in the usual scheme, and requires well known cares when the operators involved in the process anti-commute. As it will be seen that, in the conventional theories, the quantities

(V,7) $$j_B(x) = -i S^{\dagger}\frac{\delta S}{\delta B(x)}$$

are the "sources" or "currents" of the coupled fields

which tend asymptotically to the in-fields $B(x)$, hermitian by virtue of the unitarity of the S-matrix if the B-fields are real, this condition has a simple physical meaning: it states that the effect on a property of the system observed at point (y) of a disturbance applied at point (x) can only be felt if (x) is in the past light cone of (y). More generally, it is reasonable to formulate local causality in the following form:

$$\frac{\delta J(y)}{\delta A(x)} = 0 \qquad x \gtrsim y \tag{V,8}$$

where $J(y)$ is a locally measurable quantity and $\delta A(x)$, a disturbance. This condition can be taken as a postulate. It implies the ambiguities that we pointed out in the preceding section, since it leaves undetermined the response of the system to a disturbance at the point where this disturbance is applied. This undeterminacy must be taken as a proof of the incompleteness of a causal theory formulated in terms of local fields, since the high energy behaviour of all the relevant matrix elements- which are Fourier transforms of vacuum expectation values of time ordered products of current operators, by virtue of the causality and unitarity requirements- depends on unknown renormalization polynomials of the relativistic invariants which can be formed with the four-momenta involved.

C. Definition of coupled field operators.

In order to complete the specification of the local aspects of the theory, one has to state prescriptions

according the coupled field operators should be defined. One will then have a scheme comparable to the Lagrangian formalism. Given an in-field $O_{in}(x)$, we shall define the corresponding coupled field through:

$$O_H(x) = S^\dagger T(O_{in}(x), S) \tag{V,9}$$

This relationship holds in conventional theories. In order that this definition might have a meaning, it is necessary to rewrite the S-matrix in time ordered form, using the converse of Wick's theorem. Using Eq.V,2. for the definition of the time ordering function, together with the free field equations of motion and commutation rules, one readily obtains:

$$\begin{aligned} A_{\mu H}(x) &= A_{\mu in}(x) + \int D_{ret}(x-x')\left(-i S^\dagger \frac{\delta S}{\delta A_\mu(x')}\right)dx' \\ &= A_{\mu out}(x) + \int D_{adv}(x-x')\left(-i S^\dagger \frac{\delta S}{\delta A_\mu(x')}\right)dx' \qquad A_\mu^{out}(x) = S^\dagger A_\mu^{in}(x) S \\ \psi_H(x) &= \psi_{in}(x) + \int S_{ret}(x-x')\left(-i S^\dagger \frac{\delta S}{\delta \bar\psi(x')}\right)dx' \\ &= \psi_{out}(x) + \int S_{adv}(x-x')\left(-i S^\dagger \frac{\delta S}{\delta \bar\psi(x')}\right)dx' \qquad \psi^{out}(x) = S^\dagger \psi^{in}(x) S \end{aligned} \tag{V,10}$$

$$\Box A_{\mu H}(x) = i S^\dagger \frac{\delta S}{\delta A_\mu(x)} = -j_{\mu H}(x)$$

$$(\gamma^\mu \partial_\mu + m)\psi_H(x) = -i S^\dagger \frac{\delta S}{\delta \bar\psi(x)} = f_H(x).$$

These equations are essentially those proposed in (29).

Those of the first group, (Yang-Feldman equations), have been obtained by commuting the in-field operators in Eq. V,9. either to the right or to the left, whereas those of the second group make use of the differenciation of a time ordered product together with the commutation rules.

Whereas these equations assume familiar forms, they have some unsatisfactory features. Indeed, in order to decide whether these fields should be taken as the renormalized or unrenormalized operators, we should evaluate the matrix elements $\langle 0|\psi_\mu(x)||\text{particle}\rangle$, $\langle 0|A_{\mu\nu}(x)||\text{particle}\rangle$. One is tempted to write $\langle 0|\psi_\mu(x)|1\rangle = \langle 0|\psi_{in}(x)|1\rangle$, $\langle 0|A_{\mu\nu}(x)|1\rangle = \langle 0|A_{\mu\nu in}(x)|1\rangle$, in view of the stability property $\langle 0|f(x)|1\rangle = \langle 0|j_\mu(x)|1\rangle = 0$, (see (27), p.389). However, in the conventional theory, the integrals occurring in the Yang-Feldman equations assume the undetermined form 0/0, (cf. (1), p.342 and ff.), so that, after all, this definition keeps being ambiguous. As however the field operators have no direct physical meaning, we shall leave this question open.

We should like, before closing up this section, to point out that this scheme may be, so far, much broader than the usual one, since no dynamics has been defined yet, (for instance one does not have necessarily $j_{\mu\nu}(x) = S^+ T(ie\bar{\psi}_{in}(x)\gamma_\mu\psi_{in}(x)S)$, $f(x) = S^+ T(ie\gamma_\mu A^\mu_{in}(x)\psi_{in}(x)S)$, and since it has even not been specified that there exists an interaction Hamiltonian.

D. Specification of the dynamics and of an approximation scheme.

The dynamics of the system are contained in the F-functions defined in Eq.V,3. which are restricted by the usual covariance properties and the unitarity and causality requirements. The simplest three functions are:

-the fermion self energy:

$$(V,11) \qquad \Sigma^c(x,x') = \langle 0| \frac{\delta^2 S}{\delta\psi(x)\delta\bar{\psi}(x')} S^+|0\rangle$$

-the photon self energy:

$$(V,12) \qquad \Pi^c_{\mu\nu}(x,x') = \langle 0| \frac{\delta^2 S}{\delta A_\mu(x)\delta A_\nu(x')} S^+|0\rangle$$

-the vertex operator:

$$(V,13) \qquad \Gamma^{(p)}_\mu(x,x';y) = \langle 0| \frac{\delta^3 S}{\delta\psi(x)\delta\bar{\psi}(x')\delta A_\mu(y)} S^+|0\rangle$$

The stability conditions imply that the Fourier transforms of the first two vanish for momenta on the mass shell. They are the kernels involved in the definition of the propagators:

$$(V,14) \qquad i S'^c(x-x') = T\langle 0|\psi(x)\bar{\psi}(x') S|0\rangle$$

$$(V,15) \qquad -i D'^c_{\mu\nu}(y,y') = T\langle 0|A_\mu(y) A_\nu(y').S|0\rangle$$

Some physically relevant matrix element of the current operators are on-the-mass-shell Fourier transforms of the vertex function; for instance, $\langle p'|j_\mu|p\rangle$ describes the particle charge and magnetization distributions, whereas $\langle 0|j_\mu|p,p'\rangle$ describes the production of pairs by an external field. These two quantities are therefore analytic extensions of each other, ($p' \to -p'$). Similar remarks can be formulated about matrix elements of the type $\langle 0|f|p,k\rangle$ $\langle p|f|k\rangle$ .....

As all the matrix elements of products of current operators can be expanded, using a complete set of states, and since this process will never stop, one would like to set up an approximation scheme in which one might cut off this infinite expansion. We first of all decide that the terms of the expansion should not contain other matrix elements than those of the vertex operator. It seems then natural to base the approximation method on some expansion of the vertex. Since the vertex is ambiguous within a quasilocal operator which belongs to its dispersive part, we conjecture that if one takes it to be:

$$\Gamma^{\rho}_{\mu}(x,x',y) = T\langle 0|f(x),\bar{f}(x'),j_{\mu}(y)|0\rangle + ie_0\,\delta(x-y)\,\delta(x'-y)\,\gamma_{\mu} \qquad (V,16)$$

and combine the intermediate state expansion with a series expansion in powers of e_0, and suitable renormalization prescriptions for the various other matrix elements, one should be able to reproduce the results of the conventional theory. Since these finite quasilocal operators can be suspected to produce divergences, and as we would like to keep the relationships implied by unitarity and causality, we shall introduce a breakdown of the theory in the following fashion:

-assume that the S-matrix we start from describes a wider scheme than electrodynamics.

-take as a first approximation to the vertex an expression allowed by causality.

-in order to compute an effect, split the corresponding amplitudes into invariant parts, and express their coefficients in terms of matrix elements of products of current operators; then, evaluate these matrix elements by combined use of the two above mentioned expansions. (Only the states involving charged fermions and photons will be considered; we feel that this is a convenient way to violate unitarity and causality.)

E. The extended electron model.

A physically reasonable deviation from the electrodynamics of a point particle is obtained when one assumes that the particle is spread. Thus, we take for the lowest approximation to the current, the following causal expression: (see (1) p.358 ff.)

$$\langle p'|j_\mu|p\rangle = \bar{u}(p')\left[e_0 F(p'-p)^2)\gamma_\mu + i\mu_0\sigma_{\mu\nu}(p'-p)^\nu G((p'-p)^2)\right]u(p) \qquad (V,17)$$

$$F(\Delta p^2) = \bar{F}_0(\Delta p^2) + i\pi\epsilon(\Delta p)F_0(\Delta p^2) \; ; \; G(\Delta p^2) = \bar{G}_0(\Delta p^2) + i\pi\epsilon(\Delta p)G_0(\Delta p^2)$$

where the barred functions are Hilbert transforms of the unbarred ones. We shall presently see that if the form factors are sufficiently damped for large momentum transfers, the usual divergences are suppressed.

a) The current induced in vacuum by an external field.

We shall write the S-matrix in the presence of an external field by replacing $A^{in}_\mu(x)$ by $A^{in}_\mu(x) + A^{ext}_\mu(x)$. The approximation to the current induced in vacuum, linear in the external field, is:

$$(V,18)\qquad \langle 0|j_\mu(x)|0\rangle^{ind} \sim \int dx' \langle 0|\frac{\delta j_\mu(x)}{\delta A_\nu(x')}|0\rangle A_\nu^{ext}(x') = \int dx' K_{\mu\nu}^R(x,x') A_\nu^{ext}(x')$$

where, by virtue of the causality requirement:

$$(V,19)\qquad K_{\mu\nu}^R(x,x') = -i\eta(x-x')\langle 0|[j_\mu(x), j_\nu(x')]|0\rangle$$

The covariance requirements on the polarization kernel are the following:

-it is a symmetric tensor.

-it is gauge invariant (or rather has to be made gauge invariant with the help of the proper quasilocal term).

Thus, it must assume the form:

$$(V,20)\qquad K_{\mu\nu}^R(x,x') = (\Box^2 g_{\mu\nu} - \partial_\mu\partial_\nu)\Pi^R(\overline{x-x'}^2)$$

since there must not be any induced current if $A_\mu^{ext}(x) = \partial_\mu\Lambda(x)$ which corresponds to a gauge transformation. In terms of the Fourier transforms, one has:

$$(V,21)\qquad j_\mu^{ind}(p) = -\Pi^R(p) j_\mu^{ext}(p)$$

As charge conservation implies that the induced charge must vanish, we must require that

$$(V,22)\qquad j_\mu^{ind}(0) = 0$$

which is achieved by using another renormalization counter-term. Thus finally, the total observed current must be:

$$(V,23)\qquad j_\mu^{obs}(p) = [\,1 - \Pi^R(p^2) + \Pi^R(0)\,]\, j_\mu^{ext}(p)$$

(see (1) p.282). Now, it is well known that causality implies the separation of $\Pi^R(p^2)$ into an absorptive and a dispersive part:(ref.(1) p.344 & ff.)

(V,24) $$\Pi^R(p^2) = \overline{\Pi}(p^2) + i\pi\epsilon(p)\,\Pi(p^2) = -\frac{1}{3\pi^2}\,K^R_{\mu\mu}(p)$$

where $$\Pi(p^2) = -\frac{1}{3p^2}\int d\nu(n)\,\delta(p-p_n)\,\langle 0|j_\mu|n\rangle\langle n|j_\mu|0\rangle$$

and $\overline{\Pi}(p^2)$ is the Hilbert transform of $\Pi(p^2)$; $\Pi(p^2)$ essentially describes the production of real particles by the external field. The production of pairs is contained in:

(V,25) $$\Pi^{pair}(p^2) = -\frac{1}{3p^2}\int \delta(p-p'-p'')\,dp'dp''\,\theta(p')\,\delta(p'^2+m^2)\,\theta(p'')\,\delta(p''^2+m^2)\,\langle 0|j_\mu|p'p''\rangle\langle p'p''|j_\mu|0\rangle$$

and is essentially given by the perturbation theory result provided one disregards the anomalous magnetic moment term. It is then clear that, as the matrix element $\langle 0|j_\mu|pp'\rangle$ is the analytic extension of the electromagnetic form factor, if the latter is sufficiently damped for large time-like as well as space-like momenta, then the dispersive part is finite as well as $\overline{\Pi}(0)$. As now both $\Pi(p^2)$ and $\overline{\Pi}(p^2)$ go to zero when p^2 goes to infinity in the future light cone, the finite renormalization $1+\overline{\Pi}(0)$ describes the response of vacuum to an external field at the very same point where it is applied, and defines the bare charge:

(V,26) $$e_{bare} = [1+\overline{\Pi}(0)]\,e_{phys.}$$

which one would see if one could perform an experiment during an infinitely short time.

One point of interest is whether assuming a form factor would suffice to insure the finiteness of all the terms contributing to the polarization of vacuum. For instance, the one pair, one photon term, should be evaluated in the lowest approximation, in conjunction with the use of the second approximation to the vertex in the one pair term. This would take us too far and will be done in the near future.In the event that the contribution of this process is finite, one shall have to see whether it implies a modification of the Compton amplitude of the type indicated in Chapter IV, and, if it is so, connect the S-function with the fermion form factor; this, however takes us beyond the philosophy we have adopted. Before giving up this question, we should like to remark that, as there is not yet any rigorous proof of dispersion relations for Compton scattering with non vanishing momentum transfer, (30), there is no reason not to expect deviations of the above mentioned or of another type.

We shall now go back to the lowest approximation.

b) The self energy.

The electromagnetic mass is naturally defined in the Hamiltonian formalism through:

$$\delta m \sim \langle p | \int dx \; j_\mu(x) A_\mu(x) | p \rangle \qquad (V,27)$$

or, with the help of Eq.V,10.

(V,28) $$\delta m \sim \langle p| \int j_\mu(x) A_\mu^{in}(x) |p\rangle - \int D_{ret}(x-x') \langle p| j_\mu(x)\, j_\mu(x') |p\rangle$$

The first term is connected with the forward Compton amplitude, whereas the second term can be expressed, using the expansion in terms of intermediate states, as a sum of squares of matrix elements for other processes off the mass shell. So, essentially the same observations as were made about vacuum polarization hold here too.

However, this definition may not be consistent, and we have too use the alternative definition in terms of the S-matrix, that is to say, we have to interpret the mass as a parameter involved in the definition of the propagator. The stability requirement

(V,29) $$\langle 0| f(x) |p\rangle = 0$$

implies that the self energy operator destroys one particle states:

(V,30) $$\Sigma(p)\, u(p) = \int e^{ipx'} \Sigma(x-x')\, u(p)\, dx' \neq 0; \quad \Sigma(x-x') = -i\eta(x-x') \langle 0| [f(x), \bar{f}(x')] |0\rangle$$

The counter terms necessary in order to satisfy this condition must then be interpreted as self energy terms: (actually, we find here again the ambiguity stated about the definition of the coupled field operators; this is however irrelevant for the evaluation of the lowest approximation.

(V,31) $$\frac{\delta m}{Z_2} = \Sigma(p)\Big|_{ip+m=0}$$

Again, in view of the separation of the self energy operator into absorptive and dispersive part, a cut off on matrix elements of the type $\langle o|\mathcal{H}|p,k\rangle$ supplies the necessary convergence.

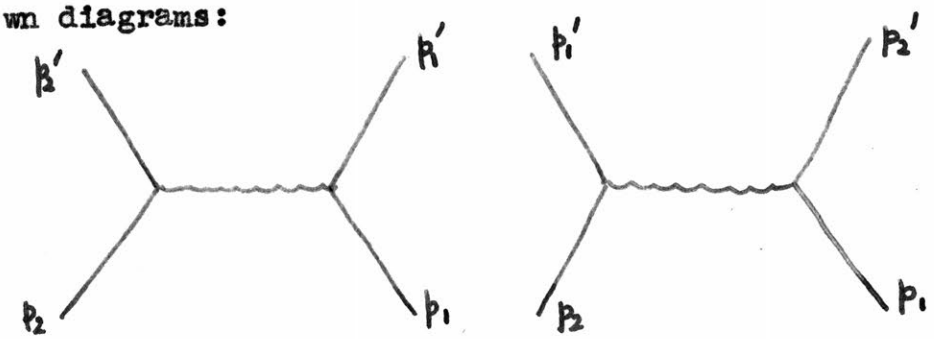
F. Conclusion.

Although we have not made a complete study of the "extended electron" model, it suppresses the divergences in the first approximation, for reasons which are physically sensible. It allows for an extension of our philosophy on cut offs, in a way which does not exclude the conclusions of practical interest we came to in the preceding chapter.

APPENDIX A. MOLLER SCATTERING.

A. Electron-Electron scattering.

The matrix element is computed in terms of the well known diagrams:



The cross section is found to be:(9)

$$(A,1)\qquad \frac{d\sigma}{d\Omega} \propto \frac{m^4}{(p_1-p_1')^4}A + \frac{m^4}{(p_2'-p_1)^4}B - \frac{m^4}{(p_1'-p_1)^2(p_2'-p_1)^2}2C$$

where A, B, C, are suitable traces of Dirac matrices and projection operators on positive energy states. The cut off correction is given by:

$$(A,2)\qquad \frac{\Delta d\sigma}{d\Omega} \propto -2\left[\frac{m^4A}{(p_1'-p_1)^2\left[(p_1'-p_1)^2+M_{ph}^2\right]} + \frac{m^4B}{(p_2'-p_1)^2\left[(p_2'-p_1)^2+M_{ph}^2\right]} - m^4C\left\{\frac{1}{(p_1'-p_1)^2\left[(p_2'-p_1)^2+M_{ph}^2\right]} + \frac{1}{(p_2'-p_1)^2\left[(p_1'-p_1)^2+M_{ph}^2\right]}\right\}\right]$$

In the center of mass frame of reference,

$$(A,3)\qquad \begin{aligned}(p_1'-p_1)^2 &= 4p^2\sin^2\frac{\theta}{2} \qquad p=\beta\gamma m\\ (p_2'-p_1)^2 &= 4p^2\cos^2\frac{\theta}{2}\end{aligned}$$

$$\frac{\Delta d\sigma}{d\Omega} \propto -\frac{1}{(2\beta\gamma)^4}\,8p^2\gamma^2\frac{m^2}{M_{ph}^2}\left[\frac{A}{\sin^2\frac{\theta}{2}} + \frac{B}{\cos^2\frac{\theta}{2}} - \frac{C}{\sin^2\frac{\theta}{2}\cos^2\frac{\theta}{2}}\right]$$

where β and γ are the common velocity and energy of the electron, and θ, the scattering angle. This expression is valid provided that $\gamma^2\frac{m^2}{M_{ph}^2}<1$, which is the condition of validity of the model. With

$$A = \frac{1}{2}\left[(2\gamma^2-1)^2 + \gamma^4(1+\beta^4\cos^2\theta) - 2(\gamma^2-1) + 2\beta^2\gamma^2(\gamma^2+1)\cos\theta\right]$$

$$B = \frac{1}{2}\left[(2\gamma^2-1)^2 + \gamma^4(1+\beta^4\cos^2\theta) - 2(\gamma^2-1) - 2\beta^2\gamma^2(\gamma^2+1)\cos\theta\right]$$

(A,4)

$$C = -\frac{1}{2}(2\gamma^2-1)(2\gamma^2-3)$$

one finds:

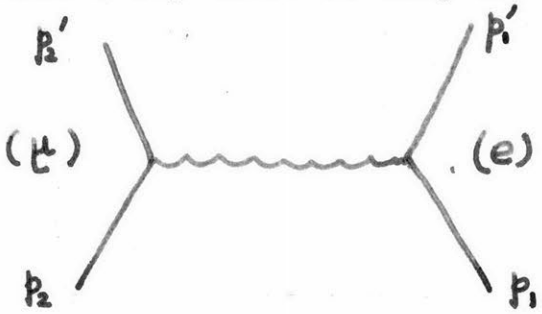
$$(A,5)\qquad \frac{\Delta d\sigma}{d\Omega} \propto -\frac{m^2}{M_{ph}^2(\gamma^2-1)}\left[\frac{12\gamma^4-16\gamma^2+5}{\sin^2\theta} - (3\gamma^2+1)(\gamma^2-1)\right]$$

Hence

$$(A,6)\qquad \frac{\Delta d\sigma}{d\sigma} = -\frac{m^2(\gamma^2-1)}{M_{ph}^2}\sin^2\theta\left[\frac{(12\gamma^4-16\gamma^2+5) - (3\gamma^2+1)(\gamma^2-1)\sin^2\theta}{\frac{(\gamma^2-1)^2}{4}\sin^4\theta - (2\gamma^4-\gamma^2+\frac{1}{4})\sin^2\theta + (2\gamma^2-1)^2}\right]$$

B. μ-meson - electron scattering.

In this case, there is only one diagram:



Thus, only the direct term survives; since in the relative correction A cancels, one has:

$$(A,7)\qquad \frac{\Delta d\sigma}{d\sigma} = -4\beta^2\gamma^2\frac{\mu^2}{M_{ph}^2}\sin^4\frac{\theta}{2}$$

provided that $\beta^2\gamma^2\frac{\mu^2}{M_{ph}^2} \ll 1$, where μ is the μ-meson mass.

APPENDIX B. GENERALIZED WARD IDENTITIES.

In order to investigate the requirements imposed by local gauge invariance, we shall study the propagation of a fermion in an external electromagnetic field; let

$$(B,1) \qquad T\langle \psi(x)\bar{\psi}(x')\rangle_0 = i S_c(x,x') = G(x,x')$$

be the fermion propagator; in the presence of an external field which we assume to be weak, we can perform an expansion:

$$(B,2) \qquad G(x,x';A^{ext}) = G(x-x') + \int A_\mu^{ext}(\eta)\, \frac{\delta G(x,x';A)}{\delta A_\mu(\eta)}\Big|_{A^{ext}=0} d^4\eta + \frac{1}{2!}\int A_\mu^{ext}(\eta) A_\nu^{ext}(\eta')\, \frac{\delta^2 G(x,x';A)}{\delta A_\mu(\eta)\,\delta A_\nu(\eta')}\Big|_{A^{ext}=0} d^4\eta\, d^4\eta'$$

In terms of diagrams, one has the following representation:

where  = 1 + + + ····

is the complete photon propagator in which one free propagation function has been omitted. We then define:

$$(B,3) \qquad \frac{\delta G(x,x')}{\delta A_\mu^{ext}(\eta)} = ie\int G(x-\xi)\,\Gamma_\mu^P(\xi,\eta,\xi')\,G(\xi'-x')\,d\xi\,d\xi'$$

where $\Gamma_\mu^P(\xi,\eta,\xi')$ is the "polarized vertex which differs from the usual one by the multiplicative factor $1+D_c\Pi+D_c\Pi\,D_c\Pi+\cdots$

Similarly, let

$$(B,4) \qquad \frac{\delta^2 G}{\delta A_\mu(\eta)\delta A_\nu(\eta')} = -e^2\int G(x-\xi)\,\Gamma_{\mu\nu}^P(\xi,\eta,\xi';\eta')\,G(\xi'-x')\,d\xi\,d\xi'$$

define the two fold polarized Compton operator. The vertex operator can also be expanded in powers of the external field:

$$(B,5)\quad \Gamma_\mu(x,y,x';A) = \Gamma_\mu(x,y,x') + \int A_\nu^{ext}(y') \left.\frac{\delta\Gamma_\mu(x,y,x';A)}{\delta A_\nu(y')}\right|_{A=0} + \cdots.$$

and define the singly polarized proper Compton operator:

$$(B,6)\quad \frac{\delta\Gamma_\mu(x,y,x')}{\delta A_\nu(y')} = ie\,\Gamma_{\mu(\nu p)}(x,y,y',x')$$

We now define the Fourier transformed quantities $\Gamma_\mu(p',p;k)$ and $C_{\mu\nu}(p',k';p,k)$ through:

$$(B,7)\quad (2\pi)^4\delta(p'-p+k)\Gamma_\mu(p',p) = \int e^{i(-p'\xi+p\xi'-k\eta)}\Gamma_\mu(\xi,\eta,\xi')$$

$$(2\pi)^4\delta(p'+k'-p-k)C_{\mu\nu}(p',k';p,k) = \int e^{-i(p'\xi+k'\eta-p\xi'-k\eta')}C_{\mu\nu}(\xi,\eta;\eta'\xi')$$

where the crossing symmetry is expressed by:

$$(B,8)\quad C_{\mu\nu}(p',k';p,k) = C_{\nu\mu}(p',-k;p,-k')$$

We further remark on the connection between the total and proper Compton operators:

$$\frac{\delta}{\delta A_\nu(\eta')} ie\int G(x,\xi)\Gamma_\mu(\xi,\eta,\xi')G(\xi',x') = -e^2\int G(x,\xi)C_{\mu\nu}(\xi\eta\eta'\xi')G(\xi',x')$$

$$(B,9) = ie^2\Big[\int G(x,\xi'')\Gamma_\nu(\xi'',\eta',\xi''')G(\xi''',\xi)\Gamma_\mu(\xi,\eta,\xi')G(\xi',x')$$
$$+\int G(x,\xi)\Gamma_\mu(\xi,\eta,\xi')G(\xi',\xi'')\Gamma_\nu(\xi'',\eta',\xi''')G(\xi''',x')$$
$$+\int G(x,\xi)\Gamma_{\mu\nu}(\xi,\eta;\eta',\xi')G(\xi',x')\Big]$$

Let us now perform a gauge transformation by putting $A_\mu^{ext} = -\partial_\mu\Lambda(x)$. Then we know that

$$(B,10)\quad G(x,x';\partial_\mu\Lambda) = e^{ie[\Lambda(x)-\Lambda(x')]}G(x,x')$$

$$\Gamma_\mu^P(x,y,x';-\partial_\mu\Lambda) = e^{ie[\Lambda(x)-\Lambda(x')]}\Gamma_\mu^P(x,y,x')$$

Thus, we obtain from Eq.B,2.

$$(B,11)\qquad \int -\partial_\mu \Lambda(\eta)\, ie\, G(x,\xi)\, \Gamma_\mu^{\rho}(\xi,\eta,\xi')\, G(\xi',x') = ie\left[\Lambda(x)-\Lambda(x')\right] G(x,x')$$

In terms of the Fourier transformed operators, and in view of the transversality of $\partial_\mu \Lambda$, one has, owing to the gauge invariance of the vacuum polarization:

$$(B,12)\qquad -i\, G(p')\,(p'^{\mu}-p^{\mu})\,(2\pi)^4\, \Lambda(p'-p)\, \Gamma_\mu(p',p)\, G(p) = (2\pi)^4 \Lambda(p'-p)\left[G(p')-G(p)\right]$$

which is essentially the first Ward identity:

$$(B,12^{a})\qquad i\,(p'^{\mu}-p^{\mu})\,\Gamma_\mu(p',p) = G^{-1}(p') - G^{-1}(p)$$

where $G(p) = \int \rho(a)\, da\, \frac{i\not{p}-a}{p^2+a^2-i\epsilon}$.

Similarly, from Eq.B,5. one deduces:

$$(B,13)\qquad \int dk\, \Lambda(k)\,(2\pi)^4 \left[\Gamma_\mu(q,p)-\Gamma_\mu(p',q)\right]\delta(p'+k'-p-k) = -i\int dk\, \Lambda(k)(2\pi)^4 \times \delta(p'+k'-p-k)\,\Gamma_{\mu\nu}(p',k';p,k)\,k_\nu$$

where the polarization part has dropped out for the same reason as in Eq.B,11., and where we have used the values of the uncrossed and crossed momenta: $q = p'+k' = p+k$; $q = p'-k = p-k'$.

Thus,

$$(B,14)\qquad \Gamma_{\mu\nu}(p',k';p,k)\, i k'_\nu = \Gamma_\mu(p',q) - \Gamma_\mu(q',p)\ .$$

and, from the crossing symmetry,

$$(B,15)\qquad i k_\mu\, \Gamma_{\mu\nu}(p',k';p,k) = \Gamma_\nu(q,p) - \Gamma_\nu(p',q')$$

Furthermore, since

$$(B,16)\qquad C_{\mu\nu}(p',k';p,k) = \Gamma_\mu(p',q)\, G(q)\, \Gamma_\nu(q,p) + \Gamma_\nu(p',q')\, G(q')\, \Gamma_\mu(q,p) + \Gamma_{\mu\nu}(p',k';p,k)$$

we also have:

(B,17)

$$ik'_\mu C_{\mu\nu}(p';k';p,k) = -\Gamma_\nu(p';q')G(q')G(p)^{-1} + G^{-1}(p')G(q)\Gamma_\nu(q,p)$$

$$C_{\mu\nu}(p';k';p,k)\, ik_\nu = \Gamma_\mu(p';q)G(q)G(p)^{-1} - G^{-1}(p')G(q')\Gamma_\nu(q';p)$$

APPENDIX C. FORMAL STRUCTURE OF THE VACUUM POLARIZATION AND SELF ENERGY DIAGRAMS.

We wish to establish here the high energy deviations of the vacuum polarization and fermion self energy in a theory where the fermions interact with some neutral regulator fields.

The equations of motion are:

(C,1)
$$\Box A_\mu(x) = -\frac{ie}{2}\left[\bar\psi(x).\,\gamma_\mu\psi(x)\right]$$
$$\left(\gamma^\mu\partial_\mu + m\right)\psi(x) = -ie\,\gamma^\mu A_\mu(x)\psi(x) + f(x)$$

where $f(x)$ is the operator which represents the effect of the non electromagnetic couplings.

The photon propagator is defined by:

(C,2)
$$-i\,D'^{c}_{\mu\nu}(x,x') = T\langle 0|A^{in}_\mu(x).A^{in}_\nu(x').S|0\rangle = \frac{\delta}{\delta J_\mu(x)}\langle 0|T[A_\nu(x').S]|0\rangle$$
$$= \frac{\delta}{\delta J_\mu(x)}\mathcal{A}_\nu(x')$$

where the differenciation is taken with an external current.

Now,

(C,3)
$$\Box\mathcal{A}_\nu(x') = J^{ext}_\nu(x') + T\langle 0|\frac{ie}{2}\left[\bar\psi(x).\,\gamma_\nu\psi(x')\right].S|0\rangle$$
$$= J^{ext}_\nu(x') + ie\,\mathrm{tr}\,\gamma_\nu G(x';x')$$

where $G(x,x') = T\langle 0|\psi(x)\bar\psi(x').S|0\rangle$ is the fermion propagator.

Now, writing
$$\frac{\delta}{\delta J_\mu(x)} = \int dx'\,\frac{\delta\mathcal{A}_\nu(x')}{\delta J_\mu(x)}\,\frac{\delta}{\delta\mathcal{A}_\nu(x')} = \int dx'(-i)\,D'^{c}_{\mu\nu}(x,x')\,\frac{\delta}{\delta\mathcal{A}_\nu(x')}$$

one obtains:

(C,4)
$$\Box\,D'^{c}_{\mu\nu}(x,x') = \delta(x-x')g_{\mu\nu} + \mathrm{Tr}\,e\int dx''\gamma_\mu\,\frac{\delta G(x;x')}{\delta\mathcal{A}_\lambda(x'')}D'^{c}_{\lambda\nu}(x'',x)$$
$$= g_{\mu\nu}\,\delta(x-x') - \int dx''\,\Pi_{\mu\rho}(x,x'')\,D'^{c}_{\rho\nu}(x'',x')$$

in which

$$\Pi_{\lambda\mu}(x',x'') = -\,\mathrm{Tr}\, e\,\gamma_\mu \frac{\delta G(x',x')}{\delta A_\lambda(x'')} \tag{C,5}$$

From the definition of the vertex operator:

$$ie\,\Gamma_\mu(x,y,x') = \frac{\delta G^{-1}(x,x')}{\delta A_\mu(y)} = -\int dx''dx'''\, G^{-1}(x,x'')\frac{\delta G(x'',x''')}{\delta A_\mu(y)} G^{-1}(x''',x') \tag{C,6}$$

one deduces:

$$\frac{\delta G(x',x')}{\delta A_\lambda(x'')} = -ie\int G(x',x''')\,\Gamma_\mu(x''',x'',x^{IV})\,G(x^{IV},x')\,dx'''\,dx^{IV} \tag{C,7}$$

Thus,

$$\Pi_{\lambda\mu}(x',x'') = ie^2\,\mathrm{Tr}\int \gamma_\mu\,\delta(x'-x'')\,G(x''',x^{III})\,\Gamma_\mu(x^{III},x'',x^{IV})\,G(x^{IV},x')\,dx'''\,dx^{IV}\,dx^{III} \tag{6,8}$$

which proves Eq.I,12 in the text, if $G(x,x')$ and $\Gamma_\mu(x,y,x')$ are approximated by their zerpth orderapproximation in the electromagnetic coupling.

The fermion propagator, on the other hand, can be written: $G(x,x') = T\langle 0|\psi(x)\bar{\psi}(x')S_{e.m.}|0\rangle$, where $\psi(x)$, $\bar{\psi}(x')$, are the fermion fields coupled to the neutral regulator fields, $S_{e.m.}$ being the electromagnetic S-matrix, in which the current operator is expressed in terms of the same fields. Correctly to the second order in the electromagnetic coupling, one has:

$$G(x,x') = G^0(x,x') + ie^2\int D^c_{\mu\nu}(y-y')\,T\langle 0|\psi(x)\bar{\psi}(x')j_\mu(y)j_\nu(y')|0\rangle$$

$$= G^0(x,x') + ie^2\int dy\,dy'\,D^c_{\mu\nu}(y,y')\,T\langle 0|\psi_{in}(x)\bar{\psi}_{in}(x')j^0_\mu(y)j^0_{in}(y')S^{abn}|0\rangle \tag{C,9}$$

The latter vacuum expectation value, in which S^{abn} is the S-matrix produced by the abnormal couplings, is closely connected with the Compton Green's function. More precisely, one has:

$$T\langle 0|\psi_{in}(x).\bar{\psi}_{in}(x').j_{\mu,in}(y)\,j_{\nu,in}(y')\,S^{abn}|0\rangle = -\frac{\delta^2 G(x,x')}{\delta A_\mu(y)\delta A_\nu(y')}$$

(C,10)
$$= \int G^{(0)}(x-\xi)\,C_{\mu\nu}(\xi,y,y',\xi')\,G^{(0)}(\xi'-x')\,d\xi\,d\xi'.$$

Hence, the self-energy is given by:

(C,11)
$$\Sigma(x,x') = \Sigma^{abn}(x,x') + ie^2\int D^c_{\mu\nu}(y-y')C_{\mu\nu}(x,y,y',x')\,dy\,dy' + ie^2\int \Sigma^{abn}(x-\xi')\,G^{o}(\xi'-\xi'')\,D^c_{\mu\nu}(y-y')\,C_{\mu\nu}(\xi'',y,y',x')\,d\xi'\,d\xi''\,dy\,dy'$$

where $\Sigma^{abn}(x,x')$ is the self energy due to the abnormal couplings, and where the electromagnetic part is expressed in terms of the Compton matrix. This justifies the result quoted p.27 of the text.(The techniques used here are those to be found in : J. Schwinger. Proc. Natl. Acad. Sci. 37, 452, 1951.)

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