Unitary Transformations on Temporal Modes using Dispersive Optics for Boson Sampling and Quantum Simulation

by

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Abstract

Conventionally, unitary transformations on optical modes have been implemented on a spatial basis set using a system of beamsplitters and phase shifters. We present methods which allow orders of magnitude increase in the number of modes in linear optics experiments by moving from spatial encoding to temporal encoding and using dispersion. This enables significant practical advantages for linear quantum optics and Boson Sampling experiments. Passing identical, consecutively heralded photons through time-independent dispersion and measuring the output time of the photons is equivalent to a Boson Sampling experiment for which no efficient classical algorithm is reported, to our knowledge. With time-dependent dispersion, it is possible to implement arbitrary single-particle unitaries. Given the relatively simple requirements of these schemes, they provide a path to realizing much larger linear quantum optics experiments including post-classical Boson Sampling machines.

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Chapter 1

Introduction

Single particle unitary transformations of photons are used to implement single qubit quantum gates [12, 19], quantum simulations [20] and Boson Sampling [1, 24, 4, 26, 6, 5, 27, 23]. Furthermore, they also form an important part of many linear optical quantum computing schemes [12, 19]. Conventionally these transformations have been implemented on discrete spatial optical modes. In this thesis we show how moving to a temporal mode set and using dispersive optics can allow us to implement a unitary on a much larger number of modes than would be possible with spatial modes. In particular, we consider the possibility of realizing a post-classical Boson Sampling machine using a temporal mode set.

This chapter describes conventional spatial mode unitary transformations and Boson Sampling.

Chapter two describes the unitary transformations using fixed dispersion and pulse-shaping.

Chapter three describes the construction of arbitrary unitaries using time-dependent dispersion.

Chapter four concludes the thesis and describes avenues for future work

1.1 Linear optics and Unitary transformations

It was shown by Reck and Zeilinger [21] that an arbitrary unitary transformation can be implemented using $m(m-1)/2$ beam splitters and phase-shifters. Given the relative ease of implementing linear optical elements in single photon experiments, such transformations are extensively used in quantum optics experiments. Furthermore, integrated photonics has allowed the fabrication of large beam splitter arrays with excellent phase stability [24, 4, 26, 6, 5]. However, for certain applications, the required number of elements in such a decomposition can still be too large to implement experimentally. In this thesis, we explore the implementation of such unitary transformations on temporal modes using dispersion.

1.2 Boson Sampling

Quantum Computing offers the interesting possibility of being able to solve classically intractable problems in polynomial time. Optics is seen as a promising physical platform for quantum information processing because photons do not decohere easily i.e. a photon retains its quantum superposition for a long time. Furthermore, photons provide a natural integration of quantum computation and quantum communication. However, optical quantum information processing suffers from a major challenge because the nonlinear interaction strength between photons in a medium is very small. There are ways to induce an effective interaction between photons by using the result of photon detection on part of the system to determine the linear optic unitary transformations acting on other parts of the system (feed-forward) [11] but this is challenging due to the fast speed of light. Finally, most quantum computing schemes require on-demand sources which are challenging to implement.

Boson Sampling refers to the process of sending unentangled photons through a system of beam splitters and phase shifters implementing a unitary transformation on spatial modes and then measuring the output of the modes using single photon detectors (Fig. 1-1)

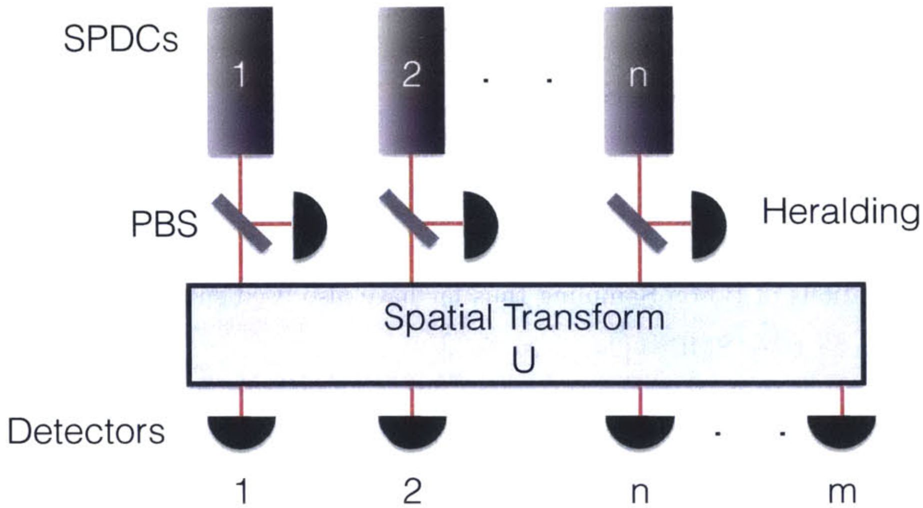

Figure 1-1: Schematic of spatial mode Boson Sampling (SMBS)

Specifically, photons are injected into input modes $1....j...n$. The system then transforms the creation operator for input spatial mode j as $\hat{a}_j^\dagger \rightarrow \sum_i U_{ij}\hat{a}_i^\dagger$, at which point photons in each mode are measured using single photon detectors [1, 24, 4, 26, 6, 5].

It was shown by Scott Aaronson and Alex Arkhipov that an efficient classical algorithm for estimating the output of such a system would, under plausible assumptions, imply a collapse of the polynomial hierarchy [1]. Furthermore, it is believed that simulating the output of a Boson Sampling machine with 30 photons is beyond the capabilities of today's classical computers [1].

A Boson Sampling machine is believed to be incapable of universal quantum computation or even universal classical computation. However, it provides a path to realizing a post-classical machine without nonlinear materials or feed-forward which are major challenges in optical quantum computing. Furthermore, unlike other quantum computing schemes that require on-demand sources, Boson Sampling with probabilistic but heralded input photons has been proposed to be computationally hard for a classical computer [15].

1.2.1 Advantages of Temporal Mode Boson Sampling over Spatial Mode Boson Sampling

In the original Boson Sampling proposal, the unitary transformation is implemented on spatial modes using an array of beamsplitters and phase-shifters. Most experimental implementations of Boson Sampling thus far have also used such systems (which we shall now refer to as SMBS) [24, 4, 26, 6, 5].

SMBS entails several difficult challenges, that, as we show, favor temporal mode encoding. First, Boson Sampling requires an extremely large number of modes to be classically computationally difficult. Strictly speaking, the complexity argument for Boson Sampling assumes that, if n is the number of photons in the system and m is the number of modes, $m \geq \Omega(n^5 \log^2 n)$. Although Aaronson and Arkhipov have conjectured that the complexity arguments still hold when $m = O(n^2)$ [1], the number of modes is still large: e.g., even with $m = n^2$ and $n = 30$, the interferometer would require 900 modes. Furthermore, the experiment would require 900 detectors and, if the photons came from heralded sources, 900 sources [15]. To date, experimental demonstrations of SMBS have been limited to 5 photons in 21 modes [5]. The number of modes in temporal mode Boson Sampling (TMBS) can be increased simply by increasing the dispersion. Even with 10000 ps/nm dispersion, which can be achieved with off-the shelf components, and 100 ps detector jitter, which can be routinely achieved with silicon avalanche photodiodes or with superconducting nanowire single photon detectors [18], the number of modes in TMBS is orders of magnitude higher than SMBS. In principle, the experiment can be implemented in a single fiber with only a single photon source and only two detectors: one to herald input photons and one to detect the output state, regardless of the number of interfering photons in the system. Given the dead time t_{dt} of single photon detectors, the output may have to be split between a larger number of detectors. However, in general, the number of detectors is smaller than required in SMBS [1].

Furthermore, uncertainty in the time when photons are injected into different modes leads to distinguishability and loss of boson interference. This is a particular problem in SMBS with heralded sources based on spontaneous parametric down conversion (SPDC). Most SMBS experiments to date have relied on downconverted photons. Temporal or spectral filtering could improve the interference, but at an exponential loss in multi-photon throughput. In TMBS, the lack of control over the input time of our photons only corresponds to a lack of control over the choice of our input modes; however, as long as the input modes are known, this does not affect the ability to perform Boson Sampling [15].

1.2.2 Previous proposals using temporal encoding

Previous proposals have considered temporal modes for Boson Sampling [17, 9] but they relied on temporarily converting temporal modes to spatial modes and then mixing the modes with beamsplitter operations. Hence, increasing the number of output modes ($m \gg n$) requires a large number of effective beamsplitter operations. They also require active elements that operate on a picosecond time scale. Furthermore, since they are based on the interference of narrow photon packets, they suffer from the same issues with temporal mismatch as SMBS. In TMBS, detector jitter can limit the accuracy with which the input mode can be heralded but this limitation can be overcome by using large dispersion.

[1] See Appendix A

Chapter 2

Unitary Transformations with Time-Independent Dispersion

In this chapter we present the simplest scheme for performing unitary transformations on temporal modes using time-independent dispersion.

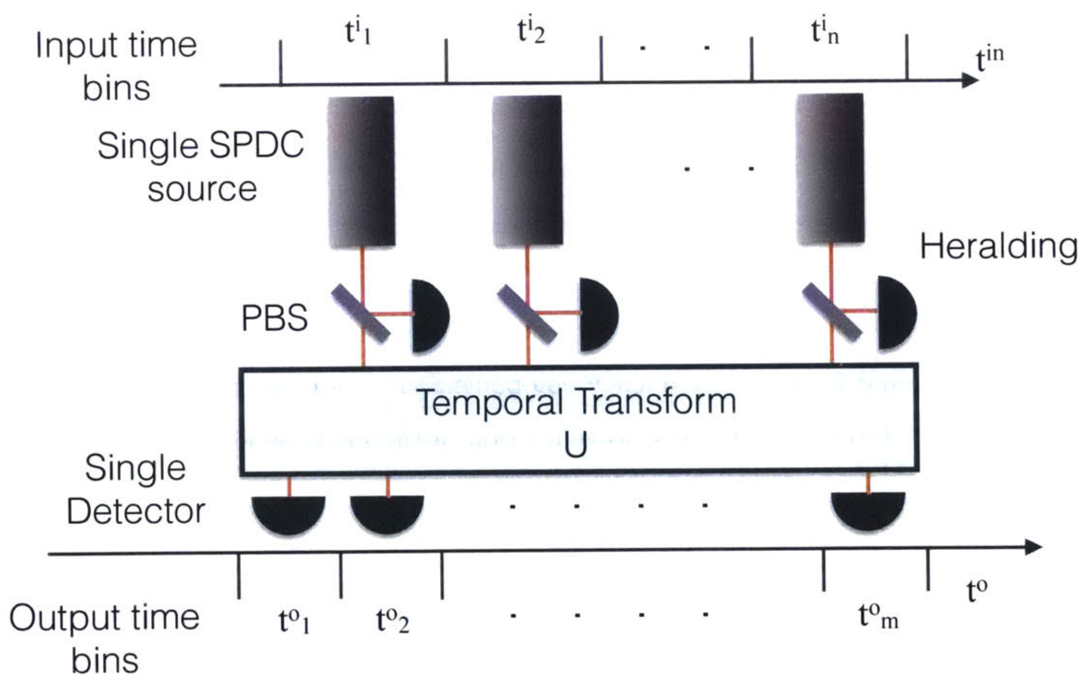

Figure 2-1: Conceptual Schematic of temporal mode Boson Sampling (TMBS)

2.1 Description of the scheme

Fig. 2-2 shows the setup for the scheme. A single SPDC source is repeatedly pumped resulting in a chain of entangled photons. The output from the SPDC is sent through a polarizing beamsplitter to split the signal and idler. Since a heralded source is used, not all the bins for which the SPDC source is pumped have photons but by heralding on the idler, it is possible to know which bins have photons and such a system is sufficient for Boson Sampling [15]. After going through the dispersion, the time of arrival of the photons at the detector is measured. We show below that such a scheme is equivalent to implementing a unitary of the form 2.2

If we use an SPDC source with idler photons heralded at times t_j and signal photons used as input photons, the input state is given by $|\Psi_{in}\rangle = \left(\prod_j \hat{a}_{Aj}^\dagger\right)|0\rangle$ where $|0\rangle$ is the multimode vacuum state and $\hat{a}_{Aj}^\dagger \equiv \int_{-\infty}^{\infty} \mathrm{d}t\ \hat{a}^\dagger(t)A(t-t_j)$ represents the creation operator for the input state centered at t_j. $\hat{a}^\dagger(t)$ is the creation operator for time t and ω_0 is the central frequency of the input photons. We assume that the photon state after heralding of the idler is a pure state of the form $A(t-t_j)$. However, a realistic detector projects the signal photon into a mixed state with t_j varying over the timescale of the detector jitter; the effect of this temporal mismatch can be made negligible with large dispersion (section 2.3.1).

$\hat{a}_{Aj}^\dagger$ can be expanded in the frequency domain as $\hat{a}_{Aj}^\dagger = \int_{-\infty}^{\infty} \mathrm{d}\omega\hat{a}^\dagger(\omega)\mathcal{F}\{A(t-t_j)\}$ where $\hat{a}^\dagger(\omega)$ is the creation operator for frequency ω and $\mathcal{F}\{A(t-t_j)\}$ is the Fourier transform of $A(t-t_j)$. After passing through a dispersive element with dispersion relation $\beta(\omega)$ and length L, frequency components at ω are multiplied by a factor $\mathrm{e}^{-i\phi(\omega)}$ where $\phi(\omega) \equiv \beta(\omega)L$. The wavefunction of the multi-photon system is then given by $|\Psi_{out}\rangle = \left(\prod_j \hat{b}_j^\dagger\right)|0\rangle$ where $\hat{b}_j^\dagger \equiv \int_{-\infty}^{\infty} \mathrm{d}\omega\hat{a}^\dagger(\omega)\mathcal{F}\{A(t-t_j)\}\mathrm{e}^{-i\phi(\omega)}$. Going back to the time domain, $\hat{b}_j^\dagger = \int_{-\infty}^{\infty} \mathrm{d}t\hat{a}^\dagger(t)U(t,t_j)$ with

$$U_{ij} = \left[\sqrt{t_s}A(t-t_j) * \mathcal{F}^{-1}\{\mathrm{e}^{-i\phi(\omega)}\}\right]_{t=t_i} \tag{2.1}$$

where '$*$' is the convolution operator.

An arbitrary functional form for the dispersion $\phi(\omega)$ can be obtained by using

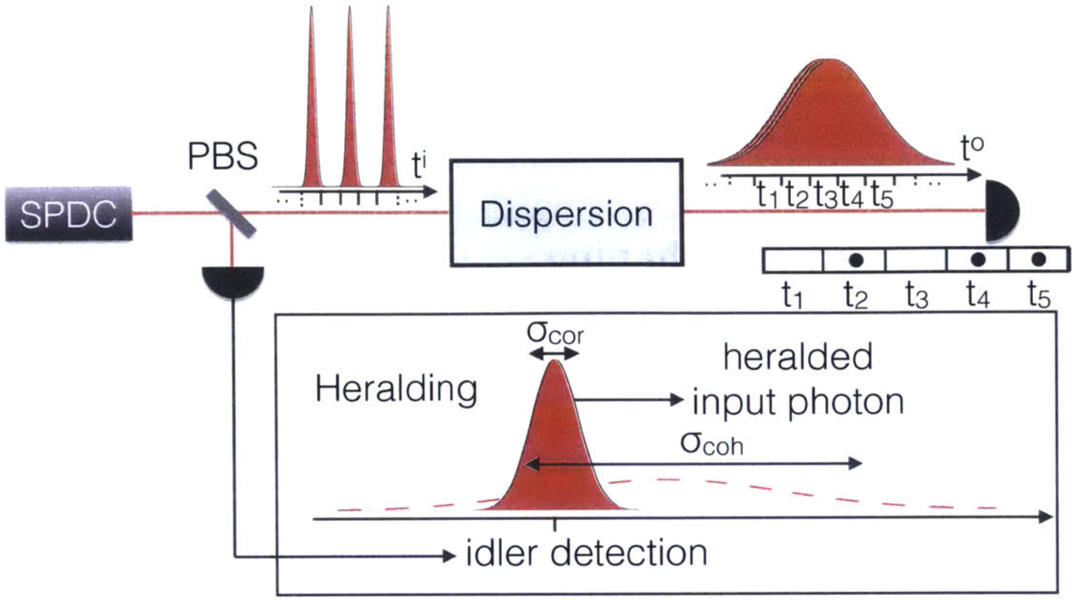

Figure 2-2: Schematic of TMBS with time-independent dispersion

approaches used in optical functional design [16] and femtosecond pulse-shaping [28]. There are even commercial products for implementing arbitrary dispersion used for pulse-shaping in telecommunication[2].

In conventional linear optics experiments with spatial modes, the modes are generally defined by waveguides and are discrete. In contrast, in our scheme, photons have a continuous wavefunction in time. However, in order to draw a parallel with spatial mode transformations, we show that the result of our experiment can be made arbitrarily close to a discrete experiment (section 2.3.2).

If dispersion parameters are chosen such that $U(t, t_j)$ does not change appreciably when t varies in a window of width t_s, the modes can be discretized so that the transformation is well approximated by $\hat{a}^\dagger_{Aj} \rightarrow \sum_i U_{ij}\hat{a}^\dagger_{ti}$ where $\hat{a}^\dagger_{ti}$ represents the creation operator at the discretized time step near t_i, i.e. $\hat{a}^\dagger_{tj} = \int_{t_j}^{t_j+t_s} \mathrm{d}t\hat{a}^\dagger(t)/\sqrt{t_s}$. We can then write the transformation as $\hat{a}^\dagger_{Aj} \rightarrow \sum_i U_{ij}\hat{a}^\dagger_{ti} = \int_{-\infty}^{\infty} \mathrm{d}tU(t, t_j)\hat{a}^\dagger(t_i)$. If we assume that $U(t, t_j)$ is approximately constant for a small time step t_s, we can approximate $U_{ij}\hat{a}^\dagger_{ti} \approx U(t_i, t_j) \int_{t_j}^{t_j+t_s} \mathrm{d}t\hat{a}^\dagger(t)$. Hence, we have $U_{ij} = \sqrt{t_s}U(t_i, t_j)$.

[2]See `https://www.finisar.com/optical-instrumentation`

$\hat{a}^\dagger_{Aj} \rightarrow \sum_i U_{ij}\hat{a}^\dagger_{ti}$ where

$$U_{ij} = \left[\sqrt{t_s}A(t-t_j) * \mathcal{F}^{-1}\{\mathrm{e}^{-i\phi(\omega)}\}\right]_{t=t_i} \tag{2.2}$$

2.2 Adding pulse-shaping

It is possible to sample from a larger class of unitaries by shaping the temporal form of the input photons. Methods for shaping single photons with arbitrary amplitude and phase in time have been proposed [10] and an experimental demonstration of shaping the spatial waveform of single photons had been reported [13]. With pulse-shaping, the input waveform $A(t-t_j)$ is replaced by a more general set of functions $A_j(t)$ so that the accessible set of unitaries becomes

$$U_{ij} = \left[\sqrt{t_s}A_j(t) * \mathcal{F}^{-1}\{\mathrm{e}^{-i\phi(\omega)}\}\right]_{t=t_i} \tag{2.3}$$

The setup for implementing such an experiment is shown in Fig. 2-3

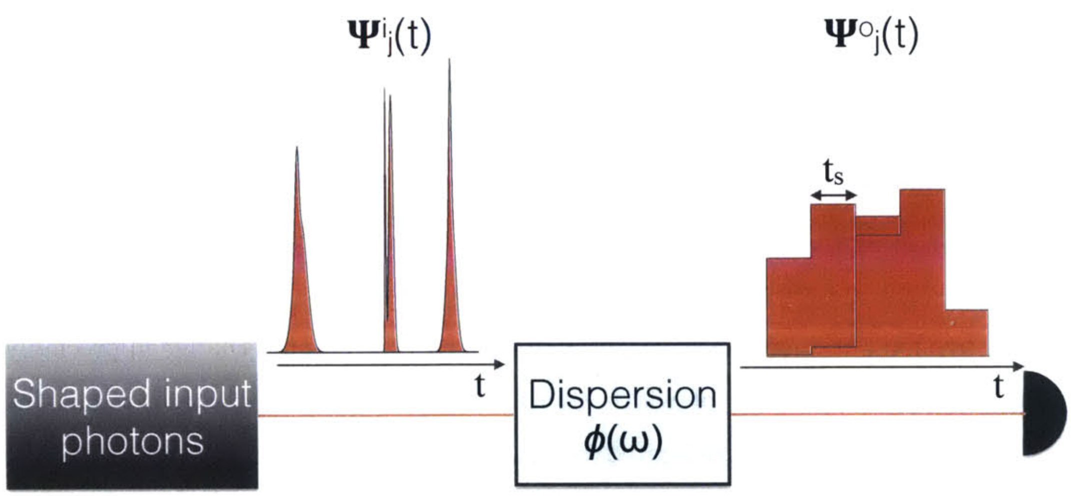

Figure 2-3: Schematic of TMBS with pulse-shaping and time-independent dispersion

If it were possible to choose any set of functions $A_j(t)$, then the unitary could be chosen column by column using A_j and simply detecting the photons without any dispersion would be equivalent to Boson Sampling. However, it is experimentally challenging to prepare multiple overlapping photons with a specific waveform. Hence, for realistic implementation, the photon wave-packets should be separated in time. This limits the possible unitaries represented by Eq. 2.3.

2.3 Error bounds

We find bounds on the error in the sampling distribution due to detector jitter and discretization. U is the ideal unitary that we wish to implement, $\tilde{U}$ is the unitary with errors and $\mathcal{D}_U$ and $\mathcal{D}_{\tilde{U}}$ are the corresponding probability distributions over outcomes.

It has been shown in [2] that if there are n photons in the system,

$$||\mathcal{D}_{\tilde{U}} - \mathcal{D}_U|| \leq n||\tilde{U} - U||_{op} \tag{2.4}$$

If the relative error of each matrix element has a upper bound of R i.e. $|\tilde{U}_{ij} - U_{ij}| \leq R|U_{ij}|$ for all i, j,

$$\begin{aligned} ||\mathcal{D}_{\tilde{U}} - \mathcal{D}_U|| &\leq n||RU||_{op} \\ ||\mathcal{D}_{\tilde{U}} - \mathcal{D}_U|| &\leq nR \end{aligned} \tag{2.5}$$

where we have used the fact that U is unitary.

Hence, in order to have $||\mathcal{D}_{\tilde{U}} - \mathcal{D}_U|| = o(1)$, $R = o(\frac{1}{n})$.

2.3.1 Error due to detector jitter

We have shown that the in the case of input photons with a fixed shape $A(t - t_j)$ sent through a dispersion $\beta(\omega)L$, the resulting unitary sampled from is

$$\begin{aligned} U_{ij} &= \sqrt{t_s}U(t_i, t_j) \qquad (2.6) \\ &= \sqrt{t_s}A(t_i - t_j) * B(t_i) \end{aligned}$$

$$= \sqrt{t_s} A(t_i) * B(t_i - t_j)$$

where $B(t) = \mathcal{F}^{-1}\{\mathrm{e}^{-i\phi(\omega)}\}$.

Due to detector jitter, there is an uncertainty in t_j. If the maximum timing error due to detector jitter is t_e,

$$\begin{aligned} R &= \left| \frac{A(t_i) * B(t_i - t_j + t_e) - A(t_i) * B(t_i - t_j)}{A(t_i) * B(t_i - t_j)} \right| \\ &\approx \left| \frac{A(t_i) * \dot{B}(t_i - t_j)}{A(t_i) * B(t_i - t_j)} t_e \right| \end{aligned} \tag{2.7}$$

If we write the dispersion in the form

$$\phi(\omega) = \phi'(\phi_s \omega) \tag{2.8}$$

where ϕ_s is used to scale the dispersion. It can be seen that $\dot{B}(t_i - t_j)/B(t_i - t_j) = o(1/\phi_s)$. Hence, if $A(t)$ is chosen independent of n, $R = o(t_e/\phi_s)$. Therefore, in order to limit the error due to detector jitter, $t_e/\phi_s = o(1/n)$.

2.3.2 Error due to discretization

In the case of time independent dispersion, the wavefunction generated after dispersion is treated discretely in order to draw a parallel with the original formalism for Boson Sampling [1].

$U_{ij} = \sqrt{t_s} U(t_i, t_j)$. The relative error can be written as

$$\begin{aligned} R &< \max_t \left\{ \left| \frac{U(t, t_j) - U(t_i, t_j)}{U(t_i, t_j)} \right| \right\} \\ &\approx \max_t \left\{ \left| \frac{\dot{U}(t, t_j)}{U(t_i, t_j)} \right| \right\} t_s \end{aligned} \tag{2.9}$$

where $t \in [t_i - t_s/2, t_i + t_s/2]$. If we define $\zeta = \max_t\{\dot{U}(t, t_j)/U(t_i, t_j)\}$, we can see that $R = o(\zeta t_s)$. As discussed previously, we have $\zeta = o(1/\phi_s)$. Hence, in order to limit the error, $t_s/\phi_s = o(1/n)$.

2.4 HOM-like interference

A central feature in spatial boson collision experiments is the bunching of bosons in the output modes i.e., Hong-Ou-Mandel interference [7]. An analogous feature appears in the temporal modes. Consider a heralded input state of two photons, $a^\dagger(t-t_1^i)a^\dagger(t-t_2^i)\left|0\right\rangle$. If this state passes through group velocity dispersion in a fiber of length L: $\beta(\omega) = \beta_0 + \beta_1(\omega-\omega_0) + 1/2\beta_2(\omega-\omega_0)^2$, $\phi_j = \beta_j L, j \in \{0,1,2\}$. For simplicity, we have assumed a Gaussian input shape of the form

$$A(t) = \left(\frac{1}{\sigma_{cor}\sqrt{2\pi}}\right)^{1/2} \exp\left[-\frac{t^2}{4\sigma_{cor}^2}\right]\exp[i\omega_0 t], \tag{2.10}$$

where we have assumed that the correlation time σ_{cor} of the photons from the heralded photon source is much shorter than the biphoton coherence time. SPDC photons are often approximated as Gaussians in time. However, the two photon interference effects would be visible with any shape in general. In the frequency domain, the wavefunction is given by

$$\begin{aligned}\mathcal{F}\{A(t)\} &= \int \mathrm{d}t A(t)\mathrm{e}^{-i\omega t} \qquad (2.11)\\ &= \left(\frac{1}{\sigma_{cor}\sqrt{2\pi}}\right)^{1/2}\int \mathrm{d}t\ \exp\left(-t^2/4\sigma_{cor}^2 - i(\omega-\omega_0)t\right)\\ &= 2^{3/4}\sqrt{\sigma_{cor}}\pi^{1/4}\exp[-(\omega-\omega_0)^2\sigma_{cor}^2]\end{aligned}$$

After passing through this dispersive element, the new wavefunction in the frequency domain is

$$B(\omega) = \mathrm{e}^{-i\phi(\omega)}\mathcal{F}\{A(t)\} \tag{2.12}$$

Going back to the time domain, we find that the dispersed wavefunction in time is given by

$$B(t) = (2\pi)^{-1/4}\sqrt{\frac{\sigma_{cor}}{\sigma_{cor}^2 + i\phi_2/2}}\mathrm{e}^{-i\phi_0}\mathrm{e}^{i\omega_0 t}\exp\left[-\frac{(t-\phi_1)^2}{4(\sigma_{cor}^4+\phi_2^2/4)}\left(\sigma_{cor}^2 - i\frac{\phi_2}{2}\right)\right] \quad (2.13)$$

Hence, after passing through the dispersion, assuming $\phi_2 \gg \sigma_{cor}^2$, the two photon wavefunction for photons input at t_1^i and t_2^i is given by

$$\begin{aligned} |\Psi\rangle \;=\; & \frac{\sigma}{\sqrt{2\pi}(\sigma_{cor}^2 + i\phi_2/2)}\int_{-\infty}^{\infty}\mathrm{d}t'\int_{-\infty}^{\infty}\mathrm{d}t''\hat{a}^\dagger(t')\hat{a}^\dagger(t'') \\ & \exp\Big[\frac{i\phi_2/2}{4(\sigma_{cor}^4+\phi_2^2/4)}\{(t'-t_1^i-\phi_1)^2 \\ & +(t''-t_2^i-\phi_1)^2\}\Big]\mathrm{e}^{i\omega_0(t'+t'')-i2\phi_0}\,|0\rangle \end{aligned} \quad (2.14)$$

Every combination of $a^\dagger(t_1)a^\dagger(t_2)$ is repeated twice under the integrals. The repetition can be removed by rewriting the expression as

$$\begin{aligned} |\Psi\rangle = & \int_{-\infty}^{\infty}\mathrm{d}t'\int_{t'}^{\infty}\mathrm{d}t''\hat{a}^\dagger(t')\hat{a}^\dagger(t'')\frac{\sigma\mathrm{e}^{i\omega_0(t'+t'')-i2\phi_0}}{\sqrt{2\pi}(\sigma_{cor}^2+i\phi_2/2)} \\ & \Big\{1+\exp\Big[\frac{i\phi_2/2}{4\sigma_{cor}^4+\phi_2^2}\{2(t_1^i-t_2^i)(t'-t'')\}\Big]\Big\} \\ & \exp\Big[\frac{i\phi_2/2}{4\sigma_{cor}^4+\phi_2^2}\{(t'-t_1^i-\phi_1)^2 \\ & +(t''-t_2^i-\phi_1)^2\}\Big]\,|0\rangle \end{aligned} \quad (2.15)$$

From this expression, we can see that, assuming $\phi_2 \gg \sigma_{cor}^2$, the probability of detecting photons at t_1^o and t_2^o vanishes when

$$\Delta t^i \Delta t^o = (2m+1)\pi\phi_2 \quad (2.16)$$

In Fig. 2-4, the joint probability of observing a photon at t_1^o and t_2^o is plotted when two photons with $\sigma_{cor} = 200$ fs and a Gaussian temporal waveform centered at $t_1^i = -\phi_1$ and $t_2^i = -\phi_1 + 100$ ps are sent through a dispersive element with a GVD parameter of magnitude $|D| = 2\pi c\phi_2/\lambda^2 = 10000$ ps/nm. The assumption $\phi_2 \gg \sigma_{cor}^2$ has not been used. At the time-scale of a few nanoseconds, the output resembles a Gaussian pulse centered at $t^o = 0$, as would be expected from single photon input at $t^i = -\phi_1$. However, at the time scale of hundreds of picoseconds, two-photon interference effects can be seen in clear dips in the two-photon output probability, as predicted by Eq. 2.16. The plot has been discretized with $t_s = 10$ ps. We chose a low value of t_s here to clearly show the shape of our interference pattern; such timing resolution is not necessarily required to resolve the two-photon interference pattern. Experimentally, these dips would be easily resolved using detectors with a jitter of 100 ps as shown in Fig. 2-4 (c). The increase in the size of each bin also increases the probability of detecting photons in each time bin by a factor of 100.

2.5 Hardness of sampling from time-independent dispersion

Eq. 2.2 shows the class of unitary transformations from which we can sample using time-independent dispersion without pulse-shaping. The unitary is band-diagonal because of the time-invariant nature of the system. Classical algorithms exist for the computation of the permanent of banded matrices with a banded inverse which is polynomial in the size of the matrix but exponential in the number of bands [25]. The inverse of a unitary banded matrix is banded. However, because the number of bands is extremely large and the number of bands/dispersion is increased with the number of photons in order to limit the effect of jitter (section 2.3.1), a classical simulation using the algorithm from [25] would be inefficient.

If we add pulse-shaping to the experiment, the class of unitaries is expanded to Eq. 2.3. Although we don't have any proof of the hardness of sampling from such a unitary, the class of unitaries is large enough such that the sampling problem is still expected to be hard. Hence, our scheme provides a realistic path to perform multi-photon interference experiments with a much larger number of modes than possible with conventional spatial schemes using beam-splitter operations.

2.6 The pulse-shape as part of the unitary

One of the apparently surprising aspects of our scheme is that the shape of the input pulses $A(t)$ is incorporated into the unitaries in Eq. 2.2 and 2.3. This is because, unlike conventional unitary implementations, the input states start of in a different basis than the measurement e.g. the input photons could start off as gaussians but the measurement is in the time basis. If the input and output basis were both defined as eigenfunctions of the time basis, the unitary would no longer be dependent on the shape of the input photons. However, the input states would be unphysical and any physically realizable input state would start of in a superposition of the input basis.

There is an easy way to see that the results of an experiment will be the same as a spatial unitary implementing Eq. 2.2 or 2.3. Suppose that in a fictitious experiment, the photons start of as eigenfunctions of time. They then go through a unitary U_1 which puts them in a superposition of the input states of the form $A(t)$ (physically, this is the state of the photons input to the dispersion). The photons then go through another unitary U_2 which is the dispersion. The unitary implemented by this system is $U_1 U_2$.

It is easy to see that the output of this system will be the same as our scheme. The only difference is that the photons were never in the input state of the initial system and instead directly have photons with U_1 applied. However, the detector, which measures in the time basis sees the same output state as if the operator $U_1 U_2$ was applied to input photons that were eigenfunctions of time.

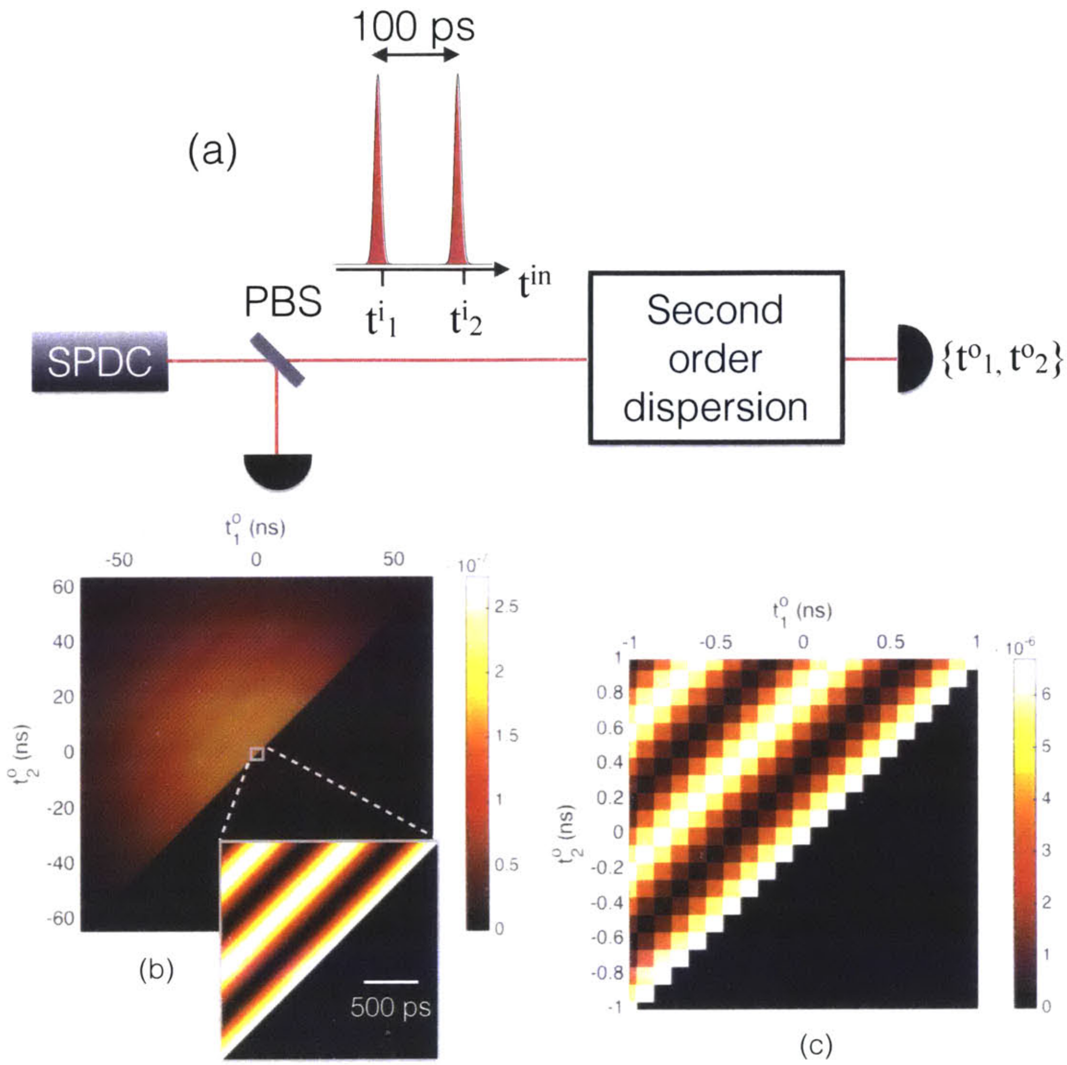

Figure 2-4: (a) Setup for seeing 'HOM-like' interference. (b) The joint probability of detecting the first photon at t_1^o and the second photon at t_2^o when two input photons near t = 0 and separated by 100 ps are sent through a second order dispersive element. σ_{cor} = 200 fs and the dispersive element has a GVD parameter of magnitude $|D| = 2\pi c\phi_2/\lambda^2$ = 10000 ps/nm. t_s = 10 ps (c) The joint probability when the bin size is increased to 100 ps.

Chapter 3

Unitary Transformations with Time-Dependent Dispersion

In section, we present a method to implement arbitrary unitaries with time dependent dispersion.

Using the time projection operator $\hat{t}_p = \int \mathrm{d}t'\, t \left|t'\right\rangle \left\langle t'\right|$ and the frequency projection operator $\hat{\omega}_p = \int \mathrm{d}\omega\, \omega \left|\omega\right\rangle \left\langle\omega\right|$ which satisfy $[\hat{t}_p, \hat{\omega}_p] = i/2$ (similar to $\hat{x}$ and $\hat{p}$ in [14, 22]), an arbitrary continuous variable unitary on temporal modes can be written as $\mathrm{e}^{-if(\hat{t}_p,\hat{\omega}_p)}$ where $f(\hat{t}_p, \hat{\omega}_p) = \int \mathrm{d}t H(\hat{t}_p, \hat{\omega}_p, t)$. $H(\hat{t}_p, \hat{\omega}_p, t)$ is the Hamiltonian of the system. It should be noted that $\hat{t}_p$ is the time projection operator corresponding to the photon's temporal waveform and not the time over which the system evolves. Hence, the variable of integration t above is distinct from the operator $\hat{t}_p$.

Given the ability to realize arbitrary continuous variable single particle unitaries over $\hat{t}_p$, any discrete unitary transformation can be implemented by making the transformation constant over the output time bins. Such an experiment with multiple photons would be equivalent to Boson Sampling and if the discrete unitary is chosen with Haar measure, the results are believed to be classically intractable [1].

3.1 Direct implementation

Such a Hamiltonian can be physically realized by making the elements of the dispersion time-dependent. In the previous section we discussed methods for implementing arbitrary dispersion which implements a Hamiltonian of the form $\phi(\hat{\omega}_p)$. Since photons reaching the dispersive element at different times see a different $\phi(\hat{\omega}_p)$, the Hamiltonian can be written as $H(\hat{t}_p, \hat{\omega}_p, t)$ where H can be any real function. Hence, we get the desired unitary $e^{-if(\hat{t}_p, \hat{\omega}_p)}$ with time-dependent elements that allow for changes in the frequency spectrum of the pulses which was not possible with time-independent dispersion.

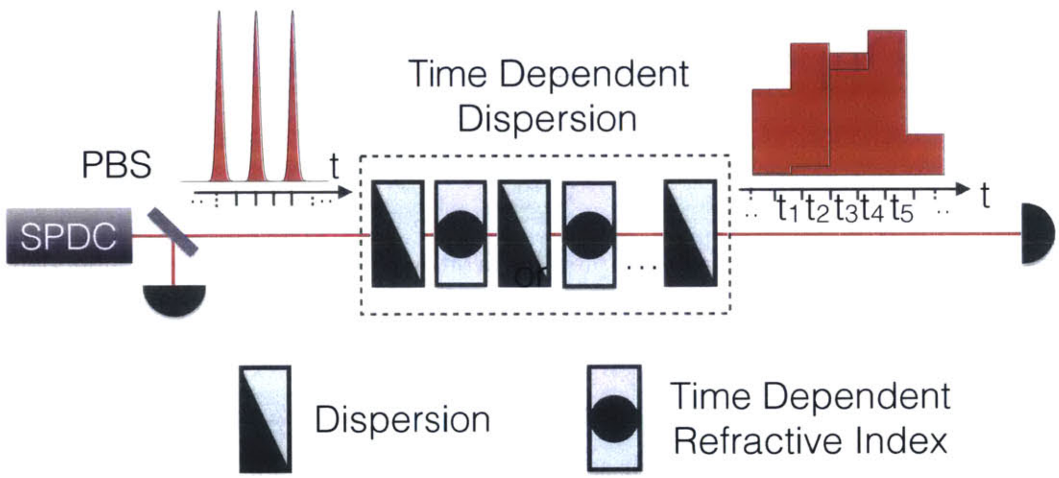

Figure 3-1: Schematic of TMBS with time-dependent dispersion

3.2 Cascading dispersion and phase modulation

Another option for realizing such a unitary would be to cascade elements that implement time-independent dispersion and a time-dependent refractive index. The Hamiltonians corresponding to dispersion and time dependent refractive index are $\phi(\hat{\omega}_p)$ and $g(\hat{t}_p)$ respectively.

Following previous work on realizing arbitrary Hamiltonians for continuous variable systems [14, 22], one can construct Hamiltonians of the form $[g_1(\hat{t}_p), \phi_1(\hat{\omega}_p)] +$

$[g_2(\hat{t}_p), [g_3(\hat{t}_p), \phi_3(\hat{\omega}_p)]] + ...$ using the properties

$$\begin{aligned} e^{-i\hat{A}\delta t}e^{-i\hat{B}\delta t}e^{i\hat{A}\delta t}e^{i\hat{B}\delta t} &= e^{[\hat{A},\hat{B}]\delta t^2} + O(\delta t^3) && (3.1) \\ e^{i\hat{A}\delta t/2}e^{i\hat{B}\delta t/2}e^{i\hat{B}\delta t/2}e^{i\hat{A}\delta t/2} &= e^{i(\hat{A}+\hat{B})\delta t} + O(\delta t^3). && (3.2) \end{aligned}$$

The following Hamiltonians can be cascaded to generate a Hamiltonian that can be any polynomial in $\hat{t}_p$ and $\hat{\omega}_p$ [14]: $\hat{t}_p$, $\hat{\omega}_p$, $\hat{t}_p^2 + \hat{\omega}_p^2$ and a Hamiltonian of the form $\hat{\omega}_p^n$ where $n \geq 3$. The Hamiltonians $\hat{\omega}_p$ and $\hat{t}_p$ are first order dispersion (inverse group velocity) and a linear varying refractive index. A high order Hamiltonian $\hat{\omega}_p^n$ can be realized with higher order dispersion. $\hat{t}_p^2 + \hat{\omega}_p^2$ can be realized with a combination of second order dispersion and a quadratically varying refractive index using Eq. 3.2. Although such a decomposition allows us to build arbitrary Hamiltonians with a number of elements which increase as a small polynomial in the number of photons [3], more efficient decompositions are often possible with fewer elements [22].

Furthermore, as opposed to the conventional construction of $f(\hat{x}, \hat{p})$ [14] which uses low-order polynomials in $\hat{x}$ and $\hat{p}$ (high order polynomials require a non-linear medium), a unitary of the form $g(\hat{t}_p)$ or $\phi(\hat{\omega}_p)$ can be of any arbitrary functional form without requiring an explicit Kerr-type nonlinearity.

Chapter 4

Conclusion and Outlook

The schemes presented in this paper present an alternative to conventional spatial implementations of linear optics unitary transformations by moving to a temporal basis. Unlike implementations of spatial mode unitaries with beam splitters and phase shifters which mix two modes at a time, each element in our scheme mixes all the modes together. As a result, a unitary transformation on temporal modes using dispersion can allow the experimental implementation of a much larger unitary transformation than would be practically possible with an approach using beam splitters and phase shifters.

However, there are certain unitaries which are harder to implement with our scheme than with a system of beamsplitters e.g. a unitary which mixes only two spatial modes and leaves the others untouched can be constructed with a single beam splitter but in our scheme, such a unitary would require time-dependent dispersion.

With time-independent dispersion, a limited set of unitary transformations can be implemented. However, to our knowledge, there is no known efficient sampling algorithm for such unitaries. With time-dependent dispersion, it is possible to sample from arbitrary unitaries.

Since our scheme could provide a path to a post-classical Boson Sampling machine, a large part of this thesis is dedicated to the application of unitaries based on dispersion to Boson Sampling. However, future work could look at other applications requiring unitary transformations with a large number of modes where this scheme could lead to experimental simplification. There has been recent work on quantum chemistry simulations using Boson Sampling [8] and it would be interesting to study whether the required unitary for such simulations can be implemented easily with dispersion.

In this thesis, we have shown that the output of our system, which is continuous, can approach the output of a discrete system i.e. unitary transformations on spatial waveguide modes. However, an interesting future direction for this scheme would be to take advantage of the continuous nature of our system which could be valuable for quantum simulations of systems with a continuous degree of freedom.

Appendix A

The Required Number of Detectors

In this section we show that the number of detectors in TMBS can be much smaller than SMBS

In Fig. 2-4, if photon detection is binned in steps of 100 ps, there are 1934 modes. The number of modes is defined as the number of time bins within which the absolute value of the dispersed wavefunction is greater than 90% of the peak value.

We assume that our detectors have a dead time of 1 ns and look at the failure rate of our boson sampling scheme with 2000 input and output modes and 30 photons when each photon output is passively and equally split between 30 detectors. We assume that each time bin on the heralding as well on the output detector bank is equally likely to receive a photon. We post-select on the cases where there are a total of 30 photons incident on each detector bank (as in conventional scattershot boson sampling).

The scheme is considered a failure if two photons are incident on any detector within the dead time. Based on a Monte-Carlo simulation of the system, we find that the probability of failure is less than 10%

The assumption of photons being equally likely to arrive at any bin is accurate for the heralding detector bank. For the detector bank which detects photons after going through the unitary, the assumption may lead to an underestimated failure rate. However, we can post-select on the number of photons detected after the unitary being equal to the number of heralded photons and hence a higher failure rate is tolerable. An accurate simulation of the failure rate with a 30 photon, 2000 mode system is expected to be close to the limit of current computing capabilities.

Hence, TMBS can allow for a 30 photon 2000 mode experiment with 60 detectors, whereas an equivalent SMBS experiment would require 4000 detectors. It is interesting to observe that for the same number of photons and dead time bins, the number of detectors required for TMBS goes down with an increase in the number of modes since there is a smaller chance of detecting multiple photons within a dead time. In SMBS, the number of detectors is equal to twice the number of modes.

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