**Coupled Modeling of Multiphase Flow and Fault Poromechanics During Geologic CO\(_2\) Storage**

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GHGT-12

Coupled modeling of multiphase flow and fault poromechanics during geologic CO₂ storage

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Abstract

Coupling between fluid flow and mechanical deformation in porous media plays a critical role in geologic storage of CO₂. One of the key issues in simulation of CO₂ sequestration is the ability to describe the mechanical and hydraulic behavior of faults, and the influence of the stress tensor and change in pressure on fault slip. Here, we present a new computational approach to model coupled multiphase flow and geomechanics of faulted reservoirs. We represent faults as surfaces embedded in a three-dimensional medium by using zero-thickness interface elements to accurately model fault slip under dynamically evolving fluid pressure and fault strength. We incorporate the effect of fluid pressures from multiphase flow in the mechanical stability of faults by defining a fault pressure. We employ a rigorous formulation of nonlinear multiphase geomechanics based on the increment in mass of fluid phases, instead of the change in porosity. Our nonlinear formulation is required to properly model systems with high compressibility or strong capillarity, as can be the case for geologic CO₂ sequestration. To account for the effect of surface stresses along fluid-fluid interfaces, we use the equivalent pore pressure in the definition of multiphase effective stress. We develop a numerical simulation tool by coupling a multiphase flow simulator with a mechanics simulator, using the unconditionally stable fixed-stress scheme for a computationally efficient sequential solution of two-way coupling between flow and geomechanics. We validate our modeling approach using several synthetic test cases that illustrate the onset and evolution of earthquakes from fluid injection.

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1. Introduction

Injection of CO₂ requires displacement and compression of the ambient groundwater, and an overpressurization of the target aquifer, which could fracture the caprock [16], trigger seismicity and cause shear slip on pre-existing faults [125,126,127,30,24,25,103,104], and potentially compromise the caprock by activating faults [162]. Overpressurization can limit the CO₂ storage capacity of the aquifer [145]. Despite the growing environmental, industrial, and economic importance of coupled flow and geomechanics, many aspects remain poorly understood. One of the fundamental unresolved issues is the ability to describe the mechanical and hydraulic behavior of faults, and the influence of the full stress tensor and change in pressure on fault slip. Injection and production of fluids from a geologic reservoir induce changes in the state of stress, both within and outside the reservoir, and these can affect the stability of preexisting faults. The effects of injection and production depend on the initial state of stress, the elastic moduli of the geologic structures, and the fault frictional properties. The effects are not always intuitively obvious and should be quantified using geomechanical models. This requires the development of a new generation of geomechanical models that include coupling between fluid flow and fault motion.

Currently, geomechanical models typically treat faults as failure zones that are discretized as three-dimensional elements where the rheology is allowed to be different (e.g., plastic with weakening failure) than in the rest of the domain (e.g., elastoplastic with hardening law) [126,24,25]. This approach has several limitations, including the inability to model actual slip along a surface of discontinuity, and the dependence of the simulation results on the level of grid refinement. Other models represent faults as surfaces using interface elements [47], but so far these models are uncoupled to flow, and they model fault slip using a penalty method [58]. Such methods require a priori selection of the penalty parameters for the fault, and therefore cannot represent dynamically evolving fault strength, such as slip-weakening or rate- and state-friction models [40].

The interactions between flow and geomechanics have been modeled computationally using various coupling schemes [39,74,75,97,101,136,137,148,149,150,82,83,84,85]. In the fully implicit method, one solves the coupled discrete nonlinear system of equations simultaneously, typically using the Newton-Raphson scheme [144,114,92,93,47]. The fully implicit method guarantees unconditional stability if the mathematical problem is well posed, but the simulation of flow and geomechanics for realistic fields becomes computationally very expensive [136,148]. Sequential approaches to modeling coupled flow and geomechanics are highly desirable because they offer the flexibility of using separate simulators for each subproblem [45,129,101,124]. The design and analysis of sequential methods with appropriate stability properties for poromechanics and thermomechanics has a long history [159,7,8,6,136,97,74]. Recently, a new sequential method for coupled flow and geomechanics, termed the “fixed-stress split,” has been proposed and analyzed [83,84,85]. Stability and convergence analyses have shown that the fixed-stress split inherits the dissipation properties of the continuum problem and is therefore unconditionally stable, both in the linear (poroelastic) and nonlinear (poroelastoplastic) regime. The analysis has shown that the fixed-stress split enjoys excellent convergence properties, even in the quasi-incompressible limit. It has also been shown recently that the stability and convergence properties of the fixed-stress split for single-phase flow carry over to multiphase systems if a proper definition of pore pressure, the “equivalent pore pressure” [32], is used [85].

In this article, we present a new computational model for coupled flow and geomechanics of faulted reservoirs. We couple a flow simulator with a mechanics simulator using the fixed-stress scheme [83]. We employ a rigorous formulation of nonlinear multiphase geomechanics [31] based on the increment in mass of fluid phases, instead of the more common, but less accurate, scheme based on the change in porosity [136,101,148,149,150,124]. Our nonlinear formulation is required to properly model systems with high compressibility or strong capillarity [31], as can be the case for geologic CO₂ sequestration [125,126], groundwater extraction from unconfined aquifers [52,59], and shale gas production [43]. To account for the effect of surface stresses along fluid-fluid interfaces, we use the equivalent pore pressure in the definition of multiphase effective stress [32,85]. We model faults as surfaces of discontinuity using interface elements [1]. This allows us to model stick-slip behavior on the fault surface for dynamically evolving fault strength.
2. Governing Equations

2.1. Balance Laws

We use a classical continuum representation in which the fluids and the solid skeleton are viewed as overlapping continua [11,33]. The governing equations for coupled flow and geomechanics are obtained from conservation of mass and balance of linear momentum. We assume that the deformations are small, that the geomaterial is isotropic, and that the conditions are isothermal. Let \( \Omega \) be our domain of interest and \( \partial \Omega \) be its closed boundary. Under the quasistatic assumption for earth displacements, the governing equation for linear momentum balance of the solid/fluid system can be expressed as

\[
\nabla \cdot \sigma + \rho_b \mathbf{g} = 0,
\]

where \( \sigma \) is the Cauchy total stress tensor, \( \mathbf{g} \) is the gravity vector, and \( \rho_b = \phi \sum_{\beta} n_{\text{phase}} \rho_{\beta} S_{\beta} + (1 - \phi) \rho_s \) is the bulk density, \( \rho_{\beta} \) and \( S_{\beta} \) are the density and saturation of fluid phase \( \beta \), and \( \rho_s \) is the density of the solid phase, \( \phi \) is the true porosity, and \( n_{\text{phase}} \) is the number of fluid phases. The true porosity is defined as the ratio of the pore volume to the bulk volume in the current (deformed) configuration. Assuming that the fluids are immiscible, the mass-conservation equation for each phase \( \alpha \) is

\[
\frac{dm_{\alpha}}{dt} + \nabla \cdot w_{\alpha} = \rho_{\alpha} f_{\alpha},
\]

where the accumulation term \( dm_{\alpha}/dt \) describes the time variation of fluid mass relative to the motion of the solid skeleton, \( w_{\alpha} \) is the mass-flux of fluid phase \( \alpha \) relative to the solid skeleton, and \( f_{\alpha} \) is the volumetric source term for phase \( \alpha \). The two balance equations (1) and (2) are coupled by virtue of poromechanics. On one hand, changes in the pore fluid pressure lead to changes in effective stress, and induce deformation of the porous material—such as ground subsidence caused by groundwater withdrawal. On the other hand, deformation of the porous medium affects fluid mass content and fluid pressure. The simplest model of this two-way coupling is Biot’s macroscopic theory of poroelasticity [15,55,31]. In the remainder of this section we provide the mathematical description of poroelasticity for multiphase fluid systems.

2.2. Multiphase poromechanics

In the multiphase or partially saturated fluid system, it is not possible to linearize the equations of poroelasticity around a reference state because [31]:

1. Gases are very compressible,
2. Capillary pressure effects are intrinsically nonlinear, and
3. Phase saturations vary between 0 and 1 and, therefore, a typical problem samples the entire range of nonlinearity.

Therefore, following [31], we use the incremental formulation of poromechanics for multiphase systems, which does not assume physical linearization of total stress from the initial state to the current (deformed) state. We make a modeling assumption that allows us to express the deformation of a multiphase porous material in terms of the increment in applied total stresses and internal fluid pressures. We adopt an effective stress formulation in the multiphase poromechanics [17,18] because constitutive modeling of porous materials is usually done in terms of the effective stress. Under this formulation, we split the total stress on the porous material into two parts: one that is responsible for deformation of the solid skeleton (the effective stress), and another component that is responsible for changes in the fluid pressures,

\[
\delta \sigma = C_{dv} : \delta \varepsilon - \sum_{\beta} b_{\beta} p_{\beta} \mathbf{1},
\]
where \( b_\beta \) are the Biot coefficients for individual phases such that \( \sum_\beta b_\beta = b \), where \( b \) is the Biot coefficient of the saturated porous material. It is common to further assume that \( b_\beta \) are proportional to the respective saturations \( S_\beta \) [91,34,90].

The effective stress concept allows us to treat a multiphase porous medium as a mechanically equivalent single-phase continuum [81,108]. The appropriate form of the effective stress equation in a multiphase system is still an active area of research [61,32,108,153,107,85]. Here we use the concept of equivalent pressure [32] in the effective stress equation (Eq. (3)),

\[
P_E = \sum_\beta S_\beta p_\beta - U,
\]

where \( U = \sum_\beta \int p_\beta dS_\beta \) is the interfacial energy computed from the capillary pressure relations [85]. The equivalent pressure accounts for the interface energy in the free energy of the system, and leads to a thermodynamically consistent and mathematically well-posed description of the multiphase fluid response to the solid deformation [85].

For a system with two phases, the wetting phase \( w \) and the non-wetting phase \( o \), the capillary pressure is

\[
P_w(S_w) \equiv P_{wo}(S_w) = p_o - p_w,
\]

and the interfacial energy is \( U = \int_{S_w}^1 P_{wo} dS \). Assuming \( b_\beta = b S_\beta \) [91,34,90], and using Eq. (4) in Eq. (3), we obtain the stress-strain relation-ship for multiphase linear poroelasticity:

\[
\delta \sigma = \delta \sigma' - b \delta p_E \mathbf{1}, \quad \delta \sigma' = C_{a \nu} : \delta \epsilon.
\]

Once we have a definition of the effective stress in multiphase systems, we now express the change in the fluid mass in terms of the mechanical deformation and the change in the fluid pressures. In the deformed configuration, the mass of phase \( \alpha \) per unit volume of porous medium is

\[
m_\alpha = \rho_\alpha S_\alpha \phi (1 + \varepsilon_\nu),
\]

Note that, by definition, the sum of all fluid phase saturations adds up to 1. For multiphase systems [31,32], we have

\[
\frac{dm}{d\rho}_\alpha = b_\alpha d\varepsilon_\nu + \sum N_{a\beta} dp_\beta,
\]

where \( N = M^{-1} \) is the inverse Biot modulus. In a multiphase system, the Biot modulus is a symmetric positive definite tensor \( M = [M_{a\beta}] \), and the Biot coefficient is a vector. To determine the coupling coefficients \( N_{a\beta} \) as a function of the primary variables (pressure, saturations, and displacements) and rock and fluid properties we develop an alternate expression for the differential increment in fluid mass. Using Eq. (7),

\[
dm = d\left(\rho_\alpha S_\alpha \phi (1 + \varepsilon_\nu)\right),
\]

which can be expanded as

\[
\frac{dm}{d\rho}_\alpha = \phi \frac{\partial S_\alpha}{\partial P_{a\beta}} dP_{\alpha\beta} + \phi S_\alpha c_\alpha dp_\alpha + \phi S_\alpha d\varepsilon_\nu + S_\alpha d\phi,
\]

where \( c_\alpha \) is the compressibility of the fluid phase \( \alpha \), and \( \partial S_\alpha / \partial P_{a\beta} \) is the inverse capillary pressure derivative. Above, repeated indices do not imply summation and we have assumed infinitesimal deformations. We can express the increment in porosity \( d\phi \) as a function of the volumetric effective stress \( d\sigma' \nu \) to obtain a closed-form expression
of Eq. (10). Let \( V_s = V_b - V_p \) be the volume of the solid matrix, and \( d\varepsilon_{sv} = d\varepsilon_v/V_s = d\sigma_{sv}/K_s \) be the volumetric dilation of the solid matrix, where \( \sigma_{sv} \) is the volumetric matrix stress. From an expansion of \( d\phi \) we can write the incremental form of strain partition as

\[
(1 - \phi)d\varepsilon_v = (1 - \phi)d\varepsilon_{sv} + d\phi.
\]

Similarly, the volumetric Cauchy total stress can be partitioned into the volumetric matrix stress and the fluid pressure as

\[
d\sigma_v = (1 - \phi)d\sigma_{sv} - \phi dp_f.
\]

Substituting \( d\sigma_{sv} \) from Eq. (12) into Eq. (11), we obtain

\[
d\phi = \frac{b - \phi}{K_{dr}} (d\sigma'_v + (1 - b)dp_f).
\]

Eq. (13) implies that an increment in porosity is related to increments in volumetric effective stress and fluid pressures. Substituting \( d\varepsilon_v \) from Eq. (6) and \( d\phi \) from Eq. (13) into Eq. (10) allows us to express the increment in the phase mass as a function of the increments in the total volumetric stress and phase pressures. Equating this to Eq. (8) yields the desired expressions for the coupling coefficients \( N_{a\beta} \).

Finally, we obtain the multiphase flow equation for phase \( \alpha \) in a poroelastic medium by substituting the two constitutive relations, the effective stress equation, Eq. (6), and the fluid mass increment equation, Eq. (8), in the mass balance equation, Eq. (2):

\[
\frac{\partial}{\partial t} \left( \rho_\alpha \sum_\beta \left( N_{a\beta} + \frac{b_\alpha b_\beta}{K_{dr}} \right) p_\beta \right) + \frac{1}{K_{dr}} \frac{\partial}{\partial t} (\rho_\alpha b_\alpha \sigma_v) + \nabla \cdot \mathbf{w}_\alpha = \rho_\alpha f_\alpha, \quad \forall \alpha = 1, ..., n_{\text{phase}}
\]

The role of \( N \) and \( b \) as the coupling coefficients among different fluid phases and the solid phase is evident from the above equation. The bulk density, \( \rho_b \), in the mechanical equilibrium equation, Eq. (1), also acts as a coupling parameter because it is a function of the porosity and the phase saturations. Because we assume that the fluids are immiscible, the mass-flux of phase \( \alpha \) is \( \mathbf{w}_\alpha = \rho_\alpha \mathbf{v}_\alpha \), where we adopt the traditional multiphase-flow extension of Darcy’s law [106,11]:

\[
\mathbf{v}_\alpha = \frac{k_{\alpha}}{\mu_\alpha} (\nabla p_\alpha - \rho_\alpha \mathbf{g}),
\]

where \( \mu_\alpha \) and \( k_{\alpha} \) are the dynamic viscosity and the relative permeability of phase \( \alpha \) in presence of other fluid phases.

### 3. Poromechanics of Faults

There are two basic approaches to represent faults in a three-dimensional medium: either as a three-dimensional fault zone (e.g., [126]) or a two-dimensional fault surface (e.g., [77,102,46]). The advantage of representing faults as surfaces of discontinuity is that they can more faithfully describe the localized (discontinuous) displacement at the fault, and that one can incorporate models of dynamic frictional strength (like the rate- and state-friction model [40]) capable of reproducing runaway fault slip characteristic of earthquakes. Moreover, introducing discrete fault surfaces does not preclude modeling an adjacent fault zone with appropriate rheology.

A central feature of our work is that we treat faults as surfaces of discontinuity embedded in the continuum, across which displacement is allowed to be discontinuous to recognize the possibility of fault slip (Fig. 1). We use zero-thickness elements, also known as interface elements or cohesive elements in the finite element literature.
[60,14,27,57,88], to represent the fault surfaces. Mathematically, the fault surface is treated as an interior boundary between the two adjacent domains. The two sides of the fault surface, which need not be planar, are designated as the ‘+’ side and the ‘−’ side, and the fault normal vector, $\mathbf{n}$, points from the negative side to the positive side. Slip on the fault is the displacement of the positive side relative to the negative side,

$$ (u_+ - u_-) - d = 0 \quad \text{on} \quad \Gamma_f, $$

where $u_+$ and $u_-$ are the displacements on the two sides of the fault surface, denoted by $\Gamma_f$, and $d$ is the fault slip vector. Fault slip is governed by the effective traction on the fault, which is a function of the effective stress tensors on both sides of the fault, the fault normal direction, and the fault constitutive law. We impose the effective traction on the fault by introducing a Lagrange multiplier, $l$, which is a force per unit area required to satisfy the equilibrium equation for a given relative displacement, $d$, across the fault. The magnitude of the effective normal traction on the fault is

$$ \sigma'_n = l \cdot \mathbf{n}. $$

A positive value of $\sigma'_n$ indicates that a tensile effective stress is transmitted across the fault surface. The Kuhn-Tucker conditions of contact mechanics are obeyed such that no penetration occurs and the effective normal traction stays compressive at the contact surface. The shear traction vector is, by definition, tangent to the fault surface and its magnitude is

$$ \tau = |l - \sigma'_n \mathbf{n}|. $$

3.1. Fault Pressure in the Failure Criterion

Traditionally, in the Andersonian faulting theory [5], fault slip is modeled in a “dry environment,” that is, in the absence of fluids. While, in some cases, the presence of fluid has been recognized through the effective stress concept, the dynamics of flow was not included for reasons of conceptual and computational simplicity, as well as for the belief that fluid flow played a secondary role in the release of tectonic stresses [69,119]. The effect of pore pressure was accounted for by modifying the coefficient of fault friction $\mu_f$ [66,67], an approach later suggested to be “unwise” [13]. In the case of mature faults, the fault core permeability can be low due to comminution of grains while the damaged host rock permeability can be high due to fractures [138,140,29,22]. In addition, the permeability
can vary substantially across the fault during the seismic cycle [139,141]. As a result, pore pressures can be significantly different across the fault [142,120,29].

A difference in fluid pressure across the fault leads to a pressure jump $[p]_{f} = p_+ - p_-$, where $p_+$ and $p_-$ are the equivalent multiphase pressures (Eq. (4)) on the “positive” and the “negative” side of the fault. One of the key features of the 2-D representation of faults is the ability to reproduce a finite jump in the pressure, $[p]_{f}$, across the fault. This pressure jump leads to a discontinuity in the effective stress across the fault, such that the total stress is continuous

$$
\sigma'_{\perp} \cdot \mathbf{n} - b p_{\perp} \mathbf{n} = \sigma'_{\perp} \cdot \mathbf{n} - b p_{\perp} \mathbf{n},
$$

(19)
a requirement for momentum balance on the fault. This gives rise to the question of how to incorporate in the formulation the pressure jump across a fault. This is important because it determines the stability of the fault.

Fault stability can be assessed by evaluating the stability criterion on both sides of the fault separately. The side of the fault where the criterion is met first determines the fault stability. Equivalently, this can be achieved by defining a fault pressure that is a function of the pressures on the two sides, $p_+$ and $p_-$. Introducing the fault pressure allows us to uniquely define the effective normal traction on the fault, $\sigma'_n$, and determine the fault friction $\tau_f$. Since the stability criterion, $\tau \leq \tau_f$, is first violated with the larger pressure, we define the fault pressure, $p_f$, as

$$
p_f = \max(p_+, p_-).
$$

(20)

Our definition of fault pressure is a natural result of our fault representation, rather than a conservative assumption. Note that estimating the fault pressure as the arithmetic average of the pressures on the two sides, as proposed in the case of tensile fractures [133,134,135], may incorrectly delay the onset of shear failure. By univocally defining the pressure at the fault (Eq. (20)), we also univocally define the effective traction at the fault (the Lagrange multiplier $l$), something that is required to evaluate the fault stability criterion.

4. Implementation into a Simulation Software

We developed a coupled multiphase flow and geomechanical simulator by coupling the General Purpose Research Simulator (GPRS) [23,113] as the flow simulator, and PyLith [1,2] as the mechanics simulator. Below we describe the major steps in the development of this coupled simulator.

4.1. The flow simulator

GPRS is a general purpose, object-oriented, reservoir simulator for multiphase/multicomponent subsurface flows. It treats element connections through a general connection list, which allows for both structured and unstructured grids. GPRS is capable of handling complex production and injection scenarios in the field, such as wells perforated at multiple depths and flowing under variable rate and pressure controls. The original simulator [23,113] does not account for coupling with the mechanical deformation, and it models the mechanical behavior of the system through a user-provided rock compressibility [9]. We modified and extended the original code to implement the coupling with the mechanics simulator. In particular, we implemented the functionality to compute the modified accumulation term in the fluid phase mass balance equations. We also modified the setup of the linear system to implement the flow step of the fixed-stress sequential solution scheme [83].

4.2. The geomechanics simulator

PyLith is a finite element code for the simulation of static and dynamic large-scale deformation problems [1,2]. Much of its development has been motivated by the modeling of earthquake physics; however, its applicability extends to problems at any other scale, such as the reservoir scale or the laboratory scale. Some of the advantages of PyLith are (1) it is an open-source code and can be modified for specific purposes; (2) it is written using C++ and Python languages and is extendable; (3) it is suitable for parallel computing; (4) it allows localized deformation...
along discrete features, such as faults; and (5) it is well integrated with meshing codes, such as LaGriT for tetrahedral meshes [87] and CUBIT for both tetrahedral and hexahedral meshes [36]. PyLith uses an implicit formulation to solve quasi-static problems and an explicit formulation to solve dynamic rupture problems. Originally, PyLith is not coupled to any fluid flow model. We modified the code of PyLith version 1.8.0 and coupled it with the flow simulator, GPRS. In particular, we implemented a C++ class, iGPRS, to allow communication between the flow and the mechanics simulators. iGPRS provides the functionality required for exchanging information (pressures, saturations, and volumetric total stress) between the two simulators.

PyLith supports distributed memory parallelization (Message Passing Interface or MPI) whereas GPRS’s parallelization is based on the shared memory architecture (Multiprocessing or OpenMP). We integrated the two such that we can run the coupled simulator on a cluster with multiple compute nodes (distributed memory) where individual nodes have multiple cores or processors (shared memory).

4.3. Grid

We use a single grid for both GPRS and PyLith. The grid is generated using CUBIT [36] or LaGriT [87] mesh generation software. We define geologic surfaces, material regions, faults, and pinch-outs during the geometry creation stage. Then we mesh the domain with hexahedral elements using a fine mesh in the reservoir domain and an increasingly coarse mesh in the overburden, underburden, and sideburden regions. We export the grid in a finite element format such as the Exodus-II format [36] for PyLith. We process the grid file using a MATLAB script to generate the equivalent finite volume grid in the domain with element centroid coordinates, element bulk volumes, and face transmissibilities in the Corner Point Geometry format [130]. Any grid elements lying outside the flow region of interest (e.g., in overburden and underburden) can be deactivated for the solution of the flow problem. GPRS uses the finite volume grid for simulating flow in the region of interest. The two simulators exchange pressures, saturations, and volumetric stress information inside this region.

4.4. Implementation of Faults

To support relative motion across fault surfaces, PyLith modifies the grid topology to create zero-thickness fault elements and adds additional degrees of freedom to hold the Lagrange multipliers and fault slip vectors at the Lagrange nodes [1,2] (Fig. 1 and 2). PyLith solves the contact problem iteratively in two steps. In the first step, the elasticity problem is solved over the entire domain to update the displacements and the fault tractions (Lagrange multipliers) corresponding to the current estimate of the slip. The Lagrange multipliers are compared with the friction stress on the fault and are adjusted to be compatible with the fault constitutive model. In the second step, the fault slip is updated corresponding to the adjustment in the Lagrange multipliers while assuming that the deformation due to slip is localized to the elements adjacent to the fault, that is, that displacements at non-fault nodes do not change from their values at the current Newton iteration. If the fault slips over the entire domain, such that the assumption of deformation being limited to the adjacent elements is not met, the convergence of the iterative scheme is poor. Also, if the fault friction coefficient changes significantly with slip (e.g., in rate- and state-dependent models), it leads to large changes in $\tau_f$ at every iteration and convergence may degrade. To improve convergence, a line-search routine is used as part of the iterative scheme to find the optimum perturbation in the Lagrange multipliers that minimizes the combined mismatch between the fault friction and the fault shear traction at all the fault nodes [2]. We modified PyLith’s original line-search routine such that the inequality constraint, $\tau \leq \tau_f$, is always honored.
Fig. 2: Exploded view of our computational representation of a fault, illustrating different node types, locations of different variables, and the zero-thickness fault element. Fluid pressures $p_i$ and saturations $S_i$ are located at the element centers as they are discretized using the finite volume method. Displacements and Lagrange multipliers at the fault are discretized using the nodal-based finite element method. There are two types of nodes in the domain: the displacement nodes and the Lagrange nodes. The displacement nodes carry the displacements $U_{br}$ at the regular nodes, the displacements $U_{b+}$ on the positive side of the fault, and the displacements $U_{b-}$ on the negative side of the fault. The Lagrange nodes carry two types of fault variables: the Lagrange multipliers $L_b$ (related to fault tractions), and the fault slip $D_b$. The displacement nodes on the positive and negative sides, and the Lagrange nodes are collocated in the initial grid.

5. Representative Numerical Simulations

We illustrate the validity and applicability of our modeling approach through a number of representative simulations [76]. Some are verification problems and others are more realistic scenarios. We conduct these simulations using our coupled simulator. The 2-D numerical example simulations are conducted with a 2-D implementation in plane strain. Note that both GPRS and PyLith have been tested previously on respective benchmark problems, i.e., validation on flow-only problems for GPRS [23] and mechanics-only problems for PyLith [1]. Fault implementation in PyLith has been validated by comparison with analytical solutions from elastic dislocation theory [109].

5.1. Faulting due to CO$_2$ injection: Plane strain

This is an example of CO$_2$ injection in a deep confined aquifer for the purpose of geologic carbon sequestration [24]. The aquifer is hydraulically compartmentalized with a sealing fault that cuts across it. Storage capacity of the aquifer is limited by overpressurization and slip on the fault. As described in [24,76], we consider a two-dimensional plane-strain model with the fault under normal faulting conditions, that is, vertical principal stress due to gravity is the largest among the three principal stresses (Fig. 3). We choose a value of 0.7 for the ratio of horizontal to vertical initial total stress. After 20 days of injection, pressure in the aquifer increases approximately uniformly by 3.6 MPa (Fig. 4). Overpressure causes volumetric expansion of the aquifer (Fig. 5). This, in turn, results in an increase in the effective normal tractions throughout the aquifer, and an increase in the magnitude of shear tractions at the top and bottom boundaries of the aquifer.

Fig. 3: Model of the CO2 injection plane strain case (adapted from [24]). The lateral compression is 0.7 times the overburden, and both increase with the lithostatic gradient. CO$_2$ is injected in the confined aquifer at a depth of 1500 m. The aquifer is bounded on the top and bottom by a low-
permeability caprock, and the fault is impermeable to flow.

![Fig. 4: CO2 injection in the plane strain case. (left) Overpressure and (right) water saturation at t = 21 days.](image)

5.2. Faulting due to CO2 injection: 3-D

This example is similar to the previous one, but here we consider a 4 x 4 x 2 km three-dimensional domain with a 200 m thick anticlinal aquifer (Fig. 6), and we use the rate and state-dependent friction model [40] for the fault friction. The rate and state constitutive parameters are $A = 0.002$, $B = 0.08$, the critical slip $d_c = 1$ cm, the static coefficient of friction $\mu_o = 0.4$, and the cohesive strength of fault $\tau_c = 0$. These values strongly favor unstable sliding on the fault. CO2 is injected at a rate of 30 million standard cubic feet per day (17.64 kg/s) leading to overpressurization of the aquifer (Fig. 7). The anticline is off-centered in $y$, leading to asymmetry in the overpressure field. Rupture nucleates at the base of the aquifer after 202 days of injection and propagates on the fault along the bottom boundary of the aquifer (Fig. 8). After approximately two months, a second rupture sequence begins along the layer just above the base of the aquifer. At $t = 320$ days, the underburden rock layer below the aquifer reaches the failure criterion and slips, and the rupture subsequently propagates in both up-dip and down-dip directions on the fault with higher slip velocity in the down-dip direction. Downward slip is favored due to the imposed normal faulting condition.
Fig. 6: CO₂ injection in a 3-D anticlinal aquifer. (left) The geomechanical domain, shown with the traction boundary conditions on the top and on the right boundaries. The lateral compression is 0.7 times the overburden, and both increase with the lithostatic gradient. Zero-normal displacement is imposed on all other boundaries. A no-flow boundary condition is imposed on all the boundaries. The flow domain is composed of the four layers marked as aquifer, and the injector is located near the center of the anticline. (top right) Plan view and (bottom right) cross-section view of the aquifer. Depth contours are marked in the plan view. The cross-section view is exaggerated in the vertical direction.

Fig. 7: CO₂ injection in a 3-D anticlinal aquifer. (left) Overpressure and (right) water saturation in the aquifer layers at $t = 202$ days, when fault slip starts.
6. Conclusions

We have presented a new computational model to simulate the coupling between multiphase flow and poromechanics of faults and developed a two-way coupled simulator that interlaces a geomechanics simulator (PyLith) with a multiphase flow simulator (GPRS). Our approach enjoys the following features:

1. It is computationally efficient because it relies on a sequential solution of the two-way coupled problem.
2. It is unconditionally stable, due to the use of the fixed-stress sequential split between multiphase flow and deformation. The model accounts rigorously for multiphase flow effects through a fully nonlinear poromechanics formulation.
3. It represents faults as surfaces embedded in a three-dimensional domain, therefore allowing for a discontinuous displacement field across the fault (fault slip). Our approach elucidates the role of the pressure discontinuity across the fault on the stability of the fault through the definition of a "fault pressure."
4. It incorporates realistic fault constitutive behavior, such as the rate- and state-dependent friction model, capable of simulating runaway fault slip typical of earthquakes.

We assume quasi-static mechanical deformation by neglecting the inertial term in the solid momentum balance equation, and we use an implicit time-marching scheme for the coupled simulation. While this is an excellent approximation prior to fault rupture, during fault slip the inertial term is not negligible due to propagation of seismic waves. We are currently extending the capabilities of our simulation tool to implement a dynamic implicit-explicit time-marching scheme that can take small time steps required to resolve the propagation of rupture on the fault, while taking orders of magnitude larger time steps during aseismic periods.

Our framework allows us to investigate fault slip and induced seismicity in underground reservoirs due to the coupled processes of fluid flow and mechanical deformation, such as those encountered during groundwater withdrawal and geologic CO\textsubscript{2} storage. In this work, we have demonstrated the effectiveness and applicability of our approach through a few synthetic, but realistic, examples. We are currently applying our computational model for the study of ground deformations detected from geodetic measurements via GPS and InSAR [64,44,146] and for the post mortem analysis of natural or induced earthquakes [59,151,80,20].
References


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