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<td><a href="http://dx.doi.org/10.1017/CBO9781139519328.011">http://dx.doi.org/10.1017/CBO9781139519328.011</a></td>
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<tr>
<td>Publisher</td>
<td>Cambridge University Press</td>
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<td>Version</td>
<td>Original manuscript</td>
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<td>Accessed</td>
<td>Sun Dec 23 00:52:13 EST 2018</td>
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Presupposition projection from Quantificational Sentences: trivalence, local accommodation, and presupposition strengthening

Danny Fox, HUJI and MIT

The main goal of this paper is to make sense of conflicting data pertaining to the way quantifier phrases project the presuppositions of their arguments. More specifically, I will try to argue that the facts can be understood within trivalent approaches to presupposition projection, when coupled with two independently needed mechanisms (a) one which strengthens presuppositions (needed to deal with the so called “proviso problem”) and (b) another which incorporates presuppositions into truth conditions at various scope positions (“local accommodation”). A secondary goal is to provide some sketchy remarks on how the trivalent predictions could be derived in a classical bivalent system with the aid of a modified bridge principle – a modification of the principle suggested in Stalnaker (1974) to connect formal presuppositions to pragmatic conditions of language use.

The paper is organized as follows. The first three sections will present empirical challenges for a theory of presupposition projection that come from quantificational sentences. We will see that the challenges come from three sources of variability in judgments. The first pertains to differences among individual speakers: some speakers report judgments that are consistent with extremely weak presuppositions whereas the judgments of other speakers require much stronger presuppositions – in some cases it looks like universal projection is required. The second source of variability pertains to the nature of the quantifier: it seems that the strong judgments require a distinction among different quantifiers (some quantifiers appear to project universal presuppositions whereas others do not). The third source of variability pertains to the nature of the presupposition trigger: all quantifiers and all speakers appear to provide evidence for universal projection in the context of so-called strong triggers.

1 Acknowledgments: to be added.
3 As is clear from Fox (2008), my perspective on the topic relies heavily on the proposal of Schlenker (2008)a, and, in particular, on the idea that the theory of presupposition projection should be divided into two components: (a) a conceptually simple theory of projection which has no left right asymmetry and (b) a principle that “incrementalizes” this theory of projection thus introducing asymmetry. The significance of Schlenker’s work is not clear from the presentation in this paper, which entirely avoids component (b) (incrementalization). But avoiding this component would have been impossible without Schlenker’s paper. Schlenker teaches us how to abstract away from left-right asymmetries and how to re-introduce them (by quantifying over “good finals”). When we wish to avoid incrementalization, we need to limit our attention to the problem of predicting the projection of the presuppositions of sentence final constituents, a problem for which the conceptually simple theory and its incrementalization are equivalent.
Section 4 will introduce the predicted presupposition of Strong Kleene. Section 5 will explain the way that judgments (the strong judgments) can be derived from these predictions if presuppositions are collapsed with assertions. Section 6 will problematize this observation noting that collapsing assertion and presupposition is not necessarily a sound move if presuppositions are viewed as providing admittance conditions.

Section 7 will characterize a very specific prediction that follows from the Strong Kleene system when presuppositions are taken to provide admittance conditions. The prediction is based on the fact that universal projection is never derived directly in Strong Kleene, but is, instead, the result of presupposition strengthening (global accommodation). We will see evidence that when presupposition strengthening is not required given contextual assumptions, universal projection is never supported by data (for any combination of speaker, trigger and quantifier). Sections 8-11 will discuss two additional mechanisms in detail (local accommodation and presupposition strengthening) and will argue that the patterns of variability can follow from the claim that local accommodation is impossible for strong triggers and is dispreferred for certain speakers (in any position but the matrix). Section 12 will outline a way of deriving the results reported here in a bivalent system with a modified bridge principle.

1. **Projection from the nuclear scope – a complicated empirical terrain**

Consider the sentences in (1) under an interpretation in which the pronoun *his* is bound by the quantificational subject.

(1) a. Some boy drives *both* of his cars to school.
   b. Every boy drives *both* of his cars to school.
   c. No boy drives *both* of his cars to school.

In all of these sentences the word *both* (a presupposition trigger) introduces a presupposition of some sort, one that affects the presupposition of the sentence. If we want to understand the way the presupposition of the sentence is affected by the presupposition of *both*, i.e. the way in which the presupposition triggered by *both* projects, it is very useful to be able to state the presupposition triggered by *both*. But how should that be done?

In analogy with simple sentences that contain the word *both*, one would be tempted to state this as the presupposition that some particular person has exactly two cars. But who is that person? It should be the person that the pronoun *his* refers to. But we know that there is no such person: the pronoun has no reference (independently of an assignment function); rather it serves as a variable bound by the quantificational subject. So the problem of presupposition projection exemplified here is one of determining ways in which an assignment dependent presupposition projects (and becomes assignment independent) once a variable is bound. Let us, then, adopt the

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4 Unless we adopt a variable free semantics. Under variable free semantics we would be presenting the problems in a different format, but, as far as I can see, the issues will not be affected.
notation of e.g. Beaver 2001 and write the presupposition of the nuclear scope as a subscript that contains a variable bound outside the subscript:

\[(1)\textsuperscript{'}a. \text{Some boy } \lambda x. \text{x drives both of x’s cars to school}_{x \text{ has two cars}}\]
\[b. \text{Every boy } \lambda x. \text{x drives both of x’s cars to school}_{x \text{ has two cars}}\]
\[c. \text{No boy } \lambda x. \text{x drives both of x’s cars to school}_{x \text{ has two cars}}\]

Our problem of presupposition projection is to determine and explain the presupposition for sentences of the form \(QP\lambda x.\) where \(p(x)\) is the (assignment dependent) presupposition of the nuclear scope.

One important challenge in addressing this problem is that the facts are far from clear. In fact, different authors have made different empirical claims about the relevant presuppositions. Heim (1983), who pointed out the significance of the problem to debates about presupposition projection, suggested that all quantificational sentences presuppose that every individual in the domain of quantification satisfies the presupposition of the nuclear scope – that \(\forall x(A(x) \rightarrow p(x))\). Beaver (2001), by contrast, argued that this universal presupposition is too strong, and suggested that all the sentences have, instead, an existential presupposition: \(\exists x(A(x) \land p(x))\).

Chierchia (1995) was, as far as I know, the first to suggest that the situation is more nuanced. Specifically, he suggested that both an existential and a universal presupposition are possible and that they follow from two available representations. Chemla (2009) presented evidence that the presupposition depends on the choice of quantifier, that for some quantifiers a universal presupposition is too strong, but for others an existential presupposition is too weak. Finally, Charlow (2009) argues that, despite initial appearances, the true presuppositions are universal and that the appearance of weaker presuppositions should be attributed to the fact that certain presuppositions can sometimes be cancelled (to the process of local accommodation).

2. Some Evidence that different quantifiers behave differently

Heim (1983) pointed out that people do not tend to draw a universal inference for existential sentences such as those in (1)a. This observation was consistent with her claims about universal projection for quantificational sentences, given her assumption that indefinites are not quantificational. However, if one treats indefinites as existential quantifiers, the observation seems to argue that a universal presupposition is not, in general, correct. Furthermore, it seems to argue against the recent proposal of Schlenker (2008a) that derives universal projection based on the meaning of the relevant sentences, independently of assumptions about the syntax and semantics of the indefinite phrase itself.

\[5\] As we will see, Heim’s position on indefinites, as in (1)a, was different, given her assumption that they are not quantificational.
It has been noted in the literature that the empirical issue (even when limited to indefinites) is not straightforward, given the availability of “implicit domain restriction”. It is well established that quantificational domains are determined not only by overt restrictors, i.e. that implicit contextual restriction is often available. So, in order to rule out universal projection, one needs to factor out domain restriction. However, it has also been pointed out (in particular in Beaver 2001) that this can be done and that the initial suspicion that a universal inference is too strong for existential sentences is corroborated. The following (based on the combination of manipulations suggested by Beaver 2001 and Schlenker 2008a) illustrates the point.

(2) Half of the ten boys wrote two papers. Furthermore, one of the ten boys is proud of both of his papers.

Cf. *Half of the ten boys wrote papers. #Furthermore, every one of the ten boys wrote excellent papers.*

Given the contradictoriness of the universal italicized sentence, it is reasonable to assume that in the context of (2) implicit domain restriction is impossible. The fact that the existential sentence in (2) is entirely coherent (at least for most speakers) demonstrates that universal projection should not be derived for existential sentences (at least not all the time).\(^6\)

For other quantificational sentences, the facts about presupposition projection are less clear. Universal sentences, such as (1)b, clearly support a universal inference, but (as pointed out by Beaver) this need not have any consequences for presupposition projection – given that a universal inference ought to follow from the assertive component on its own. Negative existentials, such as that in (1)c, should be more revealing, since a universal inference, if it were present, would probably not follow from the assertive component. The problem is that speakers’ judgments do not seem to be uniform. Nevertheless, there do seem to be some speakers who get a universal inference, as suggested by the experimental work reported in Chemla (2009). Furthermore, informal survey’s I’ve conducted indicate that at least some speakers reject sequences of sentences parallel to what was acceptable in (2):

(3) Half of the ten boys wrote two papers. (#) And/but none of the 10 boys is proud of both of his papers.

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\(^6\) The situation is complicated somewhat if one adopts Schwartzschild’s theory of wide scope indefinites (Schwartzschild 2002). Under this theory indefinites can appear to have unrestricted wide scope because they can quantify over singleton domains. However, note that theory must distinguish indefinites from definite description, in that in the former the identity of the singleton domain cannot be common ground (von Fintel 2000a). This raises an obvious question for presupposition projection, one that can be seen most clearly under von Fintel’s assumption that the singleton domain is quantified over by an island insensitive existential quantifier. The problem of presupposition projection for Schwartzschild is to figure out the way in which the existential quantifier over domain restrictions projects the presupposition of its nuclear scope. The advantage of predictive theories of projection (such as the one we will be pursing) is that they leave no wiggle room for stipulations.
So we might conclude that a universal presupposition is true for negative existentials, as in (1)c, but not for plain existentials as in (1)a. This state of affairs is rather odd. One might think that (1)c, which in a classical system (bivalent with no presuppositions) is the negation of (1)a should have the same presuppositions. More directly, the sentence in (1)a appears to support a universal inference (again, for some speakers) when it is transparently embedded under negation as in (4)a or under other “holes” for projection, such as the environment that forms polar questions (see Schlenker 2008a, p. 294; 2009, 3.1.1).

(4)  
a. Not one of these 10 students is \( t_1\) proud of both his \( t_1\) papers.

b. Is one of these 10 students \( t_1\) proud of both his \( t_1\) papers? 

(5) Half of the ten boys wrote two papers. (\#) Is one/any of the 10 boys proud of both of his papers.

The empirical conclusions seem to be rather complex. There seems to be at least some evidence that different quantifiers project their presuppositions differently. But the picture is far from clear given informal evidence for inter-speaker variation. There, thus, seems to be a clear goal for empirical research, namely to ascertain the range of variation in presupposition projection (a) across speakers, and (b) across different quantificational constructions.

But the facts at the moment seem to be the following. The judgments of some speakers appear to show no evidence for universal projection in any quantificational environment. However, the judgment of other speakers appears to distinguish among different quantificational environments: in some environments, universal inferences seem to be derived (universal quantifiers, negative existentials and existentials embedded under polar questions), but not in other environments (simple existentials).

3. Evidence that all quantifiers project universal presuppositions – Charlow (2009)

We have seen evidence that in simple existential sentences, in possible contrast to other quantificational sentences, presuppositions of the nuclear scope do not project universally. Charlow (2009), however, presented data suggesting that this holds only for certain presupposition triggers. Specifically he observes that the presupposition triggered by also appears to yield a universal inference even in the scope of an existential quantifier. This is illustrated by the contradictoriness of (6) when compared to (2), or the more minimal counterpart in (7).7

(6) Just half of these 100 boys have smoked in the past. They have all smoked Nelson 

#Unfortunately, some of these 100 boys have also smoked MarlboroF.

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7 As noted by Charlow, one has to make sure that the focus particle is interpreted within the nuclear scope of the quantifier. This can be done (at least for many speakers) by placing it after the auxiliary, as in (6).
Some of these 100 boys $\lambda x. x$ has smoked Marlboro
Some of these 100 boys $\lambda x. x$ has smoked Nelson

Just half of these 100 boys smoke. Unfortunately, some of these 100 boys advertise the fact that they smoke.

To deal with this complicated state of affairs, Charlow relies on the distinction between “soft” and “strong” triggers. It has been claimed that the presuppositions that are triggered by certain lexical items (so called soft triggers) are cancellable in certain environments more easily than the presuppositions triggered by other lexical items (strong triggers) (Abusch 2010 Chierchia and McConnell-Ginet 2000, and references therein). 

Also, and other so called anaphoric triggers, appear to be strong triggers, as illustrated by the contrast in (8):

(8) a. No, he is not interviewing the applicants; there are no applicants
    b. #No, he is not talking also to Bill; he didn’t talk to anyone else.

Charlow’s Conclusion is that to study the true projection properties of quantifiers we ought to look at sentences containing strong triggers, such as (6). For other triggers, such as both and the, presupposition cancelation (a.k.a., local accommodation) is available (as illustrated in (8)), masking the true nature of things. From (6) we can conclude, Chalow argues, that all quantifier phrases, even those headed by existential quantifiers, project the presuppositions of their arguments universally.

This is an interesting proposal, but there are some puzzles that still ought to be addressed. Charlow has argued that all quantifiers project the presuppositions of strong triggers in the same way. But he did not address the differences between different quantificational environments that arise (at least for some speakers) with soft triggers. So, for example, under his proposal, the contrast between (1)a and (1)c is still mysterious, as is the effect of embedding in polar questions. Furthermore it seems that presupposition cancellation, or local accommodation, is easier when soft triggers occur in quantificational sentences, such as (7) and (1)a, than in other environments, such as (8), and the reasons for this still need to be understood.

**4. The Presuppositions Derived by Strong Kleene**

In this section I present the predicted presuppositions of a theory that is based on a trivalent logic. The predictions will, at first, seem rather odd and my goals for the remainder of the paper will be to argue that they might nevertheless be correct once investigated in conjunction with certain auxiliary hypotheses. The relevant trivalent logic (Strong Kleene) is based on a general recipe for transforming bivalent semantic values (functions from various domains to the two
truth values 0 and 1) into trivalent ones (functions from various domains to the three truth values 0, 1, and #). The trivalent values, in turn, lead to predictions about presuppositions, once one adopts something like Stalnaker’s bridge principle in (9) (see Beaver and Krahmer, 2001).

(9) **Stalnaker’s Bridge Principle:**

A truth denoting sentence $S$ is assertable given a context set $C$ only if

$$\forall w \in C \ [\text{the denotation of } S \text{ in } w \text{ is either 0 or 1}].$$

In other words, the semantics will make predictions about the conditions under which a sentence, $S$, receives a given truth value, and these conditions, together with the bridge principle, will derive the presupposition of $S$ – the presupposition that the semantic value of $S$ is not the third value, #.

So our first step is to derive the trivalent truth conditions. For minimal sentences that contain a presupposition trigger, the truth conditions are derived based on stipulations (as in all systems), or on some independent theory (see, e.g., Abrusán in press). For example, *John drives both his cars to school* receives the third value if the number of cars John has $n_J$ is any number other than 2 and otherwise the sentence is true if John drives all of his cars to school, and false if not.

\[
(10) [[\text{John drives both of his cars to school}]] = \begin{cases} 
1 & \text{if } n_J = 2 \text{ and } \forall x \ (\text{Car-of-} \text{-} \text{John}(x) \rightarrow \text{Drive}(J,x)) \\
0 & \text{if } n_J = 2 \text{ and } \exists x \ (\text{Car-of-} \text{-} \text{John}(x) \land \neg \text{Drive}(J,x)) \\
# & \text{otherwise}
\end{cases}
\]

If we abstract over the position of *John* we derive the denotation of the nuclear scope in (1):

(11) **Trivalent denotation of the nuclear scope in (1)a,b,c:**

$$\lambda x. \begin{cases} 
1 & \text{if } x \text{ has exactly two cars and } x \text{ drives both of } x\text{'s cars to school} \\
0 & \text{if } x \text{ has exactly two cars and } x \text{ doesn’t drive at least one of } x\text{'s cars to school} \\
# & \text{if the number of cars } x \text{ has is different from 2}
\end{cases}$$

What is needed, next, is a general recipe for transforming classical lexical entries to trivalent ones. To understand the Strong Kleene recipe it is useful to assume that the system is underlyingly bivalent and that the third value stands for either 0 or 1 – we just don’t know which.\(^9\) The way the third value projects is the way that uncertainty projects.

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\(^8\) This principle will be modified in section 12 (as discussed in the introduction). The modification will make the trivalent system vacuous. Not that the principle as it stands does not apply to non-truth denoting sentences. The modified principle will apply to a sentence of any semantic type.

\(^9\) A possible objection to the trivalent system is that this epistemic justification might seem inappropriate for a theory of presupposition. This could be another motivation for the alternative alluded to in section 12.
Assume that a certain constituent receives the third value. This means that we don’t know whether its value is 0 or 1. If we can determine the bivalent value of the entire sentence without knowing the value of the relevant constituent, the sentence would have that bivalent value. Otherwise – since we do not know the bivalent value of the entire sentence – it will receive the third value.\(^\text{10}\) In order for this idea to apply more broadly, we will have to discuss cases in which trivalent functions serve as arguments of higher order predicates.\(^\text{11}\) We will focus here on the bit needed to deal with quantificational sentences, with the hope that the extension (to the general case) will be transparent.

Imagine that \(f\) is a function of type \(<e,t>\), one which assigns the third value to certain elements in its domain \((D_e)\). If we apply \(f\) to an element of type \(e\), we will receive a truth-value by simple function application. But what if \(f\) is an argument of a quantifier, \(Q\), (type \(<et,t>\))? In order to understand what happens, we need to continue to imagine that the system is underlyingly bivalent, and that cases in which \(f\) returns \# indicate lack of knowledge pertaining to the bivalent value. What we want to know is under what conditions knowledge (or lack of knowledge) projects, i.e., under what circumstances the elements of type \(e\) for which \(f\) assigns the third value could be bi-passed in determining the truth-value of \(Q(f)\). The answer is simple, elements can be bypassed when we would get the same value for \(Q(f)\) irrespective of the value that \(f\) assigns to them. And when does that happen? Whenever, we get the same value under every replacement of \(f\) with what George (2010) calls a bivalent correction of \(f\) – a bivalent function, \(g\), that agrees with \(f\) on all individuals to which \(f\) assigns a bivalent value.

\[(12)\] A function \(g:X \rightarrow \{0,1\}\) is a bivalent correction of a function \(f:X \rightarrow \{0,1,#\}\) if
\[
\forall x[(f(x)=0 \lor f(x)=1) \rightarrow g(x)=f(x)]
\]

\[(13)\] **Strong Kleene:**

The denotation of \(S\) in \(w\) is

(a) \(1\) if its denotation (in a bivalent system) would be \(1\) under every bivalent correction of sub-constituents.

(b) \(0\) if its denotation would be \(0\) under every bivalent correction of sub-constituents.

(c) \# if neither (a) nor (b) hold

Consider now the existential sentence in (1)a, and assume that there is one boy \(s_1\) who has (exactly) two cars and drives both of them to school. Since the denotation of the nuclear scope, (11), would map \(s_1\) to true, every bivalent correction of (11) would map \(s_1\) to true and the existential sentence in (1) would, thus, be true under any such correction. If there is no boy who

\(^{10}\) In this sense Strong Kleene is very similar to a trivalent logic based on Supervaluation. The difference is that a logic based on supervaluation is, so to speak, smarter in deciding whether it can ignore a third value: specifically under supervaluation you consider bivalent corrections of the entire sentence instead of bivalent correction of a particular constituent (on its own). This difference is irrelevant for our purposes the moment the system is incrementalized (see Schlenker 2008c, appendix)

\(^{11}\) In George (2008), this is generalized to expressions that do not denote truth-values. However, I think that this will not affect our point about the bridge principle (see note 8).
has two cars and drives each of them to school, there will, of course, be at least one bivalent correction of the trivalent function that would make the sentence false. So having one boy who has two cars and drives each to school is the minimal condition that must be met for the sentence to be true under all bivalent corrections. For the sentence to be false under all bivalent corrections, it must be the case that all boys have two cars and no boy drives each of them to school. The sentence in (1)a is, thus, predicted to have a disjunctive presupposition: that some boy has two car and drives them to school or that all boys have two cars and no boy drives drives each to school (equivalently: that some boy has two cars and drives each to school or that all boys have two cars).

For the sentence in (1)b to be true under all bivalent corrections of (11), it must be the case that every boy has two cars and drives each school. For it to be false under all bivalent corrections there must be at least one counter-example: a boy who has two cars and doesn’t drive each to school. The sentence will thus presuppose a disjunction as well: that some boy has two cars and doesn’t drive one to school or that all boys have two cars. The sentence in (1)c is the negation of the sentence in (1)a. It will thus be false whenever the latter is true and vice versa. Hence whenever one sentence has same truth-value under all bivalent corrections of the partial function, the other will as well. They are thus predicted to have the same presuppositions.

(14) Trivalent Predictions:¹²

a. Some boy₁ [t₁ drives both his₁ cars to school]
   Presupposes:
   Either
   [Some boy has exactly two cars and drives each to school] or
   [Every boy has two cars]

b. Every boy₁ [t₁ drives both his₁ cars to school]
   Presupposes:
   Either
   [Some boy has exactly two car and doesn’t drive each to school] or
   [Every boy has two cars]

c. No boy₁ [t₁ drives both his₁ cars to school]
   Presupposes:
   Either
   [Some boy has exactly two cars and drives each to school] or
   [Every boy has two cars]

¹² Note that these disjunctive presuppositions are weaker than the universal projections assumed by Heim and stronger than the existential projections assumed by Beaver.
OK, so these are the presuppositions predicted by the system. On the face of it, they seem rather unintuitive: we clearly do not intuit these disjunctive statements as the presupposition of the sentence. But, as argued by many authors (see in particular von Fintel 2000b), there is no reason to expect that the presuppositions of a sentence would accessible to direct introspection. So we should ask whether these unintuitive presuppositions might be empirically adequate, and in particular, whether they might be adequate in addressing the empirical challenges presented at the end of section 3.

My strategy in addressing this question will be based, at first, on the pretense that presuppositions can be collapsed with truth conditions. When this is done, we will derive differences between quantifiers that correspond, more or less, to what we’ve seen in section 2. But we will still not understand the odd property of polar questions, (5), inter-speaker variation, or the difference between soft and strong triggers discussed in section 3. My goal in the subsequent sections will be to show that the facts could be understood once we take seriously the Stalnakerian view of presuppositions as “admittance” conditions on the common ground, conditions that demand that that certain things be presupposed prior to assertion.

5. Collapsing assertion and presupposition

So let’s begin by pretending that we’re allowed to think about the inferences of a sentence as simply following from the conjunction of assertion and presupposition, ignoring important issues pertaining to the special status of presuppositions as admittance conditions. Under this pretense, we straightforwardly predict the difference between existential and negative existential sentences discussed in section 2. The two types of sentences have the same disjunctive presupposition. But, once we add the assertion, the negative existential sentence leads to a universal inference but the existential sentence does not:

(1)a Some student \([x \text{ drive both of } x\text{'s cars to school}]\) \(x \text{ has (exactly) two cars}\)
Presupposes:
Either [Some student has two cars and drives each to school] or
[Every student has two cars]

Asserts:
Some student has two cars and drives each to school.

Hence: no universal inference

(1)c No student \([x \text{ drive both of } x\text{'s cars to school}]\) \(x \text{ has (exactly) two cars}\)
Presupposes:
Either [Some student has two cars and drives each to school] or
[Every student has two cars]
Asserts:

It’s not the case that some student has two cars and drives each to school.

Hence: a universal inference follows.

Of course, also the universal inference of a universal sentence is predicted, though this is not particularly surprising (for reasons discussed in section 2):

\[\text{(Error! Reference source not found.) Every student } [x \text{ drives both of } x\text{’s cars to school}]_x \text{ has (exactly) two cars}\]

\textbf{Presupposes:}

Either [Some student has two cars and drives each to school] or
[Every student has two cars]

\textbf{Asserts:}

Every student has two cars and drives each to school.

Perhaps more surprising is the prediction that the negation of the universal statement will not be associated with a universal inference – a prediction that contradicts the assumption that universal quantifiers projected universal presuppositions, as most theorists assume. It is not obvious that this prediction holds in general, but some evidence that it might be correct comes from the following discourse.

\textbf{(15) Relevant context: May and Jane are students trying to organize an event of some sort for which they need many cars.}

Mary: There are many students around, hence many cars.

Jane: No, half of the students don’t have a car.

\begin{enumerate}
\item Furthermore, some don’t drive their car to school.
\item Furthermore, not every student drives his car to school.
\item # Furthermore, every student leaves his car at home
\end{enumerate}

The oddness of the variant of Jane’s response in (c) follows straightforwardly from the universal inference associated with this response. It also teaches us that in this kind of context, contextual restriction to individuals that satisfy the presupposition (the students that have a car) is not available. This sets the stage for our observation that the negative universal in (b), just like the plain existential in (a), is not associated with a universal inference.\(^\text{13}\)

\(^{13}\) The facts in (15) would also follow from Beaver’s assumption that the projected presuppositions are existential. Beaver’s assumption, however, will make predictions that are too weak for polar question. See section 7.
The observations we’ve made shed some positive light on the trivalent system, but there are a few questions, both empirical and conceptual, that need to be addressed.¹⁴ The first question is based on the assumption that presuppositions should be viewed as admittance conditions. If so, it is not right to simply collapse assertion and presupposition. The combined effects of assertion and presupposition are indeed predicted to be inferences of a sentence, but there might be other predictions that follow from the special status of presuppositions, and the question is whether these predictions come out right under the trivalent theory. Other obvious questions pertain to the variability in judgments mentioned in section 2 and to the strengthened presuppositions that arise when soft triggers are replaced with strong triggers, discussed in section 3. Finally, there is also the observation that presuppositions of existential sentences such as (1)a appear stronger (at least for some speakers) when these sentence are further embedded to form polar questions, as seen in (4)b and (5) (see also note 13), which has also not been addressed.

6. The oddness of the presupposition – qua presupposition

Consider again (1)a, along with its presupposition and assertion:

(1)ʻa Some student [x drive both of x’s cars to school] x has (exactly) two cars

Presupposes:
Either [Some student has two cars and drives each to school] or
[Every student has two cars]

Asserts: [Some student has two cars and drives each to school]

The presupposition is the disjunction of two propositions one of which is the universal statement that the presupposition holds of every individual in the domain of quantification and the other is the assertion itself. Conjoining this disjunction with the assertion is, of course, equivalent to the assertion. Once the conjunction is completed we, thus, loose any trace of the presupposition, and, in particular, of the universal disjunct. That was our explanation for the lack of a universal inference for existential sentences.

But we did not consider the demands that the presupposition itself makes regarding what is presupposed prior to assertion. Under the Stalnakerian view that we’ve been assuming, sentences that have presuppositions can only be asserted and evaluated against an information

¹⁴ Chemla’s (2009) presents evidence that the DP exactly 3 NP does not project the presupposition of its scope universally. This observation is problematic for the trivalent system (see George 2008 for attempted corrections). I don’t have a solution for this problem, but would like to explore it in conjunction with the yet unpublished proposal of Benjamin Spector that exactly in this context has the same meaning that it has elsewhere, namely that it serves to indicate a more stringent level of granularity than we would have otherwise. The meaning of exactly 3 is derived by an “at least” meaning accompanied with an obligatory Scalar Implicature (required by the shift in granularity). It is my hope that the projection properties would follow from an independent study of the effects of Scalar Implicatures on presuppositions.
state in which the presupposition is met (a common ground that entails the presupposition). So we must ask whether it is true that sentences such as (1)a can only be asserted when the disjunctive presupposition is entailed by the common ground. Addressing this question is no trivial matter given the ubiquity of presupposition accommodation (Lewis 1979, Karttunen 1974). In other words, the Stalnakerian claim that presuppositions must be presupposed prior to assertion is based on a non-trivial idealization (abstracting away from accommodation) and deriving predictions is not going to be straightforward. Still the question is real and needs to be addressed (see, von Fintel 2000b, 2008).

One way to begin thinking about it is to investigate the various types of information-state that would entail the disjunctive presupposition, and for each type to ask how reasonable it is to assume that the sentence would be uttered with this type of information state as common ground. The information states that entail the presupposition can be divided into three types, one that entails the first disjunct (some student has two cars and drives each to school), \( T_1 \), one that entails the second disjunct (every student has two cars), \( T_2 \), and one that entails neither, yet entails the disjunction, \( T_3 \). \( T_1 \) is inappropriate since it not only entails the presupposition but also the assertion itself: under \( T_1 \) the utterance is pointless (violating one of Stalnaker’s basic principles). \( T_2 \), which entails the universal statement, is appropriate. What about \( T_3 \)? The problem with such an information state is that it would typically involve presupposition of a non-trivial, and implausible, connection between the two disjuncts. None of the disjuncts is itself believed to be true when this information state is taken to be common ground, yet there is a belief that if one of the disjuncts is false, the other is true. This does not seem like a very plausible information state to be in.

So where does this leave us? It seems that the only plausible information state that (i) entails the disjunctive presupposition of (1)a, and (ii) can serve as common ground for the utterance of (1)a is one where the universal statement that every student has two cars is presupposed. So, once we insist on viewing presuppositions as admittance conditions, it is not at all clear that we can explain the fact that (1)a is not associated with a universal inference.

More generally, all the quantificational sentences we looked at involve disjunctive presuppositions for which a \( T_3 \) information state (one that entails the disjunction without entailing any of the disjuncts) is implausible – unless a lot of work is done introducing highly specific contextual assumptions. This means that \( T_3 \) would not be the type of information state in which the sentence would be uttered (at least not typically). Furthermore, it is easy to see that a \( T_1 \) information state – one which does not entail the universal statement (that all individuals in the domain of quantification satisfy the presupposition of the nuclear scope) – always makes the assertion either a contextual tautology or a contextual contradiction (in the cases we’ve looked

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15 This discussion owes, of course, quite a bit to earlier discussions in the literature of the proviso problem, which arises in the context of the conditional or disjunctive presuppositions predicted by Karttunen and Heim (see section 10).
at) leaving us with a $T_2$ information state (one in which the universal presupposition holds) as the only available option.\footnote{Since $(1)c,$ is the negation of $(1)a,$ $T_1$ will make the assertion a contextual contradiction. The discussion can be applied to other cases only if we are able to identify $T_1 - T_3.$ For the sentences in $(1),$ I hope my intention was clear: $T_1$ refers to contexts in which the first disjunct in $(14)$ is satisfied, $T_2$ to context in which the second disjunct is satisfied (the universal statement), and $T_3$ to contexts that satisfy the disjunction without satisfying either of the disjuncts. So $(1)b,$ will be a contextual contradiction in $T_1,$ and the negation of $(1)b,$ in $(15),$ will be a contextual tautology.

For the discussion to generalize beyond $(1),$ we have to first state the trivalent presupposition as a disjunction with the second disjunct being the universal statement (the predicted presupposition in Heim’s system). The problem is that there is no unique statement of this sort, a problem very much related to the claim that there is a need for a mechanism of presupposition strengthening (section 10).}

This discussion is, of course, incomplete in that it is not accompanied with a perspective on the nature of accommodation – the process by which participants in the conversation (are expected to) accommodate their view of the common ground in light of the presuppositions of utterances. We could consider approaches to accommodation that would allow us to circumvent this problem, but I would like to suggest that the problem noticed above is real and is at the heart of the explanation for Charlow’s observation that certain triggers appear to show evidence for universal projection. I will agree with Charlow that these triggers resist the process of local accommodation. However, I will disagree with Charlow’s conclusion that quantifiers project a universal presupposition. I will argue, instead, that a universal inference follows from considerations of plausibility of the sorts sketched above. The considerations will be a bit more complicated once the process of accommodation is discussed along with theories of presupposition strengthening (proposed to resolve the so called “proviso problem”).

Some of the argument for viewing things from the trivalent perspective is that it will allow us to address the questions that were left open for Charlow. But before we get there, I would like to present a more direct argument for the trivalent predictions by drawing what I think is the clearest prediction, namely that every quantificational sentence with a presupposition in the nuclear scope will be admissible when a $T_3$ information state is plausible and taken to be common ground. Constructing sentences and contexts that make a $T_3$ information state plausible is going to be difficult, but this can be done systematically, once the general recipe is clear. The results will corroborate the predictions of the trivalent system for every combination of quantifier and presupposition trigger.

7. Getting Rid of Oddness

In this section, I will construct contexts in which a $T_3$ information state is the common ground (and the universal presupposition is not met). Against these contexts, I will investigate those sentences that out of context appear to support a universal inference. We will see that the sentences are admissible as predicted by the trivalent system.
7.1. Existential sentences embedded under polar questions

In section 2 we have seen evidence that existential sentences when used to form polar questions lead to the inference that the presupposition of the nuclear scope holds of every individual in the domain of quantification (examples (4)b and (5)). Our goal later on will be to interpret this fact as following from the implausibility of $T_3$, something that will require a way of distinguishing the polar question from unembedded existential sentences. But before we get there, I would like to motivate this line of attack by showing that when $T_3$ is plausible, the universal inference disappears (even for those speakers who, out of context, insist on a universal inference for polar questions).

Consider the polar question in (16). When uttered out of the blue, the sentence strongly suggests (for some speakers) that each of the two players has chips. This could be explained by the claim that existential sentences in polar questions project the presupposition of the nuclear scope universally. I would like to argue that the facts should be explained based on the implausibility of a $T_3$ context, one in which the only thing that is presupposed is the relevant disjunction (either both of the players have chips or one them has chips and is ready to cash them). This, as mentioned, is implausible because it suggests a connection between the two disjuncts (if one is false the other is true).

(16) Is one of the two card players ready to cash his chips?

This line of explanation would predict that if we establish the appropriate connection between the two disjuncts, the polar question will be appropriate and would not lead to a universal inference. The following argues that the prediction is correct:

(17) John and Bill meet for a game of poker. The rules they set for their engagement are the following. They each give Jane 100 dollar and get chips in return. The game will continue until one of them has no more chips left. The moment this happens, the winner (the player that has 200 chips) can go to Jane and cash his chips.

Fred (who knows the rules of engagement) is responsible for cleaning the room the moment the game is over. He calls Jane and asks the following question:

Is one of the two players ready to cash his chips?

Another example that illustrates the same point is (18). Here it seems reasonable to believe that either all of the bankers have a fortune or one of them has a fortune that he acquired by wiping out some other banker (without believing one of the disjuncts). In other words $T_3$ is, once again, reasonable.

(18) Did anyone of these bankers acquire his fortune by wiping out one of the others?
Presupposition: if none of these bankers acquired his fortune by wiping out one of the others, they all have a fortune.

So it might seem that we have provided important evidence for the disjunctive presuppositions, but as pointed out to me by Ben George, there is a serious confound here that needs to be addressed. Specifically, it is reasonable to propose that in both (17) and (18) the putative universal inference needs to be restated in light of consideration pertaining to the temporal interpretation of noun phrase. Moreover, once restated, it is no longer obvious that it is not met. Our discussion of (17), for example, took the universal inference to be the statement that both players have chips at the time of utterance. But if instead, we take the presupposition to be that both players have chips at some time or other (see Enç 1981, 1987), we notice that it is met and the initial suggestion (from section 2) that existential sentences embedded under polar questions are associated with universal inference is not affected. It is easy to see that the same issue arises for (18).

However, this confound can be overcome. In fact, there are two ways to overcome it, and both lead to the results predicted by the trivalent system. One way involves explicit reference to time within the relevant noun phrase, (19) and (20), and the other involves contexts where the universal presupposition does not hold even after it is construed in the weak sense (with existential quantification over time), (21).

(19) Did anyone of these bankers acquire the fortune he deposited in the bank last week by wiping out one of the others?
Presupposition: if no banker acquired the fortune he deposited in the bank last week by wiping out one of the others, they each deposited a fortune last week.

(20) In the context of (17)
Is any one of the two players allowed to cash the chips that he now has in his possession?

(21) Two partners (Bill and Fred) started a new company based on a new algorithm that they developed. If neither partner reveals the algorithm, they will both earn millions once the company goes public. If, however, one of them shares the algorithm with Tom – a well known English businessman – before the company goes public, this partner will be getting millions from Tom but then the other partner will remain very poor.

*Will one of the two partners get his millions from Tom?*

7.2. Strong triggers

In section 2 we looked at Charlow’s evidence that strong triggers project universal presuppositions in quantificational sentences. Here, I would like to show that, also in the case of
strong triggers, things appear to be different once T₃ is made plausible.¹⁷ To see this, consider the following context:

(22) The TV game show “diamonds are not enough” has the following format:
Every week, there are ten contestants and one million dollars to be spent on prizes for the contestants. As in many shows of this sort, there are all sorts of ways of scoring points – irrelevant for our purposes.

What is important is that there are two possible outcomes
1. If everyone scores less than 1000 points, the million dollars will be used to purchase 10 diamonds (each for 100K) and each contestant will receive a diamond.
2. Otherwise, the top-scoring contestant (the winner) will receive 500K and the 5 highest scoring contestants (including the winner) will each receive a (100K) diamond.

Every week at least 5 of the ten contestants gets a diamond. I bet that this week one of the 10 contestants will also get 500K.

Every week at least 5 of the ten contestants gets a diamond. I wonder if this week one of the 10 contestants will also get 500K.

7.3. Universal sentences embedded under polar questions

A universal sentence, such as (1)b, leads to the universal inference that the presupposition of the nuclear scope holds of every individual in the domain of quantification. As mentioned in section 5, this ought to follow from the assertive component on its own, and, therefore, need not argue for universal projection. Furthermore, the example in (15) where the universal sentence is negated seems to argue against a universal presupposition. However, a polar question formed from universal sentence again leads to a universal inference (for example in (23) that each of the 3 boys has/had money).

(23) a. Did each of the 3 boys invest his money wisely?
   b. It matters to me whether each of the 3 boys will invest his money wisely.

Later on, I will attempt to explain this special property of polar questions (observed already with existential sentences). But here I want to argue that things change once T₃ is made plausible. The argument is based on (24).

¹⁷ One might attempt to derive all of the results of this section from local accommodation. Specifically, one might suggest that local accommodation is motivated, in the cases discussed here, by the contextual assumptions that make it clear that a universal presupposition is not met. The contrast between the facts in (22) and Chalow’s example argues against this possibility.
(24) John got money gambling in the racetrack. If he invests his money wisely, he will be able to pay Bill (who will have no money otherwise). If Bill gets money from John and invests it wisely, he will be able to pay Fred (who will have no money otherwise). If Fred has money and invests it wisely, he will be able to pay me, and otherwise I will have no money at all.

So it matters to me quite a bit whether or not each of the three fellows will invest his money wisely.

7.4. Negative Existentials

In section 5 we derived the universal inference for negative existentials by considering the combined effect of assertion and presupposition. However, when a negative existential is embedded in various environments where the assertive component is modified, things might be different. In particular, if (1)c is embedded in a polar question, the assertive component is eliminated and we no longer predict a universal inference. So in principle, we could consider exactly the same manipulations we considered in 7.1., replacing an existential with a negative existentials. Unfortunately, the results are not as natural as we would like, probably due to the special biases that are associated with negative polar questions (Romero and Han, 2004).

For this reason, I will consider the same contextual manipulations but will look at constructions in which the negative existential is embedded in the antecedent of a conditional. First note that out of the blue, such embedding leads to a universal inference (for reasons that still need to be clarified) – in the case of (25) that both partners will get millions:

(25) If neither partner gets his millions from Tom, I will be quite content.

However, once T₃ is made plausible, as in (21), this inference disappears.

(26) Two partners (Bill and Fred) started a new company based on a new algorithm that they developed. If neither partner reveals the algorithm, they will both earn millions once the company goes public. If, however, one of them shares the algorithm with Tom – a well known English businessman – before the company goes public, this partner will be getting millions from Tom, but then the other partner will remain very poor.

If neither partner gets his millions from Tom, I will be quite content.

7.5. Interim Summary
In this section, we focused on a peculiar property of the disjunctive presuppositions predicted by Strong Kleene (in conjunction with the bridge principle). This presupposition would be satisfied in three types of context, and the context which is crucial for zeroing in on the disjunctive nature of the presupposition, $T_3$, is implausible without some contextual background. The goal of this section was to provide the background and to show that when this is done a universal inference that is otherwise often attested is eliminated. This provides some support for the trivalent predictions, but as mentioned in section 6, we still need to understand how the perspective on presuppositions as admittance conditions does not lead to universal inferences in all cases in which a $T_3$ information state in not plausible. In the next sections, we will try to address this question along with the question of variability in judgments.

8. Additional Mechanisms

Our discussion to this point is incomplete for three reasons. First, as just mentioned, we seem to predict universal inferences in all but $T_3$ contexts, a prediction that appears to be wrong for certain quantifiers in combination with weak triggers. Second, we did not deal with inter-speaker variation, and third we did not deal with cases in which the presupposition is not satisfied in the actual conversational context but needs to be accommodated (global accommodation).

In this section, I will deal with the first of these open issues by adopting Charlow’s hypothesis that soft triggers (and only soft triggers) allow for local accommodation. I will view local accommodation as a process that collapses assertion and presupposition at various scope positions. The predictions described in section 5 will be derived from applying local accommodation at the matrix level. Speaker variation, the second open issue, will be dealt with by the assumption that speakers vary in how easily they allow for local accommodation, and possibly in the scope position in which they prefer the process to apply. Finally, I will deal with the third issue, discussing the process of global accommodation, in light of the proviso problem, and the need for a mechanism of “presupposition strengthening”.

8.1. Local Accommodation

Our first observation is that there are some individual differences in the distribution of universal inferences. To account for this observation, and for the contrast between strong and soft triggers, I would like to suggest that there is a process that collapses assertion and presupposition, which I will call (following Heim 1983) local accommodation. I will assume that the relevant mechanism is a covert operator $B$ (for Bochvar) which maps the third value $#$ to 0 (see Beaver and Krahmer 2001).

\[(27) \quad [B] = \lambda p. 1 \text{ if } p=1, 0 \text{ if } p \neq 1.\]
If a constituent receives the third value, applying B to it yields 0; in all other cases, B does not affect the value of the constituent to which it applies. If B applies to the nuclear scope of a quantificational expression, presuppositions get cancelled, and a universal inference will not surface unless it follows from the assertive component (as in simple universal quantification). If we apply B to the entire sentence, we collapse the assertion and presupposition of the entire sentence.

Our account in section 5 of the difference between different quantificational expressions that embed weak triggers would, thus, follow if we assume that B applies at the matrix level. For the sake of illustration, consider (1)c with the two parses in (28).

(28) Two Parses of (1)c:
   a. No boy λx. B(x drives both of x’s cars to school x has two cars)
   b. B(No boy λx. (x drives both of x’s cars to school x has two cars)

(28)a expresses a bivalent proposition which is true if and only if there is no boy for which the assertion and presupposition of the nuclear scope holds, i.e. iff every boy for which the presupposition holds, the nuclear scope is false. It is easy to see that no universal inference follows. (28)b, by contrast, is true iff the disjunctive presupposition is true (every boy has two cars or some boy has two cars and drives both to school) and the second disjuncts is false, which of course entail the first disjuncts. It thus follows that every boy has two cars. For existential quantifiers, by contrast, both parses lack a universal inference. It is easy to see that applying B at the matrix level yields the results for universal and negative quantifiers discussed in section 5.

To account for inter-speaker variation, I would like to propose that speakers vary in their ability (or tendency) to apply B in an embedded position. Some speakers can apply B within the nuclear scope while other speakers either avoid B or apply it at the matrix level. Speakers of the first sort will not report a universal inference, and speakers of the second sort will (for some quantifiers, when they apply B at the matrix level; for all quantifiers in a non T3 context, when they avoid B altogether – as we will see shortly). We might also predict a third type of speaker, one who does not apply B at all, for which a universal inference will be predicted for all quantifiers in all but the special scenarios discussed in section 7.

To account for the behavior of strong triggers, I would like to agree with Charlow that such triggers resist local accommodation. Under the implementation proposed here, this will be stated as a constraint banning application of B to a constituent that contains a strong trigger. However, given the discussion in section 7 (and in particular 7.2.), I conclude (as already

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18 In Sudo et. al. (in press), we found evidence that some speakers get a universal inference even for existential quantifiers that embed weak triggers. To accommodate that, we made a slightly different assumption about speaker variation, namely that some speakers avoid B altogether, whereas others tend to apply it, and then might select a scope position randomly.

19 Romoli (2012) points out that things need to be stated somewhat differently to accommodate cases in which a constituent embeds both a strong and a weak trigger and only the presupposition of the latter is cancelled. What it seems we will need is a process that “associates” with a particular presupposition trigger and only cancels that one. As far as I can see, this will be easier to implement in the system outlined in section 12.
mentioned) that the universal inference associated with strong triggers does not follow directly from universal projection, but rather from the oddness of the disjunctive presupposition discussed in section 6. But the discussion in section 6 did not take into account situations in which the disjunctive presupposition is not part of the common ground at the time of assertion, i.e. the process of global accommodation (as apposed to local accommodation). In the next subsection, I will argue that the empirical expectations do not change once global accommodation is taken into account.

8.2. Global Accommodation

In section 6 we discussed how a universal inference might follow for simple existential sentences that embed a presupposition trigger if we are serious in viewing presuppositions as admittance conditions (despite the fact that the universal inference does not follow directly from the trivalent truth conditions). Consider again (1)a, under the assumption that B is not part of the syntactic representation.

(1)a  Some student [x drive both of x’s cars to school]x has (exactly) two cars

Presupposes:

Either [Some student has two cars and drives each to school] or
[Every student has two cars]

We discussed three types of context where the disjunctive presupposition is met: T₁, where the first disjunct is presupposed and the sentence is a contextual tautology, T₂, where the universal disjuncts is presupposed, and T₃, where neither disjunct is presupposed yet the disjunction is presupposed. Since the sentence is unassertable in T₁ and T₃ is implausible, we concluded that speakers are likely to draw a universal inference. This, one might suggest, is the account of the universal inference drawn with strong triggers under our assumption that B is not part of the syntactic parse.

But this account ignores situations where the disjunction is not presupposed but accommodated upon assertion. If we assume that accommodation is minimal and then followed by the assertion, we would not derive the universal inference. Hence we will not be able to account for Charlow’s observation (or for better studied cases of presupposition strengthening discussed in section 10). I would therefore like to adopt the view that accommodation involves the postulation of a plausible common ground (Beaver 1999). In other words, accommodation cannot result in a T₃ context (unless a connection between the disjuncts is made plausible) and

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20 Terminology can be confusing given that the mechanism for local accommodation can apply globally (i.e., at the matrix level). In the system proposed here the results for matrix application of local accommodation and for global accommodation are quite different, given the phenomena of presupposition strengthening.

21 In earlier presentations of this material, I assumed such a model of accommodation, one in which presupposition strengthening was considered only if the result of minimal accommodation and assertion lead to an implausible information state.
some form of “presupposition strengthening” is required (as assumed in various accounts of the proviso problem, to which we will return). There are two types of strengthening that could be considered: $T_1$ and $T_2$. Since the sentence is unassertable in $T_1$, the only available option (among the two) is $T_2$. Hence, we predict a universal inference whenever $B$ is not part of the parse.23

So where are we? We have seen that if $B$ is not part of the parse, we predict a universal inference for all quantifiers and in all contexts other than the specialized contexts discussed in section 7. We accounted for Charlow’s observation (and the counter examples discussed in 7.2.) under Charlow’s assumption that strong triggers resist local accommodation, which for us is derived by $B$. When $B$ is part of the parse, we cancel the presuppositions entirely when it applies to the nuclear scope. When it applies at the matrix level we derive the differences among quantifiers discussed in section 5. Differences among speakers are predicted if speakers vary in whether they allow $B$ to apply locally.

In the next section, I would like to discuss what is predicted for polar questions (and other embeddings that modify the assertive component) given the perspective mentioned above. But first a small point about the ease of local accommodation. One issue that was mentioned for Charlow in section 3 pertains to the argument for a distinction between soft and strong based on contrasts such as (8) repeated below:

(8)  
a. No, he is not interviewing the applicants; there are no applicants  
b. #No, he is not talking also to Bill; he didn’t talk to anyone else.

Under the current implementation, we would like to say that the contrast shows that inserting $B$ below negation is possible when soft triggers are involved, (8)a, but not when strong triggers are involved, (8)b. Our question for Charlow was why insertion of $B$ below negation seems to be more marked than in the context of existential sentences. In particular, the first sentence in (8)a, if uttered in isolation, would clearly lead to the inference that there are applicants. It is only upon hearing the second sentence that we consider a parse with local accommodation – one that avoids a contradiction. With existential sentences, we don’t seem to need a continuation.

This question arises for the current proposal as well. But under the current proposal, we have two ways of addressing it that were not available for Charlow. One way is to assume that the markedness of embedding $B$ below negation should not be attributed to a general dispreference for local accommodation. Rather, it is the result of a general dispreference for embedded $B$. In existential sentences, we can avoid a universal inference by applying $B$ to the entire sentence, and this option is not dispreferred by the parser.

22 This perspective is more reasonable than minimal accommodation if we are serious in viewing presuppositions as admittance conditions. If presuppositions are admittance conditions, the presupposition of a sentence has to be part of the common ground when the sentence is evaluated – under the appropriate idealization. In cases that deviate from this idealization, hearers have to figure out what contexts speakers took to be the common ground, and the relevant contexts have to be reasonable.

23 As discussed in section 10 (based on Singh’s work), one might argue that there are other options (besides $T_1$ and $T_2$) we ought to consider, e.g. specific instantiations of $T_3$ that are nevertheless plausible.
A second possibility, under the current proposal, is to claim that local accommodation (insertion of B) must be motivated. In (8)a it is motivated after the fact by the need to avoid a contradiction. In the absence of the continuation, it would not be motivated. In quantificational sentences, by contrast, local accommodation (either at a matrix or embedded position) is motivated by the desire to avoid an odd presupposition. The second possibility (which is not incompatible with the former) needs to be further articulated by specifying the set of considerations that could motivate local insertion of B. But we can already see that it ought to have predictions for other cases that involve odd presuppositions (and in particular for their embedding), predictions that I hope to return to at some point.  

9. Embedding under polar questions

Despite our evidence that quantificational sentences are not associated with universal presuppositions, we have seen that for some speakers, at least out of the blue, these sentences do lead to universal inferences when used to form polar questions ((4)b (5), (16), (23)). We now can make sense of this observation. The trivalent presuppositions, if they are not strengthened, are odd in out of the blue contexts (since they suggest an implausible connection between two unrelated propositions, the two disjuncts). This oddness can be circumvented by two means: local accommodation and presupposition strengthening. We have postulated that universal inferences with soft triggers result from a preference to apply B globally. But in polar questions B cannot apply globally (for type considerations). This, in and of itself, could serve to explain the preference for a universal inference.  

But there is also another possible explanation. Specifically, we might propose that the relevant strategy (the one employed by speakers who get a universal inference) involves applying B as high as possible given type considerations. Even under this strategy, polar questions would require presupposition strengthening, under Karttunen’s assumption about their syntax. To see this, consider the polar question in (4)b, under the Karttunen/Hamblin assumption that questions denote sets of propositions. If B applies to what Karttunen assumes is the proto-

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24 For example it would predict that local accommodation (below negation) would be easier in (i) than in (8)a. (i) I don’t think that if he flies to Paris his sister will pick him up.

25 Note that the same reasoning applies here. T₁ is not an available strengthening because it would make the question inappropriate (in that the answer is part of the common ground).

It has been argued that questions are not always inappropriate when the answer, given the presupposition, must be part of the common ground – specifically that they can be used rhetorically (Guerzoni 2004). I would claim that this rhetorical affect can be achieved only if the answer must be part of the common ground under any type of accommodation, i.e., that one would prefer to accommodate a common ground that would make the question have a non-rhetorical interpretation. Thanks to Luka Crnić for raising this issue.

26 I haven’t discussed the way presuppositions project by the operator that forms polar questions and how this might be explained in the trivalent system. The problem is that, at the moment, no predictions are made. Appropriate predictions are made by the system outlined in section 12 (see Fox 2011). Thanks to Alexandre Cremers.
question, we will get two propositions in the denotations, one resulting from applying B to the existential sentence and the other from applying B to the negative existential.\textsuperscript{27}

\[b'] \text{Is B} \left[ \text{one of these 10 students} \left[ t_1 \text{ proud of both his} \, \text{papers} \right] \right] \text{?}
\]

\textbf{Denotation}
\[
\{ p: p \text{ is the denotation of B} \left[ \text{one of these 10 students} \left[ t_1 \text{ proud of both his} \, \text{papers} \right] \right] \text{ or } p \text{ is the denotation of B} \left[ \text{one of these 10 students} \left[ t_1 \text{ proud of both his} \, \text{papers} \right] \right] \}
\]

\{ that one of these 10 students has two papers and is proud of both of them, that all of these 10 students have two papers and none of them is proud of both of them \}

In contrast to ordinary yes no questions, there are states of affairs where neither proposition in the Hamblin denotation is true. Specifically, a proposition in the set is true iff the disjunctive presupposition of the existential is met (or equivalently of the negative existential). If we assume that a question presupposes that a member of its denotation is true,\textsuperscript{28} we would be left with the disjunctive presupposition. This presupposition would need to be strengthened in out-of-the-blue contexts, yielding a universal inference.\textsuperscript{29}

10. A mechanism for presupposition strengthening

Our discussion of presupposition strengthening is, as mentioned, parallel to various discussions in the literature of the proviso problem, illustrated by the following contrast.

\[(29) \quad \begin{array}{ll} a. & \text{If John is a scuba diver, he will bring his wetsuit.} \\
& \text{Appears to presuppose: If John is a scuba diver, he has a wetsuit.} \\
& \text{b. If John scuba diver, he will bring his car.} \\
& \text{Appears to presuppose: John has a car.} \end{array} \]

\[(29)b \text{ leads to the inference that John has a car, whereas (29)a } \text{does not appear to lead to the parallel inference that John has a wetsuit. In other words, the presupposition triggered by the definite description appears to project differently in the two sentences.} \]

\text{According to one influential account of this contrast, this appearance is misleading. The presupposition triggered by the definite description projects in the same in both cases, resulting in the presupposition that if John is a scuba diver, he has a car/wetsuit.\textsuperscript{30} The appearance of a different presupposition in (29)b is the result of presupposition strengthening. This idea is}

\textsuperscript{27} \text{The same result would be achieved if B could apply point-wise to each member of the question denotation as in Kratzer and Shimoyama (2002).}

\textsuperscript{28} \text{This would follow from the } Ans \text{ operator proposed in Dayal (1996) and further argued for in Fox and Hackl (2006) and Abrusán and Spector (2011).}

\textsuperscript{29} \text{This logic extends to the conditional in (25), which I will not go over here. The important observation is that conditionals do not entail that the antecedent is true. Hence applying B globally will not eliminate the implausibility of the inference of a connection between the two disjuncts.}

\textsuperscript{30} \text{Note that the trivalent approach predicts this uniform presupposition hence is committed to this line of analysis.}
identical in form to the proposal about presupposition projection made here. To see this, note first that the conditional presupposition is understood as a material conditional which can be re-stated as a disjunction: either John is not a scuba diver or he has a car/wetsuit.

Let’s now divide contexts in which the presupposition is satisfied to three different types (along the lines of our discussion in section 6): contexts in which the first disjunct is presupposed, T₁, contexts in which the second disjunct is presupposed, T₂, and context in which neither disjunct is presupposed yet the disjunction is presupposed. Both sentences share an important property, namely that both are not assertable in T₁ (conditionals are inappropriate when the antecedent is presupposed to be false). So, as in section 6, we only need to consider T₂ and T₃. The difference between (29)a and (29)b is that in (29)a T₃ is a plausible context but not in (29)b. The reason for this is that the two disjuncts in (29)b are not plausibly related to each other: we do not expect there to be a dependency between having a car and being a scuba diver. But in (29)a the connection is very plausible: there is obviously a connection between having a wetsuit and being a scuba diver. The implausibility of T₃ in (29)b leads to the inference that the relevant context is T₂ and hence that John has a car. In the case of (29)a, this inference is not called for and hence we do not infer that John has a wetsuit.

In other words, we saw that the disjunctive presupposition of a conditional sentence sometimes gets strengthened to one of the disjuncts (the one that it can be strengthened to). This is exactly what was assumed for the disjunctive presupposition of a quantificational sentence. The question in both cases, however, is whether we need a special mechanism for this type of strengthening. The literature on this matter is divided. Still there are reasons to believe that a special mechanism is required (Geurts (1996), Singh (2007, 2008, 2009, 2010), Katzir and Singh 2010), reasons that I think are strengthened significantly if the trivalent theory of presupposition projection is correct.

One conceptual reason (stressed by Singh) pertains to the very nature of the problem: one in which a sentence is uttered with a presupposition that is too weak for minimal accommodation. In such a scenario the accommodated common ground would have to entail some new information stronger than the presupposition itself. But the space of possible strengthenings is in principle larger than the one we considered. After all, a disjunction is entailed by many propositions besides those denoted by the disjuncts. So in order for the account of presupposition strengthening to stand on firm grounds, we should have some characterization of the space of possible strengthenings that we ought to consider.

Geurts has made a more concrete argument based on cases for which presupposition strengthening is impossible: sentences that have presuppositions that are too weak for minimal accommodations and nevertheless cannot be strengthened (yielding pragmatic deviance). The response to this argument has been to point out special properties of Geurts’ sentences that are in conflict with presupposition strengthening (in particular certain implicatures): Heim 2006, Beaver 2006, Pérez Carballo 2007. In light of this impasse, it would be good to ask whether there are pairs of sentences that have exactly the same semantic properties and the same
presupposition and nevertheless yield different forms of presupposition strengthening. If such sentences exist, that could strongly support an account in terms of a specialized mechanism.

With this in mind, consider the following pairs of questions under the assumption that the trivalent presuppositions are correct:

(30)  a. Does one of your two sons drive his car to school?
     b. #Does one of your two sons have a car and drive it to school or do both your sons have a car and neither drives it to school?

If trivalent presuppositions are correct (and if B is not inserted in (30)a at the most embedded position), the two sentences in (30) share the following presupposition: Either (p) one of your 2 children has a car and drives it to school or (q) both of your children have a car and neither drives it to school. Furthermore, they ask for exactly the same information: they have \{p,q\} as their Hamblin denotation. But still they feel very different. For (30)b, the disjunctive presupposition is perceived and the result is some form of pragmatic deviance. (30)a, by contrast, is an entirely natural question where the disjunctive presupposition is not perceived: some speakers report a universal inference, whereas others do not. To explain this, we’ve hypothesized that either B is inserted at a low position, thus cancelling the presupposition altogether, or the resulting structure yields the implausible presupposition of (30)b, which escapes pragmatic deviance through presupposition strengthening to a universal inference.

The obvious question to ask is why presupposition strengthening can’t apply to (30)b, thus eliminating deviance. My answer to this question is based on Singh’s claim that we need a theory of presupposition strengthening which specifies the space of potential strengthenings from which participants in a conversation might select the most plausible one. More specifically, I would like to adopt Schlenker’s (2011) theory of the set of alternatives. Schlenker proposes that this set is derived by computing the admittance conditions while ignoring the identity of certain sub-constituents of the relevant sentence, where ignoring a sub-constituent is interpreted in a very special way. Specifically, to ignore the identity of a sub-constituent, X, while computing admittance conditions is intended to mean that the admittance conditions must be satisfied no matter what the identity of X is. In other words:

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31 Other pairs to consider (inspired by Singh and Schlenker) are the following:
(i)  a. If John lives in Michigan and works as a corporate lawyer in Detroit, he will find a way to take advantage of his rights as a holocaust survivor.
    Likely inference: If John lives in Michigan, he has rights as a holocaust survivor.
    b. If John lives in a Midwestern state and works as a corporate lawyer in Detroit, he will find a way to take advantage of his rights as a holocaust survivor.
    Likely inference: If John lives in a Midwestern state, he has rights as a holocaust survivor.

32 The disjunctive presupposition for (b) comes from the general observation about the presuppositions of questions. See note 28.
(31) Sloppy notation: When $S(X)$ refers to a sentence that contains a constituent $X$, $S(X')$ will refer to the result of substituting $X$ with $X'$ in $S$.

Let, $S(X)$ be a sentence with presupposition $p$, $p^+$ is a possible strengthening of $p$ if

$$p^+ = \cap \{p': \exists X' X' \text{ has no presupposition trigger and } p' \text{ is the presupposition of } S(X')\}$$

I would like to add to (31) the assumption that representations such as those in (1)' are real, in particular, that a subscript on a constituent has reality (independent of the identity of the constituent). From this it follows that the part of the nuclear scope that does not include the subscript can be substituted independent of the subscript. With this assumption at hand, we derive the desired results: the universal inference turns out to be a potential strengthening of all simple quantificational sentences. Specially, the universal inference results from ignoring the identity of the (unsubscripted) nuclear scope, i.e., for every quantifier we’ve considered, $Q$, (in fact for every conservative quantifier).\(^{33}\)

(32) Every $(A) (\lambda x.p(x)) = \cap \{p: \exists Z \ p \text{ is the trivalent presupposition of } Q(A)(\lambda x.Z_{p(x)})\}$

11. Conclusions

In this paper I’ve attempted to deal with presupposition projection from the nuclear scope of quantificational sentences. My goal was to deal with a very complicated empirical picture, one that involved variance among quantificational determiners, presupposition triggers, and individual speakers. My starting point was the observation that despite this variability one fact seems to be constant, namely the acceptability of the relevant sentences in $T_3$ contexts.

I took this as important evidence for the trivalent system. In order to deal with variability, I capitalized on the oddness of the trivalent presupposition in out-of-the-blue contexts. I suggested that there are two different mechanisms that can deal with this oddness: local accommodation and presupposition strengthening. Furthermore, I assumed that the former is dispreferred for some individuals and unavailable for all individuals when strong triggers are involved.

12. Postscript: A Bivalent Perspective

Finally, I would like point out that the results reported here can be obtained in a bivalent system in which Stalnaker’s Bridge Principle in (9) is modified. Potential motivations for such a system are the following:

\(^{33}\) Another way of getting this strengthened meaning might beto consider substitution for the determiner:

Every $(A) (\lambda x.p(x)) = \cap \{p: \exists Q Q \text{ a natural language quantifier } \& p \text{ is the trivalent presupposition of } Q(A)(\lambda x.Z_{p(x)})\}$
(33) (a) it will straightforwardly extend to an account of the presuppositions of non-truth denoting expressions, e.g. questions (which Stalnaker’s principle is silent about).

(b) its conceptual grounding will not be based on the epistemic reasoning that was used to introduce the trivalent system – reasoning that is not obviously appropriate for presupposition projection.

In the system we sketched in this paper, trivalent semantic values are computed (via the Strong Kleene recipe) and these truth conditions, in turn, yield admittance conditions via Stalnaker’s bridge principle. In the alternative bivalent system, trivalent semantic values are not computed at all and instead admittance conditions are stated directly via a modification of Stalnaker’s principle. The modified principle (like Schlenker’s 2008a principle) makes direct reference to the minimal sentences dominating a presupposition trigger. Such sentences must have a two dimensional representation, which can be annotated using subscripts. The subscript plays a role in the admittance condition but not in the semantics, which is, as mentioned, standard and bivalent.

Imagine that $S_p$ is a minimal sentence dominating a presupposition trigger and that $p$ is the “atomic presupposition” (the one that results from the lexical stipulations). The problem of presupposition projection is to determine the contribution of $S_p$ to the admittance conditions of a sentence $\phi$ that dominates $S_p$, $q(S_p)$. Instead of computing trivalent truth conditions, we can encode the contribution of $S_p$ directly to admittance conditions for $q$ (conditions that are independent on the semantic type of $\phi$, (33)a).

Keeping to the format of (9), this will be implemented by a constraint on the set of worlds compatible with the common ground, which we’ve simply called the context $C$. Let’s start with the simple case in which $p$ contains no free variables that are bound outside of $p$. In such a case it is meaningful to ask whether $p$ is true in a given world in $C$. The intuition behind our constraint is based on the idea that $S_p$ is inadmissible in a world $w$ (unverifiable, or problematic for some other reasons, see (33)b) if $p$ is false in $w$. Now assume that $w$ is a member of $C$. To see whether $p$ needs to be true in $w$, we need to know whether the value of $S_p$ in $w$ is important/relevant for determining the value of $\phi$. If the answer is yes, then $p$ must be true in $w$. Otherwise it need not be.

\[(\phi(S_p)) \text{ is assertable in } C \text{ only if }\]
\[
\forall w \in C: \text{ Relevant}(S_p, \phi(S_p), w) \rightarrow p \text{ is true in } w.  \tag{34}\]

\[(\text{Rel}(S_p, \phi(S_p), w) \leftrightarrow_{\text{def}} (\llbracket \phi(T) \rrbracket^w \neq \llbracket \phi(\bot) \rrbracket^w)) \tag{35}\]

Where $\llbracket T \rrbracket^w = 1$ for all $w$ and $\llbracket \bot \rrbracket^w = 0$ for all $w$

\[\text{Henceforth: ‘Rel}(S, \phi(S), w)’). \text{ This should be read as the value of } S \text{ is relevant for the value of } \phi \text{ in } w.\]
The next two goals are to extend the system to atomic sentences that contain variables are are bound from the outside and to incrementalize the condition along the lines of Schlenker (2008a) (see Fox 2008). Unfortunately, there is no space for introducing the relevant machinery, so (for now) I will refer the reader to a detailed handout (Fox 2011: p. 19-31).35

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35 In Fox (2011) I did not define local accommodation – the operator B. What seems to be needed is something like the following: \([\text{BS}] = \lambda w. \text{S is assertable in } \{w\} \text{ and } [[\text{S}]][w]=1.\)

To derive selective cancellation of presuppositions (see note 19), we will index subscripts with B:

\([\text{BS}] = \lambda w. \text{S is assertable in } \{w\} \text{ and } [[\text{S}]][w]=1\)

where S is like S except that every subscript with an index distinct from i is deleted

We will also need to exempt subscripts from (34) if they are co-indexed with B.

\((34) \quad \forall \omega \in C: \text{Relevant}(S_p, \sigma(S_p), \omega) \rightarrow p \text{ is true in } \omega.\)
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