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Proton-Proton Fusion and Tritium $\beta$ Decay from Lattice Quantum Chromodynamics

Martin J. Savage,1,2 Phiala E. Shanahan,3,2 Brian C. Tiburzi4,5,6,2 Michael L. Wagman,7,2 Frank Winter,8 Silas R. Beane,7,2 Emmanuel Chang,1 Zohreh Davoudi,3,2 William Detmold,3,2 and Kostas Orginos9,8

(NPLQCD Collaboration)

1Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195-1550, USA
2Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA
3Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
4Department of Physics, The City College of New York, New York, New York 10031, USA
5Graduate School and University Center, The City University of New York, New York, New York 10016, USA
6RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA
7Department of Physics, University of Washington, Box 351560, Seattle, Washington 98195, USA
8Jefferson Laboratory, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA
9Department of Physics, College of William and Mary, Williamsburg, Virginia 23187-8795, USA

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The nuclear matrix element determining the $pp \to d\bar{e}^\nu$ fusion cross section and the Gamow-Teller matrix element contributing to tritium $\beta$ decay are calculated with lattice quantum chromodynamics for the first time. Using a new implementation of the background field method, these quantities are calculated at the SU(3) flavor–symmetric value of the quark masses, corresponding to a pion mass of $m_\pi \sim 806$ MeV. The Gamow-Teller matrix element in tritium is found to be $0.979(03)(10)$ at these quark masses, which is within $2\sigma$ of the experimental value. Assuming that the short-distance correlated two-nucleon contributions to the matrix element (meson-exchange currents) depend only mildly on the quark masses, as seen for the analogous magnetic interactions, the calculated $pp \to d\bar{e}^\nu$ transition matrix element leads to a fusion cross section at the physical quark masses that is consistent with its currently accepted value. Moreover, the leading two-nucleon axial counterterm of pionless effective field theory is determined to be $L_{1A} = 3.9(0.2)(1.0)(0.4)(0.9)$ fm$^3$ at a renormalization scale set by the physical pion mass, also agreeing within the accepted phenomenological range. This work concretely demonstrates that weak transition amplitudes in few-nucleon systems can be studied directly from the fundamental quark and gluon degrees of freedom and opens the way for subsequent investigations of many important quantities in nuclear physics.

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Weak nuclear processes play a central role in many settings, from the instability of the neutron to the dynamics of core-collapse supernovae. In this work, the results of the first lattice quantum chromodynamics (LQCD) calculations of two such processes are presented, namely, the $pp \to d\bar{e}^\nu$ fusion process and tritium $\beta$ decay. The $pp \to d\bar{e}^\nu$ process is centrally important in astrophysics as it is primarily responsible for initiating the proton-proton fusion chain reaction that provides the dominant energy production mechanism in stars like the Sun. Significant theoretical effort has been expended in refining calculations of the $pp \to d\bar{e}^\nu$ cross section at the energies relevant to solar burning, and progress continues to be made with a range of techniques [1–10], as summarized in Ref. [11]. This process is related to the $\tau d \to nne^+$ neutrino-induced deuteron-breakup reaction [12–14], relevant to the measurement of neutrino oscillations at the Sudbury Neutrino Observatory [15,16], and to the muon capture reaction $\mu^-d \to nne_\mu$, which is the focus of current investigation in the MuSun experiment [17]. The second process studied in this work, tritium $\beta$ decay, is a powerful tool for investigating the weak interactions of the Standard Model and plays an important role in the search for new physics. The superallowed process $^3H \to ^3He e^-\bar{\nu}$ is theoretically clean and is the simplest semileptonic weak decay of a nuclear system. In contrast to $pp$ fusion, this decay has been very precisely studied in the laboratory (see Ref. [18] for a review) and provides important constraints on the antineutrino mass [19]. Tritium $\beta$ decay is also potentially sensitive to sterile neutrinos [20,21] and to interactions not present in the Standard Model [21–24]. Although the dominant contributions to the decay rate are under theoretical control as this is a superallowed process, the Gamow-Teller (GT) contribution (axial current) is somewhat more challenging to address than the Fermi ($F$) contribution (vector current). Improved constraints on multibody contributions to the GT matrix element will translate into reduced uncertainties in predictions for decay rates of larger nuclei and are a first step towards understanding the quenching of $g_A$ in nuclei [25–27], a long-standing problem in nuclear theory.
In this Letter, LQCD is used to study the $pp \rightarrow de^+\nu_e$ fusion process and the Gamow-Teller matrix element contributing to tritium $\beta$ decay for the first time, albeit at unphysically large values of the light quark masses and neglecting the effects of isospin breaking and electromagnetism. This is accomplished using a new algorithm for implementing background fields, which is superior to existing methods. Further, the quantities of interest are extracted at high precision using a refined analysis strategy made possible by this new approach. For $pp \rightarrow de^+\nu_e$, the deviations from the single-nucleon contributions are small but are well resolved with the new technique. The leading two-nucleon axial counterterm of pionless effective field theory ($\pi$EFT), $L_{1,A}$, is determined. The axial coupling of $^3\text{H}$ that determines the matrix element for $^3\text{H} \rightarrow ^3\text{He}e^+\bar{\nu}_e$ in the isospin limit is found to be slightly smaller than that of the proton and is consistent with previous phenomenological estimates [6].

**Background axial fields.**—Background field techniques were first used in LQCD in the pioneering works of Ref. [28] and Refs. [29,30] in the cases of axial and magnetic fields, respectively. Significant effort has been applied to using background electromagnetic fields to extract magnetic moments and electromagnetic polarizabilities of hadrons [31–35] and nuclei [36–38], as well as the magnetic transition amplitude for the $np \rightarrow dy$ process [39]. Very recently, axial background fields have been employed to extract the axial charge of the proton [40,41], and generalizations to nonzero momentum transfer [42–44] have been used [45] to access the axial form factor of the nucleon.

In this work, a new method is used to generate hadronic correlation functions order by order in the background field. In the standard approach, correlation functions are constructed from the contraction of quark propagators that are modified by the presence of a background field. The same effect can be achieved by directly constructing propagators that include explicit current insertions, and then using such propagators to construct correlation functions. For the quantities studied in this work only a single insertion of the background axial field is required. To this end, the compound propagator

$$ S_{\lambda,\lambda}(x,y) = S^{(q)}(x,y) + \lambda_q \int dz S^{(q)}(x,z)\Gamma S^{(q)}(z,y) $$

is constructed for $\Gamma = \gamma_\lambda\gamma_5$ and flavors $q = \{u,d\}$, where $S^{(q)}(x,y)$ is the quark propagator of flavor $q$ and $\lambda_q$ is a constant (a similar approach is implemented in Ref. [46] in a different context). The second term in this expression is computed using standard sequential source techniques and the procedure can be repeated to produce propagators with higher-order couplings. These compound propagators are sufficient to construct the isovector matrix elements for zero momentum insertion in any hadronic or nuclear system (isoscalar responses, which also involve insertions on the sea-quark propagators, are not addressed). This work focuses on zero momentum–projected correlation functions

$$ C_{\lambda,\lambda}(t) = \sum_x \langle 0 | \chi_h(x, t) \lambda^{(0)}_{\lambda,\lambda}(0) | 0 \rangle $$

where $\langle \cdots | \lambda^{(0)}_{\lambda,\lambda} \rangle$ denotes the expectation value determined using the compound propagators. The interpolating operators for hadrons and nuclei, $\chi_h$, are those previously used to study the spectroscopy of these systems [47,48]. By construction, $C_{\lambda,\lambda}(t)$ is a polynomial of maximum order $\lambda u N u d N d$ in the field strengths, where $N(u,d)$ is the number of up (down) quarks in the particular interpolating operator.

**Details of the LQCD calculation.**—The calculations presented below used an ensemble of gauge-field configurations generated with a clover-improved fermion action [49] and a Lüster-Weisz gauge action [50]. The ensemble was generated with $N_f = 3$ degenerate light-quark flavors with masses tuned to the physical strange quark mass, producing a pion of mass $m_\pi \sim 806$ MeV, with a volume of $L^3 \times T = 32^3 \times 48$ and a lattice spacing of $a \sim 0.145$ fm (as determined from $\Upsilon$ spectroscopy). For these calculations, 437 configurations, with a spacing of ten trajectories between configurations, were used. Correlation functions were computed for $h = \{p, n, d, nn, np(1S_0), pp, ^3\text{H}, ^3\text{He}\}$ from propagators generated from a smeared source and either a smeared (SS) or point (SP) sink. Sixteen different source locations were averaged over on each configuration. Compound propagators and correlation functions were calculated at six different values of the background field strength parameter $\lambda = \{\pm0.05, \pm0.1, \pm0.2\}$. The axial current renormalization factor $Z_A = 0.867(43)$ was determined from computations of the vector current in the proton, noting that $Z_A = Z_\nu + O(a)$ and assigning a 5% systematic uncertainty associated with lattice-spacing artifacts (statistical uncertainties are negligible). A determination that removes the leading lattice-spacing artifacts leads to $Z_A = 0.8623(01)(71)$ [51,52] at a pion mass of $m_\pi \sim 317$ MeV.

**The proton axial charge.**—The simplest matrix element of the isovector axial current determines the axial charge of the proton. The correlation function $C_{\lambda,\lambda}(t)$ is at most quadratic in $\lambda_u$ and linear in $\lambda_d$ when constructed from the compound propagators $S_{\lambda_u,\lambda_d}(x,y)$ and $S_{\lambda_d,\lambda_u}(x,y)$, as the proton has two valence up quarks and one valence down quark. Consequently, using at least one (two) nonzero value(s) of $\lambda_{d(u)}$ enables extraction of the axial current matrix element as the linear response by using suitable polynomial fits. The difference of the up-quark and down-quark matrix elements can be used to construct the desired three-point function containing the isovector axial current. This can then be combined with the zero-field two-point...
The renormalization factor yields an axial charge of second is systematic (arising from choices of fit ranges and the finite extent of the time direction are suppressed by at least $e^{-2mT/3} \sim 10^{-7}$ in the signal region in the present set of calculations. The effective-$g_A$ plots resulting from the correlator differences are shown in Fig. 1, along with the result of a combined constant fit to the SS and SP ratios that extracts $g_A/Z_A$ from the late-time asymptote. The extracted value is $g_A/Z_A = 1.298(2)(7)$, where the first uncertainty is statistical (determined from a bootstrap analysis) and the second is systematic (arising from choices of fit ranges in both the field strengths and temporal separation as well as from differences in analysis techniques). Including the renormalization factor yields an axial charge of $g_A = 1.13(2)(7)$, which is consistent with previous determinations from standard three-point function techniques at this pion mass [53,54].

The GT matrix element for tritium $\beta$ decay.—The half-life of tritium, $t_{1/2}$, is related to the $F$ and GT matrix elements by [1]

$$\frac{(1 + \delta_R)f_{t/2}}{K/G_T} = \frac{1}{(F)^2 + f_A/f_{t/2}g_A(GT)^2},$$

where the factors on the left-hand side are known precisely from theory or experiment. On the right-hand side, $f_{t/2}$ denote known Fermi functions [55] and $(F)$ and $\langle GT \rangle$ are the $F$ and GT reduced matrix elements, respectively. The Ademollo-Gatto theorem [56] implies $(F) \sim 1$, modified only by second-order isospin breaking and by electromagnetic corrections. However, $\langle \beta|\bar q \gamma_5 \gamma^\tau q|^3H\rangle = \bar n\gamma_\tau \gamma^\tau u g_A(GT)$ (assuming vanishing electron mass and at vanishing lepton momentum) is less constrained, and its evaluation is the focus of this section.

By isospin symmetry, the spin-averaged GT matrix element for $^3H \rightarrow ^3He e^-\bar{\nu}$ is related to the axial charge of the triton, $g_A(\text{H})$, when the light quarks are degenerate and in the absence of electromagnetism. Analogous to $R_p(t)$ in Eq. (3), the ratio $R_H(t)$ of correlation functions in background fields is constructed such that, analogous to Eq. (4), $R_H(t)$ goes to $g_A(\text{H})/Z_A$ in the large-time limit. The analysis of these correlation functions is more complex than for the proton because the triton has four up quarks and five down quarks and the correlators are thus quartic and quintic polynomials in $\lambda_{u,d}$, respectively. Polynomial fits to the calculated correlation functions are sufficient to extract the terms linear in $\lambda_{u,d}$, respectively. Results for $R_H(t)$ are shown in Fig. 2 along with a constant fit to the asymptotic value $g_A(\text{H})/Z_A$. Also shown in Fig. 2 is $\langle GT \rangle(t) = \bar R_H(t)/R_p(t)$, which is independent of $Z_A$, and the fit to its asymptotic value $g_A(\text{H})/g_A$. Analyses of these ratios lead to

$$g_A(\text{H})/Z_A = 1.272(6)(22), \quad g_A(\text{H})/g_A = 0.979(3)(10),$$

where the first uncertainties are statistical and the second arise from systematics as described for $g_A$. The result for $g_A(\text{H})/g_A$ is quite close to the precise, experimentally determined value of $\langle GT \rangle = 0.9511(13) [6]$ at the physical quark masses. In the context of $\varphi$EFT, the short-distance two-nucleon axial-vector operator, with coefficient $L_{1A}[4]$, is expected to give the leading contribution to the difference of this ratio from unity [57].

The low-energy proton-proton fusion cross section.—The low-energy cross section for $pp \rightarrow d e^+\nu$ is dictated by the matrix element

$$\langle d; j|A_A^q|[pp] \rangle = g_A C_q \sqrt{\frac{32\pi}{3}} \Lambda(p) \delta_{jk},$$

where $A_A^q(x)$ is the axial current with isospin and spin components $a$ and $k$, respectively, $j$ is the deuteron spin index, $C_q$ is the Sommerfeld factor, and $\gamma$ is the deuteron binding momentum. The quantity $\Lambda(p)$ has been calculated.
mixing between the two-nucleon channels induced by an isovector magnetic field was treated by diagonalizing a (channel-space) matrix of correlators and determining the splittings between energy eigenvalues. This provided access to the matrix element dictating the transition matrix element in the finite lattice volume:

\[
R_{\lambda_u,\lambda_d=0}^{(3)S_1;S_0}(t) = \frac{C_{\lambda_u,\lambda_d=0}^{(3)S_1;S_0}(t)}{\sqrt{C_{\lambda_u,\lambda_d=0}^{(3)S_1;S_0}(0)C_{\lambda_u,\lambda_d=0}^{(3)S_1;S_0}(0)}}.
\]

Consequently, the difference between ratios at neighboring timeslices determines the isovector matrix element:

\[
\mathcal{R}_{\lambda_u,\lambda_d=0}^{(3)S_1;S_0}(t) = \frac{R_{\lambda_u,\lambda_d=0}^{(3)S_1;S_0}(t+1) - R_{\lambda_u,\lambda_d=0}^{(3)S_1;S_0}(t)}{t \to \infty} \left(3^3S_1; J_z = 0 | A_3^{(3)S_1;S_0}; I_z = 0 \right) / Z_A = 2.568(5)(31),
\]
FIG. 3. Ratios of correlation functions that determine the
unrenormalized isovector axial matrix element in the \( J_z = I_z = 0 \)
coupled two-nucleon system (upper panel), and the unrenormal-
ized difference between the axial matrix element in this channel
and \( 2g_A \) (lower panel). The orange diamonds (blue circles)
correspond to the SS (SP) effective correlator ratios and the bands
correspond to fits to the asymptotic plateau behavior and include
only the statistical and fitting systematic uncertainties (the additional 1% uncertainty from Wigner symmetry breaking is
not represented in the bands).

and the finite-volume effects in the matrix elements are
negligible [62,63]. At lighter values of the quark masses,
where the \( np(1S_0) \) system and/or the deuteron is unbound
or only weakly bound, the connection between finite-
volume matrix elements and transition amplitudes requires
the framework developed in Refs. [62,63].

To isolate the two-body contribution, the combination
\[ L_{1A}^{sd,2b}(t)/Z_A = \frac{[R_{S_1,S_0}(t) - 2R_p(t)]}{2} \]
form is shown in the lower panel of Fig. 3. Taking advantage of the near
degeneracy of the \( 3S_1 \) and \( 3S_2 \) two-nucleon channels at the
quark masses used in this calculation, it is straightforward
to show that this correlated difference leads directly to the
short-distance two-nucleon quantity \( L_{1A}^{sd,2b} \). Fitting a constant
to the late-time behavior of this quantity leads to

\[
\frac{L_{1A}^{sd,2b}}{Z_A} = \langle S_1; J_z = 0|A_{1A}^1 S_0; I_z = 0 \rangle - 2g_A
\]

\[
= -0.011(15),
\]

where the first uncertainty is statistical and the second
encompasses fitting and analysis systematics.

In light of the mild quark-mass dependence of the
analogous short-distance, two-body quantity contributing
to \( np \rightarrow dy \) [39], \( L_{1A}^{sd,2b} \) is likely to be largely insensitive
to the pion mass between \( m_\pi \sim 806 \) MeV and its physical
value. This approximate independence and the associated
systematic uncertainty will need to be refined in subsequent
calculations. Based on this expectation, the result obtained
here at \( m_\pi \sim 806 \) MeV is used to estimate the value of

\[
L_{1A}^{sd,2b}
\]
at a renormalization scale \( \mu = m_\pi \). The four uncertainties
are the statistical uncertainty, the fitting and analysis
systematic uncertainty, the mass extrapolation systematic
uncertainty, and a power-counting estimate of higher order
corrections in \( \pi \text{EFT} \), respectively. This value is also very
close to previous phenomenological estimates, as summa-
rized in Refs. [11,14].

Summary.—The primary results of this work are the
isovector axial-current matrix elements in two- and three-
nucleon systems calculated directly from the underlying
theory of the strong interactions using lattice QCD (see also
the Supplementary Material [64]). These matrix elements
determine the cross section for the \( pp \) fusion process \( pp \rightarrow
de^+\nu \) and the Gamow-Teller contribution to tritium \( \beta \) decay,
\( ^3H \rightarrow ^3\text{He} e^-\bar{\nu} \). While the calculations are performed at
unphysical quark masses corresponding to \( m_\pi \sim 806 \) MeV
and at a single lattice spacing and volume, the mild mass
dependence of the analogous short-distance quantity in the
\( np \rightarrow dy \) magnetic transition enables an estimate of the
\( pp \rightarrow de^+\nu \) matrix element at the physical values of
the quark masses, and the results are found to agree within
uncertainties with phenomenology. Future LQCD calcula-
tions, including electromagnetism beyond Coulomb
effects, at lighter quark masses with isospin splittings,
larger volumes, and finer lattice spacings, making use of the
new techniques that are introduced here, will enable
extractions of these axial matrix elements with fully
quantified uncertainties and will be important for phenom-
enology, providing increasingly precise values for the \( pp \)
fusion cross section and GT matrix element in tritium
\( \beta \) decay.

Beyond the current study, background axial-field
calculations also allow the extraction of second-order, as
well as momentum-dependent, responses to axial fields.
Second-order responses are important for determining nuclear $\beta\beta$-decay matrix elements, both with and without (for a light Majorana neutrino) the emission of associated neutrinos [70]. Momentum-dependent axial background fields will allow the determination of nuclear effects in neutrino-nucleus scattering. In both cases, LQCD calculations of these quantities in light nuclei will provide vital input with which to constrain the nuclear many-body methods that are used to determine the matrix elements for these processes in heavy nuclei.

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Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.062002 provides additional details pertaining to the correlation functions, axial current renormalization, effective masses, uncertainty propagation in EFT matching, and quark mass dependence and also includes Refs. [65–69].