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Packet Acquisition for Time-Frequency Hopped Asynchronous Random Multiple Access

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Abstract—Packet acquisition for a time-frequency hopped asynchronous random multiple access (RMA) system is investigated. A novel analytical approach to performance evaluation is provided, which enables the waveform designer to quickly evaluate the network performance as various waveform design and RMA protocol parameters and are chosen. The results show that the network throughput is estimated and upper-bounded by the acquisition performance. The performance evaluation procedure is broadly general, allowing for variations in waveform parameters, network topology, network population and population heterogeneity (in, e.g., rate, transmit power, and medium access behavior). To illustrate the accuracy of the analytical results, simulations are performed over a wide range of waveform, RMA protocol and network parameters.

I. INTRODUCTION

Random multiple access (RMA) allows members of a network to transmit data packets at nondeterministic and uncoordinated times. This medium access methodology can achieve low latency and simplify network management. Well known examples of RMA protocols are the Aloha [1]–[3], time-slotted Aloha [4] and carrier-sense multiple access [5]. RMA also permits time and frequency hopping to be incorporated into the physical-layer structure of the packet to provide over-the-air security, frequency diversity, and protection from interferences [6]–[8]. RMA can also be used with code division multiple access (CDMA) [9] and frequency hopped CDMA [10].

Existing performance results for RMA protocols rely on simplifying assumptions, such as infinite-population and Poisson traffic process [1]–[3]. In [9], a broader simplification is made by assuming a Poisson traffic process in a finite-population network where each user transmits packets at a given rate and is subject to a brick-wall collision model. Although such an analysis provides a simple performance approximation, practical situations fail to uphold the underpinning assumptions. For a finite-population network where each user transmits at a non-negligible rate, it is not even mathematically consistent to impose the Poisson traffic model. Furthermore, when packets are sent at random times, the receiver must first detect the arrival of each packet in order to demodulate it. This suggests that the Aloha throughput results of [1]–[3], for instance, implicitly assume that packet acquisition is perfect regardless of the noise level and network offered load. However, as shown herein, packet acquisition performance degrades as the offered load increases when multiple access interference (MAI) is taken into account.

In this paper we determine analytically the impact of MAI and noise on the acquisition performance by characterizing the MAI occurring according to the time-frequency hopped RMA protocol used. In [11], slot-synchronous random multiple access (SSRMA) is considered. Here, we investigate asynchronous RMA (ARMA). Like in the SSRMA protocol, each physical layer packet is divided into equal time segments called pulses; each pulse is transmitted in only one frequency channel whereas a packet can be transmitted over multiple frequency channels. However, the ARMA lifts the slot-synchronous restriction, and packets from different users are not assumed to be time-aligned at the receiver of interest.

Throughout the paper, $p_{\cdot\cdot}$ denotes probability mass function (PMF). For example, $p_A[a]$ is the PMF of $A$ evaluated at $a$ and $p_{A|B}[a|b]$ is the conditional PMF of $(A|B)$.

II. SIGNAL MODEL AND ARMA PROTOCOL

Consider a system in which each user independently transmits multi-pulse packets over $N_f$ frequency channels. The carrier frequency for each pulse is drawn pseudo-randomly from the set of $N_f$ frequencies. Each pulse is transmitted in a pulse slot (PS) having a time duration longer than that of the pulse, and the start time of the pulse in its PS is pseudo-randomly determined. Thus, the pulses of a packet follow a time-frequency hopping pattern. A packet addressed to a particular receiver follows a particular hopping pattern known a priori to the receiver. In addition, the ARMA protocol considered herein adopts the following:

1) Each pulse consists of $L_d$ chips of payload data, followed by $L$ synchronization (sync) chips followed by another $L_d$ chips of payload data. The pulse length is $\bar{L} \overset{\Delta}{=} 2L_d + L$ chips. The PS duration is $T_w \geq \bar{L}$ chips.
2) Two consecutive packets transmitted by the same node are separated in time by a random number of idle (non-transmit) pulse slots (IPS). For user $u$, denote this random variable by $\Psi_u \overset{\Delta}{=} 0$ with PMF $p_{\Psi_u}[\psi] = P_r[\Psi_u = \psi]$, which controls the average packet rate of the user. Random backoff time, a way to moderate network load, is thus incorporated into $\Psi_u$.
3) The number of pulses per packet for user $u$ is $N_u$, allowing the packet size to vary from user to user.
The pulse slots of a packet are connected but non-overlapping time intervals, each of duration $T_w$ and allocated to exactly one pulse.

The packet structure can be such that the sync preamble (a collection of pulses to be used for packet acquisition) precedes the payload; in this case $L_d$ in item 1) above is set to zero for the sync pulses, and each payload pulse consists of $L$ chips of data. The analysis below covers this case as well. From the perspective of each transmitter, this case is similar to the SSRMA protocol considered in [11], except that it permits random back-off and asynchronous packet arrivals.

Item 2) above permits any traffic process that can be generated by a transmitter. For example, a node transmits packets at equally spaced times when $\Psi_u$ is deterministic, i.e., held fixed at some $\psi_0$; then $p_{\Psi_u}[\psi] = \delta[\psi - \psi_0]$, where $\delta[\cdot]$ is the Kronecker delta function. In addition, when packets are sent back to back (no backoff time), $\psi_0 = 0$. As another example, if each PS following the end of a transmitted packet is associated with a Bernoulli trial with success probability $p$ that a new packet begins in that PS, then $\Psi_u$ admits the geometric distribution $(1 - p)^{\psi_0}$, $\psi_0 \in \{0, 1, 2, \ldots\}$.

We investigate the acquisition performance of a particular receiver when $N_I$ interfering users also transmit using the same $N_I$ carriers. For complexity considerations, the receiver may choose to process only the first $N_d \leq N$ pulses in determining the arrival time of a packet, where $N$ denotes the number of pulses per packet for the desired user. To acquire a packet, the receiver must tune to all the frequencies that are used to send the $N_d$ pulses. The down converted received signal is sampled at the chip rate (or higher) and shifted through a buffer at the same rate. To investigate the acquisition performance, we examine the observed signal at the time when the packet arrives at the receiver of interest. That is, for the $k$-th sync pulse (or the sync part of the $k$-th pulse when each pulse carries payload data), the sampled observed signal is represented by

$$r_k = s_kg_k + v_k + n_k \quad (2.1)$$

for $k = 1, 2, \ldots, N$. In the above, $s_k = [s_{k, 1}, s_{k, 2}, \ldots, s_{k, L}]^T$ represents the sampled version of the transmitted sync pulse known a priori to the receiver; the unknown complex scalar $g_k$ represents the channel gain; $v_k = [v_{k, 1}, v_{k, 2}, \ldots, v_{k, L}]^T$ represents the total interfering signal at this particular time and $n_k = [n_{k, 1}, n_{k, 2}, \ldots, n_{k, L}]^T$ represents the background noise, modeled as white Gaussian noise (WGN).

The MAI signal $v_k$ comes from the $N_I$ interfering users, i.e., $v_k = \sum_{u=1}^{N_I} \tilde{s}_{k,u}g_{k,u}$, where $\tilde{s}_{k,u}$ represents the interfering contribution from the $u$-th user and $g_{k,u}$ denotes the associated channel gain. Clearly, $\tilde{s}_{k,u}$ is zero when the $u$-th user does not interfere with the $k$-th pulse of the user of interest; this situation arises when there is no overlap, in both time and frequency, at the packet arrival time being considered. However, $\tilde{s}_{k,u}$ contains some zero and some nonzero elements when there is a collision in frequency and partial overlap in time. When there is a total overlap, all elements of $\tilde{s}_{k,u}$ are nonzero. A key part of the analysis is to analytically determine the probability distribution of this overlap.

Without loss of generality, we assume for convenience that the transmitted pulses for all users have unit modulus. Specifically, $|s_{k,l}| = 1$ for all $k, l$. Each nonzero element of $\tilde{s}_{k,u}$ also has magnitude 1. For all practical purposes, sync sequences belonging to different pulses are assumed independent, and each sync sequence is assumed white.

To determine the arrival time of a packet, at each chip time the receiver tests the following hypotheses against each other:

$$H_1 : r_k = s_kg_k + v_k + n_k, \quad H_0 : r_k = v_k + n_k \quad (2.2)$$

for $k = 1, 2, \ldots, N_d$. In the above, $n_k \sim CN(0, N_kI_L)$, where $I_L$ denotes the $L \times L$ identity matrix and $N_k$ represents the noise variance for the $k$-th pulse.

III. ACQUISITION FOR THE WGN CHANNEL

A. GLRT-vote detection rule

Consider the GLRT-vote detection rule described in [11], where each sync pulse is detected individually using the generalized likelihood ratio test (GLRT) [12] at each chip time. If enough pulses are declared present by the GLRT, the packet is declared present. Otherwise, the packet is declared absent. This can be described as follows:

$$d_k \triangleq \begin{cases} 1, & ||U_kr_k||^2 > \alpha||I_L - U_k||^2D \ni \ni H_1 \\ 0, & \text{otherwise} \end{cases} \quad (3.1)$$

$$D \triangleq \sum_{k=1}^{N_d} d_k \leq \frac{D}{H_0} \quad (3.2)$$

where $U_k \triangleq L^{-1}s_kg_k^H$. The threshold factor $\alpha$ can be controlled to tradeoff between false-alarm and detection probabilities, and $D$ is the vote threshold, which should be chosen jointly with $\alpha$ to achieve a given packet false-alarm probability and maximize the detection probability. When $D = [N_d/2]$, (3.2) is a majority-vote decision rule.

The $k$-th pulse detection probability is $P_{dk} \triangleq \Pr[d_k = 1|H_1]$, and the packet detection probability is $P_d \triangleq \Pr[D \ni \ni H_1]$. The detector performance is usually characterized in terms of the packet missed detection probability $P_m \triangledown 1 - P_d$.

B. Probability of detection

The variables $|g_k|^2$ and $N_k$ can be interpreted respectively as the received signal chip energy and the noise power spectral density by, without loss of generality, normalizing the chip duration to one time unit. Since the $N_k$’s are allowed to be different and, as seen below, $P_{dk}$ depends on $|g_k|^2$ only via the ratio $|g_k|^2/N_k$, it can be assumed without loss of generality that the signal chip energy, denoted by $E_c$, is the same for all the pulses of a packet, i.e., $E_c = |g_1|^2 = |g_2|^2 = \cdots = |g_N|^2$. Define

$$\lambda_k = \frac{2LE_c}{N_k} = \frac{2E_p}{N_k}, \quad (3.3)$$

where $E_p \triangleq LE_c$ represents the sync pulse’s energy. It is shown in [11] that for the WGN channel, the pulse detection
The probability is
\[ P_{1k} = 1 - F(\alpha(L-1); 2, 2(L-1), \lambda_k) \]  
where \( F(x; n, m, \lambda) \) is the cumulative distribution function (CDF) of the noncentral \( F \)-distribution \( F_{n,m}(\lambda) \) evaluated at \( x \) [13]. This function can be easily evaluated using computing software packages such as Matlab.

When \( N_1 = N_2 = \cdots = N_{N_u} \) (i.e., all pulses experience the same background noise power), \( P_{1k} = P_{11} \) for all \( k = 1, 2, \ldots, N_d \) and the probability of packet detection is
\[ P_d = 1 - \sum_{n=0}^{D-1} \binom{N_d}{n} P_{11}^n (1 - P_{11})^{N_d-n} . \]  
The case where the background noises in the pulses have different power levels requires a simple generalization and is treated in [11].

C. Probability of false alarm

It is shown in [11] that the \( k \)-th pulse false-alarm probability is independent of the noise power and is given by
\[ P_{1k} \triangleq P_r[d_k = 1|H_0] = f_T^{-1} \],
where \( f_T \triangleq \alpha + 1 \). The packet false-alarm probability is
\[ P_1 \triangleq P_r[D \geq D|H_0] = \sum_{n=D}^{N_d} \binom{N_d}{n} P_{11}^n (1 - P_{11})^{N_d-n} . \]

IV. PERFORMANCE UNDER ARMA

A. MAI characterization

We characterize the MAI to any particular desired pulse (DP) transmitted at some particular frequency, say \( f_0 \) for brevity; a DP is defined as a pulse belonging to the desired user. To this end, we examine how the DP is affected by any interfering packet (IPac) from any particular interfering user, say the \( u \)-th user. For consistency, the IPSs immediately following a packet are associated with that packet as illustrated in Fig. 1. The arrival time (AT) of a packet is defined as the AT of the first chip position of the first PS of the packet. Given that a packet is an IPac to the DP, the earliest and latest possible ATs of the DP are, respectively, the AT of the IPac and the last chip position of the last IPS (or the last chip position of the last PS when \( \Psi_u = 0 \)).

For a given IPac, the number \( T_u \) of overlapped chips between the \( u \)-th user and the DP’s sync part depends only on the arrival time difference between the DP and the IPac. Since all users transmit independently, it is simple to see that \( T_u \) and \( T_v \) are independent for \( u \neq v \) by choosing the DP’s AT as the reference and assigning to it an arbitrary fixed value.

The objective in this subsection is to determine the PMF of \( T_u \). To do so, it is more convenient to choose the IPac arrival time as the reference by assigning to it a value of 1. This is analogous to saying that the DP is "thrown" randomly into the traffic generated by the \( u \)-th interfering user, and the packet whose time span contains the AT of the DP becomes the IPac. Then, the DP’s arrival time \( \Theta \) falls in \( A_{\psi} \triangleq \{1, 2, \ldots, (N_u + \psi)T_w\} \) given \( \Psi_u = \psi \). Since no particular arrival time should be considered more likely than another, the AT of the DP is uniform, i.e.,
\[ p(\psi|\Psi_u) \triangleq \begin{cases} \frac{1}{(N_u + \psi)T_w}, & \theta \in A_{\psi} \\ 0, & \text{otherwise}. \end{cases} \]  

Let \( S \) and \( B \) be independent random variables respectively admitting the uniform distributions
\[ p_S[s] = \begin{cases} \frac{1}{N_u + \psi}, & 1 \leq s \leq N_u + \psi \\ 0, & \text{otherwise}. \end{cases} \]
\[ p_B[b] = \begin{cases} \frac{1}{T_w}, & 1 \leq b \leq T_w \\ 0, & \text{otherwise}. \end{cases} \]

It is easy to verify that \( (\Theta|\Psi_u) \) admits the PMF (4.1) if
\[ \Theta = (S - 1)T_w + B . \]  

Fig. 1 depicts a random realization in which \( \Psi_u = \psi \) and \( S = s \). Given \( S = s \), there are two pulses that can interfere with the DP: the left interfering pulse (LIP) which occurs in the \( s \)-th PS and the right interfering pulse (RIP) which occurs in the \((s+1)\)-th PS. When \( s = N_u + \psi \), the RIP is the first pulse of the packet transmitted immediately after IPac. If \( t_1 \) and \( t_2 \) are the arrival times of two pulses, then the number of chips that the sync part of one pulse overlaps with the other pulse in time (regardless of frequency) is
\[ \Omega(|\Delta t|) \triangleq \begin{cases} L, & |\Delta t| \leq L_d \\ L + L_d - |\Delta t|, & L_d < |\Delta t| < L + L_d \\ 0, & L + L_d \leq |\Delta t| \end{cases} \]
where \( \Delta t \triangleq t_1 - t_2 \). The arrival time of the \( s \)-th pulse of the IPac is \((s-1)T_w + A_s \), where \( A_s \) is uniformly distributed as determined by the time hopping pattern, i.e.,
\[ p_{A_s}[a] \triangleq \begin{cases} \frac{1}{T}, & 1 \leq a < T \\ 0, & \text{otherwise}. \end{cases} \]
where
\[ T \triangleq T_w - \bar{L} + 1 . \]  

Similarly, the arrival time of the \((s+1)\)-th pulse of the IPac is \( sT_w + A_{s+1} \), where \( A_{s+1} \) and \( A_s \) are i.i.d. since different pulses are jittered independently. Let \( i_s \) denote a binary random variable such that \( i_s = 1 \) when the \( s \)-th pulse is transmitted at frequency \( f_0 \) and \( i_s = 0 \) otherwise. Since all frequencies are equally likely to be chosen to carry a pulse, the PMF of \( i_s \) for a given \( s \) is determined by
\[ p_{i_s}[k] = \begin{cases} 1/N_t, & k = 1, 1 \leq s \leq N_u \\ 1 - 1/N_t, & k = 0, 1 \leq s \leq N_u \\ 1, & k = 0, N_u < s . \end{cases} \]  

Note that (4.8c) follows from the fact that during an IPS of a packet, no pulse is transmitted. Let \( T_u^* \) and \( T_u^* \) denote the
number of interfering chips that the DP’s sync part receives from the LIP and RIP, respectively. We have
\[
(T_u^- | S, B, i_S, \Psi_u) = i_S \Omega(|A_S - B|) \quad (4.9)
\]
for \(1 \leq S \leq N_u + \Psi_u\). Also,
\[
(T_u^+ | S, B, i_{S+1}, \Psi_u) = i_{S+1} \Omega(T_u + A_{S+1} - B) \quad (4.10)
\]
for \(1 \leq S \leq N_u + \Psi_u - 1\). For \(S = N_u + \Psi_u\), the RIP is the first pulse of the following packet, so
\[
(T_u^+ | S, B, i_1, \Psi_u) = i_1 \Omega(T_u + 1) \quad (4.11)
\]
where \(i_1\) and \(A_1\) have the same meanings and PMFs as those of \(i_t\) and \(A_t\), respectively, but for the first pulse of the following packet.

To find the PMF of \(T_u\), we first find the PMFs of \(T_u^-\) and \(T_u^+\). Define the cumulative PMF of \(p_t\), defined in (4.6), as
\[
P[t] \triangleq \Pr[A_s \leq t] = \sum_{\tau=1}^t p[\tau]. \quad (4.12)
\]
Also define
\[
p^-[t|b] \triangleq \begin{cases} 
1 - \frac{P[b+L+La]}{P[b+L+La-1]} & , \quad t = 0 \\
\frac{P[b+L+La]-P[b+L+La-1]}{P[b+L+La]} & , \quad 1 \leq t < L \\
0 & , \quad t = L \\
\end{cases} \quad (4.13)
\]
\[
p^+[t|b] \triangleq \begin{cases} 
1 - \frac{P[b+L+La-T_u^-]}{P[b+L+La-T_u^-]} & , \quad t = 0 \\
\frac{P[b+L+La-T_u^-]}{P[b+L+La-T_u^-]} & , \quad 1 \leq t < L \\
0 & , \quad t = L \\
\end{cases} \quad (4.14)
\]

By averaging the PMFs of \((T_u^- | S, B, i_S, \Psi_u)\) and \((T_u^+ | S, B, i_{S+1}, \Psi_u)\) over \(i_s\) and \(i_{s+1}\), it is straightforward to show that
\[
p_{T_u^-}[s, B, \Psi_u][t|s, b, \psi] = \begin{cases} 
p^-[t|b], & 1 \leq s \leq N_u \quad (4.15a) \\
p^+[t|b], & s > N_u \quad (4.15b)
\end{cases}
\]
\[
p_{T_u^+}[s, B, \Psi_u][t|s, b, \psi] = \begin{cases} 
p^+[t|b], & 1 \leq s \leq N_u \quad (4.16a) \\
p^-[t|b], & s > N_u \quad (4.16b)
\end{cases}
\]
Note that (4.15b) and (4.16b) come from the fact that there is no interference from the LIP or the RIP when the \(s\)-th and \((s+1)\)-th pulse slots are IPSs, respectively; (4.16c) accounts for the interference from the first pulse of the next packet when the DP arrives during the last IPS of the IPac.

The number of interfering chips that the DP receives is
\[
T_u = T_u^- + T_u^+. \quad (4.17)
\]
Since different pulses are jittered independently, \(T_u^-\) and \(T_u^+\) are independent given \(S\) and \(B\). Therefore, the PMF of \((T_u | S, B, \Psi_u)\) is
\[
Pr_{T_u}[s, B, \Psi_u][t|s, b, \psi] = (Pr_{T_u^-}[S, B, \Psi_u] \otimes Pr_{T_u^+}[S, B, \Psi_u])[t|s, b, \psi] \quad (4.18)
\]
where \(\otimes\) denotes the discrete time convolution in the first argument. Note that since the LIP and RIP occupy disjoint time intervals, \(Pr_{T_u^-}[S, B, \Psi_u][t|s, b, \psi] = 0\) for \(t \notin \{0, 1, 2, \ldots, L\}\), so the last \(L\) elements of the convolution can be ignored. Although \(Pr_{T_u^-}[S, B, \Psi_u][t|s, b, \psi]\) is a 4-variate function, \(Pr_{T_u}[t]\) can be evaluated using only several bi-variate functions. To do this, define
\[
p^+[t|b] \triangleq (p^- \otimes p^+)[t|b]. \quad (4.19)
\]
For \(s \in \{1, 2, \ldots, N_u - 1\}\), \(Pr_{T_u^-}[S, B, \Psi_u][t|s, b, \psi]\) and \(Pr_{T_u^+}[S, B, \Psi_u][t|s, b, \psi]\) are constant over \(s\) and \(\psi\), leading to
\[
Pr_{T_u}[s, B, \Psi_u][t|s, b, \psi] = p^+[t|b]. \quad (4.20)
\]
When \(\psi = 0\), (4.20) holds also for \(s = N_u\) and
\[
Pr_{T_u}[s, B, \Psi_u][t|s, b, \psi] = p^+[t|b] \equiv \frac{1}{T_w} \sum_{b=1}^{T_w} p^+[t|b]. \quad (4.21)
\]
When \(\psi = 1\), (4.15) and (4.16) yield
\[
Pr_{T_u}[S, B, \Psi_u][t|s, b, \psi] = \begin{cases} 
p^+[t|b], & 1 \leq s < N_u \quad (4.22a) \\
p^-[t|b], & s = N_u \quad (4.22b) \\
p^+[t|b], & s = N_u + 1\quad (4.22c)
\end{cases}
\]
which leads to
\[
Pr_{T_u}[\Psi_u][t|\psi] = \frac{(N_u - 1)p^+[t] + p^-[t] + p^+[t]}{N_u + 1}, \quad (4.23)
\]
where
\[
p^-[t] \triangleq \frac{1}{T_w} \sum_{b=1}^{T_w} p^-[t|b] \quad (4.24)
\]
\[
p^+[t] \triangleq \frac{1}{T_w} \sum_{b=1}^{T_w} p^+[t|b] \quad (4.25)
\]
For any $\psi > 1$

$$p_{\tau_{\psi} | s, B, \psi}(t | s, b, \psi) = \begin{cases} p^\pm[t | b], 1 \leq s < N_u & (4.26a) \\ p^\pm[t | b], s = N_u & (4.26b) \\ \delta[t], N_u < s < N_u + \psi & (4.26c) \\ p^\pm[t | b], s = N_u + \psi & (4.26d) \end{cases}$$

leading to

$$p_{\tau_{\psi} | s} (t | \psi) = \frac{(N_u - 1)p^\pm[t] + p^-[t] + (\psi - 1)\delta[t] + p^+[t]}{N_u + \psi}. \quad (4.27)$$

Finally,

$$p_{\tau_{\psi} | s}(t) = \sum_{\psi} p_{\tau_{\psi} | s}(t | \psi)p_{\psi}(\psi). \quad (4.28)$$

The highest data dimensions encountered in computing (4.28) are $(L + 1) \times T_w$, which are the spans of $t$ and $b$, occurring in (4.13) and (4.14). Thus, the evaluation of (4.28) is straightforward and computationally inexpensive.

The total number of chips that the DP receives from all interfering users is $T_\psi = \sum_{u=1}^{N_i} T_u$. Since packets from different users arrive independently, the PMF of $T_\psi$ is

$$p_{\tau_{\psi}}[t] = (p_{\tau_1} \otimes p_{\tau_2} \otimes \cdots \otimes p_{\tau_{N_i}})[t]. \quad (4.29)$$

### B. Probability of detection

To evaluate the pulse detection probability, consider first the case where all interfering users have the same chip energy, $E_1$.

Using the analysis in [11], we have

$$P_{\text{th}} = 1 - \sum_{t=0}^{N_i L} p_{\tau_{\psi}}[t] F \left( \frac{2L E_1}{\bar{N}_k}, \frac{2L E_1}{\bar{N}_k} \right). \quad (4.30)$$

The case where interferers have different chip energies can be treated with a straightforward generalization. Due to space limitation, we describe it here only briefly by noting that the noncentrality parameter now has the form $\lambda_k = \frac{E_1}{\bar{N}_k + 2L}$, where $\bar{N}_k \triangleq \sum_u e_u T_u$, with $e_u$ being the chip energy of user $u$. The probability distribution $p_{\Omega | \psi}$ of $\Omega$ can be obtained directly from the PMFs of the $T_u$’s. The result follows by taking the expectation of $F(\cdot; \cdot; \psi, \lambda_k)$ over $p_{\Omega | \psi}$.

Assuming that MAI affects different pulses independently, the packet detection probability is given by (3.5) when the $N_k$’s are equal, with $P_{\text{th}}$ given by (4.30). This independence assumption is expected to hold well for all practical purposes. We verify it in Section VI by employing simulations to evaluate $P_{\text{th}}$ directly.

### C. Probability of false alarm

If the interferers have the same chip energy $E_1$, the pulse false-alarm probability is well approximated by

$$\begin{align*}
P_{\text{fa}} &= 1 - \sum_{t=0}^{N_i L} p_{\tau_{\psi}}[t] \int_0^\infty \chi^2(w; 2) \\
&\times F \left( \frac{\alpha(L - 1) + \varphi E_1}{\bar{N}_k L}; 2, 2L - 2, \frac{E_1 w}{\bar{N}_k L} \right) dw. \quad (4.31)\end{align*}$$

where $\chi^2(w; k)$ is the $\chi^2$ PDF and $\varphi$ is a fine-tuning parameter, $0.7 < \varphi \leq 1$ (see [11]). Similarly, for the case where the interferers have different chip energies,

$$P_{\text{fa}} = 1 - \sum_{\omega} p_{\Omega | \psi}(\omega) \int_0^\infty \chi^2(w; 2) \times F \left( \frac{\alpha(L - 1) + \varphi \omega}{\bar{N}_k L}; 2, 2L - 2, \frac{\omega w}{\bar{N}_k L} \right) dw. \quad (4.32)$$

The packet false-alarm probability can be evaluated as described in [11].

### V. Network Capacity Considerations

#### A. Throughput

An estimate of the network throughput can be obtained. Consider a single-hop network with only unicast traffic, i.e., each packet is intended for just one receiver. Let $r_v$ index the receiver for which packets transmitted from user $v$ are intended. From the viewpoint of receiver $r_v$, there are $N_1$ interfering users and user $v$ is the desired user.

Let $P_{\text{det}}$ denote the packet detection probability for transmitting user $v$ at its intended receiver, $r_v$. Let $P_{\text{suc}}$ denote the probability that a packet from user $v$ is successfully demodulated (decoded) at receiver $r_v$. We have $P_{\text{suc}} = P_{\text{det}} P_{\text{bit}}$, where $P_{\text{det}}$ denotes the conditional probability that a packet is successfully demodulated given that it is detected. For a properly designed system, $P_{\text{suc}}$ should be reasonably close to 1, so $P_{\text{suc}} \approx P_{\text{det}}$.

Following [11], the throughput is

$$\text{Throughput} = \sum_{v=1}^{N_1} \left( \sum_{w=1}^{N_v} P_{\text{det}}(\psi_v) \right) = \sum_{v=1}^{N_1} \left( \sum_{w=1}^{N_v} \frac{P_{\text{det}}(\psi_v)}{N_v} \right).$$

The average packet duration is

$$T_v = \frac{\sum_{w=1}^{N_v} \frac{P_{\text{det}}(\psi_v)}{N_v}}{\sum_{v=1}^{N_1} \left( \sum_{w=1}^{N_v} \frac{P_{\text{det}}(\psi_v)}{N_v} \right)}.$$

The network capacity is

$$C_{\text{net}} = \frac{1}{\text{Throughput}}.$$

### Summing (5.1) over $v$ gives, for the single-hop case, the network throughput

$$R_{\text{th}} = \sum_{v=1}^{N_1} R_v = \sum_{v=1}^{N_1} \frac{1}{P_{\text{det}}(\psi_v)}.$$

We call $R_{\text{th}}$ the detection throughput (DTP). Inequality (5.2) shows that the DTP represents an upper bound to the throughput. The closer is $P_{\text{det}}(\psi_v)$ to 1, the tighter is the bound. For example, if $P_{\text{det}}(\psi_v) = 1 - 10^{-3}$, the throughput is within 0.1% of the bound. The offered load $\ell \triangleq \sum_{v=1}^{N_1} R_{\text{th}}$ can be chosen (e.g., by varying the network population) such that the throughput reaches the network capacity

$$C_{\text{net}} = \max_{\ell} R_{\text{th}} \approx C_{\text{det}} = \max_{\ell} R_{\text{det}}.$$

We refer to $C_{\text{det}}$ as the detection capacity (DCap).
B. Relationship between DTP and offered load

Consider a homogeneous single-hop network, where all the users are alike in every aspect. Suppose there are \( N_1 + 1 \) transmitting users generating unicast traffic to \( N_1 + 1 \) receiving (non-transmitting) nodes, each being addressed by exactly one transmitting user and seeing \( N_1 \) interferers in addition to its desired user. Examine the case where \( E_c/E_s \) and \( E_c/N_0 \) stay the same for all receiving nodes, where \( N_0 \) denotes the background noise power spectral density. Let \( R_{\text{det}} = \frac{R_{\text{pac}}}{N_0} \) and \( K_0 = K_0 \) for all transmitting nodes. Then, \( R_{\text{det}} \) in (5.2) simplifies to

\[
R_{\text{det}} = N_{\text{pop}} R_{\text{pac}} K_0 P_d(N_{\text{pop}}),
\]

where \( N_{\text{pop}} \triangleq N_1 + 1 \) and \( P_d(N_{\text{pop}}) \) as a function of \( N_{\text{pop}} \) is the packet detection probability which is the same for all users. Dividing by the single-user offered load \( R_{\text{pac}} K_0 \) we get the normalized DTP (NDTP) and normalized DCap (NDCap)

\[
r_{\text{det}} \triangleq N_{\text{pop}} P_d(N_{\text{pop}}),
\]

\[
\epsilon_{\text{det}} \triangleq \max N_{\text{pop}} P_d(N_{\text{pop}}).
\]

VI. NUMERICAL RESULTS

For all cases shown below, \( \text{INR} \triangleq N_1 E_1/N_0 \) and \( N_k = N_0 \) for all pulse index \( k \). The pulse duty factor is \( \delta_1 \triangleq L/T_n \). Fig. 2 shows that the analytically calculated \( P_m \) agrees with its simulated value. The behavior of the packet false-alarm probability \( P_f \) as a function of the threshold \( f_T \) is very similar to that of the SSRMA case of [11] and, therefore, is not shown here.

Fig. 3 shows the NDTP given by (5.5) as a function of transmitting population \( N_{\text{pop}} \). It is useful to note from the figure that the NDCap is directly proportional to the number of frequency channels. This suggests that the network capacity is in general proportional to its total bandwidth. However, the throughput at an offered load that guarantees a high quality of service (low \( P_m \)) is only sub-linear with the bandwidth.

Fig. 3: Normalized detection throughput \( r_{\text{det}} \) as a function of transmitting population \( N_{\text{pop}} \) for a homogeneous network described by eq. (5.5) with \( L = 32, N_1 = 48, N_0 = 16, D = N_0/4, L_d = 0, E_c/E_s = 10 \text{ dB}, E_c/E_s = 0 \text{ dB}, P_{\text{inr}}[\nu] = \delta_3[\nu - N/2], P_t = 10^{-5} \) for the WGN channel, and \( \delta_1 = 0.1 \).

VII. CONCLUDING REMARKS

We have developed an analytical method for quantifying the performance of packet acquisition in a time-frequency hopping RMA environment, accounting for all MAI, noise and partial-band interference. The results readily provide an estimate for and upper bound on the network throughput. We have verified the analytical results with simulations.

REFERENCES