Outage-based Throughput in Wireless Packet Networks

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Abstract—With the increased competition for the electromagnetic spectrum, it is important to characterize the impact of interference in the performance of a wireless packet network, which is traditionally measured by its throughput. This paper presents a unifying framework for characterizing the throughput in wireless packet networks. We analyze the throughput from an outage perspective, in which a packet is successfully received if the signal-to-interference-plus-noise ratio (SINR) exceeds some threshold, considering the aggregate interference generated by all emitting nodes in the network. Our work generalizes and unifies various throughput results scattered throughout the literature. Furthermore, the proposed framework encompasses all types of wireless propagation effects (e.g., Nakagami-\(m\) fading, Rician fading, and log-normal shadowing), as well as traffic patterns (e.g., slotted-synchronous, slotted-asynchronous, and exponential-interarrivals traffic).

Index Terms—Wireless networks, throughput, aggregate interference, spatial Poisson process, stable laws.

I. INTRODUCTION

The performance of a network is often quantified by its throughput, which measures the probability of successful communication. In a wireless environment, the throughput is constrained by various impairments that affect communication between nodes, namely: the wireless propagation effects, such as path loss, multipath fading, and shadowing; the network interference, due to signals radiated by other transmitters; and the thermal noise, introduced by the receiver electronics. It is therefore of interest to develop a framework that quantifies the impact of all these impairments on the throughput of the network. Such framework should also incorporate other important network parameters, such as the spatial distribution of nodes and their transmission characteristics.

The analysis of throughput in wireless networks has received considerable attention in the literature. The throughput of ALOHA channels for networks distributed in space is analyzed in [1]–[4], but not considering wireless propagation effects such as fading or shadowing. The distribution of the aggregate interference power is analyzed in [5], assuming deterministic node placement, slotted ALOHA, path loss exponent equal to 2, and Rayleigh fading. The issue of decentralized throughput maximization in wireless networks is analyzed in [6]. The characteristic function of the interference associated with a Poisson field of nodes is derived in [7], ignoring any fading or shadowing effects, and assuming a slotted ALOHA mechanism. The moments of the aggregate interference generated by a finite Poisson field of nodes subject to shadowing are analyzed in [8]. The probability of successful transmission based on the signal-to-interference-plus-noise ratio (SINR) is analyzed in [9], [10], assuming a spatial Poisson process, slotted ALOHA, and Rayleigh fading. Clearly, the analysis of throughput in the literature is largely constrained to some restrictive combination of path loss exponent, propagation model, spatial configuration of nodes, and packet traffic. Furthermore, the existing results are not easily generalizable if some of these network parameters are changed.

In this paper, we introduce a mathematical framework for the analysis of throughput in wireless packet networks, where the nodes are randomly scattered in the plane. Our work generalizes and unifies various results scattered throughout the literature, by accommodating arbitrary wireless propagation effects (e.g., Nakagami-\(m\) fading, Rician fading, or log-normal shadowing), as well as arbitrary traffic patterns (e.g., slotted-synchronous, slotted-asynchronous, or exponential-interarrivals traffic). We first provide a probabilistic characterization of the SINR of a link subject to aggregate interference and noise. Such characterization is valid regardless of the considered type of propagation scenario or packet traffic. We then obtain expressions for the throughput of a link from an outage perspective, where a packet is successfully received if the SINR exceeds some threshold, considering the aggregate interference generated by all emitting nodes in the network. Furthermore, we analyze the effect of the propagation characteristics and the packet traffic on the throughput.

This paper is organized as follows. Section II describes the system model. Section III characterizes the throughput from an outage perspective. Section IV provides numerical results to illustrate the dependence of the throughput on important network parameters. Section V concludes the paper and summarizes important findings.

II. SYSTEM MODEL

A. Spatial Distribution of Nodes

We model the spatial distribution of the nodes according to a homogeneous Poisson point process in the two-dimensional plane. Typically, the terminal positions are unknown to the
network designer a priori, so we may as well treat them as completely random according to a spatial Poisson process. This process has been successfully used in the context of wireless networks, most notably in what concerns routing [11], connectivity and coverage [12], interference [13], and physical-layer security [14], among other topics. The probability of $n$ nodes being inside a region $R$ of area $A$ is given by

$$P\{n \text{ in } R\} = \frac{(\lambda A)^n}{n!} e^{-\lambda A}, \quad n \geq 0,$$

where $\lambda$ is the (constant) spatial density of interfering nodes, in nodes per unit area. We define the interfering nodes to be the set of terminals which are transmitting within the frequency band of interest and hence are effectively contributing to the total interference. Then, irrespective of the network topology (e.g., point-to-point or broadcast) or multiple-access technique (e.g., frequency hopping), the above model depends only on the density $\lambda$ of interfering nodes. The proposed spatial model is depicted in Fig. 1. For analytical purposes, we assume there is a probe link composed of two nodes: one located at the origin of the two-dimensional plane (without loss of generality), and another one (node $i = 0$) deterministically located at a distance $r_0$ from the origin. All the other nodes ($i = 1 \ldots \infty$) are interfering nodes, whose random distances to the origin are denoted by $\{R_i\}$, where $R_1 \leq R_2 \leq \ldots$. Our goal is then to determine the throughput of the probe link subject to the effect of all the interfering nodes in the network.

### B. Wireless Propagation Characteristics

To account for the propagation characteristics of the environment, we consider that the power $P_{tx}$ received at a distance $R$

$$P_{tx} = \frac{P_{tx} \prod_{k=1}^{K} Z_k}{R^{2b}},$$

(1)

where $P_{tx}$ is the average transmitted power measured 1 m away from the transmitter; $b$ is the amplitude loss exponent; and $\{Z_k\}$ is a sequence of random variables (r.v.'s), independent in $k$, which account for other propagation effects such as multipath fading and shadowing. The term $1/R^{2b}$ accounts for the path loss with distance $R$, where the amplitude loss exponent $b$ is environment-dependent and can approximately range from 0.8 (e.g., hallways inside buildings) to 4 (e.g., dense urban environments), with $b = 1$ corresponding to free space propagation. This paper carries out the analysis generally in terms of $\{Z_k\}$, and therefore our results are valid for any wireless propagation effect. For illustration purposes, we consistently analyze four typical propagation scenarios throughout this paper:

1. **Path loss only**: $K = 1$ and $Z_1 = 1$.

2. **Path loss and Nakagami-\(m\) fading**: $K = 1$ and $Z_1 = \alpha^2$, where $\alpha^2 \sim \mathcal{G}(m, \frac{1}{m})$.

3. **Path loss and log-normal shadowing**: $K = 1$ and $Z_1 = e^{2\sigma G}$, where $G \sim \mathcal{N}(0,1)$.

4. **Path loss, Nakagami-\(m\) fading, and log-normal shadowing**: $K = 2$, $Z_1 = \alpha^2$ with $\alpha^2 \sim \mathcal{G}(m, \frac{1}{m})$, and $Z_2 = e^{2\sigma G}$ with $G \sim \mathcal{N}(0,1)$.

We emphasize that the proposed framework encompasses a wider variety of propagation effects other than these four cases, such as Ricean fading.

### C. Transmission Characteristics of Nodes

We analyze the case of half-duplex transmission, where each device transmits and receives at different time intervals, since full-duplexing capabilities are rare in typical low-cost applications. Nevertheless, the results presented in this paper can be easily modified to account for the full-duplex case. We consider interfering nodes with the same transmit power $P_1$ – a plausible constraint when power control is too complex to implement (e.g., decentralized ad-hoc networks). For generality, however, we allow the probe node to employ an arbitrary power $P_0$, not necessarily equal to that of the interfering nodes.

We further consider the scenario where all nodes transmit with the same traffic pattern. In particular, we examine three types of traffic, as depicted in Fig. 2:

1. **Slotted-synchronous traffic**: Similarly to the slotted ALOHA protocol, the nodes are synchronized and transmit in slots of duration $L_p$ seconds.\(^2\) A node transmits

\(^2\)Note that the amplitude loss exponent is $b$, while the corresponding power loss exponent is $2b$.

\(^3\)We use $\mathcal{G}(x, \theta)$ to denote a gamma distribution with mean $x\theta$ and variance $x\theta^2$.

\(^4\)We use $\mathcal{N}(\mu, \sigma^2)$ to denote a Gaussian distribution with mean $\mu$ and variance $\sigma^2$.

\(^5\)By convention, we define these types of traffic with respect to the receiver clock. In the typical case where the propagation delays with respect to the packet length can be ignored, all nodes in the plane observe exactly the same packet arrival process.
in a given slot with probability \( q \). The transmissions are independent for different slots and for different nodes.

2) Slotted-asynchronous transmission: The nodes transmit in slots of duration \( L_p \) seconds, which are not synchronized with other nodes’ time slots. A node transmits in a given slot with probability \( q \). The transmissions are independent for different slots and for different nodes.

3) Exponential-interarrivals traffic: The nodes transmit packets of duration \( L_p \) seconds. The idle time between packets is exponentially distributed with mean \( 1/\lambda_p \).

III. THROUGHPUT

In what follows, we analyze the throughput of the probe link from an outage perspective. In such approach, a node can hear the transmissions from all the nodes in the two-dimensional plane. A packet is successfully received if the SINR exceeds some threshold. Therefore, we start with the statistical characterization of the SINR, and then use those results to analyze the throughput.

A. Signal-to-Interference-Plus-Noise Ratio

Typically, the distances \( \{ R_i \} \) and propagation effects \( \{ Z_{i,k} \} \) associated with node \( i \) are slowly-varying and remain approximately constant during the packet duration \( L_p \). In this quasi-static scenario, it is insightful to define the SINR conditioned on a given realization of those r.v.’s. As we shall see, this naturally leads to the derivation of an SINR outage probability, which in turn determines the throughput. We start by formally defining the SINR.

Definition 3.1: The signal-to-interference-plus-noise ratio associated with the node at the origin is defined as

\[
\text{SINR} \triangleq \frac{S}{I + N},
\]

where \( S \) is the power of the desired signal received from the probe node, \( I \) is the aggregate interference power received from all other nodes in the network, and \( N \) is the (constant) noise power. Both \( S \) and \( I \) depend on a given realization of \( \{ R_i \} \), \( i = 1, \ldots, \infty \), and \( \{ Z_{i,k} \} \), \( i = 0, \ldots, \infty \), \( k = 1, \ldots, K \).

Using (1), the desired signal power \( S \) can be written as

\[
S = \frac{P_0 \prod_{k=1}^{K} Z_{0,k}}{\nu^{2b}}.
\]

Similarly, the aggregate interference power \( I \) can be written as

\[
I = \sum_{i=1}^{\infty} \frac{P_1 \Delta_i \prod_{k=1}^{K} Z_{i,k}}{R_i^{2b}},
\]

where \( P_1 \) is the transmitted power associated with each interferer, and \( \Delta_i \in [0, 1] \) is the (random) duty-cycle factor associated with interferer \( i \). As we shall see, the r.v. \( \Delta_i \) accounts for the different traffic patterns of nodes, and is equal to the fraction of the packet duration \( L_p \) during which interferer \( i \) is effectively transmitting. Note that since \( S \) and \( I \) depend on the random nodes positions and random propagation effects, they can be seen as r.v.’s whose value is different for each realization of those random quantities.

Furthermore, we show in [13] that the r.v. \( I \) has a skewed stable distribution [16] given by

\[
I \sim \mathcal{S}(\alpha = \frac{1}{b}, \beta = 1, \gamma = \frac{\pi \lambda C_x^{-1} P_1^{1/b} \prod_{k=1}^{K} \mathbb{E}(\Delta_i^{1/b}) \prod_{k=1}^{K} \mathbb{E}(Z_{i,k}^{1/b})}{2 \pi \Gamma(2-2b)}),
\]

where \( b > 1 \), and \( C_x \) is defined as

\[
C_x \triangleq \begin{cases} \frac{1-x}{2} & x \neq 1, \\ \frac{1}{\pi} & x = 1. \end{cases}
\]

As we shall see, the probe link throughput depends on the traffic pattern of the nodes through \( \mathbb{E}(\Delta_i^{1/b}) \) in (5).

B. Probe Link Throughput

We now use the results developed in Section III-A to characterize the throughput of the probe link, subject to the aggregate network interference. We start by defining the concept of throughput.

Definition 3.2: The outage-based throughput \( T \) of a link is the probability that a packet is successfully communicated during an interval equal to the packet duration \( L_p \). For a packet to be successfully received, the SINR of the link must exceed some threshold.

\[ S(\alpha, \beta, \gamma) \]

This is equivalent to each node using a M/D/1 queue for packet transmission, characterized by a Poisson arrival process with rate \( \lambda_p \), a constant service time \( L_p \), a single server, and a single system place.
\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = 1 - \frac{(m - 1)!}{\Gamma(m)} \left( 1 - \sum_{k=0}^{m-1} \sum_{j=0}^{k} (-\nu_1)^j (\nu_1 N)^{k-j} e^{-\nu_1 N} \left. \frac{d^j \phi_I(s)}{ds^j} \right|_{s = \nu_1} \right) \quad (11)
\]
\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = \exp \left( -\frac{r_0^2 \theta^* N}{P_0} \right) \exp \left[ -\frac{\pi \lambda C^{-1/b}_1 \Gamma \left( 1 + \frac{1}{b} \right) E_{\Delta_i}^{1/b} \left( P_{\theta_0} \theta^* \right)^{1/b}}{\cos \left( \frac{\phi}{2b} \right)} \right] \quad (12)
\]

Using the definition above, we can write the throughput \( T \) as
\[
T = \mathbb{P}\{\text{probe transmits}\} \mathbb{P}\{\text{receiver silent}\} \mathbb{P}\{\text{no outage}\}. \quad (7)
\]
The first probability term, which we denote by \( p_T \), depends on the type of packet traffic. The second term, which we denote by \( p_S \), also depends on the type of packet traffic and corresponds to the probability that the node at the origin is silent (i.e., does not transmit) during the transmission of the packet by the probe node. This second term is necessary because the nodes are half-duplex, so they cannot transmit and receive simultaneously. The third term is simply \( \mathbb{P}\{\text{SINR} \geq \theta^*\} \), where \( \theta^* \) is a predetermined threshold that ensures reliable packet communication over the probe link. Therefore, the throughput of a wireless packet network can be written as
\[
T = p_T p_S \mathbb{P}\{\text{SINR} \geq \theta^*\}. \quad (8)
\]

Using (2), (3), and the law of total probability with respect to r.v.'s \( Z_{o,k} \) and \( I \), we can write
\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = \mathbb{E}_I \left\{ \mathbb{P}\{Z_{o,k} \geq \frac{r_0^2 \theta^*}{P_0} (I + N) \mid I \} \right\} \quad (9)
\]
or, alternatively,
\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = \mathbb{E}_{(Z_{o,k})} \left\{ F_I \left( \frac{P_0 \prod_k Z_{o,k}}{r_0^2 \theta^*} - N \right) \right\}, \quad (10)
\]
where \( F_I(\cdot) \) is the c.d.f. of the stable r.v. \( I \), whose distribution is given in (5). As we shall see, both forms are useful depending on the considered propagation characteristics. Equations (8)–(10) are general and valid for a variety of propagation conditions as well as traffic patterns. As we will see in the next sections, the propagation characteristics determine only \( \mathbb{P}\{\text{SINR} \geq \theta^*\} \), while the traffic pattern determines \( p_T, p_S \), and \( \mathbb{P}\{\text{SINR} \geq \theta^*\} \).

C. Effect of the Propagation Characteristics on \( T \)

We now determine the effect of four different propagation scenarios described in Section II-B on the throughput. Recall that the propagation characteristics affect the throughput \( T \) only through \( \mathbb{P}\{\text{SINR} \geq \theta^*\} \) in (8), and so we now derive such probability for these specific scenarios.

1) Path loss only: In this case, the expectation in (10) disappears and we have
\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = F_I \left( \frac{P_0}{r_0^2 \theta^*} - N \right),
\]
where the distribution of \( I \) in (5) reduces to
\[
I \sim S \left( \alpha = \frac{1}{b}, \beta = 1, \gamma = \pi \lambda C^{-1/b}_1 P_1^{1/b} E_{\Delta_i}^{1/b} \right).
\]

Note that the characteristic function of \( I \) was also obtained in [7] using the influence function method, for the case of path loss and slotted-synchronous traffic only.

2) Path loss and Nakagami-\( m \) fading: In this case, (9) reduces to
\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = 1 - \frac{1}{\Gamma(m)} \mathbb{E}_I \left\{ \gamma_{\text{inc}} \left( m, \frac{r_0^2 \theta^* (I + N) m}{P_0} \right) \right\},
\]
where \( \gamma_{\text{inc}}(a, x) = \int_0^x t^{a-1} e^{-t} dt \) is the lower incomplete gamma function, and the distribution of \( I \) in (5) reduces to
\[
I \sim S \left( \alpha = \frac{1}{b}, \beta = 1, \gamma = \pi \lambda C^{-1/b}_1 P_1^{1/b} E_{\Delta_i}^{1/b} \frac{\Gamma \left( m + \frac{1}{b} \right)}{m^{1/b} \Gamma(m)} \right).
\]

For integer \( m \), this can be expressed in closed form [17] as (11) given at the top of this page, where
\[
\nu_1 = \frac{r_0^2 \theta^* m}{P_0},
\]
and
\[
\phi_I(s) = \exp \left( -\frac{\pi \lambda C^{-1/b}_1 P_1^{1/b} \Gamma \left( m + \frac{1}{b} \right) E_{\Delta_i}^{1/b}}{m^{1/b} \Gamma(m) \cos \left( \frac{\phi}{2b} \right)} s^{1/b} \right),
\]
for \( s \geq 0 \). For the particular case of Rayleigh fading \((m = 1)\), we obtain (12) at the top of this page.

3) Path loss and log-normal shadowing: In this case, (9) reduces to
\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = \mathbb{E}_I \left\{ Q \left( \frac{1}{2 \sigma} \ln \left( \frac{r_0^2 \theta^* (I + N)}{P_0} \right) \right) \right\},
\]
where \( Q(\cdot) \) denotes the Gaussian \( Q \)-function, and the distribution of \( I \) in (5) reduces to
\[
I \sim S \left( \alpha = \frac{1}{b}, \beta = 1, \gamma = \pi \lambda C^{-1/b}_1 P_1^{1/b} e^{2a^2/2b^2} E_{\Delta_i}^{1/b} \right).
\]
\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = 1 - \frac{(m-1)!}{\Gamma(m)} \left(1 - \sum_{k=0}^{m-1} \sum_{j=0}^{k} E_G \left\{ \frac{(-\nu_2)^j (\nu_2 N)^{(k-j)} e^{-\nu_2 N}}{(k-j)!} \frac{d^j \phi_I(s)}{ds^j} \bigg| s = \nu_2 \right\} \right)
\] (13)

\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = E_G \left\{ \exp \left( -\frac{r_0^2 \theta^* N}{P_0 e^{2\sigma_G}} \right) \exp \left( -\frac{\pi \lambda C_{1/b} e^{2\sigma^2/\theta^*} \Gamma(1 + \frac{1}{b}) \exp \left( \Delta_i^{1/b} \right)}{\cos \left( \frac{\pi}{2b} \right) \left( P_1 r_0^2 \theta^* \right)^{1/b} \Gamma(m)} \right) \right\}
\] (14)

4) Path loss, Nakagami-\(m\) fading, and log-normal shadowing: In this case, (9) reduces to

\[
\mathbb{P}\{\text{SINR} \geq \theta^*\} = 1 - \frac{1}{\Gamma(m)} E_{G_0,t} \left\{ \frac{\gamma_{\text{inc}}(m, \frac{r_0^2 \theta^* (I + N) m}{P_0 e^{2\sigma_G}})}{\Gamma(m)} \right\},
\]
where the distribution of \( I \) in (5) reduces to

\[I \sim S \left( \alpha = \frac{1}{b}, \beta = 1, \gamma = \pi \lambda C_{1/b} \frac{1}{b} e^{2\sigma^2/\theta^*} \frac{\Gamma(1 + \frac{1}{b}) \exp \left( \Delta_i^{1/b} \right)}{\cos \left( \frac{\pi}{2b} \right) \left( P_1 r_0^2 \theta^* \right)^{1/b} \Gamma(m)} \right) \).

For integer \( m \), this can be expressed in closed form \cite{17} as given in (13) at the top of this page, where

\[\nu_2 = \frac{r_0^2 \theta^* m}{P_0 e^{2\sigma_G}} \]
and

\[\phi_I(s) = \exp \left( -\frac{\pi \lambda C_{1/b} \frac{1}{b} e^{2\sigma^2/\theta^*} \Gamma(1 + \frac{1}{b}) \exp \left( \Delta_i^{1/b} \right)}{\cos \left( \frac{\pi}{2b} \right) \left( P_1 r_0^2 \theta^* \right)^{1/b} \Gamma(m)} \right) \]
for \( s \geq 0 \). For the particular case of Rayleigh fading \((m = 1)\), we obtain (14) at the top of this page.

D. Effect of the Traffic Pattern on \( T \)

We now investigate the effect of the three different types of traffic pattern on the throughput. Recall that the traffic pattern affects the throughput \( T \) through \( p_T, p_S, \) and \( \mathbb{P}\{\text{SINR} \geq \theta^*\} \) in (8). The type of packet traffic determines the statistics of the duty-cycle factor \( \Delta_i \), and in particular \( \mathbb{E}\{\Delta_i^{1/b}\} \) in (5), which in turn affects \( \mathbb{P}\{\text{SINR} \geq \theta^*\} \).

1) Slotted-synchronous traffic: In this case, \( p_T = q \) and \( p_S = 1 - q \). The duty-cycle factor \( \Delta_i \) is a binary r.v. taking the value 0 or 1, and we can show \cite{17} that \( \mathbb{E}\{\Delta_i^{1/b}\} = q \).

2) Slotted-asynchronous traffic: In this case, \( p_T = q \) and \( p_S = (1 - q)^2 \). The duty-cycle factor \( \Delta_i \) is either 0, 1, or a continuous r.v. uniformly distributed over the interval \([0, 1] \). We can show \cite{17} that \( \mathbb{E}\{\Delta_i^{1/b}\} = q^2 + 2q(1 - q) \frac{b}{b+1} \).

3) Exponential-interarrivals traffic: Considering that \( L_p \lambda_p \ll 1 \), then \( p_T \approx L_p \lambda_p \) and \( p_S = e^{-2L_p \lambda_p} \). The duty-cycle factor \( \Delta_i \) is either 0 or a uniform r.v. in the interval \([0, 1] \), and we can show \cite{17} that \( \mathbb{E}\{\Delta_i^{1/b}\} = (1 - e^{-2L_p \lambda_p}) \frac{b}{b+1} \).

IV. NUMERICAL RESULTS

Figure 3 shows the dependence of the packet throughput on the transmission probability, for various types of packet.
traffic. We observe that the throughput is higher for slotted-synchronous traffic than for slotted-asynchronous traffic, as demonstrated in Section III-E. Figure 4 plots the throughput versus the spatial density of interferers, for various wireless propagation effects. We observe that the throughput decreases monotonically with the spatial density of the interferers, as also shown in Section III-E.

V. CONCLUSION

In this paper, we introduced a mathematical framework for the characterization of throughput in wireless packet networks. Our work generalizes and unifies various results scattered throughout the literature, by accommodating arbitrary wireless propagation effects, as well as arbitrary traffic patterns. We provided a probabilistic characterization of the SINR of a link subject to aggregate interference and noise. We obtained expressions for the throughput of a link from an outage perspective, where a packet is successfully received if the SINR exceeds some threshold, considering the aggregate interference generated by all emitting nodes in the network. We then analyzed the effect of the propagation characteristics and the packet traffic on the throughput. Specifically, we showed that the throughput is higher for slotted-synchronous traffic than for slotted-asynchronous traffic, regardless of the specific propagation conditions. We also showed that the throughput degrades faster with an increase in the spatial density of the interferers, than with an increase in their transmitted power, regardless of the specific propagation conditions and traffic pattern.

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