Laboratory investigation of lateral dispersion within dense arrays of randomly distributed cylinders at transitional Reynolds number

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Laboratory investigation of lateral dispersion within dense arrays of randomly distributed cylinders at transitional Reynolds number

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Relative (effective) lateral dispersion of a passive solute was examined at transitional Reynolds numbers within a two-dimensional array of randomly distributed circular cylinders of uniform diameter $d$. The present work focuses on dense arrays, for which previously developed theory [Y. Tanino and H. M. Nepf, J. Fluid Mech. \textbf{600}, 339 (2008)] implies that the asymptotic (long-time/long-distance) dispersion coefficient, when normalized by the mean interstitial fluid velocity, $\langle \tilde{u} \rangle$, and $d$, will only exhibit a weak dependence on Reynolds number, $Re_d = \langle \tilde{u} \rangle d/\nu$, where $\nu$ is the kinematic viscosity. However, the advective distance required to reach asymptotic dispersion is expected to be controlled by pore-scale mixing, which is strongly $Re_d$-dependent prior to the onset of full turbulence. Laser-induced fluorescence was used to measure the time-averaged lateral concentration profiles of solute released continuously from a point source in arrays of solid volume fraction $\phi = 0.20$ and $0.35$ at $Re_d = 48–120$. Results are compared to previous measurements at higher $Re_d$. Lateral dispersion reaches the same rate as asymptotic dispersion in fully turbulent flow at $x = 154d$ at ($\phi, Re_d) = (0.20, 110–120)$ and at $x = 87d$ at ($\phi, Re_d) = (0.35, 300–390)$. In contrast, dispersion does not reach the fully turbulent flow limit at $Re_d < 100$ within the range of $x$ considered. Also, concentration profiles deviate further from a Gaussian distribution at $\phi = 0.35$ than at $0.20$ for similar $Re_d$ and $x\phi/d$. From these observations, it can be inferred that the pre-asymptotic regime extends farther downstream, in terms of the number of cylinders spanned, at lower $Re_d$ and at larger $\phi$. © 2009 American Institute of Physics. [DOI: 10.1063/1.3119862]

I. INTRODUCTION

Obstructed flows, i.e., flows through an array of solid bodies, are ubiquitous in the environment. Classic examples include groundwater flow and flow around buildings, trees, and aquatic plants. The simplest obstructed flow is uniform flow through a two-dimensional array of circular cylinders. Such an array is often used as an idealized porous medium to study fluid mechanics text books, e.g., Refs. 5–7, so only the key regimes relevant to this paper are identified here. As $Re_d$ is increased from Stokes flow, flow remains steady and laminar everywhere in the fluid up to $Re_d \approx 40$. At $Re_d \approx 40$, the isolated cylinder wake begins to oscillate periodically. Next, at $Re_d \approx 100$ within the range of $Re_d$ considered. Also, concentration profiles deviate further from a Gaussian distribution at $\phi = 0.35$ than at $0.20$ for similar $Re_d$ and $x\phi/d$. From these observations, it can be inferred that the pre-asymptotic regime extends farther downstream, in terms of the number of cylinders spanned, at lower $Re_d$ and at larger $\phi$. © 2009 American Institute of Physics. [DOI: 10.1063/1.3119862]

The Cartesian coordinates $x = (x, y)$ are defined such that the $x$-axis is aligned with the fluid velocity averaged over time and the fluid volume, denoted by $\langle \tilde{u} \rangle$, and the $y$-axis is in the horizontal plane and perpendicular to the $x$-axis (Fig. 1). The overbar denotes an average over a time interval much longer than those associated with turbulent fluctuations and vortex shedding. $\langle \tilde{u} \rangle$ denotes an average over infinitesimally thin fluid volume that span many cylinders or, equivalently, an ensemble average, i.e., an average over many arrays with the same $\phi$, $d$, and flow conditions, but with different cylinder configurations. We only consider constant, uniform mean flow, i.e., time-independent $\langle \tilde{u} \rangle$.

The two length scales identified above can be used to define two Reynolds numbers, $Re_d = \langle \tilde{u} \rangle d/\nu$ and $Re = \langle \tilde{u} \rangle (s_n)_{\lambda}/\nu$, where $\nu$ is the kinematic viscosity. It is well established that flow patterns around isolated cylinders are similar at the same $Re_d$. Detailed descriptions of these flows are available in classic papers and in standard fluid mechanics text books, e.g., Refs. 5–7, so only the key regimes relevant to this paper are identified here. As $Re_d$ is increased from Stokes flow, flow remains steady and laminar everywhere in the fluid up to $Re_d \approx 40$. At $Re_d \approx 40$, the isolated cylinder wake begins to oscillate periodically. Next, at
Re_d=90, vortices begin to periodically shed from the cylinder. This unsteady, laminar wake regime continues until Re_d=200, beyond which the periodic motion of the wake gradually breaks down and the wake becomes turbulent as Re_d is increased further. The wake becomes fully turbulent at roughly Re_d≈5000.6

At sufficiently small φ, flow around each cylinder in the array resembles flow past an isolated cylinder. In this limit, where ⟨s_ω⟩A tends to infinity, Re_d is the relevant Reynolds number and, for example, transitions between the flow regimes described above occur at the same Re_d as in the isolated cylinder wake. At sufficiently large φ, the array resembles a network of intersecting channels of width ⟨s_ω⟩A. Accordingly, flow in such arrays is expected to be kinematically and dynamically similar to the same Re, Arrays considered in this study fall below these limits, such that neither Re_d or Re_e captures the similarity across flows in different φ. For simplicity, results will be discussed in terms of Re_d only. In the laboratory experiments considered in the present paper, d was kept constant, and differences in Re_d result entirely from differences in ⟨u⟩/v, i.e., Re_d dependence refers specifically to the dependence on ⟨u⟩/v.

It is convenient to describe the conservation of a passive solute in a two-dimensional array in terms of temporally and spatially averaged parameters,8

\[
\frac{\partial \langle \bar{c} \rangle}{\partial t} + \langle \bar{u} \rangle \frac{\partial \langle \bar{c} \rangle}{\partial x_j} = - \frac{\partial}{\partial x_j} \left( \langle v_j \rangle \bar{c} \right) - D_m \frac{\partial^2}{\partial x_j^2} \left( \langle \bar{c} \rangle + \bar{c}^2 \right).
\]

(1)

Here, ”\(n\) denotes the spatial fluctuations of the local temporal average (denoted by ‘), ‘\(n\) denotes the temporal fluctuations, t is time, \(c(x,t)=\langle \bar{c} \rangle(x,t)+\bar{c}^2(x,t)+c^2(x,t)\) is the solute concentration, \(\mathbf{v}(x,t)=(u(x,t),v(x,t))\) is the fluid velocity, and \(D_m\) is the molecular diffusion coefficient of the solute. By definition, \(\bar{c}, \bar{v}_1, \bar{v}_2, \bar{v}_3, \bar{v}_4, \bar{v}_5\)=0. Also, by our definition of the Cartesian coordinates, \(\langle \bar{u} \rangle=0\). In uniform, time-independent mean flow through a spatially-homogeneous random array, long-range velocity correlations are not expected because of the obstructions.9,10 Accordingly, Fickian dispersion is expected after sufficiently long time after the solute is introduced to the flow, once the spatial scale over which the mean concentration gradient varies exceeds the finite scales of velocity correlations.11 Previous14,12 and present (Sec. III C) observations of dispersion support this conjecture. Then, Eq. (1) reduces to

\[
\frac{\partial \langle \bar{c} \rangle}{\partial t} + \langle \bar{u} \rangle \frac{\partial \langle \bar{c} \rangle}{\partial x_j} = - \frac{\partial}{\partial x_j} \left( K_{ij} \langle \bar{c} \rangle \right) D_m \frac{\partial^2 \langle \bar{c} \rangle}{\partial x_j^2},
\]

(2)

where \(K_{ij}\) are the coefficients for asymptotic (long-time/long-distance) net dispersion. The first equality in Eq. (2) states that the sum of the fluxes on the right-hand side of Eq. (1) obeys Fick’s law. The second equality states that \(K_{ij}\) is spatially homogeneous, which is expected in a homogeneous array.

In the present study, we are only concerned with the lateral component of the macroscopic dispersion coefficient, \(K_{yy}\). Consider an experiment in which solute particles are released continuously from a point source \((x=0)\). Then, \(\bar{c}(x)\) is the temporal average of the solute concentration observed at some point \(x\) during a single experiment, and \(\langle \bar{c} \rangle(x)\) is its ensemble average. If the solute is transported according to Eq. (2), the lateral variance of its distribution is related to \(K_{yy}\) as13

\[
\frac{K_{yy}}{\langle u \rangle d} = \frac{1}{2d} \frac{d}{dx} \left( \langle \sigma_y^2(x) \rangle + \left( \frac{m_1}{m_0}(x) \right)^2 - \left( \frac{m_1}{m_0}(x) \right)^2 \right),
\]

(3)

where \(\sigma_y^2\) and \(m_1/m_0\) are the variance and the center of mass, respectively, of the time-averaged lateral concentration distribution at a given \(x\) in a single experiment; \((m_1/m_0)\) and the expression inside [ ] are the center of mass and the variance, respectively, of \(\langle \bar{c} \rangle\) at a given \(x\). From Eq. (2), it follows that \(d(m_1/m_0)/dx=0\). Further, since \(m_1/m_0\) is the mean lateral displacement of many solute particles, \(d((m_1/m_0)^2)/dx\) also approaches zero at sufficiently large \(x\). Then, Eq. (3) reduces to

\[
\frac{K_{yy}}{\langle u \rangle d} = \frac{1}{2d} \frac{d}{dx} \langle \sigma_y^2(x) \rangle.
\]

(4)

The right-hand side of Eq. (4) is commonly referred to as the ‘relative14 or “effective”15 dispersion coefficient. Previous14,16 studies observed two properties at sufficiently high Re_d over the range of \(x\) considered (Re_d>67–320 and \(x/d>7–80\), depending on \(\phi\)): (i) \(d(\sigma_y^2)/dx\) remained constant in \(x\), consistent with a spatially homogeneous \(K_{yy}/\langle u \rangle d\), and (ii) \(d(\sigma_y^2)/dx\) was independent of Re_d. As a result, the corresponding \(K_{yy}/\langle u \rangle d\) was a function only of \(\phi\) (d was kept constant).

Tanino and Nepf4 showed that \(K_{yy}/\langle u \rangle d\) in this high-Re_d regime can be described by the linear superposition of a model for turbulent diffusion and existing models for asymptotic (long-distance) dispersion associated with the tortuous flow path that fluid is forced to follow around the cylinders. In predicting the contribution from turbulent diffusion, fully turbulent flow, defined here to refer to flow that has achieved the maximum (and therefore Re_d-independent)}
mean turbulence intensity for that \( \phi \), was assumed. Their model predicts that turbulent diffusion makes a non-negligible (specifically, \( \geq 10\% \)) contribution to \( K_{yy}/\langle \bar{u}\rangle d \) at \( d/\langle s_n \rangle_A \leq 2.6 (\phi = 0.19) \) in fully turbulent flow (Fig. 2). Since turbulent diffusion must decrease as \( Re_d \) decreases below the fully turbulent regime, the theory implies that \( K_{yy}/\langle \bar{u}\rangle d \) will also decrease as \( Re_d \) decreases in this \( \phi \) range in turbulent flow. However, in unsteady laminar flow, periodic wakes may introduce a different mechanism of dispersion.

Conversely, the model predicts that at large \( \phi > 0.19 \), turbulence does not contribute significantly to asymptotic dispersion and, consequently, that the contribution from the tortuous flow path, i.e., the time-averaged, spatially heterogeneous velocity field, dominates. In dense arrays, physical reasoning suggests that the time-averaged velocity field, and therefore its contribution to lateral dispersion, are determined primarily by the local cylinder configuration and, therefore, do not depend strongly on \( Re_d \). Therefore, \( K_{yy}/\langle \bar{u}\rangle d \) at \( \phi > 0.19 \) is not expected to depend strongly on \( Re_d \) either. To our knowledge, the \( Re_d \)-dependence of the time-averaged velocity field has not been investigated directly. Nevertheless, the good agreement between experiment and the model of Tanino and Nepf, which assumes a \( Re_d \)-independent contribution from the tortuous flow path, supports this conjecture. Indeed, Hill et al. made the same conjecture for dense arrays of randomly distributed spheres, based on their observation that the spatial variance of the transverse components of velocity normalized by \( \langle \bar{u} \rangle^2 \) differs by less than 4\% between Stokes flow and \( Re_d = 113 \) numerical simulations at \( \phi \approx 0.588 \). The same conjecture has also been made for thomohedrally distributed spheres.

In fully turbulent flow, \( d(\sigma_x^2)/dx \) is expected to be independent of \( Re_d \) at all \( x \). This implies that \( \langle \sigma_x^2 \rangle \) is also \( Re_d \)-independent at all \( x \) in this flow regime, given the same initial distribution of the solute \([\langle \sigma_x^2(x = 0) \rangle = 0]\). Indeed, \( \langle \sigma_x^2 \rangle \) was observed to be \( Re_d \)-independent above \( Re_d \approx 200 \) at selected \( \phi \) (0.091 and 0.20) and \( x \). In contrast, \( \langle \sigma_y^2 \rangle \) was observed to decrease as \( Re_d \) decreased below \( Re_d \approx 100 \) at these \( \phi \) and \( x \) (These thresholds are identified in terms of a different Reynolds number in Ref. 16; here, they are expressed in terms of \( Re_d \) for consistency with the rest of this paper). Recall that for \( \phi > 0.19 \), asymptotic (long-distance) \( d(\sigma_y^2)/dx \) is not expected to depend strongly on \( Re_d \). Therefore, the observed \( Re_d \)-dependence of \( \langle \sigma_y^2 \rangle \) at \( Re_d \approx 100 \) must arise entirely from a \( Re_d \)-dependent pre-asymptotic (short-distance) dispersion.

The objective of the present study is to use laboratory observations to evaluate two conjectures for lateral dispersion in dense arrays: (i) asymptotic \( d(\sigma_y^2)/dx \) exhibits only a weak dependence on \( Re_d \) and (ii) the advective distance required for the solute to achieve this asymptotic behavior is strongly dependent on \( Re_d \) prior to the onset of full turbulence due to the \( Re_d \)-dependence of pore-scale mixing. The latter conjecture is explained in detail in Sec. III. To the authors’ knowledge, experimental measurements of lateral dispersion at transitional \( Re_d \) within homogeneous random cylinder arrays are not currently available in literature. Shavit and Brandon measured laboratory measurements in a \( \phi \approx 0.035 \) array, but illustrations of their array suggest that the cylinder distribution was in fact not random. More recently, Serra et al. measured the lateral dispersion of solute released at the upstream edge of a random array of \( \phi = 0.10, 0.20, \) and 0.35 at \( Re_d = 11 – 120 \). Their estimates of \( K_{yy}/\langle \bar{u}\rangle d \) are independent of \( Re_d \) within experimental uncertainty over \( 32 < Re_d < 120 \), which is consistent with conjecture (i) above. However, the \( \phi \) dependence of their reported values disagrees with that reported in Ref. 4 (Fig. 2). Specifically, \( K_{yy}/\langle \bar{u}\rangle d \) at \( \phi = 0.10 \) and at \( \phi = 0.20 \) were the same within experimental uncertainty, and \( K_{yy}/\langle \bar{u}\rangle d \) was the smallest at \( \phi = 0.35 \) in the experiments of Serra et al. This disagreement may be due to their flow not being fully developed at the solute source or measurements being collected at very short distances \((x/d = 8 – 26)\). Although Serra et al. reported that their estimates of \( K_{yy} \) did not vary with \( x \) within this range, comparison with experiments reported in the present paper suggests that they were sampling in the pre-asymptotic regime (Sec. III C).

In this study, a set of laser-induced fluorescence (LIF) experiments were performed in arrays of \( \phi = 0.20 \) and 0.35 at selected \( Re_d \) in the range of \( Re_d = 48 – 120 \). Visualizations of pore-scale mixing are presented, and the observed \( Re_d \)-dependence is discussed qualitatively (Sec. III A). Time-averaged concentration profiles are discussed in Secs. III B and III C, in terms of their deviation from a Gaussian distribution, their variance, and the evolution of these two parameters with \( x \). The growth rate of the variance is also compared to the corresponding \( K_{yy}/\langle \bar{u}\rangle d \) in fully turbulent flow presented in Ref. 4, and pre-asymptotic and asymptotic regimes are identified.
II. MATERIALS AND METHODS

A. Flume and cylinder array configuration

Laboratory experiments were conducted in a recirculating Plexiglas flume with a \((x \times y) = 284 \, \text{cm} \times 40 \, \text{cm}\) test section. The volumetric flow rate was measured with an in-line flow meter; its temporal variations ranged in magnitude from 1% to 4% in the present experiments. Flow is approximately uniform within an emergent array.\(^4\)\(^,\)\(^1\) Therefore, the temporally and spatially averaged interstitial velocity \(\langle \bar{u} \rangle\) is approximated by its cross-sectional average, which was calculated as the temporal average of the volumetric flow rate divided by the width of the test section, the temporally and spatially averaged water depth \(\langle \bar{H} \rangle\), and \(1 - \phi\). Note that the mean water depth decreases along the length of the array to balance the net drag exerted on the flow.\(^2\)\(^,\)\(^3\) The maximum decrease in mean water depth between the solute source and the LIF measurement location was 17% of the depth at the solute source. For consistency, \(\langle \bar{u} \rangle\), and thus \(\text{Re}_\text{d}\), for a given experiment will be defined at the solute source in this paper. The range of experimental conditions examined is summarized in Table I.

Cylindrical maple dowels of diameter \(d = 0.64 \, \text{cm}\) (Saunders Brothers, Inc.) were inserted into perforated polyvinyl chloride sheets of either 20% or 35% hole fraction to create 280 cm-long arrays of two densities: \(\phi = 0.20\) and 0.35. These custom-made sheets were manufactured previously\(^4\) by generating uniformly distributed random coordinates at \(d \times d\) square around its center. The \(\phi = 0.35\) array was created by filling all of the holes in the 35% hole fraction sheets. Different \(\phi = 0.20\) arrays were created by selecting different combinations of the 20% and 35% hole fraction sheets. In doing so, more array realizations could be included in the ensemble average. Where 35% hole fraction sheets were used to create a \(\phi = 0.20\) array, the holes that were filled were selected using MATLAB's random number generator. The cylinders are perpendicular to the horizontal bed of the test section.

B. LIF experiments

LIF was used to measure the lateral concentration distribution at \((x, \text{Re}_d)\) selected systematically between \(\text{Re}_d = 48\) and 120 and \(x = 38\) and 234 cm \((x = 0\) is the solute source\). The procedure was essentially the same as that described in Ref. 4, so only key steps are presented here. Dilute rhodamine WT was injected continuously with a syringe pump (Orion Sage™ M362) from a needle embedded within the array. The rate of injection was matched visually to the local flow. A single horizontal beam of argon ion laser (Coherent INNOVA\textsuperscript{R} 70 and 70 C ion laser) passed laterally through the flume at a single streamwise position, \(x\), downstream of the solute source (Fig. 3). A charge-coupled device camera (Sony XCD-X710) recorded the line of fluoresced solute from above the flume in a sequence of 1024 \times 48 bitmap images. 530 and 515 nm long-pass filters (Midwest Optical Systems, Inc.) were attached to the camera to filter out the laser beam. Preliminary measurements confirmed that the recorded fluorescence intensity was linearly proportional to rhodamine WT concentration; for simplicity, the former was used in all analyses. For each experiment, cylinders were removed to create a single 1.3-cm gap in the array, which allowed the laser beam to pass through the entire width of the flume.

Instantaneous intensity profiles were extracted from the images, corrected for background and anomalous pixel intensities, and averaged over the duration of the experiment to yield a time-averaged intensity profile, \(I(y, t)\). The duration of the experiments in \(\phi = 0.20\) ranged from 46 to 213 s (4.5–18.9 frames/s). In \(\phi = 0.35\), images were collected over 94 s (10.1–10.7 frames/s) in all but six runs; the duration of the other six runs ranged from 80 to 112 s (9.0–12.5 frames/s). The high frame rate at \(\phi = 0.20\) arose because one of the two computers used was able to record images at a

---

TABLE I. Relevant parameters for previous (Ref. 4) and present experiments considered in this paper. \(d/\langle \bar{u} \rangle\) is predicted by Eq. (A16) in Ref. 4. \(x/\langle \bar{u} \rangle\) is predicted by Eq. (9), with \(\langle \bar{u} \rangle / \langle H \rangle\) as predicted for fully turbulent flow by Eq. (4.1) in Ref. 4, and \(1/d\) as predicted by Eq. (10). \(\langle \bar{u} \rangle\) was approximated by its cross-sectional average. \(n\) is the total number of LIF experiments at all \(x\) for \((\phi, \text{Re}_d) = (0.35, 300–390)\) and only at \(x\) for which ensemble-averaged values are derived for all other \((\phi, \text{Re}_d)\). Values for \(\text{Re}_d, \langle \bar{u} \rangle\), and \(\langle \bar{H} \rangle\) represent the range across these replicate runs at the solute source.

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<th>(\phi)</th>
<th>(d/\langle \bar{u} \rangle) (fully turbulent)</th>
<th>(\text{Re}_d)</th>
<th>(\langle \bar{u} \rangle) (cm/s)</th>
<th>(\langle \bar{H} \rangle) (cm)</th>
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<td>2.70</td>
<td>(-5.9)</td>
<td>(58)–(61)</td>
<td>(0.82)–(1.4)</td>
<td>(12.7)–(16.8)</td>
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<td>(1.3)–(2.0)</td>
<td>310–340</td>
<td>(5.0)–(6.5)</td>
<td>11.8–15.1</td>
<td>23(^a)</td>
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<td>110–120</td>
<td>(1.6)–(2.5)</td>
<td>12.8–16.5</td>
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<tr>
<td>300–390</td>
<td>(4.5)–(5.6)</td>
<td>13.3–20.3</td>
<td>17(^a)</td>
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<tr>
<td>0.35</td>
<td>5.95</td>
<td>(-11)</td>
<td>(48)–(51)</td>
<td>(0.93)–(1.1)</td>
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<td>13.5–15.0</td>
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<tr>
<td>97–100</td>
<td>(1.4)–(2.2)</td>
<td>13.5–15.3</td>
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</table>

\(^a\)These experiments were reported in Ref. 4.
faster frame rate. The time-averaged profile was further corrected for noise and background. Two quantitative parameters were considered in describing $I(y,t)$. First, its variance was calculated as

$$\sigma^2(x) = \frac{m_2(x)}{m_0(x)} \left[ \frac{m_1(x)}{m_0(x)} \right]^2,$$  

(5)

where $m_j(x)$ is the $j$th moment,

$$m_j(x) = \int_{x_1}^{x_2} y^j I(y,t) dy,$$

(6)

$y=0$ is defined at the lateral position of the solute source. The zeroth, first, and second moments and the corresponding $\sigma^2$ were calculated by setting the limits of integration in Eq. (6), $\kappa_{1,2}$, at the two edges of the images. Next, $\kappa_{1,2}$ were redefined as $\kappa_{1,2} = (m_1/m_0) \pm 4\sigma_y$ and the calculation was repeated. These limits were applied to prevent small fluctuations at large distances from the center of mass from unrealistically increasing the variance estimate. Second, the deviation of $I(y,t)$ from a Gaussian distribution was parameterized by

$$C_{m_y} = \frac{1}{\kappa_1 - \kappa_2} \int_{\kappa_1}^{\kappa_2} \left( \frac{I(y,t)/m_0 - I_0(y)}{I_0(y)} \right)^2 dy,$$

(7)

where $\kappa_{1,2} = (m_1/m_0) \pm 3\sigma_y$, and $I_0(y)$ is a Gaussian distribution with unit total mass and with the same center of mass ($m_1/m_0$) and $\sigma_y$ as $I(y,t)$.

### III. EXPERIMENTAL RESULTS AND DISCUSSION

By analogy with turbulent diffusion, dispersion in a homogeneous array is expected to become Fickian once the plume becomes well-mixed at, and significantly larger than, the spatial scales of all contributing mechanisms. In steady laminar flows and in turbulent flows through dense random arrays, the mechanisms that contribute to (macroscopic) lateral dispersion are molecular diffusion, turbulent diffusion, and dispersion associated with the tortuous flow path. Turbulent eddies are constrained by the local cylinder spacing, and thus the largest eddies scale with $(s_\phi)^A$ in dense arrays. Dispersion associated with the tortuous flow path can be modeled as a series of independent, discrete lateral deflections that fluid particles undergo as they flow around cylinders; each deflection is expected to scale with $d^2$. Accordingly, asymptotic dispersion is expected once $\langle \sigma^2 \rangle \geq d^2$, $(s_\phi)^A$.

The initial (small $x$) growth of the ensemble-averaged variance of the plumes may be approximated by $\langle \sigma^2 \rangle \approx 2D_{\text{pore}} t = 2D_{\text{pore}} x / \langle \bar{u} \rangle$, where the diffusion coefficient $D_{\text{pore}}$ characterizes pore-scale mixing. Then, the mean streamwise distance that the solute is advected before achieving asymptotic behavior, $x_\infty$, is expected to scale as

$$\frac{x_\infty}{d} \sim \left( \max(d, (s_\phi)^A) \right)^2 \frac{\langle \bar{u} \rangle d}{D_{\text{pore}}},$$

(8)

In steady laminar flow, $D_{\text{pore}}$ is the molecular diffusion coefficient $D_m$, and $x_\infty / d$ is linearly proportional to the Peclet number $Pe = \langle \bar{u} \rangle d / D_m$. This Pe dependence has been observed in simulations of different types of steady laminar obstructed flows, e.g., in packed beds of spheres, in periodic cylinder arrays, and in a lattice network. In contrast, in turbulent flow, the appropriate definition for $D_{\text{pore}}$ is the turbulent kinetic energy per unit mass and $l_t$ is the integral length scale of turbulence. Recall that $d > (s_\phi)^A$ in the arrays considered in the present paper (Sec. I; Table I). Under these conditions, Eq. (8) yields

$$\frac{x_\infty}{d} \sim \left( \frac{\langle k_t \rangle}{\langle \bar{u} \rangle} \right)^{-1}.$$  

(9)

It was previously shown that the integral length scale is accurately described by $l_t / d = \min(1, \langle s_\phi \rangle / d)$ in fully turbulent flow. Then, in the arrays considered in this paper,

$$\frac{l_t}{d} = \frac{(s_\phi)^A}{d},$$

(10)

which decreases monotonically as $\phi$ increases. Further, Eq. (10) is expected to be valid in transitional turbulent flow as well, because the physical reasoning behind it, i.e., that the cylinder spacing limits the size of the largest turbulent eddies, involves only a purely geometric constraint. In contrast, mean turbulence intensity, $\langle \langle k_t \rangle / \langle \bar{u} \rangle \rangle$, is expected to depend strongly on $Re_d$. In fully turbulent flow, $\langle \langle k_t \rangle / \langle \bar{u} \rangle \rangle$ can be described by scale models in Ref. 4; at smaller $Re_d$, values for $\langle \langle k_t \rangle / \langle \bar{u} \rangle \rangle$ are not available to our knowledge. Nevertheless, $\langle \langle k_t \rangle / \langle \bar{u} \rangle \rangle$ is expected to increase gradually and monotonically with $Re_d$ between laminar and fully turbulent conditions. While we are not aware of any experimental verification of this expected $Re_d$ dependence in packed beds or random cylinder arrays, it has been observed in ceramic foams and in numerical simulations of a periodic cylinder array. With this assumption, Eq. (9) implies that in turbulent flow, $x_\infty$ decreases as $Re_d$ increases until the flow becomes fully turbulent. The appropriate definition for $D_{\text{pore}}$ in unsteady laminar flow is not obvious. Nevertheless, $x_\infty$ is
expected to be continuous over all Re, which requires that \( x_c \) decrease as the flow transitions from the steady laminar flow regime to the turbulent flow regime.

In summary, dispersion in dense arrays (\( \phi > 0.19 \)) is expected to exhibit three properties. First, dispersion at large \( x \) is expected to be continuous over all Re, which necessarily increases with Re as well as in the region immediately downstream of the cylinder marked with a \( \times \). In the classic paper by Taylor, a transient decrease in the dispersion coefficient has been observed in numerical simulations of randomly packed bed of spheres and of lattice networks.

A. Pore-scale mixing and the approach to asymptotic dispersion

Pore-scale mixing was visualized using fluorescein for both solid volume fractions (\( \phi = 0.20 \) and 0.35) in steady laminar flow and in unsteady flow. The instantaneous distribution of fluorescein immediately downstream of its injection point is depicted in Fig. 5. It should be noted that the LIF measurements, discussed in Secs. III B and III C, were collected farther downstream (\( x \geq 59d \)). Beginning at the lowest Re, tracer streaklines remained stationary in time, indicating steady flow, everywhere in Figs. 5(a) and 5(e) except in the region immediately downstream of the cylinder marked with an oval in Fig. 5(a). Vortices shed from the marked cylinder, indicating that flow was unsteady in that region. This simultaneous occurrence of both steady and un-
steady flow at \((\phi, \text{Re}_d) = (0.20, 61)\) is attributed to the random distribution of the cylinders and the resulting spatial variations in the local velocity field.

In the absence of turbulence, the tracer is mixed with ambient fluid only through molecular diffusion, which is a very slow process: \(x\phi/d\) increases from left to right: (a) 21, (b) 42, (c) 58, (d) 87, and (e) 100. \(C_{ms}\) values are also in \((\text{cm}^{-1})\). The growth of \(\sigma_y^2\) with \(x\) can be discerned from the width of the profiles, which are truncated at \(y = m_y / m_\zeta \pm 3\sigma_y\).

**FIG. 6.** \(I(y,t)/m_y\) \((\text{cm}^{-1})\) (solid) and the corresponding \(I_y(x)\) \([\text{dotted; see Eq. (7)}]\) for selected runs at \((\phi, \text{Re}_d) = (0.35, 97–100)\). \(x\phi/d\) increases from left to right: (a) 21, (b) 42, (c) 58, (d) 87, and (e) 100. \(C_{ms}\) values are also in \((\text{cm}^{-1})\). The growth of \(\sigma_y^2\) with \(x\) can be discerned from the width of the profiles, which are truncated at \(y = m_y / m_\zeta \pm 3\sigma_y\).

\[
\text{Re}_d = 180 \quad \text{in the} \quad (\phi = 0.35 \text{ array, the interface between the tracer and the ambient fluid was blurred, and the tracer was well mixed at the pore scale and distributed over distances larger than} \quad d \quad \text{and} \quad \langle \nu_y \rangle \quad \text{by the time it reached the right edge of the respective photos [Figs. 5(c), 5(d), and 5(h)]}.
\]

The rapid pore-scale mixing and the absence of any apparent periodicity are indicative of turbulent flow, in which small-scale turbulence provides an additional, much faster mechanism for mixing within the pores. Recall that the tracer plume must be well mixed and much wider than \(d\) and \(\langle \nu_y \rangle\) for its dispersion to be asymptotic. Based on the observations above, we expect that beyond the steady laminar flow regime, the distance solute particles are advected before their dispersion becomes asymptotic, \(x_\phi\), is smaller at higher \(\text{Re}_d\).

**B. Deviation of the time-averaged concentration profile from a Gaussian distribution**

Fickian dispersion of solute released from a point source results in a Gaussian concentration distribution. Therefore, the deviation of the time-averaged concentration profile, \(\bar{I}(y,t)\), from a Gaussian distribution is a convenient measure of the proximity to Fickian behavior. For reference, selected \(\bar{I}(y,t)\) are presented in Fig. 6. The time-averaged profiles gradually approach a Gaussian distribution as \(x\) increases, and this trend is reflected in the corresponding reduction in the mean normalized squared deviation, \(C_{ms}\) \([\text{Eq. (7)}]\).

The ensemble-averaged \(C_{ms}\) are presented in Fig. 7 as a function of \(x\phi/d\), which is the expected number of cylinders in a \(d \times x\) area \((\text{multiplied by} \pi/4)\). Two salient trends can be identified at each \(\phi\). First, \(\langle C_{ms} \rangle\) decreases as \(x\) increases at each \(\text{Re}_d\); second, \(\langle C_{ms} \rangle\) decreases as \(\text{Re}_d\) increases for all \(x\).

**FIG. 7.** Ensemble average of mean normalized squared deviation \(C_{ms}\) \((\text{cm}^{-1})\) as defined by Eq. (7) for (a) \(\phi = 0.20\) at \(\text{Re}_d = 58–61\) (\(\bigcirc\)), 84–89 (\(\bigotimes\)), 110–120 (\(\times\)), and 310–340 [\(\square\) Ref. 4]; (b) \(\phi = 0.35\) at \(\text{Re}_d = 48–51\) (\(\bigcirc\)), 77–82 (\(\bigotimes\)), and 97–100 (\(\times\)). Each data point for these \((\phi, \text{Re}_d)\) represents an average of four or more runs: horizontal bars indicate the range in \(x\) and vertical bars indicate the standard error of the mean. (b) also includes individual \(C_{ms}\) at \((\phi, \text{Re}_d) = (0.35, 300–390)\) [\(\square\) Ref. 4]. Note that ensemble averages cannot be computed for this \((\phi, \text{Re}_d)\) because replicate measurements were not collected. Data points in gray correspond to \((\phi, \text{Re}_d, x)\) at which dispersion was observed to have occurred at the same rate as asymptotic dispersion in fully turbulent flow (Figs. 8 and 9). \((C_{ms}) = 2.5 \times 10^{-3} \text{ cm}^{-1}\) (dashed) is the proposed empirical boundary between the pre-asymptotic and asymptotic dispersion regimes. Dotted line is provided only as a guide to the eye.

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considered. The former trend is attributed to the plume having had more time to mix over the pore scale at larger $x$, and the latter trend is attributed to the enhancement of pore-scale mixing that results from the increase in turbulent intensity with increasing Re$_d$. Note that the large reduction in $(C_{ms})$ between Re$_d$=58–61 and 84–89 at $\phi=0.20$ coincides with the transition from steady to unsteady flow captured in Figs. 5(a) and 5(b). A similar reduction occurred at $\phi=0.35$ between Re$_d$=48–51 and 77–82. Note that the ranges in Re$_d$ reflect unintended small variations across replicate runs, not measurement uncertainty (Table 1).

At similar $x\phi/d$ and Re$_d$, $(C_{ms})$ was consistently larger at $\phi=0.35$ than at $\phi=0.20$ (Fig. 7). This trend implies that at larger $\phi$, a longer advective distance in terms of the number of cylinders per depth is required for dispersion to become Fickian for the same Re$_d$. This $\phi$ dependence is expected in fully turbulent flow, for which Eq. (9) predicts $x,\phi/d \sim 4$ for $\phi=0.35$ and $\sim 1$ for $\phi=0.20$ (Table 1). Unfortunately, the $\phi$ dependence of $x,\phi/d$ at transitional Re$_d$ cannot be predicted from Eq. (9) because measurements of $\langle \bar u \rangle / \tilde u$ are not available.

### C. Variance of the time-averaged concentration profile

Measurements of the ensemble-averaged variance of time-averaged concentration profiles ($\sigma^2(x)$) are presented in Figs. 8 and 9 for $\phi=0.20$ and 0.35, respectively. A subset of previous measurements at high Re$_d$ is also included ( ). As discussed in Sec. I, $\sigma^2(x)$ at a given $x$ becomes Re$_d$ independent at Re$_d=200$ in a $\phi=0.20$ array. The high Re$_d$ ($=310–340$) data for $\phi=0.20$ were selected from this regime, i.e., both $\sigma^2(x)$ and $d(\sigma^2)/dx$ are independent of Re$_d$. At $\phi=0.35$, Re$_d$-independent $\sigma^2(x)$ could not be confirmed under experimental conditions considered previously or in the present study, i.e., at Re$_d<400$. Therefore, $\sigma^2(x)$ at $\phi,Re_d=(0.35,300–390)$ is expected to be smaller than in fully turbulent flow for all $x$ (cf. Fig. 4). Unfortunately, the laboratory flume cannot accommodate Re$_d>400$ at this $\phi$.

Both $\langle \bar u \rangle$ and $d(\sigma^2)/dx$ decrease as Re$_d$ decreases below Re$_d\approx 100$ at both $\phi$ (Figs. 8 and 9). As depicted in Fig. 4, this Re$_d$ dependence of $d(\sigma^2)/dx$ at transitional Re$_d$ is attributed to pre-asymptotic effects, i.e., the plume reached the measurement location before the solute particles experienced a sufficiently large subset of velocities in the array or, equivalently, before the solute particles dispersed over scales much larger than $d$ and $\langle \bar u \rangle$ [Eq. (8)]. A qualitatively similar increase in $d(\sigma^2)/dx$ (with $\langle \bar u \rangle/\nu$ at a fixed $x$ and subsequent $\langle \bar u \rangle/\nu$ independence at high $\langle \bar u \rangle/\nu$ have been observed in ceramic foam.

If the observed Re$_d$ dependence of $d(\sigma^2)/dx$ is indeed due to pre-asymptotic effects, then $d(\sigma^2)/dx$ is expected to approach its asymptotic value in fully turbulent flow at sufficiently large $x$ (Fig. 4). At $(\phi,Re_d)=(0.20,58–61)$, flow is steady and laminar [Fig. 5(a)], and the predicted $x,\phi/d$ [Eq. (8)] is two orders of magnitude larger than the length of the laboratory array $\langle \bar u \rangle$ (d=88). Accordingly, $d(\sigma^2)/dx$ remains smaller than its asymptotic value at high Re$_d$ ($=310–340$) for all $x$ considered (Fig. 8 (a)). In contrast, at $(\phi,Re_d)=(0.20,110–120)$, $d(\sigma^2)/dx$ is equal to its asymptotic value at high Re$_d$ within uncertainty at $x,\phi/d>30$, but deviates at $x,\phi/d=20$ [Fig. 8 (b)]. This trend indicates that asymptotic dispersion is reached in the range $20<x,\phi/d\leq 30$ at this $(\phi,Re_d)$. This distance is larger than $\langle x,\phi/d \sim 1$, predicted by Eq. (9) for fully turbulent flow, presumably because the flow was not fully turbulent at this Re$_d=110–120$ and the weaker turbulence intensity relative to fully turbulent flow extended $x$ (cf. Fig. 4).

It is convenient to relate numerical values of $C_{ms}$ asymptotic/pre-asymptotic dispersion as interpreted directly from $d(\sigma^2)/dx$. A threshold defined at $C_{ms}=2.5\times 10^{-3}$ cm$^{-1}$ accurately segregates $(\phi,Re_d)=(0.20,110–120)$ data in the asymptotic regime (x, $\phi/d>30$) from those in the pre-asymptotic regime (x, $\phi/d\leq 20$),
as determined above. With this threshold, $\langle C_{ms} \rangle$ for $(\phi, \text{Re}_d) = (0.20, 58–61)$ and $(0.20, 310–340)$ are also correctly classified into the pre-asymptotic and asymptotic dispersion regimes, respectively [Fig. 7(a)].

This criterion can now be applied to classify the remaining measurements, for which the anticipated transition from pre-asymptotic to asymptotic (large $x$) dispersion cannot be easily identified by eye (Figs. 8 and 9). For example, the range of $x$ over which $d\sigma_y^2/dx$ is constant is not obvious for $(\phi, \text{Re}_d) = (0.35, 300–390)$. However, $\langle C_{ms} \rangle$ at $(\phi, \text{Re}_d) = (0.35, 300–390)$ and $(0.20, 84–89)$ fall below the threshold at $x/d > 30$ and $\approx 55$, respectively, implying that the asymptotic regime had been reached at these $x$. Least-squares fit to data in the former yields $d\sigma_y^2/dx = 0.36 \pm 0.05 \text{ cm}$ (coefficient of determination $r^2 = 0.86$, $n = 10$) (Fig. 9, dashed line). The correlation is highly significant, confirming that dispersion is indeed Fickian at $x/d > 30$. Furthermore, the best-fit $d\sigma_y^2/dx$ is equal, within uncertainty, to that predicted for fully turbulent flow by Eq. (2.21) in Ref. 4, with scaling constants proposed therein (Fig. 9, solid line). This agreement supports the conjecture that asymptotic $d(\sigma_y^2)/dx$ is independent of $\text{Re}_d$. In contrast, a linear regression at $55 < x/d < 72$ for $(\phi, \text{Re}_d) = (0.20, 84–89)$ does not yield a significant correlation ($r^2 = 0.56$, $n = 3$). Since $\langle \sigma_y^2 \rangle$ measurements are constant within standard error in this range, the weak correlation is most likely due to measurements not extending sufficiently far into the asymptotic regime. Finally, $\langle C_{ms} \rangle$ (or $\langle C_{ms} \rangle$) corresponding to $(\phi, \text{Re}_d, x)$ at which measured $d(\sigma_y^2)/dx$ (or $d\sigma_y^2/dx$) was identified above to have occurred at the same rate as asymptotic dispersion in fully turbulent flow are plotted in gray in Fig. 7. The empirical boundary $\langle C_{ms} \rangle = 2.5 \times 10^{-3} \text{ cm}^{-1}$ (dashed) accurately segregates the ensemble-averaged time-averaged concentration profiles in the asymptotic regime from those in the pre-asymptotic regime.

IV. CONCLUSIONS

The lateral distribution of passive solute released continuously from a point source was measured at $x/d = 59–370$ and $\text{Re}_d = 48–120$ in random cylinder arrays of solid volume fraction $\phi = 0.20$ and 0.35, and the results were compared to previous measurements at $\text{Re}_d = 300–390$. Previous predictions for fully turbulent flow imply that asymptotic (large $x$) lateral dispersion, $K_{yy}/(u_d d)$, will not exhibit a strong dependence on $\text{Re}_d$ at these $\phi$. Measured $d(\sigma_y^2)/dx$ reached asymptotic rates predicted for fully turbulent flow at $(\phi, \text{Re}_d) = (0.20, 110–120)$ and $(0.35, 300–390)$ at large $x$, supporting this conjecture. At the same time, it is apparent from flow visualization that turbulence substantially enhances pore-scale mixing, which in turn accelerates the approach to asymptotic dispersion. This $\text{Re}_d$ dependence explains why dispersion at lower $\text{Re}_d (<100)$ did not reach the fully turbulent flow limit within the length and width of the laboratory array.

Further, results suggest that at the same $\text{Re}_d$, the pre-asymptotic regime extends farther downstream at $\phi = 0.35$, in terms of the number of cylinders spanned $(x/d)$, than at $\phi = 0.20$. Concentration profiles within the asymptotic dispersion regime deviated from a Gaussian distribution by less than $\langle C_{ms} \rangle = 2.5 \times 10^{-3} \text{ cm}^{-1}$ in both previous and present experiments. Using this as an empirical criterion, we can evaluate from time-averaged profiles at a single streamwise location whether dispersion at that distance is pre-asymptotic or asymptotic. This method requires significantly less effort than obtaining measurements at multiple streamwise positions to evaluate $d(\sigma_y^2)/dx$.

Returning to our original motivation for this study, let us now consider typical conditions in aquatic systems with vegetation. The above discussion suggests that solute released into dense vegetation in the field undergoes pre-asymptotic dispersion through a significant portion of the system. A rough extrapolation of our laboratory data suggests that at $\text{Re}_d $= 60, solute introduced in dense $(\phi > 0.19)$ arrays must flow through a distance larger than $x/d = 100$ before its dispersion becomes asymptotic (Fig. 7). This value corresponds to $x = 30 \text{ m}$, for example, in a random array with the same $d = 3 \text{ cm}$ and $\phi = 0.1$ as measured by Lightbody et al. in a constructed wetland ($\text{Re}_d = 30–40$). This is a non-negligible distance in many aquatic systems (e.g., constructed wetlands, mangrove swamps, and salt marshes), where homogeneous vegetation conditions extend no more than about 200 m perpendicular to land, and in some systems much less. In particular, constructed wetlands typically comprise multiple cells of variable dimensions, and the shortest cell may only extend 30–300 m (Refs. 36 and 41–43) streamwise.

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