Jeffrey Rohlfs’ 1974 Model of Facebook: An Introduction

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This short essay, forthcoming in *Competition Policy International*, summarizes and evaluates Jeffrey Rohlfs’ 1974 *Bell Journal* paper, “A Theory of Interdependent Demand for a Telecommunications Service.” Rohlfs’ work helped create a large literature on markets with network externalities in which demand decisions have long-lasting consequences, a literature that has informed competition policy. But Rohlfs assumed that demand-side decisions did not have long-lasting consequences. Social networking and Internet-based markets of this sort are increasingly important but have not been extensively studied. While they may pose interesting antitrust challenges, they are almost certainly not the challenges to which the post-Rohlfs literature pointed.

I. Introduction

Jeffrey Rohlfs’ pioneering 1974 study of demand in the presence of network externalities,² which make each actor’s demand for some good or service depend in part on whether others purchase it, laid the foundation for a huge academic literature that has had a major impact on antitrust policy. The government’s case in *U.S. v. Microsoft*, for instance, relied heavily on network externality arguments.

In most of the post-Rohlfs network-effects literature, buyers are modeled as making long-term product or technology choices because those choices either involve the purchase of significant durable goods or create switching costs. Examples include the choices between VHS and Betamax VCRs or between Apple and Wintel computers, or the choice to purchase an early fax machine.

In contrast, Rohlfs presents a model that seems better suited to analysis of new Internet-based businesses that rely on network effects, like Facebook and YouTube. These businesses provide services rather than durable goods, and their customers are not required to make long-term commitments. Switching costs are at most moderate, and customers can often participate in multiple competing networks at the same time. (In the terminology of the recent, related literature

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1 Howard W. Johnson Professor of Economics and Management, Massachusetts Institute of Technology. I am indebted to David Evans and Joe Farrell for helpful comments, but errors and opinions are mine alone. It is a particular pleasure to help increase awareness of the Rohlfs paper both because its insights have shaped some of my own recent work (as discussed below) and because I remember Jeff fondly from our time together in graduate school at MIT.

on two-sided markets, they can “multi-home.” Thus I think the Rohlfs paper deserves to be read carefully on its own, apart from the literature it helped to launch.

Rohlfs’s fundamental assumption was that the amount any individual firm or household would be willing to pay for a telecommunications service would depend on the set of other individuals with whom they could use that service to communicate. This effect, what we now call a *direct* network effect or externality, seems to have been recognized from the first days of the telephone industry. (The recent two-sided markets literature, in contrast, focuses mainly on *indirect* network externalities, in which participation by one customer group makes a platform more attractive to another customer group.) Rohlfs’ was not the first formal analysis to incorporate this assumption: his paper cites earlier articles by Artle & Averous and by Squire that do so. Those papers were concerned with optimal telecommunications pricing, however, while Rohlfs’ generally took price as given and provided a deeper, more general analysis of the implications of network externalities for market demand—without, it should be noted, ever using the term “network externalities.”

The next section provides a guide to Rohlfs’ analysis. It is, I hope, a bit easier to digest than the paper it attempts to exposit, but it is intended mainly for economists and lawyers who are tolerant of formal reasoning. None of its equations are essential for Section III, which describes the impact Rohlfs’ paper has had and considers some of its implications for both economic analysis of and competition policy toward Facebook-like businesses.

II. A New Telecommunications Service

Writing as a Member of the Technical Staff at Bell Laboratories, Rohlfs was mainly concerned with the problem of launching a new telecommunications service (his main example, in 1974, was a video communications service) and thus with disequilibrium situations. His most general model (in Section 2) begins with the assumption that, all else equal, if any individual $i$ subscribes to the telecommunications service under consideration, her utility will not be decreased and may be increased if any other individual $j$ also subscribes. Since in the 1970s customers of the Bell System


could only rent telephones and other terminal equipment, it was natural for Rohlfs to assume that subscriptions to a new service would involve only a per-period price, with no fixed cost of subscribing or unsubscribing.

Under these assumptions, Rohlfs defines an equilibrium user set: taking prices and all other individuals’ status as given, each individual in such a set wishes to continue to subscribe to the service, and each individual not in the set does not wish to subscribe. This is a natural Nash equilibrium concept, but note that it rules out both explicitly coordinated behavior and non-myopic behavior that depends on expectations of others’ future actions. Since decisions to subscribe or unsubscribe could be reversed at little, if any, cost in the regulated telecommunications sector circa 1974, it is not unreasonable to assume that expectations about the future would not affect an individual’s current actions. As we discuss below, however, coordination devices could nonetheless have both private and social value.

Rohlfs notes that it is a fundamental feature of this model that equilibrium user sets are not generally unique. Consider, for example, a population of four individuals in which individuals 1 and 2 are willing to pay a lot to be able to communicate with each other but neither has any interest in individual 3 or individual 4. Suppose the situation is symmetric, so that individuals 3 and 4 are eager to communicate with each other but care nothing for 1 or 2. Then, if the service is of no value unless it enables you to communicate with somebody you care about and if its price is low enough, there will be four equilibrium user sets: the null set, individuals 1 and 2, individuals 3 and 4, and all four individuals.

Whenever the service is of no value to any individual unless at least one other individual subscribes, the null set, in which no one subscribes, is always an equilibrium user set. Similarly, in more general settings there may also be a dramatic difference between the largest and smallest equilibrium user sets, and, as Rohlfs notes, “In a practical situation, this difference may mean the difference between marketing success and failure.”5

Real markets with large numbers of participants rarely leap instantly to equilibrium, particularly when they involve novel goods or services. To reflect this, Rohlfs considers a broad class of adjustment processes in which, out of equilibrium, each individual who wishes to subscribe or unsubscribe does so with some finite lag. In general, the equilibrium set to which such processes converge depends on the initial set of subscribers and, possibly, on the details of the adjustment

5 Rohlfs, supra note 1, at p. 24.
process. In particular, Rohlfs notes, “It may be critical whether or not the disequilibrium non-users subscribe before the disequilibrium users drop out.”

In order to obtain sharper results, Rohlfs turns in his Section 3 to the special case in which the utility of the service to any one individual depends only on the number of other individuals who also subscribe, which he refers to as the uniform calling model, and the relationship is linear. This portion of the paper has perhaps been the most influential. Let $f$ be the fraction of the total relevant population that subscribes. Then for a given positive price, $p$, a typical individual $i$ will want to subscribe to the service if and only if

$$f w_i - p \geq 0, \text{ or } w_i \geq p/f.$$  

where $w_i$ is a non-negative constant. For a large population, one can treat the $w_i$ as distributed according to a smooth probability density function, $h(w)$.

Given all the assumptions in the preceding paragraph, the shape of $h(w)$ and the corresponding distribution function, $H(w)$, will determine the characteristics of market demand. Taking price as given, if the fraction of the population currently subscribing is $f$, the fraction of the population who will want to subscribe once they know this, $f^*$, is just the fraction of the population for which equation (1) is satisfied:

$$f^*(f \mid p) = 1 - H(p/f).$$

Because distribution functions are non-decreasing, the function $f^*(f)$ is non-increasing in $p$ and non-decreasing in $f$. If $f^*(f \mid p) = f$, then, given $p$, there is an equilibrium user set consisting of the fraction $f_i$ of the population with the largest values of $w$. As long as price is positive, the null set ($f = 0$) is always an equilibrium user set, since $H(\infty) = 1$ for any distribution function. There may be no equilibrium user sets with $f > 0$, or there may be one or more such sets. If $H(p) = 1$, so that everyone in the population is willing to pay price $p$, then $f^*(1 \mid p) = 1$, and the whole population is an equilibrium user set.

To analyze disequilibrium situations, Rohlfs considers a general class of adjustment mechanisms according to which $f$ increases over time if $f < f^*$, and $f$ decreases over time if $f > f^*$. It is then easy to show that if there are equilibria with $f > 0$, stable and unstable equilibria must

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6 Id.
alternate. That is, if the null set is a stable (unstable) equilibrium, the next smallest, if it exists, must be unstable (stable), and so on. Rohlfs defines the critical mass problem for the new system as the problem of somehow reaching a level of $f$ such that $f^* > f$ and the business is viable at the next highest stable equilibrium, to which the system will then tend over time if price remains fixed. He notes, however, that attaining the socially optimal equilibrium user set “may require ruinous (albeit temporary) promotional costs.” Moreover, he adds that although he naturally assumes in his analysis “that the product is viable, it is worth noting that in real life the seller would have no such guarantee.”

If there are multiple stable equilibria, it may be advantageous for both the seller and its customers (for whom participation by others adds value) if the market attains an equilibrium with high participation rather than a lower equilibrium or even the null equilibrium in which there is zero participation. For this reason, devices to coordinate behavior may have both private and social value. Thus, for instance, Glen Weyl analyzes “insulating tariffs,” in which prices are carefully set as functions of participation levels so that the market is guided to the desired equilibrium. In practice, of course, particularly with highly innovative new products, the distribution of reservation prices is unknown and thus so are the available equilibria. And, partly as a consequence, investors’ enthusiasm for spending on costly coordination attempts is typically muted. On the other hand, it is not unusual for new products with network effects to raise price over time as participation grows and the product thus becomes more attractive, and Rohlfs considers strategies of this sort. One can think of these as developing and announcing pieces of insulating tariffs on the fly.

Much of the analysis in Rohlfs’ Section 3 is devoted to an example in which $w$ is distributed uniformly between zero and one, so that $H(w) = w$ over that interval. Then (2) becomes simply

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(3) \quad f^*(p|w) = \begin{cases} 
1 - (p/f), & \text{for } 0 < p < f \leq 1, \\
0, & \text{for } p > f. 
\end{cases}
\]

7 Suppose some equilibrium point, $f_0$, is stable. Then $df^*/df$ must be less than one at that point, so that $f^* > f$ for $f$ just below $f_0$, and $f^* < f$ for $f$ just above $f_0$. But then in order for $f$ and $f^*$ to be equal for some $f_1 > f_0$, $f^*$ must rise faster than $f$ near $f_1$, since $f^*$ is approaching $f$ from below. Thus $df^*/df$ must exceed one at $f_1$, from which it follows that that equilibrium is unstable. Repeating the argument starting at an unstable equilibrium establishes the result.
8 Rohlfs, supra note 1, at p 32.
9 Id. at p. 33.
11 A common alternative in the Internet world is to keep price to consumers at zero but to degrade quality by adding advertising.
Solving equation (3) for $f = f^*$, there are at most three distinct equilibria, which are shown in Figure 1:

$$f_0 = 0, \quad f_1 = \frac{1-\sqrt{1-4p}}{2}, \quad \text{and} \quad f_2 = \frac{1+\sqrt{1+4p}}{2}.$$  

Note that if $p > 1/4$, the only equilibrium involves no subscribers, even though if everybody subscribed, $3/4$ of them would gladly pay more than $1/4$. The problem is that those willing to pay less than $1/4$ even with the entire population subscribing would leave the service, reducing its attractiveness to those who remained, inducing further defections, and so on until the business spiraled down to zero.

The arrows in Figure 1 illustrate the dynamics of this example for some $p < 1/4$. The smallest equilibrium, $f_0$, is stable; the next smallest, $f_1$, is unstable, and the largest, $f_2$, is stable. At the given price, the critical mass problem is to get the fraction of the population subscribing above $f_1$. If that can’t be done, the subscriber base will inevitably shrink to zero, but if it is done, network effects will fuel organic growth to the largest equilibrium, $f_2$.

In Section 4, Rohlfs generalizes the linear utility model in equation (1) to a situation in which the population can be divided into $k$ groups. Initially he assumes that individuals in group $i$ care only about the number of other individuals in group $i$ who subscribe; then he allows for inter-group externalities. In the latter case the analysis involves considerations of group-specific critical mass levels, and the ultimate equilibrium reached from any specific starting point depends on the details of both that point and the adjustment process.

Rohlfs’ Section 5 considers two approaches to solving the start-up problem for a new service: 1) free service to a carefully selected group of people for a limited period of time, and 2) a low introductory price that is raised over time. If being a subscriber has value only to the extent that it enables communication (rather than, say, because it is a status symbol), the hard part is getting two or more individuals with high values of $w$ to subscribe at a positive price. In theory, at least, this might be done by offering the service for free for a limited time to a targeted group, then raising price just above zero so that at least a few high-$w$ individuals find it optimal to remain subscribers.

Once this has been done successfully, however, at least in the uniform calling case, price can then be increased gradually over time according to some function $p^g(f)$. As long as $p^g(f) < f(1-f)$, $f^*$
will exceed \( f \) from (3), and the subscriber base will grow. (Once again, it doesn’t matter whether individuals anticipate later price increases or not, since myopic behavior is individually rational here.) When the optimal level of price is reached, price is constant thereafter, and the system adjusts to the higher, stable equilibrium corresponding to that price. These pricing policies are, as I noted above, broadly in the spirit of Weyl’s insulating tariffs, but Rohlf's seems to view them as generally being developed as information about demand arrives, rather than as the results of \textit{ex ante} optimization with demand known.

Rohlf's establishes a very neat result in this context. Suppose that at some point, after the price has been raised to \( p_0 > 0 \) and all those who find it optimal to unsubscribe have done so, a fraction \( f_0 > 0 \) of the population finds it optimal to remain on the system. Then as long as \( p^0(f) \) is less than \( f(1-f) \), as above, and the elasticity of \( p^0(f) \) with respect to its argument does not exceed one, the system will grow and no subscriber will ever leave.\(^{12}\)

All this rests on the extremely unrealistic assumption that the distribution of the \( w_i \) is somehow known, of course. And, as Rohlf's shows, if the uniform calling assumption does not hold, devising a startup strategy necessarily becomes more complex even if all taste distributions are known, and the details of non-uniformity matter.

\section*{III. Impact and Implications}

Jeffrey Rohlf's 1974 paper has been widely cited – 669 times according to Google Scholar – but its importance beyond telecommunications took some time to be recognized.\(^ {13}\) The first widely cited paper that I can find that cites the Rohlf's paper is a 1980 survey of the economic theory of clubs.\(^ {14}\) The literature on network externalities, and with it citations of the Rohlf's paper, exploded in the mid-1980s with the publication of influential papers by Joseph Farrell and Garth Saloner and by Michael Katz and Carl Shapiro that acknowledged Rohlf's contribution.\(^ {15}\)

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  \item[12] If at some value of \( f \), household \( i \) finds it optimal to subscribe, it must be that \( w_i \geq p^*(f)/f \). This household will remain a subscriber as \( f \) increases if \( p^*(f)/f \) does not increase, which is equivalent to the requirement that the elasticity of \( p^* \) with respect to \( f \) not exceed unity. Rohlf's requires \( p^*(f) \) to be concave because he assumes that it passes through the origin.
  \item[13] All citation counts are as of March 10, 2011.
  \item[15] Joseph Farrell and Garth Saloner, \textit{Standardization, Compatibility, and Innovation}, Rand Journal of Economics 16, 70-83 (1985) and Michael L. Katz and Carl Shapiro, \textit{Technology Adoption in the Presence of Network Externalities}, Journal of Political Economy 94, 822-841. These papers have 1662 and 1713 Google Scholar citations, respectively. Interestingly enough, the most frequently cited of the first-round network externality papers by these authors (Michael L. Katz and Carl Shapiro, \textit{Network Externalities, Competition, and Compatibility}, American Economic Review 73, 424-
\end{enumerate}
As noted above, however, the literature that grew out of these papers deals with very different market environments than those considered by Rohlfs. In a widely cited 1994 survey, Michael Katz and Carl Shapiro note that in this literature’s analysis of technology adoption and product selection decisions, expectations and coordination play important roles.\textsuperscript{16} As we noted above, expectations and coordination do not appear at all in the Rohlfs analysis, with the exception that he considers increasing price paths as a device to attain a desirable equilibrium.

Katz and Shapiro’s initial example of a technology adoption decision is the decision to buy a fax machine.\textsuperscript{17} In that decision expectations about the behavior of other potential fax machine owners are clearly important. Had fax machines existed in the early 1970s when Rohlfs was writing, however, they would have been available only for rent from the local telephone company, and expectations and coordination would have been of much less importance in decision-making about subscribing to fax services. Similarly, expectations and coordination were clearly important in making many of the product selection decisions between incompatible rival “hardware/software” systems that they describe as being “in the newspaper almost every day.”\textsuperscript{18} These include the choice between Beta and VHS video recording systems and among rival home video game systems. Again, if short-term rental of the “hardware” parts of these systems were available, expectations and coordination would be much less important.

The focus of this post-Rohlfs network-effects literature on technology and product selection decisions with long-lasting consequences sent two important messages to competition policy-makers. The first was implicit: technology adoption and product selection were generally modeled as discrete, once-and-for-all decisions that typically produced winner-take-all results. Not only is that how the choice between Beta and VHS seemed to most observers at the time, but to model multi-stage processes, in which expectations of future technologies and products (like DVDs and Blu-Ray and…) could influence today’s choices, would have involved considerable incremental complexity. The second was explicit: most theoretical analysis showed that market outcomes in markets with once-and-for-all competition and network effects could be seriously socially

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\textsuperscript{16} Michael L. Katz and Carl Shapiro, \textit{Systems Competition and Network Effects}, Journal of Economic Perspectives 8, 93-115 (1994). This article has 1621 Google Scholar citations. Katz and Shapiro also note that the literature they survey considers compatibility decisions, but these are beyond the scope of the present discussion.
\textsuperscript{17} Id. at p. 93.
\textsuperscript{18} Id. at p. 105.
\end{footnotesize}
inefficient: buyers could find themselves selecting the wrong product or technology, and society could be locked-in to those bad choices for the foreseeable future.

Katz and Shapiro do counsel caution in their 1994 survey and note that, “In short, we are far from having a general theory of when government intervention is preferable to an unregulated market outcome.” The literature they surveyed nonetheless suggested that, at the very least, competition authorities should pay particular attention to competition in industries with network effects to ensure that firms with short-run market power don’t use anticompetitive behavior and network effects to build and lock in dominant positions for the long haul. As Carl Shapiro, then Deputy Assistant Attorney General for Economics in the Antitrust Division of the U.S. Department of Justice, put it in a January, 1996 speech:

Even more so than in other areas, antitrust policy in network industries must pay careful attention to firms’ business strategies, the motives behind these strategies, and their likely effects… Furthermore, antitrust enforcers must be alert in these industries because the very nature of the “positive feedback” cycle means that monopolization may be accomplished swiftly. And, once achieved, the network effects that helped create dominance may make it more difficult for new entrants to dislodge the market leader than in other industries lacking network characteristics.

The Justice Department’s Sherman Act case against Microsoft, which rested heavily on arguments involving network effects, was filed in May, 1998.

All this suggests that the Rohlfs paper reprinted here may have become one of those classics that is often cited but rarely read. I think that is unfortunate, but not because the network-effects literature than began in the mid-1980s is wrong in any technical sense or inapplicable to some markets. Nor do I think the policy concerns it raised are not relevant in those markets or that the enforcement stance expressed by Professor Shapiro in the thoughtful speech just cited is inappropriate in those markets.

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19 Id. at p. 113.
The point is that the network-effects literature that began in the mid-1980s does not seem to have much to say about markets in which product or technology choices do not have long-lasting consequences – markets like those analyzed by Rohlfs. Without durability, expectations are not critical to decision-making, and lock-in is much less likely, particularly if multi-homing is possible.

And, as David Evans and I have argued,22 “almost every day” one now reads about markets of the sort analyzed by Rohlfs. Consider social networks, for instance. There are clearly direct network effects in these businesses: the value of being a participant in any particular social network depends on who else is participating. (There are also indirect network effects in these businesses, since participants attract advertisers.) But it is easy to switch between networks or to participate in multiple networks at the same time. Thus despite its early network-effect-enhanced advantages, MySpace has been almost totally eclipsed by Facebook, which used a university-based launch strategy that could have been cribbed from Rohlfs’ discussion of launching a new service in a population consisting of multiple groups with strong intra-group affinities. Network effects, while present, were clearly not the only important factors in competition between MySpace and Facebook.

It is particularly interesting to note that video communications services, of the sort that Rohlfs could only hypothesize in the early 1970s, are now generally available on the Internet. And, as Rohlfs assumed, network effects plainly matter in choosing which service to use. But deciding to use one or another at any point in time has essentially no long-term implications. I currently software for two such services on my computer and could no doubt access others if I saw benefits from doing so. I have no reason to care which service will prove more popular in the future, as I can use any one when it is worth using and ignore it otherwise.

As a final example, consider smart phone operating systems. As I write this, there is a vigorous struggle going on between Apple and Google, with Microsoft and RIM also engaged. But this is not the PC market in the 1990s: consumers buy new smart phones fairly often, and the costs of switching between phones based on different operating systems do not seem to be significant. Thus, even though consumers don’t generally multi-home in this market, as they easily can for social networks and video calling services, product selection does not involve a very durable commitment, and competition does not look like a one-shot, winner-take-all affair.

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22 David S. Evans and Richard Schmalensee, Failure to Launch: Critical Mass in Platform Businesses, Review of Network Economics 9, Article 1 (2010). That paper provides several brief case studies and generalizes the Rohlfs uniform calling analysis to platform businesses serving two customer groups, between which there are indirect network externalities. The nature of the critical mass constraint – “chicken and egg” or “chicken or egg” – is shown to depend on details of the adjustment process, and if there are multiple equilibria, stable equilibria and saddle-points alternate.
Thus, while Jeffrey Rohlfs’ paper has been influential in calling the attention of the economics profession to markets with network externalities, both economists and policy-makers have tended until recently to focus on a particular subset of those market—those in which it is at least arguably the case that anticompetitive behavior during critical periods of winner-take-all competition may lead to undesirable and long-lasting outcomes. In such markets, especially close attention by antitrust authorities during those critical periods is appropriate. Rohlfs, however, considered a different subset of markets with network effects, one that I believe is becoming more important in part because of the internet. In these markets switching costs are not important, and the key decisions do not involve purchases of big-ticket, long-lived durables. Accordingly, participation decisions can easily be reversed. Moreover, multi-homing is often possible in these markets, so even at the individual level competition is not a winner-take-all matter even for short periods of time.

This is not to say that the markets considered by Rohlfs may not raise novel and interesting competition policy issues, since the presence of externalities and the potential for multiple equilibria imply the possibility of departures from the textbook norm. But it is far from clear that such markets deserve especially close antitrust scrutiny.\(^{23}\) In any case, I believe Rohlfs’ analysis deserves to be read carefully and to be both extended and applied by economists and, further, that its implications should be carefully considered by antitrust enforcement agencies as they increasingly deal with markets for which that analysis is relevant.

\(^{23}\) If network effects do lead to dominance, for instance, but that dominance can be quickly eroded by a better product either because participation decisions can be costlessly reversed or through multi-homing, the general presumption that network effects create troublesome entry barriers would not hold. Joe Farrell has pointed out (personal communication) that in some respects this sort of dominance resembles market-wide exclusive dealing contracts of very short duration.
Figure 1. The Example from Rohlfs’ Section 3

\[ f^* = 1 - \left( \frac{p}{f} \right) \]