Fracture mode control: a bio-inspired strategy to combat catastrophic damage

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The excellent mechanical properties of natural biomaterials have attracted intense attention from researchers with focus on the strengthening and toughening mechanisms. Nevertheless, no material is unconquerable under sufficiently high load. If fracture is unavoidable, constraining the damage scope turns to be a practical way to preserve the integrity of the whole structure. Recent studies on biomaterials have revealed that many structural biomaterials tend to be fractured, under sufficiently high indentation load, through ring cracking which is more localized and hence less destructive compared to the radial one. Inspired by this observation, here we explore the factors affecting the fracture mode of structural biomaterials idealized as laminated materials. Our results suggest that fracture mode of laminated materials depends on the coating/substrate modulus mismatch and the indenter size. A map of fracture mode is developed, showing a critical modulus mismatch (CMM), below which ring cracking dominates irrespective of the indenter size. Many structural biomaterials in nature are found to have modulus mismatch close to the CMM. Our results not only shed light on the mechanics of inclination to ring cracking exhibited by structural biomaterials but are of great value to the design of laminated structures with better persistence of structural integrity.

Biological competition between predator and prey happens ubiquitously in nature. Through millions of years’ evolution, organisms have developed diverse strategies to enhance their chance of survival. For example, animals such as fishes, turtles and snails, tend to equip themselves with hard exoskeletons to protect their vulnerable bodies. The survival of these species through biological competitions proved the success of the adopted biological materials in protection against the mechanical attacks and may imply ingenious design conceptions potentially applicable to the synthetic protective materials in engineering. In recent decades, intensive efforts¹⁻⁴ have been devoted to the studies on bioarmors in an attempt to understand the underlying principles for material design. So far, most works have been focusing on the mechanisms accounting for the excellent mechanical properties such as higher strength and toughness compared to their building constituents⁵⁻¹³, inspiring a number of biomimetic endeavors¹⁴⁻¹⁷. Despite the strengthening and toughening mechanisms which increase the difficulty of crack formation and propagation in materials, fracture is still unavoidable when overwhelming loading is encountered. Under this circumstance, reducing the scope of damage to the minimum extent becomes a realistic choice to combat the catastrophic damage which is vital to the survival of the individuals and even the species. Such strategy is believed to be present in nature after reviewing the findings of recent studies on structural biomaterials. For instance, it has been reported that indentation on the scale of Polypterus senegalus, a ‘living-fossil’ fish species, normally causes ring cracks rather than radial ones¹. Such inclination to ring cracks was also exhibited by the shell of Crysomallon squamiferum¹⁸, a snail species inhabiting in deep sea hydrothermal environment, and teeth of Mylopharyngodon piceus (see Fig. 1a), a fish species feeding on mollusks¹⁹; but it is not often the case in synthetic systems such as glass coating on polymeric substrate²⁰ (Fig. 1b) and dental crown²¹. From the perspective of overall structural integrity, ring cracks are more localized and therefore less destructive than the radial ones. Is the inclination to ring cracking exhibited by the above-mentioned biological structural materials an accidental phenomenon or a consequence of evolution? If it results from natural selection, what are the geometrical and mechanical features of the structural materials that regulate their fracture modes under indentation? Are there any guidelines that we can follow to control the fracture modes as desired? Speculation of these questions inspires us to investigate the mechanics of fracture modes of biological materials undergoing external load represented by indentation.

For a material free of pre-existing defects, fracture due to external loadings tends to start from crack initiation, followed by crack propagation. The location of the crack initiation and the orientation of the resulting crack plane depend on the stress field developed by the external loading, and the anisotropy, inhomogeneity and “mechanical...
nature" of the material. For example, a homogeneous and isotropic cylinder specimen, when subjected to uniaxial tension, generally breaks along the plane perpendicular to the longitudinal axis if it is made by brittle material while fractures along conical surface at 45° to the longitudinal axis if it is made by ductile material. This is because brittle materials generally fail along the plane with maximum tensile stress while the ductile materials often fail along the plane having maximum shear stress. To predict the onset of crack initiation as well as the orientation of the resulting crack plane, setting a proper failure criterion is necessary. Due to the lack of a universal failure criterion, various failure criteria have been developed for materials with diverse mechanical natures. For example, the simple maximum-tensile-stress criterion, also known as Rankine criterion, states that a crack will occur when the maximum tensile principal stress (σ1) exceeds the fracture strength (σf) of the material. It applies to brittle and even quasi-brittle materials such as concrete, rocks, minerals and ceramics if the materials’ anisotropy and inhomogeneity are negligible. The plane of the resulting crack predicted by the Rankine criterion is perpendicular to the direction of the maximum principal stress. Considering that most structural biomaterials are biomineralized composites with brittle minerals (e.g. calcium carbonate and hydroxyapatite) being their major constituents, Rankine criterion is considered suitable for them.

Most structural biomaterials in nature exhibit laminated microstructures. For example, the shell of Crysomallon squamiferum consists of a compliant organic layer sandwiched by the outer iron sulfide layer and inner calcium carbonate layer. The scale of Polypterus senegalus comprises four layers including ganoine, dentine, isopedine and bone; the tooth of Mylopharyngodon piceus is composed of enamelled and dentine layers. Although these laminated structures display great diversity, common features in structure and mechanical property are still present. For example, the most outer layer in these structures is found to be notably stiffer and thinner than the adjacent layer. A rigid (non-deformable) spherical indenter is compressed by force P onto the laminated structure, simulating the external attack from, for example, predators. Friction between the indenter and coating is neglected. Following the convention in contact mechanics, the profile of the spherical indenter is approximated by a paraboloid of revolution described by \( f(r) = r^2/2R \) (see Fig. 2a), where R is the tip radius of the indenter. According to the Rankine criterion, the prediction of fracture modes entails the knowledge of the maximum principal stress \( \sigma_1 \) especially its peak value(s) and location(s) of emergence.

**Evolution of the maximum stress.** The contact problem posed above has been extensively studied in literature. Previous results indicated two possible locations of the maximum principal stress \( \sigma_1 \) on the coating surface (\( z = 0 \)) and the axis of symmetry (\( r = 0 \)) respectively, as illustrated in Fig. 2b, c. In both cases, \( \sigma_1 \) was found to be along the radial direction. Therefore, these two local peaks of \( \sigma_1 \) represent the maximum radial stresses on the coating surface (CS) and along the axis of symmetry (AoS) respectively. The existence of two local peaks of \( \sigma_1 \) on CS and AoS has also been reported in materials with continuously graded Young’s modulus. In particular, for materials with exponentially graded Young’s modulus along the depth direction, \( E \propto e^{cz} \), the maximum of \( \sigma_1 \) occurs at the contact perimeter when \( \alpha > 0 \) while below the surface along the \( z \)-axis when \( \alpha < 0 \).

It can be shown that (Supplementary Information) the radial stress components in the coating can be expressed as

\[
\frac{E^+ t}{\pi R} \theta(r, z, \theta, \phi, \psi_c, \psi_v), \quad \text{where} \quad E^c = \frac{E_c}{1 - \nu^2_c},
\]

is the reduced modulus of coating, \( t \) denotes the thickness of the coating, normalized radial stress \( \theta(\cdot) \) is a dimensionless function of the normalized coordinates \( \tilde{r} = \frac{r}{t}, \tilde{z} = \frac{z}{t} \) and four non-dimensional parameters including the normalized contact radius \( \tilde{a} = \frac{a}{t} \), modulus ratio \( E_c/E_v \), Poisson’s ratios \( \nu_c \) and \( \nu_v \). Given these four non-dimensional parameters, the normalized maximum principal stress \( \tilde{\sigma}_1 \equiv \sigma_1 R/E^+ t \) can be obtained semi-analytically for any point with normalized coordinates \( \tilde{r} \) and \( \tilde{z} \) (Supplementary Information). For example, taking \( E_c/E_v = 4.0, \nu_c = \nu_v = 0.3 \), the calculated distributions of \( \tilde{\sigma}_1 \) along AoS (\( r = 0 \)) and CS (\( z = 0 \)) at different contact radii are shown in Fig. 3a and b, respectively. It can be seen that the maximum principal stress along the AoS increases monotonically from compressive to tensile as the depth \( z \) increases. It reaches the peak value, denoted by \( \tilde{\sigma}_1^{\text{AoS}} \), at the coating/substrate interface (\( z = 0 \)). In contrast, on the CS the location of the peak \( \tilde{\sigma}_1 \) is not fixed but changes with the contact radius. Fig. 3b shows that the peak \( \tilde{\sigma}_1 \) on the CS occurs

**Results**

**Theoretical modeling.** Consider an idealized bilayer structure consisting of an elastic coating firmly attached on an elastic substrate, as shown in Fig. 2a. The elastic moduli and Poisson’s ratios of the coating and substrate are denoted by \( E_c, E_s, \nu_c, \) and \( \nu_v \), respectively. Although structural biomaterials in nature often consist of layers more than two, this simplified bilayer model is still applicable considering that the most outer layer is normally much thinner than the adjacent layer. A rigid (non-deformable) spherical indenter is compressed by force \( P \) onto the laminated structure, simulating the external attack from, for example, predators. Friction between the indenter and coating is neglected. Following the convention in contact mechanics, the profile of the spherical indenter is approximated by a paraboloid of revolution described by \( f(r) = r^2/2R \) (see Fig. 2a), where \( R \) is the tip radius of the indenter. According to the Rankine criterion, the prediction of fracture modes entails the knowledge of the maximum principal stress \( \sigma_1 \) especially its peak value(s) and location(s) of emergence.

![Figure 1](image-url) | (a) Ring cracks created by indentation on the occlusal surface of a black carp tooth. (b) Radial cracks produced by indentation on a glass coating attached on epoxy substrate.

.png
exactly at the contact perimeter ($r = a$, $z = 0$) when the contact radius $a$ is relatively small in comparison with the coating thickness $t$. With the increase of contact radius $a$, the $\sigma_1$ at the contact perimeter decreases while that outside the contact region rises and finally exceeds the former. The peak value of $\sigma_1$ on the CS, no matter where it occurs, is denoted by $(\sigma_1)^{\text{CS}}_{pk}$.

As indicated by Rankine criterion, cracking takes place if the maximum tensile principal stress $\sigma_1$ reaches the fracture strength $\sigma_f$ of the material. The presence of two local peaks of $\sigma_1$ under indentation implies two possible fracture modes. If $(\sigma_1)^{\text{CS}}_{pk}$ reaches $\sigma_f$ first, ring cracking would happen as the plane of the resulting crack should be perpendicular to the direction of the $(\sigma_1)^{\text{CS}}_{pk}$ which is along the radial direction. Otherwise, radial cracking would be initiated first as the crack should be on the plane passing through the AoS. The determination of the initial fracture mode entails the comparison between $(\sigma_1)^{\text{CS}}_{pk}$ and $(\sigma_1)^{\text{AoS}}_{pk}$. Fig. 4a shows the evolution of $(\sigma_1)^{\text{CS}}_{pk}$ and $(\sigma_1)^{\text{AoS}}_{pk}$ as functions of normalized contact radius when $E_1/E_2 = 4.0$ and $v_c = v_s = 0.3$. It can be seen that at small contact size, $(\sigma_1)^{\text{CS}}_{pk}$ is greater than $(\sigma_1)^{\text{AoS}}_{pk}$. As indentation proceeds and contact radius increases, $(\sigma_1)^{\text{AoS}}_{pk}$ exceeds $(\sigma_1)^{\text{CS}}_{pk}$. But the predominance of $(\sigma_1)^{\text{AoS}}_{pk}$ over $(\sigma_1)^{\text{CS}}_{pk}$ is not lasting. With further increase of the contact radius, $(\sigma_1)^{\text{AoS}}_{pk}$ would not increase all the way. Instead, there is a critical contact radius, at which $(\sigma_1)^{\text{AoS}}_{pk}$ reaches its maximum and then decreases. After certain contact radius, $(\sigma_1)^{\text{AoS}}_{pk}$ is surpassed by $(\sigma_1)^{\text{CS}}_{pk}$. Since the fracture mode of the coating depends on whether $(\sigma_1)^{\text{AoS}}_{pk}$ or $(\sigma_1)^{\text{CS}}_{pk}$ reaches the fracture strength first as indentation proceeds, the varying contrast between $(\sigma_1)^{\text{AoS}}_{pk}$ and $(\sigma_1)^{\text{CS}}_{pk}$ shown above implies the dependence of fracture mode on the fracture strength. The case with $E_1/E_2 = 4.0$ and $v_c = v_s = 0.3$, Fig. 4a shows that if the normalized fracture strength is either sufficiently low or high, $(\sigma_1)^{\text{CS}}_{pk}$ would reach the fracture strength first, therefore resulting in ring crack. The lower and upper thresholds of the normalized fracture strength for ring cracking can be readily determined from Fig. 4a to be around 0.05 and 0.9. Here normalized fracture strength is defined as $\sigma_{f} \equiv \sigma_f \pi R/E_1 t$. The critical modulus mismatch for ensuring ring cracking is expected to occur first irrespective of the value of $\sigma_{f}$. Recalling the definition of the normalized fracture strength $\sigma_{f} \equiv \pi \sigma_{f} R/E_1 t$, the value of $\sigma_{f}$ depends not only on the geometrical and mechanical properties of the laminated materials under indentation but also on the dimension of the indenter characterized by the tip radius $R$. If the modulus ratio $E_1/E_2$ is lower than this critical value, $(\sigma_1)^{\text{CS}}_{pk}$ is always higher than $(\sigma_1)^{\text{AoS}}_{pk}$ irrespective of the contact radius. In that case, ring cracking is expected to occur first irrespective of the value of $\sigma_{f}$. For the case of $E_1/E_2$ is lower than this critical value, $(\sigma_1)^{\text{CS}}_{pk}$ is always higher than $(\sigma_1)^{\text{AoS}}_{pk}$, our results show that the maximum tensile stress always occurs on the coating.
surface along the radial direction. Therefore, ring cracking, instead of radial cracking, occurs first for brittle materials. This conclusion is consistent with the prediction of the classic Hertzian theory.

**Fracture mode map.** The dependence of fracture mode on stiffness ratio $E_c/E_s$ and normalized fracture strength $\sigma_f$ can be systematically depicted by a map in terms of these two nondimensional parameters, as shown in Fig. 5 for the case of $\nu_c = \nu_s = 0.3$. It can be seen that a v-shaped curve divides the plane into two regions corresponding to radial cracking and ring cracking respectively. The valley point of the v-shaped boundary corresponds to the scenario with CMM. It shows again that the destructive radial cracking can be inhibited or deferred for indenters of any size as long as the modulus ratio $E_c/E_s$ is lower than the CMM.

It should be pointed out that the CMM revealed above depends on the Poisson’s ratios of both coating and substrate. To gain a deeper insight into the effect of Poisson’s ratios on the CMM, we studied a series of cases with $\nu_c$ and $\nu_s$ in the reasonably broad range from 0.1 to 0.4. Fig. 6a shows the variation of CMM as a function of $\nu_c$ and $\nu_s$. It can be seen that CMM is proportional to $\nu_s$ while inversely proportional to $\nu_c$. Therefore, CMM reaches its maximum and minimum at $\nu_c = 0.1$, $\nu_s = 0.4$, and $\nu_c = 0.4$, $\nu_s = 0.1$ respectively. The corresponding maps of fracture mode for these two limiting cases are displayed in Fig. 6b. In comparison to the case with $\nu_c = \nu_s = 0.3$ shown in Fig. 5, it can be seen that the territory of radial cracking regime shrinks in the case of $\nu_c = 0.1$, $\nu_s = 0.4$ while expands in the case of $\nu_c = 0.4$, $\nu_s = 0.1$, resulting in CMMs equal to 3.3 and 1.0 respectively.

**Discussion**

For the laminated biomaterials in nature, it may not be easy to obtain the precise Poisson’s ratio for each layer and then to predict the corresponding CMM. However, if we take 0.1–0.4 as the reasonable range of Poisson’s ratio of biomaterials, the CMM of natural laminated biomaterials, in the light of Fig. 6a, is estimated ranging from 1.0 to 3.3. It is interesting to compare this theoretical prediction with the measured value. Table 1 listed the modulus mismatch of some
laminated biomaterials reported in literature. The inclination to ring cracking has been confirmed in some of these materials such as the scale of *Polypterus senegalus* being bitten by its predator’s teeth [1]. It is interesting to notice that the measured modulus mismatch of the listed biomaterials range from 0.93 to 3.6, overlapping the range of the CMM calculated above. However, for a particular case such as the teeth of *Myloparyngodon piceus* [19], if the Poisson’s ratios of the coating (enameloid) and substrate (dentine) are both taken as 0.3, the CMM is estimated to be around 1.6, which is lower than the measured modulus mismatch 3.5 as shown in Table 1. Such discrepancy may result from our negligence of more specific features in our modeling above. For example, in our previous analysis, we assumed that the modulus across the coating/substrate interface is discontinuous. In fact, transitional interlayer with graded properties is often observed between distinct layers in natural laminated biomaterials.

Such gradient interlayer has been demonstrated to play an important role in mitigating the stress and strain concentration at the interface [1,18]. The effect of gradient transition layer on the CMM is studied by using finite element analysis (Supplementary Information). Fig. 7 shows the calculated fracture mode maps for the cases with gradient interlayer of thickness $t_{grad}$ in comparison to those without interlayer. It can be seen that the radial cracking regime shrinks and the CMM increases after introducing the gradient interlayer between the coating and substrate. The thicker the interlayer, the higher the CMM. For the teeth of *Myloparyngodon piceus*, the reduced modulus obtained by nano-indentation exhibits gradient in the vicinity of the enameloid/dentine interface [19]. Assuming that the thickness of the gradient interlayer is equal to the coating (enameloid layer) thickness, Fig. 7 implies that the CMM could rise up to 3.5, which agrees well with its measured value.

From the perspective of ensuring ring cracking as the initial fracture mode, soft coating in combination with hard substrate seems more preferable. But why do natural materials barely adopt such kind of design? A conceivable answer to this question is because fracture mode may not be the only issue that biomaterials should tackle. Consideration of other issues such as strength, toughness, wear resistance and economics of construction and maintenance [36] may raise different and even competing requirements for the stiffness layout in different layers. The modulus mismatch in the existing biomaterials is most likely a consequence of optimization of diverse design requirements including fracture mode control.

Inspired by the inclination to ring cracking observed in most natural biomaterials, in this paper we theoretically explored the factors determining the emergence of radial cracking and ring cracking in coating/substrate systems. It was revealed that the fracture mode of the coating depends on the coating/substrate modulus mismatch and indenter size. There exists a critical modulus mismatch, CMM, below which emergence of ring cracking always precedes that of radial cracking irrespective of the indenter size. For structural materials, radial cracking is much more catastrophic in comparison to the ring one. Our results imply a strategy to inhibit catastrophic damage by controlling the modulus mismatch. For structural biomaterials in nature, the CMM was estimated to be in the range from 1.0 to 3.3, consistent with the values found in many existing structural biomaterials. Recalling the strengthening and toughening mechanisms discovered in biomaterials, it can be seen that structural biomaterials in nature utilize synergic strategies to mitigate damage due to external mechanical loadings at various intensities and length scales. If the load is relatively low, the intrinsic carrying capacity with the aid of diverse strengthening mechanisms prevents the onset of fracture. If the load exceeds the fracture strength and hence causes crack initiation, various toughening mechanisms operate and resist the subsequent growth of cracks. If indeed overwhelming load is encountered and partial or even whole structural damage is unavoid-

### Table 1 | The modulus mismatch of some laminated structural materials in nature

<table>
<thead>
<tr>
<th>Laminated materials in nature</th>
<th>$E_1$ (GPa)*</th>
<th>$E_2$ (GPa)*</th>
<th>$E_1/E_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tooth of <em>M. Piceus</em> (Black carp) [19]</td>
<td>$-70^1$</td>
<td>$-20^1$</td>
<td>$-3.5$</td>
</tr>
<tr>
<td>Shell of <em>C. squamiferum</em> [8]</td>
<td>$-28.8$</td>
<td>$-8.0$</td>
<td>$-3.6$</td>
</tr>
<tr>
<td>Shell of <em>Haliotis rufescens</em> (California red abalone) [32,33]</td>
<td>$-65^2$</td>
<td>$-70^2$</td>
<td>$-0.93$</td>
</tr>
<tr>
<td>Scale of <em>Arapaimas gigas</em> [4,35]</td>
<td>$-46.8^3$</td>
<td>$-16.7^3$</td>
<td>$-2.8$</td>
</tr>
<tr>
<td>Scale of <em>Polypterus senegalus</em></td>
<td>$-33.7^4$</td>
<td>$-15.7^4$</td>
<td>$-2.15$</td>
</tr>
<tr>
<td>Scale of <em>Morone saxatilis</em> [8]</td>
<td>$-62$</td>
<td>$-29$</td>
<td>$-2.1$</td>
</tr>
<tr>
<td>Exoskeleton of <em>Homarus americanus</em> (American Lobster) [26]</td>
<td>$-1.0^5$</td>
<td>$-0.45^5$</td>
<td>$-2.2$</td>
</tr>
</tbody>
</table>

*Modulus here refers to, unless indicated otherwise, the reduced modulus obtained by nano-indentation; *
*The occlusal surfaces of black carp teeth are flat and therefore laminated plate model is applicable; *
*Young’s modulus obtained by tensile tests; *
*Dehydrated samples; *
*Hydrated samples; *
*Mean value of the reduced moduli in normal and transverse directions.*
Figure 7 | Effect of gradient transition layer on the CMM. In the gradient transition layer, the Young’s modulus is assumed to vary from the modulus of the coating to that of the substrate in a linear way. Here \( t_{\text{grad}} \) and \( t \) stand for the thicknesses of gradient layer and coating respectively. Poisson’s ratios are consistently taken as 0.3.

able, as the last resort, fracture mode may be controlled to inhibit or defer the catastrophic failure. It is the synergy of strengthening, toughening and fracture mode controlling that ensures higher survival probability of the structural biomaterials and their associated species in the intensive biological competitions. In summary, prevention of catastrophic structural failure is the most important goal of structural material design. Reducing the extent of overall structure damage caused by fracture is as important as strength and toughness to the safety and integrity of structures. Fracture mode control is one key strategy that we learned from natural biomaterials to combat catastrophic damage. Nevertheless, the existence of other mechanisms is expected and deserves further investigations.
