

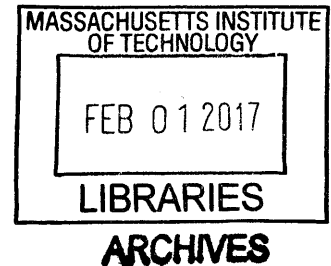
**Exploring Optimal Mixed-Asset Portfolio Allocation:
Hedge Funds and Private Equity vs. Real Assets**

by

Yoojin Sul

B.Arch, 2010

Yonsei University



Submitted to the Program in Real Estate Development in Conjunction with the Center for Real Estate in Partial Fulfillment of the Requirements for the Degree of Master of Science in Real Estate Development

at the

Massachusetts Institute of Technology

February, 2017

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ABSTRACT

The current world economy confronts investors with many challenges, especially investors managing institutional portfolios. Global GDP growth has been slowed, and the performance of traditional assets – equities and bonds – alone are often not able to satisfy the various risk and return objectives that institutional investors seek in their portfolios. Amid this challenging investment environment, investors around the world are seeking new investment strategies to lessen their reliance on those traditional asset classes. Consequently, alternative investments continue to garner greater attention of investors as an effective method to diversify their portfolios and to potentially increase overall returns and mitigate risk. However, the term “alternative investments” encompasses a broad range of investment concepts and there is no generally accepted standard definition.

A major focus of this thesis is to compare real estate and real assets with hedge funds and private equity, the four most prevalent sub-classes within alternative investments. Specifically, we address the question of whether, or to what extent, real assets including real estate can improve the performance of institutional investment portfolio, in particular in comparison with the private equity and hedge funds. Additionally, we analyze the effect of diversifying globally compared to domestically.

We first develop a common ground regarding alternative investments and their characteristics. Then, we focus primarily on traditional mean-variance optimization but also consider risk parity as the allocation criterion to explore the optimal mixed-asset allocation strategies as a function of the investor’s expected return target. Additionally, we compare the resulting allocations with institutional investors current average allocation in their portfolios.

The findings clearly indicate that adding alternative asset classes generally offers attractive diversification opportunities to a portfolio consisting of only traditional asset classes – stocks and bonds. We find that real assets and the private equity & hedge fund type of alternative assets both enhance the portfolio, and the aggregated optimal share of these alternative investments is much higher than current industry practice. However, the role of the various different types of alternative investments varies widely in a portfolio, in particular as a function of the investor’s risk/return appetite.

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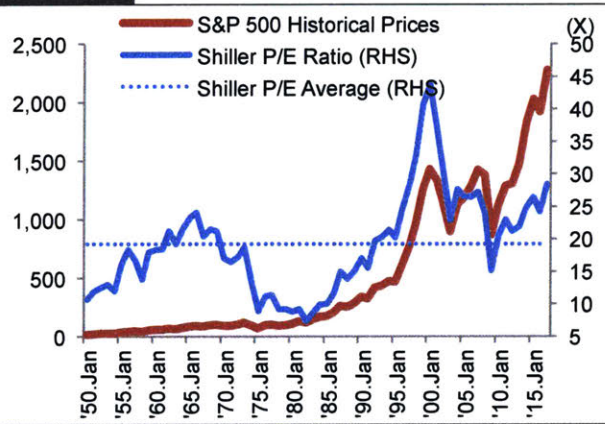
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CHAPTER 1 Introduction

1.1 Research Motivation

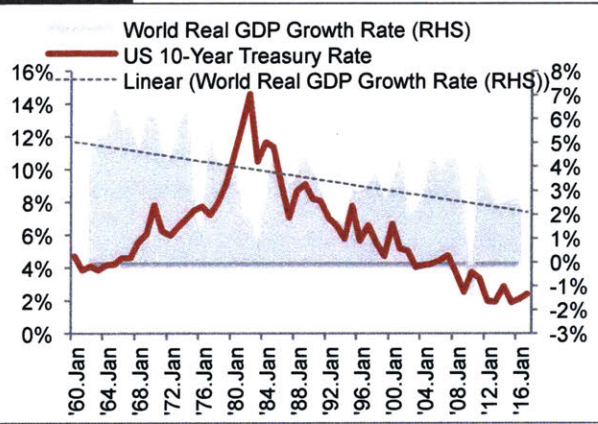
Financial markets have become more globally connected but at the same time more specialized with lots of new investment instruments coming into the market. However, despite the ever-evolving market, the current world economy confronts investors with many challenges, especially investors managing institutional portfolios. Global GDP growth has been slowed, and the performance of traditional assets – equities and bonds – alone are often not able to satisfy the various risk and return objectives that institutional investors seek in their portfolios (Exhibits 1, 2).

Exhibit 1 S&P 500 Historical Performance and Shiller P/E Ratio



Source: Standard & Poors, Robert J. Shiller
 Note: The Shiller P/E ratio is defined as price divided by the average of ten years of moving average earnings, adjusted for inflation.

Exhibit 2 Government Bond Returns and Real GDP Growth



Source: US Treasury Department, World Bank

Amid this challenging investment environment, investors around the world are seeking new investment strategies to lessen their reliance on those traditional asset classes. Consequently, alternative investments continue to garner greater attention of investors as an effective method to diversify their portfolios and to potentially increase overall returns and mitigate risk. Within alternative asset classes, there are various types of investments. Among alternative investments, hedge funds and private equity are more widely known and employed among most institutional investors. Other alternative assets, in particular real assets, have only recently become more prevalent in institutional portfolios. Although allocation to these alternative investments (i.e., hedge funds, private equity, and real assets) has grown at twice the growth rate of traditional assets from

2005 to 2013¹, their share in institutional portfolios is still small. For example, the average current allocation to real estate, a type of real assets, is 8% of AUM as of 2015². Stocks and bonds remain the majority.

While investors have been increasing their allocations to alternative investments, this begs the question of what is the actual result of adding these assets in portfolio? Is there any difference within the sub-categories of the alternative asset classes? What would be the optimal allocation for each asset class when various combinations of asset classes are available in the portfolio choice set? From the perspective of US institutional investors, is there any difference in the performance of portfolios between investing only in US domestic assets versus investing in global assets? These are some of the questions we explore in this thesis.

More specifically, a major focus of this thesis is to compare real estate and real assets with hedge funds and private equity. These are the four most prevalent sub-classes within alternative investments. Our particular focus is on the diversification effect of the various alternative assets within a traditional portfolio that starts out consisting of just stocks and bonds. We explore how a given alternative asset affects the performance of the portfolio in comparison with other alternatives in terms of risk and return. Additionally, we analyze the effect of diversifying globally compared to domestically. We focus primarily on traditional mean-variance optimization (the classical Markowitz framework and Sharpe Ratio maximizing portfolio), but also consider risk parity as the allocation criterion (which does not require any assumption about mean or expected return of the asset classes). Through the study, we explore the optimal mixed-asset allocation strategies as a function of the investor's expected return target, and we compare the resulting allocations with institutional investors current average allocation in their portfolios. The quantitative analysis is based on the historical investment performance of the asset classes during the 1996~2015 period (or 2001~2015 period for global), as exemplified by representative benchmark indices widely used in the institutional investment industry.

¹ McKinsey & Company from "McKinsey on Investing", 6pg, November, 2014, McKinsey & Company

² Preqin, 2016, "Preqin Investor Outlook: Alternative Assets H1 2016", 27pg

1.2 Thesis Outline

In this thesis, we particularly focus on the quantitative modeling using various combinations of assets and different portfolio allocation methods to explore the optimal mixed-asset allocation strategies in the framework of institutional portfolios.

However, before delving into full-blown portfolio analysis, we will develop a common ground regarding alternative investments for further discussion in later chapters. In Chapter 2, we start by defining alternative investments, as the term encompasses a broad range of investment concepts and there is no generally accepted standard definition. Then, we explore types and characteristics of alternative asset classes that we plan to focus on in this paper. We also estimate the size and growth of the alternative investment industry and the current institutional asset allocation trends of investing in these asset classes.

Chapter 3 quantifies the historical returns of both traditional asset classes and alternative asset classes. We use those returns to investigate the volatilities of the asset classes and the correlation among them. In Chapter 4, we conduct a portfolio allocation analysis to decide optimal mixed-asset portfolio allocation by using the statistical characteristics from Chapter 3 as inputs for the analysis. In portfolio analysis, we use two different methods: 1) Mean-Variance Optimization and 2) Risk Parity Asset Allocation. In each approach, we compare the performance of portfolios and optimal weights of each constituent asset classes among portfolios with various asset combinations. Chapter 5 concludes the discussion and analysis with highlighting some interesting topics for further investigation.

CHAPTER 2. Background & Current Market Trend: Alternative Investment for Institutional Investors

2.1 Definition of Alternative Investments

It is hard to define “alternative investments” clearly. The term encompasses a broad range of investment concepts, investing in so-called “alternative assets.” A commonly accepted definition is that alternative investments refer to several “assets classes” that fall outside of traditional investments.³ Traditional investments are considered to be stocks and bonds, with also cash rounding out the traditional portfolio. Mutual funds are a traditional vehicle for investing in stocks and bonds. Alternative assets can be anything that does not fall into the realm of traditional assets and that have economic value for institutions or individuals to invest in. They can even include collectibles such as comic books. Exhibit 3 shows one list of alternative assets. But for purposes of large institutional investors, it would be helpful to limit the types of alternative assets into a few major categories that can serve such sophisticated investors.

Exhibit 3 Types of Alternative Assets

- *Private Equity / Venture Capital*
- *Hedge Fund*
- *Funds of Funds*
- *Real Estate*
- *Infrastructure*
- *Energy*
- *Others: Gem Stones, Art Piece, Antiques, Musical Instruments, Wine, Classic Cars, etc.)*
- *Commodities*
- *Gold*
- *Active Currency*
- *Managed Futures*
- *Distressed Securities*
- *Forestry*

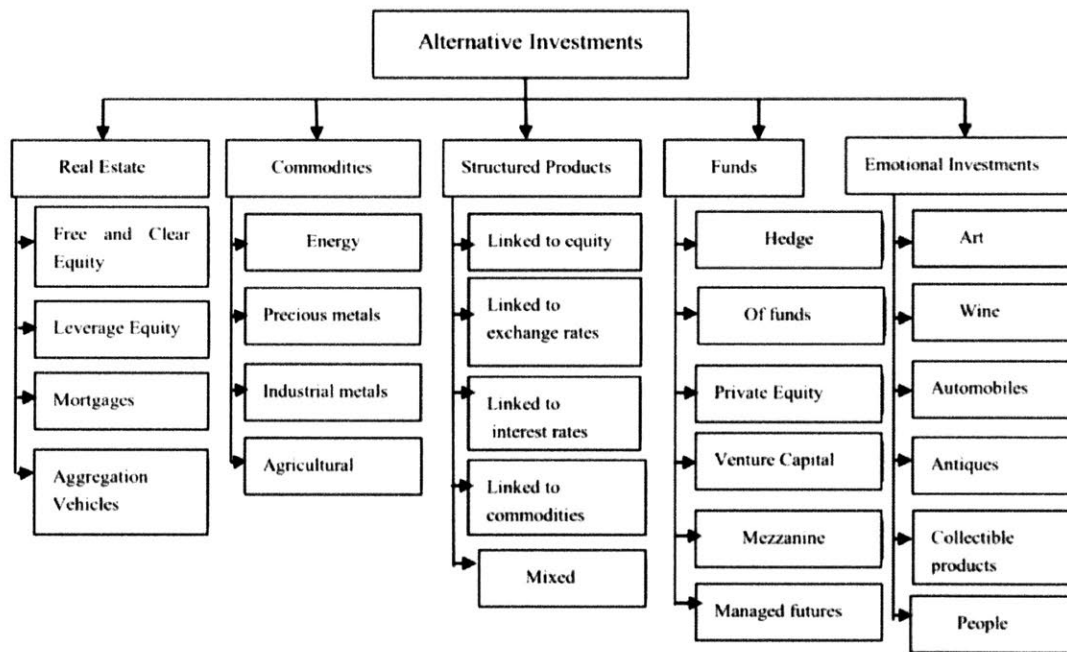
Source: Guy Fraser-Sampson, 2011, *Alternative Assets: Investments for a Post-Crisis World*, John Wiley & Sons, Inc.

As with defining alternative assets, there is also no standard rule for classifying them, in part because of the aforesaid lack of a widely accepted definition. Therefore, there are different ways of grouping alternative investments depending on the classification criterion. Broadly, Yau, Schneeweis, Robinson and Weiss (2007) divide alternative investments into two groups: 1) Traditional alternative investments – this category includes real estate, private equity/venture capital, and commodities; 2) Modern alternative investments – this includes managed futures,

³ H. Kent Baker and Greg Filbeck, 2013, *Alternative Investment: Instruments, Performance, Benchmarks, and Strategies*, Chapter 1, John Wiley & Sons, Inc.

hedge funds, and distressed securities.⁴ Ewelina Sokołowska (2016) provides a more organized classification tool to be more complete. She divides alternative investments based on two dimensions: what are the underlying assets, and what are the management and legal structures of the investments. (Exhibit 4) For example, in Sokolowsky's classification, Commodities is a major category all of whose constituent asset classes represent different types of physical commodities grouped according to how they might primarily relate to sectors of the economy, such as energy, precious metals, and so on. In contrast, Real Estate is a major category but Sokolowsky's designation of asset classes within this category is based not on differences in the physical property assets but rather on types of investment vehicles such as debt versus equity funds.

Exhibit 4 Classification of Alternative Investments



Source: Ewelina Sokołowska, 2016, *The Principles of Alternative Investments Management*, Springer International Publishing Switzerland

In this thesis we won't try to consider all of the range of alternative assets identified by Sokolowska. Rather, we limit our focus to certain major categories – private equity, hedge funds, and real assets. Real assets include real estate, infrastructures, maritime assets⁵. We limit our

⁴ Yau, Jot K., Thomas Schneeweis, Thomas R. Robinson, and Lisa R. Weiss. 2007, "Alternative Investments Portfolio Management." In John L. Maginn, Donald L. Tuttle, Dennis W. McLeavey, and Jerald E. Pinto, *Managing Investment Portfolios—A Dynamic Process*, John Wiley & Sons, Inc

⁵ Maritime investments refer to investments in the shipping industry. It can be direct investments in various vessels (such as tankers, containerhips, and bulkers) covering different routes, investing in shipping companies, or financing and chartering ships.

discussions to these asset classes because hedge fund, private equity, and real estate account for the largest portion of alternative assets in institutional portfolios and are currently being most widely considered by pension and endowment funds and sovereign wealth funds. According to a major industry information provider, Preqin, the alternative investment industry is worth US\$7.4 trillion as of 2015. Of that, hedge funds account for about 43%, private equity for 32%, and real estate for 11% based on the total assets under management⁶. Recently, these institutional investors have become increasingly interested in infrastructure, natural resources, timber and farmland, which can be grouped into “real assets” together with real estate. The present thesis delves into the implications of adding real assets to the portfolio and how real assets can perform differently from other major alternative assets, in particular, hedge funds and private equity.

2.2 Types and Characteristics of Alternative Asset Classes

As we have discussed what types of alternative asset classes we will explore in this thesis, we can now develop a common ground for the definition of each alternative asset we plan to explore and its characteristics.

Real Assets

The term “real assets” generally refers to tangible, physical assets (such as land, built property, equipment, raw materials, and infrastructure) with intrinsic value as opposed to financial assets such as stocks (equities) and bonds (fixed-income). Some argue that the “real” in real assets means real return after adjusting inflation that these assets provide.⁷ Generally, real assets provide investors with two sources of return: 1) stable bond-like cash flows from operation (only often with more inflation hedging ability than fixed-rate bonds), and; 2) equity-like upside potential through capital gain from asset value appreciation and inflation (based fundamentally on ability to grow net cash flow). In this regard, real assets bridge the gap between fixed income and equity.

There are several characteristics of real assets that can be attractive to institutional investors. First, the value of these assets tends to be less volatile than financial assets. Presumably,

⁶ Preqin, The Preqin 2016 Alternative Assets Performance Monitor, 2016 Preqin Global Private Equity & Venture Capital Report, 2016 Preqin Global Real Estate Report, 2016 Preqin Global Infrastructure Report

⁷ Hossein Kazemi, Thomas Schneeweis, Richard Spugin, George Martin, 2007, “Real Assets in Institutional Portfolios: the Role of Commodities”, *Alternative Investment Analytics*

a fundamental reason for lower volatility could be that these assets provide very basic services that are relatively less cyclical or sensitive to business cycles. But another reason could be because the transaction of these tangible, physical assets tends to be more difficult and costly than the trading of liquid assets. Private asset markets such as those in which real assets directly trade are less informationally efficient than public stock exchanges. The result could be that observable values do not change much overnight or from week to week.

A second reason real assets are attractive relates to the fundamentals noted above. Real assets tend to generate stable cash flows. In fact, not only are such assets often relatively immune from business cycles, but they also have relatively good ability to grow their cash flow with inflation. Thus, the investment income generated by real assets can hedge inflation during the investor's holding period. In some cases long-term contractual revenues are legally structured to rise with inflation. In other cases the ability to grow profits with inflation derives from the "real" nature of the asset and the service it provides (for example, real estate rents).

Finally, real assets are attractive as investments because their returns tend to show low correlation to traditional equity and bond returns. This further helps to provide investors with attractive risk-adjusted returns in their portfolios through the benefit of diversification to improve overall portfolio performance. Again, the low correlation with stocks and bonds may in part reflect fundamentals (the previously noted relative lack of sensitivity to the business cycle), and may in part reflect the type of asset market in which real assets trade (illiquid private, whole-asset search markets, as opposed to liquid public stock exchanges).

Although what precisely constitutes a real asset investment may vary, real assets typically include real estate, infrastructure, commodities and natural resources, timberland, and farmland as shown below.

- Real estate: direct or indirect exposure to commercial real estate through real estate debt, private real estate equity, public real estate securities (REIT)
- Infrastructure: debt or equity investments in hard assets that generate cash flows by providing essential services typically with substantial public sector involvement or regulation. Examples include toll roads, ports, railroads, telecommunications, airports, power plants, etc.

- Commodities and Natural Resources: exposure to gold or natural resources such as energy, metals or agricultural products via physical commodities, natural resource equities or private commingled funds, or commodity-linked derivatives⁸
- Timberland: direct or indirect ownership in timberland. Act as both factory and storage for timber and provide investors optionality to delay or harvest the products depending on the timber price
- Farmland: direct or indirect investments in farmland to capitalize increasing global demand for food. Investors can either acquire the land and lease to an external farming operation or grow widely diversified portfolio of crops themselves

Hedge Funds

Hedge funds refer to legal entities that allow investors to pool their money together, which is then managed by an investment manager who attempts to exploit pricing inefficiencies in the market to generate high returns while trying to assume as little risk as possible⁹. Academics have defined hedge funds as privately offered, relatively unregulated pools in the form of limited partnerships or limited liability companies that have the flexibility to invest in a broad range of securities and commodities using a wide variety of trading techniques¹⁰. Techniques and strategies include taking not only long but also possibly short positions, as well as using leverage, derivatives, and arbitrage trading strategies. The ability to use short positions is a particularly characteristic feature of hedge funds compared to more traditional investment approaches and vehicles. According to David Stowell¹¹, hedge funds have the following characteristics: (1) almost complete flexibility in relation to investments, including both long and short positions; (2) ability to borrow money (and further increase leverage through derivatives) to enhance returns; (3) minimal regulation; (4) somewhat illiquid since an investor's ability to get their money back is restricted through lock-up agreements.

⁸ The Economist Intelligence Unit, 2014, "The Ascent of Real Assets: Gauging growth and goals in institutional portfolios", BlackRock

⁹ Alex R. McClean, 2006, Note, "The Extraterritorial Implications of the SEC's New Rule Change to Regulate Hedge Funds," 38 Case W. Res. J. Int'l L. 105, 106

¹⁰ Mcneil, Bruce J., 2012, "Hedge Funds and Private Equity Funds As Investment Vehicles", *Journal of Pension Planning & Compliance*. Spring2012, Vol. 38 Issue 1, p38-63. 26p.

¹¹ David Stowell, 2012, *Investment banks, hedge funds, and private equity*(2nd Ed), Ch 11. Academic Press,

One of the characteristics of hedge funds, their relative lack of regulation, results from certain exemptions available under the securities laws. Hedge funds can avoid registration under the Investment Company Act of 1940 either under 15 U.S.C.S. § 80a-3(c)(1) by having 100 or fewer beneficial owners within the United States and not offering their securities to the public or, under 15 U.S.C.S. § 80a-3(c)(7) by having their investors all being qualified high net-worth individuals or institutions.¹² Exemptions from the Investment Company Act of 1940 allow hedge funds to aggressively invest in various assets using leverage and long-short positions without disclosing their positions and strategies to the SEC or their shareholders.

Private Equity

Private equity investments refer to investments in companies or securities on the private capital market, which are not listed on a public stock exchange. Capital for private equity investments is generally raised through a limited partnership, a limited liability company, or other forms of pass-through entity and invested in companies at various stages of their development to develop new business, restructure or acquire existing business, or offer additional funding for existing business.

As private equity investments target mid to long-term profit from increases in the value of capital (as opposed to current income), these investments typically have a fixed investment period, seven to ten years depending on the holding period of the portfolio investments¹³. Like other alternative investments, private equity investments encompass a wide-range of investment categories. The most common types of private equity investments include venture capital, leveraged buyout funds, and distressed securities.

2.3 The Recent Trend in Institutional Investors Asset Allocation

Amid the challenging investment environment of traditional asset classes, investors around the world are seeking new investment methods to lessen their reliance on traditional asset classes. Alternative investments continue to garner increased attention of investors as effective methods to

¹² Mcneil, Bruce J., 2012, "Hedge Funds and Private Equity Funds As Investment Vehicles", *Journal of Pension Planning & Compliance*. Spring2012, Vol. 38 Issue 1, p38-63. 26p.

¹³ Mcneil, Bruce J., 2012, "Hedge Funds and Private Equity Funds As Investment Vehicles", *Journal of Pension Planning & Compliance*. Spring2012, Vol. 38 Issue 1, p38-63. 44p.

diversify their portfolios and to potentially increase overall returns and mitigate risk. In this section, we explore current asset allocation trends among institutional investors. Then, we review major institutional investors and their allocation framework toward real estate and infrastructure, which are the major subset in real asset classes.

To study the above issues, we mainly use two sources of data; the first source is annual reports from Preqin and the second source is the annual survey result done by Pension Real Estate Association (PREA). Preqin tracks over 22,800 funds globally with performance data across private equity, private debt, private real estate, infrastructure, natural resources and hedge funds, and regularly publishes performance and fund raising data as well as institutional investors attitude toward each alternative asset class. On the other hand, PREA's survey is monitors specifically focusing on the real estate investment activities. Major players covered by PREA are public and corporate pension funds, endowments, foundations, and insurance companies, who invest in real estate either directly or indirectly. Therefore, the investor groups that participate in the PREA survey tend to have a relatively higher allocation to real estate than do non-PREA members.

Alternative Investment Industry and Overall Asset Allocation

Although it is hard to estimate the accurate size of the alternative investment industry as the definition of alternative investments encompasses a broad range of assets, the common figure is over \$7 trillion. As we discussed in the previous section, the alternative investment industry is worth \$7.4 trillion as of 2015 according to Preqin¹⁴. Exhibit 5 illustrates the breakdown of alternative investment industry based on the total assets under management (AUM). Of \$7.4 trillion, hedge funds account for about 43%, private equity for 32%, and real estate for 11% based on the total AUM.

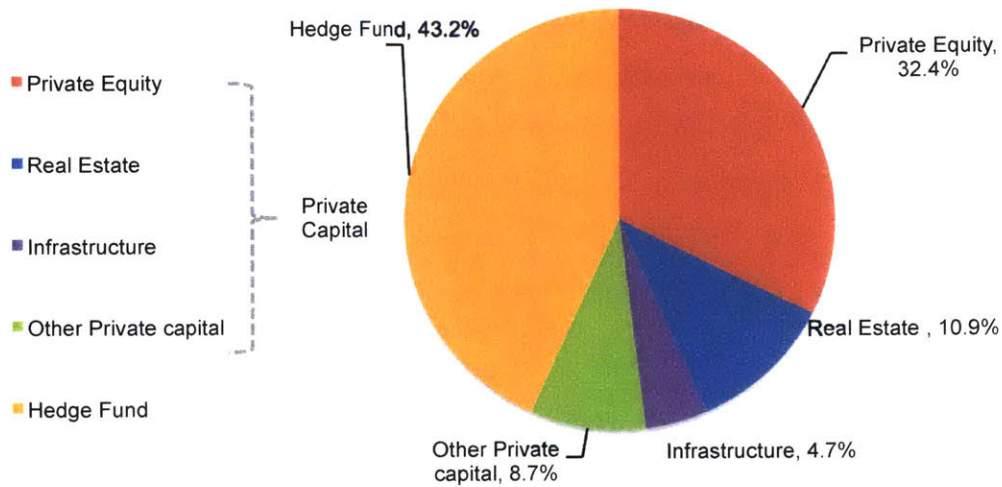
Exhibit 6 compares the growth rate of alternative investments over traditional investments. According to McKinsey & Company¹⁵ global asset under management of alternative investment industry is about \$7.2 trillion in 2013, about 13% of traditional investments. Although alternative investment assets are still small in size compare to traditional assets, notice that the alternative

¹⁴ Preqin, The Preqin 2016 Alternative Assets Performance Monitor, 2016 Preqin Global Private Equity & Venture Capital Report, 2016 Preqin Global Real Estate Report, 2016 Preqin Global Infrastructure Report

¹⁵ McKinsey & Company from "McKinsey on Investing", 6pg, November, 2014, McKinsey & Company

investments industry grew at an annualized pace of 10.7%, which is almost twice the growing rate of traditional investments. Global asset under management in alternative investments doubled its size since 2005.

Exhibit 5 Breakdown of Alternative Investment Industry by Asset Classes

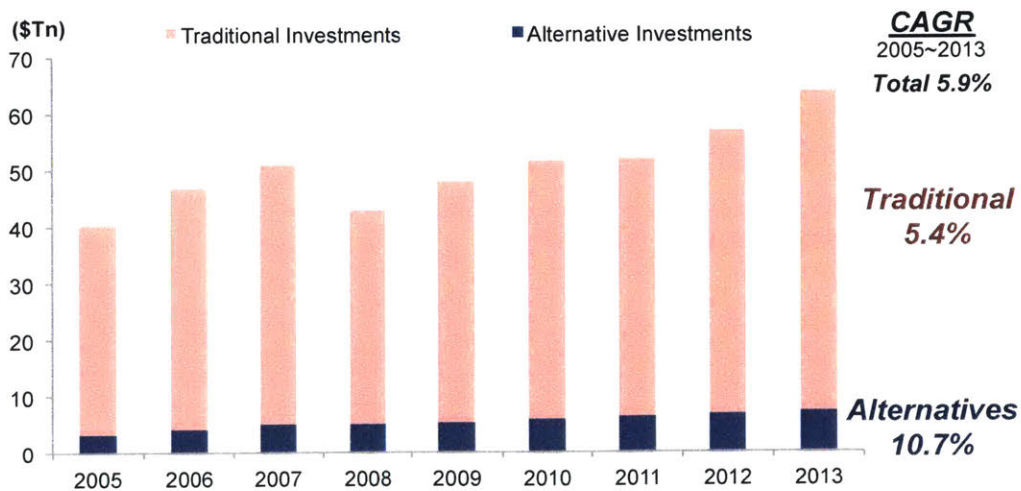


Source: Preqin, The Preqin 2016 Alternative Assets Performance Monitor, 2016 Preqin Global Private Equity & Venture Capital Report, 2016 Preqin Global Real Estate Report, 2016 Preqin Global Infrastructure Report

Note 1: Private Equity refers to the core asset class centered on the buyout and venture capital industry, together with other closely related strategies.

Note 2: Private Capital refers to the broader spectrum of private closed-end funds, including private equity, private debt, private real estate, infrastructure, and natural resources.

Exhibit 6 Global Assets Under Management: Traditional vs. Alternative Investments

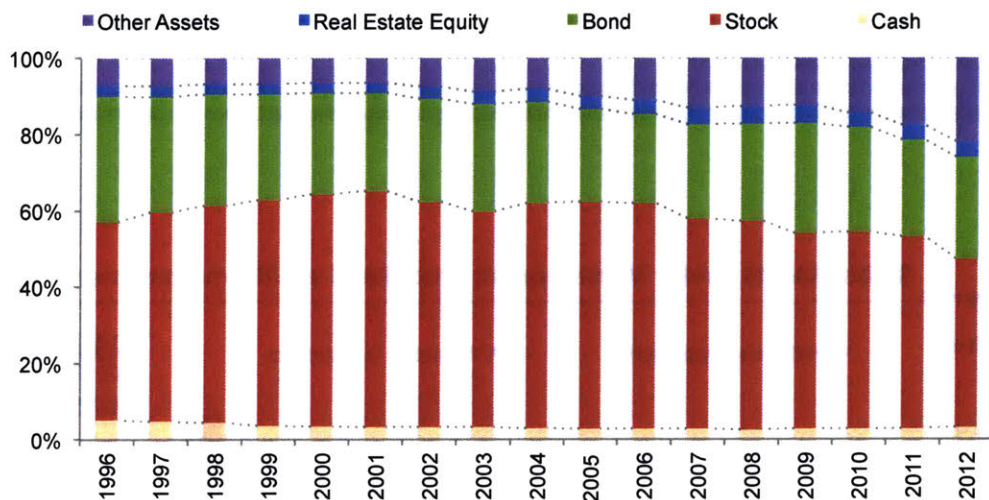


Source: Hedge Fund Research; Preqin; McKinsey & Company from "McKinsey on Investing", 6pg, November, 2014, McKinsey & Company

Note: Alternative investments do not include retail alternatives (i.e., exchange-traded funds, mutual funds, and registered closed-end funds).

Exhibit 7 illustrates breakdown of PREA’s all plan sponsors asset allocation by major asset classes. Although the time horizon of the data does not include the most recent period, we can see a long-term trend. The prominence of “other assets” class within institutional portfolio had been consistently increased. PREA explicitly states that, “the other assets class has increased in recent years due to the growing allocation by investors into alternative assets. In addition, a diversity of investments is not easily classified in major asset classes and thus also is categorized as alternatives by PREA”¹⁶. In contrast, allocation to equity had consistently dropped from 60% in 2005 to 44% in 2012, while bonds share in institutional portfolios remained stable between 24% and 28% since 2000. All plans allocation to real estate equity increased from 2.7% in 2000 to 4.1% in 2012, and the real estate equity allocation of plan sponsors with real estate equity holdings increased from 4.4% to 7.3% over the same period.

Exhibit 7 All Plan Sponsors: Major Asset Allocation → Pension Plans are increasing their Allocation to Real Estate and Other Assets

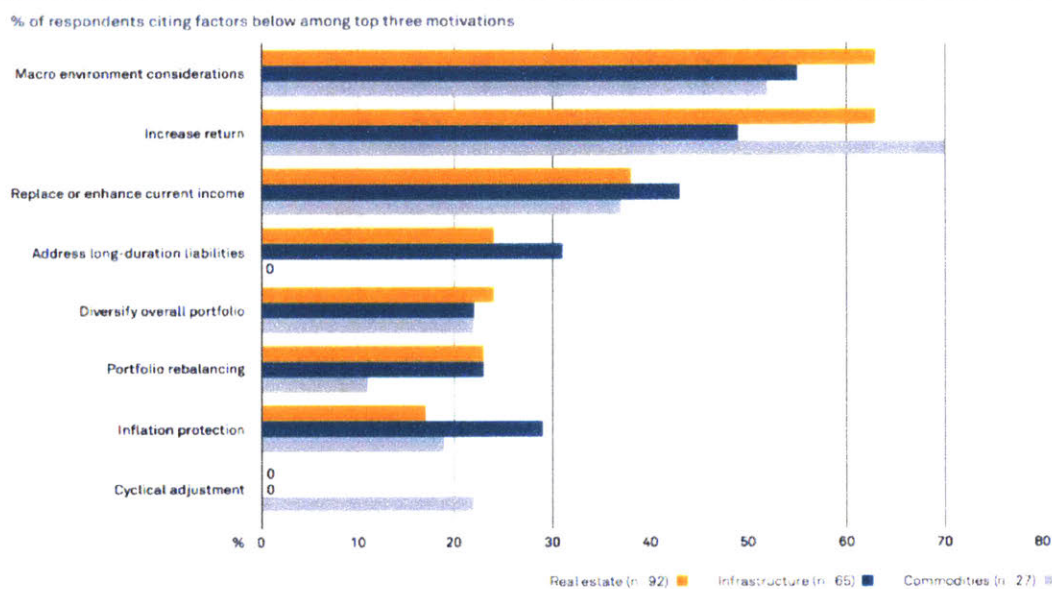


Source: PREA, Standard & Poor's Money Market Directories from PREA Investor Report August 2013
 Note: "Other Assets" include alternative investments and mutual fund investments

¹⁶ Pension Real Estate Association, PREA Investor Report August 2013, pg 13

Then, why do these institutional investors increase their allocation toward the alternative asset classes including real assets? Top three reasons why institutional investors invest in real asset classes are 1) out of macro environment considerations, 2) to increase returns, 3) to replace or enhance current income. (Exhibit 8)

Exhibit 8 Institutional Investors' Reasons for Increase in Real Asset Allocations

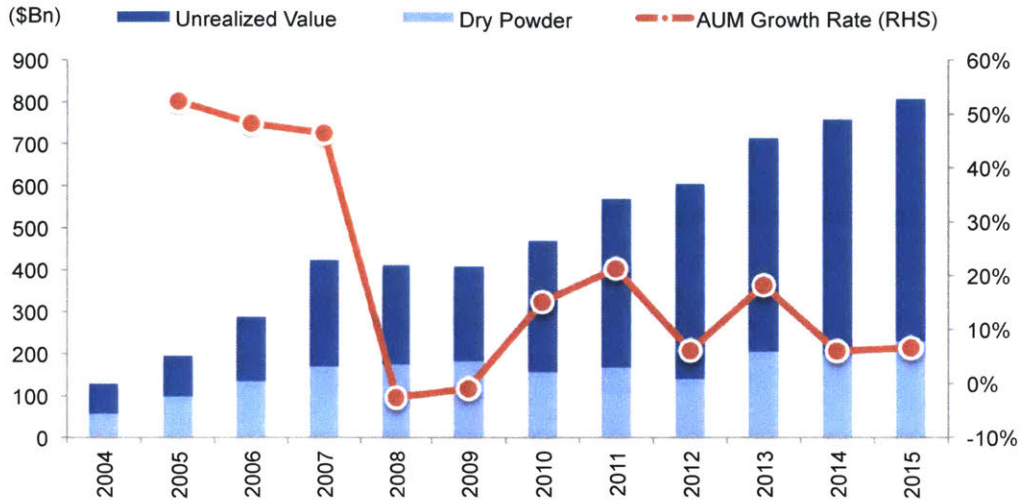


Source: The Economist Intelligence Unit, 2014, "The Ascent of Real Assets: Gauging growth and goals in institutional portfolios", BlackRock

Institutional Real Estate Investment Allocation

The size of closed-end private real estate investment based on AUM is \$743 billion as of June 2015, up from \$605 billion in December 2012 according to Preqin. Real estate AUM's 5-year compounded annual growth rate since 2010 is 11% and recently growth rate dropped to 7% in 2015. (Exhibit 9). For the year ending 2015, PREA member investors reported \$237 billion in real estate holdings (both private and public), or 9.5% of their total assets, up from 8.8% at the end of 2014.

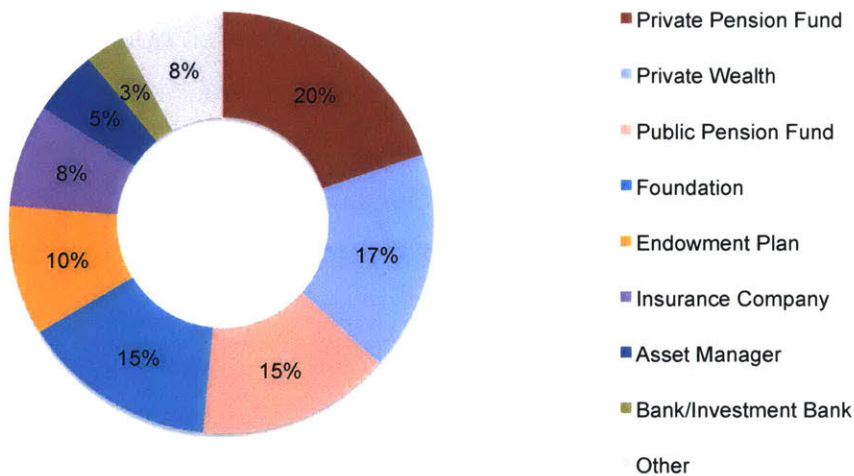
Exhibit 9 Closed-End Private Real Estate Asset Under Management



Source: Preqin, "Preqin Investor Outlook: Alternative Assets H1 2016," 2016, Preqin |

Exhibit 10 shows a breakdown of institutional investors allocation in real estate by their type. Pension funds both public and private represent over a third of real estate AUM. This is because private real estate investments require relatively long investment horizon that is suited to pension funds. Private wealth institutions, foundations, endowment plans and insurance companies collectively account for 50% of private real estate investment assets.

Exhibit 10 Breakdown of Institutional Investors Allocation in Real Estate by Type

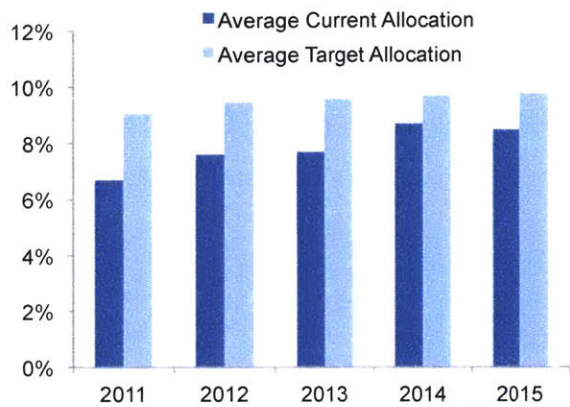


Source: Preqin, "Preqin Investor Outlook: Alternative Assets H1 2016," 2016, Preqin |

Exhibit 11 and 12 show investors' current allocation and target allocation to real estate. The average current allocation rose from 6.7% of AUM in 2011 to 8.5% in 2015, while the average

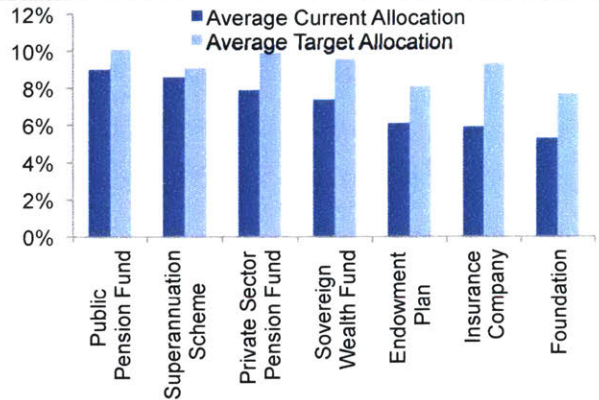
target allocation rose from 9.1% to 9.8% over the same period. When we compare the current and target allocation to real estate among different institutional investors, the top three prominent investors who have the highest average current and target allocations are all pension funds, currently allocating 8~9% of AUM.

Exhibit 11 Investors' Changing Current and Target Allocations to Real Estate



Source: Preqin, "Preqin Investor Outlook: Alternative Assets H1 2016," 2016, Preqin I

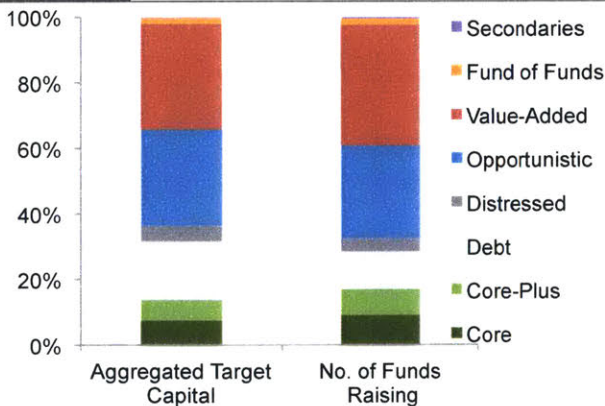
Exhibit 12 Investors' Allocations to Real Estate by Investor Type



Source: Preqin, "Preqin Investor Outlook: Alternative Assets H1 2016," 2016, Preqin

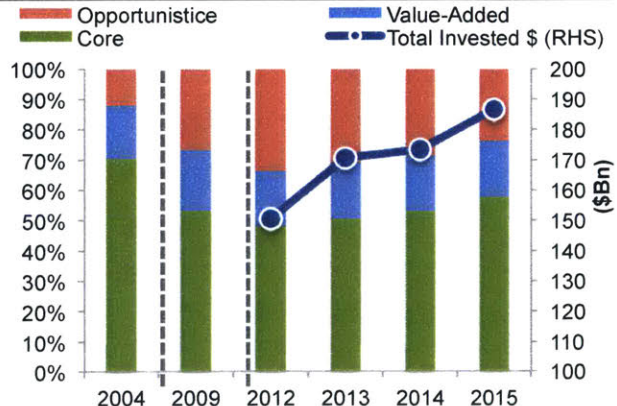
Next, we delve more into institutional asset allocation by strategies within the real estate class. Exhibit 13 illustrates the breakdown of closed-end private real estate funds by strategies and Exhibit 14 is the breakdown of PREA's all plan sponsors' real estate allocation by strategies. Notice that the size of closed-end private real estate AUM is much larger than the size of PREA member's real estate AUM as the closed-end private real estate encompasses much broader range of institutional investors than PREA does.

Exhibit 13 Closed-End Private Real Estate Funds by Strategies



Source: Preqin, "2016 Preqin Global Real Estate Report", 2016

Exhibit 14 PREA's All Plan Sponsors: Major Asset Allocation by Strategies



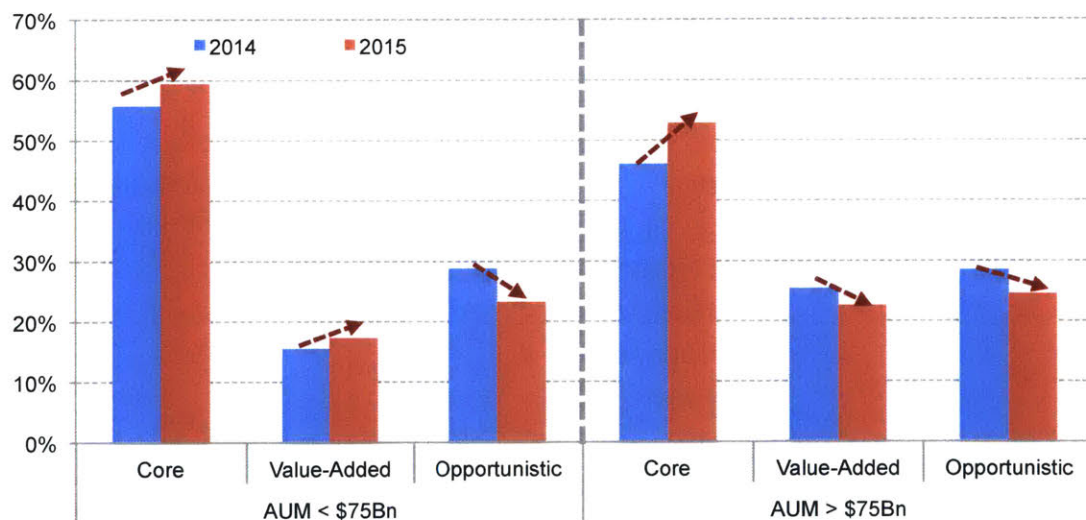
Source: PREA Investor Report 2016, 2015, 2014

Note: Excludes debt and investments not readily allocable by strategy

We can see that with the broader range of institutional investors, all closed-end funds are heavily concentrated toward opportunistic and value-added funds, together accounting for about two-thirds of funds in market and total targeted capital. Core and core-plus together target less than 10% of capital. However, among PREA members, the situation is exactly the opposite. In recent three years, PREA members' real estate allocation to core is over 50%. Their allocation to core strategy has been steadily growing from 48% of total invested amount in 2012 to 58% in 2015 at the expense of opportunistic. Allocation to value added remained stable at mid 18% over the same period. The difference in core real estate allocation may illustrate the different return objective among institutional investors as PREA member pool is heavily concentrated toward pension plans who seeks stable income yield over long term investment horizon than other types of institutional investors.

From Exhibit 15, we can see that PREA members with larger AUM over \$75 billion hold a slightly more of their asset to value-added and opportunistic strategies than members with AUM less than \$75 billion. Larger funds allocate 59% in core whereas 53% for smaller investors. Regardless of AUM size, both groups increased their allocation to core strategies in 2015.

Exhibit 15 PREA's Distribution of Private Real Estate Investments by Strategy – By Plan Size



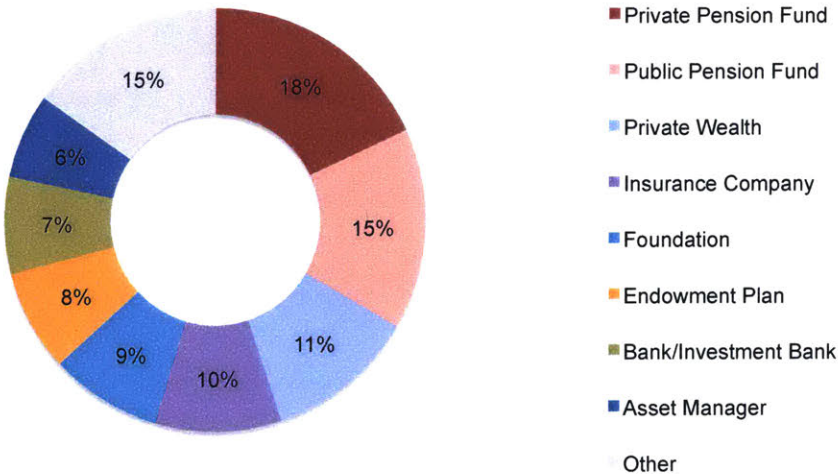
Source: PREA Investor Report 2016
 Note: Excludes debt and investments not readily allocable by strategy

Institutional Infrastructure Investment Allocation

According to Preqin, aggregate deal value of the 661 infrastructure deals completed globally in 2015 is estimated to be \$349 billion, a decline from the previous year’s estimated deal value of \$444 billion for 914 completed transactions. Average size of an unlisted infrastructure fund closed is \$858 million, and average deal value increased to \$528 million, both marking record high amount in 2015¹⁷. Decreased aggregate deal value and numbers with increased fund size and average deal value suggests that valuations are going up due to increased competition and inflow of capital.

Similar to previous players in private real estate investments, pension funds account for over a third of infrastructure investors, followed by private wealth and insurance company, both representing 20%. This is because long-term and inflation hedging characteristics of infrastructure matches well with these investors’ long-term liability. (Exhibit 16)

Exhibit 16 Infrastructure: Breakdown of Institutional Investors in Infrastructure by Type



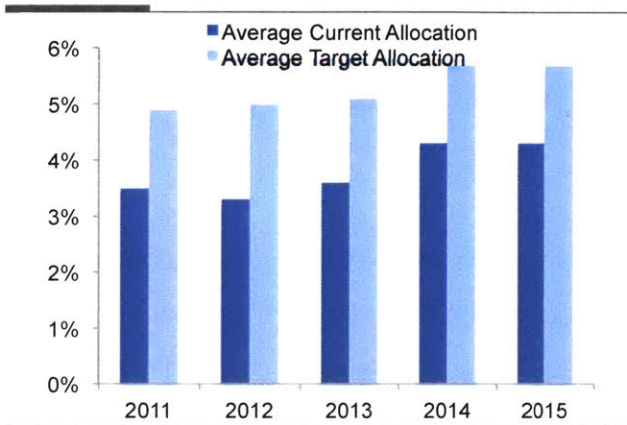
Source: Preqin, "2016 Preqin Global Infrastructure Report", 2016

Exhibit 17 and 18 show investors’ current allocation and target allocation to infrastructure investments. Investors generally allocate small portion to infrastructure when compared with other alternative asset classes as infrastructure has recently been the option for institutional portfolios. However, increase in both average target and current allocation in infrastructure with fund flow

¹⁷ Preqin, "2016 Preqin Global Infrastructure Report", 2016

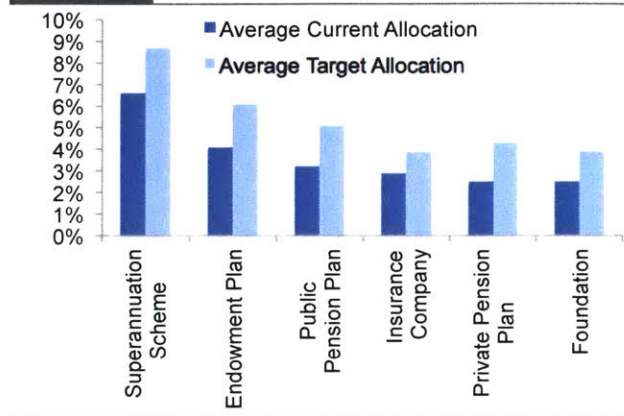
shows investors' rising appetite toward the asset class. As of 2015, average current allocation in infrastructure is 4.3% of AUM while the average target allocation is 5.7%.

Exhibit 17 Investor's Changing Current and Target Allocations to Infrastructure



Preqin, "2016 Preqin Global Infrastructure Report", 2016

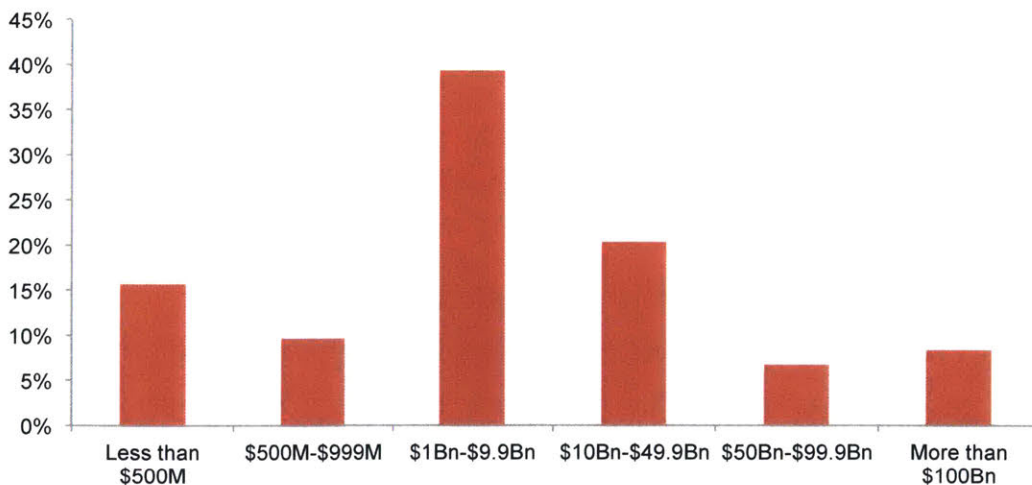
Exhibit 18 Investors' Average Current and Target Allocations to Infrastructure by Investor Type



Preqin, "2016 Preqin Global Infrastructure Report", 2016

Exhibit 19 illustrates a breakdown of institutional investors in infrastructure by AUM. We can see that investors with large AUM are investing in infrastructure. 74% of investors who invest in infrastructure have at least \$1 billion AUM. Investors with less than \$500 million AUM only represent 16%. This may possibly due to the relatively young history of asset class and the asset's characteristic that investors need monitoring on operation of the asset and government regulation.

Exhibit 19 Breakdown of Institutional Investors in Infrastructure by AUM



Preqin, "2016 Preqin Global Infrastructure Report", 2016

CHAPTER 3 Risk & Return Among the Asset Classes

In the previous chapter we introduced conceptually the major investment asset classes we will consider in the portfolio. In this chapter we will review the recent historical risk and return statistical characteristics of those asset classes, before using those statistical characteristics as inputs in our portfolio allocation analysis to be conducted in Chapter 4.

3.1 Objective

In this chapter, we first quantify the historical returns of each asset. We use those returns to investigate the volatilities of the asset classes and the correlation among them. Additionally, this chapter presents a study of the alphas (super-normal returns) of alternative assets relative to traditional assets – stocks and bonds – over the historical period covered.

By exploring the risk and return of each alternative asset class and their long-term co-movement with traditional assets, we can study how alternative assets show different characteristics compared to traditional assets, with particular focus on the potential diversification effect of the alternative investments. Admittedly, the historical period for which we have investment performance data is short. However, it is an eventful period, including the Global Financial Crisis (GFC), an event of obviously particular interest for investors. By separating the data points during the financial crisis, we will explore the impact of the GFC on each asset. The data gathered in this chapter will be the stepping stone for the next chapter where we will discover the diversification potential of alternative assets in combination with traditional assets within the investor's portfolio.

3.2 Data Resources

To study the investment performance history of different asset classes, we use the datasets shown below to represent the asset classes in the choice set for the investor's portfolio. We include representatives of all the major asset classes in a typical institutional investment portfolio. These include traditional assets including stocks and bonds, as well as alternative assets. (Exhibit 20) To compare the returns of each asset class, we use time series of simple holding period returns, gross of fees, reflecting total returns including both income yield and capital gain (return from the appreciation, whether that be positive or negative). To represent the historical returns of

each asset, we used an index that may be viewed as a plausible benchmark for that asset class, in other words, a good representative of the type of returns investors in that asset class could achieve in practice. Benchmark of each asset class represents core assets to represent assets in a typical institutional portfolio.

Most of the global assets returns are measured in local currency units because the translation to US dollars would muddy the waters of the portfolio analysis by mixing currency effects with real asset return effects. In fact, real estate and real assets returns typically face a quarterly mark to market in institutional portfolios based on NAV, and this enables institutional investors to hedge away most currency exchange rate risk at this frequency. To avoid muddying the waters with seasonal effects, and because institutional investors generally need not rebalance positions too frequently (especially in non-liquid asset classes), we use annual returns.

Exhibit 20 Data Sources Summary

Asset	Coverage	Data Source	Unit	Currency	Start Year	End Year
Equity	US	S&P 500 Total Return Index	Total Return, Gross	US\$	1996	2015
	Global	MSCI World Gross Total Return Local Index	Total Return, Gross	Local	2001	2015
Bond	US	Barclays US Aggregate Total Return	Total Return	US\$	1996	2015
	Global	Barclays Global Aggregate Total Return Index	Total Return	Local	2001	2015
Hedge Fund	Global	HFRI Funds of Fund: Diversified – Total Return Index	Total Return	US\$	1996	2015
Private Equity	Global	Burgiss, Prequin, Cambridge Associates, and Thomson Reuter	Annual Index Change	US\$	1996	2015
Real Estate	US	NCREIF ODCE Gross Total Return	Total Return, Gross,	US\$	1996	2015
	Global	JP Morgan Asset Management Global Core Real Estate Total Return Index	Total Return, Gross, Unlevered	Local	2001	2015
Real Assets	Global	JP Morgan Asset Management Real Asset Total Return Index	Total Return, Gross,	Local	2001	2015

Source: Bloomberg

Note: Private Equity Return index is a combination of data from Burgiss, Prequin, Cambridge Associates, and Thomson Reuter.

In addition to the datasets described above, we use the US Consumer Price Index to represent inflation, and the 3-month Treasury bill rate to represent the risk-free rate. Detailed descriptions regarding each dataset are shown below.

Global Equity: MSCI World Gross Total Return Local Index represents total returns on the global stock market. More specifically, the index represents developed markets and the index, with

dividends reinvested, provides an estimate of total return that would be achieved by reinvesting one-twelfth of the annual yield reported at every month-end.

US Equities: S&P 500 Total Return Index represents total returns on US stock market and is calculated by S&P based on the price changes and reinvested dividends of S&P 500. ¹⁸

Global Bonds: Barclays Global Aggregate Total Return Index measures return on global investment-grade debt from twenty-four local currency markets. This multi-currency benchmark includes treasury, government-related, corporate and securitized fixed-rate bonds from both developed and emerging markets issuers.³

US Bonds: Barclays US Aggregate Total Return Index represents investment grade, US dollar-denominated, fixed-rate taxable bond market. The index includes treasuries, government-related and corporate securities, MBS (agency fixed-rate and hybrid ARM pass-throughs), ABS and CMBS (agency and non-agency).³

Hedge Fund: HFRI Funds of Fund: Diversified – Total Return Index measures total return for funds that invests in a variety of strategies among multiple managers and tends to show a minimal loss in down markets while achieving superior returns in up markets.¹⁹

Private Equity: Private Equity Return Index represents the return on private equity. We constructed this index by combining different indexes.²⁰ This is partly because available indexes do not show appropriate return, especially during the financial crisis, and some indexes do not have data for the full historical period that we are trying to cover. Returns from 2006 to 2015 use the Burgiss PRIVATE iQ All index, which demonstrates 99% correlation with the Burgiss Private Equity Index over the same period. Returns from 2001 to 2005 are based on data from the Preqin All Private Equity Quarterly Index. From 1997 to 2000, data is retrieved from Thomson Reuter Total Return Private Equity Buyout Price Index. For 1996, the return is the average of 3 years of the Cambridge Associate's Private Equity Index. The resulting constructed index, covering a period from 1996 to 2015, shows a correlation of 87% with the Burgiss Private Equity Index and mean and standard deviation of 12.9% and 20.2% respectively, which are in-line with Burgiss Private

¹⁸ Bloomberg: Description for each indexes

¹⁹ Hedge Fund Research: HFRI Indices - Index Descriptions (<https://www.hedgefundresearch.com/hfri-indices-index-descriptions>)

²⁰ Private equity index is constructed to represent the return that is similar to the Burgiss Private Equity Index.

Equity Index, which shows a mean of 14.5% and standard deviation of 22.3% over the same period.

US Real Estate: NCREIF(National Council of Real Estate Investment Fiduciaries) ODCE VW Index measures gross total return on US real estate Open-end, Diversified Core Equity funds. The return is based on capitalization-weighted, gross of fee, time-weighted total return. The term, Diversified Core Equity style, typically reflects lower risk investment strategies utilizing low leverage and generally represented by equity ownership positions in stable U.S. operating properties diversified across regions and property types.²¹

Global Real Estate: JP Morgan Asset Management Global Core Real Estate Total Return Index represents a return on global core real estate. The index is composed of weights 50% in the US, and 25% in APAC, and 25% in Europe's core real estate. The specifics of the US real estate returns are described above, and APAC and Europe's return data is from IPD and JLL based on certain gateway cities.

Global Real Assets: JP Morgan Asset Management Global Real Assets Return Index is constructed with returns on global real estate, OECD infrastructure, and maritime assets. The global real estate has the highest weight followed by infrastructure and maritime.

3-month US Treasury Bill: Annual average of quarterly 3-month US T-bill rate announced by the U.S. Department of Treasury to represent the risk-free rate

Inflation: Annual average of monthly Consumer Price Index for all urban consumers produced by the U.S. Department of Labor Bureau of Labor Statistic to represent the U.S annual inflation

3.3 Methodology

In this section, we first report the overall risk and return of each of the above described asset classes over the historical period covered as reflected in the volatility and mean return. For a analysis time period, we use two sets of periods for the analysis; For US asset classes, we use a period between 1995 and 2015, and for the comparison between US and global asset classes, we

²¹ National Council of Real Estate Investment Fiduciaries – Description for Data Products (<http://www.ncreif.org/data-products/funds/>)

use return data from 2000 to 2015. Then, we do a correlation analysis to investigate how certain asset classes move in relation to others. Traditional asset classes – public equity and bond – are set as the base to be compared with, and examine how the alternative assets performances are linked to the performances of traditional assets. Then, we investigate how alternative assets react to changes in the CPI in order to understand their hedging potential against the inflation. In addition, by comparing the performance and correlation of US domestic assets and global assets, we will examine how these relationships among different asset classes differ in relation to the geographic market.

Finally, we will investigate alphas of each alternative asset classes using linear regression to test out whether alternative assets have provided investors with excess returns relative to public equity and bonds.

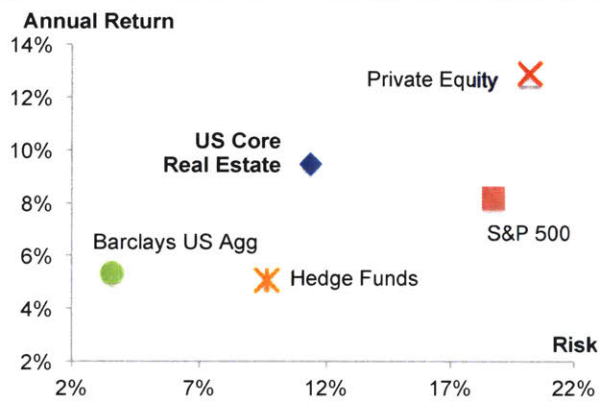
3.4 Risk & Return of Asset Classes

Exhibit 21 and 22 compare annual mean return and standard deviation of selected US asset classes for a period from 1995 to 2015 and for a period from 1995 to 2007 respectively. Later in this section, to compare the characteristics of the US and global asset classes, we use return data from 2000 to 2015. In the US, core real estate and private equity investments have dominated traditional equity and bond, whereas hedge funds did not outperform US bonds over the past twenty years.

Alternative assets return showed large swings ranging from -30% to 60% from 1996 to 2015. During the same period, S&P 500 return ranged from -37% to 33% and US bonds return was in between -2% and 12%. Among alternative investments, private equity showed the most volatile returns and the highest mean among all asset classes during the same period; Private equity's annual returns were in a range from -24% to 61% with standard deviation 18.7%. Hedge funds showed the least volatility and the lowest mean return among alternative asset classes. However, hedge funds did not dominate US bonds. Hedge funds mean return (5.1%) was only 0.2%p less than US bonds, but its volatility of 9.7% was much higher than US bonds volatility, which was only 3.6%. US core real estate performance was between stock and bond and between hedge funds and private equity; US core real estate return ranged from -30% to 21% with the mean return of 9.5% and the standard deviation of 11.4%. When we look at the risk and return

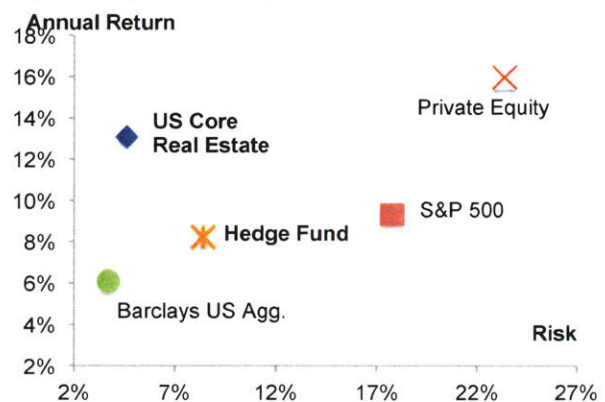
characteristic of each asset class using a period till 2007 before the impact of the GFC, mean returns of all alternative assets increase, while the changes in volatility show different direction between real estate and private equity. In a complete period, US real estate volatility increases whereas private equity volatility decreases compare with volatilities based on a period of 1996 through 2007.

Exhibit 21 Risk and Return of Selected US Assets (1996~2015)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream

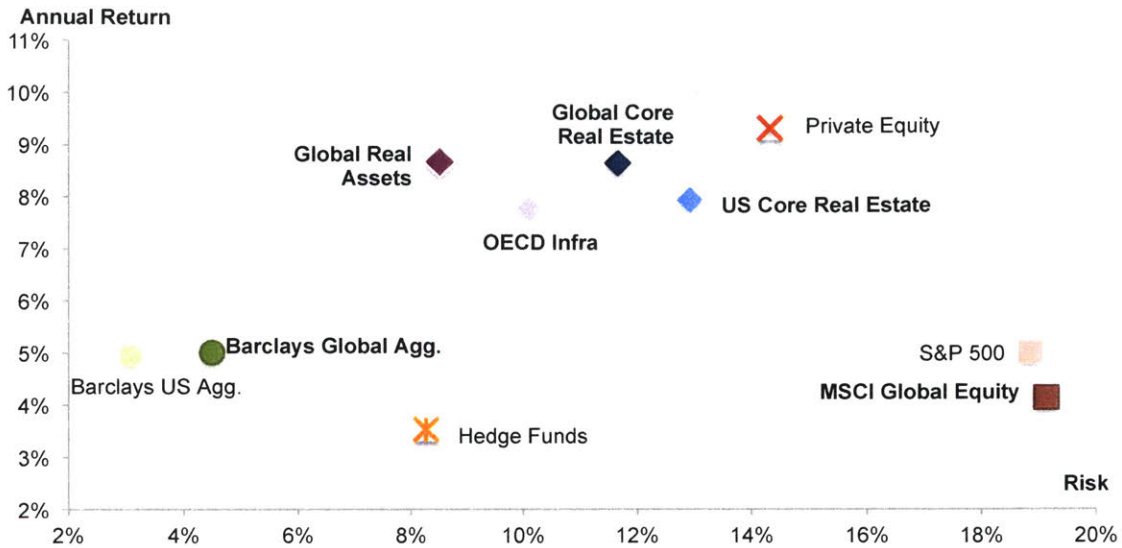
Exhibit 22 Risk and Return of Selected US Assets (1996~2007)



Source: S&P500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream

Exhibit 23 compares risk and return between US assets and global assets over the past fifteen years. From 2001 to 2015, both global and US equity show the highest volatility. However, their return was similar to fixed income asset classes, which showed the lowest volatility. Global traditional asset classes – bonds and stocks – did not outperform US traditional asset classes. Global returns of traditional asset classes tend to show the similar or lower return with higher volatility than US traditional asset classes.

Exhibit 23 Risk and Return of Selected Global Asset Classes (2001~2015): Global Real Estate and Global Real Assets Dominated Other Asset Classes



Source: S&P500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream

Over the past fifteen years, private equity showed the highest mean return of 9.3%, followed by global real asset return of 8.7% and global core real estate return of 8.6%. However, although private equity showed a slightly higher return, the volatility of global real assets and global real estate was much lower than the private equity, demonstrating the volatility of 8.5% and 11.7% respectively, whereas the private equity's volatility was 14.3%. Hedge funds showed the lowest mean return among all asset classes.

In the real estate sector, global core real estate showed higher return with lower volatility than US real estate. Because global real estate had 50% weight in the US, if we exclude US sector, international core real estate's performance was better, showing higher return and lower volatility.

3.5 Correlation Analysis of US Asset Classes

Correlation Analysis

First, we compare the correlations among US asset classes for the period from 2001 to 2015 to include a complete economic cycle (Exhibit 24). During the past fifteen years, US core real estate showed the best diversification potential with other asset classes in terms of correlation. Real Estate showed the lowest correlation with other asset classes. With public equity and hedge funds, real estate showed the correlation of 0.18 and 0.17. US core real estate showed the highest correlation with private equity partly because private equity index also includes returns of private equity real estate funds. Another possible explanation for the high correlation between US real estate and private equity is that both asset classes are traded and evaluated privately. The characteristics of private transactions may cause these two asset classes to be evaluated differently or cause their indexes to be constructed differently from the assets that are on a liquid public markets, resulting in a high correlation between the US real estate and private equity.

During the same period, US bonds are negatively correlated with all other asset classes. Hedge funds and private equity show high correlations of 0.65 and 0.79 respectively with US stock. Hedge funds show a strong correlation with stock possibly because hedge funds invest in the stock market and their relative short-term investment horizon may make them more responsive to the stock market. Private equity showing high correlation with the stock suggests that private equity is heavily affected by the performance of the stock market although they invest in a private capital market. This may possibly be due to the capital flow in the overall market. Also, hedge funds and private equity are highly correlated with one another showing a correlation of 0.67.

Exhibit 24 Correlations among Selected US Asset Classes (1996~2015)

1996~2015	S&P 500	Barclays US Aggregate	Hedge Funds	Private Equity	US Real Estate
S&P 500	1.00				
Barclays US Agg.	-0.33	1.00			
Hedge Funds	0.65	0.36	1.00		
Private Equity	0.79	0.25	0.67	1.00	
US Real Estate	0.18	-0.09	0.17	0.34	1.00

Source: MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream

Next, we compare the correlation among asset classes before the recession, dating from 1996 to 2007 (Exhibit 25).

Exhibit 25 Correlations among Selected US Asset Classes (1996~2007)

1996~2007	S&P 500	Barclays US Aggregate	Hedge Funds	Private Equity	US Real Estate
S&P 500	1.00				
Barclays US Agg.	-0.42	1.00			
Hedge Funds	0.41	0.72	1.00		
Private Equity	0.79	0.63	0.58	1.00	
US Real Estate	0.41	-0.26	0.09	0.49	1.00

Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream

US bonds still show negative correlation with all other asset classes. Private equity does not show any big difference in correlation when we exclude the recession period. Private equity still shows a high correlation of 0.79 with S&P 500. However, hedge funds show lower correlation with stock when we shorten the historical period that is before the GFC hit the market.

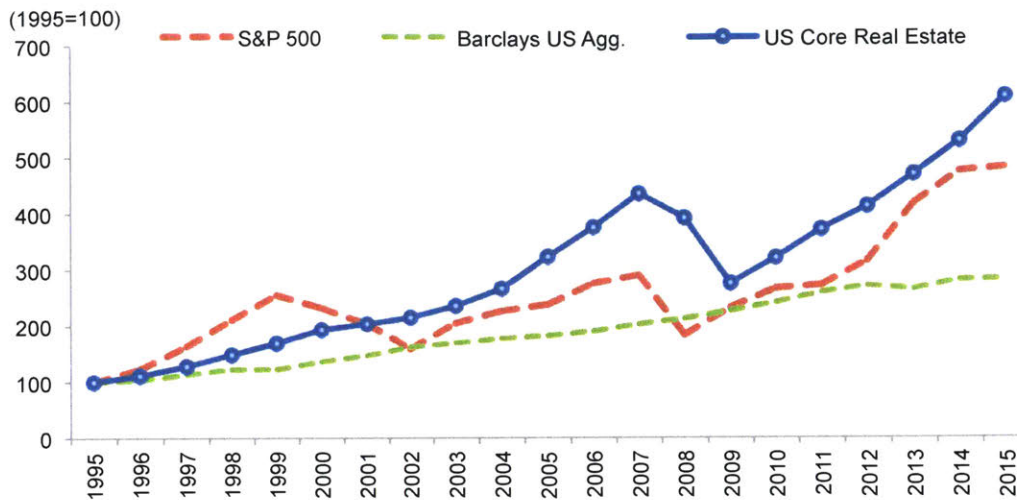
US real estate's correlation increases when we exclude the performance from 2008 and forward. This is counterintuitive when we consider that the last recession started from the collapse of US real estate market.

Real Estate vs. Stock

Exhibit 26 illustrates historical index of US core real estate, stock, and bonds over the past twenty years, and Exhibit 27 shows historical annual returns of these assets to illustrate the co-movement. After exploring annual returns and the co-movements of the two assets, real estate was much less volatile than the public equity market. On the contrary, the stock market showed several fluctuations and larger swings from the peak to trough than real estate market.

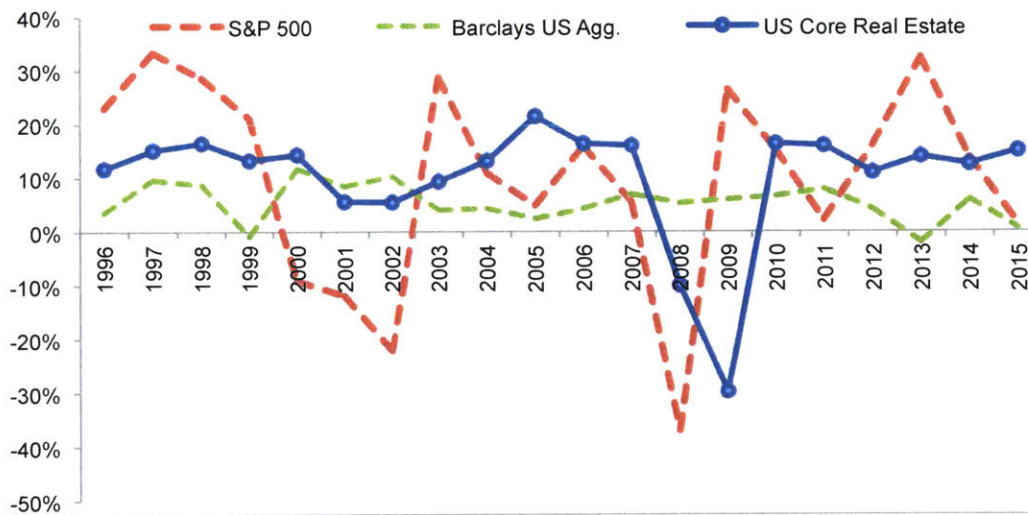
We can see that the stock market went through two depressions in the early 2000s and 2008, while US core real estate market showed a steady increase from 1995 to 2007 until the recession hit the market. From the 2008 recession, real estate was more impacted than the stock market, taking a year longer to recover from the recession. Real estate index plunged till 2009, whereas, S&P 500 started to recover right after 2008.

Exhibit 26 US Core Real Estate vs. Stock and Bonds Historical Index (1995~2015)



Source: S&P 500, MSCI, Barclays, NCREIF, Bloomberg, Datastream

Exhibit 27 US Core Real Estate vs. Stock and Bonds Historic Annual Return (1996~2015)



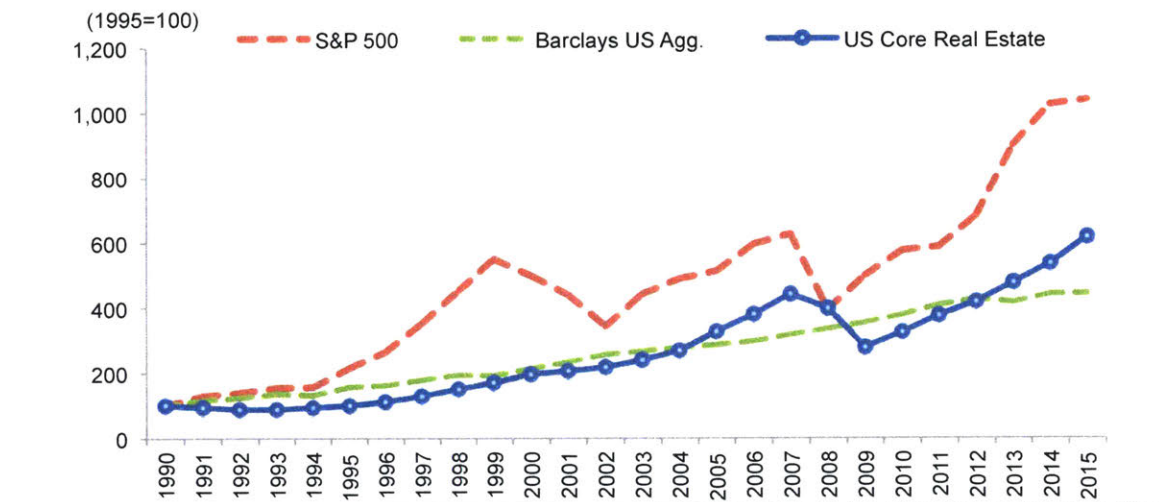
Source: S&P 500, MSCI, Barclays, NCREIF, Bloomberg, Datastream

However, after 2009, US real estate showed continuous prosperity every year, annual growth rate ranging from +10% to +16% every year. But during the same period, stock market showed a period of stagnation, partly because of the economic concern in Europe. The historical co-movement of US real estate and S&P 500 are consistent with the result in correlation analysis. In the previous correlation analysis, we were able to see that the correlation of the US real estate with S&P 500 decreased if we include the complete period of 2008 to 2015. When the overall financial market was impacted from the GFC, the real estate assets took the hardest hit and took

longer than other asset classes to recover from the recession. After the recession, once the market recovered, US core real estate showed more stabilized performance than other assets.

One thing to notice regarding the historical index is the selected time period. The historical index of the two assets starting from 1995, real estate outperformed and S&P 500 (Exhibit 26). However, if we construct the index from 1990, the result is the opposite (Exhibit 28).

Exhibit 28 US Core Real Estate vs. Stock and Bonds Historical Index (1991~2015)



Source: S&P 500, MSCI, Barclays, NCREIF, Bloomberg, Datastream

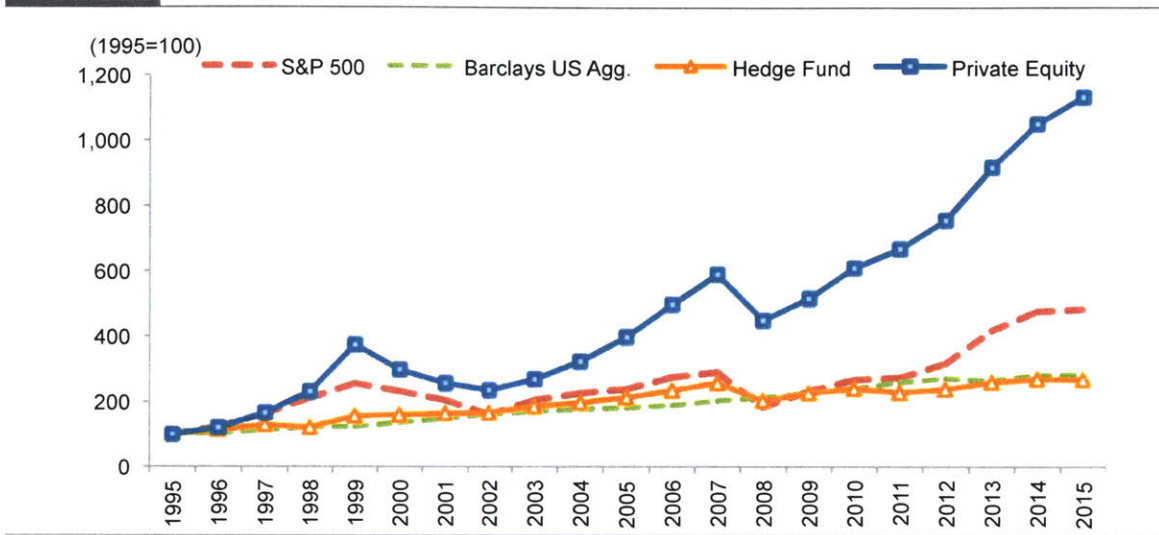
Showing different results depending on the selected period is because the stock market showed better performance in the early 1990s than the real estate, which showed stagnation. There may be a possible controversy that the each asset in the index does not represent its trough-to-peak points. However, every asset class has a different cycle, starting at a different point in time. Therefore, it is hard to initiate the analysis from the period when all of the assets were at their lowest performance.

Hedge Funds & Private Equity vs. Stock

Private equity and hedge funds showed the highest correlation with stock in the previous analysis; the correlation of private equity was 0.79, and hedge funds correlation with S&P 500 was 0.65 for a period from 1996 to 2015. Hedge funds and private equity were highly correlated to one another during the same period, displaying 0.65 of correlation. Exhibit 29 illustrates historical index

of other alternative investments – private equity and hedge funds – and traditional asset classes over the past twenty years.

Exhibit 29 Hedge Funds and Private Equity vs. Stock: Historical Index (1995~2015)

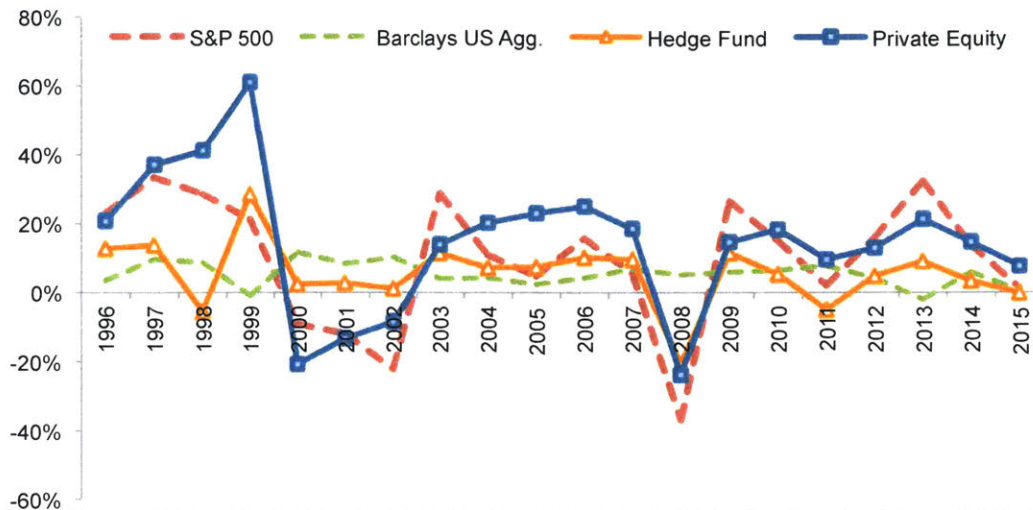


Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream

Private equity showed robust performance relative to S&P 500 and hedge funds, partly due to the private equity’s explosive growth in the late 1990s. From 1996 to 1999, private equity displayed annual growth rate ranging from +20% to +60% as we can see in Exhibit 30 describing historical annual returns of these assets. In the late 1990s, a boom in leverage buyout investments caused the expansion of private equity investments. When the stock market collapsed in the early 2000s, 2008, and 2011, private equity showed the similar movement with S&P 500.

Hedge funds showed relatively modest return if we look at the index. Hedge funds index starting from 1995, hedge funds did not outperform S&P 500, showing similar overall growth with bonds. Annual returns of hedge funds displayed much less volatility than returns of the public stock market and private equity. When both private equity and S&P 500 showed negative return in the period from 2000 to 2002, hedge funds were more defensive showing a positive return. However, hedge funds were not able to defend the market from 2008 recession. Also, in 2011, when S&P 500 showed almost flat growth, hedge funds recorded negative return. During the booming stage of the stock market, hedge funds showed relatively lower growth. It seems like hedge funds are becoming less defensive to the market downturn, while not capturing the full growth potential of the market in the booming period.

Exhibit 30 Hedge Funds and Private Equity vs. Stock: Historic Annual Return (1996~2015)



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Prequin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream

Hedge Funds, Private Equity, and Real Estate vs. Inflation

Now, we investigate how alternative assets react to the CPI in order to understand their hedging potential against the inflation. We first look at the correlation of each asset class relative to the inflation. (Exhibit 31) Real estate showed correlation of 0.36 with US inflation, higher correlation than US bonds between 1996 and 2015. Bonds and hedge funds showed mixed results depending on the selected time period. Other asset classes, stocks and private equity, showed negative correlations with the inflation. This suggests that real estate has good inflation protecting potential against the inflation. However, note that during the historical period considered here, inflation was low and not very volatile.

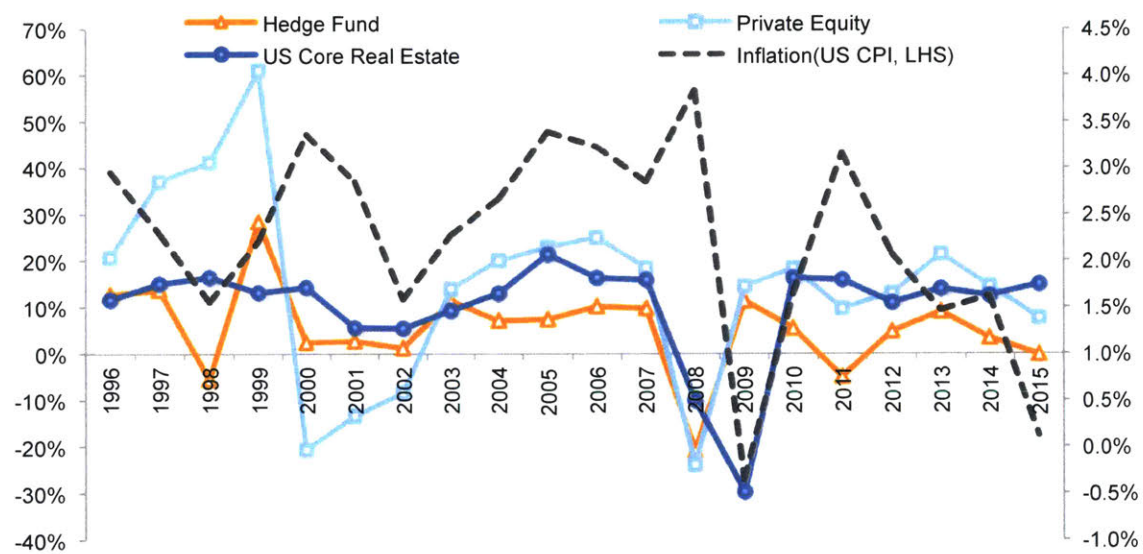
Exhibit 31 Correlations of Selected US Asset Classes with Inflation

	Inflation 1996~2015	Inflation 1996~2007	Inflation 1996~2015 (without 2008~2009)
S&P 500	-0.39	-0.17	-0.14
Barclays US Aggregate	0.20	-0.18	0.29
Hedge Fund	-0.18	0.13	0.18
Private Equity	-0.22	-0.29	-0.11
US Core Real Estate	0.36	0.40	0.16

Source: S&P 500, Barclays, NCREIF, HFR, Prequin, Thomson Reuters, Cambridge Associates, U.S. Department of Labor Bureau of Labor Statistic.

Next, Exhibit 32 looks at the co-movement of three alternative asset classes together with the inflation. US core real estate showed the least volatile movement when compared with private equity and hedge funds. Except for 2008 and 2009, real estate showed positive annual growth every year, mostly over +10%. During the same period, private equity and hedge funds displayed some negative returns. Real estate also outperformed hedge funds most of the time, showing higher annual return than hedge funds.

Exhibit 32 Hedge Funds, Private Equity, and US Core Real Estate vs. Inflation: Historic Annual Return (1996~2015)



Source: NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, U.S. Department of Labor Bureau of Labor Statistic.

Similar to the results of the previous analysis of real estate and stock, real estate took longer to recover from the recession in 2008 than other alternative assets. This is partly because that the last recession is caused by the collapse of US real estate market and the real estate assets are more illiquid than other assets that hedge funds and private equity are investing in.

3.6 Correlation Analysis of Global Asset Classes

Now we will explore the correlation among asset classes incorporating global real assets. Institutional investors diversify their portfolios not only by having different asset classes but also by having various geographic markets. It would be valuable to compare the correlation and co-

movement between US domestic assets and global assets to provide the guide to structure optimum asset allocation strategies in terms of portfolio management.

Correlation Analysis

Exhibit 33 is the summary of correlation among the annual returns of all asset classes covering US and global between 2001 and 2015. Traditional asset classes, hedge funds, and private equity exhibits similar result from the previous analysis of US asset classes. Both US and global bonds returns are mostly negatively correlated with other asset classes. Also, hedge funds and private equity are highly correlated with both US and global stock market. When we compare the traditional assets between the US and global, public equities are almost perfectly correlated to S&P 500, whereas bonds markets are less correlated to each other.

Exhibit 33 Correlations among Selected Global Asset Classes (2001~2015)

	S&P 500	MSCI Global Equities	Barclays US Agg.	Barclays Global Agg	Hedge Funds	Private Equity	US Real Estate	Global Real Estate	Infra	Maritime	Global Real Assets
S&P 500	1.00										
MSCI Global Equities	0.98	1.00									
Barclays US Agg.	-0.49	-0.55	1.00								
Barclays Global Agg.	-0.16	-0.17	0.43	1.00							
Hedge Funds	0.81	0.84	-0.20	0.10	1.00						
Private Equity	0.85	0.88	-0.44	-0.14	0.78	1.00					
US Real Estate	0.13	0.15	-0.22	-0.18	0.19	0.43	1.00				
Global Real Estate	0.30	0.35	-0.31	-0.22	0.37	0.65	0.94	1.00			
Infra	-0.05	-0.10	0.17	0.16	0.15	-0.14	0.23	0.10	1.00		
Maritime	0.24	0.21	-0.29	0.10	0.22	0.38	0.72	0.66	0.48	1.00	
Global Real Assets	0.22	0.22	-0.18	-0.06	0.36	0.44	0.87	0.84	0.62	0.84	1.00

Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream

Now we can take a closer look at the correlation among real asset classes, starting with the real estate sector. Real asset classes suggest good potential of diversification, showing a lower correlation with other asset classes. Among the real assets, infrastructure investments are least correlated with other assets during a period from 2001 to 2015.

When we compare the US and global real estate's correlation with other assets, global real estate showed a higher correlation with other asset classes than US real estate did; with the US and global stocks, global real estate showed a correlation of 0.30 and 0.35, while US real estate showed almost one-third of correlation. With hedge funds and private equity, global real estate also displayed a higher correlation than US real estate did. It can also be inferred that the US core real

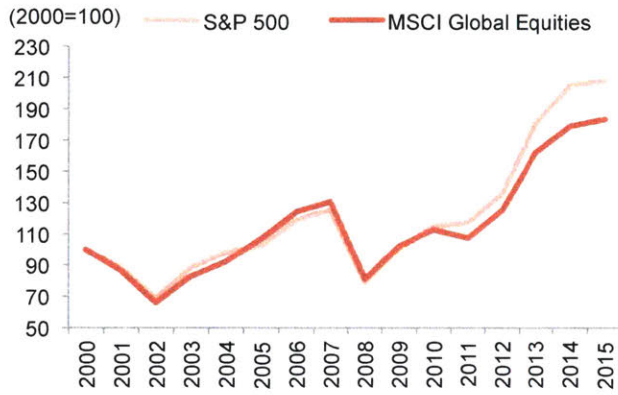
estate can add more value of diversification in investment portfolios, provided the historical experience over the period covered is indicative of future behavior.

Infrastructure investments were the least correlated asset class following bonds. They were the only assets in the dataset that were negatively correlated with public equities. With other alternative investments and other real assets, infrastructure exhibited a low correlation. Maritime displayed a correlation with equities in between of the US and global real estate. With hedge funds and private equity, maritime showed a correlation of 0.22 and 0.38, which are higher than the US real estate and infrastructure, but still lower than global real estate. Interesting point is that maritime displayed a higher correlation with real estate classes than infrastructure investments did.

US vs. Global: Traditional Asset Classes

We first look at the co-movement of traditional assets classes in the US and global. S&P 500 and global equities show the similar co-movement, suggesting that both are almost perfectly correlated. (Exhibit 26, 27) The correlation between two assets was 0.98 in the previous analysis.

Exhibit 34 US and Global Equities: Historical Index (2000~2015)



Source: S&P 500, MSCI, Barclays, Bloomberg

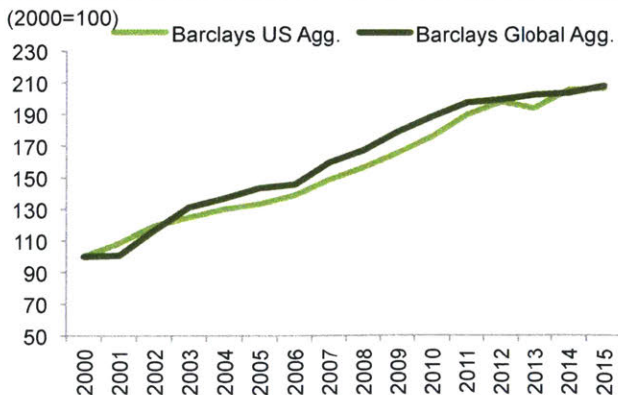
Exhibit 35 US and Global Equities: Historical Annual Return (2001~2015)



Source: S&P 500, MSCI, Barclays, Bloomberg

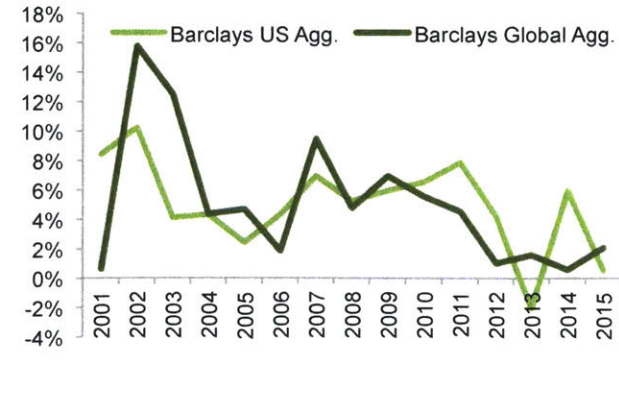
US and global bonds exhibited smaller correlation with each other than equities did, showing a correlation of 0.43. We can suggest that the equities returns are more heavily affected by global economic conditions than bonds are. Also, in 2008, both the US and global stock market were heavily impacted, whereas returns of bonds in the US and global displayed a steady growth, defending the impact from the recession.

Exhibit 36 US and Global Bonds: Historical Index (2000~2015)



Source: S&P 500, MSCI, Barclays, Bloomberg

Exhibit 37 US and Global Bonds: Historical Annual Return (2001~2015)

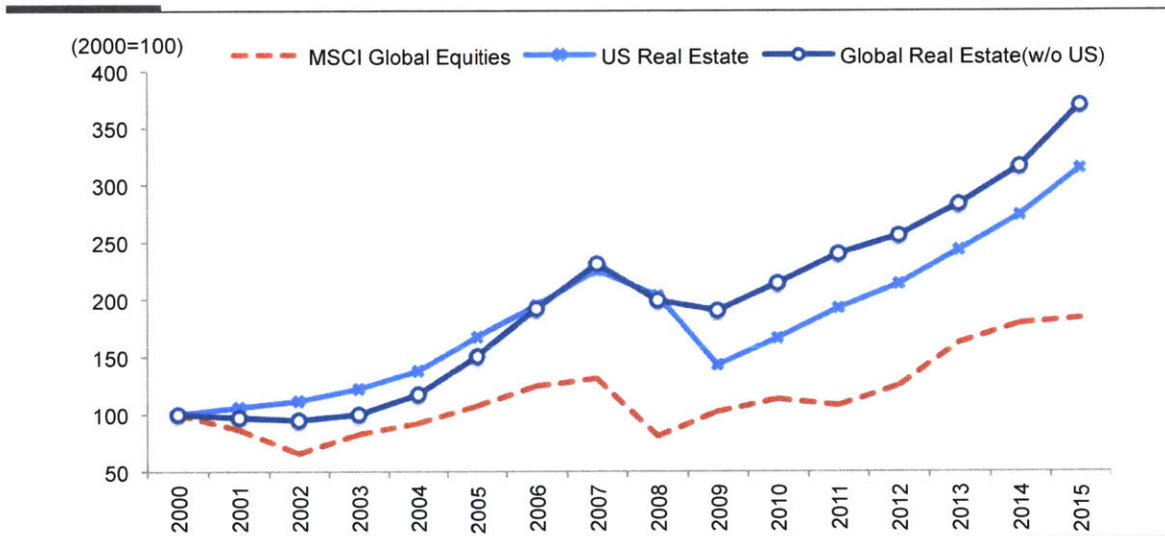


Source: S&P 500, MSCI, Barclays, Bloomberg

US vs. Global: Core Real Estate

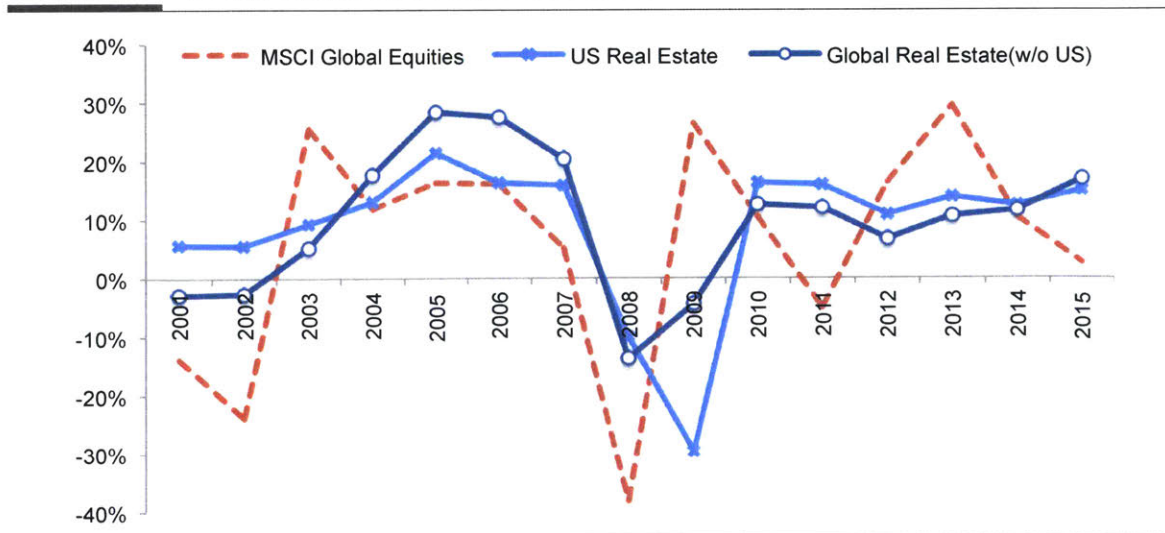
Now we compare the performances of the US and global core real estate. Here, the global real estate index does not include the US and consists of the returns from real estate in APAC and Europe to illustrate the difference in the performance between the US and non-US real estate. The global real estate outperformed the US real estate. In the early 2000s, global real estate exhibited negative to flat growth, but the explosive growth in mid 2000s made the global real estate to catch up the performance of the US real estate. In the 2008 crisis, global real estate also took a hit, but the impact was much smaller than the impact in the US.

Exhibit 38 US and Global Real Estate with Global Stock: Historical Index (2000~2015)



Source: S&P 500, MSCI, NCREIF, Bloomberg, Datastream

Exhibit 39 US and Global Real Estate with Global Stock: Annual Returns (2001~2015)

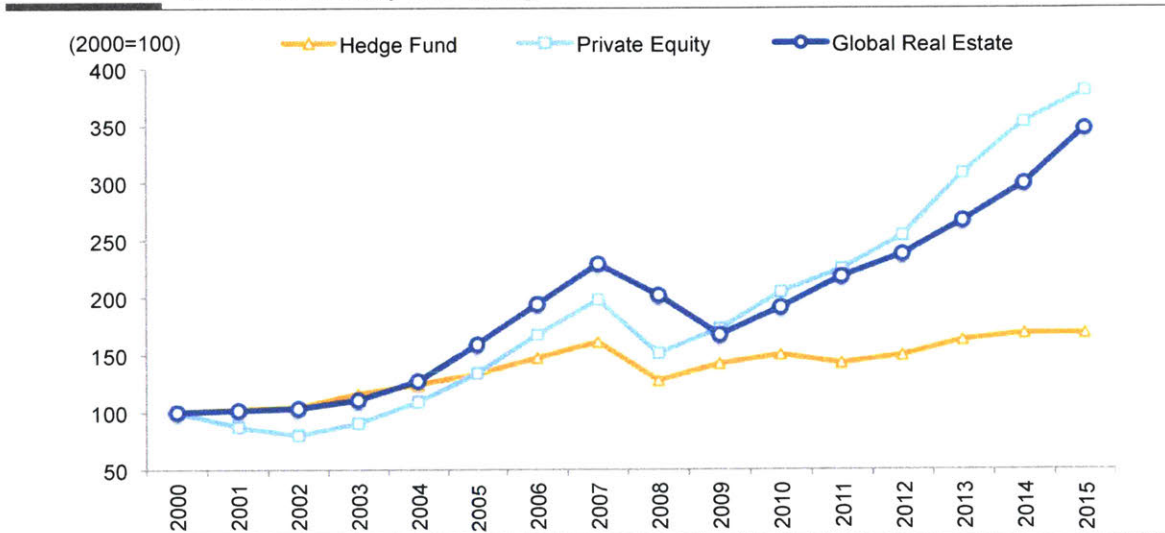


Source: S&P 500, MSCI, NCREIF, Bloomberg, Datastream

Global Real Estate vs. Hedge Funds and Private Equity

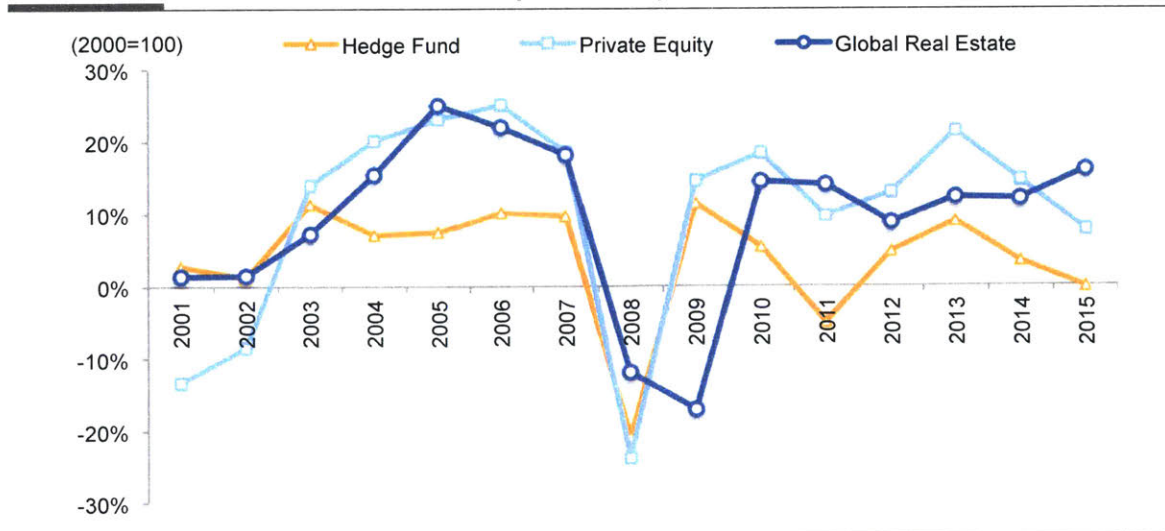
Similar to the analysis result of the US asset classes in the previous, global real estate showed overall performance that is in between of hedge funds and private equity. However, unlike US real estate, global real estate outperformed hedge funds and private equity during 2008 despite the fact that global real estate also took a year longer to recover from the crisis. The 2008 crisis that started from the US housing market also affected global real estate, however, fundamentals of the global real estate were not hardly hurt, enabling them to recover from the recession.

Exhibit 40 Global Real Estate vs. Hedge Funds and Private Equity: Historical Index (2000~2015)



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream

Exhibit 41 Global Real Estate vs. Hedge Funds and Private Equity: Historical Annual Returns (2001~2015)

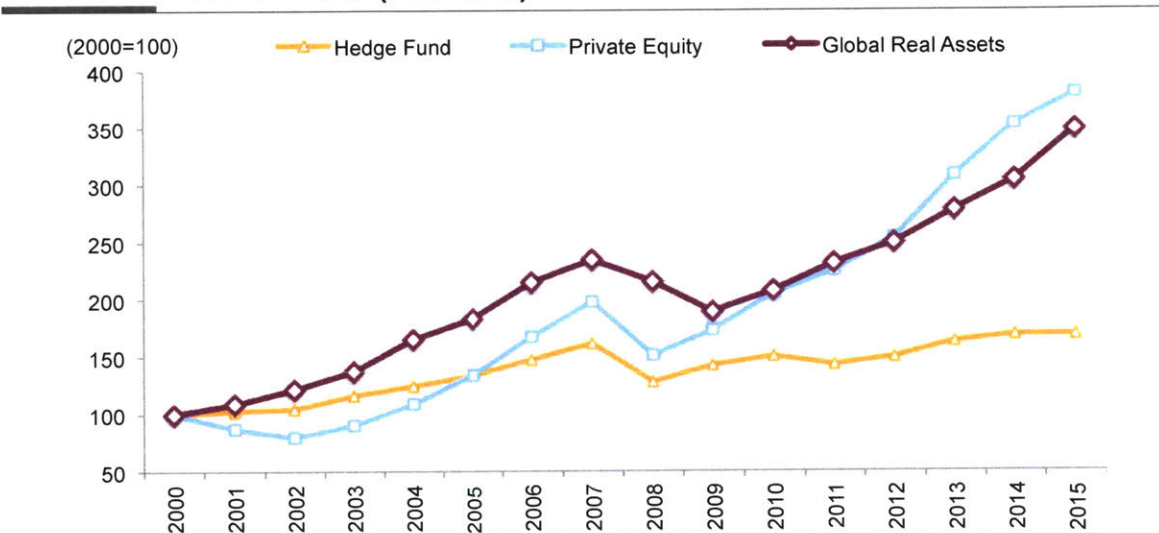


Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream

Global Real Assets vs. Hedge Funds, and Private Equity

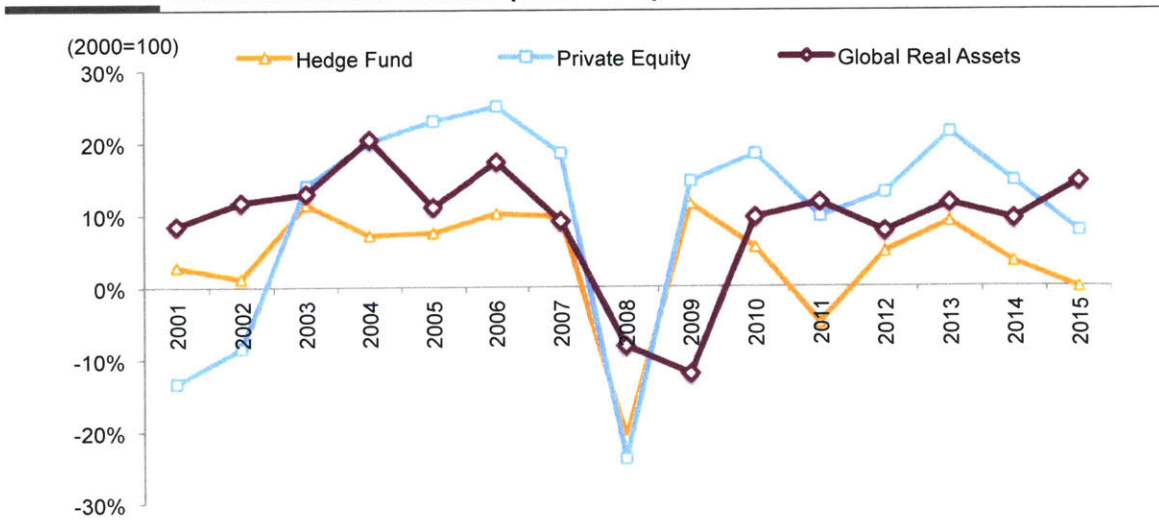
Now we compare the performances among global real assets, private equity, and hedge funds. Similar to the previous analysis, global real assets showed overall performance that is in between of hedge funds and private equity. Global real assets outperformed hedge funds and private equity during 2008 and exhibited a smaller volatility than other two assets throughout the period from 2000 to 2015. Global real assets displayed higher return than the hedge funds with lower volatility than the volatility of the private equity investments.

Exhibit 42 Global Real Assets vs. Hedge Funds, and Private Equity: Historical Index (2000~2015)



Source: NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream

Exhibit 43 Global Real Assets vs. Hedge Funds, and Private Equity: Historical Annual Returns (2001~2015)



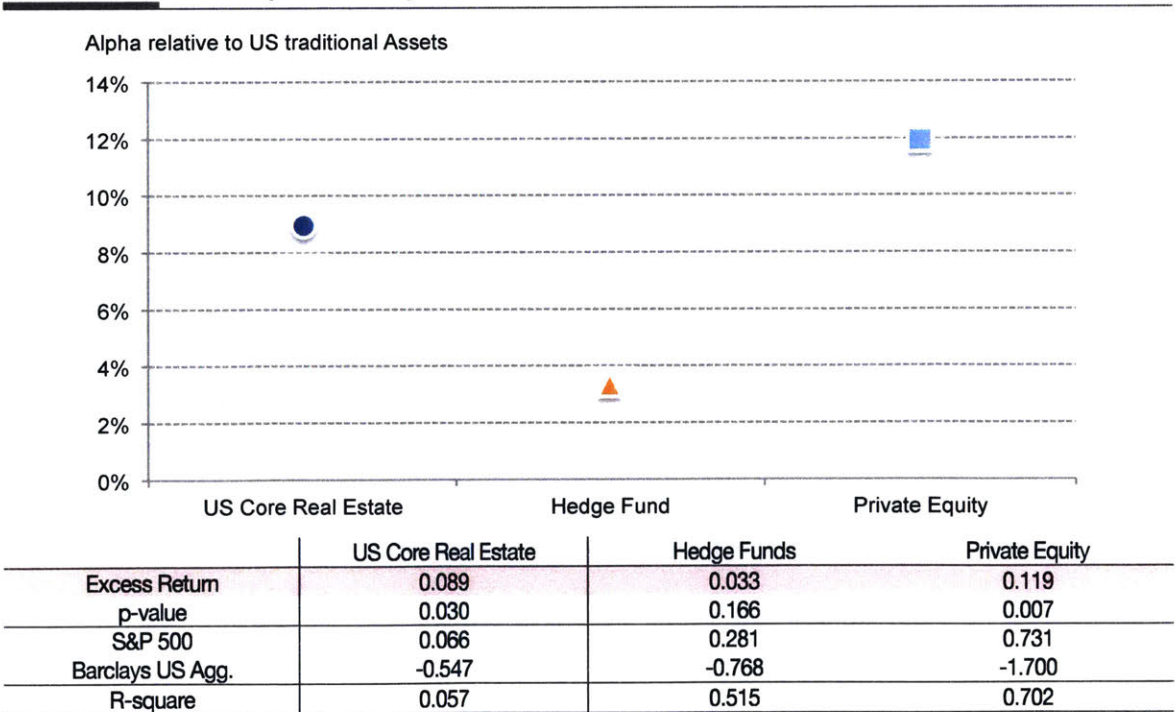
Source: NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream

3.7 Excess Return Estimates of Alternative Investments relative to Traditional Assets

In this section, we will investigate whether the alternative investments have provided investors with any excess returns relative to the performance of the traditional asset classes. We estimate alphas by regressing the annual excess returns of each alternative asset on the annual excess returns of public equities and bonds. To calculate excess returns, we subtracted total return of each asset on the 3-month Treasury bill rate. The alpha is represented by the regression coefficient on the regression constant. A positive alpha means that the alternative asset classes outperformed traditional assets, suggesting that investors would have been able to improve the performance of the portfolio by adding alternative investments to a portfolio consisting of only stocks and bonds.

Exhibit 44 and 45 shows excess return estimates of each alternative asset classes relative to the US and global traditional asset classes. Although the number of samples is not large because of relatively short history of annual returns in alternative investments, we believe we can get a glimpse regarding how alternative assets returns performed relative to stocks and bonds.

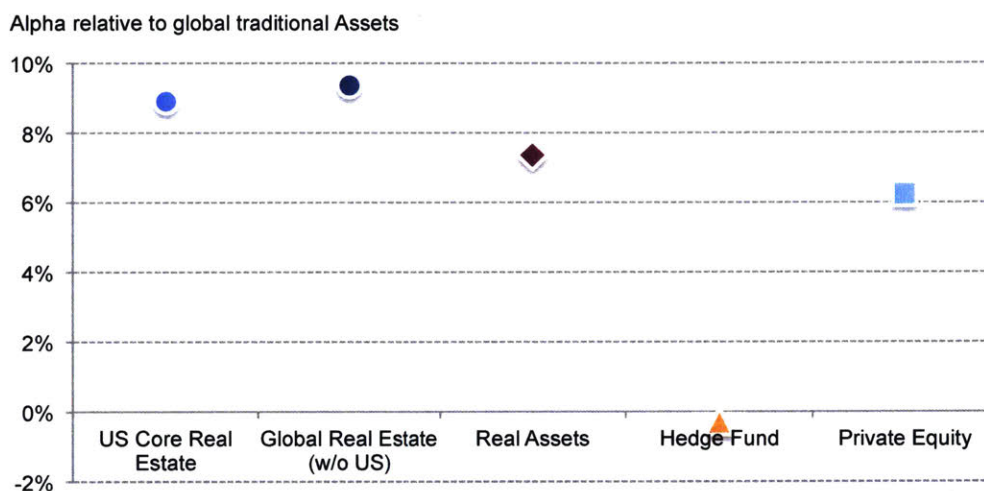
Exhibit 44 Excess Return Estimates of Alternative Assets relative to US Stocks and Bonds (1996~2015)



Source: NCREIF, HFR, Precin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream

From 1996 to 2015, private equity showed the highest alpha estimate of 12% against the US traditional assets induced by the growth of private equity industry in the late 1990s. US core real estate displayed a meaningful excessive return for the investors, suggesting that the investors may have improved the return of their portfolios if they had added US core real estate. Hedge funds displayed the smallest and statistically not significant positive value.

Exhibit 45 Excess Return Estimates of Alternative Assets relative to Global Stocks and Bonds (2001~2015)



	US Core Real Estate	Global Real Estate (w/o US)	Real Assets	Hedge Funds	Private Equity
Excess Return	0.089	0.094	0.074	-0.003	0.062
p-value	0.071	0.017	0.029	0.834	0.023
S&P 500	0.102	0.306	0.111	0.371	0.659
Barclays US Agg.	-0.560	-0.684	-0.092	0.277	-0.110
R-square	0.071	0.363	0.072	0.794	0.797

Source: NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream

From 2001 to 2015, the results of alpha estimates varied from the previous analysis, which included a longer history. Global core real estate excluding the US showed the best performance in terms of the alpha, followed by the US core real estate. Alpha estimate of the real assets was also higher than the estimate of private equity during the same period, yielding healthy excessive return. Private equity's alpha estimate against the global traditional assets was 6%, lower than previous analysis from longer period. This is partly because the earlier period when private equity yielded high growth was eliminated in the analysis. Surprisingly, hedge funds did not outperform global traditional asset classes, suggesting that investors may have not benefitted a lot by adding hedge funds into their portfolio during this period.

CHAPTER 4 Application of Modern Portfolio Theory to Alternative Investment

In the previous chapter we reviewed the recent historical risk and return statistical characteristics of major asset classes, including both the traditional assets and alternative investments. In this chapter, we conduct a portfolio allocation analysis to decide optimal mixed-asset portfolio allocation by using the statistical characteristics from the previous chapter as inputs for the analysis in the present chapter.

4.1 Objective

We explore the optimal mixed-asset portfolio allocation using traditional asset classes and alternative investments including real assets as well as hedge funds and private equity. Through exploring optimal allocation, we analyze how the return and risk of the portfolio varies if we add different alternative assets to the portfolio. Our particular focus is on the diversification effect of the various alternative assets within a traditional portfolio consisting of stocks and bonds. Additionally, we delve into how the optimal weights of asset classes in the efficient portfolios change as a function of the investor's expected return target, analyzing asset compositions in the portfolios. We focus especially on how adding real asset classes affects the performance of a portfolio differently from adding traditional alternative investments (i.e. private equity and hedge funds).

4.2 Data Resources & Methodology

We use the historical annual total return data of the selected asset classes described in the previous chapter. We structure asset allocation models to test the diversification benefit of the various assets against and we consider the effect of diversifying between globally as well as domestically. By adding different alternative investments into the portfolio, we examine how alternative asset classes enhance the returns and risk of the portfolio.

We investigate the optimal asset allocation strategies using two different approaches: 1) Mean – Variance Optimization and 2) Risk Parity. We lead off with the Mean – Variance approach, where we explore various combinations of traditional assets and alternative investments. For the

Risk Parity analysis, we look at the portfolio construction based on combinations spotlighting the role of alternative investments (i.e., private equity, hedge funds, and real assets).

In the portfolio analysis to follow, we use three sets of asset combinations. The first set consists of combinations of US domestic assets. The second consists of the global asset classes, while the third includes both global and US asset classes. The portfolio choice sets to be examined are summarized below.

Exhibit 46 List of Pairing Tests

Regions	Portfolio No.	Asset Pairing	Based Historical Annual Return Period
US	1	US Equities + US Bonds	1996~2015
	2	US Equities + US Bonds + US Real Estate	1996~2015
	3	US Equities + US Bonds + Hedge Funds + Private Equity	1996~2015
	4	US Equities + US Bonds + Hedge Funds + Private Equity + US Real Estate	1996~2015
Global	5	Global Equities + Global Bonds	2001~2016
	6	Global Equities +Global Bond +Global Real Estate	2001~2016
	7	Global Equities +Global Bond + Hedge Funds + Private Equity	2001~2016
	8	Global Equities +Global Bond +Global Real Assets	2001~2016
	9	Global Equities +Global Bond + Hedge Funds + Private Equity + Global Real Estate	2001~2016
	10	Global Equities +Global Bonds + Hedge Funds + Private Equity + Global Real Assets	2001~2016
	11	Global Equities +Global Bonds + Hedge Funds + Private Equity + Global Real Estate + Other Real Assets	2001~2016
US & Global	12	Global Equities + Global Bonds + US Real Estate	2001~2016
	13	Global Equities + Global Bonds + US Real Estate + International Real Estate	2001~2016

Note: Annual total returns are used for the selected asset classes

In constructing a portfolio, we applied several definitions, assumptions, and constraints as shown below.

Item:

1. All the weights of asset classes within the portfolio must be non-negative, meaning that we do not allow assumptions of borrowing or shorting an asset.
2. Within the zero/one range, there are no constraints on minimum or maximum weight of an asset in the portfolio. (This is for analytical clarity, although we recognize that current industry practice typically imposes a certain range of allocation to traditional or risk-free assets.)
3. The portfolio mean return is given by the following definition, multiplying the mean of each asset class and the weight of that asset:

$$E(r_p) = \sum_{i=1}^n w_i * E(r_i)$$

n denotes the number of assets in the portfolio, and i refers to a certain asset class within the portfolio.

4. Similarly, Portfolio variance is defined and computed by summing across all ordered pairwise combinations in the portfolio (including with itself), the covariance between the pair's returns multiplied times the product of the portfolio weights of the two assets (where weights are fractions of unity):

$$Var_p = \sum_{i=1}^n * \sum_{j=1}^n w_i * w_j * Cov_{i,j}$$

$$Cov_{i,j} = \sigma_i * \sigma_j * Cor_{i,j}$$

$Cov_{i,j}$ denotes the covariance between the asset i and j , σ_i denotes the standard deviation of the asset i , and $Cor_{i,j}$ denotes the correlation between the returns of asset i and j .

5. The sum of all weights among asset classes in the portfolio must equal 1.

$$\sum_{i=1}^n w_i = 1$$

4.3 Portfolio Analysis: Mean – Variance Optimization

We start with the portfolio analysis using the optimal mean-variance optimization approach. This approach, first introduced by Harry Markowitz in 1952 and often referred to as Modern Portfolio Theory, is widely known and practiced in the industry. Traditional Mean-Variance Theory (MPT) defines the “risk” that matters to the investor as the variance of the portfolio, that is, the longitudinal variance of the time series of portfolio periodic total returns. (Variance is the square of the standard deviation, which is referred to as the “volatility”.)

In this section, we first we first derive the so-called “efficient frontier”. The Efficient Frontier is the set of non-dominated portfolios. A portfolio is a combination of assets, and a dominated portfolio is such a combination for which another (different) portfolio would either have lower volatility with the same expected return or higher expected return with the same volatility. Thus, the

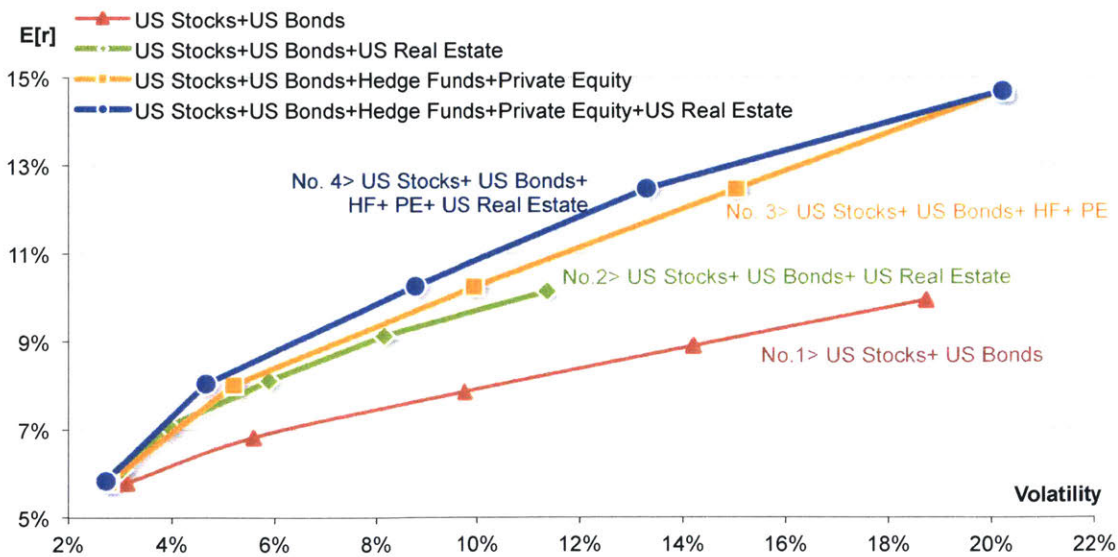
efficient frontier is constructed to allocate each constituent asset class in a way that would give the lowest-volatility (i.e. variance) of a portfolio for the mean target return range.

After we compute the efficient frontier, we explore the asset compositions in the portfolios along that frontier, using an area chart. Asset compositions tell us how the optimal weights of the asset classes in the efficient portfolios change as a function of the investor's expected return target. Here, we conduct the optimal mean-variance analysis using two sets of portfolios. The first set includes portfolios that allocate to US assets, and the second set has allocation to global assets, including real assets.

Efficient Frontiers of US Asset Classes

Exhibit 47 illustrates the efficient frontiers we find consisting of US domestic asset classes, including alternative investments, based on the historical annual returns for 1996 to 2015. We start with a base case choice set (labeled as Frontier "No.1"), consisting only of stocks and bonds. Then, we progressively add selected alternative asset classes to the choice sets, in Frontiers No. 2, 3, & 4. Each frontier consists of the non-dominated ("efficient") portfolio combinations based on the asset classes available in the respective choice sets, as identified in the Exhibit 46.

Exhibit 47 Efficient Frontier using Markowitz Portfolio: US Asset Classes



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

In Frontier No.2 we add US core real estate to the choice set in addition to stocks and bonds. We see that this moves the efficient frontier considerably up and to the left in the risk/return diagram. If instead of real estate, we allow both hedge funds and private equity to be the alternative assets added to stocks and bonds, we get Frontier No.3. When we look at the expected return objective range of these two efficient frontiers, the aggressive end of expected return objective of the portfolio that includes hedge funds and private equity is much higher (4.6%p more) than the portfolio that includes US core real estate. The minimum variance return target of the two portfolio is similar at around 3%. In the conservative return target range of 3% through 5%, the portfolio with real estate actually performs a slightly better, even though it has only three asset classes in its choice set. This is because real estate has a relatively lower expected mean return but also a lower volatility and correlation with other assets than private equity does. As noted in Chapter 3, private equity had a very high mean return during the 1996-2015 history covered. However, as the return objective increases, the portfolio with private equity and hedge funds outperforms, albeit only slightly. The comparison between the portfolios No.2 and No.3 suggests that an investor with aggressive return objective, more than about 10% based on the 1996-2015 history, would need to employ hedge funds and private equity rather than real estate, as the additions to the base portfolio of stocks and bonds. Although the correlation between private equity and stock is high (about 0.79 as seen in the previous chapter), it still gives a benefit of diversification within the portfolio especially in the higher target range, primarily because stocks alone are not able to provide a mean return above around 10% (during the source history).

Finally, Frontier No.4 allows the greatest diversification. Here, we include all five asset classes in the choice set. We can see that adding alternative investments (i.e., hedge funds, private equity, and real estate) definitely improves the performance of the portfolio when compared with a portfolio that only consists of equities and bonds. The most diversified portfolios (those on Frontier No 4.), which allocate to all asset classes, shows the most improved performance regarding its risk and return, and this is seen over a broad range of expected return targets. This is mainly because real estate has a low correlation with other asset classes. This low correlation induces the overall volatility of portfolios to decrease holding the target return constant. For example, for a mid-range target return of 8%, adding just real estate to the stock+bond choice set reduces the efficient portfolio volatility by approximately 400 basis points, from around 10% to around 6%. Adding private equity and hedge funds instead of real estate only reduces that volatility slightly more, to

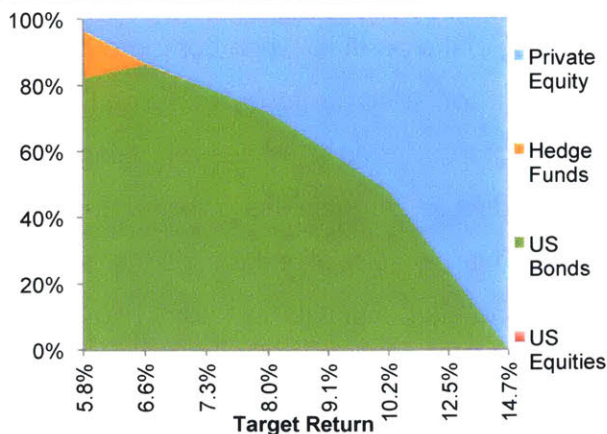
around 5%. And adding all three alternative asset classes reduces the 8%-target portfolio volatility barely more, to around 4.5%.

Asset Compositions of the Optimal Portfolios with US Asset Classes

Next, we use area charts to explore how the optimal shares of the asset classes in a portfolio vary as a function of the investor's expected return target. (Exhibit 48 and 49)

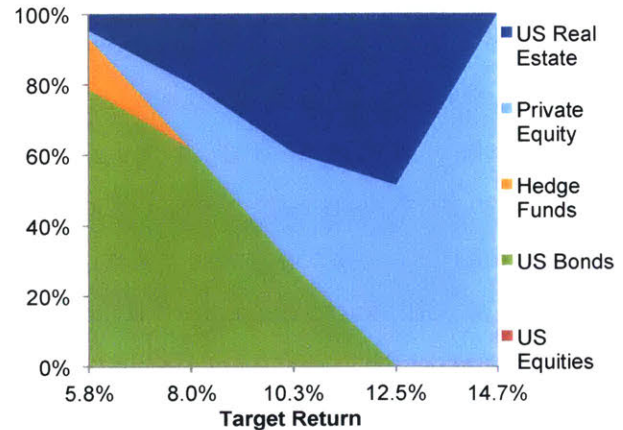
Notice that, in a portfolio with US stock, bonds, and traditional alternative investments (i.e., hedge funds and private equity), the optimal share of private equity continuously increases as the target return goes up, and at the aggressive end, the balance of the optimal portfolio goes from mostly bonds to private equity. This is because private equity has the highest expected mean return among the constituent asset classes. As we increase the target return of a portfolio, private equity earns more shares in the portfolio to increase the overall return of the portfolio. Also, when we include private equity and hedge funds in a portfolio, the optimal portfolio does not allocate any portion to stocks. This is because the correlation between private equity and bonds is much more negative than the correlation between stocks and bonds, while the expected mean return of private equity is higher than the stocks. Furthermore, as we saw in Chapter 3, stocks are highly correlated with private equity (79%), and the correlation between private equity and hedge funds is similar to the correlation between stocks and hedge funds. Thus, private equity dominates over stocks in the optimal mean-variance portfolio.

Exhibit 48 Asset Composition of the Efficient Frontier (No. 3)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996–2015 historical annual returns

Exhibit 49 Asset Composition of the Efficient Frontier (No. 4)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996–2015 historical annual returns

The optimal hedge funds share starts at 14% for a conservative end, and quickly replaced by bonds. This makes sense because hedge funds expected return is lower and its volatility is higher than the bonds. Although the hedge funds correlations with other traditional assets are lower than the private equity's correlations, the bonds mean return is similar to hedge funds and the correlations of bonds with other assets are negative, whereas hedge funds correlation with stock and private equity is over 0.65. When constructing the optimal portfolio, the hedge funds are dominated for most of the return target range not only by private equity but also by bonds. (Exhibit 48)

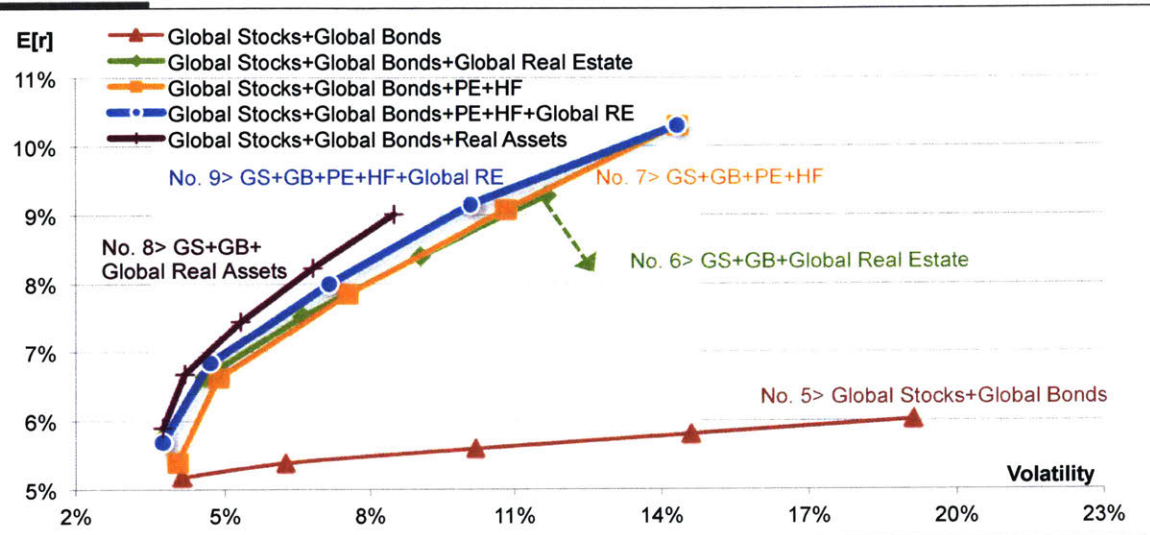
In Exhibit 49, we add US core real estate to the portfolio. We can see that private equity still dominates other asset classes in the aggressive target return range (12.5%~14.7%) of the optimal portfolio. However, in a broad range of conservative to moderately aggressive return targets (from the minimum-variance target return of around 6% through 12.5%), the optimal real estate share continuously increases up to almost a half of the portfolio, taking shares away from bonds. This is because real estate displays relatively low correlation with other asset classes in the portfolio and has a moderate expected return following stocks and private equity.

Efficient Frontiers of Global Asset Classes

Now, we look at the efficient frontiers of global asset classes based on the historical returns from 2001 to 2015. Exhibit 50 illustrates efficient frontiers for choice sets consisting of global asset classes, including traditional asset classes, private equity, hedge funds, and real estate. Notice that the optimal portfolios including alternative investments exhibit much higher expected return and much lower volatility than the portfolio consisting of only global stocks and bonds. The performance enhancement by adding alternative investments in a portfolio seem much more pronounced among global asset classes than among US asset classes. Among hedge funds / private equity, global core real estate, and global core real assets, real asset best improves the performance of the portfolio in terms of risk-return efficiency when added to the traditional stock + bonds portfolio (Nos. 6,7,8). Notice that adding only real asset to traditional stocks+bonds portfolio (No.8) performs better than adding hedge funds, private equity, and real estate all together to the same traditional portfolio (No. 9). Based on the 2001-2015 data, the greater diversification allows more than 1500 basis points reduction in volatility for a 6% target return portfolio. The volatility reduction due to added diversification, for mid-range target portfolios with just US domestic asset

classes, is less than 600 basis points, based on 1996-2015 data, as noted above. In the global portfolio, the effect of adding real estate to a portfolio with the other four asset classes (stocks, bonds, private equity & hedge funds) is to reduce the volatility by about 100 basis points for a mid-range target return of around 8% (in the 2001-15 time frame).

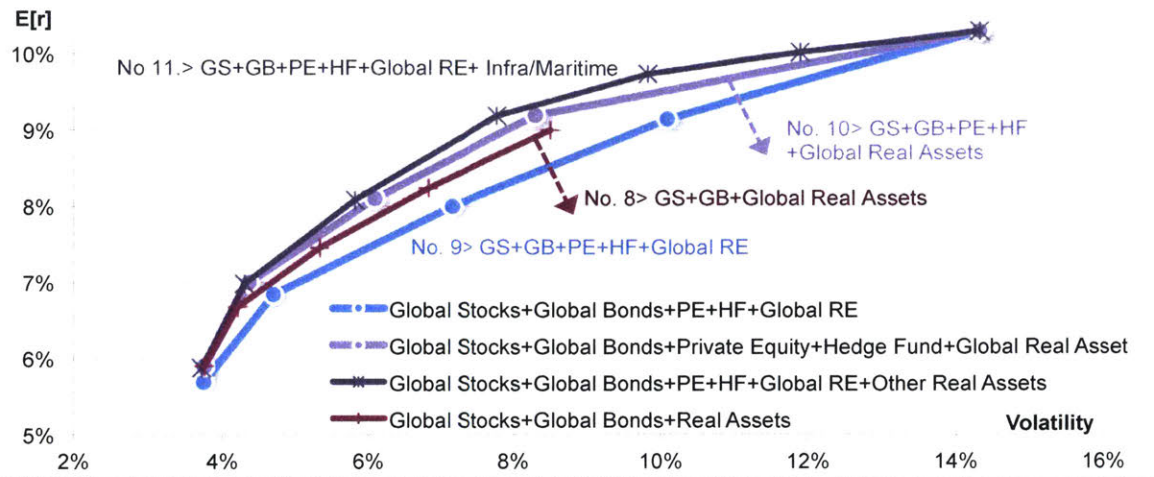
Exhibit 50 Efficient Frontier using Markowitz Portfolio: Global Asset Classes



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns
 Note: GS refers to global stocks, and GB refers to global bonds

Next, in Exhibit 51, we focus on the effect of adding real assets (not just real estate) to the portfolio. The Exhibit illustrates efficient frontiers incorporating global real asset classes. If we add real assets (which include real estate and other types of real assets as a single aggregated asset class) in a portfolio instead of real estate, the portfolio with real assets (No. 10) dominates the portfolio with just real estate (No. 9). This is because, in comparison with real estate alone, the real asset class as a whole shows a lower correlation with most of the other assets in the global portfolio choice set (except for bonds), as shown in Exhibit 52.

Exhibit 51 Efficient Frontier using Markowitz Portfolio: Global Asset Classes, including Real Asset Classes



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns
 Note: GS refers to global stocks, and GB refers to global bonds

Exhibit 52 Global Real Estate vs. Global Real Assets: Correlations with Other Asset Classes

	MSCI Global Equities	Barclays Global Agg	Hedge Funds	Private Equity
Global Real Estate	0.35	-0.22	0.37	0.65
Global Real Assets	0.22	-0.06	0.36	0.44

Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

To explore whether there is any benefit in diversifying within the real assets, we separate the real asset class into more specific categories, global core real estate and other global real asset classes, which consist of infrastructure and maritime assets. In Exhibit 42, portfolio No. 11 with separate real asset classes dominates portfolio No. 10 for a moderate to aggressive return objective. This suggests that both real estate and other real asset classes as independent asset classes can play separate roles in the optimally diversified portfolio. Other real assets (here, infrastructure and maritime investments) are less similar to global real estate regarding their risk and return performance. More importantly, the two assets are much less correlated than one might at first suppose. Specifically, infrastructure investments only have a correlation of 0.10²² with global real estate. Maritime investments exhibit a higher correlation of 0.66²² with global real estate, but they are far from perfectly so. Therefore, enabling the two forms of real asset investing complements one another in the optimal portfolio, improving the performance of the portfolio.

²² Based on historical annual returns from 1996 to 2015.

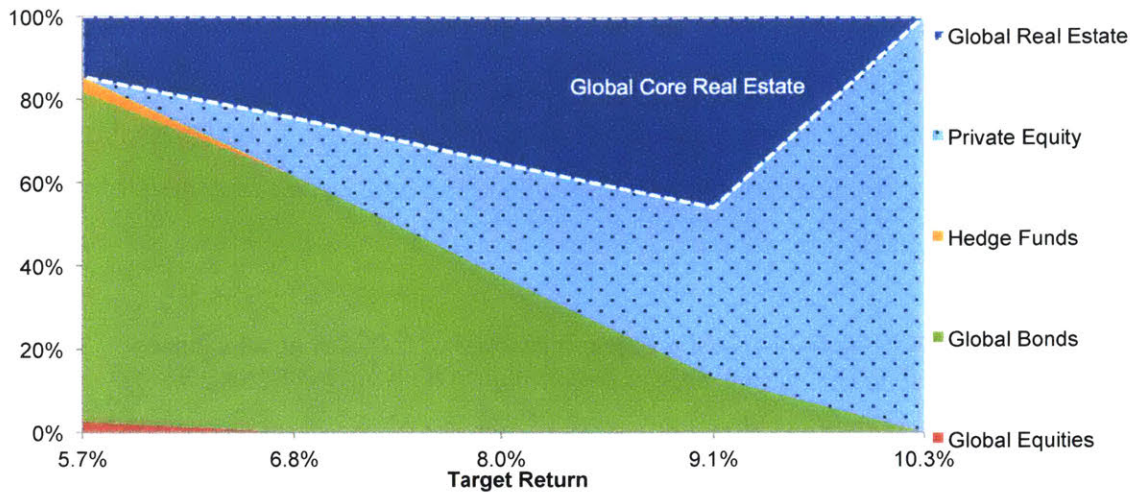
Asset Compositions of the Optimal Portfolios with Global Asset Classes

In this section, we look at asset compositions of the optimal portfolio with global assets. The asset composition of the efficient portfolio that has stocks, bonds, private equity, and hedge funds exhibit similar results compared to the corresponding choice set for the US domestic asset classes. The difference is that the overall optimal share of stocks decreases due to its lower expected return. However, it should be noted that the lower return to stocks is due not to the global coverage but only due to the different time period of the historical data (2001-15).

Exhibit 53 illustrates the asset composition of the efficient frontier that includes global core real estate, and Exhibit 54 is the asset composition with global real assets. From both charts, the expected target return range is similar and the optimal share of real estate and real assets increase for a broad range of conservative to moderately aggressive return targets (from the minimum-variance target of mid 5% through just over 9%, which is not far below the maximum all-positive weights portfolio return of 10.3%). In both cases, private equity still takes up a large share in the portfolios for the aggressive-end target return range. This is because both real estate and real asset have higher expected return than stocks and bonds but lower than private equity, and the correlations of these two assets (real estate and real assets) with other asset classes are lower than correlations between other asset classes. (Of course, as in all of the mean-variance analysis, the “expected” return is simply proxied by the mean historical return during the period covered: 1996-2015 for the domestic analysis; 2001-2015 for the global analysis.)

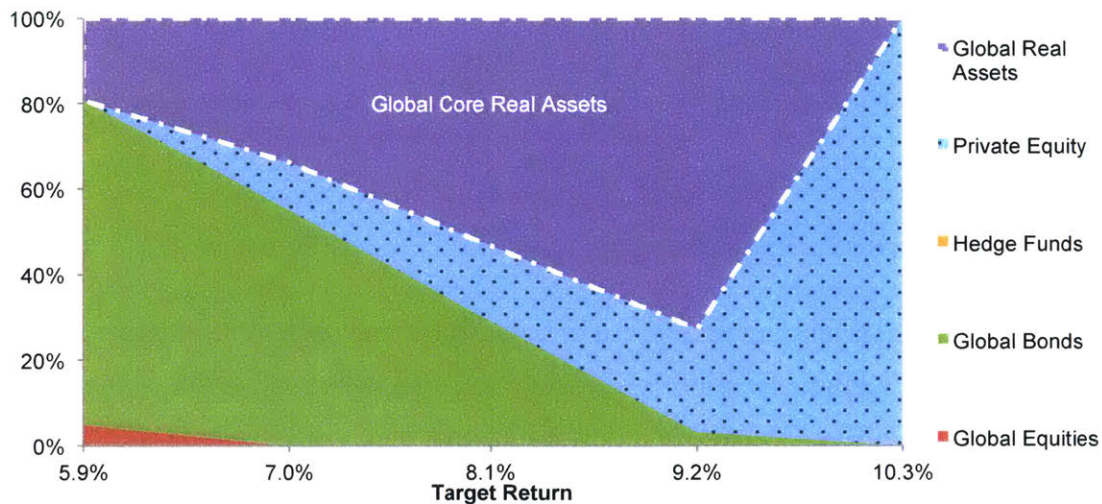
The intriguing point is that the optimal share of real assets is much higher than the share of just real estate alone, even though the target return ranges of the two portfolios are the same. For a return target of just over 9%, the weight of real estate is 46% in the Exhibit 53, while the weight of real assets is 73% in the Exhibit 54. The real asset class is allocated larger share because its expected return is higher than that of global core real estate and the correlations between real assets and other constituent asset classes are slightly lower than the correlations between real estate and others. In the aggressive return target range, the optimal real assets share is still larger than that of real estate. We can recall that the volatility of the portfolio No. 10 is lower than that of the portfolio No. 9 (Exhibit 51) This suggests that adding the real asset class as a whole improves the performance of a portfolio regarding risk and return more so than only adding real estate alone.

Exhibit 53 Asset Composition of the Efficient Frontier with Global Asset Classes, including Global Real Estate (No. 9)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 54 Asset Composition of the Efficient Frontier with Global Asset Classes, including Real Assets (No. 10)

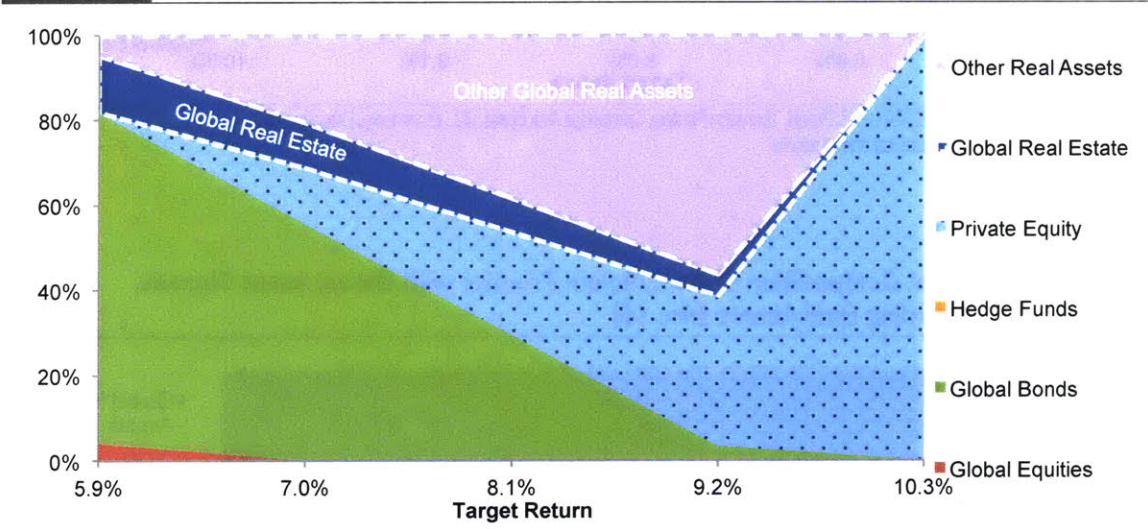


Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

As noted previously, we further explore the role of diversification within the real asset class by separating the real assets into more specific categories. Exhibit 55 shows the asset composition of the optimal portfolio that separately invests into sub-real asset categories (i.e., global real estate and other global real assets – infrastructure and maritime). Notice that, for the same expected

return objective of 9.1%, the optimal share of real estate and other global real assets together is 61% (Exhibit 55), which is in between of the 73% in Exhibit 54 and the 46% in Exhibit 53. Decreased share of real assets goes to mostly private equity and partly to bonds. This is because when we introduce real assets separately, other real assets show lower correlation with other assets than real estate does. These characteristics cause the increase in the share of other real assets and decrease in the share of real estate, resulting in overall decreased share of real assets as a whole.

Exhibit 55 Asset Composition of the Efficient Frontier with Global Asset Classes, including Global Real Assets (Separation in Real Asset Classes, No. 11)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note 1: Based on 2001-2015 historical annual returns
 Note 2: Other Global Real Assets refer to infrastructure and maritime

Among the real asset classes, the role of other global real asset class is larger than that of core real estate within the optimal portfolio. Although the expected mean return of infrastructure and maritime investments (8.75%) is slightly lower than that of global real estate (9.3%), infrastructure and maritime investments show lower correlations with other asset classes (except for bonds) than real estate does as shown in the correlation matrix (Exhibit 56). Especially the infrastructure’s correlation with private equity is lower than the correlation between real estate and private equity that plays an important role in increasing the return in a portfolio. These features make the balance of the optimal portfolio tilted more to infrastructure and maritime investments than to global core real estate.

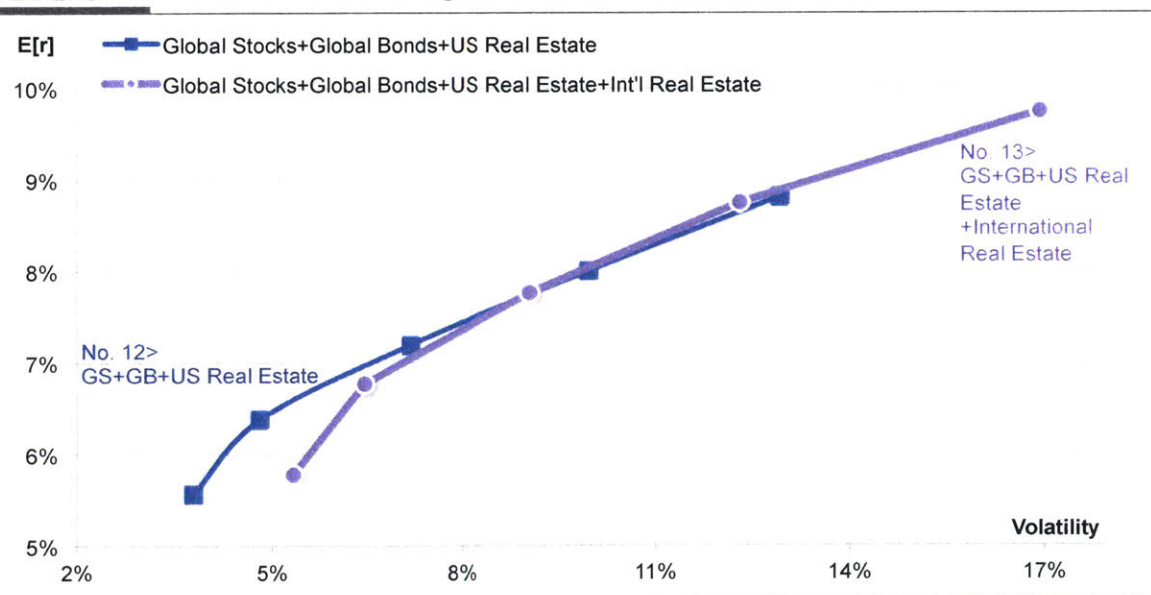
Exhibit 56 Correlations among Selected Global Asset Classes

	MSCI Global Equities	Barclays Global Agg	Hedge Funds	Private Equity	Global Real Estate	Infra
Global Real Estate	0.35	-0.22	0.37	0.65	1.00	
Infra	-0.10	0.16	0.15	-0.14	0.10	1.00
Maritime	0.21	0.10	0.22	0.38	0.66	0.48

Source: MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note 1: Based on 2001–2015 historical annual returns

Finally, we compare the role of US real estate and global real estate in the optimal portfolio. This is a perspective that might be particularly interesting to a foreign investor who is open to investing specifically in US real estate. Exhibit 57 illustrates the efficient frontiers with regional diversification within real estate class. Exhibit 58 illustrates the asset composition of the efficient frontier with US core real estate and Exhibit 59 with US and international core real estate. The shares of real estate continuously increase in both cases. Here, the international real estate includes combination of only APAC and Europe core real estate to analyze how international core real estate affects the performance of a portfolio differently from the US core real estate.

Exhibit 57 Efficient Frontier using Markowitz Portfolio: Real estate

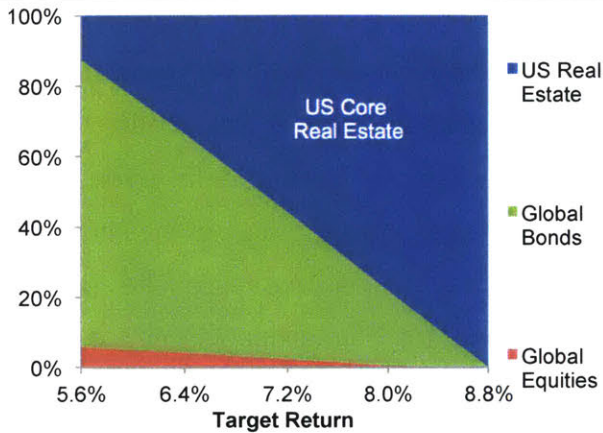


Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note: Based on 2001–2015 historical annual returns
 Note: GS refers to global stocks, and GB refers to global bonds

From the range of expected return objective, note that the optimal portfolio with regionally diversified real estate yields much more higher return range (400 basis points more for the aggressive end) than the portfolio with US real estate does. This suggests that an investor

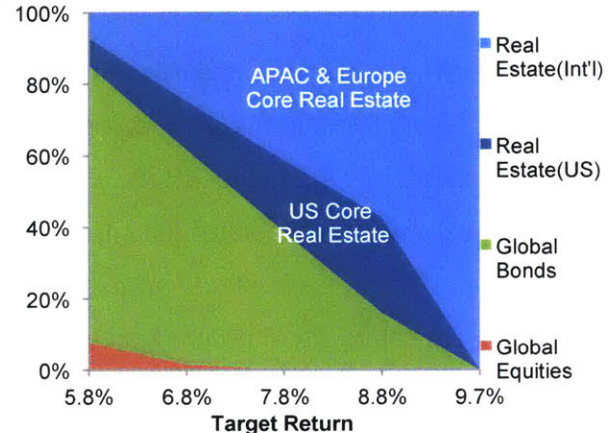
considering allocation to the real estate should think about the possibility of diversifying regionally within real estate asset classes to increase the return of a portfolio.

Exhibit 58 Asset Composition of the Efficient Frontier: Global Traditional Assets + US Real Estate (No. 12)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001–2015 historical annual returns

Exhibit 59 Asset Composition of the Efficient Frontier: Global Traditional Assets + US RE + International RE (No. 13)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001–2015 historical annual returns

Exhibit 60 Correlations of US and International Real Estate Classes with Global Stocks and bonds

	MSCI Global Equities	Barclays Global Agg.	US Real Estate	International Real Estate
US Real Estate	0.15	-0.18	1.00	
International Real Estate (APAC, Europe)	0.52	-0.23	0.75	1.00

Source: MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note 1: Based on 2001–2015 historical annual returns

Within the real estate classes (Exhibit 57, No.13), the allocation to international core real estate is larger than the allocation to US real estate. The optimal share of US real estate increases up to 27% till target return of 8.8%. Notice that the optimal share of stocks decrease as we diversify the real estate asset classes. This is because international core real estate has a relatively high correlation with global equities than US real estate does, and the expected return and risk features of international real estate dominate global equities (meaning that the expected return of international real estate is higher and volatility is lower than that of stocks). Therefore, the optimal share of stocks (move to international real estate when we diversify within the real estate classes.

4.4 Portfolio Allocation by the Risk Parity Criterion

Mean-variance optimization, the identification of non-dominated portfolios as in the preceding sections of this chapter, is the classical framework for portfolio allocation. But it has some weaknesses, and this has led decision makers in recent years to consider alternative approaches. A major such alternative is allocation according to the “Risk Parity Criterion”. In this section, we review the definition of risk parity allocation, and then apply risk parity to construct the optimal portfolio that includes various alternative investments as well as traditional asset classes, as examined in the preceding sections.

While the Harry Markowitz’s mean-variance approach is widely known and practiced in the industry, risk parity is less known. Therefore, it would be helpful to review the definition of this approach before we delve into the analysis.

Definition of Risk Parity

Risk parity is an alternative portfolio construction approach that allocates capital to each asset inversely proportional to its future expected volatility.²³ Thus, the risk parity approach allocates investment capital on a risk-weighted basis. The risk parity approach defines a well-diversified portfolio as one where all asset classes have the same marginal contribution to the total risk of the portfolio²⁴. In this sense, a risk parity portfolio is an equally weighted portfolio, where the weights refer to risk rather than dollar amount invested in each asset.

The most distinctive feature of risk parity allocation is that this approach does not require an investor to forecast the ex-ante return of the constituent asset classes as the mean variance approach does. Mean-variance allocation balances the risk and return of a portfolio, whereas risk-parity only aims to balance the distribution of risk across the constituents of a portfolio, ignoring information relating to expected returns.

The disadvantage of this approach is obvious at a theoretical level. It produces a portfolio that is dominated by the efficient frontier of the Mean-Variance Optimization described previously.

²³ Gregg S. Fisher, Philip Z. Maymin, Zakhar G. Maymin, December 2012, “Risk Parity Optimality”, NYU Polytechnic Institute, Gerstein Fisher Research Centre

²⁴

That is, the Risk Parity Portfolio (RPP) will have more risk for the same return, or less return for the same risk, compared to some asset combinations on the Markowitz Efficient Frontier. As a corollary, the RPP will have a lower Sharpe Ratio (expected return risk premium divided by the volatility) than the Sharpe-maximizing (“tangent”) portfolio on the Markowitz frontier.

But the RPP also has a crucial advantage. It is robust to the future expected (mean) return (assumption). The expected future return is generally the statistic about which the analyst has least certainty, least confidence. Both the RPP and the Mean-Variance Optimal portfolios require assumptions about the volatilities and correlations among the assets in the allocation choice set. But only the Mean-Variance optimization requires assumptions about assets’ expected returns. The RPP is thus more robust to uncertainty, as distinguished from risk. Economists define “uncertainty” as “unknown risks”(risks that are not quantifiable), and investors are arguably more averse to uncertainty than to risk.

Data and Methodologies

For our purposes in this thesis, we define the risk relevant to the RPP as the standard deviation of each asset’s annual total return (aka “volatility”), the traditional definition and the same as we did in the previous sections on the mean-variance approach. Then, we construct the risk parity portfolio by assigning weights to each constituent asset class in a way for each asset to have the same marginal contribution of risk in a portfolio.

$$\begin{aligned}
 MC_i &= (\text{Weight of Asset Class } i) * \frac{\Delta \text{ Total Risk of Portfolio}}{\Delta \text{ Weight of Asset Class } i} \\
 &= w_i * \frac{\sum_{j=1}^N w_j \text{Cov}[R_i, R_j]}{\sigma[R_p]}
 \end{aligned}$$

Where, N is number of asset classes in a portfolio, MC_i is the asset i ’s marginal risk contribution in a portfolio, σ is the standard deviation (i.e., volatility) of the portfolio and MC_i satisfies below conditions.

$$\begin{aligned}
 \text{Total Risk of Portfolio} &= MC_1 + MC_2 + \dots + MC_N \\
 MC_1 &= MC_2 = \dots = MC_N
 \end{aligned}$$

We use the following asset pairing sets to construct the risk parity portfolio.

Exhibit 61 List of Pairing Tests

Regions	Portfolio No.	Asset Pairing	Based Historical Annual Return Period
US	1	US Equities + US Bonds	1996~2015
	2	US Equities + US Bonds + US Real Estate	1996~2015
	3	US Equities + US Bonds + Hedge Funds + Private Equity	1996~2015
	4	US Equities + US Bonds + Hedge Funds + Private Equity + US Real Estate	1996~2015
Global	5	Global Equities + Global Bonds	2001~2016
	6	Global Equities + Global Bond + Global Real Estate	2001~2016
	7	Global Equities + Global Bond + Hedge Funds + Private Equity	2001~2016
	8	Global Equities + Global Bond + Global Real Assets	2001~2016
	9	Global Equities + Global Bond + Hedge Funds + Private Equity + Global Real Estate	2001~2016
	10	Global Equities + Global Bonds + Hedge Funds + Private Equity + Global Real Assets	2001~2016
	11	Global Equities + Global Bonds + Hedge Funds + Private Equity + Global Real Estate + Other Real Assets	2001~2016

Note: Annual total returns are used for the selected asset classes

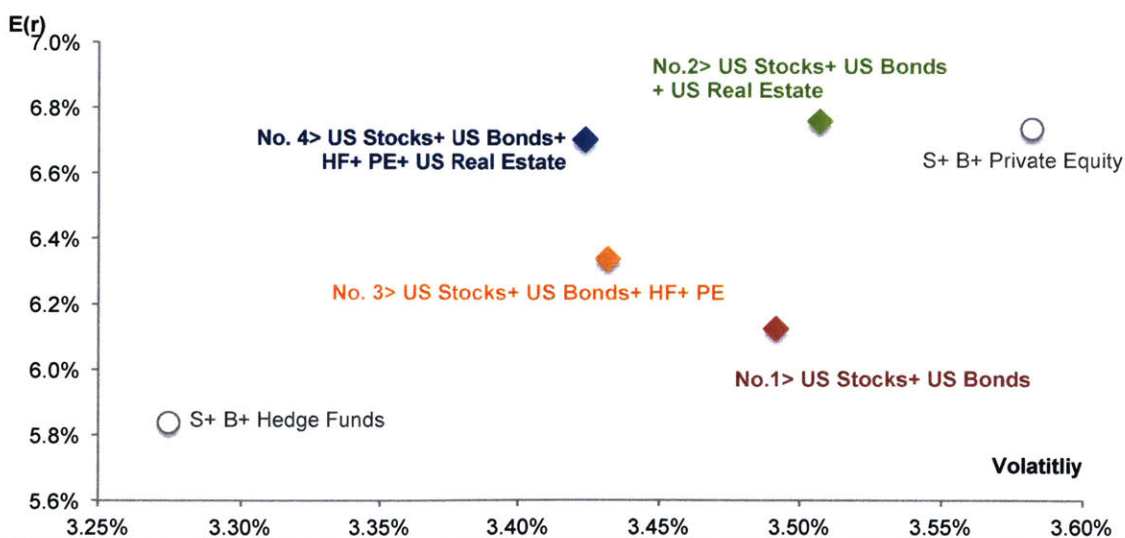
Risk Parity Portfolio Allocation: US Asset Classes

Here, we first compare the characteristics among various RPPs that have different sets of constituents. We examine if there is any different implication regarding adding alternative investments in a portfolio construction, compared to the mean-variance approach previously presented. Then, we compare the risk parity portfolios with the mean-variance efficient frontiers. It is critical for a risk parity portfolio to be not far away from the efficient frontier. In effect, we need for the RPP Sharpe Ratio to be not too far below the Sharpe Ratio of the tangent (Sharpe-maximizing) portfolio on the Mean-Variance efficient frontier.

Exhibit 62 illustrates the risk parity portfolios composed of US asset classes. Notice that the ranges of the expected return and the volatility of RPPs are much lower than the range of return and volatility we explored in the mean-variance efficient frontiers from the previous chapter. The nature of risk parity allocation is that it tends to allocate larger shares to more stable assets, and this results in less dispersion in the risk/return ranges across the various asset class choice sets examined here. In other words, from the risk parity perspective, the expansion of the choice set (increasing diversification) matters less than it seems to from the mean-variance perspective. This is intuitive, because risk parity allocation ignores the impact of differing return expectations. In other words, one of the major sources of how and why the asset class choice set would impact the risk and return of the optimal portfolio is removed from the analysis.

Although the difference is marginal, looking at how allocation to additional asset classes changes the characteristics of the portfolio still gives a valid point. By comparing RPPs that have different constituent assets, we can conclude that including all types of alternative investments on top of the traditional asset classes in a portfolio (No. 4) shows the best performance in terms of return and risk. When we compare the RPP to which only real estate is added (No. 2) and the RPP only with stock and bonds (No. 1), real estate increases both return and volatility of the portfolio. This is because the balance of RPP moves from US bonds to US real estate and US stock, asset classes that have higher volatility and correlation to each other. In contrast, when we add only hedge funds and private equity (No. 3) to the portfolio with traditional asset classes (no real estate), the portfolio's return increases but the volatility decreases. This is because private equity's high return contributes to the portfolio's increased return, compensating for the decreased weight of stocks. Regarding the decreased volatility of a portfolio, increased weight in the hedge funds decreases the overall volatility of the portfolio. The role of hedge funds and private equity in the RPPs becomes more obvious when we add each asset separately into a traditional equity + bonds portfolio as shown in grey dots Exhibit 62. Generally, the No.2 and No.4 portfolios, those that include real estate, have the highest Sharpe Ratios among the RPP optimal portfolio choice sets.

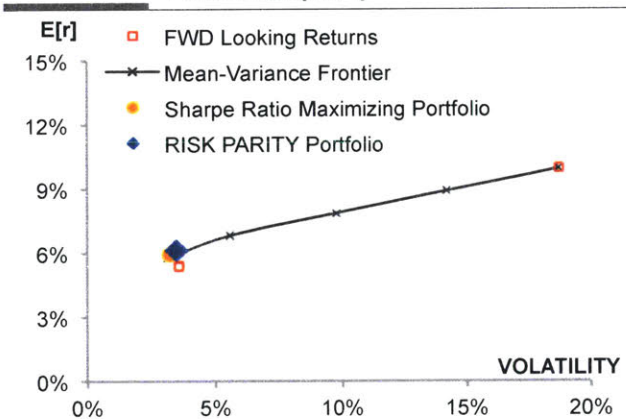
Exhibit 62 Risk Parity Portfolios Comparison: US Asset Classes



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

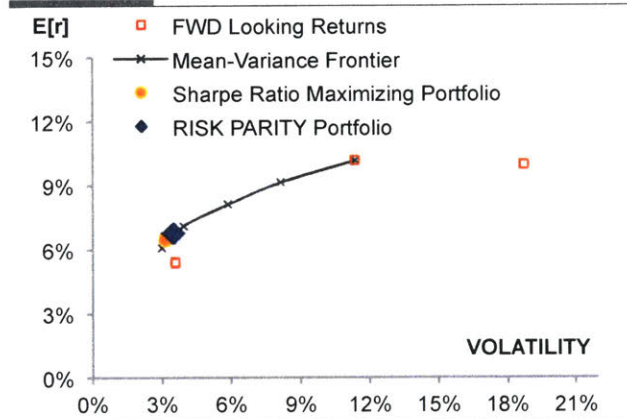
Exhibit 63 through 66 compares the mean-variance efficient frontiers with RPPs, using US asset classes. We can see that the risk parity portfolios are very close to mean-variance tangent portfolio, which is as noted is critical for the RPP to work. As risk parity portfolios make relatively large allocations to low risk asset classes, creating lower expected return of portfolios, an investor may have to use the leverage to generate a sufficient target return. If the risk parity portfolio is far away from the efficient frontier (has a significantly lower Sharpe Ratio than the tangent portfolio on the M-V efficient frontier), then applying leverage to the optimal RPP may lead to significantly poorer performance than the M-V optimal allocation. While in principle this might not matter if the reason one is considering the RPP allocation is because one does not have confidence in the mean returns inputs used in the M-V optimization, there is a comfort in finding the two approaches give similar results in terms of the Sharpe Ratio.

Exhibit 63 Risk Parity & M-V Frontier: US Stocks + US Bonds (No. 1)



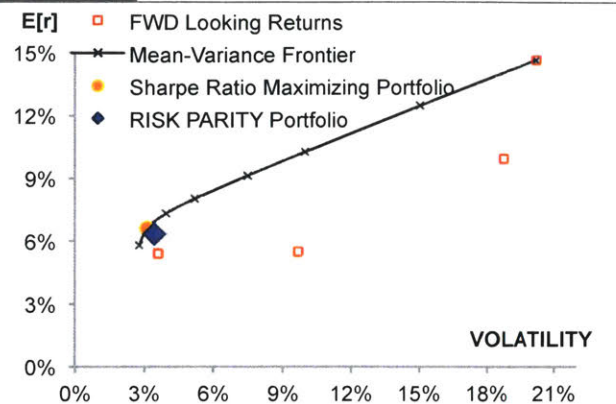
Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 64 Risk Parity & M-V Frontier: US Stocks + US Bonds + US Real Estate (No. 2)



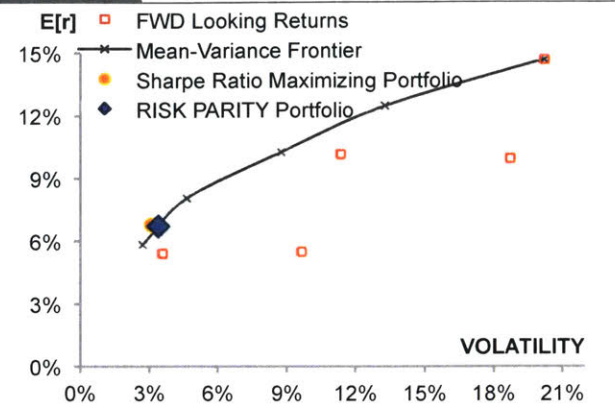
Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 65 Risk Parity & M-V Frontier: US Stocks + US Bonds + Hedge Funds + Private Equity (No. 3)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 66 Risk Parity & M-V Frontier: US Stocks + US Bonds + Hedge Funds + Private Equity + US Real Estate (No. 4)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

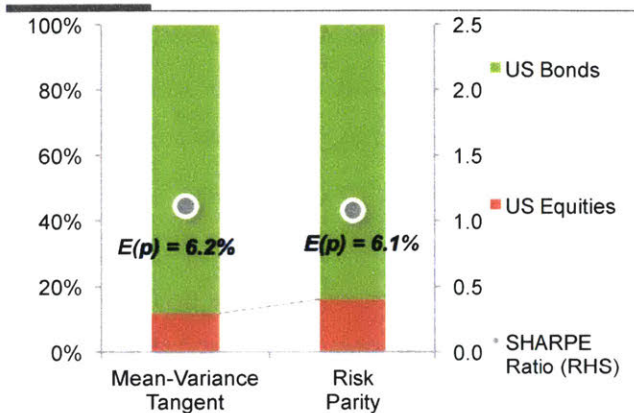
The volatility of all the modeled risk parity portfolios increases about 0.3%p when compared with mean-variance tangent portfolios, while expected returns display mixed results. Expected returns of RPPs that include both hedge funds and private equity (No. 3 and No. 4) decrease from the returns of mean-variance tangent portfolios with the same constituents. This is mainly because, in RPPs, hedge funds take weights away from private equity that was a more dominant asset class in the M-V efficient portfolios. Recall that over the history examined, private equity displayed a very high return, and hedge funds a relatively low return. These returns drive the M-V frontier results, but have no influence in the RPP. The resulting decrease in the share of private equity compared to hedge funds within the RPP compared to the M-V frontier makes the overall return of the RPPs decrease.

(Detailed comparisons of mean, standard deviation, and asset composition between RPPs and mean-variance tangent portfolios are described in the summary table at the end of this chapter, Exhibit 81 through 83)

Now we compare how the allocation share of particular asset classes varies between the mean-variance efficient frontier and the risk parity portfolio. To make such a comparison, we take

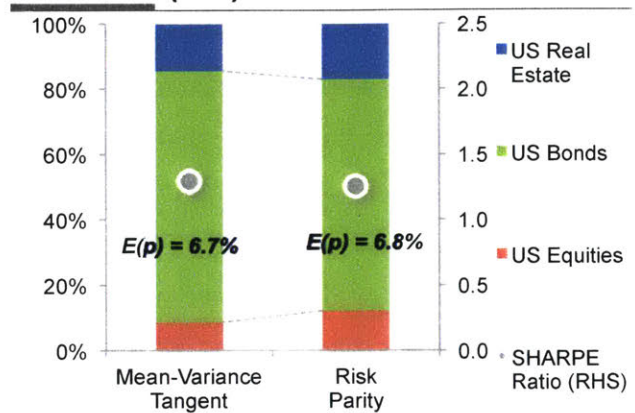
the Sharpe Ratio maximizing portfolio²⁵ (the “tangent portfolio”) from the M-V efficient frontier, to compare with the RPP.

Exhibit 67 Asset Composition Comparison: US Stocks + US Bonds (No. 1)



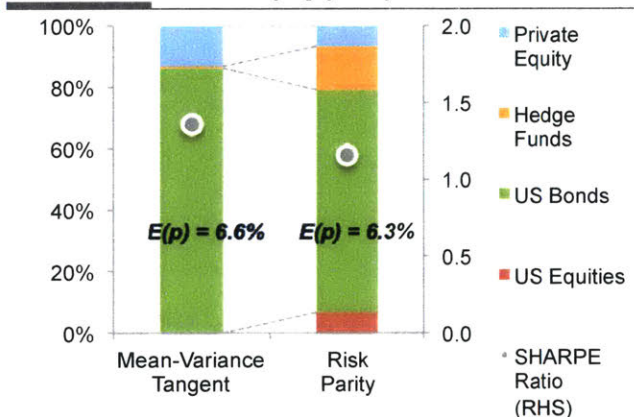
Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 68 Asset Composition Comparison: US Stocks + US Bonds + US Real Estate (No. 2)



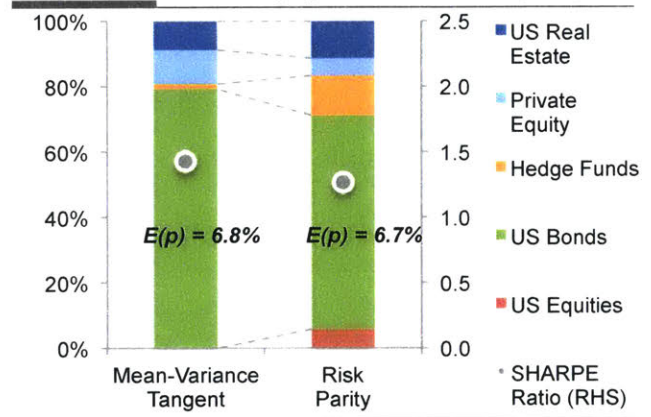
Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 69 Asset Composition Comparison: US Stocks + US Bonds + Hedge Funds + Private Equity (No. 3)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 70 Asset Composition Comparison: US Stocks + US Bonds + Hedge Funds + Private Equity + US Real Estate (No. 4)



Source: S&P 500, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

In all risk parity portfolio sets, the share of bonds decreases in comparison with tangent portfolios, which is counterintuitive as risk parity portfolios generally allocate more to less riskier assets. This is because asset allocations in efficient portfolios are less diversified over constituent.

²⁵ To construct Sharpe maximizing portfolios, we used average 3-month US Treasury Bill rate during the period, which the analysis based on.

In the mean-variance portfolios, bonds took up the majority share because it showed relatively moderate return and low volatility with negative correlation with other assets, especially replacing hedge funds. Also, private equity replaced stocks as private equity dominated stocks in terms of the return-risk feature and correlation. However, in risk parity portfolios, all asset classes have to have certain weights in order to have the same marginal risk contribution. Therefore, the shares of assets that did not have any weight or have marginal weight (in this case, such as stocks and hedge funds) increased by certain amount. In all RPPs that have allocation to one or more alternative investments (Nos. 2, 3, 4), the share of alternative investments as a whole increases compared with the mean-variance. The aggregated weights of alternative investments in the mean-variance portfolio was between 14% and 21%, whereas they increase to 21~29% in RPPs. Within alternative investments, US core real estate and hedge funds shares increase, while private equity share decreases.

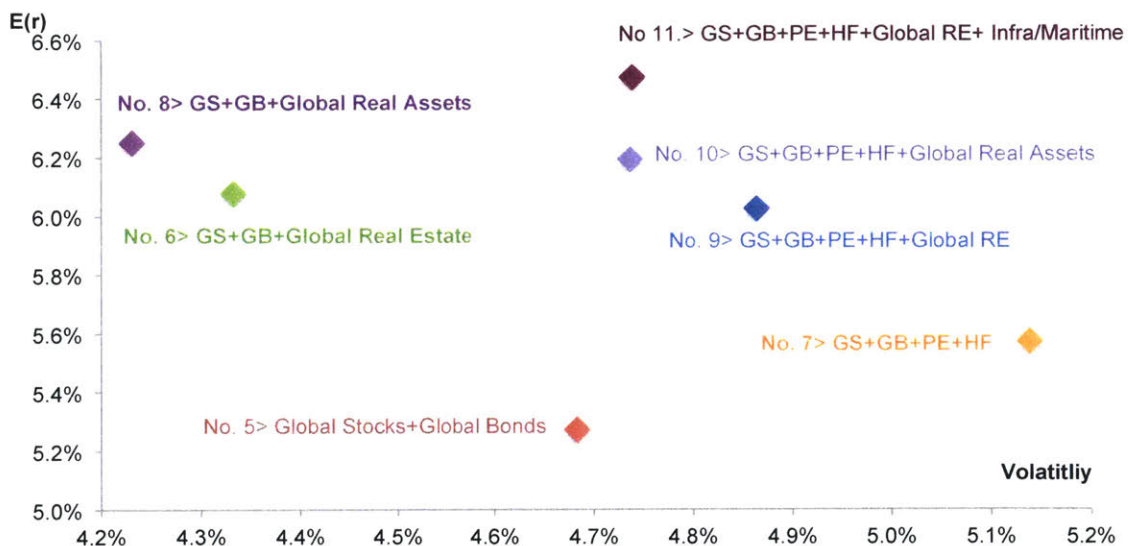
Risk Parity Portfolio Allocation: Global Asset Classes with Real Assets

Exhibit 71 illustrates the risk parity portfolios with global asset constituents. Like the analysis result from previous RPPs with US asset classes, the most diversified risk parity portfolio that consists of all types of alternative investments including real assets displays the best performance in terms of risk and return among RPPs with global asset classes. Also, as in the mean-variance optimization, the diversification within the real asset classes improves the performance of a portfolio in risk parity allocation (Nos. 9, 10, 11). When we compare the impact of adding global real estate (No.9) and adding global real assets (No.10) to a portfolio consisting of traditional assets, private equity, and hedge funds, real assets add more value to the RPP than real estate does, increasing the expected return while decreasing the volatility of the portfolio.

In a global risk parity allocation approach, adding just real estate to the stock + bond choice set (No. 6) outperforms adding private equity and hedge funds (No. 7), or adding private equity, hedge funds, and real estate all together (No. 9) to the stock + bond choice set. Adding just real estate to the stock + bond choice set reduces the volatility by 40 basis points whereas adding private equity, hedge funds, and real estate all together increases the volatility by 20 basis points. Both choices increase the return up to the similar level of 6.1% from 5.3%. When we compare the portfolio No. 7 and No. 9, global real estate increases return and reduces volatility when added to the portfolio consists of traditional assets, hedge funds, and private equity.

When we compare the RPPs consisting of global assets and RPPs consisting of US assets, the result of adding real estate to the stock + bond choice set shows the opposite direction. While adding real estate increases expected returns in both US and global portfolios, it increases volatility in US portfolio, but decreases volatility in global portfolios. This is because 1) global real estate displays lower volatility than US real estate does, and 2) the share of global real estate (21%) is larger in the global RPP than the share of US real estate (17%) in the US RPP. Additionally, the result of adding private equity and hedge funds to the stock + bond choice set shows the opposite direction between US and global RPPs. Adding private equity and hedge funds in the global RPP increases the volatility, whereas the US RPP shows the opposite result. Again, this is due to the difference in the balance of portfolios. Global RPP allocates more shares to the assets with higher volatility (here, stocks and private equity) than US RPP does.

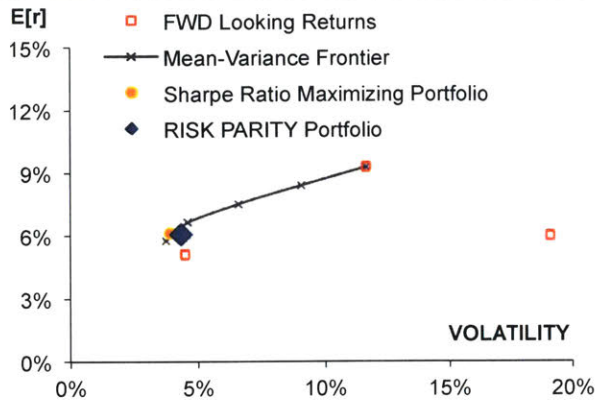
Exhibit 71 Risk Parity Portfolios Comparison: Global Asset Classes



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

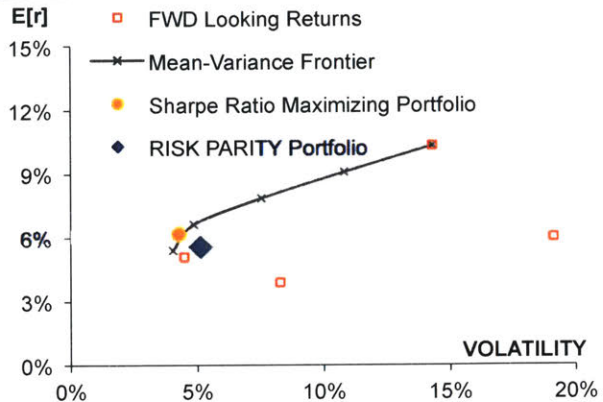
Exhibit 72 through 75 illustrates comparison between the mean-variance efficient frontiers and RPPs consisting of global asset classes. Contrary to the previous comparisons result that showed RPP being close to efficient frontier using US asset classes, there are some risk parity portfolios that are placed further away from the efficient frontier.

Exhibit 72 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + Global Real Estate (No. 6)



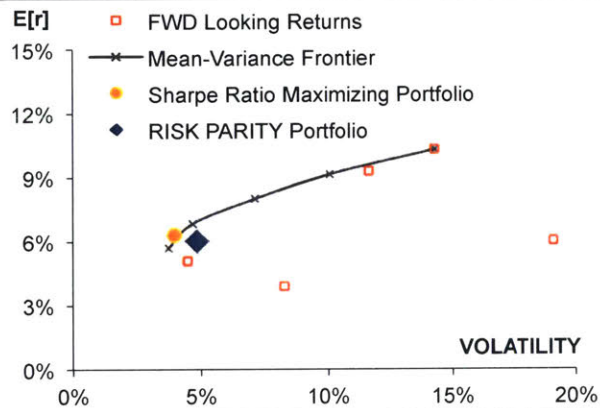
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001–2015 historical annual returns

Exhibit 73 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + Hedge Funds + Private Equity (No. 7)



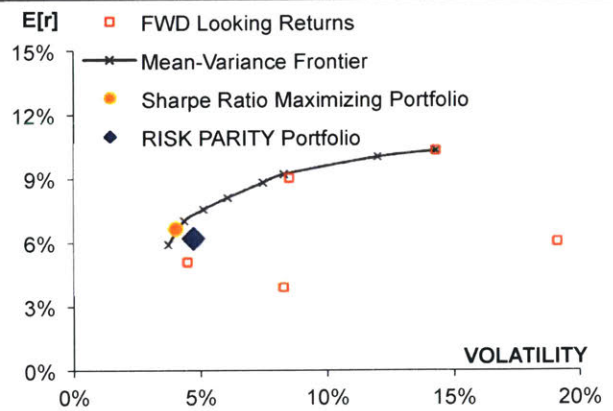
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001–2015 historical annual returns

Exhibit 74 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + HF + PE + Global Real Estate (No. 9)



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001–2015 historical annual returns

Exhibit 75 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + HF + PE + Global Real Assets (No. 10)

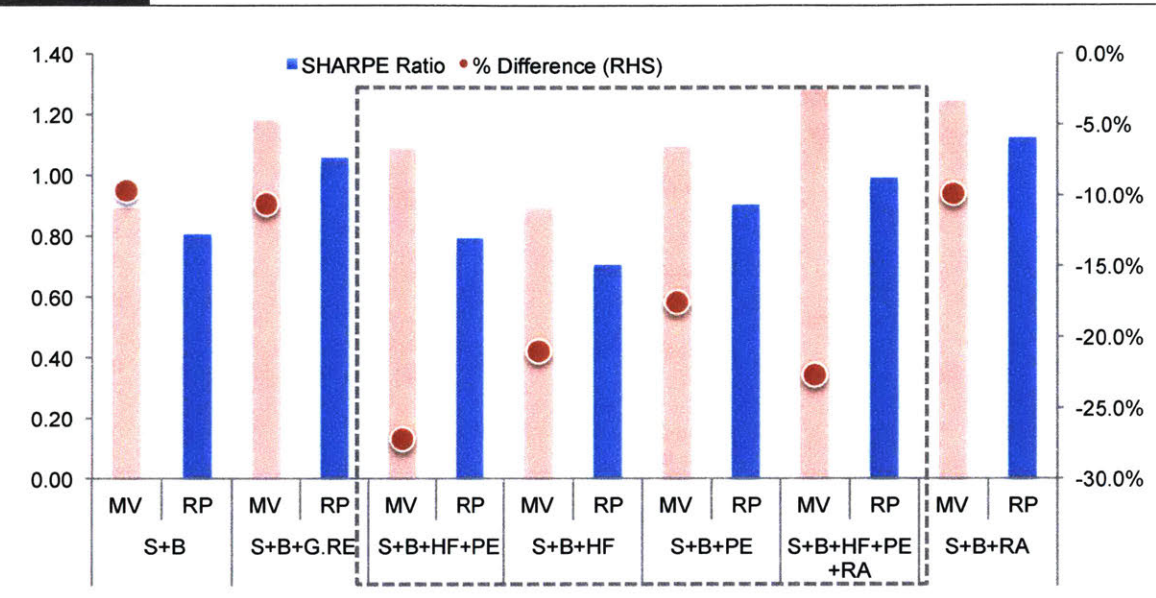


Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001–2015 historical annual returns

As we can see from the Exhibit 72 through 74, the risk parity portfolios that include private equity and hedge funds are located further away from the efficient frontiers than other portfolios do, which do not have these asset classes in constituents. Exhibit 75 shows the decrease of Sharpe ratios among some selected asset combinations. When compared with the mean-variance portfolios, the Sharpe ratio of risk parity portfolios, which include private equity, hedge funds, or both asset classes, decreased by the range of -19% through -28%, while the Sharpe ratio of other risk parity portfolio decreased by only about -7% through -11%. Compared with RPPs consisting

of US asset classes, larger increase in the aggregated share of hedge funds and private equity causes RPPs with global asset classes to have larger discrepancies from the efficient frontiers.

Exhibit 76 Sharpe Ratio Comparisons between Mean-Variance and Risk Parity



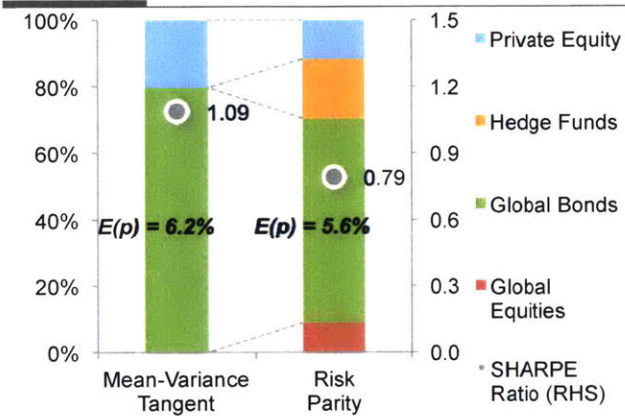
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

Exhibit 77 through 80 compares the asset compositions between mean-variance tangent portfolios and RPPs. Although the exhibits are selected to show a certain combination of assets (hedge funds, private equity, global real estate, and real assets), in all risk parity portfolios, the hedge funds share and global equities share increases. Regarding other alternative investments (i.e., private equity, real estate, and real assets), each asset class share decreases or stays the same as we construct the portfolios from mean-variance optimization to risk parity allocation.

This is because of the way risk parity is constructed. In the mean-variance allocation, bonds completely dominate over hedge funds as hedge funds are expected to display inferior performance (lower expected return and higher volatility) than bonds. This relationship also applies to stocks and private equity, private equity mostly dominating over stocks. Therefore, in the mean-variance portfolios, hedge funds share was 0 and global equities share was marginal. However, the risk parity approach has to allocate certain weights to all of the constituents. It cannot allocate 0 weight to an asset that is expected to show bad performance. Therefore, stocks and hedge funds share increases in the risk parity portfolios. Especially, the weight of hedge funds increases by

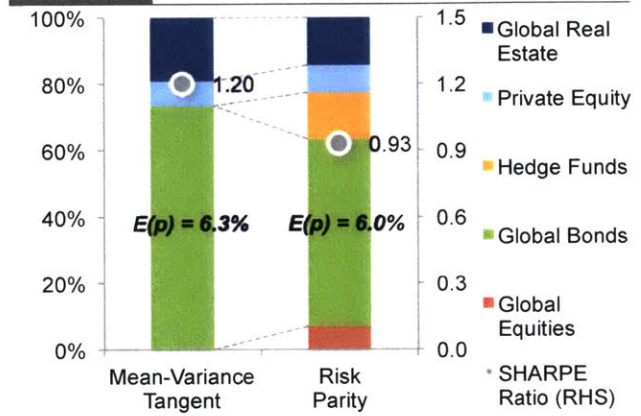
large portion (between +18%p ~ +23%p) because hedge funds expected volatility was still lower than most of other asset classes irrespective of its expected return. Increased weight of hedge funds mostly comes from bonds, and partly from private equity, real estate, and real asset classes.

Exhibit 77 Asset Composition Comparison: Global Stocks + Global Bonds + Hedge Funds + Private Equity (No. 7)



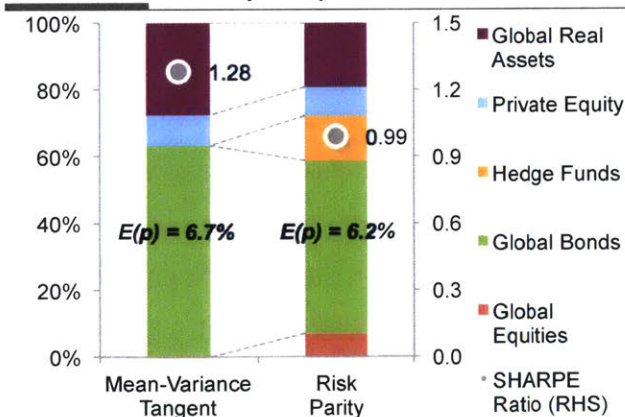
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001~2015 historical annual returns

Exhibit 78 Asset Composition Comparison: Global Stocks + Global Bonds + HF + PE + Global Real Estate (No. 9)



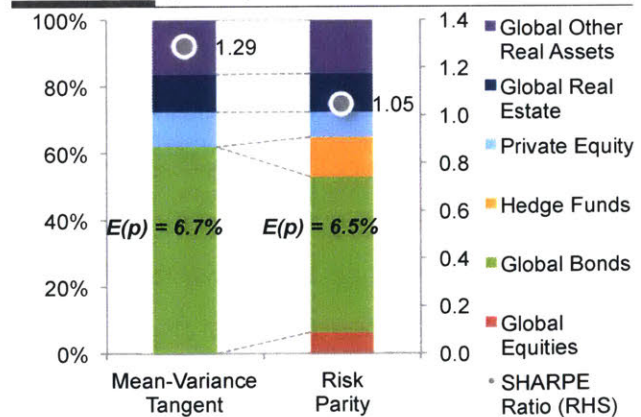
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001~2015 historical annual returns

Exhibit 79 Asset Composition Comparison: Global Stocks + Global Bonds + Hedge Funds + Private Equity + Global Real Assets (No. 10)



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001~2015 historical annual returns

Exhibit 80 Asset Composition Comparison: Global Stocks + Global Bonds + HF + PE + Global RE + Other Real Assets (No. 11)



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001~2015 historical annual returns

Exhibit 81 RPP and Mean-Variance Tangent Portfolio Comparison Summary: US Asset Classes

No.	Constituent Assets	Allocation Method	R_p	Standard Dev.	SHARPE Ratio	Weights					Weights Aggregate	
						US Equities	US Bonds	Hedge Funds	Private Equity	US Real Estate	Real Estate	Alternative Investments
1	US Equities +US Bonds	M-V Tangent	5.9%	3.2%	1.111	18%	82%					
		Risk Parity	6.1%	3.5%	1.078	16%	84%					
		<i>Difference</i>	0.2%p	0.3%p	-0.033	-2%p	2%p	-	-	-	-	-
2	US Equities +US Bonds + US Real Estate	M-V Tangent	6.5%	3.2%	1.291	9%	77%			15%	15%	
		Risk Parity	6.8%	3.5%	1.253	12%	71%			17%	17%	
		<i>Difference</i>	0.3%p	0.3%p	-0.038	4%p	-6%p	-	-	3%p	3%p	-
3	US Equities +US Bonds + Hedge Funds + Private Equity	M-V Tangent	6.6%	3.1%	1.359	0%	86%	1%	13%			14%
		Risk Parity	6.3%	3.4%	1.158	7%	72%	14%	7%			21%
		<i>Difference</i>	-0.3%p	0.3%p	-0.201	7%p	-14%p	14%p	-7%p	-	-	7%p
4	US Equities +US Bonds + Hedge Funds + Private Equity + US Real Estate	M-V Tangent	6.8%	3.1%	1.428	0%	79%	2%	10%	9%	9%	21%
		Risk Parity	6.7%	3.4%	1.268	6%	65%	12%	5%	11%	11%	29%
		<i>Difference</i>	-0.1%p	0.3%p	-0.161	6%p	-14%p	11%p	-5%p	2%p	2%p	8%p
Comparison between Hedge Funds & Private Equity												
	US Equities +US Bonds + Hedge Funds	M-V Tangent	5.7%	2.9%	1.150	7%	82%	10%				10%
		Risk Parity	5.8%	3.3%	1.061	10%	81%	19%				19%
		<i>Difference</i>	0.1%p	0.3%p	-0.088	2%p	-2%p	8%p	-	-	-	8%p
	US Equities +US Bonds + Private Equity	M-V Tangent	6.7%	3.4%	1.269	7%	82%		10%			10%
		Risk Parity	6.7%	3.6%	1.221	10%	81%		10%			10%
		<i>Difference</i>	0.0%p	0.2%p	-0.048	2%p	-2%p	-	-1%p	-	-	-1%p

Exhibit 82 RPP and Mean-Variance Tangent Portfolio Comparison Summary: Global Asset Classes

No.	Constituent Assets	Allocation Method	R_p	Standard Dev.	SHARPE Ratio	Weights								Weights Aggregate				
						Global Equities	Global Bonds	Hedge Funds	Private Equity	Global Real Estate	Global Real Assets	US Real Estate	Int'l Real Estate	Global Other Real Assets	Real Estate	Real Assets	Alternative Investments	
5	Global Equities + Global Bonds	M-V Tangent	5.2%	4.2%	0.891	9%	91%											
		Risk Parity	5.3%	4.7%	0.805	19%	81%											
		<i>Difference</i>	0.1%p	0.5%p	-0.086	10%p	-10%p	-	-	-	-	-	-	-	-	-	-	-
6	Global Equities + Global Bond + Global Real Estate	M-V Tangent	6.1%	3.9%	1.182	2%	74%			24%					24%		24%	
		Risk Parity	6.1%	4.3%	1.057	12%	67%			21%					21%		21%	
		<i>Difference</i>	-0.0%p	0.4%p	-0.126	10%p	-7%p	-	-	-3%p	-	-	-	-	-3%p	-	-3%p	
7	Global Equities + Global Bond + Hedge Funds + Private Equity	M-V Tangent	6.2%	4.3%	1.089	0%	80%	0%	20%									20%
		Risk Parity	5.6%	5.1%	0.792	9%	62%	18%	12%									30%
		<i>Difference</i>	-0.6%p	0.9%p	-0.297	9%p	-18%p	18%p	-9%p	-	-	-	-	-	-	-	-	9%p
8	Global Equities + Global Bond + Real Assets	M-V Tangent	6.4%	4.0%	1.246	3%	63%			34%					34%		34%	
		Risk Parity	6.3%	4.2%	1.123	13%	61%			27%					27%		27%	
		<i>Difference</i>	-0.2%p	0.3%p	-0.123	10%p	-3%p	-	-	-	-7%p	-	-	-	-	-7%p		-7%p
9	Global Equities + Global Bond + Hedge Funds + Private Equity + Global Real Estate	M-V Tangent	6.3%	4.0%	1.202	0%	73%	0%	7%	19%					19%		27%	
		Risk Parity	6.0%	4.9%	0.930	7%	56%	14%	8%	14%					14%		37%	
		<i>Difference</i>	-0.3%p	0.9%p	-0.272	7%p	-17%p	14%p	1%p	-5%p	-	-	-	-	-5%p	-	10%p	
10	Global Equities + Global Bonds + Hedge Funds + Private Equity + Global Real Assets	M-V Tangent	6.7%	4.0%	1.282	0%	63%	0%	9%	28%					28%		37%	
		Risk Parity	6.2%	4.7%	0.991	7%	52%	13%	9%	19%					19%		41%	
		<i>Difference</i>	-0.5%p	0.7%p	-0.291	7%p	-12%p	13%p	-1%p	-	-8%p	-	-	-	-	-8%p		4%p
11	Global Equities + Global Bonds + HF + PE + Global Real Estate + Other Real Assets	M-V Tangent	6.7%	4.1%	1.290	0%	62%	0%	10%	11%				16%	11%	28%	38%	
		Risk Parity	6.5%	4.7%	1.049	6%	47%	12%	8%	11%				16%	11%	28%	47%	
		<i>Difference</i>	-0.2%p	0.7%p	-0.241	6%p	-15%p	12%p	-3%p	0%p	-	-	-	-0%p	0%p	0%p	9%p	

Exhibit 83 RPP and Mean-Variance Tangent Portfolio Comparison Summary: Global Asset Classes (Continued)

No.	Constituent Assets	Allocation Method	R_p	Standard Dev.	SHARPE Ratio	Weights								Weights Aggregate		
						Global Equities	Global Bonds	Hedge Funds	Private Equity	Global Real Estate	Global Real Assets	US Real Estate	Int'l Real Estate	Global Other Real Assets	Real Estate	Real Assets
Comparison within Real Estate																
6	Global Equities +Global Bond +Global Real Estate	M-V Tangent	6.1%	3.9%	1.182	2%	74%			24%					24%	24%
		Risk Parity	6.1%	4.3%	1.057	12%	67%			21%					21%	21%
		<i>Difference</i>	<i>-0.0%p</i>	<i>0.4%p</i>	<i>-0.126</i>	<i>10%p</i>	<i>-7%p</i>	<i>-</i>	<i>-</i>	<i>-3%p</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-3%p</i>	<i>-</i>
	Global Equities +Global Bond +US Real Estate	M-V Tangent	5.8%	3.9%	1.122	5%	76%				19%				19%	19%
		Risk Parity	6.0%	4.3%	1.029	13%	67%				20%				20%	20%
		<i>Difference</i>	<i>0.1%p</i>	<i>0.4%p</i>	<i>-0.093</i>	<i>8%p</i>	<i>-9%p</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>1%p</i>	<i>-</i>	<i>-</i>	<i>1%p</i>	<i>-</i>
	Global Equities +Global Bond +US Real Estate + International Real estate	M-V Tangent	6.3%	5.6%	0.850	5%	69%				11%	16%			26%	26%
		Risk Parity	6.4%	6.2%	0.791	14%	56%				15%	14%			29%	29%
		<i>Difference</i>	<i>0.2%p</i>	<i>0.6%p</i>	<i>-0.059</i>	<i>10%p</i>	<i>-12%p</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>4%p</i>	<i>-2%p</i>	<i>-</i>	<i>3%p</i>	<i>-</i>
Comparison between Hedge Funds & Private Equity																
7	Global Equities +Global Bond + Hedge Funds + Private Equity	M-V Tangent	6.2%	4.3%	1.089	0%	80%	0%	20%							20%
		Risk Parity	5.6%	5.1%	0.792	9%	62%	18%	12%							30%
		<i>Difference</i>	<i>-0.6%p</i>	<i>0.9%p</i>	<i>-0.297</i>	<i>9%p</i>	<i>-18%p</i>	<i>18%p</i>	<i>-9%p</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>9%p</i>
	Global Equities +Global Bonds + Hedge Funds	M-V Tangent	5.2%	4.2%	0.891	9%	91%	0%								0%
		Risk Parity	4.9%	4.9%	0.704	12%	65%	23%								23%
		<i>Difference</i>	<i>-0.3%p</i>	<i>0.7%p</i>	<i>-0.187</i>	<i>3%p</i>	<i>-26%p</i>	<i>23%p</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>23%p</i>
	Global Equities +Global Bonds + Private Equity	M-V Tangent	6.2%	4.3%	1.094	0%	80%		20%							20%
		Risk Parity	6.0%	5.0%	0.902	12%	73%		15%							15%
		<i>Difference</i>	<i>-0.1%p</i>	<i>0.7%p</i>	<i>-0.192</i>	<i>12%p</i>	<i>-7%p</i>	<i>-</i>	<i>-5%p</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-</i>	<i>-5%p</i>

In this chapter we summarize the “big picture”, the main points from the preceding study. Then we also mention some broader considerations the reader may wish to keep in mind.

5.1 The “Big Picture”

This thesis set out to address the question of whether, or to what extent, real assets can improve the performance of institutional investment portfolio, in particular in comparison with the private equity and hedge fund classes of alternative investments that have heretofore been more popular with institutional investors.

At a broad level, the findings in this thesis clearly indicate that adding alternative asset classes generally offers attractive diversification opportunities to a portfolio consisting of only traditional asset classes – stocks and bonds. However, the role of the various different types of alternative investments varies widely. This poses some challenges, but it also brings the potential to improve the performance of institutional investment portfolios.

To address the title question of this thesis, we find that real assets and the private equity & hedge fund type of alternative assets both enhance the portfolio. The investor is leaving potential performance improvements on the table if they neglect real assets, or if they neglect private equity and hedge funds.

The low correlation between real assets, and other asset classes, and the relatively better risk-return efficiency of the real assets, are the source of attractive diversification benefits from adding real asset classes into the portfolio. We see this whether we treat real assets either as an aggregated single asset class or as several real asset classes separately, allowing diversification within them. Sharpe maximizing portfolios with expected returns ranging from 5.2% to 6.7% suggest that investors should allocate approximately between 21% and 34% to core real assets including real estate, depending on the constituent asset classes within the portfolio. This is much higher than current industry practice, where the average current allocations to real estate and infrastructure are 8.5% and 4.3% respectively as of 2015.

When we compare the role of real assets with other alternative investments – private equity and hedge funds, it is hard to determine whether real assets dominate over private equity because each asset class has a different role within the portfolio, in particular as a function of the investor's risk/return appetite. For a range of conservative to moderate return targets, adding real assets on top of the traditional portfolio consisting of stocks and bonds dominates over adding private equity. However, an investor with a more aggressive return objective, more than about 10% based on the historical period of 2001 through 2015, would need to employ private equity rather than real estate or real assets, as the additions to the base portfolio of stocks and bonds. This is because private equity displays the highest expected return among the possible constituent asset classes that we examined based on the historical returns of the selected period. Although private equity increases the volatility of a portfolio, it works sufficiently as a return enhancer, easily making its way onto the efficient frontier at the higher return targets. However, it should be noted that during the historical period used in the thesis (1996~2015 for US assets and 2001~2015 for global assets), the private equity industry experienced explosive growth, which might not be typical in the future. Therefore, one should interpret our results with this in mind.

Within the traditional asset classes, bonds have a stable role in all the possible pairing tests for a broad range of conservative to aggressive return targets. This is consistent with industry practice, as bonds' share in institutional portfolios has remained stable between 24% and 28% since 2000, according to PREA. On the other hand, the optimal allocation to equities is very marginal in the mean-variance optimization portfolios for a wide range of return targets. When alternative assets are introduced into the portfolio choice set, the optimal share of stock decreases to less than 9%. This limited allocation to stocks is more prominent when private equity is included in the portfolio. Based on the private equity return index, private equity shows a higher Sharpe Ratio than stocks, while being highly correlated with stocks, resulting in a dominant role for PE in the upper range of the efficient frontier. But such a large role for PE is contrary to actual industry practice, where public equities still take up the majority share in institutional portfolios. The wide discrepancy between the analysis result and reality may reflect legitimate concerns about liquidity and transaction/management costs (which are not explicitly included in this thesis). There may also be a concern about the reliability and meaningfulness of the historical returns observed in the PE benchmark index, given that liquid transaction prices for PE are not observable.

To some extent, some of these issues with private equity may also lie behind what we find about the roles of hedge funds and private equity as two sub-classes within the traditional main alternative investments category. Private equity plays a greater role than hedge funds when added to the stock/bond portfolios. As noted, private equity works as a return enhancement in a portfolio, especially when investors expect aggressive return target. This is because private equity has a high correlation with stocks but yields higher expected return and lower (than global equities) or slightly higher (than US equities) volatility than equities. The apparent better risk-return efficiency of private equity compared to stocks makes this asset class dominate over public equities. On the other hand, in contrast to the common notion that hedge funds effectively hedge risks, thereby reducing volatility and securing a consistent rate of return, hedge funds play a very limited diversification role in both US domestic portfolios and global asset portfolios, based on the historical index performance that we use in this study. This result is similar in both the mean-variance optimization and risk parity asset allocation method. This is because hedge funds showed similar return with bonds but higher volatility. Additionally, bonds are negatively correlated with most other asset classes while hedge funds showed slightly higher correlations with other assets. Again though, it should be noted that these characteristics of hedge funds may be idiosyncratic to the historical period for which we had data.

Between real estate and real assets (which include real estate and other real asset classes), the real asset class is allocated a larger share in a portfolio than real estate alone. This is because the expected return of real assets is higher than that of global core real estate and the correlations between real assets and other constituent asset classes are slightly lower than the correlations between real estate and others. This suggests that adding the real asset class as a whole improves the performance of a portfolio regarding risk and return more so than only adding real estate alone. Regarding the regional diversification within real estate, an investor considering allocating to real estate should think about the possibility of diversifying regionally within real estate. A portfolio with regionally diversified real estate yields higher returns than the portfolio with only US real estate.

Similar to the result in the mean-variance approach, risk parity allocation also shows that adding alternative investments to the portfolio consisting of stocks and bonds help improve risk-return efficiency of the portfolio. Within alternative investment asset classes, global real assets and real estate performs better role in diversification effect than hedge funds and private equity do. This

is more prominent in portfolios consisting of global assets. In most of RPPs that have allocation to one or more alternative investments, the share of alternative investments as a whole increases compared with the mean-variance. Although risk parity asset allocation provides robustness because investors do not have to make assumptions about future expected (mean) return, this approach often requires investors to lever up the RPPs to match their expected return target. Investors should note that the cost of using leverage is not included in the RPPs. Therefore, they should consider the cost of leverage and the risks associated with it, especially in today's environment where interest rate is expected to rise.

5.2 Closing Note

We would be remiss without mentioning a final closing note. The real estate asset classes defined in this study reflect the index weighting across all real estate sectors and locations. This point also applies to all other alternative asset classes we covered in this thesis as they are private assets. In general, these indices representing private asset classes in which “whole assets” are traded privately, cannot be exactly replicated by portfolio managers. Therefore, the optimized portfolio weights represent the point of view of an “average” investor. Actual portfolios in the real world cannot invest in the benchmark indices used to represent real estate (and, for that matter, the other alternative asset classes as well). This is in contrast to the indices representing stocks and bonds, which can effectively be replicated in the real world portfolio. This is a point of uncertainty (akin to “basis risk” or “tracking error”) that alternative investments must contend with.

Also, various features of alternative investments that are distinguished from traditional asset classes may cause institutional investors refrain from investing in alternative investments despite their apparent superior risk – return efficiency as demonstrated in this thesis. These differences include: a relatively long lock-up period, assets being highly illiquid, higher transaction/management costs, and less efficient markets. Topics regarding these so-called “pain points” of investing in alternative investments in the real world will be interesting to study in future research. Alas, they must remain beyond the scope of the current thesis. However, it is the author's hope that the present thesis has provided an intriguing starting point for such asset management research, by exploring the potential portfolio management strategies using various alternative asset classes based on the available historical performance data.

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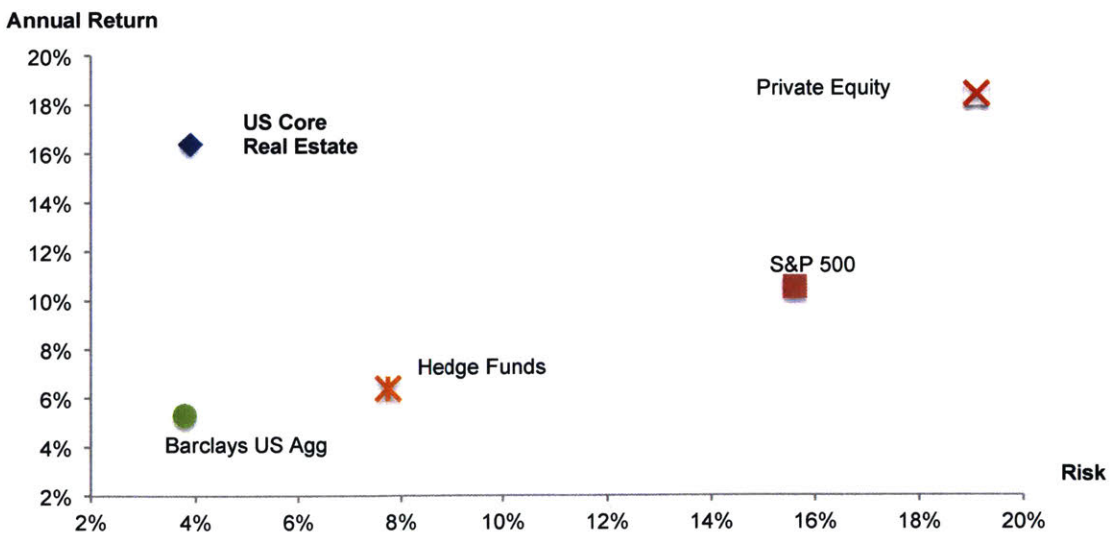
APPENDIX

US Asset Classes excluding 2008~2009

We separate the time period of recession (2008 and 2009) to explore the impact of the last recession on risk and returns of the selected asset classes.

If we take out the data from 2008 to 2009, US core real estate performed much better than other asset classes including traditional asset classes, private Equities, and hedge funds. Excluding the period of 2008 and 2009, US core real estate shows extremely low volatility that is in line with the volatility of US bonds. At the same time, the mean return of US core real estate is higher than US and almost as high as the return of private equity; for US core real estate, the volatility decreases from 11.4% to 3.9% and the mean increases from 9.5% to 16.4%. This is because real estate assets were the hardest hit from the last recession of 2008 considering that the last recession was induced by real estate and the impact was dispersed throughout the financial market, contributing to increased overall volatility of other assets. Hedge funds show another interesting result. If we exclude the data from the last recession, hedge funds outperform US bonds, which is the opposite result when we include the whole period.

Exhibit 84 Risk and Return of Selected US Assets (1996~2015 excluding 2008 ~2009)



Source: MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, JLL, Bloomberg, Datastream

Then, we review the correlation of the whole period excluding 2008 and 2009.

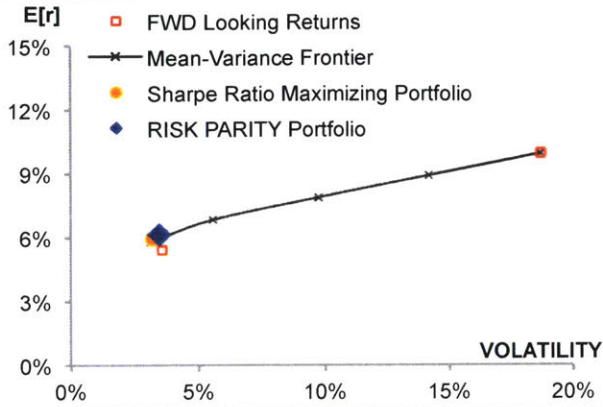
Exhibit 85 Correlations among Selected US Asset Classes (1996~2015 excluding 2008 & 2009)

1996~2015 excluding 2008~2009	S&P 500	Barclays US Aggregate	Hedge Funds	Private Equity	US Real Estate
S&P 500	1.00				
Barclays US Agg.	0.46	1.00			
Hedge Funds	0.42	0.49	1.00		
Private Equity	0.75	0.49	0.57	1.00	
US Real Estate	0.32	-0.20	0.00	0.44	1.00

Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream

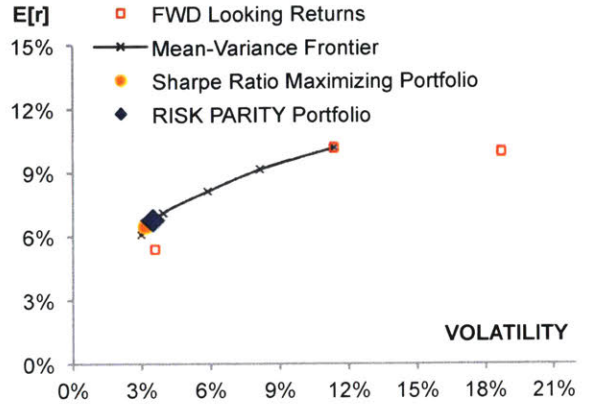
Risk Parity & M-V Frontier: US Asset Classes

Exhibit 86 Risk Parity & M-V Frontier: US Stocks + US Bonds (No. 1)



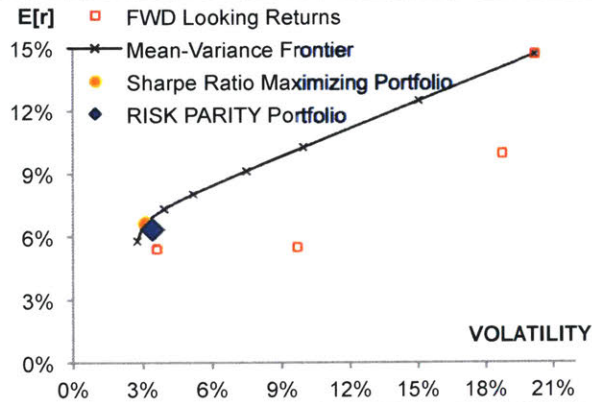
Source: S&P 500, Barclays, NCREIF, HFR, Prejin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 87 Risk Parity & M-V Frontier: US Stocks + US Bonds + US Real Estate (No. 2)



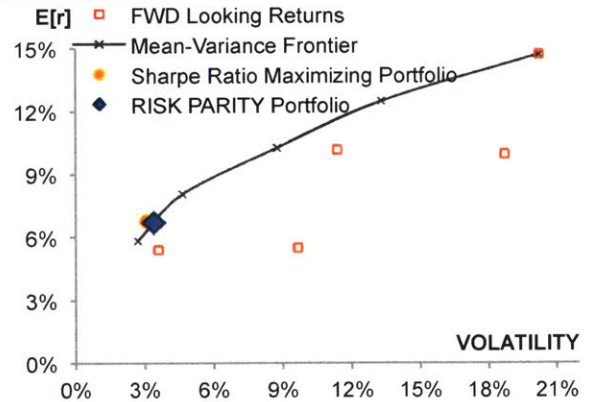
Source: S&P 500, Barclays, NCREIF, HFR, Prejin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

Exhibit 88 Risk Parity & M-V Frontier: US Stocks + US Bonds + Hedge Funds + Private Equity (No. 3)



Source: S&P 500, Barclays, NCREIF, HFR, Prejin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

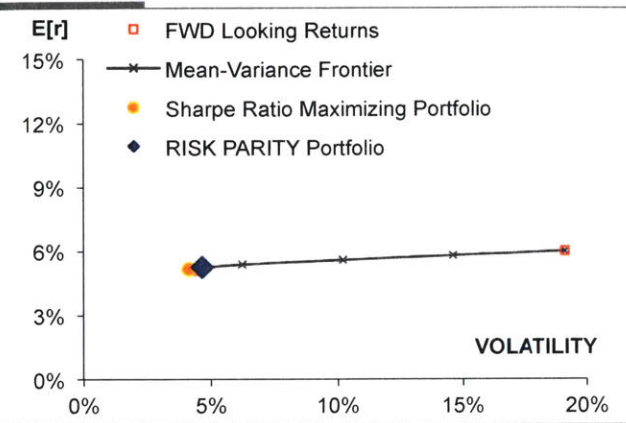
Exhibit 89 Risk Parity & M-V Frontier: US Stocks + US Bonds + Hedge Funds + Private Equity + US Real Estate (No. 4)



Source: S&P 500, Barclays, NCREIF, HFR, Prejin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 1996-2015 historical annual returns

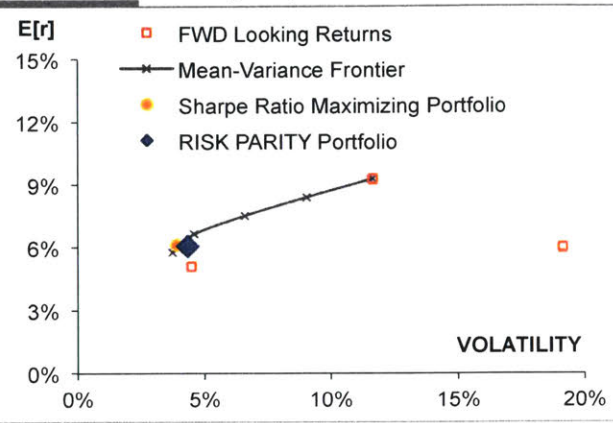
Risk Parity & M-V Frontier: Global Asset Classes

Exhibit 90 Risk Parity & M-V Frontier: Global Stocks + Global Bonds (No. 5)



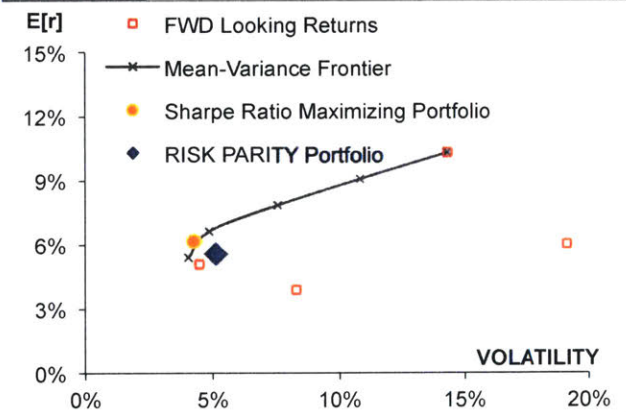
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

Exhibit 91 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + Global Real Estate (No. 6)



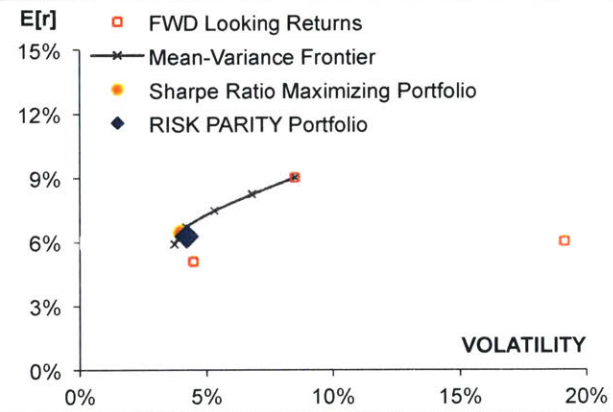
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

Exhibit 92 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + Hedge Funds + Private Equity (No. 7)



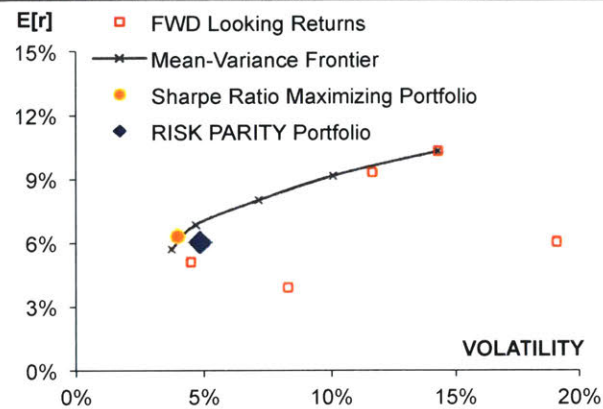
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

Exhibit 93 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + Global Real Assets (No. 8)



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

Exhibit 94 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + HF + PE + Global Real Estate (No. 9)



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

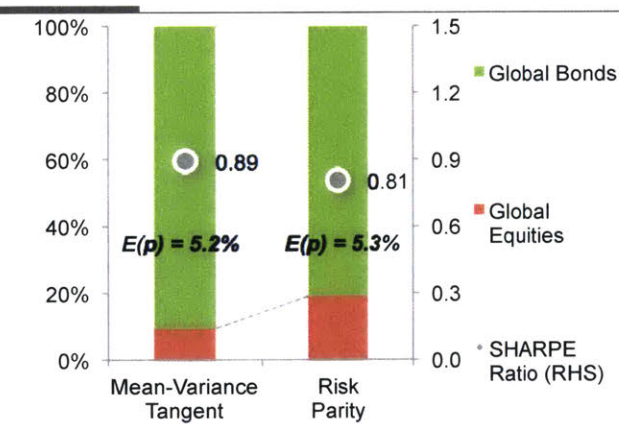
Exhibit 95 Risk Parity & M-V Frontier: Global Stocks + Global Bonds + HF + PE + Global Real Assets (No. 10)



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

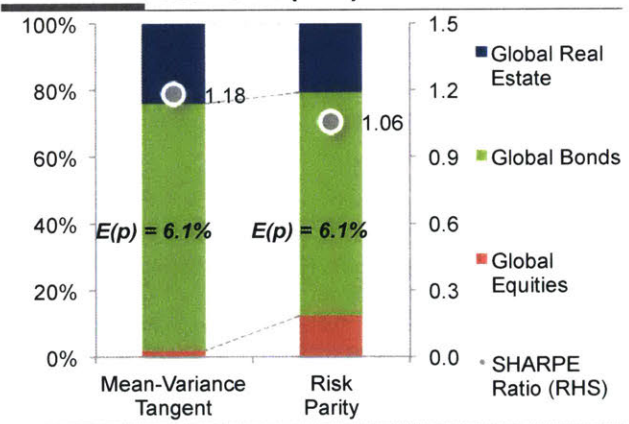
Risk Parity & M-V Asset Composition Comparisons: Global Asset Classes

Exhibit 96 Asset Composition Comparison: Global Stocks + Global Bonds (No. 5)



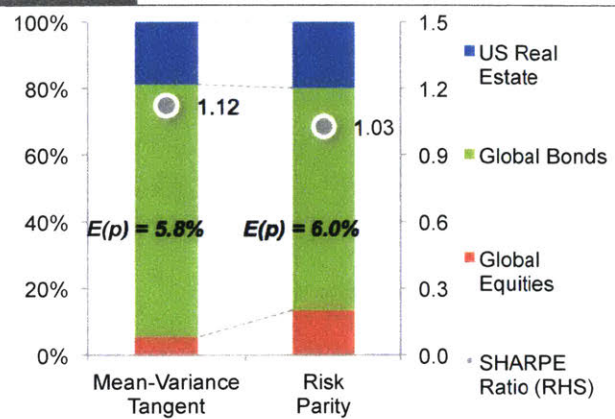
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

Exhibit 97 Asset Composition Comparison: Global Stocks + Global Bonds + Global Real Estate (No. 6)



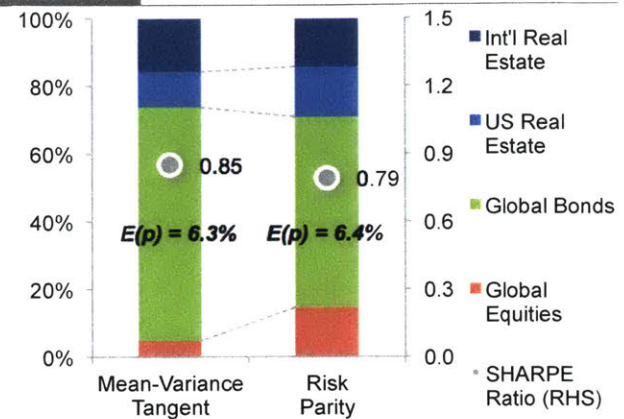
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
 Note: Based on 2001-2015 historical annual returns

**Exhibit 98 Asset Composition Comparison:
Global Stocks + Global Bonds + US
Real Estate**



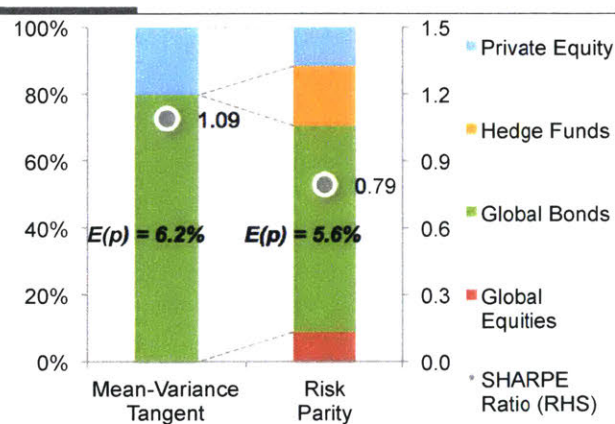
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
Note: Based on 2001–2015 historical annual returns

**Exhibit 99 Asset Composition Comparison:
Global Stocks + Global Bonds + US
Real Estate + International Real Estate**



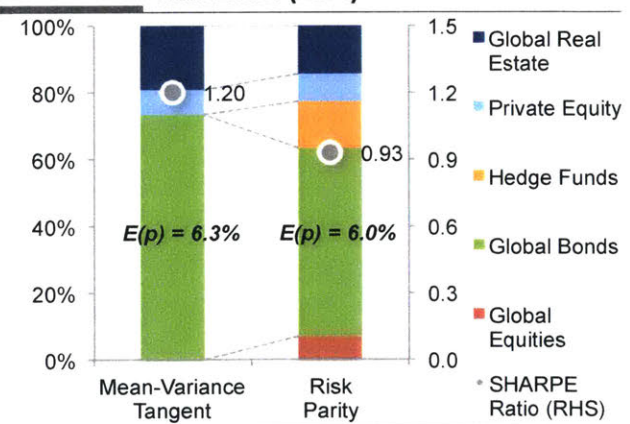
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
Note: Based on 2001–2015 historical annual returns

**Exhibit 100 Asset Composition Comparison:
Global Stocks + Global Bonds + Hedge
Funds + Private Equity (No. 7)**



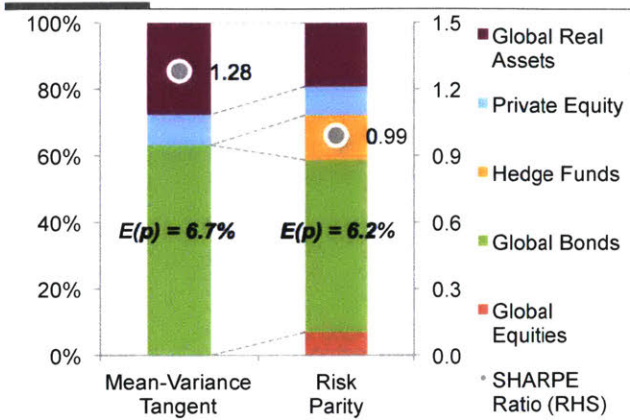
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
Note: Based on 2001–2015 historical annual returns

**Exhibit 101 Asset Composition Comparison:
Global Stocks + Global Bonds +
Hedge Funds + Private Equity + Global
Real Estate (No. 9)**



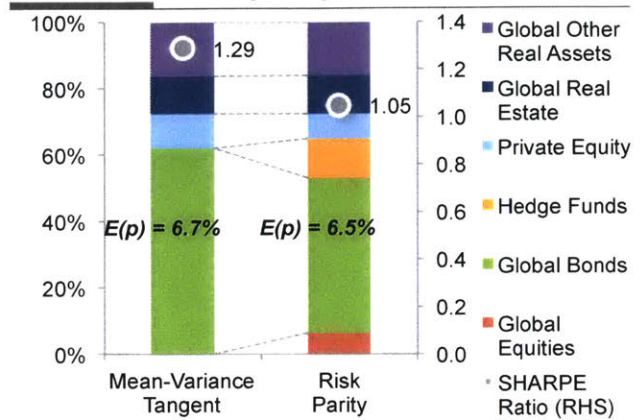
Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Preqin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
Note: Based on 2001–2015 historical annual returns

**Exhibit 102 Asset Composition Comparison:
Global Stocks + Global Bonds + Hedge
Funds + Private Equity + Global Real
Assets (No. 10)**



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Prequin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
Note: Based on 2001–2015 historical annual returns

**Exhibit 103 Asset Composition Comparison:
Global Stocks + Global Bonds +
Hedge Funds + Private Equity + Global
Real Estate + Other Global Real
Assets (No. 11)**



Source: S&P 500, MSCI, Barclays, NCREIF, HFR, Prequin, Thomson Reuters, Cambridge Associates, Bloomberg, Datastream
Note: Based on 2001–2015 historical annual returns