An Introduction to the Airline Recovery Problem

by

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ABSTRACT

Airlines are constantly faced with operational problems which develop from severe weather patterns and unexpected aircraft or personnel failures. A significant amount of computational time and effort is invested in developing an airline's operational schedule which is impacted by these irregular events. Over the last decade, airlines have become more concerned with developing an optimal flight schedule, with very little slack left in the system to accommodate for any form of variation from the optimal solution. Substantial research has been conducted on the problem of fleet assignment and aircraft routing. However, very little research has been done on the problem of addressing the impact of irregular operations, and developing potential decision systems which could aid in aircraft re-scheduling.

The primary goal of this research project has been to develop algorithms, procedures and new methodologies to be used to reschedule planned activities (flights) in the event of irregular operations in large scale scheduled transportation systems, such as airline networks. A mathematical formulation of the Airline Recovery Problem is given, along with a decision framework which is used to develop efficient solution methodologies. These heuristic procedures and algorithms have been developed for use in a comprehensive real-time decision support systems (DSS), incorporating several aspects of the tactical operations of the transport system. These include yield management, vehicle routing, maintenance scheduling, and crew scheduling. The heuristic procedures developed will enable the carrier to recover from an irregular operation and maintain an efficient schedule for the remainder of a given resolution horizon.

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Chapter 1

Introduction

1.1 Overview

Airlines are constantly faced with operational problems which develop from severe weather patterns and unexpected aircraft or personnel failures. A significant amount of computational time and effort is invested in developing an airline's operational schedule which is impacted by these irregular events. Over the last decade, airlines have become more concerned with developing an optimal flight schedule, with very little slack left in the system to accommodate for any form of variation from the optimal solution. Substantial research has been conducted on the problem of fleet assignment and aircraft routing. However, very little research has been done on the problem of addressing the impact of irregular operations, and developing potential decision support systems which could aid in aircraft re-scheduling.

The primary objective of this research was to develop algorithms, procedures and new methodologies to be used to reschedule planned activities (flights) in the event of irregular operations in large scale scheduled transportation systems, such as airline networks. These heuristic procedures and algorithms would be developed for use in a comprehensive real-time decision support systems (DSS), incorporating several aspects of the tactical operations of the transport system. These include yield management, vehicle routing, maintenance scheduling, and crew scheduling. The heuristic procedures will enable the carrier to recover from an irregular operation and maintain an efficient schedule for the remainder of a given rotation period. Having been exposed to issues relevant to the problem of irregular operations, the author is confident that these procedures when developed and implemented, will have a substantial impact on future transportation system operations.

The development of an airline's published flight schedule is one of the most important aspects of its strategic planning. Significant efforts are made to ensure that the airline is efficiently making use of its resources in order to maximize its prescribed utility, such as maximize revenue or profits. The overall scheduling process depends on an extensive array of information, and it starts several months ahead of the actual operation of a given flight. The process of deciding which aircraft type is assigned to a given flight is called the fleet assignment problem, and the process of assigning a specific tail number to a given flight is known as the aircraft rotation/routing problem.

Throughout the course of daily operations, the airline is often faced with situations that may result in substantial variations from its planned operations, and is required to make real-time decisions that can have a significant impact on the overall operations of the airline. These irregular operations impact all aspects of the airline's operations, but are most detrimental to the schedule of limited resources such as aircraft and flight crews. The cause of the irregularity may range from severe weather to aircraft breakdowns. yet it will result in the need to reschedule flights and reroute aircraft. These actions will lead to potential flight delays and cancellations. Regardless of the cause of the irregularity, its impact will have an effect on the aircraft maintenance routing decision process.

The ability of the airline to recover from such unexpected irregularities will depend on its ability to effectively make use of operational information that is readily available throughout the airline's computer databases. It is anticipated that the recovery of aircraft will dictate a combined decision making process, wherein a hybrid of the traditional fleet assignment and the aircraft routing problem is solved in an effort to resolve the irregularities. The decision maker will in effect be trying to assign aircraft to the most valuable flights, while achieving maintenance routing requirements of all operational aircraft.

1.2 Motivation

Currently, the resolution of flight irregularities is primarily a manually driven decision process, wherein the airline controller assesses all the available information, and makes an informed decision about the airline's operations. In general, this decision process is sufficient to solve the existing irregularity; however, it may have a significant impact on other activities which were not considered by the controller. The ability of a computer based decision support system to consider all relevant activities should have great benefit to the overall resolution process. It is important to underscore the role of the airline controller in the decision making process, as it is only with extensive experience in the Airline Operations Control Center, that the controller can effectively deal with resolving irregularities.

For a typical airline, approximately ten percent (10%) of its scheduled revenue flights are affected by irregularities, with a large percentage being caused by severe weather conditions. In an article published in the New York Times [January 21, 1997], it was noted that the financial impact of irregularities on the daily operations on a single major US domestic carrier can exceed $440 million per annum in lost revenue, crew overtime pay, and passenger hospitality costs. During the late spring of 1995, a severe hailstorm over Dallas-Fort Worth resulted in the damage of nearly one hundred aircraft parked at the airport terminals [Aviation Week; May 8, 1995]. In fact, eighty of these damaged aircraft belonged to American Airlines, accounting for nearly nine percent of its total fleet. In the immediate aftermath of this irregularity, American had to cancel up-to ten percent of its scheduled flights, and needed almost an entire month to return to normal operations.

In January of 1996, it was estimated that a single snow storm "The Blizzard of '96" costs the US airline industry between $50 - $100 million [Aviation Week; January 15, 1996]. On a daily basis, airlines have to cope with reduced fleet size, as a result of aircraft breakdowns, as well as external factors such as ATC flow management restrictions, which affect the planned operations of the carrier. It is important to point out that the causes of airline irregularities on daily operations are not limited to severe weather patterns during the winter season, or even to the effects of poor weather conditions. Based on data obtained from the US Department of Transportation, its was established that poor weather conditions are cited as the largest causes of irregularities in the airline system, as reported by the airlines themselves.

In recent years, airlines have invested significantly in the development of their Operations Control Centers, with extensive infrastructure improvements in communications channels, and new computer architectures which promote the free flow of information throughout the entire airline company. The presence of these centralized decision centers have allowed airline controllers to make better decisions regarding the carrier's operations, based on up-to-date and accurate information from numerous divisions within the airline, available to them on state-of-the-art information systems. But the existence of robust and efficient decision support tools to help airline controllers in the decision process is not apparent. The development of such methodology is warranted, as airlines will gain financially from the availability of such decision tools.

1.3 Problem Statement

Throughout the course of daily operations, an airline is faced with the potential of deviations in the planned flight schedule as a result of various unexpected events. The impact of these deviations on the three primary airline operational schedules (Flights, Crew Rotations, and Aircraft Rotations) will vary, depending on the flexibility and robustness of the original schedules. Any changes which may occur to the three airline system schedules are often defined as "operational deviations". Deviations that do not cause significant rerouting problems are defined as "time deviations", and deviations that lead to rerouting of airline resources are referred to as "irregular operations".

Time deviations are defined as any variation from the original scheduled times in any of the system schedules, and often result from minor delays in the airspace system. One of the main causes of time deviations is the variation in wind patterns, which affect the overall airborne time of a given flight. They usually do not have a negative impact on the airline's flight operations, but simply reflect small changes in the arrival and departures times during normal daily operations. Time deviations are distinguished from irregular operations are they do not generally require any aircraft reassignment decisions. However, there may be rescheduling of gates and other ground resources.

An "irregular operation" is defined as the aftermath of unexpected events which have a significant impact on the carrier's schedule. This often results from severe weather patterns and the resulting delays in the air traffic control system, airport closures, aircraft breakdowns, lack of adequate flight personnel (cockpit and cabin crew), problems in ground handling and support services, and equipment failures. Irregular operations generally result in aircraft rescheduling and rerouting, with the added impact of flight delays and cancellations. In addition, the aircraft rescheduling will have an impact on the scheduling of maintenance resources for the carrier.

On a daily basis, airlines suffer from irregular operations which can a significant impact on their profitability and ability to compete effectively. In fact, many carriers now see the need to address the problem of irregular operations as one issue necessary to maximize operating revenue, by reducing additional operating expenses which result from such irregularities. However, decision support systems for the purpose of aircraft operational re-scheduling do not exist, and very little research has been done on the topic. At the majority of the airline operation centers throughout the world, irregular operations are dealt with manually, with a heavier reliance on the human controller, his past experience, and his knowledge of available spare aircraft and other resources such as terminal gates, regulations and maintenance schedules. The need for real-time decision making tools to assist in the event of irregular airline operations is therefore apparent.

There are several questions that have to be considered when trying to the solve the problem of irregular airline operations. These include:

- How can flights and aircraft rotations be adjusted in the aftermath of irregular airline operations?
- What flights can be cancelled to minimize the loss of profit, based on available resources and the actual number of passenger on-board a given flight?

- Is it possible to carry out the proposed flight schedule with the available number of flight crews?
- How easy is it to develop new crew rotations in the aftermath of irregular operations?
- How will the revised flight schedule and corresponding aircraft rotations affect the scheduled maintenance program of the airline?

The availability of high-performance workstations, which are already in use in the strategic stage of airline planning could play a significant role in tactical planning. The use of these computers would give the airline controller the ability to incorporate demand and revenue data from computer reservation and yield management systems, to interact with maintenance scheduling, crew scheduling, and other elements of airline planning. Historically, little interaction exists during the tactical phase of operations between the various operational divisions (maintenance, fleet assignment, yield management, etc.), and the presence of irregular operations only adds to the problem. This has changed somewhat with the advent of the development of the centralized Airline Operations Control Center (AOCC).

It should be possible to develop a decision support system whose primary goal would be to regain the strategic schedule of the airline within a given time period, minimizing the overall impact of cancellations and delays on profitability, and on the operational schedules. The most severely impacted aspects of the planning process are fleet assignment and subsequent aircraft routing. Although these problems are generally developed independently in the strategic stage, the need to reschedule aircraft operations in real-time causes both fleet assignment and routing to be considered concurrently. The utilization of a decision support system should provide significant benefit to the airline, and potentially to the traveller (through significant reduced flight delays, and/or cancellations).

1.4 Model Development

In order to develop effective decision support tools to assist airline controllers in the resolution of irregularities, it is imperative for the researcher to establish a thorough understanding of the daily operations of the Airline Operations Control Center, and the role it plays in the airline operational activities. In addition, it is necessary to identify the operational requirements of any tool which will be developed and deployed in the AOCC, as it is essential to incorporate the experience of the controller in the decision process. Trade-offs have to been made between the level of automation in the decision process versus flexibility, and the ability of the controller to guide the decision process.

Although the overall goal of the decision process is to fully resolve any irregularities, the shear size of the airline network often dictates that the underlying problem has to be decomposed and considered in different phases. Decisions about rerouting aircraft will be affected by the availability of eligible flight crews at each station, as well as adequate ground resources to process aircraft and passengers at a station. Conversely, the allocation of these support services will be driven by the final aircraft schedule. It was established in the early phases of this research, that the problem of resolving irregular airline operations would have to be addressed through a phased approach.

The primary decision that has to be made is the reassignment of aircraft to flights, within the constraints of crew availability, the number of landing slots at a given station, and the level of ground resources. Initially, the aircraft have to be reassigned to flights based on revenue data and maintenance routing requirements, while secondarily taking into consideration, issues such as the availability of flight crews, landing slots, and in some cases, limited ground resources and passenger flow considerations. The allocation of crews, landing slots and ground resources is done after the primary aircraft reassignment problem has been solved, and if necessary, there then would be an iterative process implemented to improve upon the primary decision.

Based on discussions with airline controllers at major US carriers, it was established that one of the most important operational requirements of any decision support tool is the ability to provide real-time decision making. Throughout the course of this research project, this issue was thus placed at the forefront of the design process. However, several other requirements were incorporated into the development of the solution methodology. These include the ability to consider switching between different types of aircraft in the fleet, aircraft maintenance capabilities, crew scheduling considerations, and to make trade-offs between delaying and cancelling a given flight using a single decision model.

1.5 Overview of the Airline Operations Control Center (AOCC)

Airline operational planning is generally handled in two phases, strategic and tactical. Strategic planning is concerned with creating a flight schedule of services to be offered to passengers (called the Schedule of Services), and is established by the Commercial/ Marketing department. The Operations group then generates the Nominal Operational Schedule (NOS) for the airline's generic resources such aircraft rotations and crew rotations, and then assigns tail numbers, and individual crew members to a given flight. These activities constitute the resource schedule generation, and the resource allocation phases of the scheduling process. They are carried out by various groups which support the development of the planned schedule for all airline resources. A more comprehensive discussion of the airline scheduling process can be found in Grandeau [1995].

Given these resource schedules, the tactical side of the Operations group is responsible for the final stage of the scheduling process: Execution Scheduling. Execution scheduling is the process of executing the system resource schedules on a daily basis. This involves three main activities: executing the pre-planned schedules, updating the schedules for minor operational deviations, and rerouting for irregular operations. The tactical operations of a regular scheduled air carrier is usually under the 24 hour/day control of a central organization often referred to as the Airline Operational Control Center AOCC.

This section presents a summary of a typical AOCC, outlining its organization, primary activities within the airline, and operational facilities. The facilities and personnel of a particular AOCC will vary considerably depending on the type and size of the airline. AOCC centers can range from a single controller/dispatcher on duty to several dispatchers and hundreds of other personnel handling flights throughout the carrier's entire global network. During the process of operation control, the AOCC is supported by the Maintenance Operations Control Center (MOCC) which controls aircraft maintenance activities, and by various Station Operations Control Centers (SOCC) which control station resources (gates, refuelers, catering, ramp handling, and passenger handling facilities).

Operations Control Centers are usually linked to the Aeronautical Radio Inc. (ARINC) and the Societe International Telecommunications Aeronautiques (SITA) networks to send and receive teletype/telex messages. Communications with maintenance and engineering, customer service, and airport services are maintained to facilitate prompt contact with the appropriate personnel. Teletype, telex, facsimile, telephone, leased lines, and public data networks combine to provide an effective medium of collecting information and communicating revised operational plans developed by the AOCC center. In some cases, the AOCC has communications systems connected to VHF, HF and Satcom radio links, air traffic control centers, and other relevant locations, allowing them to effectively gather and disseminate information instantaneously.

1.5.1 Functional Groups Within the AOCC

The AOCC is organized into three functional groups, each with a distinct responsibility within the schedule execution process. The airline Operations Controllers are responsible for maintaining the operational version of all the system resource schedules (crew, aircraft and flight), and for the management of irregular operations. The final operational decisions are made by one (or more) Operation Controller(s) who are assisted by four types of support personnel: 1) The Flight Dispatch group is responsible for flight planning, flight dispatch and enroute flight following. 2) The Crew Operations group is responsible for tracking individual crew members as they move through the airline's route network, for maintaining up to date status for all crew members, and for calling in reserve crews as required. These three groups are usually located together in the AOCC Center. The other two support groups, the MOCC and SOCC's are not physically located at the centralized AOCC.

The AOCC at larger airlines may have a dedicated airline Air Traffic Control ATC coordinator, as well as specific supporting personnel for functions such as dispatch, crew scheduling, aircraft scheduling, and meteorology services within the AOCC center. Ancillary off-line services such as the maintenance of the navigation database, operations engineering (or flight technical services) are usually located nearby and serve to provide supporting resources for AOCC personnel. In addition, the crisis center to manage activities after an accident or incident is often an integrated part of the Airline's Operational Control Center.

1.5.2 Operations Controllers

The airline Operation Controllers are the center of the airline operation control process. They are the sole operational group within the AOCC with the authority and responsibility to resolve problems that develop during the course of both regular and irregular operations. Airline Operation Controllers receive information from every facet of the airline during operations (see Figure One). From these inputs, the Controllers maintain an updated version of the airline system resource schedules which includes delays, irregular routings for aircraft and crews, and additional flights. These can be called the "Current Operational Schedules " (COS). Other personnel in the AOCC are normally grouped by the geographic areas of the flights they manage and monitor. As the focal point in the AOCC for flight and schedule management, controllers interact with the following key personnel and organizations:

- Flight Dispatchers
- Crew Operations (scheduling, tracking, and rescheduling)
- Station managers and gate coordinators
- Passenger service managers
- Ramp service managers (fuelers, baggage handling, aircraft loading, catering)
- Maintenance and engineering
- Meteorology
- Operations engineering/route planning
- Air traffic control coordinator

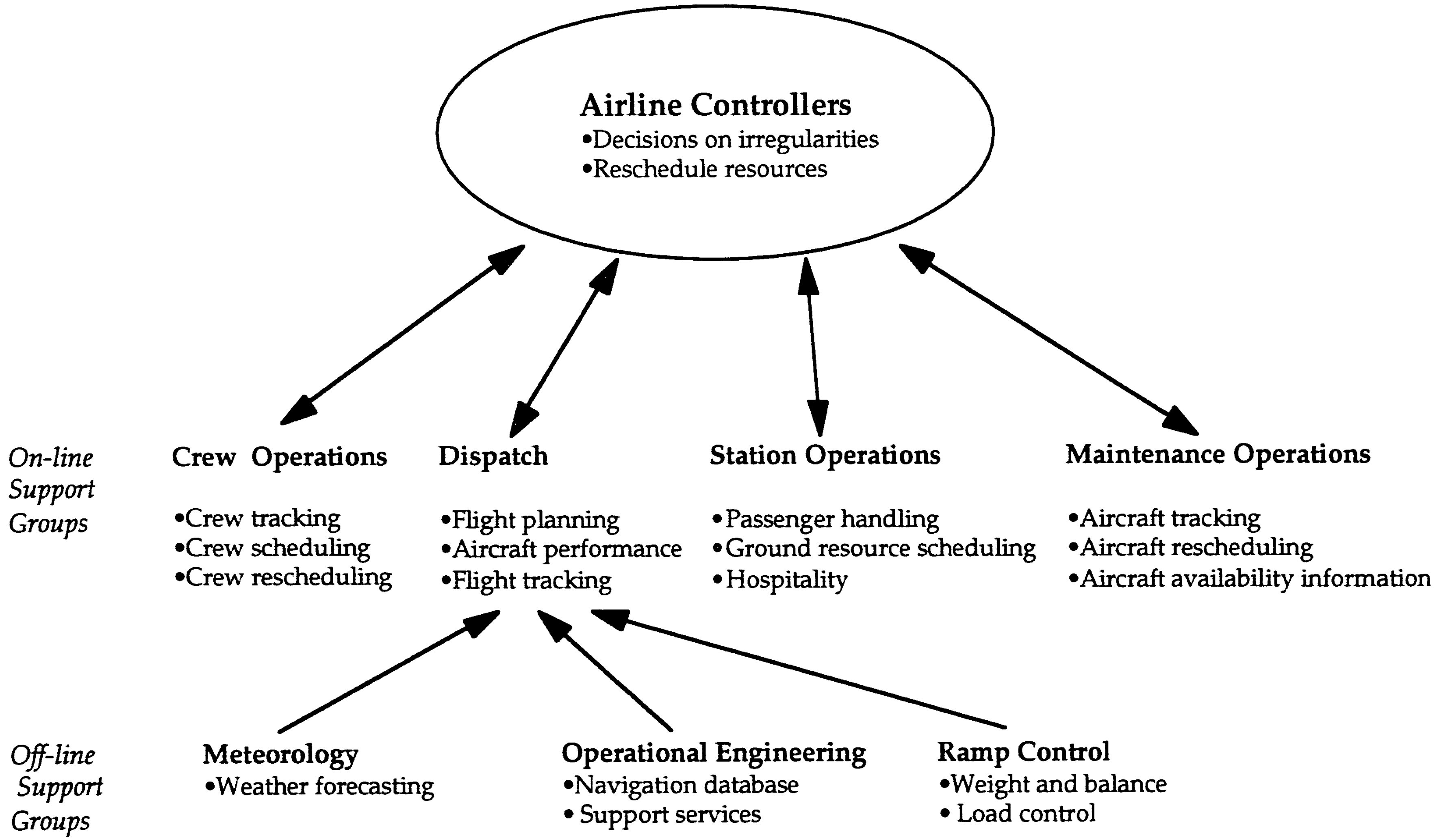

Figure 1-1 Information Flow Diagram for the Airline Operations Control Center (AOCC)

During normal operations, Dispatchers are responsible for the successful release of a flight, depending on maintenance issues (deferred maintenance equipment list [MEL] or configuration deviation list [CDL] items), aircraft restrictions (such as noise), the availability of required operational support (fuel, gates, ground power, airport facilities) at the departure, destination and alternate airports. During irregular operations and emergencies, the Dispatcher will inform the Operations Controller of the problem, and their role is to handle the additional coordination that such situations demand. If the airline is experiencing irregularities, the Operation Controllers have to devise modified operational schedules on a very short notice. The Current Operational Schedule is the plan that the airline will follow in order to return to Nominal Schedule of Services. These modified schedules are disseminated to the relevant divisions, and stations of the system.

1.6 Research Objectives

The primary objective of this research project was to develop efficient heuristic procedures for flight rescheduling in the aftermath of irregular airline operations. In the first phase of the program, the overall structure of the problem was defined, and a large-scale mathematical model was formulated to represent a complex decision process for aircraft rerouting. Based on discussions with airline controllers, potential solution methodologies were investigated, and the underlying operational requirements and capabilities of candidate decision procedures were established. In the second phase, a series of algorithms were developed to solve the established problem based on concepts of network flow theory and linear programming theory. These solution procedures have been developed and implemented in an UNIX operating system environment using the C++ programming language.

1.7 Thesis Outline

In the next chapter, a review of existing decision support tools and solution methodologies currently in use at airline operations control centers of major US domestic carriers is presented, outlining the major characteristics of these systems. In addition, an extensive literature review of airline operations is given, summarizing research that has been done on the topic of irregular airline operations, as well as work on other closely related research topics. These include the airline fleet assignment, aircraft routing, airline scheduling, crew scheduling, and the ATC slot allocation problem.

In Chapter 3, the mathematical formulation of the airline recovery problem is presented, outlining the decomposition of this complex problem. The main problem considered is the reassignment of aircraft to scheduled flights in the aftermath of irregularities. Based on this output, the residual airline network in used as the basis to assign crews, terminal gates, ATC landing slots, and for solving the passenger reaccommodation problem. Each resulting subproblem is outlined with a representative formulation of the problem.

Chapter 4 outlines the underlying linear programming theory and network flow theory which were used to develop the solution methodologies and procedures. This includes an overview of the implicit column generation procedure, and a review of a constrained shortest path algorithm, and a constrained minimum cost flow algorithm.

In Chapter 5, the solution procedures developed in the research project are discussed, incorporating concepts presented in Chapter 4, and the underlying structure of the decision model.

Chapter 6 summarizes the major contributions of this thesis, and outlines future research initiatives which are warranted to validate the algorithms, and solution procedures developed in this research project.

Chapter 2

Literature Review

2.1 Review of Exisiting Information Systems and Decision Support Tools

The overall impact of irregularities on the daily operations of an airline will depend on the level of precautionary measures the carrier has built into its schedules to deal with typical irregularities. Many carriers have developed extensive resolution procedures which are generally implemented manually in the aftermath of irregularities, with little if any dependence on automated decision support systems. Decisions regarding future operational schedules and actual operations of the airline are made based on forecasted and often out-dated data and information, and this can have a significant effect of the accuracy and benefit of the decision process. In some cases, the airline may decide to delay or even cancel flights, only to find out that these actions were unnecessary for the resolution of irregularities in the network.

Airlines have identified the need to develop robust decision support systems which have the ability to assist airline controllers in the real-time operations of the carrier. They have invested heavily in state of the art, Airline Operations Control Centres (AOCC), sometimes referred to as system operations control centers, which gather an extensive array of operational information and data. However, very little effort has been placed in developing solution procedures and methodologies which could complement the decision making capabilities of experienced airline controllers. In order to develop an appreciation for the need for such systems, the following is a summary of some of the resolution procedures and decision support systems, currently in use at operation control centers of major US domestic carriers.

United Airlines has developed and deployed the "System Operations Advisor" SOA, a real-time decision support system for use at its AOCC which they refer to as the Operations Control Center (OCC) to increase the effectiveness of its operational decisions. The SOA system consists of three primary components: the Status Monitor, the Delay and Swap Advisor, and the Delay or Cancellation Advisor. The purpose of the Status Monitor ubsystem is to alert the airline controller of potential irregularities such as delays and cancellations through a graphical user interface. The interface provides mechanisms to launch tools such as the Delay and Swap Advisor for developing solutions to existing operational problems. The Delay or Cancellation Advisor can then be deployed in order to determine potential resolution procedures to problems which have developed from irregularities in the airline's network. It is important to point out that decisions regarding delays and cancellations of scheduled flights are made independently of each other in this current system.

The AOCC at American Airlines is called the System Operations Control center (SOC), and relies on an array of decision support tools to make informed decisions about the operations of the carrier. The airline's primary goal in the aftermath of irregularities is to return to the operational schedule as soon as possible, regardless of its impact to potential revenues. The controllers consider the number of passengers booked a given flight segment instead of the actual value of the flight. In resolving irregularities, the airline controllers subjectively incorporate passenger flow issues such as connectivity, goodwill, and volume of traffic, into the decision process.

The airline has identified crew scheduling as the important parameter in the resolution of irregularities in the network and as such, most aircraft substitutions are done within a given fleet. In the aftermath of an irregularity, the carrier first establishes a reduced flight schedule, and then figures out how to implement this schedule. It takes into consideration such issues as critical departure times, mission compatibility, and system balance in the daily flight cycle. American Airlines describes mission compatibility as any decision which minimizes downstream effects in schedule variation, and provides a feasible resolution in a timely fashion. Decisions are generally made to initially delay flights, and then if necessary determine flight cancellations.

Delta Air Lines recently opened its operations control centre in Atlanta, responsible for monitoring weather, flight schedules and maintenance problems that may develop during the course of normal operations. The airline makes use of readily available operation data to fine tune its flight schedules to accommodate for prevailing weather conditions. It is apparent however, that most of the decision making regarding flight delays and cancellations at Delta is manually executed, with little if any reliance of automated decision support systems. The airline is currently in the process of developing such software, including a program named the Inconvenienced Passenger Rebooking System, which allows the airline to notify passengers of cancellations or delays and aid in passenger flow recommendations. In addition, they are reportedly in the middle of developing software to assist in the redeployment of flight crews in the aftermath of irregularities.

In recent years, many airlines have come to rely extensively on pre-emptive decision making, developing flight cancellation plans which are implemented long before an airport or region is actually impacted by severe weather conditions which would result in irregularities. At Continental Airlines, they have developed a resolution procedure referred to as the Severe Weather Action Plan, which is used to minimize the number of aircraft and crews remaining in a geographical region forecasted to have bad weather conditions. The airline controllers believe that such preemptive actions are beneficial to the carrier, as it makes schedule recovery easier, and greatly facilitates restarting normal operations. However, they may in fact compromise revenue operations, which could have occurred without the influence of the prevailing irregularities. Continental recently opened its operations control centre, similar to those existing at American, United and Delta airlines.

Northwest Airlines is currently in the process of developing decision support systems for use in the carrier's operations control center. In the interim, the airline has developed and implemented several alternative aircraft thinning procedures that incorporate both operational and economic factors in the decision making process. "Thinning of flights" is defined as the response to irregular operations, based on forecasted adverse weather conditions that are expected to reduce the operational capacity of airports in the given region. The thinning process is designed to match operations with the level of reduced airport capacity, while ensuring that net revenue contributions are maximized, as well as minimizing customer inconvenience, and disruptions to crew and maintenance scheduling. The overall guidelines for a thinning operations are to recover safely, and efficiently to normal operations as soon as physically possible, in the aftermath of the irregularity. Similar to Continental Airlines, it is Northwest's policy to pre-cancel flights in preparation for the reduced operational capacity.

2.2 Irregular Airline Operations

The problem of irregular airline operations has only been recently considered in research projects conducted by Dusan Teodorovic et. al and in work done by the Research and Development department of United Airlines. Teodorovic and Gubernic [10] discuss the problem of minimizing overall passenger delays in the aftermath of a schedule perturbation. They attempt to determine the least expensive set of aircraft routings and schedule plan using a branch and bound procedure. Their methodology is based on the assumption that all the aircraft in the fleet have the same capacity, and they only considered a marginally sized fleet of three aircraft operating a total of eight scheduled flights. Teodorovic [11] presents research on the reliability of airline scheduling as it relates to meteorological conditions, the ability to identify an indicator for quantifying the adaptability of such airline schedules to weather condition, and an overview of a potential solution procedure. The author outlines this heuristic algorithm for minimizing the number of aircraft required to accommodate a given traffic volume, while ensuring that aircraft are assigned to only one flight within a given time period.

Teodorovic and Stojkovic [8] discuss a greedy heuristic algorithm for solving a lexicographic optimization problem which considers aircraft scheduling and routing in a new daily schedule while minimizing the total number of cancelled flights in the network. The algorithm developed is based on dynamic programming, and is characterized by a sequential approach to solving the problem as flights are assigned to aircraft in sequences. The solutions obtained using this methodology are highly sensitive to the decision matrix, and the ranking of the various objective functions. The model does not consider the impact of crew scheduling in the aircraft scheduling process. Teodorovic and Stojkovic [9] outline a model for operational daily airline scheduling which incorporates all operational constraints, and is used to reduce airline schedule perturbations. Their heuristic model based on FIFO principle and a sequential approach based on dynamic programming, is developed to facilitate and incorporate the work and experience of the dispatcher in the decision process regarding traffic management. The model developed is used to determine the aircraft rotations, as well as the crew rotations, while minimizing the number of cancelled flights.

The research and development department at United Airlines has conducted several projects on the topic of irregular airline operations, and has presented material on its efforts at annual symposiums of AGIFORS (Airline Group of the International Federation of Operations Research Societies). The work at United is part of the development of a comprehensive decision support system for use in the carrier's operations control centre. Jarrah et. al [1] present an overview of a decision support framework for airline flight cancellations and delays at United. Their underlying solution methodology is based on network flow theory, as the models cast some of the problems faced by flight controllers while dealing with irregularities into minimum-cost network flow problems.

Jarrah's paper outlines two separate network flow models which provide solutions in the form of a set of flights delays (the delay model) or a set of flight cancellations (the cancellation model), while allowing for aircraft swapping among flights and the utilization of spare aircraft. The models assume that a disutility can be assigned to each flight in order to reflect the lost revenue if the flight is cancelled, and that the disutility of delaying each flight is assessable. Both models are solved using Busacker-Gowen's dual algorithm for the minimum cost flow problem in which the shortest path is solved repeatedly to achieve the necessary flow in the network. The network models presented are solved independently of each other, and does not take into consideration crew and aircraft maintenance constraints. This solution framework is deficient in that it does not allow for a trade-off between cancelling and delaying a given flight in a single decision process. In addition, the solution methodology does not allow for potential substitution of aircraft with varying capacity, and operational capabilities.

Yan and Yang [12] develop a decision support framework for handling schedule perturbations which incorporates concepts published by United Airlines. The framework is based on a basic schedule perturbation model constructed as a dynamic network (time-space network) from which several perturbed network models are established for scheduling following irregularities. The authors formulate both pure network flow problems which are solved using a network simplex algorithm, and network flow problem with side constraints, which are solved using Lagrangian relaxation with subgradient methods. They outline the basic schedule perturbation model which is designed to minimize the schedule-perturbed period after an incident, while maximizing profitability. In addition, they consider the effects of flight cancellations, flight delays and ferry flights as solution alternatives in the decision process. The framework is designed to aid airlines in handling schedule perturbations caused by aircraft breakdowns, and assumes scenarios with only one broken down aircraft and a single fleet type. In addition, the models do not incorporate aircraft maintenance and crew constraints in the formulation.

In all the published literature dealing with irregular airline operations, there is an underlying assumption that the fleet assignment problem is solved before considering the aircraft re-routing problem. There has been extensive work done on the topics of fleet assignment, aircraft routing and crew scheduling, and the following is a summary of some of the major research papers in each area. In recent years, there has been a trend towards addressing hybrid airline problems such as the combination of the aircraft assignment and routing problem, and the combined fleet assignment and crew scheduling problem. However, these hybrid problems have been considered only for the strategic phase of the airline planning process.

2.3 Fleet Assignment

Given a predetermined flight schedule, the fleet assignment problem is to determine which aircraft type is assigned to a given flight segment in the carrier's network. One of the early published articles on the topic of fleet assignment was presented by American Airlines in Interfaces in 1989. Abara [41] discusses the application of integer linear programming to the fleet assignment problem, and explains how this technique is used extensively throughout the carrier. The goal of the fleet assignment problem is to assign as many candidate flight segments as possible in a schedule pattern to specific aircraft types, based on such factors as operating costs, revenues, and operational constraints and capabilities. The problem is formulated and solved as an integer programming IP model, which permits the assignment of multiple fleet types to a flight schedule simultaneously.

Subramanian et. al [49] present a solution procedure referred to as Coldstart, which is a fleet assignment methodology developed by Delta Airlines. They discuss how recent advances in mathematical programming algorithms and computer hardware have make it possible to solve optimization problems of the scope of fleet assignment for major US domestic carriers such as Delta. The Coldstart model is a large scale mixed-integer linear program that assigns fleet types to flight legs so as to minimize a combination of operating and passenger spill costs, subject to operational constraints. The solution strategy used in Coldstart employs the OB1 interior point method to solve the problem initially as a linear program, then modifies the structure of the original problem by fixing certain variables, and solves the resulting problem as a mixed integer problem using the Optimization Subroutine Library (OSL) mixed integer programming MIP code.

Hane et. al [47] outline a model for the fleet assignment problem, solved as a large scale integer problem. The model is a large multi-commodity flow problem with side constraints defined on a time-space network. The authors discuss solution problems that often exist with such large problems including degeneracy, which leads to poor performance of standard LP solution techniques. The solution methodology presented incorporates an interior point algorithm, cost perturbation, model aggregation, branching on set-partitioning constraints, and prioritizing the order of branching, in an effort to develop more efficient solution procedures for the problem. A comparison is given on the performance of the solution procedure to standard LP based branch and bound methodology.

Clarke et. al [43] discuss maintenance and crew considerations in the basic daily fleet assignment problem of Hane et. al (1993), and implementations issues related to its solvability. The model generalizes the daily fleet assignment model to capture certain aspects of maintenance and crew scheduling. The solution methodology presented involves the use of the dual steepest edge simplex method, and solving the mixed integer problem by branch and bound. Gu et. al [46] present the fleet assignment problem modelled as a multi-commodity flow problem on a time-space network.

However, the authors discuss the complexity of the problem and the behaviour of the solution methodology as a function of the number of fleets in the airline.

2.4 Aircraft Routing

The aircraft routing problem is traditionally solved after the successful completion of the fleet assignment problem. It involves the allocation of candidate flight segments to a specific aircraft tail number within a given sub-fleet of the airline. The process of aircraft routing has traditionally been a manual activity at many carriers, but in recent years, researchers have developed efficient solution procedures that can be applied to the problem. Bard and Cunningham [52] explore aircraft routing while taking into consideration the benefits of through-flight schedules and the potential for increased revenues. The authors develop an algorithm that can be used to efficiently pair inbound and outbound routes at hub cities over the course of the day. The methodology is based on a forward searching heuristic designed to transcend the combinatorial difficulties that arise in aircraft routing problems for major domestic carriers operating a hub and spoke network.

Soumis et. al [70] present a model for large-scale aircraft routing and scheduling problems which incorporates passenger flow issues. The solution methodology proposed is a heuristic adaptation of the Frank-Wolfe algorithm for an integer problem with a special structure. The procedure involves solving alternatively the aircraft routing problem, and the passenger assignment problem until a prescribed criterion is satisfied. The authors discuss the technique used to transfer information from the passenger flow problem to the aircraft routing problem. Daskin and Panayotopoulos [44] discuss a Lagrangian relaxation approach to an integer linear program model which is used to assign aircraft to routes in a hub and spoke network. They outline the

Lagrangian solution procedure, as well as heuristics for converting the Lagrangian solutions into primal solutions to the problem. The research results suggest that the Lagrangian relaxation approach is effective at providing an upper bound on the objective function, and the heuristics can yield good solutions when there are adequate aircraft available to meet the passenger demand.

Feo and Bard [57] present a model that can be used by airline planners to both locate maintenance stations and to develop flight schedules that better meet the cyclical demand for maintenance. The model is formulated as a minimum cost, multi-commodity flow network problem with integrality constraints, and solved using a two-phase heuristics. In the first phase of the solution procedure, a single aircraft schedule is generated ignoring maintenance requirements. These maintenance requirements are then covered by choosing the best base locations so as to minimize total cost. This step involves the solution of a set covering problem using Chvatal's heuristic, which is a greedy procedure that is guaranteed to find a feasible solution to the set covering problem. The overall airline network is perturbed so that the best "K" schedules are generated and stored for phase two of the solution process. In the second phase, the K best tail number schedules are inputted to a probabilistic version of Chvatal's heuristic and the final aircraft routings are determined from the outcome of this process.

Kabbani and Patty [61] discuss aircraft maintenance routing at American Airlines, and the application of mathematical programming techniques to solve the problem. Their initial approach to the problem was to formulate it as a set partitioning model, where the columns were constructed to represent each possible week-long routing. This formulation did not take into consideration maintenance constraints, and the solution procedure had a poor performance. The authors further outline a modified solution procedure in which the decision process in divided into two separate sub-problems. The first subproblem dealing with the solution of appropriate daily aircraft turns, and the second with connections among these daily routings. The solution methodology also makes use of heuristic procedures, if the sequential approach is not successful in determining all aircraft rotations for the fleet.

Desaulniers et. al [55] outline the daily aircraft routing and scheduling problem, and present two different formulations of the problem. The first is a set partitioning type formulation, and the second a time constrained multi-commodity network flow formulation. The authors describe the solution methodology wherein the underlying network structure of the subproblems is revealed when a column generation technique is applied to solve the linear relaxation of the first model. A Dantzig-Wolfe decomposition approach is used to solve the linear relaxation of the multi-commodity flow problem. In addition, they discuss alternative branching strategies that are compatible with the column generation technique, and show the compatibility between the two different formulations. Zhu et. al [75] present a mathematical formulation for the aircraft rotation problem and discuss its similarity with the asymmetric travelling salesman problem. Their solution procedure employs Lagrangian relaxation and subgradient optimization, and is applied to a real world problem based on data from a major US domestic airline.

In recent years, researchers have started to explore so called hybrid strategic planning problems, combining different phases of the airline planning process, which have been traditionally considered in sequential order. One such problem is that of the combined aircraft fleeting and routing problem. Barnhart et. al [42] discuss a model and solution approach to solve simultaneously the fleet assignment and aircraft routing problems.

The authors state that the methodology incorporates costs associated with aircraft connections, and complicating constraints (such as maintenance requirements, and aircraft utilization restrictions) which are usually ignored in traditional fleet assignment solution procedures. The model is string-based and a branch and price solution approach is used to solve the problem (discussed below). As described by the authors, a string is a sequence of connected flights that begins and ends at a maintenance station, satisfying flow balance, and meets the required maintenance constraints. The methodology is validated using operational data from a long-haul carrier.

2.5 Aircraft Schedule Transition

During the normal operations of a carrier, situations often develop wherein modifications have to be made to the existing schedule plan. In addition, due to the inherent variation in passenger demand over the course of the week, airlines find it necessary to adjust their daily flight schedules to adequately meet demand. This will result in the need to make minor modification to aircraft routings and possibly fleet assignments. Talluri [72] describes an algorithm for making aircraft swaps that will not affect the equipment type composition overnighting at various stations throughout the airline's network. The algorithm repeatedly calls a shortest-path algorithm, and the performance of the swapping algorithm is a reflection of the availability of very fast shortest path algorithms. He also outlines the application of the swapping procedure in the airline schedule development process.

Klincewicz and Rosenwein [62] describe the Airline Exception Scheduling Problem, which involves the modification of scheduled flight legs and fleet assignments to create schedule exceptions, that meet changes in the operational environment. They outline an approach, centered around formulating and solving a network flow problem, for efficiently scheduling exceptions which occur in daily operations. The automated procedure and algorithm is able to modify a nominal daily fleet assignment to account for exceptions to the schedule on a particular day. The solution methodology uses a tree search algorithm to generate alternative sequences of flight legs that can be swapped among fleets, and then an optimal fleet assignment is then determined using a network flow algorithm. As an extension to their work, the authors considered the application of their methodology to evaluate profitable exceptions, which could then be intentionally done by the carrier.

2.6 Crew Scheduling

The problem of crew schedule planning has received substantial attention in published literature, with early works on the topic published by American Airlines in the 1980's. In an article published in Interfaces, Gershkoff [85] outlines an optimization model which uses a set-partitioning framework, wherein the rows represent flights to be covered and the columns represent candidate crew trips. The primary objective of the model is to minimize the cost of flying the published airline schedule, subject to operational crew constraints. The crew scheduling problem is formulated as an integer programming problem, and can solved using a commercial optimization software package such as CPLEX. In these early efforts on the topic, it was found that a global optimization to the problem was difficult to achieve, and a lot of research focused on the development of efficient heuristic procedures to address the problem. Concepts such as dynamic column generation and LP relaxation play a major role in the ability of researchers to tackle, and successfully solve such large-scale mathematical programs of crew pairing optimization.

Crew pairing optimization has evolved substantially in the last ten years, as advances in computer technology, CPU's time, and the availability of efficient optimization software packages, have given practitioners the ability to tackle even larger problems than before possible. The development of TRIP (trip re-evaluation and improvement program) at American reflected an advancement in the solution methodology used in the area of crew scheduling and planning. In an article published by American in Interfaces (1991), the carrier discusses some of the operational benefits of implementing such a decision support system, and its overall impact on the financies of the company. TRIP is based on a solution approach in which the crew pairings are iteratively improved by generating and solving a series of subproblems, whose performance will depend on the level of detail applied to the subproblems. In addition to the nominal role of developing crew pairing for the carrier, TRIP has been used by American as a general decision support tool to address problems related to crew staffing, such as the effects of crew base closures on operations, manpower requirements at a given crew base, the economic impact of contract negotiations and changes in operational rules. Similar systems have been developed by United Airlines, based on the idea of an iterative approach to solving the crew pairing problem.

One of the driving characteristics of the crew scheduling problem is the size of the constraint matrix in the mathematical formulation of the problem. Substantial research efforts have been spent studying the application of revolutionary solution techniques to aid in the timely solution of such large-scale problems. One of the most successful solution procedure that has been developed is the so called branch-and-price technique, which combines the standard integer programming IP solution technique of branch and bound, and column generation. Another hybrid technique of mention is the branch and cut procedure, which combines branch and bound, with cutting plane generation. In recent years, a number of articles have been published on these methodologies, and they are considered the leading edge technology in the field of crew scheduling.

Barnhart et. al [83] discuss a column generation technique for the long-haul crew assignment problem, which is solved by a two phase process. An linear programming LP relaxation is applied to the problem, and the resulting linear program is solved using column generation. Columns are efficiently determined using a specialized shortest path procedure. In the second phase of the solution procedure, an integer crew assignment is found using IP solution techniques such as branch and bound. The hybrid solution procedure of branch and price is a slight variation on the preceeding methodology in that, during the course of the solution process, there are iterations between the two phases (steps) outlined above. At each node of the branch and bound tree, an LP relaxation is solved using column generation. Information obtained from the optimization portion of the method is used to modify the underlying problem, and the resulting problem is repeatedly optimized until a prescribed decision criterion is achieved.

Hoffman and Padberg [87] outline the branch and cut methodology in a paper published in Management Science in 1993. The branch and cut solver generates cutting planes based on the underlying structure of the polytope defined by the convex hull of the feasible integer points of the problem, and incorporates these cuts into a tree-search algorithm that uses automatic reformulation procedures, heuristics and linear programming technology to assist in the solution process. The authors present case studies of both pure set partitioning problems, and set partitioning problems with side constraints.

The problem of crew reassignment (crew recovery) in the aftermath of irregular airline operations has been considered by researchers at the logistics institute of the Georgia Institute of Technology. They have recently developed a mathematical programming based solution methodology which uses an integer programming model to optimally reassign crews to flight segments. In a presentation given at the INFORMS meeting in the fall of 1995, one of the researchers outlined a model which reassigns crews to flight legs, while minimizing the additional cost and operational difficulties to the airline. The solution strategy initially identifies a set of eligible crews, whose original assigned unflown flight segments are used to form new crew pairings which are then reassigned to individual crew members through a set covering problem.

2.7 Slot Allocation Problem

The ability of an airline to recover from severe weather conditions will depend on its interaction with the air traffic control system. Under such conditions, ATC typically impose restrictions on aircraft movement at affected airports and implements what is generally referred to as a slot allocation scheme, as well as ground-delay programs. The response of the airline to these imposed conditions will be based on available data in the system operations control center. The guidelines governing such slot substitutions have been recently changed to help accommodate the operating needs of carriers in the ATC system. Most of the published literature on the topic of slot allocation has been rendered obsolete, as changes to the substitution guidelines have significantly altered recovery procedures in use at airline's operations control centers.

Vasquez-Marquez [80] discusses a network-based optimization system that was implemented at American Airlines to help reduce delays imposed by air traffic control. The arrival slot allocation system uses a network-based heuristic procedure to reschedule arrivals into affected airports, as ATC has developed slot substitution rules that allow airlines to use the arrival slots of cancelled flights to reduce delays created by the ground delay programs. The underlying model is based on a mixed-integer mathematical formulation, and is solved using network flow theory. The model is treated as a directed travelling salesman problem and the solution methodology uses a tour building heuristic approach designed to preserve aircraft, crew, and gate connections among flights at hub airports. The procedures is capable of handling multiple cancellations for which the problem becomes a multiple directed travelling salesman problem.

Research on the topic of slot allocation at Research and Development department of United Airlines has resulted in the development of the slot allocation model (SLAM) which uses an assignment problem formulation to represent the allocation of landing slots to candidate arriving aircraft. The primary objective of the model is to minimize delay costs in the airline's system, while satisfying FAA regulations. The solution procedure of SLAM is based on a minimum cost flow algorithm that achieves optimality for any number of flight cancellations. A similar assignment model has been developed by the Operations Research Group at USAirways. The solution procedure for this model (FLOW) is also based on a minimum cost flow algorithm.

Luo and Yu [4] present the airline schedule perturbation problem under the ground delay program with restrictions on the flow of resources in the network. The authors propose strategies that can be implemented to address a specific schedule perturbation such as the ground delay program. They outline solution procedures for solving the perturbation problems which are based on assignment algorithms, and more efficient algorithms which rely on the detailed structure of the problem. In another paper written by Luo and Yu [3], they consider the general case of the ground delay program and its implications to the airline schedule perturbation problem. The general problem is modelled as an integer programming formulation with no restrictions on the flow of resources, and it considers both aircraft landings and take-offs in the solution process. The solution methodology involves an LP relaxation of the problem, and the application of a heuristic procedure to find feasible integer solutions. It is based on a polynomial algorithm for minimizing delay among outbound flights in the network.

Milner [78] in his doctoral research considered the problem of dynamic slot allocation with airline participation. He investigates methods for allowing greater participation of airlines in the dynamic slot allocation process, and identifies the potential benefits of such a mechanism. The author develops a bank delay model which minimizes the total delay passengers encounter when travelling through a hub airport, which is operating under a ground delay program. The solution methodology for this model incorporates both an indexing heuristic and a Lagrangian heuristic. Milner formulates models to assist airlines in responding to dynamic slot allocations, and applies a dynamic programming procedure which is capable of solving problems involving bank structure of flights. In addition, he outlines an array of mathematical models of varying complexity, which capture the nature of typical airline operations. The author presents a network cancellation/delay model which outlines how an airline can respond to several slot allocations in a hub-and-spoke network. It is based on an integer programming formulation which incorporates a set of valid inequalities, and is solved using branch and bound procedures.

Carlson [76] recently completed a research project on the problem of ATC slot allocation, and its impact on the scheduling of flight banks at a hub airport. He outlines the formulation of a decision model which incorporates the dependencies induced by connecting passenger flow at hub airports, as well as the ability to assess the overall economic benefit of assigning arriving aircraft to available ATC slots at a given station. The model is validated using real-world scenarios from major hub airports in the US domestic market. In addition, the author discusses several guidelines which can be used to accurately estimate the cost coefficients for the decision model.

Chapter 3

The Airline Recovery Problem

3.1 Discussion of Time-Space Network

The overall framework of the mathematical model of the airline recovery problem is based on a time-space network which represents the published daily schedule of the airline's network. A representative diagram of such a network is shown in Figure 3-1. Each event (arrival and departure) at a given station is represented by a node for a specific time and location coordinate. Each flight is represented by a "flight arc" which connects the corresponding nodes at the origin and destination of the scheduled flight. In addition, replica flight arcs may exist in the network to represent potential delay alternatives for each flight during the resolution procedure. These replica arcs are referred to as "delay arcs" and are generated based on parameter settings, prior to the implementation of the solution algorithms. The number of such replica arcs would depend on the level of complexity warranted.

"Ground arcs" in the network connect chronologically successive pairs of event nodes at a given station. These arcs are necessary in order to monitor the flow of aircraft through the network and for the application of network flow algorithms. "Maintenance arcs" in the network represent the time period of a given aircraft undergoing an unplanned maintenance check within the resolution horizon. The resolution horizon is defined as the total time required to return the airline's operational schedule back to the originally

Time-Space Network Representation

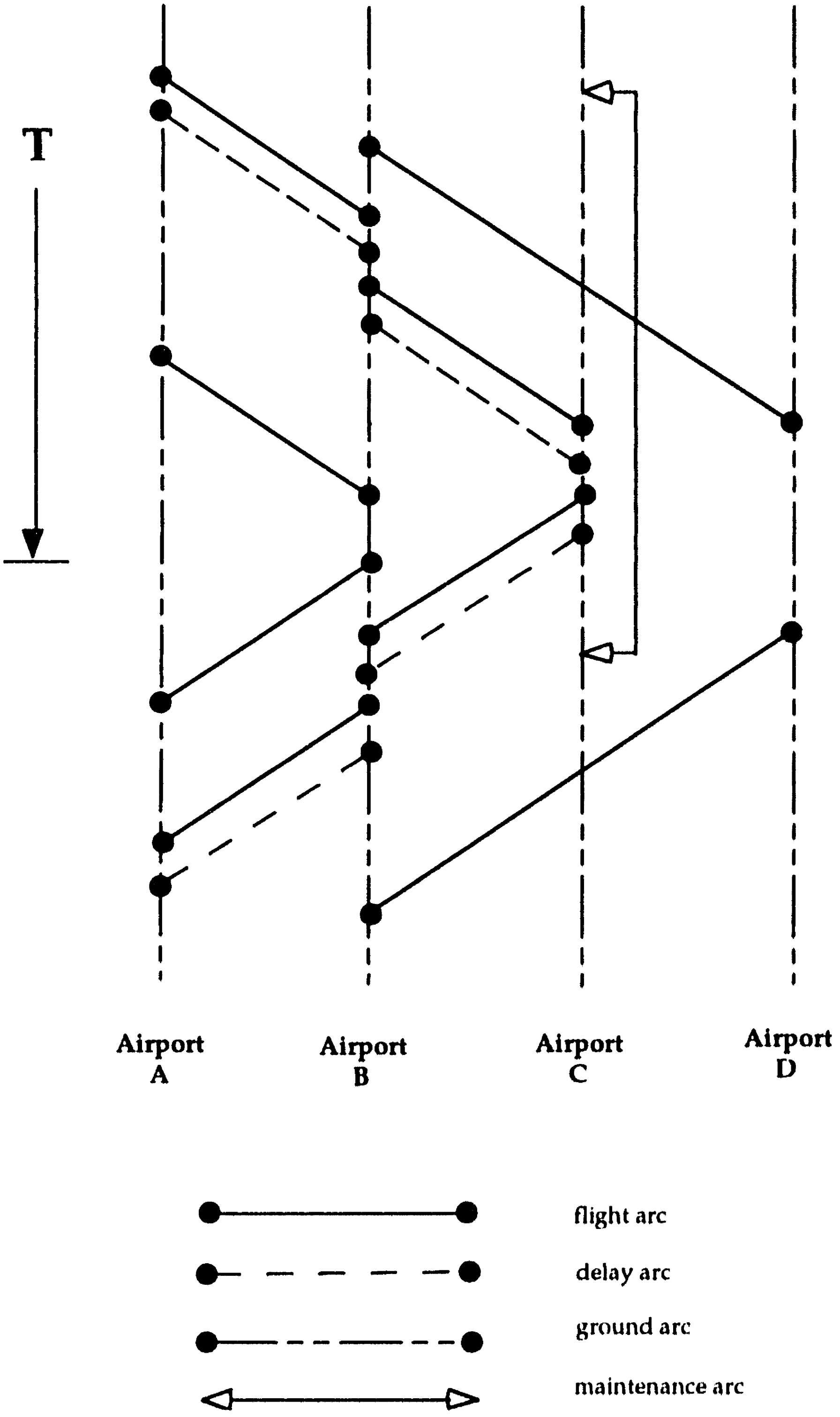

Figure 3-1 **Airline Network Representation**

planned schedule. The duration of the horizon will depend on the overall dimensions of the recovery problem, incorporating issues such as the number of aircraft in the fleet, and the number of scheduled flights being considered, as this will affect the performance of the solution methodology.

The development of the airline recovery problem based on the time-space network allows the use of efficient tree searching algorithms that can be used to quickly solve the underlying subproblem of finding the best possible aircraft routing, subject to several operating constraints. Based on concepts from network flow theory and linear programming theory, algorithms have been developed that can be used to solve the airline recovery problem in a real-time environment. In the next chapter, a brief summary of these underlying theories will be discussed, as it relates to the development of the solution methodology. In addition, a more detailed description of the time-space network will be given in Chapter 5, incorporating aspects of the solution procedures.

3.2 Mathematical Formulation

In developing the airline recovery problem, a path-based formulation was considered in which the decision variable corresponds to the assignment of a specific aircraft tail number to a predetermined sequence of flights. An aircraft would therefore not be considered for a given sequence of flights unless it meets its maintenance requirements. The overall framework of the problem is based on concepts from network flow theory, and relies on specialized tree-searching algorithms to generate the sequence of flights. These include a modified version of the out-of-kilter algorithm for constrained minimum cost flow, and the constrained shortest path multi-labelling algorithm to establish candidate flight sequences, based on anticipated revenues and aircraft operating costs. In creating these customized flight sequences, the tree-searching algorithms would incorporate operational constraints that would limit the eligibility of a specific aircraft tail number to cover a given flight segment.

The primary model solves the problem of aircraft reassignment for all operational aircraft in the fleet. It can be best described as a hybrid of the traditionally defined fleet assignment problem and the aircraft routing/rotation problem. The following terms are defined prior to the statement of the model:

Indices

F	set of all flights f
F(j,k)	subset of flights that can be assigned to aircraft k at station j
F(i,p)	subset of flights arriving/departing from station i in time period p
N	set of all feasible flight sequences for all aircraft in the fleet
N(k)	subset of all feasible sequence of flights for aircraft k
K	set of all aircraft k in the fleet
K(t)	subset of aircraft of type t in the fleet
K(i, p)	subset of aircraft scheduled to arrive at station I in time period p
K(t, i, p)	subset of aircraft of type t, scheduled to arrive at station i, in time period p

Known Variables

D_{ij}	actual passenger demand for flight (i,j)
f_{ij}	average fare per passenger on flight (i,j)
r_{ij}	goodwill value per passenger on flight (i,j)
t_{ij}	flight time for flight segment (i,j)
C_{ijk}	operating cost of assigning aircraft k to flight (i,j)
C_{ijo}	cost of cancelling flight (i,j)
M_{it}^{T}	maintenance/ground capacity for aircraft type t at station i at time T
AC_{it}^{T}	number of aircraft type t required at station i at time T

$SLOTS_{ip}$	number of landing slots available at station i during period p
$GATES_{ip}$	number of terminal gates available at station i during period p
$CREWS_{tip}$	number of crews for aircraft type t, available at station i during period p
CAP_k	seating capacity of aircraft k
$TIME_k$	legal flight time remaining on aircraft k before maintenance is required
$CYCLE_k$	maximum number of flight cycles permitted on aircraft k
α_{ijn}	equals one if flight sequence n contains flight segment (i,j)
C_{nk}	cost of assigning flight sequence n to aircraft k

The decision variables involved are:

X_{nk} = 1 if flight sequence n is assigned to aircraft k, 0 otherwise

Y_{ij} = 1 if flight (i,j) is cancelled, 0 otherwise

Z_k = 1 if aircraft k is assigned to maintenance, 0 otherwise

S_{ij} = amount of spilled passengers from flight (i,j)

The model can be expressed as:

Objective Function

$$\min \sum_{n \in N} \sum_{k \in K} C_{nk} X_{nk} + \sum_{(i,j) \in F} C_{ijo} Y_{ij}$$

where;

$$C_{nk} = \sum_{ij \in n} \left\{ C_{ijk} + \sum_{(i,j) \in F} r_{ij} S_{ij} - \min\left[D_{ij}, CAP_k \right] \cdot f_{ij} \right\} \forall k$$

subject to:

flight covering

$$\sum_{n \in N} \sum_{k \in K} \alpha_{ijn} \cdot X_{nk} + Y_{ij} = 1 \forall ij$$

aircraft covering

$$\sum_{n \in N} X_{nk} \leq 1 \forall k$$

aircraft utilization

$$\sum_{n \in N} \sum_{(i,j) \in n} t_{ij} \alpha_{ijn} X_{nk} \leq TIME_k \forall k$$

leg based demand covering

$$D_{ij} - \sum_{n \in N} \sum_{k \in K} \alpha_{ijn} \cdot CAP_k \cdot X_{nk} - S_{ij} = 0 \forall ij, S_{ij} \geq 0$$

and further, subject to additional operational constraints:

crew availability

$$\sum_{k \in K(t,i,p)} \sum_{n \in N} \sum_{ij \in F(i,p)} \alpha_{ijn} \cdot X_{nk} \leq CREW_{tip} \forall t, i, p$$

ATC slot allocation

$$\sum_{k \in K(j,p)} \sum_{n \in N} \sum_{ij \in F(j,p)} \alpha_{ijn} \cdot X_{nk} \leq SLOTS_{j,p} \forall j, p$$

Gate allocation

$$\sum_{k\in K(j,p)} \sum_{n\in N} \sum_{ij\in F(j,p)} \alpha_{ijn} \cdot X_{nk} \leq GATES_{j,p} \forall j, p$$

Conservation of flow

$$\sum_{n\in N} \sum_{k\in K(t)} \sum_{ij\in F(j,p)} \alpha_{ijn} \cdot X_{nk} \geq AC_{jt}^{T} \forall j, \forall t$$

Maintenance allocation

$$\sum_{n\in N} \sum_{k\in K(t)} \sum_{ij\in F(j,p)} \alpha_{ijn} \cdot X_{nk} \leq M_{jt}^{T} \forall j, \forall t$$

The objective function sums over all the potential flight sequences and scheduled flights implicitly, the costs associated with reassigning flights to operational aircraft within the confines of the available resources. These cost coefficients include aircraft direct operating costs, predetermined spill costs, and operating revenue. Operating revenue is determined based on the actual passenger loads for each scheduled flight, and incorporates the impact of schedule delays in terms of recapture, passenger retention, and lost passenger goodwill. Spill costs account for the predisposed impact of spilling passengers on a given flight. Direct operating costs include fuel, cockpit crew costs, direct maintenance and ownership costs, accounting for all costs that are generally allocated against the actual flying time of the aircraft.

The flight covering constraint sums over all candidate flight sequences and has a right hand side coefficient of one, to ensure that each flight is either covered by one aircraft at a given time, or is cancelled. The coefficients for each flight sequence is determined using a specialized tree-searching algorithm, and have value one if the given flight is part of the candidate sequence of flights. The aircraft covering constraint sums over all flight sequences to ensure that each aircraft is assigned to no more than one sequence at a given time. The aircraft utilization constraint ensures that for each aircraft, the potential sequence of flights does not exceed the number of available flight time left on the aircraft before scheduled maintenance. The leg based demand constraint accounts for the accommodation of passengers on each flight segment, and is implicitly considered in the solution procedure. This constraint also serves as a definition of passenger spill in the model.

In addition, there are five additional operational constraints that have been considered for the airline recovery and flight rescheduling model. These include constraints on crew availability, ATC slot allocation, gate allocation, maintenance planning allocation, and conservation of flow (aircraft balance) at the end of the resolution horizon. The crew availability constraint ensures that the number of outbound flights at a given station within a given time period does not exceed the number of crews available at the station. The ATC slot allocation constraint limits the number of arriving flights to an airport with a given period, based on slot information provided by the ATC system. The gate allocation constraint limits the number of operational aircraft at the terminal based on the maximum number of gates available at the given airport. It is likely to be satisfied by the Flight Service Schedule if all gates are available, unless earlier flights were delayed in the system.

Similarly, the maintenance allocation constraint ensures that the number of aircraft assigned to a given maintenance station (overnight) does not exceed the capacity of the base, and the corresponding manpower available. The constraint on the conservation of flow ensures that the aircraft balance for each station at the end of the resolution horizon, corresponds to the number of aircraft "positioned" in the existing maintenance routing plan, which existed before the irregularity occurred in the system.

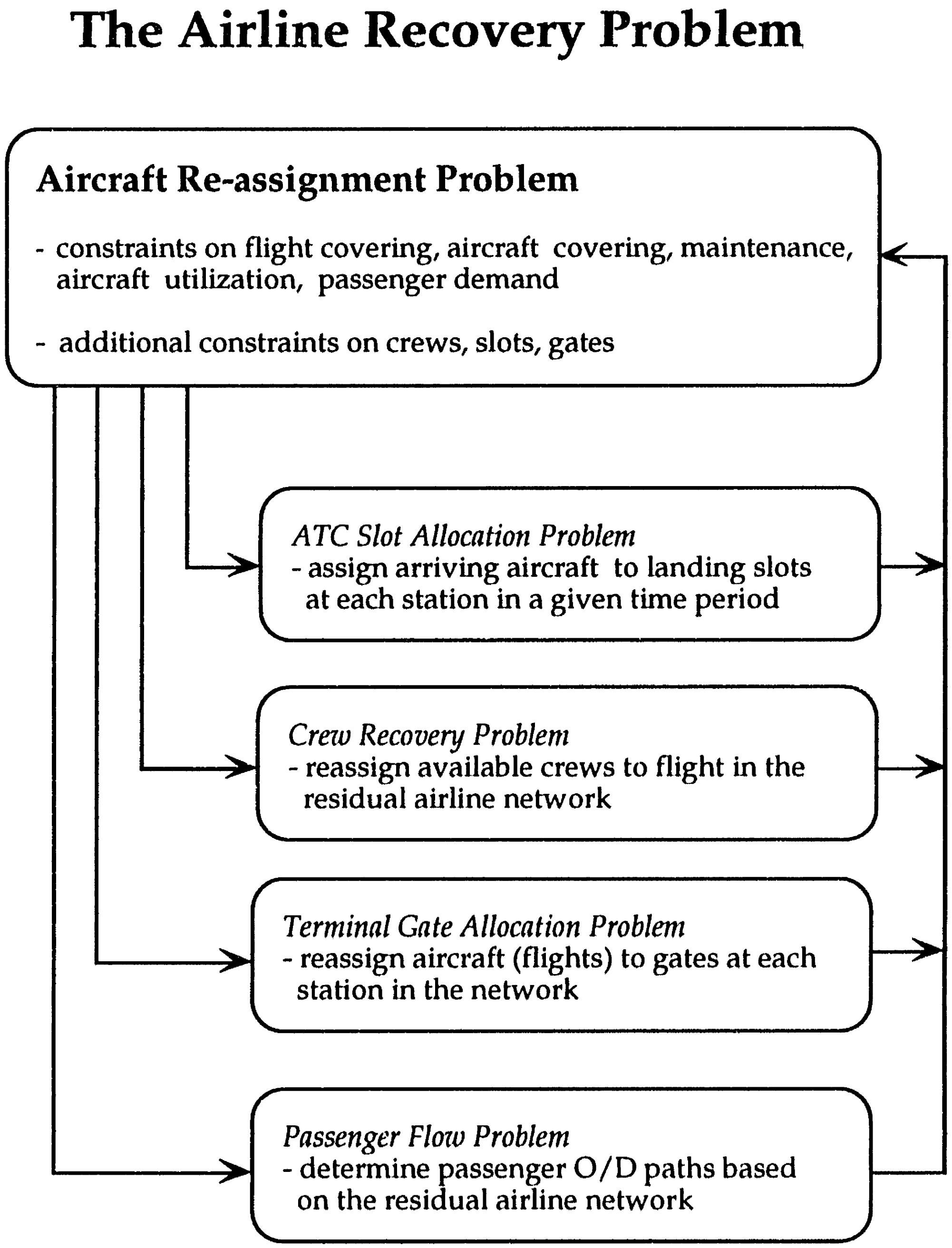

Figure 3-2 Decomposition of the Airline Recovery Problem

It is important to point out that these auxiliary constraints are best described as soft constraints, as the actual value of the right hand side coefficients should be ideally determined dynamically during the solution process.

3.3 Auxiliary Problems

Each of these additional constraints may lead to its own sub-problem for the reassignment of the given resource to each operational flight. The actual scheduled flights considered in each sub-problem would depend on the outcome of the main airline recovery problem. As outlined in Chapter 2, significant work has been done on the topics of slot allocation, crew scheduling and recovery, and on the general topic of resource allocation. The envisioned subproblems of this mathematical formulation would share many of the characteristics of such models and corresponding solution methodology. The overall framework of the decision model is outlined in Figure 5-2. The primary focus of this dissertation is to develop the formulation of the airline recovery problem with an emphasis on the aircraft rescheduling aspect of the problem.

3.4 ATC Slot Allocation Problem

The ability of US domestic carriers to freely assign individual flights to prescribed landing slots under an ATC ground delay program is an underlying assumption in the overall airline recovery problem formulation. As such, each flight has a certain value associated with it, and the assignment of flights to slots can be modelled using the classical transportation assignment problem. The following model is a representative formulation of the slot allocation problem. Under a typical operating situation, several airport stations would be affected by ATC slot restrictions, and the assignment problem would incorporate each airport in the decision process. As discussed in Chapter 2, more elaborate models for this problem and an extensive overview on this problem can be found in Carlson [76].

This model solves the problem of slot allocation for all operational flights in the airline's network. It can be expressed as:

$$\min \sum_{f\in F(j,t)} \sum_{t\in T} C_{fjt} X_{fjt}$$

subject to;

$$\sum_{t\in T(j)} X_{fjt} = 1 \forall j, t$$

$$\sum_{f\in F(j,t)} X_{fjt} \leq SLOTS_{jt} \forall j, t$$

where;

X_{ftj}	equal to one if flight f is assigned to slot t at station j, 0 otherwise
F	set of all operational flights
T(j)	set of all landing slots at station j
F(j,t)	subset of flights arriving at station j that can be assigned to time slot t
C_{fjt}	Cost of assigning flight f to landing slot t at station j
$SLOT_{tj}$	Number of arrivals possible at station j at time t

The cost parameter would reflect the value of a given flight to the airline based on issues such as the total passenger delay time, or the total operating costs. The actual form of this coefficient could be adjusted by the airline controller. The first constraint ensures that each flight is assigned to only one landing slot time, and the second places a limit on the number of flights assigned to slots at a given time t.

3.5 The Crew Recovery Problem

The rescheduling of flights in the airline network is affected by several operational constraints as outlined in the formulation, but it important to point out the level of complexity which results from the crew constraints. Crew scheduling is by far the most complex aspects of the airline planning process, and the ability to reschedule crews will depend on the actual operational flights, which in turn, will depend on the availability of crews at each station. Unlike all other resources in this system, the movement of the crew members adds significant complexity in trying to solve the flight rescheduling problem. Again, this sub-problem would be solved iteratively, and the resulting number of legal flight crews at each station within a given time period would then be updated in the main problem after each iteration. The following formulation of the crew recovery problem is based on research of Lettovsky [2] on the topic.

This model solves the problem of crew rescheduling for all legal crew members "displaced" in the network. It is based on the assumption that the airline has the ability to reassign crew members to modified bidlines without the consent of each individual, provided the crew member is able to maintain legality throughout the network. The model can be expressed as:

$$\min \sum_{m \in M} \sum_{p \in P(m)} \sum_{f \in p} C_{fm} \delta_{fp} X_{pm}$$

subject to;

$$\sum_{p \in P(m)} X_{pm} \leq 1 \forall m \in M$$

$$\sum_{m\in M}\sum_{p\in P(m)} \delta_{fp}X_{pm} \geq CREW_f \forall f \in F$$

$$\sum_{p\in P(m)}\sum_{f\in p} \tau_f\delta_{fp}X_{pm} \leq \alpha_m \forall m \in M$$

$$\sum_{p\in P(m)}\sum_{f\in p} d_f\delta_{fp}X_{pm} \leq \beta_m \forall m \in M$$

where;

X_{pm}	equal to one if crew path p is assigned to crew m
C_{fm}	Cost of assigning flight f to crew member m
δ_{fp}	equal to one if crew path p contains flight f
F	set of all operational flights
M	set of all available crew members m
P(m)	set of all possible crew paths for crew member m
α_m	amount of legal flying time remaining for crew member m
β_m	amount of legal duty time remaining for crew member m
τ_f	total flying time for flight f
d_f	total duty time for flight f
$CREW_f$	Number of crew members required for flight f

The primary objective of this subproblem is to minimize the cost of reassigning crews to operating flights in the residual airline network in the aftermath of the irregularity. The first constraint ensures that each crew member is assigned to only one crew path at a given time, and the second constraint ensures that all operating flights have the adequate number of crew members on-board the aircraft. Constraints three and four in this model ensure that each crew member does not violate established FAA operating safety requirements.

3.6 The Gate Allocation Problem

After the flight rescheduling problem has been completely solved, the reallocation of flights to terminal gates would then be addressed, as some flights have the potential of being delayed, thereby losing their originally scheduled time slot at a given gate. As the number of aircraft on the ground is restricted by the number of available gates at each station in the solution of the primary aircraft problem, all operational flights can be accommodated. The only required task would be to re-assign aircraft (flights) to gates, taking into consideration such issues as passenger connectivity, gates handling constraints, and the availability of ground support services. The following model of the gate allocation problem is solely for outlining the resulting subproblem. A more comprehensive discussion of this subproblem can be found in Svrcek [35]. It is based on the assumption that an airline has to the ability to reassign aircraft to gates at will, provided the necessary airport operational regulations are satisfied. The model can be expressed as:

$$\min \sum_{f \in F} \sum_{g \in G(f)} C_{fg} X_{fg} \forall j \in J, p \in P$$

subject to;

$$\sum_{f \in F(p,j)} X_{fg} \leq 1 \forall g \in G(j), \forall p \in P$$

$$\sum_{g \in G(f,j,p)} X_{fg} = 1 \forall f \in F$$

where;

P set of time periods p considered at a given station j

F	set of all operational flights f
$F(p,j)$	subset of flights on the ground at station j during time period p
$G(j)$	set of all gates at station j
$G(f,j,p)$	subset of gates eligible for flight f at station j during time period p
C_{fg}	"cost" index for assigning flight f to gate g
X_{fg}	equal to one if flight f is assigned to gate g, zero otherwise

The objective of this model is to minimize the "cost" of the gate allocation decision. The actual content of such a cost function would depend on the operational philosophy of the airline, and would potentially take in consideration issues such as aircraft size, passenger walking distance, baggage transfer, and aircraft servicing requirements. The first constraint ensures that each gate is assigned to only one flight which is on the ground at a given station and time period. The second constraint ensures that each flight is assigned to only one gate at a time.

3.7 The Passenger Flow Problem

Although the actual passenger itinerary issues are not explicitly considered in this model formulation, the passenger flow problem has to addressed in the aftermath of the flight rescheduling decision. Based on the residual flight network, the airline has to reassign passengers to flights in such a way that some prescribed criterion in minimized. The decision objective of the passenger flow model would depend on the operational philosophy of the carrier. Examples of such objectives range from minimizing overall passenger delay time, to maximizing the passenger revenue "recovered" in the modified flight schedule; as passengers could be potentially lost to competing carriers. The model is based on the assumption that all spilled passengers of a specific "high-valued" origin-destination itinerary are recaptured, provided there is adequate capacity to accommodate such passengers. In effect, priority is given in the model to accommodate as many valuable passengers as possible in the residual flight network. Again, the value of each passenger would depend on the operational directives of the carrier.

The following formulation of the passenger flow problem is based on research currently being done at MIT on the topic of an origin-destination based fleet assignment model by Barnhart and Kniker [48]. In this representative form, the primary objective of the model is to maximize the recovered passenger revenue in the residual flight network, through the optimal reassignment of seats to origin-destinations itineraries on each operational flight. The model can be expressed as:

$$\max \sum_{i \in I} \sum_{p \in P(i)} f_i X_{ip}$$

subject to;

$$\sum_{i \in I} \sum_{p \in P(i)} \delta_{if} X_{ip} \leq CAP_f \forall f \in F$$

$$\sum_{p \in P(i)} X_{ip} \leq D_i \forall i \in I$$

where;

F	set of all operational flights f in the residual network
I	set of all potential origin-destination itineraries i at a given time
P	set of all potential passenger travel paths p in the residual network
P(i)	subset of paths that can be considered for a passenger with itinerary i
f(i)	average passenger revenue for itinerary i

CAP_f	capacity of the aircraft assigned to flight leg f
D_i	total number of passenger booked to travel on itinerary i
δ_{if}	equal to one if itinerary i contains flight leg f, zero otherwise
X_{ip}	number of passengers for itinerary i assigned to path p

The subset of passenger paths considered in the reallocation of passenger flows in the residual flight network would be generated depending on the operational constraints employed in the decision process (such as the maximum allowable delay for a given passenger). For each itinerary, it is assumed that one fare class exists; as in practice, ticketed passengers are not generally differentiated during this phase of the airline recovery process. The ability to accommodate as many revenue passengers as possible on the residual flight network could potentially influence flight reassignment decisions made in the main aircraft problem. For example, it may possible to ensure that certain origin-destination markets are covered within a given time period, thereby guaranteeing that certain "valuable" passengers are taken to their destinations in a timely fashion.

Chapter 4

Review of Network Flow Theory and Linear Programming Theory

4.1 Overview

The overall framework for the mathematical modelling and the solution methodologies for the airline recovery and flight rescheduling problem is based on network flow theory. A comprehensive review of network theory can be found in Network Flows : Theory, Algorithms and Applications (Ahuja, Magnanti, Orlin: Prentice Hall). The following sections discuss several algorithms that have been adapted, and further enhanced for solving the schedule recovery problem. These include a specialized multi-label shortest path algorithm, a multi-label out-of-kilter OKF algorithm, and a column generation procedure which uses the revised simplex algorithm. The out-of-kilter algorithm is used to solve the constrained minimum cost flow problem, and the multi-label shortest path algorithm is used to solve the constrained shortest path problem. In the next chapter, there is an extensive discussion of the solution methodologies developed, but first it is necessary to give an introduction to the underlying theory used in creating such methodologies.

4.2 The Constrained Minimum Cost Flow Problem

The specialized algorithm developed to solve this problem is based on concepts of the out-of-kilter (OKF) algorithm, originally developed by Ford and Fulkerson for circulation flows. The primary enhancement being a modified version of the tree searching procedure within the OKF algorithm, in which multiple parameter labels are monitored during the execution process, and the resulting minimum cost flow satisfies additional constraints of the flow, such as time duration of the total flow in the network. The name out-of-kilter reflects the fact that arcs in the network either satisfy the complementary slackness optimality conditions (in-kilter) or do not (out-of-kilter).

Complementary Slackness Optimality Conditions

Theorem A feasible solution is an optimal solution of the minimum cost flow problem if and only if for some set of node potentials p, the reduced costs Cijp and flow values Xij satisfy the following complementary slackness optimality conditions for every arc (i,j) in the network:

If Cijp greater than zero, then Xij equal zero
If flow Xij within arc limits, then Cijp equal zero
If Cijp less than zero, then Xij equal upper arc limit Uij

The out-of-kilter algorithm attempts to find the minimum cost cyclic flow in a network, within the prescribed constraints of the problem. The algorithm iteratively modifies arc flows and node potentials (later referred to as node prices) in a way that decreases the infeasibility of the solution and simultaneously moves the solution closer to optimality. The procedure concentrates on a particular out-of-kilter arc and attempts to put it in kilter. The algorithm does this in such a way that all in-kilter arcs stay in-kilter, whereas the state (kilter number) for any out-of-kilter arc either decreases or stays the same after each iteration. On each such iteration, the network is scanned, and the labelling process for increasing or decreasing a particular arc flow in the circulation is found.

```
algorithm out-of-kilter
begin
    Out-of-Kilter scan
    scan all arcs in the network to determine if any out-of-kilter arc exists
    define the residual network G(x) and compute the kilter number of arcs;
    while the network contains an out-of-kilter arc do
    begin
        select an out-of-kilter arc (p, q) in G(x);
        identify target node for the labelling process;
        while  target node not labelled do
        begin
            constrained forward labelling from opened nodes in the network;
            constrained reverse labelling from opened nodes in the network;
            if target node labelled, break;
            else if new labels, continue labelling;
            else, update node prices;
            if node price update not possible, STOP, infeasible flow;
        end;
        augment flow cycle;
        update kilter number of arcs in the network;
    end;
end;
```

Figure 4 - 1 Multi-label Out-of-Kilter OKF Algorithm

It is possible to identify potential cost reduction arcs in the network, where a negative cost cycle could be found using a set of temporary node prices (potentials) and reduced arc costs (c-bar) that can be determined using optimal tree construction techniques. If the flow in some arc is infeasible (i.e. exceeds upper/lower bounds), then the out-of-kilter arc can be scanned to bring it into feasibility. By scanning only the out-of-kilter arcs, and making the appropriate flow changes, it is possible to find a minimum cost, feasible circulation flow in the network for any values of the arc attributes. It is important to re-iterate that the primary decision parameter in the minimum cost flow problem is cost, but the feasible flow has to also satisfy the time constraints of the problem, which is incorporated into the searching procedure of the algorithm.

In order to implement the OKF algorithm, it is necessary to define the various out-of-kilter states for arcs, based on the reduced arc cost, and the current arc flow relative to the flow constraints placed on the arc.

Case 0	In-Kilter (no changes done to the network flow)	
	alpha	c-bar greater than zero, and flow equal lower arc limit
	beta	c-bar equal zero, and flow within arc flow range
	gamma	c-bar less than zero, and flow equal upper arc limit
Case 1	Out-of-Kilter (increase flow in arc if possible)	
	alpha 1	c-bar greater than zero, and flow less than lower arc limit
	beta 1	c-bar equal zero, and flow less than lower arc limit
	gamma 1	c-bar less than zero, and flow less than upper arc limit
Case 2	Out-of-Kilter (decrease flow in arc if possible)	
	alpha 2	c-bar greater than zero, and flow greater than lower are limit
	beta 2	c-bar equal zero, and flow greater than upper arc limit
	gamma 2	c-bar less than zero, and flow greater than upper arc limit

If it is found that an arc is in states Case 1 or Case 2, it is required that the flow in the network be modified to bring the arc into kilter. For the states alpha one, and beta one, it is necessary to increase the arc flow to reach feasibility. In state gamma one, the negative value of the reduced cost indicates the potential for reducing the cost of the flow by increasing the arc flow. For these three states, it is necessary to determine the possibility of increasing the circulation flow in order to find a least cost feasible flow. If the arc is found to be in state alpha two, it has a positive cost, but the possibility of reducing its flow will allow a reduction of the network total flow cost. In states beta two, and gamma two, it is necessary to reduce the arc flow in order to bring it into feasibility. Figure 4-1 summarizes the out-of-kilter algorithm, as it is used to solve the constrained minimum cost flow problem.

4.3 The Constrained Shortest Path Problem

The shortest path problem is one of the fundamental problems studied in the operations research field. Extensive research has been done on the topic, and a comprehensive summary of such work can be found in an article by Deo and Pang [101]. In the case of the constrained shortest path problem, many researchers have attempted to solve this problem through the use of modified algorithms which were originally designed to solve the shortest path problem. These algorithms make use of linear programming concepts such as the relaxation of the additional and complicating constraints on the problem in order to achieve a solution to the problem. In reviewing existing solution methodology developed to solve complex problems such as the constrained shortest path problem, the generalized permanent labelling algorithm (Desrochers and Soumis, 1984) appeared to be the most efficient algorithm available to solve the problem.

The generalized permanent labelling algorithm for the shortest path problem with time windows developed by Desrochers, et. al at the GERAD Institute, is modified to efficiently solve the shortest path problem with schedule time constraints. This algorithm is a variation of the Ford-Bellman algorithm for the shortest path problem, and assigns multiple labels to each node representing the cost and time constraint. During the solution procedure, the routes have to be compared based on the multiple criterion of the problem. Several labels have to be stored at each node in the network and they are used dynamically to calculate the labels of other nodes which satisfy all the side constraints on the problem, such as a maximum cumulative time on the routing.

The algorithm stores at each node multiple labels of time and cost, until a less costly and/or less travel time route arriving at the given node is found. At a given node, a new label is said to dominate an existing label if both its time and cost parameters are better than the "best" label to date. The set of labels stored at each node is dynamically managed in such a way that unnecessary or "dominated" labels are deleted from the linked list at each node in the network, and the label list is sorted in decreasing cost order. Each label corresponds to a different path through the network from the source to the given node, and is classified as being efficient (Desrochers and Soumis, 1988). An efficient path is defined as one such that all of its labels are efficient, and such paths are used to determine the constrained shortest path from source to sink in the network.

The underlying network used for the constrained shortest path problem is designed in such a way as to prevent any cycling in the solution procedure. It is important to point out that during the solution process, there is the possibility that all paths considered into a node result in efficient labels. Depending on the structure of the time-space network, there can be an exponential number of paths in the network, an exponential number of labels may exist, and as a result, the permanent labelling algorithm can take exponential time to solve. This issue has played a substantial role in the development and implementation of the modified algorithm, especially in the design of the data structures used in the labelling procedure. The following diagram summarizes the modified version of the generalized permanent labelling algorithm based on this implementation.

```
algorithm    modified generalized permanent labelling
begin
    Initialize all label values at each node
    Set "dominance label" at each node to zero cost and zero time
    Open source node
    while the network contains "opened" nodes do
    begin
        Scan all arcs from all opened nodes in the network
        Establish candidate labels based on dominance test (cost and time parameters)
        If cost or time is less than dominant label, store label; else discard new label
        Open nodes whose labels satisfy dominance test
        Update multiple attribute label linked list at each open/unscanned node
        Close scanned nodes at end of iteration
    end
    Select shortest path from source to sink in the network that satisfies schedule
    constraints
end
```

Figure 4 - 2 Modified Generalized Permanent Labelling Algorithm

4.4 Column Generation Procedure

The column generation method is based on the decomposition principles of Dantzig-Wolfe, and it takes advantage of the premise that it is not necessary to store the complete constraint matrix during the solution process, and that columns can be generated only on a "as-needed" basis. The Dantzig-Wolfe decomposition technique was originally developed to solve large scale, structured linear programming problems. Based on the solution of the coordinating restricted master problem, the underlying subproblems are modified and iteratively solved until a prescribed criterion is satisfied in the problem.

The process of implicit column generation using the revised simplex method is based on the principle that the reduced cost of any feasible variable in the restricted master problem should be non-negative in any optimal solution to a minimization problem. The overall column generation procedure is more or less an extension of the simplex method, in which subproblems and the restricted master problem are iteratively solved until the optimal solution is achieved. The form of the subproblem will depend on the underlying characteristics of the problem being considered, and it was established during the course of the research project that both the constrained minimum cost flow problem, and the constrained shortest path problem discussed above were applicable as subproblems to the flight rescheduling problem.

During the column generation procedure, the large scale linear programming problem is classified as the master problem MP and can be represented by the following mathematical formulation (Bradley, et. al) :

$$Z^*: \quad \text{Min } z = C_1X_1 + C_2X_2 + \dots + C_nX_n$$

subject to;

$$a_{i1}X_1 + a_{i2}X_2 + \dots + a_{in}X_n = b_i \qquad (I = 1, 2, \dots, m)$$

$$Xj >= 0 \qquad (j = 1, 2, \dots, n).$$

As in decomposition, an assumption is made a priori that certain variables, $X_{k+1}, X_{k+2}, \ldots, X_n$ are nonbasic variables with value zero. The resulting linear program is described as being a restricted problem, and is referred to as the restricted master problem RMP.

$$Z^K: \quad \text{Min } z = C_1X_1 + C_2X_2 + \ldots. + C_kX_k$$

subject to;

$$a_{i1}X_1 + a_{i2}X_2 + \ldots + a_{ik}X_k = b_i \qquad (I = 1, 2, \ldots, m)$$

$$Xj >= 0 \qquad (j = 1, 2, \ldots, K).$$

where;

Π_i^K are the optimal shadow prices for each constraint equation

From linear programming theory, the solution to the restricted master problem if feasible, may be optimal to the master problem if and only if the simplex optimality conditions are satisfied. Let $\Pi_1^K, \Pi_2^K, \ldots \Pi_m^K$ denote the optimal dual variables for the restricted master problem, and as such, the reduced cost C-bar$_j$ of variable j is defined by:

$$\text{C-bar}_j = C_j - \text{ sum over m } \{\Pi_i^K a_{ij}\}$$

The simplex optimality conditions state that the solution is optimal if all reduced costs in the restricted master problem are non-negative, that is C-bar$_j$ is greater than or equal to zero. If this condition is met, the original master problem has been solved without explicitly using all the constraint data or solving the full master problem. If any of the reduced costs are negative, the corresponding variable (column) would be introduced into the basis of the restricted master problem and re-optimized using the revised simplex method. The procedure used to determine the reduced cost of each variable is itself an optimization problem, and is generally referred to as the subproblem.

An overview of the complete column generation procedure for minimization problems is summarized in the Figure 4 - 3. The efficiency of the solution methodology is a result of its ability to take advantage of the underlying structure of the subproblems, and to obtain an optimal solution before numerous columns have been added to the restricted master problem.

```
algorithm    column generation using revised simplex method
begin
    establish a restricted master problem with a feasible subset R of columns;
    while simplex optimality conditions are not met do
    begin
        solve the RMP to optimality over the restricted subset;
        obtain dual variables from existing solution;
        using the dual variable, update subproblems and solve to determine new
        variable (columns) to be added to the restricted master problem;
        if minimum reduced cost column has a non-negative reduced cost,
        STOP, global optimality.
        otherwise, add minimum reduced cost column to the restricted subset R.
    end
end
```

Figure 4 -3 Column Generation Procedure

The application of the column generation procedure in solving the airline flight rescheduling problem is complicated by the fact that each aircraft in the fleet has to represented as an individual commodity in the problem, and this has significant impact on the overall dimensions of the problem. The ability to solve such large-scale multi-commodity flow MCF problems calls for the reformulation of the generic assignment problem as a path based formulation instead of an arc based formulation, as was outlined in Chapter 3. Based on the flow decomposition theorem of network flows, it is possible to decompose optimal arc flows into path flows such that mass balance conditions are satisfied in the problem. A comprehensive discussion of the column generation procedure applied to multicommodity flow problems can be found in Network Flows: Theory, Algorithms and Applications [91].

The underlying principles are the same for the path based formulation, but there are significant benefit through constraint size reduction, and the resulting solution time for the problem being shortened. For a network with n nodes, m arcs, and K commodities, the path formulation problem contains m + K constraints, in addition to any non-negativity restrictions imposed on the path flow variables. On the other hand, the arc based formulation will have m + nK constraints since it contains one mass balance constraint for every node and commodity combination. Based on the resulting structure of the constraint matrix, it is possible to apply a specialized version of the simplex method such as the generalized upper bounding (revised) simplex method to efficiently solve the path flow formulation of the problem.

It is important to point out that the immense number of potential path possibilities for each commodity in the problem may have a negative impact on the solution time, and overall algorithm efficiency. However, from linear programming theory, it is known that at most K + m paths carry positive flow in some optimal solution to the problem. The implementation of the generalized upper bounding linear programming procedure enables one to take advantage of this observation. At each step of the revised simplex method, a basis is maintained for the problem, which is used to determine the vector of simplex multipliers for each constraint.

In the path-based formulation, there will be a dual variable wij for each arc constraint in the matrix, as well as a dual variable σk for each commodity demand constraint in the problem. The resulting reduced cost expression for each path flow variable will be given by;

$Cp_{\sigma,w}$ = Sum over all (i,j) in P $\{C_{ij}^{k} + w_{ij}\}$ - σ^{k} for each commodity k

As in the arc based formulation case, it is required for all the reduced costs to be non-negative for optimality in any minimization problem. The complementary slackness conditions for optimality require that:

1) the dual variable w_{ij} of an arc (i,j) is zero if the optimal solution does not use all of the capacity of the arc.

2) the modified path cost "sum over all P $(C_{ij}k + w_{ij})$" for each path connecting the source node sk and the sink node tk of commodity k must be at least as large as the commodity cost σ^{k}

3) the reduced cost must be zero for any path P that carries flow in the optimal solution.

Based on these optimality conditions, it can be stated (Ahuja et. al):

> σ^{k} *is the shortest path distance from source sk to node tk with respect to the modified costs* $c_{ij}k + w_{ij}$ *and in the optimal solution every path from node sk to node tk that carries a positive flow must be a shortest path with respect to the modified costs.*

This result shows that the arc costs w_{ij} permit the decomposition of the multicommodity flow MCF problem into a set of independent "modified" cost shortest path problems.

Chapter 5

Solution Approach

5.1 Overview

In developing solution methodologies for the flight rescheduling problem, the role of the airline operations controller was a constant factor in the design process. It was identified that any decision procedures and methodologies should have the ability to incorporate the high degree of uncertainty which exist in the daily operations of an airline, wherein it looked at problems from a global perspective, rather than on a localized decision level. During the development phases, several factors were considered including the ability to have multiple aircraft fleet switching, to combine the decision on flight delays and cancellations, to consider the effects of crew scheduling problems on the hybrid fleet assignment/aircraft routing problem, as well as the current solutions procedures currently in use at airline operation control centers.

The ability to solve the aircraft reassignment problem in real-time dictates very efficient solution procedures and methodologies which will provide the user with the best possible options. A trade-off has to be made between the optimality of the solution versus the solution time, as the airline operations controller may be required to address several irregularities during a given shift period. During the initial development phase, it was uncertain if the real-time decision requirements would demand heuristic procedures for the resolution process. The following section will present an overview of several solution methodologies that have been developed throughout the course of the research program, that are yet to be validated and tested with a real world case study.

Each of the solution procedures, whether heuristic or optimal, was developed around the framework of a three phase decision process. These are:

Generate

Potential aircraft rotations that met all operational constraints, using modified tree search algorithms on a sub-graph of the overall network schedule map

Assign

Aircraft rotations to each operating aircraft while optimizing specified objective (e.g. maximize profit). If there are less aircraft than rotations, some flights will be assigned to "cancellation" rotations

Revise

Overall network structure, adjusting scheduled arrival and departure times of each flight, reflecting the output of the ASSIGN module

The following solution procedures have been developed and implemented as computer algorithms using the C++ programming language. The optimization portion of the optimization methodology was developed around the CPLEX callable programming library, which consists of a wide array of mathematical programming solution procedures such as the revised simplex method, and the branch and bound method. A comprehensive discussion of these solution procedures can be found in Applied Mathematical Programming (Bradley, Magnanti, Hax: Addison-Wesly 1983) and Network Flows: Theory, Algorithms and Applications (Ahuja, Magnanti, Orlin: Prentice Hall 1993).

Option 1: Heuristic

The flight rescheduling problem is solved using specialized tree searching procedures, based on network flow theory. At each iteration, a suboptimal assignment of an aircraft to a generated sequence of flights is made using a prescribed decision matrix.

Option 2: Optimization

The flight rescheduling problem is solved as a large scale set-packing problem, in which sequence of flights are generated on an underlying structured sub-problem and optimally assigned to operational aircraft using the revised simplex method. This solution methodology is similar to state-of-the-art procedures used to the solve the airline crew scheduling problem.

5.2 Pre-Processing Procedures

The implementation of the solution procedures includes the generation of flight delay arcs (replica arcs) and ground arcs in the time-space network, based on information from the originally scheduled revenue flights in the airline network, and established operational philosophies and requirements of the carrier. These include, but would not limited to operational limitations such as the maximum allowable delay for flights at a given station and time period, passenger connectivity issues, arrival-departure bank integrity, the ability of a given aircraft to operate a specific flight based on range capability, overwater requirements, or type of aircraft originally assigned to the flight, and the ability to cancel a given flight in the resolution process. Information for all operational aircraft in the fleet and for scheduled revenue flights are inputted to the computer module, and the required arcs are automatically generated to create the directed time-space network, which was described in Chapter 3.

The generation of the delay arcs in the time-space network enables the solution procedures to efficiently make trade-offs between cancelling and delaying each individual flight in a single decision process. The number of replica arcs for a given flight would be restricted such that cycling in the network would be prohibited, i.e. to prevent multiple covering of the same flight in a generated sequence of flights. This approach to the flight delay issue was taken to allow the delay of individual flights, independent of upstream effects in the network, thereby minimizing delay propagation. Concern was also given to the impact of the increase in the number of arcs in the network to the overall size of the problem, and the resulting solution time requirements. Each replica arc would be coupled to the corresponding original flight arc such that any decisions about the flight would be reflected on all copies of the flight in the network. The network generation procedure is summarized in figure 5-1.

```
procedure    delay arcs and ground arcs generation procedure
begin
    Read in flight information from data file
    Generate replica flight arcs based on operational constraints
    Generate chronological event list of all potential aircraft movement activity at
    each station, including delay (replica) flight arcs
    Generate ground arcs between consecutive "nodes" using sorted event lists
    Build airline network of flight arcs, delay flight arcs, ground arcs and cycle arcs
    Create specialized duplicate network for each aircraft in the fleet, based on
    operational capabilities and constraints.
end
```

Figure 5-1 Network generation Procedure

It is important to re-iterate that one of the driving design parameters in developing these solution methodologies was the desire to provide "real-time" decision making capabilities to the airline controller. The following paragraphs discuss each solution procedure developed, outlining the main phases of the solution process.

5.3 Greedy Heuristic Solution Procedure

The application of network based algorithms to solve the flight rescheduling problem is a result of the underlying structure of the problem. As outlined in Chapter 3, the time-space network representing the airline's flight network is acyclic and as such, the modified multiple criterion permanent labelling algorithm for the constrained shortest path problem or the modified out-of-kilter minimum cost flow algorithm presented in Chapter 4 can be used effectively in the solution of the three dimensional assignment problem. In attempting to solve this complex problem in a real-time setting, a greedy heuristic methodology was initially developed based on concepts of network theory.

The overall greedy heuristic methodology is summarized in Figure 5-2 and Figure 5-3. In the first case, the primary concern is to assign the most "critical" aircraft first, i.e. based on an important parameter such as the amount of remaining maintenance time available of the aircraft. In the second case, the primary decision parameter is to maximize the objective function based on the prescribed decision criterion such maximizing operating revenue, incorporating both operating and potential passenger spill costs. It is important to point out the role of the decision maker in implementing these solution methodologies as it is necessary for such a person to prescribe which parameter is being used as the decision criterion.

```
methodology   greedy heuristic solution procedure one
begin
    Initialize parameters for tree searching algorithms
    Input flight and aircraft data to the data structures
    Create operational constraint decision matrix
    Create "specialized" flight networks for each aircraft
    Sort aircraft based on remaining maintenance time available
    while  any operational aircraft is not assigned to a flight sequence  do
    begin
        Determine candidate sequence of flights for most "critical" unassigned aircraft
        in the fleet using modified tree searching algorithm, which meet all
        operational constraints
        Select aircraft assignment which maximizes decision criterion
        Delete "covered" flights from residual airline network
        Update operational constraints information, e.g. gate utilization
    end
end
```

Figure 5-2 Greedy Heuristic Solution Procedure One

As an example, the primary objective of the problem could be to minimize the amount of wasted maintenance time left over on each aircraft at the end of the decision horizon. In other cases, the airline controller who would serve as the decision maker may find it necessary to minimize the overall cost of resolving the flight irregularities over the prescribed time horizon.

The assignment of operational aircraft to potential flights is restricted by several operational constraints as outlined in the mathematical formulation described in Chapter 3. These include conditions on the number of arriving flights at a given station within a given time periods because of gate capacity, and landing slot availability. On the other hand, departing flights are constrained by availability of legal crew members to staff all operating flights. Once a decision has been made to assign an aircraft to a sequence of flights using the heuristic solution methodology, the number of limited resources available at each station has to be automatically updated. This is achieved by monitoring the flight assignment procedure, and keeping track of the resulting flight covering.

```
methodology    greedy heuristic solution procedure two
begin
    Initialize parameters for tree searching algorithms
    Input flight and aircraft data to the data structures
    Create operational constraint decision matrix
    Create "specialized" flight networks for each aircraft
    while  any operational aircraft is not assigned to a flight sequence  do
    begin
        Determine candidate sequence of flights for each unassigned
        operational aircraft in the fleet using modified tree searching algorithm
        Select aircraft assignment which maximizes decision criterion
        Delete "covered" flights from residual airline network
        Update operational constraints information, e.g. gate utilization
    end
end
```

Figure 5-3 Greedy Heuristic Solution Procedure Two

5.4 Optimization Solution Procedure

A logical extension to the greedy heuristic procedure is a large-scale integer programming set-packing problem, which can solved using the branch and bound procedure. Initially, a linear programming LP relaxation of the complex assignment problem is solved using the efficient implicit column generation solution methodology outlined in Chapter 4. The underlying structure of the problem allows the utilization of the constrained shortest path problem as the subproblem in the solution process, which is solved using the multi-labelling permanent labelling algorithm given in Chapter 4. The output of each subproblem is used to generate a path (column) for addition to the restricted master problem, provided it meets the necessary conditions for inclusion.

```
methodology    integer programming optimization solution procedure
begin
    Initialize parameters for tree searching algorithms
    Input flight and aircraft data to the data structures
    Create "specialized" flight networks for each aircraft
    while   eligible columns exist for addition to the master problem   do
    begin
        Generate flight sequence for each aircraft fleet using modified tree searching
        algorithm
        Determine "aircraft" column corresponding to each variable and add to the
        restricted master problem
        Using the revised simplex method, determine the best aircraft sequence
        currently available
        Using dual variables found in revised simplex procedure, adjust costs on each
        corresponding flight arcs in each specialized aircraft network
    end
    Solve restricted master problem as an integer programming problem using the
    branch and bound solution procedure
    Determine final aircraft assignment based on output of the IP solution procedure
end
```

Figure 5-4 Optimization Solution Procedure

Each column contains information on the sequence of flights to be covered by the aircraft, and as well as information on the corresponding operational constraints within the problem, such as landing slot utilization, gate utilization, and crew allocation.

5.4.1 Column Generation Solution Procedure

As discussed in Chapter 4, the underlying linear programming theory and network flow theory allow the development of efficient solution methodologies to solve the problem of aircraft and flight rescheduling. During the column generation process, the dual variables (multipliers) are used to price out the nonbasic variables (columns) by considering their reduced costs. The dual variables ensure that the reduced cost for every variable in the basis is zero. If any reduced cost is negative in a minimization problem, the method will introduce the corresponding nonbasic variable into the basis in place of one of the current basic variables, and recompute the simplex multipliers. In order to use column generation, the columns need to have structural characteristics which allows pricing out operations without explicitly considering every possible column in the problem.

The revised simplex method determines the dual variables so that they satisfy the following expression:

$$\text{sum over all } (i,j) \text{ in } P \ \{c_{ij}k + w_{ij}\} = \sigma k \qquad \text{for every path } P \text{ in the basis}$$

In effect, the revised simplex procedure attempts to check if all reduced cost of variables are non-negative for optimality, such that:

$$Cp_{\sigma w} = \text{sum over all } (i,j) \text{ in } P \ \{c_{ij}k + w_{ij}\} - \sigma k >= 0 \qquad \text{for all } P \text{ in } Pk$$

or equivalent;

$$\text{Min [sum over all (i,j) in P } \{c_{ij}k + w_{ij}\}\text{] } >= \sigma k$$

The left hand side of this expression is the length of the time constrained shortest path connecting the source and sink nodes of commodity k with respect to the modified costs $c_{ij}k + w_{ij}$. If for all commodities k, the length of the constrained shortest path for that commodity is at least as large as its corresponding dual variable σk, the procedure will satisfy the complementary slackness conditions, and the solution will be optimal. Otherwise, based the constrained shortest path on the modified network, the reduced cost of the column (path) is less than the length σk for a given commodity k. By introducing this column into the problem basis, there will be an improvement to the objective function.

As a result, the changed basis will lead to new dual variables, and thus a modified shortest path distance σk between the source and sink nodes of the commodity k. At each iteration, the dual variables are found to ensure that the reduced cost of all basis columns is zero. Based on the new dual variables, the constrained shortest path problem would be resolved on the modified network, to determine whether any commodity path has a shorter length than its corresponding dual variable σk. If this occurs, the path is introduced into the problem basis, and the solution procedure will continue by alternatively finding new values for the dual variables for each arc constraint and for path length sigmak, and solving the constrained shortest path problem for each commodity k. The process is thus repeated iteratively until the linear programming complementary conditions are satisfied.

5.4.2 Column Generation Termination Mechanism

In order to effectively implement the column generation procedure in a real-time solution environment, the ability to prematurely stop the generation process can have a significant impact on the duration of the solution process. It is important for this mechanism to have a minimal effect on the quality of the LP relaxation solution of the problem, as this will be used as the lower bound for the integer programming branch and bound procedure. In reviewing the column generation procedure described in chapter four, one can identify several mechanisms which can be used to terminate the solution procedure, provided an aprior criterion is established within the solution module. For this research project, two such efficient stopping mechanisms were developed using concepts from linear programming theory; one being setting a tolerance on the reduced cost optimality conditions (less than zero), and the second being a variation of the lagrangian relaxation technique for the lower bound on the problem.

Based on Lagrangian relaxation theory, it is possible to establish both lower and upper bounds to the optimal solution of the resulting linear programming problem being solved by the column generation procedure, as this problem is equivalent to the LP problem that would exist during a lagrangian relaxation solution procedure (Network Flows: Ahuja, 1993). Using Z^* to denote the optimal objective function value of the multi-commodity flow problem and Z_{lp} to represent the optimal objective function value at any iteration in solving the path flow formulation of the problem by the revised simplex methodology. From linear programming theory, Z_{lp} corresponds to a feasible solution to the problem, such that $Z^* <= Z_{lp}$. From lagrangian relaxation theory, the optimal value L(w) of the lagrangian subproblem is a lower bound on Z^* for any value of the arc dual variables (prices) w. During the course of the column generation solution methodology developed to solve the airline rescheduling problem, the solution of each modified constrained shortest path subproblem at each iteration corresponds to solving the lagrangian subproblem with respect to the current arc prices w_{ij}.

The value of the lagrangian subproblem can be expressed as:

$$L(w) = \text{sum over } K \{ l^k(w) \} - \text{sum over all } (i,j) \text{ in } A \{w_{ij}\ u_{ij}\}$$

where $l^k(w)$ is the constrained shortest path length for all commodities k with respect to the modified costs $c_{ij}k + w_{ij}$, and u_{ij} is the upper bound on each arc. From the theory of Lagrangian relaxation;

$$L(w) <= Z^* <= Z_{lp}$$

It is important to point out that this stopping mechanism is based on the lower bound of the objective function value which is determined as a by-product of finding the constrained shortest path distances $l^k(w)$, as the algorithm is pricing out columns during the course of the column generation procedure. Based on an aprior tolerance range, the solution procedure can be prematurely terminated to obtain a near optimal solution to the relaxed linear program problem. The utilization of the revised simplex methodology guarantees that the objective value Z_{lp} of the LP problem (upper bound) is monotonically nonincreasing after each iteration of the algorithm. On the other hand, the value of the lagrangian subproblem L(w) need not decrease at each iteration, and as such, the stopping mechanism would use the largest value of L(w) as the best lower bound on the problem.

5.4.3 Branch and Bound Solution Procedure

After the successful completion of the column generation procedure, the resulting optimal solution to the relaxed LP problem is then used as the root node to the branch and bound procedure for solving the original flight rescheduling problem which is modelled as an integer programming problem. The branch and bound solution procedure is based on the ability to use derived lower bounds to the optimal solution as an algorithmic tool in reducing the number of computations required to solve the problem to optimality. This final phase of the solution methodology involves the solution of the integer programming problem which represents the combinatorial optimization nature of the complex reassignment problem.

During the branch and bound procedure, the feasible region F of the problem is systematically partitioned into subregions $F_1, F_2, \ldots F_k$ (Network Flows: Ahuja, 1993). If $\underline{X}$ denotes the best feasible objective function solution value after each iteration, either F_k is empty or X_k is a solution of a relaxation of the set F_k and $C\underline{X} <= CX_k$ for each subregion k. If these condition are satisfied, no point in any of the subregions can have a better objective function value than $\underline{X}$, and as such $\underline{X}$ solves the original optimization problem. If $C\underline{X} > CX_k$ for any region F_k, it would be necessary to subdivide this region by "branching" on some of the variables (i.e. dividing a subregion into two by setting $Xj = 0$ or $Xj = 1$ for some variable j to define two new subregions in the original problem). The solution procedure would then continue until the necessary optimality conditions are met, and the optimal solution is determined.

The development and implementation of an efficient branch and bound procedure can be greatly influenced by many design decisions including the branching strategy (order for choosing the subregions), the variable selection criterion for branching, the node selection in the branch and bound tree, an aprior objective solution optimality gap, the pricing algorithm, and the underlying solution algorithms. Each parameter listed above can have a significant impact on the quality of the final solution, as well as the solution time necessary for a particular problem.

Chapter 6

Summary and Conclusions

6.1 Discussion

The Airline Recovery Problem is by far one of the most complex problems facing airlines today. The ability to address flight rescheduling in the aftermath of irregular airline operations depends on the availability of up-to-date, and accurate operational information from all divisions of the airline. The underlying assumption of this research project is that an efficient information flow mechanism already exists in the airline's operation control center, and that airline controllers have full access to all relevant information and corresponding databases, in order to make informed decisions about the operations of the carrier.

The primary objective of this thesis has been to provide an introduction to the airline recovery problem, by presenting a comprehensive mathematical model formulation to the problem. Based on this formulation, solution methodologies and procedures have been developed using concepts of network flow theory, and linear programming theory. The development of the airline recovery problem has been greatly influenced by previous work on related airline scheduling topics, as well as communications with airline controllers, the potential end-users of the envisioned decision support tool.

In Chapter 2, an extensive literature review is given, discussing several topics which are closely related the problem of irregular airline operations. These include the fleet assignment problem, the aircraft routing problem, the crew scheduling problem, and the ATC slot allocation problem. Although these problems are primarily addressed during the strategic phase of the airline scheduling process, one can gain substantial insight into the problem of irregular airline operations through comparing and contrasting the airline recovery problem to these problems.

In Chapter 3, the mathematical formulation was presented, outlining the overall framework of the decision process. The primary emphasis of this thesis has been to address the main problem of the recovery process, efficiently reassigning aircraft to flights. The residual airline network is thus determined from the solution to the main problem, and this is used to determine new gate allocations, landing slot allocations and to reassign available crews to the remaining operational flights. In addition, it is necessary to determine updated passenger origin-destination paths based on the residual network of flights.

In Chapter 4, a review of the necessary linear programming theory and network flow theory is presented, to provide the foundation for the development of the solution methodologies and procedures. The time-space representation of the airline flight schedule allows the use of efficient network flow based algorithms for the solution of the underlying problem of determining a time constrained sequence of flights. Variations of the out-of-kilter algorithm [Ford and Fulkerson], and the multi-label constrained shortest path permanent labelling algorithm [Desrochers and Soumis] are discussed, underscoring their applicability to the airline recovery problem.

In Chapter 5, there is a discussion of prototype solution methodologies which can be used to efficiently solve the airline recovery problem. Based on preliminary research on the topic of irregularities, the issue of real-time solution capabilities has played a major role in the development of these methodologies and procedures. The underlying framework of the resolution process in to first generate potential aircraft flight sequences using specialized searching algorithms on customized flight networks for each individual aircraft. Using these flight sequences, aircraft are assigned to flights in order to maximize a predetermined criterion. In effect, the solution methodology calls for the solution of a three dimensional assignment problem, as it attempts to assign aircraft to flights and satisfy the maintenance compatible routings simultaneously. Once the solution process is completed, the overall airline network is updated to reflect changes in arrival and departure times based on suggested flight delays and potential flight cancellations which would result in the aftermath of the irregularity.

6.2 Directions for Future Research

Having presented an introduction to the airline recovery problem, and a framework for resolving operational irregularities, future research on these topics would warrant the validation of the solution methodology and procedures developed in this research. A comparative case study using real world airline operational data from a major US domestic carrier would provide an adequately sized dataset to test the robustness and practicality of the decision model. The results from the decision model using historical data could be compared to actual decisions made by the airline in the aftermath of an irregularity. Several statistics could be compared such as the percentage of flights delayed, cancelled, and the overall impact of the decision on the operations of the carrier, in order to establish the overall benefit of implementing such solution procedures.

Lessons learnt from such case studies may call for the development of alternative solution methodology based on the existing formulation. It is important for the researcher to be conscious of physical limitations which may exist as a result of CPU memory, and the processing speeds of existing computer systems. Another issue that may be important to address is the integration of a decision support tool based on this research into existing operations control centers. For example, how will the deployment of such a system impact the daily operations of airlines? In order to fully benefit from the use of any decision support systems, it is essential for the system to be fully compatible with existing procedures and operations.

Currently, several independent research projects are studying one of the auxiliary subproblems, but it is necessary for future researchers to consider the interaction between such subproblems, as decisions made in one problem can have a significant impact on the overall structure of other subproblems. The ability to solve these problems have warranted such problem decomposition, but with advances in computer technologies, larger and more complex hybrid problems should become more tractable and should be considered, as only then can mathematical modelling truly represent the real-world decision environment.

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