POTENTIAL ENERGY GRADIENT ON A PERFORATED PLATE

by

HOWARD M. MCDOWELL

Submitted in Partial Fulfillment
of the Requirements for the
Degree of Bachelor of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June, 1960

Signature of Author: Signature redacted ....
Department of Chemical Engineering, May 20, 1960

Certified by: Signature redacted ....
Thesis Supervisor

Accepted by: ....
Head of Department

420 Memorial Drive
Cambridge 39, Mass.
May 20, 1960

Professor T.W. Mix
Department of Chemical Engineering
Massachusetts Institute of Technology
Cambridge 39, Mass.

Dear Professor Mix:

This thesis is the result of my work with potential energy gradients on a perforated plate. I have made use of your derivations in both friction factors presented.

I would like to thank you for counsel and guidance during the work on this thesis.

Sincerely,

Signature redacted

Howard M. McDowell

CONTENTS

TABLES AND FIGURES

I. SUMMARY

Design of rectification or distillation columns which use either bubble caps or perforated plates requires consideration of fluid flow as well as vapor liquid contact. A quantitative method by which prediction of fluid flow characteristics can be made is necessary. Previous workers have used a Fanning type friction factor correlated with a plate Reynolds number to give estimates of drag losses. These friction factors required empirical correction terms to correlate the experimental data. The purpose of this thesis was to examine the flow on a plate considering potential energy as the driving force for flow.

The equipment used was a rectangular perforated plate across which water was pumped and through which air was blown. The potential energy was calculated at two places on the plate. Other properties such as foam density, velocity profile, and drag on a flat plate were also examined.

Two different friction factors were derived and were plotted against a dimensionless group which contained the variables affecting drag. The most useful term was α:

$$\alpha = \frac{\Delta\left(\frac{P.E.}{cm.^2}\right) b}{V_f \, V_g \, N\left(\rho_f \, b + 2h_{av}/g\right)}$$

This term was plotted against a turbulence factor

$$\frac{V_g \, h_{av}}{\mu \, g}$$

The six major conclusions obtained through this thesis are:

1. The effect of air momentum on the liquid was negligible.
2. There was a density gradient.
3. There was a high degree of verticle mixing on the plate.
4. There was a uniform velocity down the plate.
5. There was a drag gradient similar to the density gradient.
6. An average α can be used as to estimate the magnitude of potential energy or drag losses.

II. INTRODUCTION

A. General

The design and operation of distillation or rectification columns requires consideration of several factors balanced to achieve an optimum. In columns using either perforated plates or bubble caps, the most consideration is usually given to maximizing vapor-liquid contact. However, the most efficient contact system is usless unless the tower is also an operatable fluid flow device. A typical tower design is shown in figure (1). Vapor flows up through the perforations in the plate, making a foam from the liquid. The liquid enters onto the plate from a downcomer and the foam flows off over an exit weir. Two limitations are immediately obvious. If the resistance to flow on the plate is very high, the head of liquid in the downcomer may not exert enough force to drive the liquid across the plate. As the level of liquid in the downcomer rose over the exit weir, flooding would result. Another flow problem is pressure differential across the plate. As the pressure difference increases the vapor tends to flow through the low pressure side of the plates. The foam is no longer supported over the opposite perforations and will flow through them onto the next plate. This condition is known as dumping. In order to predict these conditions, a quantitative description of the flow process is necessary.

Several theses have been written attempting to describe the nature of the flow. Since the only resistance that the fluid could meet is drag on the walls and plate, the most common approach has been defining a Fanning type friction factor. This factor was then experimentally related to a plate Reynolds number. Knowing the variables contained in the Reynolds number, a property such as head loss can be predicted for a similar plate. Klein (1) did extensive work with bubble cap trays. His friction factor:

$$f_k = \frac{2 r_h F g_c}{V_f^2 N}$$

was correlated with a plate Reynolds number:

$$Re_k = r_h V_f \frac{\rho_f}{\mu_f}$$

Klein was able to obtain linear correlation on log-log paper if he multiplied the friction factor by a correction factor T. T was dependent on the weir height. Klein also investigated potential energy gradient for his

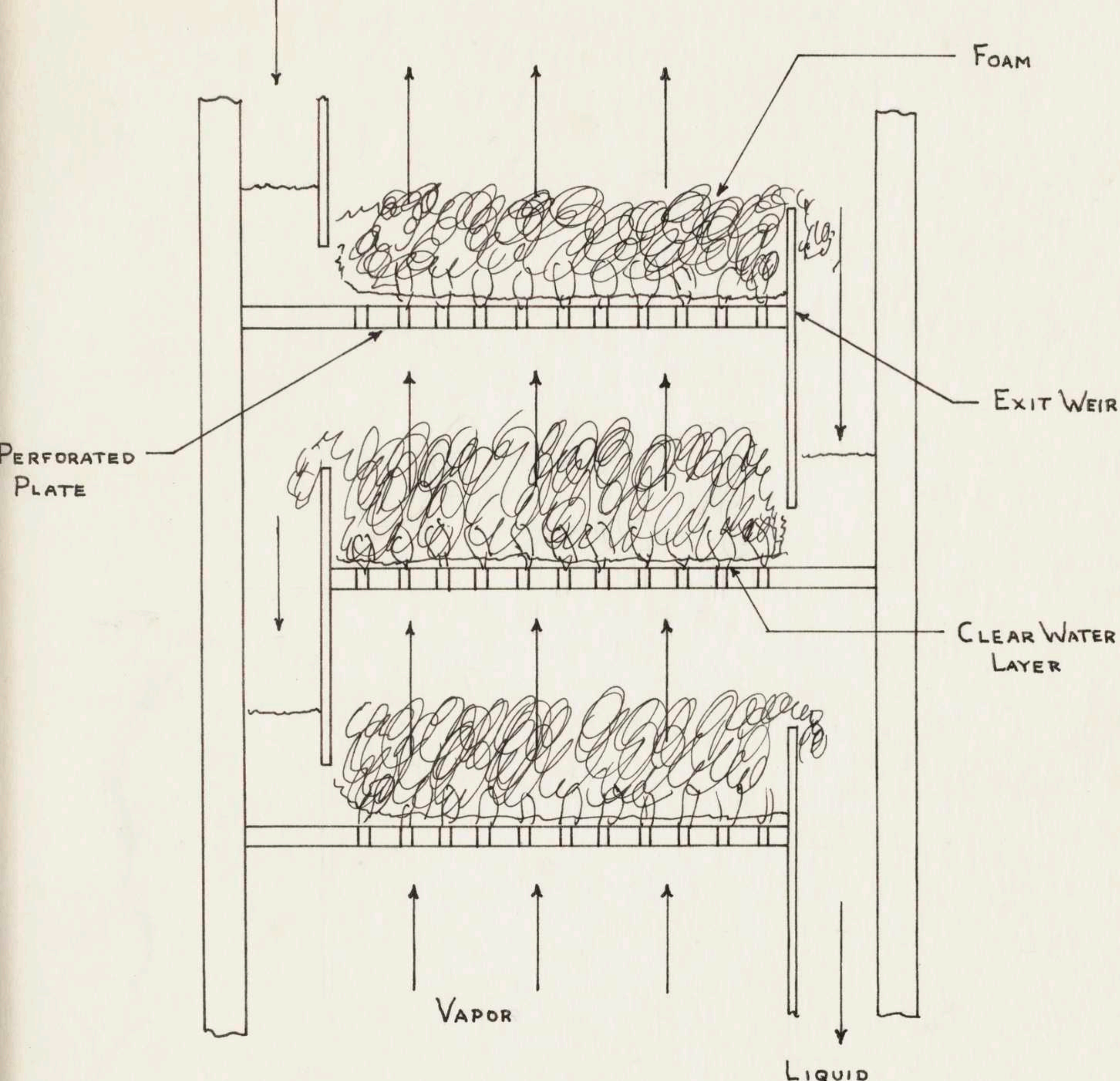


Figure 1 Tower Using Perforated Plate

systems, observing that it was the driving force of the liquid across the plate. Hucks and Thompson (2) also found correlation with this type of friction factor-correction factor vs. a plate Reynolds number for a perforated plate. Although both obtained correlations, a more basic approach is desirable. The friction factors were no more than adjusted pipe-flow terms and the correction factors were purely empirical terms to pull the results together. Mix (3) has suggested that Klein's finding of potential energy driving force be used in conjunction with two pictures of the foam flow to derive friction factors. One picture is straight drag on the wall considering uniform flow; the second is an application of the Reynolds analogy for turbulent flow. The latter considers momentum transfer to occur by circulation of fluid from the interior of the system to the walls and back.

Other interesting properties of the foam flow are the density as a function of position above the plate, the momentum transfer of gas to the liquid, and the velocity profile of the foam flow.

B. Drag Analysis

The potential energy of a particle is given by the following expression:

$$P.E. = \int_0^m gH \, dm$$

$$\frac{dm}{cm.^2} = \rho \, dH$$

so:

$$\frac{P.E.}{cm.^2} = \int_0^{H_t} \rho \, g \, H \, dH \qquad (1)$$

At any point in the fluid, since density is not constant:

$$\rho = \frac{dh}{g \, dH} \qquad (2)$$

This gives:

$$\frac{P.E.}{cm.^2} = \int_0^{h_t} H \, dh \qquad (3)$$

The drag is defined:

$$D = \tau A = \mu A \frac{dV}{dy} \qquad (4)$$

Considering the foam as a homogeneous mixture moving down the plate with a linear velocity profile and all transfer of momentum across a thin layer of liquid at the wall:

$$\frac{dV}{dy} = \frac{\Delta V}{\Delta y}$$

Since the velocity at the wall is zero:

$$\Delta V = (V_f - 0) = V_f$$

The conditions of flow can not be varied over a great range, so Δy is assumed to be proportional to a constant with the dimension of length. The proportionality is incorporated in the friction factor. Equation (4) becomes:

$$D = f' \mu A V_f = f' \mu V_f N(b + 2 H_{av}) \qquad (5)$$

Solving: $$f' = \frac{D}{\mu V_f N(b + 2 H_{av})} \qquad (6)$$

The drag must be equal to the potential energy loss:

$$D = \Delta \left(\frac{P.E.}{cm.^2} \right) b \qquad (7)$$

Using the results of equation (7) in equation (6):

$$f' = \frac{\Delta \left(\frac{P.E.}{cm.^2} \right) b}{\mu V_f N (b + 2H_{av})} \qquad (8)$$

The turbulence in the system, and hence the drag is affected primarily by the variables of superficial gas velocity, viscosity and head of liquid on the plate. An appropiate measure to plot f' against is a dimensionless group using these terms rather than the Reynolds term used in previous work. Such a group is:

$$\frac{V_g \; h_{av}}{\mu \; g} \qquad (9)$$

C. Reynolds Analogy Approach

The Reynolds analogy used in heat transfer (cf. 4) for turbulent transport of a mass or volume per unit time to the walls of the flow container is directly applicable to momentum transport. Denoting τ' as the drag per unit area:

$$\tau' = W \rho'_f \; V_f \qquad (10)$$

Where W is the volume of the foam taken to the wall per unit area per unit time. The foam on the plate is observed to be a mixture of a homogeneous foam and large slugs of gas. The material moved to the walls is the homogeneous foam with a density ρ'_f. However, only a certain fraction of the plate or walls can be in contact with this homgeneous foam at any instant. This fraction is taken to be $\frac{\rho_f}{\rho'_f}$, where ρ_f is the overall or average density on the plate including slugs. The rate of transfer is a function of V_g. The expression for W incorporates $\frac{\rho_f}{\rho'_f}$ and V_g with a proportionality constant α:

$$W = \frac{\rho_f}{\rho'_f} \; V_g \alpha \qquad (11)$$

This makes equation (10):

$$\tau' = \rho_f \, V_g \, V_f \, \alpha \qquad (12)$$

For the plate:

$$D_p = \int_0^A \tau' dA = \int_0^A \rho_f V_g V_f \alpha \, dA = \rho_{fp} V_g V_f \alpha A$$

$$= \rho_{fp} V_g V_f \alpha \, bN \qquad (13)$$

Where ρ_{fp} is the density of the foam at the plate.

For the walls:

$$D_w = 2\int_0^A \tau' \, dA = 2\int_0^A \rho_f V_g V_f \alpha \, dA = 2V_g V_f \alpha N \int_0^{H_{av}} \rho_f \, dH$$

From equation(2) $\rho dH = \dfrac{dh}{g}$

$$D_w = \frac{2 V_g V_f \alpha N}{g} \int_0^{h_{av}} dh = \frac{2V_g V_f \alpha N h_{av}}{g} \qquad (14)$$

$$\text{Total drag} = D = V_g V_f \alpha N \left(b \rho_{fp} + \frac{2h_{av}}{g} \right) \qquad (15)$$

From equation (7) the drag is also equal to $\Delta(\frac{P.E.}{cm.^2})b$. Equating and solving for α :

$$\alpha = \frac{\Delta(\frac{P.E.}{cm.^2})b}{V_g V_f N \left(b\rho_{fp} + \dfrac{2h_{av}}{g} \right)} \qquad (16)$$

This constant is analogous to a friction factor and can be plotted against the same dimensionless group which was determined in the previous section.

D. Effect of Gas Momentum

In the above analyses, the assumption was made that the head of liquid on the plate is independent of gas velocity. It is reasonable to as assume that the gas will transfer most of its momentum to the liquid during its passage. This effect can be derived using a maomentum balance (cf. Appendix) from below the plate to the space above the foam. The pressure drop across the foam is derived to be:

$$\Delta P = \rho_L g h_o - \rho_g v_g (v_g - v_h) \qquad (17)$$

The first term is the pressure loss due to the foam alone and the second term corrects for momentum loss of the gas. This correction term should equal the difference in measured head and the actual head, at any gas rate:

$$\rho_L g (h_o - h) \qquad (18)$$

E. Apparatus

Air and water were used for the purposes of this study. The equipment is outlined in figure (2). Water was fed onto the rectangular, perforated plate by pumps from feed tanks. There was a baffle immediately in front of the entrance pipe to diffuse the flow. After diffusion, the water flowed over an inlet weir, across the plate, over an exit weir and back into one of the feed tanks. Water flow was adjusted with either of the valves in the feed line. Air was supplied beneath the plate by a blower. The air rate was adjusted by the use of baffles on its inlet. Both the air and water were metered with manometers placed across orifice plates. A more complete description, with pictures, is contained in other theses (1) (5).

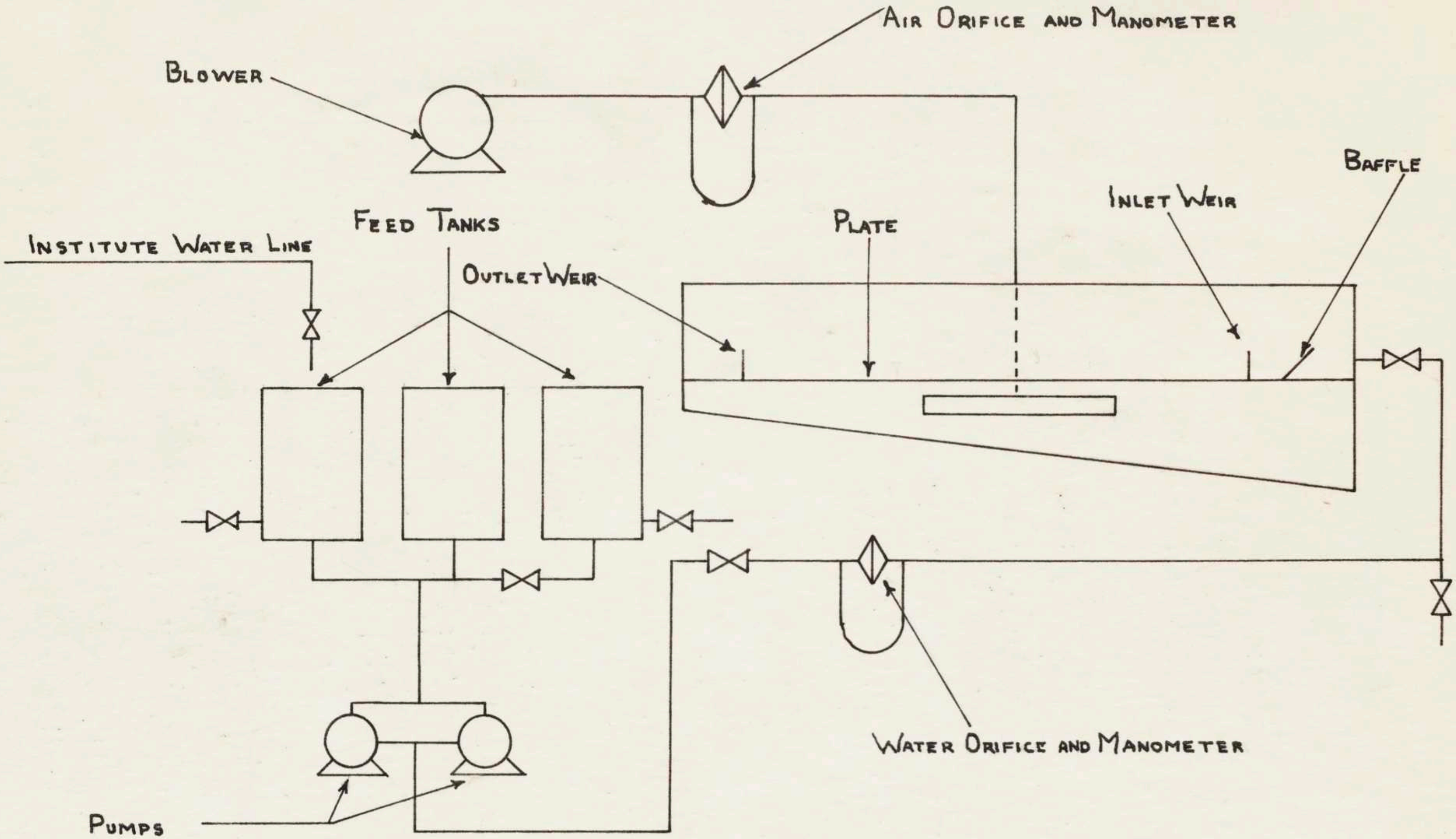


FIGURE 2 EQUIPMENT FLOWSHEET

III. PROCEDURE

A. Operation of Plate

The first step was to close all the valves on the equipment. Next the feed tanks were filled approximately three-fourths full from the Institute water line. The blower was turned on and allowed to come up to speed. The pumps were started and primed by opening small stop-cocks on top of them. The next step was to open both valves on the water feed line. Once water was flowing across the plate, trapped air was bled from the mercury manometer on the water line. Air rate was adjusted with baffles on the intake of the blower and also measured with a mercury manometer. Water rate was sdjusted with either of the two valves in the water line.

It was possible to interchange weirs on the plate, allowing for variation of weir height. Weirs of one, three, and ten centimeters in height were used.

B. Head vs. Height Measurement

To measure the head as a function of height at various places on the plate, a fritted probe was used. This device is shown in figure (3). The porous nature of the fritting allowed water to flow in and out. The flow balanced the atmospheric pressure acting on the surface of the water in the open end of the U-tube manometer. A one centimeter change in level of the manometer corresponded to a one centimeter ghange in head in the liqüid. The pipe which held the probe was clamped into position from the top of the apparatus. Height of the probe above the plate was read from calibration marks on the pipe. To insure that no air was in the line, about one and one half liters of water were forced through the system from the open end of the manometer. As an additional precaution, tygon tubing was used to conect the manometer and the pipe. This allowed observation of any bubbles which might flow into the line through the fritting. Once the lines were purged of air, the probe was moved up in definte increments and the manometer readings made. Readings were taken at two places on the plate.

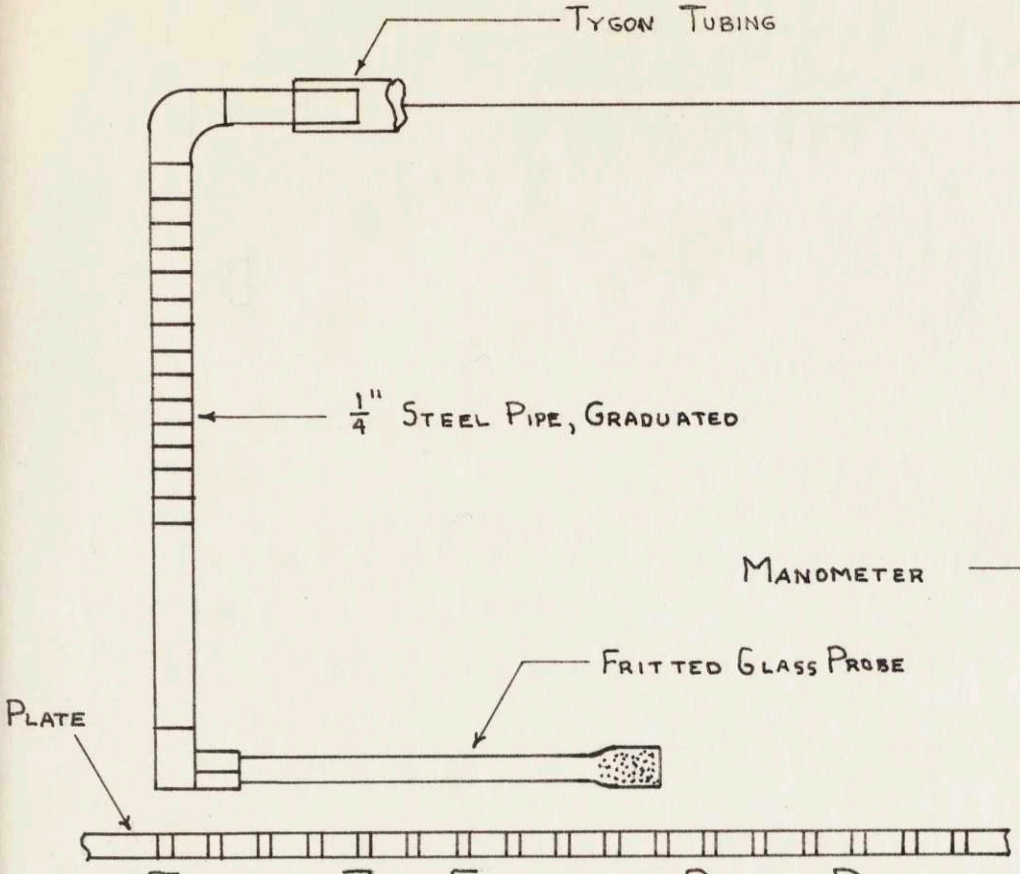


Figure 3 Fritted Probe Device

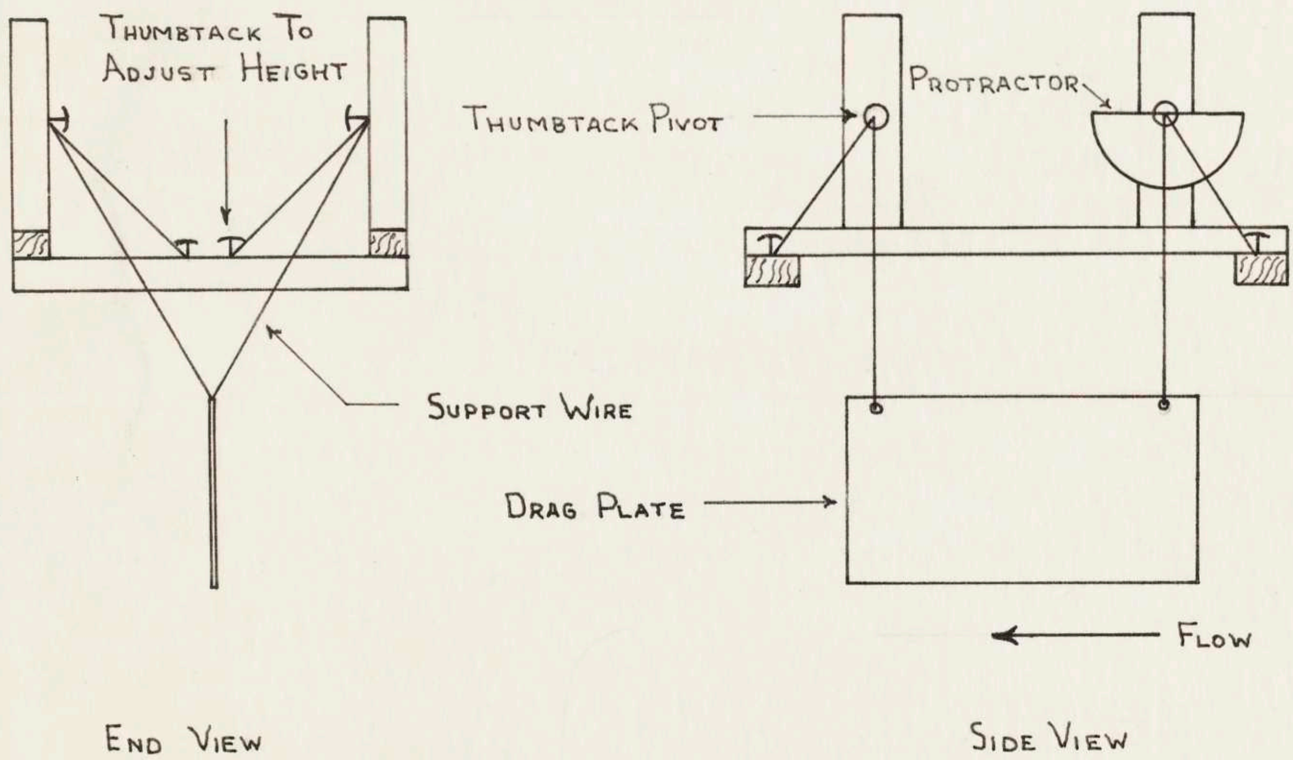


Figure 4 Drag Plate

C. Drag Plate Measurement

To measure the drag loss directly a brass plate was suspended by threads in the foam; its deflection was measured with a protractor as a function of foam velocity. Figure (4) shows the suspension arrangement. The plate was shaped on its leading and trailing edges so as to reduce impact pressure and drag due to boundary layer separation. The position of the drag plate was in the middle of the perforated plate and the height was varied by moving the thumbtacks holding the strings. Care was taken to align the plate parallel to the direction of the flow. Thread was used for suspension in most of the runs, although 0.004" and 0.008" wires were tried without much success. Deflection was measured with the plate at a fixed height at a variety of foam velocities and the three weir heights. For the 10 cm. weir the drag as a function of height was also measured.

D. Measurement of Density as a Function of Height

A sampler, shown in figure (5), was used to collect samples of the foam at varying heights above the plate. It consisted of a teflon plug fitted inside a stainless steel jacket. A hole was drilled through the jacket and the plug so that when the plug was rotated it trapped a sample in it. The device was suspended in the foam and held by a clamp. After positioning, the closing rod was pushed down, which turned the plug and trapped a sample. The sampler was then removed from the foam and the exterior dried. After placement over a 10 ml. graduate the sampler was emptied, and the volume of trapped water recorded.

E. Static Taps on the Plate

To check the total head on the plate copper tubes were inserted in a few of the perforations on the plate. These tubes were connected by rubber tubing to an open manometer similar to the one used for the fritted probe. Water was forced through the open end of the manometer to purge air as before.

F. Effect of Air Momentum

To determine the momentum loss of the air all but a section 42.5 cm.

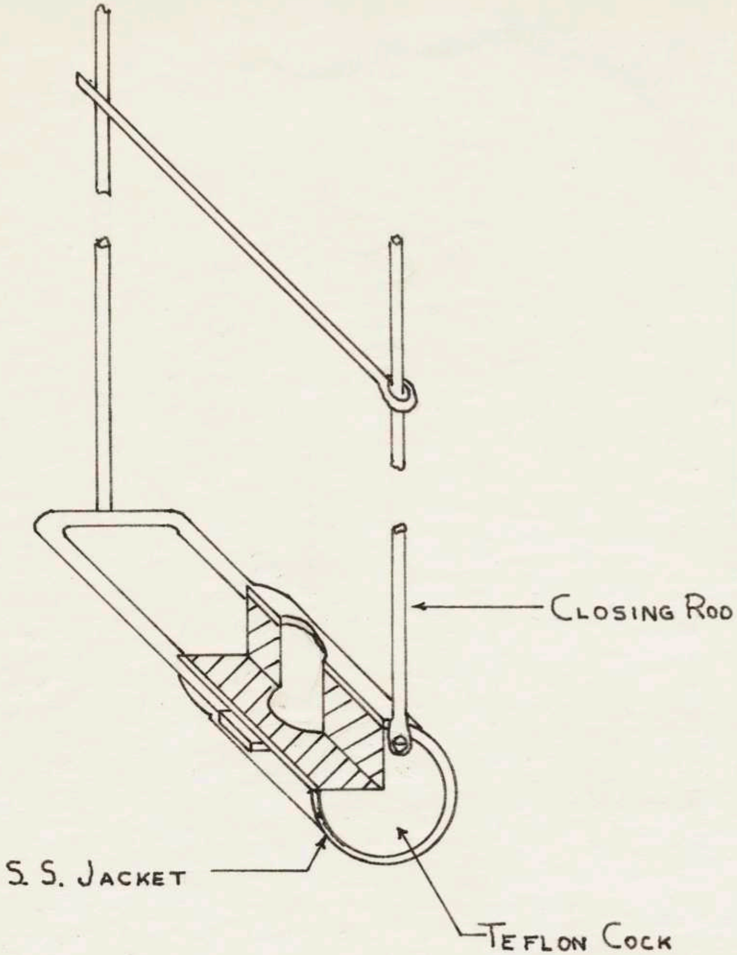


Figure 5 Foam Sampler

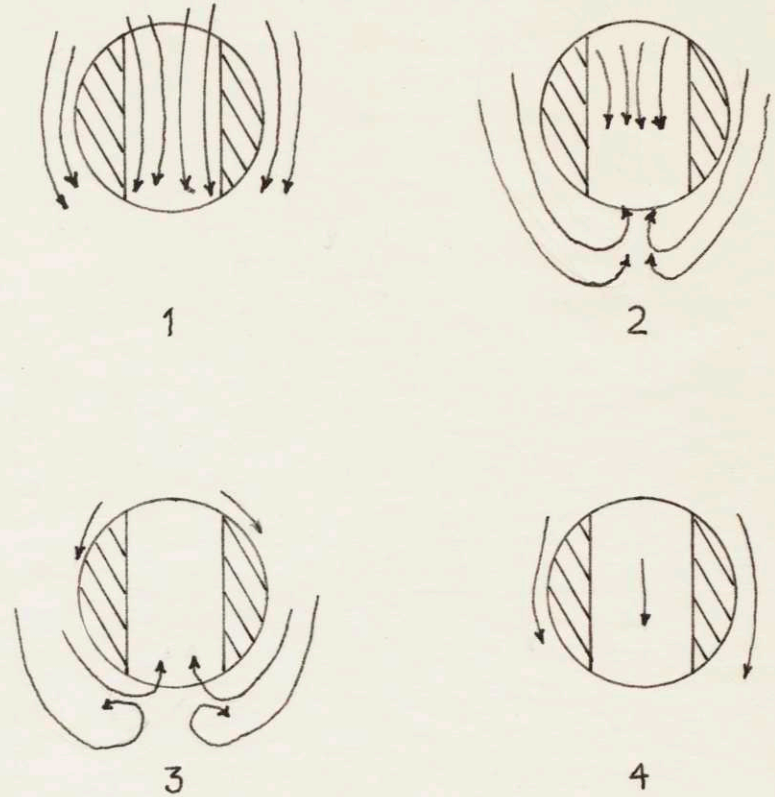


Figure 6 Flow Through Sampler

long of the plate was sealed from the air flow. A foot high weir plate was placed at each end of this section. There was one static tap in the center of this area. The blower was turned on and water was poured into the test section. The effective head shown on the manometer was read for various air rates. Care was taken to replace the water lost by entrainment and spillage.

G. Dye Injection

To check the velocity profile and mixing between a clear water layer existing near the plate and the foam, dye injection was used. A hole was drilled in the wall of the apparatus very close to the plate and sealed with tape. When the plate was in operation, a hypodermic syringe with a suitable needle was used to inject the dye into the water layer.

IV. RESULTS

A. Effect of Air Momentum

The theoretical effect of air momentum and its experimental counterpart were given in equations (17) and (18). When these terms are plotted against the superficial gas velocity, V_g, two smooth curves, shown in figure (7) are obtained. The experimental values lie below the theoretical values at all points. The maximum superficial gas velocity under normal operating conditions was 1.44×10^2 cm./sec. At this extreme the difference in the measured head and the actual head is calculated to be 0.255 cm. of water. The difference for the minimum superficial gas velocity used, 0.80×10^2 cm./sec., is calculated to be 0.153 cm. of water.

B. Measurement of Density as a Function of Height

The measurements taken with the sampler have large variations. A cycle of flow through the sampler was observed and shown in figure (6). In case (1) the foam flowed down the hole. Case (2) shows eddies which have started to form on the bottom side of the sampler. In case (3) the eddies increased to a point where they caus a slight reversal of flow and stagnation in the hole. The foam disappeared in the hole and flow started downward in the hole again as shown in case (4).

The various values for the density were plotted on arithmetic probability paper and were found to be part of a normal population. In figure (8) the mean of these values for one run is plotted vs. the height of collection. Also plotted are the values obtained from the height vs. head data. Equation (1) states that:

$$\rho = \frac{1}{g} \frac{dh}{dH} \qquad (1)$$

The density at any point is the differential or slope of the head vs. height curve divided by the gravitational constant. The curve obtained with the sampler shows a hump while the head data shows a smooth decrease. The sampler data shows a higher average density.

C. Drag Plate Analysis

In the runs at constant height the drag was found to increase with increasing foam velocity. The readings for the 10 cm. weir were fairly inaccurate due to the extreme turbulence which tended to pick up the plate and throw it.

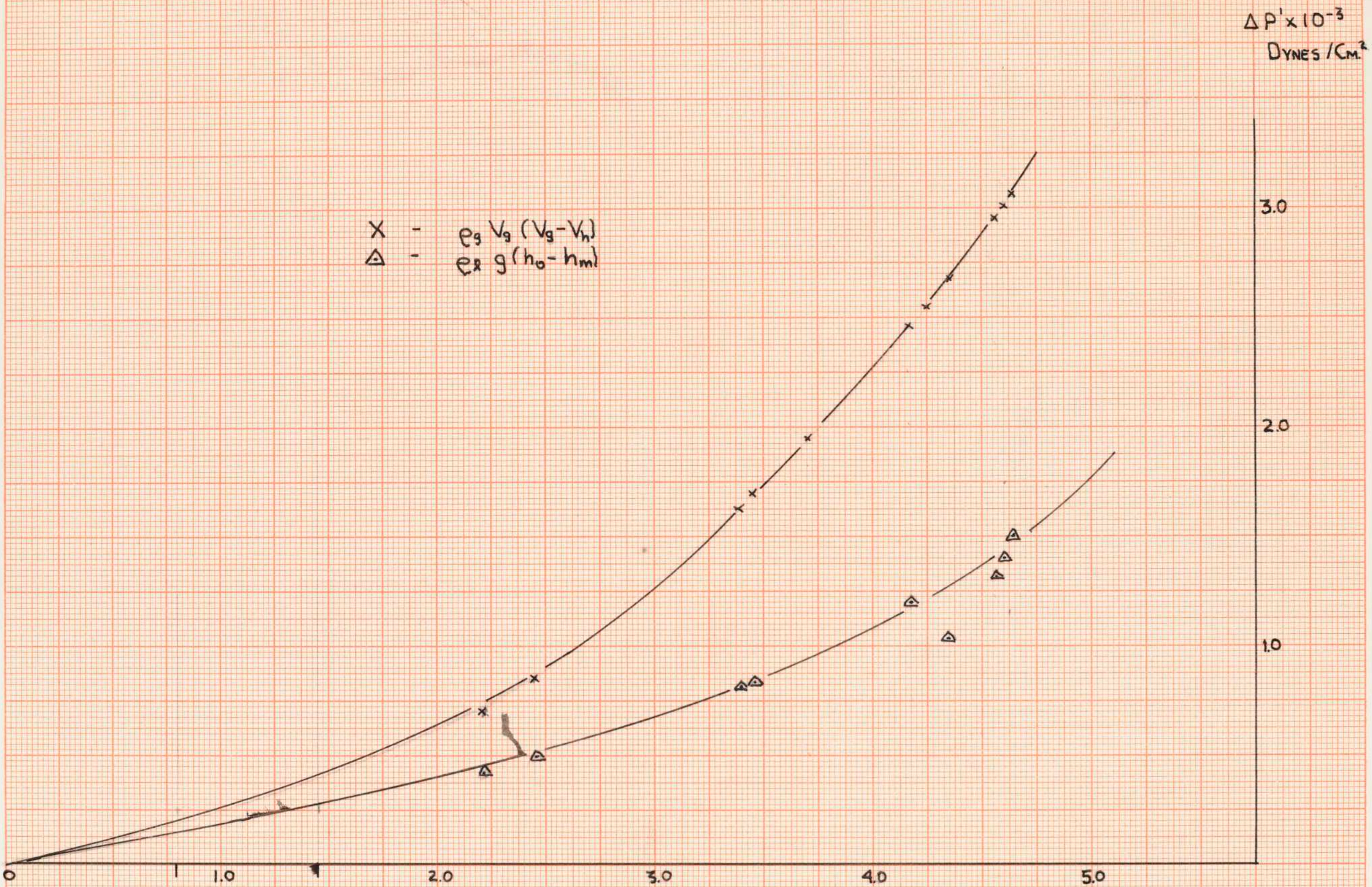


Figure 7 Efect of Air Momentum

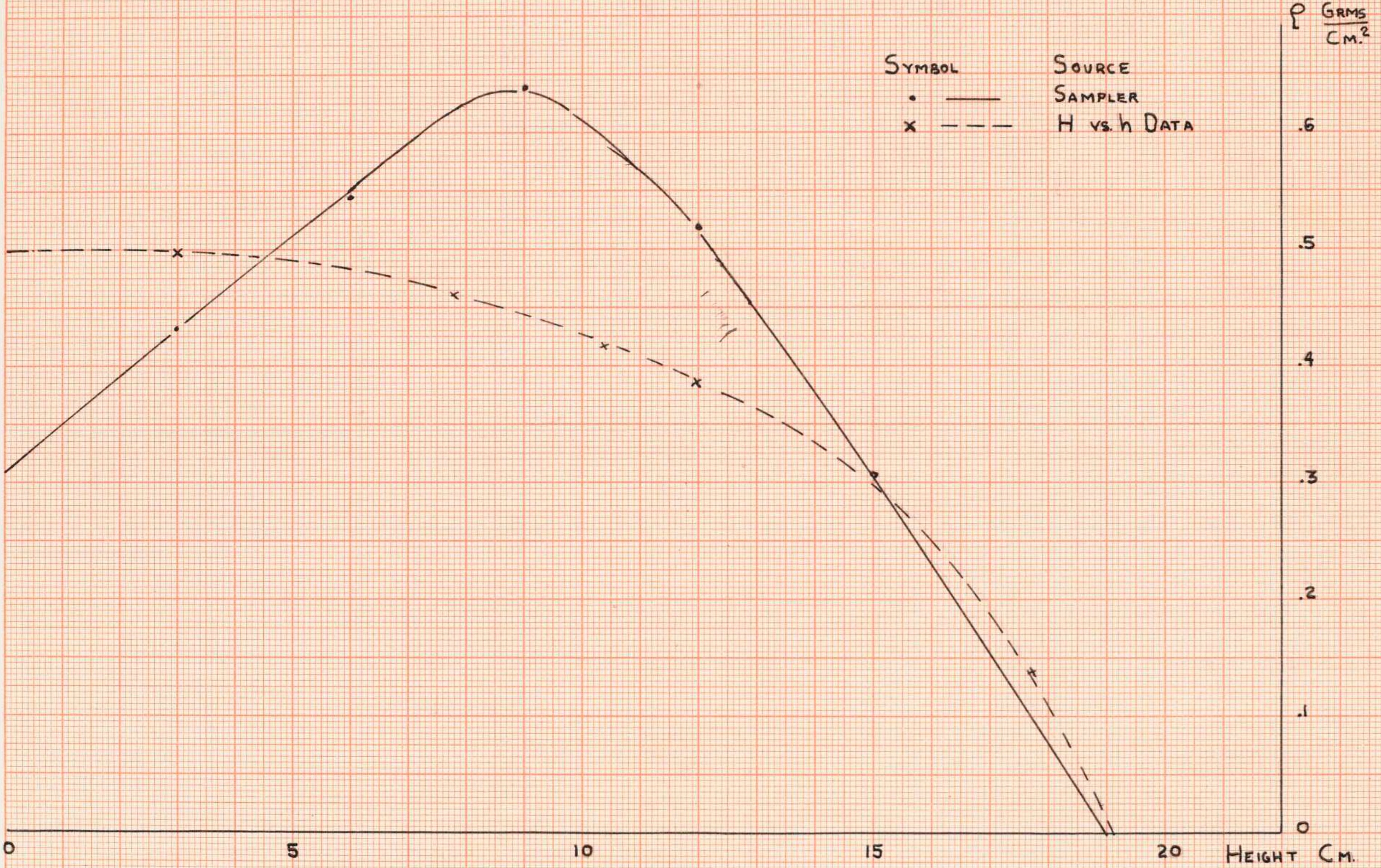


Figure 8 Density as a Function of Height

Table I.

Drag On a Plate, Constant Height

Run	Weir Ht, (cm.)	h_{av} (cm.)	V_f (cm./sec.)	V_g (cm./sec.)	Deflection(°)	D/A($\frac{dyne}{cm.^2}$)
D 1	10	6.55	12.0	132	6 ± 1	62 ± 10
D 2	"	6.05	9.3	132	4 "	41 "
D 3	"	6.95	12.6	132	10 "	104 "
D 4	5	5.40	18.3	128	6 ± ½	62 ± 5
D 5	"	4.80	14.7	132	4 "	41 "
D 6	"	4.00	11.3	132	2 "	21 "
D 7	"	4.10	11.0	128	2 "	21 "
D 8	"	4.60	9.8	110	1 "	10 "
D 9	"	5.00	11.6	107	2 "	21 "
D10	"	5.15	13.7	107	4 "	41 "
D11	"	4.65	7.6	90	½ "	5 "
D12	"	5.05	11.1	90	2½ "	25 "
D13	"	4.35	8.5	100	1½ "	16 "
D14	1	2.50	14.8	94	1 "	10 "
D15	"	2.95	17.6	98	2½ "	25 "
D16	"	3.80	24.8	98	6 "	62 "
D17	"	3.90	25.0	98	6½ "	67 "
D18	"	3.55	27.2	120	5 "	52 "
D19	"	3.15	22.4	123	3½ "	36 "
D20	"	2.35	17.9	123	2 "	21 "
D21	"	2.30	18.2	136	1 "	10 "
D22	"	3.60	27.7	138	7 "	73 "

Top of the drag plate was 9.0 cm. above the perforated plate for all runs.

Data for the runs at various heights are presented in table II. As stated above, the data for the 10 cm. weir are inaccurate due to turbulence and the height of the foam. There is a distinct drag gradient for variable heights with constant foam velocity.

D. Dye Injection

When the dye was injected in the clear water layer near the plate, a line of dye appeared almost instantaneously from the top to the bottom of the foam. The line of dye stayed well defined and perpendicular to the plate for the entire lenght of the plate.

E. Drag Analysis

Figure (9) shows the correlation obtained plotting the drag friction factor, f' (equation 7) against the turbulence factor. Although the points are scattered, three lines corresponding to the drag friction factors for the three weir heights may be drawn. The drag friction factors calculated from the drag plate are included with the potential energy calculations.

F. Reynolds Analogy

The values for the drag constant are plotted against the turbulence factor as shown in figure (10). Again both methods of measuring the drag, potential energy loss and the drag plate, give nearly identical values for the friction factor. A line for guide only was drawn for the points obtained from the potential energy loss. The equation corresponding to this line is:

$$\alpha = 1.15 \times 10^{-6} \frac{V_g h_{av}}{\mu g} \qquad (19)$$

Both α and the turbulence factor vary over a small range.

Table II.

Drag on a Plate, Varying Height

Run	V_f(cm./sec)	Ht. above plate (cm.)	Deflection($\pm$ 1°)	D/A(dynes/cm.^{2})
D38	9.3	9.1	$2\frac{1}{2}$	25 $\pm$ 5
D32	"	11.0	2	21 "
D35	"	12.9	$1\frac{1}{2}$	16 "
D41	"	18.5	$\frac{1}{2}$	5 "
D37	12.0	9.1	$3\frac{1}{2}$	36 "
D31	"	11.0	3	31 "
D34	"	12.1	$2\frac{1}{2}$	25 "
D40	"	18.5	$1\frac{1}{2}$	16 "
D39	12.6	9.1	5	52 "
D33	"	11.0	$4\frac{1}{2}$	47 "
D36	"	12.9	$3\frac{1}{2}$	36 "
D42	"	18.5	$2\frac{1}{2}$	25 "

The weir height was 10.0 cm. and the superficial gas velocity was constant at 132 cm./sec.

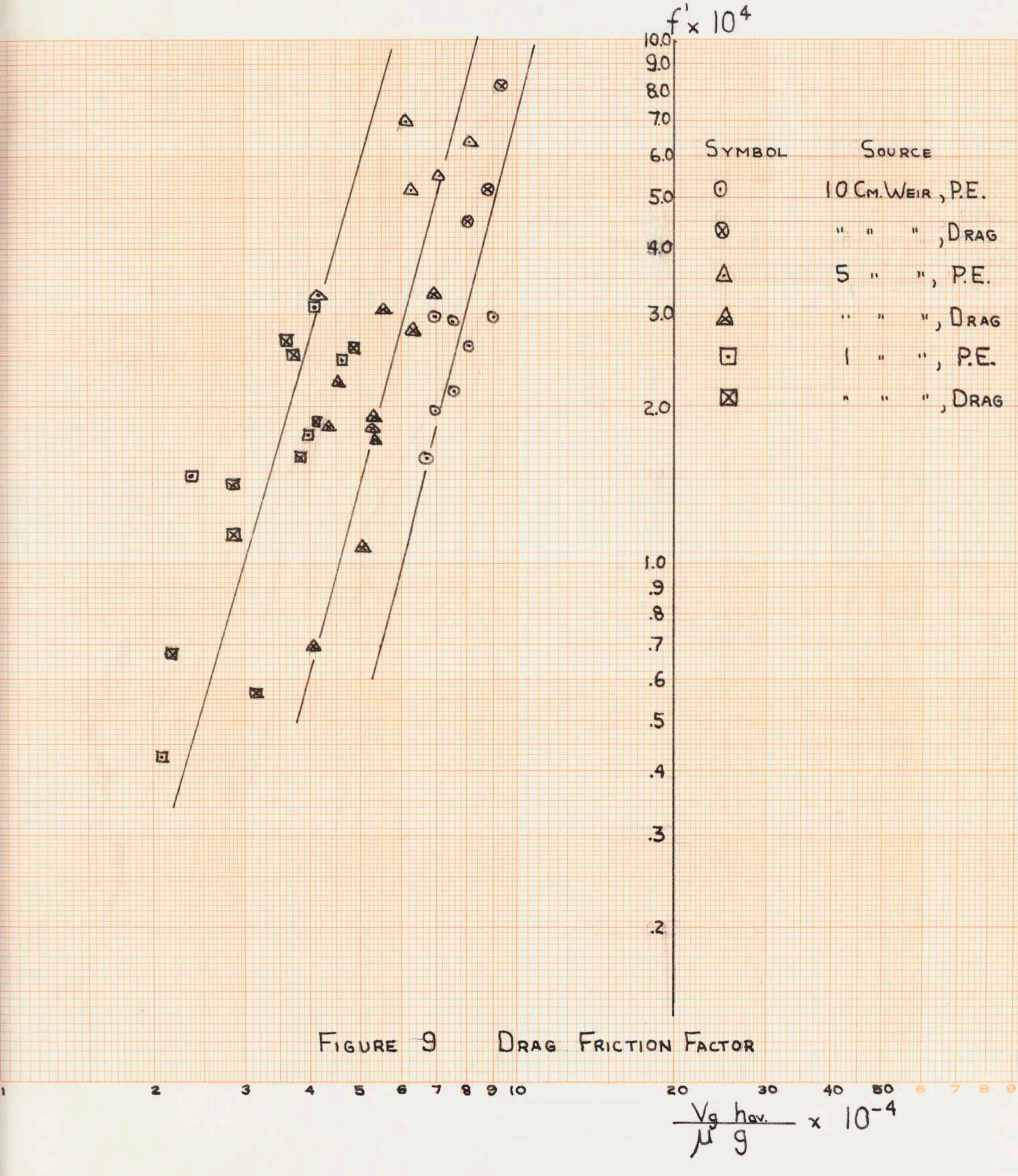


Figure 9 Drag Friction Factor

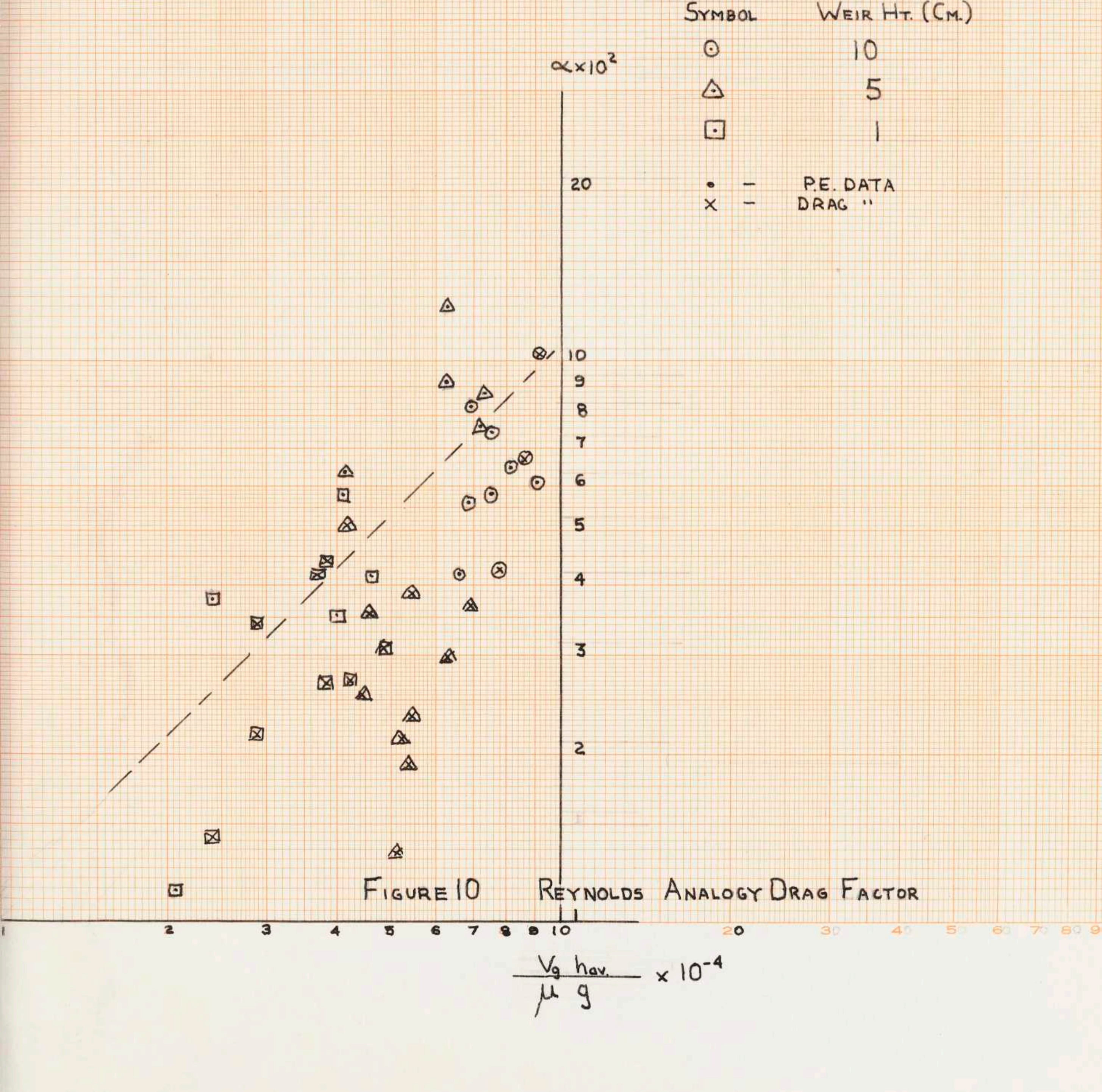


FIGURE 10 REYNOLDS ANALOGY DRAG FACTOR

V. DISCUSSION OF RESULTS

A. Effect of Air Momentum

Over the operating range of the plate, as marked in figure (8), the effect of air momentum on the measured head can be assumed negligible. The maximum calculated difference was 0.255 cm. of water and the experimental curve falls below the calculated curve at all points. The actual effect was less than 4% of the smallest head which occured on the plate.

B. Measurement of Density as a Function of Height

The data from the sampler was not accurate. Its magnitude compared with the values obtained from differentiating the height vs. head curve. Due to the flow characteristics around the sampler the mean of the values could not be assumed correct. The flow and buildup of water films are inherent in any device of this sort. Sampling yielded a poor estimation of density.

The calculated density curve as shown in figure (8) was a good estimation of the density. It is the average density, i.e. the average of small bubbles which have a long residence time in the foam, and slugs which pass through rapidly.

C. Dye Injection

The results of the dye injection showed two things:

1. There was a high degree of vertical mixing.
2. The velocity profile was linear and vertical.

These facts are implied in the derivation of f' and α.

D. Drag Analysis

The drag analysis friction factor has the same fault as the Fanning-type factors in the work of Klein (1) and Hucks and Thompson (2). It shows a different dependency on the turbulence factor for each weir height. The points also have a wide degree of scatter, showing that the dependency is not consistent. f' varies one hundred fold in a short range of the turbulence factor. This lessens its utility for flow prediction.

E. Reynolds Analogy

The α factor, although it shows more scatter, is more homogeneous than f' with respect to values for different weir heights. This indicates that a better picture of the flow was used in the derivation of α. The values for the drag plate and the potential energy loss fit together well considering that two of the terms in α had to be estimated for the drag plate. The agreement between the two methods confirms the approach. Equation (18) is an approximation as no dependency of α on the turbulence factor can be estimated. Most of the scatter can be accounted for in the experimental error. The reliability of α is only to 15%. Since varies only tenfold over the range investigated, it could be used for predicting the flow. An average value of 4×10^{-2} can be used. Using this value and estimating the density at the plate to be about 0.50 grms./cm.^{3}, potential energy loss can be predicted if the operating variables are known. Solving equation (18) for Δ(P. E./cm.^{2}) with the variables for run (P5):

$$\Delta\left(\frac{\text{P. E.}}{\text{cm.}^2}\right) = \frac{V_f V_g N}{b}\left(b\rho_{fP} + \frac{2h_{av}}{g}\right)$$

$$= \frac{(4\text{x}10^{-2})(13.7)(110)(94.4)(33.3\text{x}0.50 + 2\text{x}6.9\text{x}10^3/981)}{33.3}$$

$$= 5.21\text{x}10^3 \text{ ergs/cm.}^2$$

The experimental value is 6.5 ± 0.5 ergs/cm.^{2} The estimation comes within twenty percent of the correct value. It would be closer than this on some runs. The usefulness of α for design purposes would be as an indication of magnitude rather than the exact value.

F. Drag Plate and Its Applications

The results of the drag plate experiments show that the basic picture of drag on the walls is correct. The friction factor values fit in well with the values obtained for potential energy loss. The fact that a drag gradient existed in the foam was not considered when the magnitudes of the drag on the plate were calculated. The results can be considered only as orders of magnitude, especially for the 10 cm. weir height.

The existence of the drag gradient verified the assumption used in the derivation of α, i. e. there is a drag gradient proportional to the density gradient. Quantitative comparison is impossible due to the inaccurate drag plate measurements.

G. General Conclusions and Recommendations

The work done in this thesis has shown (1) the effect of air momentum on the liquid was negligible; (2) there was a density gradient; (3) there was a high degree of vertical mixing on the plate; (4) there was a uniform velocity down the plate; (5) there was a drag gradient similar to the density gradient; and (6) an average α can be used as to estimate the magnitude of potential energy or drag losses. Instead of striving to obtain friction factors which correlate with Reynolds numbers or other dimensionless groups, a constant value for would be superior. As in Klein's and Huck's and Thompson's work (1,2), the data obtained in this thesis gave a linear relationship when plotted as a Fanning type friction factor verses a Reynolds number. The utility and accuracy of such a plot is questionable. It must be noted that in this type of plot a hydraulic radius term appears in both terms. This tends to minimize variations due to the other variables.

It is recommended that (1) modification of α be attempted, and (2) that other systems of liquid and gas be tried. It is to be noted that the contains no viscosity term and it may vaty considerably for other systems. The dependence on the turbulence factor might be observed.

VI. CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

1. The effect of air momentum on the liquid was negligible.
2. There was a density gradient.
3. There was a high degree of vertical mixing on the plate.
4. There was a uniform velocity down the plate.
5. There was a drag gradient similar to the density gradient.
6. An average α can be used as to estimate the magnitude of potential energy or drag losses.

B. Recommendations

1. Modification of α should be attempted.
2. Other systems of gas and liquid should be tried.

VII. APPENDIX

A. Supplementary Details

1. Detail of Equipment Measurements

a. Perforated Plate:

Plate width	33.3 cm.
Distance from wall to first holes	4.4 cm.
Hole diameter	0.125 in.
Distance between holes in one row	0.875 in.

Staggered rows of the holes were 0.875 in. appart.

Total length of plate (weir to weir)	184 cm.
Distance between points of measurement:	94.4 cm.

b. Drag Plate

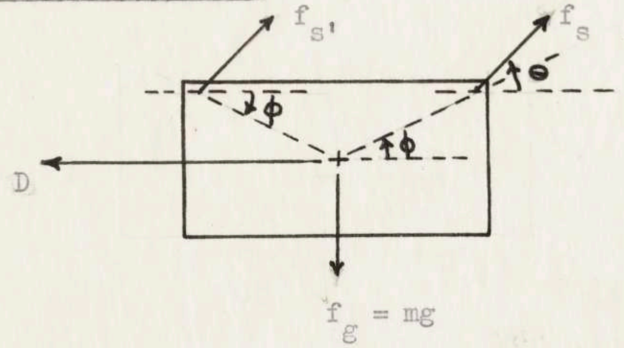
Material; 1/16 " brass plate

Length: 12.0 cm.

Width: 7.3 cm.

Weight: 104.3 grms.

2. Drag Plate Force Derivation



Using a standard static force balance on the plate:

$$\Sigma f_x = 0 \qquad \Sigma f_y = 0 \qquad \Sigma (\text{moments}) = (\vec{f} \times \vec{r}) = 0$$

Defining the force exerted by the strings as f_s and $f_{s'}$ and the drag force as D:

$$f_x = (f_s + f_{s'}) \cos \theta - D = 0$$

$$f_y = (f_s + f_{s'}) \sin\theta - f_g = 0$$

$$(\vec{f} \times \vec{r}) = r \left[f_s \sin (180 - \{\theta - \phi\}) - f_{s'} \sin (\phi + \theta) \right] = 0$$

Solving for D :

$$D = f_g \frac{\cos\theta}{\sin\theta} = mg \cot\theta$$

This derivation assumes an equal drag force over the whole plate. It also postulates a static position of the plate when the drag force is acting.

3. Air Momentum Derivation

Using a momentum balance, in the form of pressures, from the space directly below the plate to a point just above the foam the following equation is easily arrived at:

$$(\rho_g V_g) V_h + P_p = P_T + \rho_g V_g^2 + \rho_L g h_o$$

Where
ρ_g = density of the air
V_g = superficial gas velocity
V_h = air velocity through the holes
P_p = pressure below the plate
P_T = " above the foam
h_o = head of pure liquid present on the plate

Solving:

$$\Delta P = \rho_L g h_o - \rho_g V_g (V_h - V_g)$$

It is seen that with no gas or a negligible momentum effect, that the pressure drop observed would be $\rho_L g h_o$. The correction term for momentum is $\rho_g V_g (V_h - V_g)$, and this should equal $\rho_L g (h_o - h_m)$ where h_m is the measured head at a specific gas velocity.

B. Summary of Data and Calculated Values

1. Potential Energy Loss

Run	Weir	H_{av}(cm.)	h_{av}(cm.H_2O)	$V_f(\frac{cm.}{sec})$	$V_g(\frac{cm.}{sec})$	$(\frac{P.E.}{cm.^2})$	$f' \times 10^4$	$\alpha \times 10^2$	$\frac{V_g h_{av}}{\mu_g} \times 10^{-4}$
P4	1	21	6.5	11.3	116	7.1	2.94	7.47	7.55
P5	1	23	6.9	13.7	109	6.5	2.12	5.80	7.52
P6	1	23	7.4	12.2	94	5.4	1.99	5.61	6.92
P7	1	22	7.0	10.5	99	6.8	2.97	8.27	6.93
P8	1	22	6.7	7.5	99	2.7	1.58	4.18	6.64
P9	1	19	6.2	3.3	122	7.0	10.50	22.80	7.51
P10	1	24	6.7	11.0	122	6.6	2.64	6.41	8.17
P11	1	22	7.4	13.6	123	8.8	2.96	6.14	9.04
P15	2	13	4.4	6.4	96	3.5	3.26	6.36	4.17
P16	2	13	6.4	6.6	96	7.7	7.00	12.50	6.14
P17	2	14	6.6	8.0	96	9.0	5.15	9.17	6.34
P18	2	14	6.3	9.9	113	9.5	5.51	8.75	7.14
P19	2	15	7.2	11.4	113	12.9	6.38	7.71	8.10
P20	3	7	2.8	22.2	82	4.5	1.48	3.76	2.39
P21	3	8	2.6	16.7	79	1.0	0.43	1.13	2.06
P22	3	9	3.1	19.2	133	8.7	3.11	5.85	4.05
P23	3	11	3.2	30.3	144	13.4	2.47	4.17	4.60
P24	3	11	3.2	30.7	127	9.2	1.71	3.53	4.00

Weir 1 = 10 cm., weir 2 = 5 cm., and weir 3 = 1 cm.

B. Summary of Data and Calculated Values, cont.

2. Drag Plate

Run	Weir	ρ_{fp} est.(grm./cm.^{3})	f' x 10^4	α x 10^2	$\frac{V_g h_{av}}{\mu_g}$ x 10^{-4}
D1	1	0.38	5.2	6.70	8.65
D2	1	0.40	4.3	4.21	7.99
D3	1	0.38	8.2	10.30	9.17
D4	2	0.65	3.2	3.62	6.92
D5	2	0.66	2.8	2.95	6.34
D6	2	0.70	1.8	1.90	5.28
D7	2	0.61	1.9	2.10	5.25
D8	2	0.65	1.1	1.31	5.06
D9	2	0.65	1.8	2.34	5.35
D10	2	0.65	3.0	3.84	5.51
D11	2	0.67	0.7	5.12	4.18
D12	2	0.65	2.3	3.51	4.54
D13	2	0.68	1.8	2.62	4.35
D14	3	0.51	0.7	1.42	2.35
D15	3	0.45	1.4	3.43	2.89
D16	3	0.60	2.5	4.06	3.72
D17	3	0.60	2.7	4.32	3.82
D18	3	0.55	1.9	2.74	4.26
D19	3	0.43	1.6	2.70	3.58
D20	3	0.43	1.2	2.20	2.89
D21	3	o.42	0.6	0.95	3.13
D22	3	0.60	2.6	3.10	4.97

Weir 1 = 10 cm., weir 2 = 5 cm., weir 3 = 1 cm.

C. Sample Calculations

1. Potential Energy Loss

For run P5 the following data were taken directly:

air rate: 1.6" Hg ≙ 6.7×10^5 cm.^{3}/sec.

water rate: 3.0" Hg ≙ 1.8×10^5 cm.^{3}/min.

The readings on the U-tube manometer as a fuction of height were as follows:

Ht.(cm.)	Tap 1	Tap 3
2	11.6	11.5
5	13.7	13.4
8	15.5	15.1
11	17.2	16.9
14	19.2	18.9
17	21.3	20.9
20	23.8	23.5
23	26.5	26.2
26	29.5	29.1

To relate the readings to the datum of the probe, the difference in reading between the probe and the manometer at a point above the foam was subtracted from each reading. The difference was (29.5 - 26.0) = 3.5 for tap 1 and (29.1 - 26.0) for tap 3. The effective head at any point was the difference between the corrected reading and the height of the probe:

Ht.(cm.)	Tap 1		Tap 2	
	Corr,reading	head	Corr.reading	head
2	8.1	6.1	8.4	6.4
5	10.2	5.2	10.3	5.3
8	11.0	3.0	12.0	4.0
11	13.7	2.7	13.8	2.8
14	15.7	1.7	15.8	1.8
17	17.8	0.8	17.8	0.8
20	20.8	0.8	20.4	0.4
23	23.0	0.0	23.1	0.1
26	26.0	0.0	26.0	0.0

To obtain the effective head at the plate, the values at various heights were plotted vs. height and the curve extrapolated to zero height. This gave a value of 6.7 for tap 1 and 7.1 for tap 3. Since the equations defines the head as the pressure caused by any amount of liquid up to the point considered, the values at each height were subtracted from the zero point values. The head also had to be converted to dynes/cm.^{2} from cm. of H_2O. This was accomplished by multiplying each value by $\rho_L g = 1(\text{grm./cm.}^3)\ 981(\text{cm./sec.}^2)$.

The head thus calculated was plotted vs. height as shown in figure (11). Equation (3) states that the area under each curve is the P.E./$cm.^2$. The results of the integration are 64.3×10^3 ergs/$cm.^2$ for tap 1 and 67.4×10^3 ergs/$cm.^2$ for tap 3.

Since tap 3 is upstream from tap 1, the P.E. loss per $cm.^2$ is $(67.4 - 64.3) \times 10^3 = 3.1 \times 10^3$ ergs/$cm.^2$ However, the platewas not level so a correction term had to be added. The difference in level was 0.5 cm. so the amount of energy per $cm.^2$ equal to $(\rho H) g \Delta H = h_{av} \Delta H$ must be added to the calculated loss. This equal $6.9 \times 10^3 \times 0.5 = 3.4 \times 10^3$. The total loss of P.E./ $cm.^2$ is $(3.1 + 3.4) = 6.5 \times 10^3$ ergs/ $cm.^2$

2. Calculation of f'

f' was given in equation (6) as:

$$\frac{D}{\mu N V_f (2H_{av} + b)}$$

i. For P.E. loss:

$$D = \Delta(\text{P.E.}/cm.^2)\, b$$

for run P5:

P.E. loss $\Delta(\text{P.E.}/cm.^2) = 6.5 \pm 0.5 \times 10^3 \frac{ergs}{cm.^2}$

Plate width $b = 33.3 \pm 0.1$ cm.

Plate length $N = 94.4 \pm 0.1$ cm.

Foam velocity $V_f = 13.7 \pm 0.5$ cm./sec.

Avg. foam ht. $H_{av} = 23 \pm 1$ cm.

$$f' = \frac{(6.5 \pm 0.5) \times 10^3 (33.3 \pm 0.1)}{10^{-2}(13.7 \pm 0.5)(94.4 \pm 0.1) \quad 2(23 \pm 1) + (33.3 \pm 0.1)}$$

$$= (2.12 \pm 0.04) \times 10^{-4}$$

ii. For the drag plate:

$$D = (D/A)A = (D/A)(2H_{av} + b)$$

$$f' = \frac{f'(D/A)}{\mu V_f N}$$

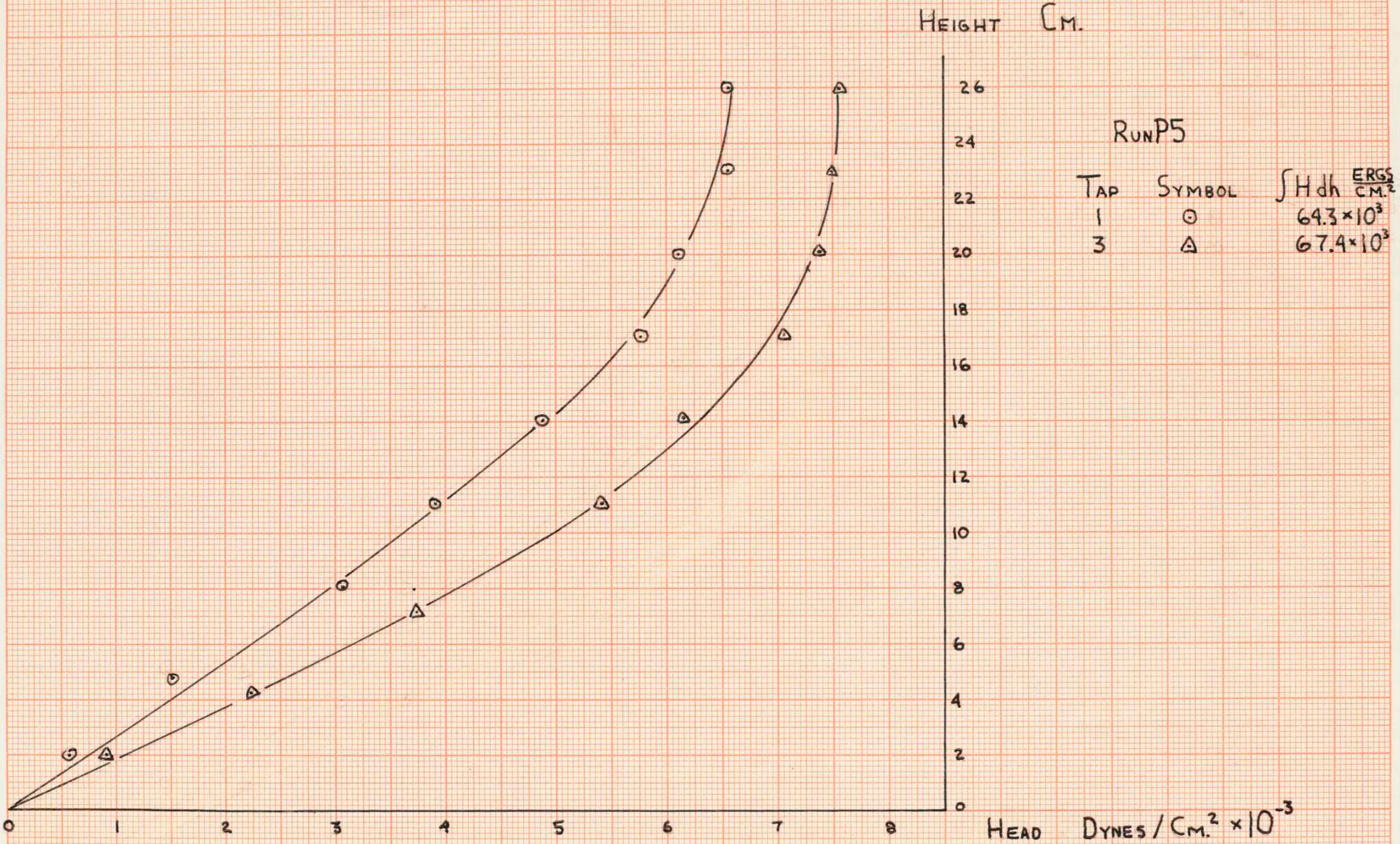


Figure 11 Calculation of Potental Energy

for run D4:

Plate length $N = 94.4 \pm 0.1$ cm.
Drag per unit area $(D/A) = 62 \pm 5$ dynes/cm.^{2}
Foam velocity $V_f = 18.3 \pm 0.5$cm./sec.

$$f' = \frac{62 \pm 5}{10^{-2}(18.3 \pm 0.5)(94.4 \pm 0.1)}$$
$$= (3.2 \pm 0.9) \times 10^{-4}$$

3. Calculation of α:

i. Potential Energy:

$$\alpha = \frac{(P.E./cm.^2)\, b}{V_g V_f N\, (\rho_{fp} b + 2h_{av}/g)}$$

for runP5:

P.E. loss $(P.E./cm.^2) = (6.5 \pm 0.5) \times 10^3$ ergs/cm.^{2}
Plate width $b = 33.3 \pm 0.1$ cm.
Superficial gas velocity $V_g = 110 \pm 5$ cm./sec.
Foam velocity $V_f = 13.7 \pm 0.5$ cm./sec.
Plate length $N = 94.4 \pm 0.1$ cm.
Density at the plate $\rho_{fp} = 0.38 \pm 0.04$ grm./cm.^{3}
Avg. head $h_{av} = (6.9 \pm 0.1) \times 10^3$ dynes/ cm.^{2}

$$\alpha = \frac{(6.5 \pm 0.5) \times 10^3\, (33.3 \pm 0.1)}{(110 \pm 5)(13.7 \pm 0.5)(94.4 \pm 0.1)\, (0.38 \pm 0.04)(33.3 \pm 0.1) + 2\frac{(6.9 \pm 0.1) \times 10^3}{981}}$$

$$\alpha = (5.80 \pm 0.70) \times 10^{-3}$$

ii. Drag plate:

$$\alpha = \frac{(D/A)(2H_{av} + b)}{V_g V_f\, (\rho_{fp} b + 2h_{av}/g)}$$

for run D4:

Drag/unit area $D/A = 62 \pm 5$ dynes/cm.^{2}

Avg. foam ht. $H_{av} = 12 \pm 1$ cm.

Plate width $b = 33.3 \pm 0.1$ cm.

Superficial

gas velocity $V_g = 128 \pm 5$ cm./sec.

Foam Velocity $V_f = 18.3 \pm 0.5$ cm./sec.

Avg. head $h_{av} = (5.4 \pm 0.1) x 10^3$ dynes/cm.^{2}

Density at plate $\rho_{fp} = 0.65 \pm 0.10$ grm./cm.^{3}

$$\alpha = \frac{-\quad(62 \pm 5) \quad 2(12 \pm 1) + (33.3 \pm 0.1)}{(128 \pm 5)(18.3 \pm 0.5)(33.3 \pm 0.1)(0.65 \pm 0.10) + \frac{2(5.4 \pm 0.1) x 10^3}{981}}$$

$$\alpha = (3.62 \pm 1.80) \times 10^{-3}$$

4. Calcualtion of turbulence factor

$$\frac{V_g \, h_{av}}{\mu \, g}$$

For run D4:

Superficial gas velocity $V_g = 128 \pm 5$ cm./sec.

Avg. head $h_{av} = (5.40 \pm 0.1) \times 10^3$ dynes/cm.^{2}

$$\text{Factor} = \frac{(128 \pm 5)(5.40 \pm 0.1)}{10^{-2} \quad 981}$$

$$= (6.92 \pm 0.40) \times 10^2$$

D. Limits of Literature Survey

The literature survey covered two books by Prantl (4)(6) and one by Schlicting (7) for a general background in fluid dynamics. Two theses (1)(2) were read concerning the specific topic.

E. Location of Original Data

The original data is in the thesis notebook of the author. Carbon copies of each page are in the possession of Prof. T.W. Mix, Department of Chemical Engineering, M.I.T.

F. Nomenclature

1.English

A	area, $cm.^2$
b	plate width, cm.
D	drag force, dynes
f'	drag friction factor
f_k	Klein's friction factor
f	force, dynes
g	gravitational acceleration, $cm./sec.^2$
h	head, dynes/ $cm.^2$
h_{av}	average head on plate
h_o	head present if all foam collapsed to pure water
h_m	effective head measured with plate taps
m	mass, gram
N	plate length or length of test section
P	pressure, dynes/ $cm.^2$
P_p	pressure below plate
P_T	pressure above foam
P.E.	potential energy
T	correlation factor used by Klein
V_f	foam velocity, cm./sec.
V_g	superficial gas velocity
V_h	velocity of gas through holes
W	volume of fluid taken to wall per unit area per unit time, cm./sec.
y	distance from wall, cm.

2.Greek

α	Reynolds analogy friction factor
θ	angle
μ	viscosity, poises
ρ_f	density of foam mixture, $grm./cm.^3$

ρ_{fp} density of foam mixture at the plate

ρ_g density of gas or air

ρ_L density of liquid or water

τ drag force per unit area, dynes/ cm.^{2}

τ' " " " " " " "

ϕ angle

G. Literature Citations

1. Klein, J.H. Sc.D. Thesis, Dept. of Chem. Eng. M.I.T. 1950
2. Hucks and Thompson M.S. Thesis Dept. of Chem. Eng. M.I.T. 1954
3. Mix, T.W. personal communications November 1959 through May 1960
4. Prantl, L. Essentials of Fluid Dynamics, Hafner Publishing Company (NewYork 1952)
5. Mix, T.W. Sc.D. Theis, Dept. of Chem. Eng. M.I.T. 1959
6. Prantl, L. Applied Hydro and Aeromechanics, McGraw-Hill Book Company (NewYork 1934)
7. Schlicting, Boundary Layer Theory, McGraw-Hill Book Company