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# Dynamic Region Following Formation Control for a Swarm of Robots

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**Abstract**—This paper presents a dynamic region following formation control method for a swarm of robots. In this control strategy, a swarm of robots shall move together as a group inside a dynamic region that can rotate or scale to enable the robots to adjust the formation. Various desired shapes can be formed by choosing appropriate functions. Unlike existing formation control methods, the proposed method do not need to have specific identities or orders in the group but yet dynamic formation can be formed for a large group of robots. This enables a swarm of robots to adjust the formation during the course of maneuver. The system is also scalable in the sense that any robot can move into the formation or leave the formation without affecting the other robots. Lyapunov-like function is presented for convergence analysis of the multi-robot systems. Simulation results are presented to illustrate the performance of the proposed controller.

## I. INTRODUCTION

Rapid advances in sensing, computing and communication technologies have led to the development of autonomous robots functioning in outdoor environment. In many applications, a given task is too complex to be achieved by a single robot acting alone. As a result, multi-robot systems working cooperatively are required to complete the task. Formation control is one of the most important research area in multi-robot systems. In some situations, formation maintaining alone is not enough as a team of robots may need to change its formation to suit the environment. There are three major approaches in formation control namely behavior-based formation control, leader-following approach and virtual structure method. In behavior-based formation control [1]-[6], a desired set of behaviors is implemented onto individual robots. By defining the relative importance of all the behaviors, the overall behavior of the robot is formed. In leader-following control strategy [7]-[11], the leaders are identified and the follower are defined to follow their respective leaders. In virtual structure method [12]-[15], the entire formation is considered as a single entity and desired motion is assigned to the structure. The formation of the group in virtual structure approach is very rigid as the geometric relationship among the robots in the system must be rigidly maintain during the movement. Therefore, it is generally not possible for the formation to change with time.

Formation switching control strategies have been imple-

mented in some studies of leader-following approach. Das et. all [8] described a framework for cooperative control of a group of mobile robots that allows one to build a complex system from simple controllers. The switching between simple decentralized controllers allows formation switching while following a leader or performing a specific task. Desai et. all [9] used nonlinear control and graph theory approach to study the formation control of mobile robots. Formation changing can be achieved by adding or deleting edges in the formation graph. The problem with the control strategies in [8], [9] is that the control problems get complicated as the number of robots in the formation increase. Fredslund and Mataric [11] used only minimal local communication between robots to achieve the formation establishment and maintenance. A simple switch between line to diamond, line to column and diamond to wedge was presented with simulation as well as with real robots. One obvious problem in this control strategy [11] is that if any one of the robot in the chain fails, the entire system fails.

Although some formation changing have been studied in leader-following approach, these studies are not suitable for controlling a swarm of robots because the constraint relationships among robots become more complicated as the number of robots in the group increases. For controlling a swarm of robots, behavior-based formation control is the most suitable strategy. However, this control strategy does not allow us to analyze the system mathematically. Belta and Kumar [16] proposed a control method for a swarm of robots to move along a specified path. However, this proposed control strategy has no control over the desired formation since the shape of the whole group is dependent on the number of the robots in the group. For large number of robots, the formation is fixed as an elliptical shape whereas for a small number of robots the formation is fixed as a rectangular shape. Moreover, this method does not consider the effects of dynamics on formation control. Recently Cheah et. all [17] proposed a region following formation control for swarm of robots with the consideration of robots dynamics. In this new formation control method, various formation can be formed by choosing appropriate objective functions. However, only static or fixed formation is considered and hence the multi-robot systems cannot change its formation to cope with changing terrain during movement.

In this paper, we propose a dynamic region following formation controller for a swarm of robots. The main challenge comes from the fact that, for a swarm of robots, it is difficult to adjust its formation by specifying the changes in positions or constraint relationships of the robots. Unlike existing formation control methods, our proposed methodology does not require specific orders or roles of the robots inside the dynamic formation. Yet, the swarm of robots are able to coordinate with their neighbors to form a formation and also adjust its formation by rotating and scaling. Communication is only done between the neighboring robots and a specific shape can be formed by choosing appropriate objective functions. The dynamics of the robots are also considered in the stability analysis of the formation control system. The system is scalable in the sense that any robot can move into the formation or leave the formation without affecting the other robots. Lyapunov-like theory is used to show the stability of the multi-robot systems. Simulation results are presented to illustrate the performance of the proposed formation controller.

## II. DYNAMIC FORMATION CONTROL OF MULTI-ROBOT SYSTEM

We consider a group of  $N$  fully actuated mobile robots whose dynamics of the  $i^{th}$  robot with  $n$  degrees of freedom can be described as [18], [19]:

$$M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + D_i(x_i, \dot{x}_i)x_i + g_i(x_i) = u_i \quad (1)$$

where  $x_i \in R^n$  is a generalized coordinate,  $M_i(x_i) \in R^{n \times n}$  is an inertia matrix,  $C_i(x_i, \dot{x}_i) \in R^{n \times n}$  is a matrix of Coriolis and centripetal terms,  $D_i(x_i, \dot{x}_i) \in R^{n \times n}$  represents the damping force,  $g_i(x_i) \in R^n$  denotes a gravitational force vector, and  $u_i \in R^n$  denotes the control inputs.

Several properties of the dynamic equation described by equation (1) are given as follows [18], [19]:

**Property 1:** The inertia matrix  $M_i(x_i)$  is symmetric and positive definite for all  $x_i \in R^n$ .

**Property 2:** The Coriolis and centripetal matrix  $C(x, \dot{x})$  is characterized by the following property  $y^T[M_i(x_i) - 2C_i(x_i, \dot{x}_i)]y = 0$  for any  $y \in R^n$ .

**Property 3:** The damping matrix  $D_i(x_i, \dot{x}_i)$  is positive definite for all  $x_i \in R^n$ .

**Property 4:** The dynamic model described by equation (1) is linear in a set of unknown parameters  $\theta_i \in R^p$  as

$$\begin{aligned} M_i(x_i)\ddot{x}_i + C_i(x_i, \dot{x}_i)\dot{x}_i + D_i(x_i, \dot{x}_i)x_i + g_i(x_i) \\ = Y_i(x_i, \dot{x}_i, \ddot{x}_i)\theta_i \end{aligned} \quad (2)$$

where  $Y_i(x_i, \dot{x}_i, \ddot{x}_i) \in R^{n \times p}$  is a known regressor matrix. Next, we present a dynamic region following formation controller for the group of mobile robots. The dynamic formation control is necessary in some situations where the team of robots needs to adjust its formation to suit the environment. In this paper, we consider rotation and scaling of the formation. Illustrations of formation rotation and formation scaling are shown in figure 1.

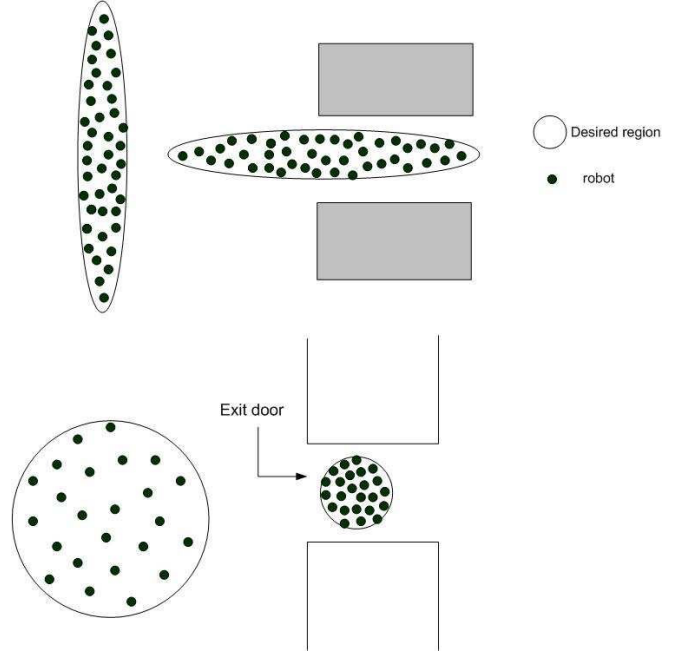


Fig. 1. Examples of Formation Rotation and Formation Scaling

First we define a dynamic region of specific shape for all the robots to stay inside. This can be viewed as a global objective of all robots. Second, a time-varying minimum distance is specified between each robot and its neighboring robots. This can be viewed as a local objective of each robot. Thus, the group of robots will be able to move in a desired formation while maintaining minimum distance among themselves. It should be noted that the minimum distance between robots is time-varying as the robots may need to adjust the minimum distance between each other when the formation is adjusted. Let us define a global objective function by the following inequality:

$$\begin{aligned} f_G(\Delta x_{Ri}) &= [f_{G1}(\Delta x_{Ri}), f_{G2}(\Delta x_{Ri}), \dots, f_{GM}(\Delta x_{Ri})]^T \\ &\leq 0 \end{aligned} \quad (3)$$

where  $\Delta x_{Ri} = x_{Ri} - x_o = RS\Delta x_i$ ,  $\Delta x_i = x_i - x_o$ ,  $x_o(t)$  is a reference point inside the desired region,  $l = 1, 2, \dots, M$ ,  $M$  is the total number of objective functions,  $R(t)$  is a time-varying rotation matrix,  $S^{-1}(t)$  is a time-varying scaling matrix that is not singular,  $f_{Gl}(\Delta x_{Ri})$  are continuous scalar functions with continuous partial derivatives that satisfy  $f_{Gl}(\Delta x_{Ri}) \rightarrow \infty$  as  $\|\Delta x_{Ri}\| \rightarrow \infty$ .  $f_{Gl}(\Delta x_{Ri})$  is chosen in such a way that the boundedness of  $f_{Gl}(\Delta x_{Ri})$  ensures the boundedness of  $\frac{\partial f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}}$ ,  $\frac{\partial^2 f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}^2}$ . Each desired region should rotate about the common centroid  $x_o$ . Various formations such as circle, ellipse, ring, square etc. can be formed by choosing the appropriate functions.

Let us define an elliptical region specified by the following inequality:

$$f_1(\Delta x_{Ri}) = \frac{(x_{Ri1} - x_{o1})^2}{a^2} + \frac{(x_{Ri2} - x_{o2})^2}{b^2} - 1 \leq 0 \quad (4)$$

where  $a$  and  $b$  are positive constant,  $x_i = [x_{i1}, x_{i2}]^T$ ,

$$\begin{bmatrix} x_{Ri1} \\ x_{Ri2} \end{bmatrix} = RS \begin{bmatrix} x_{i1} \\ x_{i2} \end{bmatrix},$$

$$R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix},$$

and

$$S^{-1} = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix},$$

$\theta(t)$  is an angle of rotation,  $s_1(t)$  and  $s_2(t)$  are scaling factors. It should be noted when the scaling factor increases, the desired region also increases.

Though the global objective function  $f_G(\Delta x_{Ri}) \leq 0$  can be separated into position function  $f_p \leq 0$  and orientation function  $f_o \leq 0$ , the rotation matrix  $R$  need not be specified for orientation function in general.

Next, we define a distance between robots by the following inequality:

$$g_{Lij}(\Delta x_{Rij}) = r^2 - \|S\Delta x_{ij}\|^2 \leq 0 \quad (5)$$

Note from the above inequality that the function  $g_{Lij}(\Delta x_{Rij})$  is twice partially differentiable. From equation (5), it is clear that

$$g_{Lij}(\Delta x_{Rij}) = g_{Lji}(\Delta x_{Rji}) \quad (6)$$

and

$$\frac{\partial g_{Lij}(\Delta x_{Rij})}{\partial \Delta x_{Rij}} = -\frac{\partial g_{Lji}(\Delta x_{Rji})}{\partial \Delta x_{Rji}} \quad (7)$$

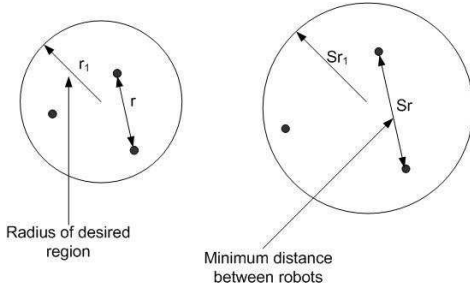


Fig. 2. Minimum Distance between Robots before and after scaling

Next, we define a vector  $\dot{x}_{ri}$  as

$$\begin{aligned} \dot{x}_{ri} &= \dot{x}_o - S^{-1}R^T(\dot{R}S + R\dot{S})\Delta x \\ &\quad - S^{-1}R^T(\alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij}) \end{aligned} \quad (8)$$

where

$$\Delta\xi_i = \sum_{l=1}^M k_l \max(0, f_{Gl}(\Delta x_{Ri})) \left( \frac{\partial f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}} \right)^T \quad (9)$$

$$\Delta\rho_{ij} = \sum_{j \in N_i} k_{ij} \max(0, g_{Lij}(\Delta x_{Rij})) \left( \frac{\partial g_{Lij}(\Delta x_{Rij})}{\partial \Delta x_{Rij}} \right)^T \quad (10)$$

$N_i$  is the set of robots around robot  $i$ ,  $\alpha_i$  and  $\gamma$  are positive constants. Let  $\Delta\epsilon_i = \alpha_i\Delta\xi_i + \gamma\Delta\rho_{ij}$ , we have

$$\dot{x}_{ri} = \dot{x}_o - S^{-1}R^T(\dot{R}S + R\dot{S})\Delta x - S^{-1}R^T\Delta\epsilon_i \quad (11)$$

Differentiating equation (8) with respect to time we get

$$\begin{aligned} \ddot{x}_{ri} &= \ddot{x}_o - (S^{-1}\dot{R}^T + S^{-1}\dot{R}^T)(\dot{R}S + R\dot{S})\Delta x_i \\ &\quad - (S^{-1}R^T)(\ddot{R}S + 2\dot{R}\dot{S} + R\ddot{S})\Delta x_i \\ &\quad - (S^{-1}R^T)(\dot{R}S + R\dot{S})\Delta \dot{x}_i \\ &\quad - (S^{-1}\dot{R}^T + S^{-1}\dot{R}^T)\Delta\epsilon_i - (S^{-1}R^T)\Delta\dot{\epsilon}_i \end{aligned} \quad (12)$$

A sliding vector for robot  $i$  is then defined as:

$$\begin{aligned} s_i &= \dot{x}_i - \dot{x}_{ri} \\ &= \Delta\dot{x}_i + S^{-1}R^T(\dot{R}S + R\dot{S})\Delta x \\ &\quad + S^{-1}R^T\Delta\epsilon_i \end{aligned} \quad (13)$$

where  $\Delta\dot{x}_{Ri} = \dot{x}_{Ri} - \dot{x}_o$ . Differentiating equation (13) with respect to time yields

$$\dot{s}_i = \ddot{x}_i - \ddot{x}_{ri} \quad (14)$$

where  $\Delta\ddot{x}_i = \ddot{x}_i - \ddot{x}_o$ . Substituting equations (13) and (14) into equation (1), and using property 4 we have

$$\begin{aligned} M_i(x_i)\dot{s}_i + C_i(x_i, \dot{x}_i)s_i + D_i(x_i, \dot{x}_i)s_i \\ + Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\theta_i = u_i \end{aligned} \quad (15)$$

where  $Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\theta_i = M_i(x_i)\ddot{x}_{ri} + C_i(x_i, \dot{x}_i)\dot{x}_{ri} + D_i(x_i, \dot{x}_i)\dot{x}_{ri} + g_i(x_i)$ . The region following controller for multi-robot systems is proposed as

$$u_i = -K_{si}s_i - S^T R^T K_p \Delta\epsilon_i + Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\hat{\theta}_i \quad (16)$$

where  $K_{si}$  are positive definite matrices,  $K_p = k_p I$ ,  $k_p$  is a positive constant and  $I$  is an identity matrix. The estimated parameters  $\hat{\theta}_i$  are updated by

$$\dot{\hat{\theta}}_i = -L_i Y_i^T(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})s_i \quad (17)$$

where  $L_i$  are positive definite matrices.

The closed-loop dynamic equation is obtained by substituting equation (16) into equation (15):

$$\begin{aligned} M_i(x_i)\dot{s}_i + C_i(x_i, \dot{x}_i)s_i + D_i(x_i, \dot{x}_i)s_i \\ + K_{si}s_i + Y_i(x_i, \dot{x}_i, \dot{x}_{ri}, \ddot{x}_{ri})\Delta\theta_i \\ + S^T R^T K_p \Delta\epsilon_i = 0 \end{aligned} \quad (18)$$

where  $\Delta\theta_i = \theta_i - \hat{\theta}_i$ . Let us define a Lyapunov-like function for multi-robot systems as

$$\begin{aligned} V &= \sum_{i=1}^N \frac{1}{2} s_i^T M_i(x_i) s_i + \sum_{i=1}^N \frac{1}{2} \Delta\theta_i^T L_i^{-1} \Delta\theta_i \\ &\quad + \sum_{i=1}^N \frac{1}{2} \alpha_i k_p \sum_{l=1}^M k_l [\max(0, f_{Gl}(\Delta x_{Ri}))]^2 \\ &\quad + \frac{1}{2} \sum_{i=1}^N \frac{1}{2} \gamma k_p \sum_{j \in N_i} k_{ij} [\max(0, g_{Lij}(\Delta x_{Rij}))]^2 \end{aligned} \quad (19)$$

Differentiating equation (19) with respect to time and using equation (17), (18), (9), (6), (7), (10), (13) and property 2 we can show that

$$\dot{V} = -\sum_{i=1}^N s_i^T K_{si} s_i - \sum_{i=1}^N s_i^T D_i(x_i, \dot{x}_i) s_i - \sum_{i=1}^N k_p \Delta\epsilon_i^T \Delta\epsilon_i \quad (20)$$

We are ready to state the following theorem:

**Theorem:** Consider a group of  $N$  robots with dynamics described by equation (1), the adaptive control law (16) and the parameter update laws (17) give rise to the convergence of  $\Delta\epsilon_i \rightarrow 0$  and  $s_i \rightarrow 0$  for all  $i = 1, 2, \dots, N$ , as  $t \rightarrow \infty$ .

**Proof:** Since  $M_i(x_i)$  are uniformly positive definite,  $V$  in equation (19) is positive definite in  $s_i$ ,  $\Delta\theta_i$ ,  $\sum_{i=1}^N [\max(0, f_{Gl}(\Delta x_{Ri}))]^2$  and  $\sum_{i=1}^N \sum_{j \in N_i} [\max(0, g_{Lij}(\Delta x_{Rij}))]^2$ . Hence,  $s_i$ ,  $\Delta\theta_i$ ,  $f_{Gl}(\Delta x_{Ri})$  and  $g_{Lij}(\Delta x_{Rij})$  are bounded. The boundedness of  $f_{Gl}(\Delta x_{Ri})$  ensures the boundedness of  $\frac{\partial f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}}$ ,  $\frac{\partial^2 f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}^2}$ . Therefore,  $\Delta\xi_i$  is bounded. From equation (5),  $\max(0, g_{Lij}(\Delta x_{Rij})) (\frac{\partial g_{Lij}(\Delta x_{Rij})}{\partial \Delta x_{Rij}})^T$  is always bounded. Hence,  $\Delta\rho_{ij}$  is bounded. Next,  $\dot{x}_{ri}$  is bounded if  $\dot{x}_o$  is bounded as can be seen from equation (8). From equation (13)  $\Delta\dot{x}_i$  is bounded since  $s_i$ ,  $\Delta\xi_i$  and  $\Delta\rho_{ij}$  are bounded. Hence  $\Delta\dot{x}_{Ri}$  is bounded. The boundedness of  $\Delta\dot{x}_{Ri}$  implies the boundedness of  $\dot{x}_{Ri}$  for all  $i = 1, 2, \dots, N$  if  $\dot{x}_o$  is bounded. Differentiating equation (9) with respect to time yields

$$\begin{aligned} \Delta\dot{\xi}_i = & \sum_{l=1}^M k_l f_{Gl}(\Delta x_{Ri}) \left( \frac{\partial f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}} \right)^T \\ & + \sum_{l=1}^M k_l \max(0, f_{Gl}(\Delta x_{Ri})) \frac{\partial^2 f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}^2} \Delta\dot{x}_{Ri} \end{aligned} \quad (21)$$

where

$$f_{Gl}(\Delta x_{Ri}) = \begin{cases} 0, & f_{Gl}(\Delta x_{Ri}) \leq 0 \\ \left( \frac{\partial f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}} \right) \Delta\dot{x}_{Ri}, & f_{Gl}(\Delta x_{Ri}) > 0 \end{cases} \quad (22)$$

Since  $\frac{\partial f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}}$ ,  $\Delta\dot{x}_{Ri}$  and  $\frac{\partial^2 f_{Gl}(\Delta x_{Ri})}{\partial \Delta x_{Ri}^2}$  are bounded,  $\Delta\dot{\xi}_i$  is therefore bounded. Similarly, differentiating equation (10) with respect to time yields

$$\begin{aligned} \Delta\dot{\rho}_{ij} = & \sum_{j \in N_i} k_{ij} \dot{g}_{Lij}(\Delta x_{Rij}) \left( \frac{\partial g_{Lij}(\Delta x_{Rij})}{\partial \Delta x_{Rij}} \right)^T \\ & + \sum_{j \in N_i} k_{ij} \max(0, g_{Lij}(\Delta x_{Rij})) \frac{\partial^2 g_{Lij}(\Delta x_{Rij})}{\partial \Delta x_{Rij}^2} \Delta\dot{x}_{Rij} \end{aligned} \quad (23)$$

where

$$\dot{g}_{Lij}(\Delta x_{Rij}) = \begin{cases} 0, & g_{Lij}(\Delta x_{Rij}) \leq 0 \\ \left( \frac{\partial g_{Lij}(\Delta x_{Rij})}{\partial \Delta x_{Rij}} \right) \Delta\dot{x}_{Rij}, & g_{Lij}(\Delta x_{Rij}) > 0 \end{cases} \quad (24)$$

From equation (5),  $\Delta x_{Rij}$  is bounded if  $g_{Lij}(\Delta x_{Rij}) > 0$ .  $\Delta\dot{x}_{Rij}$  is bounded since  $\dot{x}_{Ri}$  is bounded for all  $i$ . Hence,  $\dot{g}_{Lij}(\Delta x_{Rij}) (\frac{\partial g_{Lij}(\Delta x_{Rij})}{\partial \Delta x_{Rij}})^T$  is always bounded. Therefore,  $\Delta\dot{\rho}_{ij}$  is bounded since  $\frac{\partial^2 g_{Lij}(\Delta x_{Rij})}{\partial \Delta x_{Rij}^2}$  is bounded (from equation (5)). From equation (12),  $\dot{x}_{ri}$  is bounded if  $\dot{x}_o$  is bounded. From the closed-loop equation (18), we can conclude that  $\dot{s}_i$  is bounded. Differentiating equation (20)

with respect to time we get

$$\begin{aligned} \ddot{V} = & - 2 \sum_{i=1}^N \dot{s}_i^T K_{si} s_i \\ & - 2 \sum_{i=1}^N k_p (\alpha_i \Delta\dot{\xi}_i + \gamma \Delta\dot{\rho}_{ij})^T (\alpha_i \Delta\xi_i + \gamma \Delta\rho_{ij}) \\ & - 2 \sum_{i=1}^N \dot{s}_i^T D_i(x_i, \dot{x}_i) s_i - \sum_{i=1}^N s_i^T \dot{D}_i(x_i, \dot{x}_i) s_i \end{aligned} \quad (25)$$

Hence,  $\ddot{V}$  is bounded since  $\Delta\xi_i$ ,  $\Delta\dot{\xi}_i$ ,  $\Delta\rho_{ij}$ ,  $\Delta\dot{\rho}_{ij}$ ,  $s_i$  and  $\dot{s}_i$  are bounded. Therefore,  $\dot{V}$  is uniformly continuous. Applying Barbalat's lemma [19], we have  $\Delta\epsilon_i \rightarrow 0$  and  $s_i \rightarrow 0$  as  $t \rightarrow \infty$ . Since

$$\Delta\epsilon_i = \alpha_i \Delta\xi_i + \gamma \Delta\rho_{ij} = 0 \quad (26)$$

as  $t \rightarrow \infty$ , therefore summing all the error terms yields

$$\sum_{i=1}^N \alpha_i \Delta\xi_i + \sum_{i=1}^N \gamma \Delta\rho_{ij} = 0 \quad (27)$$

Note that the interactive forces between robots are bi-directional and these forces cancel out each other and the summation of all the interactive forces in the multi-robot systems is zero (i.e.  $\sum_{i=1}^N \Delta\rho_{ij} = 0$ ). From equation (27), we have

$$\sum_{i=1}^N \alpha_i \Delta\xi_i = 0 \quad (28)$$

One trivial solution of the above equation is that  $\Delta\xi_i = 0$  for all  $i$ . If all the robots are initially inside the desired region, then they will remain in the desired region for all time because  $\dot{V} \leq 0$  as seen from (20). Hence from equation (26), we have  $\Delta\rho_{ij} = 0$ . This means that each robot is inside the desired region and at the same time they maintain minimum distance among themselves. Next, assume to the contrary that  $\Delta\xi_i \neq 0$  is the solution of (28). If  $\Delta\xi_i \neq 0$ , then the robots are outside the desired region. If the robots are on one side of the desired region then  $\Delta\xi_i$  have the same sign along one axis and hence they cannot cancel out each other. This contradicts with the fact that  $\sum_{i=1}^N \alpha_i \Delta\xi_i = 0$ . Therefore, the only possibility that  $\sum_{i=1}^N \alpha_i \Delta\xi_i = 0$  is when each term  $\Delta\xi_i = 0$ . From equation (26), we have  $\Delta\rho_{ij} = 0$ . Hence  $\sum_{i=1}^N \alpha_i \Delta\xi_i = 0$  if and only if all the forces  $\Delta\xi_i$  are zero or cancel out each other. This means that some robots must be on the opposite sides of the desired region. Since the desired region is large, when the subgroups of robots are on opposite sides of the region, there is usually no interaction between the subgroups. Hence, similar argument can be applied to conclude that  $\Delta\xi_i = 0$ . When there are interactions or coupling among the robots from different side of the desired region, a reasonable weightage can be obtained for  $\Delta\xi_i$  by adjusting  $\alpha_i$ .  $\Delta\xi_i = 0$  implies that  $f_G(\Delta x_{Ri}) \leq 0$  if the function  $f_G(\Delta x_{Ri})$  is chosen to have a unique minimum at the desired region.  $\Delta\rho_{ij} = 0$  implies that  $g_{Lij}(\Delta x_{Rij}) \leq 0$ .

### III. SIMULATION

In this simulation a group of 100 robots are required to go through an exit door by adjusting the formation. The path of the desired regions is specified as a straight line such that  $x_{o1} = t$ , and  $x_{o2} = 0$ . The scaling matrix for the desired region is defined as:

$$S^{-1} = \begin{bmatrix} s_1 & 0 \\ 0 & s_2 \end{bmatrix},$$

For the first 4 seconds, the scaling factors are set as  $s_2(t) = s_1(t) = 1$ . The scaling factors are then reduced from 1 to 0.5 over a period from 4 to 8 seconds. From 8 to 10 seconds, the scaling factors are constant at 0.5. Next, from 10 to 14 seconds,  $s_1(t)$  and  $s_2(t)$  start to increase from 0.5 to 1. Finally, the factors remain at 1 from 14 to 16 seconds. The control gains are set as  $K_{si} = \text{diag}\{1, 1\}$ ,  $k_p = 100$ ,  $k_{ij} = 1$ ,  $k_1 = 1$ ,  $\gamma = 15$  and  $\alpha_i = 5$ . The minimum distance between robots is initially set to  $0.3 m$ . The desired region is scaled by half of the original size so that all the robots can pass through the exit door. When the size of the region is reduced by half, the minimum distance between robots is also reduced by half to  $0.15m$ .

The dynamic desired region is first specified as a circle with initial radius of  $1.5m$ . The simulation results are shown in figure 3. During the first 4 seconds, the robots move toward the desired region as seen from figure 3(a) and 3(b). After 4 seconds, the region starts to shrink and the final size of the region is half of its original size after 8 seconds (see figure 3(c)). Figure 3(d) shows that the robots have passed through the tunnel and the region starts to grow to its original size. Figure 3(f) shows the final position of the robots at 16 seconds.

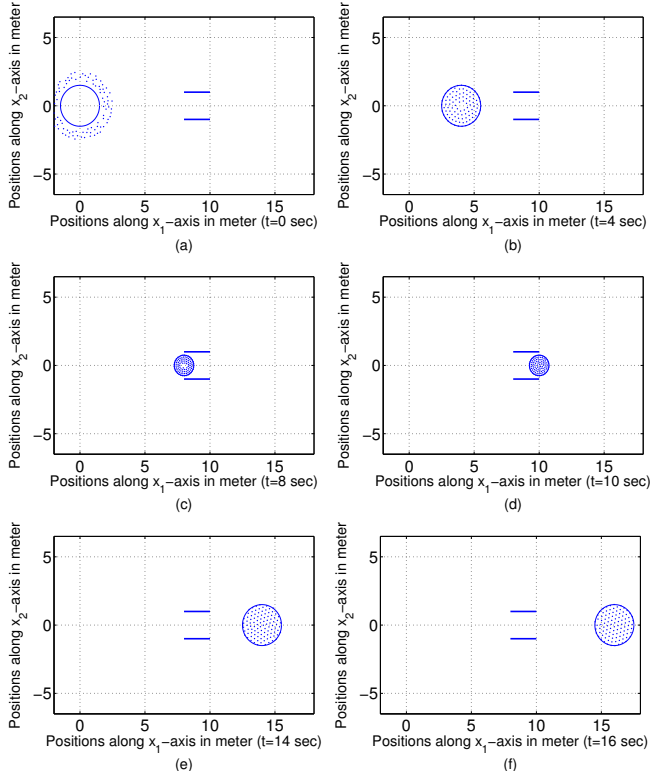


Fig. 3. A group of 100 robots move together in a dynamic circular region

The dynamic region is then specified as a square with the original length of  $2.8 m$ . The simulation results are shown in figure 4.

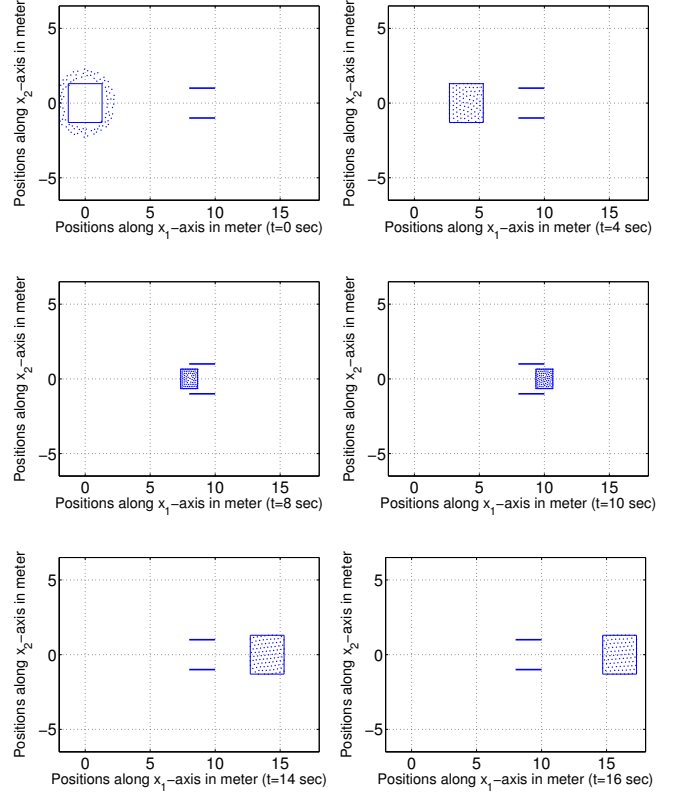


Fig. 4. A group of 100 robots move together in a dynamic square region

Next, we introduce a rotation matrix for the desired region as:

$$R = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix},$$

The rotation angle  $\theta$  is set to 0 for the first 2 seconds which means that the region does not rotate during this period. For the next 2 seconds,  $\theta$  starts to increase from 0 to  $\pi/2$ .  $\theta$  is then kept constant at  $\pi/2$  for the next 4 seconds. From 8 to 10 seconds,  $\theta$  reduces from  $\pi/2$  to 0. In the last 2 seconds,  $\theta$  remains at 0. In the simulations, the control gains are set as  $K_{si} = \text{diag}\{1, 1\}$ ,  $k_p = 100$ ,  $k_{ij} = 1$ ,  $k_1 = 1$ ,  $\gamma = 15$  and  $\alpha_i = 5$ . The minimum distance between robots is set to  $0.3 m$ . The desired region is next specified as an ellipse given by the following inequality function

$$f_1(\Delta x_{Ri}) = \frac{(x_{Ri1} - x_{o1})^2}{a^2} + \frac{(x_{Ri2} - x_{o2})^2}{b^2} - 1 \leq 0 \quad (29)$$

where  $a = 1m$  and  $b = 2.25m$ . Figure 5(a) shows the initial position of all the robots. The desired region begins to rotate after 2 seconds. After 4 second, the region has been rotated by 90 degree as seen in figure 5 (c). Figure 5(d) shows that the robots have passed through the tunnel and the region starts to rotate back to its original shape. The final positions of the robots at 12 seconds is shown in figure 5(f).

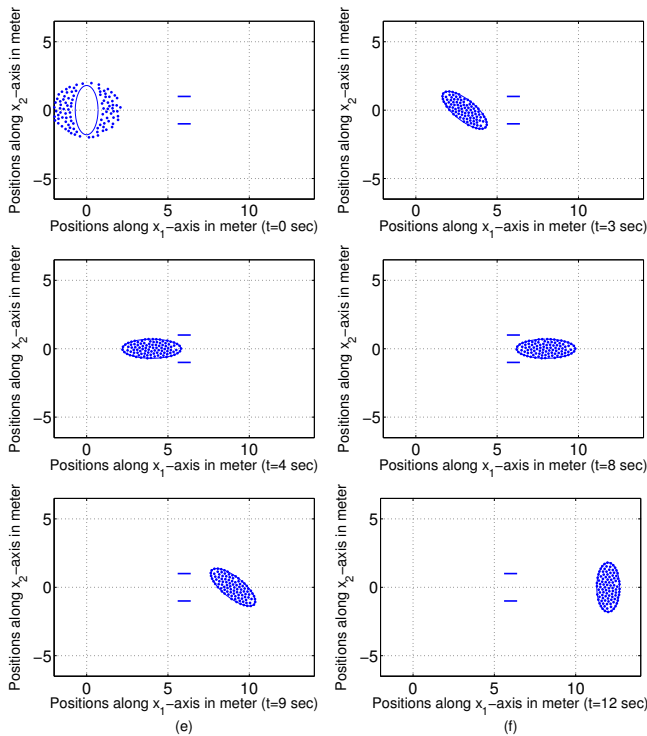


Fig. 5. A group of 100 robots move together in an ellipse formation  
 Finally, the desired region is specified as a ring using the following inequality functions

$$f_1(\Delta x_{Ri}) = \frac{(x_{Ri1} - x_{o1})^2}{a^2} + \frac{(x_{Ri2} - x_{o2})^2}{b^2} - 1 \leq 0$$

$$f_2(\Delta x_{Ri}) = 1 - \frac{(x_{Ri2} - x_{o1})^2}{c^2} - \frac{(x_{Ri2} - x_{o2})^2}{d^2} \leq 0$$

where  $a = 1.2m$ ,  $b = 2.7m$ ,  $c = 0.5m$  and  $d = 2m$ . The simulation results are shown in figure 6.

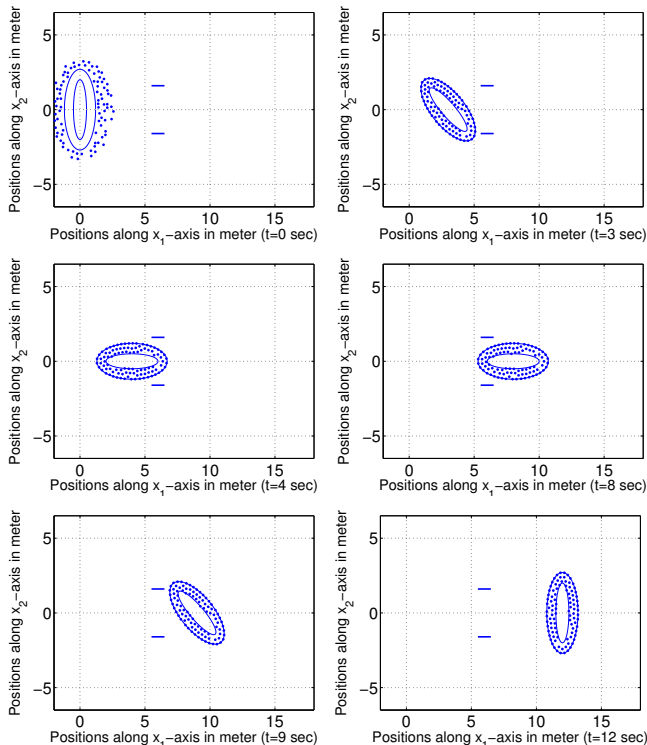


Fig. 6. A group of 100 robots move together in a ring formation

## IV. CONCLUSION

In this paper, we have proposed a dynamic region following formation control method for multi-robot systems. It has been shown that the swarm of robots are able to adjust the formation by rotating and scaling while moving together. Lyapunov-like function has been proposed for the stability analysis of the multi-robot systems. Simulation results have been presented to illustrate the performance of the proposed formation controller.

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