A PROBABILISTIC APPROACH TO THE STABILITY OF ROCK SLOPES

by

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ABSTRACT

Probabilistic analyses provide an alternate approach to assessing the stability of rock slopes. In a probabilistic or reliability analysis the integrity of the slope is expressed as its probability of failure rather than its factor of safety. The primary advantage of reliability techniques is that they explicitly consider the uncertainty associated with engineering parameters.

There are a number of probabilistic techniques currently available; however, none of them provide a comprehensive treatment of the problem. This thesis presents a new method that eliminates some of the earlier limitations. In particular, the thesis concentrates on a few specific points and develops some tools to use in examining rock slope stability from a probabilistic standpoint.

The thesis presents a procedure that enables one to use the data from joint surveys i.e., the probability density functions (pdf's) of joint orientation to derive the pdf's of parameter that characterize the shape and orientation of possible rock wedges bounded by the joints.

The thesis developed an improved analytical model which describes the failure mode that consists of sliding along both joint planes. The model examines the influence of both in situ stresses and joint stiffnesses. It extablishes the relationship between the two effects and extends the concepts to cover asymmetric wedges.

The thesis also presents a possible solution to one of the major unsolved problems in rock stability--joint persistence. The stochastic model developed in Chapter 5 provides a rational method for treating joint persistence. It uses Monte Carlo simulation to generate jointing patterns and then analyzes each pattern to identify the weakest failure path. The strength of the rock mass is characterized by a random "apparent persistence" parameter that can be used in probabilistic stability calculations.

Name and Title of Thesis Supervisor: Herbert H. Einstein
Associate Professor of Civil Engineering

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Elizabeth Manzi typed the final manuscript of the thesis and David MacNeill drafted most of the figures. I appreciate the long nights they spent in helping me meet my deadlines.

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CHAPTER 1

INTRODUCTION

1.0 Scope of Thesis

The idea of analyzing the stability of rock slopes in a probabilistic framework is not new. McMahon developed a practicable approach in the early 1970's. His original model had a relatively limited range of applicability but it did introduce the notion of using the probability of failure rather than the factor of safety in rock slope stability analyses. Reliability techniques are now gaining popularity as their sophistication and range of applicability increases. As will be discussed in Chapter 2, the current models use a number of different approaches. Some combine deterministic models with Monte Carlo simulation so that some of the input parameters can be treated as random variables. Other models treat stability as a stochastic process that can be described mathematically. At the present time there is a wide gap between the two contrasting approaches. This thesis endeavors to establish an intermediate approach that is basically stochastic but does use Monte Carlo simulation to treat aspects of the problem that cannot presently be modeled stochastically.

A rigorous analysis should consider every aspect of the problem including the correlation among random variables and the interaction of failure bodies in the slope. It should also consider all possible modes of failure; toppling and

rotation as well as sliding should be examined. Some of the problems like correlation are probabilistic. Others like toppling are largely mechanical. None of these problems are well understood at the present time; furthermore, they may remain so for many years to come. This thesis does not address all the above problems but concentrates on a number of specific points. In particular, it presents a computer program that enables one to determine the shape and orientation of tetrahedral wedges through a numerical integration technique that uses the probability density functions of joint plane orientations. It also presents a detailed description of the mechanics of the failure mode that involves sliding along two planes. The mechanical model is incorporated into two computer programs that analyze the stability of wedges deterministically and probabilistically. The thesis introduces the concept of "apparent persistence" to characterize the shearing resistance of en echelon joint sets. Finally, the thesis shows how the individual techniques can be combined into a complete reliability analysis.

1.1 Outline of Thesis

The structure of the thesis reflects the same fundamental principle that underlies virtually all rock slope stability analyses. Slope stability is essentially a problem in discontinuum mechanics wherein both the wedge and its parent rock mass are treated as rigid bodies. Two conditions must

be met before failure can occur:

1. The wedge must be oriented in such a manner that movement towards a free face is physically possible - a kinematic condition.

2. The destabiling forces must exceed the stabilizing forces - a kinetic condition.

The idea that failure depends on both the kinematic and kinetic conditions is reiterated throughout the thesis.

Chapter 2 presents a brief introduction to the concept of reliability analysis. It also reviews the state of the art of reliability analysis as applied to rock slope stability.

Chapter 3 discusses the kinematic aspects of the stability problem. It describes the computer program DAYLITE that enables the user to determine the probability density function of the orientation and shape of possible failure wedges.

Chapter 4 discusses the kinetic aspects of the stability problem. In particular, it discusses how the deterministic model for computing the factor of safety can be improved by incorporating joint stiffnesses and in situ lateral stresses into the analysis. The chapter presents two computer programs: SWARS-2PM, a modified version of SWARS-2P that includes the effects of joint stiffness and in situ stresses; and SWARS-2MC, a probabilistic version of SWARS-2PM that treats the input parameters as random variables through Monte Carlo simulation.

Chapter 5 presents a stochastic model that simulates the resistance of "en echelon" joints. It characterizes the resistance of jointed rock through an "apparent persistence"

parameter. This apparent persistence can be input directly into stability calculations.

Chapter 6 shows how the techniques described in Chapters 3 to 5 can be integrated into a complete reliability analysis. As indicated in Section 1.1, there are still many aspects of the analysis that are difficult to evaluate. The chapter outlines the limitations of the proposed analysis and notes the steps in the analysis that require additional refinement.

Chapter 7 presents a summary and conclusions.

CHAPTER 2

ROCK SLOPE RELIABILITY ANALYSIS: STATE OF THE ART

2.0 Introduction

The term "reliability analysis" is a relatively recent addition to the rock mechanics vocabulary. Engineers working in rock mechanics, like their counterparts in soil mechanics, are becoming aware of the advantages of using reliability theory. Unlike conventional approaches, reliability analyses explicitly consider the uncertainty associated with engineering judgments. The integrity of a structure is expressed in terms of its probability of failure rather than its factor of safety. Engineering parameters such as shear strength or pore pressure are treated as random variables i.e., quantities characterized by probability distributions.

The use of probability is not a totally radical departure from traditional approaches. Joint orientation data have always been analyzed statistically. Contour diagrams of joint poles correspond to probability density functions portrayed on a projected surface of a hemisphere. A deterministic analysis would be concerned primarily with the modal or central values of the distributions. The contour plots would serve to identify the modal values and perhaps influence the selection of a design factor of safety. (Diffuse diagrams may dictate a more conservative design criterion.) On the other hand, a probabilistic analysis would incorporate the dispersion or uncertainty of the

orientation data directly into the calculations.

Reliability theory is a useful tool in treating the innate variability of natural materials. It may prove to be even more valuable in examining another important source of uncertainty--incomplete information on design parameters. An engineer seldom has enough data to thoroughly describe all the inputs to his analysis. The shear strength parameters are frequently estimated from a limited number of tests. The location of the phreatic surface may be based on a very brief observation period. This same type of uncertainty pervades almost all the parameters on which an engineer predicates his design. In a reliability analysis he can quantify this uncertainty and treat the parameters as random variables with estimated probability density functions. The probability of failure will reflect the natural variability of material as well as the engineer's subjective assessment of parameters with scant data bases*.

The problem of rock slope stability provides an ideal subject for reliability studies. Most of the critical parameters such as joint persistence and cleft water pressures are difficult to evaluate deterministically. These parameters must be considered random variables to arrive at any realistic

* The distinction between natural variability and estimation error is a matter of intepretation. Natural variability can be considered a measure of the error associated with estimating the value of a parameter at a specific point. Theoretically, one could evaluate the parameter at every point and eliminate natural variability. Engineering analyses are based on a limited amount of data and should consider estimation error.

conclusions concerning the stability of a rock slope. A number of individuals have recognized the promise of such an approach and formulated probabilistic models. Most of the current models are relatively crude with limited applicability. These models may, however, form the foundations for more sophisticated versions.

The current models can be grouped into four broad catagories:

Graphical Techniques

Monte Carlo Simulations

Composite Models

Stochastic Models

The groupings are rather arbitrary. Most of the models are based on the same fundamental principles and they share many features. The groupings serve primarily as a convenience to emphasize the similarity of certain models.

Many of the models are quite innovative. They can treat parameters like joint spacing or analyze stepped failure surfaces--effects which are difficult to assess deterministically. However, none of the current models provide a complete solution to slope stability problems. The question of which one to use in a particular situation is largely a matter of personal preference.

In examining the models one should evaluate them with respect to some objective criteria. An ideal model should fulfill three requirements:

Practicality The model should rely on input data which can be obtained from reasonably simple and inexpensive field surveys or laboratory techniques.

Rigorousness The model should be mathematically and probabilistically consistent. All bias in the input should be systematically eliminated. Correlation among the random variables should be explicitly considered.

Completeness The model should recognize the three-dimensional nature of rock wedges and investigate all possible modes of failure. It should be capable of handling diverse ground water conditions and wedge geometries.

None of the present models satisfy all the criteria. In fact no one is preeminent in all three categories. Most of them strike a compromise between practicality and rigorousness.

This section will examine a number of models or techniques. The discussions will highlight the advantages and limitations of each method. They will emphasize the probabilistic aspects of the models because engineers familiar with deterministic analyses may not be aware of the subtle probabilistic assumptions inherent to each model.

2.1 Graphical Techniques

McMahon (1971, 1974, 1975) has done an extensive amount of work in this field. His method is best suited for dry slopes where sliding occurs along a single discontinuity. The model does not require plane strain conditions. It can be used in cases where the strike of the joint plane is not parallel to the strike of the open face as long as there are tension joints present to isolate blocks and permit sliding along the major joints (Figure 2.1).

The technique developed by McMahon uses a lower hemisphere equal-area projection. The two critical steps in the analysis are to establish regions of kinematic and kinetic stability in the projection. The kinematic tests indicate whether a block is physically capable of movement. They consider the orientations of the joints relative to the slope. The kinetic tests indicate whether the driving forces exceed the resisting forces. They consider the shear strength of individual joints.

A block will not slide along a plane unless the line representing its direction of movement daylights on the slope. Figure 2.2 illustrates this concept for joint planes parallel (same strike) to the slope. Only those joints whose poles lie between A and B are joints on which movement can occur. The same type of analysis can be performed on any family of joints that share the same strike. In each case the locus of kinematically unstable joints includes all joints whose dip is undercut by the slope. (The analysis implicitly assumes

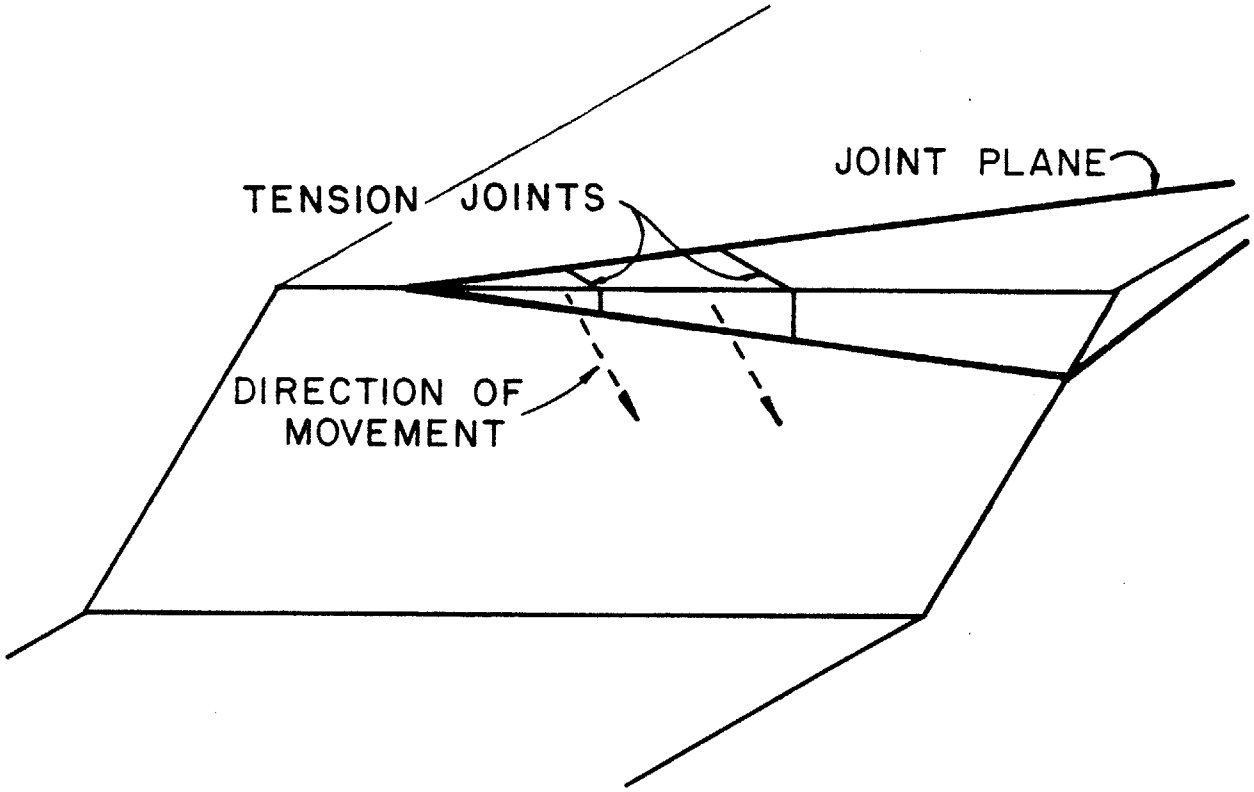


Figure 2.1 Sliding on a Single Plane

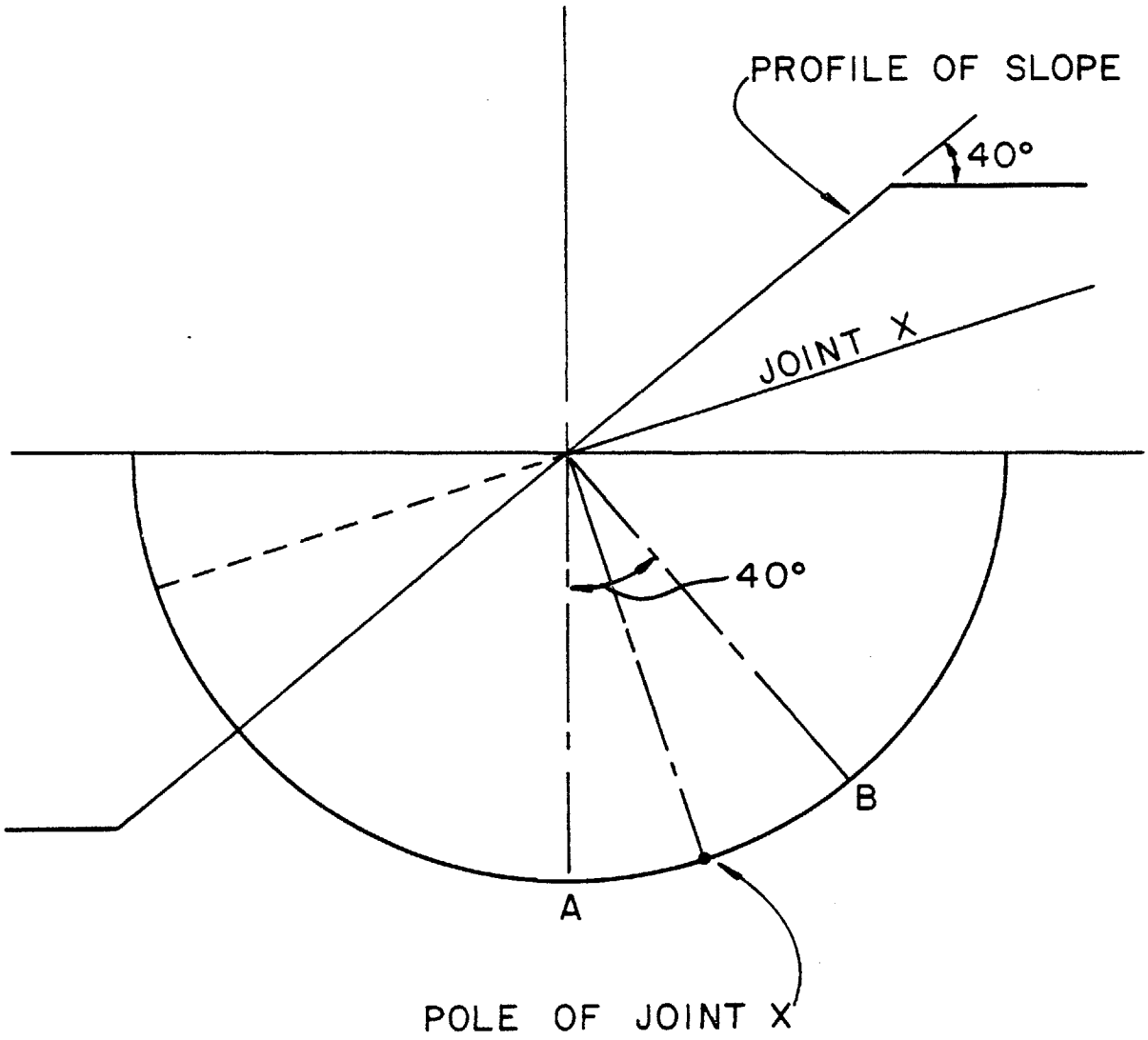


Figure 2.2 Kinematic Test for Sliding

that the dip of the joint corresponds to the direction of movement i.e., only hydrostatic and gravitational forces are present.) Figure 2.3 shows the locus of all unstable joints for slopes of various inclinations. The great circle on the right side of the figure are traces of the slopes and the closed curves on the left define the corresponding regions of kinematic instability; any joint whose pole lies within the curves will daylight. Points A and B correspond to those in Figure 2.2.

The fact that a joint may daylight does not imply that movement must occur. The shear strength of the joint may be sufficient to restrain movement. Kinetic tests must be imposed to resolve the question of stability. The case analyzed in Figure 2.4 involves a wedge whose resultant load consists solely of its weight. The wedge will not slide as long as the joint has a dip less than $\theta_{cr} = \Phi$. All joints with poles between A and C are kinetically stable. The only joints along which sliding will occur in this simple case are those whose poles plot between C and B. The three dimensional representation of the region of kinetic stability is the area inside a circle of angular radius Φ centered at the center of the projection (Figure 2.5). This circle is generated by revolving the circular sector defined by arc A-C (Figure 2.4) around the pole of the projections.

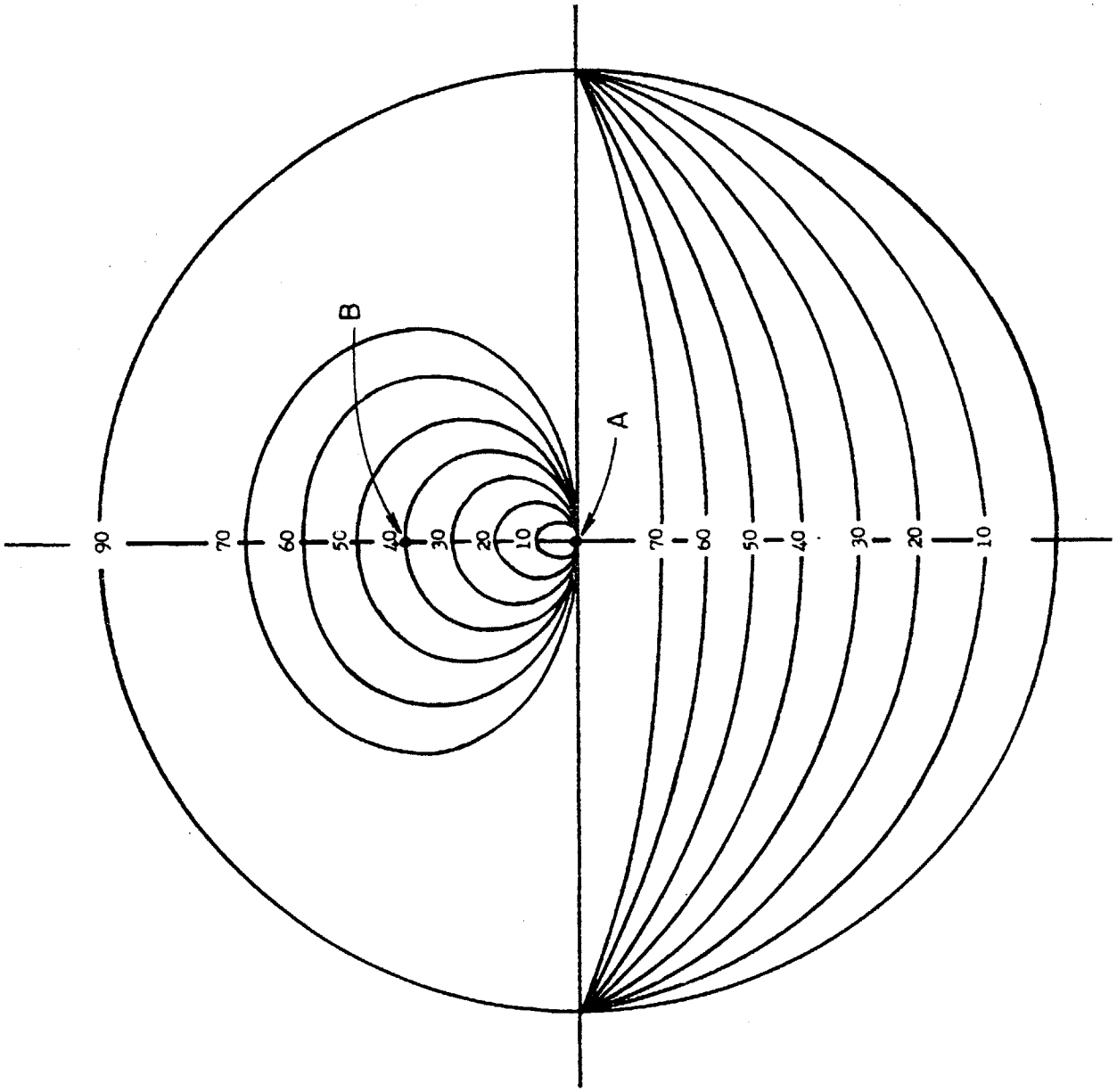
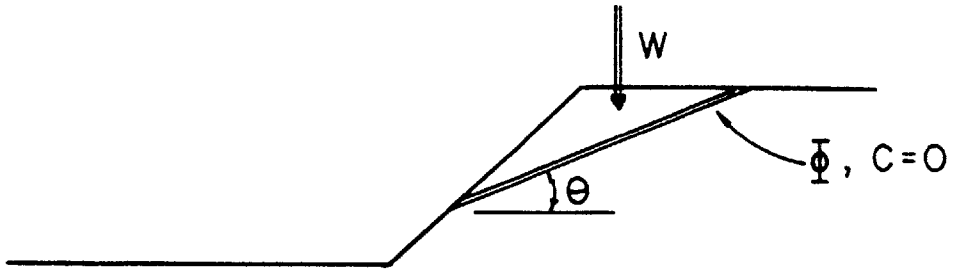
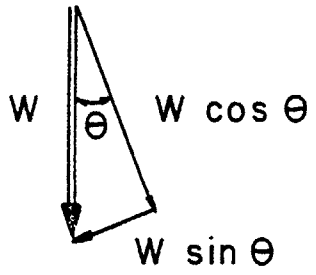


Figure 2.3 Areas of Kinematic Instability on Stereonet





$$F.S. = \frac{W \cos\theta \tan\phi}{W \sin\theta}$$

$$= \frac{\tan\phi}{\tan\theta}$$

$$\theta_{critical} = \phi$$

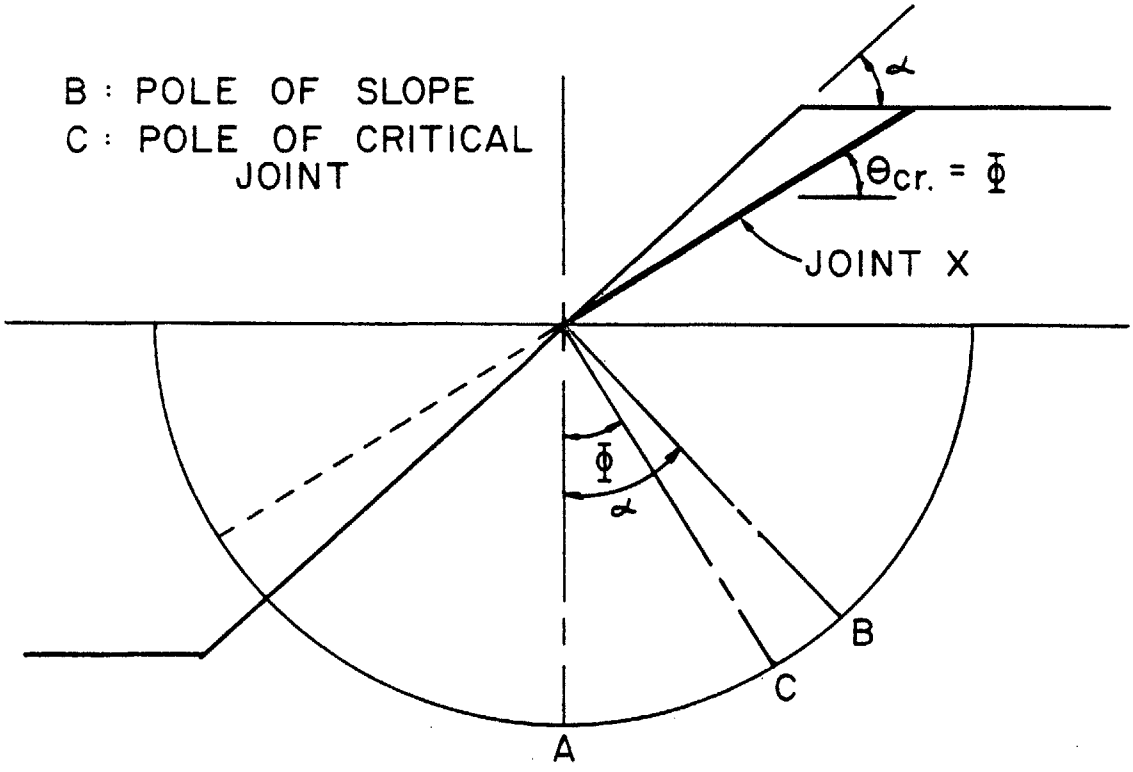


Figure 2.4 Kinetic Test for Sliding

The two stability regions just described provide a basis for applying probability theory. The reliability analysis of the slope reduces to a single decisive question: What is the probability that the slope will contain a joint (or joints) whose pole lies within the area of kinematic instability and outside the area of kinetic stability? This crucial crescent shaped zone is accentuated in Figure 2.5. McMahon (1971) discusses two limiting cases.

1. Uniform Distribution If the joints are uniformly distributed over the surface of the sphere P_J', the probability of the occurrence of an unstable joint, is

$$P_J' = \frac{Au}{A_t}$$

Where Au is the area of the critical zone shown in Figure 2.5 and A_t is the total area of the projection.

2. Indeterminate Distribution If the joint survey represents the best estimate of the sample population, P_J can be calculated as

$$P_J = \frac{Nu}{N_T}$$

Where Nu is the number of poles in the critical zone and N_T is the total number of poles or joints in the survey.

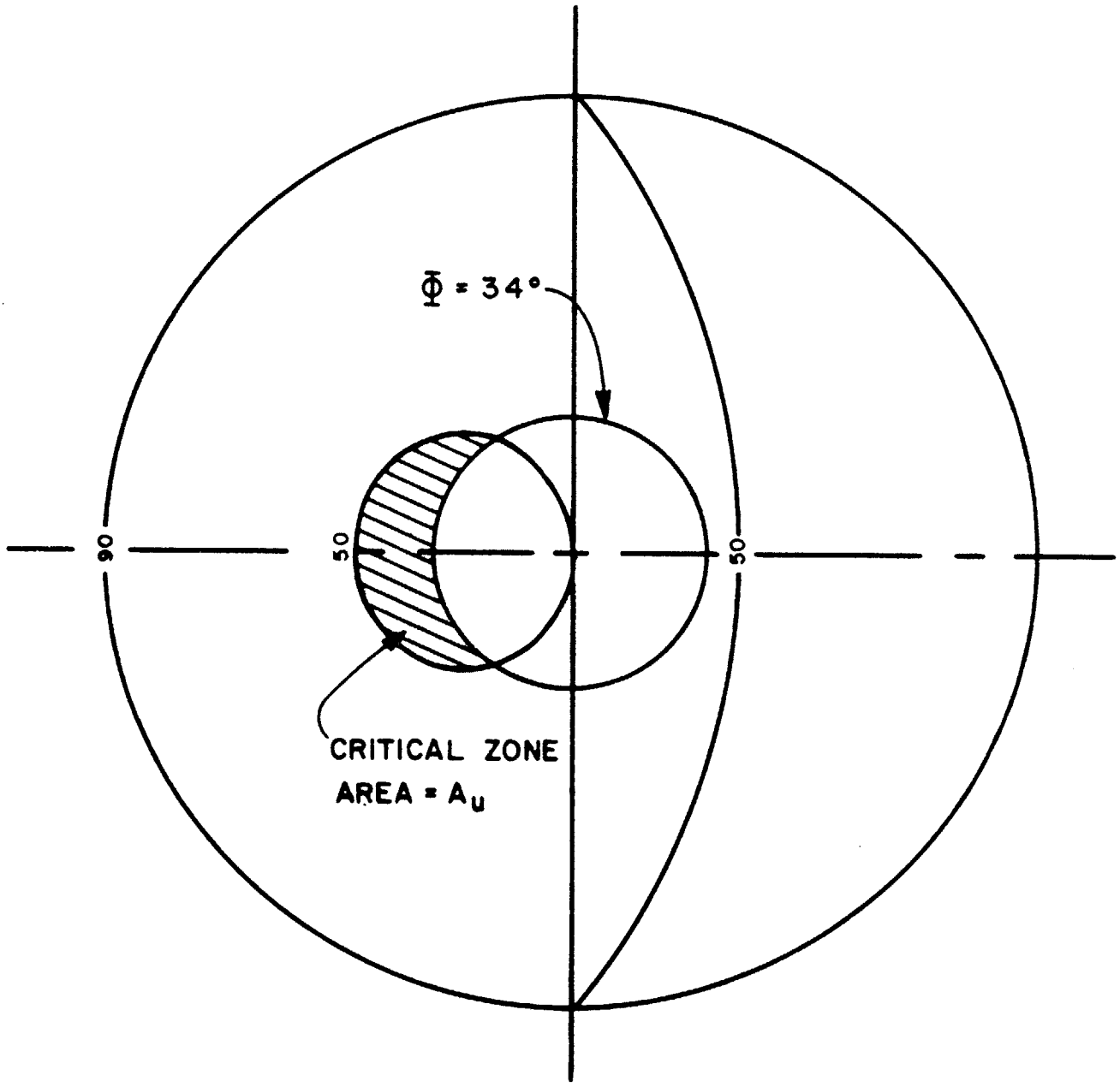


Figure 2.5 Critical Zone for Sliding

Most joint distributions do not correspond to either of these two extreme cases. If, however, the joint orientation has a mathematically describable probability density function (See Appendix A) P_J' can be found through numerical intergration techniques. McMahon (1971) describes a procedure for evaluating bevariate normal distributions with a rectangular normal probability chart. Herget (1978) also examined the bivariate normal distribution. He suggests sketching contours of constant standard deviation and then estimating the portion of each "ring" that lies in the critical zone.

P_J' is a conditional probability based on the assumption that only one joint is present. (P_J' = Probability of an unsafe joint given that only one joint is present.) If the joints are independently oriented P_J, the probability of the unsafe joint occurring, is:

$$P_J = 1 - (1 - P_J')^N \tag{2.1}$$

where N is the number of joints in the slope. (In a rigorous analysis N should be treated as a random variable.) P_J corresponds to P_f, the probability of failure, for slopes in which the joint lengths are large relative to the size of the slope i.e., the joints will form a continuous failure plane. If only a portion of the joints are long enough to cause failure, P_f is computed as

$$P_f = P_J P_C \tag{2.2}$$

where P_C is the proportion of joints with the requisite

continuity to cause failure.

The graphical approach is not restricted to cases involving only gravitational loads. McMahon (1974) introduced the concept of critical dip (β_c) to treat hydrostatic and pseudo-static seismic loadings.* The critical dip is the dip of the flattest joint along which sliding takes place at limiting equilibrium. Dip is measured in the direction of movement. Figure 2.6 shows how β_c can be determined for the two loading conditions.** β_c is used in the same manner as Φ in Figure 2.5. β_c is not necessarily a deterministic quantity. It can be treated as a function of time (temporal variations in ground water levels) or space (lateral variations in joint friction angles.)

McMahon's graphical approach is well suited to evaluate planar failures in cases where both the loads and the resistances are proportional to the weight of the block. When the proportionality occurs β_c is independent of H, i.e., the size

*McMahon's formulation of β_c includes a term to correct for lateral restrain in wedge failures:

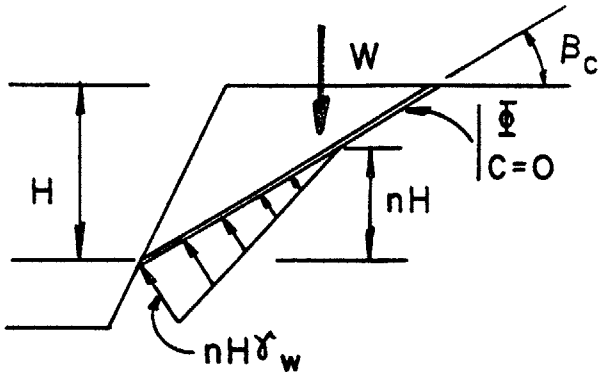
$$\tan\beta_c = K_s \tan(\Phi - \alpha) \qquad (2.3)$$

$$K_s = \frac{\sin\theta_1 + \sin\theta_2}{\sin(\theta_1 + \theta_2)}$$

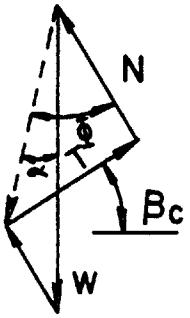
θ_1 and θ_2 are the apparent dips of the two planes bounding the wedge measured in a plane normal to their line of intersection. K_s is difficult to evaluate because both θ_1 and θ_2 are random variables. It may be more efficient to use McMahons's technique solely for planar failures (K_s = 1.0) and rely on other methods to investigate wedge failures.

**Note that the hydrostatic pressure is defined in terms of the quantity nH. β_c will be a function of n but not H because H^2 will appear in both the numerator (U) and the denominator (W). Also, U is a function of β_c. Since $\beta_c = \Phi - \alpha$, the solution for α will require a reiterative approach.

HYDROSTATIC LOAD



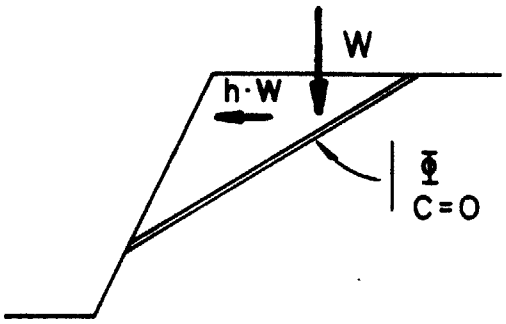
$$U = \frac{n}{2} H^2 \gamma_w \csc \beta_c$$

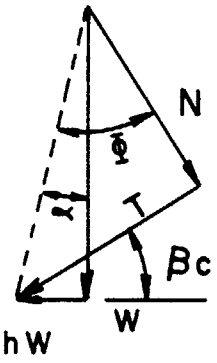


$$\text{SIN } \alpha = \frac{U \sin \Phi}{W}$$

$$\beta_c = \Phi - \alpha$$

PSEUDO - STATIC SEISMIC LOAD





$$\text{SIN } \alpha = \frac{h}{1.0}$$

$$\beta_c = \Phi - \alpha$$

Figure 2.6 Calculation of β_c

of the block does not affect its stability (Figure 2.6). Unfortunately β_c is a function of H for most hydrostatic loadings and for all cases in which the resistance includes a cohesive component. A varying β_c implies that the zone of kinetic stability changes with every combination of H and Φ. The graphical approach is difficult to implement under these conditions. Stability problems involving cohesion or water pressures can, however, be analyzed with the methods outlined in the succeeding sections.

2.2 Monte Carlo Simulations

Monte Carlo simulations constitute an almost purely numerical approach to reliability analysis. They resemble sensitivity analyses cast in a probabilistic setting. In theory, the technique can be applied to virtually any problem that can be modelled deterministically. The factor of safety (FS) is computed for many sets of randomly selected input parameters. The probability of failure (P_f) is the proportion of FS values that fall below 1.0. A more formal definition of P_f is shown in Figure 2.7. The FS values are grouped to form a histogram which is used to delineate the probability density function (pdf) of FS. P_f is the area under the curve to the left of FS = 1.0.

The selection of input values is a critical step in the simulation technique. The values for each variable are chosen from the pdf of that variable. Figure 2.8 describes the procedure. $F_X(x)$, the cumulative distribution function

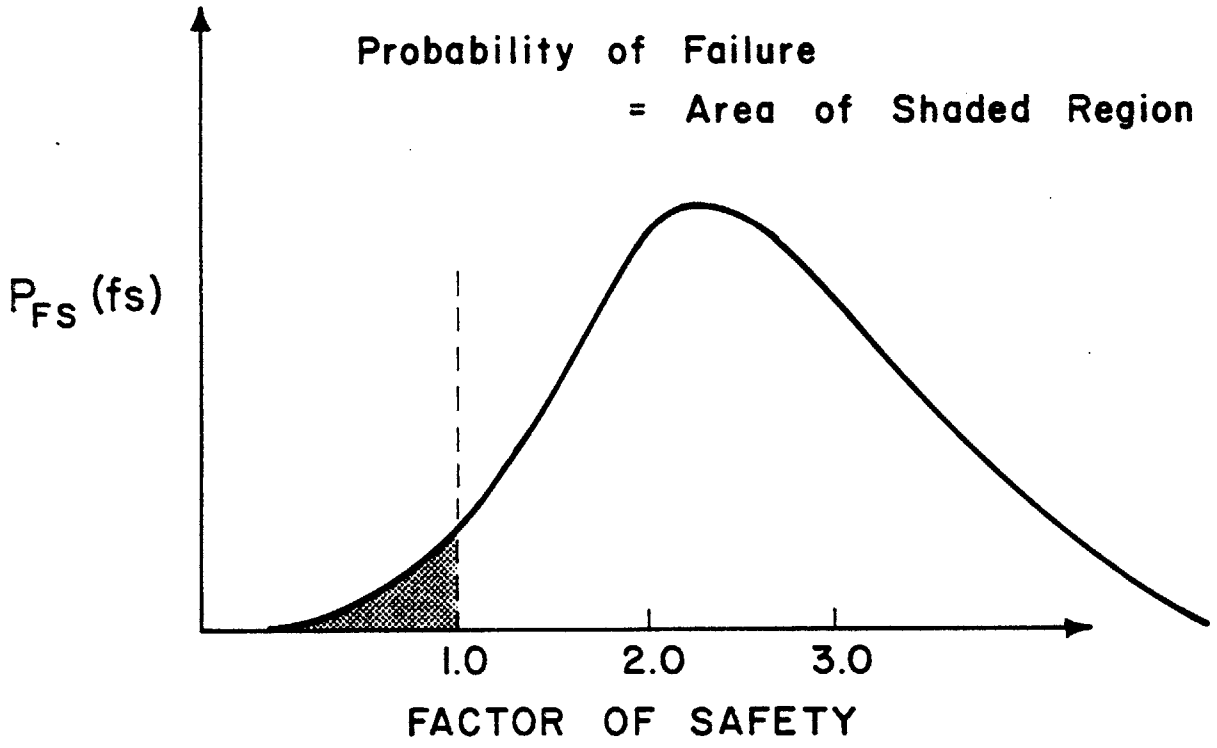


Figure 2.7 Probability of Failure

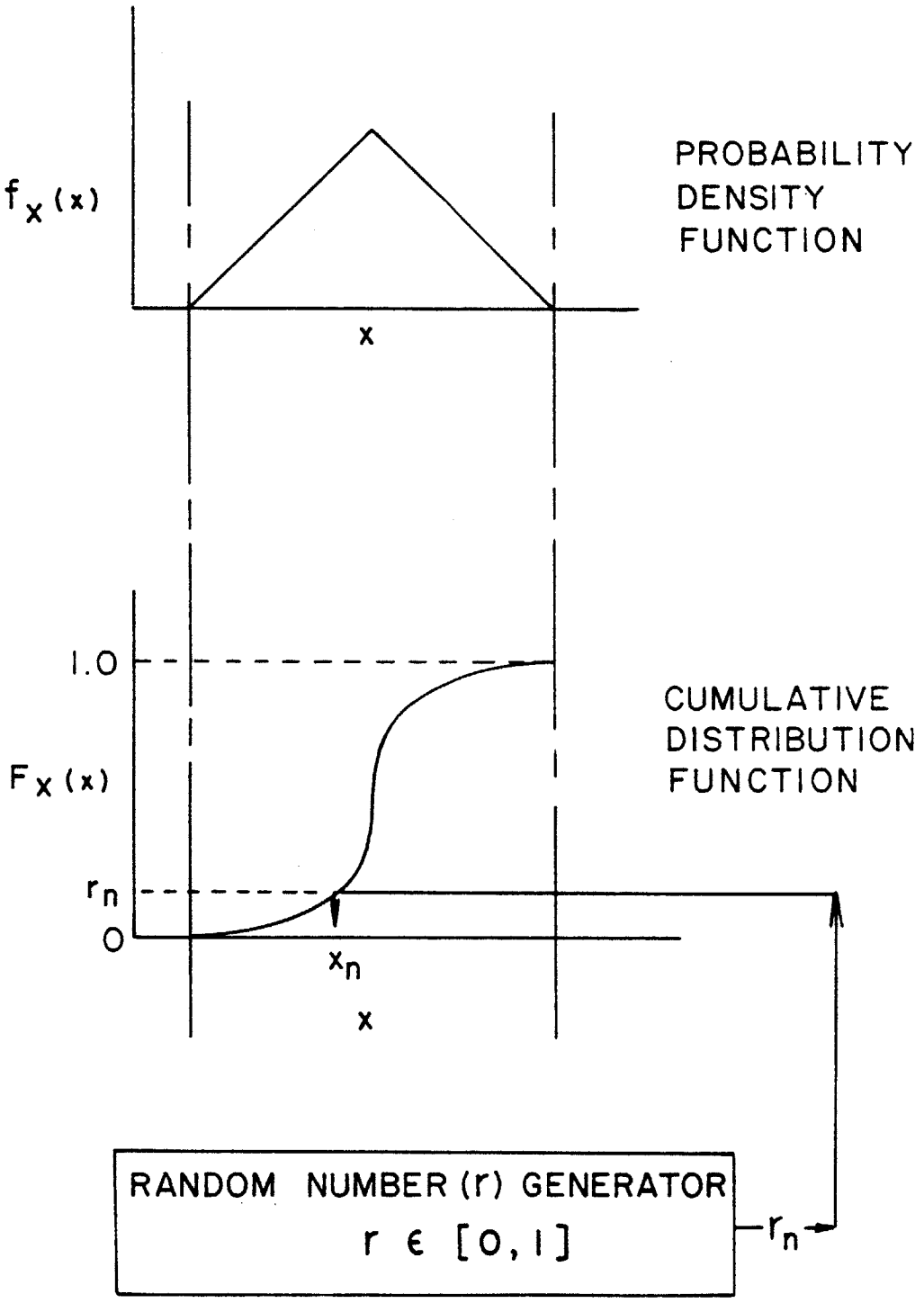


Figure 2.8 Use of Probability Density Function in Monte Carlo Simulation

of x, is the integral of $f_X(x)$. By definition, the domain of $F_X(x)$ is limited to the interval [0,1]. $F_X(x)$ will map any number r_n in that same interval [0,1] to a value of x. If the r_n's are the outputs of a random number generator the values of x will be randomly distributed i.e., the relative freqeuncy with which values of x in the interval $[x + \Delta x, x - \Delta x]$ appear will be proportional to the area under the pdf in that same interval. A histogram of many x's would resemble $f_X(x)$ in Figure 2.8.

Most Monte Carlo simulations involve several random variables. Figure 2.9 is a schematic representation of the technique applied to a rock slope stability problem. In general, the deterministic model is a limiting equilibrium analysis. Input parameters include the attitude and strength of the joint planes as well as ground water conditions and external loadings like rock bolts or surcharges. The analysis must be repeated for every set of input parameters. The volume of computations can become enormous; however, the simulation procedure can be readily programmed for a computer.

Monte Carlo techniques have been applied to numerous problems in rock mechanics. Su et al. (1970), used Monte Carlo simulations in conjunction with finite element methods to evaluate stress distributions around underground openings in rock strata. Pariseau (1973) extended their work to investigate the stability of roofs and pillars in mines. In the last few years Monte Carlo methods have been used to analyze rock slopes. In fact virtually all the reliability analyses to be discussed rely at least partially on simulation techniques.

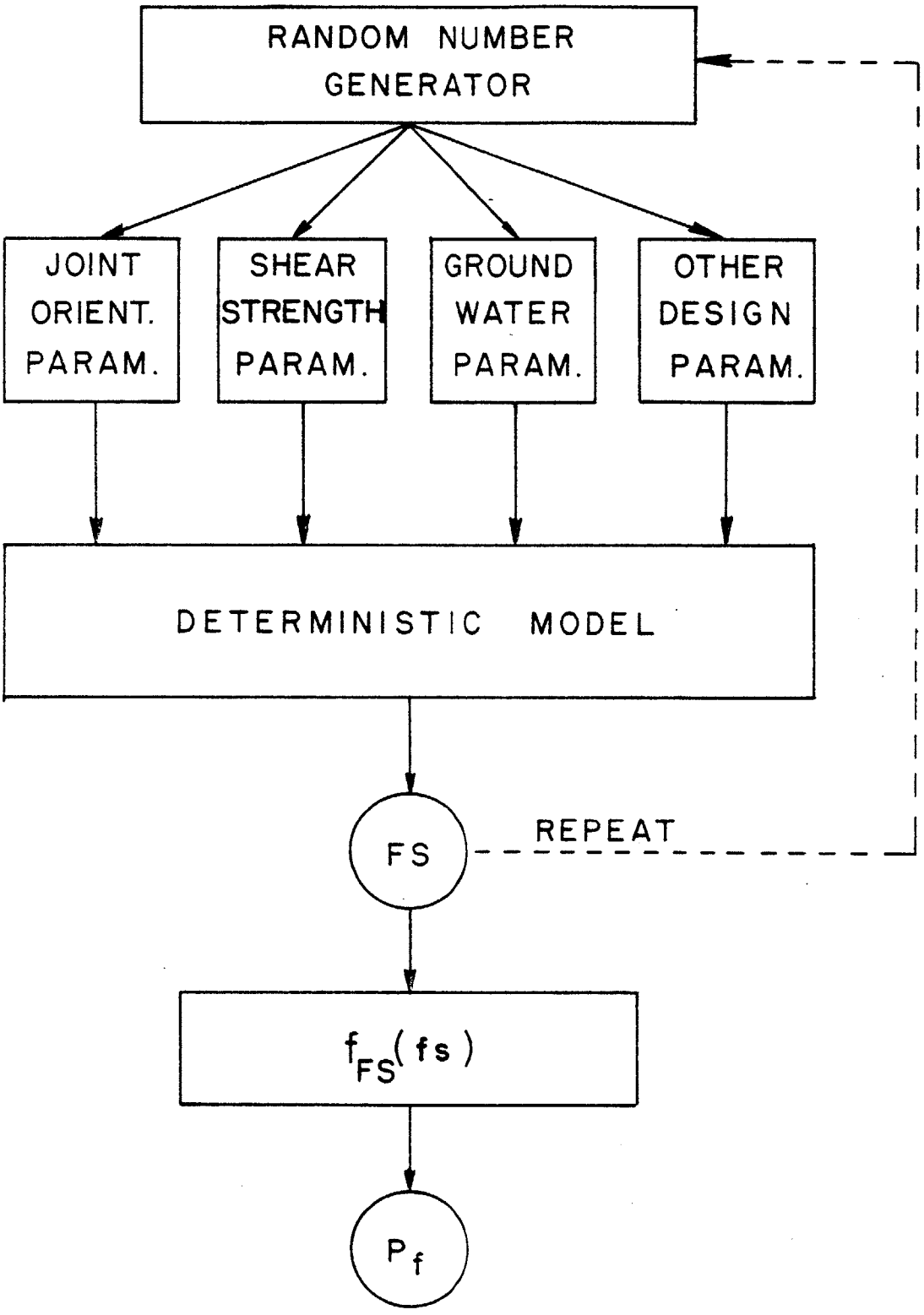


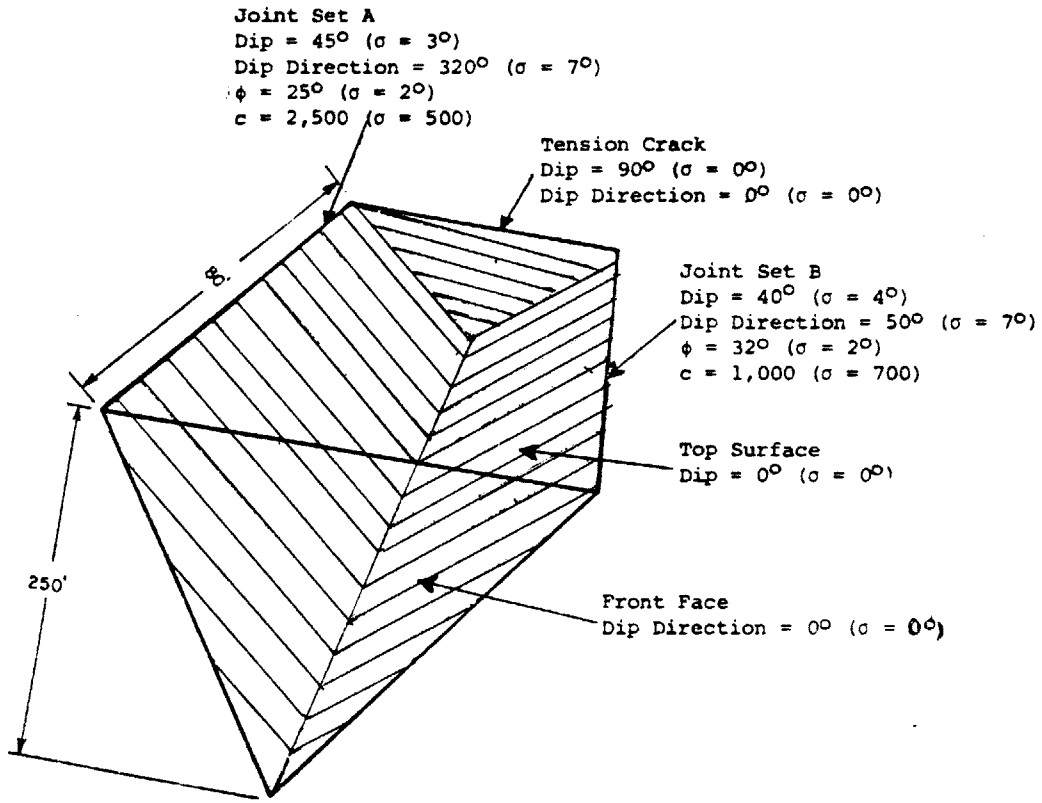
Figure 2.9 Rock Slope Stability by Monte Carlo Simulation

Major et al., (1978) have developed a probabilistic model that relies exclusively on Monte Carlo simulation. The core of their model is a deterministic algorithm that evaluates the stability of rock wedges as their FS against sliding. The wedges are formed by two intersecting discontinuities. The resulting tetrahedron may be truncated by a tension crack (Figure 2.10). Stability calculations are based on the conventional model that examines translational movement of a rigid body.* The analysis involves a maximum of 22 input parameters. Any combination (or all) of these parameters can be treated as random variables. Each random variable is characterized by one of five distributional forms: normal, truncated normal, uniform, triangular, and exponential. Figure 2.10 shows a sample problem presented by Major et al., with ten random variables. Two hundred sets of input parameters were used to find the distribution of FS. The simulations suggest that P_f is in the order of 0.07. (The authors did not document any attempt to estimate the standard error of P_f which may be fairly large.)

The value for P_f derived from the Major et al. computer program is actually a conditional probability based on the assumption that one and only one wedge forms i.e. precisely one joint from each set is present.** The true probability of

* In a companion paper (Kim et al., 1978) the authors briefly discuss other possible modes of failure.

** A similar conditioning problem was discussed earlier in connection with McMahon's graphical approach.



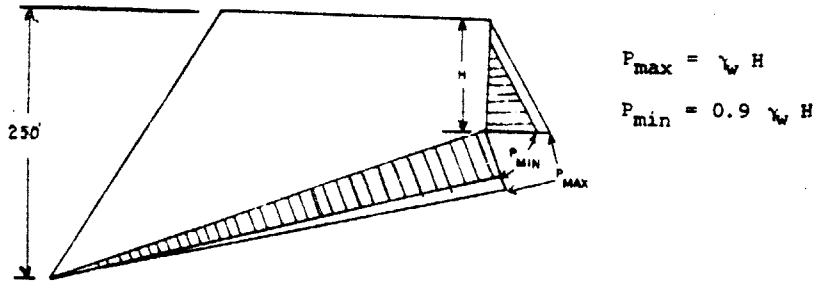


Figure 2.10 Sample Problem--Major et al.

failure is a function of the intensity of jointing. It may be lower than P_f if jointing is a relatively rare event. On the other hand, it may be considerably higher than P_f if the slope is highly jointed. Major et al. do not address this issue. However, the problem is not unique to their method. Virtually all the current reliability analyses must contend with this same problem. There is no simple solution. As will be shown, the joint model prepared by Veneziano (1978) may provide a tentative approach to the multiple wedge problem.

The Major et al. method can be a valuable tool in conducting reliability analyses. It does, however, have some difficulties which are common to all Monte Carlo methods. Simulation can be a computationally inefficient technique for highly reliable slopes. The number of trials required to determine P_f can be very large if P_f is small. The two hundred trials used in the example may be an optimistic figure. A P_f in the order of 0.001 may require several thousand simulations.*

Most Monte Carlo analyses neglect correlation among random variables. They treat each variable as if it were independently distributed. This assumption is frequently not justified. For example, strike and dip of joints might be correlated or extreme values of strength parameters might be

* P_f values in the order of 0.001 (or less) are very sensitive to the tails of the distributions of the input parameters. The tail behavior of these parameters is difficult to estimate at the present time.

associated with certain joint orientations. This correlation structure can become very complicated and difficult to incorporate in the analysis; however, neglecting the correlations can lead to erroneous results.

Monte Carlo methods are relatively easy to implement. Unfortunately this tractability often leads to their indiscriminate use. Monte Carlo simulation is not a panacea for probability analyses. It is subject to the same limitations as its deterministic algorithm. In addition, it requires a rigorously defined pdf for each random variable. The critical issue is whether one can rationally assign a pdf to all the important input parameters. For example, rock slope problems may have an extensive amount of information on joint orientation but virtually no data base for joint persistence or cleft water pressures. Yet, FS is much more sensitive to variations in joint persistence than to variations in joint strike. A poorly selected pdf for persistence will result in an unrealistic estimate of P_f. Major et al. present many options for selecting pdf's but give no hint as to typical distributions for each parameter. (Baecher et al., (1977) present a summary of current research on estimating pdf's for joint spacing and joint length.)

In using Mone Carlo techniques one tends to concentrate on the random variables with well defined pdf's. The ill-defined random variables are arbitrarily assigned disperse pdf's - often a uniform distribution is used. However, each random

variable contributes to the dispersion of FS values. The variable with uniform pdf's may have a much greater influence on the shape and location of the pdf of FS than the other variables. The sheer volume of computations and computer outputs can give one a false sense of security about the results.

2.3 Composite Models

Composite models include all the methods which endeavor to decompose the probability of failure into discrete components that can be analyzed independently. All the models have the same general form:

$$P_f = \Sigma P[A]P[B] \tag{2.5}$$

A is the event that a potentially unstable condition exists. B is the event that the destablizing force exceeds the resistance. The product of their probabilities is summed over all possible unstable conditions. The two events are analagous to the kinematic and kinetic criteria in McMahon's model. In general A and B are treated as probabilistically independent events.

Composite models present an attractive concept. P_f is calculated as the joint probability of several conditions that lead to instability. A complex process is resolved into many simple components and each component is examined as a potential source of uncertainty. Many of the early reliability

models used this approach. Those models have evolved over the past few years to the point where they are now legimate design tools. The two best examples of composite models are those developed by Marek and Savely (1978) and Serrano and Castillo (1974).

The Marek and Savely model examines two dimensional plane shear failures (Figure 2.11). The height is a deterministic quantity (H). The first step in the analysis is to discretise the dip angles (θ's). The Gaussian distribution for θ shown in Figure 2.11 is divided into twelve cells.* The angle at the midpoint of each cell is considered representative of all dips within that cell. The probability that the dip will assume any one of the 12 discrete values is equal to the area of that particular cell. P_f is caluclated as:

$$P_f = \sum_{j=1}^{n} P_{D_j} P_{L_j} P_{S_j} \qquad (2.6)$$

n = number of cells

D_j = dip at midpoint of cell

P_{D_j} = P[dip is D_j]

P_{L_j} = P[discontinuity is continuous from A to B]

P_{S_j} = P[shear resistance is less than the driving force]

*Not all values of θ need to be analyzed. Values greater than the slope angle can be excluded--a kinematic restraint. Marek and Savely suggest neglecting all values less than Φ minus 2 standard deviations of Φ --a kinetic restraint.

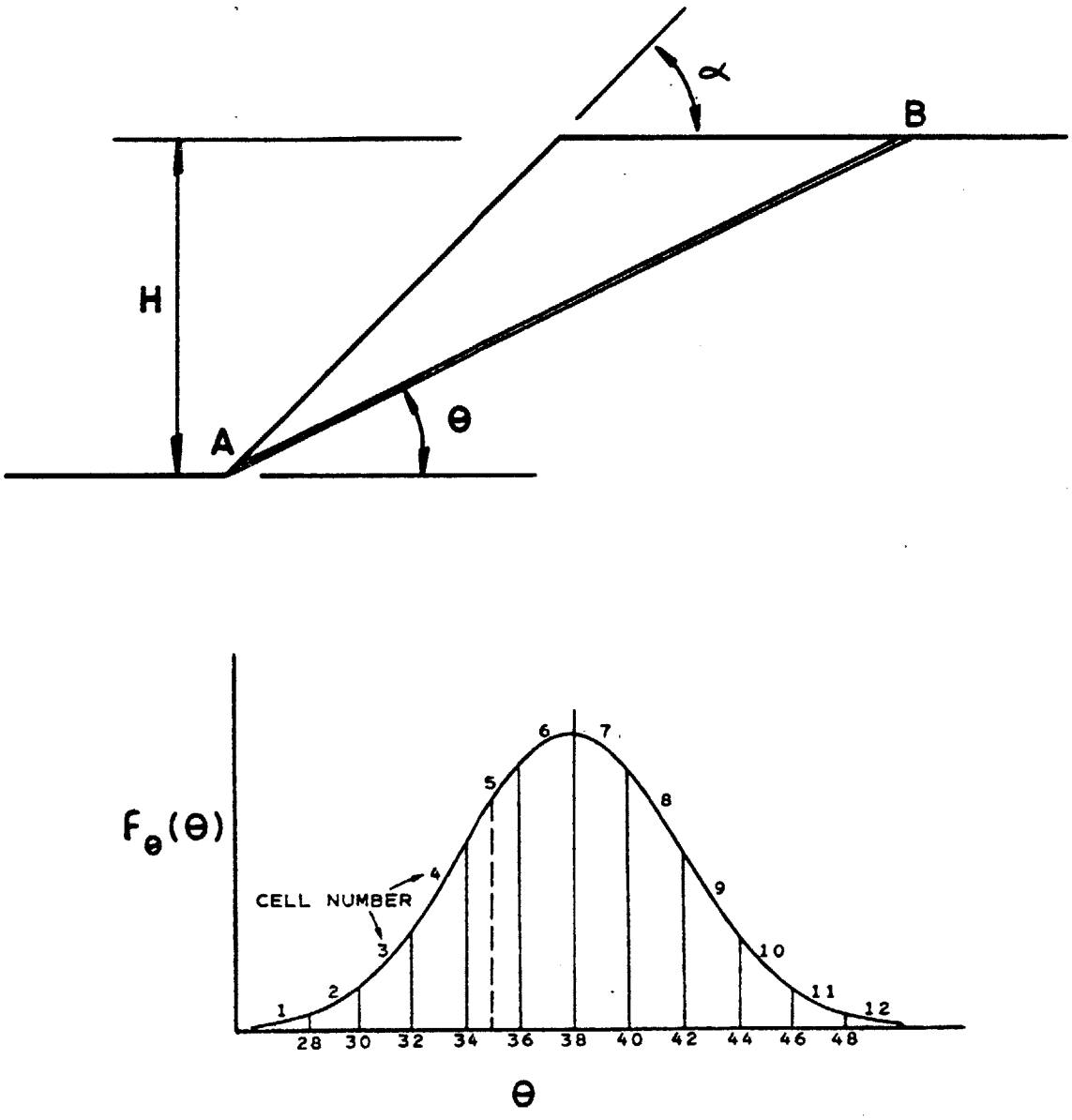


Figure 2.11 Marek & Savely Model

P_{L_j} can be determined from a cumulative distribution function of joint lengths. P_{S_j} is found through Monte Carlo simulation. Since α and H are deterministic, any value $\theta = D_j$ will completely describe the geometry and the weight (W) of the failure block as well as the shear (W sin D_j) and normal forces (W cos D_j) on the failure plane. Marek and Savely use Monte Carlo techniques to sample various sets of shear parameters (joint friction, Φ, and joint roughness, i). P_{S_j} is the proportion of trials in which the shear force (W sin D_j) exceeds the resistance W cos D_j tan (Φ + i).

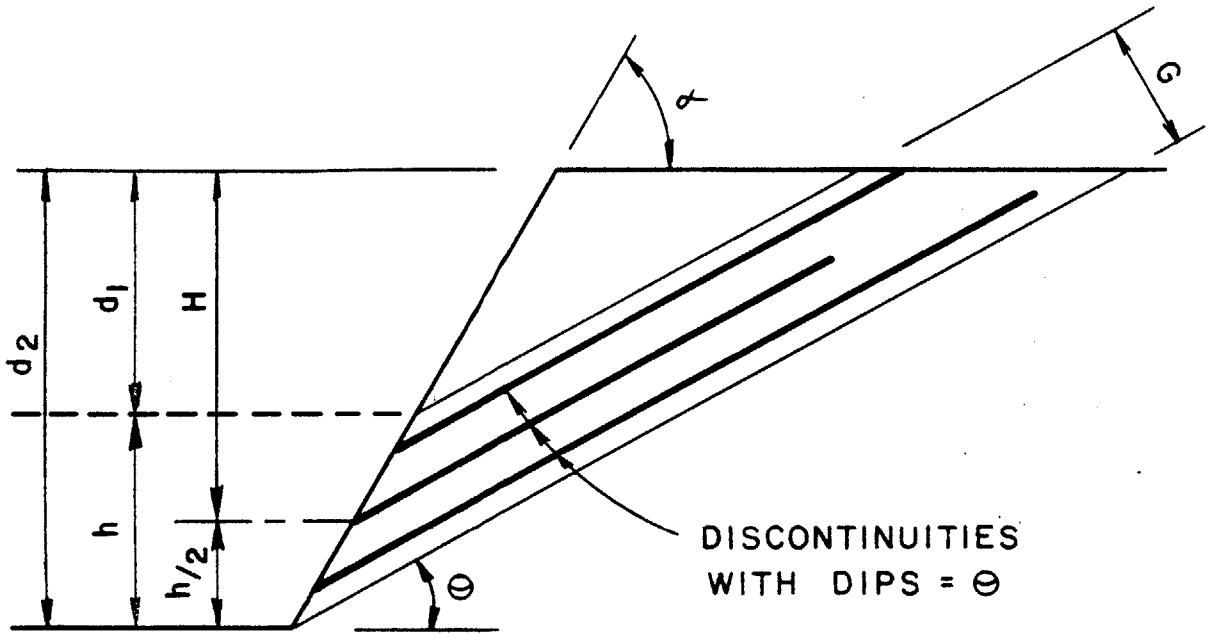
Call and Kim (1978) extended the Marek-Savely method to analyze slopes with numerous discontinuities. They considered the profile shown in Figure 2.12. It corresponds to an open pit mine which is to be deepened from d_1 to d_2--a mining increment of h. N is the number of discontinuities that will be exposed during excavation. The expected value of N can be computed:

$$N = G/\lambda \tag{2.7}$$

$$G = \frac{h \sin(\alpha - \theta)}{\sin\alpha} \tag{2.8}$$

λ = mean fracture spacing

Call and Kim define PI, the probability of instability, as the probability that at least one unstable joint will be exposed. PI can be determined through a binomial expression:



$$G = \frac{h \sin(\alpha - \Theta)}{\sin \alpha}$$

λ = MEAN JOINT SPACING

$\overline{N}$ = MEAN # OF JOINTS

$$= \frac{G}{\lambda}$$

Figure 2.12 Call & Kim Model

$$PI = 1-(1 - P_f)^N \tag{2.9}$$

P_f = probability of failure for a single fracture as computed by Marek and Savely.

The fact that N is actually a random variable can easily be incorporated into the analysis. If joint spacing is exponentially distributed N has a Poisson distribution:

$$f_N(n) = \frac{(G/\lambda)^n e^{-G/\lambda}}{n!} \tag{2.10}$$

PI can be conditioned on n and summed over all possible values of N.

$$PI = \sum_{\text{all } n} \left[f_N(n) \cdot (1 - (1-P_f)^n) \right] \tag{2.11}$$

The Call-Kim model relies heavily on the assumption involving independence. It treats joint stability as a series of Bernoulli trials i.e., mutually independent experiments with a constant probability of failure. The fact that the n^{th} joint is stable does not influence the reliability (P_f) of the $(n + 1)^{th}$ joint. Even Marek and Savely's formulation of P_f is based on the assumption that joint orientation, length and frictional properties are independent parameters.* These assumptions do not necessarily invalidate the model. They should, however, be thoroughly investigated before the model is applied

*A careful search of the data bases for each pair of parameters may reveal correlations that can be introduced into the P_f calculations.

to a specific problem.

Serrano and Castillo (1974) use the same principles in their 3-dimensional model. The principal features of their model are shown in Figure 2.13. The figure is a front view of a rock slope that contains two sets of discontinuities.* Every intersection defines a potentially unstable wedge. Serrano and Castillo characterize the wedges by their surface areas or volumes. The triangles ABC and A'B'C' typify the largest wedges that can form. All wedges of that size will have a vertex that lies along the line BB'. Small wedges such as the family that is typified by ADE and D'E'C' are much more numerous. All the members of that particular family have vertices between D and D'.

The critical feature of the computerized model is a routine that identifies all possible combinations of joints that will produce a wedge whose area lies between the prescribed limits δ_n and δ_{n+1}. The problem is by no means trivial because the joint spacing is a random variable. P_f can be computed through the techniques described earlier. $P(\delta,F_o)$ is the probability that a wedge of volume δ will have a factor of safety less than F_o.

$$P(\delta,F_o) = \sum_{i,j} \sum_{R} f_i^1 f_j^1 f_i^2 f_j^2 f_i^3 f_j^3 \qquad (2.12)$$

* A more general version of the Serrano-Castillo model can accommodate 3 sets of discontinuities.

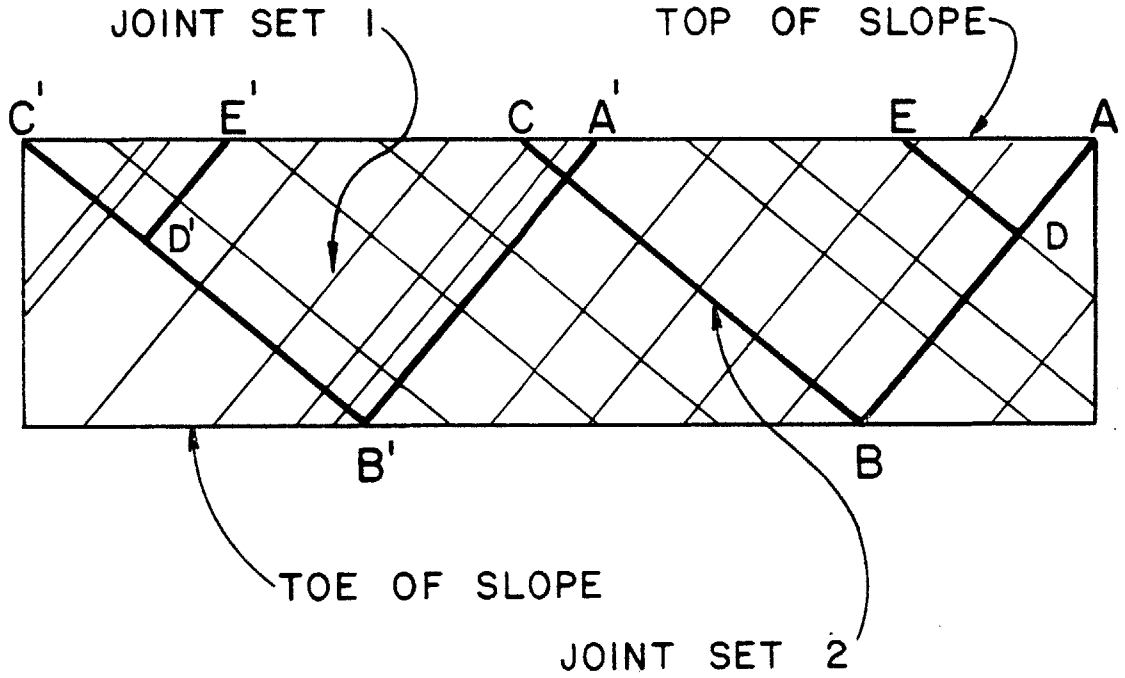


Figure 2.13 Serrano & Castillo Model

i,j = all pairs of joints i and j that satisfy the volume = δ condition.

R = subspace of all combinations of strength parameters that yield FS < F_o

f_i^1, f_j^1 = density functions of the joint strength parameters

f_i^2, f_j^2 = density functions of joint persistence

f_i^3, f_j^3 = density functions of the intact rock strength parameters

The last step in the analysis is to construct the cumulative distribution function of F_o for various values of δ. $F(\delta_o, F_o)$ is defined as the probability that some wedge with a volume equal to or larger than δ_o will have a FS less than or equal to F_o.

$$F(\delta_o, F_o) = 1 - \prod_{d \geq \delta_o} (1 - P(\delta, F_o))^{n(\delta)} \qquad (2.13)$$

$n(\delta)$ = number of wedges that have a volume of δ

Figure 2.14 shows some results that Serrano and Castillo obtained in their example. P_f is the value of $F(\delta_o, F_o)$ at FS = 1.0. It is a function of δ_o and increases as the failure volume decreases.

The Serrano-Castillo model can be a versatile tool in the planning of pit mine slopes. The concept of explicitly

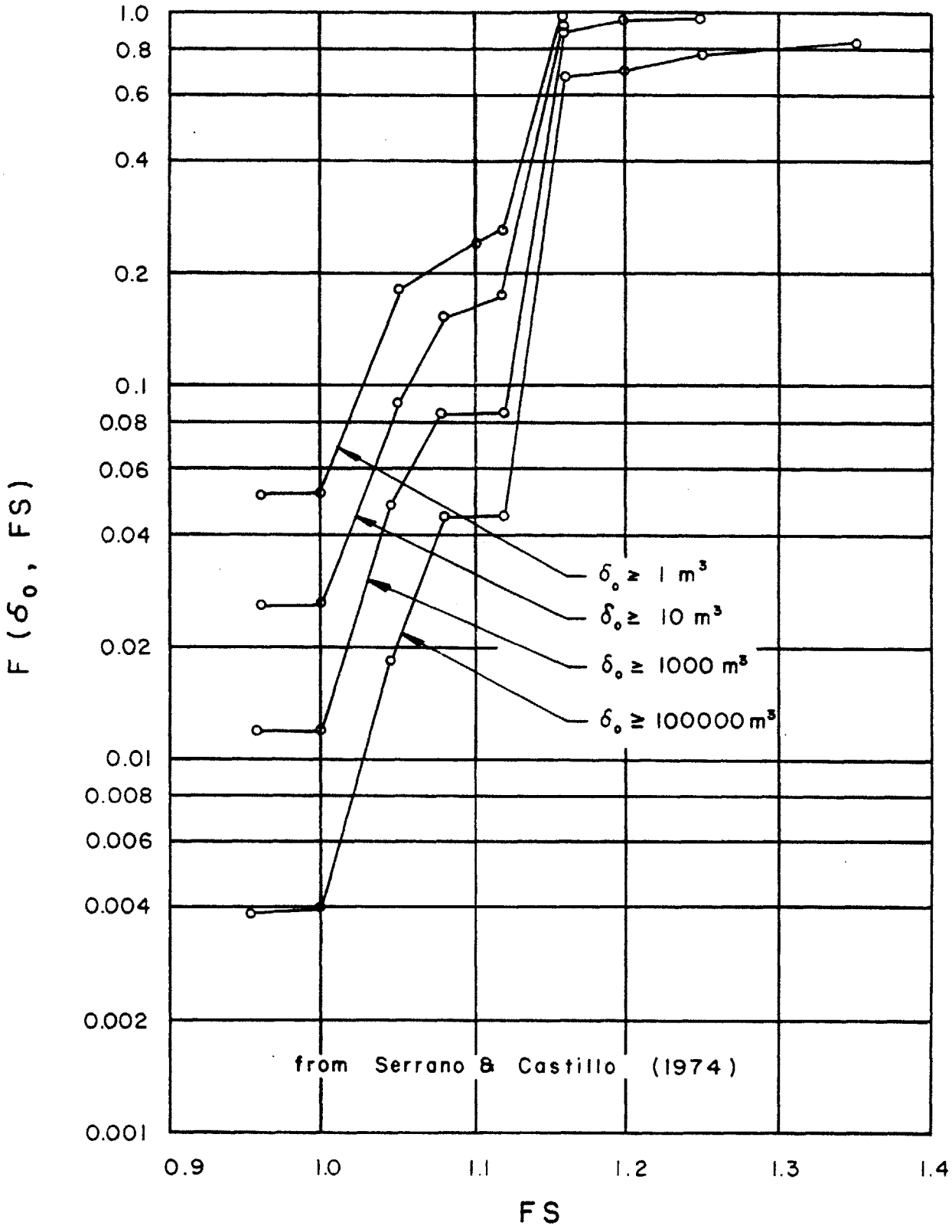


Figure 2.14 Typical Results--Serrano & Castillo Model

considering the size of the failure zone is an attractive approach because it enables a planner to evaluate the consequences of failure. However, the model is based on a number of assumptions concerning independence among parameters. As discussed earlier, these assumptions must be verified before the model is implemented. Finally, the model computes $F(\delta_o, F_o)$ through a long series of conditional probabilities. The volume of computations can become overwhelming for heavily jointed rock masses.

2.4 Stochastic Models

In many ways stochastic models constitute the most interesting approach to reliability analysis because they attempt to describe jointing as a random process. The models do not consider the mode of origin of the discontinuities but they do consider their attitude and location. The idea of treating any aspect of jointing as an analytically describable process is an innovative concept. Until recently joint data (orientation, length and spacing) has been analyzed on a strictly empirical basis. Various authors (Bridges, 1976; Barton, 1977; Call et al., 1976) have perceived some general properties i.e., the exponential nature of joint spacing, but no one has tried to interpret this behavior. Stochastic models present a unifying theory that accounts for these observed properties. However, the models do more than just enable one to visualize jointing patterns in a rock mass. They can be used to derive additional parameters which can

be valuable in stability analyses e.g., the mean number of wedge vertices on an excavated slope. The validity of the model can be measured by comparing predicted values with observed data.

Stochastic models can be either inductive or deductive. Inductive models find the pdf's for the inclination and persistence of failure surfaces by examining many continua--each of which contains a single failure path. Each path is pieced together from many individual joints whose length, orientation and spacing are treated as random variables. Inductive models infer the general properties of failure paths by investigating many specific examples. On the other hand, deductive models infer these properties by first generating joint patterns and then searching the pattern for possible failure surfaces. Deductive models populate a multi-dimensional space with a family of randomly generated joint planes. The entire joint system is examined to locate the path of minimum resistance.

Call and Nicholas (1978) developed an inductive stochastic model to investigate 2-dimensional stepped or "en echelon" failure surfaces. This failure mode occurs in slopes that contain two approximately orthogonal joint sets (Figure 2.15). The failure path is composed of many joint segments that coalesce to form a continuous surface. In some instances the joints may not form a complete path; the remaining rock bridges must rupture before the block can

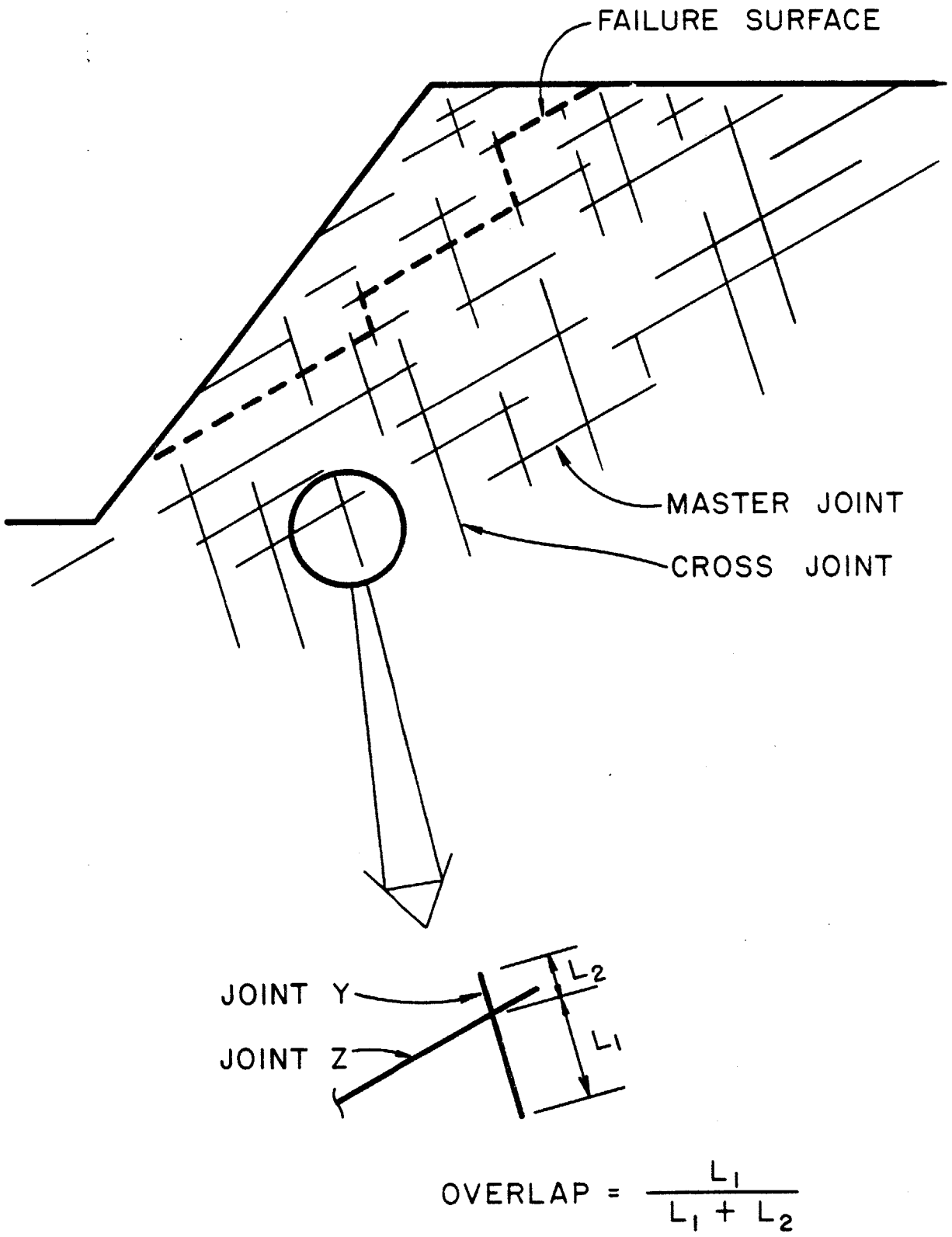


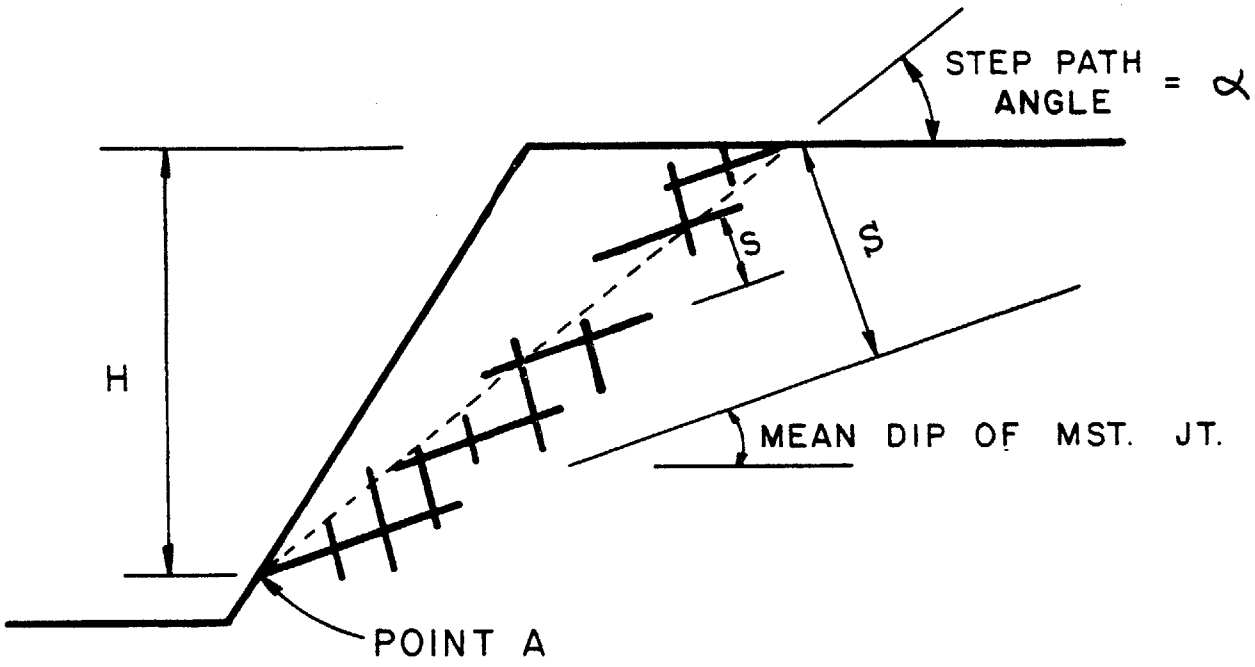
Figure 2.15 Call & Nicholas Model--Definitions

detach from its parent rock mass.

Call and Nicholas term the two joint sets "master joints" and "cross joints" (Figure 2.15). Each set is characterized by four random variables: dip, length, spacing and overlap. The last parameter describes the interaction between the two sets; it is defined in Figure 2.15. The overlap of joint Y is the ratio of L_1, the distance that Y extends beyond joint Z to $(L_1 + L_2)$, the total length of Y*.

The major features of the model are shown in Figure 2.16. The object is to fabricate a hypothetical failure path from an arbitrarily chosen point A to the top of the slope. Point A presumably locates a daylighting master joint and is defined by the distance H. A dip and length for this master joint is sampled from the appropriate pdf's through Monte Carlo techniques. A family of cross joints will intersect the master joint; the points of intersection are simulated by sampling random values from the pdf of cross joint spacing. Values are sampled until the sum of the N spacings exceeds the length of the master joint. The $(N-1)^{th}$ cross joint is the last one that actually intersects the master joint. The exact location of the $(N-1)^{th}$ joint is found by sampling values of dip, length, and overlap from the pdf's. The next master joint (the 2^{nd} "stair") is located by sampling values

* Call and Nicholas indicate that their data suggest that overlap is uniformly distributed over the interval [0,1].



$$\% \text{ TENSILE FAILURE} = \frac{s}{S} 100$$

RANDOM VARIABLES

DIP	MASTER JOINT SET
LENGTH	
SPACING	CROSS JOINT SET
OVERLAP	

Figure 2.16 Call & Nicholas Model--General Features

of spacing, length, dip and overlap from the pdf's of master joints. The complete failure path is produced by alternately generating cross and master joints.

Occasionally the joints will not intersect. Whenever the path is interrupted the simulation process begins again as a new master joint is established. The interruption corresponds to a bridge of intact rock which must rupture before complete failure can occur. The size of the bridge(s) is expressed as "% tensile failure" (Figure 2.16). The entire failure path (including rock bridges) is characterized by α, its average inclination or "step path angle."

In their complete analysis Call and Nicholas generate a large number of failure surfaces and derive the pdf's of α and % tensile failure. The entire procedure is then repeated for various values of H. (In general, α decreases asymptotically with increasing H.) Call and Nicholas used this model to predict the inclination of failure slopes in an open pit mine. They found a good correlation between the distributions for predicted and observed values.

The Call-Nicholas approach is not a true reliability model in that it does not directly provide information on P_f. It can, however, be useful in predicting failure geometries in situations where the mere existence of a continuous failure path ensures failure. The restriction is important because the model ignores the kinetic conditions--it presumes that none of the joint surfaces develop enough resistance to

maintain stability along a continuous failure path. Under simple conditions (no water, no external loads) the master joints must have a mean dip that exceeds Φ, the joint friction angle. (The mean dip of the master joints in the field study was 43° and the "% tensile failure" approached zero.) The kinetic conditions should be considered in analyzing slopes with flatter master joints. One practicable technique would involve sampling a value of Φ from the corresponding pdf for each failure path. If $\alpha < \Phi$ the path would not be considered in determining the pdf of α. The implementation of kinetic restraints would tend to shift the pdf of α to higher values because many potential failure paths with small step path angles would be stable.

The previous comments regarding Monte Carlo simulations also apply to the Call-Nicholas model. The selection of the proper pdf for each of the eight random variables is a critical requirement. The close comparison between predicted and observed failure paths suggests that the authors have chosen reasonable distributions.

Baecher et al., (1977) developed a deductive stachastic model for rock jointing. Their goal was to formulate a conceptual model that could explain two empirical observations:

1. joint lengths are lognormally distributed*.
2. joint spacings are exponentially distributed.

* In a later work (Baecher and Lanney, 1978) the authors indicate that the underlying distribution for joint length may possibly be exponential. They identify a number of sampling biases which filter data from field surveys so that an underlying exponential (or gamma) distribution may appear to be lognormal.

Baecher et al., idealized joints as 2-dimensional convex disks randomly and independently positioned in space (Figure 2.17). The disks are circular with lognormally distributed radii. Joint radius and dip as well as joint radii and location are treated as statistically independent variables. (The independence assumptions are easier to justify if one only examines joints from a particular set of joints from a small zone.) The plan and profile shown in Figure 2.17 suggest what the joint traces might look like. The joint spacing and length characteristics are consistent with empirical observations.

The model is an aid in visualizing the configuration of joints. However, it also describes a random process from which one can infer other properties. Baecher et al., have derived expressions for the mean and variance of the area of joints per unit volume. This parameter could be used as an index property in assessing the competency of a rock mass.

The Baecher et al. model is not a reliability model; in fact, it is not even concerned with slope stability. It does, however, provide a logical basis for approaching stability analyses. The model may be extended to furnish information on the location and shape of wedges at excavated faces.

The Veneziano model (Veneziano, 1978) is similar to the one proposed by Baecher et al. The latter model treats joints as isolated phenomena. Each disk is a solitary feature; the probability that any two disks will be coplanar is zero. The

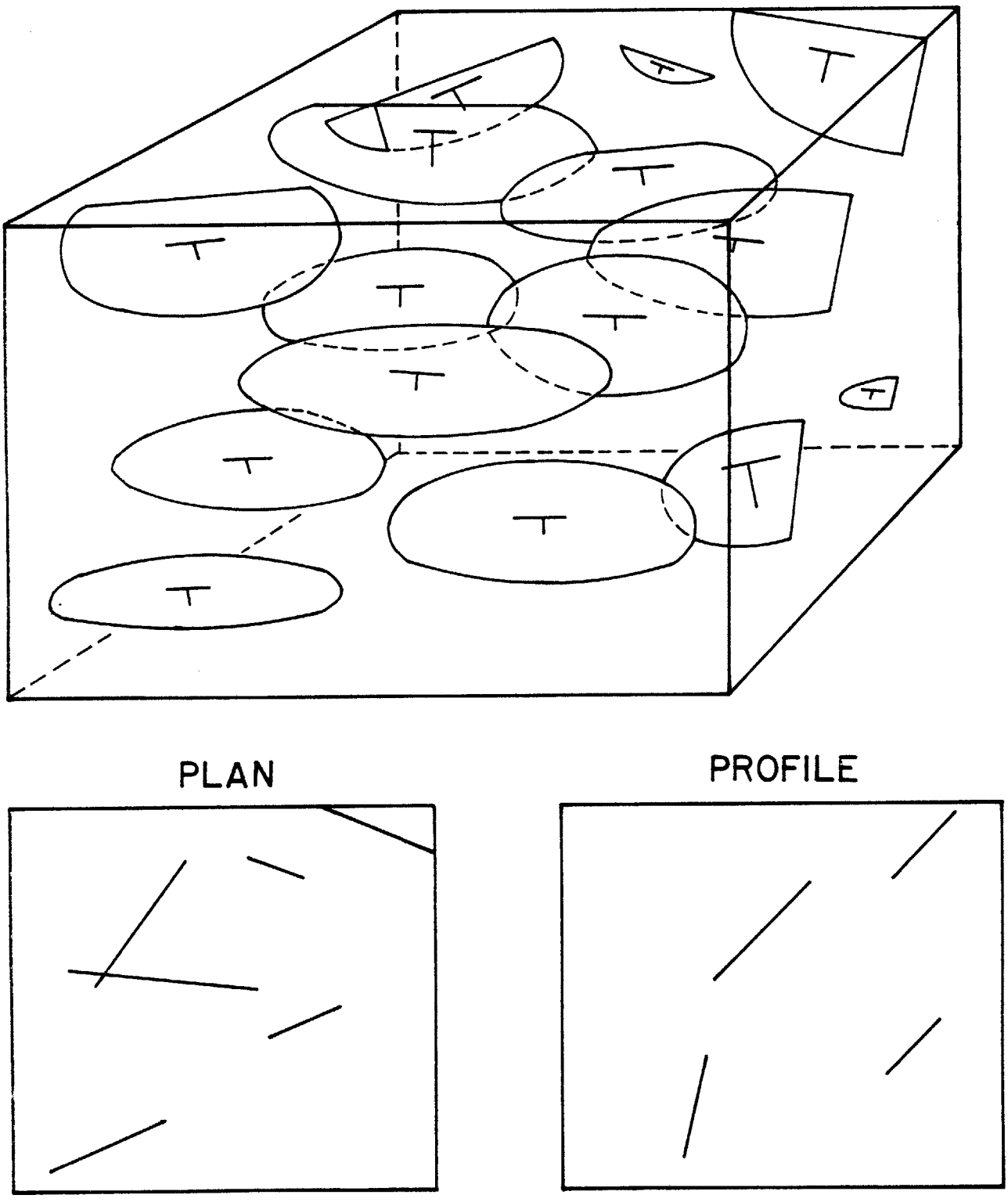


Figure 2.17 Baecher et al. Model

Veneziano model treats joints as bounded figures on Euclidean planes that extend to the borders of the rock mass. Each plane may contain several joints that are separated by zones of intact rock. The Baecher model is actually a special case of the Veneziano model in which each plane contains a single joint.

A detailed description of the Veneziano model will be presented in Chapter 5 so only the general features will be outlined here. Veneziano idealizes networks of joints as a Poisson process of planes in 3-dimensional space. The joints are generated through a three step sequence of random operations. First, Poisson planes or Poisson flats are generated in space to represent joint planes (Figure 2.18). Next, each plane is partitioned into random polygons by a family of Poisson lines. Finally, a specified proportion of the figures are shaded or colored. Shaded polygons correspond to joints while the remaining portions of the planes correspond to intact rock. In its most general form the model has five sets of input parameters: two density parameters (one of Poisson planes in space, the other of Poisson lines in each joint plane), two probability distributions (one of the orientation of planes and the other of the orientation of lines), and the shading probability. The parameters need not be constant; for example, the density of lines and the shading probability may depend upon the attitude of the joint plane. The same parameters may be uncertain and probabilistically dependent.

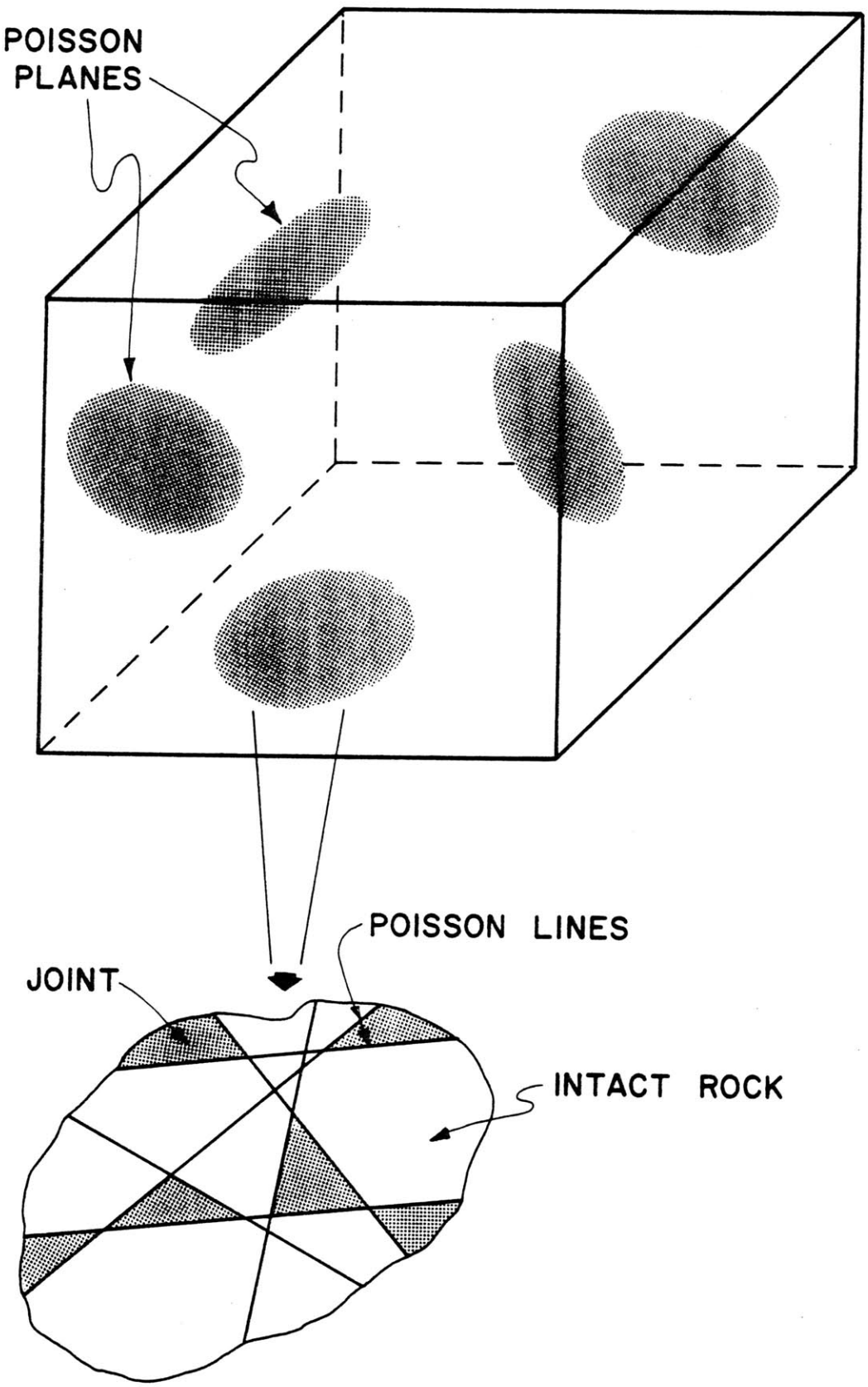


Figure 2.18 Veneziano Model

Veneziano used the model to derive a number of properties that are significant in stability analyses: the mean number of wedges that daylight in any particular region of a free face, the pdf's of the poles of the planes that define the wedges, the persistence along a specified failure surface and the area of joints per unit volume of rock. The first two properties are essentially the same parameters that Serrano and Castillo use in their analysis. Veneziano, however, has a closed form solution that greatly reduces the computational effort. These two parameters may provide the key to solving the multiple-wedge conditioning problem discussed earlier. The general approach could follow the technique developed by Serrano and Castillo: divide the excavated face into regions that contain wedges with similar sizes or volumes. Then each region can be analyzed individually and the resulting P_f values will actually be conditional probabilities based on the failure volume. The present version of the Veneziano model is not designed to compute P_f values. It is a conceptual model that hopefully will lead to a better understanding of the random process that control slope stability. The model may prove difficult to implement because of the unconventional nature of its input parameters. The five parameters that describe the Poisson and coloring processes must be derived from the data obtained in field surveys. Finally, the model may never realize its full potential. The model can handle correlations between various parameters. However, even the most

detailed field survey cannot provide enough information to define all these correlations. The model is simply much more powerful than the current state of the art in data acquisition and reduction.

2.5 Summary

All of the methods described in this chapter are recent additions to the rock mechanics literature and are currently undergoing improvements and modifications. The various authors have approached the reliability problem from many different directions. Despite the variety of techniques there are a number of observations that pertain to all the approaches:

1. All the methods demand accurately defined pdf's for the input parameters. Much more information is needed on typical distributional forms for such parameters as joint orientation and joint strength.

2. Information on correlation between random variables is almost totally lacking. For example, does Φ depend on the joint length? This point raises two questions: How can field surveys be modified to deduce these correlations (if they exist)? How can the correlations be introduced into the analyses?

3. The assumption concerning the independence of joint planes must be carefully investigated. Do joints with particular orientations tend to congregate?

If a tetrahedral wedge is formed by one joint from set X and one joint from set Y does the fact that joint X_n from set X constitutes one boundary affect the marginal distribution of the member from set Y?

4. The multi-wedge problem needs to be examined as an exercise in system reliability. The binomial approach (Equation (2.11)) for calculating P_f may be too conservative. The P_f's may be correlated i.e., adjacent wedges may have similar P_f's because they share the same hydrostatic loads, etc.

5. Should all the uncertainty associated with a particular random variable be lumped together as if it had a common source? For example, a series of laboratory tests could be used to develop a pdf for Φ. The pdf would reflect the imprecision of the tests as well as changes in the composition of the samples. However, this pdf is not necessarily the most appropriate one to use in stability calculations. The joint planes are so large that they tend to even out variations in composition. The pdf in the stability analysis should be less disperse than the laboratory one. (Vanmarcke (1977) examined this same problem in connection with fluctuations in soil properties.)

The comments outlined above span a wide range of disciplines. There are questions involving geology, random process

theory, systems engineering and geotechnical engineering. Some of the questions may not be answered for years to come. However, they should serve as a constant reminder that improvements in analytic techniques alone will never provide a complete solution to the problem of rock slope reliability.

CHAPTER 3

KINEMATIC CONDITIONS--COMPUTER PROGRAM DAYLITE

3.0 Introduction

The first step in analyzing the stability of a slope is to examine the kinematic restraints. Every pair of joints that intersect define a wedge; however, only a small number of these wedges may pose stability problems. The purpose of the kinematic tests is to isolate those potentially unstable wedges for further study. The tests are performed on a stereonet with constructions similar to the ones used in McMahon's (1971) approach.

A wedge will not be kinematically unstable unless the dip of its line of intersection is less than the apparent dip of the slope in the direction of the line's strike. From a graphical standpoint, only those lines of intersection which plot in the shaded area in Figure 3.1 will define potentially unstable wedges.

In a deterministic analysis where one is only interested in two specific planes the test is simple. Each pair of planes (and their corresponding line of intersection) can be checked individually. The tests will eliminate many wedges from further consideration.

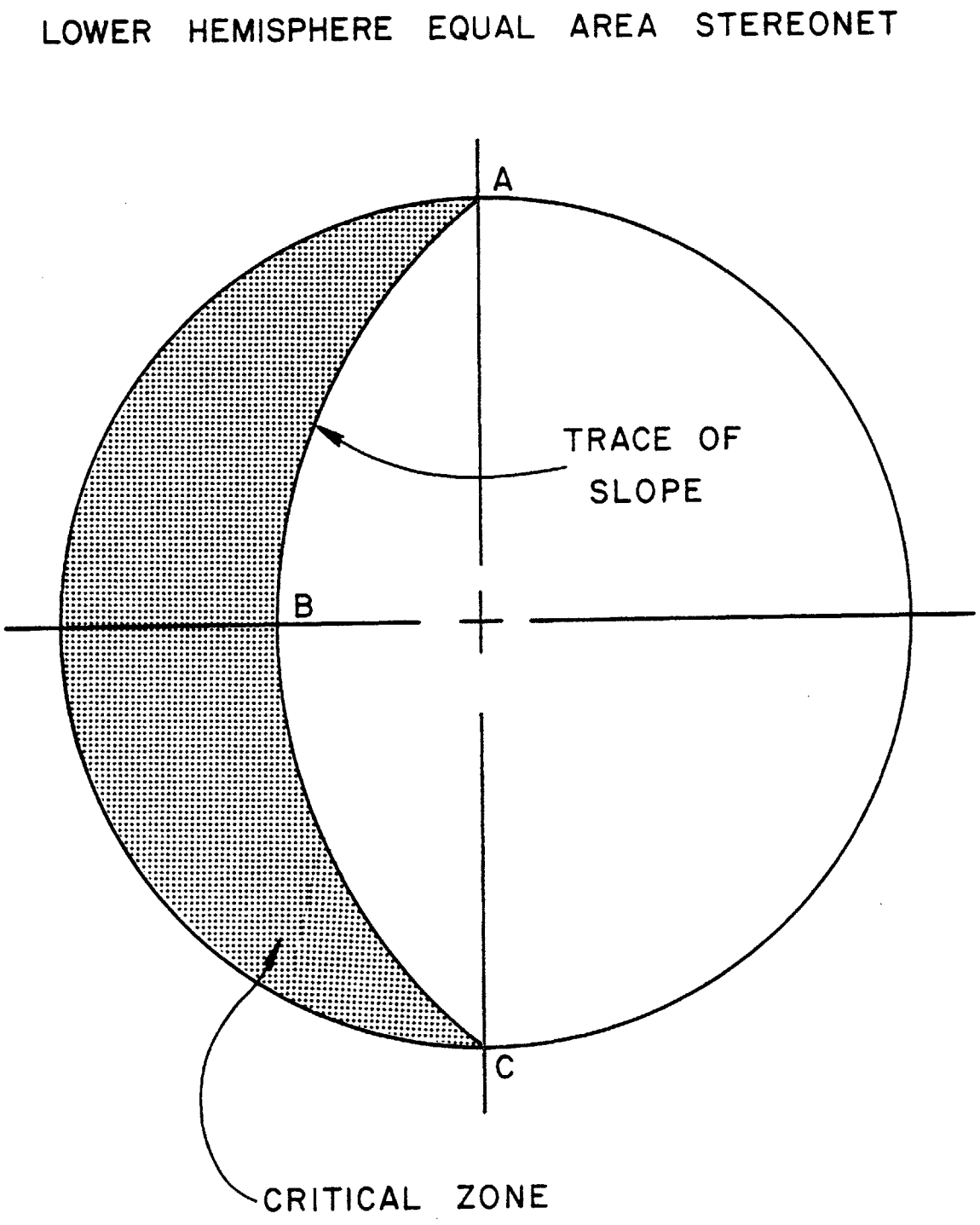


Figure 3.1 Critical Zone for Kinematic Instability

The kinematic tests are much more difficult to implement in reliability analyses because the orientations of planes are random variables. If, however, the pdf's of orientation can be described analytically it may be possible to derive the pdf of orientation for the lines of intersection. Knowing the position (and form) of this pdf in relation to the orientation of the slope face, one can compute some probabilities that may be useful in reliability analyses. For example, one can find the probability that any one wedge formed by two planes will be kinematically unstable--it is the volume* of the pdf that extends over the shaded area in Figure 3.1.

Unfortunately there is no closed form solution for obtaining the pdf of the lines of intersection given any arbitrary pair of pdf's that describe the orientation of joints. Therefore, the computer program DAYLITE was developed.

3.1 General Features of DAYLITE

DAYLITE derives the intersection pdf by computing $f(\theta,\phi)$ at specific points** in the critical zone. Each one of these points represents an orientation; the program identifies all possible combinations of joint planes which will have a line of intersection at that orientation. In essence, DAYLITE finds

* The pdf defined on the surface of a sphere is a 3 dimensional function. The integral of the curve over a region of the sphere ($\theta_1<\theta<\theta_2$; $\phi_1<\phi<\phi_2$) is the probability that the random variables (θ,ϕ) will be within the region.

**The program actually examines small incremental areas that can be characterized by a single point.

the pdf by addressing the question: Given an orientation (θ_o,ϕ_o), what is the probability that two joints will combine to form a wedge whose line of intersection points in the (θ_o, ϕ_o) direction?

The solution procedure involves 3 steps:

1. Defining and partitioning the critical zone into cells.
2. Examining each cell to find pairs of joint planes that will have the appropriate line of intersection.
3. Calculating the probability that the slope will contain a requisite pair of joints.

Figure 3.2 lists the individual operations.

The remainder of this section presents an overview of the general procedure. Section 3.2 presents the details of each operation.

DAYLITE derives the intersection pdf numerically, but many of the mathematical operations can be visualized as graphical constructions on a stereonet. As indicated in Figure 3.1, the kinematically critical zone for wedge instability is bounded by the great circle that represents the trace of the slope on a lower hemisphere projection. The program partitions the zone into equal area spherical rectangles or cells. The boundaries of the rectangles constitute small circles of constant dip and great circles of constant strike so that the geometric figures

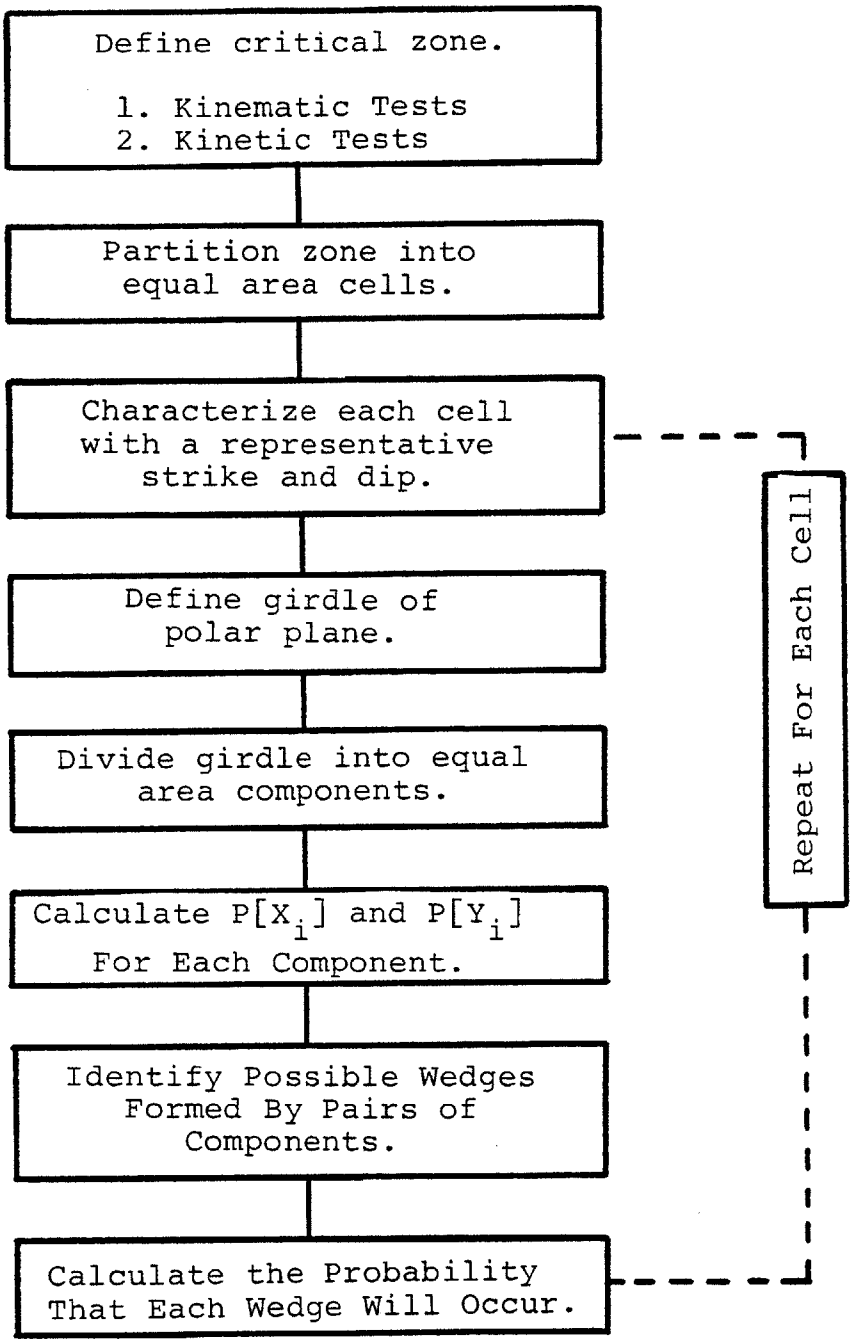


Figure 3.2 Flowchart for DAYLITE

correspond to the elements of a polar equal area stereonet (Figure 3.3). The purpose of the partitioning is to discretise the range of critical orientations into a number of representative values that can be studied on an individual basis. The value at the center of the cell characterizes all orientations within it. The program examines each of these cells and computes the probability that the line of intersection of two joint planes will be in that cell.

DAYLITE uses joint poles rather than joint planes. The line of intersection between two joints is the pole of the plane that contains the two joint poles (Figure 3.4). (This plane is termed the "polar plane".) AOB, the angle between the poles measured in the polar plane, is the dihedral angle between the joint planes. This dihedral angle is directly related* to the central angle or ψ angle of the wedge (Figure 3.5). There is a unique relationship between lines of intersection and polar planes. Each line of intersection corresponds to a different plane.

The colatitude** and latitude that characterize each cell define polar planes. Each cell represents a small range of orientations. If all the polar planes from a particular cell defined by (θ_o, ϕ_o) were plotted they would fall into a narrow

* In general, AOB and ψ are supplementary angles. However, under certain conditions, ψ = AOB. This descrepancy arises because all poles are plotted in the lower hemisphere. As will be shown in Section 3.2.6, the problem can be avoided by reverting back to planes to measure ψ angles.

**It is convenient to use spherical coordinates in developing computerized solutions. θ is the colatitude and corresponds to the azimuth. ϕ is the latitude and corresponds to the complement of the plunge angle.

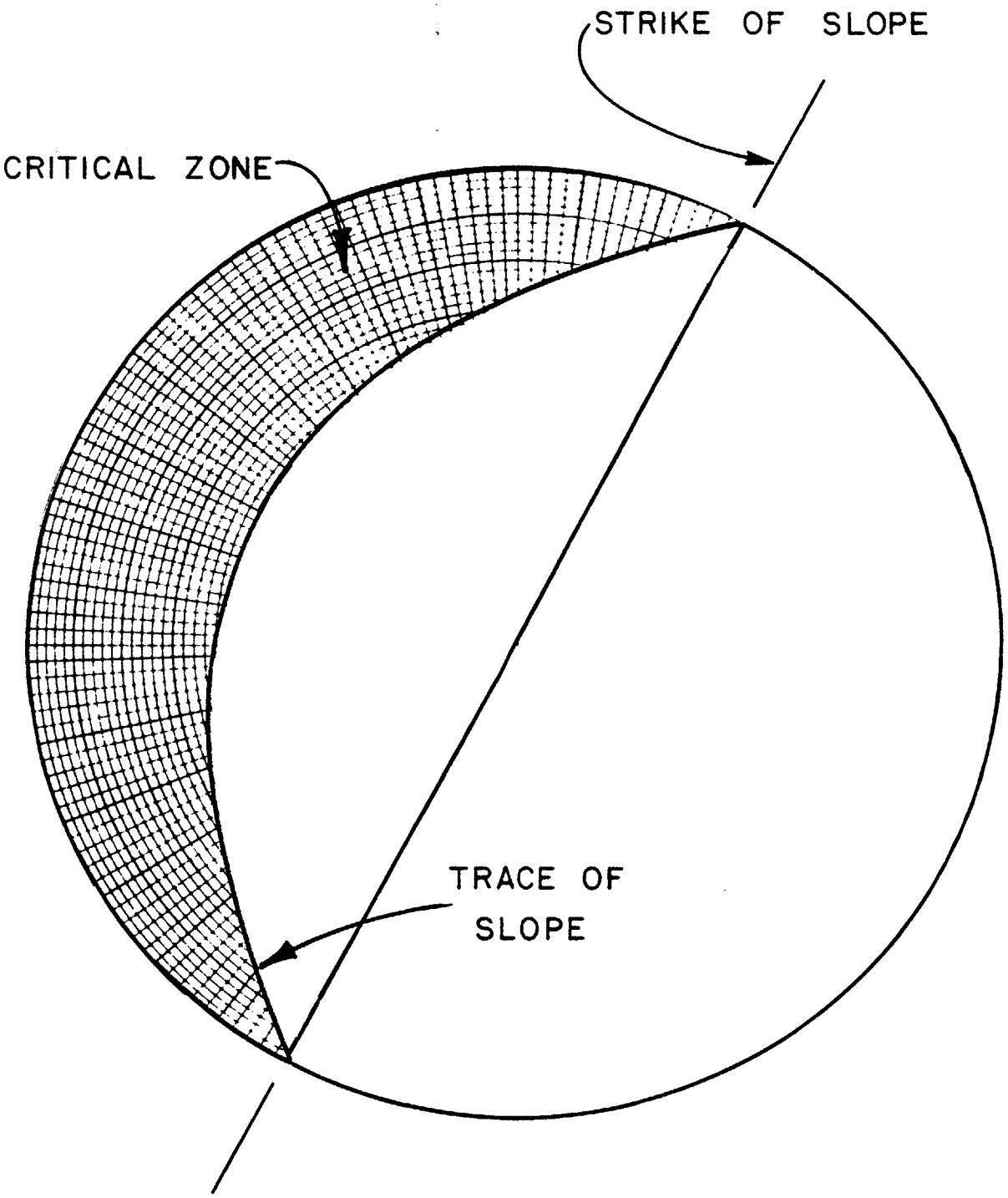


Figure 3.3 Critical Zone Partitioned into Equal Area Cells

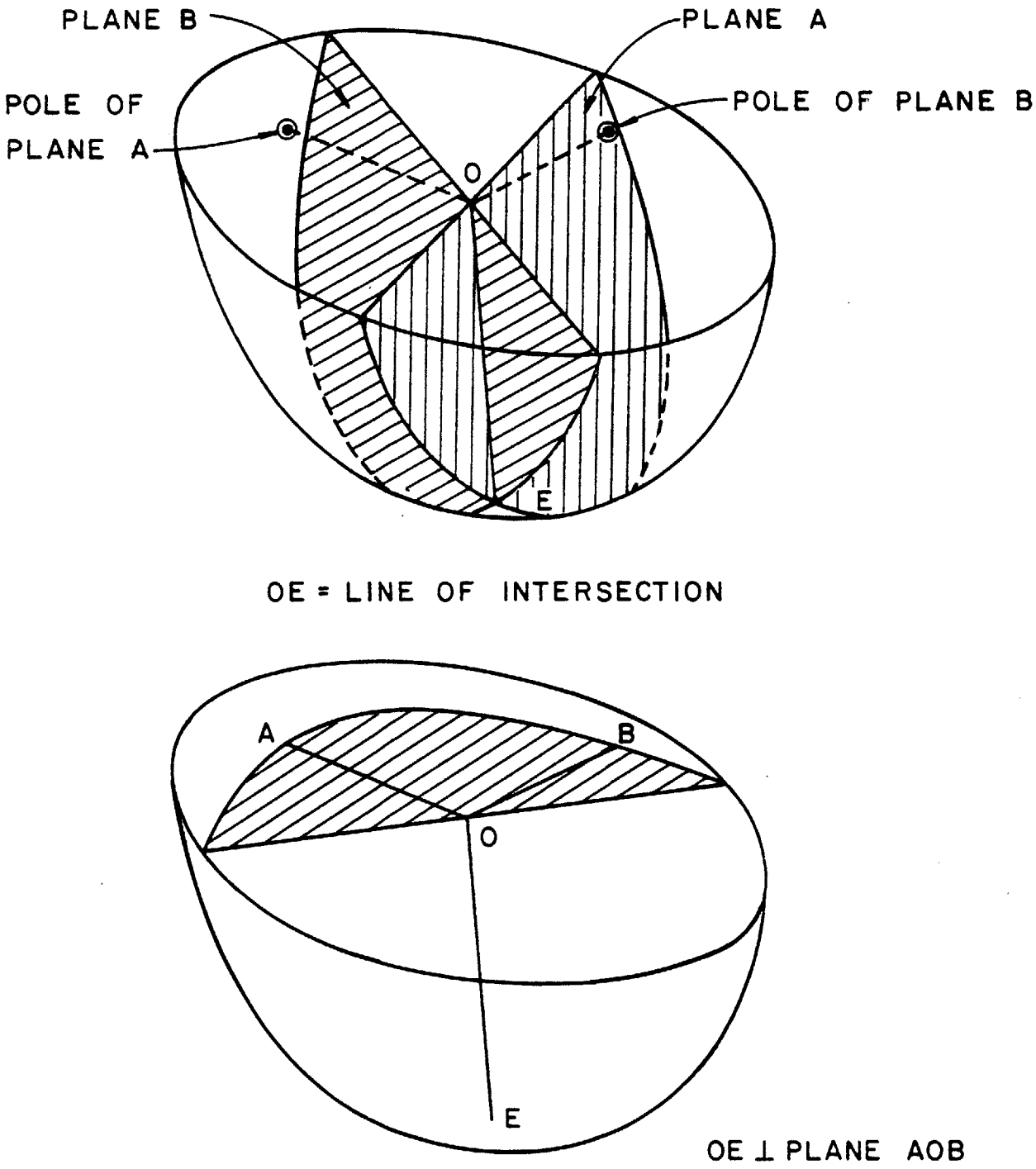


Figure 3.4 Two Intersecting Planes

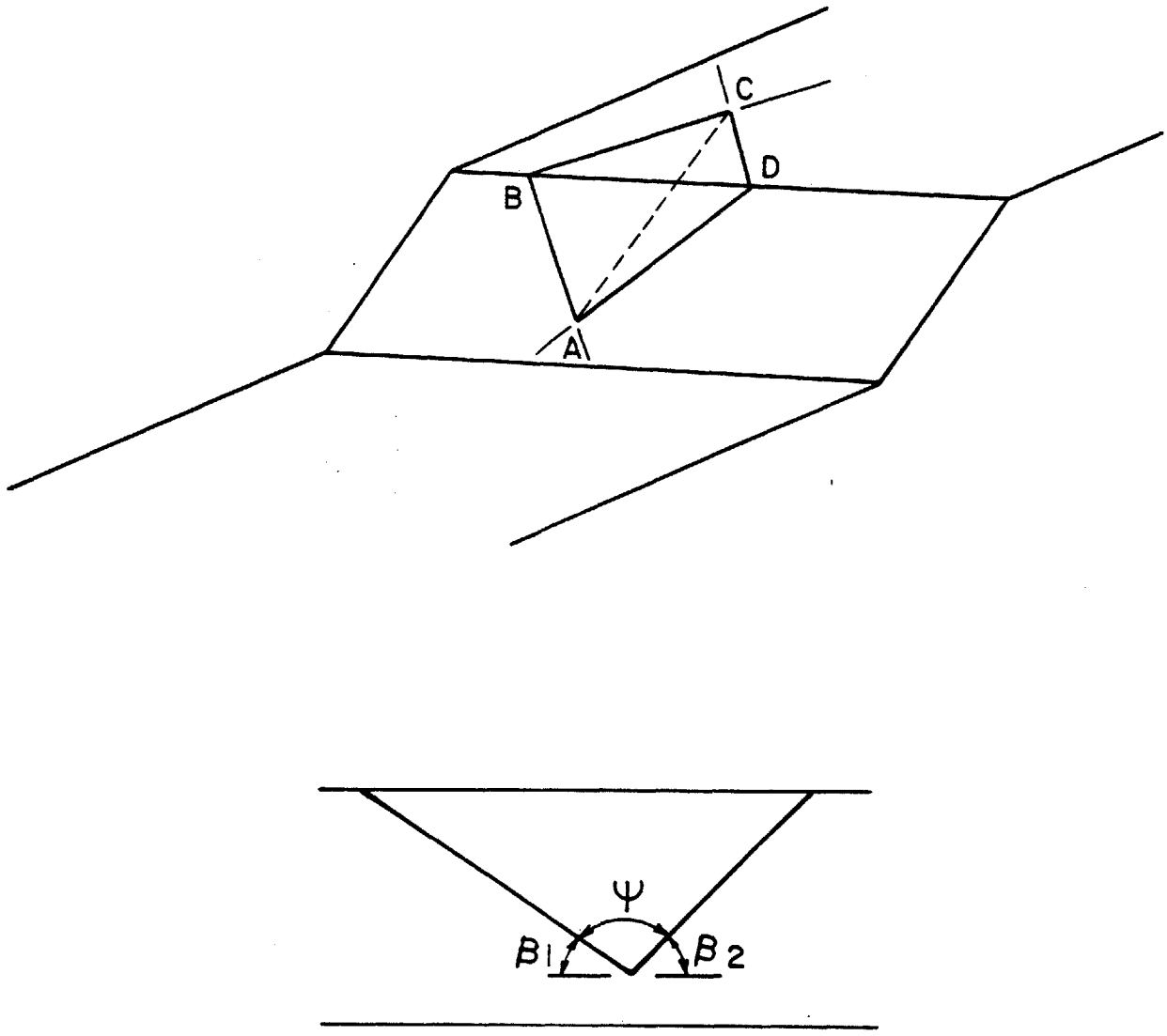


Figure 3.5 Definition of ψ

band or girdle* centered around the (θ_o, ϕ_o) polar plane (Figure 3.6). Figure 3.7 depicts the girdle from Figure 3.6 after it has been removed from the hemisphere and discretised into equal area components. Each component corresponds to a small area on the hemisphere. DAYLITE uses the pdf's of joint orientation to compute the probability that a joint pole will lie in any one component. $P[X_i]$ is defined as the probability that any one joint from set X will have a pole in component i; it is the integral of the pdf of set X over the spherical area of component i.

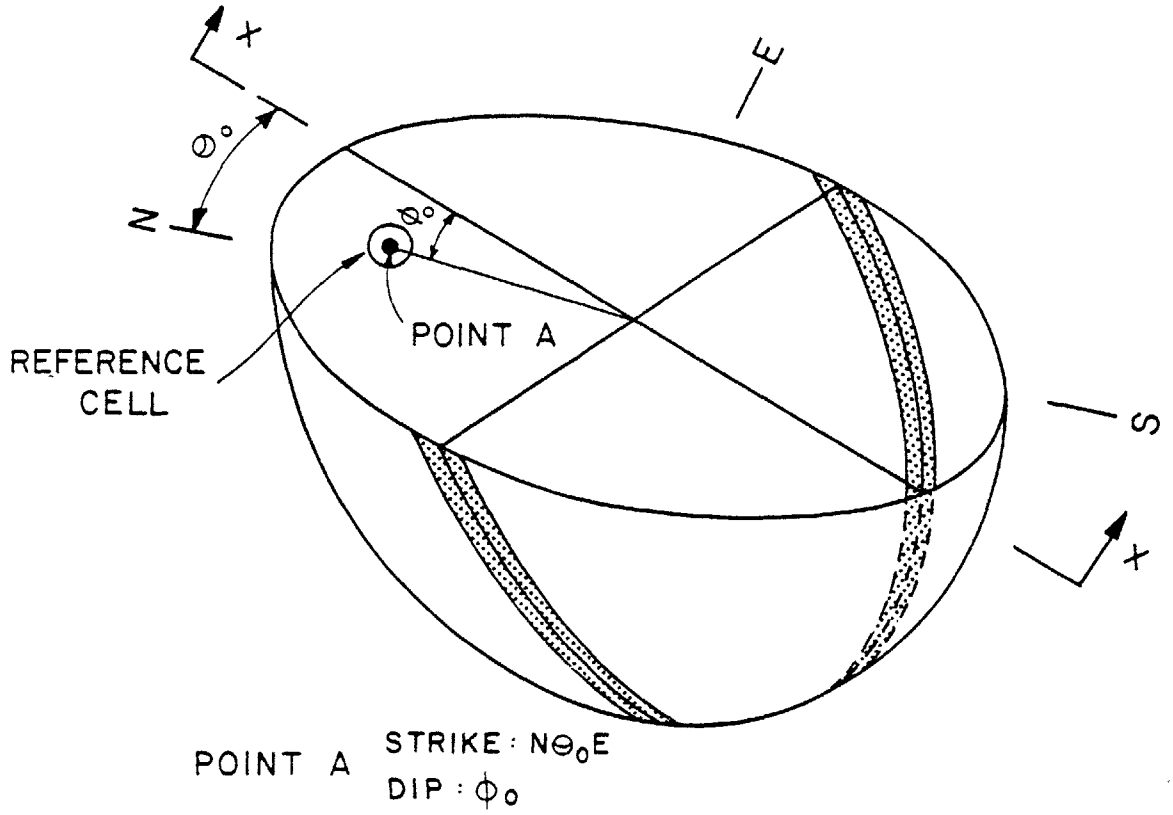
The partitioning operation on the girdle establishes n representative poles--one for each component. Each pair of poles (i,j) defines a wedge. The probability that a wedge defined by (i,j) will form is:

$$P[X_i]P[Y_j] + P[X_j]P[Y_i] \qquad (3.1)$$

One joint must lie in component i while the other lies in component j.

As will be shown, the smallest wedge the program will consider has a ψ of 180/n. The largest wedge has a ψ of (n-1)180/n. The program examines these extremes as well as all intermediate size wedges; in fact, the program investigates every possible combination of the n joint poles.

* Poles from a rectangular cell will not generate a true girdle i.e., a band of constant width. The actual band will be slightly wider at the ends.



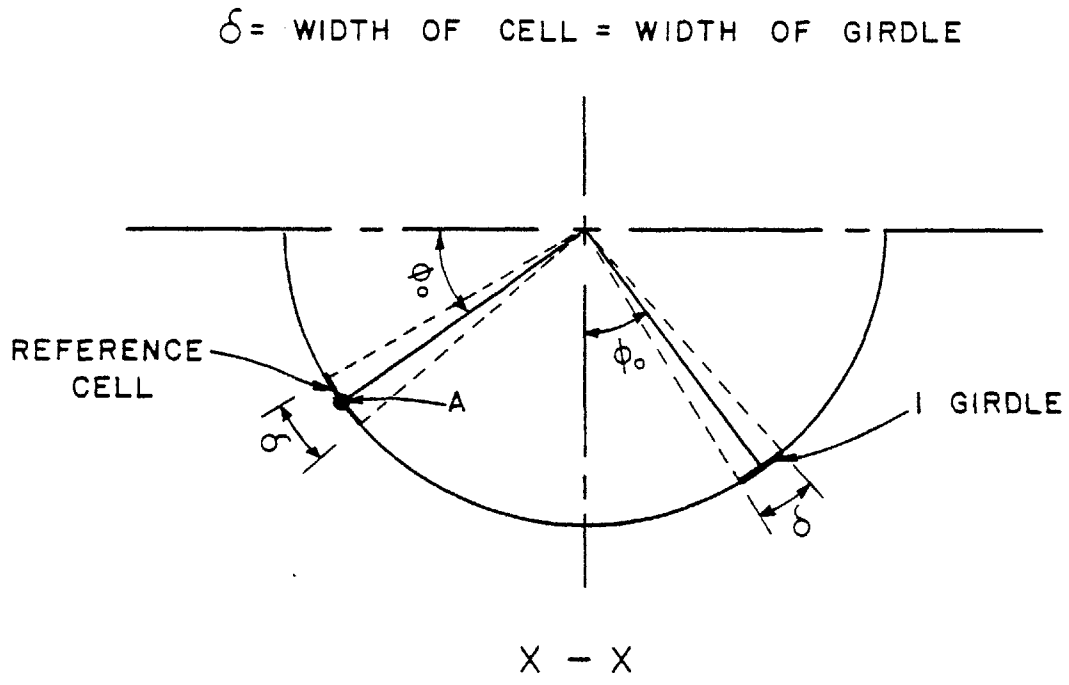
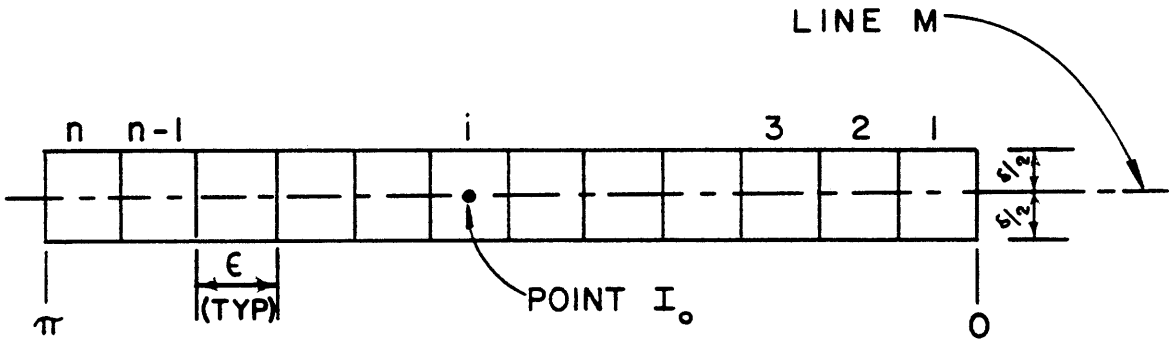


Figure 3.6 Reference Cell and Its Girdle



$$n = \text{NUMBER OF COMPONENTS} = \frac{\pi}{\epsilon}$$

Figure 3.7 Typical Girdle

The results of the analysis are expressed as $P[\theta_K, \phi_K, \psi_1]$, the probability that a wedge of central angle ψ_1 will have a line of intersection that daylights at (θ_K, ϕ_K). (θ_K, ϕ_K) are the characteristic coordinates of cell in the critical zone and ψ_1 is a discrete variable with values $\frac{180}{n}$, $\frac{2(180)}{n}$, $\frac{3(180)}{n}$ .... $\frac{n-1}{n}$ 180.

The two joints do not necessarily have to come from different sets. DAYLITE will calculate the probability that a particular wedge $(\theta_K, \phi_K, \psi_1)$ will be formed by two joints from set X (or two joints from set Y). The resulting probabilities are conditioned on the premise that only two joints from set X are present in the slope. The orientations of the two joints are treated as independent random variables i.e., the fact that one joint lies in a particular cell does not affect the pdf of the other joint. This same assumption of independence underlies the analysis involving joints from different sets. The assumption might be easier to justify in the latter case.

The critical step in the analysis is to assign values of $P[X_i]$ and $P[Y_i]$ to each component of the girdle. The operation requires that the pdf of joint orientation have an analytic form. Once the forms are determined the probability calculations become exercises in numerical integration. $P[X_i]$ is the volume of the pdf of X that lies over the i^{th} component.

DAYLITE will accept six different distributional forms:* Uniform, Fisher, Arnold, Bivariate Normal, Bingham, and Dershowitz. All these distributions are described in Appendix A. At the present time each set of joint orientation data must be studied to find the distribution which provides the closest fit. No one distribution appears to be universally applicable. Baecher et. al. (1978) made some preliminary studies on this subject and tentatively concluded that the Bivariate Normal and Bingham distributions are the most useful forms in describing orientation data.** Lanney (1978) and Dershowitz (19xx) provide some examples of fitting curves to actual data.

3.2.1 Defining the Critical Zone

The kinematically unstable zone consists of the entire region above the great circle ABC in Figure 3.1 because every point in that region represents a daylighting line of intersection. However, the fact that a line (or wedge) daylights does not necessarily imply that the wedge is unstable. Indeed, the second step in the reliability analysis is to examine resisting and driving forces to determine the probability that a daylighting wedge will fail. (This subject will be discussed in Chapters 4 and 6.) Nevertheless, wedges with shallow dips may have relatively small driving forces and, consequently,

* The current version of DAYLITE uses the six forms listed. The program can be modified to accept any spherical distribution that can be expressed analytically.

**The Dershowitz Distribution is a modified version of the Bingham and was formulated after the completion of the Baecher et al. study.

negligible probabilities of failure. The reliability analysis can be simplified by eliminating these wedges from further consideration at this early stage. The DAYLITE user can specify a minimum dip (α) which will reduce the critical zone. Figure 3.8 indicates that even a modest value of α can significantly reduce the critical zone.

In a rigorous reliability analysis α should be zero. Every wedge regardless of its orientation has a finite probability of failure; hence, every line of intersection should be examined. In choosing a non-zero α, the DAYLITE user opts to ignore some possible (albeit improbable) failure geometries involving wedges with shallow dips. The user elects to introduce some error into the analysis in order the reduce the volume of calculations. The selection of α requires some judgment. The parameter might be considered the lower bound of joint friction angles. From a physical standpoint it could represent the lowest practical coefficient of friction for mineral to mineral contact. From an analytical standpoint, α depends on the pdf of Φ. If Φ were normally distributed α might be defined as Φ_{mean} minus 2 standard deviations of Φ; in this case, $P[\Phi<\alpha] = 0.0228$.

3.2.2 Partitioning the Critical Zone

As mentioned in Section 3.1 and shown in Figure 3.3, the critical zone is partitioned into equal area cells to help one discern the location and shape of $f(\theta,\phi)$, the pdf of the line of intersection. θ and ϕ are continuous random variables;

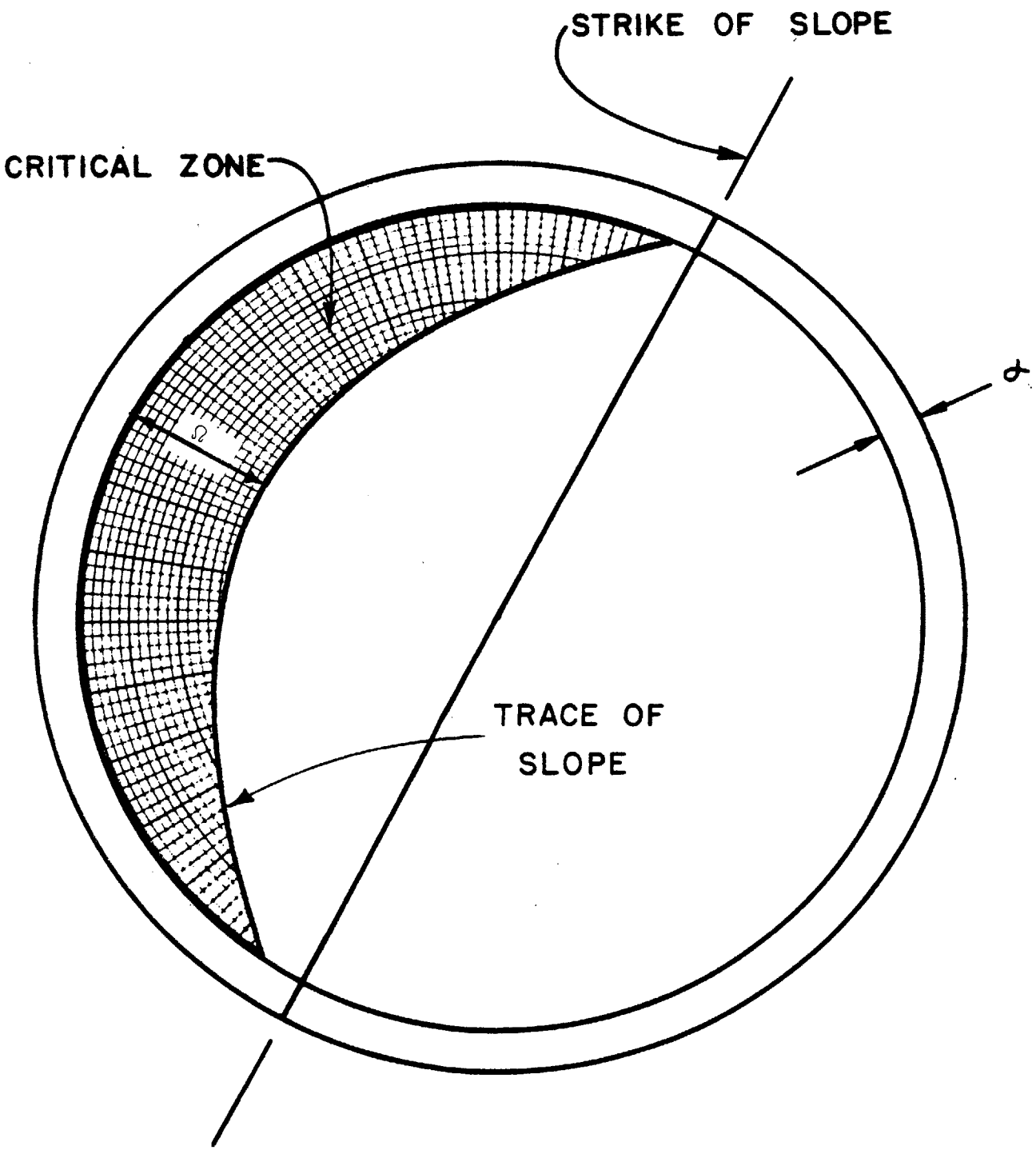


Figure 3.8 Reduction of Critical Zone with α

DAYLITE treats them as discrete random variables defined only at the characteristic coordinates of each cell, (θ_i,ϕ_i). The program uses numerical techniques to compute $P[\theta_i,\phi_i]$, the probability that the line of intersection will lie in cell i. The method approximates the integral of $f(\theta,\phi)$ over A_i, the area of cell i.

$$P[\theta_i\phi_i] = \int_{Ai} f(\theta,\phi)dA \qquad (3.2)$$

As long as A_i is small,

$$P[\theta_i\phi_i] = f(\theta_i\phi_i)A_i \qquad (3.3)$$

Or,

$$P[\theta_i\phi_i]/A_i = f(\theta_i\phi_i) \qquad (3.4)$$

If A_i is a constant for all i,

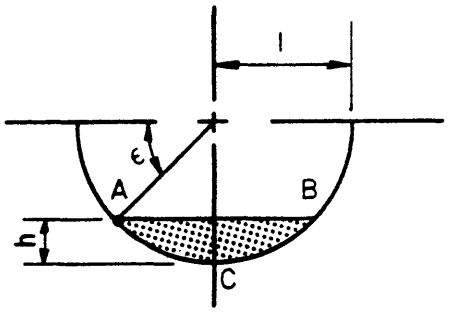
$$P[\theta_i\phi_i] \propto f(\theta_i\phi_i) \qquad (3.5)$$

for all cells. Thus, the maximum(a) of $P[\theta_i\ \phi_i]$ correspond to the mode(s) of $f(\theta,\phi)$.

The partitioning of the critical area is based on two parameters: Ω and ε. Ω, the maximum width of the zone, is defined as the dip of the slope (ϕ_s) minus α (Figure 3.8). ε is an angular value that controls the size of the unit cells. DAYLITE divides the spherical segment that lies between ϕ_s and α into $\frac{\Omega}{\varepsilon}$ segments bounded by small circles.

The procedure is illustrated in Figure 3.9 which is a section through the hemisphere. The area of the spherical segment DEFG defined by Ω and α is $2\pi(\sin(\phi_s)-\sin\alpha)$; hence the area between any adjacent small circles should be:

3.9A



AREA OF SEGMENT ABC

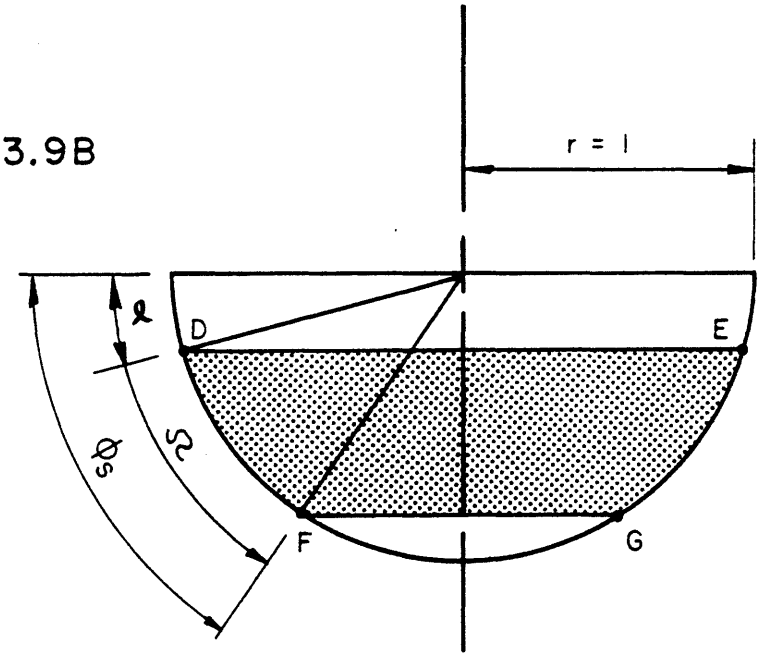
$= 2\pi r h$

$= 2\pi r^2(1 - \sin\omega)$

$= 2\pi (1 - \sin\omega)$

for $r = 1$

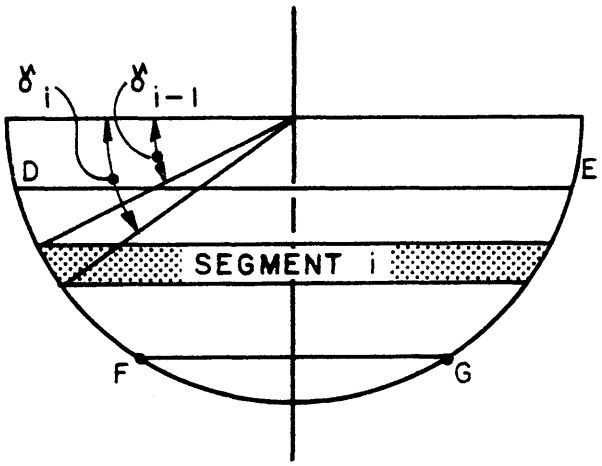
3.9B



AREA OF SEGMENT DEFG

$= 2\pi (\sin(\phi_s) - \sin(\alpha))$

3.9C



AREA OF SEGMENT i

$$= \frac{2\pi (\sin(\phi_s) - \sin(\alpha))}{\pi/\epsilon}$$

Figure 3.9 Areas of Spherical Segments--Definition of δ

$$\Delta A = 2\pi \frac{\mathrm{Sin}(\phi_s) - \mathrm{Sin}\alpha}{\frac{\Omega}{\varepsilon}} \qquad (3.6)$$

The dips of the small circles can be computed from the surface areas. Let γ_{i-1} and γ_i ($\gamma_{i-1} < \gamma$) be the dips of the circles that bound the i^{th} spherical segment (Figure 3.9C):

$$A_t - (A_o + (i)\Delta A) = 2\pi(1 - \sin\gamma_i) \qquad (3.7)$$

A_t = total area of the unit hemisphere
= 2π

A_o = area of the spherical segment above the small circle DE in Figure 3.9B
= $2\pi\sin\alpha$

Equation (3.7) can be rewritten as:

$$\gamma_i = \sin^{-1}\left(\sin\alpha + \frac{(i)\Delta A}{2\pi}\right) \qquad (3.7A)$$

Let n = the number of segments. From Equation (3.7A):

$$\gamma_o = \alpha \qquad (3.8)$$

$$\gamma_n = \Omega + \alpha \qquad (3.9)$$

$$= \phi_s$$

The spherical segments of area ΔA are subdivided into cells of equal area by lines of constant strike. The angular

increment between adjacent strike lines is $\frac{\pi}{\varepsilon}^*$. Figure 3.10 shows a segment bounded by α and ϕ_s that has been partitioned into cells. The area of each cell is $\frac{\Delta A}{\pi/\varepsilon}$.

The method outlined above will partition the entire spherical segment. However, only those cells in the critical zone (Figure 3.8) must be studied. The great circle that defines the slope disects some of the peripheral cells. DAYLITE examines all cells that lie even partially in the critical zone.

Each cell is characterized by a value of colatitude and latitude. The characteristic colatitude is the mean of the two colatitudes (or strikes) that border the cell. The characteristic latitude is the latitude that divides the cell into two parts with equal surface areas. It is computed in the same manner as γ_i:

ϕ_i: = characteristic latitude of cell bounded by γ_{i-1} and γ_i

$$= \pi/2 - \text{characteristic dip} = \pi/2 - \hat{\gamma}_i \qquad (3.10)$$

$$= \pi/2 - \sin^{-1}\left(\sin\alpha + \frac{(i)\Delta A}{2\pi} - \frac{\Delta A}{2\pi}\,\frac{1}{2}\right) \qquad (3.11)$$

3.2.3 Defining the Girdle

Each point in a cell defines a polar plane. All the polar planes from a single cell are located in a relatively

* ε should be chosen so that the quantities π/ε and $\frac{\Omega}{\varepsilon}$ are integers. In addition, π/ε should be a positive integer. Smaller values of ε will define smaller cells; hence, result in better defined pdf's.

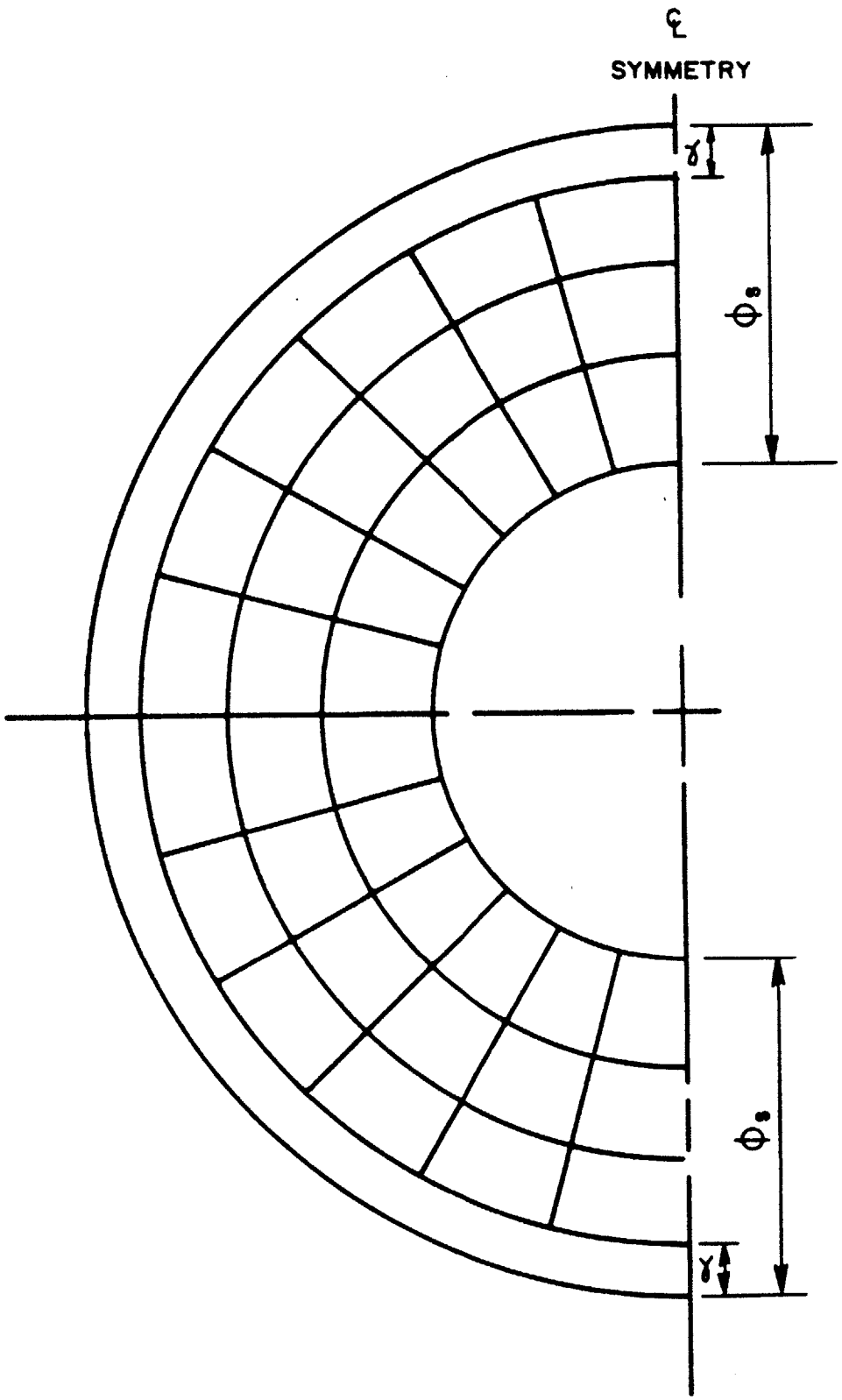


Figure 3.10 Spherical Segment Partitioned into Equal Area Cells

narrow band around the plane associated with the characteristic latitude and colatitude of the cell. If the cells were circular the locus of planes would form a girdle. In fact, the cells are spherical rectangles with aspect ratios that approach unity*. DAYLITE approximates the rectangles as circles with an equivalent area. The area of the cells is a constant:

$$A_{cell} = \frac{\Delta A}{(\pi/\varepsilon)} \qquad (3.12)$$

The diameter of the equivalent circle expressed as the spherical angle δ is shown in Figure 3.12:

$$\delta = 2 \cos^{-1}(1 - \frac{A_{cell}}{2\pi}) \qquad (3.13)$$

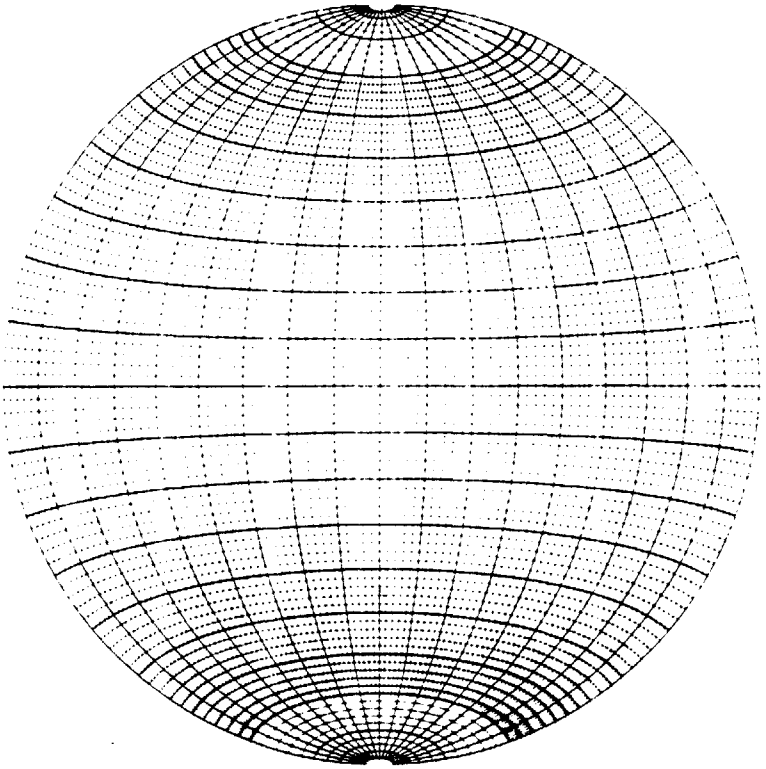
δ is also the width of the girdle that contains all the planes whose poles lie within the circle (see Figure 3.7).

3.2.4 Dividing the Girdle into Components

As shown in Figure 3.7, the dimensions of the girdle are δ by π. The girdle contains all the polar planes from a single cell in the critical zone. Line M which bisects the girdle is the trace of the polar plane defined by the characteristic colatitude and latitude of the cell (θ_o, ϕ_o). Every

* The ratio is not a constant. The cells in some regions of the hemisphere are more highly distorted. In a polar projection the maximum distortion occurs at the center whereas it occurs at the edge (along the strike direction) in meridonal projections (cf., Figure 3.11). The highly distorted region in a polar plot will not lie in the critical zone unless the slope is steep (>60°).

Meridonal:



Polar:

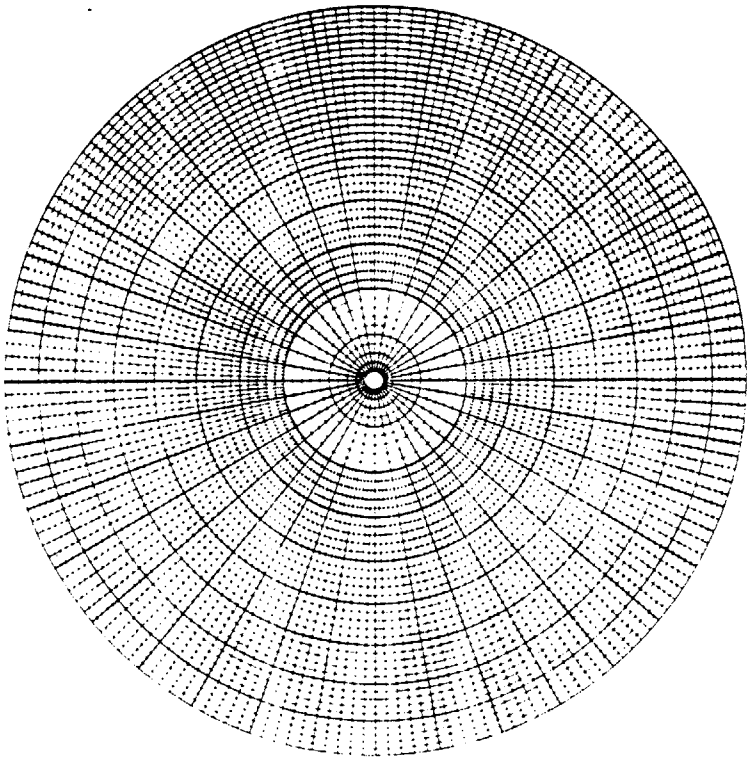
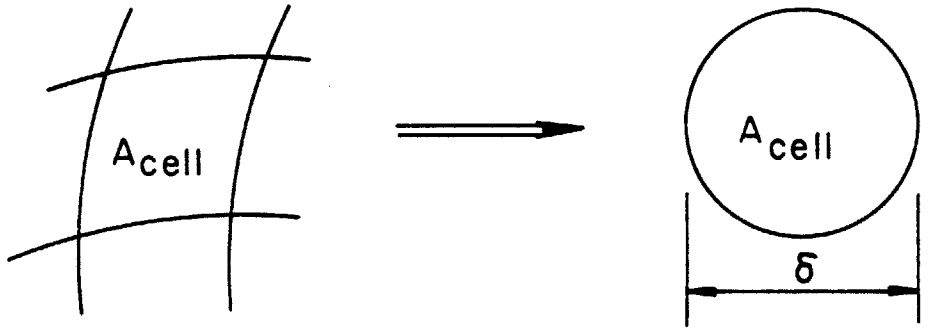
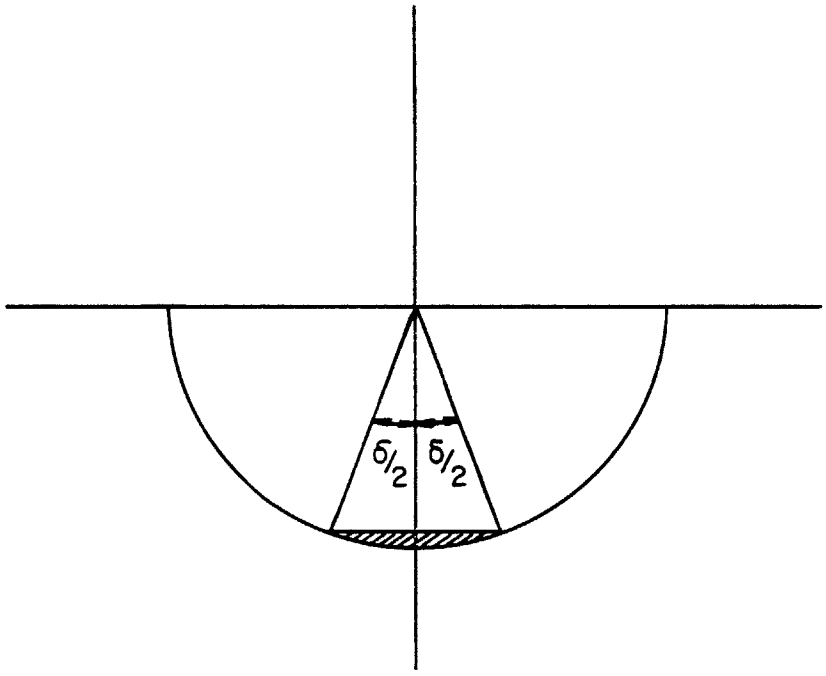


Figure 3.11 Meridonal vs. Polar Projection





$$A_{cell} = 2\pi \left(1 - \cos\frac{\delta}{2}\right)$$

Figure 3.12 Definition of δ

pair of points in the girdle represents a pair of joint orientations that could produce a requisite line of intersection i.e., any two joints whose poles lie in the girdle will define a wedge that has a line of intersection that pierces the unit hemisphere in the (θ_o, ϕ_o) cell.

Like the critical zone, the girdle is divided into a few discrete components that can be examined on an individual basis. The operation is based on ε, the parameter used to partition the critical zone. The girdle is separated into n equal area components where $n = \frac{\pi}{\varepsilon}$; hence, each component has the dimensions ε by δ^*. The components are characterized by the coordinates of their midpoints e.g., point I_o characterizes component i in Figure 3.7. All the midpoints lie along line M.

Figure 3.13 depicts the polar plane defined by the characteristic coordinates $(\theta_o \phi_o)$ or (x_o, y_o, z_o)** of a particular cell; this plane corresponds to line M of Figure 3.7. Point I_o, the midpoint of the i^{th} component (Figure 3.14), is η radians from the strike direction $(x_1, y_1, 0)$ of the polar plane.

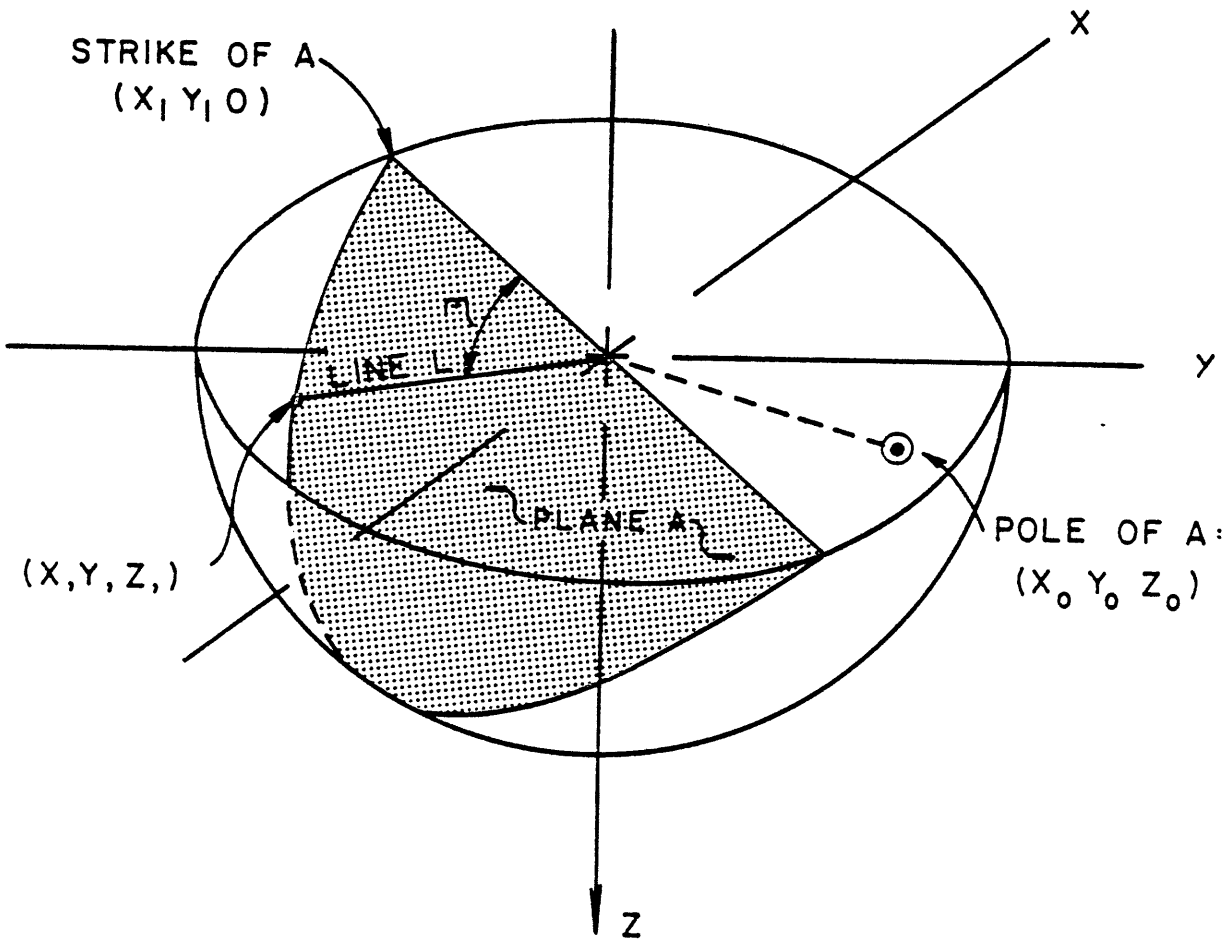
---

* The components will not be equidimensional. δ will always exceed ε, never by more than 13%. This disparity occurs because the rectangular cell in the critical zone is approximated by a circle.

** The transformation from spherical to Cartesian coordinates can be made through the following relations:

$$x = \sin \theta \sin \phi$$
$$y = \cos \theta \sin \phi$$
$$z = \cos \phi$$

The positions of the x,y and z axis are shown in Figure 3.13

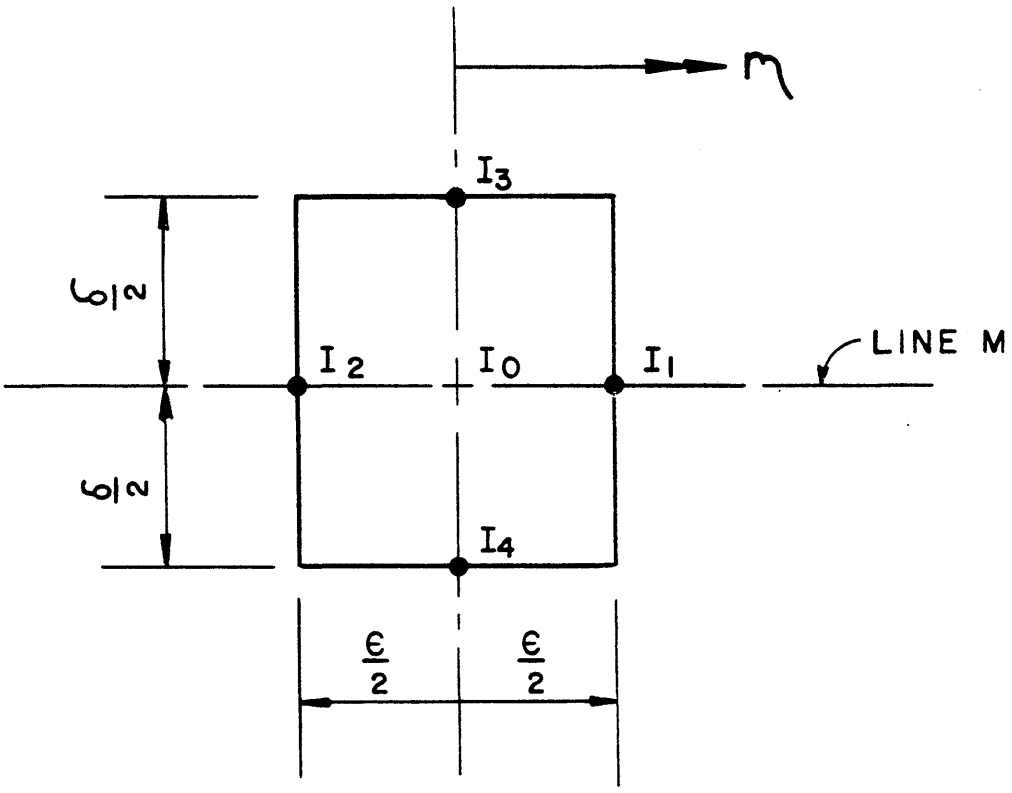


$$A = (X_0\ Y_0\ Z_0)$$

$$B = (X_1\ Y_1\ 0)$$

$$I_0 = (X\ Y\ Z)$$

Figure 3.13 Location of I_o on Polar Plane--Definition of η



i^{th} COMPONENT OF GIRDLE

Figure 3.14 Typical Component of Girdle

The x,y and z coordinates of I_o can be found by solving three simultaneous equations:

$$(x_o,y_o,z_o)\cdot(x,y,z) = x_ox + y_oy + z_oz = \cos(\pi/2) = 0 \quad (3.14)$$

$$(x_1,y_1,0)\cdot(x,y,z) = x_1x + y_1y = \cos\eta \quad (3.15)$$

$$y^2 + y^2 + z^2 = 1 \quad (3.16)$$

The solution to the equations is presented in Appendix B. Points I_1 and I_2 (Figure 3.14) can be located by replacing η in Equation (3.15) with $\eta - \frac{\varepsilon}{2}$ and $\eta + \frac{\varepsilon}{2}$.

A similar set of equations can be used to determine the coordinates of points I_3 and I_4 (Figure 3.14) at the top and bottom of the girdle. This solution is also presented in Appendix B.

Thus, DAYLITE computes the coordinates of five points for every component of the girdle.

3.2.5 Calculating $P[X_i]$

$P[X_i]$ is the probability that any one joint pole from set X will be in the spherical area defined by component i of the girdle. It is the integral of the pdf of joint pole orientations over the area of the component. DAYLITE approximates this integral as

$$P[X_i] = \frac{A_c}{6}\,[2f(I_o) + f(I_1) + f(I_2) + f(I_3) + f(I_4)] \quad (3.17)$$

where A_c is the area of the girdle component - a constant for all components and $f(I_k)$ is the value of the pdf of joint pole orientations at point I_k shown in Figure 3.14.

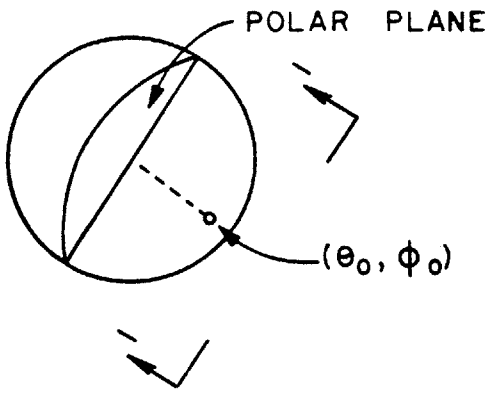
3.2.6 Identifying Wedges

The operations outlined in Sections 3.2.4 and 3.2.5 divide the girdle into n components and characterize each component by a single point. Thus, the area of the girdle is discretised into n representative points. Therefore all wedges which intersect the hemisphere in the (θ_o, ϕ_o) cell are formed by planes with poles coincident with two of the n points.

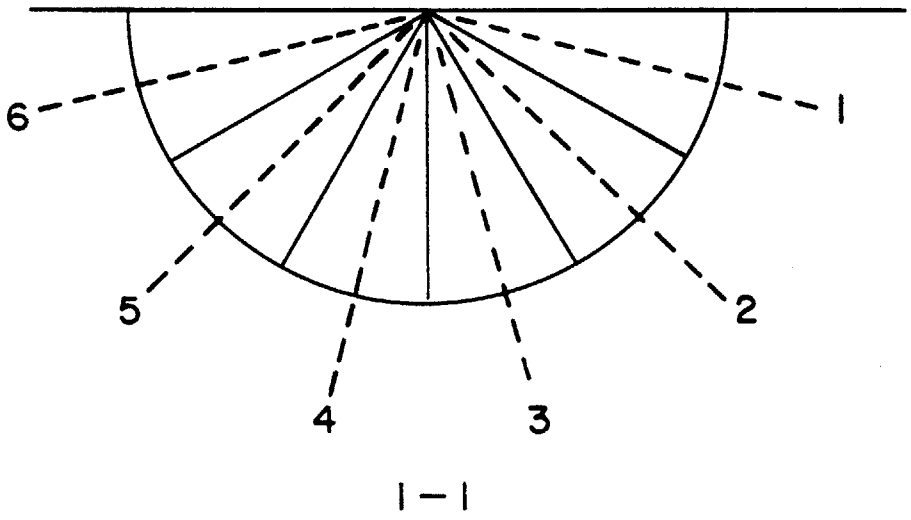
The easiest way to visualize the wedges is to examine the planes themselves rather than their poles. Figure 3.15B is a section perpendicular to the line of intersection that daylights at (θ_o, ϕ_o). It depicts a typical polar plane that has been divided into 6 components*. The heavy lines labeled 1 through 6 indicate directions of representative poles for the respective components. Figure 3.15C shows the planes that correspond to the poles. (The planes are perpendicular to the section so only their linear traces appear.) The probability that plane i will occur is identical to the probability that its pole will intersect the sphere at point i.

* A plane with 6 components is used to illustrate the concept. The actual number of components (n) depends on ε i.e., $n = \pi/\varepsilon$. n should be an even integer to facilitate the computations.

3.15A



3.15B



3.15C

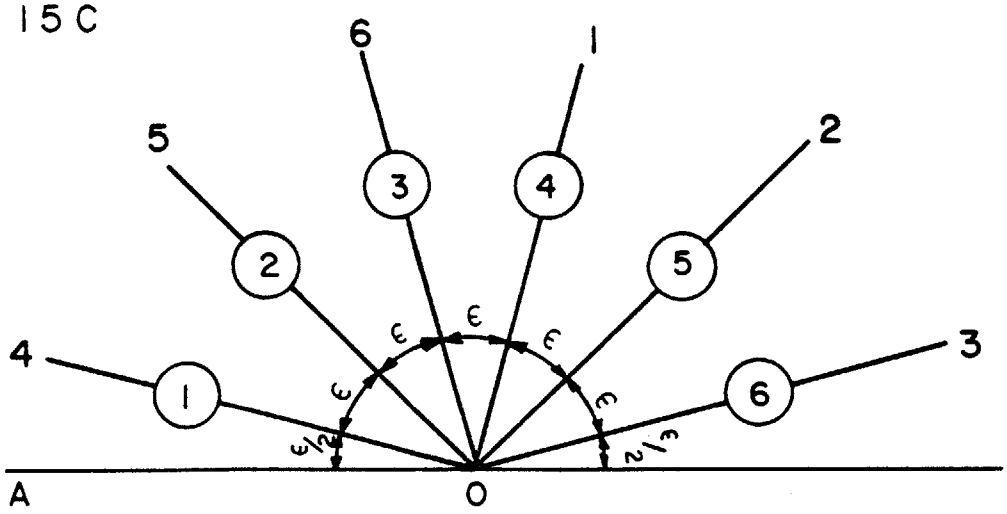


Figure 3.15 Renumbering of Characteristic Points on Girdle

The planes can be renumbered so that they become consecutive in a clockwise direction from the line segment OA. (The new numbering system is shown in Figure 3.15C as (i) .) n is the number of poles (or planes) and $(\ )_{old}$ and $(\ )_{new}$ refer to the plane designations in the two respective systems.

$$1 \leq (j)_{new} \leq \frac{n}{2} \quad (j)_{new} = \left[(j)_{new} + \frac{n}{2}\right]_{old} \tag{3.18}$$

$$\frac{n}{2} < (j)_{new} \leq n \quad (j)_{new} = \left[(j)_{new} - \frac{n}{2}\right]_{old} \tag{3.19}$$

The algorithm is different for the two quadrants.

The largest wedge that can form has a central angle or ψ of $\frac{n-1}{n}\,180°$ while the smallest wedge has a ψ of $\frac{180°}{n}$. If $\bar{\varepsilon}$ is ε expressed in degrees i.e., $\bar{\varepsilon} = \varepsilon\frac{180}{n}$:

$$\psi_{max} = (n-1)\bar{\varepsilon} \tag{3.20}$$

$$\psi_{mim} = \bar{\varepsilon} \tag{3.21}$$

The angle between any adjacent pair of planes is $\bar{\varepsilon}$; hence, the angle between any two planes must be $k\bar{\varepsilon}$ where k is an integer between 1 and n-1. Thus, the wedges defined by DAYLITE have ψ's that are multiples of $\bar{\varepsilon}$.

$$\psi_{min} = \psi_1 = \bar{\varepsilon} \tag{3.22}$$

$$\psi_2 = 2\bar{\varepsilon} \tag{3.23}$$

$$\psi_k = k\bar{\varepsilon} \tag{3.24}$$

$$\psi_{max} = \psi_{n-1} = (n - 1)\bar{\varepsilon} \tag{3.25}$$

DAYLITE computes each ψ_k and identifies all possible combinations of planes that will produce a wedge with a central angle of ψ_k. Each combination consists of two planes that are separated by an angle of $k\bar{\varepsilon}$. Thus, ψ_k will have n-k combinations:

$$(1,1 + k)$$

$$(2,2 + k)$$

$$(n - k,n)$$

The larger the ψ_k, the fewer the combinations. ψ_{max} has only one pair of points i.e., (1,n) whereas ψ_{min} has n-1 pairs.

DAYLITE also computes the β_1 angle (Figure 3.5) associated with each wedge. As shown in Figure 3.15, the incremental angle between any adjacent planes is $\bar{\varepsilon}$. Hence, for any wedge:

$$\beta_1 = \frac{\bar{\varepsilon}}{2} + \left[(j)_{new} - 1\right]\bar{\varepsilon} \tag{3.26}$$

where $(j)_{new}$ is the number of the plane which forms the right boundary of the wedge.

The four parameters ϕ, θ, ψ and β_1 completely define the orientation of a wedge.

3.2.7 Calculating $P[(\theta_o, \phi_o, \psi_o, (\beta_1)_o)]$

The results of the analysis performed by DAYLITE are expressed as the probability that a particular wedge $(\theta_o, \phi_o, \psi_o, (\beta_1)_o)$ will form. The program examines every pair of girdle components and, in doing so, identifies every possible wedge geometry. For example, all wedges in Figure 3.15C have a line of intersection oriented at (θ_o, ϕ_o) but the only two joints which form a wedge with $\psi = 2\varepsilon$ and $\beta_1 = \frac{3\varepsilon}{2}$ are joints 2 and 4. One joint pole must lie in girdle component 5 while the other lies in component 1. Thus, the probability that one joint from set X and one joint from set Y will combine to form a $(\theta_o, \phi_o, \psi_o, (\beta_1)_o)$ wedge is:

$$P[(\theta_o, \phi_o, \psi_o, (\beta_1)_o] = P[X_1]P[Y_5] + P[X_5]P[Y_1] \qquad (3.27)$$

$$P[X_i] = P[\text{set X will have a pole in girdle compent i}]$$

The output from DAYLITE includes three different probability calculations:

$P[\theta_i,\phi_i]$ - the probability that a wedge (regardless of its shape) will have a line of intersection oriented in the (θ_i,ϕ_i) direction.

$P[\theta_i,\phi_i,\psi_j]$ - the probability that a wedge with a central angle of ψ_j (regardless of β_1) will have a line of intersection oriented in the (θ_i,ϕ_i) direction.

$P[\theta_i,\phi_i,\psi_j,(\beta_1)_k]$ - the probability that a wedge with a central angle of ψ_j and a β_1 of $(\beta_1)_k$ will have a line of intersection oriented in the (θ_i,ϕ_i) direction.

The probabilities are related:

$$P[\theta_i,\phi_i,\psi_j] = \sum_{\text{all } k} P[\theta_i,\phi_i,\psi_j,(\beta_1)_k] \qquad (3.28)$$

$$P[\theta_i,\phi_i] = \sum_{\text{all } j} P[\theta_i,\phi_i,\psi_j] \qquad (3.29)$$

As indicated in Section 3.1, the user can specify that the two joints belong to different sets (as in Equation (3.27)) or that both joints belong to the same set--either set X or set Y.

$P[\theta_i,\phi_i]$ is useful in the early stages of the analysis because it provides an overview of the problem. It enables one to determine the most likely orientations for wedges independent of their shape. The other two probabilities are used in the detailed analysis. They give specific information on the shapes of wedges.

3.3 Program Information

The input parameters to DAYLITE include the orientation of the open face and the distributional parameters of the two joint sets. The user must also prescribe the accuracy desired in the numerical integration procedure. He must specify four angular values: $\varepsilon, \alpha, \hat{\psi}$ and $\Delta\psi$.

> ε controls the size of the unit cell in the critical zone and the width of girdle components. The parameter was discussed in Section 3.2.2.
>
> α reduces the critical zone. The program will not examine any lines of intersection that have a dip less than α. The parameter was discussed in Section 3.2.1.
>
> $\hat{\psi}$ is a parameter that controls the minimum size wedge. The smallest wedge the program identifies has a central angle of $\bar{\varepsilon}$ (Section 3.2.6). The user may not be interested in very narrow wedges because they tend to have high FS's (or low P_f's) The narrow geometry induces high normal stresses in the joint planes - thus, high resistances. The user can opt to ignore these narrow wedges by specifying a value for $\hat{\psi}$. The program will not examine any wedge with a central angle smaller than $\hat{\psi}$. (The program will investigate all wedges larger than $\bar{\varepsilon}$ if $\hat{\psi}$ is not specified.)
>
> $\Delta\psi$ enables the user to reduce the volume of output without sacrificing too much information. The program will sum values of $P[\theta_i, \phi_i, \psi_j]$ and express the results as $P[\theta_i, \phi_i, \psi_A \leq \psi_j \leq \psi_B]$ where $\psi_B - \psi_A = \Delta\psi$ thus,

$$P[\theta_i,\phi_i,\psi_A \leq \psi_j \leq \psi_B] = \sum_{\substack{\text{all } \psi_j \\ \text{such that} \\ \psi_A \leq \psi_j \leq \psi_B}} P[\theta_i,\phi_i,\psi_j] \qquad (3.30)$$

The program computes $P[\theta_i,\phi_i,\psi_j]$ for ψ_j values of $\psi_A, \psi_A + \bar{\varepsilon}, \psi_A + 2\bar{\varepsilon}, \ldots\ldots \psi_B$.

It then sums the probabilities as prescribed by Equation (3.30). Thus, the output describes ψ in terms of a range of central angles rather than specific angles.

$\varepsilon, \alpha, \hat{\psi}$ and $\Delta\psi$ cannot be chosen arbitrarily. DAYLITE partitions various angular dimensions and there must be an integral number of subdivisions in each instance. The 4 parameters must conform to the following criteria:

1. $\frac{\Omega}{\varepsilon}$ = an integer (3.31)
2. $\frac{\pi}{\varepsilon}$ = an even integer (3.32)
3. $\frac{\psi}{\varepsilon}$ = an integer (3.33)
4. $\frac{\Delta\psi}{\hat{\psi}}$ = an integer (3.34)

The program will notify the user if any of the criteria are violated.

Appendix C presents a user's manual and a program listing for DAYLITE. A sample output is shown in Appendix I.

CHAPTER 4

KINETIC CONDITIONS - COMPUTER PROGRAM SWARS-2PM

4.0 Introduction

Chapter 3 discussed the kinematic aspects of rock slope stability. The kinematic tests only provide part of the answer to the question of stability. They narrow the scope of the problem by focusing attention on the wedges which daylight. However, the fact that a wedge is physically capable of movement does not necessarily imply that it will fail. The resistance along joint planes may inhibit movement. The critical question is: What are the relative magnitudes of the driving and resisting forces? This chapter will examine this kinetic aspect of wedge stability.

A number of individuals (Wittke (1965A, 1965B), John (1968), Hendron et al. (1971), Hoek and Bray (1974)) have developed deterministic models to assess the stability of wedges. They all use a limiting equilibrium approach and define the factor of safety as the ratio of resisting to driving forces. All the techniques treat both the wedge and its parent rock mass as rigid bodies and assume the two bodies are separated by discontinuities that can be idealized as thin zones of relatively deformable material. The factor of safety is a function of the stresses acting on the discontinuities or joint planes. The reactions are indeterminate so all the techniques make some assumptions to simplify the problem. The

most common assumptions are:

1. Moments can be neglected - failure occurs as translational sliding.

2. In the joint plane there are no shear stresses that act perpendicular to the line of intersection of the wedge - all the reactions on the wedge are either parallel to the line of intersection or perpendicular to joint planes.

3. Lateral in situ stresses do not affect the reactions in the joint planes.

4. The strength parameters which characterize the joint plane can be expressed as deterministic values which are essentially constant over the entire joint surface.

The first three assumptions concern the mechanics of the model while the last one concerns the uncertainty (or certainty) associated with geologic parameters.

This chapter presents a model that relaxes some of these assumptions--particularly the last three. The new model modifies the analytic technique developed by Hendron et al.* by incorporating the effects of joint stiffness and in situ stresses into the stability calculations (Sections 4.2 through 4.5). This improved deterministic model is then converted into a probabilistic model by treating the input parameters as random variables through Monte Carlo simulation (Section 4.6).

The chapter begins by reviewing the state of the art in stiffness and stress effects.

---

* Campbell (1974) programmed the Hendron et al. technique for a computer. The Campbell program (SWARS-2P) will serve as a basis for a computerized version of the modified analysis.

4.1 State of The Art - Stiffness and Stress Effects

Most present approaches to rigid body analysis assume that the reactions between the wedge and the parent rock mass are perpendicular to the two** joint planes (Figure 4.1). Figure 4.2 shows a typical wedge and two sections - one perpendicular to the line AC, the line of intersection, and one parallel to line AC. The assumption regarding the orientation of the reactions in the joint planes leads to the force diagram in Figure 4.3. The two reactions, R_1 and R_2, can be derived by solving two equations of force equilibrium:

$$W \cos \theta = R_1 \cos \beta_1 + R_2 \cos \beta_2 \tag{4.1}$$

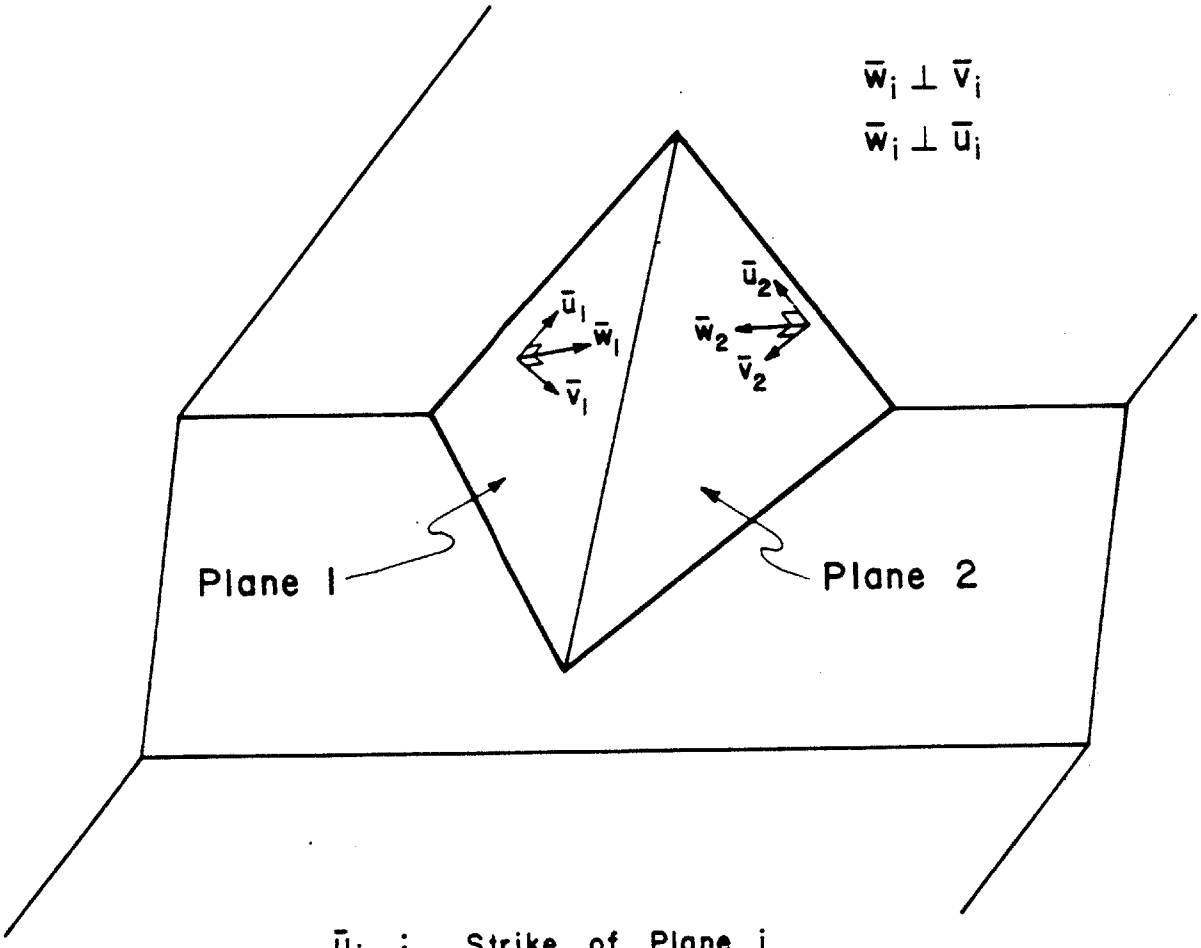
$$R_1 \sin \beta_1 = R_2 \sin \beta_2 \tag{4.2}$$

If $\beta_1 = \beta_2 = \beta$:

$$R_1 = R_2 = \frac{w \cos \theta}{2 \cos \beta} \tag{4.3}$$

The assumption concerning the normality of the reactions simplifies the analysis, but is it a realistic approach? The shear stresses on the joint planes have been investigated by Mahtab and Goodman (1970) and St. John (1971) through

** All the wedges discussed in this chapter are "two-joint wedges" i.e., tetrahedrons formed by two intersecting joints and two free surfaces.

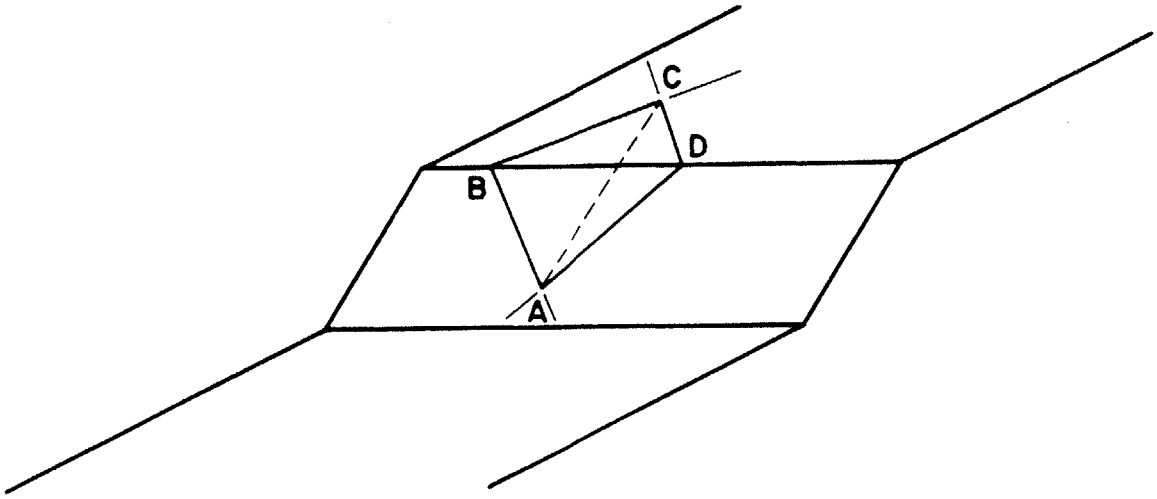


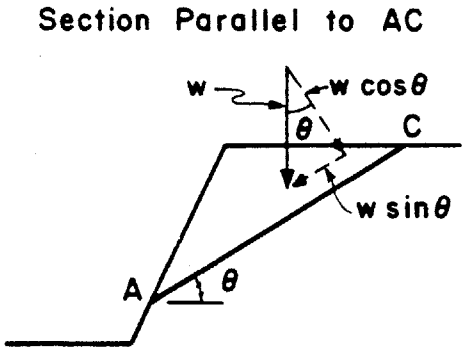
$\bar{u}_i$: Strike of Plane i

$\bar{v}_i$: Dip of Plane i

$\bar{w}_i$: Direction of Reaction from Plane i

Figure 4.1 Sliding Wedge--Reactions Perpendicular to the Joint Planes





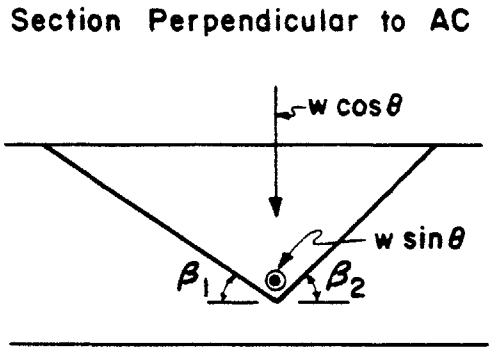


Figure 4.2 Resolution of Weight Vector into Components

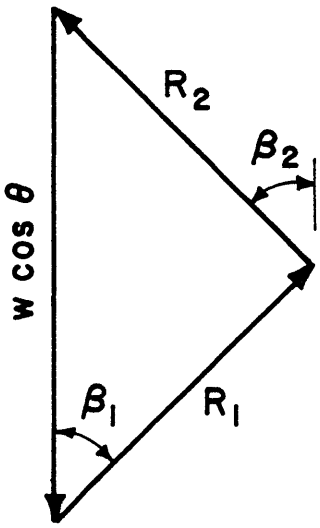


Figure 4.3 Force Diagram--Reactions Perpendicular to Joint Planes

the use of joint shear and normal stresses and by Steiner (1977) through the use of the ratio of horizontal to vertical stresses.

The stiffness approach introduces (Figure 4.4A) the tangential force T_i in addition to the normal forces N_i to represent the reactions on the sliding planes. N_i and T_i can be determined if the shear stiffness (k_s)* and normal stiffness (k_n)* of the joint or their ratio $R = \frac{k_n}{k_s}$ is known. For a symmetric wedge the ratio T_1/N_1 can be calculated as

$$\frac{T_1}{N_1} = \frac{\tau_1}{\sigma n_1} = \frac{k_s \sin \beta\delta_r}{k_n \cos \beta\delta_r} = \frac{\tan \beta}{R} \qquad (4.4)$$

From the force diagram shown in Figure 4.4C:

$$W \cos \theta = N_1 \cos \beta + N_2 \cos \beta + T_1 \sin \beta + T_2 \sin \beta \qquad (4.5)$$

and

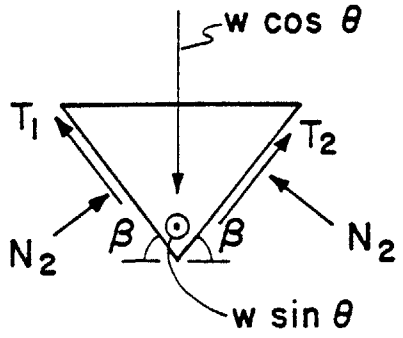
$$N_1 = N_2 = \frac{R \cos \beta}{2(\sin^2 \beta + R \cos^2 \beta)} W \cos \theta \qquad (4.6)$$

$$T_1 = T_2 = \frac{\sin \beta}{2(\sin^2 \beta + R \cos^2 \beta)} W \cos \theta \qquad (4.7)$$

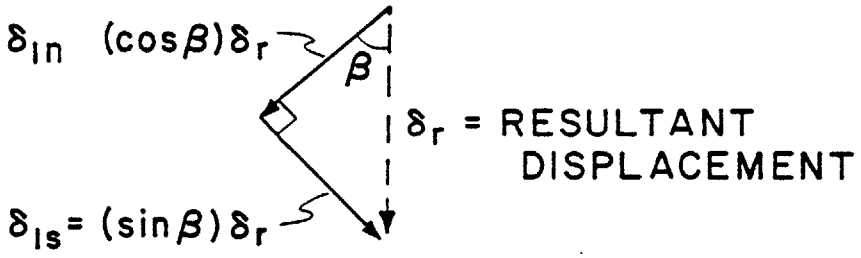
For $R = \infty$, i.e., for zero shear stiffness:

$$N_1 = \frac{W \cos \theta}{2 \cos \beta} \text{ and } T_1 = 0$$

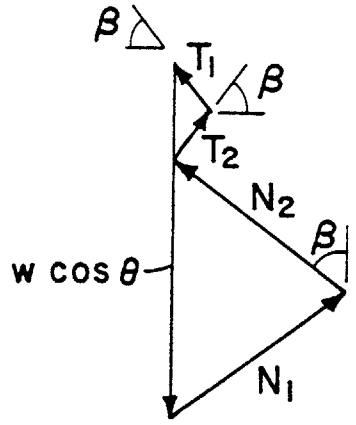
* $k_s = \frac{\tau}{\delta_s}$; $k_n = \frac{\sigma_n}{\delta_n}$ where τ and σ_n are the shear and normal stresses; δ_s and δ_n are the shear and normal displacements.



4.4 A



4.4 B



4.4 C

Figure 4.4 Symmetric Wedge--Reactions with Components Tangent to Joint Planes

These are the values derived earlier with the standard assumption regarding the normality of reactions.

The forces T_i and N_i can be used in the limit equilibruim equation for the wedge:

$$\text{Factor of Safety} = FS = \frac{2N_1 \tan \Phi}{T} \tag{4.8}$$

where $2N_1 \tan \Phi$ is the resisting factor (assuming only frictional resistance is present) and T is the driving force which is the vectorial addition of the shear forces perpendicular and parallel to the line of intersecting (Figure 4.4A).

$$T = \sqrt{(2T_1)^2 + (W \sin \theta)^2} \tag{4.9}$$

If only the shear force in the direction of wedge movement, i.e., parallel to the line of interaction AD is considered,*

$$FS = \frac{2N_1 \tan \Phi}{W \sin \theta} \tag{4.10}$$

To show the effect of different shear and normal stiffnesses Φ_R (the "required friction angle" to obtain FS = 1.0) has been plotted versus the angle β (for a symmetrical wedge) in Figure 4.5.

Decreasing R, i.e., increasing k_s relative to k_n, leads to a greater required friction angle.

* Section 4.4 examines the question of what constitutes failure and how to define the factor of safety.

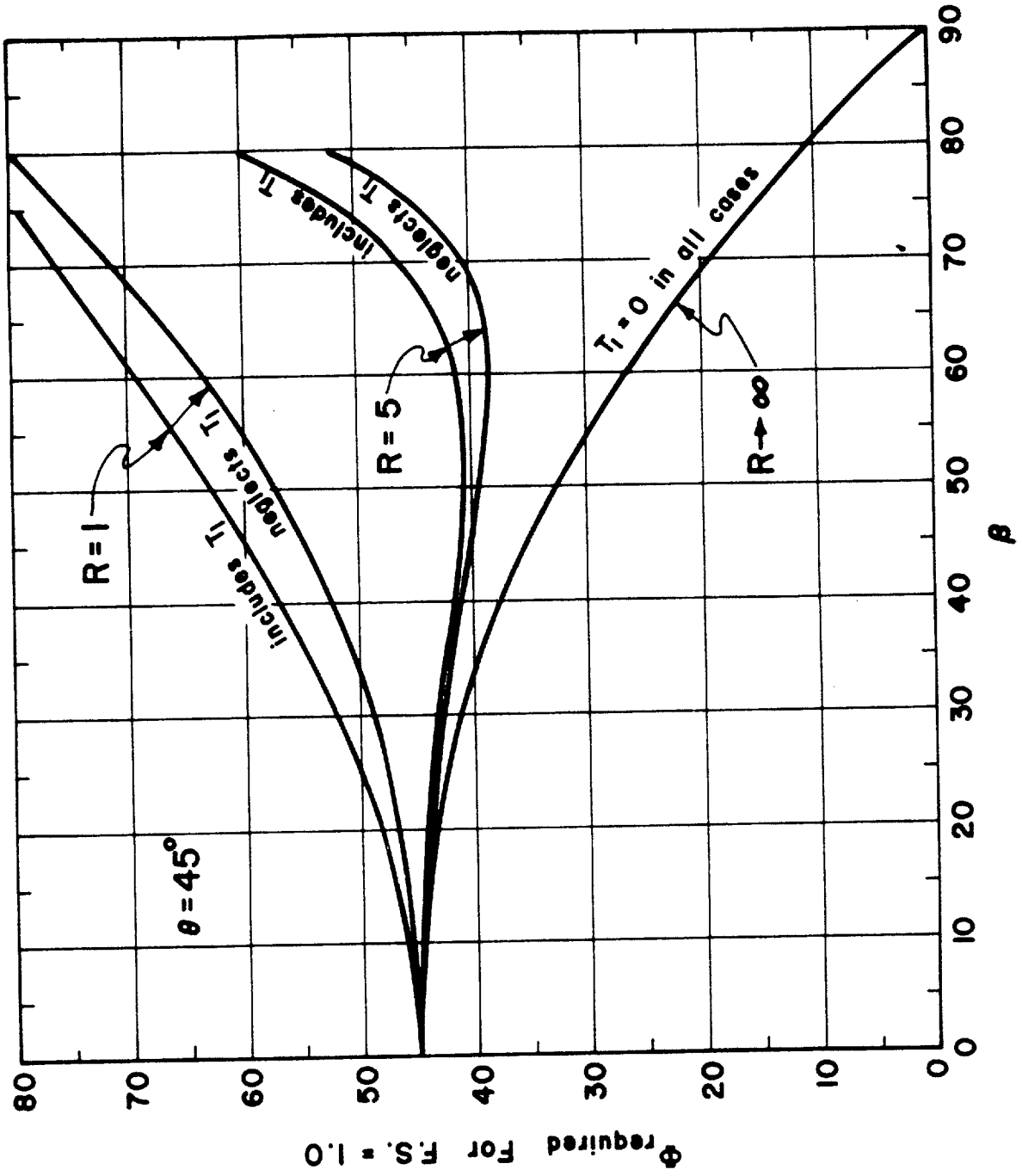


Figure 4.5 Φ Required For FS = 1.0 vs. β

The importance of R raises the question what values of R do occur in nature? Several investigations have been conducted by Goodman (1972), Kulhawy (1975), Rosso (1976) and Barton (1972). Although the number of test results (especially concerning normal stiffness) has been increasing it is not yet possible to correlate stiffness values with particular rocks, with particular surface characteristics or with a combination of both. The references listed above quote R values that range from 0.5 to 1000. Unweathered joints usually fall in the upper (500-1000) portion of the range while joints with fillers usually have R values below 50. Additional research on joint stiffness values and possible correlations with other rock joint parameters is necessary.

Several improvements of the stiffness approach should be made. The St. John model is based on a symmetric wedge with identical stiffness ratios on the two sliding planes. The strength of the joint is characterized solely in terms of frictional resistance with no provisions for cohesive resistance. Some of these limitations have been eliminated in an extended version of the model that will be discussed in Section 4.2.

The stress approach as introduced by Steiner relaxes the force direction assumptions by explicitly including the horizontal stress acting on the wedge. From the stresses acting on a small element of the wedge (Figure 4.6), one can establish a force equilibrium:

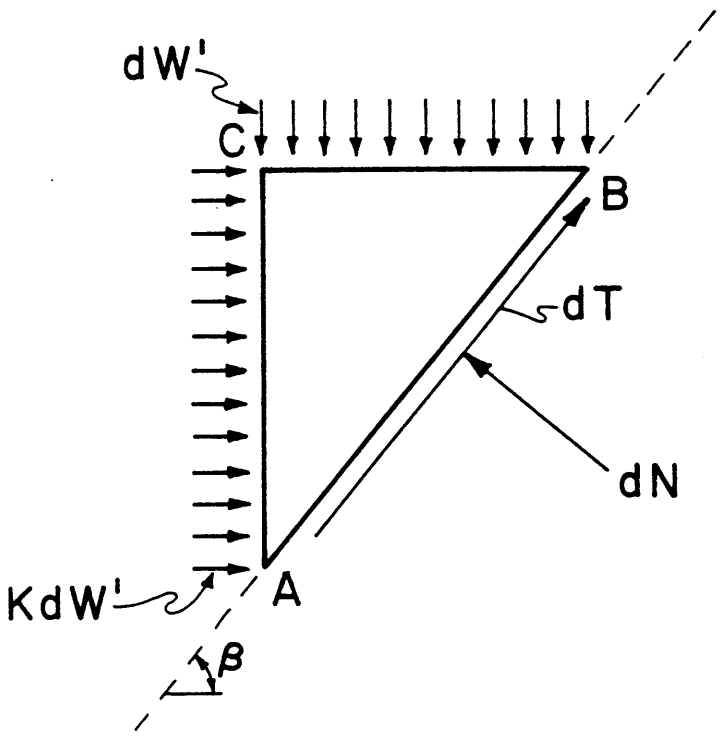


Figure 4.6 Stresses on Differential Area of the Joint Plane

$$dN = (dW') \cos \beta(L_{CB}) + (KdW') \sin \beta(L_{CA}) \quad (4.11)$$

where L_{CB} = distance CB

and L_{CA} = distance CA,

when $L_{CB} = 1$

$$L_{CA} = \tan \beta \quad (4.12)$$

and thus

$$dN = dW' \cos \beta + KdW' \sin \beta \tan \beta \quad (4.13)$$

Since dW' and KdW' are constant along the entire length of the joint the equation can be integrated to yield the normal force on the joint.

$$N = W' (\cos \beta + K \sin \beta \tan \beta) \quad (4.14)$$

(Instead of the differential force equilibrium described above, one could arrive at the same result using the Mohr stress circle, Figure 4.7.) If $W' = \frac{1}{2}W \cos \theta$, i.e., one half the weight component in a direction perpendicular to the line of intersection (see Figure 4.3),

$$N_1 = N_2 = \frac{1}{2}W \cos \theta(\cos \beta + K \sin \beta \tan \beta) \quad (4.15)$$

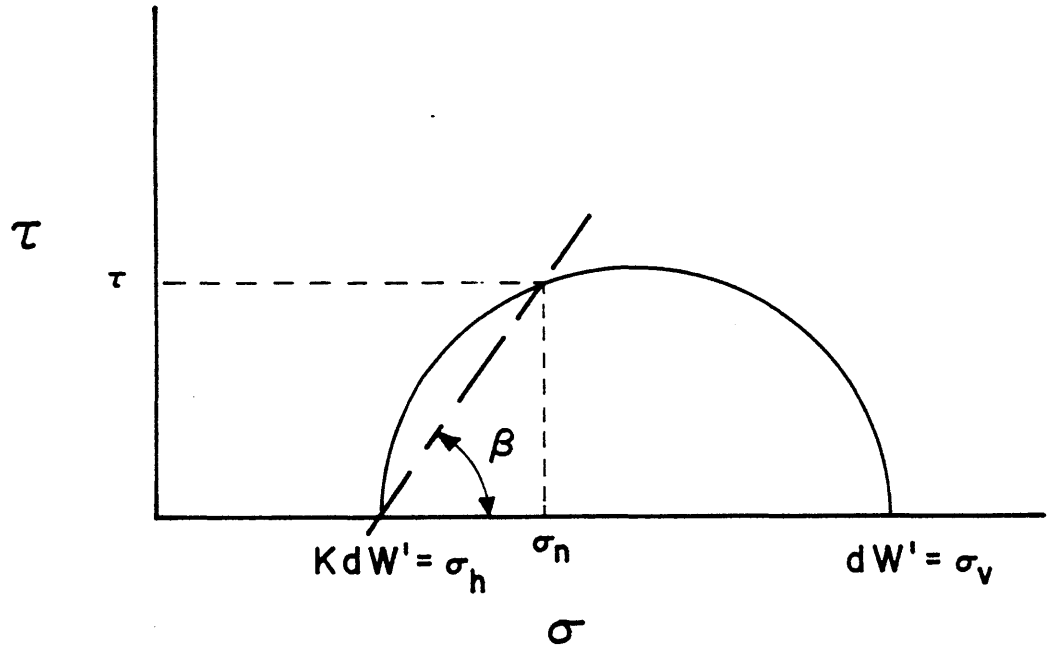


Figure 4.7 Mohr Stress Circle for the Stress State Shown in Figure 4.6

and

$$T_1 = T_2 = \frac{1}{2} W \cos \theta (\sin \beta - K \cos \beta \tan \beta) \qquad (4.16)$$

for K = 1, the forces become:

$$N_1 = \frac{W \cos \theta}{2 \cos \beta} \text{ and } T_1 = 0 \qquad (4.17)$$

i.e., the values for the standard normal reaction assumption. This standard assumption includes implicitly the equality of vertical and horizontal stresses.

Factors of safety can be derived either by considering the tangential forces parallel and perpendicular to the wedge movement direction or by only considering those parallel to the movement:

$$FS = \frac{2N_1 \tan \theta}{\sqrt{(2T_1)^2 + (W \sin \theta)^2}} \qquad (4.18)$$

$$FS = \frac{2N_1 \tan \Phi}{W \sin \theta} \qquad (4.19)$$

The effect of introducing a factor $K \neq 1$ in the computation of factors of safety is presented in Figure 4.8, where

$$\frac{FS \text{ with } K \neq 1}{FS \text{ with } K = 1} = \frac{\frac{W \cos \theta}{W \sin \theta} \tan \Phi}{\frac{W \cos \theta}{W \sin \theta} \tan \Phi} \cdot \frac{\frac{1}{\cos \beta}(\cos^2 \beta + K \sin^2 \beta)}{\frac{1}{\cos \beta}} \qquad (4.20)$$

$$= \cos^2 \beta + K \sin^2 \beta \qquad (4.21)$$

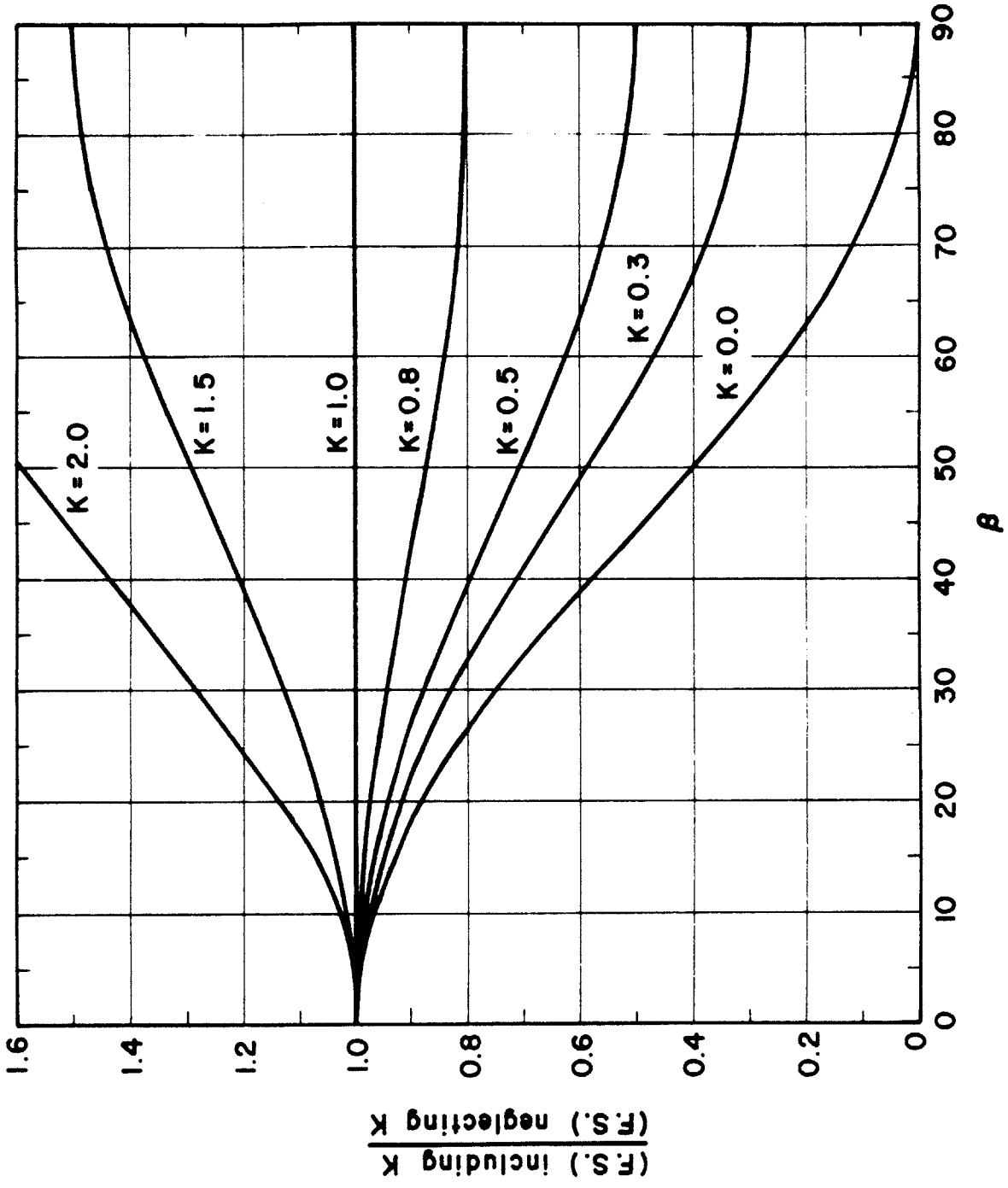


Figure 4.8 $\frac{\text{(FS) neglecting K}}{\text{(FS) including K}}$ vs. β

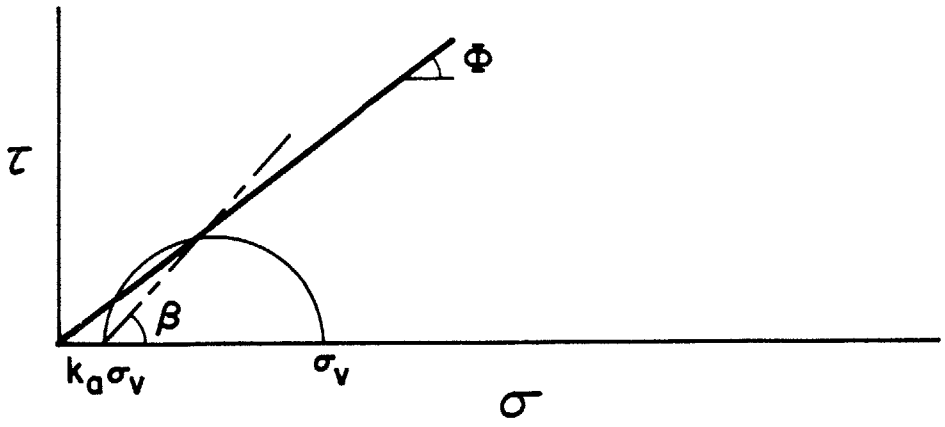
Figure 4.8 shows that with $K > 1$ the traditional assumption ($K = 1$ and thus a reaction normal to the sliding plane only) underestimates the factor of safety and for $K < 1$ it overestimates it.

The notion of the stress ratio K needs some further clarification particularly in the context of a jointed rock mass. The K used by Steiner refers to the state of stress within the wedge which may or may not be equal to the stress state in the remainder of the rock mass. A change or discontinuity in the material properties of a medium does not necessarily create a discontinuity in the state of stress existing in the medium. The question that arises is, what created the discontinuity in the first place or (more generally formulated) what is the stress history at the particular location? Depending on the stress history of the joint, one can often assume that K in the wedge is identical to K in the entire rock mass.

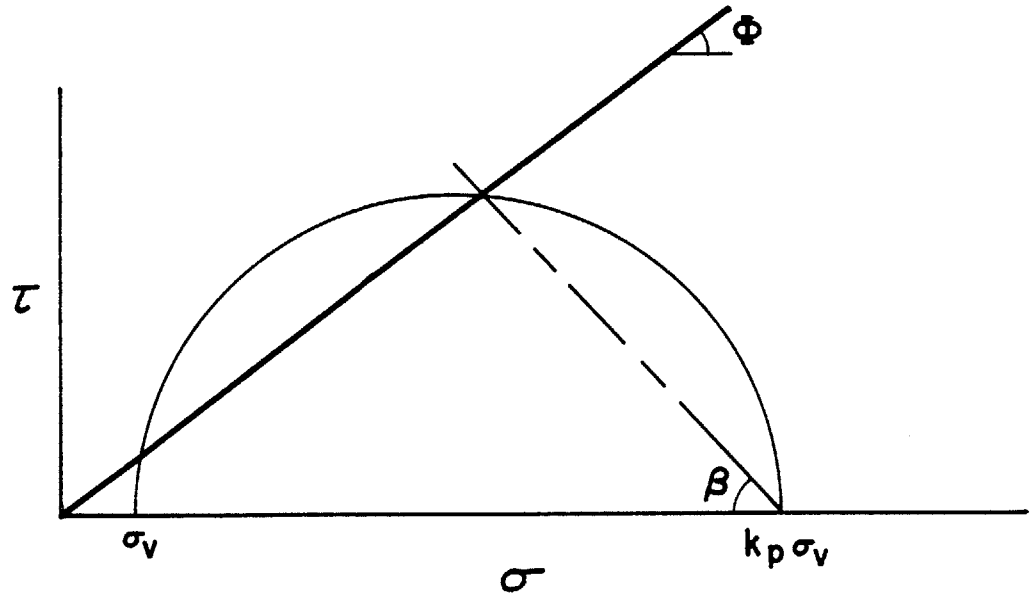
The use of the stress ratio K in wedge stability analysis introduces the possibility of active or passive failure modes of a wedge in the direction perpendicular to the line of intersection. As shown in Figure 4.9A, the active case is:

$$\tan \Phi = \frac{\tau}{\sigma_n} = \frac{T}{N} = \frac{\sin \beta - K \sin \beta}{\cos \beta + K \sin \beta \tan \beta} = \frac{(1 - K)}{\text{ctn } \beta + K \tan \beta} \quad (4.22)$$

Thus,



4.9 A



4.9 B

Figure 4.9 Mohr Stress Circles for Active and Passive Cases

$$K_a = \frac{1 - \tan \Phi \operatorname{ctn} \beta}{1 + \tan \Phi \tan \beta} \tag{4.23}$$

In the passive case (Figure 4.9B) the direction of T reverses; therefore, a minus sign must be introduced into Equations (4.16) and (4.22).

$$\tan \Phi = \frac{-T}{N} = \frac{-(1 - K)}{\operatorname{ctn} \beta + K \tan \beta} \tag{4.24}$$

Thus,

$$K_p = \frac{1 + \tan \Phi \tan \beta}{1 - \tan \Phi \operatorname{ctn} \beta} \tag{4.25}$$

The practical significance of these two failure modes will be discussed in Section 4.4. Although the joint shear strength is exceeded in both cases it does not imply sliding failure along the line of intersection. It simply means that in the active case the wedge will settle and in the passive case the wedge will be lifted up until equilibrium is reached. These represent cases where the overall stress field in the rock mass will be different from the stress field in the wedge. In the wedge, K can only vary between the limits K_a and K_p as determined by Φ and β (see Equations 4.23 and 4.25) whereas in the rock mass K values smaller and greater respectively than these K_a and K_p can exist; K in such a rock mass will be inhomogeneously distributed.

In this section both the stiffness and the stress

approaches were presented as methods for determining reactions at the joint planes. Section 4.3 discusses the relationship between the two approaches.

4.2 Generalization of the Stiffness Approach

The stiffness approach outlined by St. John can be generlized to include asymmetric wedges and arbitrary loads such as those shown in Figure 4.10. (The figure is a cross-section taken perpendicular to the line of intersection between two joint planes.) The generalized problem can be treated in a manner very similar to the one used by St. John. The fundamental assumption in the approach is that the wedge behaves as a rigid body resting on a deformable medium (the joint planes). The deformational characteristics of the joints can be expressed in terms of two stiffness values: k_n and k_s.

$$k_n = \frac{\text{normal stress}}{\text{normal displacement}} = \frac{\sigma_n}{\delta_n} \tag{4.26}$$

$$k_s = \frac{\text{shear stress}}{\text{shear displacement}} = \frac{\tau}{\delta_s} \tag{4.27}$$

k_n and k_s are material properties which are assumed constant throughout the range of stresses encountered in engineering practice. The method implicitly assumes that σ_n and δ_s as well as τ and δ_n are unrelated, i.e., the off-diagonal terms in the stiffness matrix are zero.

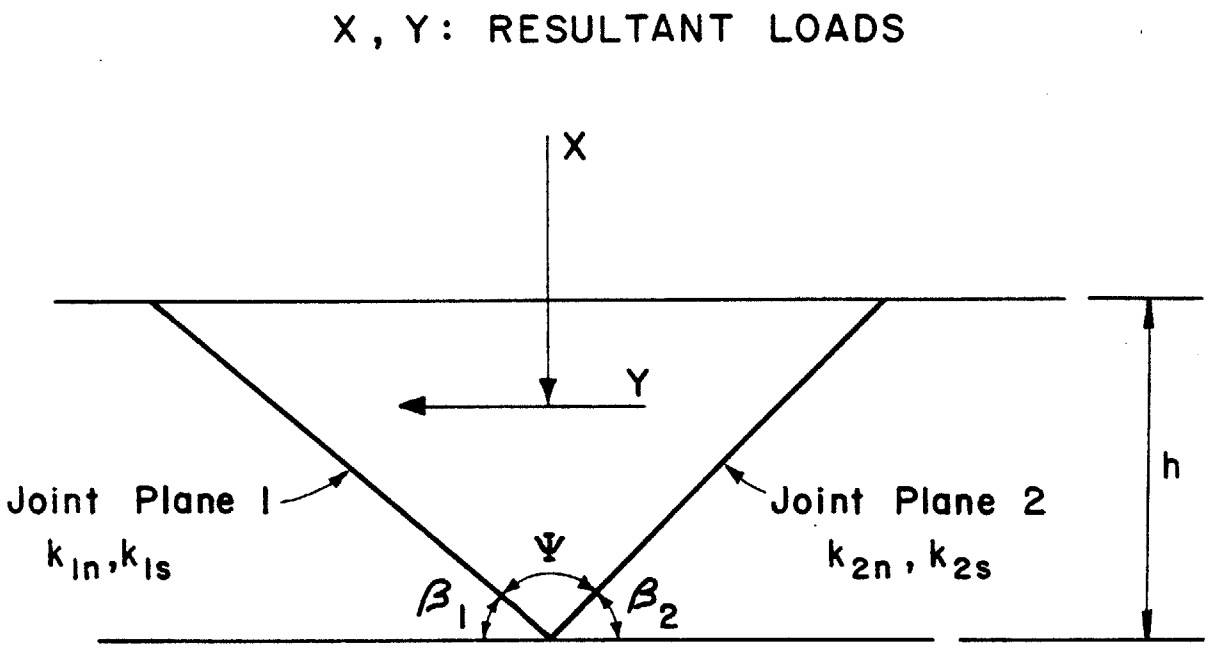


Figure 4.10 Generalized Stiffness Approach--Known Parameters

$$\begin{Bmatrix} \sigma_n \\ \tau \end{Bmatrix} = \begin{bmatrix} k_n & 0 \\ 0 & k_s \end{bmatrix} \begin{Bmatrix} \delta_n \\ \delta_s \end{Bmatrix} \tag{4.28}$$

As indicated in Figure 4.11, the problem involves six unknowns: N_1, T_1, N_2, T_2, δ_t and α. There are also six independent equations. Two equations are the force equilibrium equations in the x and y directions:

$$N_1 \cos \beta_1 + T_1 \sin \beta_1 + N_2 \cos \beta_2 + T_2 \sin \beta_2 = X \tag{4.29}$$

$$N_1 \sin \beta_1 - T_1 \cos \beta_1 - N_2 \sin \beta_2 + T_2 \cos \beta_2 = Y \tag{4.30}$$

The other four equations relate the shear and normal stresses to the magnitude and direction of the resultant displacement:

$$\delta_t h = \frac{N_1}{(\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1) k_{1n}} \tag{4.31}$$

$$= \frac{T_1}{(\cos \alpha - \sin \alpha \operatorname{ctn} \beta_1) k_{1s}} \tag{4.32}$$

$$= \frac{N_2}{(\cos \alpha \operatorname{ctn} \beta_2 - \sin \alpha) k_{2n}} \tag{4.33}$$

$$= \frac{T_2}{(\cos \alpha + \sin \alpha \operatorname{ctn} \beta_2) k_{2s}} \tag{4.34}$$

δ_T = Resultant Displacement

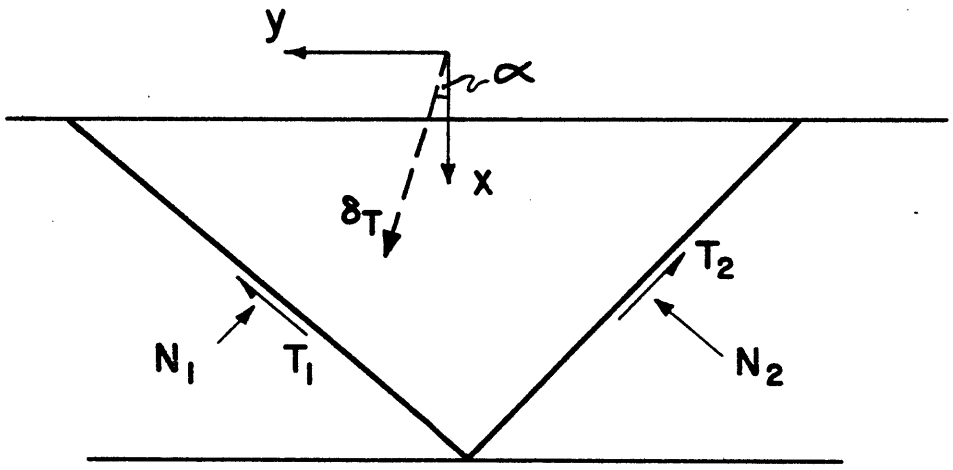


Figure 4.11 Generalized Stiffness Approach--Unknown Parameters

All of the above expressions are derived in Appendix D.

The six equations can be reduced to two if T_1, T_2 and N_2 are expressed in terms of T_1 and substituted into the force equilibrium equations:

$$\cos \beta_1 + \frac{ak_{1s} \sin \beta_1 + bk_{2n} \cos \beta_2 + ck_{2s} \sin \beta_2}{(\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1)k_{1n}} = \frac{X}{N_1} \qquad (4.35)$$

$$\sin \beta_1 - \frac{ak_{1s} \cos \beta_1 + bk_{2n} \sin \beta_2 - ck_{2s} \cos \beta_2}{(\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1)k_{1n}} = \frac{Y}{N_1} \qquad (4.36)$$

where $a = \cos \alpha - \sin \alpha \operatorname{ctn} \beta_1$

$b = \cos \alpha \operatorname{ctn} \beta_2 - \sin \alpha$

$c = \cos \alpha + \sin \alpha \operatorname{ctn} \beta_2$

The two equations involve only two unknowns (N_1 and α). They can be solved for α:

$$\alpha = \tan^{-1} \left[\frac{YB - XA}{XC - YA} \right] \qquad (4.37)$$

$$A = (k_{1n} - k_{1s})\cos \beta_1 + (k_{2s} - k_{2n})\cos \beta_2$$

$$B = k_{1n} \frac{\cos^2 \beta_1}{\sin \beta_1} + k_{1s} \sin \beta_1 + k_{2n} \frac{\cos^2 \beta_2}{\sin \beta_2} + k_{2s} \sin \beta_2$$

$$C = k_{1n} \sin \beta_1 + k_{1s} \frac{\cos^2 \beta_1}{\sin \beta_1} + k_{2n} \sin \beta_2 + k_{2s} \frac{\cos^2 \beta_2}{\sin \beta_2}$$

All the remaining unknowns can be found by substituting back into the various equations. The solution procedure is a

cumbersome operation to perform by hand, but it can be easily programmed for a computer.

All the expressions shown above were based on a two dimensional cross section; hence, the analysis implicitly assumed the wedge has a length of unity along the line of interaction. Strictly speaking, the equations are valid only for cases involving an infinitely long wedge of constant dimensions in an infinitely high rock slope (Figure 4.12). The joint planes bounding the wedge form a notch in the slope; X and Y correspond to loads per unit length along the notch. All of the reactions at the joint planes are constant along the slope. Obviously, these conditions are seldom even approximated in the field.

Fortunately, the physical interpretation of the results need not be quite so restrictive. The left side of Equations (4.35) and (4.36) depend only on stiffnesses and the shape of the wedge; they do not contain any terms involving h, the height of the wedge. Thus, the ratios X/N_1 and Y/N_1 are constant for geometrically similar cross sections even though X,Y and N_1 may each be functions of h. This independence can be used to generalize the applicability of Equations (4.35) and (4.36). They are valid for finite wedges as long as all cross sections perpendicular to the line of intersection are geometrically similar. This situation occurs whenever the

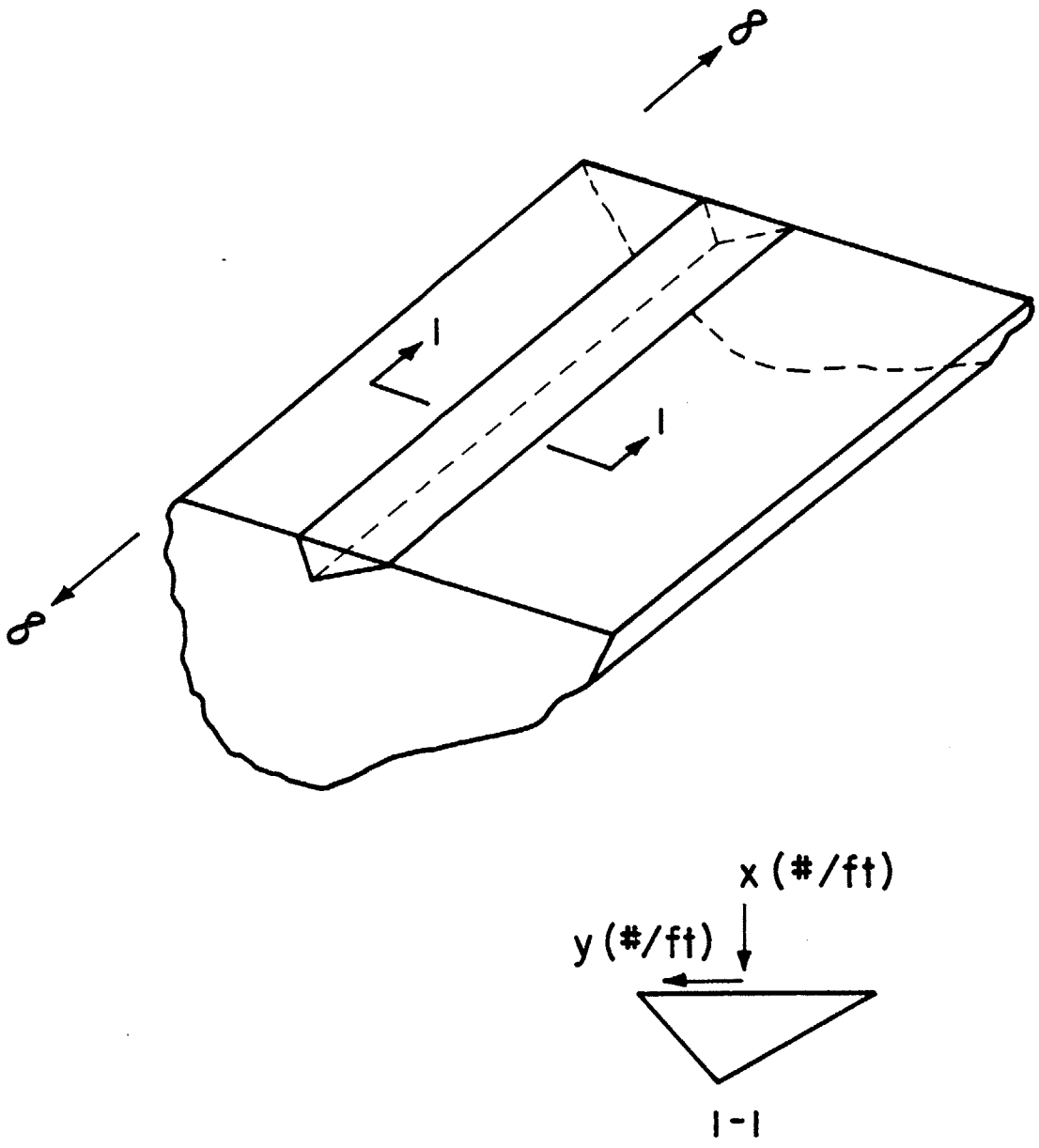


Figure 4.12 Notch in Infinite Slope

line of intersection of the joint planes intersects the slope at a right angle (Figure 4.13). The proof of this generalized relationship is relatively straightforward:

Equation (4.35) can be rewritten in terms of differentials

$$dN_1 = \frac{dX}{e} \tag{4.38}$$

where $e = (\cos \beta_1 + \frac{ak_{1s} \sin \beta_1 + bk_{2n} \cos \beta_2 + ck_{2s} \sin \beta_2}{(\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1)k_{1n}})$

N_1 is the total force in joint plane 1.

$$N_1 = \int_{\substack{\text{entire} \\ \text{wedge}}} dN_1 = \frac{1}{e} \int_{\substack{\text{entire} \\ \text{wedge}}} dX \tag{4.39}$$

$$N_1 = \frac{X}{e} \tag{4.40}$$

Similarly, $N_1 = \frac{Y}{f}$ (4.41)

where $f = (\sin \beta_1 - \frac{ak_{1s} \cos \beta_1 + bk_{2n} \sin \beta_2 - ck_{2s} \cos \beta_2}{(\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1)k_{1n}}$

Equations (4.40) and (4.41) are mathematically identical to (4.35) and (4.36); however, X and Y represent the <u>total</u> applied forces in the x and y direction rather than unit loads as in Equations (4.35) and (4.36).

Analyses involving asymmetrical wedges must be handled on an individual basis. The gross contact areas between the wedge and its parent rock mass must be calculated for each

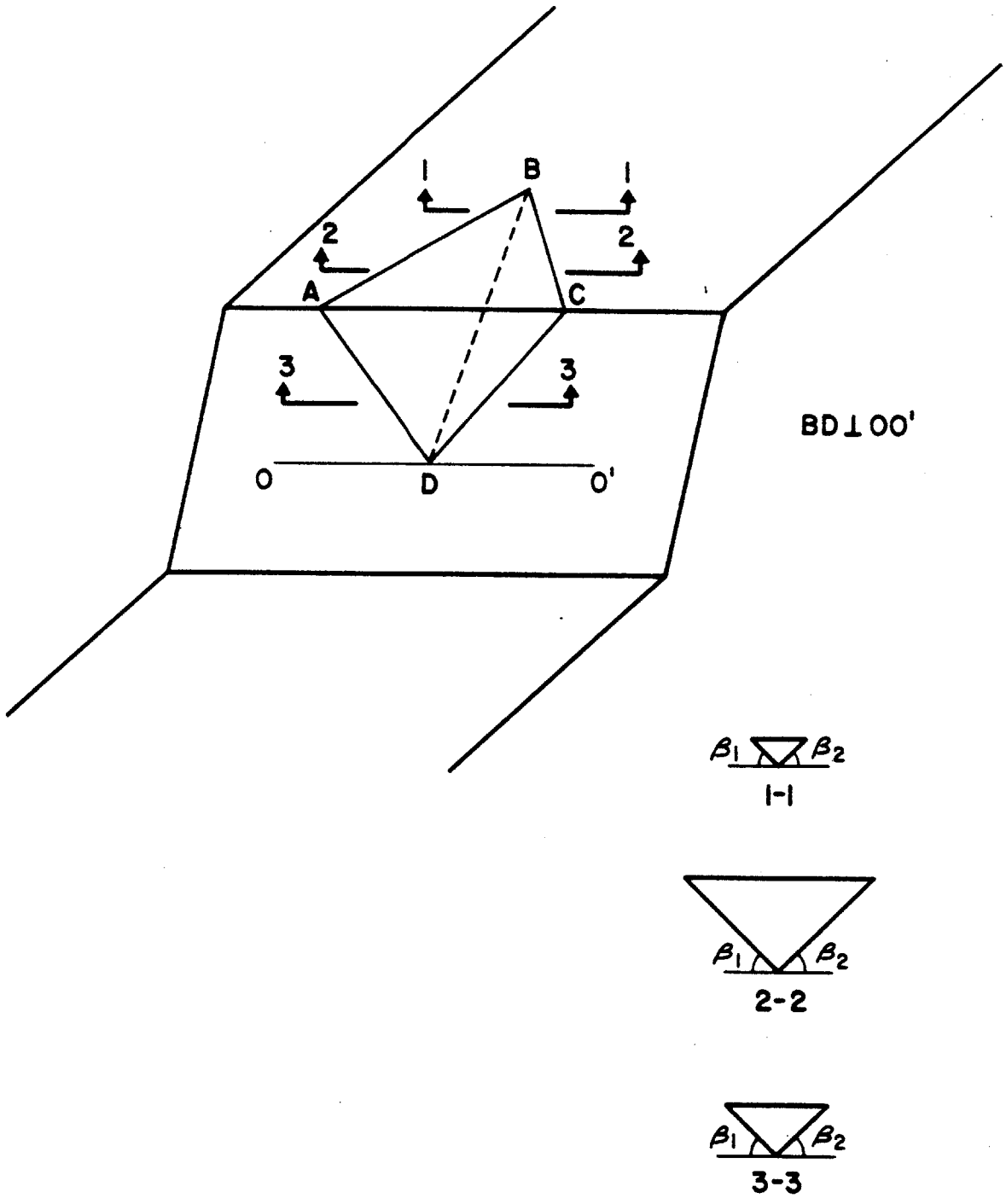


Figure 4.13 Wedge With Geometrically Similar Cross Sections

joint plane. As will be discussed later, the analysis can easily be programmed for computerized solutions.

Special Conditions

There are some special situations where it is possible to simplify the analysis. If there is no load in the y direction, i.e., Y = 0 the expression for α (Equation (4.37)) simplifies to:

$$\alpha = \tan^{-1}\left[\frac{(k_{1s} - k_{1n})\cos\beta_1 + (k_{2n} - k_{2s})\cos\beta}{k_{1n}\sin\beta_1 + k_{1s}\frac{\cos^2\beta_1}{\sin\beta_2} + k_{2n}\sin\beta_2 + k_{2s}\frac{\cos^2\beta_2}{\sin\beta_2}}\right] \qquad (4.42)$$

Case 1: $Y = 0 \quad k_{1n} = k_{2n} = k_{1s} = k_{2s}$

When all the stiffnesses are equal the numerator of the equation reduces to 0 and α becomes 0. The equality of stiffness (together with Y = 0) implies that the resultant displacement of the wedge will be in the x direction.

Case 2: $Y = 0 \quad k_{1n} = k_{2n} >> k_{1s} = k_{2s}$

These conditions correspond to the assumptions implicitly made in conventional stability analyses. The equation for α becomes:

$$\alpha = \tan^{-1}\left[\frac{\cos\beta_2 - \cos\beta_1}{\sin\beta_1 + \sin\beta_2}\right] = \frac{\beta_1 - \beta_2}{2} \qquad (4.43)$$

Thus, the resultant displacement will occur at an inclination equal to half the difference between β values (Figure 4.14).

Results

The solution procedure outlined above has been used to develop several plots that illustrate the sensitivity of the results (particularly N_1 and N_2) to changes in the input parameters ($\beta_1, \beta_2, k_{1n}, k_{2n}, k_{1s}, k_{2s}$). The results are valid for a wedge in a infinitely long notch or a wedge whose line of intersection is orthogonal to the strike of the slope face. In all cases, Y = 0 and X = weight of the wedge. The results are expressed in the same manner as those presented in Section 4.1 i.e., the dependent variable is Φ_R (the value of Φ required to produce a factor of safety equal to 1.0):

$$\Phi_R = \tan^{-1} \frac{W \sin \theta}{N_1 + N_2} \tag{4.44}$$

θ = angle between the line of intersection of joint planes and the horizontal (Figure 4.15).

W = weight of wedge.

Figure 4.15 shows a typical wedge and identifies the parameters selected for study in the sensitivity analyses.

Case 1: $k_{1n} = k_{2n}$; $k_{1s} = k_{2s}$; $\psi = 90°$

$0° \leq \beta_1 \leq 90°$

$0° \leq \beta_2 \leq 90°$

$$k_{1n} = k_{2n} >> k_{1s} = k_{2s}$$

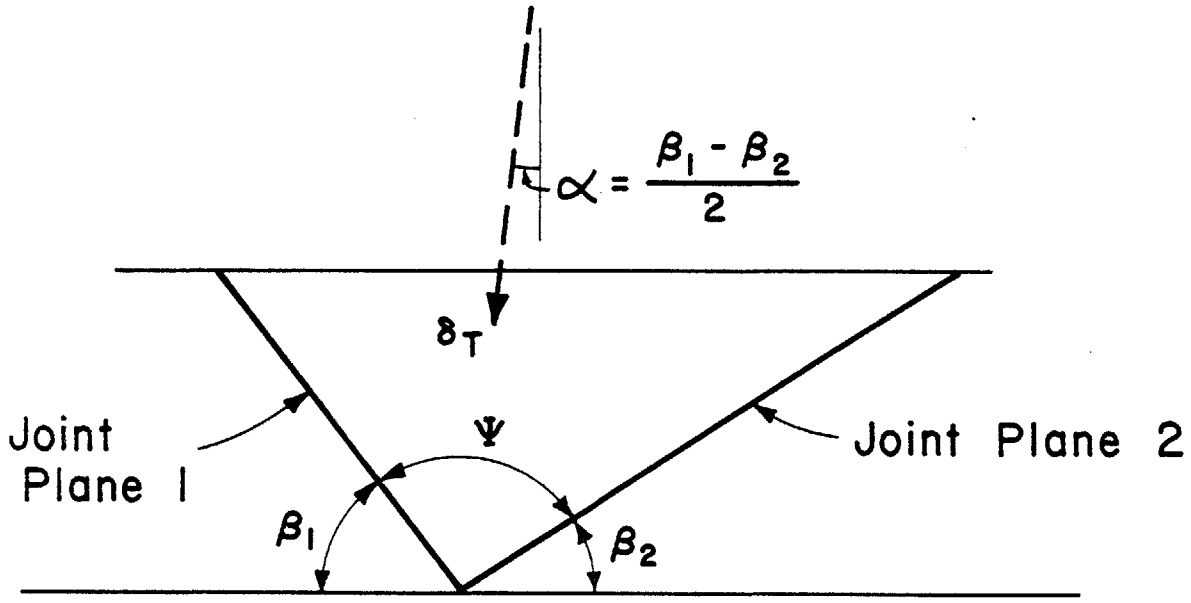


Figure 4.14 Direction of Displacement When $K_{1n} = K_{2n} >> K_{1s} = K_{2s}$

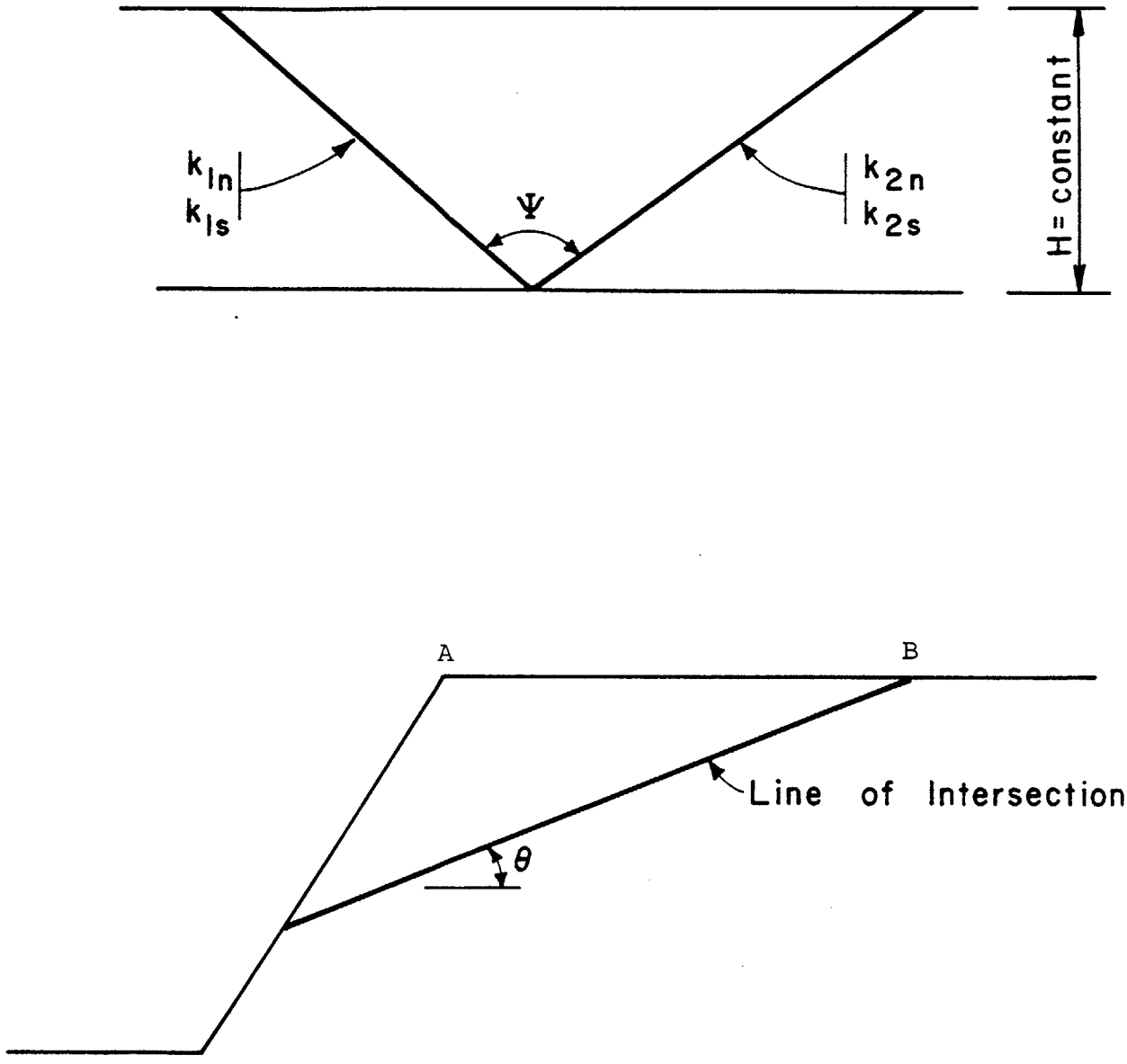


Figure 4.15 Parameters Varied in Sensitivity Analyses

Figure 4.16 illustrates the functional relationship between β_1 and Φ_R (β_1 and β_2 are complementary angles as $\beta_2 = 180° - \psi - \beta_1 = 90° - \beta_1$). Two families of curves were developed for $\theta = 30°$ and $\theta = 45°$ respectively. Each family is composed of several members that represent different values of R, the stiffness ratio:

$$R = \frac{k_{1n}}{k_{1s}} = \frac{k_{2n}}{k_{2s}} \tag{4.45}$$

All the curves are symmetrical about the line $\beta_1 = \beta_2 = 45°$.

$R = \infty$ and $R = 1$ represent conditions that are tentatively considered to be the upper and lower bounds for natural rock joints. The curve for $R = \infty$ reflects the effect of "wedging" (Hoek and Bray, 1972) as Φ_R decreases with increasing β_1. The curve for $R = 1$ shows just the opposite trend as Φ_R increases strongly with β_1. (The two curves seem to be symmetrical about the line $\Phi_R = \theta$. However, this apparent symmetry has not been analytically proven. There is no readily discernible explanation for this effect from a mechanical standpoint). A stiffness ratio of 2.5 yields a virtually constant value of friction angle that is approximately equal to θ.

Case 2 : $k_{1n} = k_{2n}$; $k_{1s} = k_{2s}$ $\psi = 120°$

$0° \leq \beta_1 \leq 60°$

$0° \leq \beta_2 \leq 60°$

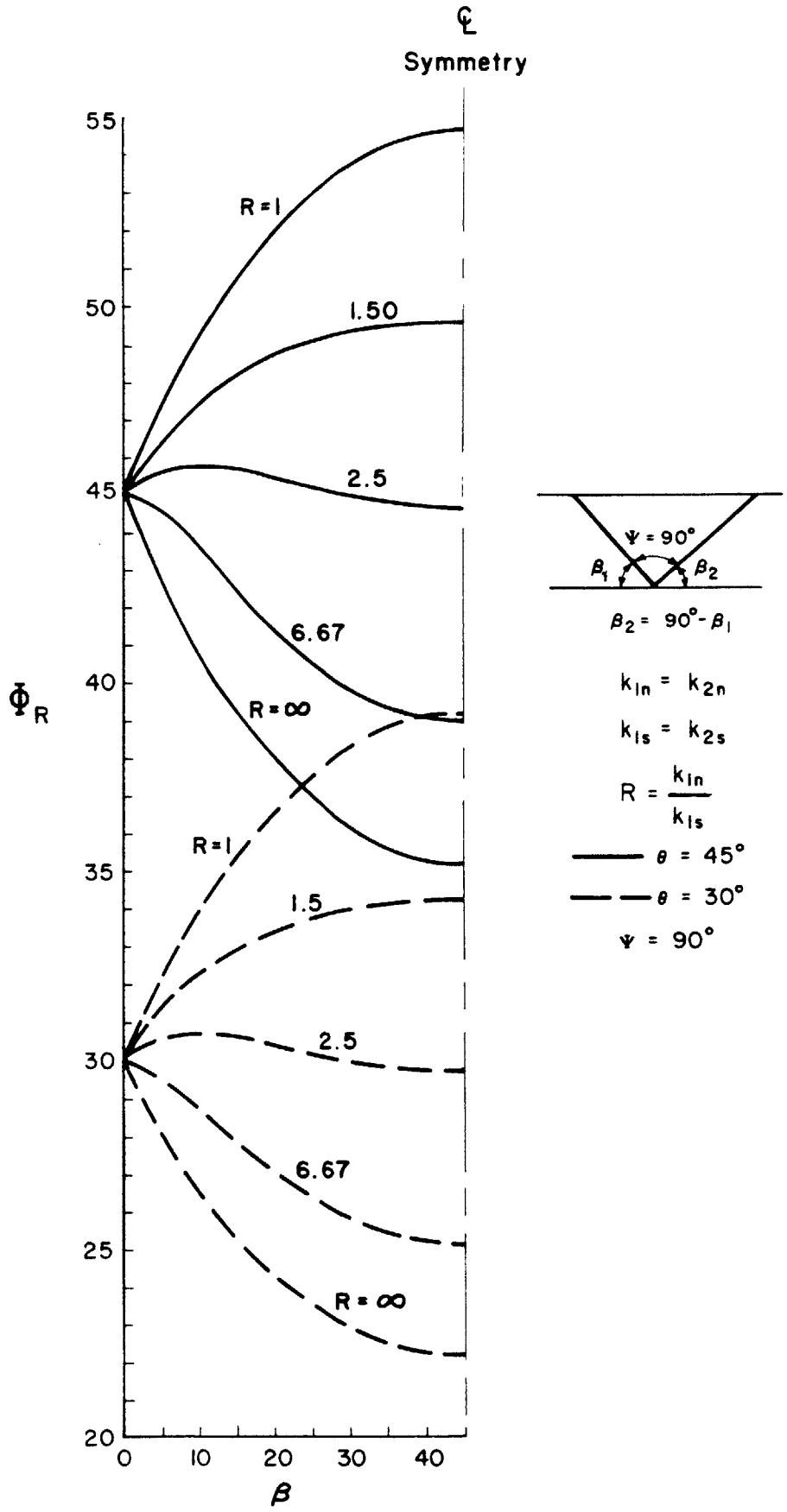


Figure 4.16 Φ_R vs. β_1 For $\Psi = 90^\circ$

The curves for $\psi = 120°$ (Figure 4.17) look quite similar to those for $\psi = 90°$ (Figure 4.16). They have an axis of symmetry and they diverge as β_1 increases. However, the divergence is not as pronounced for $\psi = 120°$. The difference in Φ_R values is approximately 8.2 at $\beta = \beta_1 = 30°$. Once again, a value of R = 2.5 yields $\Phi_R \sim \theta$ regardless of the value of β_1.

Case 3: $k_{1n} = k_{2n}$; $k_{1s} = k_{2s}$; $\psi = 60°$

$0° \leq \beta_1 \leq 120°$

$0° \leq \beta_2 \leq 120°$

Figure 4.18 presents results for the case $\psi = 60°$. Near the axis of symmetry ($\beta_1 = 60°$) the curves behave much like the ones examined earlier. However, as β_1 approaches 0 the curves become irregular. A problem arises in that region because β_2 ($\beta_2 = 120° - \beta_1$) becomes greater than 90°, i.e., joint plane 2 overhangs the wedge (Figure 4.19). The overhang does not pose any difficulties from an analytical standpoint; however, the physical interpretation of the results demands some close scrutiny. For every R there is a critical value of β_1 (defined as β_{1cr}) such that for all $\beta_1 < \beta_{1cr}$ the normal force on joint plane 2 is tensile. This value β_{1cr} occurs when the direction of the wedge's resultant displacement is parallel to joint plane 2 (Figure 4.19). Figure 4.20 shows the relationship between β_{1cr} and R. If $\beta_1 \leq \beta_{1cr}$ the

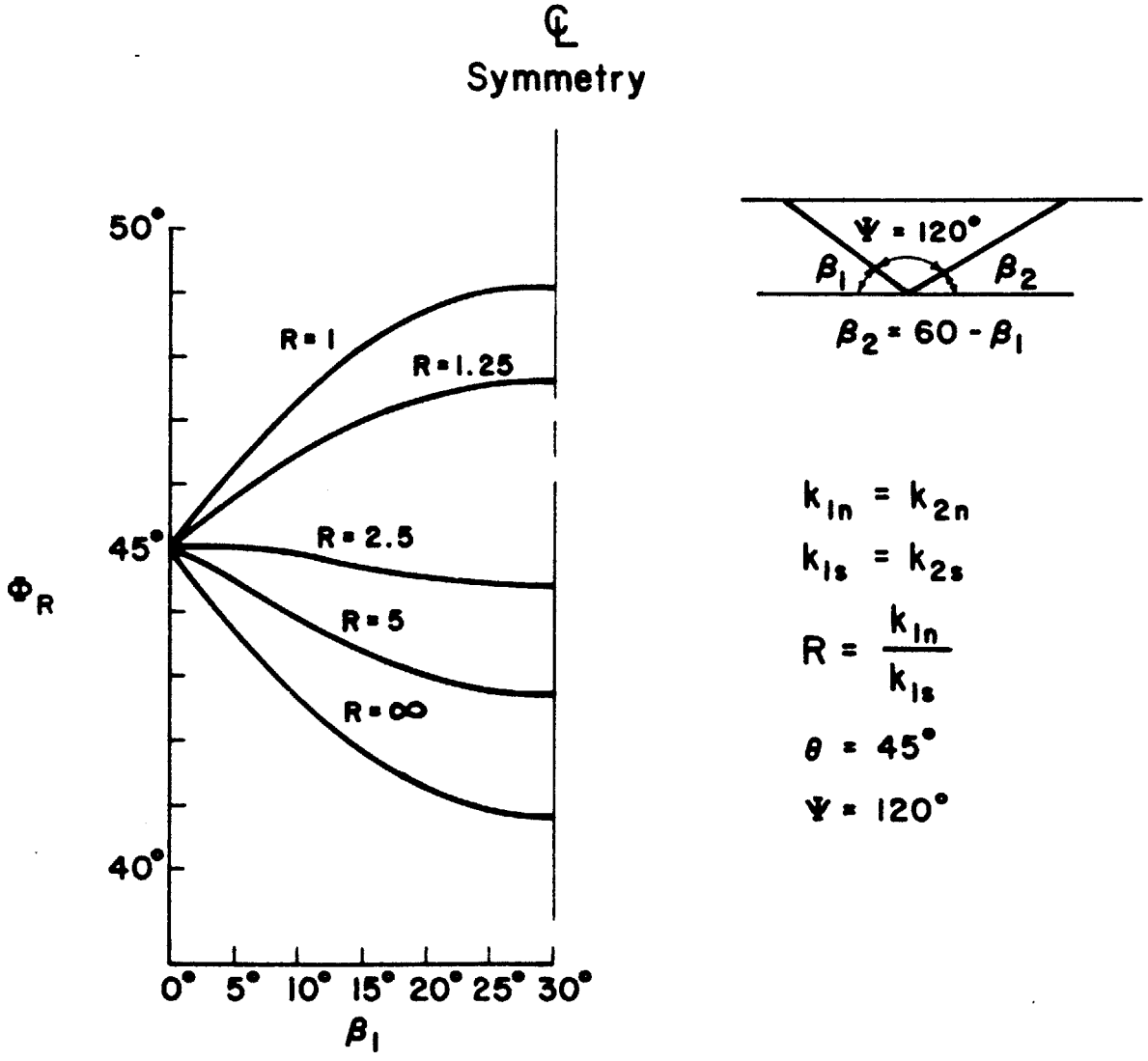


Figure 4.17 Φ_R vs. β_1 For $\Psi = 120^\circ$

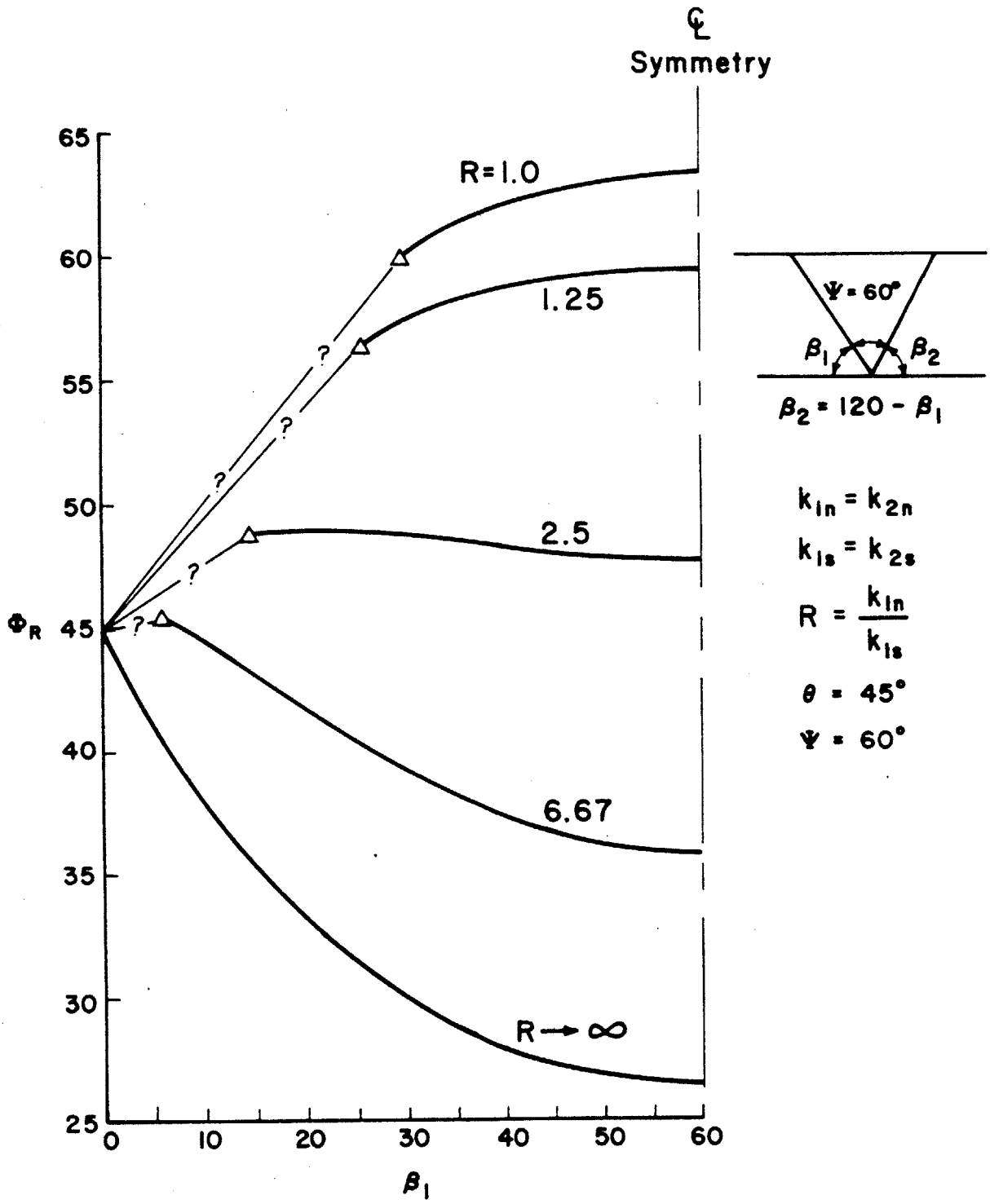


Figure 4.18 Φ_R vs. β_1 For $\Psi = 60^\circ$

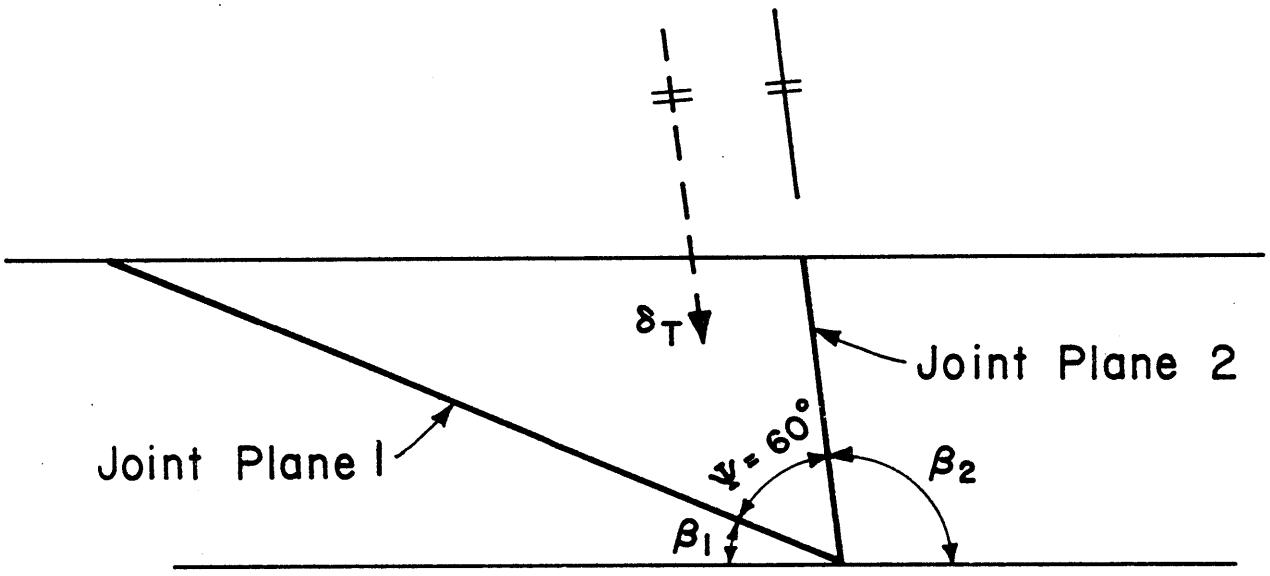


Figure 4.19 Overhanging Joint Plane

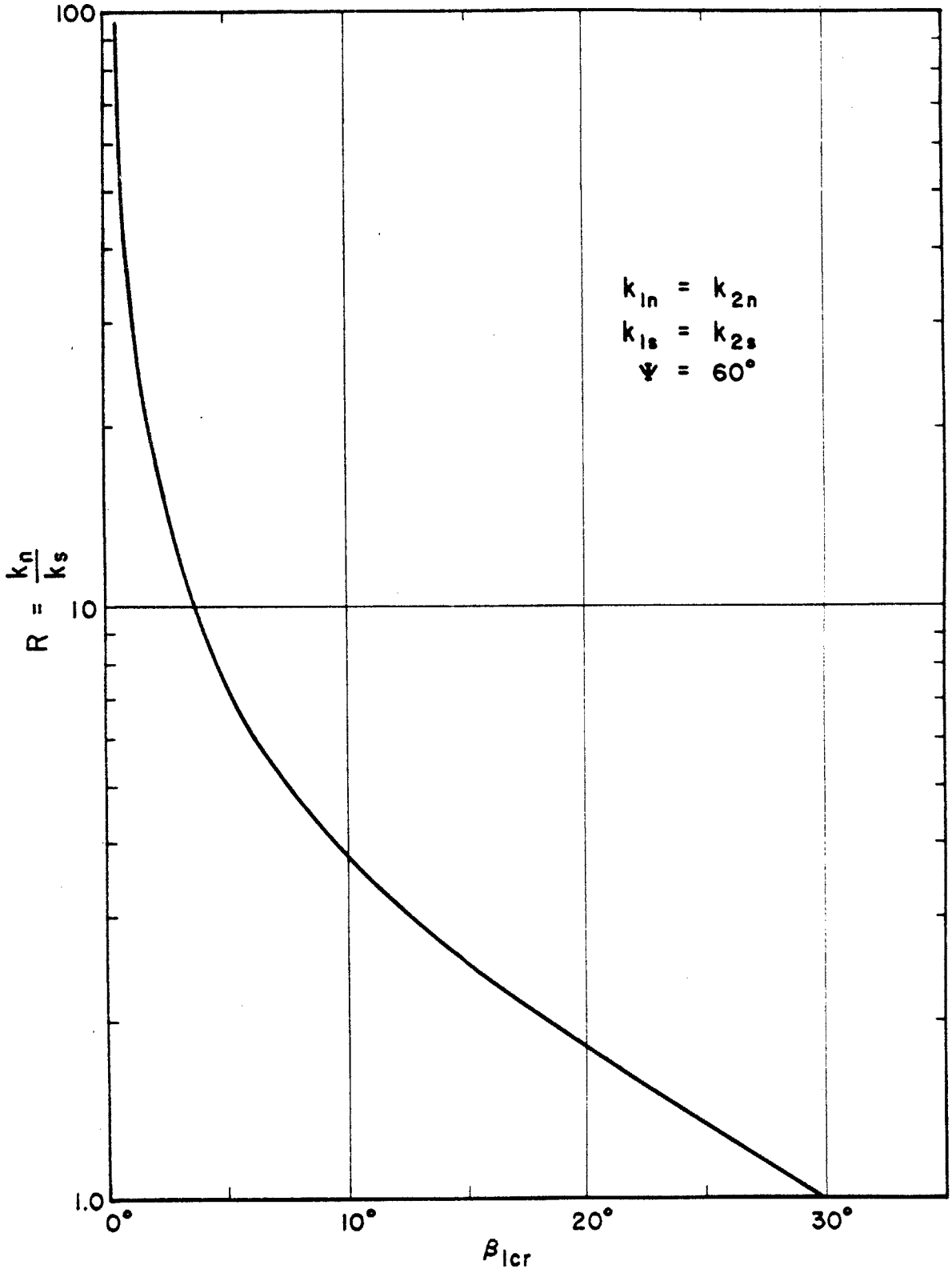


Figure 4.20 R vs. β_{1cr} for $\Psi = 60^{\circ}$

wedge tends to pull away from joint plane 2. Continuous rock joints cannot sustain a tensile load so the curves for Φ_R must be modified to reflect the change in N_2. The exact form of the curve depends on the shape and deformational characteristics of the asperties in the joint plane. It is conceivable that even in the absence of a normal force some asperties may interlock and generate a shear force in the overhanging joint plane. On the other hand, if the plane is relatively smooth, T_2 (as well as N_2) may reduce to 0 as soon as β_1 is less than β_{1cr}. Figure 4.21 illustrates schematically some of the typical forms the curve for R = 1 may assume. Curve B ignores the tension problem. It is merely a mathematical extension of Curve A into tensile region of N_2. Curve C represents the other extreme. A severe discontinuity occurs at $\beta_1 = \beta_{1cr} = 30^o$ as both N_2 and T_2 drop to 0. The portion of Curve C that lies to the right of point 0 represents sliding along a single plane--plane 1. Curve D recognizes the existence of some shear forces in the joint plane for $0 < \beta < \beta_{1cr}$. Most natural joint planes would probably have curves that plot between C and D. It is interesting to note that the inability of overhanging joints to maintain tensile loads can be helpful from a stability standpoint in that it reduces Φ_R (compare Curves B and D in Figure 4.21). (Joints with bridges of

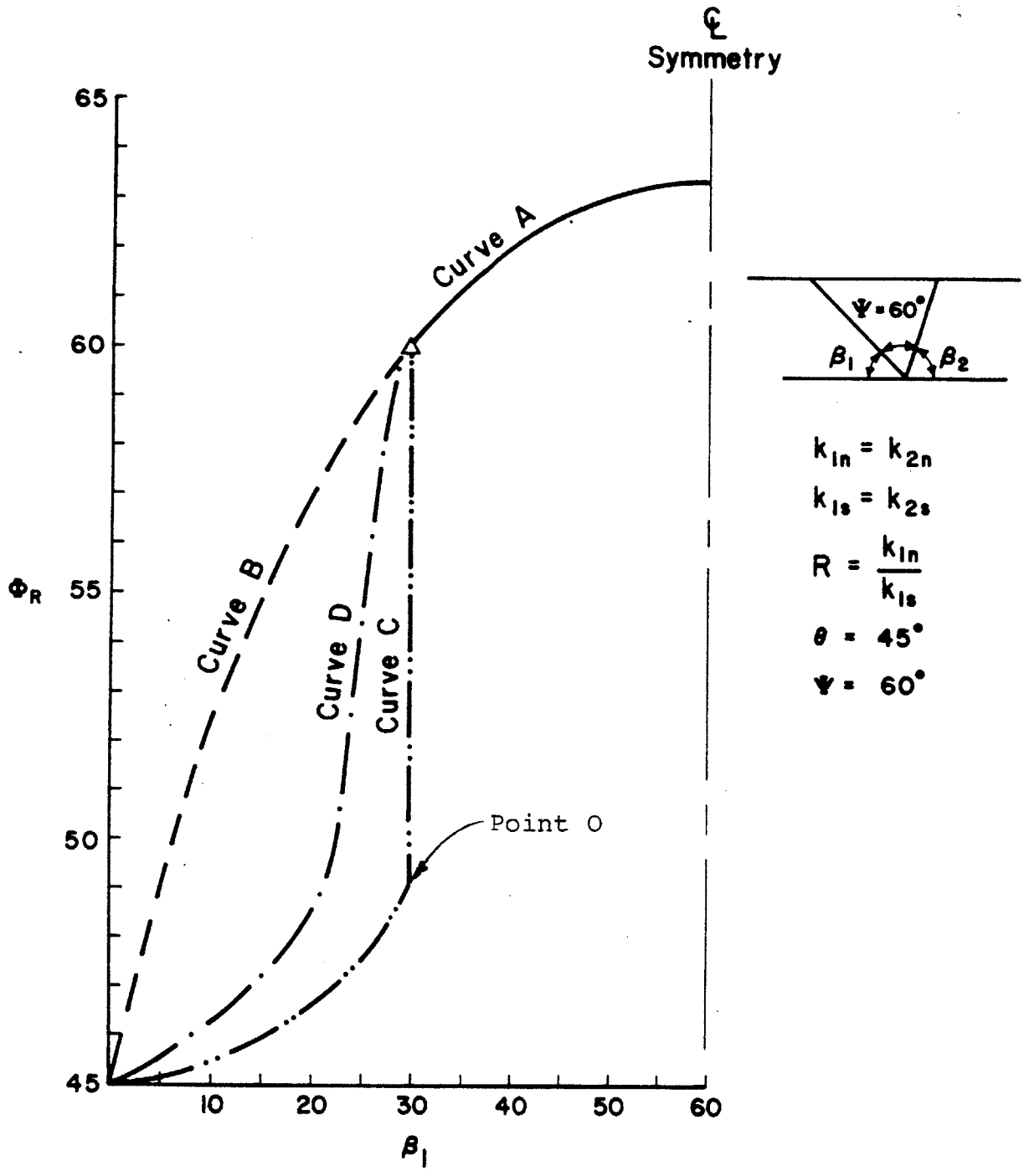


Figure 4.21 Φ_R vs. β_1 For $\Psi = 60^\circ$ and R = 1--Curves Reflect Different Reactions on Joint Plane 2

intact rock may be capable of maintaining tensile forces; the tensile capacity would depend on the persistence of the respective joints.)

When $\beta_1 > \beta_{1cr}$, the curves for R = 1 and R = ∞ seem to be symmetrical--a property which is apparently shared by all such pairs of curves. For $\psi = 60°$, the maximum difference ($\Delta\Phi_{max}$) between the curves is 30°. This difference always reaches a maximum at $\beta_1 = \beta_2$ and is a function of ψ. Figure 4.22 shows that $\Delta\Phi_{max}$ is a very strong function of ψ and decreases rapidly as ψ increases.

Case 4: $k_{1s} = k_{1n}$; $k_{2s} = k_{2n}$ $\Psi = 90°$

$0° \leq \quad _1 \leq 90°$

$0° \leq \quad _2 \leq 90°$

$$S = \frac{k_{2s}}{k_{1s}} = \frac{k_{2n}}{k_{1n}}$$

The effect of different stiffnesses in the joint planes is shown in Figure 4.23. The curves become more skewed as S increases. The changes in shape reflect the fact that in Equation (4.44) the numerator (W sin θ) is a constant while the denominator ($N_1 + N_2$) varies with geometry of the wedge. A high value of S suggests that joint plane 2 bears a disproportionately large share of the weight of the wedge. The reaction at joint plane 2 consists of N_2 and T_2. Since β_1

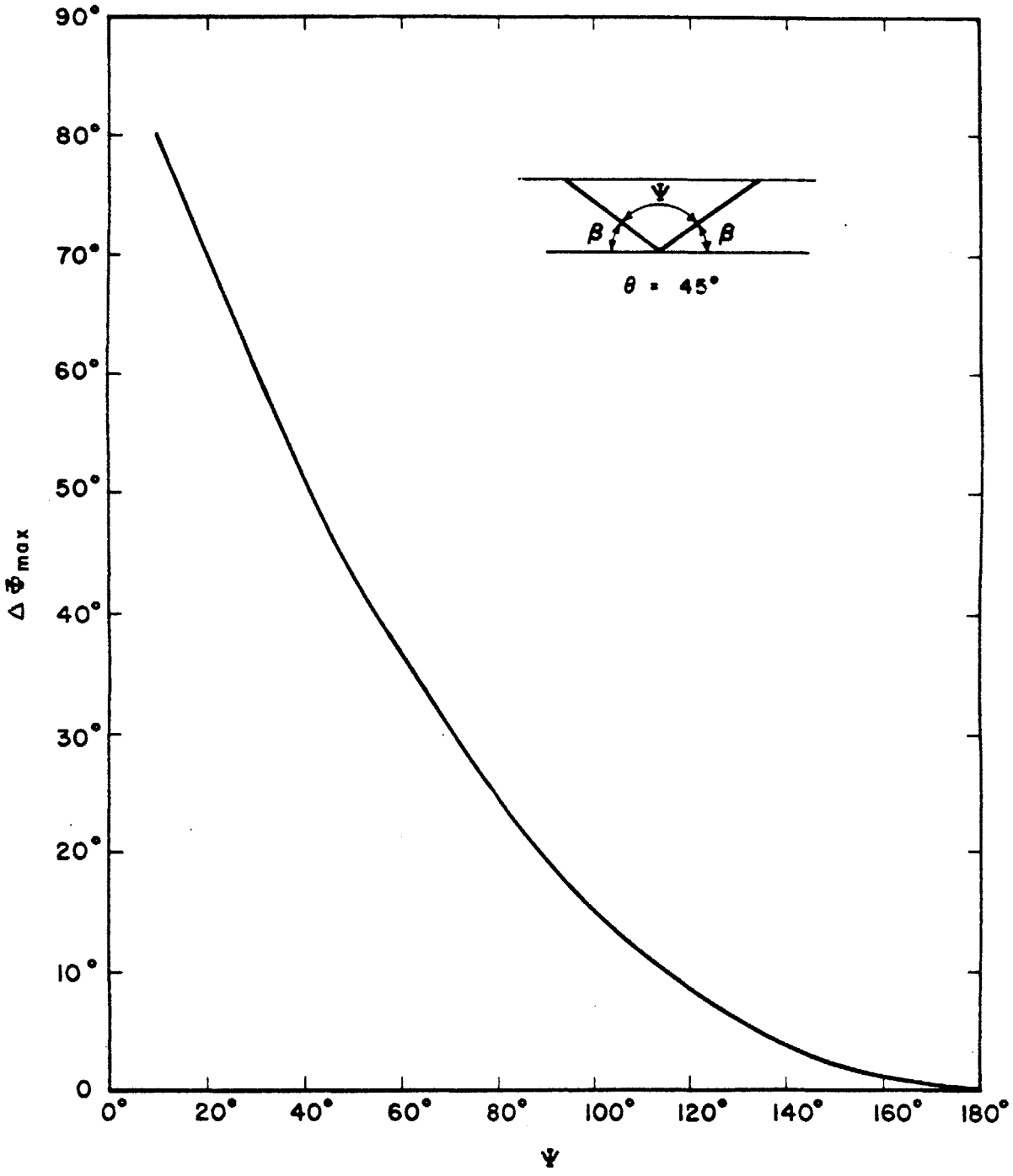


Figure 4.22 $\Delta\Phi_{max}$ vs. Ψ for $\Theta = 45^{\circ}$

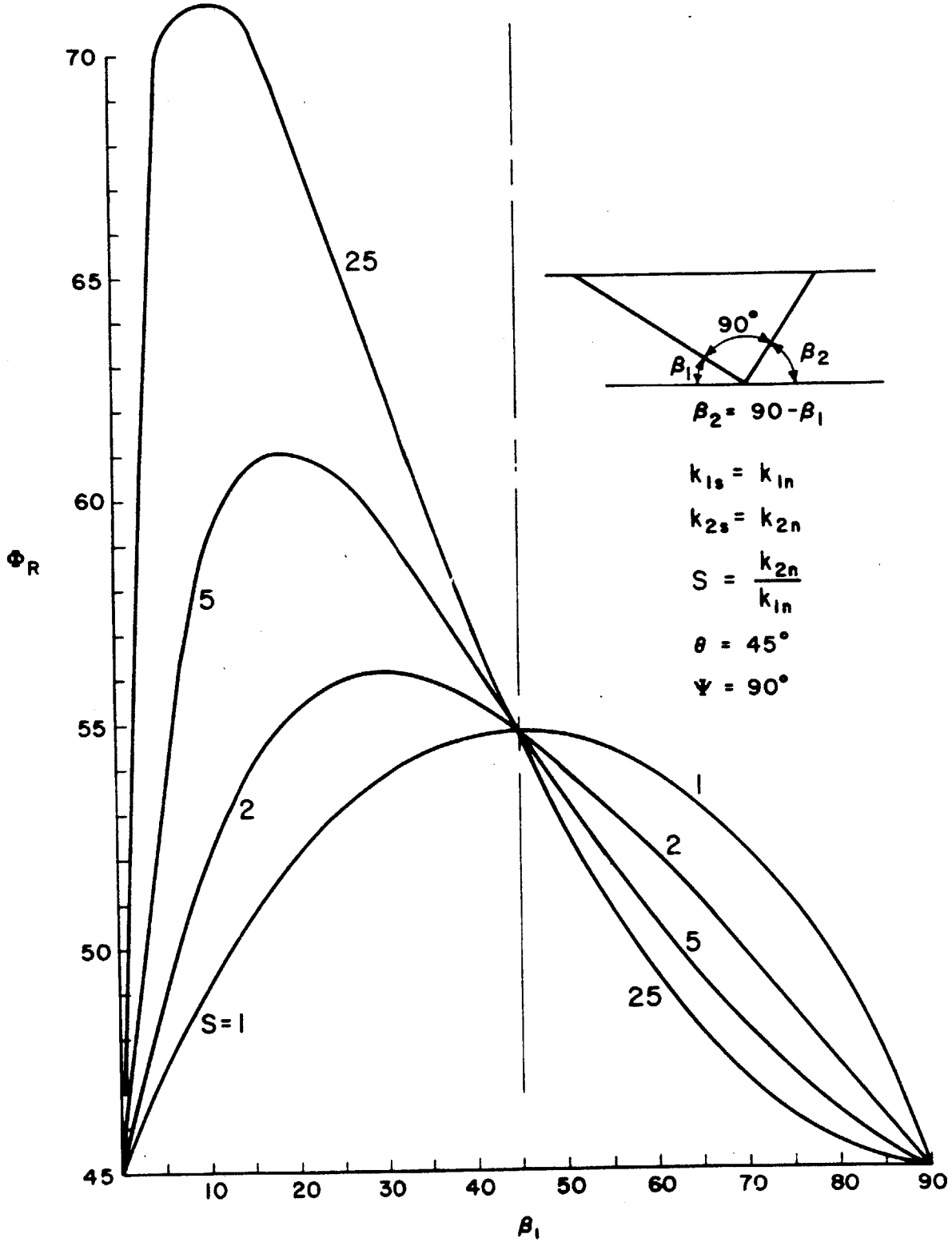


Figure 4.23 Φ_R vs. β_1 For $\Psi = 90^\circ$ and Varying S

and β_2 are complementary angles, β_1 values near 0° are associated with β_2 values near 90°. When β_2 approaches 90° most of the reaction is supplied by T_2. Under these conditions (low β_1 and high S), N_1 and N_2 are low and a high Φ_R is required to maintain equilibrium. The argument can be reversed for cases involving β_1 near 90°. β_2 near 0° implies a high N_2 and a correspondingly low Φ_R.

All of the curves pass through the point $\beta_1 = 45°$ and $\Phi_R = 54.75$. Since ψ equals 90°, β_1 of 45° defines a symmetrical wedge. Under this special condition, the sum $N_1 + N_2$ is a constant regardless of the value of S.

All of the results summarized in this section are based on wedges that have geometrically similar cross sections along their entire lengths. The physical significance of this assumption was discussed earlier. The method can be generalized to handle wedges with arbitrary geometries. The technique is discussed in Section 4.5 in connection with the computer program SWARS-2PM.

4.3 Stress Approach Vs. Stiffness Approach

4.3.1 Introduction

Section 4.1 showed that the stresses on the joint planes is a critical input to the stability analysis. That same section discussed two alternate methods for computing the stresses. An obvious question arises: How does one determine the stresses on the joint planes? The question is difficult to answer because the stresses depend on

cumulative effects of loads and joint stiffnesses - both of which can change as time progresses. Figure 4.24 illustrates the concept. It is a schematic representation of the temporal variations in loads, joint stiffnesses and stresses that occurred during the existence of a hypothetical wedge.

The stresses are a function of the loads and stiffnesses. During any small time increment Δt there is a direct relationship between incremental loads (ΔL) and incremental stresses (ΔS). ΔS is related to ΔL through the stiffness equations presented in Section 4.3. The precise relationship between ΔS and ΔL depends on the values of the stiffnesses during that time interval. An incremental load applied at t_1 will not necessarily produce the same change in stresses as the same load applied at t_2 because the stiffnesses at t_1 and t_2 may be different. The stresses at any time t reflect the original stresses (at t_0) plus all the changes that have occurred since t_0. (The model presumes that the principle of superposition is valid at least in a crude sense.) Thus, S_p, the present state of stress, is a function of the cumulative interaction between loads and stiffnesses; S_p is not directly related to the current stiffnesses. The current stiffnesses are instrumental in determining the effects of additional loads, but yield no information concerning S_p.

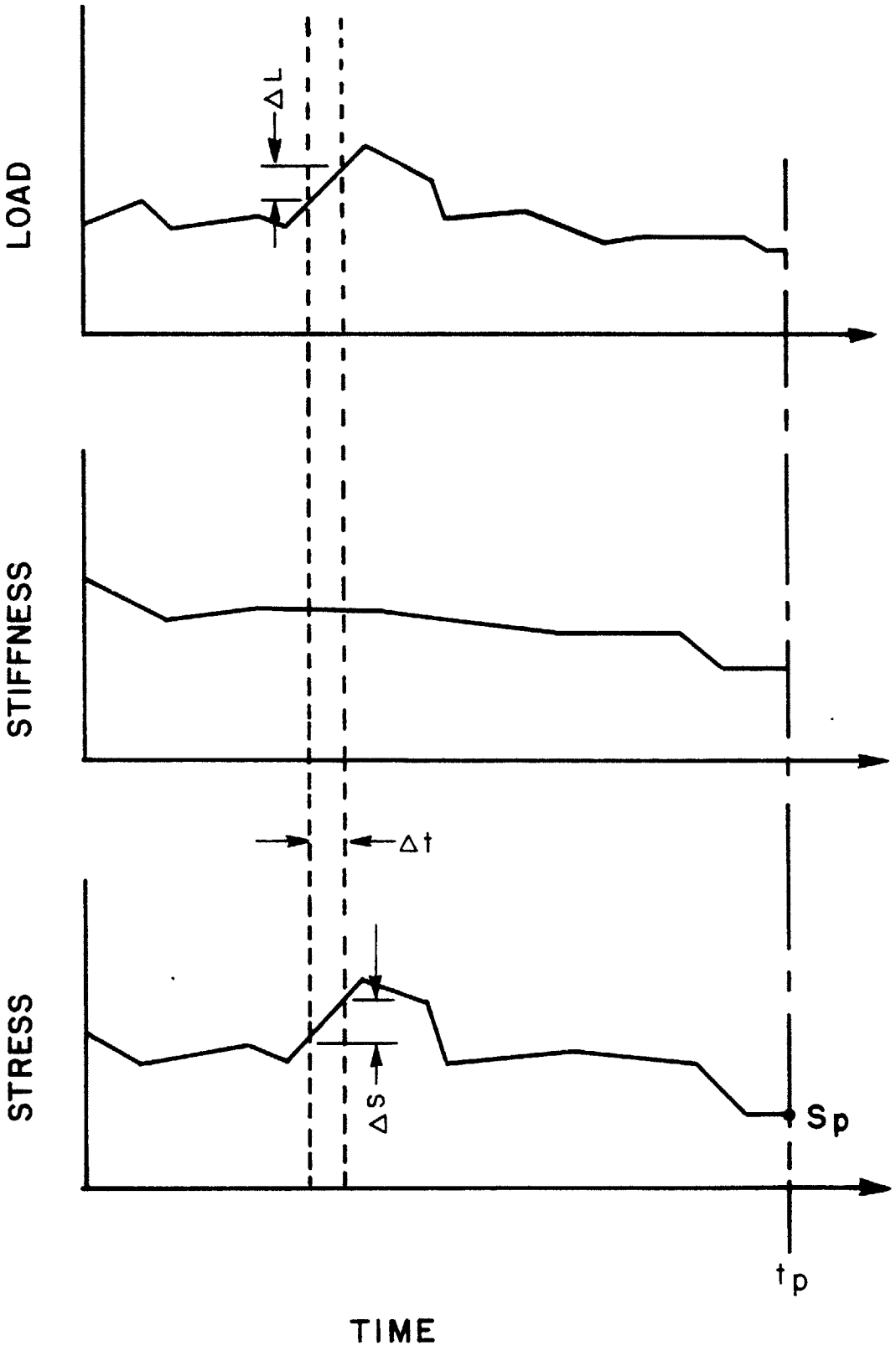


Figure 4.24 Load, Stiffness and Stress vs. Time

This discussion raises an interesting point about the stiffness plots presented in Section 4.1 (Figures 4.16 through 4.18). These plots describe a highly idealized situation that is portrayed in Figure 4.25. They treat the weight of the wedge as an external load that is not applied until after the stiffnesses have reached their current values. In effect, the wedge is placed in a preconditioned notch at t_p, the present time. Nevertheless, the plots can provide some useful information that will be discussed later in this section.

4.3.2 Detailed Discussion

The joints which define a rock wedge formed when the intact rock mass became overstressed.* The creation of the joints changed the stress field near the joints as the rock mass re-established equilibrium. Once formed, the joints may have been subjected to a variety of geologic processes. Some processes such as glaciation or erosion can be active for thousands of years. Others such as seismic activity or rainwater infiltration can be intermittent. Regardless of the process, the joints interact with their evolving environment. The stresses on the joints change to satisfy new loading conditions. The stresses at any one time reflect not only the current conditions but also the entire loading history of the joints.

K, the ratio of horizontal to vertical stresses in

* The joints may not have formed simultaneously. The time of formation is not an important feature of the argument.

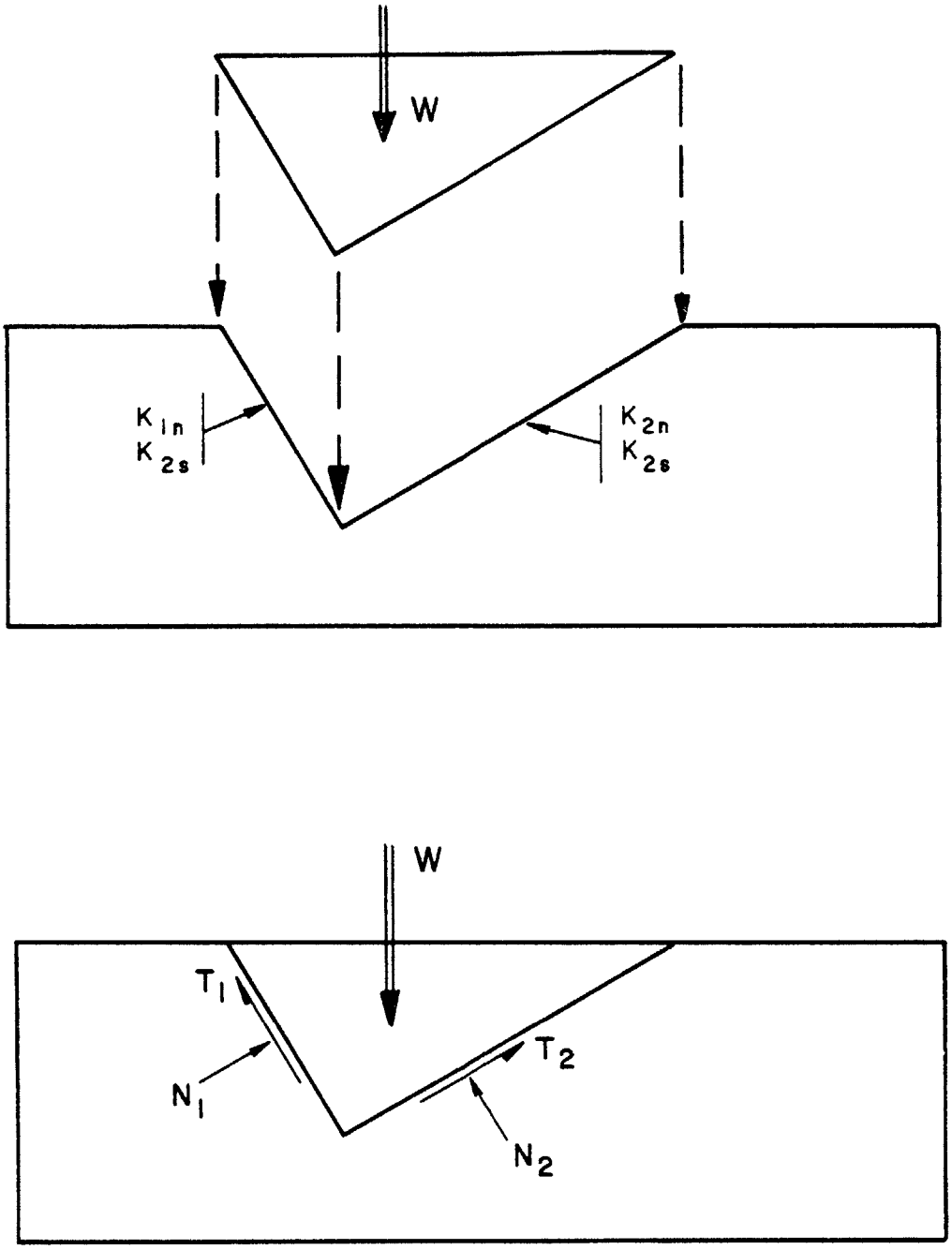


Figure 4.25 Stiffness Approach--Idealized Case

the wedge bounded by the joints, can be considered a parameter that describes the net effect of all the loadings during the wedge's existence. It is not necessarily identical to the ratio of in situ stresses in the parent rock mass. The joint is a discontinuity in the physical properties of the rock and may create irregularities in the ambient stress field that characterize the rock mass. The joints may isolate the wedge from the remainder of the rock mass. On the other hand, the stress field may be continuous across the joint.

The idea that K characterizes the stresses in the joint planes provides some insight into the parameter, but the critical question remains: How does one measure K? K is intimately linked to the geologic history of the wedge so it is almost impossible to determine the value a priori. However, there are limits to the range of values that K can assume. K must lie between K_a and K_p* which represent the active and passive states of failure described in Section 4.1. The only way to determine K for a specific wedge is to actually measure the stresses in the wedge. Unfortunately, the techniques for measuring in situ stresses are too complicated and too expensive to use for all wedges. At the present time K is usually estimated. The most realistic approach is to treat K as a random variable and perform a reliability analysis on the wedge. The uncertainty associated

* The derivations in Section 4.1 assumed that the strength of the joint consisted solely of frictional resistance.

with K can be incorporated directly into the stability analysis.

The discussion in Section 4.1 treated symmetric wedges. The concept can be extended to handle asymmetric wedges. Figure 4.26 is a section taken perpendicular to the line of intersection of a typical asymmetrical wedge. Each differential segment along the joint plane can be treated like the element shown in Figure 4.6. The K parameter relates the shear and normal stresses on the joint planes (Equations (4.15) and (4.16)). The stresses can be integrated to yield the following expressions:*

$$\frac{T_1}{N_1} = \frac{(1-K)}{\text{ctn } \beta_1 + K \tan \beta_1} = \lambda_1 \qquad (4.46)$$

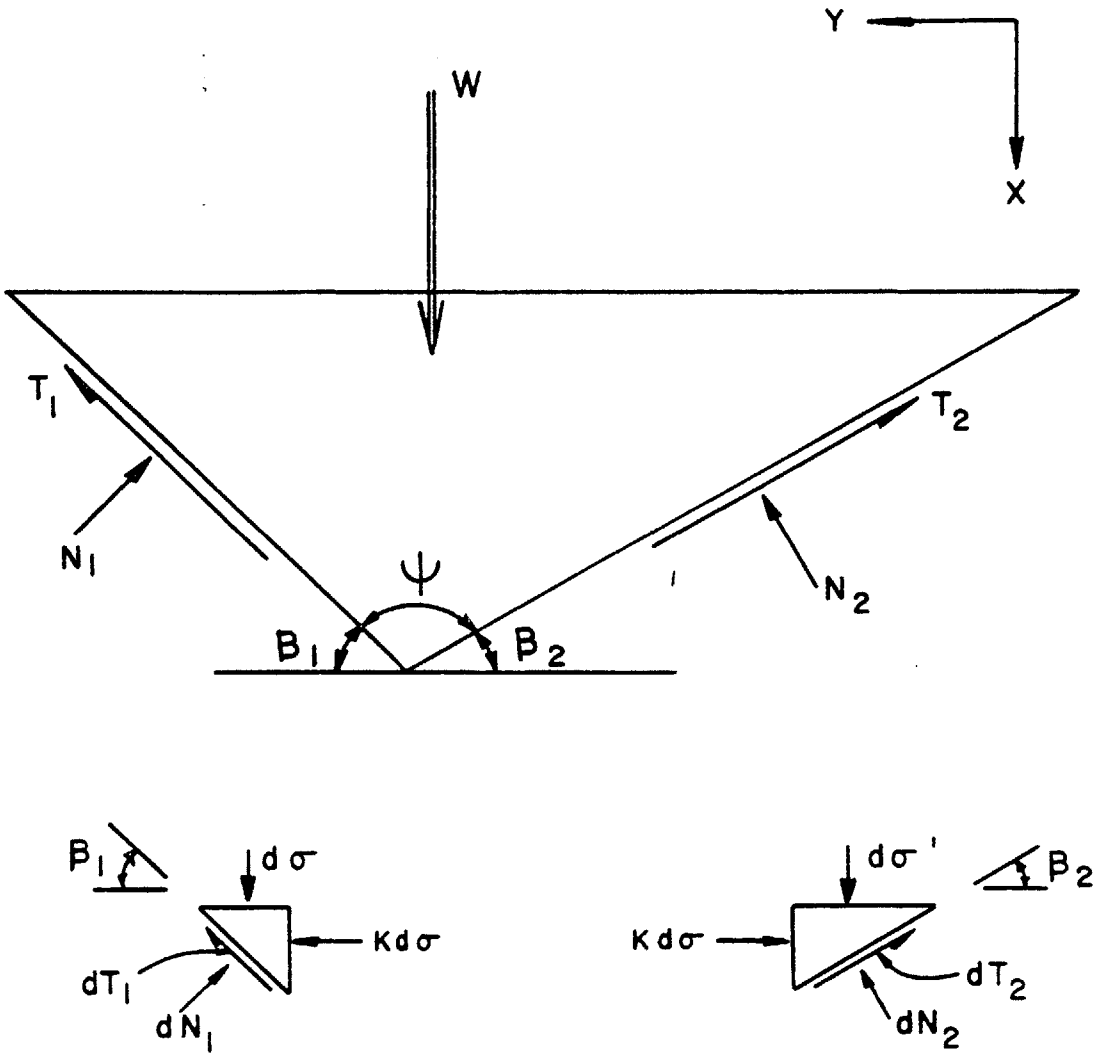
$$\frac{T_2}{N_2} = \frac{(1-K)}{\text{ctn } \beta_2 + K \tan \beta_2} = \lambda_2 \qquad (4.47)$$

Equations (4.46) and (4.47) can be used in conjunction with the equations of force equilibrium to evaluate N_1, T_1, N_2, T_2.

Summing forces in the x direction (Figure 4.26):

$$N_1 \cos \beta_1 + \lambda_1 N_1 \sin \beta_1 + N_2 \cos \beta_2 + \lambda_2 N_2 \sin \beta_2 = W^1 \qquad (4.48)$$

* K may vary over the height of the wedge. If K does vary, the parameter shown in Equations (4.46) and (4.47) is the mean value of K for the entire height.



$$\frac{dT_i}{dN_i} = \frac{1 - K}{\text{ctn } \beta_i + K \tan \beta_i}$$

Figure 4.26 Stresses on Differential Areas of the Joint Planes

W^1 is the component of the weight acting perpendicular to the line of intersection. As shown in Figure 4.2, $W^1 = W \cos \theta$. Summing forces in the y direction:

$$N_1 \sin \beta_1 - \lambda_1 N_1 \cos \beta_1 - N_2 \sin \beta_2 + \lambda_2 N_2 \cos \beta_2 = 0 \qquad (4.49)$$

Rearranging Equation 4.49:

$$\frac{N_1}{N_2} = \frac{(\sin \beta_2 - \lambda_2 \cos \beta_2)}{(\sin \beta_1 - \lambda_1 \cos \beta_1)} = \lambda_3 \qquad (4.49A)$$

Equations (4.49A) and (4.48) can be solved for N_2:

$$N_2 = \frac{W \cos \theta}{\lambda_3(\cos \beta_1 + \lambda_1 \sin \beta_1) + (\cos \beta_2 + \lambda_2 \sin \beta_2)} \qquad (4.50)$$

Finally,

$$N_1 = \lambda_3 N_2 \qquad (4.49B)$$

$$T_1 = \lambda_1 N_1 \qquad (4.46A)$$

$$T_2 = \lambda_2 N_2 \qquad (4.47A)$$

The equilibrium analysis outlined above did not explicitly consider joint stiffnesses. It was concerned solely with the current state of stress on the joint planes. The stiffnesses enter the problem indirectly through the K parameter.

K reflects the response of the wedge to a series of loadings. The response for any particular load depends on the material properties of the joints. However, these properties (the shear and normal stiffnesses) can change with time.* The incremental stresses that develop during each loading are a function of the joint properties at the time of the loading. There is no unique relationship between the current stresses and current stiffnesses.

Although the stiffnesses are not helpful in determining the current state of stress they can be used to determine the additional stresses due external loadings like rock bolts or surcharges. In fact any "current loading" can be analyzed with the stiffness approach presented in Section 4.2. The procedure is consistent with the concept proposed earlier: loads should be analyzed in terms of the stiffnesses that existed at the time of their application.

Thus, the stability analysis that will be described in Sections 4.4 and 4.5 combines the stress and stiffness approaches in the following manner:

1. The stress approach is used to calculate the reactions (N_1,N_2,T_1,T_2) due to the weight of the wedge.

2. The stiffness approach is used to calculate the reactions due to all other loads.

* There are many mechanisms that can change the joint properties. The stiffnesses may decrease during extreme loadings because asparities shear off. Ground water percolation through the joints can either decrease (through leaching) or increase (through cementation) the stiffnesses.

The procedure is admittedly a simplification of actual conditions but it does present a consistent methodology for treating rock wedges.

The weight of the wedge should be analyzed through the stress approach. The stiffness plots shown as Figures 4.16 to 4.18 are a direct violation of this principle. As shown earlier, the stiffness approach should only treat "current" loads. Although the plots should not be used to examine gravitational loads they can be used to examine any other loading that is directly proportional to the size of the wedge. The plots were based on cross sections with different shapes (β_1 varied) but with a constant height (Figure 4.15). N_1 and N_2 are directly proportional to the weight of the unit cross section (Equation (4.40); $X = W \cos \theta$). Equation (4.44) indicates that Φ_R, the ordinate of the plots, is a function of $\frac{W}{N_1 + N_2}$; hence, any loading that is directly proportional to the weight should yield the same values of Φ_R. Since the wedges have a constant height the distance AB in Figure 4.15 is directly related to the area or to the weight of the section. A uniform loading along AB would be directly related to the weight of the wedge. Therefore, Figures 4.16 to 4.19 show the effect of a uniform surcharge on the top (and front) faces of the wedge.

4.4 The Factor of Safety

The presence of shear forces on the joint planes in a direction perpendicular to the line of intersection presents some difficulties in defining the factor of safety (FS). This problem was alluded to in Section 4.1 where FS was calculated two different ways. In rock slope analyses FS is usually defined as the ratio of resisting to driving forces. However, the definition assumes that both forces act in the same direction. The condition is seldom fulfilled in real problems.

Figure 4.27 shows the shear forces that should be considered in a typical stability analysis. As derived in Figure 4.2, the shear force along the line of intersection is W sin θ*. T_1 and T_2 are shear forces perpendicular to the line of intersection. In the special case where K = 1 (the situation considered in conventional analyses) T_1 and T_2 are both zero; therefore, the entire shear resistance along the joint planes counteracts W sin θ. Whenever K does not equal unity the shear resistance must counteract T_1 and T_2 in addition to W sin θ. In fact, the effect is magnified because the presence of T_1 and T_2 actually decreases the shear resistance. As indicated in Section 4.1, T_1 and T_2 increase at the expense of N_1 and N_2. As N_1 and N_2 decrease the frictional resistance declines. The crucial questions are:

* This simplified model presumes that gravity is the only force acting on the wedge.

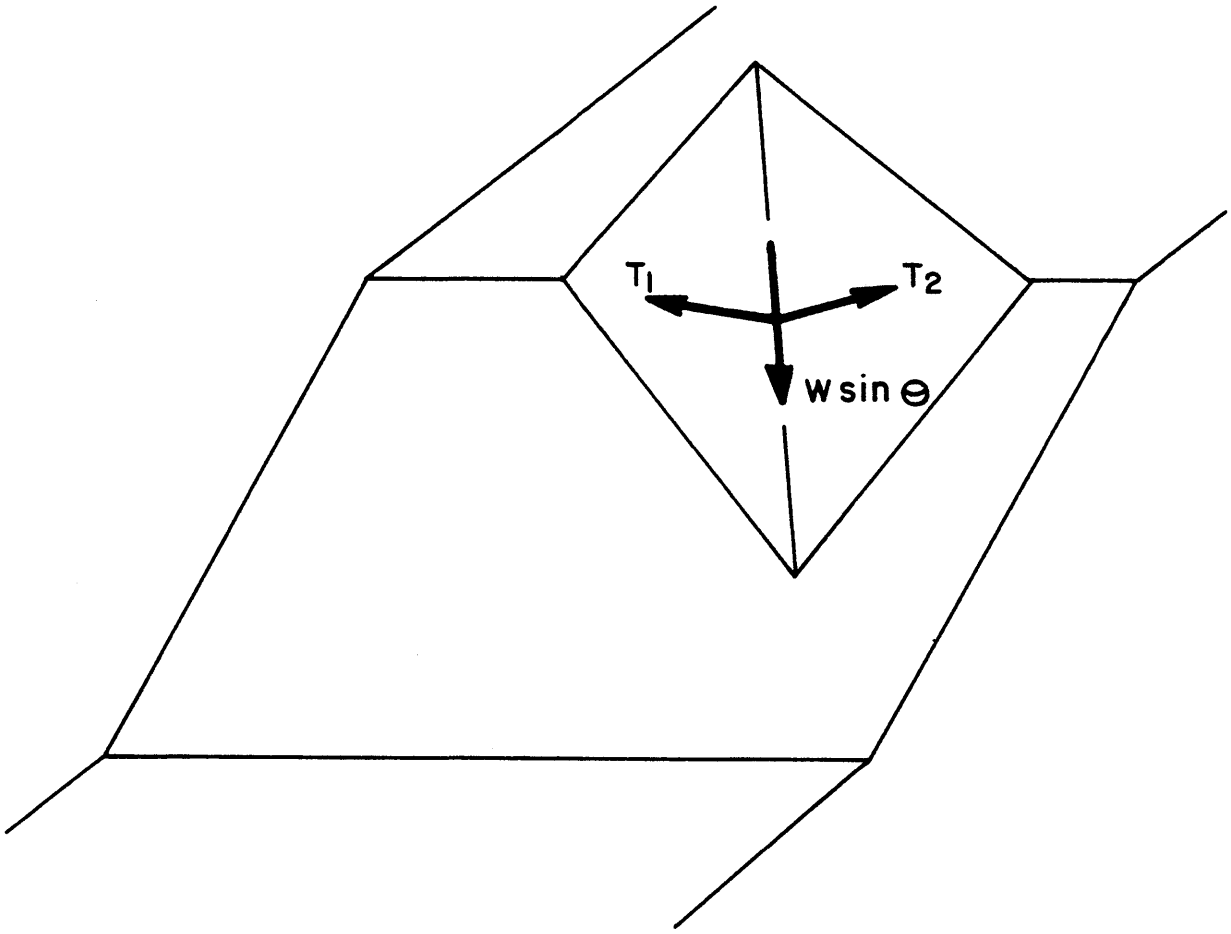


Figure 4.27 Shear Forces on Joint Planes

How do T_1 and T_2 affect the stability of the block and how can they be incorporated into stability calculations?

St. John addressed this problem in analyzing symmetric wedges with purely frictional resistance. As indicated in Section 4.1, St. John defined the factor of safety as:

$$FS = \frac{2N \tan \Phi}{\sqrt{(2T)^2 + (W \sin \theta)^2}} \qquad (4.51)$$

Thus, the driving force was considered the vectorial sum of the shear forces* in the joint planes. The resultant shear forces in the two planes are mirror images of one another because the wedge is symmetric. At limiting equilibrium the direction of impending motion is down the line of intersection and into the notch i.e., the wedge has a tendency to "settle" in the notch as well as move down the line of intersection.

The direction of motion is an important consideration in examining stability problems. What are the practical implications of a wedge settling in its notch? Surely there is a limit to the amount of movement that can occur in this direction. A minor amount of settlement may not be significant from an engineering standpoint. The important question is what happens after failure? Settlement may be tolerable but sliding down the line of intersection

* St. John computed the T forces through a stiffness analysis; however, the derivation of T is not a salient feature of the current discussion.

is clearly unacceptable. Strictly speaking, limiting equilibrium cannot provide any insight into post-failure behavior because it is concerned solely with the condition of the system just at failure. Baligh et al. (19xx) have done some interesting work in this area.

Baligh et al. investigated the influence of K, the stress ratio, on the stability of symmetric wedges. They used the stress approach developed by Steiner (as presented in Section 4.1) to determine the forces on the joint planes; hence, both T and N are functions of K. Like St. John, Baligh et al. treated the driving force as the vectorial sum of the two shear components on each plane. They define the factor of safety as F_3:

$$FS = F_3 = \frac{2N \tan \Phi}{\sqrt{(2T)^2 + (W \sin \theta)^2}} \qquad (4.51)$$

The equation can be rewritten as:

$$\frac{F_3}{\tan \Phi} = \frac{2N}{\sqrt{(2T)^2 + (W \sin \theta)^2}} \qquad (4.51A)$$

As shown in Figures 4.28 and 4.29, the quantity $F_3/\tan \Phi$ can be plotted as a function of K. The three curves in each figure correspond to wedges whose lines of intersection plunge at 5°, 15° and 45°. Figure 4.28 and Figure 4.29 were

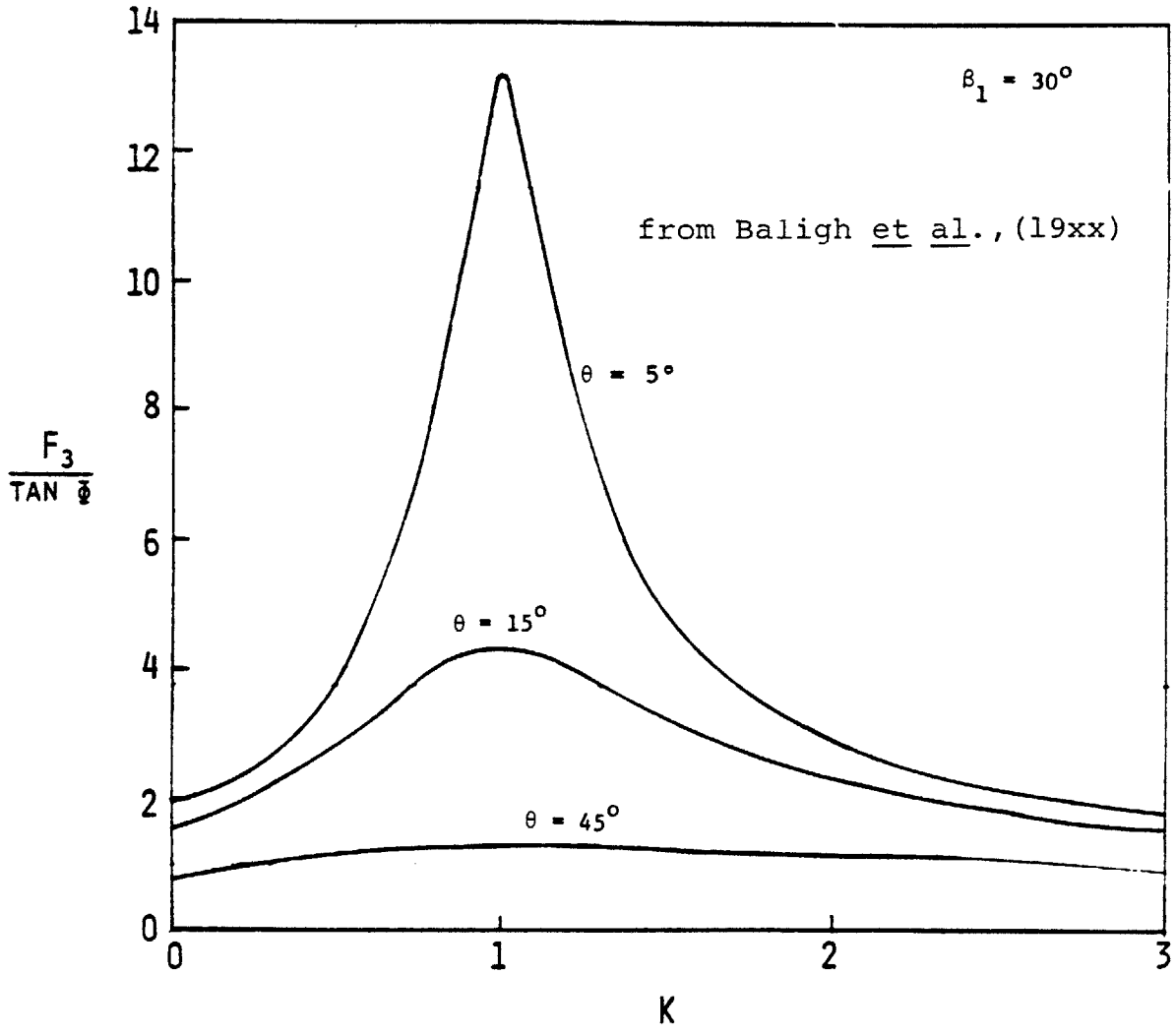


Figure 4.28 $F_3/\tan \Phi$ vs. K; $\beta_1 = 30^\circ$

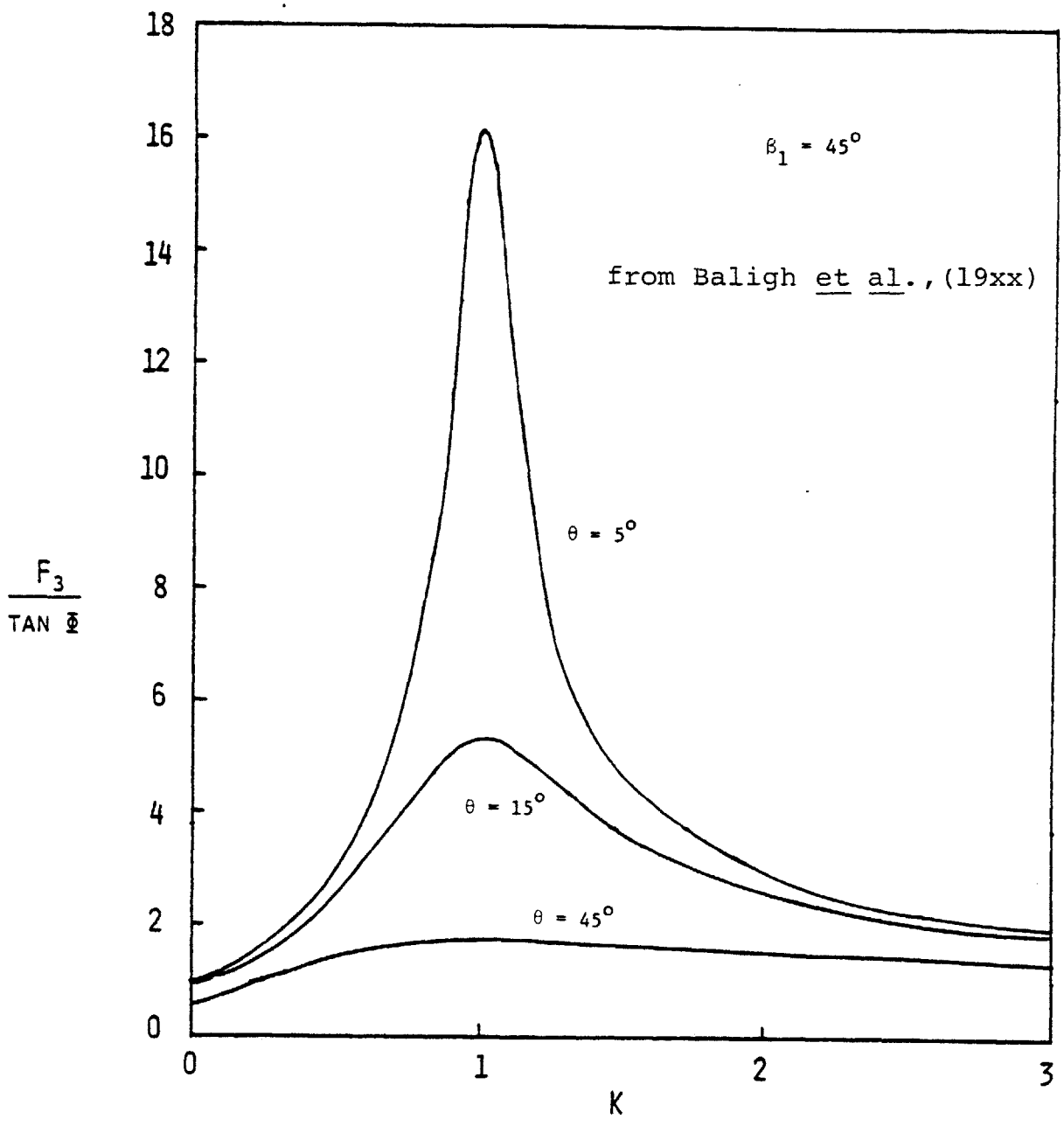


Figure 4.29 $F_3/\tan \Phi$ vs. K; $\beta_1 = 45^\circ$

developed for β's of 30° and 45° respectively.

All of the curves have a maximum at K = 1.0. When K = 1.0 , T is zero and the impending motion is down the line of intersection. Also,

$$\frac{F_3}{\tan \Phi} = \frac{1}{\tan \theta \cos \beta} \qquad (4.52)$$

The plots are helpful in visualizing how rock wedges fail. Figure 4.30 is a schematic representation of an F curve similar to the ones in Figures 4.28 and 4.29. (The term tan Φ will be considered a constant. It can be factored into the ordinate scale.) At some arbitrary time (say t_p in Figure 4.24) the stresses on the joint planes correspond to point A. If some additional loads are placed on the wedge the stresses on the joint planes will change - the incremental stresses will depend on the stiffnesses as discussed in Section 4.3. Let the new stress state correspond to point B. At B the wedge is at limiting equilibrium; any additional loads will initiate movement. The direction of movement will be along the line of intersection and into the notch. However, as soon as the wedge begins to settle into its notch the lateral stresses will increase. This "wedging" action increases K which in turn increases N and decreases T. According to this conceptual model the wedge can sustain additional loadings and pass from point B to C

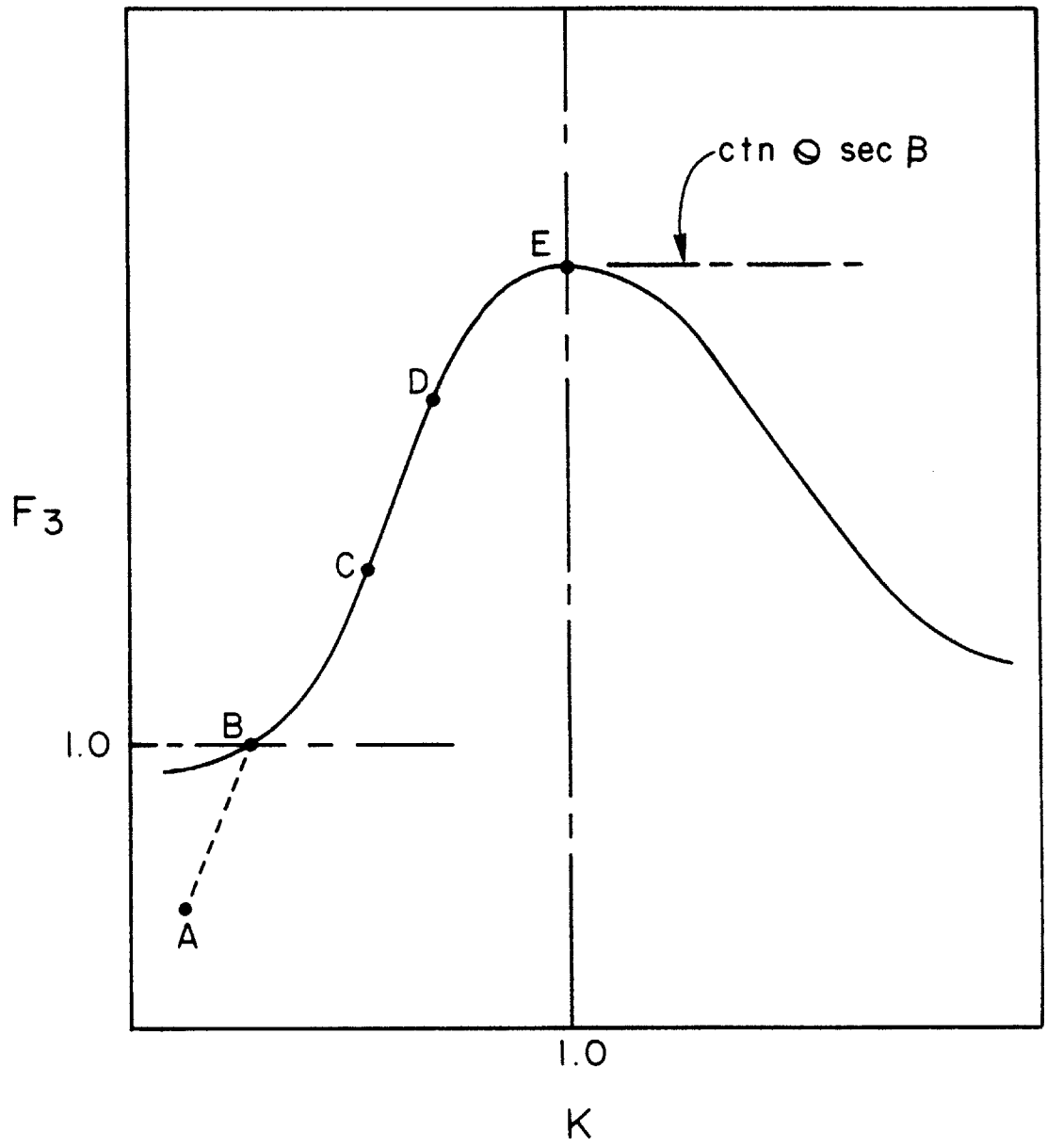


Figure 4.30 Schematic Failure Sequence

and D.* With each incremental load the wedge settles a little more and K increases. Finally, the wedge reaches point E where T is zero. At E i.e., K = 1.0 the direction of movement is exclusively along the line of intersection. Theoretically, there is nothing to prevent the wedge from sliding along the line of intersection out of its notch.

A similar effect occurs when K > 1.0 i.e., to the right of the maximum. This situation would correspond to passive conditions and the initial movement would be out of the notch and along the line of intersection. K would eventually reach 1.0 as the load increased.

Thus, it is possible to define two factors of safety: FS_I, the initial factor of safety, corresponds to point B; FS_U, the ultimate factor of safety, corresponds to point E. FS_U is the value used in traditional stability calculations. The engineer must use his judgement in deciding whether to use FS_I or FS_U as a design criterion in a particular situation. If movements must be kept to an absolute minimum FS_I would be the more appropriate choice. If he merely wanted to design against catastrophic movements, FS_U would be the better choice.

* In the model tan Φ is assumed to be constant even after initial failure (point B).

The distinction between the two factors of safety is clear in Figure 4.30. However, that figure is a highly idealized model that ignores some important effects. In particular, it assumes that the shear strength of the joint planes consists solely of frictional resistance that can be expressed with a constant Φ. In reality Φ may be composed of interlocking asperities as well as mineral to mineral friction. The asperities may shear off as movement occurs. In effect, Φ should decrease as the wedge is loaded beyond initial failure. Also, the model neglects cohesion along the joints. This cohesion may consist of bridges of intact rock which will shear when movement commences. One simplistic method for treating these effects is to neglect the cohesion and asperities in computing FS_U. (This is the approach that is used in the computer program that will be described in Section 4.5.)

All the wedges considered in the discussion of factor(s) of safety have been symmetric with respect to their geometric, stiffness, and frictional characteristics. Under these conditions i.e.

$$\beta_1 = \beta_2 \tag{4.53}$$

$$\Phi_1 = \Phi_2 \tag{4.54}$$

$$k_{1s} = k_{2s} \tag{4.55}$$

$$k_{1n} = k_{2n} \tag{4.56}$$

the impending motion is in the vertical plane that contains the line of intersection. The resultant of all the driving forces and the resultant of the shear resistances are in that same plane; in fact, the two resultants are colinear. Thus, the FS used by St. John and Baligh et al. is a direct comparison of colinear forces. Given the assumption of rigid body movement FS is rigorously defined from the standpoint of mechanics or statics.

FS is much more difficult to define if the wedge is asymmetric i.e., any of the conditions prescribed by Equations (4.53) to 4.56) are violated. Most asymmetric conditions will introduce a displacement component out of the vertical plane that contains the line of intersection. (The situation is shown in Figure 4.11. α is generally non zero for asymmetric conditions.) Also, the resultant of all the driving forces will no longer be colinear with the resultant of all the resisting forces.

The forces (and resultants) are shown in Figure 4.31 which is a plan view of the notch. Each joint plane has a shear component in the direction parallel to the line of intersection. The two components add up to W sin θ. The components in planes 1 and 2 are $n \cdot W \sin \theta$ and $(1 - n) \cdot \sin \theta$ respectively. Each plane also has a shear force perpendicular to the line of intersection (T_1 and T_2).

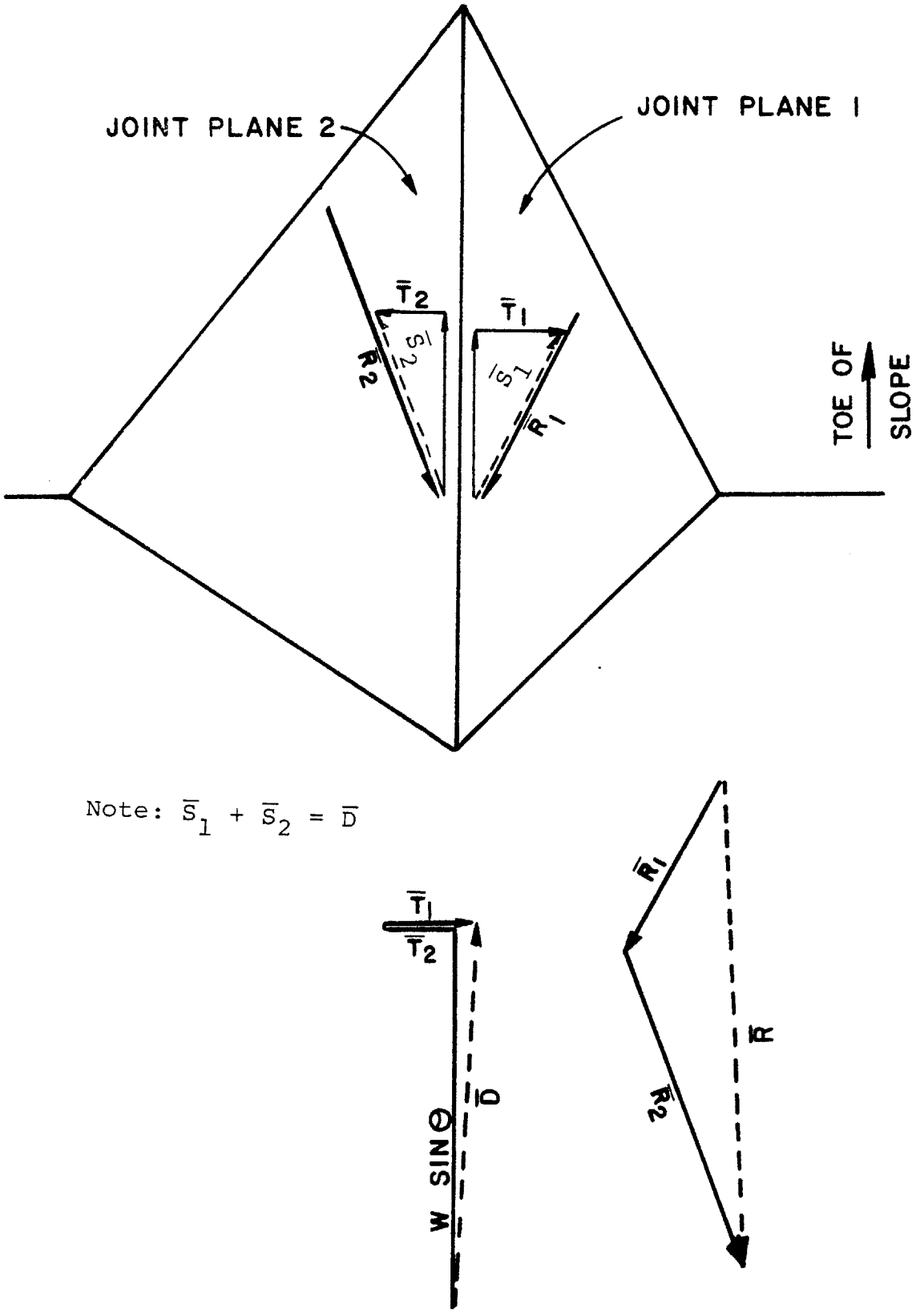


Figure 4.31 General Case--$\bar{D}$ and $\bar{R}$ Not Parallel

The resultants of (T_1 + n w sin θ = $\bar{S}_1$) and (T_2 + (1-n) W sin θ = $\bar{S}_2$) are shown in Figure 4.31. The shear resistance in each plane is in the direction of the respective resultant. Thus, $\bar{R}_1$ is in the direction of $\bar{S}_1$ and $\bar{R}_2$ is in the direction of $\bar{S}_2$. Both $\bar{R}_1$ and $\bar{R}_2$ represent the maximum possible shear resistance that can be mobilized; therefore, their magnitudes (but not directions) may differ from $\bar{S}_1$ and $\bar{S}_2$. Figure 4.31 indicates that the sum of the resistances ($\bar{R}_1$ + $\bar{R}_2$) is <u>not</u> in the same direction as the sum of the driving forces ($\bar{S}_1$ + $\bar{S}_2$). ($\bar{R}_1$ + $\bar{R}_2$) cannot be compared directly with ($\bar{S}_1$ + $\bar{S}_2$) because of the disparity in directions. Thus, FS must be defined in terms of some arbitrary criterion; there is no unequivocal definition.

The computer program that will be described in Section 4.5 uses the following definition:

$$FS = \frac{\bar{R} \cdot \bar{\Delta}}{\bar{D} \cdot \bar{\Delta}} \qquad (4.57)$$

$\bar{R}$ is the resultant vector of all resisting forces

$\bar{D}$ is the resultant vector of all driving forces

$\bar{\Delta}$ is the direction of impending motion

FS is considered the ratio of the components of $\bar{R}$ and $\bar{D}$ in the direction of $\bar{\Delta}$.

The definition of $\bar{\Delta}$ is somewhat ambiguous because the "direction of impending motion" is considered the direction of displacement due to an imposed load. Thus, $\bar{\Delta}$ will not

correspond to failure conditions unless the load creates a condition where FS = 1.0. Different loads will produce different Δ's. The program offers the user two options:

1. $\overline{\Delta}$ is displacement that would occur under a purely vertical load i.e., in what direction would the wedge move if its unit weight suddenly increased by a small percentage? $\overline{\Delta}$ is based on the current stiffness.

2. $\overline{\Delta}$ is the displacement that occurs under all the "current loads." Current loads include all loads except the weight of block; they are the loads that are analyzed using the stiffness approach.

In the special case where there is total symmetry Equation (4.57) will yield the same FS as Equation (4.51).

4.5 Computer Program SWARS-2PM

The stiffness and stress approaches discussed in Sections 4.1 to 4.4 and in Appendix D have been incorporated into a computer program, SWARS-2PM. SWARS-2PM is actually a modified version of SWARS-2P which was originally developed by Campbell (1974) using the conventional assumption regarding the absence of T_1 and T_2 forces on the joint planes. SWARS-2PM retains all the versatility of its predecessor with respect to

loading conditions and ground water options; however, it determines the reactions on the joint planes according to the method prescribed in Section 4.3. If the failure mode involves sliding on both planes*, the program computes the two factors of safety defined in Section 4.4: FS_I and FS_U.

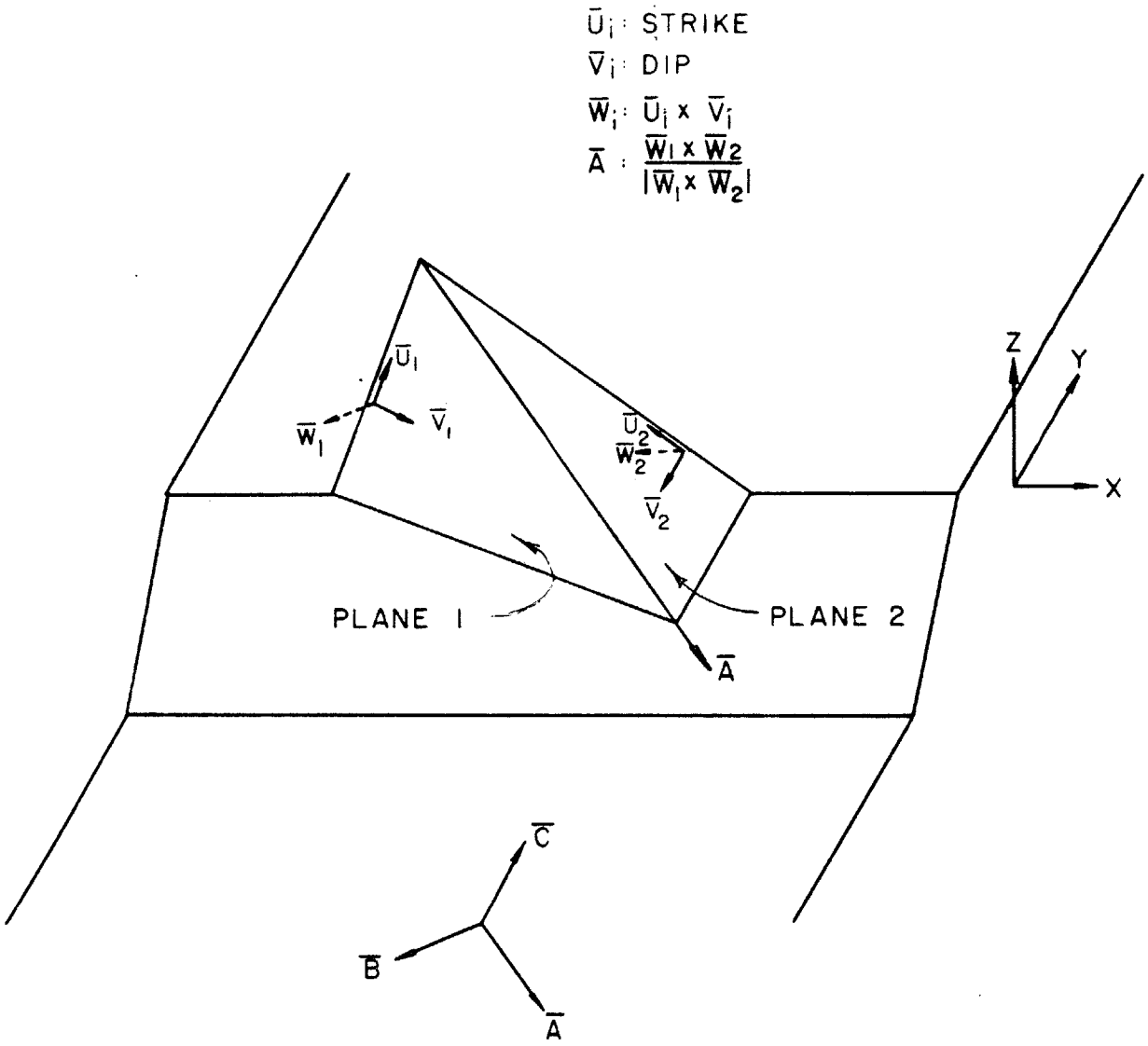
The stress approach is used to calculate the reactions on the joint planes due to the weight of the wedge. The generalized stiffness approach is used to calculate the reactions due to all other loads. In implementing the generalized stiffness approach, SWARS-2PM utilizes the relations developed in Appendix D with two modifications:

1. A_1, the area of joint plane 1, replaces the term h csc β_1 in Equation (D.3).
2. A_2, the area of joint plane 2, replaces the term h csc β_2 in Equation (D.6).

The program resolves the resultant of all "stiffness" loads into components along the A,B and C axes shown in Figure 4.32. The B and C components (Figure 4.33) correspond to the respective Y and X forces in Equations (D.10) and (D.9). The A component that lies along the line of intersection is part of $\bar{D}$, the driving vector, that appears in the denominator of the FS equation:

$$FS = \frac{\bar{R} \cdot \bar{\Delta}}{\bar{D} \cdot \bar{\Delta}} \qquad (4.57)$$

* If the failure mode involves sliding on a single plane, SWARS-2PM will perform exactly the same analysis as SWARS-2P.



$\bar{A}, \bar{B}, \bar{C}$: UNIT VECTORS FORMING SETS OF ORTHAGONAL AXES

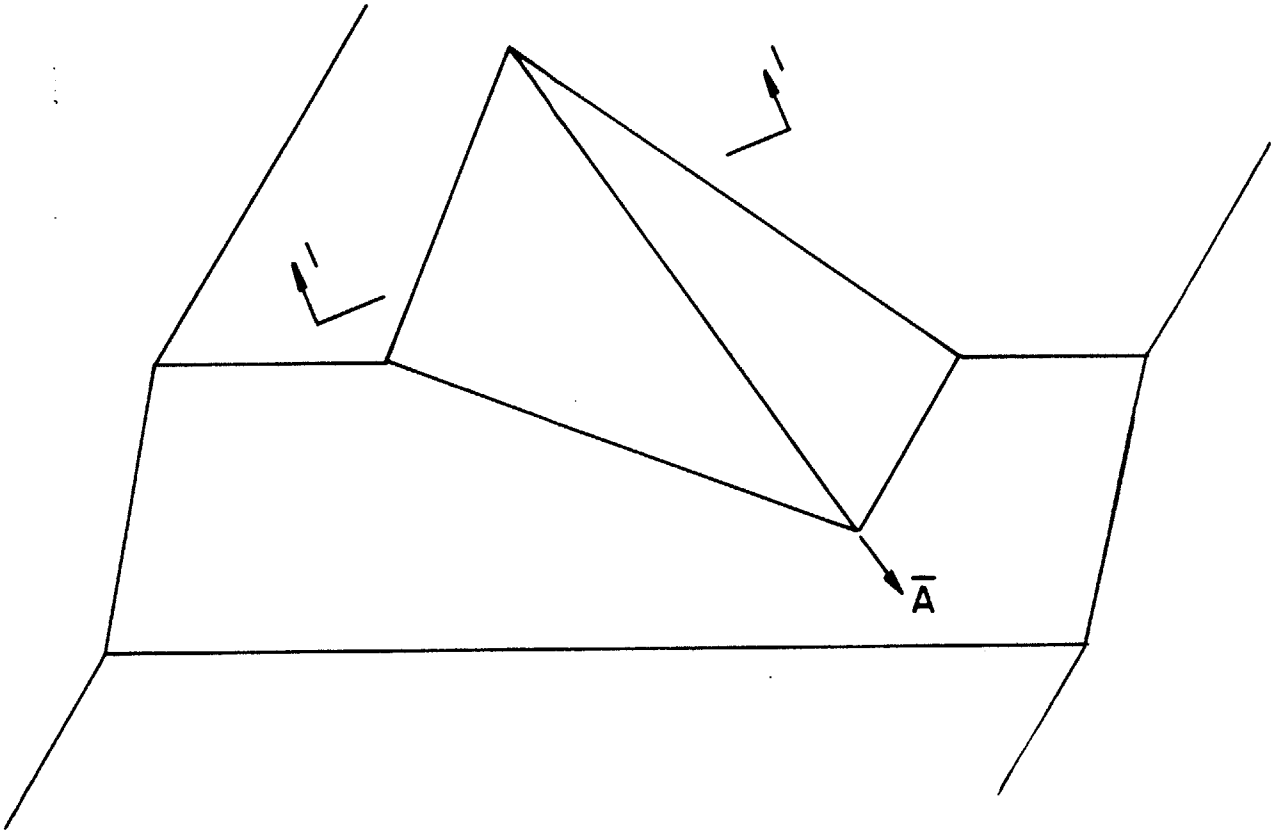
$\bar{A}$: LINE OF INTERSECTION

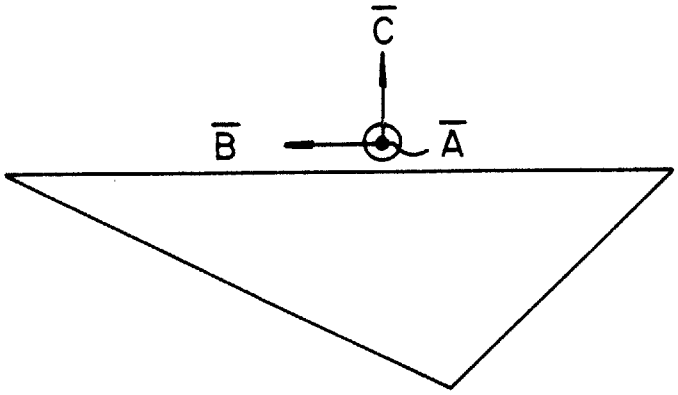
$\bar{B}$: $\bar{B} \perp \bar{A}$; $\bar{B} \perp$ Z AXIS

$$\bar{B} = \frac{\bar{A} \times (0,0,1)}{|\bar{A} \times (0,0,1)|}$$

$$\bar{C} = \bar{B} \times \bar{A}$$

Figure 4.32 A, B, C Coordinate System





SECTION I-I $\perp$ $\bar{A}$

Figure 4.33 Cross Section Perpendicular to $\bar{A}$

$\overline{\Delta}$, the direction of impending motion, is composed of components along the A, B and C axes. Δ_B and Δ_C are related to δ_t and δ, the magnitude and direction of the displacement, in the generalized stiffness approach:

$$\Delta_B = \delta_t \sin \alpha \tag{4.58}$$

$$\Delta_C = -\delta_t \cos \alpha \tag{4.59}$$

In computing Δ_A, the program assumes that the shear stiffness in the direction paralled to the line of intersection is identical to the shear stiffness in the direction perpendicular to the line of intersection.* Thus,

$$\Delta_A = \frac{F_A}{A_1 K_{1s} + A_2 K_{2s}} \tag{4.60}$$

where F_A is the load component in the A direction

If the user selects the option wherein Δ is considered the displacement that would occur in response to a purely vertical load, the program computes the Δ_A, Δ_B and Δ_C that would occur under an imaginary vertical load of unity. (The imaginary load is not considered in the $\overline{R}$ or $\overline{D}$ computations.)

Appendix E contains a complete listing of SWARS-2PM as well as a user's manual.

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* The program can easily be modified to accept two shear stiffness for each joint plane but such a refinement is probably unwarranted given the state of the art concerning the measurement of stiffness values.

4.6 Computer Program SWARS-2MC

SWARS-2MC is a modified version of SWARS-2PM that enables the user to determine the probability of failure (P_f) through Monte Carlo simulation. The program will compute the factor of safety for many sets of input parameters -- P_f is the proportion of FS values that fall below 1.0. Any (or all) of the parameters that define the shape and size of the wedge or characterize the shear resistance along the joint planes can be considered random variables. The values for these random variables should be selected in accordance with their respective probability density functions.*

SWARS-2PM and SWARS-2MC use precisely the same algorithms for computing the factor(s) of safety. There are, however, some differences in the manner in which information is input to the two programs. In particular, SWARS-2MC is designed so that its input parameters are compatible with the output from DAYLITE i.e., wedges are defined in terms of θ_o, ϕ_o, ψ and β_1 (Section 3.2.6). Thus, the conditional probabilities of ψ and β_1 calculated in DAYLITE can serve as a basis for Monte Carlo simulation.

Appendix F describes the features of SWARS-2MC and summarizes the differences between the SWARS-2MC and SWARS-2PM. Appendix F also contains a users' manual and program listing for SWARS-2MC.

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* Section 2.2 reviews the principles of Monte Carlo simulation.

CHAPTER 5

INTERACTION BETWEEN DISCONTINUITIES AND INTACT ROCK-- COMPUTER PROGRAM JOINTSIM

5.0 Introduction

A typical rock mass is characterized by several sets of discontinuities. The spacings between discontinuities within each set vary; each discontinuity is a planar feature that consists of intact rock bridges as well as truly discontinuous zones. Traditional slope stability analyses are performed by examining the stability of bodies that are bounded by planes that are oriented in the mean direction of each set of discontinuities (Figure 5.1). Persistence* and (to a lesser extent) spacing are considered by assigning different cohesion and friction parameters to the intact and discontinuous portions of the discontinuity plane as proposed by Jennings (1970). The real failure surface may actually have the shape shown in Figure 5.1 or an "en echelon" shape as shown in Figure 5.3. The failure surface forms along the path of minimum resistance. Clearly the interaction between discontinuities and intact rock plays a major role. The "intact portions" of the failure surface usually control its shape and location because of the high resistance of intact rock. Traditional slope stability analyses recognize this fact and frequently assign persistences of 100% to the failure planes to be conservative. Such an approach is

* Persistence of a joint plane is defined in Figure 5.2.

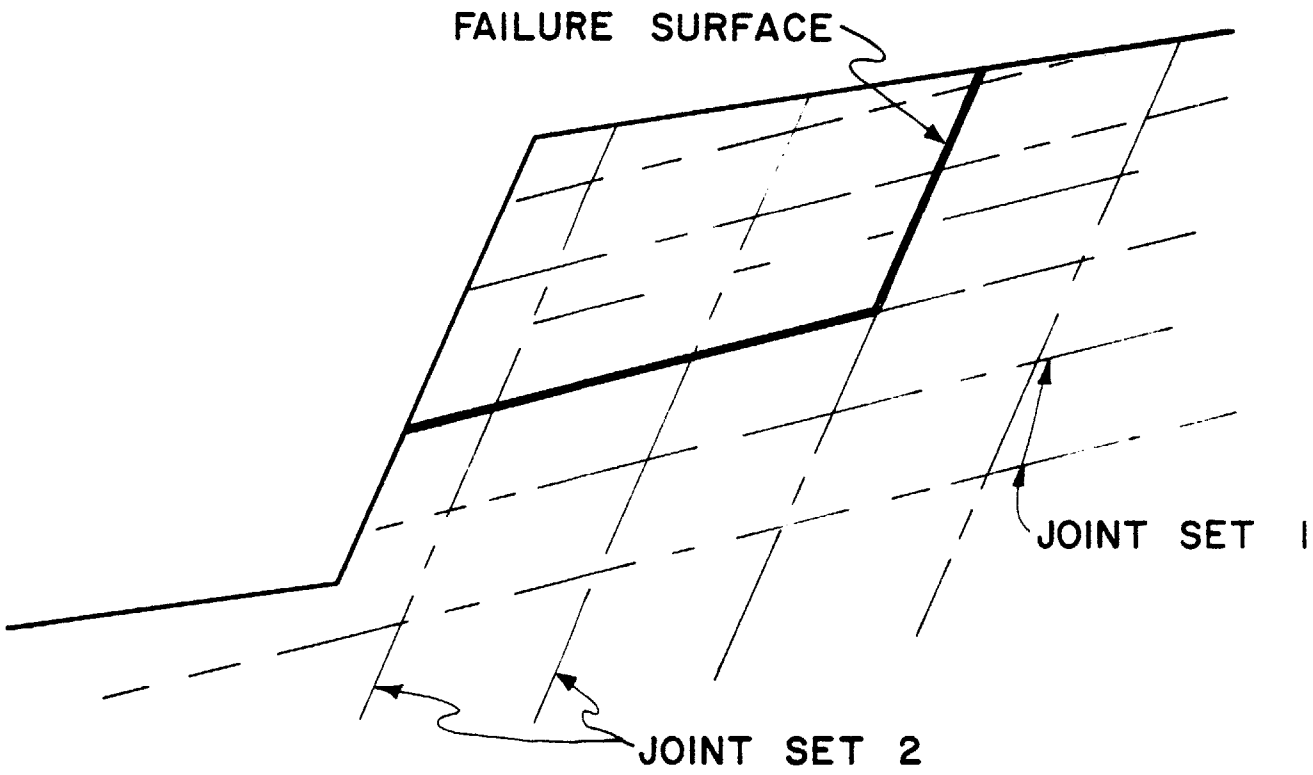
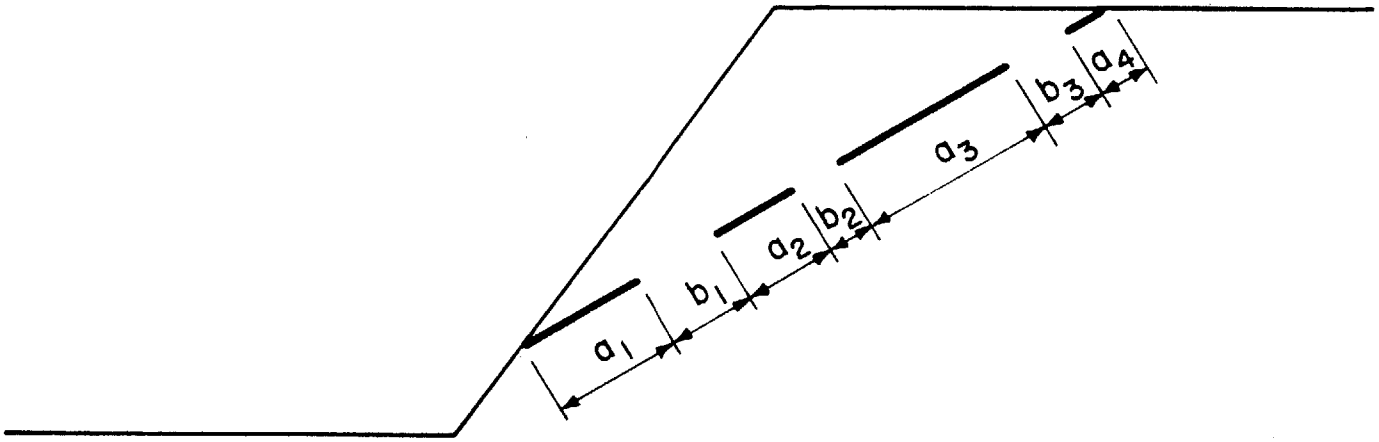


Figure 5.1 Typical Failure Surface



$$k = \text{PERSISTENCE} = \frac{\Sigma a_i}{\Sigma a_i + \Sigma b_i}$$

$$c_a = (1 - k)c_r + (k)c_j$$

$$\tan \phi_a = (1 - k)\tan \phi_r + (k)\tan \phi_j$$

Figure 5.2 Persistence--Jennings' Relations

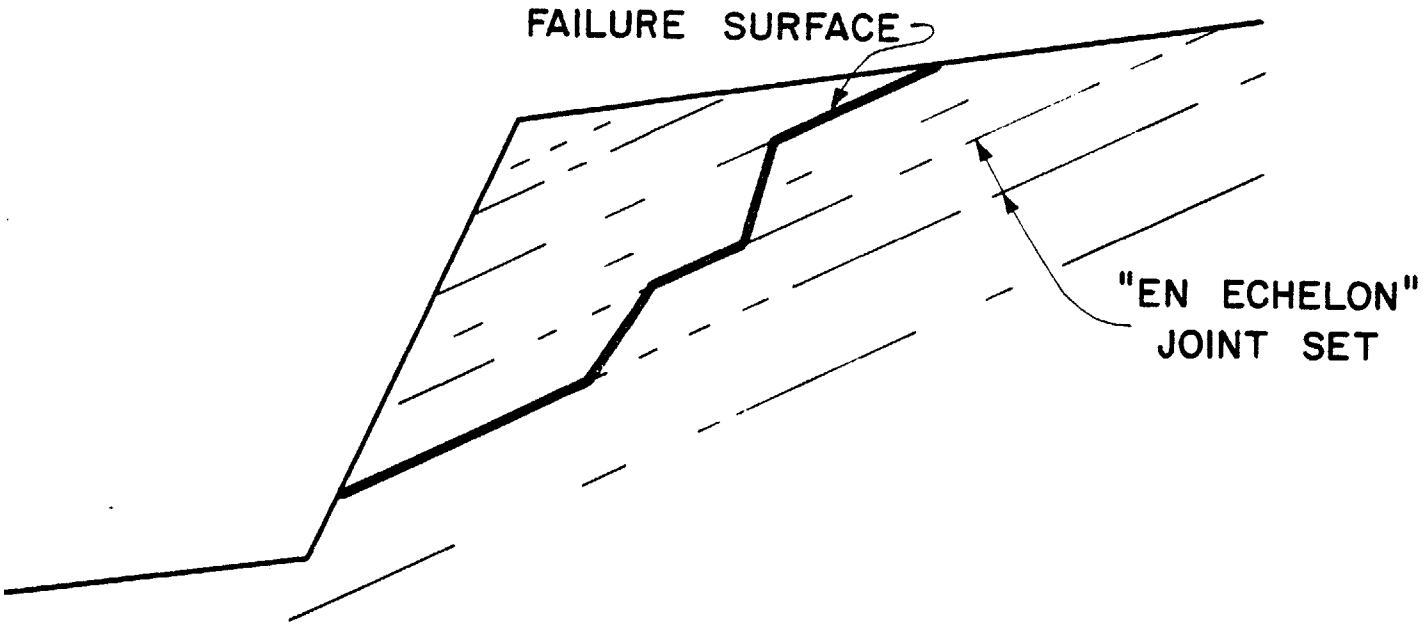


Figure 5.3 "En Echelon" Failure Surface

overly conservative and frequently uneconomical. However, it is the only rational procedure because information on persistence is seldom available. Even if information on persistence were available it would have only limited value because the persistence of the various possible "en echelon" failure surfaces would remain unknown. The inability of deterministic approaches to treat the interaction between intact rock and discontinuities is a major weakness in traditional rock slope stability analysis. This interaction must be rationally incorporated into any model that attempts to simulate the real behavior of rock slopes. The probabilistic approach that will be presented in this chapter can treat the discontinuity-intact rock interaction in a satisfactory manner.

5.1 General Features--Probabilistic Approach to Resistance of Jointed Rock

The rock mass under consideration is analyzed by first simulating joint geometries from a stochastic model. The geometries represent a distribution of joint patterns that might occur in a particular rock mass. Each geometric realization has many possible failure surfaces. The resistances of the various failure surfaces to a standard loading sequence are computed and the minimum resistance is determined. The minimum resistance characterizes that particular realization of joints. By repeating this operation for many simulated joint networks one can estimate the probability of

distribution of minimum resistance and hence, the probability of failure.

Specifically, each simulation generates a single set of parallel discontinuities with intact and discontinuous portions (Figure 5.4). The spacing and the persistence of the discontinuities are random variables. The jointing patterns are simulated through the stochastic model described in Section 5.2. The distribution of minimum resistance is found by analyzing each pattern with the mechanical model described in Section 5.3. These resistance distributions can then be expressed as disrtibutions of strength parameters e.g. cohesion and friction in a Mohr-Coulomb expression.

5.2 Stochastic Model of Joint Geometry

The network of discontinuities is idealized as a Poisson process of planes in space. The abstract mathematical process was conceived and refined by a number of individuals over the past fifteen years (Matheron, 1975; Miles, 1964, 1969, 1971, 1972; Switzer, 1965). Veneziano (19xx) adapted it to simulate rock jointing and used it to derive other properites of geotechnical significance.

Discontinuities and particularly joints within a rock mass are generated through a three step sequence of random operations. The primary process generates Poisson planes which are taken to represent joint planes (Figure 5.5). The

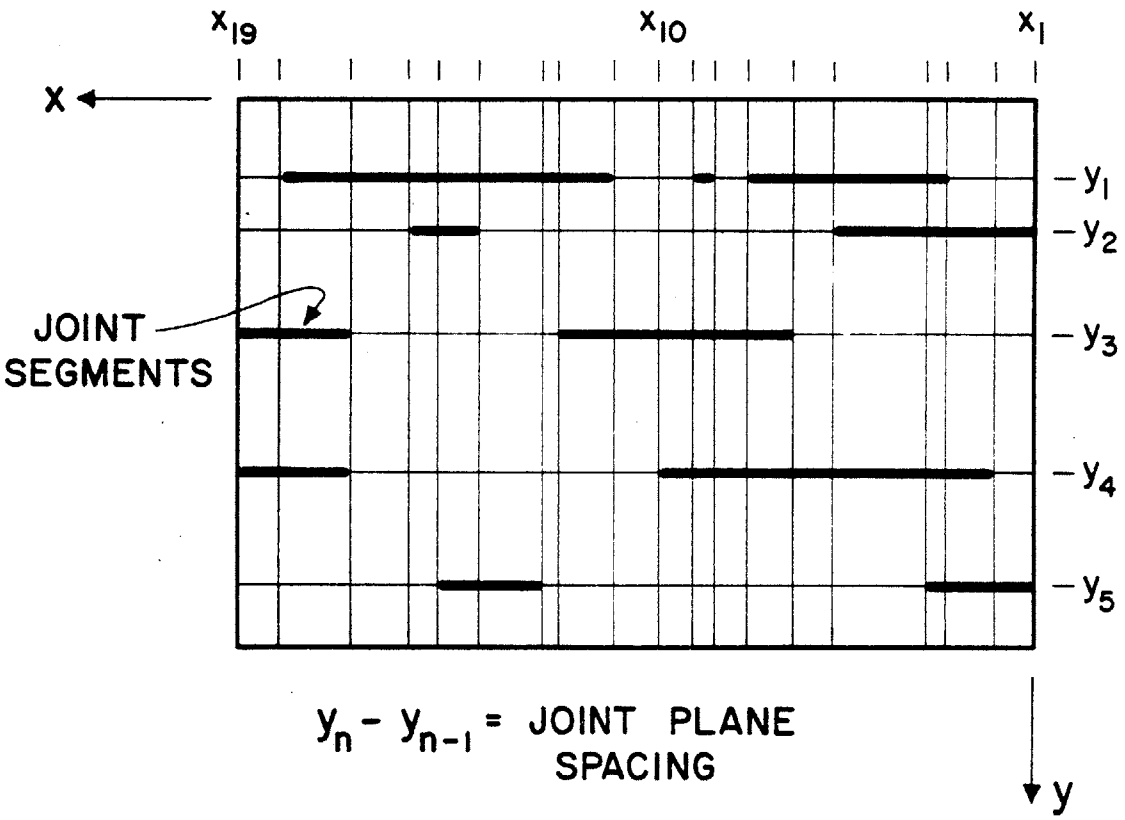


Figure 5.4 Coordinate System and Typical Jointing Pattern

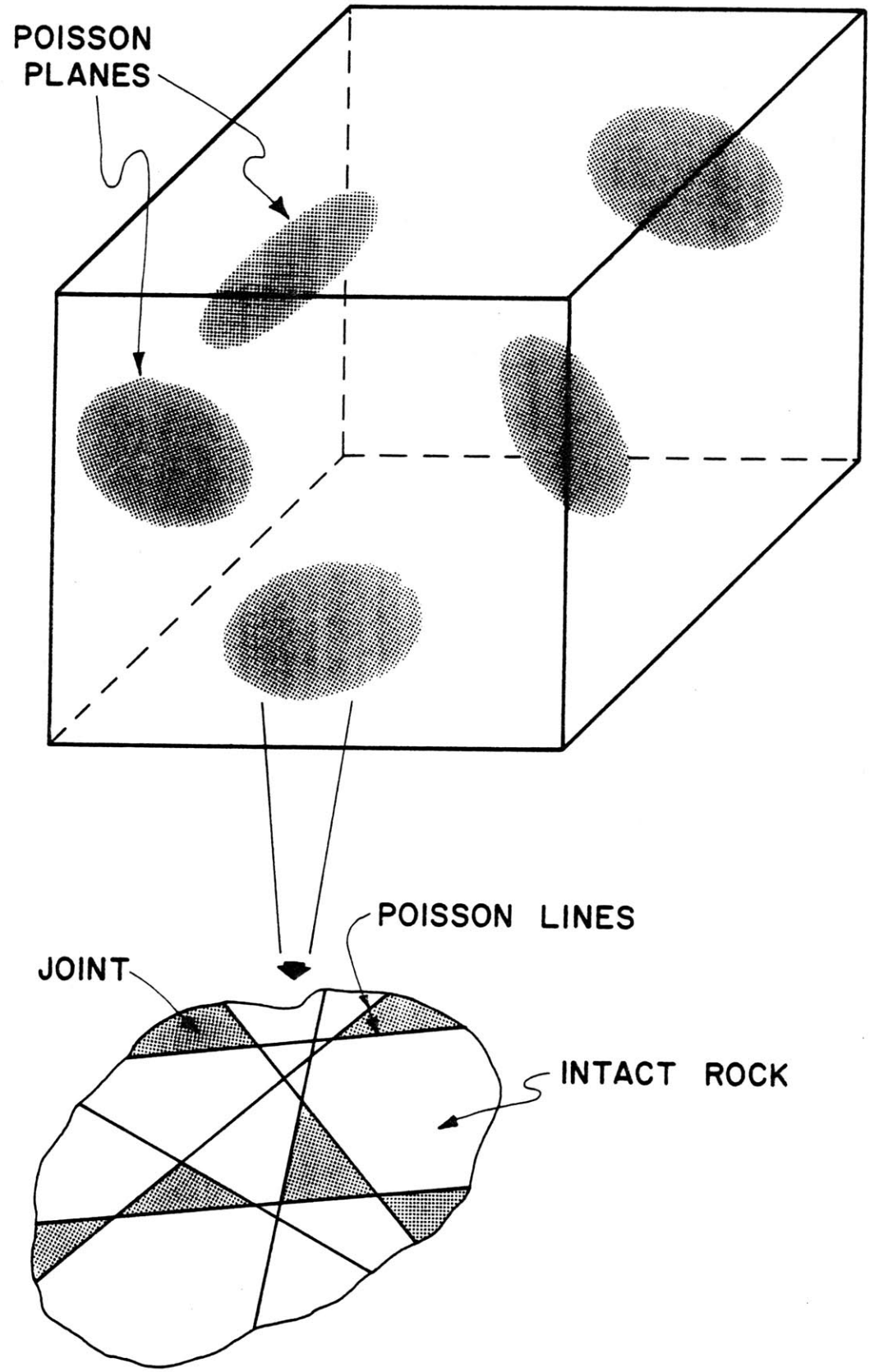


Figure 5.5 Stochastic Model of Joint Geometry

secondary process partitions each plane into random polygons by a family of Poisson lines. The tertiary process shades (or colors) a certain proportion of these figures. Shaded polygons represent joints while the remaining portions of the plane represent intact rock.

Primary Process

The stochastic model treats joint planes as Poisson flats in R^3, the three dimensional space that corresponds to the rock mass. Let X_1, X_2 and X_3 be orthogonal axes in R^3 that meet at point 0. Any arbitrary plane in R^3 can be expressed as an equation of the form:

$$X_1 \sin \psi_i \cos \Omega_i + X_2 \sin \psi_i \sin \Omega_i + X_3 \cos \psi_i = d_i \quad (5.1)$$

As shown in Figure 5.6, d_i represents the signed distance from the plane to the origin while ψ_i and Ω_i are orientation angles.

$$-\infty < d_i < \infty$$

$$0 \leq \psi_i \leq \pi$$

$$0 \leq \Omega_i \leq \pi$$

Each plane in R^3 is characterized by a unique set of parameters (d_i, ψ_i, Ω_i).

If the d_i's are marked on the real line (the R^1 space) the resulting point process (Figure 5.7) is Poisson with constant intensity $1/\rho_3$ where ρ_3 is the mean distance between neighboring points.

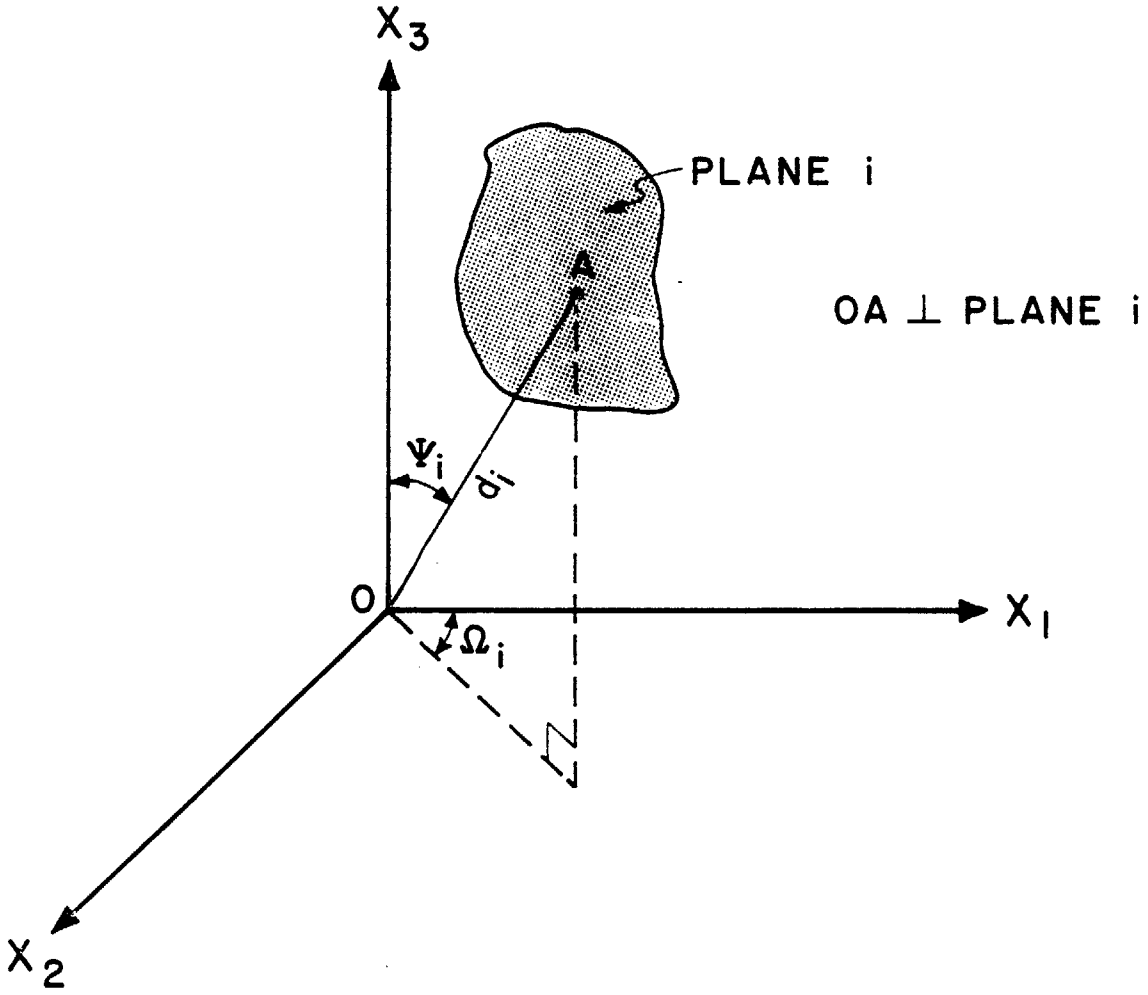


Figure 5.6 Definitions of ψ_i, Ω_i, and d_i

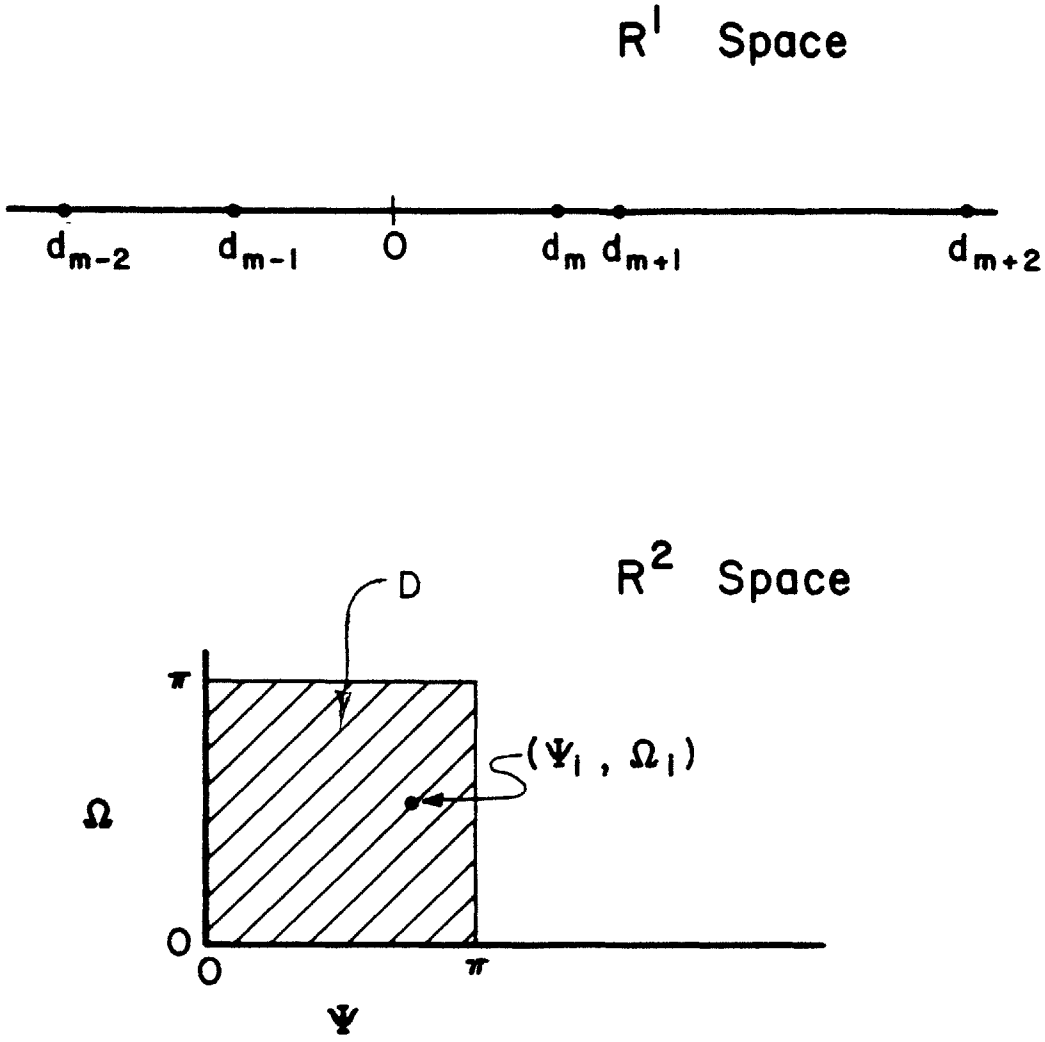


Figure 5.7 Stochastic Model--Primary Process--R^1 and R^2 Spaces

In a similar manner, the pair of orientation angles can be mapped in D, a square region of the real plane (the R^2 space), as shown in Figure 5.7. Each point in the region represents a unique orientation.

The plane region D can be combined with the real line R^1 to form P^3, a cylinder in R^3 (Figure 5.8). Miles (1972) showed that homogeneous Poisson networks in R^3 can be generated from Poisson point processes in P^3. Homogeneity implies that the process is invariant with respect to translation of the origin.

Secondary Process

The Poisson process of lines on each joint plane is similar to the Poisson process of planes in R^3. Figure 5.9 shows a typical joint plane that has been partitioned by a family of Poisson lines. Let Y_1 and Y_2 be orthoganal axes that meet at 0'. Any line in the plane can be expressed as a linear equation:

$$Y_1 \cos \Gamma_i + Y_2 \sin \Gamma_i = f_i \qquad (5.2)$$

The distance f_i and the angle Γ_i are shown in Figure 5.9.

$$-\infty < f_i < \infty$$

$$0 \leq \Gamma_i \leq \pi$$

Each line is identified by a unique pair of parameters (f_i, Γ_i). The ordered sequence

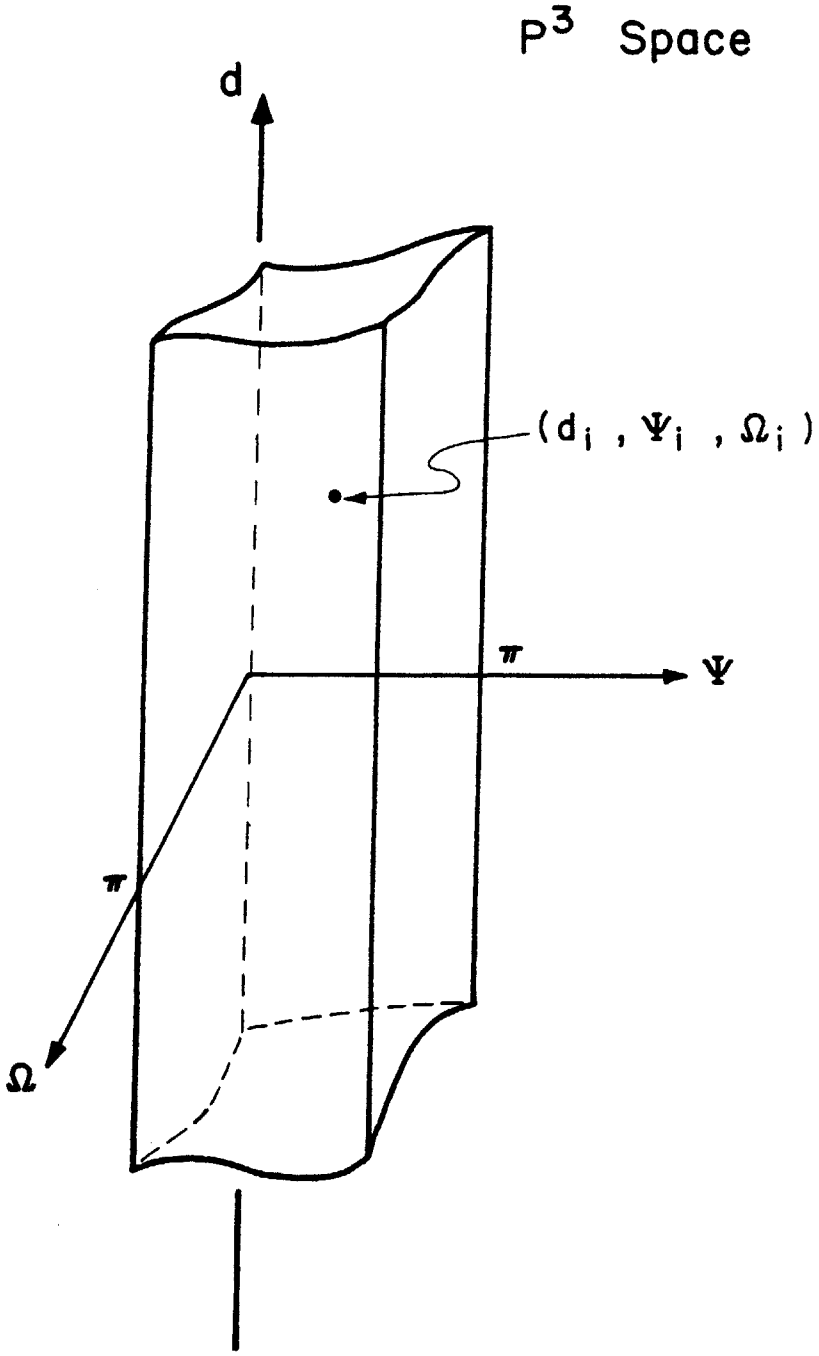


Figure 5.8 Stochastic Model--Primary Process--P^3 Space

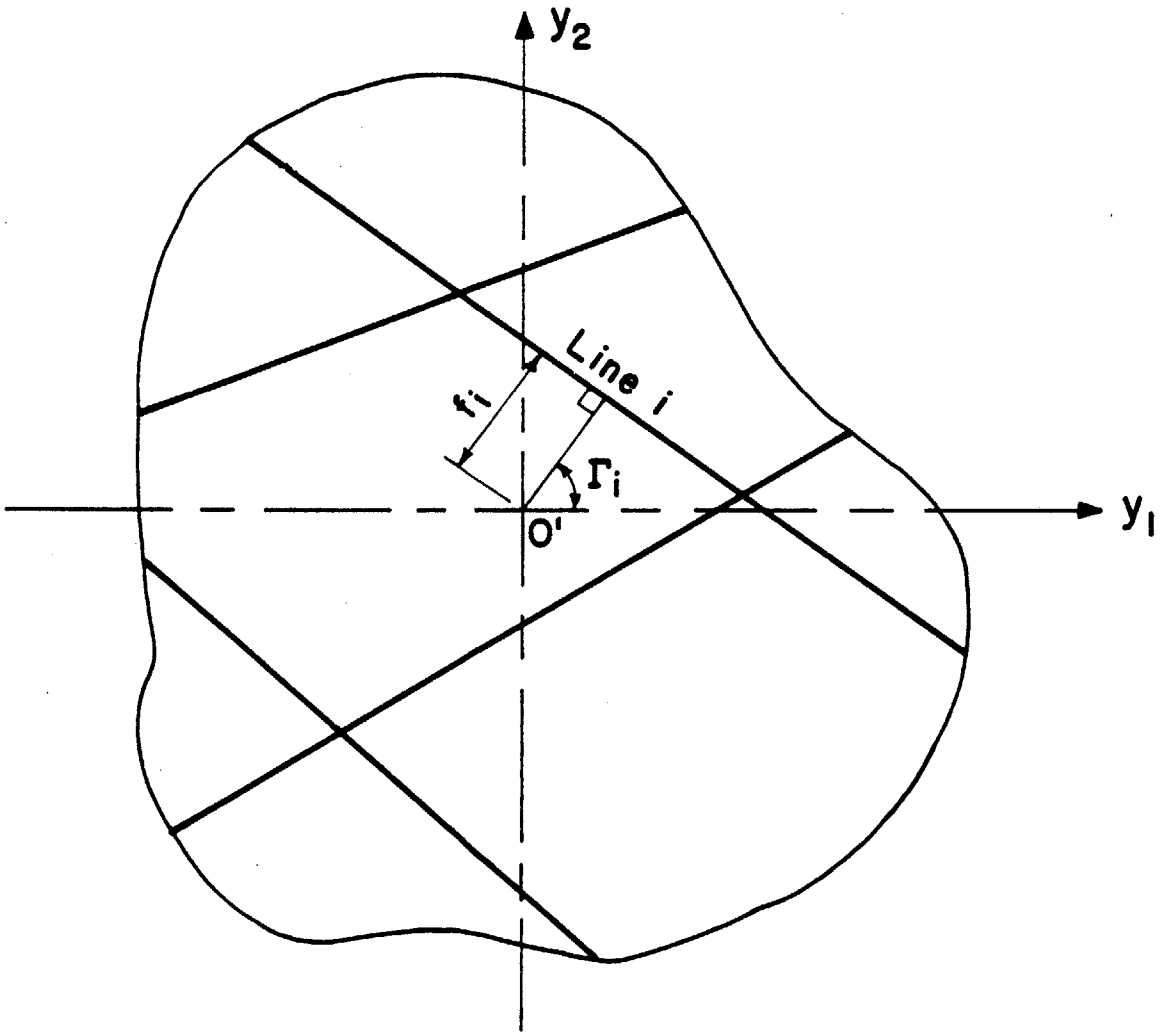


Figure 5.9 Definitions of f_i and Γ_i

$$\cdots \quad f_{m-2} < f_{m-1} < f_m < f_{m+1} < f_{m+2} \quad \cdots$$

corresponds to a Poisson process of points along the R^1 space with intensity $1/\rho_2$ (Figure 5.10). The distances between neighboring points are exponentially distributed with mean value ρ_2. The points (f_i, Γ_i) constitute a Poisson process of points within the infinite rectangle shown in Figure 5.10. If Γ is uniformly distributed over the interval $0 \leq \Gamma \leq \pi$ the Poisson process has a constant density, $\frac{1}{\rho_2 \pi}$

Miles (1964) studied the associated isotropic process of Poisson lines on the plane and developed a number of useful relationships. In particular, he found that:

1. The expected number of intersections between Poisson lines per unit area is $\frac{1}{\pi(\rho_2)^2}$
2. The intersections between any arbitrary line and the Poisson lines constitute a Poisson process with constant intensity $\frac{2}{\rho_2 \pi}$,
3. The angles of intersection (Figure 5.11) are independently distributed with identical pdf's:

$$f_A(a) = \frac{1}{2} \sin a \quad 0 \leq a \leq \pi \qquad (5.3)$$

The process may not be isotropic. Anisotropy can be treated in a fairly straighforward manner. It involves cases where the pdf for Γ is non-uniform. Miles (1964) considered this case and developed some expressions similar to those for the isotropic case.

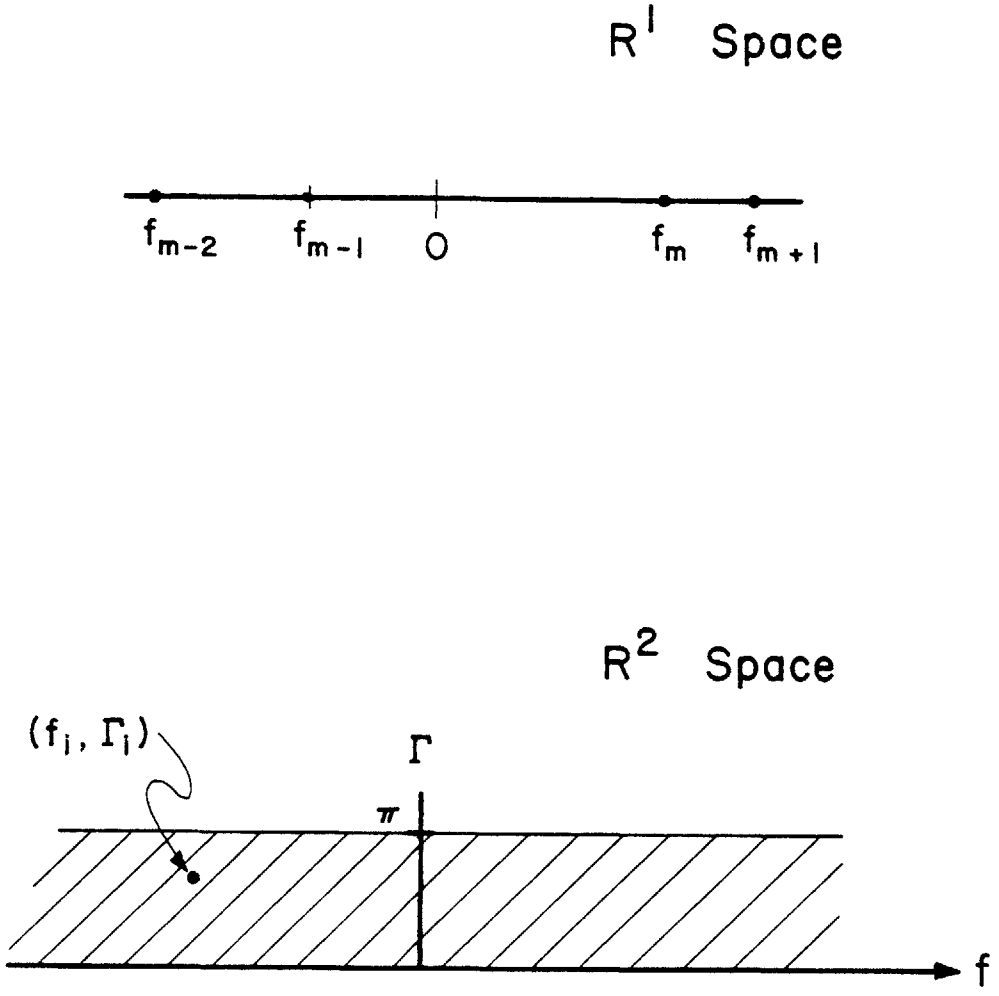


Figure 5.10 Stochastic Model--Secondary Process--R^1 and R^2 Spaces

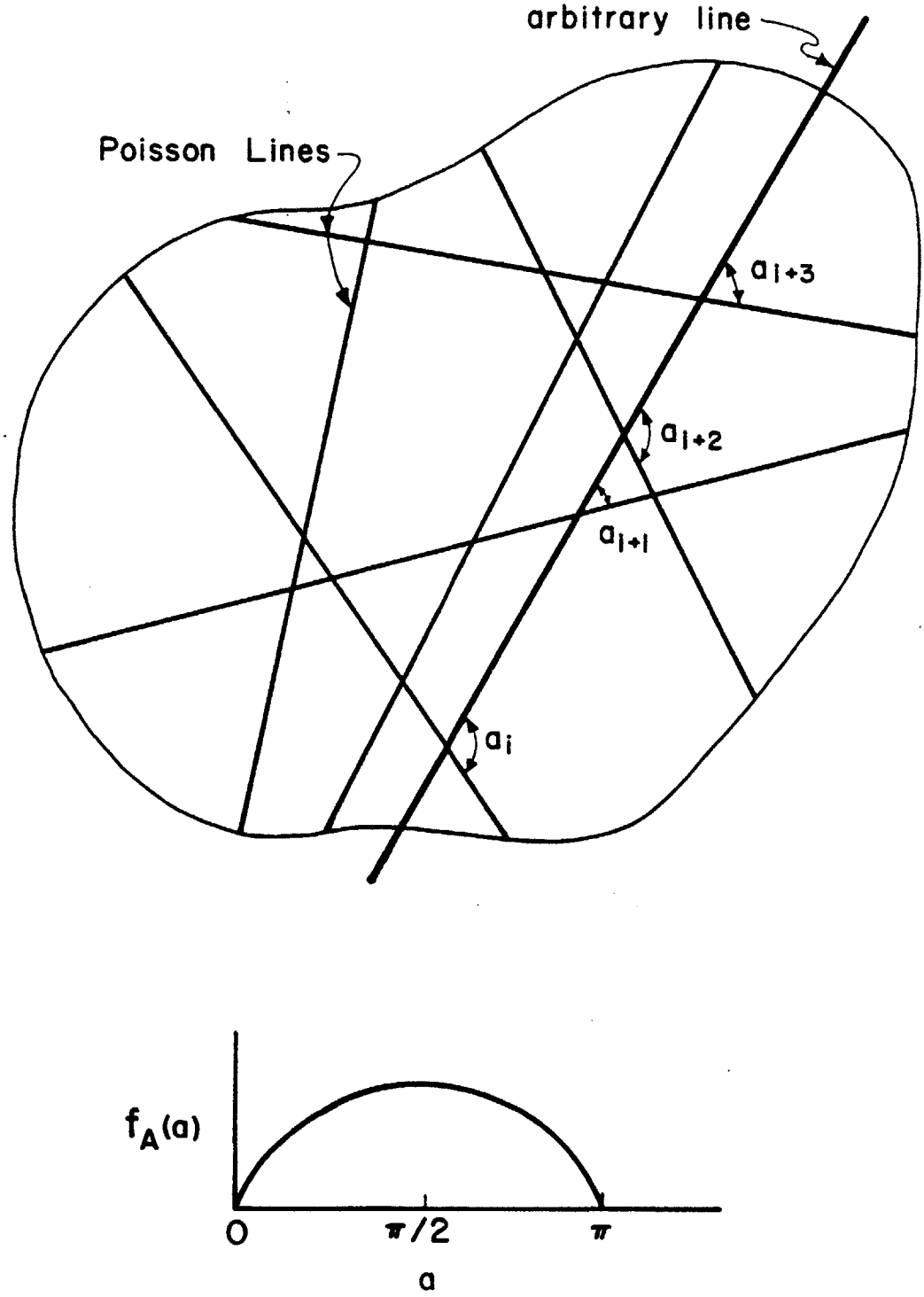


Figure 5.11 Angles of Intersection Between Poisson Lines and an Arbitary Line

Tertiary Process

The last operation involves the shading or "jointing" of individual polygons. The probability of shading has a constant value (p) and the shading operation is performed independently on each polygon i.e., the fact the one polygon is shaded does not affect the probability that an adjacent polygon will be shaded. Switzer (1965) has shown that the resulting binary random field has Markovian properties along straight lines.

Figure 5.12 shows a typical realization on a joint plane. The arbitrary line A-A' intersects numerous Poisson lines and traverses both shaded and unshaded regions. As mentioned earlier, the distance between consecutive intersections is exponentially distributed with mean value $\frac{\rho_2\pi}{2}$. However, only those intersections that involve a transition between jointed and unjointed polygons have practical significance. For example, the polygons "G" and "H" in Figure 5.12 are both shaded so that the aggregate joint length along A-A' extends from point m to point n. Thus, the shading probability can be considered a "random selection" parameter for the Poisson polygons so that only those intersections involving color transitions are observed. Therefore, the mean values for jointed and unjointed lengths become

$$\delta_j = \frac{\rho_2\pi}{2(1-p)} \qquad (5.4)$$

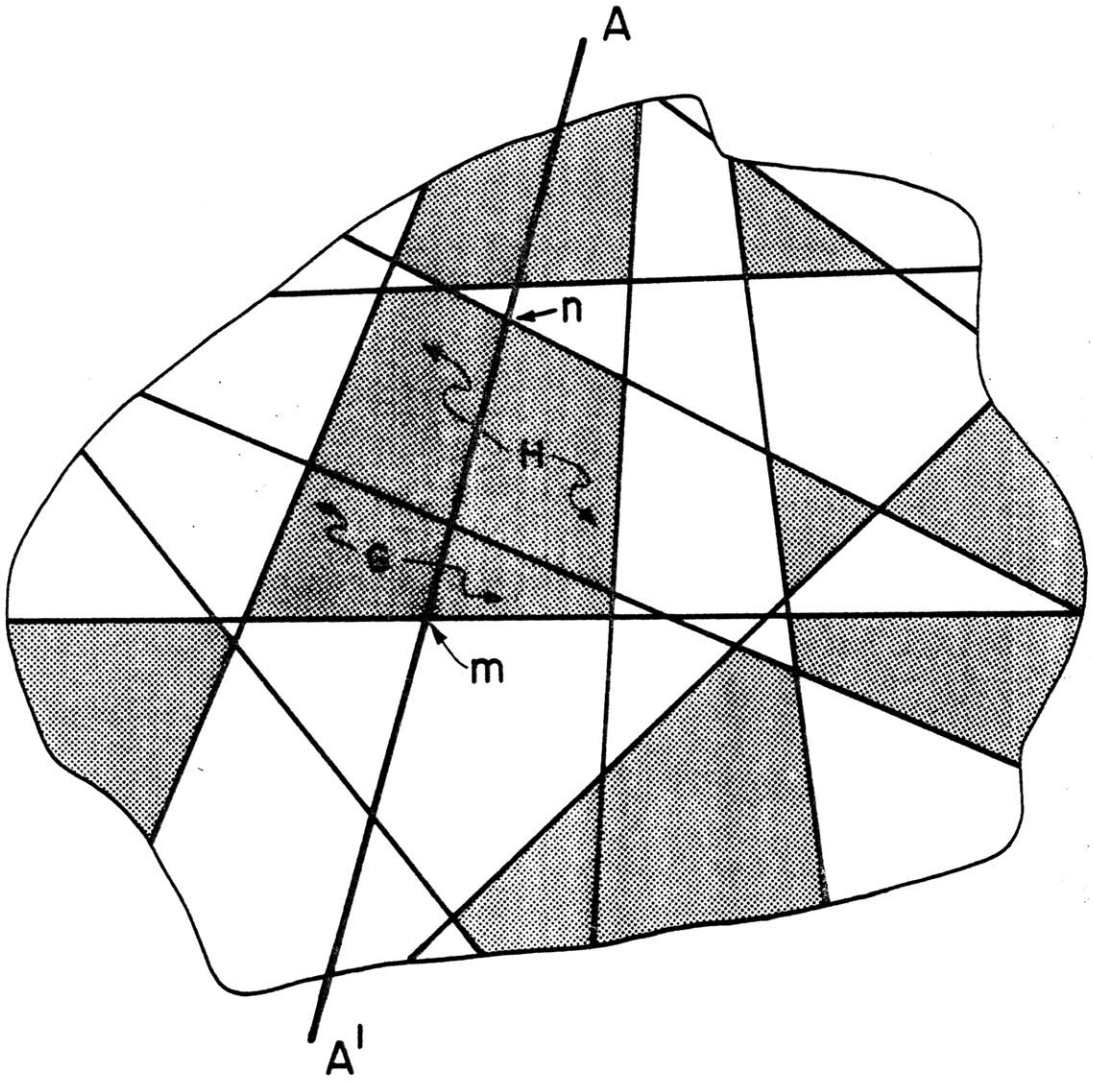


Figure 5.12 Length of Joint Segment (Secondary and Tertiary Processes)

and $\delta_r = \frac{\rho 2\pi}{2p}$ respectively, where p is (5.5)
the shading probability of each polygon.

The stochastic model for joint geometry consists of the three processes just described. Thus, in its most general form, the model has five sets of input parameters: two density parameters (one of Poisson planes in space, the other of Poisson lines in each joint plane), two probability distributions (one of the orientation of planes and the other of the orientation of lines), and the shading probability. The parameters need not be constant; for example, the density of lines and the shading probability may depend upon the orientation of the joint plane. The same parameters may be uncertain and probabilistically dependent.

"En Echelon" Joints

The number of variables can be reduced if one concentrates on the problem of "en echelon" jointing. Since joint planes are assumed to be parallel and to have a known orientation their orientation distribution becomes trivial and the density parameter of planes is simply $1/\lambda$ where λ is the mean spacing between planes. In addition, $1/\rho$, the density of lines in a plane, is assumed the same for all planes and the distribution of line orientations is considered uniform. Under these assumptions the model requires three scalar parameters: λ, ρ and p.

Figure 5.4 is a cross-section through a hypothetical rock mass whose jointing patterns can be generated by the model

just described. On a cross-section perpendicular to the joint planes (like Figure 5.4), the planes are exponentially distributed with mean value λ. The jointed and unjointed segments within each plane have identical independent exponential distributions with mean values $\delta_j = \frac{\rho\pi}{2(1-p)}$ and $\delta_r = \frac{\rho\pi}{2p}$, respectively.

5.3 Mechanical Model

An analytical method has been developed to evaluate the strength of a rock mass containing "en echelon" joints. Input parameters include the ambient stress field and the strength characteristics of both the intact rock and of the joints, as well as the joint network simulation.

Path of Minimum Resistance

Figure 5.4 can be considered a typical example of the joints in one realization. The objective is to find the path of minimum resistance for all possible failure paths which proceed from the left boundary upwards across the jointed mass and reach the right boundary. Hence paths A, B, and C in Fugure 5.13 are acceptable failure routes whereas D and E are not. Continuous paths along joint planes (such as A in Figure 5.13) constitute one obvious and relatively small family of failure paths. Routes that involve transitions between planes (such as B and C) comprise a much larger family.

The critical path is found by means of dynamic programming. Each jointing pattern is characterized by a grid formed

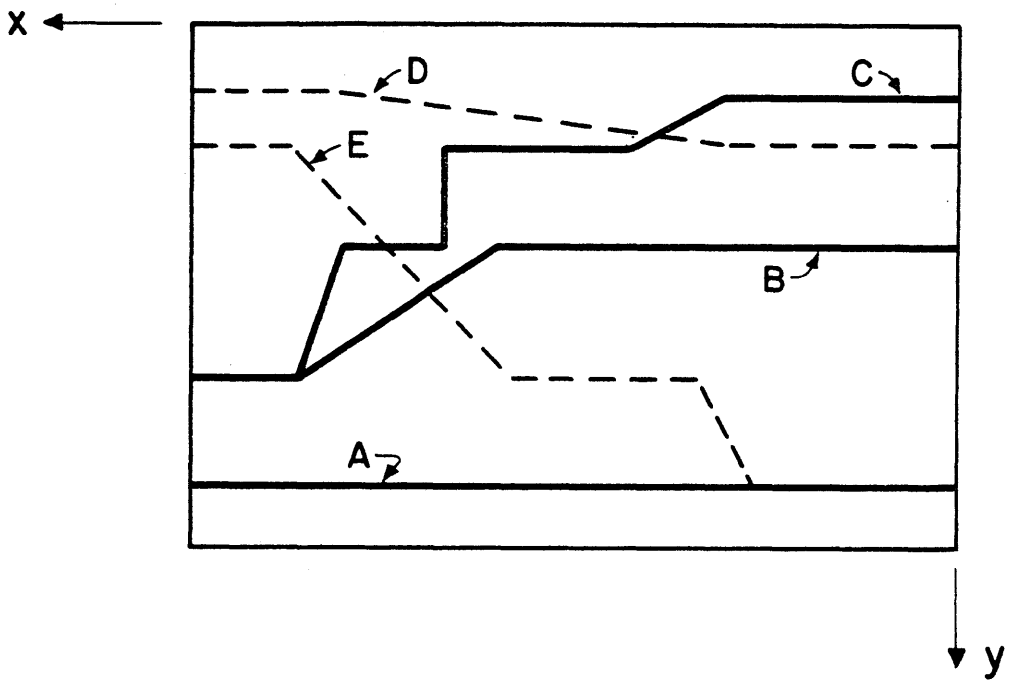
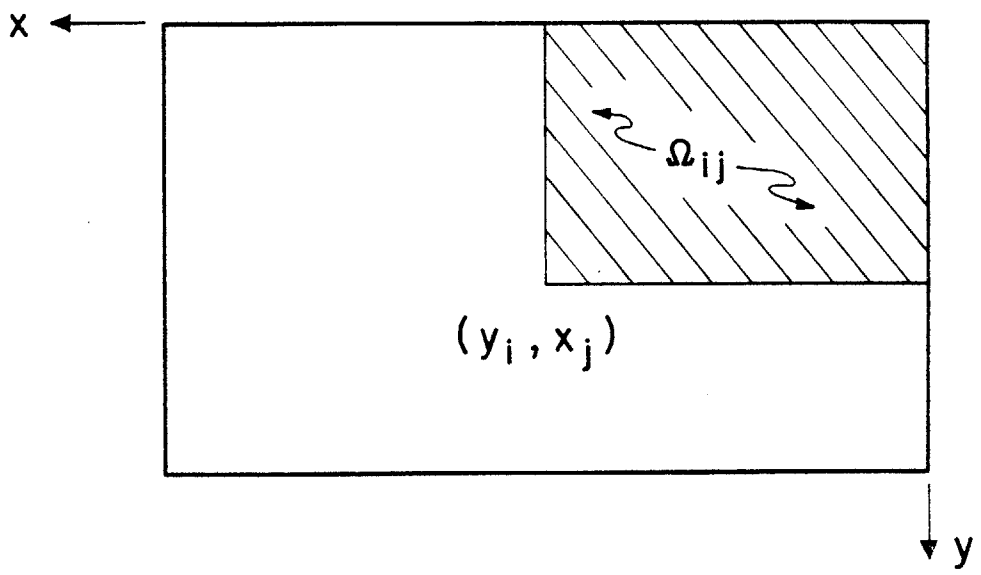


Figure 5.13 Typical Failure Paths

by pairing all possible combinations of x and y where the y's are the coordinates of the joint planes and the x's are the coordinates of all tips of joint segments (Figure 5.4). The grid points represent possible junction points for failure paths between joint sections on one plane and between planes (see Figure 5.4). Transitions always occur from (x_i,y_i) to (x_j,y_j) such that $x_i \geq x_j$ and $y_i \geq y_j$. In order to identify the critical path each point is examined to find the minimum strength path between that point and the right boundary of the jointed mass. The analysis begins at the upper right corner. The first point, $x = x_1 = 0$ and $y = y_1$ has zero strength because it lies on the right boundary. The algorithm proceeds to check all the points along the top row where $y = y_1$ and then examines progressively higher y's always working from right to left along the rows. For each point all paths through grid points above and to the right are screened. In effect, the program constructs a resistance matrix, $\underline{R}$, of order n by m where n is the number of planes and m is the total number of joint tips. The (i,j)th element, $R_{i,j}$, is the resistance of the upper right portion of the rock mass, $\Omega_{i,j}$, with respect to (i,j) given that the failure route passes through (i,j) and terminates somewhere along the right edge (Figure 5.14). Once all the coefficients in the matrix $\underline{R}$ have been evaluated, the minimum resistance R, can be found:

$$R = \min R_{i,m} \text{ for } i = 1,n \qquad (5.6)$$



$R_{i,j}$ = minimum resistance of block Ω_{ij}

Figure 5.14 Principle of Minimum Resistance

Strength Derivation

The strength of the material between grid points is calculated using a Griffith-Modified Griffith envelope for the intact rock and a linear Mohr-Coulomb envelope for the joints. The Griffith-Modified Griffith envelope is parabolic in the tensile range and linear in the compressive range (Figure 5.15C). It is completely defined by two parameters: σ_t, the tensile strength of the rock, and Φ_r, the friction angle. The Griffith-Modified Griffith envelope defines the critical stresses for crack initiation and thus gives a lower bound to the resistance of intact rock. An upper bound would be defined by the stress conditions which characterize complete failure with cracks propagating through the entire specimen. The most appropriate envelope for the problem under study may be one between these two extremes; however, the lower bound was chosen as a realistic albeit conservative estimate of resistance.

The details of the resistance determination and particularly the analytical formulations are presented in Appendix G. Figure 5.15 summarizes the resistance determination for failure through intact rock. The jointing pattern and the orientation of the joints relative to the initial state of stress are shown in Figure 5.15A. (In general, the joints are not parallel to either of the principal stress axes.) The objective is to find the resistance which must be overcome to

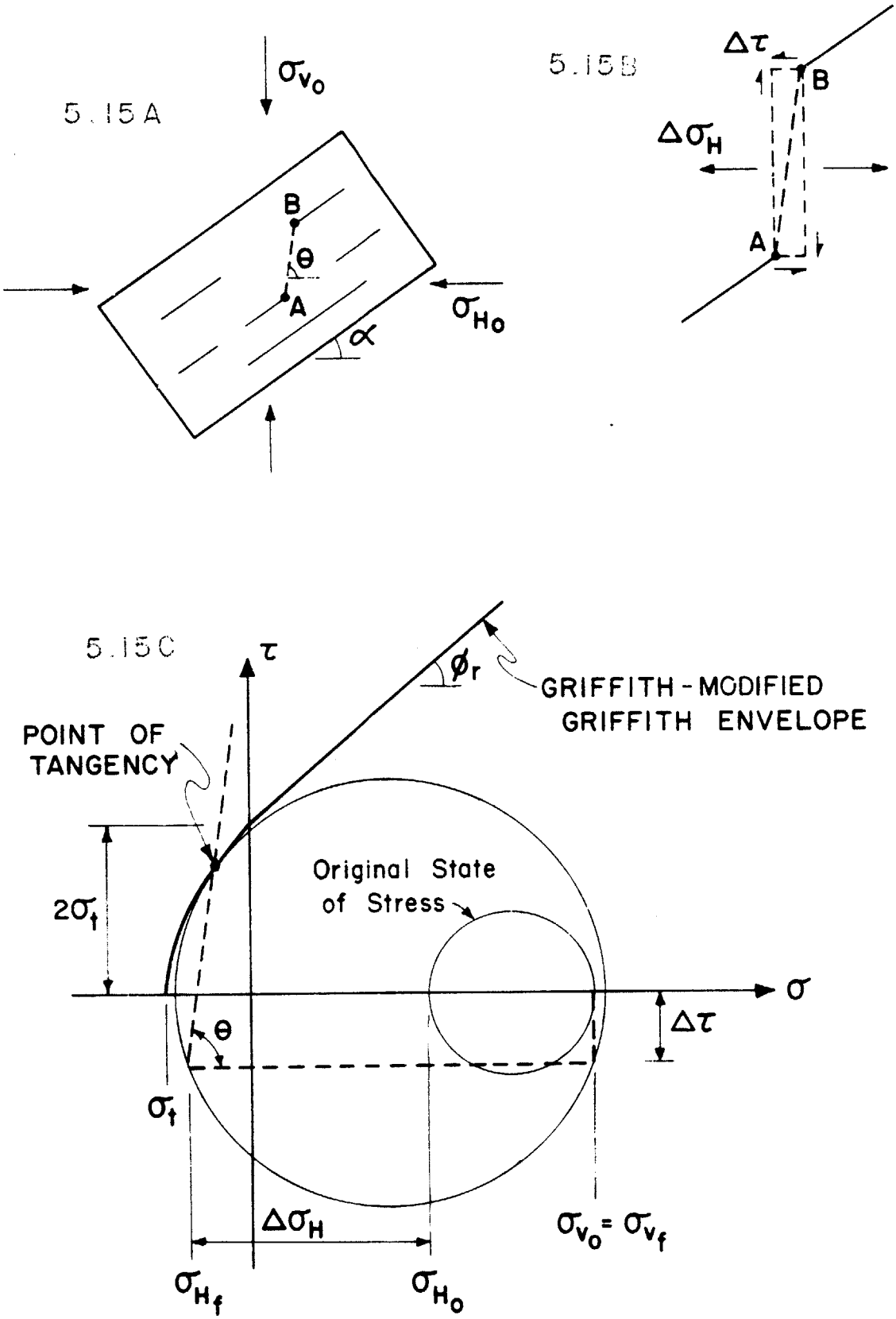


Figure 5.15 Strength Determination--Path Through Intact Rock

create a failure path from point A to point B. This path will eventually be combined with other segments to form failure routes through the entire "rock mass". Figure 5.15B is a more detailed illustration of the region near points A and B. The rectangular area bounded by the dashed lines represents a small mass of intact rock about to undergo failure. The prescribed failure path (A to B) has a definite orientation with respect to the stress field. The additional stresses required to cause failure must be applied is such a manner that failure occurs along A-B (Figure 5.15B). Specifically, the Mohr circle for the state of stress at failure must be tangent to the Griffith-Modified Griffith envelope at a point such that the failure plane develops in the direction A-B. The critical stress condition is obtained by superimposing upon the ambient stress field both negative horizontal stresses ($\Delta\sigma_h$) and shear stresses ($\Delta\tau$) in the horizontal and vertical planes (Figure 5.15B). Intact rock will generally undergo tensile failure in the stress ranges normally encountered in engineering practice.

If the two points under consideration lie in the same joint segment the strength computation is somewhat simpler (Figure 5.16). Failure along a joint (C to D) occurs when one of the points of intersection between the Mohr circle and the failure envelope is the point that represents the state of stress at the orientation of the joint. In general, the initial horizontal stress will be reduced through the

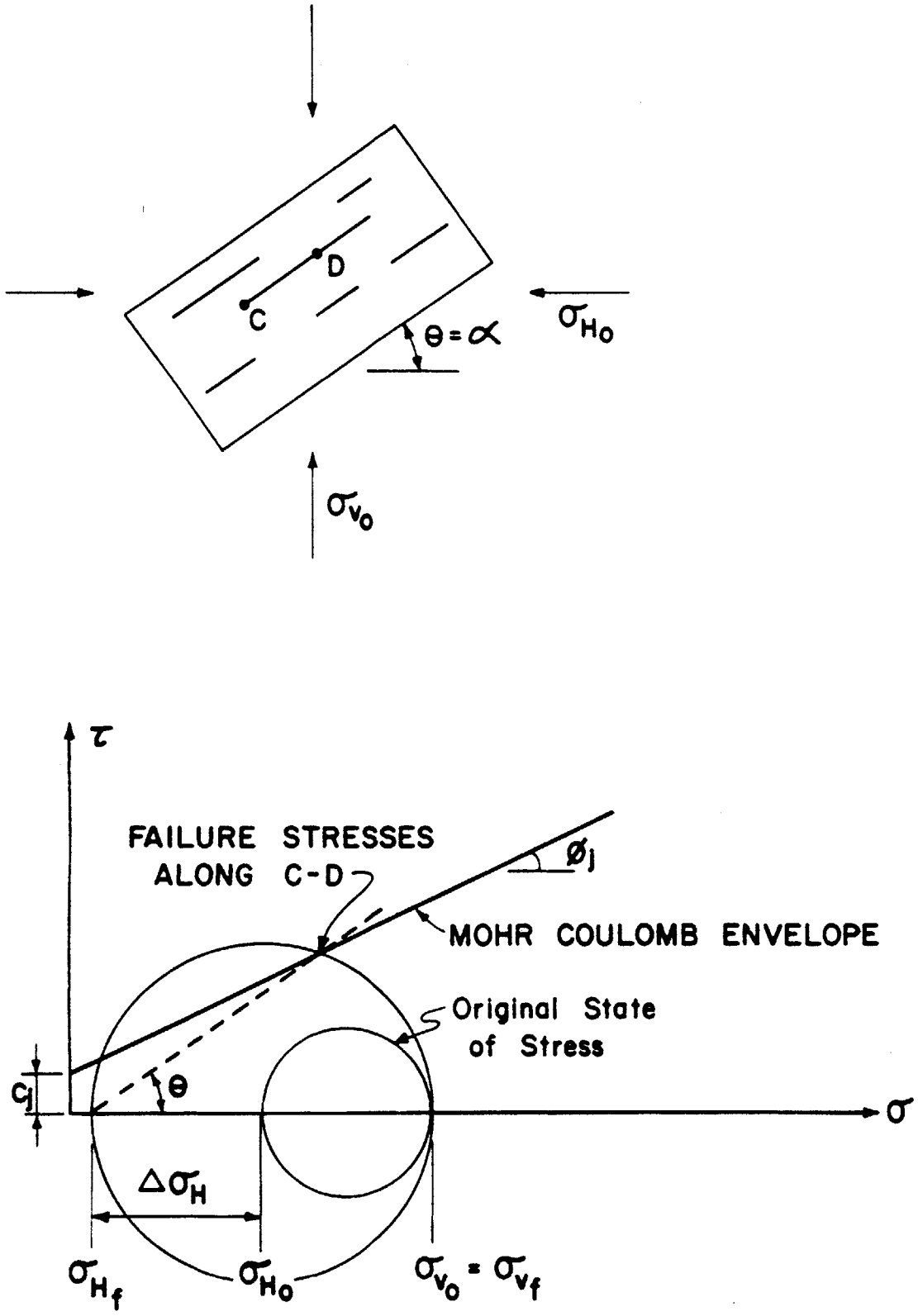


Figure 5.16 Strength Determination--Path Along Joint Segment

application of a negative horizontal stress ($\Delta\sigma_h$) in order to produce these failure conditions.

Once the strength of segments such as A-B and C-D have been calculated they are combined to determine the resistance for composite paths. The primary concern of the entire analysis is to find the composite path with the minimum strength. The strengths for individual path segments are combined by transforming the superimposed stresses into force components parallel and perpendicular to the orientation of the joint planes. Forces in the same direction are added to yield two aggregate forces: S (parallel to the joint plane) and N (perpendicular to the joint plane). Finally, S and N are combined to give the Apparent Resistance, AR, where,

$$AR = S + N(\tan \Phi_j) \tag{5.7}$$

The path with the minimum AR is the critical one. In essence, the resistance of a failure path (not necessarily coplanar with one of the joint planes) is characterized by the shear force which must be applied in the direction of the joint planes to cause failure.

Apparent Persistence

The minimum AR of a jointed rock mass is not a particularly useful parameter for engineering calculations. A better way to express the results is in terms of apparent strength

parameters. Jennings (1970) proposed that a planar failure surface involving both intact rock and joints be treated as if it were composed of a single hypothetical material with an apparent cohesion, c_a, and an apparent friction angle, Φ_a. Jennings' relations can be modified to yield the following expressions:

$$c_a = (1 - k)c_r + (k)c_j \tag{5.8}$$

$$\tan \Phi_a = (1 - k)\tan \Phi_r + (k)\tan \Phi_j \tag{5.9}$$

where subscript "r" denotes rock

subscript "j" denotes joint

k = persistence (Figure 5.2)

The concept of apparent persistence is proposed in this paper. Apparent persistence is the value of persistence along a joint plane that has the same value of AR as the critical failure path. One can compute the AR_0 and the AR_{100} which are the values of AR for a failure path that extends along planes having a persistence of 0% and 100% respectively. AR_{cr}, the value for the critical failure surface (which is typically nonplanar), is between the two extremes. The apparent persistence, w, is defined as

$$w = \frac{AR_0 - AR_{cr}}{AR_0 - AR_{100}}(100) \qquad (5.10)$$

Apparent persistence is a measure of the proximity of AR_{cr} to AR_{100}. It is concerned solely with the strength along the critical path and ignores the location and configuration of the failure surface. Apparent persistence compares the strength of the critical path to the strengths of paths solely through intact rock or solely along a joint. It expresses the strength of the critical path in terms of apparent persistence. This concept is more fully developed in Section 5.5 .

5.4 Computer Program DAYLITE

The analytical models derived in Sections 5.2 and 5.3 and Appendix G are incorporated in a computer program. The primary input parameters to the program are those listed in Table 5.1. They include the general attitude of the joints, the distributional parameters of the joints, the strength characteristics of both the joints and intact rock and the in situ stress field. The program generates a specified number of random jointing patterns (or "realizations") and analyzes each pattern to locate the path of minimum resistance. Finally, it computes the value of apparent persistence. (The program uses three different techniques to interpret the minimum resistance strength. The "Reduction Option #1" corresponds to apparent persistence as defined in the previous section.)

Table 5.1

Generic Group	Symbol	Definition	Value*
Geometry of Block	α	Angle of Inclination	30°
	h	Height	40°
	ℓ	Width	80°
Strength Parameters	Φ_r	Friction Angle of Intact Rock	30°
	c_r	Cohesion of Intact Rock	80**
	Φ_r	Friction Angle of Joint	30°
	c_r	Cohesion of Joint	2**
Distribution Parameters	ρ	Mean Joint Plane Spacing	4**
	δ_j	Mean Length of Joint Segment	10**
	δ_r	Mean Length of Rock Bridge	20**
Original Stress	σ_{v_o}	Vertical Stress	varies
	σ_{h_o}	Horizontal Stress	varies

* The values listed were used in the sensitivity analyses discussed in Section 5.5. Unless otherwise noted in plots the values listed were used in all computer runs.

** Any set of dimensionally consistent units will yield the same values of apparent persistence.

TABLE 5.1 Inputs to computer program JOINTSIM

The program is designed to handle up to 400 realizations; however, the capacity can easily be expanded. The basic output consists of a recapitulation of the inputs, a listing of the more important output parameters for each realization and a statistical summary of the inclinations of the failure path, the maximum persistence and the apparent persistence for all realizations. The program has a series of options that enable one to obtain a much more detailed output. For example, one can get the coordinates of each joint segment, a listing of the strengths of all possible failure paths from each point (x_i, y_i) or the precise location of the minimum resistance path.

A typical output (in its abreviated form) is presented in Appendix H. The appendix also contains a complete listing of the program as well as a users' manual.

5.5 Results

The analytical procedure and the computer program were tested by investigating the sensitivity of the resulting strength parameters relative to the input parameters. In particular, the effect of changes in the magnitude and in the horizontal to vertical ratio (σ_{ho}/σ_{vo}) of original stresses and the effects of variations in the joint orientation were examined. The set of input parameters used as a basis for comparison is listed in Table 5.1.

En Echelon or Multi-Planar Failure Paths

The sensitivity study was concerned primarily with parameter values that are likely to generate "en echelon" failures, i.e., failures that involve at least one transition between joint planes. Jointed rock masses which have relatively large mean spacings between planes or relatively high strength parameters for intact rock tend to fail along a single joint plane. Figure 5.17 suggests this relationship between mean spacing and frequency of "en echelon" failures. Since the number of realizations (1 realization = 1 jointing pattern) is quite small (40) the curve serves to identify a trend rather than a definitive relationship.

Maximum Persistence Vs. Apparent Persistence

The number of joint planes in each realization of the simulation procedure is a Poisson random variable, S*, with a probability mass function:

$$P_S(s) = \frac{(h/\rho)^s e^{-(h/\rho)}}{s!} \qquad (5.11)$$

and mean $m_S = h/\rho$.

From the parameters of Table 5.1: $m_S = 40/4 = 10$.

The persistence of each joint plane is also a random variable, T, with mean:

* Upper case notation is used for random variables and a lower case notation for specific values of the variable.

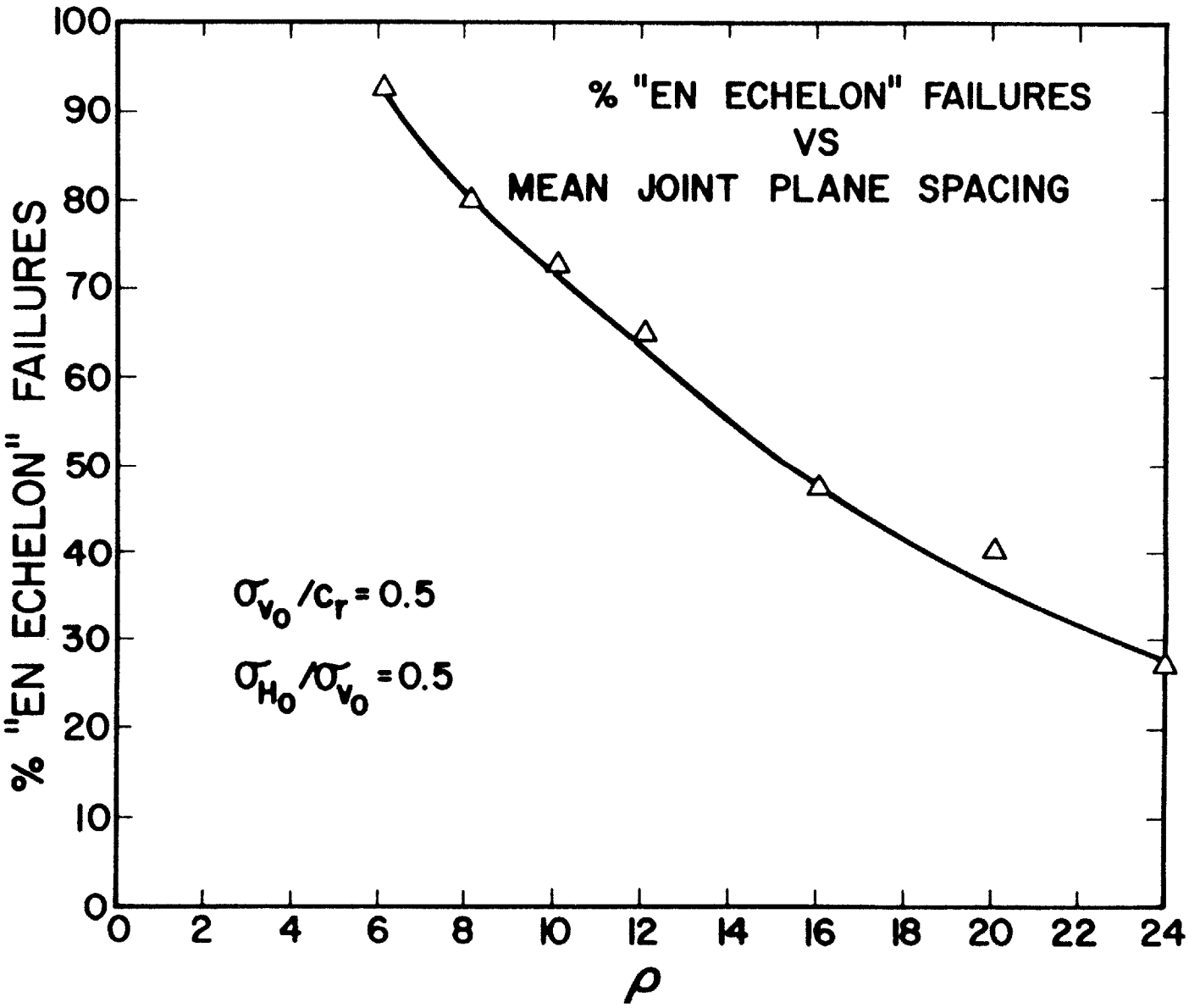


Figure 5.17 % "En Echelon" Failures vs. ρ

$$m_T = \frac{\delta_j}{\delta_j + \delta_j}(100) \qquad (5.12)$$

where δ_j is the mean length of joint segments

δ_r is the mean length of rock bridges.

From Table 5.1: $m_T = \frac{10}{10 + 20}(100) = 33.3\%$

The maximum presistence along the joint planes in each realization is a random variable, Q. There is no simple analytical expression for the mean, m_Q. It depends on the dimensions of the jointed rock block as well as on the distributional parameters of the jointing process. In any case, m_Q is not less than m_T. Using the parameter values in Table 5.1 one finds from simulation an m_Q of 63.1%.

The apparent persistence, W, defined in Section 5.4 reflects both the persistence of individual joint planes and possible transitions between joint planes. If the joint planes are widely spaced, the critical path will follow a single joint plane--the one with the maximum persistence and w = q. w is actually a lower bound for q because if there were no transitions between joint planes the critical path would follow the joint plane with the maximum persistence. When the critical path involves a transition it must have "found" a route that has less resistance than the plane with maximum persistence. In general, w > q with the difference $m_W - m_Q$ reflecting plane to plane transitions in critical paths.

Figure 5.18 shows a pair of histograms that summarize the results of 250 realizations in terms of W and Q. The histograms of apparent persistence is shifted to the right with respect to that of maximum persistence because of the aforementioned effect (causing a different $m_W - m_Q$) of transitions between planes. The maximum persistence histogram is more dispersed and has a more pronounced peak.

Magnitude of Stress

The magnitude of the stress field does not appear to exert a major influence on the apparent persistence. Figure 5.19 presents the results of a series of computer runs in which the stress ratio (σ_{ho}/σ_{vo}) was held constant (0.25 or 1.0) while the magnitude of stress (σ_{vo}/c_r) varied from 0.125 to 1.00. Each run involved 250 simulations. The curve for $\sigma_{ho}/\sigma_{vo} = 0.25$ indicates that m_W increases as σ_v increases. However, the curve for isotropic stresses suggests the m_W is virtually independent of σ_{vo}. It seems that the influence of magnitude of stress increases as the stress ratio decreases. The histogram of W for the six parameter combinations have approximately the same dispersion with sample standard deviations in the range of 11.3 to 12.7.

Stress Ratio

The effect of stress ratio on apparent persistence is shown in Figure 5.20. There is a small but steady decrease in m_W (and increase in σ_W^2) as the stress ratio increases from

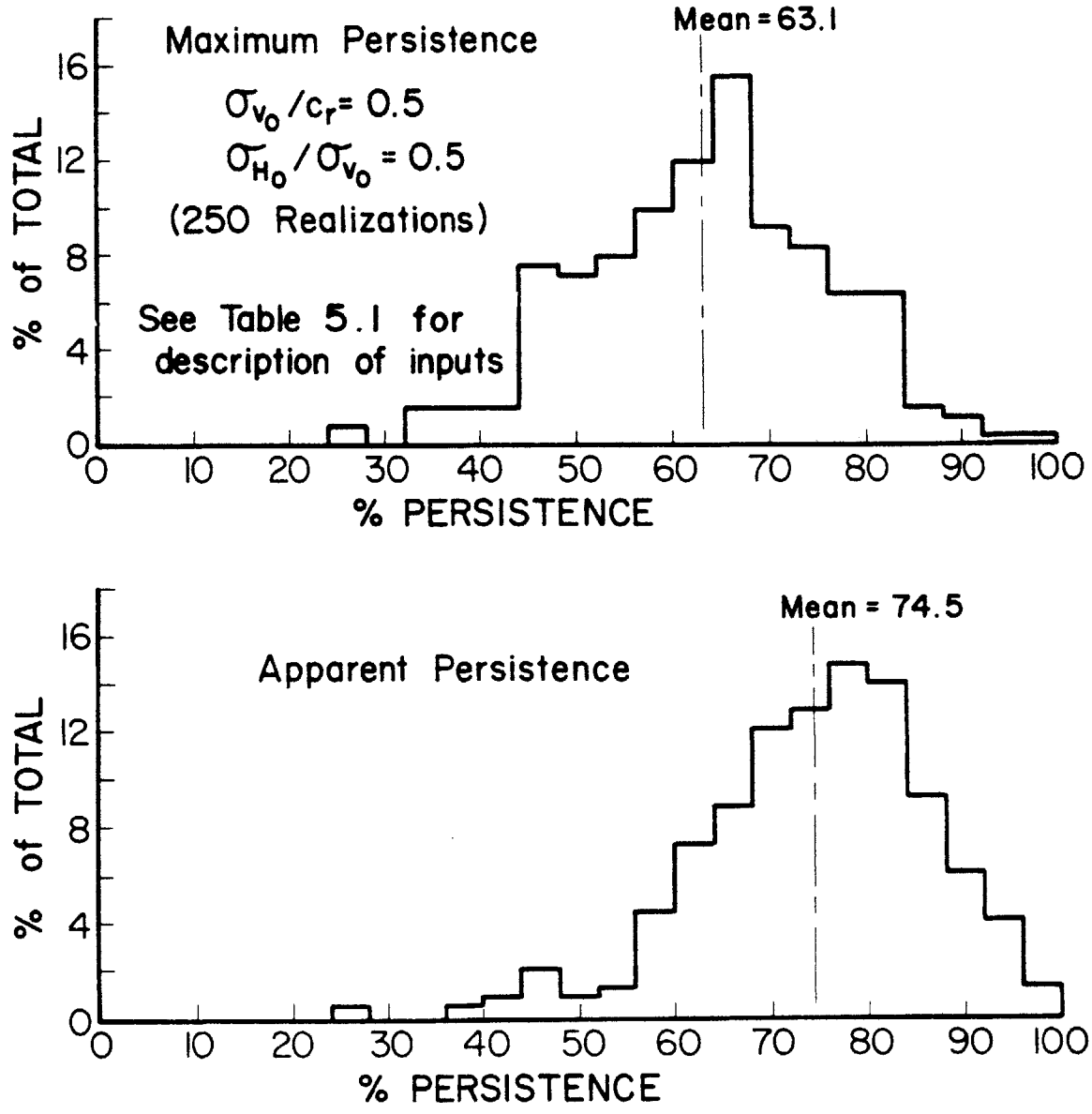


Figure 5.18 Histograms of Maximum Persistence and Apparent Persistence

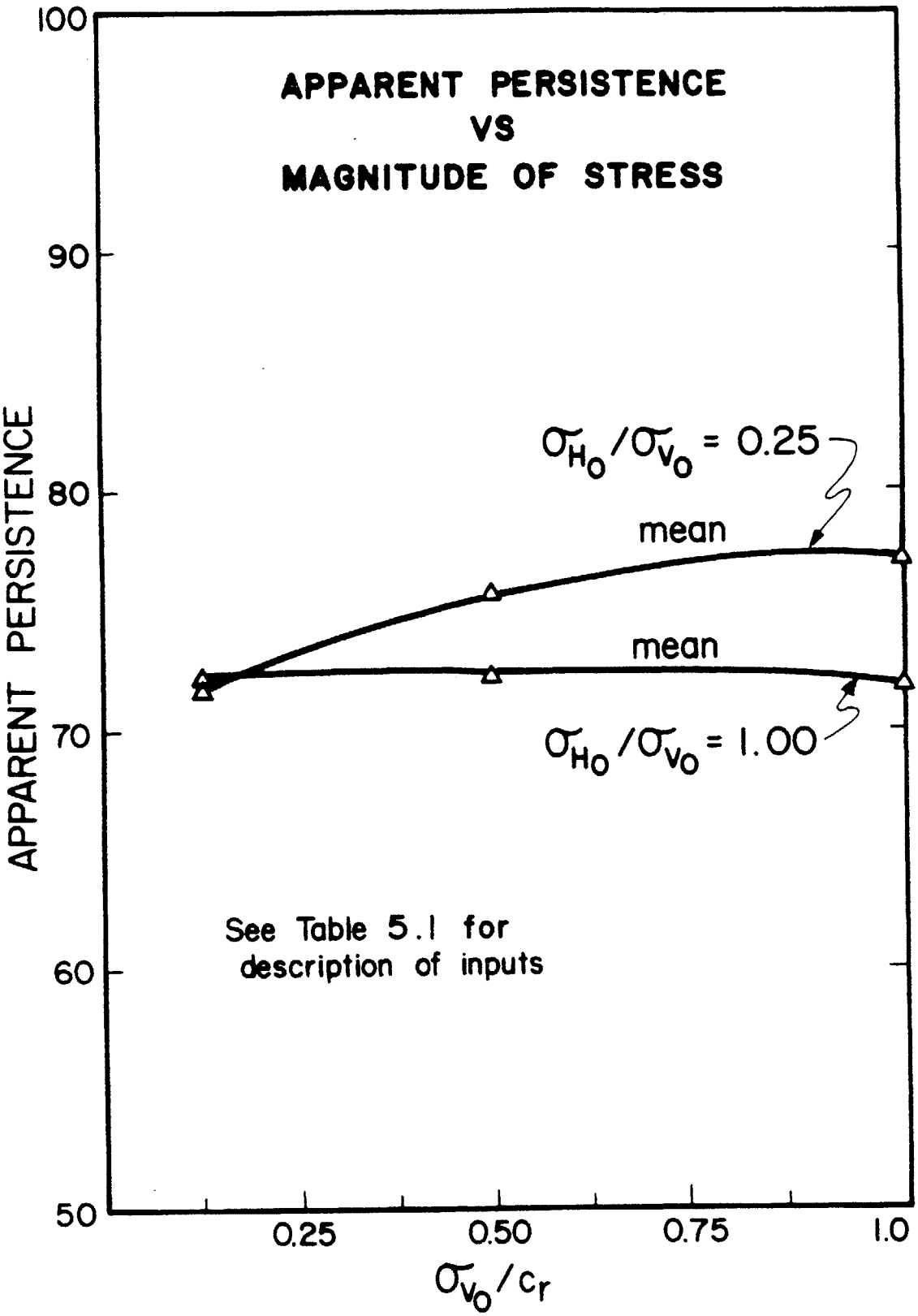


Figure 5.19 Apparent Persistence vs. Magnitude of Stress

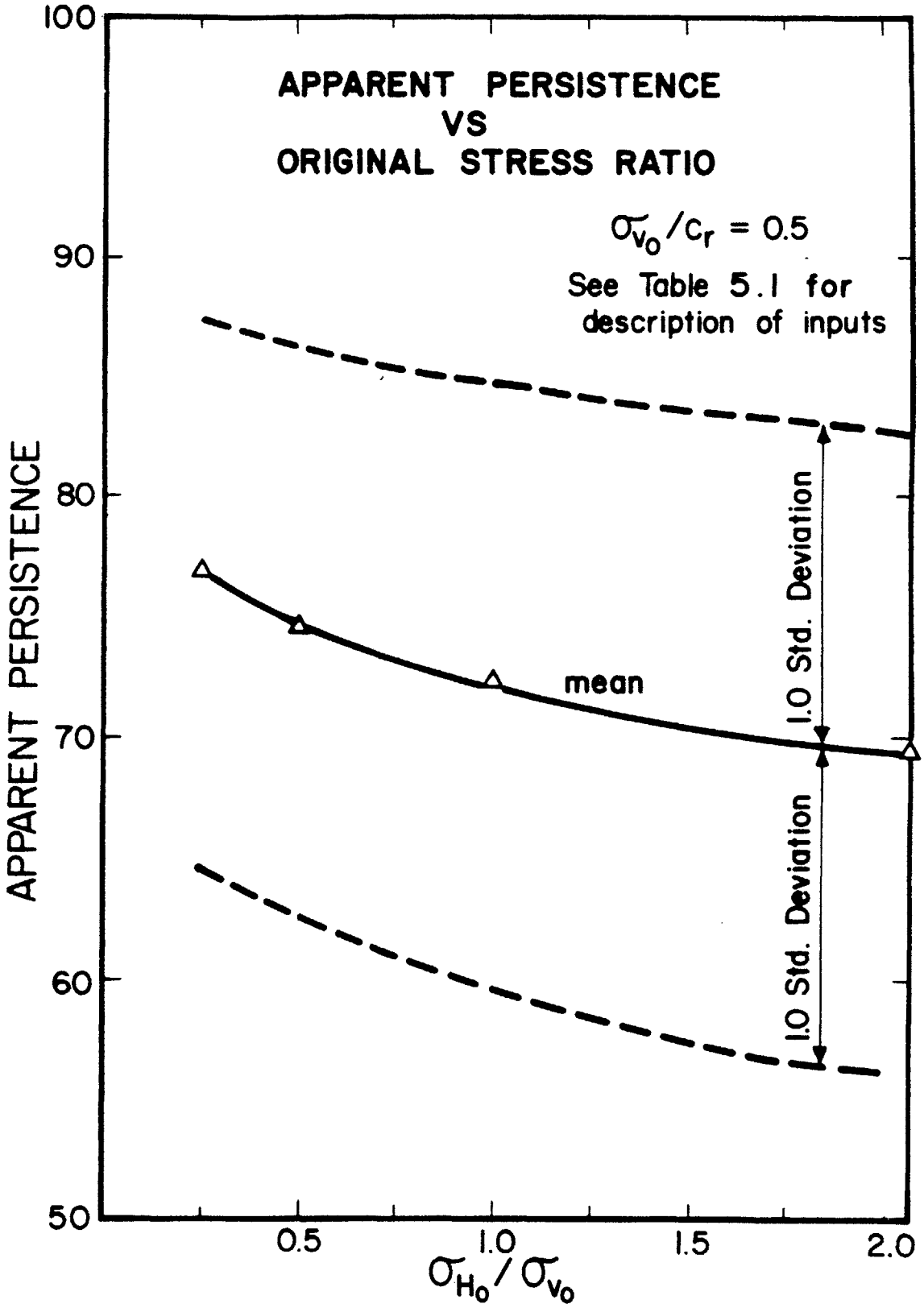


Figure 5.20 Apparent Persistence vs. Original Stress Ratio

0.25 to 2.0. The total change in m_W over that range is approximately 9%.

Orientation of Joints

Three computer runs were used to investigate the relationship between apparent persistence and the joint inclination, α. This limited data suggests that m_W is only moderately sensitive to changes in α (Figure 5.21); m_W decreases with increasing α whereas σ_W^2 is essentially constant. The change of m_W for the range of $15° < \alpha < 45°$ does not exceed the standard deviation. In this sense one may say that the apparent persistence, W, is not very sensitive to α.

This insensitivity can be a very useful result. It suggests that computations of m_W for a particular joint inclination can be used for other α's without introducing appreciable error. The fact that m_W is not a strong function of α does not imply that the absolute strength of rock masses with dissimilar joint orientations is the same. As defined in Equation (5.10), apparent persistence is a ratio of strengths. Thus, two realizations with different α's may have the same apparent persistence even though their respective A_o's, A_{cr}'s and A_{100}'s are different. As long as the respective values are proportionate, the two apparent persistences will be identical. Figure 5.21 suggests that the proportionality (almost) occurs as α varies as long as all the other parameters are identical.

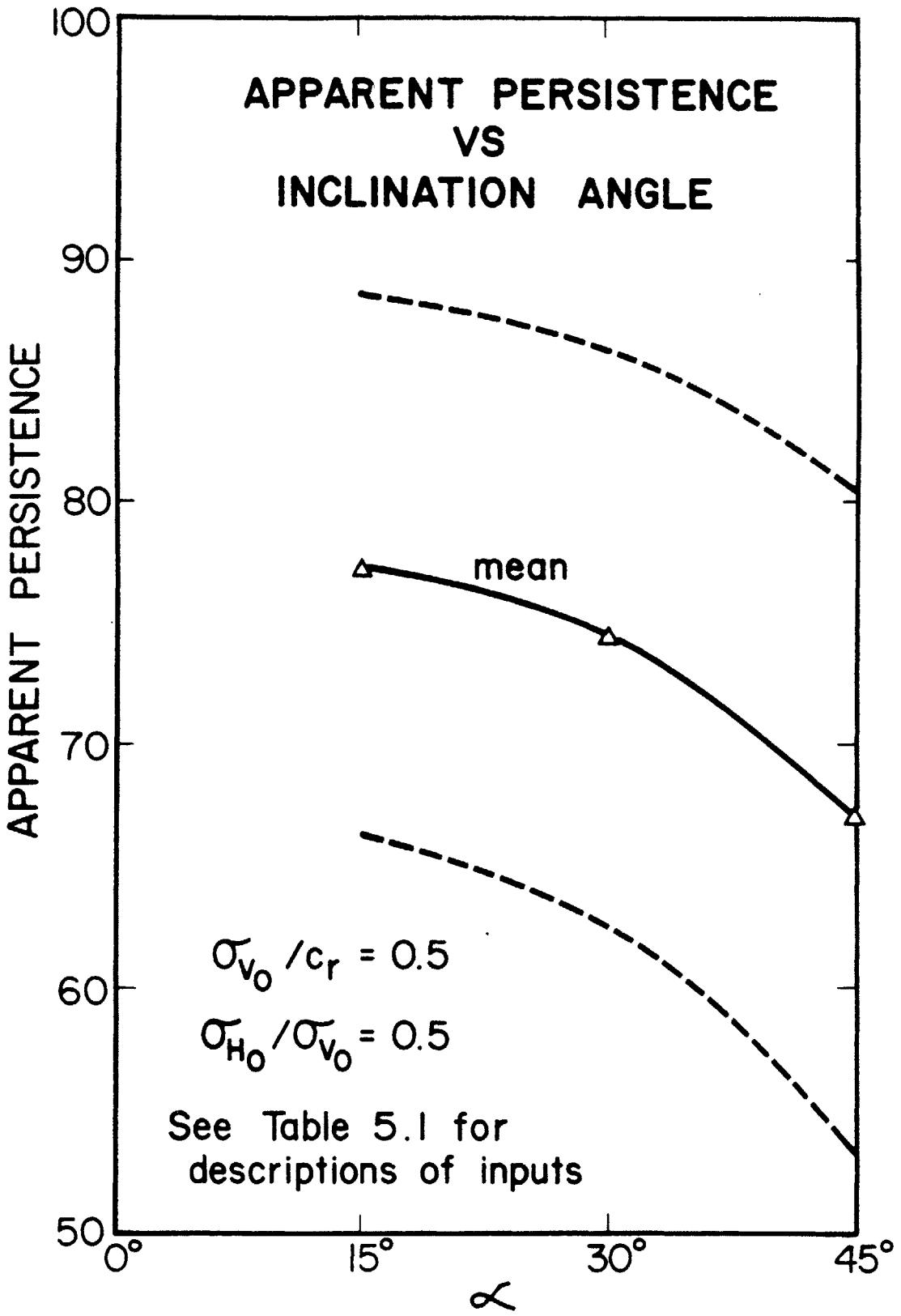


Figure 5.21 Apparent Persistence vs. Inclination Angle

In stability analyses, values of apparent persistence would be used in conjunction with the proper α's to yield different strengths.

5.6 Summary

The probabilistic model for shearing resistance of jointed rock is a step in the development of a comprehensive technique for analyzing the reliability of rock slopes. It represents a possible solution to one of the major unsolved problems in rock slope stability analysis -- the interaction between intact rock and discontinuities. The model provides a rational method for treating joint persistence; the method is based on information that can be obtained from field surveys.

The most important feature of the model is that it characterizes the strength of a jointed rock mass through an apparent persistence parameter. Apparent persistence is not a measure of absolute strength; it is an index that compares the strength of the critical path to the strength of a path through intact rock. Apparent persistence is a geometric interpretation of the interaction that occurs between the jointed and unjointed segments of en echelon joint planes.

Apparent persistence can be used in conjunction with Jennings' relations to describe the strength along failure surfaces in stability analyses involving en echelon failure planes. In analyses that consider tetrahedral wedges bounded by two joint planes the apparent persistences on the two planes are

treated as independent random variables.

The current model as described in this chapter is merely the first step in treating the persistence problem. The model examines a fairly simple (albeit common) jointing system. Some of the limitations in the model can be eliminated through conditioning; however, other restrictions can only be relaxed by modifying and extending the present version of the model.

The apparent persistence calculations presented in the chapter were based on a deterministic set of strength parameters. It is possible to treat the strength parameters as random variables by conditioning the apparent persistence on the strength parameters. For example, the pdf's of Φ_r and c_r could be discretised into a limited number of values each of which would have a finite probability of occurrence. A series of simulations (and apparent persistence computations) could be performed on each pair of Φ_r and c_r values. If Φ_r and c_r were independent random variables, the (unconditional) apparent persistence would be:

$$w = \Sigma_i \Sigma_j W_{ij} \; P[\Phi_{r_i}] \; P[c_{r_j}] \qquad (5.13)$$

where W_{ij} is the apparent persistence based on Φ_{r_i} and c_{r_j}
$P[\Phi_{r_i}]$ is the probability that Φ_r is Φ_{r_i}
$P[c_{r_j}]$ is the probability that c_r is c_{r_j}

The shape of the strength for intact rock is not a critical feature of the model. The model defined strength in terms of a Griffith-Modified Griffith envelope, but other strength criteria could be used. In particular, one might want to use the upper bound envelope that characterizes crack propagation through the entire specimen (Section 5.3). Since apparent persistence is a relative measure of strength it should not be highly sensitive to the shape of the intact failure envelope*.

The stochastic model can (and should) be expanded to consider the three dimensional aspects of persistence i.e., the model should examine the joint planes as three-dimensional features rather than just their traces on a two-dimensional cross section. Also, the model should consider cases involving non-parallel joints and recognize possible correlations between the geometric parameters of different joint sets.

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* Apparent persistence is virtually insenstitive to the strength parameters along the joints because the intact segments have such a dominant effect on the location of the critical path. The resistance along jointed segments in almost negligible compared to the resistance through intact material regardless of Φ_j and c_j.

CHAPTER 6

AN INTEGRATED APPROACH TO RELIABILITY ANALYSIS

6.0 Introduction

The earlier chapters discussed various aspects of reliability analysis in rock slopes. Chapters 3 to 5 presented some tools to use in reliability analysis but each technique was developed as an independent component. This chapter will endeavor to show how the individual elements can be combined into a complete analysis.

The reliability analysis is based on a concept that underlies all rock slope analyses: Failure cannot occur unless both kinematic and kinetic conditions are met. Chapter 3 discussed the kinematic aspects and presented the computer program DAYLITE. Chapter 4 discussed the kinetic aspects and presented the computer program SWARS-2MC. The results from the two programs can be combined to provide a complete reliability analysis. The two operations are combined in a manner similar to that used in composite models (Section 2.3) i.e.,

$$P_f = P[A]P[B] \tag{6.1}$$

P[A]: the probability that a wedge daylights

P[B]: the probability that the driving forces exceed the resisting forces.

The analysis proceeds as follows:

1. Examine each cell in the critical zone and determine $P[\theta_i, \phi_i]$, the probability that a

line of intersection for any two joints will be in the cell characterized by the coordinates (θ_i,ϕ_i).

2. Use Monte Carlo simulation to determine $P[U_i|\theta_i,\phi_i]$ the probability that a wedge daylighting in cell (θ_i,ϕ_i) will be kinetically unstable.

3. Calculate $\overline{P}_f$ for any two joints as;

$$\overline{P}_f = \sum_{\text{all } i} P[U_i|\theta_i,\phi_i]P[\theta_i,\phi_i] \qquad (6.2)$$

4. Adjust $\overline{P}_f$ to reflect the fact that the slope may contain more than one potentially unstable wedge, i.e.,

P_f = Probability of failure for the entire slope

$$= f(\overline{P}_f,k) \qquad (6.3)$$

where $\overline{P}_f$ is the probability of failure for a single wedge and k is the number of wedges on the slope.

The four-step procedure outlined above will provide a P_f value that approximates the reliability of the slope. The procedure does not provide a rigorous solution to the stability problem; there are a number of areas that require development. The purpose of this chapter is not to present a definitive solution but rather to show where the techniques developed in the earlier chapters fit into the solution. The remainder of this chapter examines each of the four steps

and in doing so emphasizes two points:

1. What assumptions are involved in each step?
2. Which steps require additional development?

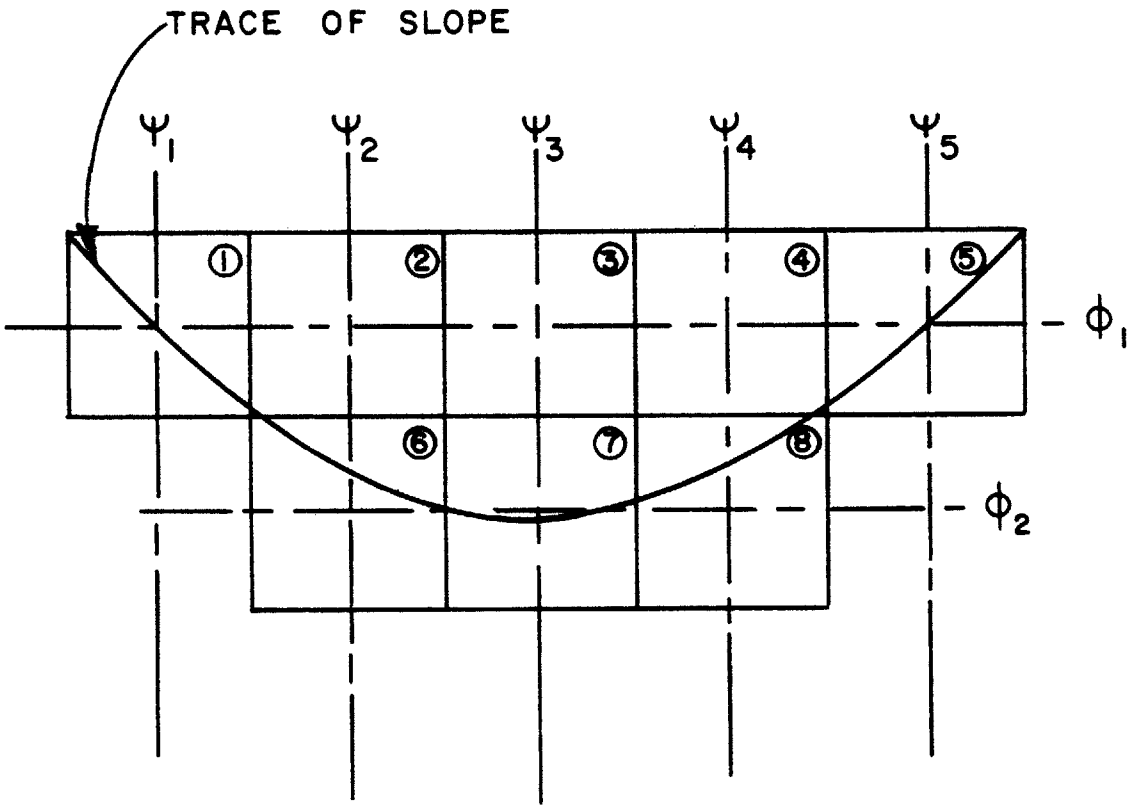
6.1 Kinematic Restraints: $P[\theta_i,\phi_i]$

Figure 6.1 shows the results from a hypothetical DAYLITE run in which the critical zone was partitioned into 8 cells* each of which is characterized by a pair of spherical coordinates (θ_i,ϕ_i). Values of $P[\theta_i,\phi_i]$ for each cell are tabulated in Figure 6.1. The values represent the probability that a joint from set 1 will combine with a joint from set 2 to form a wedge whose line of intersection daylights in the respective cell.** The values are the kinematic probabilities that will be used in reliability analysis. They were computed in accordance with the procedures described in Chapter 3 which assumed that the orientations of joint planes were independent random variables.

Some of the cells may have very low $P[\theta_i,\phi_i]$'s. The low values indicate that it is highly unlikely that a wedge will daylight at those orientations. The analysis can be shortened if these cells are eliminated from further

* 8 cells are used to illustrate the concept. A considerably larger number of cells would be used in any real problem.

**DAYLITE will also compute the probability that two joints from the same set will combine to form wedges.



Cell	$P[\theta_i, \phi_i]$
1	0.056
2	0.078
3	0.096
4	0.032
5	0.009
6	0.066
7	0.088
8	0.044

Figure 6.1 Typical Output from DAYLITE

consideration at this early stage in the analysis. In ignoring cells, one introduces some errors into the analysis because every cell in the critical zone, regardless of its orientation, has a finite probability of representing a potentially unstable wedge. However, if the cell has a $P[\theta_i, \phi_i]$ that is several orders of magnitude lower than the modal value of $P[\theta, \phi]$, the error will be small. (The decision to ignore a cell is actually a judgement on $P_f = P[U_i | \theta_i, \phi_i] \cdot P[\theta_i, \phi_i]$ rather than just on the $P[\theta_i, \phi_i]$ term. If $P[\theta_i, \phi_i]$ is very small, P_f will also be very small.)

6.2 Kinetic Restraints: $P[U_i | \theta_i, \phi_i]$

The discussion on kinetic restraints will first present some general comments on how $P[U_i | \theta_i, \phi_i]$ is obtained for each cell in the critical zone. Next, the section examines one of the inputs to that calculation, the pdf of the height of failure wedges. Finally, the section shows that the calculations for kinetic instability must be performed on each cell in the critical zone.

$P[U_i | \theta_i, \phi_i]$ for each cell is obtained through Monte Carlo simulation with the computer program SWARS-2MC (Section 4.6). θ and ϕ are the only two deterministic parameters in the analysis. ψ and β_1 define the shape of the wedge; their pdf's can be obtained from the DAYLITE output for the (θ_i, ϕ_i) cell. The mechanical properties (stiffness and strength) of

each joint set and the location of the piezometric surface must be characterized in terms of pdf's.* If the joints in a particular set are relatively closely spaced the strength of en echelon failure paths can be expressed through the apparent persistence parameters introduced in Chapter 5.

The height of the wedge, H, is the vertical distance from the top of the slope to the vertex of the wedge (Figure 6.2). If there are no cleft water pressures in the joints and there is no cohesion on either joint, the factor of safety is independent of H. If either cleft water pressure or cohesion is present H must be treated as a random variable. One reasonable approximation of the pdf of H would be the uniform distribution over the entire height of the slope. The derivation is based on the assumption that joint planes can be treated as Poisson planes in space (Section 5.2).** Figure 6.3A depicts a rock slope that contains two sets of discontinuities. Line OA is the trace on the slope of joint A from set 1. Joints from set 2 intersect joint A to form various size wedges. If the joints from set 2 are Poisson their spacing in exponentially distributed in all directions--

* At the present time little is known about typical pdf's for the mechanical properties of joints or ground water conditions. Research is currently underway at M.I.T. to establish typical pdf's for strength parameters. The results will be published in a report authored by Baecher and Einstein, et al., (19xx).

**A number of authors (e.g., Baecher et al., 1977) have presented empirical evidence that suggests that the assumption is valid.

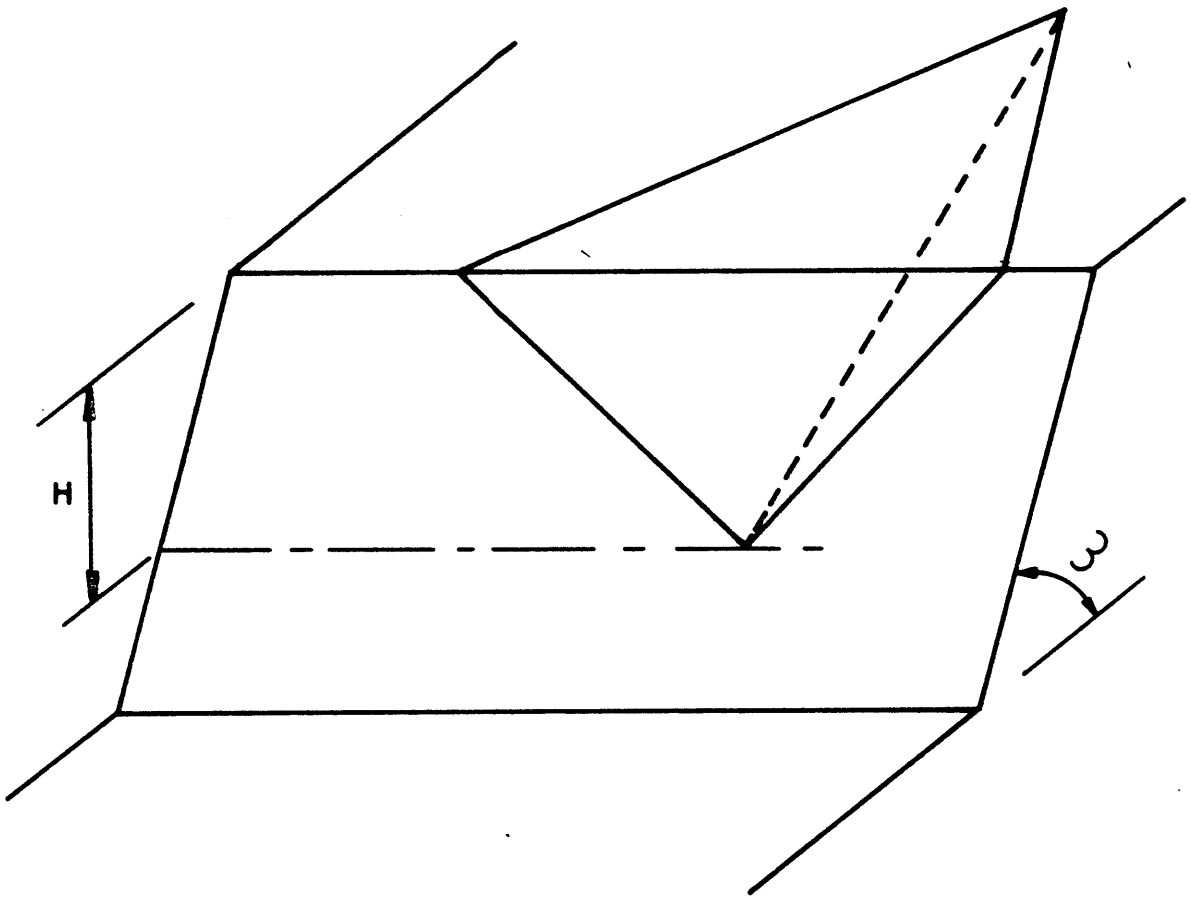
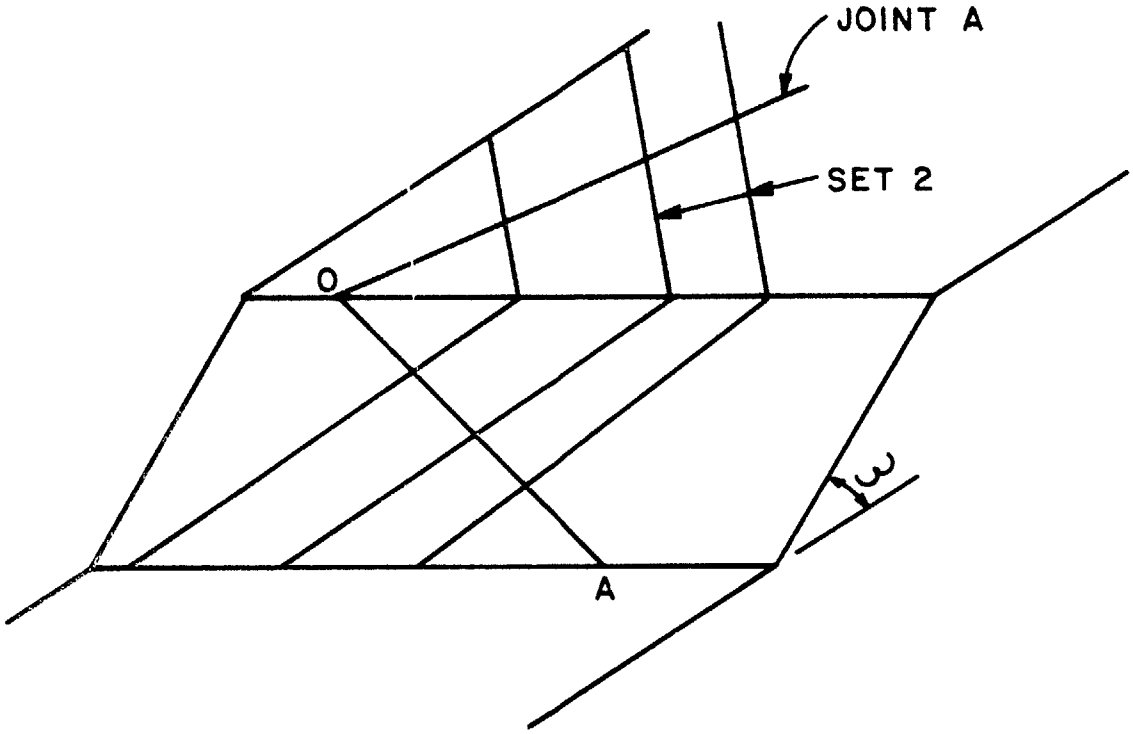
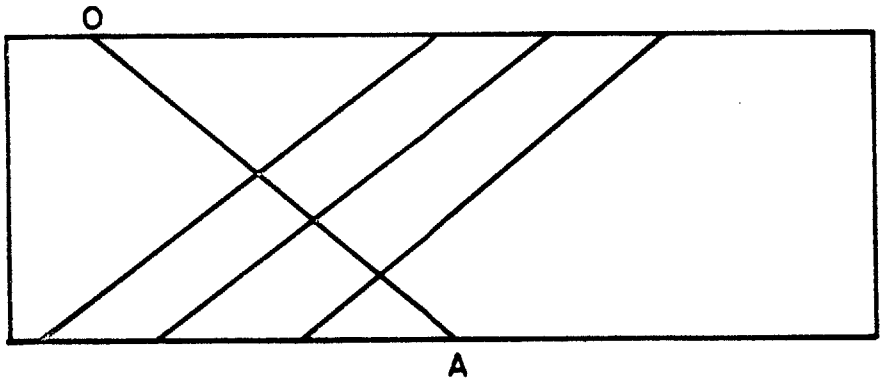


Figure 6.2 Definition of H



6.3 A



6.3 B

Figure 6.3 Intersecting Poisson Planes--Derivation of $f_H(H)$

including the direction along OA. (The mean spacing will vary with direction but the exponential nature of the distribution will not.) Thus, the points of intersection between joints from set 2 and the line OA constitute a Poisson process (Figure 6.3B). Since the intersections are Poisson every differential segment on OA has an equal probability of containing an intersection. In a similar manner, every distance d_i from point 0 to the i^{th} differential segment has an equal probability of locating an intersection. Thus, $f_D(d)$ has a uniform distribution. The intersection points represent the vertices of wedges and the d_i distances are directly related to the wedge heights (H's) through a trigonometric constant. Since $f_D(d)$ is uniform, $f_H(H)$ must also be uniform.

The pdf for H is not perfectly uniform because the preceeding argument ignored the lateral boundaries of the slope. Figure 6.4 is a front view of a slope. None of the joints from set 1 that lie to the right of joint B can form a wedge of height H because joint set 2 is restricted by the right boundary. As joints from set 1 approach the boundary they form smaller and smaller wedges. Thus, for the whole slope, small wedges are somewhat more likely to occur than large ones. Nevertheless, the uniform distribution should provide a reasonable approximation* for the pdf of H.

* Veneziano (19xx) used his model (Sections 1.5 and 5.1) to examine a related problem--the distribution of vertices on the plane of the slope. He concluded that the location of vertices did not constitute a Poisson process.

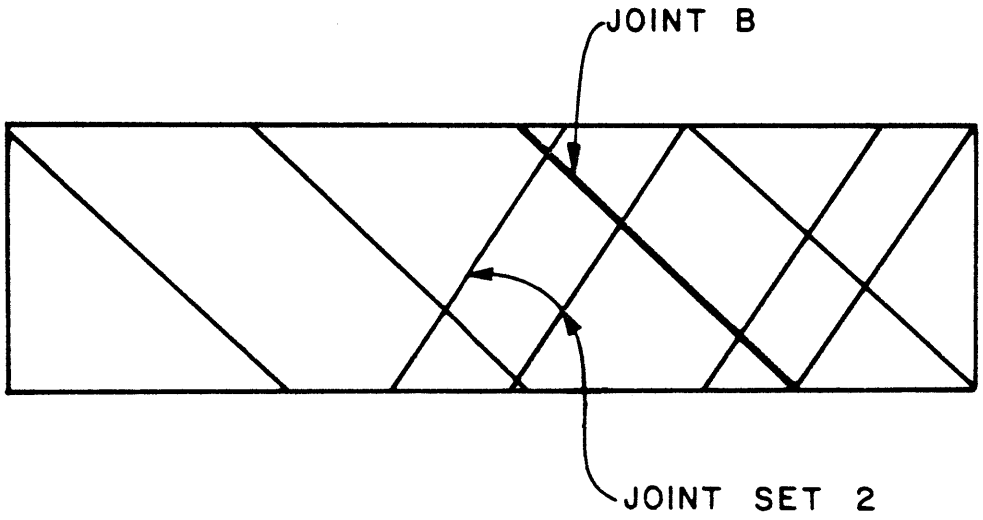


Figure 6.4 Influence of Slope Boundary

Once all the pdf's for the appropriate input parameters have been established $P[U_i|\theta_i,\phi_i]$ can be found through Monte Carlo simulation. $P[U_i|\theta_i,\phi_i]$ is the proportion of trials that have an FS that falls below 1.0. In defining failure, one must differentiate between FS_I and FS_U. As discussed in Section 4.4, FS_I corresponds to initial movement (in any direction) while FS_U corresponds to movement down the line of intersection. FS_U may be more suitable for simulation because there are fewer variables (and fewer pdf's) to define; however, either definition can be used in the analysis.

Every cell in the critical zone is characterized by a unique pair of θ and ϕ values. The simulation procedure that examines kinetic instability must be repeated for each of these orientations. The pdf's for ψ and β_1 (as determined by DAYLITE) will vary for each (θ_i,ϕ_i); the pdf's for the other random variables may be varied at the discretion of the individual performing the analysis.

As discussed in Section 2.2, Monte Carlo simulations have a number of limitations. In particular, the simulations are usually based on a number of assumptions concerning the independence of random variables. The procedure described above (and implemented in the SWARS-2MC program) relies on many independence assumptions. The most important ones are:

1. Joints from different sets are uncorrelated. The fact that a particular joint from set A is present does not affect the marginal distribution of set B.

(DAYLITE is based on this assumption.)

2. Joint orientation and joint strength parameters are uncorrelated. Low strengths are not associated with any particular joint orientation.
3. Joint length and joint spacing are uncorrelated. (JOINTSIM, the program that computes apparent persistence, is based on this assumption; however, the program can be modified to accommodate such correlation.)

Very little information is available to verify (or contradict) these assumptions. In some instances (like strength parameter) the data base is too small to discern correlations. In other instances (like orientation) the data base is adequate but the data is not usually gathered or recorded in a manner that would disclose correlations. Improvements in sampling techniques may provide some insight into the correlation structure of the random variables that describe the orientation and shape of wedges. Correlations involving the mechanical properties of joints present a much more difficult problem that may remain unsolved for the forseable future.

6.3 Calculating $\overline{P}_f$

$\overline{P}_f$ is defined as the probability that any two intersecting joints will form an unstable wedge. The wedge must be both kinematically and kinetically unstable; hence, $\overline{P}_f$ is computed as the product of the kinematic ($P[\theta_i,\phi_i]$) and

kinetic ($P[U_i|\theta_i,\phi_i]$) probabilities. The product is summed over all cells in the critical zone:

$$\bar{P}_f = \sum_{\text{all } i} P[U_i|\theta_i,\phi_i]P[\theta_i,\phi_i] \qquad (6.4)$$

$\bar{P}_f$ is actually a conditional probability based on the premise that two and only two joints intersect to form a wedge. P_f, the probability of failure for the entire slope, could be either less than or more than $\bar{P}_f$. If the rock mass has widely spaced joints relative to the dimensions of the slope there may be a high probability that no wedges form. On the other hand, there are usually many intersections and each one defines a potentially unstable wedge. $\bar{P}_f$ must be adjusted to reflect the fact that there are many possible failures.

6.4 Calculating P_f

P_f, the probability of failure for the entire slope, is a function of $\bar{P}_f$ as well as k, the number of wedges in the slope. Calculating P_f is a two-fold problem:

1. Determining the functional relationship between k and $\bar{P}_f$.
2. Determining the distribution of the random variable k.

The first problem is the more difficult one to solve because it is a problem in system reliability in which the entire rock slope is treated as an engineering system. The second problem can be handled (at least crudely) with the approximation technique presented later in this chapter.

Figure 6.5 shows two hypothetical slopes each of which contains joints from two sets. In Figure 6.5A there are two joints from each set and they form two distinct wedges. If the wedges were independent and their $\overline{P}_f$'s were $\overline{P}_{f1}$ and $\overline{P}_{f2}$ respectively, P_f, the reliability of the entire slope, would be

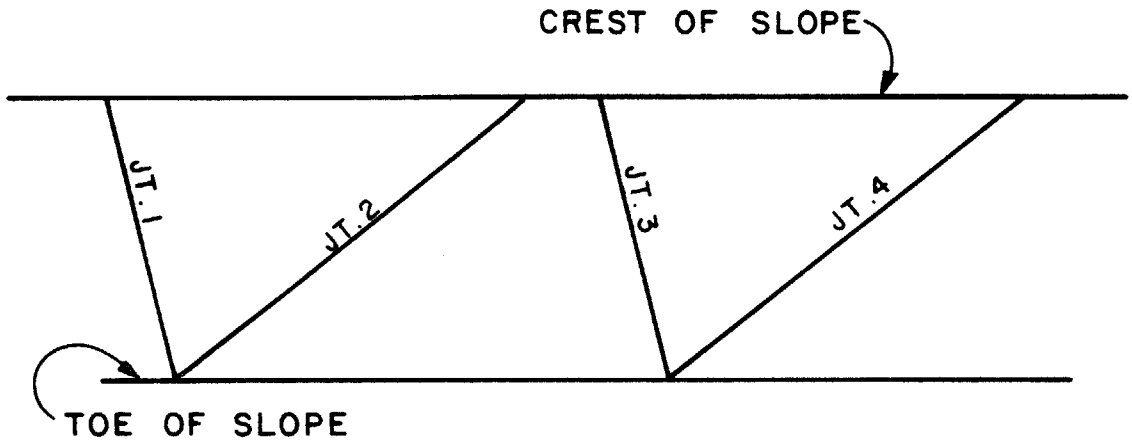
$$P_f = 1 - (1 - \overline{P}_{f1})(1 - \overline{P}_{f2}) \tag{6.5}$$

if $\overline{P}_{f1} = \overline{P}_{f2} = \overline{P}_f$:

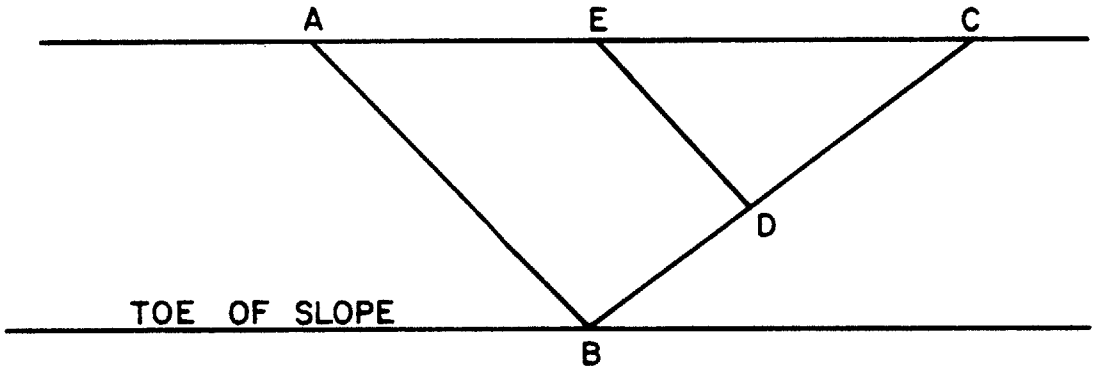
$$P_f = 1 - (1 - \overline{P}_f)^2 \tag{6.6}$$

On the other hand, if the wedges were perfectly correlated i.e. joints 1 and 3 (as well as the pair 2 and 4) were identical in orientation and strength parameters,

$$P_f = \overline{P}_f = (1 - \overline{P}_f)^1 \tag{6.7}$$



6.5 A



6.5 B

Figure 6.5 System Reliability of Slope

The fact that one wedge fails necessarily implies that its twin must fail.

Equations (6.6) and (6.7) represent two limiting cases. If the wedges were imperfectly correlated:

$$P_f = 1 - (1 - \bar{P}_f)^r \qquad (6.8)$$

where $1 < r < 2$

The argument can be extended to systems involving n wedges. The two limiting values for P_f would be:

$$P_f = 1 - (1 - \bar{P}_f)^n \quad \text{(independent wedges)} \qquad (6.9)$$

$$P_f = \bar{P}_f \quad \text{(perfect correlation)} \qquad (6.10)$$

There are at least two forms of correlation: spatial and functional. Spatial correlation implies that the variation in properties between two different joints is a function of the distance between the two joints. In other words, given joint A and its attendant properties, the marginal distribution of the properties of joint B depends on the distance between A and B. At the present time little is known about the spatial correlation of mechanical properties. They are generally assumed to be uncorrelated.

Even if the joints are not spatially correlated the wedges they form may be functionally correlated. Figure 6.5B shows a typical case. Wedges ABC and CDE are related because they are both bounded by the same joint plane. The $\overline{P}_f$'s for the wedges are also related; the degree of dependence depends on the relative importance of joint plane BDC in determining the stability of the wedge. If, for example, BCD contributes most of the resistance to both wedges, the $\overline{P}_f$'s may be highly correlated. If BCD contributes little to the respective resistances, the $\overline{P}_f$'s may be virtually uncorrelated.

As evidenced by Equations (6.9) and (6.10), correlation among wedges can have an enormous impact on reliability calculations. The difference between the upper bound (Equation (6.9)) and the lower bound (Equation (6.10)) of P_f can easily reach orders of magnitude in a heavily jointed rock mass i.e., large n. The system reliability of a slope is a complicated problem that depends on wedge geometries as well as the correlations among individual parameters. At the present time there is no rational method for assessing the system reliability of a rock slope. The solution to this problem is one of the crucial steps in developing a rigorous reliability analysis technique.

A conservative approach would be to assume independence and use the upper bound i.e.,

$$P_f = 1 - (1 - \bar{P}_f)^n \tag{6.9}$$

However, n is not a deterministic parameter; it is a random variable. The term $(1 - (1 - \bar{P}_f)^n)$ can be conditioned on n:

$$P_f = (1 - (1 - \bar{P}_f)^n)\ P[n] \tag{6.11}$$

where P[n] is the probability that the number of vertices is n.

Determining the pdf of n is no trivial matter. The following procedure illustrates how the problem might be approached. The procedure should provide a rough approximation of the pdf for some rather restrictive conditions. There are a number of assumptions:

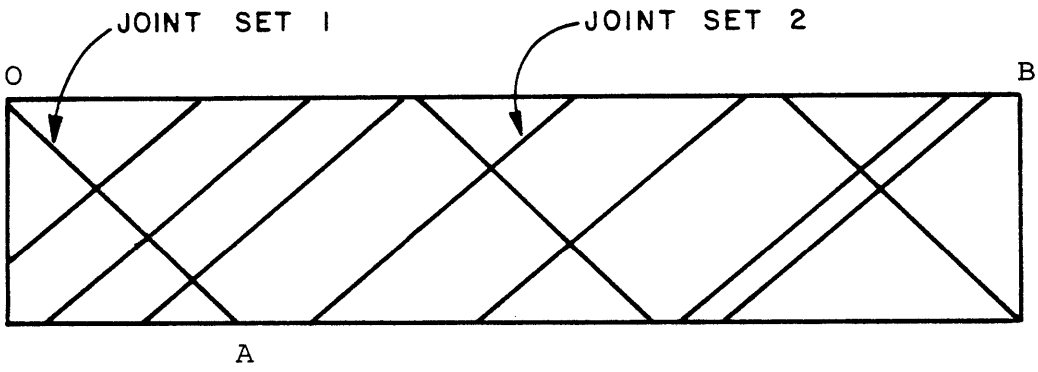
1. Joint planes are Poisson.
2. The pdf of joint plane orientation (for both joint sets) has a relatively small dispersion.
3. The joint sets are roughly orthogonal.
4. The slope is relatively lightly jointed.

The pdf for n can be approximated as follows (Figure 6.6):

ρ_1 = mean spacing of joint set 1 along the line OB

ρ_2 = mean spacing of joint set 2 along the line OA which is the trace of the mean joint from set 1.

x, the number of intersections of joints from set 2 with the line OA, has a Poisson distribution.



Distance O-A = L

Distance O-B = b

Figure 6.6 Derivation of $f_X(x)$ and $f_Y(y)$

$$f_X(x) = \frac{(L\rho_2)^x e^{-L\rho_2}}{x!} \tag{6.12}$$

where L is the distance along OA from the crest to the toe of the slope

y, the number of traces that appear on the slope also has a Poisson distribution:

$$f_Y(y) = \frac{(b\rho_1)^y e^{-b\rho_1}}{y!} \tag{6.13}$$

where b is the width of the slope.

Finally,

$$n \sim x \cdot y \text{ *} \tag{6.14}$$

Knowing the pdf's for X and Y one can compute the pdf for n. The approximation is crude and can become quite inaccurate for closely jointed rock masses. Equation (6.14) assumes that x, the number of intersections along any particular joint in set 1, is an independent random variable for all joints in set 1. If, however, two joints from set 1 are closely spaced (Figure 6.7) the number of intersections for the two joints are highly correlated.

* The derivation suggests that all joints must extend from the crest to the toe of the slope. The assumption can be justified if one treats joints as planar features with persistences less than 100% i.e., all joints in set 1 have a length L but their persistences vary. The persistence is a random variable that would be incorporated into the kinetic stability calculations.

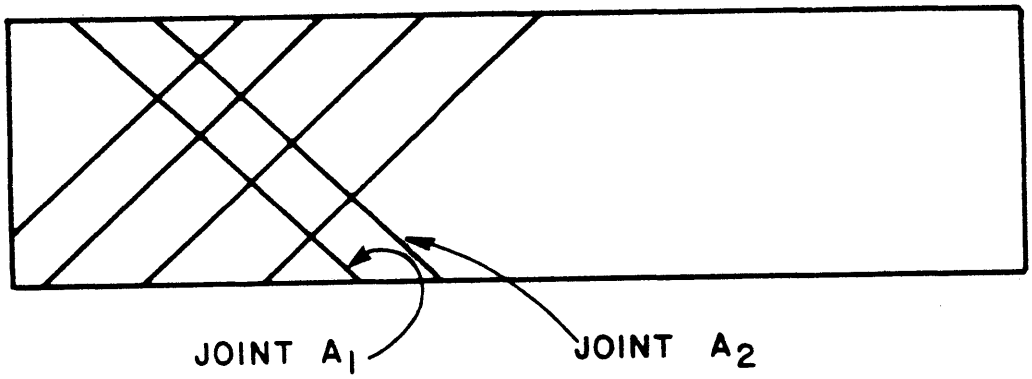


Figure 6.7 Closely Spaced Joints From Set 1--
Correlation Between x's

The procedure outlined above is not intended to serve as a design guide; it merely illustrates the concept. The method can (and should) be generalized to relax some of the limitations.* However, even a precise knowledge of $f_N(n)$ will not guarantee a reasonable estimate of P_f. $F_N(n)$ can be used to calculate the upper bound of P_f but provides no information as to the proximity of P_f to that upper bound. P_f cannot be accurately determined until the system reliability problem is resolved.

6.5 Summary

The purpose of this chapter was to show how the techniques developed in Chapters 3 to 5 could be incorporated into a reliability analysis. The techniques in themselves do not provide a complete treatment of the problem; however, they do represent an improvement over existing methods. In particular, the DAYLITE program provides information on the shape and orientation of potentially unstable wedges. The SWARS-2PM program gives some insight into the failure mode that involves sliding along two planes. Finally, the JOINTSIM program presents a rational method for evaluating the resistance of jointed rock masses.

There are still many problems in reliability analyses that are not well understood. As indicated in the earlier sections

* The method presented in this section was concerned with intersections between joints from different sets. Veneziano (19xx) used his model to examine the distribution of vertices when both of the intersecting joints come from the same set.

of this chapter, two related problems are particularly troublesome: possible correlations among random variables and a systems approach to the reliability of a multiply jointed rock slope. The problems are actually twofold: how can the correlation information be obtained from survey data and how can it be incorporated into the analysis? None of the current reliability analyses (including those presented in Chapter 2) directly address these questions. All the methods rely on assumptions (usually involving independence of parameters) to render the problem manageable. The P_f value calculated in each case in necessarily an approximation of the reliability of the slope. The credibility of P_f is directly related to the validity of the respective assumptions i.e., how closely they conform to reality. Unfortunately it is difficult to judge the validity of some of the assumptions at the present time. Many of the parameters used in rock slope stability have very limited data bases. It is hard to establish typical distributions or correlation coefficients for these variables. The statistical properties of these parameters should become more evident as additional data (and better procurement techniques) become available. The system reliability problem can be examined once the underlying correlation structure is found.

CHAPTER 7

SUMMARY AND CONCLUSIONS

Probabilistic analyses provide an alternate approach to assessing the stability of rock slopes. In a probabilistic or reliabilty analysis the integrity of the slope is expressed as its probability of failure rather than its factor of safety. The primary advantage of reliabilty techniques is that they explicitly consider the uncertainty associated with engineering parameters. The parameters are treated as random variables that can be characterized with probability distributions. The method is ideally suited for rock slope analysis because many of the critical parameters such as joint persistence are difficult to evaluate deterministically. These parameters must be considered random variables to arrive at any realistic conclusions concerning the stability of a rock slope.

Reliability analysis for rock slopes is still a relatively new concept. As indicated in Chapter 2, there are a number of analysis techniques currently available; however, none of the methods provide a comprehensive treatment of the problem. This thesis presents a new method that eliminates some of the limitations of earlier techniques. In particular, the thesis concentrates on a few specific points and develops some tools to use in examining slope stability from a probabilistic standpoint.

The method presented in this thesis uses a limiting equilibrium approach as both the rock wedge and its parent

rock mass are treated as rigid bodies. In such an approach, two conditions must be met before failure can occur:

1. The wedge must be oriented in such a manner that movement towards a free face is physically possible--a kinematic condition.
2. The destabilizing forces must exceed the stabilizing forces--a kinetic condition.

The problem can be formulated probabistically as:

$$P_f = \text{Probability of Failure}$$
$$= \Sigma\ P[A]P[B] \qquad (7.1)$$

where P[A] is the probability that a wedge is kinematically unstable and P[B] is the probability that the wedge is kinetically unstable. The product of the probabilities is summed over all potentially unstable conditions.

P[A] is determined through a numerical procedure that examines the zone of kinematic instability as defined on a stereographic projection. The technique partitions the zone into equal area cells and calculates the probability that the line of intersection of two joint planes will pierce the reference hemisphere in each cell. The technique also provides information on the shapes of possible tetrahedrons that are associated with each line of intersection.

P[B] is calculated through Monte Carlo simulation. The simulation is based on an improved version of the conventional

deterministic model for wedge stability. The conventional model was modified by incorporating the effects of joint stiffnesses and in situ stresses into the stability calculations. In a Monte Carlo simulation the input parameters must be characterized by probability density functions (pdf's). The pdf's for the orientation and shape of wedges can be derived from the algorithm that examines kinematic instability. The strength of a jointed rock mass (specifically, a rock mass with en echelon joints) can be described through an "apparent persistence" parameter which is obtained through a stochastic model discussed in the thesis.

P[A] and P[B] must be calculated for each cell in the zone of kinematic instability. The product summed over all cells is P_f--the probability that two joints will intersect to form an unstable wedge. P_f is conditioned on the premise that two and only two joints occur on the slope. The final step in the analysis is to "adjust" P_f to reflect the fact that the slope may contain many potentially unstable wedges. Evaluating the behavior of a multiply jointed rock mass is actually a problem in system reliabilty in which the entire slope is considered an engineering system. The problem is discussed briefly in Chapter 6.

As indicated earlier, this thesis addresses a number of specific points that must be considered in formulating a general method to evaluate the reliabilty of rock slopes. The techniques presented in the thesis do not, in themselves, constitute a complete analysis; however, they do clarify certain aspects of

the reliability problem. Each individual technique provides some new insight into the behavior of jointed rock masses. The cumulative effect of all the techniques is to advance the state of the art of rock slope stability analysis.

The procedure developed in Chapter 3 enables one to use the data from joint surveys i.e., the pdf's of joint orientation, to derive the pdf's of parameters that characterize the shape and orientation of tetrahedral wedges. The latter distribution is particularly useful because the orientation of the wedge is an input to the test for kinematic stability. The procedure involves a series a repetitive calculations because the pdf's are derived through numerical integration; however, the entire procedure has been programmed for use with a computer.

The thesis also presents an improved analytical model which describes the failure mode that consists of sliding along both joint planes. The model examines the influence of both in situ stresses and joint stiffnesses. As indicated in Chapter 4, earlier researchers had investigated the effects of both stresses and stiffnesses, but no one had established the relationship between the two. The model unifies the two approaches and extends the concepts to cover asymmetric wedges. The model distinguishes between two factor of safety: FS_I, the factor of safety against initial movement, and FS_U, the factor of safety against ultimate failure wherein the wedge slides down its line of intersection. In general, the initial

movement is not parallel to the line of intersection; it has a component into or out of the notch bounded by the joint planes.

The thesis also presents a possible solution to one of the major unsolved problems in rock stability--joint persistence. The stochastic model developed in Chapter 5 provides a rational method for treating joint persistence. The model uses Monte Carlo simulation to generate jointing patterns and then analyzes each pattern to identify the weakest failure path. The strength of the jointed rock mass is characterized by a random "apparent persistence" parameter that can be used in probabilistic stability calculations. Apparent persistence is not a measure of absolute strength, it is an index that compares the strength of the weakest path to the strength of paths through intact rock. Apparent persistence appears to be relatively insensitive to variations in the magnitude and orientation of the stress field; however, it is sensitive to the cohesion of intact rock as well as the distribution parameters of joint spacing and joint length.

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CISRM: Congress of the International Society for Rock Mechanics

USSRM: U.S. Symposium on Rock Mechanics

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Appendix A

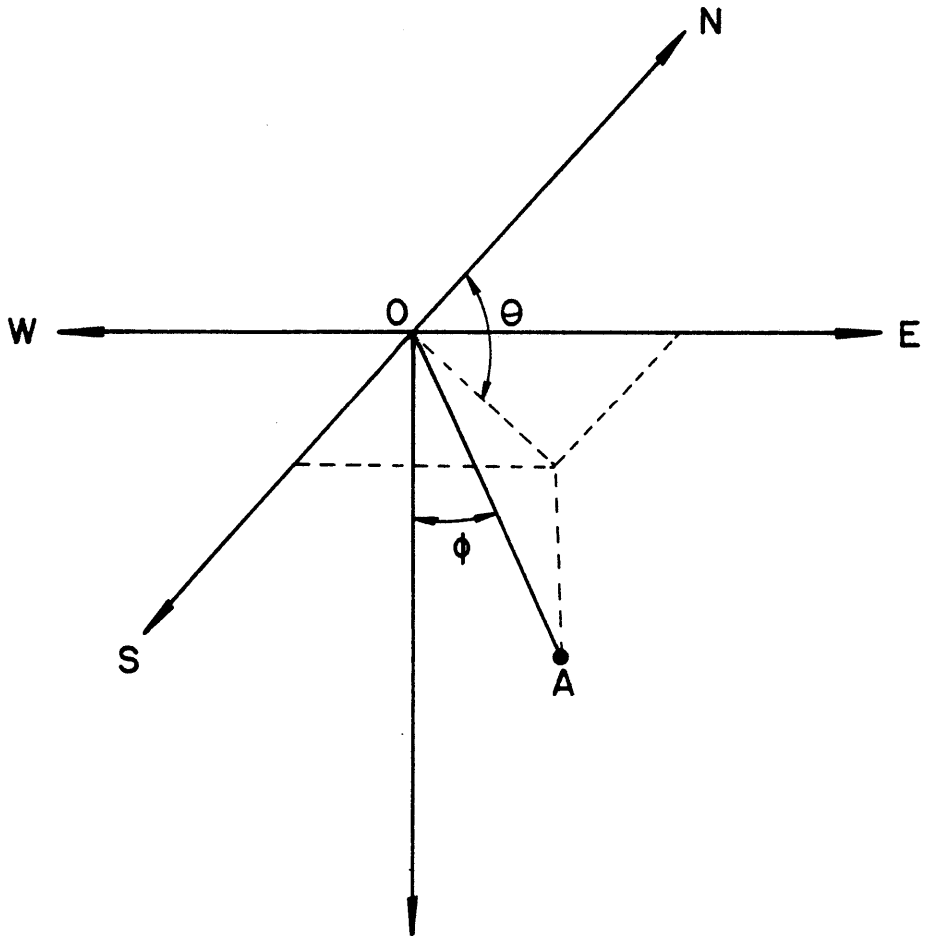
Spherical Distributions

A.0 Introduction

One of the inputs to DAYLITE is the pdf of joint orientation for each joint set. The user can choose from the six distributional forms described in this appendix. None of the six seem to be general enough to provide a good fit for any arbitrary compilation of real orientation data. The choice of an analytic form for a particular joint set should be based on the specific characteristics of its data base. Dershowitz (19xx) has developed a computer program that examines orientation data to find the best fitting distribution (and its requisite constants) from the six general forms.

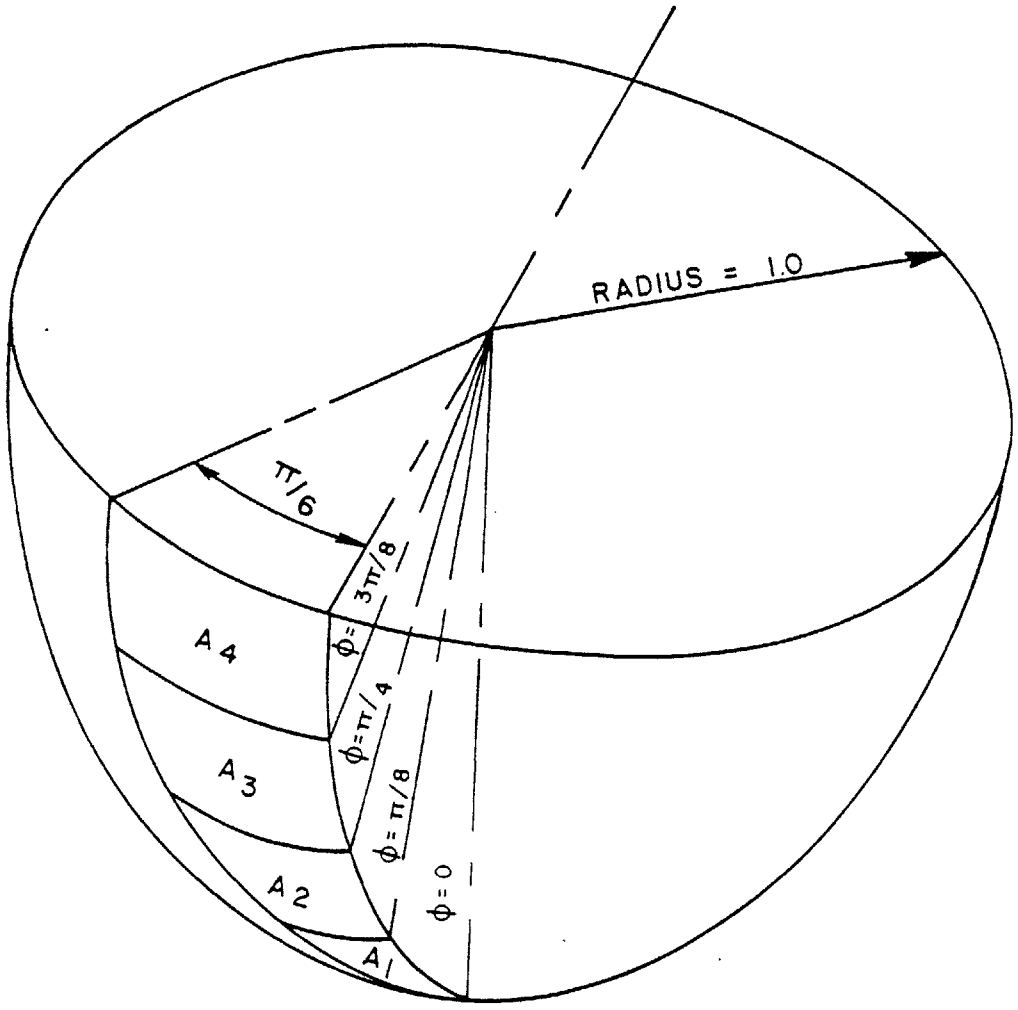
Figure A.1 shows the coordinate system used to describe orientation data. Joint planes are defined by the lower hemisphere projection of their poles. ϕ is the latitude while θ is the colatitude. θ is usually measured clockwise from North but other conventions are admissible.

Spherical coordinates are useful for describing joint orientations; however, they can become troublesome in interpreting or visualizing pdf's. The problem involves spherical surface areas: equal increments of latitude do not produce equal increments of surface areas. Figure A.2 depicts a spherical wedge of width $\frac{\pi}{6}$. The wedge is divided into four sections by small circles corresponding to ϕ values of $\frac{\pi}{8}$, $\frac{\pi}{4}$



OA = JOINT POLE

Figure A.1 Coordinate System For Joint Orientation



$A_1 = 0.040$

$A_2 = 0.114$

$A_3 = 0.170$

$A_4 = 0.200$

Figure A.2 Spherical Surface Areas

and $\frac{3\pi}{8}$. $\Delta\phi$ is a constant between any two circles, but the area between adjacent circles increases as ϕ increases.

Figure A.3 derives the general formula for the area of a spherical segment on a unit hemisphere:

$$A_s = 2\pi(1 - \cos \phi) \tag{A.1}$$

A_s is the area of the entire segment ($\theta = 2\pi$). A, the area of a wedge of width θ, is:

$$A = 2\pi(1 - \cos \phi)\frac{\theta}{2\pi} \tag{A.2}$$

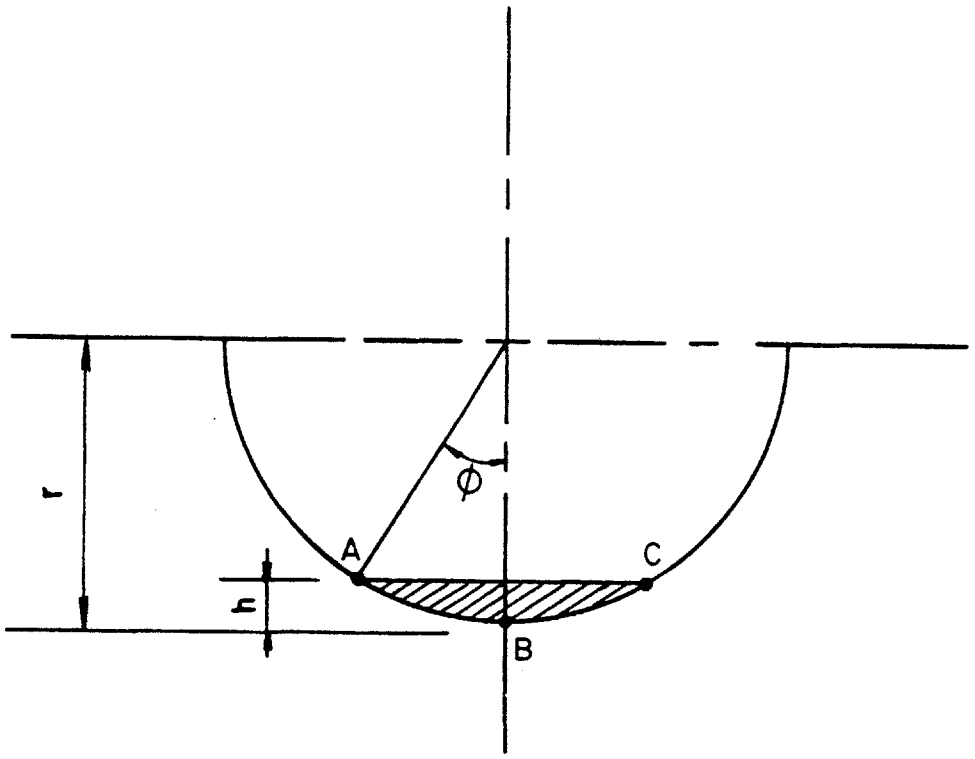
Taking the differential of each side:

$$dA = \sin \phi \, d\phi d\theta \tag{A.3}$$

Thus, a differential increment of surface area is not merely the product of $d\phi$ and $d\theta$. It is a function of ϕ i.e., where the differential area is located on the hemisphere.

Sin ϕ is a correction factor that compensates for the fact that differential spherical segments defined by constant $d\phi$'s do not have equivalent areas. The sin ϕ term appears in almost all the distributions discussed in this appendix. The lone exception is the bivariate normal which is actually defined on a plane rather than on a sphere.

The implications of the sin ϕ term become apparent when one examines specific pdf's. The simplest pdf to examine is



AREA OF SPHERICAL SEGMENT A B C =

$$2\pi r h = 2\pi r^2(1-\cos\phi)$$

FOR $r = 1$: AREA $= 2\pi(1-\cos\phi)$

Figure A.3 Surface Area of a Spherical Segment

the uniform distribution. In the uniform distribution every infinitesimal element on the surface of the hemisphere has an equal probability of containing a joint pole. The pdf has a constant value of $\frac{1}{2\pi}$ over the entire surface. Consider the marginal pdf of ϕ. By definition:

$$f(\phi) = \lim_{\Delta\phi \to 0} \frac{P[\phi - \Delta\phi < \phi < \phi + \Delta\phi]}{\Delta\phi} \qquad (A.4)$$

Table A.1 approximates $f(\phi)$ by dividing the domain of ϕ into 5° intervals and treating $f(\phi)$ as a constant over each interval. $P[\phi - \Delta\phi < \phi < \phi + \Delta\phi]$ is found by multiplying the area between the small circles defined by $\phi - \Delta\phi$ and $\phi + \Delta\phi$ by $1/(2\pi)$. (As explained earlier, this area increases with ϕ.) The results are plotted in Figure A.4 as a histogram. If the size of the interval were reduced to the point where $\Delta\phi$ approached zero, the histogram would become $f(\phi)$ -- the curve shown in Figure A.4.

$$f(\phi) = \sin \phi \qquad (A.5)$$

The fact that $f(\phi = 0) = 0$ does <u>not</u> imply that there is zero probability that a joint pole will be situated in the differential area located at the bottom of the hemisphere. That area, like any other area, has a probability of $\frac{dA}{2\pi}$. $\phi = 0$ is a singularity because that one coordinate completely defines the point; all other points require both a ϕ and a θ.

Table A.1 Numerical Approximation of $f(\phi)$--Uniform Distribution

Interval		$\Delta\phi$	Surface Area	Density of pdf	$P[\phi_{MIN} < \phi < \phi_{MAX}]$	$f(\phi)$
ϕ_{MIN}	ϕ_{MAX}		(10^{-1})		(10^{-2})	
0	5	5	0.239	$1/(2\pi)$	0.381	0.044
5	10	"	0.716	"	1.139	0.131
10	15	"	1.187	"	1.889	0.217
15	20	"	1.648	"	2.623	0.301
20	25	"	2.097	"	3.338	0.383
25	30	"	2.531	"	4.028	0.462
30	35	"	2.945	"	4.687	0.537
35	40	"	3.337	"	5.311	0.609
40	45	"	3.703	"	5.894	0.675
45	50	"	4.041	"	6.432	0.737
50	55	"	4.349	"	6.921	0.793
55	60	"	4.623	"	7.358	0.843
60	65	"	4.862	"	7.738	0.887
65	70	"	5.064	"	8.060	0.924
70	75	"	5.228	"	8.320	0.953
75	80	"	5.351	"	8.517	0.976
80	85	"	5.434	"	8.649	0.991
85	90	"	5.476	"	8.715	0.999
			$\Sigma = 2\pi$		$\Sigma = 1.00$	

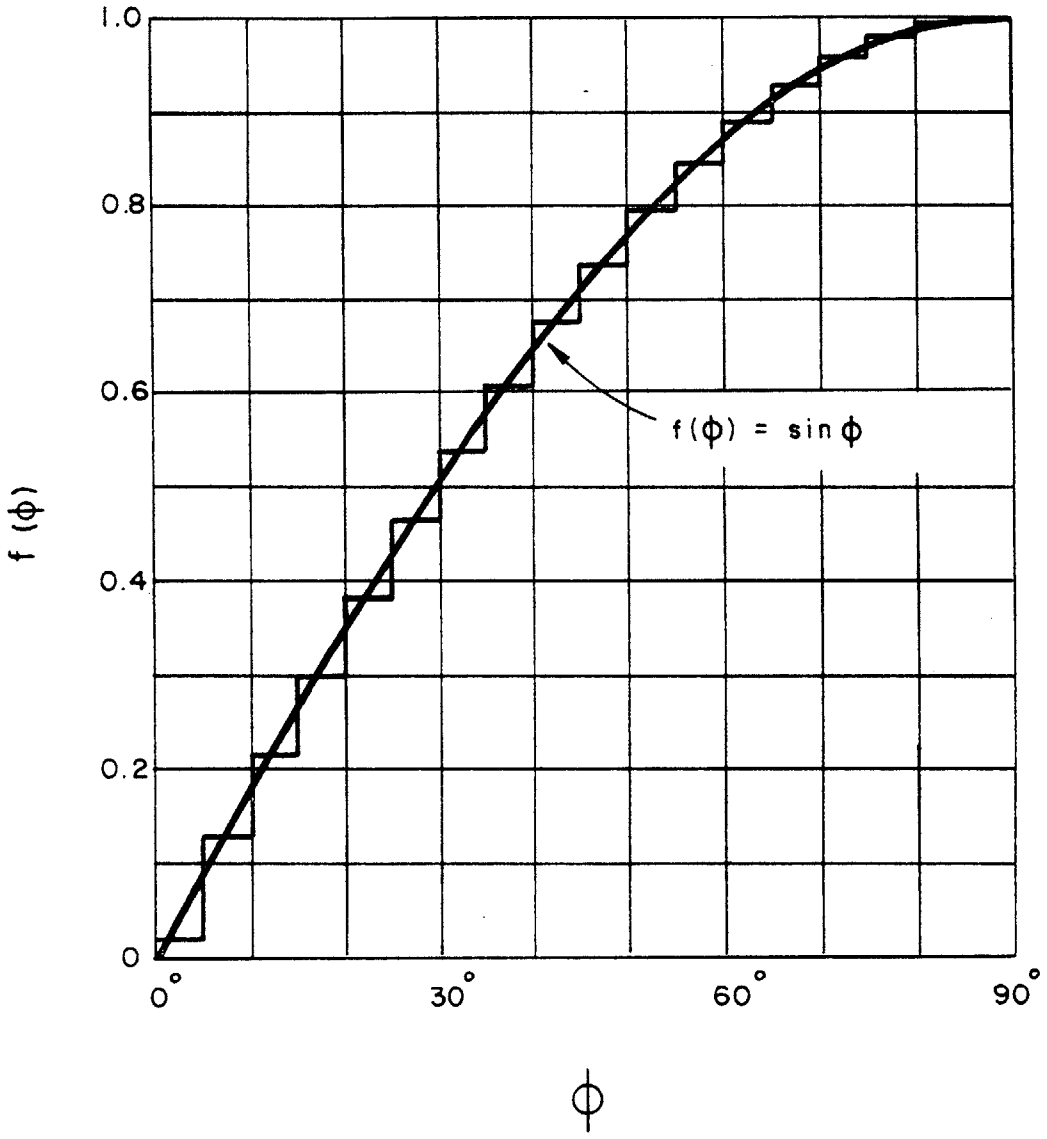


Figure A.4 $f(\phi)$ vs. ϕ for the Uniform Distribution

$\phi = 0$ is a unique point and the probability of finding a joint pole at that one point (or any other point) is zero. All other values of ϕ define small circles and there is a finite value of $f(\phi)$ for each circle. As ϕ increases the circles enlarge and $f(\phi)$ increases.

The remainder of the appendix will discuss the characteristics of specific distributions.

A.1 Uniform Distribution

The uniform distribution suggests that joint poles have no preferred orientation. The distribution has limited applicability with respect to real data but it does serve as a research tool because it represents the extreme limit of dispersion in orientation data.

As derived in the previous section:

$$f(\phi) = \sin \phi \tag{A.5}$$

Since ϕ and θ are independent random variables:

$$f(\phi,\theta) = f(\phi)f(\theta) = \frac{1}{2\pi} \sin \phi \tag{A.6}$$

Figure A.5 is a 3-dimensional representation* of the uniform distribution. It was developed by revolving the $\frac{1}{2\pi} \sin \phi$ curve through a θ angle of 2π about the $\phi = 0$ axis.

* For the sake of clarity, the curve is shown on an upper hemisphere. This same convention will be used for other pdf's in the appendix.

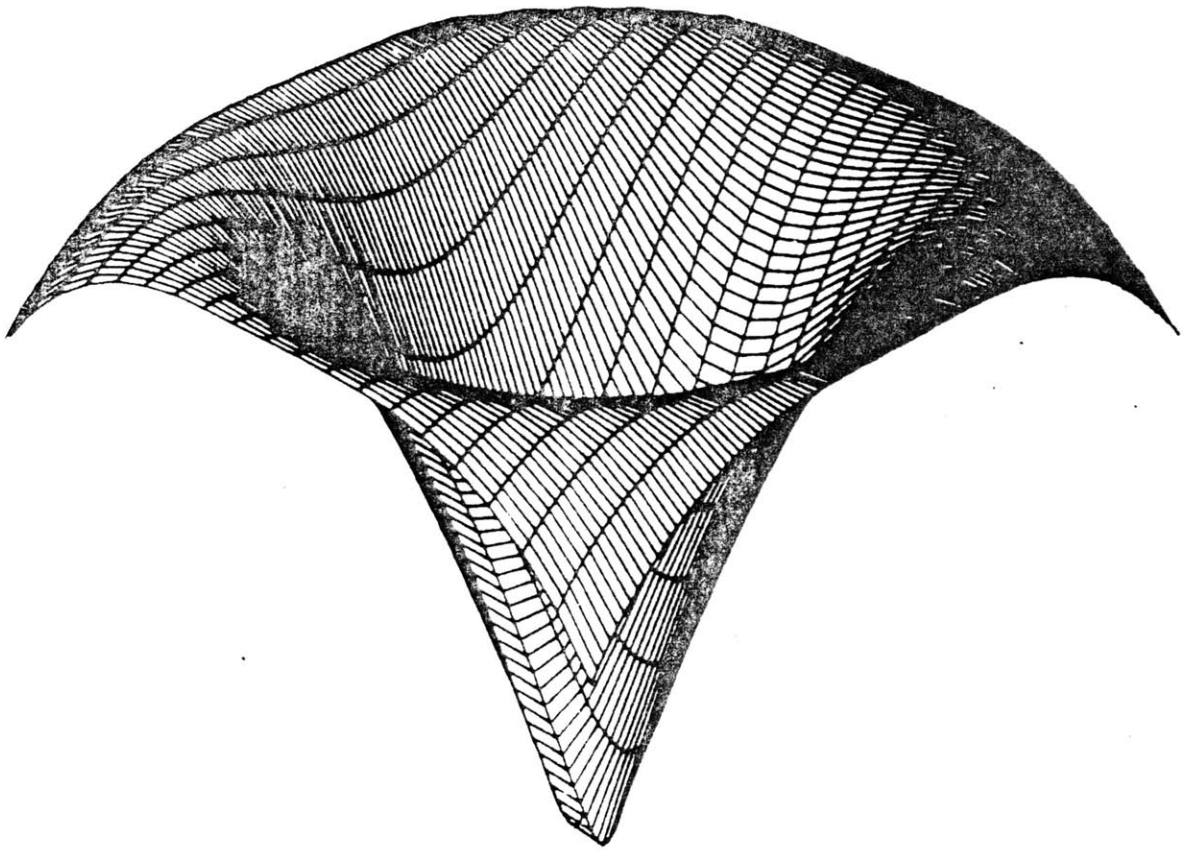


Figure A.5 Uniform Distribution

A.2 Fisher Distribution

The Fisher distribution is axially symmetric about its mean pole (θ_o, ϕ_o).

$$f(\theta,\phi) = \frac{k}{4\pi \sinh (k)} e^{k \cos \eta} \sin \eta \qquad (A.7)$$

k is a dispersion coefficient and η is the angle between (θ,ϕ) and (θ_o,ϕ_o)*. As indicated in Figure A.6, the density function begins at the origin, reaches a peak and then decays to 0 at high values of η. η can assume values up to π i.e., the distribution is defined over the entire sphere (not just a hemisphere). From a theoretical standpoint, the Fisher distribution is improper for joint data because pole diagrams only utilize a hemipshere i.e., $0 \leq \eta \leq \pi/2$. However, the distribution can be truncated at $\eta = \pi/2$ and normalized so that its integral over the hemisphere equals unity. λ, the normalizing constant, is the inverse of the cumulative distribution function at $\eta = \pi/2$.

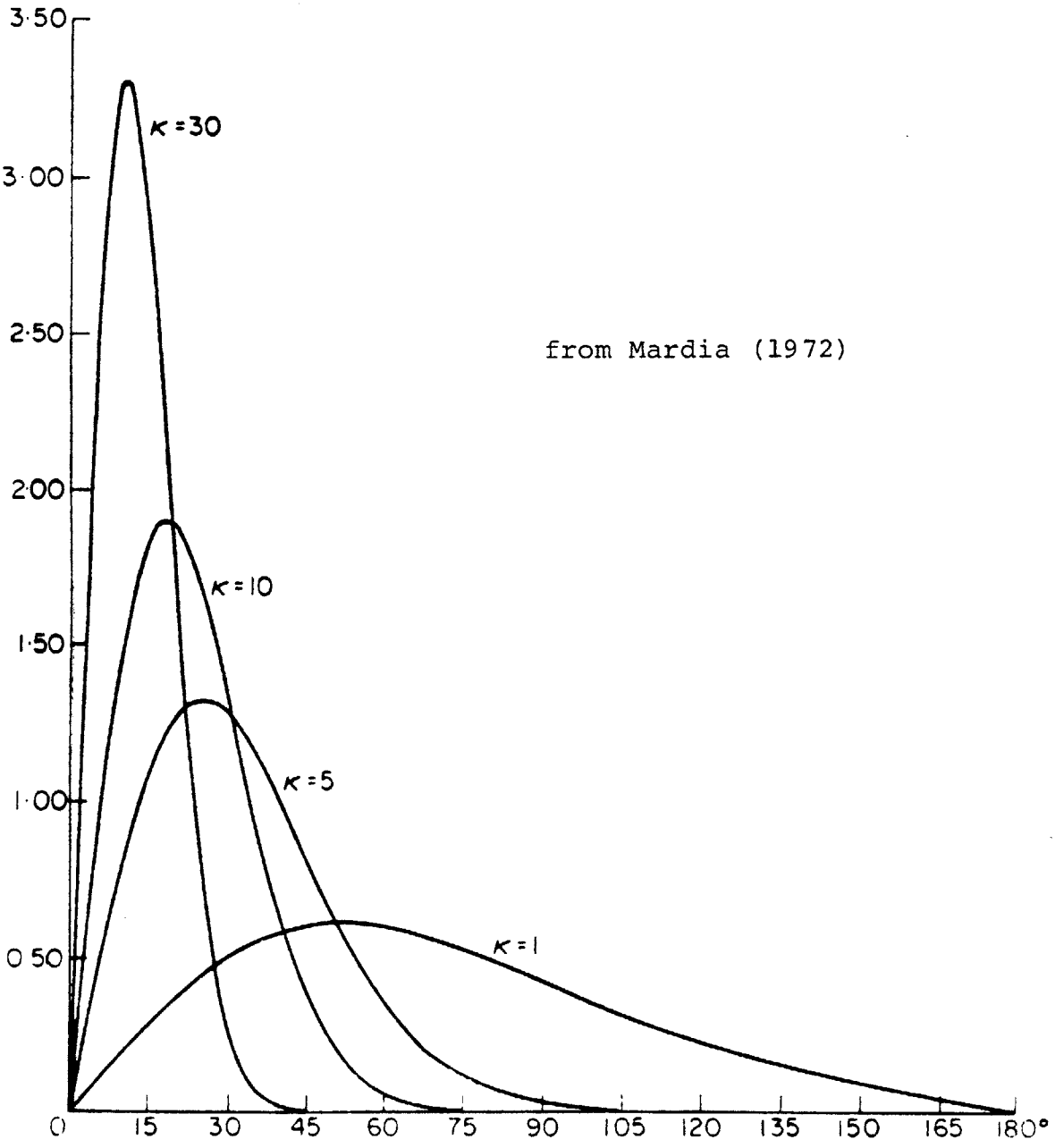
$$\lambda = \frac{1}{1 - e^{-2k}} \qquad (A.8)$$

λ does not appreciably affect the pdf for values of k greater than 3. (A k of 3 corresponds to a λ of 1.0025.)

Figure A.7 shows a Fisher distribution with a k of 5.

* η can be found through vector algebra:

$$\eta = \cos^{-1}\left((\theta_o,\phi_o) \cdot (\theta,\phi)\right)$$



from Mardia (1972)

Figure A.6 $f(\eta)$ vs. η for the Fisher Distribution

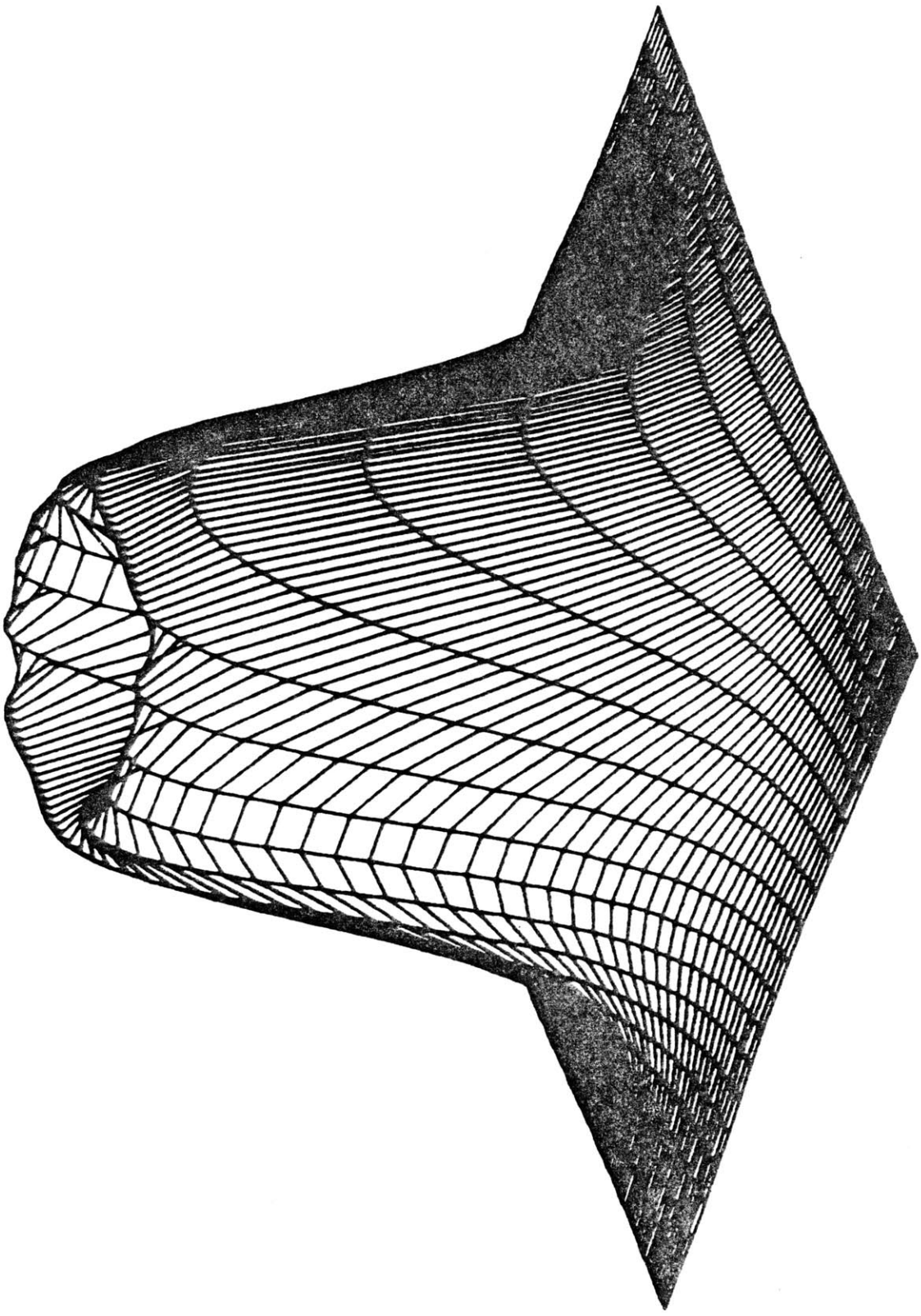


Figure A.7 Fisher Distribution (k = 5)

A.3 Arnold Distribution

The Arnold distribution is quite similar to the Fisher. Like the Fisher, it is axially symmetric about its mean pole; it does, however, have a slightly different form:

$$f(\theta,\phi) = \frac{k}{2\pi(e^{k} - 1)} e^{k \cos \eta} \sin \eta \qquad (A.9)$$

The Arnold distribution is antipodally symmetric* so only one half of the distribution is needed to describe orientation data. The normalizing constant is 2 and has been included in Equation (A.9).

Bridges (1977) pointed out that the Arnold and Fisher distributions are virtually identical for dispersion coefficients (k's) greater than 10.

A.4 Bingham Distribution

In general, the Bingham distribution is axially asymmetric; however, it is always symmetric with respect to two great circles termed the "major" and "minor" axes. The axes are orthogonal to each other as well as to the mean pole. The Bingham has five parameters: the orientation of the mean pole (θ_o,ϕ_o), the orientation of the major axis (θ_1 or ϕ_1)** and two dispersion coefficients (k_1 and k_2).

* Antipodal symmetry implies that the distribution is symmetric with respect to its equator.

** Since the mean pole and major axis are orthogonal, only θ_1 or ϕ_1 must be specified.

It is easier to work with the Bingham if the reference axes are rotated so that the new axes coincide with the major axis, minor axis and mean pole. (θ and ϕ define a direction with respect to the axes shown in Figure A.1; that same direction is described in the new system as θ^1 and ϕ^1.) The direction ($\theta^1 = 0$, $\phi^1 = 90$) corresponds to the major axis while $\phi^1 = 0$ corresponds to the mean pole. In this new reference system the pdf has the following form:

$$f(\theta^1,\phi^1) = \lambda e^{(k_1 \cos^2 \theta^1 + k_2 \sin^2 \theta^1) \sin^2 \phi^1} \sin \phi^1 \qquad (A.10)$$

where λ is a normalizing constant.

The Bingham is antipodally symmetric; thus, λ should reflect the fact that joint poles are plotted in only one hemisphere.

Figure A.8 depicts an anisotropic Bingham distribution ($k_1 = -5, k_2 = -3$).

A.5 Dershowitz Distribution

From an analytic standpoint, the Dershowitz distribution resembles the Bingham*. The Dershowitz distribution decays as cos ϕ rather than $\sin^2 \phi$. Like the Bingham, the Dershowitz involves a rotation of reference axes.

* From a probabilistic standpoint the Dershowitz distribution bears a closer resemblance to the Fisher than to the Bingham. Dershowitz (19xx) introduced the ($k_1 \cos^2 \theta^1 + k_2 \sin^2 \theta^1$) term to make the Fisher anistropic.

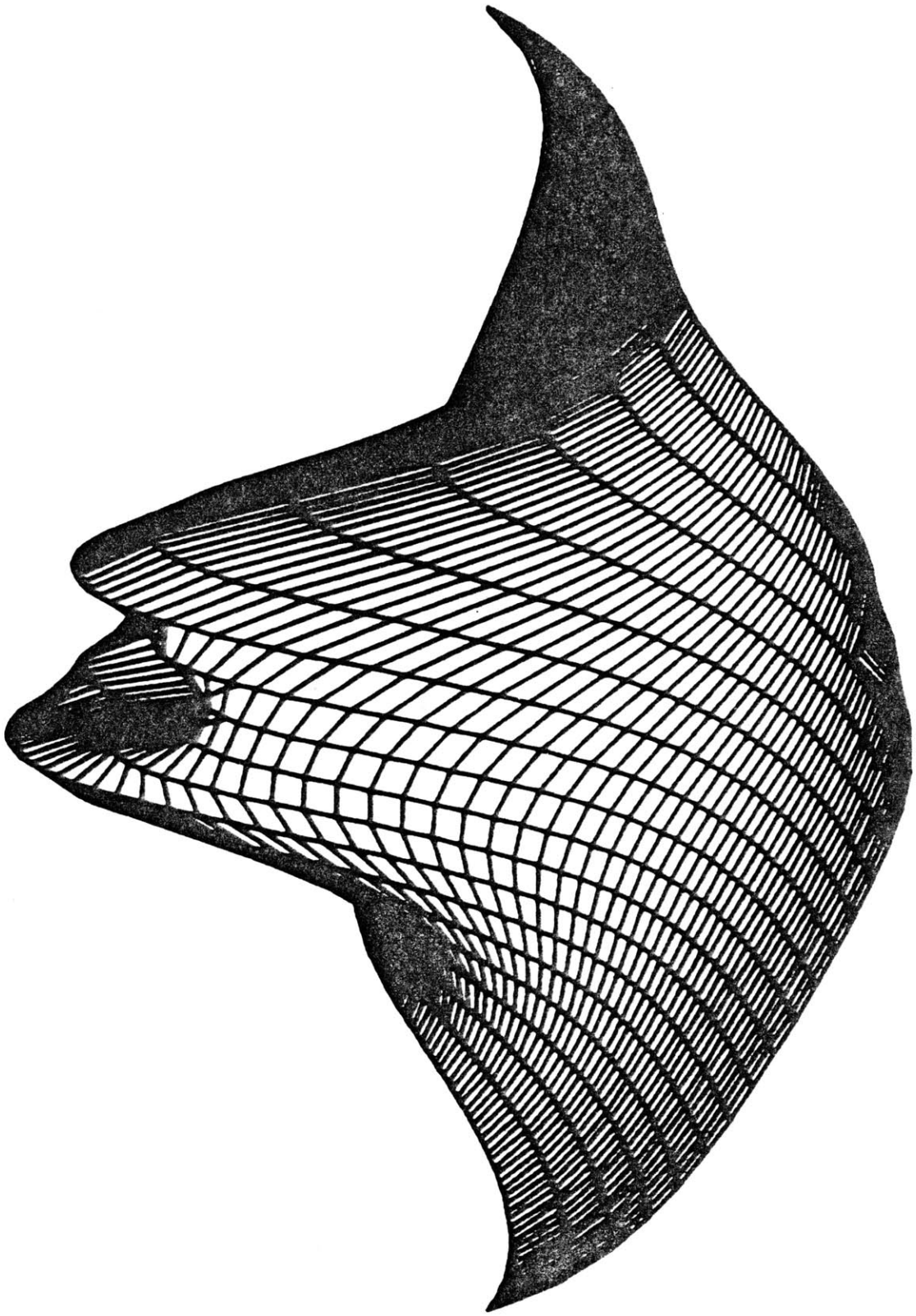


Figure A.8 Bingham Distribution ($k_1 = -5$; $k_2 = -3$)

$$f(\theta^1, \phi^1) = \lambda e^{(k_1 \cos^2 \theta^1 + k_2 \sin^2 \theta^1) \cos \phi^1 \sin \phi^1} \quad \text{(A.11)}$$

Figure A.9 shows a Dershowitz distribution that has the same dispersion coefficients (k_1 and k_2) as the Bingham in Figure A.8.

A.6 Bivariate Normal Distribution

The bivariate normal is the Gaussian or normal distribution that describes the interrelationship of two random variables.

The distribution is not defined in terms of the points on the surface of a reference hemisphere but rather in terms of the projection of the points on a reference plane*. The distribution is first rotated so that its mean pole lies at the bottom of the reference hemisphere (Figure A.10). Next, all the points on the hemisphere are projected on to a horizontal reference plane through the relations

$$x = \cos \theta \quad \text{(A.12)}$$

$$y = \sin \theta \quad \text{(A.13)}$$

Finally, the x and y axes are rotated so that $\bar{x}$ and $\bar{y}$ coincide with the two axes of symmetry -- the major and minor axes.

* The distribution can be defined directly on ϕ and θ. For example, see Steffen and Jennings (1974).

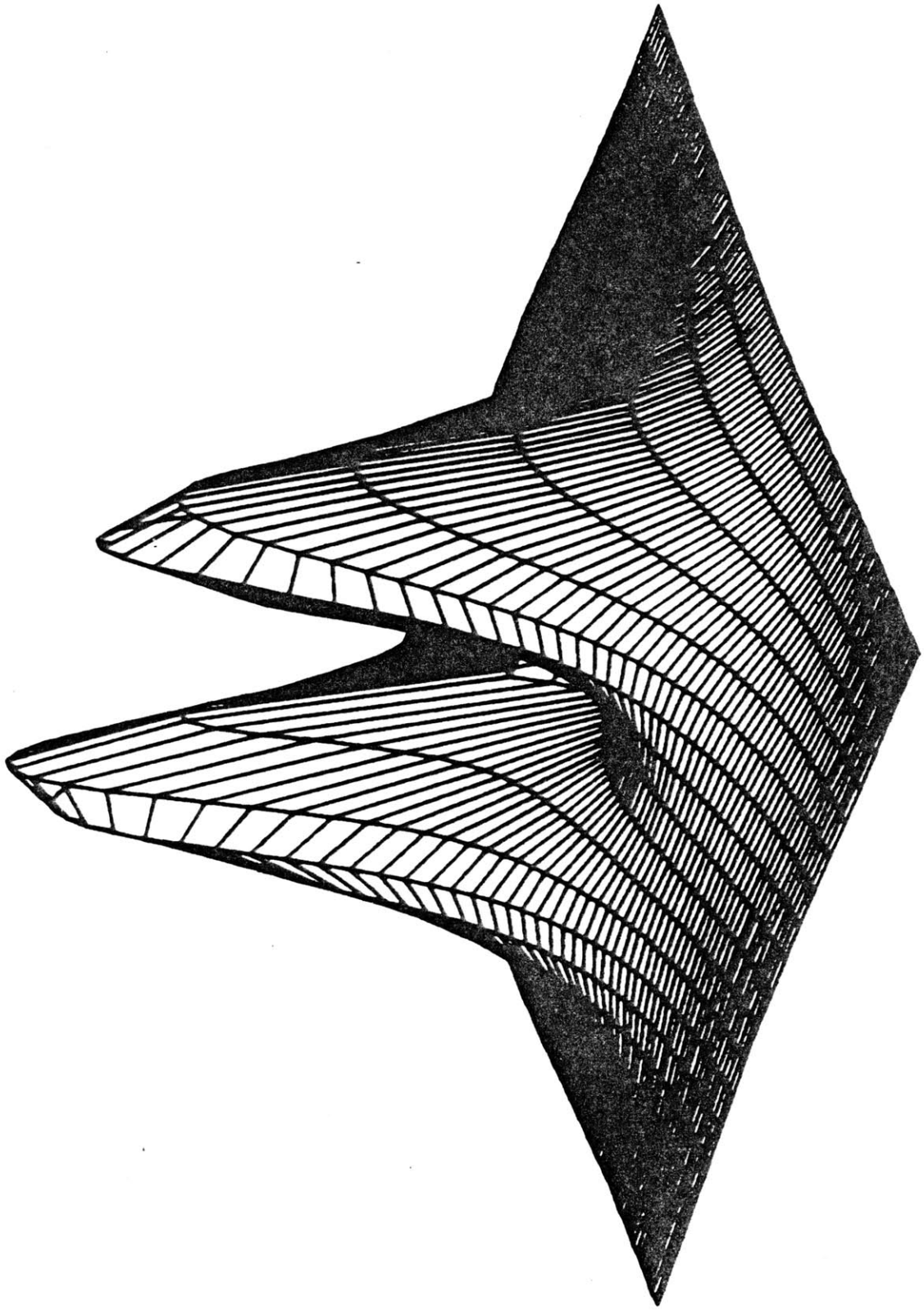


Figure A.9 Dershowitz Distribution ($k_1 = 5$; $k_2 = 3$)

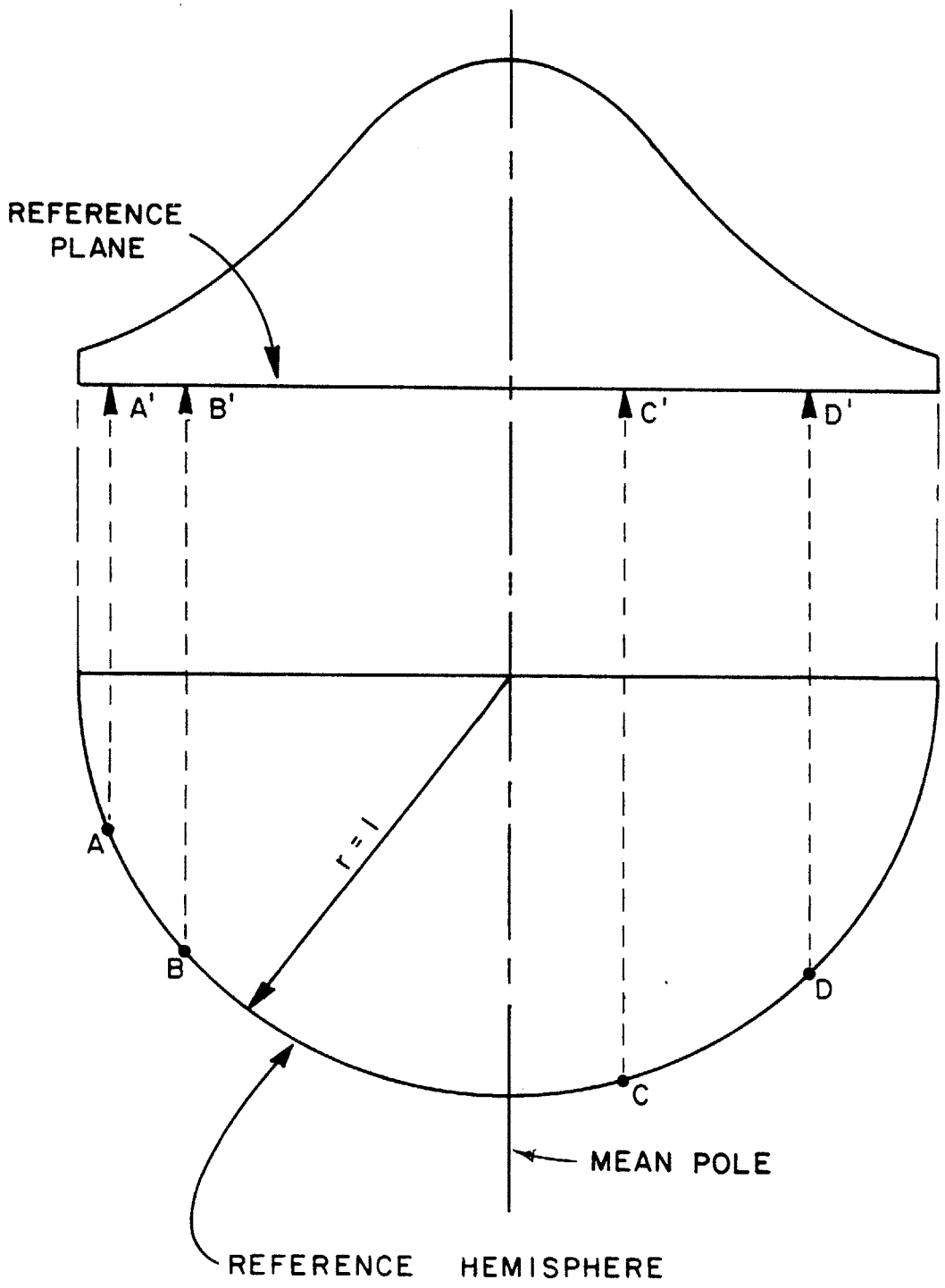


Figure A.10 Bivariate Normal Distribution--Projection on Plane

The distribution has the following form:

$$f(\bar{x},\bar{y}) = \frac{1}{2\pi\sigma_{\bar{x}}\sigma_{\bar{y}}} e^{-\frac{1}{2}\left\{\left(\frac{\bar{x}}{\sigma_{\bar{x}}}\right)^2 + \left(\frac{\bar{y}}{\sigma_{\bar{y}}}\right)^2\right\}} \qquad (A.14)$$

The bivariate normal distribution involves five parameters: the orientation of the mean pole (θ_o,ϕ_o), the orientation of the major axis (θ_1 or ϕ_1) and the variances in the $\bar{x}$ and $\bar{y}$ directions ($\sigma_{\bar{x}}^2$, $\sigma_{\bar{y}}^2$).

The bivariate normal is not an ideal distribution to use on orientation data because it assumes that both $\bar{x}$ and $\bar{y}$ are unbounded i.e., both $\bar{x}$ and $\bar{y}$ can range from $-\infty$ to ∞. As suggested in Figure A.10, the distribution must be truncated at the locus of points $x^2 + y^2 = 1$. The normalization procedure is tedious because there is no closed form solution for the cumulative distribution function of a normal distribution. The pdf must be integrated numerically over the appropriate limits.

A.7 Summary

Table A.2 summarizes the six distributions described in this appendix.

Table A.2 Summary of Distributions

Distribution	Parameters			Total Number of Parameters	Antipodal Symmetry
	Vectors		Scalars		
	Mean Pole	Major Axis			
Uniform	--	--	--	0	yes
Fisher	θ_o , ϕ_o	--	k	3	no
Arnold	θ_o , ϕ_o	--	k	3	yes
Bivariate Normal	θ_o , ϕ_o	θ_1 or ϕ_1	$\sigma^2_{\bar{x}}$, $\sigma^2_{\bar{y}}$	5	no
Bingham	θ_o , ϕ_o	θ_1 or ϕ_1	k_1 , k_2	5	yes
Dershowitz	θ_o , ϕ_o	θ_1 or ϕ_1	k_1 , k_2	5	no

Appendix B

Two Problems in Vector Algebra

This appendix solves two problems in vector algebra; the solutions are used in DAYLITE to locate points along the joint pole girdle (Section 3.2.4).

Problem 1

Given: the strike (x_1,y_1,o) and lower hemisphere pole (x_o,y_oz_o) of plane A.

Find: the point (x,y,z) on A where a line L with a rake angle η pierces the unit hemisphere. (Figure B.1)

$\overline{A}$,$\overline{B}$ and $\overline{C}$ are shown in Figure B.1 as the pole of the plane A, strike of plane A and direction of line L respectively. The three vectors have the following relationships:

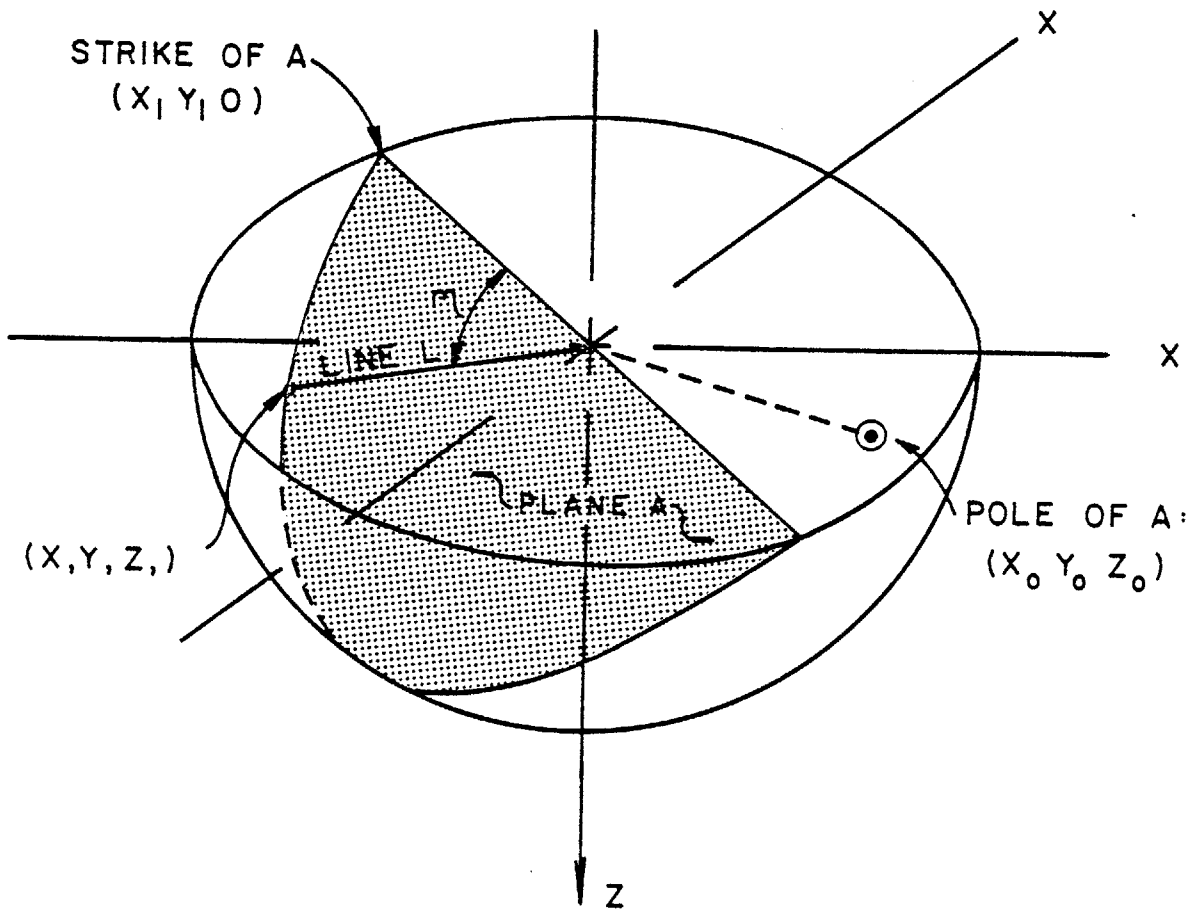
$$\overline{A} \cdot \overline{C} = x_o x + y_o y + z_o z = \cos \pi/2 = 0 \qquad (B.1)$$

$$\overline{B} \cdot \overline{C} = x_1 x + y_1 y = \cos \eta \qquad (B.2)$$

Also, by definition, $\overline{C}$ is a unit vector:

$$x^2 + y^2 + z^2 = 1 \qquad (B.3)$$

The problems can be solved by using the law of sines for spherical triangles. From Figure B.2:

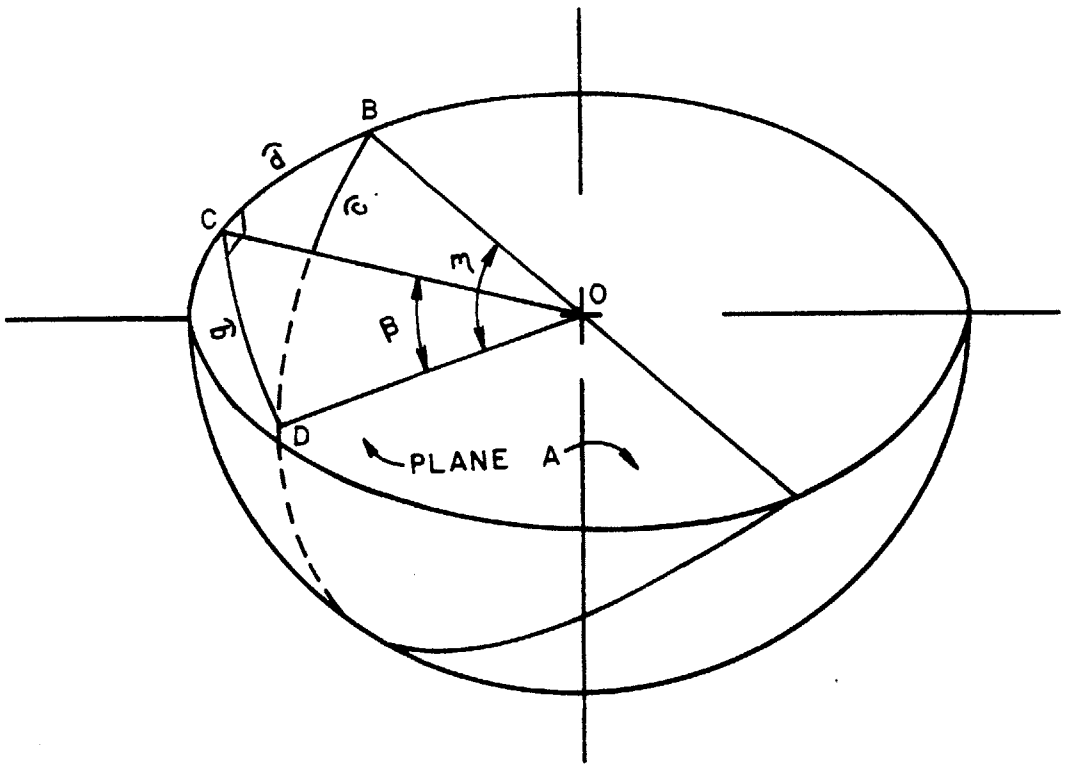


$A = (X_0\ Y_0\ Z_0)$

$B = (X_1\ Y_1\ 0)$

$C = (X\ Y\ Z)$

Figure B.1 Definition of η



SPHERICAL TRIANGLE BCD

B, C, D, : Spherical Angles at Corresponding Vertices

b, c, d, : Arc Lengths of sides

Law of Sines:

$$\frac{\sin B}{\hat{b}} \qquad \frac{\sin C}{\hat{c}} \qquad \frac{\sin D}{\hat{d}}$$

Figure B.2 Law of Sines For Spherical Triangles

$$\frac{\sin B}{\tilde{b}} = \frac{\sin C}{\tilde{c}} = \frac{\sin D}{\tilde{d}} \qquad (B.4)$$

B, C and D are the spherical angles at the corresponding vertices of spherical triangle BCD. $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ are the arc lengths (in radians) of the respective opposite legs of the triangle; they are measured by their angles subtended at the center O of the sphere e.g., $\tilde{c} = \eta$

By definition (Figure B.2):

$$C = \pi/2 \qquad (B.5)$$

$$B = \text{Dip of Plane A} = (A)_{Dip} \qquad (B.6)$$

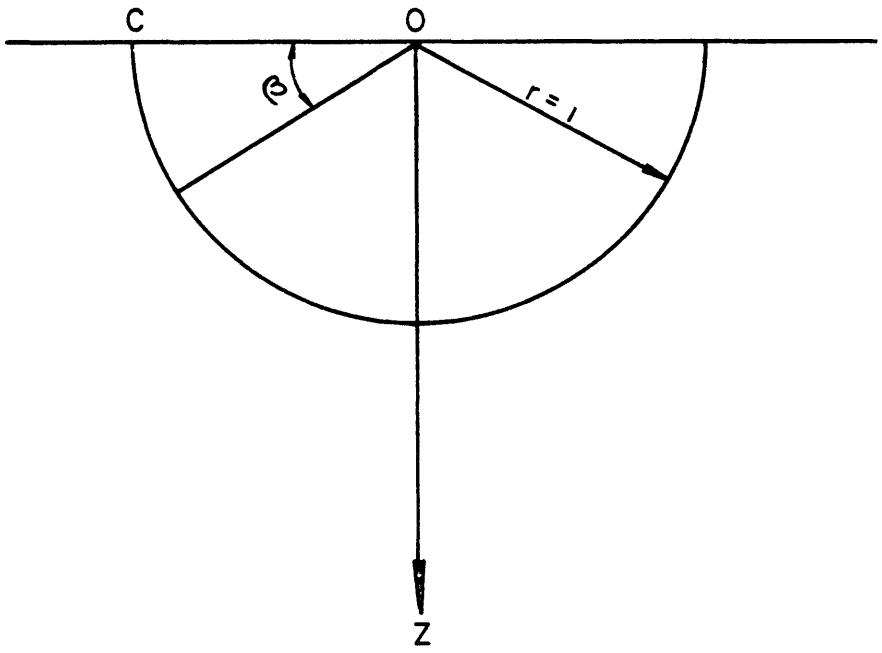
$$\frac{\sin C}{\tilde{c}} = \frac{1}{\eta} = \frac{\sin B}{\tilde{b}} = \frac{\sin (A)_{Dip}}{\tilde{b}} \qquad (B.7)$$

$$\tilde{b} = \sin (A)_{Dip} = \beta \qquad (B.7A)$$

Figure B.3 is a section along the line CO. From Figure B.3:

$$z = \sin \beta = \sin (\eta \sin (A)_{Dip}) \qquad (B.8)$$

From Equation B.2:



$$Z = \sin \beta$$

Figure B.3 Derivation of z

$$y = \frac{\cos \eta - x_1 x}{y_1} \qquad \text{(B.2A)}$$

Substituting into Equation B.3:

$$x^2 + \frac{(\cos \eta - x_1 x)^2}{y_1^2} + z^2 = 1 \qquad \text{(B.9)}$$

$$x^2 + \frac{\cos^2 \eta - 2x_1 x \cos \eta + x_1^2 x^2}{y_1^2} + z^2 = 1 \qquad \text{(B.10)}$$

Collecting terms:

$$x^2\left(1 + \frac{x_1^2}{y_1^2}\right) + x\left(\frac{-2x_1 \cos \eta}{y_1^2}\right) + z^2 - 1 + \frac{\cos^2 \eta}{y_1^2} = 0 \qquad \text{(B.10A)}$$

Equation (B.10A) can be solved with the quadratic formula.

$$x = \frac{-b \pm (b^2 - 4ac)^{\frac{1}{2}}}{2a} \qquad \text{(B.11)}$$

$$a = (1 + x_1^2 / y_1^2) \qquad \text{(B.12)}$$

$$b = (-2x_1 \cos \eta) / y_1^2 \qquad \text{(B.13)}$$

$$c = z^2 - 1 + \frac{\cos^2 \eta}{y_1^2} = \sin^2\left(\eta \sin(A_{Dip})\right) - 1 + \frac{\cos^2 \eta}{y_1^2} \qquad \text{(B.14)}$$

Finally, y can be found from Equation B.2A:

$$y = \frac{\cos \eta - xx_1}{y_1} \qquad \text{(B.2A)}$$

(The sign of the square root in Equation (B.11) can be determined by substituting the two roots into the relation $xx_o + yy_o + zz_o = 0$ i.e., $A \cdot \overline{C} = 0$. The root which fulfills the equation is the correct one.)

Problem 2

Given: the point B with coordinates (x_b, y_b, o) and a point D with coordinates (x_d, y_d, z_d) that is η radians from point B on a unit hemisphere.

Find: the point E with coordinates (x,y,z) that is η radians from B and $\delta/2$ radians from D (Figure B.4).

Points B,D and E are shown in Figure B.1. $\overline{B}$,$\overline{D}$ and $\overline{E}$ are the vectors from the origin to the respective points. The three vectors have the following relationships:

$$\overline{B} \cdot \overline{E} = x_b x + y_b y = \cos \eta \qquad \text{(B.15)}$$

$$\overline{D} \cdot \overline{E} = x_d x + y_d y + z_d z = \cos \delta/2 \qquad \text{(B.16)}$$

Also, by definition, E is a unit vector:

$$x^2 + y^2 + z^2 = 1 \qquad \text{(B.17)}$$

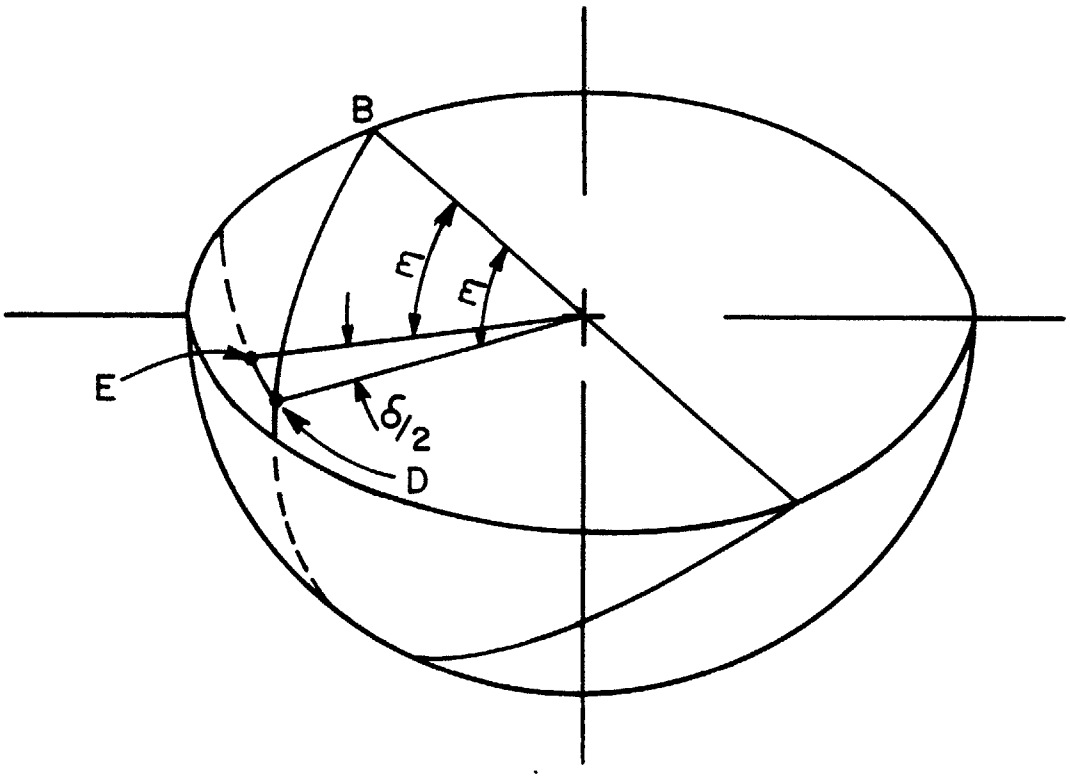


Figure B.4 Location of Points B, D and E

Multiplying Equation (B.15) by x_d and Equation (B.16) by x_b:

$$x_b x x_d + y_b y x_d = x_d \cos \eta \qquad \text{(B.15A)}$$

$$x_d x x_b + y_d y x_b + z_d z x_b = x_b \cos \delta/2 \qquad \text{(B.16A)}$$

Subtracting:

$$y(y_b x_d - y_d x_b) - z_d z x_b = x_d \cos \eta - x_b \cos \delta/2 \qquad \text{(B.18)}$$

Letting $P = x_d \cos \eta - x_b \cos \delta/2$ and solving for y;

$$y = \frac{z_d z x_b + P}{(y_b x_d - y_d x_b)} \qquad \text{(B.18A)}$$

Multiplying Equation (B.15) by y_d and Equation (B.16) by y_b:

$$x_b x y_d + y_b y y_d = y_d \cos \eta \qquad \text{(B.15B)}$$

$$x_d x y_b + y_d y y_b + z_d z y_b = y_b \cos \delta/2 \qquad \text{(B.16B)}$$

Subtracting:

$$x(x_b y_d - x_d y_b) - z_d z y_b = y_d \cos \eta - y_b \cos \delta/2 \qquad \text{(B.19)}$$

Letting $q = y_d \cos \eta - y_b \cos \delta/2$ and solving for x:

$$x = \frac{z_d z y_b + q}{(x_b y_d - x_d y_b)} \qquad \text{(B.19A)}$$

Defining r as

$$r = \frac{1}{(y_b x_d - y_d x_b)^2} = \frac{1}{(x_b y_d - x_d y_b)^2} \qquad \text{(B.20)}$$

Substituting Equations (B.18A), (B.19A) and (B.20) into Equation (B.17):

$$r(z_d z x_b + P)^2 + r(z_d z y_b + q)^2 + z^2 = 1 \qquad \text{(B.21)}$$

$$r P^2 + r2P z_d z x_b + r\, z_d^2 z^2 x_b^2$$

$$+ rq^2 + r2q z_d z y_b + r z_d^2 z^2 y_b^2$$

$$+ z^2 - 1 = 0 \qquad \text{(B.21A)}$$

Grouping terms:

$$z^2\left[r z_d^2 (x_b^2 + y_b^2) + 1\right] + z\left[2r z_d (P x_b + q y_b)\right] + (rP^2 + rq^2 - 1)$$

$$= 0 \qquad \text{(B.21B)}$$

$$\text{Since } z_b = 0,\ x_b^2 + y_b^2 = 1 \qquad \text{(B.22)}$$

Thus, Equation (B.20) can be simplified to:

$$z^2(rz_d^2 + 1) + z\left[2rz_d(Px_b + qy_b)\right] + (rp^2 + rq^2 - 1) = 0 \qquad \underline{\text{(B.21C)}}$$

Equation (B.21C) can be solved with the quadratic formula:

$$z = \frac{-b \pm (b^2 - 4_{ac})^{\frac{1}{2}}}{2a} \qquad \text{(B.23)}$$

$$a = rz_d^2 + 1 \qquad \text{(B.24)}$$

$$b = 2rz_d(Px_b + qy_b) \qquad \text{(B.25)}$$

$$c = rp^2 + rq^2 - 1 \qquad \text{(B.26)}$$

x and y can be found by substituting z into Equations (B.19A) and (B.18A) respectively.

The positive and negative roots of Equation (B.23) correspond to solutions for point E $\delta/2$ radians above point D and $\delta/2$ radians below point D.

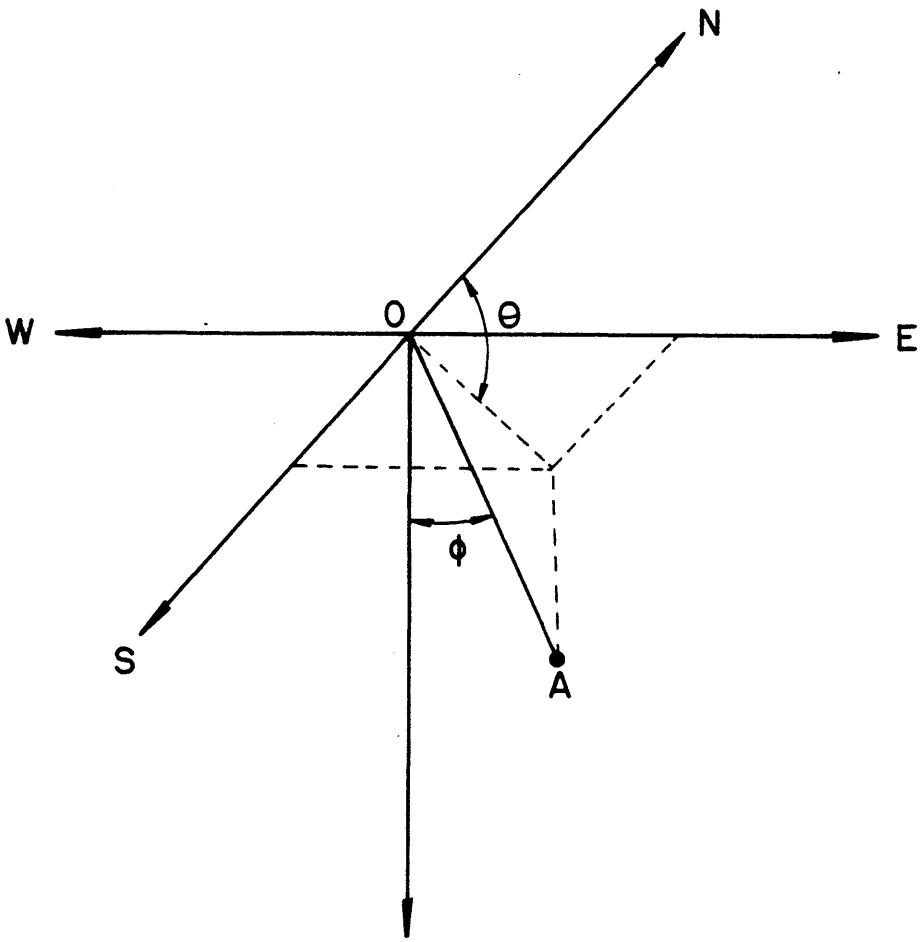
Appendix C

Computer Program DAYLITE

DAYLITE calculates the probability density function (pdf) for the orientation of the line of intersection of two joint planes. The orientations of the planes are characterized in terms of the spherical distributions discussed in Appendix A. The program uses the numerical integration technique that is presented in Chapter 3. The primary input parameters to the program are the pdf's of the two joint sets and the orientation of the slope face. The output consists of the various probabilities defined in Section 3.2.7:

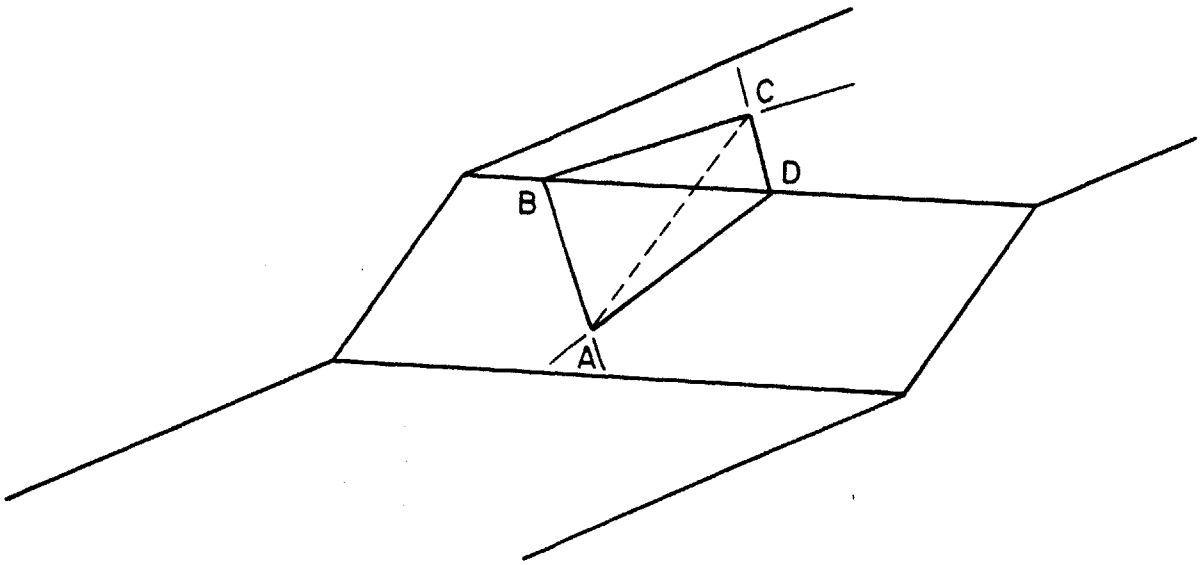
> $P[\theta_i,\phi_i]$ - the probability that a wedge (regardless of its shape) will have a line of intersection oriented in the (θ_i,ϕ_i) direction.
>
> $P[\theta_i,\phi_i,\psi_j]$ - the probability that a wedge with a central angle of ψ_j (regardless of β_1) will have a line of intersection oriented in the (θ_i,ϕ_i) direction.
>
> $P[\theta_i,\phi_i,\psi_j,(\beta_1)_k]$ - the probability that a wedge with a central angle of ψ_j and a β_1 of $(\beta_1)_k$ will have a line of intersection oriented in the (θ_i,ϕ_i) direction.

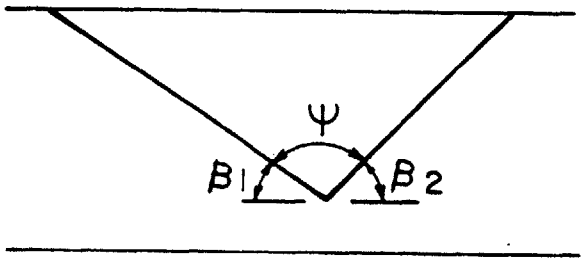
θ,ϕ,ψ and β_1 are defined in Figures C.1 and C.2.



OA = JOINT POLE

Figure C.1 Definitions of θ and ϕ





SECTION PERPENDICULAR TO AC

Figure C.2 Definitions of Ψ and β_1

C.1 Coordinate System

All orientations are expressed in geologic notation: strike and dip for a plane; strike and plunge for a line. The directions of the lines of intersection are also defined in terms of the x,y,z coordinate system shown in Figure C.3. The computer program SWARS-2MC (Appendix F) uses this same Cartesian system so that the output from DAYLITE will be compatible with the input to SWARS-2MC.

C.2 Input

Unless otherwise noted the inputs do not have to be right justified in their respective columns. The term "integer" implies that the number is expressed without a decimal point.

Angular input is always in degrees.

Card #1

Strike of the Slope (S1S, STRS, S2S) is entered in geologic notation in columns 1 to 6. Geologic strike must be expressed in the northern quadrants and the angle is expressed with one decimal place. (Example: N45.0E)

Dip of the Slope (DIPS, D1S, D2S) is entered in geologic notation in columns 11 to 16. Dip directions must indicate a particular quadrant i.e., NE, NW, SE, SW. The angle is expressed with one decimal place. (Example: 36.ONE)

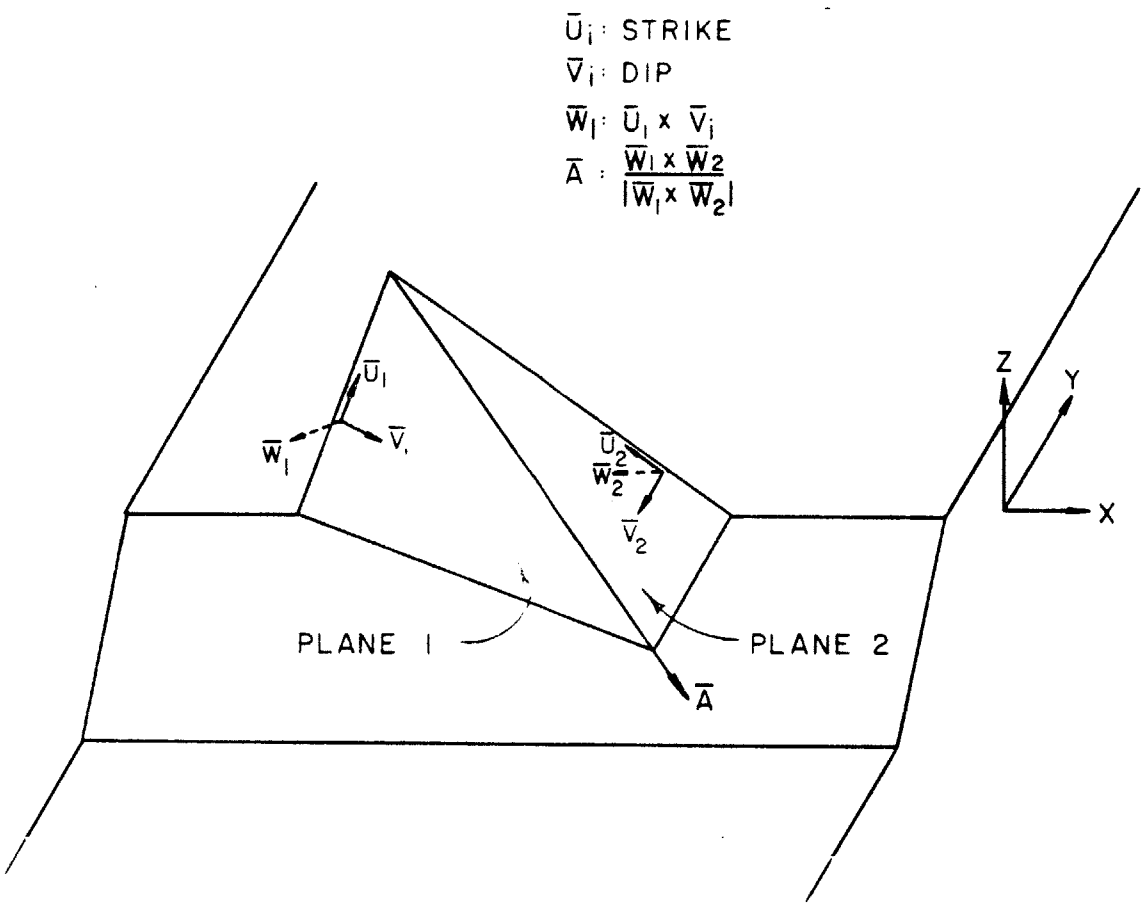


Figure C.3 Orientation of X, Y and Z Axes

Card #2

Card #2 inputs the analysis criteria discussed in Section 3.3.

ε (DEGN) is an angular dimension which controls the size of the cells in the critical zone. It is entered in columns 1 to 10 with a maximum of 2 decimal places.

α (FRIC) is an angular dimension which enables the user to shorten the analysis by ignoring lines of intersection with very shallow plunges. The program will not examine any lines of intersection that plunge at an angle less than α (Section 3.2.1). FRIC is entered in columns 11 to 20 with a maximum of two decimal places. The program will default to FRIC = 0.0 if those columns are left blank.

$\overline{\psi}$ (DEGMAN) is an angular dimension that enables the user to ignore very narrow wedges. The program will not examine any wedges that have a central angle less than $\overline{\psi}$ (Section 3.3). DEGMAN is entered in columns 21 to 30 with a maximum of 2 decimal places. The program will default to DEGMAN = DEGN if those columns are left blank.

$\Delta\psi$ (WEDINC) is an angular dimension that enables the user to consolidate the output into ranges of β_1 angles rather than individual β_1 angles (Section 3.3). WEDINC is entered in columns 31 to 40 with a maximum of 2 decimal places. The program will default to WEDINC = DEGN if those columns are left blank.

Card #3

The first two parameters on card #3 specify the distributional forms* of the two joint sets. The code is as follows:

0 : Uniform Distribution

1 : Fisher Distribution

2 : Arnold Distribution

3 : Bingham Distribution

4 : Dershowitz Distribution

5 : Bivariate Normal Distribution

Pdf Code for Joint Set 1 (NDIST1) is entered as an integer in column 10.

Pdf Code for Joint Set 2 (NDIST2) is entered as an integer in column 20.

Wedge Code (NCODE) determines which sets of joints should be combined to form wedges. If a '1' is entered in column 30 the program will calculate the pdf of the line of intersection of wedges formed by two joints from set 1. If a '2' is entered in column 30 the program will calculate the pdf of wedges formed by two joints from set 2. If a '3' is entered in column 30 the program will calculate the pdf of wedges formed by a joint from set 1 and a joint from set 2. If any integer greater than 3 is entered in column 30 the program will examine all three cases.

* The six distributions are discussed in Appendix A.

Cards #4 and #5

Cards #4 and #5 must contain all the parameters required to analytically define the pdf's specified on Card #3. Card #4 contains information relating to joint set 1 while Card #5 contains information relating to joint set 2. The information on the cards depends on the respective pdf's:

Uniform Distribution: The uniform distribution is nonparametric so the program needs no additional information. The input card i.e., Card #4 or Card #5 should be eliminated. (If joint set 1 has a uniform distribution the program will use the parameters on Card #4 to define joint set 2.)

Fisher Distribution: The Fisher distribution has the following form:

$$f(\theta,\phi) = \frac{K}{4 \sinh (K)} e^{K \cos \eta} \sin \eta \qquad (C.1)$$

η is the angle between the mean pole of the distribution and the (θ,ϕ) direction. The distribution requires three input parameters - one is K, the dispersion coefficient, the other two define the orientation of the mean plane.

Mean Strike: (FA1S, STRFA, FA2S) is entered in geologic notation in columns 1 to 6 of Cards #4 and #5. Geologic strike must be expressed in the northern quardrants

---

* The program will compute the normalizing constant to ensure that the integral of the distribution over the surface of the hemisphere is unity.

and the angle is expressed with one decimal place. (Example: N45.0E)

Mean Dip: (DIPFA, D1FA, D2FA) is entered in geologic notation in columns 11 to 16 of Cards #4 and #5. Dip directions must indicate a particular quadrant i.e., NE, NW, SE, SW. The angle is expressed with one decimal place. (Example: 36.0NE)

Dispersion Coefficient: (FAK) is entered in columns 21 to 30 of Cards #4 and #5 with a maximum of 5 decimal places.

Arnold Distribution: The Arnold distribution is very similar to the Fisher:

$$f(\theta,\phi) = \frac{K}{2\ (e^K - 1)}\ e^{K \cos \eta}\ \sin \eta \qquad (C.2)$$

Like the Fisher, the Arnold has three parameters: a dispersion coefficient and two parameters to define the orientation of the mean plane. The format for inputting the parameters is identical to that of the Fisher distribution.

Bingham Distribution: The Bingham distribution has the following form:

$$f(\theta',\phi') = K_1 e^{(K_2 \cos^2 \theta' + K_3 \sin^2 \theta')\ \sin^2 \phi'\ \sin \phi'} \qquad (C.3)$$

where θ' and ϕ' are the colatitude and latitude in the spherical coordinate system in which $\phi' = 0$ is the mean pole and

($\theta' = 0$, $\phi' = 90°$) is the major axis. In general, the (θ',ϕ') system will not coincide with the conventional coordinate system ie., the (θ,ϕ) system shown in Figure C.1. The program develops transformation relations so that the pdf in the direction expressed in the (θ,ϕ) system can be evaluated through Equation (C.3). The input parameters include three coefficients (K_1, K_2 and K_3 as shown in Equation (C.3)), the orientations (in the (θ,ϕ) system) of the mean plane and the major plane. The major plane is the plane whose pole forms the major axis.

Mean Strike: (B1S, BS, B2S) is entered in geologic notation in columns 1 to 6 of cards #4 and #5. Geologic strike must be expressed in the northern quadrants and the angle is expressed with one decimal place. (Example: N45.OE)

Mean Dip: (BD, B1D, B2D) is entered in geologic notation in columns 11 to 16 of cards #4 and #5. Dip directions must indicate a particular quadrant i.e., NE, NW, SE, SW. The angle is expressed with one decimal place. (Example: 36.ONE)

Major Strike:* (AB1S, BAS, AB2S) is entered in columns 21 to 26 of cards #4 and #5 in a manner similar to the mean strike.

Major Dip:* (ABD, ABD1, ABD2) is entered in colums 31 to 36 of cards #4 and #5 in a manner similar to the mean dip.

* Since the mean pole and major axis are orthogonal, either the major strike or the major dip (in addition to the mean pole) will completely define the reference axes. As a precaution against inadvertent errors the user must input both the major dip and strike. The program will check that the input is consistent.

K_1: (DISCO (I,2)) is a normalizing constant that is entered in columns 41 to 50 of Cards #4 and #5 with a maximum of 3 decimal places. In selecting a value for K_1 the user should assure himself that the integral of the pdf over the unit hemisphere in unity. (See Section A.4.)

K_2: (DISCO (I,3)) is a dispersion coefficient that is entered in columns 51 to 60 of Cards #4 and #5 with a maximum of 3 decimal places.

K_3: (DISCO (I,4)) is a dispersion coefficient that is entered in columns 61 to 70 of Cards #4 and #5 with a maximum of 3 decimal places.

Dershowitz Distribution: The analytic form of the Dershowitz distribution is very similar to the Bingham:

$$f(\theta',\phi') = K_1 \, e^{(K_2 \cos^2 \theta' + K_3 \sin^2 \theta')\cos \phi' \sin \phi'} \qquad (C.4)$$

Like the Bingham, the Dershowitz has 3 coefficients, a mean plane and a major plane. The format for inputting the parameters is identical to that of the Bingham distribution.

Bivariate Normal Distribution: The bivariate normal distribution has the following form:

$$f(\bar{X},\bar{Y}) = K \, e^{-\frac{1}{2}\left((\bar{X}/\sigma_{\bar{X}})^2 + (\bar{Y}/\sigma_{\bar{Y}})^2 \right)} \qquad (C.5)$$

As discussed in Appendix A (Section A.6) $\bar{X}$ and $\bar{Y}$ represent coordinates of projections from the reference hemisphere on to a plane. $\bar{X}$ is the planar distance measured along the projection of the major axis while $\bar{Y}$ is the distance along the projection of the minor axis. The input parameters include the 3 coefficients (K , $\sigma_{\bar{X}}$, and $\sigma_{\bar{Y}}$), the orientation of the mean plane and the orientation of the major plane. The major plane is the plane whose pole forms the major axis.

Mean Strike: (B1S, BS, B2S) is entered in geologic notation in columns 1 to 6 of cards #4 and #5. Geologic strike must be expressed in the northern quadrants and the angle is expressed with one decimal place. (Example: N45.0E)

Mean Dip: (BD, B1D, B2D) is entered in geologic notation in columns 11 to 16 of cards #4 and #5. Dip directions must indicate a particular quadrant i.e., NE, NW, SE, SW. The angle is expressed with one decimal place. (Example: 36.ONE)

Major Strike:* (AB1S, BAS, AB2S) is entered in columns 21 to 26 of cards #4 and #5 in a manner similar to the mean strike.

Major Dip:* (ABD, ABD1, ABD2) is entered in columns 31 to 36 of cards #4 and #5 in a manner similar to the mean dip.

K: (DISCO (I,2)) is a normalizing constant that is entered in columns 41 to 50 of cards #4 and #5 with a maximum

* Specifying both the major strike and the major dip is redundant. The subject is discussed in a footnote relating to the Bingham distribution.

of 3 decimal places. In selecting a value for K the user should assure himself that the integral of the pdf over the unit hemisphere is unity. As mentioned in Appendix A, the bivariate normal must be truncated when used to describe orientation data. (See Section A.6.)

$\underline{\sigma_{\bar{X}}^2}$: (DISCO (I,3)) is the variance along the $\bar{X}$ axis. It is entered in columns 51 to 60 of Cards #4 and #5 with a maximum of 3 decimal places.

$\underline{\sigma_{\bar{Y}}^2}$: (DISCO (I,4)) is the variance along the $\bar{Y}$ axis. It is entered in columns 61 to 70 of Cards #4 and #5 with a maximum of 3 decimal places.

DAYLITE Listing

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C                                   DAYLITE                                    DYLT0001
C     THIS PROGRAM CALCULATES THE PROBABILITY DENSITY FUNCTION                 DYLT0002
C     OF THE ORIENTATION OF THE LINE OF INTERSECTION OF TWO                    DYLT0003
C     JOINT PLANES.  THE PROGRAM UTILIZES A LOWER HEMISPHERE                   DYLT0004
C     PROJECTION OF JOINT POLES.  THE COLATITUDE (AZIM) IS                     DYLT0005
C     MEASURED CLOCKWISE FROM NORTH.  THE LATITUDE (BETA)                      DYLT0006
C     EQUALS 90 - DIP.                                                         DYLT0007
      REAL NO                                                                  DYLT0008
      DIMENSION DISCO(2,13), PJT(10,20), ZZ(91), PLINED(3,10,20)               DYLT0009
      DIMENSION X(5), Y(5), Z(5), PROB(2,5), SNBET(180)                        DYLT0010
      DATA NO,SO,ET,WT/'N','S','E','W'/                                        DYLT0011
      NRD = 5                                                                  DYLT0012
      NWR = 6                                                                  DYLT0013
      NOPT = 0                                                                 DYLT0014
      PI = 3.14159265                                                          DYLT0015
      PI2 = PI/2.0                                                             DYLT0016
      PIF = PI/180.                                                            DYLT0017
      TPRO1 = 0.0                                                              DYLT0018
      TPRO2 = 0.0                                                              DYLT0019
      TPRO3 = 0.0                                                              DYLT0020
      PRO1M = 0.0                                                              DYLT0021
      PRO2M = 0.0                                                              DYLT0022
      PRO3M = 0.0                                                              DYLT0023
      NN1 = 1                                                                  DYLT0024
      NN2 = 2                                                                  DYLT0025
   10 FORMAT (A1,F4.1,A1,4X,F4.1,A1,A1,4X,F10.5)                               DYLT0026
   11 FORMAT (A1,F4.1,A1,4X,F4.1,A1,A1)                                        DYLT0027
   15 FORMAT (4F10.2)                                                          DYLT0028
   20 FORMAT (3I10)                                                            DYLT0029
   60 FORMAT (A1,F4.1,A1,4X,F4.1,A1,A1,4X,A1,F4.1,A1,   4X,F4.1,A1,A1,         DYLT0030
     1 4X,3F10.3)                                                              DYLT0031
 1000 FORMAT ('1',29X,'**********  SLOPE  DATA  **********',//                 DYLT0032
     1 19X,'STRIKE:',15X,'N',F4.1,A1, 3X,'DIP:',15X,F4.1,A1,A1)               DYLT0033
 1002 FORMAT (////,30X,'*******  ANALYSIS CRITERIA  *******',//,34X,           DYLT0034
     1 'REFERENCE INTERVAL:',5X,F5.2,//,34X,'MINIMUM DIP:',12X,F5.2,//        DYLT0035
     2 34X,'MINIMUM WEDGE ANGLE:',4X,F5.2,//,34X,'WEDGE ANGLE ',              DYLT0036
                                                                    PAGE    1
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     3 'INCREMENT:',2X,F5.2)                                                    DYLT0037
1003 FORMAT (///,10X,'INJUDICIOUS CHOICE OF ANALYSIS CRITERIA - ',              DYLT0038
     1 '(180)/(REFERENCE INTERVAL) MUST BE AN EVEN INTEGER - ',                 DYLT0039
     2 'RUN TERMINATED')                                                        DYLT0040
1004 FORMAT (///,10X,'INJUDICIOUS CHOICE OF ANALYSIS CRITERIA - ',              DYLT0041
     1 '(DIP - MINIMUM DIP)/(REFERENCE INTERVAL) MUST BE AN INTEGER - ',        DYLT0042
     2 'RUN TERMINATED')                                                        DYLT0043
1005 FORMAT (///,10X,'INJUDICIOUS CHOICE OF ANALYSIS CRITERIA - ',              DYLT0044
     1 '(MINIMUM WEDGE ANGLE)/(REFERENCE INTERVAL) MUST BE AN INTEGER ',        DYLT0045
     2 '- RUN TERMINATED')                                                      DYLT0046
1006 FORMAT (///,10X,'INJUDICIOUS CHOICE OF ANALYSIS CRITERIA - ',              DYLT0047
     1 '(WEDGE ANGLE)/(REFERENCE INTERVAL) MUST BE AN INTEGER - ',              DYLT0048
     2 'RUN TERMINATED')                                                        DYLT0049
1008 FORMAT (////, 32X,'******     JOINT SET ',I1,'     ******',//             DYLT0050
     1 38X,'UNIFORM  DISTRIBUTION',/,'+',37X,'_______  ___________')            DYLT0051
1010 FORMAT (////, 32X,'******     JOINT SET ',I1,'     ******',//             DYLT0052
     1 39X,'FISHER DISTRIBUTION',/,'+',38X,'______ ____________',//             DYLT0053
     2 18X,'MEAN STRIKE:',11X,'N',F4.1,A1,3X,'MEAN DIP:',13X,F4.1,A1,A1)        DYLT0054
1020 FORMAT (////, 32X,'******     JOINT SET ',I1,'     ******',//             DYLT0055
     1 39X,'ARNOLD DISTRIBUTION',/,'+',38X,'______ ____________',//             DYLT0056
     2 18X,'MEAN STRIKE:',11X,'N',F4.1,A1,3X,'MEAN DIP:',13X,F4.1,A1,A1)        DYLT0057
1026 FORMAT (/,40X,'K FACTOR:',F8.2)                                           DYLT0058
1030 FORMAT (/////,20X,'****************   LINE OF INTERSECTION:   ',           DYLT0059
     1 '****************',//,18X,'STRIKE:',15X,A1,F5.2,A1,3X,'PLUNGE:',        DYLT0060
     2 20X,F5.2,//,18X,'X = ',F7.4,14X,'Y = ',F7.4,14X,'Z = ',F7.4,//           DYLT0061
     3 33X,'APPARENT DIP OF SLOPE:',F9.2,//,23X,'PDF INCLUDES ALL ',            DYLT0062
     4 'LINES THAT INTERSECT SPHERE FROM:',//,30X,'(',F5.2,' , ',F5.2,         DYLT0063
     5 ')  ***  (',F5.2,' , ',F5.2,')',//,37X,'*',21X,'*',/,37X,              DYLT0064
     6 '*  (STRIKE : PLUNGE)  *',/,37X,'*',21X,'*',//,30X,'(',                 DYLT0065
     7 F5.2,' , ',F5.2,')  ***  (',F5.2,' , ',F5.2,')' )                       DYLT0066
1035 FORMAT (////, 32X,'******     JOINT SET ',I1,'     ******',//             DYLT0067
     1 38X,'BINGHAM  DISTRIBUTION',/,'+',37X,'_______ ____________',//         DYLT0068
     2 18X,'MEAN STRIKE:',11X,'N',F4.1,A1,3X,'MEAN DIP:',13X,F4.1,A1,A1,       DYLT0069
     3 //,18X,'MAJOR STRIKE:',10X,'N',F4.1,A1,3X,'MAJOR DIP:',12X,            DYLT0070
     4 F4.1,A1,A1,//,18X,'K1 = ',F7.2,13X,'K2 = ',F7.2,11X,'K3 = ',F7.2)       DYLT0071
1036 FORMAT (//,18X,'DISTRIBUTION ',I1,': MEAN POLE AND MAJOR ',                DYLT0072
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     1 'AXIS ARE NOT ORTHAGONAL')                                          DYLT0073
1040 FORMAT (////, 32X,'******     JOINT SET ',I1,'     ******',//         DYLT0074
     1 37X,'DERSHOWITZ DISTRIBUTION',/,'+',                                DYLT0075
     1 36X,'_____________________',//                                      DYLT0076
     2 18X,'MEAN STRIKE:',11X,'N',F4.1,A1,3X,'MEAN DIP:',13X,F4.1,A1,A1,  DYLT0077
     3 //,18X,'MAJOR STRIKE:',10X,'N',F4.1,A1,3X,'MAJOR DIP:',12X,        DYLT0078
     4 F4.1,A1,A1,//,18X,'K1 = ',F7.2,13X,'K2 = ',F7.2,11X,'K3 = ',F7.2)   DYLT0079
1045 FORMAT (////, 32X,'******     JOINT SET ',I1,'     ******',//         DYLT0080
     1 34X,'BIVARIATE NORMAL DISTRIBUTION',/,'+',                          DYLT0081
     1 33X,'_____________________',//                                      DYLT0082
     2 18X,'MEAN STRIKE:',11X,'N',F4.1,A1,3X,'MEAN DIP:',13X,F4.1,A1,A1,  DYLT0083
     3 //,18X,'MAJOR STRIKE:',10X,'N',F4.1,A1,3X,'MAJOR DIP:',12X,        DYLT0084
     4 F4.1,A1,A1,//,18X,'K1 = ',F7.2,13X,'K2 = ',F7.2,11X,'K3 = ',F7.2)   DYLT0085
1055 FORMAT (//,18X,'WEDGE   ANGLE',5X,'P (WEDGE FORMING ; TWO ',          DYLT0086
     1 'JOINTS ARE PRESENT)',/,19X,'MIN      MAX',4X,'SET 1 & SET 1   ',   DYLT0087
     2 'SET 2 & SET 2   SET 1 & SET 2',//)                                 DYLT0088
1060 FORMAT (18X,F5.1,3X,F5.1,3X,G13.7,3X,G13.7,3X,G13.7)                  DYLT0089
1065 FORMAT ('+',34X,'_____________ _____________  _____________',//       DYLT0090
     1 25X,'TOTAL:',3X,G13.7,3X,G13.7,3X,G13.7)                            DYLT0091
1070 FORMAT (///,21X,'********************** SUMMARY   ',                  DYLT0092
     1 '**********************')                                           DYLT0093
1075 FORMAT (///,29X,'*********   SET ',I1,' & SET ',I1,'   *********',    DYLT0094
     1 /,31X,'TOTAL PROBABILITY:  ',G13.7,/,33X,'FOR ALL LINES',//,        DYLT0095
     2 30X,'MAXIMUM PROBABILITY: ',G13.7,//,35X,'AT STRIKE: ',A1,          DYLT0096
     3 F5.2,A1,//,36X,'AND  DIP: ',F5.2)                                   DYLT0097
1080 FORMAT (87X,'BETA 1    BETA 2    PROB.       COND.',/                 DYLT0098
     1 88X,'MIN.      MAX.                PROB.')                          DYLT0099
1085 FORMAT (87X,F6.2,4X,F6.2,2X,G10.4,2X,G10.4)                           DYLT0100
      READ (NRD,11) S1S,S2RS,S2S,DIPS,D1S,D2S                              DYLT0101
      WRITE (NWR,1000) S1RS,S2S,D1PS,D1S,D2S                               DYLT0102
      READ (NRD,15) DEGN, FRIC, DEGMAN, WEDINC                             DYLT0103
      IF ( WEDINC .LT. DEGN ) WEDINC = DEGN                                DYLT0104
      IF ( DEGMAN .LT. DEGN ) DEGMAN = DEGN                                DYLT0105
      WRITE (NWR,1002) DEGN, FRIC, DEGMAN, WEDINC                          DYLT0106
      READ (NRD,20) NDIST1, NDIST2, NCODE                                  DYLT0107
      IF ( NCODE .GT. 4 ) NCODE = 4                                        DYLT0108
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      DISCO(1,1) = NDIST1                                                    DYLT0109
      DISCO(2,1) = NDIST2                                                    DYLT0110
      DO 200 I = 1, 2                                                        DYLT0111
      IF ( DISCO(I,1) .EQ. 0.0 ) WRITE (NWR,1008) I                          DYLT0112
      IF ( DISCO(I,1) .EQ. 0.0 ) GO TO 200                                   DYLT0113
      IF ( DISCO(I,1) .GT. 2.0 ) GO TO 150                                   DYLT0114
      READ (NRD,10) FA1S, STRFA, FA2S, DIPFA, D1FA, D2FA, FAK                DYLT0115
      IF (DISCO(I,1).EQ.1.0) WRITE(NWR,1010)I,STRFA,FA2S,DIPFA,D1FA,D2FA     DYLT0116
      IF (DISCO(I,1).EQ.2.0) WRITE(NWR,1020)I,STRFA,FA2S,DIPFA,D1FA,D2FA     DYLT0117
      WRITE (NWR,1026) FAK                                                   DYLT0118
      DISCO(I,2) = FAK                                                       DYLT0119
      FFACT1 = (EXP(FAK) - 1.0)/(EXP(FAK) - EXP(-FAK))                       DYLT0120
      DISCO(I,3) = FAK/(2.0*PI*FFACT1*(EXP(FAK) - EXP(-FAK)))                DYLT0121
      DISCO(I,4) = FAK/(2.0*PI*(EXP(FAK) - 1.0))                             DYLT0122
      CALL STRIKE(PIF,STRFA,FA2S,DIPFA,D1FA,D2FA,DISCO(I,5),DISCO(I,6),      DYLT0123
     1 DISCO(I,7))                                                           DYLT0124
      GO TO 200                                                              DYLT0125
  150 READ(NRD,60) B1S,BS,B2S,BD,B1D,B2D,AB1S,BAS,AB2S,ABD,                  DYLT0126
     1 ABD1,ABD2,DISCO(I,2),DISCO(I,3),DISCO(I,4)                            DYLT0127
      IF ( DISCO(I,1) .EQ. 3.0 ) WRITE (NWR,1035) I,BS,B2S,BD,B1D,B2D,       DYLT0128
     1 BAS,AB2S,ABD,ABD1,ABD2,DISCO(I,2),DISCO(I,3),DISCO(I,4)               DYLT0129
      IF ( DISCO(I,1) .EQ. 4.0 ) WRITE (NWR,1040) I,BS,B2S,BD,B1D,B2D,       DYLT0130
     1 BAS,AB2S,ABD,ABD1,ABD2,DISCO(I,2),DISCO(I,3),DISCO(I,4)               DYLT0131
      IF ( DISCO(I,1) .EQ. 5.0 .AND. DISCO(I,4) .EQ. 0.0 )                   DYLT0132
     1 DISCO(I,4) = 1.0/(2.0*PI*DISCO(I,2)*DISCO(I,3))                       DYLT0133
      IF ( DISCO(I,1) .EQ. 5.0 ) WRITE (NWR,1045) I,BS,B2S,BD,B1D,B2D,       DYLT0134
     1 BAS,AB2S,ABD,ABD1,ABD2,DISCO(I,2),DISCO(I,3),DISCO(I,4)               DYLT0135
      CALL STRIKE(PIF,BS,B2S,BD,B1D,B2D,DISCO(I,5),DISCO(I,6),               DYLT0136
     1 DISCO(I,7))                                                           DYLT0137
      CALL STRIKE(PIF,BAS,AB2S,ABD,ABD1,ABD2,DISCO(I,8),                     DYLT0138
     1 DISCO(I,9),DISCO(I,10))                                               DYLT0139
      TESTO = DISCO(1,5)*DISCO(I,8) + DISCO(I,6)*DISCO(I,9) +                DYLT0140
     1 DISCO(I,7)*DISCO(I,10)                                                DYLT0141
      IF ( TESTO .GT. 0.01 ) WRITE (NWR,1036) I                              DYLT0142
      DISCO(I,11) = DISCO(I,6)*DISCO(I,10) - DISCO(I,7)*DISCO(I,9)           DYLT0143
      DISCO(I,12) = DISCO(I,7)*DISCO(I,8) - DISCO(I,5)*DISCO(I,10)           DYLT0144
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      DISCO(I,13) = DISCO(I,5)*DISCO(I,9) - DISCO(I,6)*DISCO(I,8)          DYLT0145
      IF ( DISCO(I,13) .LT. 0.0 ) DISCO(I,11) = -DISCO(I,11)               DYLT0146
      IF ( DISCO(I,13) .LT. 0.0 ) DISCO(I,12) = -DISCO(I,12)               DYLT0147
      IF ( DISCO(I,13) .LT. 0.0 ) DISCO(I,13) = -DISCO(I,13)               DYLT0148
  200 CONTINUE                                                             DYLT0149
      IF (S2S .EQ. ET .AND. D2S .EQ. ET) AZIMTH = STRS*PI/180.0            DYLT0150
      IF (S2S .EQ. ET .AND. D2S .EQ. WT) AZIMTH = (180.0 + STRS)*PI/180.   DYLT0151
      IF (S2S .EQ. WT .AND. D2S .EQ. WT) AZIMTH = (180.0 - STRS)*PI/180.   DYLT0152
      IF (S2S .EQ. WT .AND. D2S .EQ. ET) AZIMTH = (360.0 - STRS)*PI/180.   DYLT0153
      IF (STRS .EQ. 90.0 .AND. D1S .EQ. NO) AZIMTH = 1.5*PI                DYLT0154
      IF (STRS .EQ. 90.0 .AND. D1S .EQ. SO ) AZIMTH = PI/2.0               DYLT0155
      FRQC = FRIC*PI/180.                                                  DYLT0156
      NIDC = 1                                                             DYLT0157
      NAZIM = 180./DEGN                                                    DYLT0158
      FLT1 = NAZIM                                                         DYLT0159
      IF (FLT1*DEGN .NE. 180. ) NIDC = 2                                   DYLT0160
      IF ( (-1.0)**NAZIM .LT. 0.0 ) NIDC = 2                               DYLT0161
      NMAX = (DIPS - FRIC)/DEGN                                            DYLT0162
      FLT2 = NMAX                                                          DYLT0163
      IF ( DIPS - FRIC .NE. FLT2*DEGN ) NIDC = 3                           DYLT0164
      NMAXW = NAZIM - 1                                                    DYLT0165
      NMINW = DEGMAN/DEGN                                                  DYLT0166
      FLT3 = NMINW                                                         DYLT0167
      IF ( FLT3*DEGN .NE. DEGMAN ) NIDC = 4                                DYLT0168
      NPERWD = WEDINC/DEGN                                                 DYLT0169
      FLT4 = NPERWD                                                        DYLT0170
      IF ( FLT4*DEGN .NE. WEDINC ) NIDC = 5                                DYLT0171
      IF ( NIDC .EQ. 1 ) GO TO 205                                         DYLT0172
      IF ( NIDC .EQ. 2 ) WRITE (NWR,1003)                                  DYLT0173
      IF ( NIDC .EQ. 3 ) WRITE (NWR,1004)                                  DYLT0174
      IF ( NIDC .EQ. 4 ) WRITE (NWR,1005)                                  DYLT0175
      IF ( NIDC .EQ. 5 ) WRITE (NWR,1006)                                  DYLT0176
      GO TO 9000                                                           DYLT0177
  205 DEGN = DEGN*PI/180.                                                  DYLT0178
      DEGN2 = DEGN/2.0                                                     DYLT0179
C     AREAG IS THE SURFACE AREA OF THE GIRDLE BETWEEN FRIC AND DIPS        DYLT0180
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      AREAG = 2.0*PI*(SIN(DIPS*PI/180.) - SIN(FRIC*PI/180.))                DYLT0181
      AREAIN = AREAG/FLT2                                                   DYLT0182
      COZETA = 1.0 - (AREAIN*DEGN)/(4.0*PI**2)                              DYLT0183
      SIZETA = SQRT(1.0 - COZETA**2)                                        DYLT0184
      ZETA = ATAN(SIZETA/COZETA)                                            DYLT0185
C     AREAF IS ONE SIXTH THE AREA OF A GIRDLE COMPONENT                     DYLT0186
      AREAF = 2.0*COS(PI2 - ZETA)*DEGN/6.0                                  DYLT0187
      AREA = 2.0*PI*(1.0 - COS(PI2-FROC)) + AREAIN/2.0                      DYLT0188
      BETA1 = FRIC                                                          DYLT0189
      DO 500 J = 1, NMAX                                                    DYLT0190
      AREA = AREA - AREAIN                                                  DYLT0191
      COSDIP = (1.0 - AREA/(2.0*PI))                                        DYLT0192
      SINDIP = SQRT(1.0 - COSDIP**2)                                        DYLT0193
      DIP = ATAN(SINDIP/COSDIP)                                             DYLT0194
      BETA2 = (PI2 - DIP)*180./PI                                           DYLT0195
      AREAX = AREA - AREAIN/2.0                                             DYLT0196
      FIECOS = 1.0- AREAX/(2.0*PI)                                          DYLT0197
      FIESIN = SQRT(1.0 - FIECOS**2)                                        DYLT0198
      BETA3 = (PI2 - ATAN(FIESIN/FIECOS))*180./PI                           DYLT0199
      CHI = -DEGN2                                                          DYLT0200
      DO 206 K = 1, 91                                                      DYLT0201
      CHI = CHI + DEGN2                                                     DYLT0202
      IF ( CHI .GT. PI2 ) GO TO 207                                         DYLT0203
      ZZ(K) = SIN(CHI*SINDIP)                                               DYLT0204
  206 CONTINUE                                                              DYLT0205
  207 CONTINUE                                                              DYLT0206
      ALPHAS = TAN(FROC)/TAN(DIP)                                           DYLT0207
      ALPHAC = SQRT(1.0 - ALPHAS**2)                                        DYLT0208
      ALPHA = ATAN(ALPHAS/ALPHAC) - DEGN2                                   DYLT0209
      NEGL = ALPHA/DEGN                                                     DYLT0210
      NNEGL = NAZIM - NEGL                                                  DYLT0211
      AZIM = AZIMTH - DEGN/2.0                                              DYLT0212
      ZETA = -DEGN/2.0                                                      DYLT0213
      DO 400 I = NEGL, NNEGL                                                DYLT0214
      ZETA = ZETA + DEGN                                                    DYLT0215
      AZIM = AZIM + DEGN                                                    DYLT0216
```

```
      ADIP = ATAN(TAN(DIPS*PI/180.)*SIN(AZIM-AZIMTH))*180./PI                  DYLT0217
  208 IF ( AZIM .LT. PI2 ) STRPT = AZIM*180./PI                                DYLT0218
      IF ( AZIM .LT. PI2 ) STRPT1 = NO                                         DYLT0219
      IF ( AZIM .LT. PI2 ) STRPT2 = ET                                         DYLT0220
      IF ( AZIM .LT. PI2 ) GO TO 210                                           DYLT0221
      IF ( AZIM .LT. PI ) STRPT = (PI - AZIM)*180./PI                          DYLT0222
      IF ( AZIM .LT. PI ) STRPT1 = SO                                          DYLT0223
      IF ( AZIM .LT. PI ) STRPT2 = ET                                          DYLT0224
      IF ( AZIM .LT. PI ) GO TO 210                                            DYLT0225
      IF ( AZIM .LT. 3.0*PI2 ) STRPT = (AZIM - PI)*180./PI                     DYLT0226
      IF ( AZIM .LT. 3.0*PI2 ) STRPT1 = SO                                     DYLT0227
      IF ( AZIM .LT. 3.0*PI2 ) STRPT2 = WT                                     DYLT0228
      IF ( AZIM .LT. 3.0*PI2 ) GO TO 210                                       DYLT0229
      STRPT = (2.0*PI - AZIM)*180./PI                                          DYLT0230
      STRPT1 = NO                                                              DYLT0231
      STRPT2 = WT                                                              DYLT0232
      IF ( AZIM .GT. 2.0*PI ) AZIM = AZIM - 2.0*PI                             DYLT0233
      IF ( AZIM .GT. 2.0*PI ) GO TO 208                                        DYLT0234
  210 STRMN = STRPT - DEGN*90./PI                                              DYLT0235
      STRMX = STRPT + DEGN*90./PI                                              DYLT0236
C     XPT, YPT, ZPT ARE THE UNIT VECTORS OF THE DAYLIGHTING LINE               DYLT0237
      XPT = SIN(DIP)*COS(AZIM)                                                 DYLT0238
      YPT = SIN(DIP)*SIN(AZIM)                                                 DYLT0239
      ZPT = COS(DIP)                                                           DYLT0240
C     XR, YR ARE THE UNIT VECTORS OF THE STRIKE OF THE "JOINT POLE" PLANE      DYLT0241
      XR = COS(AZIM-PI2)                                                       DYLT0242
      YR = SIN(AZIM-PI2)                                                       DYLT0243
      IF ( NOPT .EQ. 1 ) GO TO 215                                             DYLT0244
      XSW = SIN(DIP)*COS(ZETA)                                                 DYLT0245
      YSW = -SIN(DIP)*SIN(ZETA)                                                DYLT0246
C     XSW, ETC. ARE UNIT VECTORS OF DAYLT. LINE W.R.T. SWARS COOR. SYS.        DYLT0247
      WRITE (NWR, 1030) STRPT1, STRPT, STRPT2, BETA2, XSW, YSW, ZPT,           DYLT0248
     1 ADIP,STRMN,BETA1,STRMX,BETA1,STRMN,BETA3,STRMX,BETA3                    DYLT0249
      WRITE ( NWR, 1055)                                                       DYLT0250
  215 AA = (1.0 + (XR/YR)**2)                                                  DYLT0251
      DO 250 K = 1, NAZIM                                                      DYLT0252
```

309

```
      G = K - 1                                                          DYLT0253
      CHI = -DEGN2 + G*DEGN                                              DYLT0254
      DO 225 L = 1, 3                                                    DYLT0255
      CHI = CHI + DEGN2                                                  DYLT0256
      IF ( K .NE. 1 .AND. L .EQ. 1 ) GO TO 225                           DYLT0257
      COSCHI = COS(CHI)                                                  DYLT0258
      IF ( L .EQ. 2 ) COSIN = COSCHI                                     DYLT0259
      CHO = CHI                                                          DYLT0260
      IF ( CHO .GT. PI2 ) CHO = PI - CHO                                 DYLT0261
      NZ = 1.0 + CHO/DEGN2                                               DYLT0262
      Z(L) = ZZ(NZ)                                                      DYLT0263
      BB = -2.0*XR*COSCHI/YR**2                                          DYLT0264
      CC = (COSCHI/YR)**2 + Z(L)**2 - 1.0                                DYLT0265
      DD = SQRT(ABS(BB**2 - 4.0*AA*CC))                                  DYLT0266
      X(L) = (-BB-DD)/(2.0*AA)                                           DYLT0267
      Y(L) = (COSCHI - XR*X(L))/YR                                       DYLT0268
      EE = ATAN(X(L)*XPT + Y(L)*YPT + Z(L)*ZPT)                          DYLT0269
      IF ( EE .LT. 0.0100 ) GO TO 225                                    DYLT0270
      X(L) = (-BB+DD)/(2.0*AA)                                           DYLT0271
      Y(L) = (COSCHI - XR*X(L))/YR                                       DYLT0272
  225 CONTINUE                                                           DYLT0273
      XRR = X(2)                                                         DYLT0274
      YRR = Y(2)                                                         DYLT0275
      ZRR = Z(2)                                                         DYLT0276
      H= 1.0/((XRR*YR - XR*YRR)**2)                                      DYLT0277
      S = XRR*COSIN - XR*COZETA                                          DYLT0278
      T = YRR*COSIN - YR*COZETA                                          DYLT0279
      A = H*ZRR**2 + 1.0                                                 DYLT0280
      B = 2.0*H*ZRR*(XR*S + YR*T)                                        DYLT0281
      C = H*(T**2 + S**2) - 1.0                                          DYLT0282
      D = SQRT(ABS(B**2 - 4.0*A*C))                                      DYLT0283
      Z(4) = (-B+D)/(2.0*A)                                              DYLT0284
      Y(4) = (XR*ZRR*Z(4) + S)/(XRR*YR - XR*YRR)                         DYLT0285
      X(4) = (COSIN - Y(4)*YR)/XR                                        DYLT0286
      Z(5) = (-B-D)/(2.0*A)                                              DYLT0287
      Y(5) = (XR*ZRR*Z(5) + S)/(XRR*YR - XR*YRR)                         DYLT0288
```

310

```
      X(5) = (COSIN - Y(5)*YR)/XR                                          DYLT0289
      DO 245 L = 1, 2                                                      DYLT0290
      IF ( DISCO(L,1) .EQ. 0.0 ) PJT(L,K) = AREAF*3.0/PI                   DYLT0291
      IF ( DISCO(L,1) .EQ. 0.0 ) GO TO 245                                 DYLT0292
      DO 240 LL = 1, 5                                                     DYLT0293
      IF ( K .NE. 1 ) PROB(L,1) = PROB(L,3)                                DYLT0294
      IF ( K .NE. 1 .AND. LL .EQ. 1 ) GO TO 240                            DYLT0295
      IF ( DISCO(L,1) .GT. 2.0 ) GO TO 230                                 DYLT0296
      GAMMA =      (DISCO(L,5)*X(LL)+DISCO(L,6)*Y(LL)+DISCO(L,7)*Z(LL))    DYLT0297
      GAMMA = ATAN((SQRT(1.0 - GAMMA**2))/GAMMA)                           DYLT0298
      IF ( GAMMA .LT. 0.0 ) GAMMA = PI - GAMMA                             DYLT0299
      IF ( DISCO(L,1) .EQ. 1.0 ) PROB(L,LL) = DISCO(L,3)*                  DYLT0300
     1 SIN(GAMMA)*EXP(DISCO(L,2)*COS(GAMMA))                               DYLT0301
      IF ( DISCO(L,1) .EQ. 2.0 ) PROB(L,LL) = DISCO(L,4)*                  DYLT0302
     1 SIN(GAMMA)*EXP(DISCO(L,2)*COS(GAMMA))                               DYLT0303
      GO TO 240                                                            DYLT0304
C     CMJ: COOR MAJOR AXIS, CMR: COOR MINOR AXIS, CMN: COOR MEAN AXIS      DYLT0305
  230 CMJ = X(LL)*DISCO(L,8) + Y(LL)*DISCO(L,9) + Z(LL)*DISCO(L,10)        DYLT0306
      CMR = X(LL)*DISCO(L,11) + Y(LL)*DISCO(L,12) + Z(LL)*DISCO(L,13)      DYLT0307
      IF ( DISCO(L,1) .EQ. 5.0 ) PROB(L,LL) = DISCO(L,4)*                  DYLT0308
     1 EXP(0.5*(CMJ/DISCO(L,2) + CMR/DISCO(L,3)))                          DYLT0309
      IF ( DISCO(L,1) .EQ. 5.0 ) GO TO 240                                 DYLT0310
      CMN = X(LL)*DISCO(L,5) + Y(LL)*DISCO(L,6) + Z(LL)*DISCO(L,7)         DYLT0311
      ANGPH = ATAN(CMN)                                                    DYLT0312
      IF ( ANGPH .LT. 0.0 ) ANGPH = PI - ANGPH                             DYLT0313
      ANGTH = ATAN(CMR/CMJ)                                                DYLT0314
      IF ( DISCO(L,1) .EQ. 3.0 ) PROB(L,LL) = DISCO(L,2)*SIN(ANGPH)*       DYLT0315
     1 EXP((SIN(ANGPH)**2)*(DISCO(L,3)*COS(ANGTH)**2 + DISCO(L,4)*         DYLT0316
     2 SIN(ANGTH)**2))                                                     DYLT0317
      IF ( DISCO(L,1) .EQ. 4.0 ) PROB(L,LL) = DISCO(L,2)*SIN(ANGPH)*       DYLT0318
     1 EXP(COS(ANGPH)*(DISCO(L,3)*COS(ANGTH)**2 + DISCO(L,4)*              DYLT0319
     2 SIN(ANGTH)**2))                                                     DYLT0320
  240 CONTINUE                                                             DYLT0321
      PJT(L,K) = (PROB(L,1) + PROB(L,3) + PROB(L,4) + PROB(L,5) +          DYLT0322
     1 PROB(L,2)*2.0)*AREAF                                                DYLT0323
  245 CONTINUE                                                             DYLT0324
```

```
  250 CONTINUE                                                          DYLT0325
  270 NAZIM2 = NAZIM/2                                                  DYLT0326
      DO 280 L = 1, 2                                                   DYLT0327
      DO 278 K = 1, NAZIM2                                              DYLT0328
      NA2K = NAZIM2 + K                                                 DYLT0329
      DUMP = PJT(L,K)                                                   DYLT0330
      PJT(L,K) = PJT(L,NA2K)                                            DYLT0331
      PJT(L,NA2K) = DUMP                                                DYLT0332
  278 CONTINUE                                                          DYLT0333
  280 CONTINUE                                                          DYLT0334
      PRO1S = 0.0                                                       DYLT0335
      PRO2S = 0.0                                                       DYLT0336
      PRO3S = 0.0                                                       DYLT0337
      PRO1 = 0.0                                                        DYLT0338
      PRO2 = 0.0                                                        DYLT0339
      PRO3 = 0.0                                                        DYLT0340
      DEGMIN = DEGMAN                                                   DYLT0341
      NNNN = 0                                                          DYLT0342
      DO 325 N = NMINW, NMAXW                                           DYLT0343
      NNNN = NNNN + 1                                                   DYLT0344
      NWEDGS = NAZIM - N                                                DYLT0345
      IF ( NCODE .EQ. 1 .OR. NCODE .EQ. 4 ) NSET1 = 1                   DYLT0346
      IF ( NCODE .EQ. 1 .OR. NCODE .EQ. 4 ) NSET2 = 1                   DYLT0347
  296 IF ( NCODE .EQ. 2 .OR. NCODE .EQ. 5 ) NSET1 = 2                   DYLT0348
      IF ( NCODE .EQ. 2 .OR. NCODE .EQ. 5 ) NSET2 = 2                   DYLT0349
  298 IF ( NCODE .EQ. 3 .OR. NCODE .EQ. 6 ) NSET1 = 1                   DYLT0350
      IF ( NCODE .EQ. 3 .OR. NCODE .EQ. 6 ) NSET2 = 2                   DYLT0351
      PRO = 0.0                                                         DYLT0352
      NJEG = 1                                                          DYLT0353
      MMM = 0                                                           DYLT0354
      NBET1 = 0                                                         DYLT0355
      PROBET = 0.0                                                      DYLT0356
      SUMBT1 = 0.0                                                      DYLT0357
      DO 300 M = 1, NWEDGS                                              DYLT0358
      N1 = NSET1                                                        DYLT0359
      N2 = NSET2                                                        DYLT0360
```

```
      MN = M + N                                                        DYLT0361
      PROPT = PJT(N1,M)*PJT(N2,MN) + PJT(N1,MN)*PJT(N2,M)               DYLT0362
      PRO = PROPT + PRO                                                 DYLT0363
      IF ( NCODE .GT. 3 ) GO TO 300                                     DYLT0364
      NBET1 = NBET1 + 1                                                 DYLT0365
      PROBET = PROBET + PROPT                                           DYLT0366
      IF ( NBET1 .NE. NPERWD ) GO TO 300                                DYLT0367
      MMM = MMM + 1                                                     DYLT0368
      SNBET(MMM) = PROBET                                               DYLT0369
      SUMBT1 = SUMBT1 + PROBET                                          DYLT0370
      PROBET = 0.0                                                      DYLT0371
      NBET1 = 0                                                         DYLT0372
  300 CONTINUE                                                          DYLT0373
      IF ( NCODE .GT. 3 .OR. PROBET .EQ. 0.0 ) GO TO 305                DYLT0374
      NJEG = 2                                                          DYLT0375
      MMM = MMM + 1                                                     DYLT0376
      SNBET(MMM) = PROBET                                               DYLT0377
      SWEGS = NWEDGS                                                    DYLT0378
      EEGMAX = SWEGS*DEGN*180./PI                                       DYLT0379
      SUMBT1 = SUMBT1 + PROBET                                          DYLT0380
  305 IF ( NCODE .EQ. 3 .OR. NCODE .EQ. 6 ) PRO3 = PRO + PRO3           DYLT0381
      IF ( NCODE .EQ. 6 ) GO TO 310                                     DYLT0382
      IF ( NCODE .EQ. 2 .OR. NCODE .EQ. 5 ) PRO2 = PRO + PRO2           DYLT0383
      IF ( NCODE .EQ. 5 ) NCODE = 6                                     DYLT0384
      IF ( NCODE .EQ. 6 ) GO TO 298                                     DYLT0385
      IF ( NCODE .EQ. 1 .OR. NCODE .EQ. 4 ) PRO1 = PRO + PRO1           DYLT0386
      IF ( NCODE .EQ. 4 ) NCODE = 5                                     DYLT0387
      IF ( NCODE .EQ. 5 ) GO TO 296                                     DYLT0388
  310 IF ( NCODE .EQ. 6 ) NCODE = 4                                     DYLT0389
      IF ( NNNN .NE. NPERWD ) GO TO 325                                 DYLT0390
      PRO1S = PRO1S + PRO1                                              DYLT0391
      PRO2S = PRO2S + PRO2                                              DYLT0392
      PRO3S = PRO3S + PRO3                                              DYLT0393
      DEGMAX = DEGMIN + WEDINC                                          DYLT0394
      IF (NOPT .NE. 1) WRITE (NWR,1060) DEGMIN,DEGMAX,PRO1,PRO2,PRO3    DYLT0395
      IF ( NCODE .GT. 3 ) GO TO 316                                     DYLT0396
```

```
      WRITE (NWR,1080)                                                    DYLT0397
      DO 315 IIII = 1, MMM                                                DYLT0398
      IF ( IIII .EQ. 1 ) EGMIN = 0.0                                      DYLT0399
      IF ( IIII .NE. 1 ) EGMIN = EGMAX                                    DYLT0400
      BET1 = NPERWD*IIII                                                  DYLT0401
      EGMAX = BET1*DEGN/PIF                                               DYLT0402
      SSBET = SNBET(IIII)/SUMBT1                                          DYLT0403
      IF ( IIII .EQ. MMM .AND. NJEG .EQ. 2 ) EGMAX = EEGMAX               DYLT0404
      WRITE(NWR,1085) EGMIN,EGMAX,SNBET(IIII),SSBET                       DYLT0405
  315 CONTINUE                                                            DYLT0406
  316 CONTINUE                                                            DYLT0407
      PRO1 = 0.0                                                          DYLT0408
      PRO2 = 0.0                                                          DYLT0409
      PRO3 = 0.0                                                          DYLT0410
      DEGGIN = DEGMAX                                                     DYLT0411
      NNNN = 0.0                                                          DYLT0412
  325 CONTINUE                                                            DYLT0413
      IF ( NOPT .NE. 1 ) WRITE (NWR,1065) PRO1S, PRO2S, PRO3S             DYLT0414
      PLINED(1,J,K) = PRO1S                                               DYLT0415
      PLINED(2,J,K) = PRO2S                                               DYLT0416
      PLINED(3,J,K) = PRO3S                                               DYLT0417
      TPRO1 = TPRO1 + PRO1S                                               DYLT0418
      TPRO2 = TPRO2 + PRO2S                                               DYLT0419
      TPRO3 = TPRO3 + PRO3S                                               DYLT0420
      IF ( PRO1S .LT. PRO1M ) GO TO 380                                   DYLT0421
      PRO1M = PRO1S                                                       DYLT0422
      PRO1S1 = STRPT1                                                     DYLT0423
      PRO1S2 = STRPT                                                      DYLT0424
      PRO1S3 = STRPT2                                                     DYLT0425
      PRO1D = BETA2                                                       DYLT0426
  380 IF ( PRO2S .LT. PRO2M ) GO TO 390                                   DYLT0427
      PRO2M = PRO2S                                                       DYLT0428
      PRO2S1 = STRPT1                                                     DYLT0429
      PRO2S2 = STRPT                                                      DYLT0430
      PRO2S3 = STRPT2                                                     DYLT0431
      PRO2D = BETA2                                                       DYLT0432
```

```
  390 IF ( PRO3S .LT. PRO3M ) GO TO 400                                  DYLT0433
      PRO3M = PRO3S                                                      DYLT0434
      PRO3S1 = STRPT1                                                    DYLT0435
      PRO3S2 = STRPT                                                     DYLT0436
      PRO3S3 = STRPT2                                                    DYLT0437
      PRO3D = BETA2                                                      DYLT0438
  400 CONTINUE                                                           DYLT0439
      BETA1 = BETA3                                                      DYLT0440
  500 CONTINUE                                                           DYLT0441
      WRITE (NWR,1070)                                                   DYLT0442
      IF ( NCODE .EQ. 1 .OR. NCODE .EQ. 4 ) WRITE (NWR,1075) NN1, NN1,   DYLT0443
     1 TPRO1, PRO1M, PRO1S1, PRO1S2, PRO1S3, PRO1D                       DYLT0444
      IF ( NCODE .EQ. 2 .OR. NCODE .EQ. 4 ) WRITE (NWR,1075) NN2, NN2,   DYLT0445
     1 TPRO2, PRO2M, PRO2S1, PRO2S2, PRO2S3, PRO2D                       DYLT0446
      IF ( NCODE .EQ. 3 .OR. NCODE .EQ. 4 ) WRITE (NWR,1075) NN1, NN2,   DYLT0447
     1 TPRO3, PRO3M, PRO3S1, PRO3S2, PRO3S3, PRO3D                       DYLT0448
 9000 CONTINUE                                                           DYLT0449
      STOP                                                               DYLT0450
      END                                                                DYLT0451
      SUBROUTINE STRIKE(PIF,STRFA,FA2S,DIPFA,D1FA,D2FA,A,B,C)            DYLT0452
      REAL NO                                                            DYLT0453
      DATA NO,SO,ET,WT/'N','S','E','W'/                                  DYLT0454
      IF (FA2S .EQ. WT .AND. D2FA .EQ. ET) THOM = (270. - STRFA)*PIF     DYLT0455
      IF (FA2S .EQ. WT .AND. D2FA .EQ. WT) THOM = (90. - STRFA)*PIF      DYLT0456
      IF (FA2S .EQ. ET .AND. D2FA .EQ. ET) THOM = (270. + STRFA)*PIF     DYLT0457
      IF (FA2S .EQ. ET .AND. D2FA .EQ. WT) THOM = (90. + STRFA)*PIF      DYLT0458
      IF (STRFA .EQ. 90.0 .AND. D1FA .EQ. NO) THOM = 180.0*PIF           DYLT0459
      IF (STRFA .EQ. 90.0 .AND. D1FA .EQ. SO) THOM = 0.0                 DYLT0460
      PHOM = (90. - DIPFA)*PIF                                           DYLT0461
      A = SIN(PHOM)*COS(THOM)                                            DYLT0462
      B = SIN(PHOM)*SIN(THOM)                                            DYLT0463
      C = COS(PHOM)                                                      DYLT0464
      RETURN                                                             DYLT0465
      END                                                                DYLT0466
```

Appendix D

A Generalization of the Stiffness Approach

The approach used by St. John to examine symmetric wedges can be extended to handle more general conditions. In particular, the extended solution considers asymmetrical wedges and arbitrary loadings as pictured in Figure D.1. The approach is predicated on a number of basic assumptions:

1. The rock wedge and parent rock mass are rigid bodies compared to the joints. All the deformations in the system occur at joints which are idealized as thin, pliant mediums.

2. There is a linear relationship between the stresses and displacements in the joint planes. The shear and normal stresses can be related to the corresponding displacements through the following equations:

$$k_s = \frac{\text{shear stress}}{\text{shear displacement}} = \frac{\tau}{\delta_s} \qquad (D.1)$$

$$k_n = \frac{\text{normal stress}}{\text{normal displacement}} = \frac{\sigma_n}{\delta_n} \qquad (D.2)$$

3. When subjected to loads the wedge will experience translational (but not rotational) movements. These movements can be expressed in

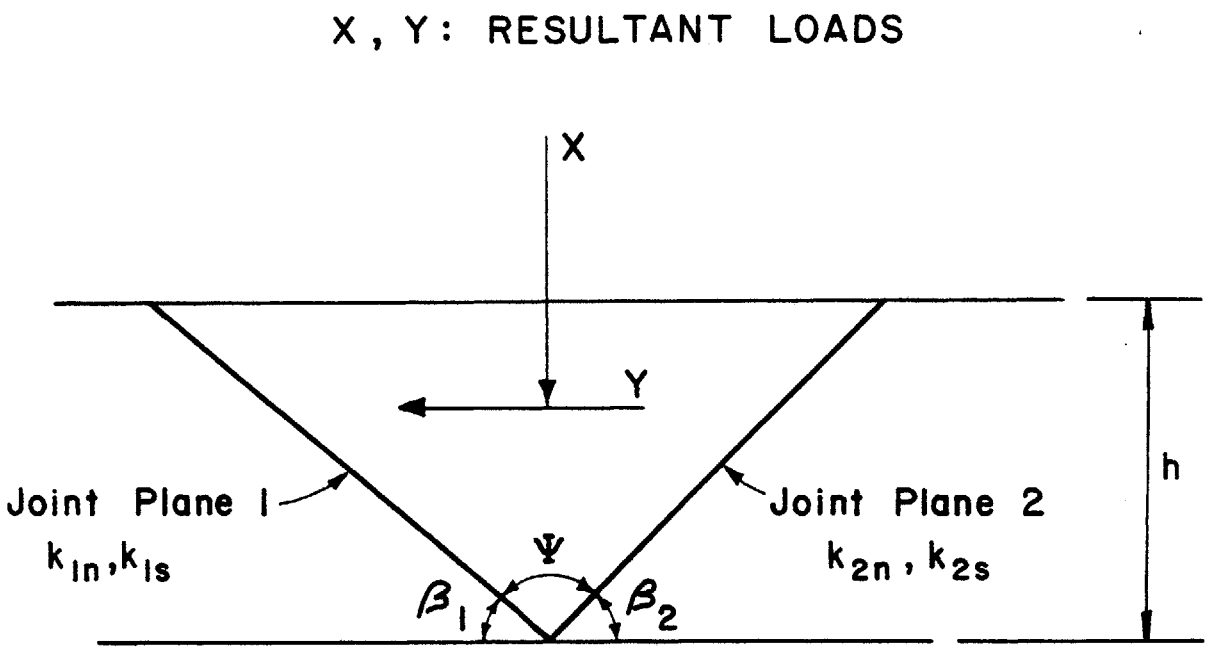


Figure D.1 Generalized Stiffness Approach--Known Parameters

terms of two parameters: δ_t, the magnitude of resultant displacement and α, the direction of resultant displacement (Figure D.2).

These assumptions make the problem statically determinate. The various reactions at the joint planes (as well as δ_t and α) can be obtained by solving a set of six simultaneous equations. The six unknowns ($N_1, T_1, N_2, T_2, \delta_t$ and α) are shown in Figure D.2).

δ_t can be resolved into horizontal and vertical components: $\delta_v = \delta_t \cos \alpha$ and $\delta_h = \delta_t \sin \alpha$.

The displacement pattern at the left joint (joint plane '1') is shown in Figure D.3. One can derive the following relationships:

$$\text{area of joint '1'} = h \csc \beta_1 \tag{D.3}$$

$$\begin{aligned} T_1 &= \tau_1 h \csc \beta_1 \\ &= (\delta_v \sin \beta_1 - \delta_h \cos \beta_1) k_{1s} h \csc \beta_1 \\ &= \delta_t (\cos \alpha \sin \beta_1 - \sin \alpha \cos \beta_1) k_{1s} h \csc \beta_1 \\ &= \delta_t (\cos \alpha - \sin \alpha \operatorname{ctn} \beta_1) k_{1s} h \end{aligned} \tag{D.4}$$

positive T_1 ∴ ↘

$$\begin{aligned} N_1 &= \sigma_1 h \csc \beta_1 \\ &= (\delta_h \sin \beta_1 + \delta_v \cos \beta_1) k_{1n} h \csc \beta_1 \\ &= \delta_t (\sin \alpha \sin \beta_1 + \cos \alpha \cos \beta_1) k_{1n} h \csc \beta_1 \\ &= \delta_t (\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1) k_{1n} h \end{aligned} \tag{D.5}$$

positive N_1 ∴ ↗

δ_T = Resultant Displacement

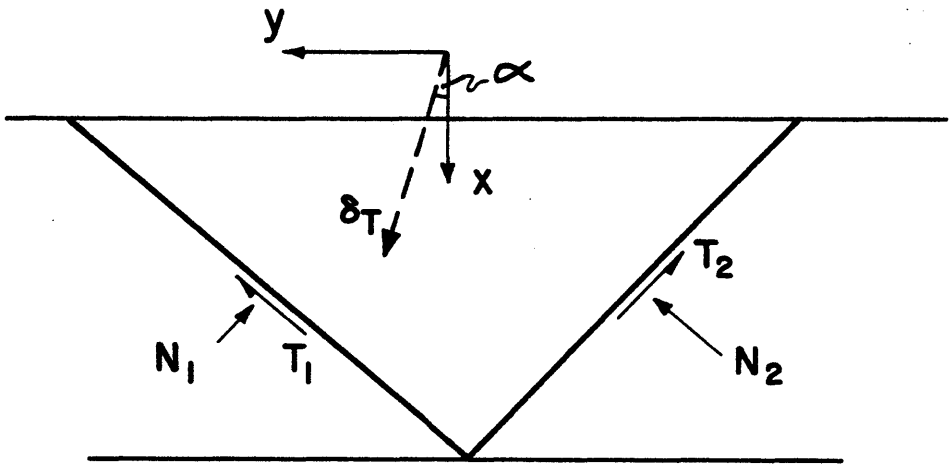
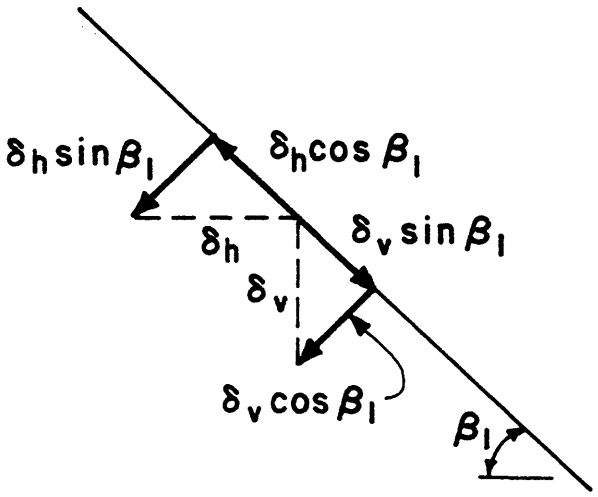


Figure D.2 Generalized Stiffness Approach--Unknown Parameters

Left Joint



$$\delta_v = \delta_T \cos\alpha$$
$$\delta_h = \delta_T \sin\alpha$$

Figure D.3 Displacements--Left Joint

The right joint (joint plane '2') can be analyzed in the same manner (Figure D.4):

$$\text{area of joint '2'} = h \csc \beta_2 \tag{D.6}$$

$$\begin{aligned} T_2 &= \tau_2 h \csc \beta_2 \\ &= (\delta_v \sin \beta_2 + \delta_h \cos \beta_2) k_{2s} h \csc \beta_2 \\ &= \delta_t (\cos \alpha \sin \beta_2 + \sin \alpha \cos \beta_2) k_{2s} h \csc \beta_2 \\ &= \delta_t (\cos \alpha + \sin \alpha \operatorname{ctn} \beta_2) k_{2s} h \end{aligned} \tag{D.7}$$

positive T_2 ∴ ↗

$$\begin{aligned} N_2 &= \sigma_2 h \csc \beta_2 \\ &= (\delta_v \cos \beta_2 - \delta_h \sin \beta_2) k_{2n} h \csc \beta_2 \\ &= \delta_t (\cos \alpha \cos \beta_2 - \sin \alpha \sin \beta_2) k_{2n} h \csc \beta_2 \\ &= \delta_t (\cos \alpha \operatorname{ctn} \beta_2 - \sin \alpha) k_{2n} h \end{aligned} \tag{D.8}$$

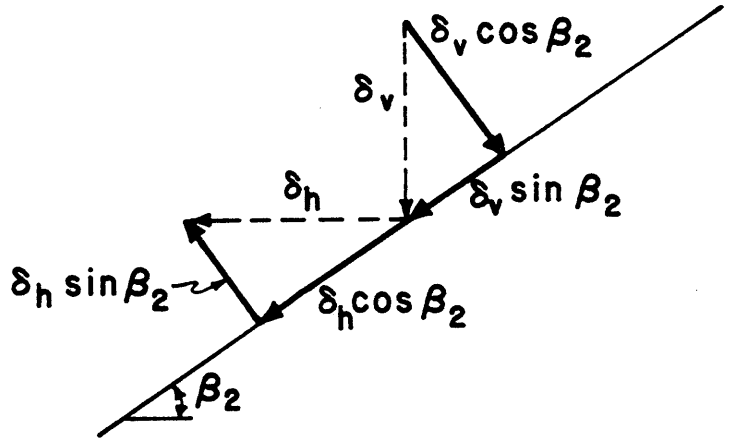
positive N_2 ∴ ↖

The term $\delta_t h$ is common to Equations D.4, D.5, D.7, and D.8. The equations can be rewritten to isolate that term:

$$\delta_t h = \frac{T_1}{(\cos \alpha - \sin \alpha \operatorname{ctn} \beta_1) k_{1s}} \tag{D.4A}$$

$$= \frac{N_1}{(\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1) k_{1n}} \tag{D.5A}$$

Right Joint



$$\delta_v = \delta_T \cos \alpha$$
$$\delta_h = \delta_T \sin \alpha$$

Figure D.4 Displacements--Right Joint

$$\delta_t h = \frac{T_2}{(\cos \alpha + \sin \alpha \operatorname{ctn} \beta_2) k_{2s}} \tag{D.7A}$$

$$= \frac{N_2}{(\cos \alpha \operatorname{ctn} \beta_2 - \sin \alpha) k_{2n}} \tag{D.8A}$$

Next, one can examine the equations for force equilibrium and sum forces in the x and y directions (Figure D.2):

$$N_1 \cos \beta_1 + T_1 \sin \beta_1 + N_2 \cos \beta_2 + T_2 \sin \beta_2 = X \tag{D.9}$$

$$N_1 \sin \beta_1 - T_1 \cos \beta_1 - N_2 \sin \beta_2 + T_2 \cos \beta_2 = Y \tag{D.10}$$

Equations D.4A, D.5A, D.7A D.8A, D.9 and D.10 constitute a set of six linear equations involving six unknowns: N_1, T_2 T_1, N_2, δ_t and α. The solution procedure can be simplified by expressing T_1, N_2 and T_2 in terms of N_1 and substituting into D.9 and D.10.

From D.5A and D.4A:

$$T_1 = \frac{(\cos \alpha - \sin \alpha \operatorname{ctn} \beta_1) k_{1s}}{(\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1) k_{1n}} N_1 \tag{D.11}$$

From D.5A and D.8A

$$N_2 = \frac{(\cos \alpha \operatorname{ctn} \beta_2 - \sin \alpha) k_{2n}}{(\sin \alpha + \cos \alpha \operatorname{ctn} \beta_1) k_{1n}} N_1 \tag{D.12}$$

From D5A and D.7A

$$T_2 = \frac{(\cos\alpha + \sin\alpha \operatorname{ctn}\beta_2)k_{2s}}{(\sin\alpha + \cos\alpha \operatorname{ctn}\beta_1)k_{1n}} N_1 \qquad (D.13)$$

Substituting into D.9 and D.10:

$$N_1\left(\cos\beta_1 + \frac{(\cos\alpha - \sin\alpha\operatorname{ctn}\beta_1)k_{1s}\sin\beta_1 + (\cos\alpha\operatorname{ctn}\beta_2 - \sin\alpha)k_{2n}\cos\beta_2 + (\cos\alpha + \sin\alpha\operatorname{ctn}\beta_2)k_{2s}\sin\beta_2)}{(\sin\alpha + \cos\alpha\operatorname{ctn}\beta_1)k_{1n}}\right) = X \qquad (D.14)$$

And

$$N_1\left(\sin\beta_1 + \frac{(\sin\alpha\operatorname{ctn}\beta_1 - \cos\alpha)k_{1s}\cos\beta_1 + (\sin\alpha - \cos\alpha\operatorname{ctn}\beta_2)k_{2n}\sin\beta_2 + (\cos\alpha + \sin\alpha\operatorname{ctn}\beta_2)k_{2s}\cos\beta_2)}{(\sin\alpha + \cos\alpha\operatorname{ctn}\beta_1)k_{1n}}\right) = Y \qquad (D.15)$$

Equations D.14 and D.15 are of the form

$$N_1(f(\alpha)) = X \qquad (D.14A)$$

$$N_1(g(\alpha)) = Y \qquad (D.15A)$$

where $f(\alpha)$ and $g(\alpha)$ are functions of the single unknown and numerous constants. The term N_1 can be isolated in both equations:

$$N_1 = \frac{X}{f(\alpha)} \qquad (D.14B)$$

$$N_1 = \frac{Y}{g(\alpha)} \qquad (D.15B)$$

Equating the right sides of Equations D.14B and D.15B:

$$\frac{X}{f(\alpha)} = \frac{Y}{g(\alpha)} \tag{D.16}$$

or

$$\frac{X}{Y} = \frac{f(\alpha)}{g(\alpha)} \tag{D.16A}$$

Thus,

$$\frac{X}{Y} = \frac{\cos\beta_1 + \dfrac{(\cos\alpha - \sin\alpha \operatorname{ctn}\beta_1)k_{1s}\sin\beta_1 + (\cos\alpha \operatorname{ctn}\beta_2 - \sin\alpha)k_{2n}\cos\beta_2 + (\cos\alpha + \sin\alpha \operatorname{ctn}\beta_2)k_{2s}\sin\beta_2}{(\sin\alpha + \cos\alpha \operatorname{ctn}\beta_1)k_{1n}}}{\sin\beta_1 + \dfrac{(\sin\alpha \operatorname{ctn}\beta_1 - \cos\alpha)k_{1s}\cos\beta_1 + (\sin\alpha - \cos\alpha \operatorname{ctn}\beta_2)k_{2n}\sin\beta_2 + (\cos\alpha + \sin\alpha \operatorname{ctn}\beta_2)k_{2s}\cos\beta_2}{(\sin\alpha + \cos\alpha \operatorname{ctn}\beta_1)k_{1n}}} \tag{D.16B}$$

Multiplying by $\dfrac{(\sin\ \alpha + \cos\ \alpha\ \operatorname{ctn}\ \beta_1)k_{1n}}{(\sin\ \alpha + \cos\ \alpha\ \operatorname{ctn}\ \beta_1)k_{1n}}$ and collecting terms:

$$\frac{X}{Y} = \frac{\sin\alpha(k_{1n}\cos\beta_1 - k_{1s}\cos\beta_1 - k_{2n}\cos\beta_2 + k_{2s}\cos\beta_2) + \cos\alpha\left(k_{1n}\dfrac{\cos^2\beta_1}{\sin\beta_1} + k_{1s}\sin\beta_1 + k_{2n}\dfrac{\cos^2\beta_2}{\sin\beta_2} + k_{2s}\sin\beta_2\right)}{\sin\alpha\left(k_{1n}\sin\beta_1 + k_{1s}\dfrac{\cos^2\beta_1}{\sin\beta_1} + k_{2n}\sin\beta_2 + k_{2s}\dfrac{\cos^2\beta_2}{\sin\beta_2}\right) + \cos\alpha(k_{1n}\cos\beta_1 - k_{1s}\cos\beta_1 - k_{2n}\cos\beta_2 + k_{2s}\cos\beta_2)} \tag{D.17}$$

Therefore, $\dfrac{X}{Y} = \dfrac{A\ \sin\ \alpha + B\ \cos\ \alpha}{C\ \sin\ \alpha + A\ \cos\ \alpha}$ (D.17A)

where $A = (k_{1n} - k_{1s})\cos\ \beta_1 + (k_{2s} - k_{2n})\cos\ \beta_2$

$$B = k_{1n}\frac{\cos^2\ \beta_1}{\sin\ \beta_1} + k_{1s}\ \sin\ \beta_1 + k_{2n}\frac{\cos^2\ \beta_2}{\sin\ \beta_2} + k_{2s}\ \sin\beta_2$$

$$C = k_{1n}\ \sin\ \beta_1 + k_{1s}\frac{\cos^2\ \beta_1}{\sin\ \beta_1} + k_{2n}\ \sin\beta_2 + k_{2s}\frac{\cos^2\ \beta_2}{\sin\ \beta_2}$$

Finally,

$$\alpha = \tan^{-1}\left[\frac{YB - XA}{XC - YA}\right] \qquad (D.18)$$

Once α is obtained the remaining unknowns can be evaluated through substitution:

Equation D.14 $\rightarrow$ N_1

Equation D.11 $\rightarrow$ T_1

Equation D.12 $\rightarrow$ N_2

Equation D.13 $\rightarrow$ T_2

Equation D.4A $\rightarrow$ δ_t

All of the equations derived in this appendix are also valid for wedges in which β_1 or β_2 is greater than 90°. The results for these cases must be examined quite closely because one of the normal forces may become tensile. This problem is discussed in Section 4.3.

Special Case

When Y = 0, Equation D.18 simplifies to:

$$\alpha = \tan^{-1} - \frac{XA}{XC} \qquad (D.19)$$

or

(D.19A)

$$\alpha = \tan^{-1} \frac{(k_{1s} - k_{1n}) \cos \beta_1 + (k_{2n} - k_{2s}) \cos \beta_2}{k_{1n} \sin \beta_1 + k_{1s} \frac{\cos^2 \beta_1}{\sin \beta_1} + k_{2n} \sin \beta_2 + k_{2s} \frac{\cos^2 \beta_2}{\sin \beta_2}}$$

Section 4.2 examines this special case in connection with some limiting values for the stiffnesses.

Appendix E

Computer Program SWARS-2PM

SWARS-2PM is an extended version of SWARS-2P that includes the effects of stiffnesses and in situ stresses as discussed in Chapter 4. The two programs have precisely the same options available for analyzing rock bolts, seismic loads, point loads, and groundwater conditions. Campbell (1974) documented the SWARS-2PM program; the SWARS-2P user should consult Campbell's work for additional information on the program logic and a more detailed description of the loading and groundwater options. The only major difference between SWARS-2P and SWARS-2PM is the manner in which they calculate the factor(s) of safety (FS). SWARS-2PM computes two FS's; FS_I, the factor of safety against initial movement, and FS_U, the factor of safety against ultimate failure. As indicated in Sections 4.3 and 4.4, FS_I is a function of the in situ stresses and the joint stiffnesses. FS_U applies to movement down the line of intersection and is independent of the original stresses and of the joint stiffness.* FS_U is analytically identical to the residual factor of safety (FS_R) computed by SWARS-2P. (SWARS-2P also computes another FS which is similar to (FS_R) but includes the effects of cohesion and asperities in calculating the resistance.)

* FS_I and FS_U are only defined for failure modes that consist of sliding on both planes. If the failure modes involve sliding along a single plane, SWARS-2PM will compute the factors of safety in exactly the same manner as SWARS-2P, i.e., a peak FS and a residual FS.

E.1 Coordinate System

The coordinate system used in SWARS-2PM is shown in Figure E.1. The positive X axis is horizontal to the right. The positive Y axis is horizontal and 90^{o} counterclockwise from the positive X axis. (It is directed into the slope.) The positive Z axis is vertical upward.

E.2 Units

All units used in the input must be consistent. Units are generally in feet or pounds, but any other system may be employed as long as all other input is in consistent units. Angular input is always in degrees.

Unless otherwise noted, input data does not have to be right justified. The term "integer" implies that the parameter is expressed without a decimal point.

E.3 Inputs

Card #1

Case Number: (ICASE) is entered as an integer in columns 1 to 5. It must be right justified.

Number of Planes: (N) is entered as an integer in column 10. N may range from 2 to 8.

Cards #2 through #(N + 1)

Note: A separate card with the following data is entered for each joint plane being analyzed.

Strike of the Joint Plane: (H1STK, STK, H2STK) is entered in geologic notation in columns 1 to 4. Geologic strike must be expressed in the northern quadrants and the

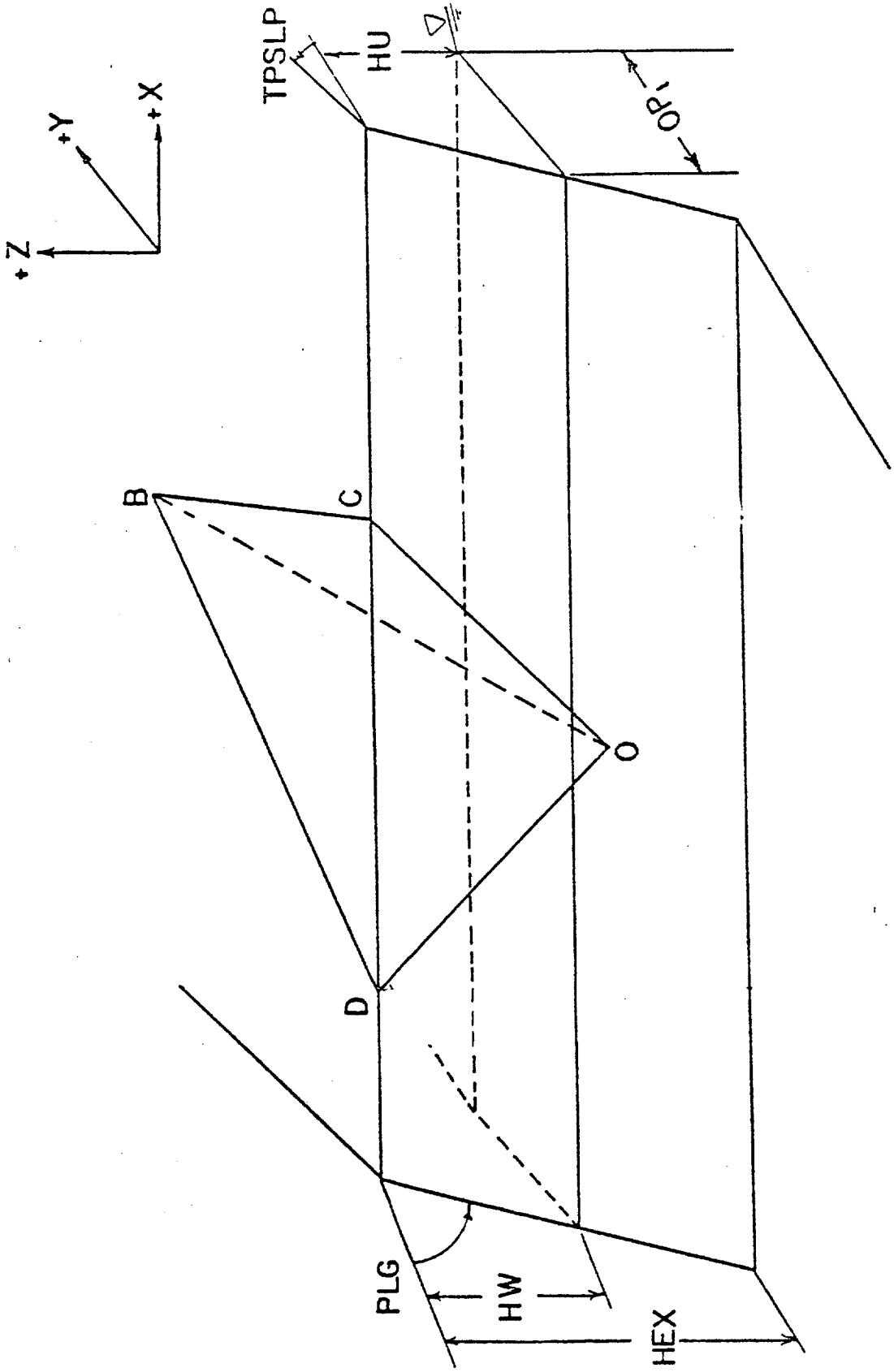


Figure E.1 Model Slope

angle must be expressed as a two digit integer. (Examples: N45E, N06W)

Dip of the Joint Plane: (DIP, H1DIP, H2DIP) is entered in geologic notation in columns 6 to 9. Dip directions must indicate a particular quadrant i.e., NE, NW, SE, SW. The angle must be expressed as a two digit integer. (Examples: 30NW, 09NE)

Friction Angle: (PHX) on the joint planes is entered in columns 11 to 15 with a maximum of two decimal places.

Asperties: (AQ), the angular value of additional shearing resistance due to larger size irregularities of the joint planes, is entered in columns 16 to 20 with a maximum of two decimal places.

Cohesion of Intact Rock: (TAU) is entered in columns 21 to 30 with a maximum of two decimal places.

Normal Stiffness: (STIFFN) is entered in columns 31 to 40 with a maximum of three decimal places. STIFFN can also be entered in scientific notation with a maximum of three decimal places. (Examples: 173.2, 1.732E2) If columns 31 to 40 are left blank the program will default to STIFFN = 1000000.0.

Shear Stiffness: (STIFFS) is entered in columns 41 to 50 with a maximum of three decimal places. STIFFS can also be entered in scientific notation with a maximum of 3 decimal places. (Examples: 6789.6, 6.79E2) If the columns 41 to 50 are left blank the computer will default to STIFFS = 1.0.

Joint Persistence: (PCTJTG) is the percent of the area of the total joint that is not intact rock. PCTJTG is entered in columns 51 to 55 with a maximum of two decimal places.

Depth to Water: (HU) is measured from the crest of the excavation to a horizontal piezometric level (Figure E.1). It is used in conjunction with Piezometric Options 3 and 4 (Ref.: Piezometric Option). HU is entered in columns 56 to 60 with a maximum of one decimal place.

Uniform Water Pressure: (UAV) is entered in columns 61 to 70 with a maximum of three decimal places. UAV is used in conjunction with Piezometric Option 2 only (Ref.: Piezometric Option).

Piezometric Option: (IU) is a code that indicates how cleft water effects are to be incorporated into the analysis. 1: No effect from water; 2: Uniform pressure acting on the joint; 3: Pressure determined by a horizontal phreatic surface; 4: Pressure determined by an arbitrary phreatic surface. The code is entered in column 71 as an integer.

Card #(N + 2)

Strike of the Slope: (H1AZ1, AZ1, H2AZI) is entered in geologic notation in columns 1 to 4. Geologic strike must be expressed in the northern quadrants and the angle must be expressed as a two digit integer. (Examples: N45E, N06W)

Dip of the Slope: (PLG, H1PLG, H2PLG) is entered in geologic notation in columns 6 to 9. Dip directions must indicate a particular quadrant, i.e., NE, NW, SE, SW. The angle is expressed as a two digit integer. (Examples: 16NW, 07NE)

Height of the Slope: (HEX) is entered in columns 11 to 20 with a maximum of three decimal places. HEX is measured from the crest of the slope to the bottom of the slope (Figure E.1).

Length of the Slope: (XLEN) is entered in columns 21 to 30 with a maximum of three decimal places.

Angle of the Top Slope: (TPSLP) is entered in columns 31 to 40 with a maximum of three decimal places. TPSLP is shown in Figure E.1.

Unit Weight of Rock: (UNIWGT) is entered in columns 41 to 50 with a maximum of three decimal places.

Unit Weight of Water: (WWGT) is entered in columns 51 to 60 with a maximum of three decimal places.

Uniform Vertical Surcharge: (SUR) is the uniform vertical load acting per unit area on the top face of the wedge (face BCD in Figure E.1). SUR is entered in columns 61 to 70 with a maximum of three decimal places.

Surcharge Option: (IS) There are two options available to analyze the effects of the surcharge on the stability of the tetrahedron in the dynamic case. Option 1 assumes that the surcharge is a mass and that it is accelerated in the

same directions as the wedge under dynamic loading. The code 1 or 2 is entered in column 71 without a decimal.

Card # (N + 3)

Stress Ratio: (RATIO) is entered in columns 1 to 5 with a maximum of three decimal places. RATIO is the ratio of horizontal to vertical stresses within the wedge.

FS Option #1: (NDOP) is a code that indicates how the program should compute FS_I. As indicated in Section 4.4 (q.v.), FS_I is defined as

$$FS_I = \frac{\bar{R} \cdot \bar{\Delta}}{\bar{D} \cdot \bar{\Delta}} \qquad \text{(E.1)}$$

where $\bar{R}$ is the resultant of all resisting forces, $\bar{D}$ is the resultant of all driving forces, and $\bar{\Delta}$ is the direction of impending movement. If a ' 0 ' is placed in column 15 $\bar{\Delta}$ is the displacement that would occur in response to a vertical load. If any other integer is placed in columns 11 to 15 $\bar{\Delta}$ is the displacement that occurs in response to the resultant of all external loads, i.e., all loads except the weight.

FS Option #2: (LOPT) is an option that allows the user to analyze all loads (including the weight of the wedge) with the stiffness approach. If a ' 1 ' is entered in column 20 the program will use this option. If any other integer is placed in colums 16 to 20 the program will analyze loads

according to the "standard" procedure presented in Section 4.3, i.e., stress approach for gravitational loads, stiffness approach for all other loads. If columns 16 to 20 are left blank the program will default to the standard procedure.

Card # (N + 4)

Depth to Piezometric Level on the Slope: (HW) is entered in columns 1 to 10 with a maximum of three decimal places. HW is shown in Figure E.1 and is used in conjunction with Piezometric Option 4.

Distance OPI: (OP1) is used in conjunction with Piezometric Option 4. As shown in Figure E.1, it is the horizontal distance from the face of the slope where the phreatic surface exits to the point where the phreatic surface becomes horizontal. OP1 is entered in columns 11 to 20 with a maximum of three decimal places.

Seismic Coefficients X, Y, Z: (AKX, AKY, AKZ) Seismic forces are entered as decimal fractions of the gravitational force. Positive values will be forces acting at the centroid of the wedge in the direction of the appropriate positive coordinate, negative values act in the direction of the appropriate negative coordinate. AKX, AKY and AKZ are entered in columns 21 to 30, 31 to 40, and 41 to 50 respectively. The three coefficents are expressed with a maximum of three decimal places. If no seismic loads are to be ocnsidered columns 21 to 50 may be left blank--the computer will default

to AKX = AKY = AKZ = 0.

Point Loads: (PX, PY, PZ) can be used in the program. PX, PY, and PZ as point loads acting at the centroid of the wedge. PX, PY, and PZ are entered in columns 51 to 60, 61 to 70, and 71 to 80 respectively. They are expressed with a maximum of three decimal places. If no point loads are to be considered columns 51 to 80 may be left blank--the computer will default to PX = PY = PZ = 0.

Card # (N + 5)

Note: If no rock bolts are to be analyzed, a blank card is inserted.

Plane Numbers Supporting a Tetrahedron: (III,JJJ) It is anticipated that a user will run the program initially without any rock bolts. After the primary stability analysis, certain tetrahedrons may prove to need rock bolts. III and JJJ and the respective joint plane numbers specified by the user that support a particular tetrahedron that requires rock bolting. They are entered in columns 5 and 10 respectively as integers.

Number of Different Bolt Orientations: (NNN) The program is designed to accommodate up to sixteen different bolt orientations simultaneously; eight different orientations for face bolting, and eight different orientations for top bolting. The value of NNN may range from 1 to 8 and is entered in column 15 as an integer.

Card # (N + 6) Through # (NNN + N + 6)

Note: If no rock bolts are to be analyzed, these cards need not be entered.

Number of Top Bolts: (BTOP) is the total number of rock bolts at any particular orientation acting on the top side of the tetrahedron. BTOP may range in value from 1.0 to 9999.0 and is entered in columns 1 to 10 with a maximum of three decimal places.

Azimuth of Top Bolts: (ORT) is the angle projection in the XY plane of the angle the bolt set makes with the strike of the slope measured from 0 to 360 counterclockwise from the positive X-axis (Figure E.2). The value is entered in degrees with a maximum of three decimal places in columns 11 to 20.

Plunge of Top Bolts: (DPT) is the angle the bolt set makes with the positive Y axis measured from 0 to 360 in a clockwise direction (Figure E.2). The value is entered in degrees with a decimal in columns 21 to 30.

Load on a Top Bolt: (TEN) is the total working load of a top bolt. It is entered with a maximum of three decimal places in columns 31 to 40.

Number of Face Bolts: (BFAC) is the total number of rock bolts at any particular orientation load acting on the slope face of the tetrahedron. BFAC may range in value from 1.0 to 9999.0 and is entered in columns 41 to 50 with a maximum of three decimal places.

Azimuth of Face Bolts: (ORF) is defined in the same manner as ORT. It is entered in columns 51 to 60 with a maximum of three decimal places.

Plunge of Face Bolt: (DPF) is defined in the same manner as DPT. It is entered in columns 61 to 70 with a maximum of three decimal places.

Load on a Face Bolt: (FTEN) is the total working load of a face rock bolt. It is entered with a maximum of three decimal places in columns 71 to 80.

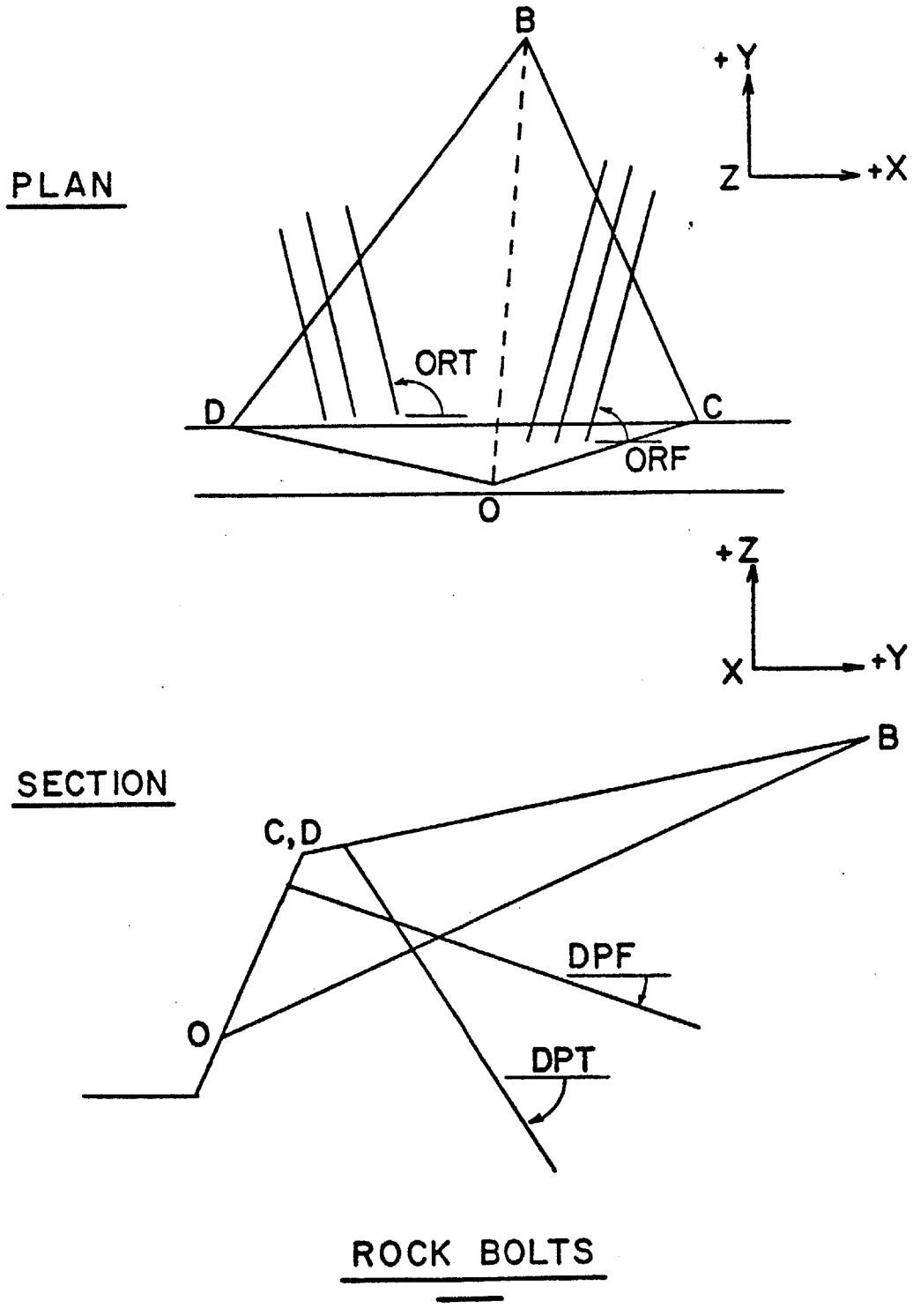


Figure E.2 Parameters For Rock Bolt Option

SWARS-2PM Listing

```
C                                  SWARS-PM                                                    SWPM0001
C        SLIDING WEDGE ANALYSIS OF ROCK SLOPES -- TWO PLANES                  26920030 SWPM0002
C      THIS IS THE ROUTINE TO TREAT FAILURE WHEN TWO PLANES ARE               26920040 SWPM0003
C      INVOLVED.                                                              26920050 SWPM0004
       DIMENSION BETA(8),C(8),GAMMA(8),VX(8),VY(8),VZ(8),TAU(8),PCTJTG(8)     26920060 SWPM0005
       DIMENSION UX(8),UY(8),UZ(8),WX(8),WY(8),WZ(8),B(8)                     26920070 SWPM0006
       DIMENSION T(8),TX(8),TY(8),TZ(8),EN(8),PHI(8)                          26920080 SWPM0007
       DIMENSION SX(8),SY(8),SZ(8),RSS(8),RRX(8),RRY(8),RRZ(8)                26920090 SWPM0008
       DIMENSION H1STK(8),STK(8),H2STK(8),DIP(8),H1DIP(8),H2DIP(8)            26920100 SWPM0009
       DIMENSION INDEX(8),DUMMY(8),PHX(8),AP(8),AQ(8),CH(8)                   26920110 SWPM0010
       DIMENSION WWX(8),WWY(8),WWZ(8),BET(8)                                  26920120 SWPM0011
       DIMENSION UAV(8),HU(8),UF(8),AREA(8),IU(8)                             26920130 SWPM0012
       DIMENSION BTOP(8),ORT(8),DPT(8),TEN(8)                                 26920140 SWPM0013
       DIMENSION BFAC(8),ORF(8),DPF(8),FTEN(8)                                26920150 SWPM0014
       DIMENSION  STIFFN(8), STIFFS(8), RATIOK(8)                                      SWPM0015
       DIMENSION UI(8)                                                                 SWPM0016
       REAL NORD, HXX, HXY                                                             SWPM0017
       LOGICAL FLIM                                                                    SWPM0018
       DATA NORD,SOTH,EAST,WEST/'N','S','E','W'/                              26920160 SWPM0019
       NWR = 5                                                                         SWPM0020
       NRD = 8                                                                         SWPM0021
       FLIM = .TRUE.                                                                   SWPM0022
       PI=3.141593                                                            26920180 SWPM0023
       PI2=PI/2.                                                              26920190 SWPM0024
       FACT=180./PI                                                           26920200 SWPM0025
    10 FORMAT(3I5)                                                            26920210 SWPM0026
    11 FORMAT (2I5)                                                                 SWPM0027
    15 FORMAT(A1,F2.0,A1,1X,F3.0,A1,A1,6G10.3,I1)                             26920220 SWPM0028
    16 FORMAT (F5.3,2I5)                                                            SWPM0029
    20 FORMAT (8G10.3)                                                        26920230 SWPM0030
    25 FORMAT (A1,F2.0,A1,1X,F2.0,A1,A1,1X,2F5.2,F10.3,2G10.3,2F5.2,                SWPM0031
      1 F10.3,I1)                                                                   SWPM0032
  1001 FORMAT('0',T20,'FS =',F6.2,' FOR SLIDING ON PLANE ',I1)                26920250 SWPM0033
  1002 FORMAT ('1',T27,'********   CASE NUMBER IS ',I5,3X,'********',////         SWPM0034
     1 T30,'*****  JOINT SETS ARE ',I1,' AND ',I1,2X,'*****',///                 SWPM0035
     2 T30,'********     JOINT  DATA     ********',///)                          SWPM0036
                                                                                  PAGE   1
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1004 FORMAT (29X,'********     JOINT SET ',I1,5X,'********',///                    SWPM0037
    1 18X,'STRIKE:',16X,'N',F3.0,A1,3X,'DIP:',18X,F3.0,A1,A1,//                  SWPM0038
    2 18X,'PHI:',19X,F5.2,3X,'ASPERITIES:',11X,F5.2,//                           SWPM0039
    3 18X,'COHESION:',9X,F10.2,3X,'PERSISTENCE:',10X,F5.2,//                     SWPM0040
    4 18X,'NORMAL STIFFNESS: ',G10.2,'   SHEAR STIFFNESS: ',G10.2,//             SWPM0041
    5 34X,'STIFFNESS RATIO: ',F10.2,//                                           SWPM0042
    6 18X,'PIEZOMETRIC OPTION:',8X,I1)                                           SWPM0043
1006 FORMAT (37X,'SEISMIC  ACCELERATIONS:',//,18X,'AKX =',F7.3,12X,              SWPM0044
    1 'AKY =',F7.3,11X,'AKZ =',F7.3,/)                                           SWPM0045
1007 FORMAT (40X,'POINT LOADS:',//,18X,'PX =',F9.2,12X,'PY =',F9.2,13X,          SWPM0046
    1 'PZ =',F9.2,/)                                                             SWPM0047
1008 FORMAT('0',T20,'RESULTANT TENDS TO LIFT BLOCK OFF SUPPORTS')       26920480 SWPM0048
1009 FORMAT('0',T20,'FS IS NEG. SINCE THE DIRECTIONS OF ADDITIONAL'/    26920490 SWPM0049
    1T20,'LOADS OR SEISMIC FORCES TEND TO DISPLACE THE WEDGE'/           26920500 SWPM0050
    2T20,'UP THE LINE OF INTERSECTION OF THE TWO PLANES')                26920510 SWPM0051
1010 FORMAT('0',T20,'CHECK FOR TOPPLING ABOUT POINT O')                  26920520 SWPM0052
1011 FORMAT('0',42X,'SEISMIC DATA'//                                     26920530 SWPM0053
    1T20,'MINIMUM SEISMIC FORCE NEEDED IS ',F6.3/                        26920540 SWPM0054
    2T20,'ACTING AT AN ORIENTATION OF ',F6.3,'  ',F6.3,'  ',F6.3)        26920550 SWPM0055
1012 FORMAT(T20,'NO ROTATIONS OCCUR ON EITHER PLANE')                    26920560 SWPM0056
1013 FORMAT(T20,'FS =',F6.2,' FOR ROTATIONS ON PLANE ',I2)               26920570 SWPM0057
1014 FORMAT('0',T20,'LINE OF INTERSECTION AND EXCAVATION',               26920580 SWPM0058
    1' FACE ARE PARALLEL')                                               26920590 SWPM0059
1015 FORMAT('0',41X,'ROCK BOLT DATA')                                    26920600 SWPM0060
1016 FORMAT (T20,'FS(ULTIMATE) =',F6.2)                                           SWPM0061
1018 FORMAT ('0',T20,'FS(INITIAL) =',F6.2,' FOR SLIDING ON BOTH ',                SWPM0062
    1 'PLANES')                                                                   SWPM0063
1019 FORMAT('0',T20,'LINE OF INTERSECTION DOES NOT INTERSECT',           26920750 SWPM0064
    1' TOP OF SLOPE')                                                    26920760 SWPM0065
1020 FORMAT('0',T20,'DIP OF THE LINE OF INTERSECTION',/                  26920770 SWPM0066
    1T20,'IS GREATER THAN THE DIP OF THE EXCAVATION')                    26920780 SWPM0067
1021 FORMAT('0',T20,'STRIKE OF PLANE AND EXCAVATION ARE THE SAME')       26920790 SWPM0068
1022 FORMAT('0',T20,'DIP AND STRIKE OF JOINT ARE IN THE SAME',           26920800 SWPM0069
    1' DIRECTION')                                                       26920810 SWPM0070
1023 FORMAT('0',T20,'DIP AND STRIKE OF THE EXCAVATION ARE IN THE',       26920820 SWPM0071
    1' SAME DIRECTION')                                                  26920830 SWPM0072
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1024 FORMAT('0',T20,'BOLT SET NO. ',I1,//                                  26920840 SWPM0073
    1T20,'TOP BOLTS  ',F5.0,'  LOAD = ',F10.0,/                           26920850 SWPM0074
    2T20,'ORT = ',F5.1,'  DPT = ',F5.1,/                                  26920860 SWPM0075
    3T20,'FACE BOLTS ',F5.0,'  LOAD = ',F10.0,/                           26920870 SWPM0076
    4T20,'ORF = ',F5.1,'  DPF = ',F5.1)                                   26920880 SWPM0077
1030 FORMAT ('+',48X,'NO EFFECT FROM WATER',///)                                SWPM0078
1031 FORMAT ('+',48X,'UNIFORM PRESSURE:',2X,F8.3,///)                          SWPM0079
1032 FORMAT ('+',48X,'DEPTH TO WATER:',4X,F8.3,///)                            SWPM0080
1033 FORMAT ('+',48X,'DEFINED PHREATIC SURFACE',///)                           SWPM0081
1034 FORMAT (//,29X,'********     WEDGE  DATA     ********',//                 SWPM0082
    1 18X,'BETA 1 = ',F6.2,8X,'PSI = ',F6.2,8X,'BETA 2 = ',F6.2,//             SWPM0083
    2 18X,'AREA: JT. PLANE 1:',F10.2,3X,'AREA: JT. PLANE 2:',F10.2,//          SWPM0084
    3 18X,'AREA: TOP FACE:',3X,F10.2,3X,'AREA: EXCAV. FACE:',F10.2,//          SWPM0085
    4 37X,'VOLUME:',F17.2,//                                                   SWPM0086
    5 18X,'HEIGHT TO CREST:',2X,F10.2,3X,'HEIGHT ABOVE CREST:',F9.2,//         SWPM0087
    6 37X,'LINE OF INTERSECTION:',//,18X,'X = ',F6.3,15X,'Y = ',F6.3,          SWPM0088
    7 14X,'Z = ',F6.3,//)                                                      SWPM0089
1035 FORMAT (29X,'********      LOAD DATA      ********',//                   SWPM0090
    1 18X,'UNIT WEIGHT OF ROCK:',F8.2,3X,'UNIT WEIGHT OF WATER: ',F6.2,        SWPM0091
    2 //,34X,'IN SITU STRESS RATIO:',F6.3,//                                  SWPM0092
    3 34X,'VERTICAL SURCHARGE:'F8.2,/)                                         SWPM0093
1036 FORMAT (29X,'*******    EXCAVATION DATA    *******',//                   SWPM0094
    1 18X,'STRIKE:',16X,'N',F3.0,A1,3X,'DIP:',18X,F3.0,A1,A1,//                SWPM0095
    2 18X,'HEIGHT OF FACE:',F13.2,3X,'LENGTH OF FACE:',F12.2,//                SWPM0096
    3 31X,'INCLINATION OF TOP SLOPE:',F8.2,///)                                SWPM0097
200  CONTINUE                                                              26920890 SWPM0098
     READ (8,11) ICASE, N                                                       SWPM0099
     IF (ICASE.EQ.0) GO TO 9000                                            26920910 SWPM0100
     DO 30 I=1,N                                                           26920920 SWPM0101
     READ (8,25) H1STK(I),STK(I),H2STK(I),DIP(I),H1DIP(I),H2DIP(I),             SWPM0102
    1 PHX(I),AQ(I),TAU(I),STIFFN(I),STIFFS(I),PCTJTG(I),HU(I),                  SWPM0103
    2 UAV(I),IU(I)                                                              SWPM0104
     UI(I) = IU(I)                                                              SWPM0105
     IF (PCTJTG(I) .EQ. 0.) PCTJTG(I)=100.                                 26920950 SWPM0106
  30 CONTINUE                                                              26920960 SWPM0107
     READ(8,15) H1AZI,AZI,H2AZI,PLG,H1PLG,H2PLG,                           26920970 SWPM0108
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     1 HEX,XLEN,TPSIP,UNIWGT,WWGT,SUR,IS                                    26920980 SWPM0109
       READ (NRD,16) RATIO, LOPT, NDOP                                              SWPM0110
       READ(8,20) HW,OP1,AKX,AKY,AKZ,PX,PY,PZ                               26920990 SWPM0111
       READ(8,10) III,JJJ,NNN                                               26921000 SWPM0112
       IF(III.EQ.0) GO TO 65                                                26921010 SWPM0113
       DO 80 I=1,NNN                                                        26921020 SWPM0114
       READ(8,20) BTOP(I),ORT(I),DPT(I),TEN(I),BFAC(I),ORF(I),              26921030 SWPM0115
     1 DPF(I),FTEN(I)                                                       26921040 SWPM0116
  80 CONTINUE                                                               26921050 SWPM0117
  65 IF (UNIWGT.EQ.0.) UNIWGT=1.0                                           26921060 SWPM0118
       IF (UNIWGT.EQ.1.0) WWGT=0.0                                          26921070 SWPM0119
       S=AZI+90.                                                            26921080 SWPM0120
       IF (H2AZI.EQ.WEST) S=90.-AZI                                         26921090 SWPM0121
       DO 35 I=1,N                                                          26921100 SWPM0122
       BETA(I)=STK(I)+90.                                                   26921110 SWPM0123
       IF (H2STK(I).EQ.WEST) BETA(I)=90.-STK(I)                             26921120 SWPM0124
       GAMMA(I)=DIP(I)                                                      26921130 SWPM0125
       BET(I)=S-BETA(I)                                                     26921140 SWPM0126
       IF (BETA(I).GE.S) BET(I)=180.-BETA(I)+S                              26921150 SWPM0127
       IF (H1PLG.EQ.WEST.OR.H2PLG.EQ.WEST) GO TO 40                         26921160 SWPM0128
       IF (BETA(I).LT.S.AND.(H1DIP(I).EQ.WEST.OR.H2DIP(I).EQ.WEST))         26921170 SWPM0129
     1 GAMMA(I)=180.-DIP(I)                                                 26921180 SWPM0130
       IF (BETA(I).GT.S.AND.(H1DIP(I).EQ.EAST.OR.H2DIP(I).EQ.EAST))         26921190 SWPM0131
     1 GAMMA(I)=180.-DIP(I)                                                 26921200 SWPM0132
       GO TO 45                                                             26921210 SWPM0133
  40 IF (BETA(I).LT.S.AND.(H1DIP(I).EQ.EAST.OR.H2DIP(I).EQ.EAST))           26921220 SWPM0134
     1 GAMMA(I)=180.-DIP(I)                                                 26921230 SWPM0135
       IF (BETA(I).GT.S.AND.(H1DIP(I).EQ.WEST.OR.H2DIP(I).EQ.WEST))         26921240 SWPM0136
     1 GAMMA(I)=180.-DIP(I)                                                 26921250 SWPM0137
  45 CONTINUE                                                               26921260 SWPM0138
       B(I)=BET(I)/FACT                                                     26921270 SWPM0139
       C(I)=GAMMA(I)/FACT                                                   26921280 SWPM0140
       PHI(I)=PHX(I)/FACT                                                   26921290 SWPM0141
       AP(I)=AQ(I)/FACT                                                     26921300 SWPM0142
  35 CONTINUE                                                               26921310 SWPM0143
       P=PLG/FACT                                                           26921320 SWPM0144
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D=TPSLP/FACT                                                                  26921330 SWPM0145
CALL ORDER(N,B,INDEX)                                                         26921340 SWPM0146
CALL SWITCH(N,C,INDEX,DUMMY)                                                  26921350 SWPM0147
CALL SWITCH(N,BET,INDEX,DUMMY)                                                26921360 SWPM0148
CALL SWITCH(N,PHX,INDEX,DUMMY)                                                26921370 SWPM0149
CALL SWITCH(N,PHI,INDEX,DUMMY)                                                26921380 SWPM0150
CALL SWITCH(N,AQ,INDEX,DUMMY)                                                 26921390 SWPM0151
CALL SWITCH(N,AP,INDEX,DUMMY)                                                 26921400 SWPM0152
CALL SWITCH(N,TAU,INDEX,DUMMY)                                                26921410 SWPM0153
CALL SWITCH(N,PCTJTG,INDEX,DUMMY)                                             26921420 SWPM0154
CALL SWITCH(N,UI,INDEX,DUMMY)                                                          SWPM0155
CALL SWITCH(N,UAV,INDEX,DUMMY)                                                26921440 SWPM0156
CALL SWITCH(N,HU,INDEX,DUMMY)                                                 26921450 SWPM0157
CALL SWITCH(N,STK,INDEX,DUMMY)                                                26921460 SWPM0158
CALL SWITCH(N,H2STK,INDEX,DUMMY)                                              26921470 SWPM0159
CALL SWITCH(N,DIP,INDEX,DUMMY)                                                26921480 SWPM0160
CALL SWITCH(N,H1DIP,INDEX,DUMMY)                                              26921490 SWPM0161
CALL SWITCH(N,H2DIP,INDEX,DUMMY)                                              26921500 SWPM0162
CALL SWITCH(N,STIFFN,INDEX,DUMMY)                                                      SWPM0163
CALL SWITCH(N,STIFFS,INDEX,DUMMY)                                                      SWPM0164
DO 105 I=1,N                                                                  26921510 SWPM0165
IU(I) = UI(I)                                                                          SWPM0166
UX(I)=COS(B(I))                                                               26921520 SWPM0167
UY(I)=SIN(B(I))                                                               26921530 SWPM0168
UZ(I)=0.                                                                      26921540 SWPM0169
VX(I)=COS(C(I))*UY(I)                                                         26921550 SWPM0170
VY(I)=-COS(C(I))*UX(I)                                                        26921560 SWPM0171
VZ(I)=-SIN(C(I))                                                              26921570 SWPM0172
WX(I)=(UY(I)*VZ(I))-(UZ(I)*VY(I))                                             26921580 SWPM0173
WY(I)=(UZ(I)*VX(I))-(UX(I)*VZ(I))                                             26921590 SWPM0174
WZ(I)=(UX(I)*VY(I))-(UY(I)*VX(I))                                             26921600 SWPM0175
UF(I)=0.                                                                      26921610 SWPM0176
IF ( STIFFS(I) .NE. 0.0 ) RATIOK(I) = STIFFN(I)/STIFFS(I)                              SWPM0177
IF ( STIFFS(I) .EQ. 0.0 ) RATIOK(I) = 1000000.                                         SWPM0178
IF ( RATIOK(I) .EQ. 1000000. ) STIFFS(I) = 1.                                          SWPM0179
IF ( RATIOK(I) .EQ. 1000000. ) STIFFN(I) = 1000000.                                    SWPM0180
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105 CONTINUE                                                                    26921620 SWPM0181
    TAREX=0.                                                                    26921630 SWPM0182
    FAREX=0.                                                                    26921640 SWPM0183
    TBX=0.                                                                      26921650 SWPM0184
    TBY=0.                                                                      26921660 SWPM0185
    TBZ=0.                                                                      26921670 SWPM0186
    FBX=0.                                                                      26921680 SWPM0187
    FBY=0.                                                                      26921690 SWPM0188
    FBZ=0.                                                                      26921700 SWPM0189
    IF(III.NE.0) CALL BOLT(III,JJJ,NNN,WX,WY,WZ,HEX,C,B,P,XLEN,                26921710 SWPM0190
   1 PLG,D,TPSLP,BTOP,ORT,DPT,TEN,TAREX,TBX,TBY,TBZ,BFAC,ORF,DPF,FTEN,          26921720 SWPM0191
   2 FAREX,FBX,FBY,FBZ,FACT)                                                    26921730 SWPM0192
    NN=N-1                                                                      26921740 SWPM0193
    DO 500 I=1,NN                                                               26921750 SWPM0194
    II=I+1                                                                      26921760 SWPM0195
    DO 510 J=II,N                                                               26921770 SWPM0196
    ID=J-I                                                                      26921780 SWPM0197
    WRITE (5,1002) ICASE,I,J                                                    26921790 SWPM0198
    DO 210 K=I,J,ID                                                             26921800 SWPM0199
    WRITE (5,1004) K, STK(K), H2STK(K), DIP(K), H1DIP(K), H2DIP(K),                      SWPM0200
   1 PHX(K), AQ(K), TAU(K), PCTJTG(K), STIFFN(K), STIFFS(K),                            SWPM0201
   2 RATIOK(K), IU(K)                                                                   SWPM0202
    IF ( IU(K) .EQ. 1 ) WRITE (NWR,1030)                                                SWPM0203
    IF ( IU(K) .EQ. 2 ) WRITE (NWR,1031) UAV                                            SWPM0204
    IF ( IU(K) .EQ. 3 ) WRITE (NWR,1032) HU                                             SWPM0205
    IF ( IU(K) .EQ. 4 ) WRITE (NWR,1033)                                                SWPM0206
    IF(H2STK(K).EQ.EAST.AND.H1DIP(K).EQ.SOTH.AND.H2DIP(K).EQ.WEST)                      SWPM0207
   1 WRITE (5,1022)                                                                     SWPM0208
    IF(H2STK(K).EQ.EAST.AND.H1DIP(K).EQ.SOTH.AND.H2DIP(K).EQ.WEST)                      SWPM0209
   1 GO TO 510                                                                          SWPM0210
    IF(H2STK(K).EQ.EAST.AND.H1DIP(K).EQ.NORD.AND.H2DIP(K).EQ.EAST)                      SWPM0211
   1 WRITE (5,1022)                                                                     SWPM0212
    IF(H2STK(K).EQ.EAST.AND.H1DIP(K).EQ.NORD.AND.H2DIP(K).EQ.EAST)                      SWPM0213
   1 GO TO 510                                                                          SWPM0214
    IF(H2STK(K).EQ.WEST.AND.H1DIP(K).EQ.NORD.AND.H2DIP(K).EQ.WEST)                      SWPM0215
   1 WRITE (5,1022)                                                                     SWPM0216
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      IF(H2STK(K).EQ.WEST.AND.H1DIP(K).EQ.NORD.AND.H2DIP(K).EQ.WEST)     26921870 SWPM0217
     1 GO TO 510                                                                    SWPM0218
      IF(H2STK(K).EQ.WEST.AND.H1DIP(K).EQ.SOTH.AND.H2DIP(K).EQ.EAST)               SWPM0219
     1 WRITE (5,1022)                                                               SWPM0220
      IF(H2STK(K).EQ.WEST.AND.H1DIP(K).EQ.SOTH.AND.H2DIP(K).EQ.EAST)               SWPM0221
     1 GO TO 510                                                                    SWPM0222
      IF(BET(K).EQ.0..OR.BET(K).EQ.180.) WRITE (5,1021)                             SWPM0223
      IF(BET(K).EQ.0..OR.BET(K).EQ.180.) GO TO 510                                  SWPM0224
  210 CONTINUE                                                              26921920 SWPM0225
      WRITE (NWR,1036) AZI, H2AZI,PLG, H1PLG, H2PLG, HEX, XLEN, TPSLP               SWPM0226
      WRITE (5,1035) UNIWGT, WWGT, RATIO, SUR                                       SWPM0227
      IF(AKX.NE.0.0.OR.AKY.NE.0.0.OR.AKZ.NE.0.0)                                    SWPM0228
     1 WRITE (NWR,1006) AKX, AKY, AKZ                                               SWPM0229
      IF(PX.NE.0.0.OR.PZ.NE.0.0.OR.PZ.NE.0.0) WRITE (NWR,1007) PX,PY,PZ             SWPM0230
      IF(H2AZI.EQ.EAST.AND.H2PLG.EQ.EAST.AND.H1PLG.EQ.NORD) WRITE(5,1023)           SWPM0231
      IF(H2AZI.EQ.EAST.AND.H2PLG.EQ.EAST.AND.H1PLG.EQ.NORD) GO TO 510               SWPM0232
      IF(H2AZI.EQ.EAST.AND.H2PLG.EQ.WEST.AND.H1PLG.EQ.SOTH) WRITE(5,1023)           SWPM0233
      IF(H2AZI.EQ.EAST.AND.H2PLG.EQ.WEST.AND.H1PLG.EQ.SOTH) GO TO 510               SWPM0234
      IF(H2AZI.EQ.WEST.AND.H2PLG.EQ.WEST.AND.H1PLG.EQ.NORD) WRITE(5,1023)           SWPM0235
      IF(H2AZI.EQ.WEST.AND.H2PLG.EQ.WEST.AND.H1PLG.EQ.NORD) GO TO 510               SWPM0236
      IF(H2AZI.EQ.WEST.AND.H2PLG.EQ.EAST.AND.H1PLG.EQ.SOTH) WRITE(5,1023)           SWPM0237
      IF(H2AZI.EQ.WEST.AND.H2PLG.EQ.EAST.AND.H1PLG.EQ.SOTH) GO TO 510               SWPM0238
      IF ( III .EQ. 0 ) GO TO 230                                                   SWPM0239
      WRITE(5,1015)                                                                 SWPM0240
      DO 255 K=1,NNN                                                    26922080 SWPM0241
      WRITE(5,1024) K,BTOP(K),TEN(K),ORT(K),DPT(K),BFAC(K),FTEN(K),     26922090 SWPM0242
     1 ORF(K),DPF(K)                                                    26922100 SWPM0243
  255 CONTINUE                                                          26922110 SWPM0244
  230 XX=(WY(J)*WZ(I)-WZ(J)*WY(I))                                      26922190 SWPM0245
      XY=(WZ(J)*WX(I)-WX(J)*WZ(I))                                      26922200 SWPM0246
      XZ=(WX(J)*WY(I)-WY(J)*WX(I))                                      26922210 SWPM0247
      X=SQRT(XX**2+XY**2+XZ**2)                                         26922220 SWPM0248
      XX1 = XX/X                                                                    SWPM0249
      XY1 = XY/X                                                                    SWPM0250
      XZ1 = XZ/X                                                                    SWPM0251
      ODX = -HEX/(TAN(C(I))*SIN(B(I))) + HEX/(TAN(P)*TAN(B(I)))                     SWPM0252
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      OCX = -HEX/(TAN(C(J))*SIN(B(J)))  + HEX/(TAN(P)*TAN(B(J)))                SWPM0253
      ODY = HEX/TAN(P)                                                         SWPM0254
      DC=OCX-ODX                                                    26922260   SWPM0255
      IF(DC.LE.XLEN) GO TO 95                                       26922270   SWPM0256
      H=XLEN*HEX/DC                                                 26922280   SWPM0257
      ODX = -H   /(TAN(C(I))*SIN(B(I)))  + H   /(TAN(P)*TAN(B(I)))             SWPM0258
      OCX = -H/(TAN(C(J))*SIN(B(J)))  + H/(TAN(P)*TAN(B(J)))                   SWPM0259
      ODY = H/TAN(P)                                                           SWPM0260
      DC=OCX-ODX                                                    26922320   SWPM0261
      GO TO 100                                                     26922330   SWPM0262
   95 H=HEX                                                         26922340   SWPM0263
  100 CONTINUE                                                      26922350   SWPM0264
      OCY=ODY                                                       26922360   SWPM0265
      ODZ=H                                                         26922370   SWPM0266
      OCZ=ODZ                                                       26922380   SWPM0267
      IF(XY.EQ.0.)GO TO 50                                          26922390   SWPM0268
      TANE=XZ/XY                                                    26922400   SWPM0269
      TAND=TAN(D)                                                   26922410   SWPM0270
      IF(TANE.LE.TAND) GO TO 125                                    26922420   SWPM0271
      E=ATAN(TANE)                                                  26922430   SWPM0272
      IF(E.LT.0.) E=E+PI                                            26922440   SWPM0273
      IF(E.GT.P) GO TO 155                                          26922450   SWPM0274
      GO TO 60                                                      26922460   SWPM0275
  125 WRITE(5,1019)                                                            SWPM0276
      GO TO 510                                                     26922480   SWPM0277
   50 WRITE(5,1014)                                                            SWPM0278
      GO TO 510                                                     26922500   SWPM0279
  155 WRITE(5,1020)                                                            SWPM0280
      GO TO 510                                                     26922520   SWPM0281
   60 AAA=1.                                                        26922530   SWPM0282
      IF (PLG.NE.90.) AAA=1.-TANE/TAN(P)                            26922540   SWPM0283
      HH=H*AAA*TAND/(TANE-TAND)                                     26922550   SWPM0284
      CCC=(H+HH)/XZ                                                 26922560   SWPM0285
      OBX = CCC*XX                                                  26922570   SWPM0286
      OBY = CCC*XY                                                  26922580   SWPM0287
      OBZ = CCC*XZ                                                  26922590   SWPM0288
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OD=SQRT(ODX**2+ODY**2+ODZ**2)                                    26922600 SWPM0289
OC=SQRT(OCX**2+OCY**2+OCZ**2)                                    26922610 SWPM0290
OB=SQRT(OBX**2+OBY**2+OBZ**2)                                    26922620 SWPM0291
OBBY=OBY-ODY                                                     26922630 SWPM0292
IF(TPSLP.NE.0.) OBBY=HH/SIN(D)                                   26922640 SWPM0293
TAREA=OBBY*DC/2.                                                 26922650 SWPM0294
AZ=H/SIN(P)                                                      26922660 SWPM0295
FAREA=DC*AZ/2.                                                   26922670 SWPM0296
BDX=ODX-OBX                                                      26922680 SWPM0297
IF(BDX.LT.0.) BDX=-BDX                                           26922690 SWPM0298
DB=OBBY                                                          26922700 SWPM0299
R1=ATAN2(BDX,OBBY)                                               26922710 SWPM0300
IF(TAN(R1).NE.0.) DB=OBBY/COS(R1)                                26922720 SWPM0301
CBX=OCX-OBX                                                      26922730 SWPM0302
IF(CBX.LT.0.) CBX=-CBX                                           26922740 SWPM0303
CB=OBBY                                                          26922750 SWPM0304
R=ATAN2(CBX,OBBY)                                                26922760 SWPM0305
IF(TAN(R).NE.0.) CB=OBBY/COS(R)                                  26922770 SWPM0306
CCP=H/XZ                                                         26922780 SWPM0307
DBPY=CCP*XY-ODY                                                  26922790 SWPM0308
VOL=DBPY*DC*(H+HH)/6.                                            26922800 SWPM0309
OSX=(OBX+OCX+ODX)/4.                                             26922810 SWPM0310
OSY=(OBY+OCY+ODY)/4.                                             26922820 SWPM0311
OSZ=(OBZ+OCZ+ODZ)/4.                                             26922830 SWPM0312
TEMP=VOL*UNIWGT                                                  26922840 SWPM0313
DUMMYT = TEMP*COS(E)*1.0E-10                                              SWPM0314
SURG=SUR*TAREA/COS(D)                                            26922850 SWPM0315
SUM1=(OD+OB+DB)/2.                                               26922860 SWPM0316
SUM2=(OC+OB+CB)/2.                                               26922870 SWPM0317
AREA(I)=SQRT(SUM1*(SUM1-OD)*(SUM1-OB)*(SUM1-DB))                 26922880 SWPM0318
AREA(J)=SQRT(SUM2*(SUM2-OC)*(SUM2-OB)*(SUM2-CB))                 26922890 SWPM0319
CH(I)=TAU(I)*(100.-PCTJTG(I))*(.01)*AREA(I)                      26922900 SWPM0320
CH(J)=TAU(J)*(100.-PCTJTG(J))*(.01)*AREA(J)                      26922910 SWPM0321
CH1=CH(I)                                                        26922920 SWPM0322
CH2=CH(J)                                                        26922930 SWPM0323
TAR=TAREA                                                        26922940 SWPM0324
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      IF(TAREX.LT.TAREA) TAR=TAREX                                      26922950 SWPM0325
      FAR=FAREA                                                         26922960 SWPM0326
      IF(FAREX.LT.FAREA) FAR=FAREX                                      26922970 SWPM0327
      RBX=TBX*TAR+FBX*FAR                                               26922980 SWPM0328
      RBY=TBY*TAR+FBY*FAR                                               26922990 SWPM0329
      RBZ=TBZ*TAR+FBZ*FAR                                               26923000 SWPM0330
      RX = RBX + PX + TEMP*AKX                                                   SWPM0331
      RY = RBY + PY + TEMP*AKY                                                   SWPM0332
      RZ = RBZ + PZ -SURG - TEMP*(1.0 - AKZ)                                     SWPM0333
      IF ( IS .EQ. 2 ) RX = RX + SURG*AKX                                        SWPM0334
      IF ( IS .EQ. 2 ) RY = RY + SURG*AKY                                        SWPM0335
      IF ( IS .EQ. 2 ) RZ = RZ + SURG*AKZ                                        SWPM0336
      CALL WATER(RX,RY,RZ,H,E,P,PLG,HU,IU,UAV,OP1,HW,UF,HH,PI,          26923100 SWPM0337
     1 AREA,WWGT,I,J,ID,WX,WY,WZ,TPSLP)                                 26923110 SWPM0338
      OIRX=OSY*RZ-OSZ*RY                                                26923120 SWPM0339
      OIRY=OSZ*RX-OSX*RZ                                                26923130 SWPM0340
      OIRZ=OSX*RY-OSY*RX                                                26923140 SWPM0341
      EMX=XX*OIRX+XY*OIRY+XZ*OIRZ                                       26923150 SWPM0342
      DO 110 K=I,J,ID                                                   26923160 SWPM0343
      EN(K)=RX*WX(K)+RY*WY(K)+RZ*WZ(K)                                  26923170 SWPM0344
      TX(K)=RX-EN(K)*WX(K)                                              26923180 SWPM0345
      TY(K)=RY-EN(K)*WY(K)                                              26923190 SWPM0346
      TZ(K)=RZ-EN(K)*WZ(K)                                              26923200 SWPM0347
      T(K)=SQRT(TX(K)**2+TY(K)**2+TZ(K)**2)                             26923210 SWPM0348
  110 CONTINUE                                                          26923220 SWPM0349
      ENN=0.                                                            26923230 SWPM0350
      IF (EN(I).GT.0.0.OR.EN(J).LT.0.0) GO TO 120                       26923240 SWPM0351
      WRITE (5,1008)                                                             SWPM0352
      GO TO 400                                                         26923260 SWPM0353
  120 DO 115 K=I,J,ID                                                   26923270 SWPM0354
      SX(K)=(XY*WZ(K)-XZ*WY(K))                                         26923280 SWPM0355
      SY(K)=(XZ*WX(K)-XX*WZ(K))                                         26923290 SWPM0356
      SZ(K)=(XX*WY(K)-XY*WX(K))                                         26923300 SWPM0357
      RSS(K)=RX*SX(K)+RY*SY(K)+RZ*SZ(K)                                 26923310 SWPM0358
  115 CONTINUE                                                          26923320 SWPM0359
      T12 = RX*XX1 + RY*XY1 + RZ*XZ1                                             SWPM0360
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      T12P = RX*XX1 + RY*XY1 + (RZ+TEMP)*XZ1                                    SWPM0361
      SILEN = SQRT(SX(I)**2 + SY(I)**2 + SZ(I)**2)                              SWPM0362
      SJLEN = SQRT(SX(J)**2 + SY(J)**2 + SZ(J)**2)                              SWPM0363
      ETACOS = (SX(J)*SX(I) + SY(J)*SY(I) + SZ(J)*SZ(I))/(SILEN*SJLEN)          SWPM0364
      ETA = ACOS(ETACOS)                                                        SWPM0365
      IF ( ETA .LT. 0.0 ) ETA = PI + ETA                                        SWPM0366
      ETO = ETA*180./PI                                                         SWPM0367
      HORIZL = SQRT(XX**2 + XY**2)                                              SWPM0368
      COSBT2 = (XY*SX(J) - XX*SY(J))/(SJLEN*HORIZL)                             SWPM0369
      SINBT2 = SQRT(1.0 - COSBT2**2)                                            SWPM0370
      BETA2 = ATAN(SINBT2/COSBT2)                                               SWPM0371
      BETO2 = BETA2*180./PI                                                     SWPM0372
      BETA1 = PI - BETA2 - ETA                                                  SWPM0373
      BETO1 = BETA1*180./PI                                                     SWPM0374
      SINBT1 = SIN(BETA1)                                                       SWPM0375
      COSBT1 = COS(BETA1)                                                       SWPM0376
      WRITE (NWR,1034) BETO1, ETO, BETO2, AREA(I), AREA(J), TAREA,              SWPM0377
     1 FAREA, VOL, H, HH, XX1, XY1, XZ1                                         SWPM0378
      IF (RSS(I).LT.0.0) GO TO 130                                  26923330    SWPM0379
      IF (RSS(J).LT.0.0) GO TO 140                                  26923340    SWPM0380
      PERP1 = 0.0                                                               SWPM0381
      TANG1 = 0.0                                                               SWPM0382
      PERP2 = 0.0                                                               SWPM0383
      TANG2 = 0.0                                                               SWPM0384
      IF ( LOPT .EQ. 1 ) GO TO 78                                               SWPM0385
C     THIS SECTION COMPUTES THE EFFECT OF IN SITU STRESSES                      SWPM0386
      WTCO = TEMP*COS(E)                                                        SWPM0387
      IF ( RATIO .NE. 1.0 ) GO TO 76                                            SWPM0388
      PERP2 = WTCO/(COSBT2 + SINBT2*COSBT1/SINBT1)                              SWPM0389
      PERP1 = PERP2*SINBT2/SINBT1                                               SWPM0390
      GO TO 78                                                                  SWPM0391
   76 RAT1=(RATIO+(1.0-RATIO)*COSBT1**2)/((1.0-RATIO)*SINBT1*COSBT1)            SWPM0392
      RAT2=(RATIO+(1.0-RATIO)*COSBT2**2)/((1.0-RATIO)*SINBT2*COSBT2)            SWPM0393
      RAT12 = (RAT2*SINBT2 - COSBT2)/(RAT1*SINBT1 - COSBT1)                     SWPM0394
      TANG2 = WTCO/(RAT12*(RAT1*COSBT1 + SINBT1)+(RAT2*COSBT2+SINBT2))          SWPM0395
      TANG1 = RAT12*TANG2                                                       SWPM0396
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      PERP1 = RAT1*TANG1                                                    SWPM0397
      PERP2 = TANG2*RAT2                                                    SWPM0398
C     THIS PART COMPUTES THE EFFECT OF JOINT STIFFNESSES                    SWPM0399
   78 HXX = XY1/(SQRT(XX1**2 + XY1**2))                                     SWPM0400
      HXY = -XX1/(SQRT(XX1**2 + XY1**2))                                    SWPM0401
      RHORIZ = RX*HXX + RY*HXY                                              SWPM0402
      RVERT =  -1.0*(RX*HXY*XZ1 - RY*HXX*XZ1 + RZ*(HXX*XY1 - XX1*HXY) )     SWPM0403
      UR1A = RVERT/(COSBT1 +COSBT2*SINBT1/SINBT2)                           SWPM0404
      UR2A = UR1A*SINBT1/SINBT2                                             SWPM0405
      UR1B = RHORIZ/(SINBT1 + SINBT2*COSBT1/COSBT2)                         SWPM0406
      UR2B = UR1B*COSBT1/COSBT2                                             SWPM0407
C     FSLR IS THE 'ULTIMATE' F.S. FOR SLIDING DOWN THE LINE OF INTERSECTION SWPM0408
      FSLR = ((UR1A + UR1B)*TAN(PHI(I)) +(UR2A + UR2B)*TAN(PHI(J))) /T12    SWPM0409
      RVERT1 = RVERT - TEMP*(HXX*XY1 - XX1*HXY)                             SWPM0410
      IF ( LOPT .EQ. 1 ) RVERT1 = RVERT                                     SWPM0411
      IF ( RVERT1 .NE. 0.0 .OR. RHORIZ .NE. 0.0 ) GO TO 79                  SWPM0412
  167 RVERT1 = DUMMYT                                                       SWPM0413
   79 ST1N = STIFFN(I)*AREA(I)                                              SWPM0414
      ST1S = STIFFS(I)*AREA(I)                                              SWPM0415
      ST2N = STIFFN(J)*AREA(J)                                              SWPM0416
      ST2S = STIFFS(J)*AREA(J)                                              SWPM0417
      FACTAA = ST1N*SINBT1 + ST1S*COSBT1**2/SINBT1 +                        SWPM0418
     1 ST2N*SINBT2 + ST2S*COSBT2**2/SINBT2                                  SWPM0419
      FACTBB = (ST1N-ST1S)*COSBT1 + (ST2S-ST2N)*COSBT2                      SWPM0420
      FACTCC = ST1N*COSBT1**2/SINBT1 + ST1S*SINBT1 +                        SWPM0421
     1 ST2N*COSBT2**2/SINBT2 + ST2S*SINBT2                                  SWPM0422
      ALPHA = ATAN((FACTCC*RHORIZ - FACTBB*RVERT1)/                         SWPM0423
     1 (FACTAA*RVERT1 - FACTBB*RHORIZ))                                     SWPM0424
      COSALP = COS(ALPHA)                                                   SWPM0425
      SINALP = SIN(ALPHA)                                                   SWPM0426
      FACTDD = ST1N*(SINALP + COSALP*COSBT1/SINBT1)                         SWPM0427
      FACTEE = ST1S*(COSALP*SINBT1 - SINALP*COSBT1)                         SWPM0428
      FACTFF = ST2N*(COSALP*COSBT2**2/SINBT2 - SINALP*COSBT2)               SWPM0429
      FACTGG = ST2S*(COSALP*SINBT2 + SINALP*COSBT2/SINBT2)                  SWPM0430
      PERP11 = RVERT1*FACTDD/(COSBT1 + FACTEE + FACTFF + FACTGG)            SWPM0431
      DISP = PERP11/FACTDD                                                  SWPM0432
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      PERP1 = PERP11 + PERP1                                                    SWPM0433
      TANG1 = PERP11*FACTEE/(SINDT1*FACTDD) + TANG1                             SWPM0434
      PERP2 = PERP11*FACTFF/(COSBT2*FACTDD) + PERP2                             SWPM0435
      TANG2 = PERP11*FACTGG/(SINBT2*FACTDD) + TANG2                             SWPM0436
      IF (.NOT.FLIM) GO TO 166                                                  SWPM0437
      IF ( NDOP .NE. 0 ) GO TO 166                                              SWPM0438
      PPERP1 = PERP1                                                            SWPM0439
      PPERP2 = PERP2                                                            SWPM0440
      TTANG1 = TANG1                                                            SWPM0441
      TTANG2 = TANG2                                                            SWPM0442
      FLIM = .FALSE.                                                            SWPM0443
       RHORIZ = 0.0                                                             SWPM0444
      GO TO 167                                                                 SWPM0445
C     THIS PART COMPUTES THE FACTOR OF SAFETY: THE RATIO OF THE                 SWPM0446
C     COMPONENTS OF DRIVING TO RESISTING FORCES IN THE DISPLACEMENT DIRECTION   SWPM0447
  166 IF ( RVERT1 .EQ. DUMMYT ) T12P = DUMMYT*TAN(E)                            SWPM0448
      ADISP = T12P/(ST1S + ST2S)                                                SWPM0449
      BDISP = DISP*SINALP                                                       SWPM0450
      CDISP = -DISP*COSAIP                                                      SWPM0451
      DISP = SQRT(ADISP**2 + BDISP**2 + CDISP**2)                               SWPM0452
      ADISP = ADISP/DISP                                                        SWPM0453
      BDISP = BDISP/DISP                                                        SWPM0454
      CDISP = CDISP/DISP                                                        SWPM0455
      IF (FLIM) GO TO 168                                                       SWPM0456
      PERP1 = PPERP1                                                            SWPM0457
      PERP2 = PPERP2                                                            SWPM0458
      TANG1 = TTANG1                                                            SWPM0459
      TANG2 = TTANG2                                                            SWPM0460
  168 FLIM = .TRUE.                                                             SWPM0461
      ADRF1 = T12*ST1S/(ST1S + ST2S)                                            SWPM0462
      BDRF1 = -TANG1*COSBT1                                                     SWPM0463
      CDRF1 = -TANG1*SINBT1                                                     SWPM0464
      ADRF2 = T12 - ADRF1                                                       SWPM0465
      BDRF2 = TANG2*COSBT2                                                      SWPM0466
      CDRF2 = -TANG2*SINBT2                                                     SWPM0467
      DRF = (ADRF1+ADRF2)*ADISP+(BDRF1+BDRF2)*BDISP+(CDRF1+CDRF2)*CDISP         SWPM0468
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      FACTHH = (ADRF1*ADISP+BDRF1*BDISP+CDRF1*CDISP)/                    SWPM0469
     1 (SQRT(ADRF1**2+BDRF1**2+CDRF1**2))                                SWPM0470
      FACTJJ = (ADRF2*ADISP+BDRF2*BDISP+CDRF2*CDISP)/                    SWPM0471
     1 (SQRT(ADRF2**2+BDRF2**2+CDRF2**2))                                SWPM0472
      REST2 = PERP2*TAN(PHI(J)+AP(J))+ CH2                               SWPM0473
      REST1 = PERP1*TAN(PHI(I)+AP(I))+CH1                                SWPM0474
      FSL = (REST1*FACTHH + REST2*FACTJJ)/DRF                            SWPM0475
      WRITE(5,1018) FSL                                                  SWPM0476
      IF(FSL.LT.0.) WRITE(5,1009)                                        SWPM0477
      WRITE (NWR,1016) FSLR                                              SWPM0478
  245 IF(FSL.LT.1.) GO TO 300                                  26923500 SWPM0479
      WWX(I)=-WX(I)                                            26923510 SWPM0480
      WWY(I)=-WY(I)                                            26923520 SWPM0481
      WWZ(I)=-WZ(I)                                            26923530 SWPM0482
      WWX(J)=WX(J)                                             26923540 SWPM0483
      WWY(J)=WY(J)                                             26923550 SWPM0484
      WWZ(J)=WZ(J)                                             26923560 SWPM0485
      DO 410 K=I,J,ID                                          26923570 SWPM0486
      COSP=COS(PHI(K)+AP(K))                                   26923580 SWPM0487
      SINP=SIN(PHI(K)+AP(K))                                   26923590 SWPM0488
      RRX(K)=WWX(K)*COSP-XX*SINP/X                             26923600 SWPM0489
      RRY(K)=WWY(K)*COSP-XY*SINP/X                             26923610 SWPM0490
      RRZ(K)=WWZ(K)*COSP-XZ*SINP/X                             26923620 SWPM0491
  410 CONTINUE                                                 26923630 SWPM0492
      CRPX=RRY(I)*RRZ(J)-RRZ(I)*RRY(J)                         26923640 SWPM0493
      CRPY=RRZ(I)*RRX(J)-RRX(I)*RRZ(J)                         26923650 SWPM0494
      CRPZ=RRX(I)*RRY(J)-RRY(I)*RRX(J)                         26923660 SWPM0495
      CRR=SQRT(CRPX**2+CRPY**2+CRPZ**2)                        26923670 SWPM0496
      CRRX=-CRPX/CRR                                           26923680 SWPM0497
      CRRY=-CRPY/CRR                                           26923690 SWPM0498
      CRRZ=-CRPZ/CRR                                           26923700 SWPM0499
      ENN=(RX*CRRX+RY*CRRY+RZ*CRRZ)/(TEMP)                     26923710 SWPM0500
      GO TO 300                                                26923720 SWPM0501
  130 IF (EN(I).LT.0.0) GO TO 150                              26923730 SWPM0502
      FLS = 0.0                                                          SWPM0503
      K=I                                                      26923750 SWPM0504
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      GO TO 145                                                                  26923760 SWPM0505
  140 IF (EN(J).GT.0.0) GO TO 150                                                26923770 SWPM0506
      FSL=0.                                                                     26923780 SWPM0507
      K=J                                                                        26923790 SWPM0508
      EN(K)=-EN(K)                                                               26923800 SWPM0509
  145 CONTINUE                                                                   26923810 SWPM0510
      FSS=(EN(K)*TAN(PHI(K)+AP(K))+CH(K))/T(K)                                   26923820 SWPM0511
      WRITE(5,1001) FSS,K                                                                 SWPM0512
      IF(CH1.NE.0..OR.CH2.NE.0..OR.AP(I).NE.0..OR.AP(J).NE.0.) GO TO 70          26923840 SWPM0513
      GO TO 240                                                                  26923850 SWPM0514
   70 FSSR=EN(K)*TAN(PHI(K))/T(K)                                                26923860 SWPM0515
      WRITE(5,1016) FSSR                                                                  SWPM0516
      GO TO 240                                                                  26923880 SWPM0517
  150 WRITE (5,1010)                                                                      SWPM0518
      GO TO 400                                                                  26923900 SWPM0519
  240 IF(FSS.LT.1.) GO TO 300                                                    26923910 SWPM0520
      ENN=SIN(PHI(K)+AP(K)-ATAN2(T(K),EN(K)))                                    26923920 SWPM0521
      AA=B(K)-90./FACT                                                                    SWPM0522
      IF(TX(K).NE.0.) AA=B(K)-ATAN2(TY(K),TX(K))                                 26923940 SWPM0523
      AB=C(K)                                                                    26923950 SWPM0524
      IF(DIP(K).NE.90.) AB=ATAN(TAN(C(K))*SIN(AA))                               26923960 SWPM0525
      PLUN=PHI(K)-AB                                                             26923970 SWPM0526
      CRRH=SQRT(TX(K)**2+TY(K)**2)                                               26923980 SWPM0527
      CRRZ=CRRH*TAN(PLUN)/T(K)                                                   26923990 SWPM0528
      CRRX=TX(K)/T(K)                                                            26924000 SWPM0529
      CRRY=TY(K)/T(K)                                                            26924010 SWPM0530
  300 IF(FSL.NE.0.) FSS=0.                                                       26924020 SWPM0531
      IF(FSS.NE.0.) GO TO 400                                                    26924030 SWPM0532
      IF(IU(I).NE.1 .AND.IU(J).NE.1 ) GO TO 400                                           SWPM0533
      AK1COS =  ((ODX*OBX+ODY*OBY+ODZ*OBZ)/(OD*OB))                                       SWPM0534
      AK10 = ATAN(SQRT(1.0 - AK1COS**2)/AK1COS)                                           SWPM0535
      AK2COS =  ((OCX*OBX+OCY*OBY+OCZ*OBZ)/(OC*OB))                              26924070 SWPM0536
      AK20 = ATAN(SQRT(1.0 - AK2COS**2)/AK2COS)                                           SWPM0537
      EM10=-(WX(I)*OIRX+WY(I)*OIRY+WZ(I)*OIRZ)                                   26924080 SWPM0538
      EM20=-(WX(J)*OIRX+WY(J)*OIRY+WZ(J)*OIRZ)                                   26924090 SWPM0539
      IF (EMX.LE.0.0.OR.EM10.LE.0.0) GO TO 310                                   26924100 SWPM0540
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      IF (AK20.GT.PI2) GO TO 310                                        26924110 SWPM0541
      IF (ETA.LT.PI2.AND.AK10.GT.PI2.AND.                               26924120 SWPM0542
     1  (TAN(AK20)/TAN(PI-AK10)).GT.(1./COS(PI-ETA))) GO TO 310         26924130 SWPM0543
      K=I                                                               26924140 SWPM0544
  305 TOP=OSX*WX(K)+OSY*WY(K)+OSZ*WZ(K)                                 26924150 SWPM0545
      BOT= RX*WX(K)+ RY*WY(K)+ RZ*WZ(K)                                 26924160 SWPM0546
      VJP=-TOP/BOT                                                      SWPM0547
      OQX=OSX+VJP*RX                                                    SWPM0548
      OQY=OSY+VJP*RY                                                    SWPM0549
      OQZ=OSZ+VJP*RZ                                                    SWPM0550
      OQWX=OQY*WZ(K)-OQZ*WY(K)                                          26924210 SWPM0551
      OQWY=OQZ*WX(K)-OQX*WZ(K)                                          26924220 SWPM0552
      OQWZ=OQX*WY(K)-OQY*WX(K)                                          26924230 SWPM0553
      DET=-OQY*OQWZ+OQZ*CQWY                                            26924240 SWPM0554
      C2=(-TZ(K)*CQY+TY(K)*OQZ)/DET                                     26924250 SWPM0555
      TT=C2*SQRT(OQWX**2+OQWY**2+OQWZ**2)                               26924260 SWPM0556
      FSR=(EN(K)*TAN(PHI(K)+AP(K))+CH(K))/TT                            26924270 SWPM0557
      WRITE(5,1013) FSR,K                                               SWPM0558
      IF(CH1.NE.0..OR.CH2.NE.0..OR.AP(I).NE.0..OR.AP(J).NE.0.) GO TO 75 26924290 SWPM0559
      GO TO 420                                                         26924300 SWPM0560
   75 FSRR=EN(K)*TAN(PHI(K))/TT                                         26924310 SWPM0561
      WRITE(5,1016) FSRR                                                SWPM0562
      GO TO 420                                                         26924330 SWPM0563
  310 IF (EMX.GE.0.0.OR.EM20.LE.0.0) GO TO 320                          26924340 SWPM0564
      IF (AK10.GT.PI2) GO TO 320                                        26924350 SWPM0565
      IF (ETA.LT.PI2.AND.AK20.GT.PI2.AND.                               26924360 SWPM0566
     1  (TAN(AK10)/TAN(PI-AK20)).GT.(1./COS(PI-ETA))) GO TO 320         26924370 SWPM0567
      K=J                                                               26924380 SWPM0568
      EN(K)=-EN(K)                                                      26924390 SWPM0569
      GO TO 305                                                         26924400 SWPM0570
  320 WRITE(5,1012)                                                     SWPM0571
      FSR=0.                                                            26924420 SWPM0572
      GO TO 400                                                         26924430 SWPM0573
  420 IF(FSR.LT.1.) GO TO 400                                           26924440 SWPM0574
      ENNR=SIN(PHI(K)+AP(K)-ATAN2(TT,EN(K)))                            26924450 SWPM0575
      CRRRX=C2*OQWX/TT                                                  26924460 SWPM0576
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      CRRRY=C2*OQWY/IT                                                  26924470 SWPM0577
      CRRRZ=C2*OQWZ/IT                                                  26924480 SWPM0578
      IF(ENNR.LT.ENN) GO TO 405                                         26924490 SWPM0579
      GO TO 400                                                         26924500 SWPM0580
  405 ENN=ENNR                                                          26924510 SWPM0581
      CRRX=CRRRX                                                        26924520 SWPM0582
      CRRY=CRRRY                                                        26924530 SWPM0583
      CRRZ=CRRRZ                                                        26924540 SWPM0584
  400 CONTINUE                                                          26924550 SWPM0585
      DOX=-ODX                                                          26924560 SWPM0586
      DBX=OBX-ODX                                                       26924570 SWPM0587
      IF(AKX.NE.0..OR.AKY.NE.0..OR.AKZ.NE.0.) GO TO 510                 26924620 SWPM0588
      IF(ENN.EQ.0.) GO TO 510                                           26924630 SWPM0589
      WRITE(5,1011) ENN,CRRX,CRRY,CRRZ                                           SWPM0590
  510 CONTINUE                                                          26924650 SWPM0591
  500 CONTINUE                                                          26924660 SWPM0592
      GO TO 200                                                         26924670 SWPM0593
 9000 CONTINUE                                                          26924680 SWPM0594
      STOP                                                              26924690 SWPM0595
      END                                                               26924700 SWPM0596
      SUBROUTINE ORDER (N,B,INDEX)                                      26924710 SWPM0597
      DIMENSION B(8),INDEX(8)                                           26924720 SWPM0598
      LOGICAL FIN                                                       26924730 SWPM0599
      DO 10 I=1,N                                                       26924740 SWPM0600
   10 INDEX(I)=I                                                        26924750 SWPM0601
      NN=N-1                                                            26924760 SWPM0602
      DO 15 J=1,NN                                                      26924770 SWPM0603
      M=N-J                                                             26924780 SWPM0604
      FIN=.TRUE.                                                        26924790 SWPM0605
      DO 20 I=1,M                                                       26924800 SWPM0606
      IF (B(I).LE.B(I+1)) GO TO 20                                      26924810 SWPM0607
      FIN=.FALSE.                                                       26924820 SWPM0608
      BB=B(I+1)                                                         26924830 SWPM0609
      B(I+1)=B(I)                                                       26924840 SWPM0610
      B(I)=BB                                                           26924850 SWPM0611
      IN=INDEX(I)                                                       26924860 SWPM0612
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      INDEX(I)=INDEX(I+1)                                                26924870 SWPM0613
      INDEX(I+1)=IN                                                      26924880 SWPM0614
   20 CONTINUE                                                           26924890 SWPM0615
      IF (FIN) RETURN                                                    26924900 SWPM0616
   15 CONTINUE                                                           26924910 SWPM0617
      RETURN                                                             26924920 SWPM0618
      END                                                                26924930 SWPM0619
      SUBROUTINE SWITCH (N,A,INDEX,DUMMY)                                26924940 SWPM0620
      DIMENSION A(8),INDEX(8),DUMMY(8)                                   26924950 SWPM0621
      DO 10 I=1,N                                                        26924960 SWPM0622
   10 DUMMY(I)=A(I)                                                      26924970 SWPM0623
      DO 15 I=1,N                                                        26924980 SWPM0624
      II=INDEX(I)                                                        26924990 SWPM0625
   15 A(I)=DUMMY(II)                                                     26925000 SWPM0626
      RETURN                                                             26925010 SWPM0627
      END                                                                26925020 SWPM0628
      SUBROUTINE WATER(RX,RY,RZ,H,E,P,PLG,HU,IU,UAV,OP1,HW,UF,HH,PI,     26925030 SWPM0629
     1 AREA,WWGT,I,J,ID,WX,WY,WZ,TPSLP)                                  26925040 SWPM0630
      DIMENSION HU(8),IU(8),UAV(8),AREA(8),UF(8),WX(8),WY(8),WZ(8)       26925050 SWPM0631
      DO 15 K=I,J,ID                                                     26925060 SWPM0632
      KK=IU(K)                                                           26925070 SWPM0633
      HHH=HU(K)                                                          26925080 SWPM0634
      IF(TPSLP.LT.0..AND.HHH.LT.-HH) HHH=-HH                             26925090 SWPM0635
      DH=H-HHH                                                           26925100 SWPM0636
      IF(DH.LE.0.) GO TO 15                                              26925110 SWPM0637
      TEM=(DH/H)*(DH/(H+HH))                                             26925120 SWPM0638
      GO TO (15,20,25,30),KK                                             26925130 SWPM0639
   20 UF(K)=AREA(K)*UAV(K)                                               26925140 SWPM0640
      GO TO 35                                                           26925150 SWPM0641
   25 PER=1.                                                             26925160 SWPM0642
      GO TO 40                                                           26925170 SWPM0643
   30 AZZ=DH/SIN(P)                                                      26925180 SWPM0644
      AXX=0.                                                             26925190 SWPM0645
      IF(PLG.NE.90.) AXX=DH/TAN(P)                                       26925200 SWPM0646
      AX=DH/TAN(E)-AXX                                                   26925210 SWPM0647
      PA=PI-P                                                            26925220 SWPM0648
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      ART=AZZ*AX*SIN(PA)/2.                                          26925230 SWPM0649
      IF(HW.NE.0.) GO TO 45                                          26925240 SWPM0650
      EA=P/2.                                                        26925250 SWPM0651
      IF(EA.LE.E) GO TO 15                                           26925260 SWPM0652
      ARA=AZZ*AZZ*SIN(PA)/2.                                         26925270 SWPM0653
      ARB=ART-ARA                                                    26925280 SWPM0654
      PER=ARB/ART                                                    26925290 SWPM0655
      GO TO 40                                                       26925300 SWPM0656
   45 HP=HW-HU(K)                                                    26925310 SWPM0657
      AY=OP1-AXX                                                     26925320 SWPM0658
      AZW=HP/SIN(P)                                                  26925330 SWPM0659
      IF(HP.GT.DH) GO TO 50                                          26925340 SWPM0660
      ARA1=AZW*AY*SIN(PA)/2.                                         26925350 SWPM0661
      ARB1=ART-ARA1                                                  26925360 SWPM0662
      IF(AY.GT.AX) GO TO 55                                          26925370 SWPM0663
      PER=ARB1/ART                                                   26925380 SWPM0664
      GO TO 40                                                       26925390 SWPM0665
   55 IF(HP.EQ.DH.AND.AY.GE.AX) GO TO 15                             26925400 SWPM0666
      AYY=AXX*AZW/AZZ                                                26925410 SWPM0667
      ZA=ATAN2(HP,(AY+AYY))                                          26925420 SWPM0668
      ZE=AY-AX                                                       26925430 SWPM0669
      ZC=PI-E                                                        26925440 SWPM0670
      ZD=PI-ZC-ZA                                                    26925450 SWPM0671
      ARF=ZB*ZB*SIN(ZA)*SIN(ZC)/(2.*SIN(ZD))                         26925460 SWPM0672
      PER=(ARB1+ARF)/ART                                             26925470 SWPM0673
      GO TO 40                                                       26925480 SWPM0674
   50 IF(AY.GE.AX) GO TO 15                                          26925490 SWPM0675
      ZG=AX-AY                                                       26925500 SWPM0676
      ZE=ATAN2(OP1,HP)+PI/2.                                         26925510 SWPM0677
      ZF=PI-ZE-ZG                                                    26925520 SWPM0678
      ARB2=ZG*ZG*SIN(ZE)*SIN(E)/(2.*SIN(ZF))                         26925530 SWPM0679
      PER=ARB2/ART                                                   26925540 SWPM0680
   40 CONTINUE                                                       26925550 SWPM0681
      UF(K)=PER*TEM*AREA(K)*WWGT*DH/3.                               26925560 SWPM0682
   35 IF(K.EQ.J) UF(K)=-UF(K)                                        26925570 SWPM0683
      UFX=WX(K)*UF(K)                                                26925580 SWPM0684
```

```
      UFY=WY(K)*UF(K)                                                    26925590 SWPM0685
      UFZ=WZ(K)*UF(K)                                                    26925600 SWPM0686
      RX=RX-UFX                                                          26925610 SWPM0687
      RY=RY-UFY                                                          26925620 SWPM0688
      RZ=RZ-UFZ                                                          26925630 SWPM0689
   15 CONTINUE                                                           26925640 SWPM0690
      RETURN                                                             26925650 SWPM0691
      END                                                                26925660 SWPM0692
      SUBROUTINE BOLT(III,JJJ,NNN,WX,WY,WZ,HEX,C,B,P,XLEN,PLG,D,TPSLP,   26925670 SWPM0693
     1 BTOP,ORT,DPT,TEN,TAREX,TEX,TEY,TBZ,BFAC,ORF,DPF,FTEN,FAREX,       26925680 SWPM0694
     2 FBX,FBY,FBZ,FACT)                                                 26925690 SWPM0695
      DIMENSION BTOP(8),ORT(8),DPT(8),TEN(8),BFAC(8),ORF(8)              26925700 SWPM0696
      DIMENSION DPF(8),FTEN(8),WX(8),WY(8),WZ(8),C(8),B(8)               26925710 SWPM0697
      IF(III.GT.JJJ) GO TO 20                                            26925720 SWPM0698
      GO TO 25                                                           26925730 SWPM0699
   20 TE=III                                                             26925740 SWPM0700
      III=JJJ                                                            26925750 SWPM0701
      JJJ=TE                                                             26925760 SWPM0702
   25 CONTINUE                                                           26925770 SWPM0703
      XY=WZ(JJJ)*WX(III)-WX(JJJ)*WZ(III)                                 26925780 SWPM0704
      XZ=WX(JJJ)*WY(III)-WY(JJJ)*WX(III)                                 26925790 SWPM0705
      ODX = -HEX/(TAN(C(III))*SIN(B(III))) + HEX/(TAN(P)*TAN(B(III)))             SWPM0706
      OCX = -HEX/(TAN(C(JJJ))*SIN(B(JJJ))) + HEX/(TAN(P)*TAN(B(JJJ)))             SWPM0707
      ODY = HEX/TAN(P)                                                            SWPM0708
      DC=OCX-ODX                                                         26925830 SWPM0709
      IF(DC.LE.XLEN) GO TO 30                                            26925840 SWPM0710
      H=XLEN*HEX/DC                                                      26925850 SWPM0711
      ODX = -H  /(TAN(C(III))*SIN(B(III))) + H  /(TAN(P)*TAN(B(III)))             SWPM0712
      OCX = -H  /(TAN(C(JJJ))*SIN(B(JJJ))) + H  /(TAN(P)*TAN(B(JJJ)))             SWPM0713
      ODY = H/TAN(P)                                                              SWPM0714
      DC=OCX-ODX                                                         26925890 SWPM0715
      GO TO 35                                                           26925900 SWPM0716
   30 H=HEX                                                              26925910 SWPM0717
   35 CONTINUE                                                           26925920 SWPM0718
      TANE=XZ/XY                                                         26925930 SWPM0719
      TAND=TAN(D)                                                        26925940 SWPM0720
```

```
      AAA=1.                                                            26925950 SWPM0721
      IF (PLG.NE.90.) AAA=1.-TANE/TAN(P)                                26925960 SWPM0722
      HH=H*AAA*TAND/(TANE-TAND)                                         26925970 SWPM0723
      CCC=(H+HH)/XZ                                                     26925980 SWPM0724
      OBY = CCC*XY                                                      26925990 SWPM0725
      OBBY=OBY-ODY                                                      26926000 SWPM0726
      IF(TPSLP.NE.0.) OBBY=HH/SIN(D)                                    26926010 SWPM0727
      TAREX=OBBY*DC/2.                                                  26926020 SWPM0728
      AZ1=H/SIN(P)                                                      26926030 SWPM0729
      FAREX=DC*AZ1/2.                                                   26926040 SWPM0730
      AX=0.                                                             26926050 SWPM0731
      AY=0.                                                             26926060 SWPM0732
      AZ=0.                                                             26926070 SWPM0733
      AFX=0.                                                            26926080 SWPM0734
      AFY=0.                                                            26926090 SWPM0735
      AFZ=0.                                                            26926100 SWPM0736
      DO 10 K=1,NNN                                                     26926110 SWPM0737
      BT=BTOP(K)*TEN(K)                                                 26926120 SWPM0738
      IF(BT.EQ.0.) GO TO 15                                             26926130 SWPM0739
      BXY=BT*COS(DPT(K)/FACT)                                           26926140 SWPM0740
      IF(BXY.LT.0.) BXY=-BXY                                            26926150 SWPM0741
      BX=BXY*COS(ORT(K)/FACT)                                           26926160 SWPM0742
      BY=BXY*SIN(ORT(K)/FACT)                                           26926170 SWPM0743
      BZ=BT*SIN(DPT(K)/FACT)                                            26926180 SWPM0744
      AX=AX+BX                                                          26926190 SWPM0745
      AY=AY+BY                                                          26926200 SWPM0746
      AZ=AZ-BZ                                                          26926210 SWPM0747
   15 CONTINUE                                                          26926220 SWPM0748
      BF=BFAC(K)*FTEN(K)                                                26926230 SWPM0749
      IF(BF.EQ.0.) GO TO 10                                             26926240 SWPM0750
      BFXY=BF*COS(DPF(K)/FACT)                                          26926250 SWPM0751
      IF(BFXY.LT.0.) BFXY=-BFXY                                         26926260 SWPM0752
      BFX=BFXY*COS(ORF(K)/FACT)                                         26926270 SWPM0753
      BFY=BFXY*SIN(ORF(K)/FACT)                                         26926280 SWPM0754
      BFZ=BF*SIN(DPF(K)/FACT)                                           26926290 SWPM0755
      AFX=AFX+BFX                                                       26926300 SWPM0756
```

```
      AFY=AFY+BFY                                                  26926310 SWPM0757
      AFZ=AFZ-BFZ                                                  26926320 SWPM0758
   10 CONTINUE                                                     26926330 SWPM0759
      TBX=AX/TAREX                                                 26926340 SWPM0760
      TBY=AY/TAREX                                                 26926350 SWPM0761
      TBZ=AZ/TAREX                                                 26926360 SWPM0762
      FBX = AFX/FAREX                                                       SWPM0763
      FBY = AFY/FAREX                                                       SWPM0764
      FBZ = AFZ/FAREX                                                       SWPM0765
                                                                            SWPM0766
      RETURN                                                       26926400 SWPM0767
      END                                                          26926410 SWPM0768
```

Appendix F

Computer Program SWARS-2MC

SWARS-2MC is a modified version of SWARS-2PM that enables the user to determine the probability of failure (P_f) through Monte Carlo simulation. The program will compute the factor of safety (FS) for many sets of input parameters--P_f is the proportion of FS values that fall below 1.0. Any (or all) of the parameters that define the shape and size of the wedge or characterize the shear resistance along the joint planes can be considered random variables. The values for these random variables should be selected in accordance with their respective probability density functions. (Section 2.2 outlines the principles of Monte Carlo simulation.)

SWARS-2MC is virtually identical to SWARS-2PM with respect to its computational algorithms. Both programs compute two FS's: FS_I, the factor of safety against initial movement, and FS_U, the factor of safety against ultimate failure. (The distinction between the two values is discussed in Section 4.4.)

There are three major differences between SWARS-2PM and SWARS-2MC.

1. In SWARS-2PM the user inputs <u>individual planes</u>. He can specify up to 8 planes. Some combinations of planes will form kinematically stable wedges; the program will

indicate that these combinations do not constitute stability problems. The user does not specify the size of the wedges. The program identifies (and analyzes) the largest wedge that can form given the overall height and width of the slope. In SWARS-2MC the user inputs pairs of planes and the height of the resulting wedge. In effect, the user specifies the shape and size of the wedge to be analyzed. Thus, SWARS-2MC is based on wedges rather than individual planes. SWARS-2MC is designed to complement the computer program DAYLITE (Appendix C) in that the input for SWARS-2MC uses the same format as the output from DAYLITE. Since DAYLITE only considers kinematically unstable wedges, the input for SWARS-2MC will not identify any wedges which are kinematically stable.

2. In SWARS-2PM the user identifies planes by their strike and dip. As indicated above, SWARS-2MC is concerned with wedges rather than planes, so the user inputs the orientation of the line of intersection of planes (XX,XY, XZ) and the central angle of the

wedge (ψ). (XX, XY, and XZ) and ψ can be obtained from DAYLITE.

3. The input from SWARS-2PM includes a detailed description of the wedges formed by each pair of planes and the two FS values for each wedge. Since the SWARS-2MC user prescribes the size and slope of each wedge, the output from the program consists of just the two FS values (as well as a recapitulation of the input parameters).

SWARS-2MC maintains most of the versatility of SWARS-2PM with respect to loading conditions and ground water conconditions but not all the conditions can be varied in each set of input parameters. In particular,

1. SWARS-2MC will not consider rock bolt loads.
2. SWARS-2MC will compute cleft water pressures with respect to a specified phreatic surface; however, (unlike SWARS-2PM) both planes which bound the wedge must have the same surface. In addition, all the wedges in any one computer run* must have the same phreatic surface. (The user can specify "Piezometric Option 2" i.e., the same uniform hydrostatic pressure on both joints, and then let the pressure vary for each wedge.)

---

* A computer run consists of a series of FS calculations based on a single set of computer cards as prescribed in Section F.3.

3. SWARS-2MC will consider pseudo-static seismic loadings but the same seismic coefficients must be used for all wedges in a particular run.
4. SWARS-2MC (unlike SWARS-2PM) will not compute the minimum seismic accelerations required to cause failure.
5. SWARS-2MC will consider point loads but the same loads will be applied to all wedges in a particular run.

Some of these restrictions can be circumvented by conditioning the results on the loading condition. For example, suppose there is a 0.05 probability that a design earthquake will occur. Two computer runs can be made: one with the appropriate seismic accelerations and one without. If one assumes that the design earthquake is the only earthquake that can occur, P_f can be calculated as:

$$P_f = (0.05)P_f^1 + (0.95)P_f^2 \qquad (F.1)$$

where P_f^1 is the probability of failure with the seismic loads and P_f^2 is the probability of failure without the seismic loads.

The user must specify the number of wedges to be analyzed and he must provide input data for each wedge. This input is listed on cards #5 and #6 (see Section F.3) so a pair of

cards must be provided for each wedge (or "realization"). The lone exception to this rule occurs when the user only wishes to investigate FS_U. If this is the case, he inputs ' 1 ' for ISTIFF (card #4) and can disregard card #6 for each realization. (Card #6 contains information on the in situ stress ratio and joint stiffnesses--none of the data is required for the FS_U computations.)

The format for inputting data is designed so that the user can vary one parameter at a time. In analyzing the i^{th} wedge the program will use all the parameters from the $(i - 1)^{th}$ wedge except those parameters which are specified on the i^{th} pair of cards #5 and #6. For example, if H, the height of the wedge, is specified as 15.0 for the first realization, the computer will continue to use H = 15.0 for all subsequent wedges until the user specifies another value for H. If in analyzing two consecutive realizations the user wishes to change the value of a parameter from a non-zero number to zero he must input ' -1.0 '. For example, in the i^{th} realization the uniform hydrostatic pressure might be 124.8. If the uniform hydrostatic pressure in the $(i + 1)^{th}$ realization should be zero, the user must input ' -1.0 ' in the appropriate columns in the $(i + 1)^{th}$ pair of cards #5 and #6.

F.1 Coordinate System

The coordinate system used in SWARS-2MC is shown in Figure F.1. The positive X axis is horizontal to the right.

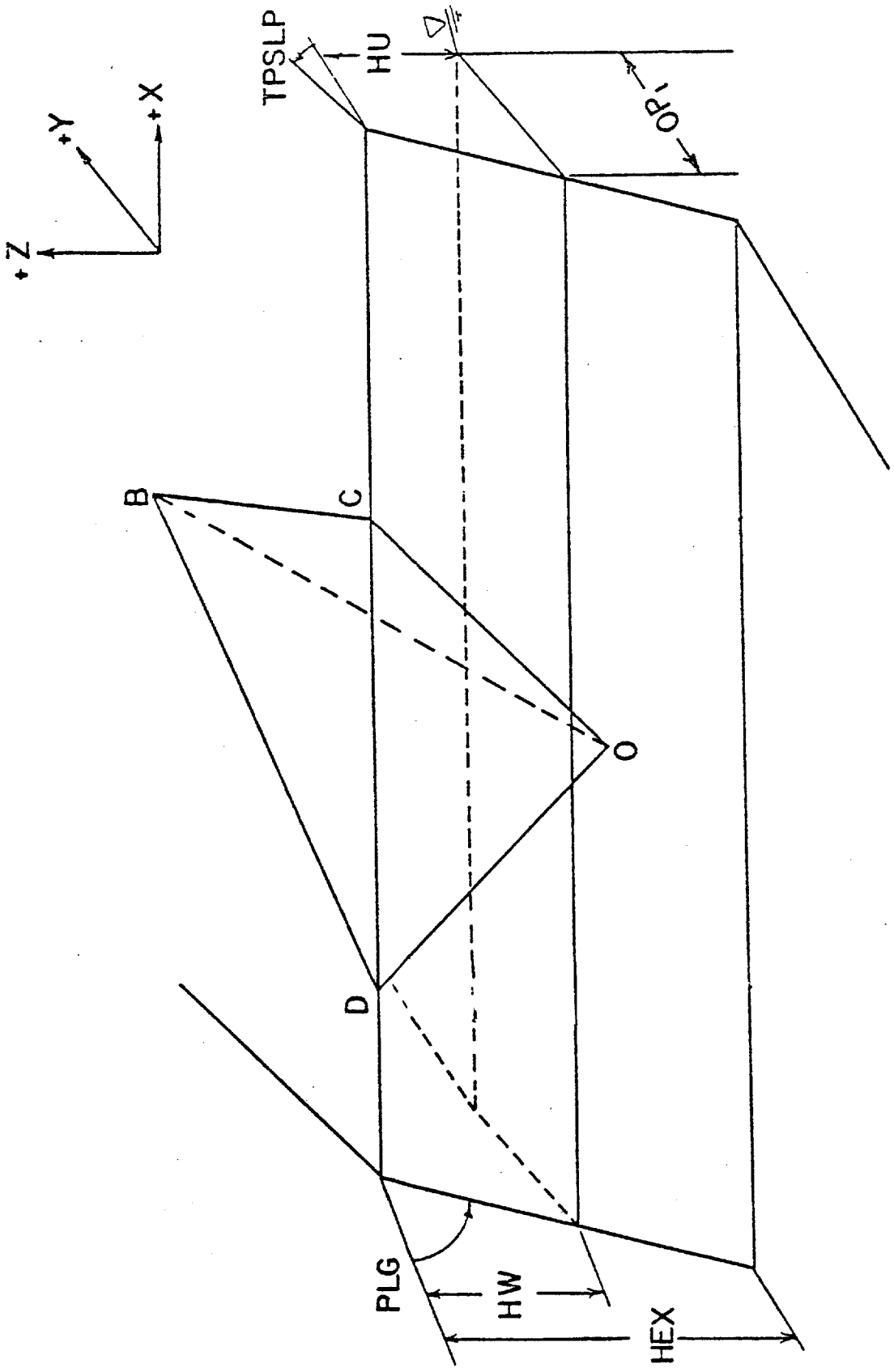


Figure F.1 Model Slope

The positive Y axis is horizontal and 90° counterclockwise from the positive X axis. (The positive Y direction is directed into the slope.) The positive Z axis is vertical upward.

DAYLITE uses this same coordinate system in describing the orientation of the line of intersection.

F.2 Units

All units used in the input must be consistent. Units are generally in feet or pounds but any other system may be employed as long as all the other input is in consistent units. Angular input is always in degrees.

Unless otherwise noted, input data does not have to be right justified. The term "integer" implies that the parameter is expressed without a decimal point.

F.3 Inputs

Card #1

Strike of the Slope: (H1AZI, AZI, H2AZI) is entered in geologic notation in columns 1 to 4. Geologic strike must be expressed in the northern quadrants and the angle must be expressed as a two digit integer. (Examples: N45E, N06W)

Dip of the Slope: (PLG, H1PLG, H2PLG) is entered in geologic notation in columns 6 to 9. Dip directions must indicate a particular quadrant i.e., NE, NW, SE, SW. The angle is expressed as a two digit integer. (Examples: 16NW, 07NE)

Height of the Slope: (HEX) is entered in columns 11 to

20 with a maximum of three decimal places. HEX is measured from the crest of the slope to the bottom of the slope (Figure F.1).

Length of the Slope: (XLEN) is entered in columns 21 to 30 with a maximum of three decimal places.

Angle of the Top Slope: (TPSLP) is entered in columns 31 to 40 with a maximum of three decimal places. TPSLP is shown in Figure F.1.

Unit Weight of Rock: (UNIWGT) is entered in columns 41 to 50 with a maximum of three decimal places.

Unit Weight of Water: (WWGT) is entered in columns 51 to 60 with a maximum of three decimal places.

Uniform Vertical Surcharge: (SUR) is the uniform vertical load acting per unit area on the top face of the wedge (face BCD in Figure F.1). SUR is entered in columns 61 to 70 with a maximum of three decimal places.

Surcharge Option: (IS) There are two options available to analyze the effects of the surcharge on the stability of the tetrahedron in the dynamic case. Option 1 assumes that the surcharge is accelerated only in the vertical direction. Option 2 assumes that the surcharge is a mass and that it is accelerated in the same directions as the wedge under dynamic loading. The code 1 or 2 is entered in column 71.

Card #2

X Coordinate of the Line of Intersection: (XX) is entered in columns 1 to 10 with a maximum of five decimal places.

Y Coordinate of the Line of Intersection: (XY) is entered in columns 11 to 20 with a maximum of 5 decimal places.

Z Coordinate of the Line of Intersection: (XZ) is entered in columns 21 to 30 with a maximum of 5 decimal places.

Note: (XX, XY, XZ) should be a unit vector i.e., $(XX^2 + YX^2 + XZ^2)^{\frac{1}{2}} = 1.0$.

Card #3

Piezometric Option: (IU) is a code that indicates how cleft water effects are to be incorporated into the analysis. 1. No effect from water; 2. Uniform pressure acting on the joint; 3. Pressure defined by a horizontal pheatic surface; 4. Pressure determined by an arbitrary phreatic surface. IU is entered in column 5 as an integer. (Unlike SWARS-2P or SWARS-2PM, SWARS-2MC requires that the same code be used for both planes. In addition, the phreatic surfaces defined in Options 3 and 4 must be identical for both planes.)

Depth to Water: (HU) is measured from the crest of the slope to a horizontal phreatic surface (Figure F.1). HU is used in conjunction with Piezometric Options 3 and 4. HU is entered in columns 6 to 10 with a maximum of 1 decimal place.

Depth to Piezometric Level on the Slope: (HW) is entered in columns 11 to 15 with a maximum of 1 decimal place. HW is shown in Figure F.1 and is used in conjunction with Piezometric Option 4.

Distance OP1: (OP1) is used in conjunction with Piezometric Option 4. As shown in Figure F.1, it is the horizontal distance from the face of the slope where the phreatic surface exits to the point where the phreatic surface becomes horizontal. OP1 is entered in columns 16 to 20 with a maximum of 1 decimal place.

Seismic Coefficients X,Y,Z: (AKX, AKY, AKZ) Seismic forces are entered as decimal fractions of the gravitational force. Positive values will be forces acting at the centroid in the direction of the appropriate positive coordinate, negative values set in the direction of the appropriate negative coordinates. AKX is entered in columns 21 to 25, AKY is entered in columns 26 to 30, and AKZ is entered in colmns 31 to 35. The three values are expressed with a maximum of three decimal places. If no seismic loads are to be considered columns 21 to 35 may be left blank--the computer will default to AKX = AKY = AKZ = 0.

Point Loads: (PX, PY, PZ) can be used in the program. PX, PY, and PZ are the X, Y, and Z components of a point load. The program treats PX, PY and PZ as point loads acting at the centroid of the wedge. PX is entered in columns 36 to 45, PY is entered in columns 46 to 55 and PZ is entered in columns 56 to 65. The three values are expressed with a maximum of three decimal places. If not point loads are to be considered columns 36 to 65 may be left blank--the computer will default to PX = PY = PZ = 0.

Card #4

Number of Realizations: (NOREAL) is entered as an integer in columns 1 to 5. NOREAL must be right justified. Note: The user must provide a set of input parameters i.e., cards #5 and #6 for each realization.

FS Option #1: (ISTIFF) If a ' 1 ' is entered in column 10 the program will compute FS_U only. If any integer greater than 1 is entered in columns 6 to 10 the program will compute both FS_I and FS_U.

FS Option #2: (NDOP) is a code that indicates how the program should compute FS_I. As indicated in Section 4.4 (q.v.), FS_I is defined as:

$$FS_I = \frac{\bar{R} \cdot \bar{\Delta}}{\bar{D} \cdot \bar{\Delta}} \qquad (F.2)$$

where $\bar{R}$ is the resultant of all resisting forces, $\bar{D}$ is the resultant of all driving forces, and $\bar{\Delta}$ is the direction of impending movement. If a ' 0 ' is placed in column 15, $\bar{\Delta}$ is the displacement that would occur in response to a vertical load. If any other integer is placed in columns 11 to 15 $\bar{\Delta}$ is the displacement that occurs in response to the resultant of all external loads i.e., all loads except the weight.

FS Option #3: (LOPT) is an option that allows the user to analyze all loads (including the weight of the wedge) with

the stiffness approach. If a ' 1 ' is entered in column 20 the program will use this option. If any other integer is placed in columns 16 to 20 the program will treat loads according to the "standard" procedure presented in Section 4.3 i.e., stress approach for gravitational loads, stiffness approach for all other loads. If columns 16 to 20 are left blank the program will default to the standard procedure.

Card #5

Note: A separate card is used for each of the NREAL realizations--see the introductory discussion to this appendix.

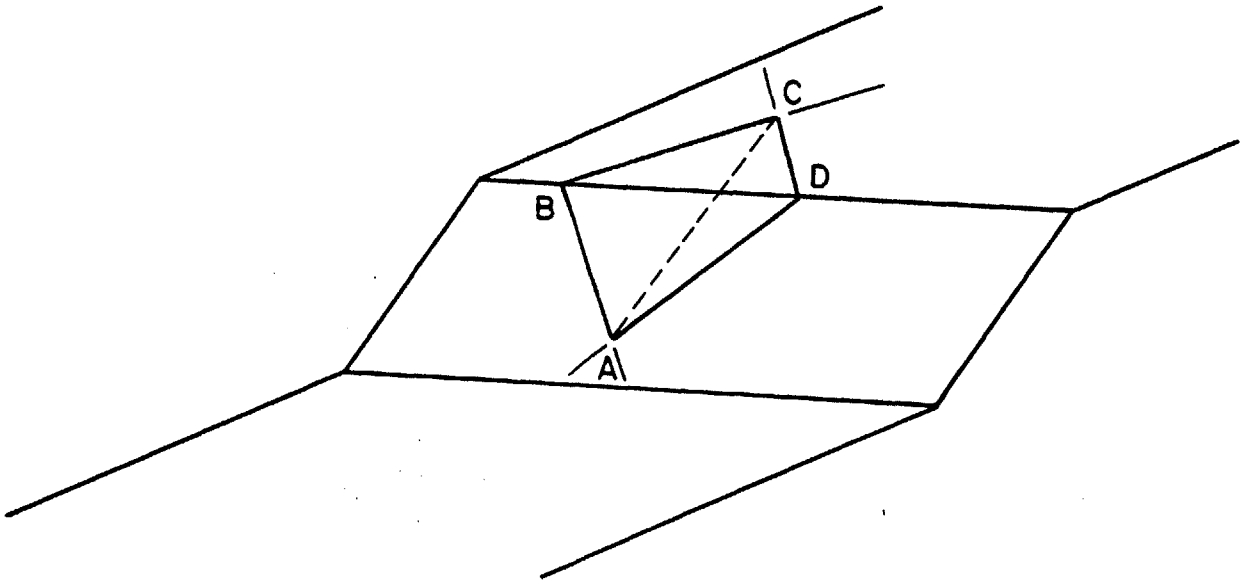
Central Wedge Angle: (PSI) is entered in columns 1 to 5 with a maximum of 1 decimal place. PSI is shown in Figure F.2 as ψ.

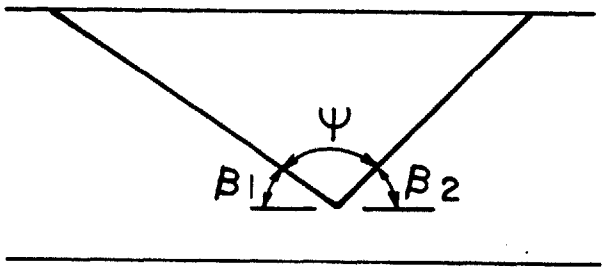
Orientation Angle #1: (BETA1) is entered in columns 6 to 10 with a maximum of 1 decimal place. BETA1 is shown in Figure F.2 as β_1.

Height of Wedge: (H) is entered in columns 11 to 15 with a maximum of 1 decimal place. H is measured from the crest of the slope to the vertex of the wedge i.e., in the same manner as HEX in Figure F.1.

Friction Angle on Joint Plane 1: (PHX(1)) is entered in columns 16 to 20 with a maximum of 1 decimal place.

Asperities on Joint Plane 1: (AQ(1)), the angular value of additional shearing resistance due to larger size





SECTION PERPENDICULAR TO AC

Figure F.2 Definitions of Ψ and β_1

irregularities on the joint plane, is entered in columns 21 to 25 with a maximum of 1 decimal place.

Cohesion of Intact Rock on Joint Plane 1: (TAU(1)) is entered in columns 26 to 33 with a maximum of 1 decimal place.

Joint Persistence on Joint Plane 1: (PCTJTG(1)) is the percent of the area of the total joint plane that is not intact rock. PCTJTG(1) is entered in columns 33 to 38 with a maximum of 1 decimal place.

Uniform Water Pressure on Joint Plane 1: (UAV(1)) is entered in columns 39 to 45 with a maximum of 1 decimal place. UAV(1) is used in conjunction with Piezometric Option 2.

Friction Angle on Joint Plane 2: (PHX(2)) is entered in columns 46 to 50 with a maximum of 1 decimal place.

Asperities on Joint Plane 2: (AQ(2)), the angular value of additional shearing resistance due to larger size irregularities on the joint plane, is entered in columns 51 to 55 with a maximum of 1 decimal place.

Cohesion of Intact Rock on Joint Plane 2: (TAU(2)) is entered in columns 56 to 62 with a maximum of 1 decimal place.

Joint Persistence on Joint Plane 2: (PCTJTG(2)) is the percent of the area of the total joint plane that is not intact rock. PCTJTG(2) is entered in columns 63 to 67 with a maximum of 1 decimal place.

Uniform Water Pressure on Joint Plane 2: (UAV(2)) is entered in columns 68 to 72 with a maximum of 1 decimal place. UAV(2) is used in conjunction with Piezometric Option 1.

Card #6

Note: A separate card is used for each of the NREAL realizations unless ISTIFF = 1 in which case Card #6 is deleted for each realization.

Stress Ratio: (RATIO) is entered in columns 1 to 10 with a maximum of 5 decimal places. RATIO is the ratio of horizontal to vertical stresses within the wedge.

Shear Stiffness on Joint Plane 1: (STIFFS(1)) is entered in columns 11 to 20 with a maximum of two decimal places. STIFFS(1) can also be entered in scientific notation with a maximum of two decimal places. (Examples: 678.54, 6.78E2)

Normal Stiffness on Joint Plane 1: (STIFFN(1) is entered in columns 21 to 30 with a maximum of two decimal places. STIFFN(1) can also be entered in scientific notation with a maximum of two decimal places. (Examples: 45678.2, 4.57E4)

Shear Stiffness on Joint Plane 2: (STIFFS(2)) is entered in columns 31 to 40 with a maximum of two decimal places. STIFFS(2) can also be entered in scientific notation with a maximum of two decimal places. (Examples: 567.9, 5.68E3)

Normal Stiffness on Joint Plane 2: (STIFFN(2)) is entered in columns 41 to 50 with a maximum of two decimal places. STIFFN(2) can also be entered in scientific notation with a maximum of two decimal places. (Examples: 10000.0, 10.0E4)

SWARS-2MC Listing

```
C                                  SWARS-MC                                                SWMC0001
C        SLIDING WEDGE ANALYSIS OF ROCK SLOPES -- MONTE CARLO                              SWMC0002
      DIMENSION BTOP(8),ORT(8),DPT(8),TEN(8),UAV(8),UF(8),AREA(8),IU(8)                    SWMC0003
      DIMENSION TAU(2), PCTJTG(2), PUFFY(30), WX(2), WY(2), WZ(2)                          SWMC0004
      DIMENSION T(8),TX(8),TY(8),TZ(8),EN(8),PHI(8)                          26920080      SWMC0005
      DIMENSION SX(8),SY(8),SZ(8),RSS(8),RRX(8),RRY(8),RRZ(8)                26920090      SWMC0006
      DIMENSION INDEX(8),DUMMY(8),PHX(8),AP(8),AQ(8),CH(8)                   26920110      SWMC0007
      DIMENSION WWX(8),WWY(8),WWZ(8),BET(8), B(2), C(2)                                    SWMC0008
      DIMENSION BFAC(8),ORF(8),DPF(8),FTEN(8)                                26920150      SWMC0009
      DIMENSION  STIFFN(8), STIFFS(8), RATIOK(8)                                           SWMC0010
      REAL NORD, HXX, HXY                                                                  SWMC0011
      LOGICAL FLIM                                                                         SWMC0012
      DATA NORD,SOTH,EAST,WEST/'N','S','E','W'/                              26920160      SWMC0013
      NWR = 6                                                                              SWMC0014
      NRD = 5                                                                              SWMC0015
      FLIM = .TRUE.                                                                        SWMC0016
      PI=3.141593                                                            26920180      SWMC0017
      PI2=PI/2.                                                              26920190      SWMC0018
      FACT=180./PI                                                           26920200      SWMC0019
   10 FORMAT (4I5)                                                                         SWMC0020
   11 FORMAT (2I5)                                                                         SWMC0021
   12 FORMAT (3F10.5)                                                                      SWMC0022
   13 FORMAT (5F5.1,F8.1,F5.1,F7.1,2F5.1,F7.1,F5.1,F7.1)                                   SWMC0023
   14 FORMAT (F10.5,4G10.2)                                                                SWMC0024
   15 FORMAT (A1,F2.0,A1,1X,F2.0,A1,A1,1X,6F10.3,I1)                                       SWMC0025
   16 FORMAT (F5.3,2I5)                                                                    SWMC0026
   17 FORMAT (6F10.5)                                                                      SWMC0027
   20 FORMAT (I5,3F5.1,3F5.3,3F10.3)                                                       SWMC0028
   25 FORMAT (A1,F2.0,A1,1X,F2.0,A1,A1,1X,2F5.2,F10.3,2G10.3,2F5.2,                        SWMC0029
     1 F10.3,I1)                                                                           SWMC0030
 1001 FORMAT('0',T20,'FS =',F6.2,' FOR SLIDING ON PLANE ',I1)             26920250      SWMC0031
 1006 FORMAT (37X,'SEISMIC  ACCELERATIONS:',//,18X,'AKX =',F7.3,12X,                     SWMC0032
     1 'AKY =',F7.3,11X,'AKZ =',F7.3,///)                                                SWMC0033
 1007 FORMAT (40X,'POINT LOADS:',//,18X,'PX =',F9.2,12X,'PY =',F9.2,13X,                 SWMC0034
     1 'PZ =',F9.2,///)                                                                  SWMC0035
 1008 FORMAT('0',T20,'RESULTANT TENDS TO LIFT BLOCK OFF SUPPORTS')        26920480      SWMC0036
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1010 FORMAT('0',T20,'CHECK FOR TOPPLING ABOUT POINT O')                  26920520 SWMC0037
1013 FORMAT(T20,'FS =',F6.2,' FOR ROTATIONS ON PLANE ',I2)               26920570 SWMC0038
1016 FORMAT (3F6.2)                                                              SWMC0039
1018 FORMAT (3F6.2)                                                              SWMC0040
1019 FORMAT('0',T20,'LINE OF INTERSECTION DOES NOT INTERSECT',           26920750 SWMC0041
    1' TOP OF SLOPE')                                                    26920760 SWMC0042
1030 FORMAT ('+',48X,'NO EFFECT FROM WATER',///)                                 SWMC0043
1031 FORMAT ('+',48X,'UNIFORM PRESSURE',///)                                     SWMC0044
1032 FORMAT ('+',48X,'DEPTH TO WATER:',4X,F8.3,///)                              SWMC0045
1033 FORMAT ('+',48X,'DEFINED PHREATIC SURFACE: HU = ',F6.2,'    HW = ',         SWMC0046
    1 F6.2,'    OP1 = ',F6.2,///)                                                SWMC0047
1035 FORMAT (29X,'********      LOAD DATA      ********',//                     SWMC0048
    1 18X,'UNIT WEIGHT OF ROCK:',F8.2,3X,'UNIT WEIGHT OF WATER: ',F6.2,          SWMC0049
    2 //,34X,'VERTICAL SURCHARGE:',F8.2,/)                                       SWMC0050
1036 FORMAT (29X,'*******    EXCAVATION DATA    *******',//                     SWMC0051
    1 18X,'STRIKE:',16X,'N',F3.0,A1,3X,'DIP:',18X,F3.0,A1,A1,//                  SWMC0052
    2 18X,'HEIGHT OF FACE:',F13.2,3X,'LENGTH OF FACE:',F12.2,//                  SWMC0053
    3 31X,'INCLINATION OF TOP SLOPE:',F8.2,//                                    SWMC0054
    4 37X,'LINE OF INTERSECTION:',//,18X,'X = ',F6.3,15X,'Y = ',F6.3,            SWMC0055
    5 14X,'Z = ',F6.3,//,18X,'PIEZOMETRIC OPTION:',8X,I1)                        SWMC0056
1040 FORMAT (' NO.      PSI     BETA1      H      PHI(1)   AQ(1)     TAU',      SWMC0057
    1 '(1)    PJ(1)    UA(1)    PHI(1)  AQ(1)     TAU(1)    PJ(2)',           SWMC0058
    2 '    UA(2)    FS(I)    FS(U)')                                             SWMC0059
1041 FORMAT (I5,3X,F5.1,3X,F5.1,3X,F5.1,3X,F5.1,3X,F5.1,3X,F5.1,3X,F7.1,       SWMC0060
    1 3X,F5.1,2X,F7.1,2X,F5.1,3X,F5.1,3X,F7.1,3X,F5.1,2X,F7.1,                   SWMC0061
    2 2X,F6.4,2X,F6.4)                                                           SWMC0062
1045 FORMAT (29X,'*  NUMBER OF REALIZATIONS:    ',I5,'  *',//,                 SWMC0063
    1 29X,'*  ISTIFF:                        ',I5,'  *',//,                    SWMC0064
    2 29X,'*  NDOP:                          ',I5,'  *',//,                    SWMC0065
    3 29X,'*  LOPT:                          ',I5,'  *',////)                  SWMC0066
     DO 1 I = 1, 30                                                              SWMC0067
   1 PUFFY(I) = 0.0                                                              SWMC0068
     READ(NRD,15) H1AZI, AZI, H2AZI, PLG, H1PLG, H2PLG,                          SWMC0069
    1 HEX, XLEN, TPSLP, UNIWGT, WWGT, SUR, IS                                    SWMC0070
     READ (NRD,12) XX, XY, XZ                                                    SWMC0071
     READ (NRD,20) IU(1), HU, HW, OP1, AKX, AKY, AKZ, PX, PY, PZ                 SWMC0072
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      READ (NRD,10) NOREAL, ISTIFF, NDOP, LOPT                                  SWMC0073
      READ (NRD,13) PSI,BETA1,H,PHX(1),AQ(1),TAU(1),PCTJTG(1),UAV(1),           SWMC0074
     1 PHX(2),AQ(2),TAU(2),PCTJTG(2),UAV(2)                                     SWMC0075
      IF (ISTIFF .NE. 1 ) READ (NRD,14) RATIO, STIFFS(1), STIFFN(1),            SWMC0076
     1 STIFFS(2), STIFFN(2)                                                     SWMC0077
   65 IF (UNIWGT.EQ.0.) UNIWGT=1.0                                  26921060 SWMC0078
      IF (UNIWGT.EQ.1.0) WWGT=0.0                                   26921070 SWMC0079
      IU(2) = IU(1)                                                            SWMC0080
      WRITE (NWR,1036) AZI, H2AZI,PLG, H1PLG, H2PLG, HEX, XLEN, TPSLP,         SWMC0081
     1 XX, XY, XZ, IU(1)                                                       SWMC0082
      IF ( IU(1) .EQ. 1 ) WRITE (NWR,1030)                                     SWMC0083
      IF ( IU(1) .EQ. 2 ) WRITE (NWR,1031)                                     SWMC0084
      IF ( IU(1) .EQ. 3 ) WRITE (NWR,1032) HU                                  SWMC0085
      IF ( IU(1) .EQ. 4 ) WRITE (NWR,1033) HU, HW, OP1                         SWMC0086
      WRITE (NWR,1035) UNIWGT, WWGT, SUR                                       SWMC0087
      IF(AKX.NE.0.0.OR.AKY.NE.0.0.OR.AKZ.NE.0.0)                               SWMC0088
     1 WRITE (NWR,1006) AKX, AKY, AKZ                                          SWMC0089
      IF(PX.NE.0.0.OR.PZ.NE.0.0.OR.PZ.NE.0.0) WRITE (NWR,1007) PX,PY,PZ        SWMC0090
      WRITE (NWR,1045) NOREAL, ISTIFF, NDOP, LOPT                              SWMC0091
      IF ( ISTIFF .EQ. 1 ) RATIO = 1.0                                         SWMC0092
      P=PLG/FACT                                                    26921320 SWMC0093
      D=TPSLP/FACT                                                  26921330 SWMC0094
      TANE=XZ/XY                                                    26922400 SWMC0095
      TAND=TAN(D)                                                   26922410 SWMC0096
      IF ( TANE .LT. TAND ) WRITE (NWR,1018)                                   SWMC0097
      IF ( TANE .LT. TAND ) GO TO 9000                                         SWMC0098
      E=ATAN(TANE)                                                  26922430 SWMC0099
      IF(E.LT.0.) E=E+PI                                            26922440 SWMC0100
      NREAL = 0                                                                SWMC0101
      WRITE (NWR,1040)                                                         SWMC0102
      DO 5000 IJOHN =1, NOREAL                                                 SWMC0103
      NREAL = NREAL + 1                                                        SWMC0104
      IF ( IJOHN .EQ. 1 ) GO TO 50                                             SWMC0105
      READ (NRD,13) (PUFFY(I),I=1,13)                                          SWMC0106
      IF ( PUFFY(1) .NE. 0.0 ) PSI = PUFFY(1)                                  SWMC0107
      IF ( PUFFY(2) .NE. 0.0 ) BETA1 = PUFFY(2)                                SWMC0108
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      IF ( PUFFY(3) .NE. 0.0 ) H = PUFFY(3)                              SWMC0109
      IF ( PUFFY(4) .NE. 0.0 ) PHI(1) = PUFFY(4)                         SWMC0110
      IF ( PUFFY(5) .NE. 0.0 ) AQ(1) = PUFFY(5)                          SWMC0111
      IF ( PUFFY(6) .NE. 0.0 ) TAU(1) = PUFFY(6)                         SWMC0112
      IF ( PUFFY(7) .NE. 0.0 ) PCTJTG(1) = PUFFY(7)                      SWMC0113
      IF ( PUFFY(8) .NE. 0.0 ) UAV(1) = PUFFY(8)                         SWMC0114
      IF ( PUFFY(9) .NE. 0.0 ) PHI(2) = PUFFY(9)                         SWMC0115
      IF ( PUFFY(10) .NE. 0.0 ) AQ(2) = PUFFY(10)                        SWMC0116
      IF ( PUFFY(11) .NE. 0.0 ) TAU(2) = PUFFY(11)                       SWMC0117
      IF ( PUFFY(12) .NE. 0.0 ) PCTJTG(2) = PUFFY(12)                    SWMC0118
      IF ( PUFFY(13) .NE. 0.0 ) UAV(2) = PUFFY(13)                       SWMC0119
      IF ( BETA1 .EQ. -1.0 ) BETA1 = 0.0                                 SWMC0120
      IF ( PHI(1) .EQ. -1.0 ) PHI(1) = 0.0                               SWMC0121
      IF ( PHI(2) .EQ. -1.0 ) PHI(2) = 0.0                               SWMC0122
      IF ( AQ(1) .EQ. -1.0 ) AQ(1) = 0.0                                 SWMC0123
      IF ( AQ(2) .EQ. -1.0 ) AQ(2) = 0.0                                 SWMC0124
      IF ( TAU(1) .EQ. -1.0 ) TAU(1) = 0.0                               SWMC0125
      IF ( TAU(2) .EQ. -1.0 ) TAU(2) = 0.0                               SWMC0126
      IF ( PCTJTG(1) .EQ. -1.0 ) PCTJTG(1) = 0.0                         SWMC0127
      IF ( PCTJTG(2) .EQ. -1.0 ) PCTJTG(2) = 0.0                         SWMC0128
      IF ( UAV(1) .EQ. -1.0 ) UAV(1) = 0.0                               SWMC0129
      IF ( UAV(2) .EQ. -1.0 ) UAV(2) = 0.0                               SWMC0130
      IF ( ISTIFF .EQ. 1 ) GO TO 50                                      SWMC0131
      READ (NRD,14) (PUFFY(I),I=14,18)                                   SWMC0132
      IF ( PUFFY(14) .NE. 0.0 ) RATIO = PUFFY(14)                        SWMC0133
      IF ( PUFFY(15) .NE. 0.0 ) STIFFS(1) = PUFFY(15)                    SWMC0134
      IF ( PUFFY(16) .NE. 0.0 ) STIFFN(1) = PUFFY(16)                    SWMC0135
      IF ( PUFFY(17) .NE. 0.0 ) STIFFS(2) = PUFFY(17)                    SWMC0136
      IF ( PUFFY(18) .NE. 0.0 ) STIFFN(2) = PUFFY(18)                    SWMC0137
      IF ( RATIO .EQ. -1.0 ) RATIO = 0.0                                 SWMC0138
      IF ( STIFFS(1) .EQ. -1.0 ) STIFFS(1) = 0.0                         SWMC0139
      IF ( STIFFS(2) .EQ. -1.0 ) STIFFS(2) = 0.0                         SWMC0140
      IF ( STIFFN(1) .EQ. -1.0 ) STIFFN(1) = 0.0                         SWMC0141
      IF ( STIFFN(2) .EQ. -1.0 ) STIFFN(2) = 0.0                         SWMC0142
   50 CONTINUE                                                           SWMC0143
      DO 60 I = 1, 2                                                     SWMC0144
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      IF ( I .EQ. 1 ) ANGLE = BETA1/FACT                                SWMC0145
      IF ( I .EQ. 2 ) ANGLE = PI - (BETA1 + PSI)/FACT                   SWMC0146
      SINGLE = SIN(ANGLE)*SQRT(XX**2 + XY**2)/XY                        SWMC0147
      AA = XX/XY                                                        SWMC0148
      BB = XX*AA + XY                                                   SWMC0149
      CC = BB**2 + XZ**2 +XZ**2*AA**2                                   SWMC0150
      DD = 2.0*SINGLE*(BB*XX + AA*XZ**2)                                SWMC0151
      EE = (XX*SINGLE)**2 - XZ**2 + (XZ*SINGLE)**2                      SWMC0152
      WY(I) = (-DD - SQRT(DD**2 - 4.0*CC*EE))/(2.0*CC)                  SWMC0153
      WX(I) = SINGLE + WY(I)*AA                                         SWMC0154
      WZ(I) =-SQRT(1.0 - WX(I)**2 - WY(I)**2)                           SWMC0155
      IF ( I .EQ. 1 .AND. ANGLE .GT. PI2 ) WZ(I) = -WZ(I)               SWMC0156
      IF ( I .EQ. 2 .AND. ANGLE .LT. PI2 ) WZ(I) = -WZ(I)               SWMC0157
      ATEST = ABS(XX*WX(I) + XY*WY(I) + XZ*WZ(I))                       SWMC0158
      IF ( ATEST .LT. 0.01 ) GO TO 59                                   SWMC0159
      WY(I) = (-DD + SQRT(DD**2 - 4.0*CC*EE))/(2.0*CC)                  SWMC0160
      WX(I) = SINGLE + WY(I)*AA                                         SWMC0161
      WZ(I) =-SQRT(1.0 - WX(I)**2 - WY(I)**2)                           SWMC0162
      IF ( I .EQ. 1 .AND. ANGLE .GT. PI2 ) WZ(I) = -WZ(I)               SWMC0163
      IF ( I .EQ. 2 .AND. ANGLE .LT. PI2 ) WZ(I) = -WZ(I)               SWMC0164
   59 CONTINUE                                                          SWMC0165
      B(I) = ATAN((-WX(I))/WY(I))                                       SWMC0166
      IF ( WY(I) .LT. 0.0 ) B(I) = PI - ATAN(WX(I)/WY(I))               SWMC0167
      C(I) = ATAN((SQRT(1.0 - WZ(I)**2))/(-WZ(I)))                      SWMC0168
      IF ( WZ(I) .GT. 0.0 ) C(I) = ATAN(WZ(I)/(SQRT(1.0-WZ(I)**2)))     SWMC0169
      PHI(I)=PHX(I)/FACT                                                SWMC0170
      AP(I)=AQ(I)/FACT                                      26921300    SWMC0171
      IF ( ISTIFF .EQ. 1 ) GO TO 60                                     SWMC0172
      IF ( STIFFS(I) .NE. 0.0 ) RATIOK(I) = STIFFN(I)/STIFFS(I)         SWMC0173
      IF ( STIFFS(I) .EQ. 0.0 ) RATIOK(I) = 1000000.                    SWMC0174
      IF ( RATIOK(I) .EQ. 1000000. ) STIFFS(I) = 1.                     SWMC0175
      IF ( RATIOK(I) .EQ. 1000000. ) STIFFN(I) = 1000000.               SWMC0176
   60 CONTINUE                                                          SWMC0177
      I = 1                                                             SWMC0178
      J = 2                                                             SWMC0179
      XX1 = XX                                                          SWMC0180
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      XY1 = XY                                                                                                  SWMC0181
      XZ1 = XZ                                                                                                  SWMC0182
      HEX = H                                                                                                   SWMC0183
      ODX = -HEX/(TAN(C(I))*SIN(B(I))) + HEX/(TAN(P)*TAN(B(I)))                                                 SWMC0184
      OCX = -HEX/(TAN(C(J))*SIN(B(J))) + HEX/(TAN(P)*TAN(B(J)))                                                 SWMC0185
      ODY = HEX/TAN(P)                                                                                          SWMC0186
      DC=OCX-ODX                                                                                    26922260    SWMC0187
      IF(DC.LE.XLEN) GO TO 95                                                                       26922270    SWMC0188
      H=XLEN*HEX/DC                                                                                 26922280    SWMC0189
      ODX = -H  /(TAN(C(I))*SIN(B(I))) + H  /(TAN(P)*TAN(B(I)))                                                 SWMC0190
      OCX = -H/(TAN(C(J))*SIN(B(J))) + H/(TAN(P)*TAN(B(J)))                                                     SWMC0191
      ODY = H/TAN(P)                                                                                            SWMC0192
      DC=OCX-ODX                                                                                    26922320    SWMC0193
   95 CONTINUE                                                                                                  SWMC0194
      OCY=ODY                                                                                       26922360    SWMC0195
      ODZ=H                                                                                         26922370    SWMC0196
      OCZ=ODZ                                                                                       26922380    SWMC0197
      AAA = 1.0                                                                                                 SWMC0198
      IF (PLG.NE.90.) AAA=1.-TANE/TAN(P)                                                            26922540    SWMC0199
      HH=H*AAA*TAND/(TANE-TAND)                                                                     26922550    SWMC0200
      CCC=(H+HH)/XZ                                                                                 26922560    SWMC0201
      OBX = CCC*XX                                                                                  26922570    SWMC0202
      OBY = CCC*XY                                                                                  26922580    SWMC0203
      OBZ = CCC*XZ                                                                                  26922590    SWMC0204
      OD=SQRT(ODX**2+ODY**2+ODZ**2)                                                                 26922600    SWMC0205
      OC=SQRT(OCX**2+OCY**2+OCZ**2)                                                                 26922610    SWMC0206
      OB=SQRT(OBX**2+OBY**2+OBZ**2)                                                                 26922620    SWMC0207
      OBBY=OBY-ODY                                                                                  26922630    SWMC0208
      IF(TPSLP.NE.0.) OBBY=HH/SIN(D)                                                                26922640    SWMC0209
      TAREA=OBBY*DC/2.                                                                              26922650    SWMC0210
      AZ=H/SIN(P)                                                                                   26922660    SWMC0211
      FAREA=DC*AZ/2.                                                                                26922670    SWMC0212
      BDX=ODX-OBX                                                                                   26922680    SWMC0213
      IF(BDX.LT.0.) BDX=-BDX                                                                        26922690    SWMC0214
      DB=OBBY                                                                                       26922700    SWMC0215
      R1=ATAN2(BDX,OBBY)                                                                            26922710    SWMC0216
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      IF(TAN(R1).NE.0.)  DB=OBBY/COS(R1)                                26922720 SWMC0217
      CBX=OCX-OBX                                                       26922730 SWMC0218
      IF(CBX.LT.0.)  CBX=-CBX                                           26922740 SWMC0219
      CB=OBBY                                                           26922750 SWMC0220
      R=ATAN2(CBX,OBBY)                                                 26922760 SWMC0221
      IF(TAN(R).NE.0.)  CB=OBBY/COS(R)                                  26922770 SWMC0222
      CCP=H/XZ                                                          26922780 SWMC0223
      DBPY=CCP*XY-ODY                                                   26922790 SWMC0224
      VOL=DBPY*DC*(H+HH)/6.                                             26922800 SWMC0225
      OSX=(OBX+OCX+ODX)/4.                                              26922810 SWMC0226
      OSY=(OBY+OCY+ODY)/4.                                              26922820 SWMC0227
      OSZ=(OBZ+OCZ+ODZ)/4.                                              26922830 SWMC0228
      TEMP=VOL*UNIWGT                                                   26922840 SWMC0229
      DUMMYT = TEMP*COS(E)*1.0E-10                                               SWMC0230
      SURG=SUR*TAREA/COS(D)                                             26922850 SWMC0231
      SUM1=(OD+OB+DB)/2.                                                26922860 SWMC0232
      SUM2=(OC+OB+CB)/2.                                                26922870 SWMC0233
      AREA(I)=SQRT(SUM1*(SUM1-OD)*(SUM1-OB)*(SUM1-DB))                  26922880 SWMC0234
      AREA(J)=SQRT(SUM2*(SUM2-OC)*(SUM2-OB)*(SUM2-CB))                  26922890 SWMC0235
      CH(I)=TAU(I)*(100.-PCTJTG(I))*(.01)*AREA(I)                       26922900 SWMC0236
      CH(J)=TAU(J)*(100.-PCTJTG(J))*(.01)*AREA(J)                       26922910 SWMC0237
      CH1=CH(I)                                                         26922920 SWMC0238
      CH2=CH(J)                                                         26922930 SWMC0239
      RX = PX + TEMP*AKX                                                         SWMC0240
      RY = PY + TEMP*AKY                                                         SWMC0241
      RZ = PZ - SURG - TEMP*(1.0 - AKZ)                                          SWMC0242
      IF ( IS .EQ. 2 ) RX = RX + SURG*AKX                                        SWMC0243
      IF ( IS .EQ. 2 ) RY = RY + SURG*AKY                                        SWMC0244
      IF ( IS .EQ. 2 ) RZ = RZ + SURG*AKZ                                        SWMC0245
      CALL WATER(RX,RY,RZ,H,E,P,PLG,HU,IU,UAV,OP1,HW,UF,HH,PI,          26923100 SWMC0246
     1 AREA,WWGT,WX,WY,WZ,TPSLP)                                                 SWMC0247
      OIRX=OSY*RZ-OSZ*RY                                                26923120 SWMC0248
      OIRY=OSZ*RX-OSX*RZ                                                26923130 SWMC0249
      OIRZ=OSX*RY-OSY*RX                                                26923140 SWMC0250
      EMX=XX*OIRX+XY*OIRY+XZ*OIRZ                                       26923150 SWMC0251
      DO 110 K=1,2                                                               SWMC0252
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      EN(K)=RX*WX(K)+RY*WY(K)+RZ*WZ(K)                                  26923170 SWMC0253
      TX(K)=RX-EN(K)*WX(K)                                              26923180 SWMC0254
      TY(K)=RY-EN(K)*WY(K)                                              26923190 SWMC0255
      TZ(K)=RZ-EN(K)*WZ(K)                                              26923200 SWMC0256
      T(K)=SQRT(TX(K)**2+TY(K)**2+TZ(K)**2)                             26923210 SWMC0257
  110 CONTINUE                                                          26923220 SWMC0258
      ENN=0.                                                            26923230 SWMC0259
      IF (EN(I).GT.0.0.OR.EN(J).LT.0.0) GO TO 120                       26923240 SWMC0260
      WRITE (5,1008)                                                             SWMC0261
      GO TO 5000                                                                 SWMC0262
  120 DO 115 K=1,2                                                               SWMC0263
      SX(K)=(XY*WZ(K)-XZ*WY(K))                                         26923280 SWMC0264
      SY(K)=(XZ*WX(K)-XX*WZ(K))                                         26923290 SWMC0265
      SZ(K)=(XX*WY(K)-XY*WX(K))                                         26923300 SWMC0266
      RSS(K)=RX*SX(K)+RY*SY(K)+RZ*SZ(K)                                 26923310 SWMC0267
  115 CONTINUE                                                          26923320 SWMC0268
      T12 = RX*XX1 + RY*XY1 + RZ*XZ1                                             SWMC0269
      T12P = RX*XX1 + RY*XY1 + (RZ+TEMP)*XZ1                                     SWMC0270
      IF (RSS(I).LT.0.0) GO TO 130                                      26923330 SWMC0271
      IF (RSS(J).LT.0.0) GO TO 140                                      26923340 SWMC0272
      SINBT1 = SIN(BETA1/FACT)                                                   SWMC0273
      COSBT1 = COS(BETA1/FACT)                                                   SWMC0274
      BETA2 = 180. - BETA1 - PSI                                                 SWMC0275
      SINBT2 = SIN(BETA2/FACT)                                                   SWMC0276
      COSBT2 = COS(BETA2/FACT)                                                   SWMC0277
      PERP1 = 0.0                                                                SWMC0278
      TANG1 = 0.0                                                                SWMC0279
      PERP2 = 0.0                                                                SWMC0280
      TANG2 = 0.0                                                                SWMC0281
      IF ( LOPT .EQ. 1 ) GO TO 78                                                SWMC0282
C     THIS SECTION COMPUTES THE EFFECT OF IN SITU STRESSES                       SWMC0283
      WTCO = TEMP*COS(E)                                                         SWMC0284
      IF ( RATIO .NE. 1.0 ) GO TO 76                                             SWMC0285
      PERP2 = WTCO/(COSBT2 + SINBT2*COSBT1/SINBT1)                               SWMC0286
      PERP1 = PERP2*SINBT2/SINBT1                                                SWMC0287
      GO TO 78                                                                   SWMC0288
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   76 RAT1=(RATIO+(1.0-RATIO)*COSBT1**2)/((1.0-RATIO)*SINBT1*COSBT1)          SWMC0289
      RAT2=(RATIO+(1.0-RATIO)*COSBT2**2)/((1.0-RATIO)*SINBT2*COSBT2)          SWMC0290
      RAT12 = (RAT2*SINBT2 - COSBT2)/(RAT1*SINBT1 - COSBT1)                   SWMC0291
      TANG2 = WTCO/(RAT12*(RAT1*COSBT1 + SINBT1)+(RAT2*COSBT2+SINBT2))        SWMC0292
      TANG1 = RAT12*TANG2                                                     SWMC0293
      PERP1 = RAT1*TANG1                                                      SWMC0294
      PERP2 = TANG2*RAT2                                                      SWMC0295
C     THIS PART COMPUTES THE EFFECT OF JOINT STIFFNESSES                      SWMC0296
   78 HXX = XY1/(SQRT(XX1**2 + XY1**2))                                       SWMC0297
      HXY = -XX1/(SQRT(XX1**2 + XY1**2))                                      SWMC0298
      RHORIZ = RX*HXX + RY*HXY                                                SWMC0299
      RVERT =  -1.0*(RX*HXY*XZ1 - RY*HXX*XZ1 + RZ*(HXX*XY1 - XX1*HXY) )       SWMC0300
      UR1A = RVERT/(COSBT1 +COSBT2*SINBT1/SINBT2)                             SWMC0301
      UR2A = UR1A*SINBT1/SINBT2                                               SWMC0302
      UR1B = RHORIZ/(SINBT1 + SINBT2*COSBT1/COSBT2)                           SWMC0303
      UR2B = UR1B*COSBT1/COSBT2                                               SWMC0304
C     FSLR IS THE 'ULTIMATE' F.S. FOR SLIDING DOWN THE LINE OF INTERSECTION   SWMC0305
      FSLR = ((UR1A + UR1B)*TAN(PHI(I)) +(UR2A + UR2B)*TAN(PHI(J))) /T12      SWMC0306
      IF ( ISTIFF .EQ. 1 ) GO TO 400                                          SWMC0307
      RVERT1 = RVERT - TEMP*(HXX*XY1 - XX1*HXY)                               SWMC0308
      IF ( LOPT .EQ. 1 ) RVERT1 = RVERT                                       SWMC0309
      IF ( RVERT1 .NE. 0.0 .OR. RHORIZ .NE. 0.0 ) GO TO 79                    SWMC0310
  167 RVERT1 = DUMMYT                                                         SWMC0311
   79 ST1N = STIFFN(I)*AREA(I)                                                SWMC0312
      ST1S = STIFFS(I)*AREA(I)                                                SWMC0313
      ST2N = STIFFN(J)*AREA(J)                                                SWMC0314
      ST2S = STIFFS(J)*AREA(J)                                                SWMC0315
      FACTAA = ST1N*SINBT1 + ST1S*COSBT1**2/SINBT1 +                          SWMC0316
     1 ST2N*SINBT2 + ST2S*COSBT2**2/SINBT2                                    SWMC0317
      FACTBB = (ST1N-ST1S)*COSBT1 + (ST2S-ST2N)*COSBT2                        SWMC0318
      FACTCC = ST1N*COSBT1**2/SINBT1 + ST1S*SINBT1 +                          SWMC0319
     1 ST2N*COSBT2**2/SINBT2 + ST2S*SINBT2                                    SWMC0320
      ALPHA = ATAN((FACTCC*RHORIZ - FACTBB*RVERT1)/                           SWMC0321
     1 (FACTAA*RVERT1 - FACTBB*RHORIZ))                                       SWMC0322
      COSALP = COS(ALPHA)                                                     SWMC0323
      SINALP = SIN(ALPHA)                                                     SWMC0324
```

```
      FACTDD = ST1N*(SINALP + COSALP*COSBT1/SINBT1)                          SWMC0325
      FACTEE = ST1S*(COSALP*SINBT1 - SINALP*COSBT1)                          SWMC0326
      FACTFF = ST2N*(COSALP*COSBT2**2/SINBT2 - SINALP*COSBT2)                SWMC0327
      FACTGG = ST2S*(COSALP*SINBT2 + SINALP*COSBT2/SINBT2)                   SWMC0328
      PERP11 = RVERT1*FACTDD/(COSBT1 + FACTEE + FACTFF + FACTGG)             SWMC0329
      DISP = PERP11/FACTDD                                                   SWMC0330
      PERP1 = PERP11 + PERP1                                                 SWMC0331
      TANG1 = PERP11*FACTEE/(SINBT1*FACTDD) + TANG1                          SWMC0332
      PERP2 = PERP11*FACTFF/(COSBT2*FACTDD) + PERP2                          SWMC0333
      TANG2 = PERP11*FACTGG/(SINBT2*FACTDD) + TANG2                          SWMC0334
      IF (.NOT.FLIM) GO TO 166                                               SWMC0335
      IF ( NDOP .NE. 0 ) GO TO 166                                           SWMC0336
      PPERP1 = PERP1                                                         SWMC0337
      PPERP2 = PERP2                                                         SWMC0338
      TTANG1 = TANG1                                                         SWMC0339
      TTANG2 = TANG2                                                         SWMC0340
      FLIM = .FALSE.                                                         SWMC0341
      RHORIZ = 0.0                                                           SWMC0342
      GO TO 167                                                              SWMC0343
C     THIS PART COMPUTES THE FACTOR OF SAFETY: THE RATIO OF THE              SWMC0344
C     COMPONENTS OF DRIVING TO RESISTING FORCES IN THE DISPLACEMENT DIRECTION SWMC0345
  166 IF ( RVERT1 .EQ. DUMMYT ) T12P = DUMMYT*TAN(E)                         SWMC0346
      ADISP = T12P/(ST1S + ST2S)                                             SWMC0347
      BDISP = DISP*SINALP                                                    SWMC0348
      CDISP = -DISP*COSALP                                                   SWMC0349
      DISP = SQRT(ADISP**2 + BDISP**2 + CDISP**2)                            SWMC0350
      ADISP = ADISP/DISP                                                     SWMC0351
      BDISP = BDISP/DISP                                                     SWMC0352
      CDISP = CDISP/DISP                                                     SWMC0353
      IF (FLIM) GO TO 168                                                    SWMC0354
      PERP1 = PPERP1                                                         SWMC0355
      PERP2 = PPERP2                                                         SWMC0356
      TANG1 = TTANG1                                                         SWMC0357
      TANG2 = TTANG2                                                         SWMC0358
  168 FLIM = .TRUE.                                                          SWMC0359
      ADERF1 = T12*ST1S/(ST1S + ST2S)                                        SWMC0360
```

```
      BDRF1 = -TANG1*COSBT1                                                    SWMC0361
      CDRF1 = -TANG1*SINBT1                                                    SWMC0362
      ADRF2 = T12 - ADRF1                                                      SWMC0363
      BDRF2 = TANG2*COSBT2                                                     SWMC0364
      CDRF2 = -TANG2*SINBT2                                                    SWMC0365
      DRF = (ADRF1+ADRF2)*ADISP+(BDRF1+BDRF2)*BDISP+(CDRF1+CDRF2)*CDISP        SWMC0366
      FACTHH = (ADRF1*ADISP+BDRF1*BDISP+CDRF1*CDISP)/                          SWMC0367
     1 (SQRT(ADRF1**2+BDRF1**2+CDRF1**2))                                      SWMC0368
      FACTJJ = (ADRF2*ADISP+BDRF2*BDISP+CDRF2*CDISP)/                          SWMC0369
     1 (SQRT(ADRF2**2+BDRF2**2+CDRF2**2))                                      SWMC0370
      REST2 = PERP2*TAN(PHI(J)+AP(J)) + CH2                                    SWMC0371
      REST1 = PERP1*TAN(PHI(I)+AP(I))+CH1                                      SWMC0372
      FSL = (REST1*FACTHH + REST2*FACTJJ)/DRF                                  SWMC0373
  245 IF(FSL.LT.1.) GO TO 300                                    26923500 SWMC0374
      WWX(I)=-WX(I)                                              26923510 SWMC0375
      WWY(I)=-WY(I)                                              26923520 SWMC0376
      WWZ(I)=-WZ(I)                                              26923530 SWMC0377
      WWX(J)=WX(J)                                               26923540 SWMC0378
      WWY(J)=WY(J)                                               26923550 SWMC0379
      WWZ(J)=WZ(J)                                               26923560 SWMC0380
      DO 410 K=I,J,ID                                            26923570 SWMC0381
      COSP=COS(PHI(K)+AP(K))                                     26923580 SWMC0382
      SINP=SIN(PHI(K)+AP(K))                                     26923590 SWMC0383
      RRX(K)=WWX(K)*COSP-XX*SINP/X                               26923600 SWMC0384
      RRY(K)=WWY(K)*COSP-XY*SINP/X                               26923610 SWMC0385
      RRZ(K)=WWZ(K)*COSP-XZ*SINP/X                               26923620 SWMC0386
  410 CONTINUE                                                   26923630 SWMC0387
      CRPX=RRY(I)*RRZ(J)-RRZ(I)*RRY(J)                           26923640 SWMC0388
      CRPY=RRZ(I)*RRX(J)-RRX(I)*RRZ(J)                           26923650 SWMC0389
      CRPZ=RRX(I)*RRY(J)-RRY(I)*RRX(J)                           26923660 SWMC0390
      CRR=SQRT(CRPX**2+CRPY**2+CRPZ**2)                          26923670 SWMC0391
      CRRX=-CRPX/CRR                                             26923680 SWMC0392
      CRRY=-CRPY/CRR                                             26923690 SWMC0393
      CRRZ=-CRPZ/CRR                                             26923700 SWMC0394
      ENN=(RX*CRRX+RY*CRRY+RZ*CRRZ)/(TEMP)                       26923710 SWMC0395
      GO TO 300                                                  26923720 SWMC0396
```

```
  130 IF (EN(I).LT.0.0) GO TO 150                                                    26923730 SWMC0397
      FLS = 0.0                                                                              SWMC0398
      K=I                                                                            26923750 SWMC0399
      GO TO 145                                                                      26923760 SWMC0400
  140 IF (EN(J).GT.0.0) GO TO 150                                                    26923770 SWMC0401
      FSL=0.                                                                         26923780 SWMC0402
      K=J                                                                            26923790 SWMC0403
      EN(K)=-EN(K)                                                                   26923800 SWMC0404
  145 CONTINUE                                                                       26923810 SWMC0405
      FSS=(EN(K)*TAN(PHI(K)+AP(K))+CH(K))/T(K)                                       26923820 SWMC0406
      WRITE(5,1001) FSS,K                                                                    SWMC0407
      IF(CH1.NE.0..OR.CH2.NE.0..OR.AP(I).NE.0..OR.AP(J).NE.0.) GO TO 70              26923840 SWMC0408
      GO TO 240                                                                      26923850 SWMC0409
   70 FSSE=EN(K)*TAN(PHI(K))/T(K)                                                    26923860 SWMC0410
      GO TO 240                                                                      26923880 SWMC0411
  150 WRITE (5,1010)                                                                         SWMC0412
      GO TO 400                                                                      26923900 SWMC0413
  240 IF(FSS.LT.1.) GO TO 300                                                        26923910 SWMC0414
  300 IF(FSL.NE.0.) FSS=0.                                                           26924020 SWMC0415
      IF(FSS.NE.0.) GO TO 400                                                        26924030 SWMC0416
      IF(IU(I).NE.1 .AND.IU(J).NE.1 ) GO TO 400                                              SWMC0417
      AK1COS =  ((ODX*OBX+ODY*OBY+ODZ*OBZ)/(OD*OB))                                          SWMC0418
      AK10 = ATAN(SQRT(1.0 - AK1COS**2)/AK1COS)                                              SWMC0419
      AK2COS =  ((OCX*OBX+OCY*OBY+OCZ*OBZ)/(OC*OB))                                  26924070 SWMC0420
      AK20 = ATAN(SQRT(1.0 - AK2COS**2)/AK2COS)                                              SWMC0421
      EM10=-(WX(I)*OIRX+WY(I)*OIRY+WZ(I)*OIRZ)                                       26924080 SWMC0422
      EM20=-(WX(J)*OIRX+WY(J)*OIRY+WZ(J)*OIRZ)                                       26924090 SWMC0423
      IF (EMX.LE.0.0.OR.EM10.LE.0.0) GO TO 310                                       26924100 SWMC0424
      IF (AK20.GT.PI2) GO TO 310                                                     26924110 SWMC0425
      IF (ETA.LT.PI2.AND.AK10.GT.PI2.AND.                                            26924120 SWMC0426
     1  (TAN(AK20)/TAN(PI-AK10)).GT.(1./COS(PI-ETA))) GO TO 310                      26924130 SWMC0427
      K=I                                                                            26924140 SWMC0428
  305 TOP=OSX*WX(K)+OSY*WY(K)+OSZ*WZ(K)                                              26924150 SWMC0429
      BOT= RX*WX(K)+ RY*WY(K)+ RZ*WZ(K)                                              26924160 SWMC0430
      VJP=-TOP/BOT                                                                           SWMC0431
      OQX=OSX+VJP*RX                                                                         SWMC0432
```

```
      OQY=OSY+VJP*RY                                                                  SWMC0433
      OQZ=OSZ+VJP*RZ                                                                  SWMC0434
      OQWX=OQY*WZ(K)-OQZ*WY(K)                                          26924210      SWMC0435
      OQWY=OQZ*WX(K)-OQX*WZ(K)                                          26924220      SWMC0436
      OQWZ=OQX*WY(K)-OQY*WX(K)                                          26924230      SWMC0437
      DET=-OQY*OQWZ+OQZ*OQWY                                            26924240      SWMC0438
      C2=(-TZ(K)*OQY+TY(K)*OQZ)/DET                                     26924250      SWMC0439
      TT=C2*SQRT(OQWX**2+OQWY**2+OQWZ**2)                               26924260      SWMC0440
      FSR=(EN(K)*TAN(PHI(K)+AP(K))+CH(K))/TT                            26924270      SWMC0441
      WRITE(5,1013) FSR,K                                                             SWMC0442
      IF(CH1.NE.0..OR.CH2.NE.0..OR.AP(I).NE.0..OR.AP(J).NE.0.) GO TO 75 26924290      SWMC0443
      GO TO 420                                                         26924300      SWMC0444
   75 FSRR=EN(K)*TAN(PHI(K))/TT                                         26924310      SWMC0445
      GO TO 420                                                         26924330      SWMC0446
  310 IF (EMX.GE.0.0.OR.EM20.LE.0.0) GO TO 320                          26924340      SWMC0447
      IF (AK10.GT.PI2) GO TO 320                                        26924350      SWMC0448
      IF (ETA.LT.PI2.AND.AK20.GT.PI2.AND.                               26924360      SWMC0449
     1  (TAN(AK10)/TAN(PI-AK20)).GT.(1./COS(PI-ETA))) GO TO 320          26924370      SWMC0450
      K=J                                                               26924380      SWMC0451
      EN(K)=-EN(K)                                                      26924390      SWMC0452
      GO TO 305                                                         26924400      SWMC0453
  320 CONTINUE                                                                        SWMC0454
  400 CONTINUE                                                                        SWMC0455
  405 CONTINUE                                                                        SWMC0456
  420 CONTINUE                                                                        SWMC0457
  510 CONTINUE                                                          26924650      SWMC0458
  500 CONTINUE                                                          26924660      SWMC0459
      WRITE ( NWR,1041) NREAL,PSI,BETA1,H,PHX(1),AQ(1),TAU(1),                        SWMC0460
     1 PCTJTG(1),UAV(1),PHX(2),AQ(2),TAU(2),PCTJTG(2),UAV(2),                         SWMC0461
     2 FSL,FSLR                                                                       SWMC0462
 5000 CONTINUE                                                                        SWMC0463
 9000 CONTINUE                                                          26924680      SWMC0464
      STOP                                                              26924690      SWMC0465
      END                                                               26924700      SWMC0466
      SUBROUTINE WATER(RX,RY,RZ,H,E,P,PLG,HU,IU,UAV,OP1,HW,UF,HH,PI,                  SWMC0467
     1 AREA,WWGT,I,J,ID,WX,WY,WZ,TPSLP)                                               SWMC0468
```

```
      DIMENSION       IU(8),UAV(8),AREA(8),UF(8),WX(8),WY(8),WZ(8)                SWMC0469
      DO 15 K = 1,2                                                               SWMC0470
      KK=IU(K)                                                          26925070  SWMC0471
      HHH=HU(K)                                                         26925080  SWMC0472
      IF(TPSLP.LT.0..AND.HHH.LT.-HH) HHH=-HH                            26925090  SWMC0473
      DH=H-HHH                                                          26925100  SWMC0474
      IF(DH.LE.0.) GO TO 15                                             26925110  SWMC0475
      TEM=(DH/H)*(DH/(H+HH))                                            26925120  SWMC0476
      GO TO (15,20,25,30),KK                                            26925130  SWMC0477
   20 UF(K)=AREA(K)*UAV(K)                                              26925140  SWMC0478
      GO TO 35                                                          26925150  SWMC0479
   25 PER=1.                                                            26925160  SWMC0480
      GO TO 40                                                          26925170  SWMC0481
   30 AZZ=DH/SIN(P)                                                     26925180  SWMC0482
      AXX=0.                                                            26925190  SWMC0483
      IF(PLG.NE.90.) AXX=DH/TAN(P)                                      26925200  SWMC0484
      AX=DH/TAN(E)-AXX                                                  26925210  SWMC0485
      PA=PI-P                                                           26925220  SWMC0486
      ART=AZZ*AX*SIN(PA)/2.                                             26925230  SWMC0487
      IF(HW.NE.0.) GO TO 45                                             26925240  SWMC0488
      EA=P/2.                                                           26925250  SWMC0489
      IF(EA.LE.E) GO TO 15                                              26925260  SWMC0490
      ARA=AZZ*AZZ*SIN(PA)/2.                                            26925270  SWMC0491
      ARB=ART-ARA                                                       26925280  SWMC0492
      PER=ARB/ART                                                       26925290  SWMC0493
      GO TO 40                                                          26925300  SWMC0494
   45 HP=HW-HU(K)                                                       26925310  SWMC0495
      AY=OP1-AXX                                                        26925320  SWMC0496
      AZW=HP/SIN(P)                                                     26925330  SWMC0497
      IF(HP.GT.DH) GO TO 50                                             26925340  SWMC0498
      ARA1=AZW*AY*SIN(PA)/2.                                            26925350  SWMC0499
      ARB1=ART-ARA1                                                     26925360  SWMC0500
      IF(AY.GT.AX) GO TO 55                                             26925370  SWMC0501
      PER=ARB1/ART                                                      26925380  SWMC0502
      GO TO 40                                                          26925390  SWMC0503
   55 IF(HP.EQ.DH.AND.AY.GE.AX) GO TO 15                                26925400  SWMC0504
```

```
      AYY=AXX*AZW/AZZ                                                   26925410 SWMC0505
      ZA=ATAN2(HP,(AY+AYY))                                             26925420 SWMC0506
      ZB=AY-AX                                                          26925430 SWMC0507
      ZC=PI-E                                                           26925440 SWMC0508
      ZD=PI-ZC-ZA                                                       26925450 SWMC0509
      ARF=ZB*ZB*SIN(ZA)*SIN(ZC)/(2.*SIN(ZD))                            26925460 SWMC0510
      PER=(ARB1+ARF)/ART                                                26925470 SWMC0511
      GO TO 40                                                          26925480 SWMC0512
   50 IF(AY.GE.AX) GO TO 15                                             26925490 SWMC0513
      ZG=AX-AY                                                          26925500 SWMC0514
      ZE=ATAN2(OP1,HP)+PI/2.                                            26925510 SWMC0515
      ZF=PI-ZE-ZG                                                       26925520 SWMC0516
      ARB2=ZG*ZG*SIN(ZE)*SIN(E)/(2.*SIN(ZF))                            26925530 SWMC0517
      PER=ARB2/ART                                                      26925540 SWMC0518
   40 CONTINUE                                                          26925550 SWMC0519
      UF(K)=PER*TEM*AREA(K)*WWGT*DH/3.                                  26925560 SWMC0520
   35 IF(K.EQ.J) UF(K)=-UF(K)                                           26925570 SWMC0521
      UFX=WX(K)*UF(K)                                                   26925580 SWMC0522
      UFY=WY(K)*UF(K)                                                   26925590 SWMC0523
      UFZ=WZ(K)*UF(K)                                                   26925600 SWMC0524
      RX=RX-UFX                                                         26925610 SWMC0525
      RY=RY-UFY                                                         26925620 SWMC0526
      RZ=RZ-UFZ                                                         26925630 SWMC0527
   15 CONTINUE                                                          26925640 SWMC0528
      RETURN                                                            26925650 SWMC0529
      END                                                               26925660 SWMC0530
```

Appendix G

Strength Derivations for Mechanical Model

The mechanical model described in Chapter 5 uses a Griffith-Modified Griffith envelope to define the strength of intact rock. The orientation of the failure surface is a critical feature in the strength determination. The model investigates numerous paths with different orientations in order to find the one with the minimum strength. Figure G.1 shows a typical path from Point A to Point B inclined at an angle θ. The objective is to find the resistance which must be overcome to create a failure path between the two points. The Mohr stress circle is used in solving this problem. The Mohr circle for the state of stress at failure must be tangent to the Griffith-Modified Griffith envelope at a point such that the failure plane develops in the direction θ (Figure G.1). The critical stress condition is obtained by superimposing upon the ambient stress field both negative horizontal normal stresses ($\Delta\sigma_h$) and shear stresses ($\Delta\tau$) in the horizontal and vertical planes.* (The vertical normal stress is assumed to be constant.) The problem is essentially a geometrical exercise in which the Mohr circle at failure is the unknown while θ, the failure envelope, σ_{vo} and σ_{ho} (the ambient stress field) are the knowns.

* Intact rock will generally fail in tension given the relatively low range of stresses encountered in most engineering problems.

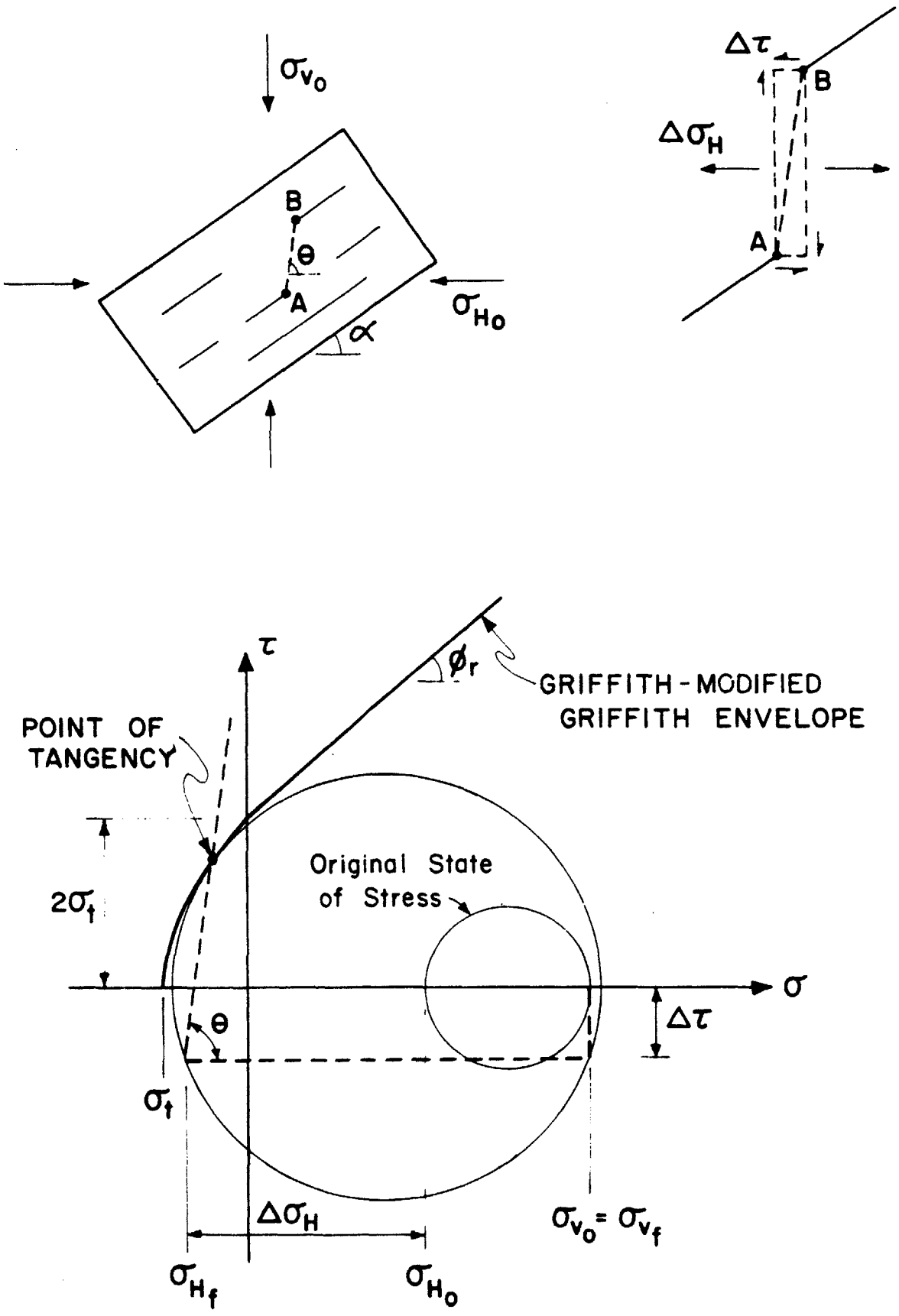


Figure G.1 Failure Conditions--Intact Rock

The mechanical model places some constraints on the values that θ can assume. The angle the failure path makes with the lower joint plane must be less than or equal to 90° (Figure G.2). However, the joint planes are inclined at an angle α relative to the horizontal so that the maximum value that θ can reach is $90° + \alpha$.

The formation of a failure path potentially lets the upper portion of the rock mass slide down the direction of the joint planes. When $\theta \leq 90°$ this impending movement implies that the shear stresses on the failure path are counterclockwise; hence, the point of tangency for the Mohr circle at failure will be above the σ axis. Similarly, when $\theta \geq 90°$, the point of tangency will be below the σ axis.

G.1 Griffith-Modified Griffith Envelope (Figure G.1)

The Griffith-Modified Griffith Envelope is composed of a parabola in the tensile range and a straight line in the compression range. It is completely defined by two parameters: σ_t, the tensile strength of the intact rock, and Φ_r, the friction angle. The envelope intercepts the τ axis at $2|\sigma_t|$.

Circles tangent to the parabola can be characterized by an angle γ where γ is the acute angle between the horizontal and line of tangency (Figure G.3). There is a unique one-to-one correspondence between γ values and failure circles.

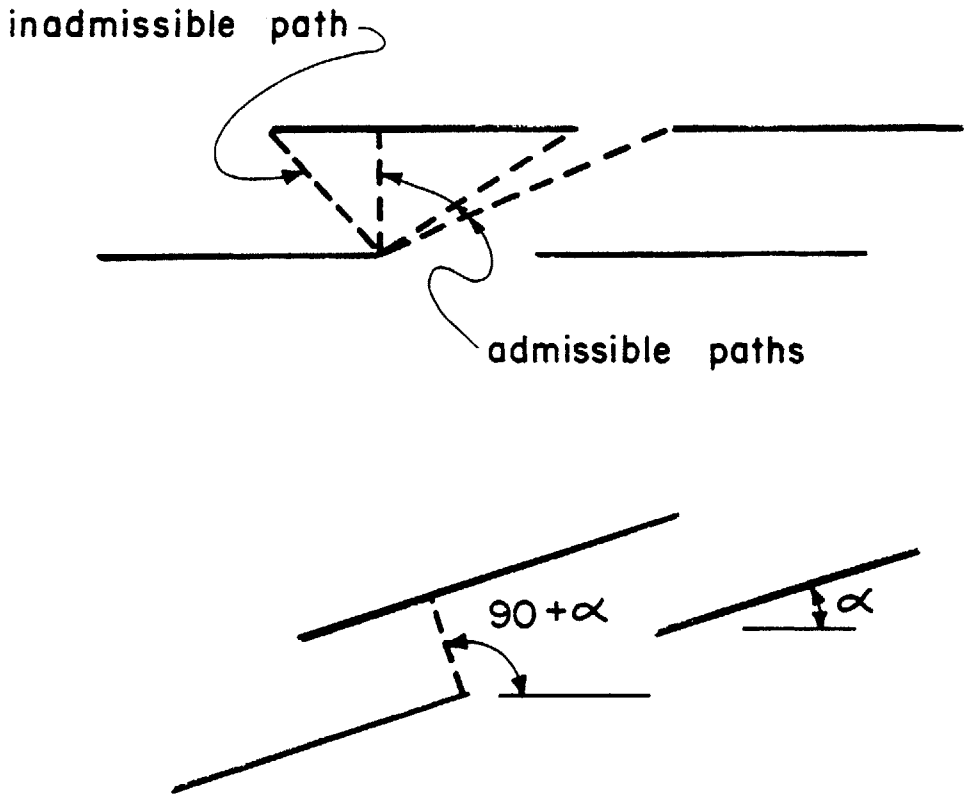


Figure G.2 Limitation on the Maximum Value of θ

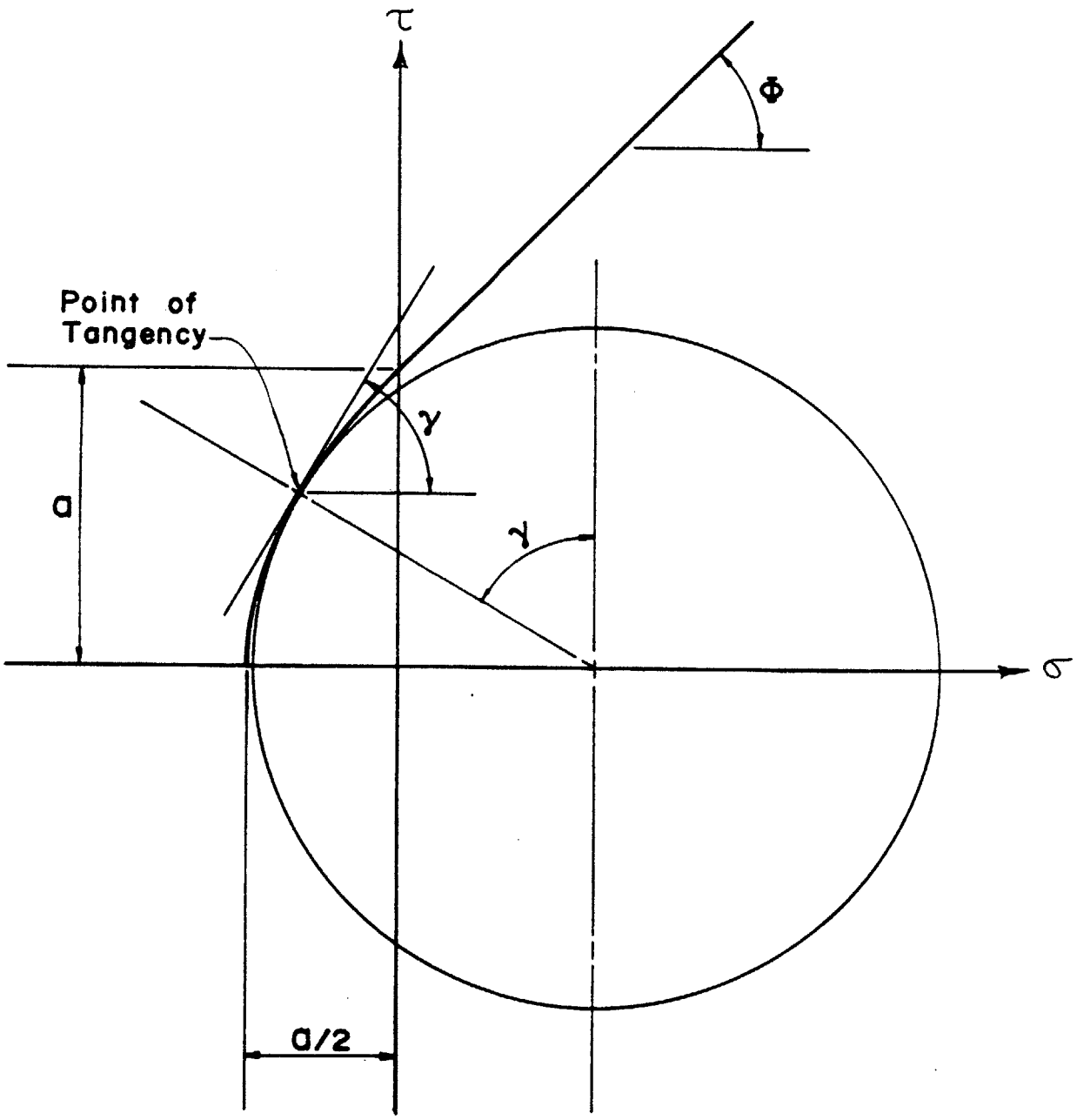


Figure G.3 Definition of γ

The circle tangent at $\sigma = \sigma_t$ and $\tau = 0$ has a $\gamma = 90^{\circ}$. Similarly, the circle tangent at $\sigma = 0$ and $\tau = 2|\sigma_t|$ has a $\gamma = 45^{\circ}$.

The point $(0,2|\sigma_t|)$ is common to both the linear and parabolic portions of the envelope. The change from parabola to line is not necessarily a smooth transition. The tangent to the parabola at $(0,2|\phi_t|)$ is inclined at 45°. If $\Phi_r = 45^{\circ}$, the envelope will be smooth, i.e., its first derivative will be continuous. On the other hand, if $\Phi_r \neq 45°$, the first derivative will be discontinuous and the envelope will be kinked. The implications of the singularity are discussed in the two succeeding sections.

G.1.1 $\Phi_r < 45^{\circ}$

For $\Phi < 45°$*, there is one circle which is tangent to both the straight line and the parabola. It is important to analytically define this circle because it serves as a transition between two distinct families of failure circles--those that are tangent to the parabola and those that are tangent to the line. The circle with dual tangency can be characterized by two parameters: r, the radius, and x, the distance along the σ axis from the origin to the center of the circle.

* For the sake of convenience the "r" subscript will subsequently be dropped from Φ_r.

Figure G.4 shows a typical circle with dual tangency. As noted in Figure G.4, $a = -2\sigma_t$. The equation of the parabola is:

$$\tau^2 = 4\sigma_t(\sigma_t - \sigma_n) \qquad (G.1)$$

since $\sigma_t = -a/2$

$$\tau^2 = a^2 + 2a\sigma_n \qquad (G.1A)$$

Hoek (1969) derived the following relation:

$$x - \sigma_n = 2\sigma_t \qquad (G.2)$$

or

$$x - \sigma_n = -a$$

or

$$\sigma_n = x - a \qquad (G.2A)$$

Substituting into Equation (G.1A):

$$\tau^2 = a^2 + 2a(x - a) = 2ax - a^2 \qquad (G.3)$$

Using the Pythagorean Theorum:

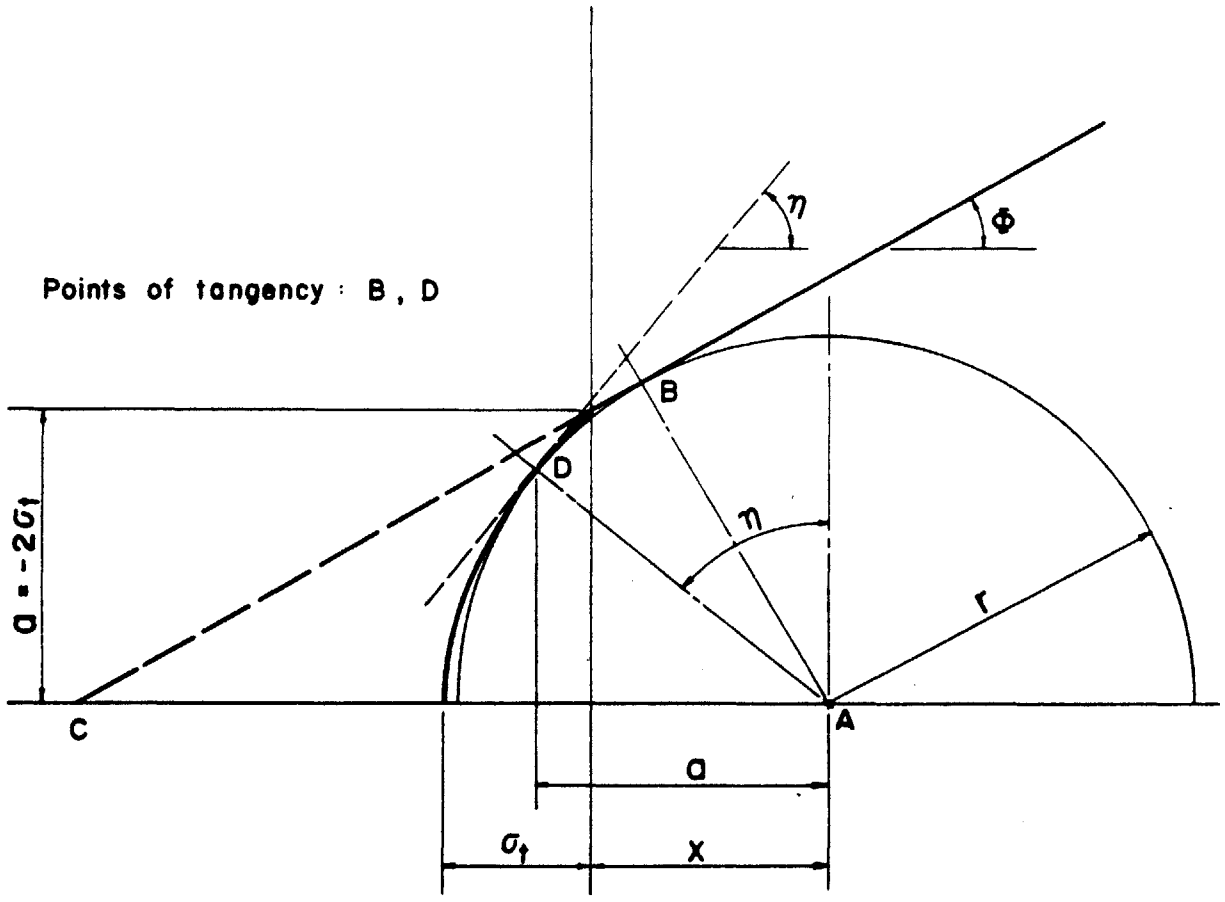


Figure G.4 Circle With Dual Tangency

$$r^2 = \tau^2 + (x - \sigma_n)^2 \qquad (G.4)$$

or

$$r^2 = \tau^2 + a^2 \qquad (G.4A)$$

Combining Equations (G.4A) and (G.3):

$$r^2 = 2ax - a^2 + a^2 = 2ax \qquad (G.5)$$

r and x are also related to the parameters defining the linear envelope:

Examining the triangle ABC:

$$r = \sin \Phi \, (x + a/\tan \Phi) \qquad (G.6)$$

$$r^2 = \sin^2 \Phi \, (x + a/\tan \Phi)^2 \qquad (G.6A)$$

Combining Equations (G.5) and (G.6A):

$$2ax = \sin^2 \Phi \, (x + a/\tan \Phi)^2 \qquad (G.7)$$

Equation (G.7) is a quadratic equation which can be solved for x:

$$x = \frac{a[(1 - \sin \Phi \cos \Phi) - (1 - 2 \sin \Phi \cos \Phi)^{\frac{1}{2}}]}{\sin^2 \Phi} \qquad (G.8)$$

The γ value for the circle with dual tangency has particular interest and is defined as η.

From Figure G.4:

$$\sin \eta = \frac{a}{r} \tag{G.9}$$

From Equation (G.5):

$$r = (2ax)^{\frac{1}{2}} \tag{G.5A}$$

Substituting for r in Equation (G.9):

$$\sin \eta = \frac{a}{(2ax)^{\frac{1}{2}}} \tag{G.10}$$

$$= \left(\frac{a}{2x}\right)^{\frac{1}{2}} \tag{G.10A}$$

Φ and η are uniquely related:

Φ	η
25°	54.93°
30°	52.05°
35°	49.49°
40°	47.16°
45°	45.00°

G.1.2 $\Phi > 45°$

For $\Phi > 45°$, the failure envelope has a cusp (Figure G.5). As indicated in the figure there is a family of failure circles which touch the envelope at the point (0,a). The family includes all circles whose centers lie between the points x_p and x_L

Φ	x_P^*	x_L^*
45°	1.0	1.0
50°	1.0	1.192
55°	1.0	1.428
60°	1.0	1.732
65°	1.0	2.145
70°	1.0	2.747

*x_P and x_L are expressed as multiples of a.

G.2 Classes of Failure Circles

All failure circles fall within one of four generic classes:

1. Circles tangent to the linear envelope.
2. Circles tangent to the parabolic envelope ($\gamma < 90°$)
3. Circles tangent to the parabolic envelope at (σ_t,0), i.e., $\gamma = 90°$.
4. Circles touching the cusp.

Class 4 is applicable only when $\phi > 45°$. Circles with dual tangency are exceptional cases in that they qualify for

both Classes 1 and 2.

The succeeding sections will examine each of the classes and derive general solutions to the fundamental problem:

Given: θ (the inclination of the failure surface)

σ_v (vertical stress)

Find: x, r (center and radius of the failure circle)

G.2.1 Circles Tangent to the Linear Envelope

Figure G.6 shows a failure circle with $45 + \Phi/2 \leq \theta \leq 90°$. From triangle ABC:

$$r = (a \text{ ctn } \Phi + x) \sin \Phi \tag{G.11}$$

From triangle ADE:

$$r^2 - r^2(\sin^2(2\theta - 90 - \Phi)) = (\sigma_v - x)^2 \tag{G.12}$$

$$r^2(1 - \sin^2(2\theta - 90° - \Phi)) = (\sigma_v - x)^2 \tag{G.12A}$$

$$(1 - \sin^2(2\theta - 90 - \Phi)) = \cos^2(2\theta - 90 - \Phi)) = \sin^2(2\theta - \Phi) \tag{G.13}$$

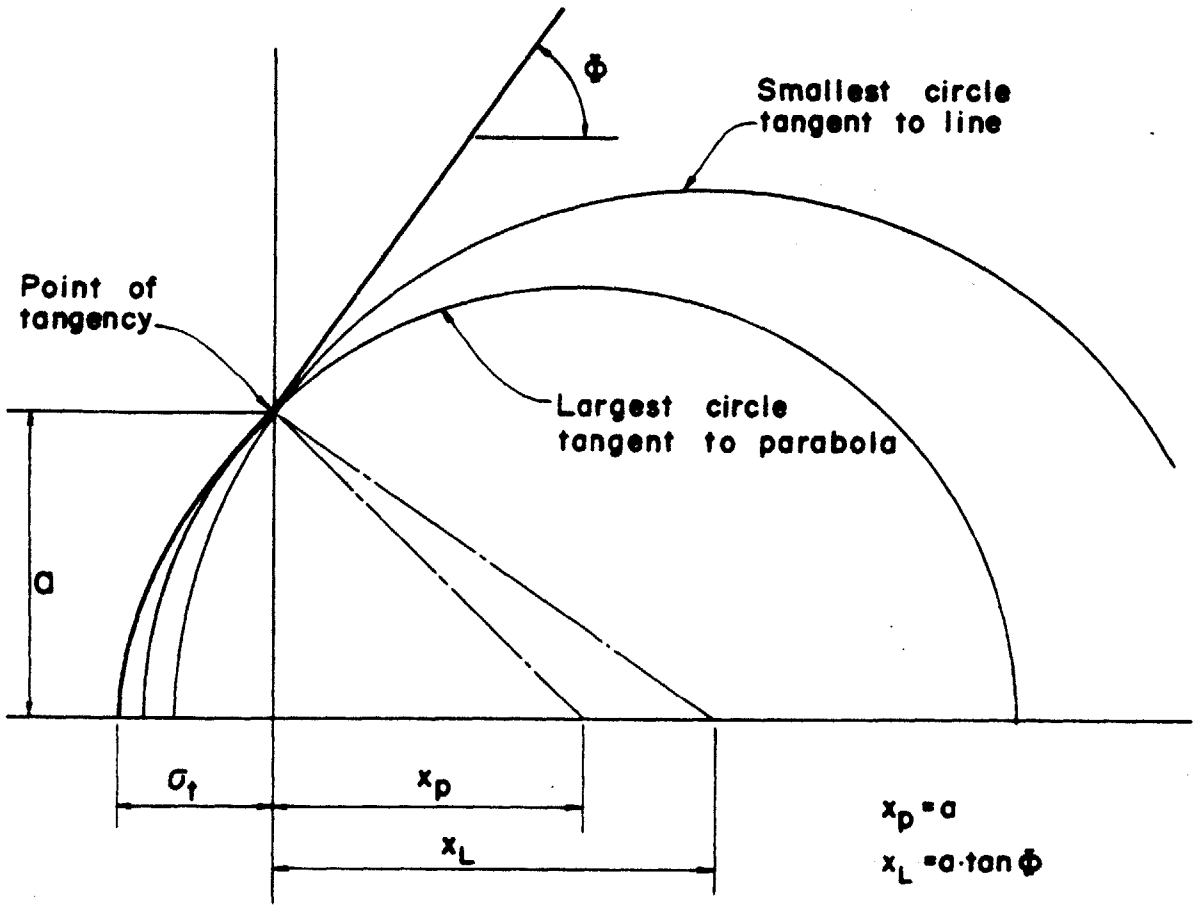


Figure G.5 Failure Envelope With a Cusp

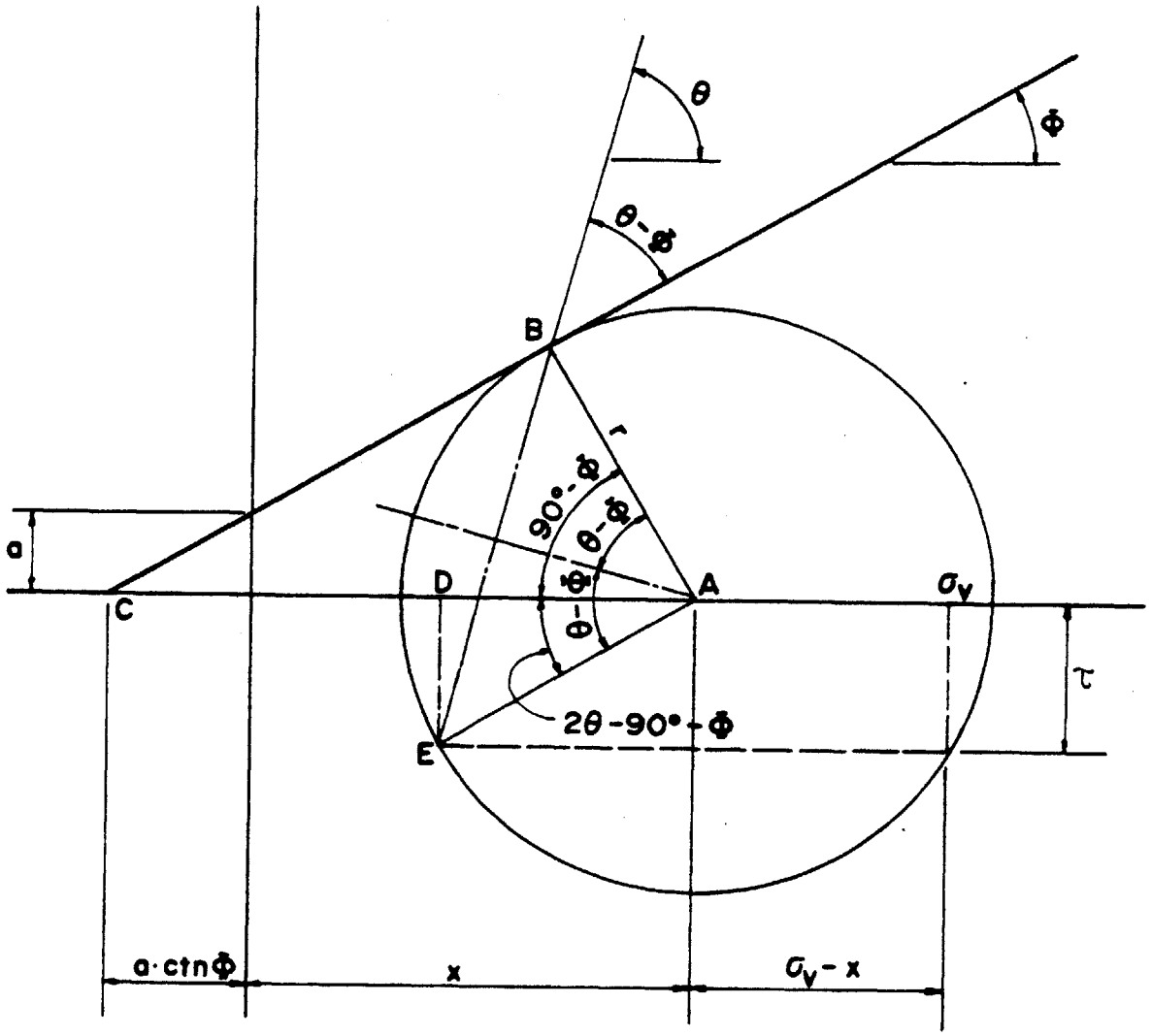


Figure G.6 Failure Circle Tangent to Linear Envelope

Letting $\lambda = \sin(2\theta - \Phi)$ and substituting into (G.12A):

$$\lambda^2 r^2 = (\sigma_v - x)^2 \qquad \text{(G.14)}$$

$$\lambda r = \sigma_v - x \qquad \text{(G.14A)}$$

Combining Equations (G.11) and (G.14A):

$$\lambda(a \operatorname{ctn} \Phi + x) \sin \Phi = \sigma_v - x \qquad \text{(G.15)}$$

Finally,

$$x = \frac{\sigma_v - \lambda a \cos \Phi}{1 + \lambda \sin \Phi} \qquad \text{(G.15A)}$$

$$\tau^2 = r^2 + (\sigma_v - x)^2 \qquad \text{(G.16)}$$

A similar analysis can be performed for other ranges of θ. In all cases, the solution has the same general form as Equation (G.16); however, the expression for λ varies.

range of θ	λ	sign of τ*
$0 \leq \theta \leq 45 + \Phi/2$	$\sin(2\theta - \Phi)$	+
$45 + \Phi/2 \leq \theta \leq 90$	$\sin(2\theta - \Phi)$	-
$90 \leq \theta \leq 135 - \Phi/2$	$-\sin(2\theta + \Phi)$	+
$135 - \Phi/2 \leq \theta \leq 180$	$-\sin(2\theta + \Phi)$	-

G.2.2 Circles Tangent to the Parabolic Envelope at $(\sigma_t, 0)$; $\gamma = 90°$

Figure G.7 shows a typical failure circle tangent to the envelope at $(\sigma_t, 0)$. The geometric relations shown in Figure G.7 and the equations derived below are valid for θ between 0° and 45°.

From triangle ABC:

$$2r = \frac{\tau}{\sin\theta \cos\theta} \tag{G.17}$$

$$\text{diameter of circle} = \sigma_1 + \frac{a}{2} = \frac{\tau}{\sin\theta \cos\theta} \tag{G.18}$$

From Figure G.7:

$$x - \sigma_v = \frac{\tau}{\tan\theta} - \frac{\tau}{2 \sin\theta \cos\theta} = \frac{\tau}{2 \sin\theta \cos\theta} - \frac{a}{2} - \sigma_v \tag{G.19}$$

* A positive τ implies shear stresses on horizontal planes that tend to rotate an element counterclockwise.

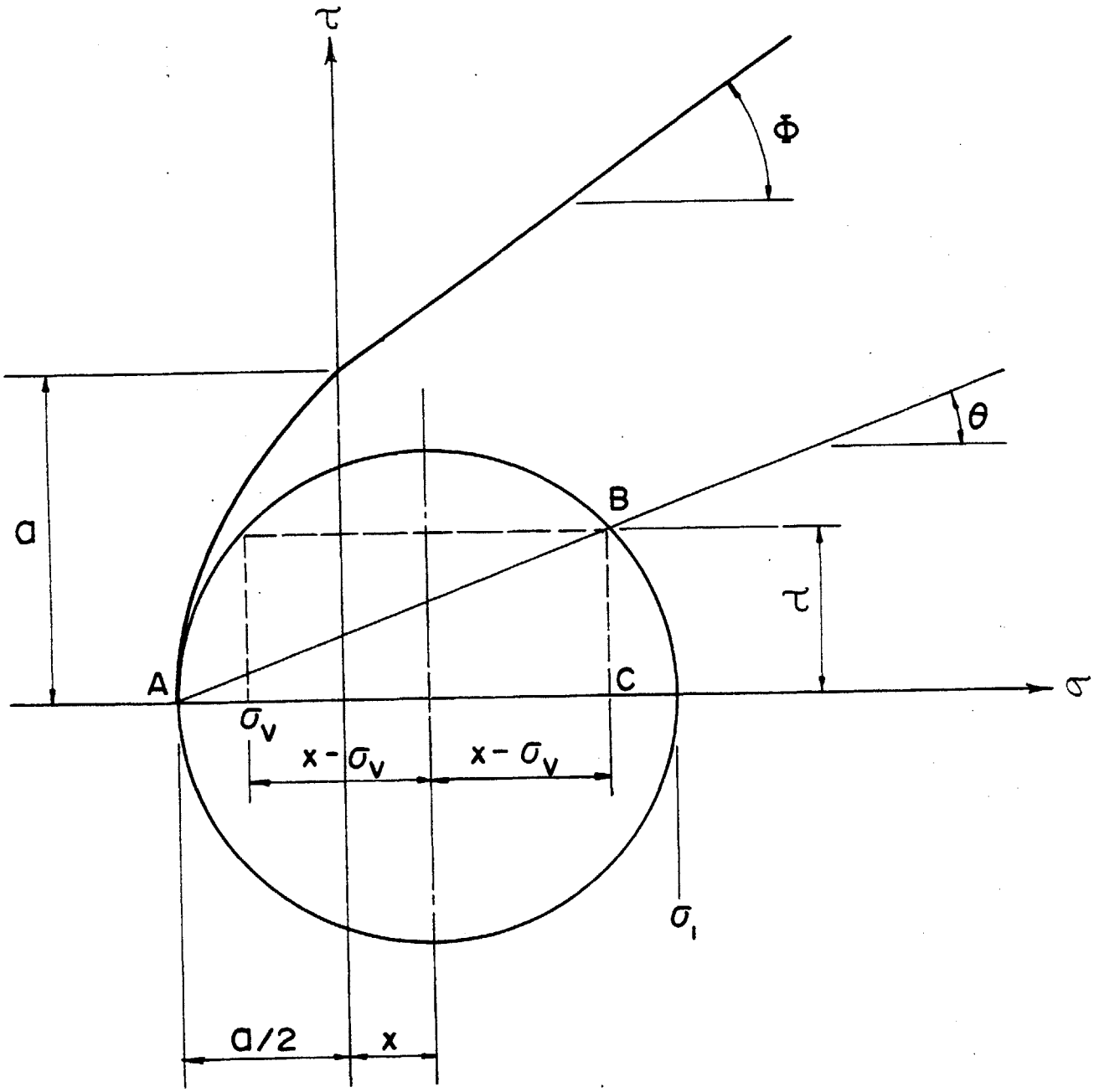


Figure G.7 Failure Circle Tangent to Parabolic Envelope at $\sigma = -a/2$ and $\tau = 0$

$$\tau = \frac{\frac{a}{2} + \sigma_v}{\left(\frac{1}{\sin\theta \cos\theta} - \frac{1}{\tan\theta}\right)} \qquad (G.20)$$

$$r = x + \frac{a}{2} = \frac{\tau}{2 \sin\theta \cos\theta} \qquad (G.21)$$

Substituting τ from Equation (G.20) into Equation (G.21):

$$x + \frac{a}{2} = \frac{1}{2 \sin\theta \cos\theta} \frac{\frac{a}{2} + \sigma_v}{\left(\frac{1}{\sin\theta \cos\theta} - \frac{1}{\tan\theta}\right)} \qquad (G.22)$$

$$x + \frac{a}{2} = \frac{\frac{a}{2} + \sigma_v}{2 \sin^2\theta} \qquad (G.22A)$$

Finally,

$$x = \frac{1}{2}\left(\frac{\frac{a}{2} + \sigma_v}{\sin^2\theta} - a\right) \qquad (G.23)$$

A similar analysis can be performed for other ranges of θ. The results indicate that Equation (G.23) is valid for all values of θ between 0° and 180°. The general expression for τ, the shear stress on a horizontal plane is

$$\tau = \frac{\sigma_v + \frac{a}{2}}{\left(\frac{1}{|\sin\theta \cos\theta|} - \frac{1}{|\tan\theta|}\right)} \qquad (G.24)$$

Range of θ	Sign of τ
$0 \leq \theta \leq 90°$	+
$90 \leq \theta \leq 180°$	-

G.2.3 Circles Tangent to the Parabolic Envelope $\eta < \gamma < 90°$

There is no closed form solution to relate x to θ and σ_v when the circle is tangent to the parabola. The problem must be solved by using iterative techniques. Various points of tangency and their corresponding failure circles are examined. For a given θ value one can calculate the "σ_v" associated with each circle. The circle whose "σ_v" corresponds to the true σ_v is the requisite circle. The convergence on the solution is very rapid and a good ($\pm$ 0.1%) approximation can be found within 5 trials.

The parameter γ is a convenient way of identifying the trial circles. As noted earlier, $\eta < \gamma < 90°$. Once a trial value for γ is chosen the problem reduces to an analysis involving a circle tangent to a straight line whose slope is tan γ. This analysis was discussed in Section G.2.1. The relations used to obtain the results below are borrowed in part from that section with the modification that η replaces Φ and that the unknown is "σ_v" rather than x.

From Figure G.6 and as derived in Section G.2.1:

$$r^2 - r^2(\sin^2 (2\theta - 90 - \Phi))^{\frac{1}{2}} = (\sigma_v - x)^2 \qquad \text{(G.12)}$$

$$r \sin (2\theta - \phi) = \sigma_v - x \qquad \text{(G.14A)}$$

Substituting γ for Φ and "σ_v" for σ_v,

$$r\sin(2\theta - \gamma) = "\sigma_v" - x \qquad (G.25)$$

From Figure G.4:

$$\sin\gamma = \frac{a}{r} \qquad (G.26)$$

$$r = a\csc\gamma \qquad (G.26A)$$

As shown earlier,

$$r^2 = 2ax \qquad (G.5)$$

$$\sin^2\gamma = \frac{a^2}{r^2} = \frac{a^2}{2ax} = \frac{a}{2x} \qquad (G.27)$$

Solving for x:

$$x = \frac{a}{2\sin^2\gamma} = \frac{a\csc^2\gamma}{2} \qquad (G.28)$$

Substituting Equations (G.26A) and (G.28) into Equation (G.25):

$$a\csc\gamma\sin(2\theta - \gamma) = "\sigma_v" - \frac{a\csc^2\gamma}{2} \qquad (G.29)$$

Rearranging terms:

$$"\sigma_v" = \frac{a}{2} \csc^2 \gamma + a \csc \gamma \sin (2\theta - \gamma) \qquad \text{(G.29A)}$$

Equation (G.29A) is valid for θ between 0 and 90°. Equation (G.30) is valid for 0 between 90° and 180°.

$$"\sigma_v" = \frac{a}{2} \csc^2 \gamma - a \csc \gamma \sin (2\theta + \gamma) \qquad \text{(G.30)}$$

Once the proper γ is found, all the remaining properties of the failure circle can be found. Equation (G.26) can be used to find r while (G.28) will yield x. τ is a function of r:

$$\tau^2 = r^2 + a^2 \qquad \text{(G.31)}$$

$$\tau = (r^2 + a^2)^{\frac{1}{2}} \qquad \text{(G.31A)}$$

The sign of τ depends on θ and γ:

θ	Sign of γ
$0 \leq \theta \leq 45 + \gamma/2$	+
$45 + \gamma/2 \leq \theta \leq 90$	-
$90 \leq \theta \leq 135 - \gamma/2$	+
$135 - \gamma/2 \leq \theta \leq 180$	-

G.2.4 Circle Touching the Cusp

If the requisite failure circle touches the cusp its center must lie between the limits x_p and x_L discussed in Section G.2.1. The solution requires iterative techniques. Numerous circles must be examined to find the one with the proper "σ_v." Figure G.8 shows a typical trial circle. Once a value for x is chosen the procedures developed for linear envelopes are applicable. The parameter ω replaces Φ of the linear envelope solution.

$$\tan \omega = x/a \tag{G.32}$$

Since x can vary from a to a·tan Φ, ω will vary from 45° to Φ. All the equations of Section G.2.1 are valid:

$$x = \frac{(\text{"}\sigma_v\text{"} - a\lambda \cos \Phi)}{(1 + \lambda \sin \Phi)} \tag{G.15A}$$

Substituting ω for Φ and rearranging the equations to reflect the fact that "σ_v" rather than x is the unknown:

$$\text{"}\sigma_v\text{"} = a\lambda \cos \omega + x(1 + \lambda \sin \omega) \tag{G.33}$$

θ	λ
$0° < \theta \leq 90°$	$\sin (2\theta - \omega)$
$90° \leq \theta \leq 180°$	$-\sin (2\theta + \omega)$

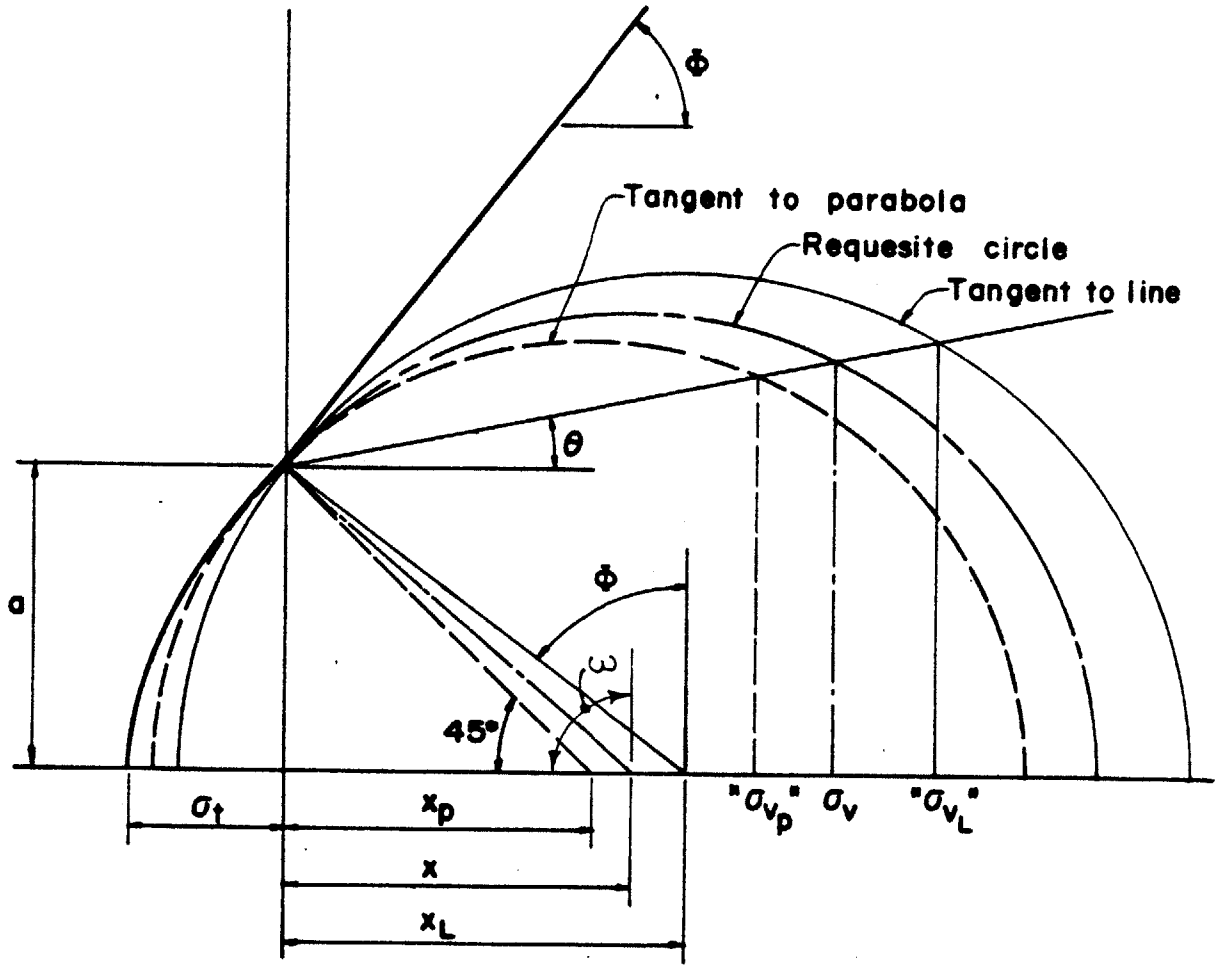


Figure G.8 Failure Circle Touching Cusp

Once the proper ω is obtained r can be calculated.

$$r = (a \operatorname{ctn} \omega + x) \sin \omega \qquad (G.34)$$

G.3 General Solution Procedure

As indicated in Section G.1, the most general form of the problem is:

Given: $a, \Phi, \sigma_v, \theta$

Find: x and r (or x and τ)

One typically does not know a priori which part of the envelope the requisite circle touches. Part of the solution is to determine into which of the four categories listed in Section G.2 the problem falls.

The solution procedure outlined below provides a logical means of identifying the critical circle based solely on the "given" information described above.

**** For $\phi \leq 45^{\circ}$ ****

Step 1: Find the circle with dual tangency.

x_η = center of circle with dual tangency

$$x_\eta = \frac{a[(1 - \sin \Phi \cos \Phi) - (1 - 2 \sin \Phi \cos \Phi)^{\frac{1}{2}}]}{\sin^2 \Phi} \qquad (G.35)$$

$$\eta = \sin^{-1} \left(\frac{a}{2x_\eta}\right)^{\frac{1}{2}} \qquad \text{(G.36)}$$

Step 2: Compute "σ_v" for the circle with dual tangency. The equations are similar to Equations (G.29) and (G.30) except η replaces γ.

$$"\sigma_v" = (a/2) \csc^2 \eta - a\delta \csc \eta \qquad \text{(G.37)}$$

θ	δ
$0 \leq \theta \leq 90$	$\sin (2\theta - \eta)$
$90 \leq \theta < 180$	$-\sin (2\theta + \eta)$

Step 3: If $\sigma_v > "\sigma_v"$ the requisite circle must be tangent to the linear envelope. Therefore, go to Step 6.

Step 4: Examine circles tangent to the parabola at $x = -a/2$; $y = 0$.

$$x = 1/2 \left(\frac{\sigma_v + \frac{a}{2}}{\sin^2 \theta} - a\right) \qquad \text{(G.38)}$$

Compute the maximum principal stress for that circle (see Figure G.7).

$$\sigma_1 = x + (x + \tfrac{a}{2}) = 2x + \tfrac{a}{2} \qquad \text{(G. 39)}$$

If $\sigma_1 \leq 1.5\ a$, the circle centered at x is the requisite circle. τ can be found from Equation (G.24).

If $\sigma_1 > 1.5\ a$, go to Step 5.

Step 5: The requisite circle must be tangent to the parabola with $\eta < \gamma < 90°$. Use the iterative technique described in Section G.2.3 to find γ and x.

Step 6: The requisite circle is tangent to the linear envelope.

$$x = \frac{(\sigma_v - a\lambda \cos \Phi)}{(1 + \lambda \sin \Phi)} \qquad \text{(G.40)}$$

θ	λ
$0 < \theta \leq 90°$	$\sin (2\theta - \Phi)$
$90 \leq \theta \leq 180°$	$-\sin (2\theta + \Phi)$

$$r = (a \operatorname{ctn} \Phi + x) \sin \Phi \qquad \text{(G.6)}$$

**** For $\Phi > 45°$ ****

Step 1: Compute "σ_{v_p}" for the largest circle tangent to the parabola. The circle has a γ of 45°.

$$"\sigma_{v_p}" = \frac{a}{2} \csc^2 45° - \delta a \csc 45° \qquad \text{(G.41)}$$

$$= a - (1.414)a\delta \qquad \text{(G.41A)}$$

θ	δ
$0 < \theta \leq 90$	$\sin(2\theta - 45)$
$90 \leq \theta < 180$	$-\sin(2\theta + 45)$

Compute the "σ_{v_L}" for the smallest circle tangent to the straight line. Use Equation (G.33) with $\omega = \Phi$.

$$x = a \cdot \tan \Phi \qquad \text{(G.42)}$$

$$"\sigma_{v_L}" = a\lambda \cos \Phi + x(1 + \lambda \sin \Phi) \qquad \text{(G.43)}$$

θ	λ
$0 < \theta < 90$	$\sin(2\theta - \Phi)$
$90 \leq \theta \leq 180$	$-\sin(2\theta + \Phi)$

Step 2: If $\sigma_v >$ "σ_{v_L}" the requisite circle must be tangent to the linear envelope. Go to Step 6 described for $\phi \leq 45°$. If $\sigma_v <$ "σ_{v_p}" the requisite circle must be tangent to the parabola. Go to Step 4 described for $\phi \leq 45°$. If "σ_{v_p}" $\leq \sigma_v \leq$ "σ_{v_L}", the requisite circle must touch the cusp. Use the iterative technique described in Section G.2.3 to find x and r.

G.4 Failure Along Joints

Failure along joints is easier to analyze than failure through intact rock. Continuous joints cannot maintain a tensile stress so the failure envelope is defined solely in the compression region. The failure envelope is assumed to be linear and is characterized by a friction angle, Φ_j, and a cohesion, c_j (Figure G.9). The problem is to find the failure circle whose failure plane is oriented at the proper θ angle. For joints, θ equals α (Figure G.9). The requisite circle is found by reducing the horizontal stress while holding the vertical stress (σ_v) constant. (Unlike the intact rock, no shear stresses need to be introduced in the horizontal and vertical planes.) The problems can be stated as:

Given: $\Phi_j, c_j, \theta = \alpha, \sigma_v = \sigma_{v_o}$

Find: the failure circle.

Figure G.10 shows a typical failure circle.

$$u + v + w = c \text{ ctn } \Phi_j + \sigma_v \qquad (G.44)$$

Examining the line AB, one can express its length two ways:

$$A - B = w\tan(90 - \theta) \qquad (G.45)$$

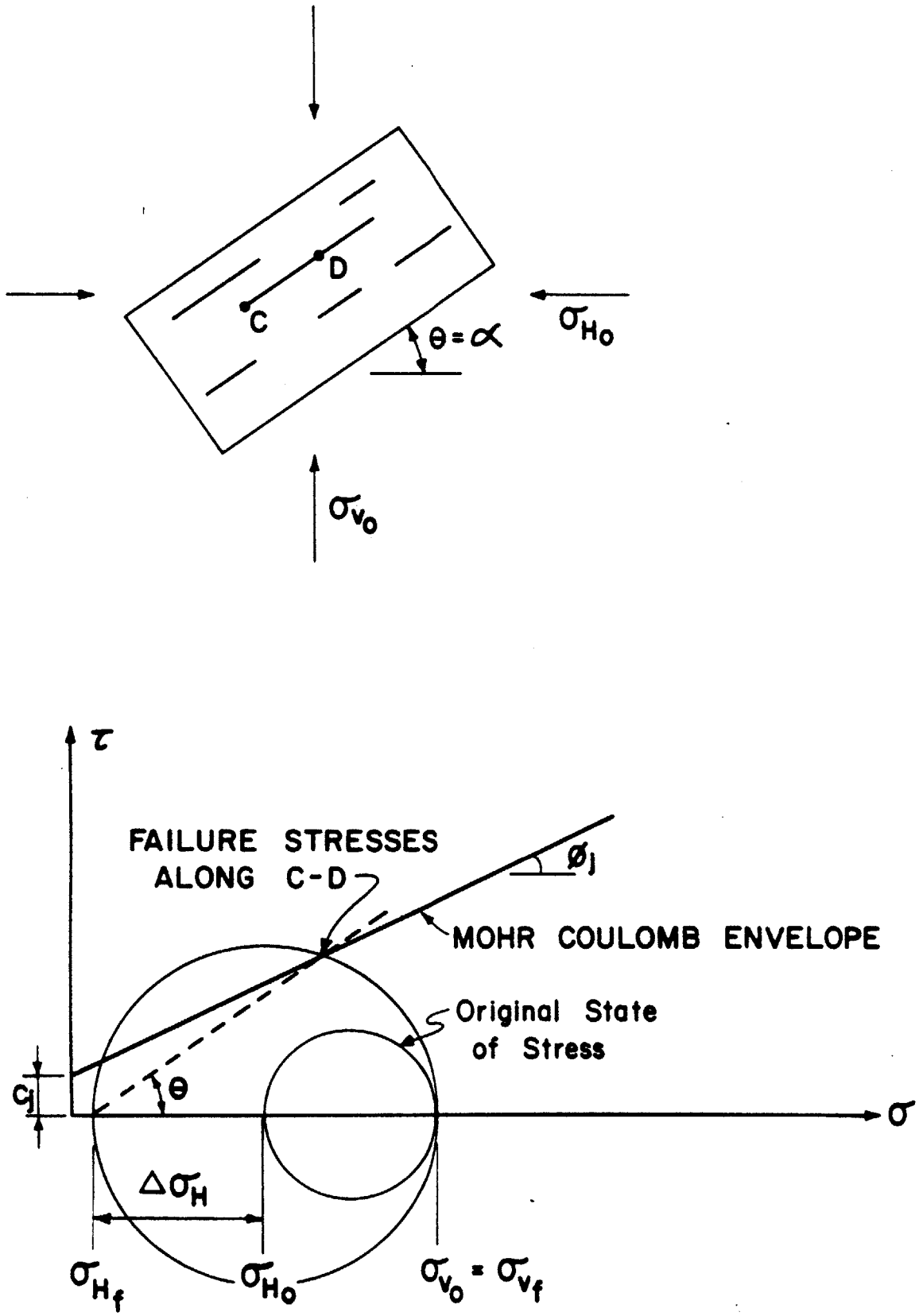


Figure G.9 Failure Conditions--Joint

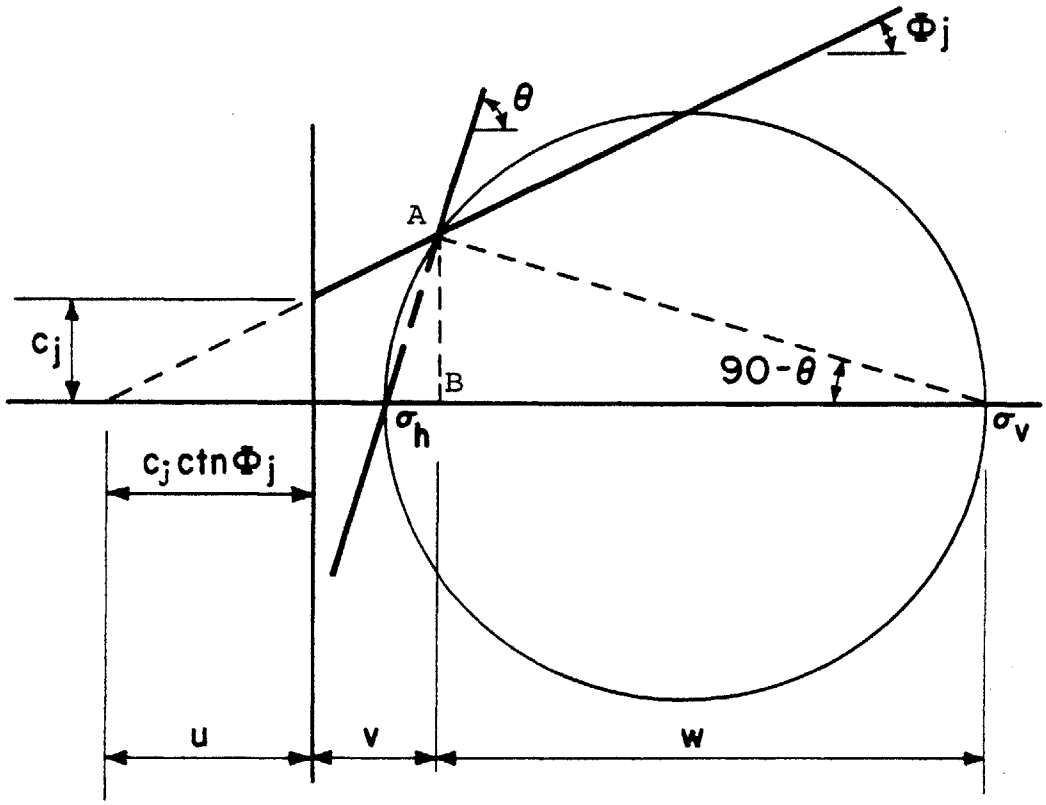


Figure G.10 Typical Failure Circle for Joint

$$A - B = (c_j \text{ ctn } \Phi_j + \sigma_v - w) \tan \Phi_j \qquad (G.46)$$

Combining Equations (G.45) and (G.46) and solving for w:

$$w = \frac{(c_j \text{ ctn } \Phi_j + \sigma_v) \tan \Phi_j}{\tan (90 - \theta) + \tan \Phi_j} \qquad (G.47)$$

$$\sigma_v - \sigma_h = \text{diameter} = \frac{w}{\cos^2 (90 - \theta)} \qquad (G.48)$$

Substituting w into Equation (G.48):

$$(\sigma_v - \sigma_h) \cos^2 (90 - \theta) = \frac{(c_j \text{ ctn } \Phi_j + \sigma_v) \tan \Phi_j}{\tan (90 - \theta) + \tan \Phi_j} \qquad (G.49)$$

Rearranging terms:

$$\sigma_h = \sigma_v - \frac{(c_j \text{ ctn } \Phi_j + \sigma_v) \tan \Phi_j}{(\tan (90 - \theta) + \tan \Phi_j) \cos^2 (90 - \theta)} \qquad (G.50)$$

Equation (G.50) is valid for all values of θ between 0° and 90°.

Appendix H

Computer Program JOINTSIM

JOINTSIM (Joint Simulation) is designed to assess the strength of a jointed rock mass. The geometry of the jointing pattern is treated as a random variable. The results of the analysis are expressed in terms of the mean and standard deviation of an "apparent persistence" parameter. The program simulates a specified number of jointing patterns and calculates the minimum strength of each pattern. Jointing is modeled as a multi-dimensional Poisson process that can be characterized by three parameters: mean joint plane spacing, mean joint length, mean length of rock bridges. The dimensions and orientation of the block, the strength parameters of the joint and intact rock and the in situ stress field are deterministic inputs.

H.1 Coordinate System

JOINTSIM uses a coordinate system as shown in Figure H.1. The origin is located at the upper right corner of the block. α is the inclination angle of the joints.

H.2 Units

All units used in the input must be consistent. Units are generally in feet or pounds, but any other consistent system may be employed.

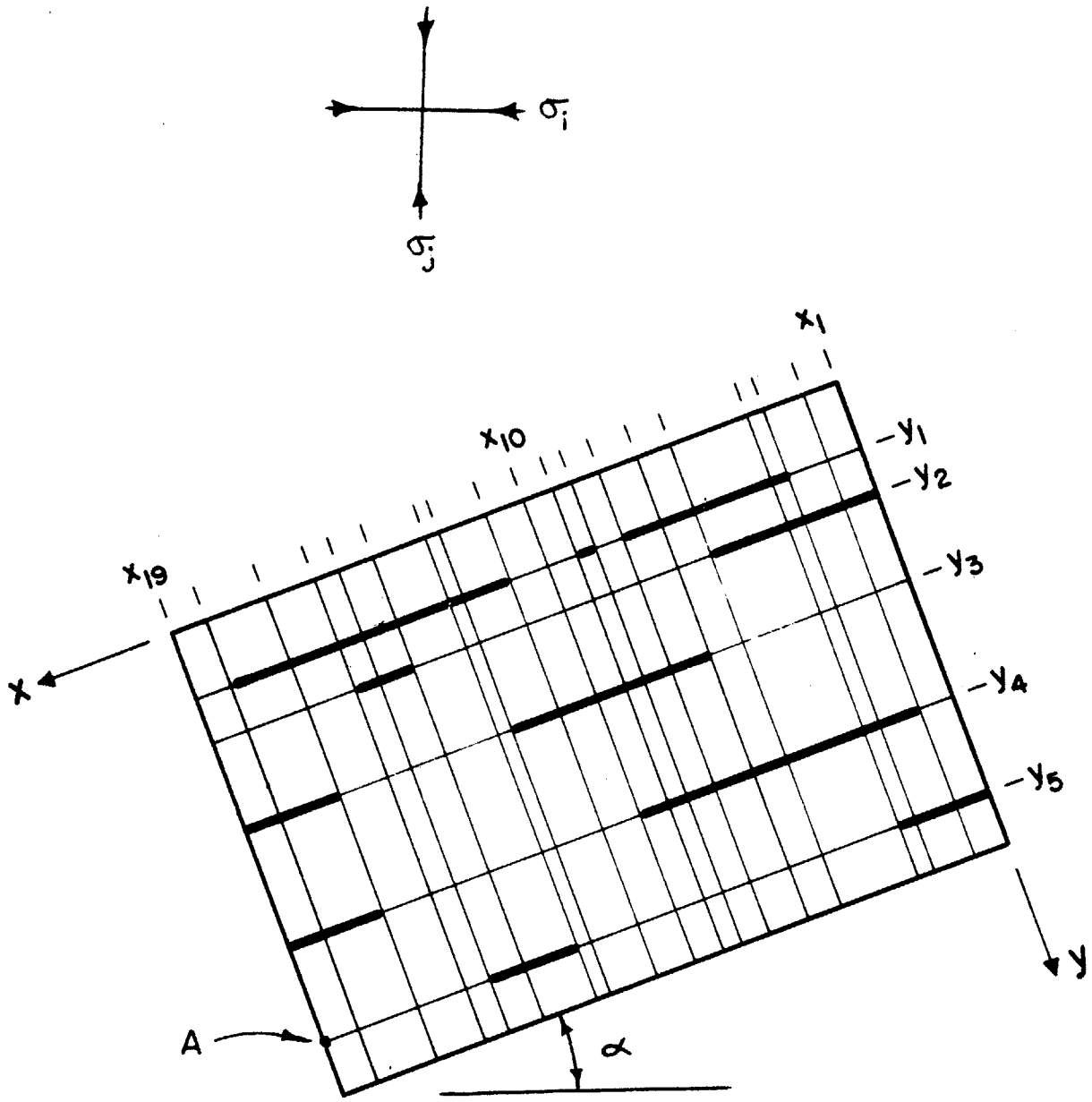


Figure H.1 Coordinate System for JOINTSIM

H.3 Inputs

Unless otherwise noted the inputs do not have to be right justified in their respective columns. The term "integer" implies the number is expressed without a decimal point.

Card #1:

Width of Block (XDIM) is entered in Columns 1 through 10 with a maximum of two decimal places.

Height of Block (YDIM) is entered in Columns 11 through 20 with a maximum of two decimal places.

Inclination of Block (ALPHA) is entered in Columns 21 through 30 with a maximum of two decimal places. The angle is expressed in degrees.

Card #2:

Joint Friction Angle (PHIJT) is entered in Columns 1 through 10 with a maximum of two decimal places. The angle is expressed in degrees.

Joint Cohesion (COJT) is entered in Columns 11 through 20 with a maximum of two decimal places.

Intact Rock Friction Angle (PHIRK) is entered in Columns 21 through 30 with a maximum of two decimal places. The angle is expressed in degrees.

Intact Rock Cohesion (CORK) is entered in Columns 31 through 40 with a maximum of two decimal places. In the 'Griffith-Modified Griffith' envelope used by the program the cohesion is twice the tensile strength of the rock.

Card #3:

Initial Vertical Stress (SIG1) is entered in Columns 1 through 10 with a maximum of 2 decimal places.

Initial Horizontal Stress (SIG3) is entered in Columns 11 through 20 with a maximum of 2 decimal places.

NOTE: The program assumes that the initial principal stresses are in the horizontal and vertical directions (see Figure H.1). The major principal stress can be in either direction.

Card #4:

Mean Joint Spacing (SP3) is entered in Columns 1 through 10 with a maximum of two decimal places.

Mean Joint Length (SPJTLN) is entered in Columns 11 through 20 with a maximum of two decimal places.

Mean Length of Rock Bridges (SPRKBR) is entered in Columns 21 through 30 with a maximum of two decimal places.

Card #5:

This card designates which technique or techniques should be used to evaluate the results. The user can select any one (or all) of three different methods.

Method #1 (Enter '-1' in Columns 9 and 10). The results are expressed in terms of the percent continuity (or persistence) required on a hypothetical joint if that joint were to have the same resistance as the weakest path

through the rock mass. The hypothetical joint is inclined at the same angle as the real joint planes.

Method #2: (Enter '0' in Column 10). This method also computes the persistence of a hypothetical joint with the same minimum resistance as the rock mass. However, the hypothetical joint is inclined at the same angle as the net failure path.

Method #3: (Enter '1' in Column 10). This method is based on the concept that the jointed rock mass can be treated as a homogenous medium whose strength parameters can be related to those of the intact rock through a reduction factor. This material has the same friction angle as the intact rock. Method #3 computes the ratio of cohesion of homogeneous material to cohesion of intact rock in order for the homogeneous material to have the same minimum resistance as the jointed rock mass.

NOTE: The program will compute the results according to all three methods if any positive integer greater than 1 is entered in Columns 1 through 10.

Card #6:

Seed Number (ISEED) for the routine to generate random numbers is entered in Columns 1 through 10. The number can be chosen arbitrarily but must be an integer less than 2147473647.

Card #7:

This card controls the amount of information given in the output. The user must enter 3 integers in Columns 1 through 10, 11 through 20 and 21 through 30 respectively. In deciding which options to use, the user should refer to the sample output included in this appendix.

Integer #1 (NOTPOP): If a 1 is entered in Column 10 the program will print the coordinates of the left and right ends of each joint segment and a sequential listing of the X coordinates. (See "Section A1" and Section A2" in the sample output.) None of this information will be printed if any other number or numbers are printed in Columns 1 through 10. (Blanks in Columns 1 through 10 will also suppress the information.)

Integer #2 (NOTPOT): If a 2 is entered in Column 20 the program will print the detailed strength data labeled "Section B1" , "Section B2" and "Section B3" in the sample output. "Section B1" lists the strengths for all points along the row y = 1 while "Section B3" details the location of the path with the minimum strength. "Section B2" is a summary of all possible failure paths that originate from the point at the lower left corner of the joint grid. (See Point A in Figure H.1). If a 1 is entered in Column 20 the program will print failure path summaries similar to

"Section B2" for all points in the joint grid. None of this information will be printed if any other number or numbers are printed in Columns 11 through 20. (Blanks in Columns 11 through 20 will also suppress the information.)

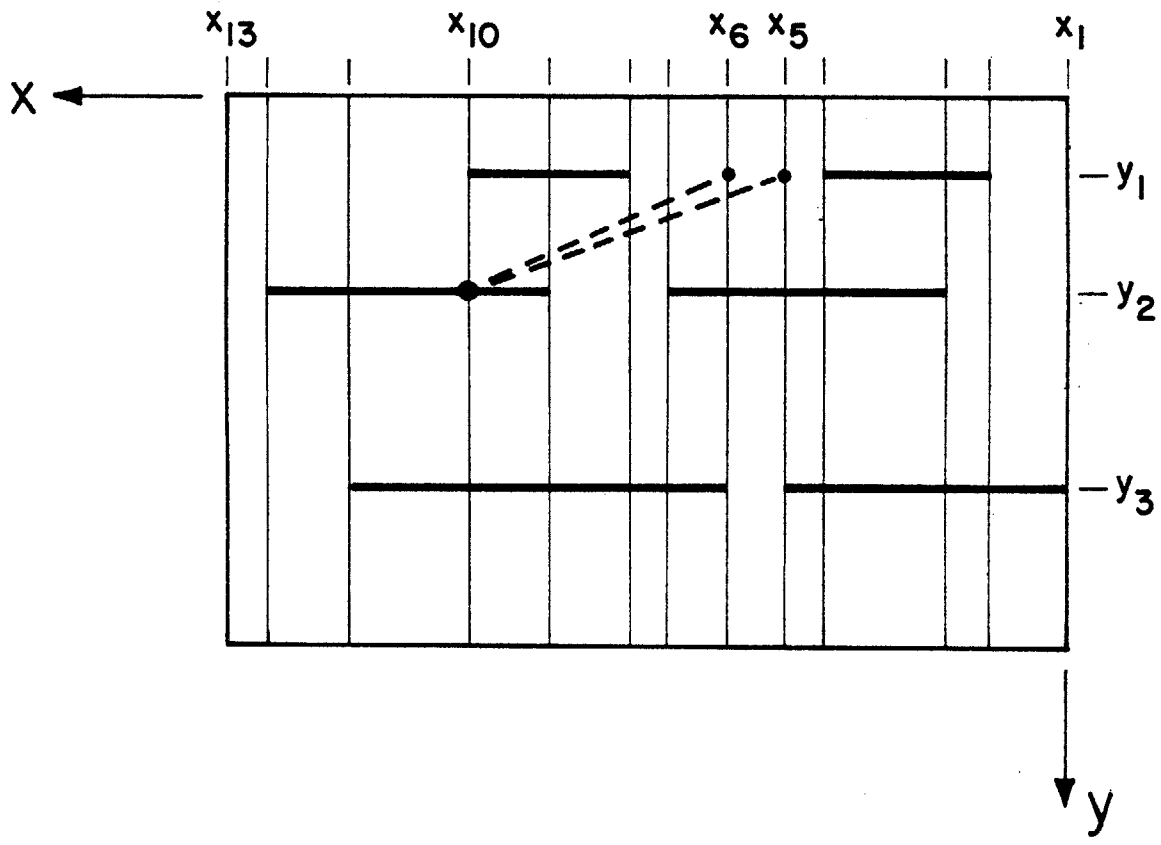
Integer #3 (NOTPOD): Serves as a safety mechanism to eliminate undesired output. The volume of output can be enormous if the number of realizations (see card #8) is large and Integers #1 and #2 are both 1. A 1 entered in Column 30 will print all information as prescribed by Integer #1 and Integer #2. None of the information will be printed if any other number or numbers is printed in Columns 21 through 30. (Blanks in Columns 21 through 30 will also suppress the information.)

Card #8:

Number of Realizations (NOREAL) may range from 1 to 400. This integer must be right justified in Columns 8, 9, and 10. A minimum of 200 realizations is usually required to get a good estimate of the mean and standard deviation of apparent persistence.

The next two parameters (DMIN and NJUMP) are used to reduce the number of failure paths the program must examine at each grid point.

Minimum Distance (DMIN) is entered in Columns 11 through 20 with a maximum of 2 decimal places. The program will not



(y_2, x_{10}) : Point Under Consideration

Program will not examine path to

(y_1, x_6) unless $x_6 - x_5 > DMIN$

Figure H.2 Definition of DMIN

examine a failure path whose transition point is within DMIN of the previous transition point (Figure H.2). If, however, the point happens to be the left or right end of a joint segment the program will check it anyway. If Columns 11 through 20 are left blank the program will default to DMIN = 0.00001 and check essentially every point.

Maximum Joint Plane Transition (NJUMP) is entered as an integer in Column 30. The program will not examine any failure paths that jump more than NJUMP planes in one transition. If Column 30 is left blank the program will check failure paths with transitions to all the eligible joint planes.

Sample Output

THE DIMENSIONS OF THE BLOCK ARE 10.00 HIGH BY 20.00 WIDE

THE INCLINATION OF THE JOINTS IS 40.0 DEGREES

STRENGTH PARAMETERS:

PHI (JOINT) =	30.00	DEGREES
COHESION (JOINT) =	5.00	
PHI (ROCK) =	30.00	DEGREES
COHESION (ROCK) =	50.00	

THE VERTICAL CONFINING STRESS IS	50.00
THE HORIZONTAL CONFINING STRESS IS	50.00

DISTRIBUTIONAL PARAMETERS FOR JOINTS:

MEAN SPACING BETWEEN PLANES =	2.00
MEAN SPACING BETWEEN JOINTS =	5.00
MEAN JOINT LENGTH =	2.50

THE INITIAL RANDOM NUMBER IS	124513
THE NUMBER OF REALIZATIONS IS	1

PATH CRITERIA:

MINIMUM SPACING =	2.00
MAXIMUM TRANSITION =	3

JOINT PLANE	Y COOR	NUMBER OF JOINTS	PERCENT CONTINUITY	RANDOM NUMBERS USED
1	1.06	3	3.4	8
2	6.38	3	42.3	8
3	7.31	4	56.0	10
4	8.66	3	22.5	8

Y COOR IS	1.055	X COOR OF RIGHT END IS	4.110
		X COOR OF LEFT END IS	4.263
Y COOR IS	1.055	X COOR OF RIGHT END IS	9.768
		X COOR OF LEFT END IS	10.232
Y COOR IS	1.055	X COOR OF RIGHT END IS	16.218
		X COOR OF LEFT END IS	16.284
Y COOR IS	6.377	X COOR OF RIGHT END IS	0.000
		X COOR OF LEFT END IS	3.824
Y COOR IS	6.377	X COOR OF RIGHT END IS	11.816
		X COOR OF LEFT END IS	14.368
Y COOR IS	6.377	X COOR OF RIGHT END IS	15.058
		X COOR OF LEFT END IS	17.144
Y COOR IS	7.314	X COOR OF RIGHT END IS	0.000
		X COOR OF LEFT END IS	0.738
Y COOR IS	7.314	X COOR OF RIGHT END IS	5.233
		X COOR OF LEFT END IS	8.289
Y COOR IS	7.314	X COOR OF RIGHT END IS	9.801
		X COOR OF LEFT END IS	13.971
Y COOR IS	7.314	X COOR OF RIGHT END IS	15.914
		X COOR OF LEFT END IS	19.559
Y COOR IS	8.658	X COOR OF RIGHT END IS	3.179
		X COOR OF LEFT END IS	7.390
Y COOR IS	8.658	X COOR OF RIGHT END IS	13.648
		X COOR OF LEFT END IS	13.770
Y COOR IS	8.658	X COOR OF RIGHT END IS	13.897
		X COOR OF LEFT END IS	14.056

*** THE NUMBER OF INDEPENDENT X COORDINATES IS 26 ***

THE ORDERED X COORDINATES:

0.000
0.738
3.179
3.824
4.110
4.263
5.233
7.390
8.289
9.768
9.801
10.232
11.816
13.648
13.770
13.897
13.971
14.056
14.368
15.058
15.914
16.218
16.284
17.144
19.559
20.000

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 0.000 IS 0.00

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 0.738 IS 48.70

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 3.179 IS 209.82

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 3.824 IS 252.36

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 4.110 IS 271.24

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 4.263 IS 276.42

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 5.233 IS 340.48

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 7.390 IS 482.84

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 8.289 IS 542.12

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 9.768 IS 639.75

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 9.801 IS 640.85

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 10.232 IS 655.48

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 11.816 IS 760.00

Section A2

Section B1

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 13.643 IS 880.90

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 13.770 IS 888.97

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 13.897 IS 897.36

THE MINIMUM TOTAL STLNGTH FOR Y COOR = 1.055 AND X COOR = 13.971 IS 902.21

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 14.056 IS 907.84

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 14.368 IS 928.40

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 15.058 IS 973.94

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 15.914 IS 1030.48

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 16.218 IS 1050.51

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 16.284 IS 1052.74

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 17.144 IS 1109.52

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 19.559 IS 1268.89

THE MINIMUM TOTAL STENGTH FOR Y COOR = 1.055 AND X COOR = 20.000 IS 1298.00

POINT CONSIDERED:

JOINT PLANE = 4
Y COORDINATE = 8.658
X COORDINATE = 20.000

PATH NO.	-- JUNCTION POINT -- JT. PL.	Y COOR	X COOR	ANGLE TO JCT.	STRENGTH TO JT PT	STRENGTH TO EDGE	STRENGTH TOTAL	MINOR PRIN. STRESS	TANGENT TO
1	4	8.658	19.559	40.60	29.10	966.21	995.31	-24.54	PARABOLA
2	1	1.055	0.000	60.81	1436.22	0.00	1436.22	-25.00	POINT
3	1	1.055	3.179	64.32	1296.55	209.82	1508.37	-25.00	POINT
4	1	1.055	4.110	65.57	1258.26	271.24	1529.50	-25.00	POINT
5	1	1.055	4.263	65.78	1251.64	276.42	1528.05	-25.00	POINT
6	1	1.055	7.390	71.09	1116.20	482.84	1599.04	-25.00	POINT
7	1	1.055	9.768	76.61	1013.26	639.75	1653.00	-25.00	POINT
8	1	1.055	10.232	77.90	993.14	655.48	1648.62	-25.00	POINT
9	1	1.055	13.648	90.12	845.24	880.90	1726.14	-25.00	POINT
10	2	6.377	0.000	46.51	1340.18	0.00	1340.18	-24.68	PARABOLA
11	2	6.377	3.179	47.72	1132.10	107.67	1239.77	-24.91	PARABOLA
12	2	6.377	3.624	48.03	1090.07	129.50	1219.57	-24.92	PARABOLA
13	2	6.377	7.390	50.25	857.48	364.90	1222.38	-24.96	PARABOLA
14	2	6.377	9.768	52.57	702.99	521.80	1224.79	-24.99	PARABOLA
15	2	6.377	11.816	55.57	569.74	656.99	1226.73	-25.00	PARABOLA
16	2	6.377	13.897	60.49	435.31	727.48	1162.79	-25.00	POINT
17	2	6.377	14.368	62.05	414.95	743.40	1158.35	-25.00	POINT
18	2	6.377	15.058	64.77	385.07	788.95	1174.02	-25.00	POINT
19	2	6.377	17.144	78.61	294.73	859.61	1154.33	-25.00	POINT
20	3	7.314	0.000	43.84	1329.19	0.00	1329.19	-24.78	PARABOLA
21	3	7.314	0.738	43.99	1280.61	24.99	1305.60	-24.79	PARABOLA
22	3	7.314	3.179	44.57	1120.17	183.80	1303.97	-24.81	PARABOLA
23	3	7.314	5.233	45.20	985.14	260.81	1245.95	-24.84	PARABOLA
24	3	7.314	7.390	46.06	843.62	333.87	1177.48	-24.87	PARABOLA
25	3	7.314	8.289	46.55	784.84	364.28	1149.12	-24.88	PARABOLA
26	3	7.314	9.801	47.51	685.94	464.07	1150.01	-24.91	PARABOLA
27	3	7.314	11.816	49.33	554.31	532.33	1086.64	-24.95	PARABOLA
28	3	7.314	13.897	52.42	418.98	602.82	1021.80	-24.98	PARABOLA
29	3	7.314	13.971	52.57	414.22	605.31	1019.53	-24.99	PARABOLA
30	3	7.314	15.914	56.21	287.23	733.57	1020.81	-25.00	PARABOLA
31	3	7.314	19.559	111.83	119.89	857.01	976.90	-25.00	POINT

THE MINIMUM TOTAL STENGTH FOR Y COOR = 8.658 AND X COOR = 20.000 IS 976.90

THE MINIMUM STRENGTH IS: 886.11

HE NUMBER SHOULD BL BETWELN:

946.99 (THE VALUE FOR A 58.0% CONTINUOUS JOINT)
AND 677.35 (THE VALUE FOR A CONTINUOUS JOINT)

THE FAILURE PATH PROCLEDS FROM:

(LEFT END)		Y COOR =	7.314	X COOR =	20.000
	TO	Y COOR =	7.314	X COOR =	19.559
	TO	Y COOR =	7.314	X COOR =	17.144
	TO	Y COOR =	7.314	X COOR =	16.284
	TO	Y COOR =	7.314	X COOR =	16.218
	TO	Y COOR =	7.314	X COOR =	15.914
	TO	Y COOR =	7.314	X COOR =	15.058
	TO	Y COOR =	7.314	X COOR =	14.368
	TO	Y COOR =	7.314	X COOR =	14.056
	TO	Y COOR =	7.314	X COOR =	13.971
	TO	Y COOR =	7.314	X COOR =	13.897
	TO	Y COOR =	7.314	X COOR =	13.770
	TO	Y COOR =	7.314	X COOR =	13.648
	TO	Y COOR =	7.314	X COOR =	11.616
	TO	Y COOR =	7.314	X COOR =	10.232
	TO	Y COOR =	7.314	X COOR =	9.801
	TO	Y COOR =	7.314	X COOR =	9.766
	TO	Y COOR =	7.314	X COOR =	8.289
	TO	Y COOR =	7.314	X COOR =	7.390
	TO	Y COOR =	7.314	X COOR =	5.233
	TO	Y COOR =	6.377	X COOR =	3.824
	TO	Y COOR =	6.377	X COOR =	3.179
	TO	Y COOR =	6.377	X COOR =	0.738
	TO	Y COOR =	6.377	X COOR =	0.000
(RIGHT END)	TO	Y COOR =	6.377	X COOR =	0.000

THE AVERAGE INCLINATION OF THE FAILURE PATH IS 42.68 DEGREES

REALIZATION NUMBER	NO. OF JT PLS	PERSISTENCE MIN.	PERSISTENCE MAX.	ANGLE OF FAIL. PATH	MINIMUM STRENGTH	REDUCTION OPTIONS (1)	(2)	(3)	UPDATED MEAN VALUES (1)	(2)	(3)	TRANSITIONS UNIT::	TOTAL
1	4	3.4	58.0	42.7	886.11	67.5	68.1	29.0	67.5	68.1	29.0	1	1

CODE FOR REDUCTION OPTIONS:

(1) PERCENT CONTINUITY FOR A FAILURE SURFACE INCLINED AT THE SAME ANGLE AS THE JOINTS
(2) PERCENT CONTINUITY FOR A FAILURE SURFACE INCLINED AT THE SAME ANGLE AS THE FAILURE PATH
(3) PERCENT COHESION OF INTACT ROCK FOR A FAILURE SURFACE INCLINED AT THE SAME ANGLE AS THE FAILUREPATH

JOINTSIM Listing

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C                              JOINTSIM                                          JTSM0001
C        THIS PROGRAM EVALUATES THE MINIMUM STRENGTH OF A RANDOMLY GENERATED     JTSM0002
C        JOINTING PATTERN AS DEFINED IN D. VENEZIANO'S MODEL                    JTSM0003
C        PROGRAM LIMIT: 400 REALIZATIONS                                        JTSM0004
      DIMENSION R(400), IIR(400), REALBD(400), PSI(400), JJPL(400),             JTSM0005
     1 APPER1(400), APPER2(400), APPER3(400), PERMIN(400), PERMAX(400),         JTSM0006
     2 AP1MEN(400), AP2MEN(400), AP3MEN(400), STAISM(400), NTRANS(400),         JTSM0007
     3 NTRAN(400)                                                               JTSM0008
C        PROGRAM LIMIT: 20 JOINT PLANES                                         JTSM0009
      DIMENSION YRAND(20), Y(20), NPT(20), IR(21)                               JTSM0010
C        PROGRAM LIMIT: 50 X COORDINATES PER JOINT PLANE                        JTSM0011
      DIMENSION CJOINT(20,50)                                                   JTSM0012
C        PROGRAM LIMIT: ( 20 X 300 ) PATH GRID                                  JTSM0013
      DIMENSION AISM(20,300), TAUSM(20,300), MINPTH(20,300), XCOOR(300)         JTSM0014
      EQUIVALENCE (IIR(400), REALBD(400)), (XCOOR(300), IR(21))                 JTSM0015
      LOGICAL FIN, FAM, FLIM, FLAM, FAD, FUD                                    JTSM0016
      READ (5, 1000) XDIM, YDIM, ALPHA                                          JTSM0017
      READ (5, 1001) PHIJT, COJT, PHIRK, CORK                                   JTSM0018
      READ (5, 1002) SIG1, SIG3                                                 JTSM0019
      READ (5, 1000) SP3, SPJTLN, SPRKBR                                        JTSM0020
      READ (5, 1004) NREDOP                                                     JTSM0021
      READ (5, 1004) ISEED                                                      JTSM0022
      READ (5, 1006) NOTPOP, NOTPOT, NOTPOD                                     JTSM0023
      READ (5, 1007) NOREAL, DISTMN, NJUMP                                      JTSM0024
 1000 FORMAT (F10.2, F10.2, F10.2)                                              JTSM0025
 1001 FORMAT (F10.2, F10.2, F10.2, F10.2)                                       JTSM0026
 1002 FORMAT (F10.2, F10.2)                                                     JTSM0027
 1004 FORMAT (I10)                                                              JTSM0028
 1006 FORMAT (I10,I10,I10)                                                      JTSM0029
 1007 FORMAT (I10, F10.2, I10)                                                  JTSM0030
 1010 FORMAT ('1',10X,'THE DIMENSIONS OF THE BLOCK ARE ',F8.2,                  JTSM0031
     1 ' HIGH BY ',F8.2,' WIDE')                                                JTSM0032
 1011 FORMAT ('0',10X,'THE INCLINATION OF THE JOINTS IS ',F4.1,                 JTSM0033
     1 ' DEGREES')                                                              JTSM0034
 1012 FORMAT ('0',10X,'STRENGTH PARAMETERS:'//                                  JTSM0035
     1 T20,'PHI (JOINT) = ',T50,F10.2,' DEGREES'/                               JTSM0036
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     2 T20,'COHESION (JOINT) = ',T50,F10.2,/                                  JTSM0037
     3 T20,'PHI (ROCK) = ',T50,F10.2,' DEGREES'/                              JTSM0038
     4 T20,'COHESION (ROCK) = ',T50,F10.2)                                    JTSM0039
  1013 FORMAT ('0',10X,'THE VERTICAL CONFINING STRESS IS ',T50,F10.2,/        JTSM0040
     1 11X,'THE HORIZONTAL CONFINING STRESS IS ',T50,F10.2)                   JTSM0041
  1014 FORMAT ('0',10X,'DISTRIBUTIONAL PARAMETERS FOR JOINTS:'//             JTSM0042
     1 T20,'MEAN SPACING BETWEEN PLANES =',T50,F10.2,/                        JTSM0043
     2 T20,'MEAN SPACING BETWEEN JOINTS = ',T50,F10.2,/                       JTSM0044
     3 T20,'MEAN JOINT LENGTH = ',T50,F10.2)                                  JTSM0045
  1015 FORMAT (//,10X,'PATH CRITERIA:',//                                    JTSM0046
     1 T20,'MINIMUM SPACING =',T50,F10.2,/                                    JTSM0047
     2 T20,'MAXIMUM TRANSITION =',T50,I10)                                    JTSM0048
  1016 FORMAT (//,10X,'THE NUMBER OF REALIZATIONS IS ',T50,I10)              JTSM0049
  1017 FORMAT (//,10X,'THE INITIAL RANDOM NUMBER IS ',T50,I10)               JTSM0050
  1018 FORMAT ('1',T10,'JOINT',14X,'Y',12X,'NUMBER OF',10X,' PERCENT ',      JTSM0051
     1 10X,'RANDOM NUMBERS'/                                                  JTSM0052
     2 T10,'PLANE',13X,'COOR',11X,'JOINTS',12X,'CONTINUITY',14X,'USED')       JTSM0053
  1020 FORMAT ('0',T10,I3,14X,F6.2,11X,I3,17X,F4.1,17X,I3)                   JTSM0054
  1021 FORMAT (T10,'REALIZATION NO. ',I4,':  NOT ENOUGH RANDOM NUMBERS',     JTSM0055
     1 ' AVAILABLE TO DEFINE ALL THE JOINT SEGMENTS ON A PLANE')              JTSM0056
  1022 FORMAT (T10,'REALIZATION NO. ',I4,':  NOT ENOUGH RANDOM NUMBERS',     JTSM0057
     1 ' AVAILABLE TO DEFINE ALL THE JOINT PLANES')                           JTSM0058
  1024 FORMAT (T10,'REALIZATION NO. ',I4,': NO JOINT PLANES WITHIN THE ',    JTSM0059
     1 'BLOCK')                                                               JTSM0060
  1025 FORMAT (T10,'REALIZATION NO. ',I4,':  REDUCTION OPTION NO. 2 ',       JTSM0061
     1 'REQUIRES THE TENSILE STRENGTH OF THE JOINT - NOT COMPUTED')           JTSM0062
  1050 FORMAT ('0','Y COOR IS ',F10.3,'     X COOR OF RIGHT END IS ',        JTSM0063
     1 F10.3)                                                                 JTSM0064
  1060 FORMAT (T27,'X COOR OF LEFT  END IS ',F10.3)                          JTSM0065
  1061 FORMAT ('0','Y COOR IS ',F10.3,5X,'*** NO JOINTS IN THIS ',           JTSM0066
     1 'PLANE ***')                                                           JTSM0067
  1070 FORMAT (///,27X,'***   THE NUMBER OF INDEPENDENT X COORDINATES',      JTSM0068
     1 ' IS ',I4,'   ***',///,44X,'THE ORDERED X COORDINATES:',//)            JTSM0069
  1072 FORMAT (T50,F10.3)                                                    JTSM0070
  1080 FORMAT (///,'FAILURE ALONG JOINT REQUIRES TENSILE STRENGTH OF',       JTSM0071
     1 ' JOINT - UNDEFINED CONDITION - RUN TERMINATED')                       JTSM0072
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1090 FORMAT ('1',10X,'POINT CONSIDERED:'//,T20,'JOINT PLANE = ',12X,          JTSM0073
    1 I5,/,T20,'Y COORDINATE = ', 9X,F10.3,/,T20,'X COORDINATE = ',9X,        JTSM0074
    2 F10.3,///,' PATH NO.',8X,'-- JUNCTION POINT --',7X,'ANGLE ',           JTSM0075
    3 4X,'STRENGTH',4X,'STRENGTH',4X,'STRENGTH',06X,'MINOR PRIN.',4X,        JTSM0076
    4 'TANGENT TO',/,14X,'JT. PL.',3X,'Y COOR',4X,'X COOR',3X,               JTSM0077
    5 'TO JCT.',4X,'TO JT PT',4X,'TO EDGE',6X,'TOTAL',10X,'STRESS',/)        JTSM0078
1092 FORMAT (I5,7X,I5,2X,F10.3,F10.3,5X,F6.2,4X,F8.2,4X,F8.2,4X,F8.2,        JTSM0079
    1 7X,F7.2)                                                               JTSM0080
1093 FORMAT ('+',T110,'POINT')                                               JTSM0081
1094 FORMAT ('+',T109,'PARABOLA')                                            JTSM0082
1095 FORMAT ('+',T111,'LINE')                                                JTSM0083
1096 FORMAT ('+',T109,'* JOINT *')                                           JTSM0084
1097 FORMAT ('+',T111,'CUSP')                                                JTSM0085
1098 FORMAT ('0','THE MINIMUM TOTAL STENGTH FOR Y COOR = ',                  JTSM0086
    1 F10.3,' AND X COOR = ',F10.3,' IS ',F10.2)                             JTSM0087
1100 FORMAT ('1','THE MINIMUM STRENGTH IS: ',F10.2,//,'THE NUMBER ',         JTSM0088
    1 'SHOULD BE BETWEEN:',//,T20,F10.2,' (THE VALUE FOR A ',F4.1,           JTSM0089
    2 '% CONTINUOUS JOINT)',/,T16,'AND ',F10.2,                              JTSM0090
    3 ' (THE VALUE FOR A CONTINUOUS JOINT)')                                 JTSM0091
1101 FORMAT ('0','THE FAILURE PATH PROCEEDS FROM:'//                         JTSM0092
    1 T10,'(LEFT END)',T31,'Y COOR = ',F10.3,T61,'X COOR = ',F10.3)          JTSM0093
1102  FORMAT (T25,'TO',T31,'Y COOR = ',F10.3,T61,'X COOR = ',F10.3)          JTSM0094
1103 FORMAT ('+',T10,'(RIGHT END)')                                          JTSM0095
1104 FORMAT ('0','THE AVERAGE INCLINATION OF THE FAILURE PATH IS ',          JTSM0096
    1 F6.2,' DEGREES',//)                                                    JTSM0097
1110 FORMAT (//,' REALIZATION    NO. OF    PERSISTENCE    ANGLE OF ',       JTSM0098
    1 '    MINIMUM    REDUCTION  OPTIONS    UPDATED MEAN VALUES',           JTSM0099
    2 '   TRANSITIONS'/                                                      JTSM0100
    3 '   NUMBER        JT PLS    MIN.    MAX.    FAIL. PATH    ',          JTSM0101
    4 'STRENGTH    (1)    (2)    (3)      (1)    (2)    (3)',               JTSM0102
    5 '     UNIT::TOTAL',//)                                                 JTSM0103
1111 FORMAT (3X,I4,10X,I2,6X,F5.1,2X,F5.1,6X,F4.1,7X,F7.2,3X,F5.1,2X,        JTSM0104
    1 F5.1,2X,F5.1,4X,F5.1,1X,F5.1,1X,F5.1,7X,I2,2X,I2)                      JTSM0105
1112 FORMAT (3X,I4,T30,'***** SEE BELOW  *****')                             JTSM0106
1113 FORMAT (///,T40,'CODE FOR REDUCTION OPTIONS:',//,T10,                   JTSM0107
    1 '(1) PERCENT CONTINUITY FOR A FAILURE SURFACE INCLINED AT',            JTSM0108
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     2 ' THE SAME ANGLE AS THE JOINTS',/,T10,'(2) PERCENT CONTINUITY',      JTSM0109
     3 ' FOR A FAILURE SURFACE INCLINED AT THE SAME ANGLE AS THE ',         JTSM0110
     4 'FAILURE PATH',/,T10,'(3) PERCENT COHESION OF INTACT ROCK FOR',      JTSM0111
     5 ' A FAILURE SURFACE INCLINED AT THE SAME ANGLE AS THE FAILURE',      JTSM0112
     6 'PATH',/////)                                                        JTSM0113
1122 FORMAT (///,45X,'*****  INCLINATION OF FAILURE PATH  *****',///        JTSM0114
     1 52X,'NO. OF REALIZATIONS:    ',I4,//                                JTSM0115
     2 52X,'MEAN VALUE:',13X,F4.1,/,52X,'MINIMUM VALUE:',10X,F4.1,/         JTSM0116
     3 52X,'MAXIMUM VALUE:',10X,F4.1,//,52X,'STANDARD DEVIATION:',5X,       JTSM0117
     4 F4.1,//////////)                                                     JTSM0118
1123 FORMAT ('0',45X,'*****      MAXIMUM PERSISTENCE      *****',///       JTSM0119
     1 52X,'NO. OF REALIZATIONS:    ',I4,//                                JTSM0120
     2 52X,'MEAN VALUE:',13X,F4.1,/,52X,'MINIMUM VALUE:',10X,F4.1,/         JTSM0121
     3 52X,'MAXIMUM VALUE:',10X,F4.1,//,52X,'STANDARD DEVIATION:',5X,       JTSM0122
     4 F4.1)                                                                JTSM0123
1124 FORMAT (///,45X,'******  REDUCTION OPTION NUMBER ',I2,'  ******',       JTSM0124
     1 ///,52X,'NUMBER OF REALIZATIONS:    ',I4,//                          JTSM0125
     2 52X,'MEAN VALUE:',13X,F4.1,/,52X,'MINIMUM VALUE:',10X,F4.1,/         JTSM0126
     3 52X,'MAXIMUM VALUE:',10X,F4.1,//,52X,'STANDARD DEVIATION:',5X,       JTSM0127
     4 F4.1,//,52X,'COEFF. OF CORRELATION',/,52X,                           JTSM0128
     5 'WITH INCLINATION ANGLE:    ',F6.3)                                  JTSM0129
      PI = 3.14159265                                                       JTSM0130
      PI4 = PI/4.0                                                          JTSM0131
      PI2 = PI/2.0                                                          JTSM0132
      ALPHO = ALPHA*PI/180.                                                 JTSM0133
      PHOJT = PHIJT*PI/180.                                                 JTSM0134
      TANJT = TAN(PHIJT*PI/180.)                                            JTSM0135
      PHORK = PHIRK*PI/180.                                                 JTSM0136
      SINRK = SIN(PHORK)                                                    JTSM0137
      COSRK = COS(PHORK)                                                    JTSM0138
      TANRK = TAN(PHORK)                                                    JTSM0139
      P = SPJTLN/(SPJTLN + SPRKBR)                                          JTSM0140
      CORK2K = COHK/2000.                                                   JTSM0141
      IF ( NJUMP .EQ. 0 ) NJUMP = 1000                                      JTSM0142
      IF ( DISTMN .EQ. 0.0 ) DISTMN = 0.00001                               JTSM0143
C        ETA AND XCTETA DEFINE THE CIRCLE WITH DUAL TANGENCY                JTSM0144
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      XCTETA = CORK*((1.0-SINRK*COSRK) - (1.0-(2.0*COSRK*SINRK))**0.5)          JTSM0145
     1 /SINRK**2.0                                                             JTSM0146
      IF ( PHIRK .GT. 45.0 ) ETA = PI4                                         JTSM0147
      IF ( PHIRK .GT. 45.0 ) ETASIN = SIN(PI4)                                 JTSM0148
      IF ( PHIRK .LE. 45.0 ) ETASIN = (CORK/(2.0*XCTETA))**0.5                 JTSM0149
      IF ( PHIRK .LE. 45.0 ) ETA = ATAN(ETASIN/((1.0 - ETASIN**2.0)            JTSM0150
     1 **0.5))                                                                 JTSM0151
      AFACT = 0.5*CORK/ETASIN**2                                               JTSM0152
      BFACT = CORK/ETASIN                                                      JTSM0153
      XCTPTF = CORK/2.0                                                        JTSM0154
      IF ( COJT .EQ. 0.0 ) THETAJ = PI2                                        JTSM0155
      IF ( COJT .NE. 0.0 ) THETAJ = ATAN(SIG1/COJT)                            JTSM0156
      IF ( ALPHO .GT. THETAJ ) WRITE (6, 1080)                                 JTSM0157
      IF ( ALPHO .GT. THETAJ ) GO TO 2000                                      JTSM0158
      WRITE (6, 1010) YDIM, XDIM                                               JTSM0159
      WRITE (6, 1011) ALPHA                                                    JTSM0160
      WRITE (6, 1012) PHIJT, COJT, PHIRK, CORK                                 JTSM0161
      WRITE (6, 1013) SIG1, SIG3                                               JTSM0162
      WRITE (6, 1014) SP3, SPRKBR, SPJTLN                                      JTSM0163
      WRITE (6, 1017) ISEED                                                    JTSM0164
      WRITE (6, 1016) NOREAL                                                   JTSM0165
      WRITE (6,1015) DISTMN, NJUMP                                             JTSM0166
      FLAM = .TRUE.                                                            JTSM0167
      IF ( NOTPOD .EQ. 1 ) FLAM = .FALSE.                                      JTSM0168
      CALL GGUB(ISEED,NOREAL,R)                                                JTSM0169
      DO 17 I = 1, NOREAL                                                      JTSM0170
   17 IIR(I) = R(I)*10**8.0                                                    JTSM0171
      TAN90 = 5.0                                                              JTSM0172
      NREAL = 0                                                                JTSM0173
      NBUM = 0                                                                 JTSM0174
      NNBUM = 0                                                                JTSM0175
      SUMAP1 = 0.0                                                             JTSM0176
      SUMAP2 = 0.0                                                             JTSM0177
      SUMAP3 = 0.0                                                             JTSM0178
      PSISUM = 0.0                                                             JTSM0179
      CONSUM = 0.0                                                             JTSM0180
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      AP1MAX = 0.0                                                          JTSM0181
      AP2MAX = 0.0                                                          JTSM0182
      AP3MAX = 0.0                                                          JTSM0183
      PSIMAX = 0.0                                                          JTSM0184
      CONMAX = 0.0                                                          JTSM0185
      AP1MIN = 100.0                                                        JTSM0186
      AP2MIN = 100.0                                                        JTSM0187
      AP3MIN = 100.0                                                        JTSM0188
      PSIMIN = 180.                                                         JTSM0189
      CONMIN = 100.0                                                        JTSM0190
      DO 1950 MM = 1, NOREAL                                                JTSM0191
      YSUM = 0.0                                                            JTSM0192
      PERCMN = 100.0                                                        JTSM0193
      PERCMX = 0.0                                                          JTSM0194
      NREAL = NREAL + 1                                                     JTSM0195
      ISEED = IIR(MM)                                                       JTSM0196
      CALL GGUB(ISEED,21,R)                                                 JTSM0197
      DO 20 I = 1, 21                                                       JTSM0198
   20 IR(I) = R(I)*10**8.0                                                  JTSM0199
      ISEED = IR(21)                                                        JTSM0200
      CALL GGUB(ISEED,20,R)                                                 JTSM0201
      DO 30 I = 1, 20                                                       JTSM0202
   30 YRAND(I) = R(I)                                                       JTSM0203
      IF ( .NOT. FLAM ) WRITE ( 6, 1018)                                    JTSM0204
   31 DO 200 I = 1, 20                                                      JTSM0205
      YSUM = YSUM + (SP3)*ALOG(1.0/(1.0-YRAND(I)))                          JTSM0206
      Y(I) = YSUM                                                           JTSM0207
      IF ( Y(I) .GT. YDIM ) GO TO 250                                       JTSM0208
      ISEED = IR(I)                                                         JTSM0209
      CALL GGUB(ISEED,51,R)                                                 JTSM0210
      IF ( R(51) .LT. P) CJOINT(I,1) = 0.0                                  JTSM0211
      IF ( R(51) .GE. P )  CJOINT(I,1) = SPRKBR*ALOG(1.0/R(1))              JTSM0212
      IF ( CJOINT(I,1) .GE. XDIM )  CJOINT(I,1) = XDIM                      JTSM0213
C                  FOR ODD N: CJOINT(I,N) IS THE RIGHT END OF THE (N+1)/2 JOINT  JTSM0214
C                  IN THE I TH JOINT PLANE                                JTSM0215
C                  FOR EVEN N: CJOINT(I,N) IS THE LEFT END OF THE N/2 JOINT      JTSM0216
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C                    IN THE I TH JOINT PLANE                                    JTSM0217
      DO 80 J = 2, 50, 2                                                        JTSM0218
      CJOINT(I,J) = CJOINT(I,J-1) + SPJTLN*ALOG(1.0/R(J))                       JTSM0219
      IF ( CJOINT(I,J) .GE. XDIM ) GO TO 90                                     JTSM0220
      CJOINT(I,J+1) = CJOINT(I,J) + SPRKBR*ALOG(1.0/R(J+1))                     JTSM0221
      IF ( CJOINT(I,J+1) .GE. XDIM ) GO TO 120                                  JTSM0222
   80 CONTINUE                                                                  JTSM0223
      REALBD(MM) = 1.0                                                          JTSM0224
      GO TO 1948                                                                JTSM0225
   90 CJOINT(I,J) = XDIM                                                        JTSM0226
      NPT(I) = J                                                                JTSM0227
      NJOINT = J/2.0                                                            JTSM0228
      NRN = J + 1                                                               JTSM0229
      GO TO 150                                                                 JTSM0230
  120 CJOINT(I,J+1) = XDIM                                                      JTSM0231
      NPT(I) = J + 1                                                            JTSM0232
      NJOINT = J/2.0                                                            JTSM0233
      NRN = J + 2                                                               JTSM0234
  150 CONTINUE                                                                  JTSM0235
      SUMJTL = 0.0                                                              JTSM0236
      DO 160 J = 1, NJOINT                                                      JTSM0237
      NNJT = 2*J                                                                JTSM0238
  160 SUMJTL = SUMJTL + CJOINT(I,NNJT) - CJOINT(I,NNJT - 1)                     JTSM0239
      PERCON = SUMJTL* 100.0/XDIM                                               JTSM0240
      IF ( PERCON .GE. PERCMX ) PERCMX = PERCON                                 JTSM0241
      IF ( PERCON .LE. PERCMN ) PERCMN = PERCON                                 JTSM0242
  190 IF (FLAM) GO TO 200                                                       JTSM0243
      IF ( CJOINT(I, 1) .EQ. XDIM ) NJOINT = 0                                  JTSM0244
      WRITE (6, 1020) I, Y(I), NJOINT, PERCON, NRN                              JTSM0245
  200 CONTINUE                                                                  JTSM0246
      REALBD(MM) = 2.0                                                          JTSM0247
      GO TO 1948                                                                JTSM0248
  250 JJPL(MM) = I - 1                                                          JTSM0249
      IF ( JJPL(MM) .EQ. 0 ) REALBD(MM) = 4.0                                   JTSM0250
      IF ( JJPL(MM) .EQ. 0 ) GO TO 1948                                         JTSM0251
      JPL = JJPL(MM)                                                            JTSM0252
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  255 IF (FLAM) GO TO 310                                                    JTSM0253
      IF ( NOTPOP .NE. 1 ) GO TO 310                                         JTSM0254
      DO 300 I = 1, JPL                                                      JTSM0255
      NOPT = NPT(I)                                                          JTSM0256
      DO 280 J =1, NOPT                                                      JTSM0257
      IF ( (-1.0)**J ) 260, 260, 270                                         JTSM0258
  260 IF ( CJCINT(I,1) .EQ. XDIM ) WRITE (6, 1061) Y(I)                      JTSM0259
      IF ( CJOINT(I,J) .EQ. XDIM ) GO TO 280                                 JTSM0260
      IF ( CJOINT(I,1) .NE. XDIM ) WRITE (6, 1050) Y(I), CJOINT(I,J)         JTSM0261
      GO TO 280                                                              JTSM0262
  270 IF ( CJOINT(I,1) .NE. XDIM ) WRITE (6, 1060) CJOINT(I,J)               JTSM0263
  280 CONTINUE                                                               JTSM0264
  300 CONTINUE                                                               JTSM0265
C         THE FOLLOWING ROUTINE ARRANGES THE X COORDINATES IS ASCENDING ORDER JTSM0266
  310 NXCORS = 0                                                             JTSM0267
      DO 345 I = 1, JPL                                                      JTSM0268
      NPTI = NPT(I) - 1                                                      JTSM0269
      DO 345 J = 1, NPTI                                                     JTSM0270
      IF ( CJOINT(I,J) .EQ. 0.0 .OR. CJOINT(I,J) .EQ. XDIM ) GO TO 345       JTSM0271
      NXCORS = NXCORS + 1                                                    JTSM0272
      XCOOR(NXCORS) = CJOINT(I,J)                                            JTSM0273
  345 CONTINUE                                                               JTSM0274
      NXCORS = NXCORS + 1                                                    JTSM0275
      XCOOR(NXCORS) = XDIM                                                   JTSM0276
      NN = NXCORS - 1                                                        JTSM0277
      DO 390 J = 1, NN                                                       JTSM0278
      M = NXCORS - J                                                         JTSM0279
      FIN = .TRUE.                                                           JTSM0280
      DO 380 I = 1, M                                                        JTSM0281
      IF ( XCOOR(I) .LE. XCOOR(I+1) ) GO TO 380                              JTSM0282
      FIN = .FALSE.                                                          JTSM0283
      XX = XCOOR(I+1)                                                        JTSM0284
      XCOOR(I+1) = XCOOR(I)                                                  JTSM0285
      XCOOR(I) = XX                                                          JTSM0286
  380 CONTINUE                                                               JTSM0287
      IF (FIN) GO TO 400                                                     JTSM0288
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  390 CONTINUE                                                          JTSM0289
  400 DO 410 I = 1, NXCORS                                              JTSM0290
      L = NXCORS + 1 - I                                                JTSM0291
  410 XCOOR(L+1) = XCOOR(L)                                             JTSM0292
      XCOOR(1) = 0.0                                                    JTSM0293
      NXCORS = NXCORS + 1                                               JTSM0294
      PERMIN(MM) = PERCMN                                               JTSM0295
      PERMAX(MM) = PERCMX                                               JTSM0296
      IF ( PERCMX .LE. CONMIN ) CONMIN = PERCMX                         JTSM0297
      IF ( PERCMX .GE. CONMAX ) CONMAX = PERCMX                         JTSM0298
      IF (FLAM) GO TO 431                                               JTSM0299
      WRITE (6, 1070) NXCORS                                            JTSM0300
      DO 427 I = 1, NXCORS                                              JTSM0301
  427 WRITE (6, 1072) XCOOR(I)                                          JTSM0302
C        THE FOLLOWING ROUTINE COMPUTES THE VALUES OF AISM(I,J) - THE   JTSM0303
C        MINIMUM APPARENT INCREMENTAL SAFETY MARGIN REQUIRED TO INITIATE JTSM0304
C        A FAILURE CRACK FROM THE ITH JOINT PLANE AT THE JTH X COORDINATE. JTSM0305
  431 DO 450 I = 1, JPL                                                 JTSM0306
      TAUSM(I,1) = 0.0                                                  JTSM0307
  450 AISM(I,1) = 0.0                                                   JTSM0308
C              THIS PART COMPUTES THE AISM FOR A UNIT LENGTH OF         JTSM0309
C              JOINT (AISMJT) AND INTACT ROCK (AISMRK).                 JTSM0310
      IF ( NREAL .GT. 1 .AND. PSISUM .NE. 0.0 ) GO TO 499               JTSM0311
  457 SINALP = SIN(ALPHO)                                               JTSM0312
      COSALP = COS(ALPHO)                                               JTSM0313
      DENOMJ = SINALP*COSALP + (SINALP**2)*TANJT                        JTSM0314
      SIG3FJ = SIG1*(1.0 - TANJT/DENOMJ) - COJT/DENOMJ                  JTSM0315
      DLTNJT = (SIG3 - SIG3FJ)*SINALP*COSALP                            JTSM0316
      DLNOJT = (SIG3 - SIG3FJ)*SINALP**2                                JTSM0317
      AISMJT = DLNOJT*TANJT + DLTNJT                                    JTSM0318
      CALL ROCK(CORK,PHORK,SINRK,COSRK,TANRK,SIG1,SIG3,PI4,PI2,         JTSM0319
     1CORK2K,ALPHO,AFACT,BFACT,ETA,XCTPTF,DELTA3,TAUFR,TANIDY,PRIM3R)   JTSM0320
      F3FR = DELTA3*SINALP + TAUFR*COSALP                               JTSM0321
      F1FR = TAUFR*SINALP                                               JTSM0322
      DLNORK = F3FR*SINALP + F1FR*COSALP                                JTSM0323
      DLTNRK = F3FR*COSALP - F1FR*SINALP                                JTSM0324
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      AISMRK = DLNORK*TANJT + DLTNRK                                    JTSM0325
C               THIS PART COMPUTES THE VALUES OF AISM(1,J)               JTSM0326
  499 STR = 0.0                                                         JTSM0327
      TAU = 0.0                                                         JTSM0328
      NPT1 = NPT(1)                                                     JTSM0329
      INDX = 1                                                          JTSM0330
      DO 570 I = 2, NXCORS                                              JTSM0331
      IF ( XCOOR(I) .GT. CJOINT(1,1) ) GO TO 510                        JTSM0332
      STR = STR + AISMRK*(XCOOR(I) - XCOOR(I-1))                        JTSM0333
      TAU = TAU + DLTNRK*(XCOOR(I) - XCOOR(I-1))                        JTSM0334
      AISM(1,I) = STR                                                   JTSM0335
      TAUSM(1,I) = TAU                                                  JTSM0336
      GO TO 570                                                         JTSM0337
  510 CONTINUE                                                          JTSM0338
      NPT0 = INDX                                                       JTSM0339
      DO 560 J = NPT0, NPT1                                             JTSM0340
      IF ( XCOOR(I) .GT. CJOINT(1,J) ) GO TO 560                        JTSM0341
      IF ( (-1.0)**J ) 520, 520, 530                                    JTSM0342
  520 STR = STR + AISMRK*(XCOOR(I) - XCOOR(I-1))                        JTSM0343
      TAU = TAU + DLTNRK*(XCOOR(I) - XCOOR(I-1))                        JTSM0344
      GO TO 550                                                         JTSM0345
  530 STR = STR + AISMJT*(XCOOR(I) - XCOOR(I-1))                        JTSM0346
      TAU = TAU + DLTNJT*(XCOOR(I) - XCOOR(I-1))                        JTSM0347
  550 AISM(1,I) = STR                                                   JTSM0348
      TAUSM(1,I) = TAU                                                  JTSM0349
      INDX = J                                                          JTSM0350
      GO TO 570                                                         JTSM0351
  560 CONTINUE                                                          JTSM0352
  570 CONTINUE                                                          JTSM0353
      STRXXX =  AISM(1,NXCORS)                                          JTSM0354
      INDY = 1                                                          JTSM0355
      IF (FLAM) GO TO 590                                               JTSM0356
      IF ( NOTPOT .NE. 1 .AND. NOTPOT .NE. 2 ) GO TO 590                JTSM0357
      DO 580 J = 1, NXCORS                                              JTSM0358
  580 WRITE (6, 1098) Y(1), XCOOR(J), AISM(1,J)                         JTSM0359
  590 IF ( JPL .EQ. 1 ) GO TO 703                                       JTSM0360
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C             THIS PART COMPLETES THE AISM(I,J) MATRIX. EACH POINT (I,J)          JTSM0361
C             HAS N FAILURE PATHS - EACH PATH HAS AN APPARENT INCREMENTAL          JTSM0362
C             SAFETY MARGIN, STRCSE, ASSOCIATED WITH IT.  AISM(I,J) IS            JTSM0363
C             THE MINIMUM OF ALL THE STRCSE VALUES                                JTSM0364
      NTRAN(MM) = 0                                                               JTSM0365
      DO 700 J = 2, JPL                                                           JTSM0366
      IF ( CJOINT(J,1) .EQ. XDIM ) GO TO 700                                      JTSM0367
      DO 680 I = 2, NXCORS                                                        JTSM0368
      FLIM = .TRUE.                                                               JTSM0369
      IF ( NOTPOT .EQ. 1 ) FLIM = .FALSE.                                         JTSM0370
      IF ( NOTPOT .EQ. 2 .AND. J .EQ. JPL .AND. I .EQ. NXCORS )                   JTSM0371
     1 FLIM = .FALSE.                                                             JTSM0372
      IF ( .NOT.FLIM .AND. .NOT.FLAM ) WRITE (6,1090) J, Y(J), XCOOR(I)           JTSM0373
C                  THE FIRST PATH TO CONSIDER IS THE ONE THAT PROCEEDS FROM     JTSM0374
C                  (J,I) TO (J,I-1) -- IT IS OFTEN THE CRITICAL PATH --         JTSM0375
C                  IT IS ALSO THE ONLY PATH THAT NEED BE EVALUATED              JTSM0376
C                  IN THE JTH ROW                                               JTSM0377
      NCASE = 1                                                                   JTSM0378
      IF ( XCOOR(I) .GT. CJOINT(J,1) ) GO TO 593                                  JTSM0379
      PRIMIN = PRIM3R                                                             JTSM0380
      STR = AISMRK*(XCOOR(I) - XCOOR(I-1))                                        JTSM0381
      STRLOW = STR + AISM(J,I-1)                                                  JTSM0382
      TAUCSE = TAUSM(J,I-1) + DLTNRK*(XCOOR(I) - XCOOR(I-1))                      JTSM0383
      ITHCSE = J*NXCORS + I - 1                                                   JTSM0384
      GO TO 603                                                                   JTSM0385
  593 NPTI = NPT(J)                                                               JTSM0386
      DO 602 K = 1, NPTI                                                          JTSM0387
      IF ( XCOOR(I) .GT. CJOINT(J,K) ) GO TO 602                                  JTSM0388
      IF ( (-1.0)**K ) 600, 600, 598                                              JTSM0389
  598 STR = AISMJT*(XCOOR(I) - XCOOR(I-1))                                        JTSM0390
      PRIMIN = SIG3FJ                                                             JTSM0391
      GO TO 601                                                                   JTSM0392
  600 STR = AISMRK*(XCOOR(I)-XCOOR(I-1))                                          JTSM0393
      PRIMIN = PRIM3R                                                             JTSM0394
  601 STRLOW = STR + AISM(J,I-1)                                                  JTSM0395
      IF (PRIMIN .EQ. SIG3FJ )                                                    JTSM0396
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     1 TAUCSE = TAUSM(J,I-1) + DLTNJT*(XCOOR(I) - XCOOR(I-1))                    JTSM0397
       IF (PRIMIN .EQ. PRIM3R )                                                  JTSM0398
     1 TAUCSE = TAUSM(J,I-1) + DLTNRK*(XCOOR(I) - XCOOR(I-1))                    JTSM0399
       ITHCSE = J*NXCORS + I - 1                                                 JTSM0400
       GO TO 603                                                                 JTSM0401
   602 CONTINUE                                                                  JTSM0402
   603 IF ( FLIM .OR. FLAM ) GO TO 604                                           JTSM0403
       WRITE ( 6, 1092) NCASE, J, Y(J), XCOOR(I-1), ALPHA, STR,                  JTSM0404
     1 AISM(J,I-1), STRLOW, PRIMIN                                               JTSM0405
       IF ( PRIMIN .EQ. SIG3FJ ) WRITE (6,1096)                                  JTSM0406
       IF ( PRIMIN .NE. SIG3FJ .AND. TANIDY .EQ. -1.0 ) WRITE (6,1093)           JTSM0407
       IF ( PRIMIN .NE. SIG3FJ .AND. TANIDY .EQ.  0.0 ) WRITE (6,1094)           JTSM0408
       IF ( PRIMIN .NE. SIG3FJ .AND. TANIDY .EQ.  1.0 ) WRITE (6,1095)           JTSM0409
       IF ( PRIMIN .NE. SIG3FJ .AND. TANIDY .EQ.  2.0 ) WRITE (6,1097)           JTSM0410
   604 J1 = J - 1                                                                JTSM0411
       I1 = I                                                                    JTSM0412
       IF ( I .EQ. NXCORS ) I1 = I - 1                                           JTSM0413
       DO 635 N = 1, J1                                                          JTSM0414
       IF ( J - N .GT. NJUMP ) GO TO 635                                         JTSM0415
       IF ( CJOINT(N,1) .EQ. XDIM ) GO TO 635                                    JTSM0416
       INDEX = 1                                                                 JTSM0417
       DO 634 M = 1, I1                                                          JTSM0418
       IF ( M .EQ. I .AND. N .NE. J1 ) GO TO 635                                 JTSM0419
       IF ( STRLOW .LT.  AISM(N,M)) GO TO 634                                    JTSM0420
       IF ( M .EQ. I1 .AND. N .EQ. J1 ) GO TO 608                                JTSM0421
       NPT0 = INDEX                                                              JTSM0422
       NPTN = NPT(N)                                                             JTSM0423
       IF ( M .EQ. 1 ) GO TO 608                                                 JTSM0424
       DO 605 K = NPT0, NPTN                                                     JTSM0425
       IF ( CJOINT(N,K) .EQ. XCOOR(M) )  INDEX = K                               JTSM0426
       IF ( CJOINT(N,K) .EQ. XCOOR(M) ) GO TO 608                                JTSM0427
   605 CONTINUE                                                                  JTSM0428
       IF ( (XCOOR(M) - XCOOR(INXLPT) ) .LT. DISTMN ) GO TO 634                  JTSM0429
   608 NCASE = NCASE + 1                                                         JTSM0430
       XDIST = XCOOR(I) - XCOOR(M)                                               JTSM0431
       IF ( XDIST .EQ. 0.0 .AND. TAN90 .NE. 5.0 ) GO TO 627                      JTSM0432
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      YDIST  = Y(J) - Y(N)                                                    JTSM0433
      IF ( XDIST .NE. 0.0 ) CHI = ATAN((Y(J) - Y(N))/XDIST)                   JTSM0434
      IF ( XDIST .EQ. 0.0 ) CHI = PI2                                         JTSM0435
      XYDIST = (Y(J) - Y(N))/SIN(CHI)                                         JTSM0436
      BETAR = CHI + ALPHO                                                     JTSM0437
      VERTZ = XYDIST*SIN(BETAR)                                               JTSM0438
      HORIZ = XYDIST*COS(BETAR)                                               JTSM0439
      IF ( XDIST .NE. 0.0 ) GO TO 610                                         JTSM0440
      CALL ROCK(CORK,PHORK,SINRK,COSRK,TANRK,SIG1,SIG3,PI4,PI2,               JTSM0441
     1CORK2K,BETAR,AFACT,BFACT,ETA,XCTPTF,DELTA3,TAUFR,TAN90,PRI90)           JTSM0442
      TAU90 = ((DELTA3*VERTZ+TAUFR*HORIZ)*COSALP - TAUFR*VERTZ*SINALP)/       JTSM0443
     1 YDIST                                                                  JTSM0444
      STR90 = ((DELTA3*VERTZ+TAUFR*HORIZ)*SINALP + TAUFR*VERTZ*COSALP)*       JTSM0445
     1 TANJT/YDIST + TAU90                                                    JTSM0446
      GO TO 627                                                               JTSM0447
  610 BETA = (CHI + ALPHO)*180.0/PI                                           JTSM0448
      CALL ROCK(CORK,PHORK,SINRK,COSRK,TANRK,SIG1,SIG3,PI4,PI2,               JTSM0449
     1CORK2K,BETAR,AFACT,BFACT,ETA,XCTPTF,DELTA3,TAUFR,TANIDX,PRIMIN)         JTSM0450
      F3FR = DELTA3*VERTZ + TAUFR*HORIZ                                       JTSM0451
      F1FR = TAUFR*VERTZ                                                      JTSM0452
      TAU = F3FR*COSALP - F1FR*SINALP                                         JTSM0453
      PER = F3FR*SINALP + F1FR*COSALP                                         JTSM0454
      STR = PER*TANJT + TAU                                                   JTSM0455
      GO TO 628                                                               JTSM0456
  627 BETA = 90.0 + ALPHA                                                     JTSM0457
      PRIMIN = PRI90                                                          JTSM0458
      TANIDX = TAN90                                                          JTSM0459
      STR = STR90*YDIST                                                       JTSM0460
      TAU = TAU90*YDIST                                                       JTSM0461
  628 STRCSE = STR + AISM(N,M)                                                JTSM0462
      IF ( STRCSE .LE. STRLOW ) STRLOW = STRCSE                               JTSM0463
      IF ( STRCSE .EQ. STRLOW ) TAUCSE = TAU + TAUSM(N,M)                     JTSM0464
      IF ( STRCSE .EQ. STRLOW ) ITHCSE = N*NXCORS + M                         JTSM0465
      IF ( STRCSE .EQ. STRLOW .AND. J-N .GT. NTRAN(MM) ) NTRAN(MM) = J-N      JTSM0466
      INXLPT = M                                                              JTSM0467
      IF ( FLIM .OR. FLAM ) GO TO 634                                         JTSM0468
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      WRITE(6, 1092) NCASE, N, Y(N), XCOOR(M), BETA, STR, AISM(N,M),          JTSM0469
     1 STRCSE, PRIMIN                                                         JTSM0470
      IF ( TANIDX .EQ. -1.0 ) WRITE (6,1093)                                  JTSM0471
      IF ( TANIDX .EQ.  0.0 ) WRITE (6,1094)                                  JTSM0472
      IF ( TANIDX .EQ.  1.0 ) WRITE (6,1095)                                  JTSM0473
      IF ( TANIDX .EQ.  2.0 ) WRITE (6,1097)                                  JTSM0474
  634 CONTINUE                                                                JTSM0475
  635 CONTINUE                                                                JTSM0476
      AISM(J,I) = STRLOW                                                      JTSM0477
      MINPTH(J,I) = ITHCSE                                                    JTSM0478
      TAUSM(J,I) = TAUCSE                                                     JTSM0479
      IF ( I .EQ. NXCORS .AND. AISM(J,I) .LE. STRXXX ) STRXXX=AISM(J,I)       JTSM0480
      IF ( I .EQ. NXCORS .AND. AISM(J,I) .LE. STRXXX ) INDY =.J               JTSM0481
       IF ( FLIM .OR. FLAM ) GO TO 680                                        JTSM0482
      WRITE (6, 1098) Y(J), XCOOR(I), AISM(J,I)                               JTSM0483
  680 CONTINUE                                                                JTSM0484
  700 CONTINUE                                                                JTSM0485
C                   THIS PART FINDS THE MINIMUM VALUE OF STRENGTH FOR THE     JTSM0486
C                   SECTION AND THE CORRESPONDING FAILURE PATH.               JTSM0487
  703 FIN = .FALSE.                                                           JTSM0488
      FAM = .FALSE.                                                           JTSM0489
      STAISM(MM) = STRXXX                                                     JTSM0490
      INDYY = INDY                                                            JTSM0491
      INDX = NXCORS                                                           JTSM0492
      IF (FLAM) GO TO 720                                                     JTSM0493
      STRREF = AISMJT*XDIM                                                    JTSM0494
      STREG = STRREF*PERCMX/100.0 + AISMRK*XDIM*(1.0-PERCMX/100.0)            JTSM0495
      WRITE (6, 1100) STRXXX, STREG, PERCMX, STRREF                           JTSM0496
      WRITE (6,1101) Y(INDY), XCOOR(NXCORS)                                   JTSM0497
  720 IF ( INDY .EQ. 1 ) GO TO 725                                            JTSM0498
      INDPTH = MINPTH(INDY,INDX)                                              JTSM0499
      INDY = INDPTH/NXCORS                                                    JTSM0500
      INDX = INDPTH - INDY*NXCORS                                             JTSM0501
      IF ( .NOT. FLAM ) WRITE (6,1102) Y(INDY), XCOOR(INDX)                   JTSM0502
  725 IF ( INDY .EQ. 1 ) FIN = .TRUE.                                         JTSM0503
      IF ( INDX .EQ. 1 ) FAM = .TRUE.                                         JTSM0504
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      IF ( FIN .AND. .NOT. FLAM ) WRITE (6,1102) Y(1), XCOOR(1)          JTSM0505
      IF (FIN) GO TO 775                                                 JTSM0506
      IF ( FAM .AND. .NOT. FLAM ) WRITE (6,1102) Y(INDY), XCOOR(1)       JTSM0507
      IF (FAM) GO TO 775                                                 JTSM0508
      GO TO 720                                                          JTSM0509
  775 IF (.NOT.FLAM) WRITE (6, 1103)                                     JTSM0510
      DO 780 I = 1, JPL                                                  JTSM0511
      IF ( Y(INDY) .EQ. Y(I) ) NTRAN1 = I                                JTSM0512
      IF ( Y(INDYY) .EQ. Y(I) ) NTRAN2 = I                               JTSM0513
  780 CONTINUE                                                           JTSM0514
      NTRANS(MM) = NTRAN2 - NTRAN1                                       JTSM0515
      RHO = ATAN((Y(INDYY) - Y(INDY))/XDIM)                              JTSM0516
      PSO = RHO + ALPHO                                                  JTSM0517
      SINPSO = SIN(PSO)                                                  JTSM0518
      COSPSO = COS(PSO)                                                  JTSM0519
      PSI(MM) = PSO*180./PI                                              JTSM0520
      IF ( PSI(MM) .LE. PSIMIN ) PSIMIN = PSI(MM)                        JTSM0521
      IF ( PSI(MM) .GT. PSIMAX ) PSIMAX = PSI(MM)                        JTSM0522
      PSISUM = PSISUM + PSI(MM)                                          JTSM0523
      CONSUM = CONSUM + PERMAX(MM)                                       JTSM0524
      IF (.NOT. FLAM) WRITE (6,1104) PSI(MM)                             JTSM0525
      PER = (STAISM(MM) - TAUSM(INDYY,NXCORS))/TANJT                     JTSM0526
      FORTAN = TAUSM(INDYY,NXCORS)*COS(RHO) - PER*SIN(RHO)               JTSM0527
      FORNOR = TAUSM(INDYY,NXCORS)*SIN(RHO) + PER*COS(RHO)               JTSM0528
      DISTFD = XDIM/COS(RHO)                                             JTSM0529
      APPER1(MM) = 0.0                                                   JTSM0530
      APPER2(MM) = 0.0                                                   JTSM0531
      APPER3(MM) = 0.0                                                   JTSM0532
      AP1MEN(MM) = 0.0                                                   JTSM0533
      AP2MEN(MM) = 0.0                                                   JTSM0534
      AP3MEN(MM) = 0.0                                                   JTSM0535
      IF ( NREDOP .GT. 1 ) GO TO 790                                     JTSM0536
      IF (NREDOP) 790, 820, 850                                          JTSM0537
  790 NRED1 = -1                                                         JTSM0538
      APPER1(MM) = 100.*((STRXXX   /XDIM)-AISMRK)/(AISMJT-AISMRK)        JTSM0539
      SUMAP1 = SUMAP1 + APPER1(MM)                                       JTSM0540
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      AP1MEN(MM) = SUMAP1/(NREAL-NBUM)                                    JTSM0541
      IF ( APPER1(MM) .LE. AP1MIN ) AP1MIN = APPER1(MM)                   JTSM0542
      IF ( APPER1(MM) .GE. AP1MAX ) AP1MAX = APPER1(MM)                   JTSM0543
      IF ( NREDOP .LT. 1 ) GO TO 860                                      JTSM0544
  820 NRED2 = 0                                                           JTSM0545
      IF ( PSO .GT. THETAJ ) REALBD(MM) = 3.0                             JTSM0546
      IF ( PSO .GT. THETAJ ) GO TO 840                                    JTSM0547
      DENOMJ = SINPSO*COSPSO   + (SINPSO**2)*TANJT                        JTSM0548
      SIG3JJ = SIG1*(1.0 - TANJT/DENOMJ) - COJT/DENOMJ                    JTSM0549
      AISMJJ = (SIG3 - SIG3JJ)*(TANJT*SINPSO**2 + SINPSO*COSPSO)          JTSM0550
      CALL ROCK(CORK,PHORK,SINRK,COSRK,TANRK,SIG1,SIG3,PI4,PI2,           JTSM0551
     1CORK2K,PSO,AFACT,BFACT,ETA,XCTPTF,DELTA3,TAUFR,TANIDX,PRIMIN)       JTSM0552
      DLNORR = DELTA3*SINPSO**2 + 2.0*TAUFR*SINPSO*COSPSO                 JTSM0553
      DLTNRR = DELTA3*SINPSO*COSPSO + TAUFR*(COSPSO**2 - SINPSO**2)       JTSM0554
      AISMRR = DLNORR*TANJT + DLTNRR                                      JTSM0555
  825 STR2DX = FORTAN + FORNOR*TANJT                                      JTSM0556
      APPER2(MM) = 100.*((STR2DX/DISTFD)-AISMRR)/(AISMJJ-AISMRR)          JTSM0557
      SUMAP2 = SUMAP2 + APPER2(MM)                                        JTSM0558
      AP2MEN(MM) = SUMAP2/(NREAL-NBUM-NNBUM)                              JTSM0559
      IF ( APPER2(MM) .GE. AP2MAX ) AP2MAX = APPER2(MM)                   JTSM0560
      IF ( APPER2(MM) .LE. AP2MIN ) AP2MIN = APPER2(MM)                   JTSM0561
  840 IF ( NREDOP .LE. 1 ) GO TO 860                                      JTSM0562
  850 NRED3 = 1                                                           JTSM0563
      STRTAN = FORTAN/DISTFD                                              JTSM0564
      STRNOR = FORNOR/DISTFD                                              JTSM0565
      TANPSO = (SIG1 - SIG3)*SINPSO*COSPSO                                JTSM0566
      PERPSO = SIG1*COSPSO**2 + SIG3*SINPSO**2                            JTSM0567
      TSTTAN = TANPSO + STRTAN                                            JTSM0568
      TSTPER = PERPSO - STRNOR                                            JTSM0569
      PHOCMB = P*PHOJT + (1.0 - P)*PHORK                                  JTSM0570
      APPER3(MM) =(TSTTAN - TSTPER*TAN(PHOCMB))*100./CORK                 JTSM0571
      SUMAP3 = SUMAP3 + APPER3(MM)                                        JTSM0572
      AP3MEN(MM) = SUMAP3/(NREAL - NBUM)                                  JTSM0573
      IF ( APPER3(MM) .GE. AP3MAX ) AP3MAX = APPER3(MM)                   JTSM0574
      IF ( APPER3(MM) .LE. AP3MIN ) AP3MIN = APPER3(MM)                   JTSM0575
  860 IF ( PSO .LE. THETAJ .OR. NREDOP .EQ. -1 .OR. NREDOP .EQ. 1 )       JTSM0576
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     1 REALBD(MM) = 0.0                                                          JTSM0577
       GO TO 1950                                                                JTSM0578
1948   NBUM = NBUM + 1                                                           JTSM0579
1950   CONTINUE                                                                  JTSM0580
       WRITE ( 6, 1110)                                                          JTSM0581
       DO 1960 MM = 1, NOREAL                                                    JTSM0582
       IF ( REALBD(MM) .NE. 0.0 ) WRITE (6,1112) MM                              JTSM0583
       IF ( REALBD(MM) .EQ. 0.0 .OR. REALBD(MM) .EQ. 3.0 )                       JTSM0584
      1 WRITE (6,1111) MM, JJPL(MM), PERMIN(MM), PERMAX(MM), PSI(MM),            JTSM0585
      2 STAISM(MM), APPER1(MM), APPER2(MM), APPER3(MM), AP1MEN(MM),              JTSM0586
      3 AP2MEN(MM), AP3MEN(MM), NTRAN(MM), NTRANS(MM)                            JTSM0587
1960   CONTINUE                                                                  JTSM0588
       WRITE (6, 1113)                                                           JTSM0589
       DO 1970 MM = 1, NOREAL                                                    JTSM0590
       IF ( REALBD(MM) .EQ. 0.0 ) GO TO 1970                                     JTSM0591
       IF ( REALBD(MM) .EQ. 1.0 ) WRITE (6, 1021) MM                             JTSM0592
       IF ( REALBD(MM) .EQ. 2.0 ) WRITE (6, 1022) MM                             JTSM0593
       IF ( REALBD(MM) .EQ. 3.0 ) WRITE (6, 1025) MM                             JTSM0594
       IF ( REALBD(MM) .EQ. 4.0 ) WRITE (6, 1024) MM                             JTSM0595
1970   CONTINUE                                                                  JTSM0596
       NREAL1 = NOREAL - NBUM                                                    JTSM0597
       NREAL2 = NOREAL - NBUM - NNBUM                                            JTSM0598
       IF ( NREAL2 .LT. 3 ) GO TO 2000                                           JTSM0599
       IF ( NREDOP .EQ. -1 .OR. NREDOP .GT. 1 ) AP1MAN = SUMAP1/(NREAL1)         JTSM0600
       IF ( NREDOP .EQ. 0 .OR. NREDOP .GT. 1 ) AP2MAN = SUMAP2/(NREAL2)          JTSM0601
       IF ( NREDOP .GE. 1 ) AP3MAN = SUMAP2/(NREAL1)                             JTSM0602
1974   PSIMAN = PSISUM/(NREAL1)                                                  JTSM0603
       CONMAN = CONSUM/NREAL1                                                    JTSM0604
       VARAP1 = 0.0                                                              JTSM0605
       VARAP2 = 0.0                                                              JTSM0606
       VARAP3 = 0.0                                                              JTSM0607
       VARPSI = 0.0                                                              JTSM0608
       VARCON = 0.0                                                              JTSM0609
       CCAP1 = 0.0                                                               JTSM0610
       CCAP2 = 0.0                                                               JTSM0611
       CCAP3 = 0.0                                                               JTSM0612
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      DO 1980 I = 1, NOREAL                                                    JTSM0613
      IF ( REALBD(I) .EQ. 1.0 .OR. REALBD(I) .EQ. 2.0 .OR.                     JTSM0614
     1 REALBD(I) .EQ. 4.0 ) GO TO 1980                                         JTSM0615
      VARPSI = VARPSI + (PSI(I)  - PSIMAN)**2                                  JTSM0616
      VARCON = VARCON + (CONMAN - PERMAX(I))**2                                JTSM0617
      IF ( NREDOP .EQ. -1 .OR. NREDOP .GT. 1 )                                 JTSM0618
     1VARAP1 = VARAP1 + (APPER1(I) -AP1MAN)**2                                 JTSM0619
      IF ( NREDOP .EQ. -1 .OR. NREDOP .GT. 1 )                                 JTSM0620
     1CCAP1 = (APPER1(I) - AP1MAN)*(PSI(I) - PSIMAN) + CCAP1                   JTSM0621
      IF ( NREDOP .GE. 1 )                                                     JTSM0622
     1VARAP3 = VARAP3 + (APPER3(I) -AP3MAN)**2                                 JTSM0623
      IF ( NREDOP .GE. 1 )                                                     JTSM0624
     1CCAP3 = (APPER3(I) - AP3MAN)*(PSI(I) - PSIMAN) + CCAP3                   JTSM0625
      IF ( REALBD(I) .EQ. 3.0 ) GO TO 1980                                     JTSM0626
      IF ( NREDOP .EQ.  0 .OR. NREDOP .GT. 1 )                                 JTSM0627
     1VARAP2 = VARAP2 + (APPER2(I) -AP2MAN)**2                                 JTSM0628
      IF ( NREDOP .EQ.  0 .OR. NREDOP .GT. 1 )                                 JTSM0629
     1CCAP2 = (APPER2(I) - AP2MAN)*(PSI(I) - PSIMAN) + CCAP2                   JTSM0630
 1980 CONTINUE                                                                 JTSM0631
      SD1 = (VARAP1/NREAL1)**0.5                                               JTSM0632
      SD2 = (VARAP2/NREAL2)**0.5                                               JTSM0633
      SD3 = (VARAP3/NREAL1)**0.5                                               JTSM0634
      SDPSI = (VARPSI/NREAL1)**0.5                                             JTSM0635
      SDCON = (VARCON/NREAL1)**0.5                                             JTSM0636
      IF ( SDPSI .EQ. 0.0 ) GO TO 1983                                         JTSM0637
      IF ( VARAP1 .NE. 0.0 ) CC1 = CCAP1/(SD1*SDPSI*NREAL1)                    JTSM0638
      IF ( VARAP2 .NE. 0.0 ) CC2 = CCAP2/(SD2*SDPSI*NREAL2)                    JTSM0639
      IF ( VARAP3 .NE. 0.0 ) CC3 = CCAP3/(SD3*SDPSI*NREAL1)                    JTSM0640
 1983 WRITE (6,1122) NREAL1, PSIMAN, PSIMIN, PSIMAX, SDPSI                     JTSM0641
      WRITE (6,1123) NREAL1, CONMAN, CONMIN, CONMAX, SDCON                     JTSM0642
      IF ( NREDOP .EQ. -1 .OR. NREDOP .GT. 1 )                                 JTSM0643
     1WRITE (6, 1124) NRED1, NREAL1, AP1MAN, AP1MIN, AP1MAX, SD1, CC1          JTSM0644
      IF ( NREDOP .EQ. 0 .OR. NREDOP .GT. 1 )                                  JTSM0645
     1WRITE (6, 1124) NRED2, NREAL2, AP2MAN, AP2MIN, AP2MAX, SD2, CC2          JTSM0646
      IF ( NREDOP .GE. 1 )                                                     JTSM0647
     1WRITE (6, 1124) NRED3, NREAL1, AP3MAN, AP3MIN, AP3MAX, SD3, CC3          JTSM0648
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 2000 CONTINUE                                                          JTSM0649
      STOP                                                              JTSM0650
      END                                                               JTSM0651
      SUBROUTINE ROCK(CORK,PHORK,SINRK,COSRK,TANRK,SIG1,SIG3,PI4,PI2,   JTSM0652
     1CORK2K,BETAR,AFACT,BFACT,ETA,XCTPTF,DELTA3,TAUFR,TANIDX,PRIMIN)   JTSM0653
      LOGICAL FAM                                                       JTSM0654
      FAM = .FALSE.                                                     JTSM0655
      IF ( BETAR .LE. PI2 ) XMNTLN = AFACT + BFACT*SIN(2.0*BETAR - ETA) JTSM0656
      IF ( BETAR .GT. PI2 ) XMNTLN = AFACT - BFACT*SIN(2.0*BETAR + ETA) JTSM0657
      IF ( SIG1 .GT. XMNTLN ) GO TO 619                                 JTSM0658
C             THIS PART EXAMINES CIRCLES WITH PURE TENSILE FAILURES     JTSM0659
      TANIDX = -1.0                                                     JTSM0660
      IF ( BETAR .EQ. PI2 ) XCTR = (SIG1 - XCTPTF)*0.5                  JTSM0661
      IF ( BETAR .NE. PI2 ) XCTR = 0.5*((SIG1 + XCTPTF)/                JTSM0662
     1 ((SIN(BETAR))**2.0) - CORK)                                      JTSM0663
      IF ( XCTR .GT. XCTPTF ) GO TO 614                                 JTSM0664
      RADIUS = XCTPTF + XCTR                                            JTSM0665
      IF ( BETAR .GT. PI2 ) FAM = .TRUE.                                JTSM0666
      GO TO 625                                                         JTSM0667
C             THIS PART EXAMINES CIRCLES TANGENT TO THE PARABOLA        JTSM0668
  614 TANIDX = 0.0                                                      JTSM0669
      ZETA1 = PI2                                                       JTSM0670
      VERT1 = 2.0*CORK*(SIN(BETAR))**2.0 - XCTPTF                       JTSM0671
      ZETA2 = ETA                                                       JTSM0672
      VERT2 = XMNTLN                                                    JTSM0673
  615 ZETA = (ZETA1 + ZETA2)/2.0                                        JTSM0674
      XCTR = (CORK*0.5)/((SIN(ZETA))**2.0)                              JTSM0675
      IF ( BETAR .LE. PI2 ) VERT = XCTR + CORK*SIN(2.0*BETAR - ZETA)/   JTSM0676
     1 SIN(ZETA)                                                        JTSM0677
      IF ( BETAR .GT. PI2 ) VERT = XCTR - CORK*SIN(2.0*BETAR + ZETA)/   JTSM0678
     1 SIN(ZETA)                                                        JTSM0679
      IF ( SIG1 .GT. VERT .AND. SIG1 .LE. VERT2 ) VERT1 = VERT          JTSM0680
      IF ( SIG1 .GT. VERT .AND. SIG1 .LE. VERT2 ) ZETA1 = ZETA          JTSM0681
      IF ( SIG1 .GT. VERT1 .AND. SIG1 .LE. VERT ) VERT2 = VERT          JTSM0682
      IF ( SIG1 .GT. VERT1 .AND. SIG1 .LE. VERT ) ZETA2 = ZETA          JTSM0683
      ABVERT = ABS(VERT - SIG1)                                         JTSM0684
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      IF ( ABVERT .GE. CORK2K ) GO TO 615                                   JTSM0685
      RADIUS = CORK/SIN(ZETA)                                               JTSM0686
      ZETA45 = ZETA*0.5 + PI4                                               JTSM0687
      ZETA35 = 3.0*PI4 - ZETA*0.5                                           JTSM0688
      IF ( BETAR .GT. ZETA45 .AND. BETAR .LE. PI2 ) FAM = .TRUE.            JTSM0689
      IF ( BETAR .GT. ZETA35 ) FAM = .TRUE.                                 JTSM0690
      GO TO 625                                                             JTSM0691
C             THIS PART EXAMINES CIRCLES TANGENT TO THE STRAIGHT LINE       JTSM0692
  619 IF ( BETAR .LE. PI2 ) QFACT = SIN(2.0*BETAR - PHORK)                  JTSM0693
      IF ( BETAR .GT. PI2 ) QFACT = -SIN(2.0*BETAR + PHORK)                 JTSM0694
      XMXTLN = CORK*QFACT*COSRK + CORK*TANRK*(1.0 + QFACT*SINRK)            JTSM0695
      IF ( PHORK .LE. PI4 .OR. SIG1 .GT. XMXTLN ) GO TO 624                 JTSM0696
      TANIDX = 2.0                                                          JTSM0697
      ZETA1 = PI4                                                           JTSM0698
      VERT1 = XMNTLN                                                        JTSM0699
      ZETA2 = PHORK                                                         JTSM0700
      VERT2 = XMXTLN                                                        JTSM0701
  622 ZETA = (ZETA1 + ZETA2)/2.0                                            JTSM0702
      IF ( BETAR .LE. PI2 ) RFACT = SIN(2.0*BETAR - ZETA)                   JTSM0703
      IF ( BETAR .GT. PI2 ) RFACT = -SIN(2.0*BETAR + ZETA)                  JTSM0704
      VERT = CORK*RFACT*COS(ZETA)+CORK*TAN(ZETA)*(1.0+RFACT*SIN(ZETA))      JTSM0705
      IF ( SIG1 .GT. VERT .AND. SIG1 .LE. VERT2 ) VERT1 = VERT              JTSM0706
      IF ( SIG1 .GT. VERT .AND. SIG1 .LE. VERT2 ) ZETA1 = ZETA              JTSM0707
      IF ( SIG1 .GT. VERT1 .AND. SIG1 .LE. VERT ) VERT2 = VERT              JTSM0708
      IF ( SIG1 .GT. VERT1 .AND. SIG1 .LE. VERT ) ZETA2 = ZETA              JTSM0709
      ABVERT = ABS(VERT - SIG1)                                             JTSM0710
      IF ( ABVERT .GE. CORK2K ) GO TO 622                                   JTSM0711
      XCTR = CORK*TAN(ZETA)                                                 JTSM0712
      RADIUS = (XCTR**2.0 + CORK**2.0)**0.5                                 JTSM0713
      ZETA45 = ZETA*0.5 + PI4                                               JTSM0714
      ZETA35 = 3.0*PI4 - ZETA*0.5                                           JTSM0715
      IF ( BETAR .GT. ZETA45 .AND. BETAR .LE. PI2 ) FAM = .TRUE.            JTSM0716
      IF ( BETAR .GT. ZETA35 ) FAM = .TRUE.                                 JTSM0717
      GO TO 625                                                             JTSM0718
  624 TANIDX = 1.0                                                          JTSM0719
      XCTR = (SIG1 - CORK*QFACT*COSRK)/(1.0 + QFACT*SINRK)                  JTSM0720
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      RADIUS = (CORK/TANRK + XCTR)*SINRK                                  JTSM0721
      IF ( BETAR .GT. PI4 +PHORK*0.5 .AND. BETAR .LE. PI2 ) FAM = .TRUE.  JTSM0722
      IF ( BETAR .GT. 3*PI4 - PHORK*0.5 ) FAM = .TRUE.                    JTSM0723
  625 TAUFR = ( ABS(RADIUS**2 - (SIG1-XCTR)**2))**0.5                     JTSM0724
      PRIMIN = XCTR - RADIUS                                              JTSM0725
  626 IF (FAM) TAUFR = (-1.0)*TAUFR                                       JTSM0726
      DELTA3 = SIG3 - (2.0*XCTR - SIG1)                                   JTSM0727
      RETURN                                                              JTSM0728
      END                                                                 JTSM0729
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