

Investors' Horizon and Stock Prices

by

Sahar Parsa

B.A., Université libre de Bruxelles (2004)

Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Economics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2011

© Sahar Parsa. All Rights Reserved

The author hereby grants to Massachusetts Institute of Technology permission to
reproduce and
to distribute copies of this thesis document in whole or in part.

Signature of Author

.....
Department of Economics
May 13, 2011

Certified by

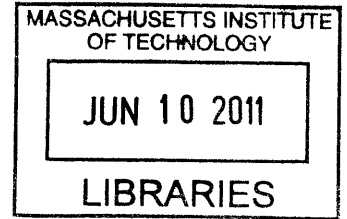
.....
George-Marios Angeletos
Professor of Economics
Thesis Supervisor

Certified by

.....
Guido Lorenzoni
Associate Professor of Economics
Thesis Supervisor

Accepted by

.....
Esther Dufflo
Abdul Latif Jameel Professor of Poverty Alleviation and Development Economics.
Chairman, Departmental Committee on Graduate Studies



ARCHIVES

Investors' Horizon and Stock Prices

by

Sahar Parsa

Submitted to the Department of Economics
on May 13, 2011, in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy in Economics

Abstract

This dissertation consists of three essays on the relation between investors' trading horizon and stock prices. The first chapter explores the theoretical relation between the horizon of traders and the negative externality generated by their activity on the information revealed by stock prices. The last two chapters focus on the empirical relation between institutional investors trading frequency and stock prices behaviour.

The first chapter examines how short term trading impacts the aggregation of information in financial markets. I develop a model where short-term traders, in an attempt to learn about the average beliefs of future market participants, make the price relatively more noisy. This typically introduces a negative informational externality on long-term investors. I show that (i) as the horizon of the informed traders decreases, the price becomes relatively less precise; (ii) an inflow of informed traders in the market can decrease the informativeness of the price when the traders have a relatively short horizon or the market is expected to be thin in the future; (iii) finally, as rational informed short-term traders have access to an extra source of information about the future price, they end up creating more noise and a decrease in the informativeness of the price might result. Thus, paradoxically, more informed trading could lead to a less informative price.

Among scholars, practitioners and policy makers, investor short-termism and high frequency trading have been associated with excess volatility in financial markets and with a disconnect between asset prices and fundamentals. Motivated by this observation, in Chapter 2 I construct a novel measure of the intrinsic frequency of trading for each of the large US institutional investors (13-F institutions) using Thomson-Reuters Institutional Holdings quarterly data for the period 1980-2005. This measure controls for the market and portfolio characteristics and identifies an investor-specific fixed effect in the frequency of trading. I then study how the composition of these fixed effects impacts stock price behavior through their forecasting role in explaining the return and the return on equity (cash flow of a company) in the short run as well as the long run. I show that (i) the securities in which investors exhibit higher intrinsic trading frequency exhibit higher volatility, but (ii) this volatility is mainly driven by the cash-flow component of the security prices. Further, (iii) the prices of the securities held by investors with a higher intrinsic trading frequency do not forecast the long-run return as opposed to the securities held by investors with a lower intrinsic trading frequency. As such, the prices mainly respond to the long-run return on equity. Overall, the results challenge the view that higher frequency of trading—a commonly used proxy for investor short-termism—causes a disconnect

between asset prices and fundamentals.

Finally, in Chapter 3 (co-authored with Fernando Duarte) we show a novel relation between the institutional investors' intrinsic trading frequency—a commonly used proxy for the investors's investment horizon— and the cross-section of stock returns. We show that the 20% of stocks with the lowest trading frequency earn mean returns that are 6 percentage points per year higher than the 20% of stocks that have the highest trading frequency. The magnitude and predictability of these returns persist or even increase when risk-adjusted by common indicators of systematic risks such as the Fama-French, liquidity or momentum factors. Our results show that the characteristics of stockholders affect expected returns of the very securities they hold, supporting the view that heterogeneity among investors is an important dimension of asset prices.

Thesis Supervisor: George-Marios Angeletos
Title: Professor of Economics

Thesis Supervisor: Guido Lorenzoni
Title: Associate Professor of Economics

Contents

1	Short-Term Traders, Learning and Informational Externalities	8
1.1	Introduction	8
1.2	Model	14
1.3	Equilibrium	18
1.3.1	Definition	18
1.3.2	Intertemporal two-ways feedback	20
1.3.3	Characterization of the equilibrium	27
1.4	Information aggregation	30
1.5	Beliefs signal	34
1.6	Alternative interpretation	40
1.7	Conclusion	42
1.8	Appendix	44
1.8.1	Appendix A	44
1.8.2	Appendix B	45
1.8.3	Appendix C	49
2	Institutional Investors' Short-Termism, Trading Frequency and Firm-Level Stock Price Volatility	56
2.1	Introduction	56
2.2	Literature Review	63
2.3	Data Description and Methodology	66

2.3.1	Institutional Investor's Trading Frequency Fixed Effect	67
2.3.2	A Security Trading Frequency Index	74
2.4	Institutions' Trading Frequency and Forecastability	77
2.4.1	Forecastability of the Return and the Return on Equity	78
2.4.2	Return Forecastability for the securities held by high trading frequency investors	83
2.4.3	Return Decomposition	86
2.5	Conclusion	95
2.6	Appendix: Data Description	97
3	Institutional Investors' Intrinsic Trading Frequency and the Cross-Section of Stock Returns¹	112
3.1	Introduction	112
3.2	Literature Review	115
3.3	Data Description and Methodology	118
3.3.1	Data Description	119
3.3.2	Methodology	120
3.3.3	Security Specific Trading Frequency	121
3.3.4	Cross-Section of Expected Stock Return	123
3.4	Results	124
3.4.1	Is there a relation between trading frequency and returns?	124
3.4.2	Can we explain trading frequency returns by systematic risks?	125
3.4.3	Can we explain trading frequency returns by a trading frequency index?	128
3.5	Conclusion	129

¹Joint with Fernando Duarte

Acknowledgements

I am deeply grateful to my advisors - George-Marios Angeletos, Victor Chernozhukov, Leonid Kogan and Guido Lorenzoni. They allowed me to think about questions outside my boundaries and helped me become a much better researcher. I hope I will continue learning from them for many years in the future. Marios has been a great advisor. Without his feedback and confidence in me, my research life at MIT would have been very different. Victor was always willing to listen to me and helped me in surprising ways. Leonid very generously agreed to help in the very last stretch and gave me the type of feedback I needed at exactly the right moment. His impeccable attitude towards research will forge my own standards. Finally, I thank Guido for his patience and willingness to listen to my ideas at various stages of my research.

I am greatly indebted to Nancy Rose for her support on the job market and Esther Duflo to keep us grounded semester after semester. I would also like to thank the staff at the Department of Economics at MIT for all their help during these years, in particular Emily Gallagher and Linda Woodbury. I am also grateful for the financial support from the Belgian American Education Foundation and from the Department of Economics at MIT.

I also want to thank the priceless feedback from my classmates, in particular Sergi Basco and Fernando Duarte, who are not only great colleagues, but also amazing friends. I am lucky to have received the amazing support from my friends of the old and the new world. First, I wish to thank Aurian, Chul, Dana and Julie for reminding me that life is more colorful than research. Chul, I appreciate your effortless friendship and making my life in Boston a bit more like holidays. I want to thank Aurian for our long philosophical conversations and the Korean Chorus. I am so grateful to Julie for allowing me to be myself in such a light way. Finally, I want to thank Dana for being Dana. The last 6 years have also been rich in lasting friendships from

the new world. In particular, I want to thank Nicolas Arregui, Suman Basu, Laura Feiveson, JuanRi, Mariangi, Negris, Samuel Pienknagura, Vincent Pons, Johannes Spinnewijn, Karen Verschoren and Tom Wilkening who put up with my existential crisis and my whining as well as my good times. I want to thank Johannes for his mentorship. Despite being 2 months younger than me, he kept me grounded and pragmatic. Finally, I am fortunate to have met Jim, whose friendship I value and proved essential at a crucial crossroads.

Last but not least, I thank my family for all their love and support. They always make me feel close to home and part of a bigger team. This thesis is dedicated to Nounou (the newest arrival), Pablo, Saba, Sepi as well as my mom and my father who made me who I am today, and gave me the strength to never give up, stand up for myself and never settle for a compromised life. They make me a better person. Finally, I would like to thank Pablo Querubin, who helped me keep my faith in the profession. His help, support and our conversations have been priceless. Writing this dissertation would have been a very different experience without them.

Chapter 1

Short-Term Traders, Learning and Informational Externalities

1.1 Introduction

Financial markets are populated by traders with different horizons that meet at a given time. Some traders have short horizons. They enter and exit the market within a day or a week. Other traders have a longer horizon. Typically, they buy an asset from a firm and hold on to the asset for much longer periods. Hence, long term traders are likely to care relatively more about the underlying fundamental value of the stock they invest in than about predicting short-term movements. In contrast, short term traders who decide to buy a risky asset today to sell it tomorrow, will care about forecasting the price in the market tomorrow. Nevertheless, the price tomorrow will depend on the forecast of the price the day after tomorrow, and so on. Using backward induction, short term traders should also care about forecasting the fundamental value of the asset they will buy.

In a world with symmetric information, by applying the law of iterated expectations, today's forecast of tomorrow's forecast of the day after tomorrow's price is given by today's forecast of the day after tomorrow's price. Generalizing, the intertemporal higher order forecast boils down to today's forecast. Hence, even though a trader cares about the future price, ultimately the only object of interest reduces to the fundamental. Short-term traders' forecast of the price

at which they will sell the asset comes down to the forecast of the fundamental value just as with long term traders.

On the other hand, in the presence of asymmetric information, traders with different horizons often behave differently and try to predict very different objects. In particular, short term traders seem to gather and use different information than long term traders who mainly use information about the underlying value of the company. In an economy with asymmetric information, traders' inference of the price at which they will sell the asset differs from their inference of the fundamental value.¹ Rational short term traders will want to learn about the average forecast of future traders in order to forecast the future price. Typically, this will give an important role to the horizon of traders in financial markets.

In this paper, I study the incidence of the horizon of traders on the informational efficiency of financial markets. One of the major roles of financial markets appears to be the allocation of scarce resources from investors to firms with production-investment decisions. The information revealed by the price appears to be crucial in the optimality of the allocation decision of investors. In a world with uncertainty and idiosyncratic information, the price can be seen as a vehicle, which aggregates and reveals this dispersed information to market participants. In particular, uninformed long term investors might want to use the information revealed by the price to allocate their resources to the best investment opportunity.² The negative externality that short-term traders generate on the informational efficiency of financial markets, has often been at the center of the policy debate on the role of short-termism (e.g. Bombay Stock Exchange market regulations: Abolition of the Badla facility (1993)). This has been particularly striking in the aftermath of the financial crisis. Understanding the related mechanism behind this friction and its respective positive implications is a necessary step for the underlying debate.

¹DeLong, Summers, Shleifer and Waldmann (1990b) first introduced the mechanism, where short-term rational traders try to forecast the beliefs of future irrational traders reflected in the price at which they sell the asset. More recently, Allen, Morris and Shin (2006) exploits the same idea by highlighting how the failure of the law of iterated expectations in the presence of asymmetric information leaves a special role to higher order beliefs as traders try to forecast the future price. Alternative mechanisms have been highlighted that give a role for the horizon in financial markets, e.g. DeLong, Summers, Shleifer and Waldmann (1990a).

²Another channel through which the informativeness of the price will matter, is the information it conveys to managers about the value of the investment they want to undertake. Informed investors in financial markets might gather a lot of important information about the value of an investment and not convey it directly to managers for strategic reasons. Hence, the price of the asset of their own firm can be a good signal of the value of the investment.

In this paper, I look at a market constituted by some traders who care only about the fundamental of the underlying firm, i.e. long term investors, and by other traders who care only about the future price, i.e. short term traders. I consider the horizon of the traders as an exogenous characteristic. Rational, short term, and risk neutral informed traders generate a negative externality on uninformed long term investors because their behavior will make the price less informative. This externality is precisely due to the informational friction created by short term traders, as they are trying to forecast the average forecast of future market participants. The central mechanism lies in the informational friction created by shorter horizon traders. This friction precisely results from the fact that in the process of forecasting the price in the future, short term traders may want to learn about the average beliefs of the market in which they'll be selling their assets. Such information about average beliefs will not necessarily be perfectly correlated with the information about the fundamental and hence, a negative informational externality on non-informed traders might result. First, I show that as the horizon of the traders decreases, the price becomes less informative. The shorter the horizon of the trader, the more he cares about near-term price movements, which are determined in the future market. Forecasting the future market price makes the traders use their information in such a way that the price ends up less accurate about the fundamental. Secondly, I show that an inflow of informed traders can be associated with a decrease in the quality of the information revealed by the price. A market characterized by very short term traders and/or a very thin market in the future, is more likely to experience a negative effect following an inflow of informed traders.

Third, I take a closer look at the inference problem of short term traders. I extend the model and explicitly allow the informed short term traders to have access to an information technology, which is informative only about the average beliefs of the market participants in the future. This signal is independent of the fundamental of the underlying asset.³ One could look at this new signal as investor sentiments. Rational short term traders might want to use the beliefs signal as it will give them valuable information about the average beliefs of future market participants and ultimately future prices. This signal is closely related to a recent paper

³This embodies the idea that traders in financial markets are interested in forecasting the forecast of other market participants. Ultimately, this will translate in them using the beliefs signal which is not related to the fundamental value of the asset.

by Angeletos and La'o (2011) where they assess the role played by sentiments and psychology in real business cycles in a world with rational agents. The value of introducing the beliefs signal is that it allows me to study a change in the structure of traders' beliefs about the average beliefs of future traders, independently of a change in their first order beliefs about the fundamental. I show that, in equilibrium, the use of this signal can lead to a negative informational externality such that the price will end up being less informative. Relatedly, increasing the precision about future agents' beliefs is not associated with an increase in the precision of first order beliefs about the fundamental. Conditional on short-term traders using the beliefs signal, an increase in its precision may end up creating a negative informational externality on long term investors.

Related literature. This paper builds on and relates to various existing literature. It builds on the literature that formalizes Keynes' higher order beliefs in asset pricing models in understanding financial markets' inefficiencies.⁴ On one hand, Harrison and Kreps (1978) introduce a heterogeneous beliefs based asset pricing models with short sales constraints (as a source of limit to arbitrage), where the stock price is generally higher than its fundamental value.⁵ The difference is given by an option value to resell the asset to future investors with a higher valuation. Higher order beliefs matter through the option value, as it depends on the opinions of other investors. On the other hand, in DeLong, Summers, Shleifer and Waldmann (1990b), short-term rational traders generates excess movements in the prices as they try to forecast the beliefs of future irrational traders, i.e. positive-feedback traders extrapolating past price trends, reflected in the price at which they will sell the asset. This paper uses this channel – trading based on forecasting the behavior of others – in understanding the effect of traders' horizon for stock prices informativeness. However, the focus is on rational investors forecasting rational investors.

As such, this paper builds on the more recent literature that analyzes the higher order expectations in noisy rational expectation asset pricing models, where traders have a common prior, but different information over the fundamentals of the asset; Allen, Morris and Shin

⁴Keynes compares investors to beauty-contest judges who vote based on what they believe other judges believe....about contestants' beauty (higher order beliefs) instead on their opinion about contestants' absolute beauty.

⁵An example of heterogenous beliefs is given by adding overconfident agents as in Scheinkman and Xiong (2003). See also Allen, Morris, and Postlewaite (1993), Biais and Boessarts (1998), Zhou (1998) and Banerjee, Kaniel and Kremer (2009) for more articles related to this strand of the literature.

(2006), Bacchetta and Van Wincoop (2008), Cespa and Vives (2009), Grisse (2009).⁶ In particular, Allen, Morris and Shin (2006) show that in the presence of dispersed information and short-lived traders, higher order expectations have a specific role to play, leading traders to overweight the public signals relative to their private signals as the public signals have an extra commonality dimension. The main channel comes from the failure of the law of iterated expectations for average expectations in the presence of asymmetric information. First order expectations differ from higher order expectation and the price deviates systematically from the average expectations of the fundamental value of the asset. By considering a simplified framework, I highlight novel positive implications and bring more insights on the role of the horizon of the traders on the behavior of the traders in the presence of asymmetric information as well as the frictions it generates.

A number of articles have stressed the role of short-term speculation in generating inefficiencies in financial markets; Tirole (1982), DeLong, Summers, Shleifer and Waldmann (1990a), Holden and Subrahmanyam (1996), Kondor (2009), Cespa and Vives (2011) among others.⁷ Tirole (1982) calls attention to the role of myopic traders in asset market bubbles as they break down the backward induction argument. DeLong, Summers, Shleifer and Waldmann (1990a) provides an alternative to the higher order beliefs channel for the horizon to be associated to a disconnect between the stock price and its fundamental value. The short-investment horizon limits risk-averse speculators' capacity to arbitrage the mispricing generated by noisy traders due to the unpredictability of the noise traders mispricing, which introduces a systematic risk in the short-run.⁸

⁶The noisy rational expectation equilibrium models have been introduced in Grossman (1976), Hellwig (1980), and Diamond and Verrecchia (1981), Admati (1985). Even though mentioned in He and Wang (1995) or Foster and Viswanathan (1996), the role of higher order beliefs had not been examined in this class of models until recently with Allen, Morris and Shin (2006), Bacchetta and Van Wincoop (2008).

⁷Holden and Subrahmanyam (1996) find that in an economy with long run and short run information, if investors are sufficiently risk-averse, they focus on the short run information. Kondor (2009) show that public announcements can feed the disagreement among short-term Bayesian traders and hence, increase the trading volume in equilibrium. By introducing heterogeneity across traders in terms of their trading frequency, Cespa and Vives (2011) connect asset illiquidity and over-reliance on public information through higher order expectations (HOE) about the fundamentals and the liquidity trades. They find that if asset prices are dominated by HOE's about fundamentals (liquidity trades), they over(under)-rely on public information, the market is illiquid (liquid) and the volume of informational trading is low (high).

⁸Due to their short horizon, arbitrageurs buying an asset with a price lower than its fundamental, would have to resell it in the near future and bear the risk that the mispricing increases. This risk, which they assume to be systematic, limits the aggressiveness of arbitrageurs

By relying on the presence of short-lived traders to highlight their source of inefficiency (e.g. higher order beliefs, noise traders), these studies bring to light an implicit difference between short and long horizon agents. Nevertheless, they don't directly address the horizon of traders in generating these inefficiencies. I explore explicitly the role of the horizon on the deviation of the price from its fundamentals by modeling the horizon as the probability of trading in the market in the next period.⁹ To that extent, this paper is related to Froot, Scharfstein and Stein (1992), Dow and Gorton (1994), Vives (1995). In particular, Froot, Scharfstein and Stein (1992) show how short-termism induces complementarities in the acquisition of information. Short-term investors have an incentive to collect/pay attention to the same (or correlated) pieces of information about the fundamentals, because this helps them speculate on short-run price movements. These positive information spillovers reduce the informational efficiency of prices and increase the level of "noise". On the other hand, in the present paper the horizon introduces intergenerational complementarities between traders today and tomorrow. As a consequence, short-term traders want to put more weight on information future traders will be using even though no other same period traders are using this information. This behavior decreases the information aggregated by the price.

This paper also connects to the behavioral finance literature; see Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Hong and Stein (1999), and Barberis and Thaler (2003) among others.¹⁰ The behavioral finance literature offers explanations for the financial market anomalies using under-reaction, over-reaction phenomenon. The disconnect is generated in a model where rational short-term agents are over-reacting to their public signal or their beliefs signal, in an attempt to forecast the forecasts of other rational agents. I exploit the use of information as opposed to noisy or irrational traders as the central mechanism. The assumption of risk neutral agents precisely allows me to focus on the use of information to forecast the forecasts of future traders by ignoring any risk aversion consideration.

⁹Later, I show how this probability can be micro-founded by the cost of portfolio re-balancing that an investor is facing.

¹⁰Barberis, Shleifer, and Vishny (1998) use a learning model where the earnings follow a random walk. However, as individuals believe they either follow a steady growth trend or are mean-reverting, it generates over and under-reaction. Identically, in Daniel, Hirshleifer, and Subrahmanyam (1998) investors under-react to public news as they are characterized by overconfidence and attribution bias. Finally, Hong and Stein (1999) use a model of gradual information diffusion where the heterogeneity among investors positively correlates with momentum. Barberis and Thaler (2003) provide a summary of the literature

As such, the approach in this paper is complementary as it provides similar phenomenon (e.g. overreaction, excess volatility, prices disconnected from the fundamental) without behavioral traders (expect for the noisy supply).¹¹

Finally, this paper is connected to the recent empirical literature on the relation between the trading frequency of institutional investors and its interaction with stock price behavior motivated by the effect of the investment horizon of institutional investors; Ke, Ramalingegowda and Yu (2006), Jin and Kogan (2007), Khan, Kogan and Serafeim (2010), Parsa (2010), Duarte and Parsa (2011), Yan and Zhang (2009). By parameterizing the horizon of the investors, this paper highlights a positive relation between the horizon and stock price behavior, which can be linked back to the previous literature. In particular, the horizon is modeled as the probability of trading in the market in the next period. It is shown that this probability can be micro-founded by the cost of portfolio re-balancing that an investor is facing. As such, there is a tight link between the notion of horizon used in this paper and the investment horizon as well as the trading frequency.

The remainder of the paper is structured as follows. Section 2 introduces the baseline model. Section 3 characterizes the equilibrium. Section 4 defines a measure for the informativeness of the market, presents and illustrates the main comparative statics. Section 5 introduces the beliefs signal, characterizes the new equilibrium and shows the comparative statics on the effect of the precision of the beliefs signal on the informativeness of the price. Section 6 discusses an alternative interpretation for the horizon of the traders. Finally, section 7 concludes. All proofs are in the Appendix.

1.2 Model

In this section, I describe in details the general model that I am going to use to study the role of short-term trading on the informational efficiency of the market.

¹¹As will be discussed later, in the class of noisy rational expectation models, the limit to arbitrage alongside the noisy supply's main purpose is the existence of an equilibrium by allowing the information to remain dispersed and the price to not be fully revealing. The limit to arbitrage is provided through a cost of investing in the risky asset, given the risk neutrality assumption. The inefficiency is derived from higher order beliefs and the use of information and does not directly depend on the noisy supply. This is in contrast to the behavioral finance framework discussed above.

Timing, Actions and Information

There are 3 periods in the economy indexed by $t = 1, 2, 3$. There are two assets. A safe asset yielding a risk-free rate of zero as it is taken to be the numeraire and its price remains at 1. A risky asset with a liquidation value of θ unknown to the traders until $t = 3$.

At $t = 1$, two groups of agents of measure-one and measure- λ_1 continuum enter the market, denoted L and S_1 respectively. A trader in the S_1 group enters the market at $t = 1$ and decides how much to trade in the risky asset at the price p_1 . When making the decision to trade in the risky asset, the trader faces uncertainty about being hit by a "liquidity shock" at $t = 2$ with probability $(1 - r) \in [0, 1]$ which will force him to sell the asset at p_2 . A "liquidity shock" is identical to a preference shock which will make traders want to consume at $t = 2$. If the trader is not hit by the liquidity shock which will occur with probability r , he will hold on to the asset and receive the liquidation value θ at $t = 3$. One can think of r as the horizon of the trader.¹² For $r = 0$, traders are short-termist and they decide the amount they are going to invest in the risky asset given they will consume at $t = 2$. As a consequence, traders care about their capital gain, p_2 . For $r = 1$, traders are long-termist and invest in the asset given they will receive the fundamental value θ . For $r \in (0, 1)$, an increase in r makes traders less sensitive to p_2 and more sensitive to θ as it becomes less likely to be hit by the liquidity shock. One can summarize the ex ante return of the S_1 group trader at $t = 1$:

$$R_i = r\theta + (1 - r)p_2, \forall i \in S_1$$

The L group enters the market at $t = 1$. An agent in the L group is defined as an investor, i.e. he is purely long-termist. He needs to decide how much to invest in the risky asset at the price p_1 given he will receive θ at $t = 3$. His return is given by:

$$R_i = \theta, \forall i \in L$$

¹²In the last section, I will provide an alternative interpretation for r . I will associate r with the cost an agent is facing of rebalancing its portfolio. This cost can be defined by real costs such as taxes, administrative costs as well as subjective costs related to preferences.

At $t = 2$, the traders hit by the liquidity shock will sell their position to a new group of traders denoted S_2 of measure- λ_2 continuum that enters the market. A trader in the S_2 group decides how much to invest in the risky asset at p_2 given he will receive the liquidation value θ . Trader's return at $t = 2$ is given by:

$$R_i = \theta, \forall i \in S_2$$

At $t = 3$, all the remaining market participants receive and observe the liquidation value, θ .

The supply of the assets is noisy at $t = 1, 2$, $K_t^S = u_t$,¹³ is uncertain and not observed where $u_t \sim N(0, \sigma_u^2)$ is serially uncorrelated and independent of θ .

At the time of making their decision, each group of agents respectively has access to an information set constituted of private and public signals. At $t = 1$, each trader in the S_1 group and each investor in the L group is using respectively the following information set

$$I_{i_1} = \{x_{i1}, z_1, p_1\}, \forall i \in S_1 \quad (1.1)$$

$$I_{i_1} = \{z_1, p_1\}, \forall i \in L \quad (1.2)$$

where $x_{i1} = \theta + \varepsilon_{i1}$, $\varepsilon_{i1} \sim N(0, \alpha_{x_1}^{-1})$ iid; $z_1 = \theta + \varepsilon_1$, $\varepsilon_1 \sim N(0, \alpha_{z_1}^{-1})$; and they use the information provided by the price p_1 .

At $t = 2$, each trader in the S_2 group is using the following signals

$$I_{i_2} = \{x_{i2}, z_1, z_2, p_1, p_2\}, \forall i \in S_2 \quad (1.3)$$

where $z_2 = \theta + \varepsilon_2$, $\varepsilon_2 \sim N(0, \alpha_{z_2}^{-1})$; $x_{i2} = \theta + \varepsilon_{i2}$, $\varepsilon_{i2} \sim N(0, \alpha_{x_2}^{-1})$ iid; and they use the information in the history of prices p_1, p_2 .

As we can notice, the traders in the S_1 group have access to additional idiosyncratic information, the L group does not have. The price will appear typically as a vehicle that will aggregate the idiosyncratic information and reveal it to the market. One can interpret the

¹³The assumption of noisy supply is often made in rational expectations models for existence reasons in order to prevent the price from fully revealing the fundamental. Noisy supply can be seen as a simplifying device which captures alternative source of uncertainty which affects the price but not the fundamental. Noisy supply can be the result of noisy traders or liquidity traders who would trade for reasons exogenous to the model and have a noisy demand.

information structure as follow. The economy at $t = 1$ is composed of two groups of agents. The first group invested in information gathering and built an additional private signal. The second group is long-termist and has access to no additional signal.

Payoffs

All market participants are risk neutral. Let's define k_{it} by the individual demand of agent i at time t . There is an additional cost associated to investing in the risky asset which is given by $\gamma \frac{k_{it}^2}{2}$ where $\gamma \in \mathbb{R}^+$. The cost can be seen as a borrowing cost or a participation cost which is a function of the amount each agent is investing in the risky asset. The payoffs of the different groups can be summarized as follows.

At $t = 1$: The trader payoff is given by

$$E((1 - r)p_2 + r\theta - p_1|I_{i1})k - \gamma k^2/2, \forall i \in S_1$$

The investor payoff is given by

$$E(\theta - p_1|I_{i1})k - \gamma k^2/2, \forall i \in L$$

At $t = 2$: The payoff of the trader is given by

$$E(\theta - p_2|I_{i2})k - \gamma k^2/2, \forall i \in S_2$$

The bigger γ , the higher the cost of investing and the smaller the demand for risky asset. For $\gamma = 0$, the demand for the risky asset is infinitely elastic. Throughout the paper, I am going to consider cases where $\gamma > 0$. This assumption must hold for obvious existence reasons. The cost of investing in the risky asset alongside the noisy supply is necessary for the price to not be fully revealing and consequently the equilibrium to exist as the price will be a function of the noise and the fundamental. From an intuitive point of view, one can interpret the cost of investing in the risky asset as an obstacle to arbitrage tightly linked to the limits of arbitrage literature. The limit of arbitrage is often an important channel for the existence of inefficiencies in financial markets to be formalized. It is an essential component in behavioral finance, absence of which rational arbitrageur would eliminate irrational agents mispricing. In the absence of costs, the

demand of rational traders fully absorbs the noisy supply. As soon as they expect the price to be lower than the expected future payoffs of their investment, they take a negative position up to the point where the supposed mispricing disappears. On the other hand, if the price is higher, they buy as long as the mispricing exists and completely absorb it. If the investors are facing a cost of investing in the risky asset, which limits their ability to buy the risky asset, then they can not fully absorb any mispricing they estimate. Similar to the behavioral literature, in the absence of the limit to arbitrage, noise traders won't matter in the equilibrium stock prices. Nevertheless, in the class of noisy rational expectation models, the limit to arbitrage alongside the noisy supply's main purpose is the existence of an equilibrium by allowing the information to remain dispersed. The inefficiency I emphasize in this paper is derived from higher order beliefs and the use of information. It does not directly depend on the noisy supply. This is in contrast to the behavioral finance framework discussed above.

Finally, notice that the cost of investing in the risky asset is in slight contrast to usual limit of arbitrage used. The noisy rational expectations literature often uses risk aversion. More generally, risks, agency issues or market frictions (short-selling costs) are used as potential limit to arbitrage; DeLong, Summers, Shleifer and Waldmann (1990a), Shleifer and Vishny (1997), Scheinkman and Xiong (2003). One could interpret the risk neutral agents with cost of investing in the risky asset as institutional investors facing regulations in terms of their holding of risky assets rather than individual investors who could suffer more from risk aversion.

1.3 Equilibrium

In this section, I am going to define and characterize the equilibrium.

1.3.1 Definition

At $t = 1, 2$ individual asset demands are a function of the realizations of their respective signals and the price, $\forall i \in S_1, k_{i_1} : R^3 \rightarrow R$; $\forall i \in L, k_{i_1} : R^2 \rightarrow R$ and $\forall i \in S_2, k_{i_2} : R^5 \rightarrow R$ given by $k_{S_1}(x_{i_1}, z_1, p_1)$; $k_L(z_1, p_1)$ and $k_{S_2}(x_{i_2}, z_1, z_2, p_1, p_2)$. Their corresponding aggregates are then functions of (θ, z_1, p_1) ; (z_1, p_1) and $(\theta, z_1, z_2, p_1, p_2)$ respectively. They are denoted K_{S_1} ; K_L and K_{S_2} . Formally, the equilibrium is defined as :

Definition 1 An (symmetric) equilibrium is a price function for $t = 1, 2$, $P_1(\theta, z_1, u_1)$ and $P_2(\theta, z_1, z_2, p_1, u_1, u_2)$ and individual investment strategies for $t = 1, 2$, $k_{S_1}(x_{i1}, z_1, p_1)$; $k_L(z_1, p_1)$ and $k_{S_2}(x_{i2}, z_1, z_2, p_1, p_2)$ as well as their corresponding aggregates $K_{S_1}(\theta, z_1, p_1)$; $K_L(z_1, p_1)$ and $K_{S_2}(\theta, z_1, z_2, p_1, p_2)$ such that:

$$\begin{aligned}
k_{S_1}(x_{i1}, z_1, p_1) &\in \arg \max_{k \in \mathbb{R}} E[(1-r)p_2 + r\theta - p_1 | x_{i1}, z_1, p_1] k - \gamma \frac{k^2}{2} \quad (1.4) \\
k_L(z_1, p_1) &\in \arg \max_{k \in \mathbb{R}} E[\theta - p_1 | z_1, p_1] k - \gamma \frac{k^2}{2} \\
K_{S_1}(\theta, z_1, p_1) &= E[k_{S_1}(x_{i1}, z_1, p_1) | \theta, z_1, p_1] \\
K_L(z_1, p_1) &= k_L(z_1, p_1) \\
\lambda_1 K_{S_1}(\theta, z_1, p_1) + K_L(z_1, p_1) &= K_1^S(u_1)
\end{aligned}$$

$$\begin{aligned}
k_{S_2}(x_{i2}, z_1, z_2, p_1, p_2) &\in \arg \max_{k \in \mathbb{R}} E[\theta - p_2 | x_{i2}, z_1, z_2, p_1, p_2] k - \gamma \frac{k^2}{2} \quad (1.5) \\
K_{S_2}(\theta, z_1, z_2, p_1, p_2) &= E[k_{S_2}(x_{i2}, z_1, z_2, p_1, p_2) | \theta, z_1, z_2, p_1, p_2] \\
\lambda_2 K_{S_2}(\theta, z_1, z_2, p_1, p_2) &= K_2^S(u_2) + \lambda_1 (1-r) K_1^S(u_1)
\end{aligned}$$

The first set of conditions defines a rational expectations competitive equilibrium for the first period. The second set of conditions defines a rational expectations competitive equilibrium for the second period. The individual demands embody that each agent uses all available information including the one he infers from both the past and present price realization. For tractability reason, I am going to focus on linear rational expectation equilibrium where the price is a linear function.¹⁴

Definition 2 A linear equilibrium is an equilibrium in which the prices, $P_1(\theta, z_1, u_1)$ and $P_2(\theta, z_1, z_2, p_1, u_1, u_2)$ are linear in (θ, z_1, u_1) and $(\theta, z_1, z_2, p_1, u_1, u_2)$:

$$p_1 = \beta_\theta \theta + \beta_{z_1} z_1 + \beta_u u_1 \quad (1.6)$$

¹⁴This restriction is common in the literature in a Gaussian information structure given the sum of jointly normally distributed variables remains normal.

and

$$p_2 = \beta_\theta^* \theta + \beta_{z_1}^* z_1 + \beta_{z_2}^* z_2 + \beta_{p_1}^* p_1 + \beta_u^* (u_1 (1 - r) \lambda_1 + u_2) \quad (1.7)$$

1.3.2 Intertemporal two-ways feedback

In this section, I am going to shed more light on the intertemporal relation between the equilibrium at $t = 1$ and the equilibrium at $t = 2$. In particular, I show how the price at $t = 1$ and $t = 2$ are going to be jointly determined in equilibrium. For that purpose, the first step consists in deducing the information structure of the price at $t = 1, 2$.

Endogenous Information

In any linear equilibrium as defined above, observing p_1 and p_2 is informationally equivalent to observing the following two Gaussian sufficient statistics :

$$\tilde{p}_1 = \theta + \frac{\beta_u}{\beta_\theta} u_1 \quad (1.8)$$

and

$$\tilde{p}_2 = \theta + \frac{\beta_u^*}{\beta_\theta^*} (u_1 (1 - r) \lambda_1 + u_2) \quad (1.9)$$

where $\tilde{p}_1 \equiv \frac{1}{\beta_\theta} (p_1 - \beta_{z_1} z_1)$; $\tilde{p}_2 \equiv \frac{1}{\beta_\theta^*} (p_2 - \beta_{p_1}^* p_1 - \beta_{z_1}^* z_1 - \beta_{z_2}^* z_2)$ and their precision is given respectively by $\alpha_{p_1} = \frac{\beta_\theta^2}{\sigma_u^2 \beta_u^2}$; $\alpha_{p_2} = \frac{\beta_\theta^{*2}}{\beta_u^{*2} \sigma_u^2 (1 + (1 - r)^2 \lambda_1^2)}$.

One can notice that the precision of the two endogenous signals is an increasing function of the sensitivity of the price to the fundamentals, i.e. β_θ and β_θ^* . β_θ and β_θ^* are associated respectively with the equilibrium average use of the private signal by the traders at $t = 1$ and $t = 2$. It embodies the informational externality the traders have on the investors. In particular, as short term traders rely more on their private signal to infer their return, they make the information revealed by the price to investors and future market participants more accurate, i.e. β_θ increases. In addition to β_θ and β_θ^* , α_{p_1} and α_{p_2} are, ceteris paribus, a decreasing function of the volatility in the supply noise (σ_u^2) and a decreasing function of β_u , β_u^* . The more the price will respond to their respective supply noise, the less informative about θ it will be. In particular, p_2 depends on the supply noise at $t = 1$, i.e. u_1 . This results from

the presence of the short term traders who import the supply noise of the first period in the second period. The more short termist is the market, (r, λ_1) , the more noisy will be the price at $t = 2$.

Backward feedback

As long as $r \in [0, 1)$, there exists an intergenerational feedback from the market at $t = 2$ to the market at $t = 1$. This comes from the fact that short term traders will sell their position at $t = 2$ with probability $(1 - r)$ and care about their capital gains. At $t = 1$, traders make their decision on the basis of their forecast of p_2 which depends on the average beliefs of future market participants about the fundamental θ . Formally, short term traders' return is given by $R = r\theta + (1 - r)p_2$, where for any given p_2 satisfying condition 1.7, the individual demand for the traders and the investors at $t = 1$ satisfy the following optimality conditions:

$$k_{S_1}(x_{i1}, z_1, p_1) = \frac{E(R|x_{i1}, z_1, p_1) - p_1}{\gamma}$$

and

$$k_L(z_1, p_1) = \frac{E(\theta|z_1, p_1) - p_1}{\gamma}$$

Aggregating the individual demands and applying the market clearing condition, p_1 is going to be an increasing function of traders' average forecast of p_2 . Denote $\bar{E}_{S_1}(\cdot) = \int_0^1 E(\cdot|x_{i1}, z_1, p_1) di$ and $\bar{E}_L(\cdot) = \int_0^1 E(\cdot|z_1, p_1) di$ the average forecast of traders and investors respectively for a measure-one continuum in each group.

Lemma 3 *In any linear equilibrium, p_1 satisfies the following condition:*

$$p_1 = \frac{\lambda_1 \bar{E}_{S_1}(r\theta + (1 - r)p_2(\theta, z_1, z_2, p_1, u_1, u_2))}{1 + \lambda_1} + \frac{\bar{E}_L(\theta)}{1 + \lambda_1} - \frac{\gamma}{1 + \lambda_1} u_1 \quad (1.10)$$

Proof. *The aggregate demand is given by*

$$K_1(\theta, z_1, p_1) = \frac{\int_0^{\lambda_1} (E(R|x_{i1}, z_1, p_1) - p_1) di + \int_0^1 (E(\theta|z_1, p_1) - p_1) di}{\gamma}$$

Using the notation above and applying the market clearing condition,

$$\frac{\lambda_1 \bar{E}_{S_1} (r\theta + (1-r)p_2(\theta, z_1, z_2, p_1, u_1, u_2)) + \bar{E}_L(\theta) - (\lambda_1 + 1)p_1}{\gamma} = u_1$$

■

The previous lemma states that p_1 is given by a weighted average of the average forecast of the traders and the investors as well as the supply noise. The main point to notice is that p_1 is an increasing function of traders's average forecast of p_2 at $t = 1$. The higher is the traders' average forecast of the future price, p_2 , the higher is going to be their aggregate demand and the equilibrium price p_1 . An increase in the horizon of the traders (r) or an inflow of traders (λ_1) makes p_1 more sensitive to p_2 as on average market participants are likely to care more about the price.

Using condition 1.7, p_2 is a linear function of the fundamental θ and the public signals used by future traders as well as the noise supply in the market at $t = 2$.

$$p_2 = \beta_\theta^* \theta + \beta_{z_1}^* z_1 + \beta_{z_2}^* z_2 + \beta_{p_1}^* p_1 + \beta_u^* (u_1 (1-r) \lambda_1 + u_2)$$

The sensitivity of the price, p_2 , on $(\theta, z_1, p_1, z_2, u_1, u_2)$ is defined by future market participants' average forecast of the fundamental. As traders at $t = 2$, rely relatively more on a certain signal to infer θ , the price will end up being relatively more sensitive to the given signal. Substituting p_2 in R , the return of the short term traders can be written as a linear combination of $(\theta, z_1, p_1, z_2, u_1, u_2)$.

$$R = (r + (1-r)\beta_\theta^*)\theta + (1-r)\left(\beta_{z_1}^* z_1 + \beta_{z_2}^* z_2 + \beta_{p_1}^* p_2 + \beta_u^* (u_1 (1-r) \lambda_1 + u_2)\right)$$

The main point to highlight is that short term traders' forecast of R differs from their forecast of θ and requires a different use of their information set than if they had to infer only θ . Precisely, the difference depends endogenously on the average forecast of future traders. Let $\kappa_\theta \equiv r + (1-r)\left(\beta_\theta^* + \beta_{z_2}^* - (1-r)\lambda_1\beta_u^*\frac{\beta_\theta}{\beta_u}\right)$; $\kappa_{z_1} \equiv (1-r)\left(\beta_{z_1}^* - (1-r)\lambda_1\beta_u^*\frac{\beta_{z_1}}{\beta_u}\right)$; $\kappa_{p_1} \equiv (1-r)\left(\beta_{p_1}^* + (1-r)\lambda_1\frac{\beta_u^*}{\beta_u}\right)$, this point is summarized in the following lemma:

Lemma 4 *In any linear equilibrium, for any given β' s defined by conditions 1.6-1.7, the average forecast of the return of the informed traders at $t = 1$ is given by:*

$$\bar{E}_{S_1}(R) = \kappa_\theta \bar{E}_{S_1}(\theta) + \kappa_{z_1} z_1 + \kappa_{p_1} p_1 \quad (1.11)$$

where let $\delta_i^{s_1} = \frac{\alpha_i}{\sum_{i \in I_{S_1}} \alpha_i}$, $i \in I_{s_1} = \{x_1, z_1, p_1\}$, $\bar{E}_{S_1}(\theta|I_{s_1}) = \delta_{x_1}^{s_1} \theta + \delta_{z_1}^{s_1} z_1 + \delta_{p_1}^{s_1} \tilde{p}_1$.

Proof. *It follows from taking traders average conditional expectation of $R = r\theta + (1-r)p_2$ after substituting p_2 by condition 1.7:*

$$\bar{E}_{S_1}(R) = (r + (1-r)\beta_\theta^*) \bar{E}_{S_1}(\theta) + (1-r)^2 \lambda_1 \beta_u^* \bar{E}_{S_1}(u_1) + (1-r)\beta_{z_2}^* \bar{E}_{S_1}(z_2) + (1-r)(\beta_{z_1}^* z_1 + \beta_{p_1}^* p_1)$$

where (i) $\bar{E}_{S_1}(z_2) = \bar{E}_{S_1}(\theta)$, given $\bar{E}_{S_1}(\epsilon_2) = 0$ and (ii) $\bar{E}_{S_1}(u_1) = \bar{E}_{S_1}\left(\frac{p_1 - (\beta_\theta \theta + \beta_{z_1} z_1)}{\beta_u}\right) = \frac{p_1}{\beta_u} - \frac{\beta_{z_1} z_1}{\beta_u} - \frac{\beta_\theta}{\beta_u} \bar{E}_{S_1}(\theta) \neq 0$. Hence, substituting in the condition above:

$$\bar{E}_{S_1}(R) = \kappa_\theta \bar{E}_{S_1}(\theta) + \kappa_{z_1} z_1 + \kappa_{p_1} p_1$$

where $\kappa_\theta \equiv r + (1-r)\left(\beta_\theta^* + \beta_{z_2}^* - (1-r)\lambda_1 \beta_u^* \frac{\beta_\theta}{\beta_u}\right)$; $\kappa_{z_1} \equiv (1-r)\left(\beta_{z_1}^* - (1-r)\lambda_1 \beta_u^* \frac{\beta_{z_1}}{\beta_u}\right)$ and $\kappa_{p_1} \equiv (1-r)\left(\beta_{p_1}^* + (1-r)\lambda_1 \frac{\beta_u^*}{\beta_u}\right)$. ■

Notice that unless $\kappa_{z_1} z_1 = \kappa_{p_1} p_1 = 0$ and $\kappa_\theta = 1$, $\bar{E}_{S_1}(R) \neq \bar{E}_{S_1}(\theta)$. In particular, the weight short term traders give on their private signal to forecast p_2 differs from their use of the private signal to forecast θ . The private signal helps short term traders predict p_2 only to the extent that it helps predict the fundamental. This difference depends on κ_θ which is an increasing function of the average use of x_{i2} and z_2 by traders at $t = 2$ in forecasting θ , i.e. β_θ^* and $\beta_{z_2}^*$. The more future traders will use their private signal in order to forecast θ , the more p_2 will be sensitive to θ . Furthermore, as future traders increase the use of z_2 in forecasting θ , short term traders increase the use of their private signal. This is because $z_2 = \theta + \epsilon_2$ where short term traders signal is informative about z_2 only through θ .¹⁵

¹⁵Notice that in order to infer p_2 , short term traders will put an additional weight on intergenerational common signals, $\{z_1, p_1\}$ as these signals reveal information about p_2 beyond the information they reveal about θ . This dimension has been highlighted by Allen-Morris-Shin (2006). Given the focus of the paper is on the incidence of

Given the optimal use of information by traders to infer R depends on the average beliefs of future market participants, p_1 's sensitivity to the fundamentals and the public signals depends on p_2 's sensitivity to the fundamental and the public signals. In other words, $(\beta_\theta, \beta_{z_1}, \beta_u)$ is a function of $(\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*)$. Let $\delta_i^L = \frac{\alpha_i}{\sum_{i \in I_L} \alpha_i}$, $i \in I_L = \{z_1, p_1\}$,

Lemma 5 *In any linear equilibrium, for any $(\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*) \in R^5$,*

$$p_1 = \beta_\theta \theta + \beta_{z_1} z_1 + \beta_u u_1$$

where

$$\beta_\theta = \frac{\kappa_\theta \lambda_1 \delta_{x_1}^{s_1}}{\kappa}, \quad \beta_{z_1} = \frac{\kappa_\theta \lambda_1 \delta_{z_1}^{s_1} + \lambda_1 (1-r) \beta_{z_1}^* + \delta_{z_1}^L}{(1 + \lambda_1 (1 - \lambda_1 (1-r) \beta_{p_1}^*))} \quad \text{and} \quad \beta_u \equiv -\frac{\gamma}{\kappa}$$

$$\text{where } \kappa = \left((1 + \lambda_1) - \kappa_\theta \lambda_1 \frac{\delta_{p_1}^{s_1}}{\beta_\theta} - \lambda_1 \kappa_{p_1} - \frac{\delta_{p_1}^L}{\beta_\theta} \right)^{16}.$$

Proof. See in Appendix A ■

Forward feedback

So far, the focus has been on the market at $t = 1$ taking as given p_2 as well as the impact of p_1 on p_2 . There exists also a forward feedback from the market at $t = 1$ to the market at $t = 2$. In the presence of asymmetric information, as long as p_1 is relatively informative from the point of view of future market participants, traders at $t = 2$ are going to use the price as an endogenous signal of the fundamental. This is illustrated more formally in what follows. At $t = 2$, for any given p_1 satisfying condition 1.6, the individual demand for the traders entering the market satisfies the following optimality condition:

$$k_{S_2}(x_{i2}, z_1, z_2, p_1, p_2) = \frac{E(\theta | x_{i2}, z_1, z_2, p_1, p_2) - p_2}{\gamma}$$

the horizon of traders on the precision of the information revealed by the price. The main discussion will be on the optimal use of the private signal x_{i1} . The more short term traders will use their private signal, the more the price will depend on it and reveal it.

¹⁶ κ adjusts the market clearing price to endogenize its informational role. As seen earlier, the market clearing price is given by the average forecast of traders and investors as well as the supply noise. However, the price is used as a signal in the average forecast. The sensitivity of the price to the different signals is given by the use of the different signals in the forecasting process adjusted by the information brought by the price in the inference process. This is precisely κ . The bigger the informative role of the price, the less sensitive the aggregate demand is going to be to the price. As a consequence, κ will become smaller.

This intergenerational information externality exists independently of the horizon of the traders. Aggregating the individual demands and applying the market clearing condition, p_2 is going to be a function of p_1 . Let $\delta_i^{s_2} = \frac{\alpha_i}{\sum_{i \in I_{S_2}} \alpha_i}$, $i \in I_{S_2} = \{x_2, z_1, z_2, p_1, p_2\}$ and denote $\bar{E}_{S_2}(\cdot) = \int_0^1 E(\cdot | x_{i2}, z_1, z_2, p_1, p_2) di$ the traders' average forecast at $t = 2$, the next lemma summarizes this relation:

Lemma 6 *In any linear equilibrium, the equilibrium price p_2 satisfies the following condition*

$$p_2 = \bar{E}_{S_2}(\theta) - \frac{\gamma(u_1(1-r)\lambda_1 + u_2)}{\lambda_2} \quad (1.12)$$

where $\bar{E}_{S_2}(\theta) = \delta_{x_2}^* \theta + \delta_{z_1}^* z_1 + \delta_{z_2}^* z_2 + \delta_{p_1}^* \tilde{p}_1 + \delta_{p_2}^* \tilde{p}_2$.

The equilibrium price at $t = 2$ is given by traders' average forecast of θ and the supply noises. There are two points to highlight from the previous lemma. First, p_2 is an increasing function of p_1 given p_1 is positively correlated to the fundamental, i.e. $\beta_\theta > 0$. This relation illustrates the informational externality from $t = 1$ to $t = 2$ as the price is informative about the fundamental. A higher p_1 is likely to signal a higher fundamental to traders at $t = 2$ which increases their demand and p_2 . Second, the sensitivity of p_2 to the fundamental, the exogenous public signals and p_1 , i.e. $(\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*)$, depends on the sensitivity of p_1 to the fundamentals, the public signal and the supply noise, i.e. $(\beta_\theta, \beta_{z_1}, \beta_u)$. A more informative p_1 , i.e. higher $\frac{\beta_\theta^2}{\sigma_u^2 \beta_u^2}$, makes traders at $t = 2$ increase the use of p_1 to forecast θ at the expense of the other signals.

Lemma 7 *In any linear equilibrium, for any $(\beta_\theta, \beta_{z_1}, \beta_u) \in \mathbb{R}^3$,*

$$p_2 = \beta_\theta^* \theta + \beta_{z_1}^* z_1 + \beta_{z_2}^* z_2 + \beta_{p_1}^* p_1 + \beta_u^* (u_1(1-r)\lambda_1 + u_2)$$

where

$$\beta_\theta^* = \delta_{x_2}^* + \delta_{p_2}^*; \quad \beta_{z_1}^* = \delta_{z_1}^* - \delta_{p_1}^* \frac{\beta_{z_1}}{\beta_\theta}; \quad \beta_{z_2}^* = \delta_{z_2}^*$$

$$\beta_{p_1}^* = \frac{\delta_{p_1}^*}{\beta_\theta} \quad \text{and} \quad \beta_u^* = - \frac{\gamma}{\left(1 - \delta_{p_2}^* \frac{1}{\beta_\theta^*}\right) \lambda_2}$$

Proof. See in Appendix A ■

Two-ways feedback

The previous two lemmas highlighted the existence of a forward feedback taking as given the impact of p_2 on p_1 as well as a backward feedback taking as given the impact of p_1 on p_2 . In any linear equilibrium, p_1 and p_2 are jointly determined internalizing the intertemporal two-ways feedback and solving for a fixed-point problem. A higher price today is likely to be associated with a higher expected price tomorrow which raises the demand of short term traders and the price today. This is summarized in the following proposition:

Proposition 8 *Any linear equilibrium is the solution of a system of equations given by the following two conditions:*

$$p_1 = \frac{\lambda_1 \bar{E}_{S_1}(r\theta + (1-r)p_2)}{1 + \lambda_1} + \frac{\bar{E}_L(\theta)}{1 + \lambda_1} - \frac{\gamma}{1 + \lambda_1} u_1$$

and

$$p_2 = \bar{E}_{S_2}(\theta) - \frac{\gamma(u_1(1-r)\lambda_1 + u_2)}{\lambda_2}$$

where $\bar{E}_{S_2}(\theta) = \delta_{x_2}^* \theta + \delta_{z_1}^* z_1 + \delta_{z_2}^* z_2 + \delta_{p_1}^* \tilde{p}_1 + \delta_{p_2}^* \tilde{p}_2$

More formally, the two-ways feedback in the prices is associated to the two-ways feedback in the inference process of traders at $t = 1$ and $t = 2$. The optimal use of information by short term traders to forecast p_2 will affect the relation between p_1 and the fundamentals and ultimately the informativeness of the price (α_{p_1}). The informativeness of the price has an impact on the optimal use of information by traders at $t = 2$ which affect the sensitivity of the price (p_2) to the fundamentals and the different signals. As a result, the optimal use of information by short term traders to forecast p_2 will change. Concretely, the fixed point problem will be solved in term of the sensitivity of the prices at $t = 1, 2$.

Corollary 9 *Any linear equilibrium is the solution of the following system of equations:*

$$\beta = B(\beta^*) \text{ and } \beta^* = B^*(\beta)$$

where $\beta = [\beta_\theta, \beta_{z_1}, \beta_u]$ and $\beta^* = [\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*]$. Furthermore, $B : \mathbb{R}^5 \rightarrow \mathbb{R}^3$ and $B^* : \mathbb{R}^3 \rightarrow \mathbb{R}^5$.

1.3.3 Characterization of the equilibrium

In this section, I am going to characterize the equilibrium. As we have just seen in the previous section, the equilibrium is the solution of a fixed-point problem in the β 's. The intertemporal two-ways feedback relies on two assumptions: (i) traders at $t = 1$ care about their capital gains, i.e. $r < 1$ and (ii) traders at $t = 2$ use p_1 in order to infer the fundamental. Suppose that traders at $t = 1$ only care about the fundamental value θ , then the equilibrium at $t = 1$ will be independent of the equilibrium at $t = 2$. On the other hand, suppose traders at $t = 2$ find p_1 uninformative about θ , then p_2 will be independent of p_1 . This could occur whenever either of $\alpha_{x_2}, \alpha_{z_2} \rightarrow \infty$ at $t = 2$. Before proceeding, I am going to shed more light on the two-ways feedback by deriving the equilibrium for 3 extreme benchmark cases: (i) No feedback, (ii) No forward feedback and (iii) No backward feedback.

Benchmark

Suppose $r = 1$ and $\alpha_{x_2} \rightarrow \infty$. For $r = 1$, the traders at $t = 1$ have a long horizon, they care about the fundamental value of the stock they are buying. On the other hand, at $t = 2$, as $\alpha_{x_2} \rightarrow \infty$ traders' average forecast of the fundamental is independent of p_1 . Hence, the equilibrium price at $t = 0$ is independently determined of the equilibrium at $t = 1$ where the equilibrium prices are given by:

$$p_1 = \frac{\lambda_1}{1 + \lambda_1} \bar{E}_{S_1}(\theta) + \frac{1}{1 + \lambda_1} \bar{E}_L(\theta) - \frac{\gamma}{1 + \lambda_1} u_1 \text{ and } p_2 = \theta - \frac{(u_1(1 - r)\lambda_1 + u_2)\gamma}{\lambda_2}$$

$$\text{Given } \bar{E}_{S_1}(\theta) = \delta_{x_1}^{s_1} \theta + \delta_{z_1}^{s_1} z_1 + \delta_{p_1}^{s_1} \tilde{p}_1 \text{ and } \bar{E}_L(\theta) = \delta_{z_1}^L z_1 + \delta_{p_1}^L \tilde{p}_1 :$$

$$p_1 = \beta_\theta \theta + \beta_{z_1} z_1 + \beta_u u_1$$

$$\text{where } \beta_\theta \equiv \frac{\lambda_1 \delta_{x_1}^{s_1}}{\kappa}; \beta_{z_1} \equiv \frac{\lambda_1}{1 + \lambda_1} \delta_{z_1}^{s_1} + \frac{1}{1 + \lambda_1} \delta_{z_1}^L; \beta_u \equiv -\frac{\gamma}{\kappa} \text{ and } \kappa \equiv 1 - \frac{\delta_{p_1}^L}{\beta_\theta} + \lambda_1 \left(1 - \frac{\delta_{p_1}^{s_1}}{\beta_\theta}\right).$$

The solution is determined in term of $b \equiv -\frac{\beta_\theta}{\beta_u}$, i.e. the negative of the sensitivity of the price to the fundamental relative to the supply noise. b is the unique solution to the following

equation: $b = \frac{\lambda_1 \alpha_{x_1}}{\gamma \left(\alpha_{x_1} + \alpha_{z_1} + \frac{b^2}{\sigma_u^2} \right)}$ ¹⁷. One can express all the coefficients, $(\beta_\theta, \beta_{z_1}, \beta_u)$, as a function of b . The existence and the uniqueness of the equilibrium will follow.

Suppose $r < 1$, p_1 will be a function of p_2 .

$$p_1 = \frac{\lambda_1 \bar{E}_{S_1} (r\theta + (1-r)p_2)}{1 + \lambda_1} + \frac{\bar{E}_L(\theta)}{1 + \lambda_1} - \frac{\gamma \sigma_u}{1 + \lambda_1} u_1 \text{ and } p_2 = \theta - \frac{(u_1(1-r)\lambda_1 + u_2)\gamma}{\lambda_2}$$

Nevertheless, as future market participants' beliefs are given by θ , traders' average forecast of p_2 boils down to the average forecast of the fundamental and the supply noises at $t = 2$. As earlier the equilibrium at $t = 1$ will be independent of future market participants' beliefs. The main difference comes from the fact that short term traders internalize the effect of the supply noise at $t = 1$ (u_1) on the future price.¹⁸

Finally, suppose α_{x_1} is bounded but $r = 1$. At $t = 2$, traders are going to use p_1 to forecast θ . As a result, traders' optimal use of information depends on the informativeness of p_1 . Formally,

$$p_2 = \beta_\theta^* \theta + \beta_{z_1}^* z_1 + \beta_{z_2}^* z_2 + \beta_{p_1}^* p_1 + \beta_u^* (u_1(1-r)\lambda_1 + u_2)$$

$$\text{where } \beta_\theta^* = \delta_{x_2}^* + \delta_{p_2}^*; \beta_{z_1}^* = \delta_{z_1}^* - \delta_{p_1}^* \frac{\beta_{z_1}}{\beta_\theta}; \beta_{z_2}^* = \delta_{z_2}^*; \beta_{p_1}^* = \frac{\delta_{p_1}^*}{\beta_\theta} \text{ and } \beta_u^* = -\frac{\gamma}{\left(1 - \delta_{p_2}^* \frac{1}{\beta_\theta^*}\right) \lambda_2}.$$

The solution is determined in term of $b^* \equiv -\frac{\beta_\theta^*}{\beta_{u_1}^*}$. b^* is the unique solution to the following equation: $b^* = \frac{\lambda_2 \alpha_{x_2}}{\gamma \left(\alpha_{x_2} + \alpha_{z_1} + \alpha_{z_2} + \frac{b^2}{\sigma_u^2} + \frac{b^{*2}}{\sigma_u^2 (1 + (1-r)^2 \lambda_1^2)} \right)}$. One can express all the coefficients, $(\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*)$, as a function of b^* for any $(\beta_\theta, \beta_{z_1}, \beta_\theta)$.

Equilibrium characterization

As noted earlier, completing the equilibrium characterization requires solving a fixed-point problem. On the one hand, p_1 is defined by how traders use their available information to infer

¹⁷ b^* is the unique solution to the cubic expression: $b(\alpha_{x_1} + \alpha_{z_1}) + \frac{b^3}{\sigma_u^2} - \frac{\lambda_1}{\gamma} \alpha_{x_1}$.

¹⁸ One way to completely cancel any feedback from $t = 2$ to $t = 1$ will be to have in addition to $\alpha_{x_1} \rightarrow \infty$, $\sigma_u^2 \rightarrow 0$ or $\gamma \rightarrow 0$. In both cases, p_2 will be independent of the supply noise.

R which depends on p_2 and on how future traders use their available information to infer the fundamentals. On the other hand, how future traders use their available information depends on the informativeness of p_1 which is determined by how traders at $t = 1$ use their information. Concretely, solving the fixed-point problem boils down in solving a fixed-point problem in term of the β' s, the sensitivity of p_1 and p_2 to the different signals. The equilibrium is reduced to the solution of the equation in the following lemma.

Lemma 10 \exists functions $G : \mathbb{R}^{+8} \times \mathbb{R}_0^+ \times [0, 1] \rightarrow \mathbb{R}^+$ and $F : \mathbb{R}^{+8} \times \mathbb{R}_0^+ \times [0, 1] \rightarrow \mathbb{R}^8$ such that let $b = -\frac{\beta_\theta}{\beta_u}$, in any equilibrium, b solves

$$b = G(b, \alpha_{x_1}, \alpha_{x_2}, \alpha_{z_1}, \alpha_{z_2}, \sigma_u^2, \lambda_1, \lambda_2, r, \gamma) \quad (1.13)$$

while $(\beta_\theta, \beta_u, \beta_{z_1}, \beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*; \beta_{p_1}^*; \beta_u^*) \equiv F(b, \alpha_{x_1}, \alpha_{x_2}, \alpha_{z_1}, \alpha_{z_2}, \sigma_u^2, \lambda_1, \lambda_2, r, \gamma)$.

Proof. Proof in Appendix B ■

The equilibrium is the solution of a fixed-point problem in term of b which is defined by the sensitivity of the price (p_1) to the fundamental relative to the supply noise. It also characterizes the informativeness of the price p_1 . Given the informativeness of the price, b , one can solve for the sensitivity of the price at $t = 2$ to the fundamental relative to the supply noise, i.e. $b^* = \frac{\beta_\theta^*}{\beta_u^*}$, in the same fashion as for the forward feedback benchmark case. As a consequence, one can determine $(\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*)$ as a function of b given b^* is also characterized by b . This allows me to define the function G . The next proposition states the existence and the uniqueness of the equilibrium.

Proposition 11 *There exists at least one equilibrium for any $\gamma > 0$, i.e. \exists at least one $b \in \mathbb{R}^+$ that solves*

$$b \in \{b \in \mathbb{R}^+ : b = G(b, \alpha_{x_1}, \alpha_{x_2}, \alpha_{z_1}, \alpha_{z_2}, \sigma_u^2, \lambda_1, \lambda_2, r, \gamma)\}$$

Furthermore, the equilibrium is unique.

Proof. See in Appendix B ■

1.4 Information aggregation

In this section, I am going to examine more precisely the relation between the horizon of the traders and the informational efficiency of the market. The focus of this paper is on the information revealed by the price and the extent in which the average horizon of the traders is going to affect the information conveyed by the price about the fundamental to the investors at $t = 1$ and the traders at $t = 2$.¹⁹

In the model we are considering, a natural measure of the prices' informativeness is given by α_{p_1} and α_{p_2} .²⁰ α_{p_1} and α_{p_2} are both functions of $\frac{\beta_\theta}{\beta_u}$ and $\frac{\beta_\theta^*}{\beta_u^*}$, i.e. the sensitivity of the price to the fundamental relative to the supply noise at $t = 1$ and $t = 2$. In what follows, I am going to present the extent in which a change in the horizon of the traders affects the behavior of the traders and their optimal use of information which ultimately changes α_{p_1} . As we know, from the previous section, $\frac{\beta_\theta}{\beta_u}$ and $\frac{\beta_\theta^*}{\beta_u^*}$ are both associated to traders' relative use of the private signal in inferring the unknown object of interests.

The first natural measure of the horizon of the trader is given by r . If $r = 1$, each trader is long-termist and at the time of making his decision he will forecast θ . Any increase in r raises the importance of θ at the expense of p_2 for each trader making him more long-termist. At the same time, as the traders are all identical, on average the horizon of the traders in the market increases. In a first step, I am going to explore the effect of a change in r on α_{p_1} .

In a second step, I am going to explore the effect of an inflow of informed traders in the market at $t = 1$ on α_{p_1} , i.e. λ_1 . λ_1 is a measure of the informed short term traders entering the market at $t = 1$. As we are going to see, a higher λ_1 is not always characterized by a higher α_{p_1} . It will typically depend on r among others. An increase in λ_1 is characterized by an arrival of continuum of traders at $t = 1$ having private information. However, for any given horizon $r < 1$, it also increases the share of traders with shorter horizon r . Hence, the average horizon of the market decreases. For the rest of this section, I am going to focus on α_{p_1} .

¹⁹The informational efficiency of the market is an important measure that has been used extensively in Finance. Prices are informationally efficient if they fully and correctly reflect the relevant information. The focus of this section will be on the information revelation of the price. In particular, on the extent investors can infer the information which is dispersed in the economy.

²⁰From the point of view of an econometrician who would have access to the same common signal as the traders, α_{p_1} and α_{p_2} would be as the R-squared of a regression of the price on the liquidation value, θ .

Proposition 12 *In any linear equilibrium,*

$$(i) \frac{\partial \alpha_{p_1}}{\partial r} \geq 0$$

Proof. *See in Appendix B* ■

The previous proposition states that there is an increasing monotone relation between the horizon of the traders (r) and the information revealed by p_1 about the fundamental. As the horizon of the informed trader increases, the share of their gross return which depends on the fundamental increases, making the traders use more their private information.²¹ Overall, a market associated with relatively more long term traders tends to reveal better quality of information through its price. In particular, in an economy with only long term traders, i.e. traders that only care about the fundamental of the company they are investing in ($r = 1$), the price is more informative than in an economy where traders care also about their capital gains, i.e. for any $r < 0$. The logic behind holds as follow. On one hand, a long term informed trader who tries to forecast θ to make his optimal decision to invest in an asset will attach more weight on his private signal than a trader who cares less about the fundamental and more on his capital gains, i.e. $r < 1$ ²². It is translated by $\kappa_\theta \in [0, 1]$. This is because the private signal is informative only about θ . Hence, traders with a smaller horizon, will use their private signal relative to the other signals to forecast p_2 only to the extent that p_2 is correlated to θ . On the other hand, as each trader uses more their private signal, the aggregate demand is relatively more sensitive to θ and on aggregate the price is more informative about the fundamental.²³ Overall the presence of short-term traders creates a negative informational externality on the market. The same intuition goes for any r such that a decrease in the horizon of the traders decreases the informativeness of the price. As all informed traders care relatively more about the fundamental and relatively less about forecasting the average beliefs of future

²¹There exist several negative indirect effects but overall the direct positive effect of an increase in the horizon of the traders is stronger. (i) As the horizon of the traders increases today, there will be less supply noise tomorrow in the market and the price tomorrow will be more informative decreasing the relation between p_2 and θ . Ultimately this leads traders to use less their private signal; (ii) An increase in α_{p_1} makes p_1 more informative which induce traders at $t = 1$ and $t = 2$, to increase p_1 at the expense of the other signals in inferring θ . This leads to further decreases in α_{p_1} . All these indirect effects will make the $\frac{\partial \alpha_{p_1}}{\partial r}$ smaller but it will remain positive.

²²This result has been illustrated by Allen-Morris-Shin (2003).

²³There is an additional effect which goes on the other direction. As the price becomes more informative, the informed traders uses relatively more the price as a signal at the expense of their private signal. In equilibrium, this effect is always smaller than the original increase in the use of their private signal.

market participants when making their decision, they all use relatively more their private signal and the price becomes a more precise information aggregator.

Next, I am going to look at the effect of an inflow of informed traders in the market on α_{p_1} . As we mentioned earlier, an increase in λ_1 is associated with an increase in the number of informed traders but also by an increase in the average horizon of the market as long as $r \neq 1$. There will be on average more short-term traders. As we are going to see, having more traders with private information does not automatically increase the informativeness of the price.

Proposition 13 *In any linear equilibrium,*

(i) For $r = 1$, $\frac{\partial \alpha_{p_1}}{\partial \lambda_1} \geq 0$.

(ii) For $r \neq 1$, α_{p_1} is a non monotonic function of λ_1 .

Proof. See in Appendix B ■

The first part states that in an economy with only long term informed traders, an increase in the number of traders increases the informativeness of the price. This is simply due to the fact that the aggregate demand is composed of a bigger share of informed traders who have access to private information. As a consequence the price is more precise. Whenever, $r \neq 1$, this effect still exists. However, an increase in the share of informed traders can lead each trader to decrease their relative use of private signal so much that on aggregate the price may end up being relatively less revealing about the fundamental.

For r relatively small, informed traders anticipate that an inflow of short term traders in the market at $t = 1$ will be associated with a bigger transfer of the supply noise in the market at $t = 2$. Hence, informed traders anticipate the fact that the correlation between the supply noise and p_2 is more important. As a consequence, in forecasting p_2 they will use relatively less their private signal as we have seen earlier. The same occurs whenever there are less traders at $t = 2$ in the market, i.e. λ_2 small. If one considers a market to be thin whenever there are buyers and sellers in small numbers, then a small λ_2 can be associated with a thinner market.²⁴ Hence, a thinner market defined as above is more likely to have a decrease in the informativeness of the

²⁴A thin market is often associated with a more volatile and less liquid asset.

market price as the share of informed traders increases.²⁵ The reasoning is similar as for small r .

The figures 1a-1d illustrate the last 2 propositions. The first figure represents the relation between α_{p_1} and r . The parameters value for the precision of the different exogenous signals are equal to 1, i.e. $\alpha_{x_1} = \alpha_{x_2} = \alpha_{z_1} = \alpha_{z_2} = \sigma_u^2 = 1$. Furthermore, $\gamma = 1$ and $\lambda_2 = 0.1$. One of the reason I look at low values of λ_2 is to be able to highlight as we are going to see the interaction between a thinner market and a higher λ_1 . The remaining points remain the same for λ_2 relatively larger. I look at two different cases for λ_1 : $\lambda_1 = 0.1$ and $\lambda_1 = 0.5$. The first thing to notice is the fact that for both cases, α_{p_1} is monotonically increasing in r .

There are three main comments to make when comparing the two cases. First, the higher the share of informed traders in the market (λ_1), the greater the increase in the information revealed by the market price as the horizon of the traders increases. In particular, a market characterized by a low proportion of informed traders, e.g. $\lambda_1 = 0.1$, does not see an important effect of the horizon of the traders on the quality of the information revealed by the price. Second, for low enough r , a market with more informed traders is not associated with more precise information aggregated by the price. This shows the point made above where a thin market with shorter horizon traders can be related with a more precise information revealed by the price when there is less informed traders in the market. Finally, for high enough r more informed traders is associated with a higher α_{p_1} . The limit where $r = 1$ is stated in proposition 11.

Figures 1.b-d. explore more precisely proposition 11. Figure 1.b-c focus on the relation between α_{p_1} and λ_1 while figure 1.d. take a closer look at the impact of λ_2 on α_{p_1} . Figure 1.b. compares two cases for the thinness of the market at $t = 2$, i.e. λ_2 : $\lambda_2 = 0.1$ and $\lambda = 1$, while keeping r constant at 0.1. On the other hand, Figure 1.c. keeps λ_2 constant at 0.1 and compares the two cases: $r = 0.1$ and $r = 0.5$. All the other parameters value remain the same as in figure 1.a.

In figure 1.b. two points should be highlighted. The first one is the fact that a thinner market at $t = 2$, i.e. lower λ_2 is associated with a lower α_{p_1} . The difference increases as the

²⁵In the discussion section, I will present an alternative way to interpret the model where λ_2 is linked to the share of short term traders.

proportion of informed traders in the market increases. In other words, when the share of informed traders in the market is small, a higher or a lower λ_2 won't have much impact on α_{p_1} . This is the analogue of the point made earlier where the effect of a change in r on α_{p_1} is small if λ_1 is small. Second, for low enough λ_2 , one can see that an increase in λ_2 at first raises α_{p_1} but it reaches a maximum and decreases from then on. When there are few informed traders in the market, an inflow of informed traders increases α_{p_1} as there is simply a bigger share of the aggregated demand composed of traders having access to the private signal. There is however a second effect. More short-term traders transfers a higher share of the supply noise at $t = 1$ in the market at $t = 2$. It results in a decrease in the use of the private information as explained earlier. As λ_1 increases the second effect becomes bigger. Higher λ_1 is associated with more traders hit by the shock selling in the market at $t = 2$. This leads to a decrease in the quality of the information revealed by the price.

Likewise in figure 1.c., a market characterized by relatively short-term traders can be related to a non monotonic relation between α_{p_1} and λ_1 . For small λ_1 , an increase in the proportion of informed traders raises α_{p_1} . However, as λ_1 increases the share of short-term traders in the market increases so much that informed traders use less their private signal to infer p_2 . Each trader anticipates the fact that the price p_2 will as a consequence be more related to the supply noise u_1 .

Finally, as shown in figure 1.d which illustrates the relation between α_{p_1} and λ_2 for two values of λ_1 : $\lambda_1 = 0.1$ and $\lambda_1 = 1$, a thinner market is associated with a loss in the informativeness of the price, α_{p_1} mainly when the number of short term traders in the market is relatively high. As explained earlier, this is due to the sensitivity of p_2 on the supply noise.

1.5 Beliefs signal

So far, the information the informed traders have about the average beliefs of the traders at $t = 2$, was correlated to the fundamental. Everything they inferred about the market was through their signals about θ where some signals are more correlated to the beliefs of the agents at $t = 2$, e.g. z_1, p_1 and others are only correlated through θ , e.g. x_{i1} . However, one could imagine short-term traders, $r \neq 1$, trying to forecast the direction of the average beliefs of

the traders at $t = 2$ through some information set that are not correlated to the fundamental. Nevertheless, rational short-term traders will use this information as it will help them infer the future price through the average beliefs of future market participants. In this section, I am extending the previous analysis by allowing the informed traders to have access to a signal about the noise in the public signal of the traders born the next period: $y = \epsilon_2 + \omega$ where $\omega \sim N(0, \sigma_\omega^2)$ ²⁶ that I am going to refer to beliefs signal. Hence, the information set of the informed traders is updated to $I_{i_1} = \{x_{i_1}, y, p_1\}$, $i \in S_1$. Given y is only informative about the average beliefs of future market participants, the benchmark equilibrium, i.e. $r = 1$, with the new information set will be the same as in an economy with y uninformative. Contrary to an economy without the beliefs signal, the introduction of y , makes possible the analysis of the effect of a change in the precision of higher order beliefs independently of the precision of the information about θ .²⁷

The beliefs signal can be seen as investor sentiment, i.e. a measure of the mood of the market. Any change in y can be interpreted as a change in the level of optimism or pessimism of the traders at $t = 1$ about the future prices or the future market independently of their beliefs of the fundamentals. The role of investor sentiments is widely acknowledged in financial markets both by practitioners and academicians²⁸. In this section, the role played by the investor sentiment is directly connected to the importance of higher order beliefs which raises in the presence of short term traders and asymmetric information. Rational short term traders will decide how much to invest in the risky asset in the basis of investor sentiment as it is informative about the future price. Hence, the price will depend on investor sentiments. The beliefs signal is closely related to recent work by Angeletos-La'o (2011) which shows in a rational set up how the business cycle can be driven by shocks independent of fundamental shocks and the expectation of fundamental shocks. However, these shocks affect the business cycle through the expectation of aggregate activity as they are related to higher order beliefs. They refer to these shocks as market sentiments.

²⁶The beliefs signal is not common knowledge. Uninformed traders and traders at $t = 2$ do not observe it. This is a crucial hypothesis in the derivation of the endogenous information structure which follows.

²⁷One can notice that y helps traders forecast p_2 to the extent that it helps them forecast ϵ_2 . On the other hand, the private signal, x_{i_1} , helps traders predicts p_2 only to the extent that it is informative about the fundamental. This is in contrast to p_1 which is common to the traders at $t = 1, 2$.

²⁸For a recent paper about investor sentiments and stock markets, I refer to Wurgler and Baker (2007).

For the remaining of the analysis, suppose $\alpha_{z_1} = 0$, $\alpha_{x_2} = 0$. Hence, as $\alpha_{x_2} = 0$, the price at $t = 2$ won't carry extra information to traders at $t = 2$. These simplifications allow me to highlight the main intuition behind the effect of the introduction of the beliefs signal. All the results hold in the more general case.

Equilibrium characterization

At $t = 1$, as rational short-term traders have access to information about ϵ_2 ,

$$E(z_2|I_{i_1}) = E(\theta|I_{i_1}) + \frac{\alpha_\omega}{(\alpha_\omega + \alpha_{z_2})}y, \quad i \in S_1$$

One can notice that in the presence of y , $E(z_2|I_{i_1}) \neq E(\theta|I_{i_1})$, $i \in S_1$, precisely because $E(\epsilon_2|I_{i_1}) = \frac{\alpha_\omega}{(\alpha_{z_2} + \alpha_\omega)}y \neq 0$, $i \in S_1$. As a consequence, the forecast of p_2 and ultimately the equilibrium price, p_1 , will depend on y . An increase in the relative precision of the beliefs signal, $\frac{\alpha_\omega}{\alpha_{z_2}}$, makes rational short-term traders put relatively more weight on their beliefs signal to infer z_2 and ultimately forecast p_2 .

Let $p_2 = \beta_{z_2}^* z_2 + \beta_{p_1}^* p_1 + \beta_u^* (u_1 (1-r) \lambda_2 + u_2)$, for any $(\beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*) \in \mathbb{R}^3$ given, define $\kappa_\theta \equiv r + (1-r) \left(\beta_{z_2}^* - (1-r) \lambda_1 \beta_u^* \frac{\beta_\theta}{\beta_u} \right)$; $\kappa_{p_1} \equiv (1-r) \left(\beta_{p_1}^* + (1-r) \lambda_1 \frac{\beta_u^*}{\beta_u} \right)$ and $\kappa_y \equiv (1-r) \left(\beta_{z_2}^* - \lambda_1 (1-r) \beta_u^* \frac{\beta_y}{\beta_u} \right)$. The following lemma summarizes the dependence of the short-term traders' forecast of R to the beliefs signal, y .

Lemma 14 *In any linear equilibrium, for any $(\beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*) \in \mathbb{R}^3$, the forecast of the return of the informed agents at $t = 1$ is given by:*

$$\bar{E}_{S_1}(R) = \kappa_\theta \bar{E}_{S_1}(\theta) + \kappa_y \bar{E}_{S_1}(\epsilon_2) + \kappa_{p_1} p_1 \quad (1.14)$$

where let $\delta_i^{s_1} = \frac{\alpha_i}{\sum_{i \in I_{s_1}} \alpha_i}$, $i \in I_{s_1} = \{x_1, p_1\}$, $\tilde{p}_1 = \frac{(p_1 - \beta_y y)}{\beta_\theta}$ and $\alpha_{p_1} = \frac{\beta_\theta^2}{\sigma_u^2 \beta_u^2}$:

$$\bar{E}_{S_1}(\theta) = \delta_x^{s_1} \theta + \delta_{p_1}^{s_1} \tilde{p}_1 \quad \text{and} \quad \bar{E}_{S_1}(\epsilon_2) = \frac{\alpha_\omega}{(\alpha_{z_2} + \alpha_\omega)}y$$

Proof. See in Appendix C ■

For any $(\beta_{z_1}^*, \beta_{p_0}^*, \beta_u^*) \in \mathbb{R}^3$ given, the main difference with the previous case is $\kappa_y \neq 0$. The traders' average forecast of R , depends on the beliefs signal as it helps them predict future price. κ_y measures the sensitivity of the traders' average beliefs of R on y . One can notice that that as $r \rightarrow 1$, κ_y tends to zero. Furthermore, the weight short term traders give to both their private signal and their beliefs signal is increasing in the sensitivity of the future price to z_2 , i.e. $\beta_{z_2}^*$. As earlier, $\beta_{z_2}^*$ is defined by future market participants' use of z_2 in forecasting θ which depends on the informativeness of the price and ultimately $(\beta_\theta, \beta_y, \beta_u)$.

As earlier, the equilibrium is the solution of a fixed-point problem which internalizes the two-ways feedback. For more details about the derivations please refer to the appendix.

Information aggregation

In this subsection, I am exploring the extent in which the quality of the information revealed by the price is affected by the use of the beliefs signal. Before proceeding, it is worth mentioning the existence of the asymmetry in the information revealed by the price p_1 between investors at $t = 1$ and future market participants. Given the linear structure of the equilibrium, the information revealed by p_1 is summarized by the Gaussian signal :

$$\bar{p}_1 = \theta + \frac{\beta_u}{\beta_\theta} u_1 + \frac{\beta_y}{\beta_\theta} y$$

where $\bar{p}_1 \equiv \frac{p_1}{\beta_\theta}$ with precision $\alpha_{\bar{p}_1} = \frac{\beta_\theta^2}{\beta_u^2 \sigma_u^2 + \beta_y^2 \sigma_y^2}$; $\sigma_y^2 \equiv \sigma_\omega^2 + \sigma_{\epsilon_1}^2$.²⁹ From the point of view of the traders at $t = 2$, as short term informed traders use the beliefs signal, p_1 and z_2 become correlated with covariance $\sigma_{zp} = \frac{\beta_y}{\beta_\theta} \sigma_{z_2}$. Overall the precision of the information set of future market participants will internalize the dependence between the two signals.

Lemma 15 *In any linear equilibrium,*

(i) *The information revealed by the price at $t = 1$ to uninformed trader is given by*

$$\alpha_{\bar{p}_1} = \frac{\beta_\theta^2}{\beta_u^2 \sigma_u^2 + \beta_y^2 \sigma_y^2} \tag{1.15}$$

²⁹ Given y is unobserved by the investors and future traders, it appears as an extra source of noise in addition to the supply noise.

(ii) The information revealed by the price at $t = 1$ to traders at $t = 2$ is given by

$$\alpha_{pZ} = \alpha_{z_2} \lambda_{z_1} + \alpha_{\bar{p}_1} \lambda_{p_1} \quad (1.16)$$

where $0 \leq \lambda_{z_2}, \lambda_{p_1} \leq 1$.

Proof. See in Appendix C ■

This asymmetry in the informational externality results precisely from the nature of the use of the beliefs signal. As explained earlier, traders will use the beliefs signal precisely because they want to infer the average forecast of traders at $t = 2$. As much as for long term investors it appears as an additional source of noise, as the short term traders are using y to forecast p_2 , the endogenous information provided by the market to future market participants is going to change in two dimensions: (i) covariance between p_1 and z_2 and (ii) variance of \bar{p}_1 . The precision of the information set of future traders is not anymore given by the sum of the precision of the two signals, but it endogenizes the covariance between the two signals. Intuitively, by using y they transfer information about $z_2 = \theta + \epsilon_2$ through the information the price imports about ϵ_2 . This indirectly affects the information future market participants get out of z_2 about θ . In addition, the use of y affects directly the informativeness of p_1 ($\alpha_{\bar{p}_1}$).

If $\sigma_{zp} = 0$, the effect of the use of y on the information set of future market participants will only be through $\alpha_{\bar{p}_1}$ where $\alpha_{pZ} = \alpha_{z_1} + \alpha_{\bar{p}_1}$. However, the introduction of the beliefs signal decreases α_{pZ} for given $\alpha_{\bar{p}_1}$ as $0 \leq \lambda_{z_2}, \lambda_{p_1} \leq 1$. The positive correlation between the two signals makes the information set of future market participants more noisy in equilibrium for given $\alpha_{\bar{p}_1}$. As a result, whenever the use of y by short term traders decreases the informativeness of the price from the point of view of long term investors at $t = 0$, the overall precision of future traders' information set decreases also. Let $\alpha_{\bar{p}_1}^*$ be the precision of the signal when y is uninformative about θ .

Lemma 16 *If $\alpha_{\bar{p}_1} \leq \alpha_{\bar{p}_1}^*$, then $\alpha_{pZ} \leq \alpha_{z_1} + \alpha_{\bar{p}_1}^*$.*

Proof. See in Appendix C ■

In the remainder of this section, I am going to illustrate graphically the effect of a change in the precision of y on the information revealed by the price to uninformed traders and future traders in equilibrium.

Figures 2.a.-2.b. illustrate $\alpha_{\bar{p}_1}, \alpha_{p_z}$ respectively as a function of α_ω where the pink line represents the benchmark case where y is uninformative $\alpha_\omega \rightarrow 0$. The parameters value for the precision of the different exogenous signals are equal to 1, i.e. $\alpha_{x_1} = \alpha_{z_2} = \sigma_u^2 = 1$. and $r = \lambda_1 = \lambda_2 = \gamma = 0.5$

From figure 2.a., one can notice that the use of the beliefs signal decreases the informativeness of the price from the point of view of long term investor. In general, the introduction of the beliefs signal generates two effects: (i) relatively more noise (y) and (ii) change in the sensitivity of the price to the supply noise volatility $\left(\frac{\sigma_u^2 \beta_u^2}{\beta_\theta^2}\right)$. The first one is the consequence of using the beliefs signal which appears as a noise from the point of view of long term investors who want to infer the fundamental. The second one is related to a change in the use of the private signal which follows the introduction of the beliefs signal. The use of y will make the price relatively more noisy from the point of view of future market participants. As a consequence, traders at $t = 2$ will use relatively more their public signal, z_2 , in inferring the fundamental. As short term traders anticipate tomorrow's price to be more sensitive to z_2 , they will rely more on their private signal and the price will be relatively less sensitive to the supply noise. The overall outcome depends on the size of the two effects which go in opposite directions. As illustrated in the figures 2.c.-2.d., it will depend on γ : for relatively low γ the price is less informative while for relatively high, the overall effect is almost inexistent.

In addition, an increase in α_ω can be associated with a decrease in the information revealed by the price to investors.³⁰ In other words, an economy where short term traders have a more precise signal about the price p_1 , can result in an economy with less accurate information revealed by the price about the fundamentals. The logic remains the same as earlier, an increase in the precision of the beliefs signal leads the price to be more noisy as $\left(\frac{\beta_y}{\beta_\theta}\right)^2 \sigma_y^2$ increases. However, the price depends relatively less on the supply noise. The overall effect depends on γ .

Figure 2.b. on the other hand, illustrates the relation between α_{p_z} and α_ω . As we can notice, the use of the beliefs signal has a negative effect on the informativeness of future market participants' information set.³¹ This is a direct consequence of the lemma above. As short term

³⁰Notice that even though the higher the horizon, the more likely it is to have a negative relation between α_ω and $\alpha_{\bar{p}_1}$. The $\alpha_{\bar{p}_1}$ is overall higher for higher horizon.

³¹As future traders have only 2 signals, a smaller α_{p_z} is associated with a loss in the precision of their information set.

traders use the beliefs signal, not only for given precision, the positive correlation between the two signals generates a loss in term of the precision of future market participants information set, i.e. $\lambda_{p_1}, \lambda_{z_2} \in [0, 1]$, but the precision of p_1 tends to decrease. As a result, they end up with a less accurate information about the fundamental.

Figures 2.c-2d. shed more light on the role played by γ in the impact of the use of the beliefs signal. They look more closely at the relation between the $\alpha_{\bar{p}_1}, \alpha_{P_z}$ and γ respectively. The parameters value for the precision of the different exogenous signals are equal to 1, i.e. $\alpha_{x_1} = \alpha_{z_2} = \sigma_u^2 = 1$ and $\lambda_2 = \lambda_1 = 0.2$; $r = 0.1$. Two cases are being considered: $\alpha_\omega \rightarrow 0$ and $\alpha_\omega = 1$. As mentioned earlier, the effect of the introduction of y depends on γ . Three main points are to be noted from figure 2.c-2.d.: (i) The information revealed by the price when $\alpha_\omega \rightarrow 0$ is more precise than when $\alpha_\omega = 1$ from the point of view of investors and future market participants; (ii) As γ increases, the difference in the informativeness of the price between $\alpha_\omega \rightarrow 0$ and $\alpha_\omega = 1$ decreases and (iii) The informativeness of the price is a decreasing function of γ .

A low γ is associated with a high demand. In equilibrium, the price becomes less sensitive to the supply noise as γ decreases. At the limit, $\gamma \rightarrow 0$, the price is not a function of the supply noise. This explains the negative slope. In addition, as we explained earlier, the use of the beliefs signal makes the price more noisy as $\left(\frac{\beta_y}{\beta_\theta}\right)^2 \sigma_y^2$ as well as less dependent on the supply noise. The second effect is more important for high γ than low γ . The information gain from the traders relying more on their private signal is more beneficial in an economy with high supply noise (higher γ) than an economy with low supply noise (lower γ). As a consequence, for relatively low γ , the negative effect of introducing y is bigger than the positive one while for relatively high γ , the effect is almost inexistent.

1.6 Alternative interpretation

Before concluding, I would like to provide another interpretation to the horizon of the traders, i.e. r . So far, I have defined r as the probability of being hit by a liquidity or preference shock and exiting the market at $t = 2$. r summarized the extent in which the traders care about the θ at the expense of p_2 . Consider the following alternative economy, where there is a measure one

of long term investors born at $t = 1$ identical to the long term investors defined earlier where $U = k_1 (\theta - p_1) - \frac{k_1^2}{2}$. In addition, there is a measure λ_1 of short term traders characterized by the following utility function:

$$U = k_1 (p_2 - p_1) - \frac{k_1^2}{2} + k_2 (\theta - p_2) - \frac{k_2^2}{2} - \frac{c(k_1 - k_2)^2}{2}$$

In words, the traders behave as myopic short term traders. They decide how much to invest in the risky asset given that the market will be open at $t = 2$ and they will re-trade the asset at p_2 . Hence, the short term traders can reenter the market at $t = 2$ and readjust their position while the long term investors hold on to the asset until the end. The information set of the traders is defined as in the section 2.³² Suppose there are no traders born at $t = 2$. Typically, the informed traders will trade among each others at $t = 2$. Finally, suppose there is an extra cost c of being active in the market at $t = 2$, i.e. changing position. The higher is c , the more costly it is to participate in the market at $t = 1$.³³ When $c \rightarrow \infty$, $k_2 = k_1$, i.e. agents only care about the fundamental. It is so costly to change their position that the traders decide their respective positions at $t = 1$. They know they will have to keep their position until $t = 3$ and receive θ . Let the first order condition at $t = 2$ be given by

$$k_{i2} = \frac{E_{i2}(\theta - p_1)}{1 + c} + \frac{c}{(1 + c)} k_{i1}$$

As we can notice, the higher is k_{i1} , the higher is k_{i2} as long as $c > 0$. Traders at $t = 1$ will anticipate that their optimal decision at $t = 1$ will constrain their optimal position at $t = 2$ because of the cost of changing their position. Substituting k_{i2} in the payoff function we have:

$$U = k_1 \left(\left(1 - \frac{c}{(1 + c)} \right) p_2 + \frac{c}{(1 + c)} \theta - p_1 \right) - \frac{1 + 2c}{1 + c} \frac{k_1^2}{2}$$

³²In particular, suppose that $\alpha_{x_2} = \alpha_{x_1} + \alpha_{x_2}^*$ where x_2 can be seen as a sufficient statistic for the two signals x_1, x_2^* .

³³Wang-Huang (2008) develop a model where they associate the liquidity of the market to the cost of participation in the market. Typically, in their model they associate hedge funds as permanent participants in financial market who improves the liquidity of the market. One could think of short term traders in this variation of the model as traders who are active all the time in the market because they are facing low enough participation cost. One plausible interpretation would be taxes of engaging in short term trading or administrative costs. However, one can also look at c as subjective cost related to preferences which make an investor rebalancing its portfolio costly.

As a result, in this economy, the choice of traders at $t = 1$ can be summarized by the following utility function:

$$U = k_1 (r\theta + (1 - r) p_2 - p_1) - \gamma \frac{k_1^2}{2}$$

where $r \equiv \frac{c}{1+c}$ and $\gamma \equiv \frac{1+2c}{1+c}$. One can notice that the payoff function of the short term traders in this alternative model is identical to the payoff function of the short term traders in the main text where r can be seen as the relative cost of changing their portfolio at $t = 2$. The higher the cost, the more the short term trader is going to expect it to be unlikely to change his portfolio and the more he will care about θ when making his decision.³⁴

1.7 Conclusion

In this paper, I studied an economy where short-term traders introduce an informational friction in financial markets. The present study can be divided in two parts where I first look at the extent in which a change in the composition of the market between informed and uninformed traders on one hand, short versus long term traders on the other, affect the informational efficiency of the market. Then, I focus on the process by which traders try to learn the beliefs of future market participants, by introducing a beliefs signal. A beliefs signal is defined as a signal which is informative about future price as it is informative about the future market participants average beliefs but independent of the fundamental.

The results can be summarized as follow: (i) a decrease in the horizon of the informed traders deteriorate the precision of the price; (ii) an inflow of informed traders in the market can decrease the informativeness of the price when the traders have a relatively short horizon or the market is expected to be thin in the future; (iii) the use of the beliefs-related information can decrease the precision of the information revealed by the price; (iv) as the informed traders have a more precise information about the average beliefs, i.e. a more precise beliefs signal, long term investors have overall a less precise information; (v) this phenomenon is more likely the smaller the cost of participation in the market.

³⁴One point to mention is that γ is also a function of c . In other word, the higher is c the higher is the cost of investing at $t = 1$. The more long-termist, the smaller their position or the more the price is sensitive to the supply shock.

Further investigation needs to be made in understanding better the extent to which short-term trading affects the market equilibrium outcome. In particular, empirically, few studies have looked at the effect of short-term trading on financial markets. In line with Berkman and Eleswarapu (1998), Parsa (2010), ³⁵, future work needs to be made in trying to assess empirically the effect of short-term trading on the informational efficiency of the market and in general financial markets outcome.

³⁵Berkman and Eleswarapu (1998) study the effect of short term trading on share prices and liquidity. To do so, they look at the Bombay Stock Exchange market before and after the abolition of the reinstatement of the Badla (forward trading facility). They argue that short term traders have on average a positive effect on the market. Parsa (2010) assess the horizon of investors through their intrinsic trading frequency. Then it explores the decomposition of the price between the cash flow and discount factor component as a function of the trading frequency of institutional investors. It shows that overall the prices of the securities held by investors trading more frequently mainly forecast the long run cash flow component of the securities.

1.8 Appendix

1.8.1 Appendix A

Lemma 3

For any given $(\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*) \in \mathbb{R}^5$, we know from lemma 2 that:

$$p_1 = \frac{\lambda_1 (\kappa_\theta \bar{E}_{S_1}(\theta) + \kappa_{z_1} z_1 + \kappa_{p_1} p_1)}{1 + \lambda_1} + \frac{\bar{E}_L(\theta)}{1 + \lambda_1} - \frac{\gamma u_1}{1 + \lambda_1}$$

where let $\delta_i^{s_1} = \frac{\alpha_i}{\sum_{i \in I_{S_1}} \alpha_i}$, $i \in I_{s_1} = \{x_1, z_1, p_1\}$ and $\delta_i^L = \frac{\alpha_i}{\sum_{i \in I_L} \alpha_i}$, $i \in I_L = \{z_1, p_1\}$.

Hence, $\bar{E}_{S_1}(\theta) = \delta_{x_1}^{s_1} \theta + \delta_{z_1}^{s_1} z_1 + \delta_{p_1}^{s_1} \tilde{p}_1$ and $\bar{E}_L(\theta) = \delta_{z_1}^L z_1 + \delta_{p_1}^L \tilde{p}_1$. Substituting and solving for p_1 and rearranging, we have:

$$p_1 = \beta_\theta \theta + \beta_{z_1} z_1 + \beta_u u_1$$

where $\beta_\theta = \frac{\lambda_1 \kappa_\theta \delta_{x_1}^{s_1}}{\kappa}$; $\beta_{z_1} = \frac{\lambda_1 \kappa_\theta \delta_{z_1}^{s_1} + \lambda_1 \kappa_{z_1} + \delta_{z_1}^L}{((1 + \lambda_1) - \lambda_1 \kappa_{p_1})}$; $\beta_u = -\frac{\gamma}{\kappa}$ and $\kappa_\theta \equiv r + (1 - r) \left(\beta_\theta^* + \beta_{z_2}^* - (1 - r) \lambda_1 \beta_u^* \frac{t}{f} \right)$
 $\kappa_{z_1} \equiv (1 - r) \left(\beta_{z_1}^* - (1 - r) \lambda_1 \beta_u^* \frac{\beta_{z_1}}{\beta_u} \right)$; $\kappa_{p_1} \equiv (1 - r) \left(\beta_{p_1}^* + (1 - r) \lambda_1 \frac{\beta_u^*}{\beta_u} \right)$; $\kappa = \left((1 + \lambda_1) - \lambda_1 \kappa_\theta \frac{\delta_{p_1}^{s_1}}{\beta_\theta} - \lambda_1 \kappa_{p_1} \right)$

Substituting κ_{z_1} and κ_{p_1} in β_{z_1} and solving the equation for β_{z_1} :

$$\beta_{z_1} = \frac{\lambda_1 \kappa_\theta \delta_{z_1}^{s_1} + \lambda_1 (1 - r) \beta_{z_1}^* + \delta_{z_1}^L}{(1 + \lambda_1 (1 - (1 - r) \beta_{p_1}^*))}$$

Lemma 5

For any given $(\beta_\theta, \beta_{z_1}, \beta_{p_1}) \in \mathbb{R}^3$, we know from lemma 4 :

$$p_2 = \frac{\bar{E}_{S_2}(\theta)}{\lambda_2} - \frac{(u_1 (1 - r) \lambda + u_2) \gamma}{\lambda_2}$$

where $\delta_i^{s_2} = \frac{\alpha_i}{\sum_{i \in I_{S_2}} \alpha_i}$, $i \in I_{S_2} = \{x_2, z_1, z_2, p_1, p_2\}$, $\bar{E}_{S_2}(\theta) = \delta_{x_2}^* \theta + \delta_{z_1}^* z_1 + \delta_{z_2}^* z_2 + \delta_{p_1}^* \tilde{p}_1 + \delta_{p_2}^* \tilde{p}_2$ and $\tilde{p}_1 \equiv \frac{1}{\beta_\theta} (p_1 - \beta_{z_1}^* z_1)$ and $\tilde{p}_2 \equiv \frac{1}{\beta_\theta^*} (p_2 - \beta_{p_1}^* p_1 - \beta_{z_1}^* z_1 - \beta_{z_2}^* z_2)$. Substituting and solving for p_1 and rearranging, we have:

$$p_2 = \beta_\theta^* \theta + \beta_{z_1}^* z_1 + \beta_{z_2}^* z_2 + \beta_{p_1}^* p_1 + \beta_u^* (u_1 (1 - r) \lambda_1 + u_2)$$

$$\begin{aligned}
\text{where } \beta_\theta^* &= \frac{\delta_{x_2}^*}{\left(1 - \delta_{p_2}^* \frac{1}{\beta_\theta^*}\right)}; \beta_{z_1}^* = \frac{\left(\delta_{z_1}^* - \delta_{p_1}^* \frac{\beta_{z_1}}{\beta_\theta} - \delta_{p_2}^* \frac{1}{\beta_\theta^*} \beta_{z_1}^*\right)}{\left(1 - \delta_{p_2}^* \frac{1}{\beta_\theta^*}\right)}; \beta_{z_2}^* = \frac{\left(\delta_{z_2}^* - \delta_{p_2}^* \frac{1}{\beta_\theta^*} \beta_{z_2}^*\right)}{\left(1 - \delta_{p_2}^* \frac{1}{\beta_\theta^*}\right)}; \\
\beta_{p_1}^* &= \frac{\left(\frac{\delta_{p_1}^*}{\beta_\theta} - \delta_{p_2}^* \frac{1}{\beta_\theta^*} \beta_{p_1}^*\right)}{\left(1 - \delta_{p_2}^* \frac{1}{\beta_\theta^*}\right)}; \beta_u^* = -\frac{\gamma}{\left(1 - \delta_{p_2}^* \frac{1}{\beta_\theta^*}\right) \lambda_2}. \text{ Rearranging, we obtain: } \beta_\theta^* = \delta_{x_2}^* + \delta_{p_2}^*, \\
\beta_{z_1}^* &= \delta_{z_1}^* - \delta_{p_1}^* \frac{\beta_{z_1}}{\beta_\theta}, \beta_{z_2}^* = \delta_{z_2}^*, \beta_{p_1}^* = \frac{\delta_{p_1}^*}{\beta_\theta}, \beta_u^* = -\frac{\gamma}{\left(1 - \delta_{p_2}^* \frac{1}{\beta_\theta^*}\right) \lambda_2}.
\end{aligned}$$

Corrolary 1

The equilibrium is concretely the solution of a system of equations in terms of the sensitivity of the price at $t = 1, 2$. The system of equations are defined from lemma 3 and lemma 5. Precisely, one can define from lemma 3 $(\beta_\theta, \beta_{z_1}, \beta_u)$ as an implicit function of $(\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*)$ by solving the system of equations for $(\beta_\theta, \beta_{z_1}, \beta_u)$, denoted $B(\cdot)$. Furthermore, one can define from lemma 5 $(\beta_\theta^*, \beta_{z_1}^*, \beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*)$ as an implicit function of $(\beta_\theta, \beta_{z_1}, \beta_u)$, denoted $B^*(\cdot)$. The solution will be the joint solution of the system of equations defined by $B(\cdot)$ and $B^*(\cdot)$.

1.8.2 Appendix B

Lemma 6

Let $b = -\frac{\beta_\theta}{\beta_u}$ and $b^* = -\frac{\beta_\theta^*}{\beta_u^*}$. One can define the system above to depend on b and b^* :

$$\begin{aligned}
\beta_\theta^* &= \delta_{p_2}^* + \delta_{x_2}^* \\
\beta_{z_1}^* &= \delta_{z_1}^* - \delta_{p_1}^* \beta_{z_1} \\
\beta_{z_2}^* &= \delta_{z_2}^* \\
\beta_{p_1}^* &= \frac{\delta_{p_1}^*}{\beta_\theta} \\
\beta_u^* &= -\beta_\theta^*/b^*
\end{aligned}$$

and

$$\begin{aligned}
\beta_\theta &= \frac{\kappa_\theta \lambda_1 \delta_{x_1}^{s_1}}{\lambda_1 \kappa_\theta \delta_{z_1}^{s_1} + \lambda_1^2 (1-r) \beta_{z_1}^* + \delta_{z_1}^L} \\
\beta_{z_1} &= \frac{\lambda_1 \kappa_\theta \delta_{z_1}^{s_1} + \lambda_1^2 (1-r) \beta_{z_1}^* + \delta_{z_1}^L}{(1 + \lambda_1 (1 - (1-r) \beta_{p_1}^*))} \\
\beta_u &= -\beta_\theta/b
\end{aligned}$$

where $\alpha_{p_1} = \frac{b^2}{\sigma_u^2}$ and $\alpha_{p_2} = \frac{b^{*2}}{(1 + (1-r)^2 \lambda_1^2) \sigma_u^2}$. Furthermore, using the system of equation above, one can redefine κ_θ as functions of only b and b^* ,

$$\kappa_\theta = r + (1-r) \left(\frac{(\delta_{p_2}^* + \delta_{x_2}^* + \delta_{z_2}^*) b^* - (\delta_{p_2}^* + \delta_{x_2}^*) \lambda_1 (1-r) b}{b^*} \right)$$

As a consequence, Hence, b and b^* must satisfy the following two conditions:

$$b = f_1(b, b^*)$$

$$b^* = f_2(b, b^*)$$

where $f_1(b, b^*) = \frac{\kappa_\theta \lambda_1 \delta_{x_1}^{s_1}}{\gamma}$ and $f_2(b, b^*) = \frac{\lambda_2 \delta_{x_2}^*}{\gamma}$ given $\delta_{x_1}^{s_1} = \frac{\alpha_{x_1}}{\alpha_{x_1} + \alpha_{z_1} + \alpha_{p_1}}$; $\delta_{x_2}^* = \frac{\alpha_{x_2}}{\alpha_{x_2} + \alpha_{z_1} + \alpha_{z_2} + \alpha_{p_1} + \alpha_{p_2}}$; $\alpha_{p_1} = \frac{b^2}{\sigma_u^2}$ and $\alpha_{p_2} = \frac{b^{*2}}{(1 + (1-r)^2 \lambda_1^2) \sigma_u^2}$. Notice that for notational purpose, I am abstracting the reliance of all the functions on (b, b^*) until it becomes necessary.

One can use $b^* = f_2(b, b^*)$ to solve for b^* as a function of b given b^* is the unique solution of the depressed cubic :

$$\phi(b, b^*) = 0$$

where $\phi(b, b^*) \equiv b^* \left(\alpha_{x_2} + \alpha_{z_1} + \alpha_{z_2} + \frac{b^2}{\sigma_u^2} \right) + \frac{b^{*3}}{(1 + (1-r)^2 \lambda_1^2) \sigma_u^2} - \frac{\lambda_2}{\gamma} \alpha_{x_2}$; $\phi(b, b^*)$ is continuous in (b, b^*) ; for any $b \in \mathbb{R}$: (i) $\phi(b, 0) = -\frac{\lambda_2}{\gamma} \alpha_{x_2} \leq 0$; (ii) $\phi'(b, b^*) = \left(\alpha_{x_2} + \alpha_{z_1} + \alpha_{z_2} + \frac{b^2}{\sigma_u^2} \right) + 3 \frac{b^{*2}}{(1 + (1-r)^2 \lambda_1^2) \sigma_u^2} \geq 0$ and (iii) $\lim_{b^* \rightarrow +\infty} \phi(b, b^*) = +\infty$. Hence, there is a unique b^* that solves $\phi(b, b^*) = 0$ for any $b \in \mathbb{R}$. Denote the solution $b^* = b^*(b)$. Now substituting the solution $b^* = b^*(b)$ in $f_1(b, b^*)$, one can define the function $G(b, \cdot) \equiv f_1(b, b^*(b))$.

Finally, I still need to show that the system of equations above can be written as a function of b only. To start with, notice that $\beta_\theta^*, \beta_{z_2}^*, \beta_u^*$ are functions of b after substituting $b^* = b^*(b)$, given they depend on (b, b^*) . Furthermore, $\beta_\theta = \frac{\kappa_\theta \lambda_1 \delta_{x_1}^{s_1}}{\kappa}$ can be rewritten as a function of

(b, b^*) and hence, as a function of b :

$$\beta_\theta = \frac{\kappa_\theta \lambda_1 \delta_{x_1}^{s_1}}{\kappa}$$

where $\kappa = \left((1 + \lambda_1) - \lambda_1 \kappa_\theta \frac{\delta_{p_1}^{s_1}}{\beta_\theta} - \lambda_1 \left((1 - r) \left(\frac{\delta_{p_1}^*}{\beta_\theta} + (1 - r) \lambda_1 \frac{\beta_\theta^* b / b^*}{\beta_\theta} \right) \right) - \frac{\delta_{p_1}^L}{\beta_\theta} \right)$ and $\kappa_\theta = \kappa_\theta(b, b^*)$ from above. Solving for β_θ , we have:

$$\beta_\theta = \frac{\kappa_\theta \lambda_1 (\delta_{x_1}^{s_1} + \delta_{p_1}^{s_1}) + \lambda_1 ((1 - r) (\delta_{p_1}^* + (1 - r) \lambda_1 (\delta_{p_2}^* + \delta_{x_2}^*) b / b^*)) + \delta_{p_1}^L}{(1 + \lambda_1)}$$

where the previous condition only depends on (b, b^*) . As a consequence, $\beta_{p_1}^*, \beta_u$ are determined by b . Finally, one can solve the following system for linear equations in $(\beta_{z_1}, \beta_{z_1}^*)$ to get them as a function of b :

$$\begin{aligned} \beta_{z_1}^* &= \delta_{z_1}^* - \delta_{p_1}^* \beta_{z_1} \\ \beta_{z_1} &= \frac{\lambda_1 \kappa_\theta \delta_{z_1}^{s_1} + \lambda_1^2 (1 - r) \beta_{z_1}^* + \delta_{z_1}^L}{(1 + \lambda_1 (1 - (1 - r) \beta_{p_1}^*))} \end{aligned}$$

where $\beta_{z_1} = \frac{\lambda_1 \kappa_\theta \delta_{z_1}^{s_1} + \lambda_1^2 (1 - r) \delta_{z_1}^* + \delta_{z_1}^L}{(1 + \lambda_1 (1 - (1 - r) \beta_{p_1}^*)) + \lambda_1^2 (1 - r) \delta_{p_1}^*}$. Hence, one can define the function $F(b, \cdot)$.

Proposition 2

I need to prove the existence and the uniqueness of the equilibrium. The proof is given for $\gamma > 0$.

(i) Uniqueness: $G(b, \cdot)$ is a continuous function over R^+ given I am looking at a positive solution. Furthermore, $G(0, \cdot) = \frac{\kappa_\theta \lambda_1 \delta_{x_1}^{s_1}}{\gamma} \geq 0$

because (i) $\kappa_\theta = r + (1 - r) \left(1 - \frac{\alpha_{z_1}}{\alpha_{x_2} + \alpha_{z_1} + \alpha_{z_2} + \bar{\alpha}_{p_2}} \right) \geq 0$ where $\bar{\alpha}_{p_2} = \frac{b^{*2}(0)}{(1 + (1 - r)^2 \lambda_1^2) \sigma_u^2}$ and

(ii) $\delta_{x_1}^{s_1} = \frac{\alpha_{x_1}}{\alpha_{x_1} + \alpha_{z_1}} \geq 0$. On the other hand, $\lim_{b \rightarrow +\infty} G(b, \cdot) = 0$ because (i) $\lim_{b \rightarrow +\infty} \delta_{x_1}^{s_1} = 0$,

(ii) $\lim_{b \rightarrow +\infty} (\delta_{p_2}^* + \delta_{x_2}^* + \delta_{z_2}^*) = 0$ and (iii) $\lim_{b \rightarrow +\infty} \delta_{x_1}^{s_1} \frac{(\delta_{p_2}^* + \delta_{x_2}^*) \lambda_1 (1 - r) b}{b^*} = 0$

Hence, there is at least one $b \in \mathbb{R}^+$ such that $b = G(b, \alpha_{x_1}, \alpha_{x_2}, \alpha_{z_1}, \alpha_{z_2}, \sigma_u^2, \lambda_1, \lambda_2, r, \gamma)$.

(ii) In order to prove uniqueness, we need to show that $G'(b, \cdot) \leq 0$.

$$G(b, \cdot) = \frac{\partial f_1(b, b^*(b))}{\partial b} + \frac{\partial f_1(b, b^*(b))}{\partial b^*} \frac{db^*(b)}{b}$$

where (a) $\frac{\partial f_1(b, b^*(b))}{\partial b} = -2\delta_{p_1}^{s_1} - \frac{\lambda_1 \delta_{x_1}^{s_1} \left((1-r) \left(\left(2 \frac{(\delta_{p_2}^* + \delta_{x_2}^* + \delta_{z_2}^*)}{b} \delta_{p_1}^* \right) + \gamma \frac{(\alpha_{p_2} + \alpha_{x_2}) \lambda_1 (1-r)}{\lambda_2 \alpha_{x_2}} \right) \right)}{\gamma}$

(b) $\frac{\partial f_1(b, b^*(b))}{\partial b^*} = \frac{(1-r) \left(2 \frac{(\delta_{p_1}^* + \delta_{z_1}^*) \delta_{p_2}^*}{b^*} - 2\gamma \frac{\alpha_{p_2}^* \lambda_1 (1-r) b}{b^* \lambda_2 \alpha_{x_2}^*} \right) \lambda_1 \delta_{x_1}^{s_1}}{\gamma}$

(c) $\frac{db^*(b)}{b} = \frac{f_{21}(b, b^*)}{1 - f_{22}(b, b^*)} = -b^* \frac{2\delta_{p_1}^*/b}{1 - 2\delta_{p_2}^*} \leq 0$

Hence, one can show that $G'(b, \cdot) \leq 0$ which shows uniqueness.

Proposition 3

We need to show that: $\frac{\partial \alpha_{p_1}}{\partial r} \geq 0$. Notice that $\alpha_{p_1} = \frac{b^2}{\sigma_u^2}$. Hence, $\frac{\partial \alpha_{p_1}}{\partial r} \Leftrightarrow \frac{\partial b}{\partial r}$. So, we need to show that the sensitivity of the price to the fundamental relative to the noise increases with the horizon of the traders.

$$\frac{\partial b}{\partial r} = \frac{\frac{\partial f_1(b, b^*(b), r)}{\partial r} + \frac{\partial f_1(b, b^*(b), r)}{\partial b^*} \frac{\partial b_\theta^*(b, r)}{\partial r}}{1 - \left(\frac{\partial f_1(b, b^*(b), r)}{\partial b} + \frac{\partial f_1(b, b^*(b), r)}{\partial b^*} \frac{db^*(b, r)}{b} \right)} \geq 0$$

We know from above that the previous proposition that the denominator is positive. We still need to show that:

(a) $\frac{\partial f_1(b, b^*(b), r)}{\partial r} = \frac{(\delta_{p_1}^* + \delta_{z_1}^*) \lambda_1 \delta_{x_1}^{s_1}}{\gamma} \geq 0$

(b) $\frac{\partial f_1(b, b^*(b), r)}{\partial b^*} = \frac{\left(r + (1-r) \left(2 \frac{(\delta_{p_1}^* + \delta_{z_1}^*) \delta_{p_2}^*}{b^*} - 2\gamma \frac{\alpha_{p_2}^* \lambda_1 (1-r) b}{b^* \lambda_2 \alpha_{x_2}^*} \right) \right) \lambda_1 \delta_{x_1}^{s_1}}{\gamma}$

(c) $\frac{\partial b_\theta^*(b, r)}{\partial r} = -2b^* \delta_{p_2}^* \frac{(1-r) \lambda_1^2}{(1 + (1-r)^2 \lambda_1^2)} \leq 0$

Hence, the condition above is satisfied.

Proposition 4

In any linear equilibrium,

$$\frac{\partial b}{\partial \lambda_1} = \frac{\frac{\partial f_1(b, b^*(b), r)}{\partial \lambda_1} + \frac{\partial f_1(b, b^*(b), r)}{\partial b^*} \frac{\partial b^*(b, r)}{\partial \lambda_1}}{1 - \left(\frac{\partial f_1(b, b^*(b), r)}{\partial b} + \frac{\partial f_1(b, b^*(b), r)}{\partial b^*} \frac{db^*(b, r)}{b} \right)} \geq 0$$

See graphs for the proof. For $r = 1$, the proof is trivially given by: $\frac{\partial b}{\partial \lambda_1} = \frac{\frac{\delta_{x_1}^{s_1}}{\gamma}}{1 + \delta_{p_1}^{s_1}} \geq 1$.

1.8.3 Appendix C

I am going to solve the problem in the presence of the beliefs signal in more details.

Lemma 7

Given the linear structure of equilibrium,

$$\bar{p}_1 = \theta + \frac{\beta_u}{\beta_\theta} u_1 + \frac{\beta_y}{\beta_\theta} y$$

where $\bar{p}_1 \equiv \frac{p_1}{\beta_\theta}$ with precision $\alpha_{\bar{p}_1} = \frac{\beta_\theta^2}{\beta_u^2 \sigma_u^2 + \beta_y^2 \sigma_y^2}$; $\sigma_y^2 \equiv \sigma_\omega^2 + \sigma_{\epsilon_2}^2$. We can notice that p_1 and z_2 conditional on θ are distributed as a bivariate normal distribution with covariance $\sigma_{zp} = \frac{\pi_y}{\pi_\theta} \sigma_{z_1}^2$. As a consequence :

$$\theta | z_2, p_1 \sim N(P_Z, \alpha_{P_Z}^{-1})$$

Denote $P_Z \equiv \bar{p}_1 \bar{\delta}_{p_1} + \bar{\delta}_{z_2} z_2$ the sufficient Gaussian statistic of z_2, \bar{p}_1 for θ with precision $\alpha_{P_Z} = \alpha_{z_2} \frac{1 - \frac{\sigma_{zp}}{\sigma_p^2}}{1 - \rho^2} + \alpha_{\bar{p}_1} \frac{1 - \frac{\sigma_{zp}}{\sigma_z^2}}{1 - \rho^2}$ where $\bar{\delta}_{p_2} = \frac{\alpha_{z_2}}{\alpha_{z_2} + \tilde{\alpha}_{p_0}}$; $\bar{\delta}_{z_2} = 1 - \bar{\delta}_{p_1}$ and $\tilde{\alpha}_{p_1} = \alpha_{p_1} \frac{\left(1 - \frac{\sigma_{zp}}{\sigma_z^2}\right)}{\left(1 - \frac{\sigma_{zp}}{\sigma_p^2}\right)}$.

Furthermore, $\rho^2 = \frac{\left(\frac{\beta_y}{\beta_\theta}\right)^2 \sigma_{z_2}^4}{\sigma_u^2 + \beta_y^2 \sigma_y^2 \sigma_{z_2}^2}$; $\frac{\sigma_{zp}}{\sigma_{z_2}^2} = \frac{\beta_y}{\beta_\theta}$ and $\frac{\sigma_{zp}}{\sigma_p^2} = \frac{\frac{\beta_y}{\beta_\theta} \sigma_{z_2}^2}{\sigma_u^2 + \beta_y^2 \sigma_y^2}$. Notice that for $\sigma_{zp} = 0$,

$$\alpha_{P_Z} = \alpha_{z_2} + \alpha_{\bar{p}_1}.$$

Typically, as short term traders are using y , p_1 transfers information about ϵ_2 to future market participants which affects the information they extract from z_2 about θ . The weight given to both signals and ultimately to their respective precision is such that $\left(\frac{\sigma_{zp}}{\sigma_{z_2}^2}\right) \frac{\sigma_{zp}}{\sigma_p^2} \leq \rho^2$ iff the weight given to α_{z_2} ($\alpha_{\bar{p}_1}$) is bigger than 1.³⁶ Intuitively, one way to look at it is as follows. The variance decomposition of P_Z between z_2 and \bar{p}_1 is such that there is a higher dependence on the precision of z_2 if in a linear regression of $z_2 \sim \bar{p}_1$, the R^2 is higher than the regression coefficient. Hence, whenever \bar{p}_1 explains a significant share in ϵ_2 , a weight higher than 1 will be given to it. As a consequence, the decomposition of precision of P_Z will depend relatively more on the precision of z_2 .

From the point of view of short term traders, for any given $(\beta_{z_2}^*, \beta_{p_1}^*, \beta_u^*) \in \mathbb{R}^3$, we know that

$$p_2 = \beta_{z_2}^* z_2 + \beta_{p_1}^* p_1 + \beta_u^* ((1-r) \lambda_1 u_1 + u_2)$$

Given $R = r\theta + (1-r)p_2$ and $I_{i_1} = \{x_{i_1}, p_1, y\}$, $i \in S_1$, we have:

$$\bar{E}_{S_1}(R) = \kappa_\theta \bar{E}_{S_1}(\theta) + \kappa_y \bar{E}_{S_1}(\epsilon_1) + \kappa_{p_1} p_1$$

where $\bar{E}_{S_1}(R) = \bar{E}_{S_1}(r\theta + (1-r)p_2) = (r + (1-r)\beta_{z_1}^*) \bar{E}_{S_1}(\theta) + (1-r)\pi_{p_0}^* p_0 - (1-r)^2 \lambda_1 \bar{E}_{S_1}(u_1) + (1-r)\pi_{z_1}^* \bar{E}_{S_1}(\epsilon_1)$. Let $\delta_i^{S_1} = \frac{\alpha_i}{\sum_{i \in I_{S_1}} \alpha_i}$, $i \in I_{S_1} = \{x_1, p_1\}$, (i) $\bar{E}_{S_1}(\theta) = \delta_x^{S_1} \theta + \delta_{p_1}^{S_1} \tilde{p}_1$; (ii) $\bar{E}_{S_1}(u_1) = -\frac{\beta_\theta \bar{E}_{S_1}(\theta) - \beta_{p_1} p_1 + \beta_y y}{\beta_u}$ and (iii) $\bar{E}_{S_1}(\epsilon_1) = \frac{\alpha_\omega}{\alpha_\omega + \alpha_{z_2}} y$. Notice that from the point of the view of the informed traders, $\tilde{p}_1 = \frac{1}{\beta_\theta} \left(p_1 - \frac{\beta_y}{\beta_\theta} y \right)$ and $\alpha_{p_1} = \frac{\beta_\theta^2}{\beta_u^2 \sigma_u^2}$. This is because informed traders observe y . Hence, it does not appear as noise.

Equilibrium characterization

From the point of view of the market at $t = 2$, the equilibrium price is given by:

$$p_2 = \beta_{z_2}^* z_2 + \beta_{p_1}^* p_1 + \beta_u^* ((1-r) \lambda_1 u_1 + u_2)$$

³⁶Notice that by construction we can not have both $\frac{\sigma_{zp}}{\sigma_{z_2}^2}, \frac{\sigma_{zp}}{\sigma_p^2} \leq \rho^2$ at the same time. This is because $\rho^2 \in [0, 1]$ and $\frac{\sigma_{zp}^2}{\sigma_{z_2}^2 \sigma_p^2} = \rho^2 \leq \rho^4$ can not be.

where $\beta_{z_2}^* \equiv \bar{\delta}_{z_2}$; $\beta_{p_1}^* = \frac{\bar{\delta}_{p_1}}{\beta_\theta}$ and $\beta_u^* \equiv -\frac{\gamma}{\lambda_2}$; $\bar{\delta}_{p_{z_2}} = \frac{\alpha_{z_2}}{\alpha_{z_2} + \bar{\alpha}_{p_0}}$; $\bar{\delta}_{z_2} = 1 - \bar{\delta}_{p_1}$ and $\bar{\alpha}_{p_1} = \alpha_{p_1} \frac{\left(1 - \frac{\sigma_{zp}}{\sigma_z^2}\right)}{\left(1 - \frac{\sigma_{zp}}{\sigma_p^2}\right)}$.

At $t = 1$, on the other hand, we have:

$$p_1 = \frac{\lambda_1 (\kappa_\theta \bar{E}_{S_1}(\theta) + \kappa_y \bar{E}_{S_1}(\epsilon_1) + \kappa_{p_1} p_1)}{1 + \lambda_1} + \frac{\bar{E}_L(\theta)}{1 + \lambda_1} - \frac{\gamma}{1 + \lambda_1} u_1$$

where $\kappa_\theta \equiv r + (1-r) \left(\beta_{z_2}^* - (1-r) \lambda_1 \beta_u^* \frac{\beta_\theta}{\beta_u} \right)$; $\kappa_{p_1} \equiv (1-r) \left(\beta_{p_1}^* + (1-r) \lambda_1 \frac{\beta_u^*}{\beta_u} \right)$ and $\kappa_y \equiv (1-r) \left(\beta_{z_2}^* - \frac{\gamma \lambda_1 (1-r) \beta_y \alpha_\omega + \alpha_{z_2}}{\lambda_2 \beta_u \alpha_\omega} \right)$ and $\bar{E}_{S_1}(\theta) = \delta_x^{s_1} \theta + \delta_{p_1}^{s_1} \tilde{p}_1$; (ii) $\bar{E}_{S_1}(\epsilon_1) = \frac{\alpha_\omega}{\alpha_\omega + \alpha_{z_2}} y$ and (iii) $\bar{E}_L(\theta) = \frac{p_1}{\beta_\theta}$. Hence, denote $\kappa \equiv \left(1 + \lambda_1 - \lambda_1 \kappa_{p_1} - \lambda_1 \kappa_\theta \frac{\delta_{p_1}^{s_1}}{\beta_\theta} - \frac{1}{\beta_\theta} \right)$

$$\beta_\theta = \frac{\lambda_1 \delta_x^{s_1} \kappa_\theta}{\kappa}; \beta_y = \frac{\lambda_1 \left(\kappa_y \frac{\alpha_\omega}{\alpha_\omega + \alpha_{z_2}} - \kappa_\theta \frac{\delta_{p_1}^{s_1}}{\beta_\theta} \beta_y \right)}{\kappa}$$

and $\beta_u = -\frac{\gamma}{\kappa}$

In the same fashion as earlier, denote $b_\theta = -\frac{\beta_\theta}{\beta_u}$ and $b_y = -\frac{\beta_y}{\beta_u}$, the equilibrium will be characterized by a fixed point of this two dimensional object.

Proposition 17 \exists functions $G : \mathbb{R}^{2+} \times \mathbb{R}_0^{4+} \times [0, 1]^3 \rightarrow \mathbb{R}^{2+}$ and $F : \mathbb{R}^{2+} \times \mathbb{R}_0^{4+} \times [0, 1]^3 \rightarrow \mathbb{R}^5$ such that there exists at least one equilibrium i.e. \exists at least one $(b_\theta, b_y) \in \mathbb{R}^{2+}$ that solves the following system of equations:

$$b_\theta = G_1(b_\theta, b_y, \alpha_{x_1}, \alpha_{z_2}, \alpha_\omega, \sigma_u^2, \lambda_1, \lambda_2, r, \gamma)$$

$$b_y = G_2(b_\theta, b_y, \alpha_{x_1}, \alpha_{z_2}, \alpha_\omega, \sigma_u^2, \lambda_1, \lambda_2, r, \gamma)$$

where $G = [G_1, G_2]$ and $(\beta_\theta, \beta_y, \beta_u, \beta_{z_1}^*, \beta_u^*) \equiv F(b_\theta, b_y, \alpha_{x_0}, \alpha_{z_1}, \alpha_\omega, \sigma_u^2, \lambda, \mu, r, \gamma)$.

Proof. We know from above that $\beta_{z_2}^* \equiv \bar{\delta}_{z_2}$ depends only on (b_θ, b_y) . Furthermore, $\beta_\theta = \frac{\lambda_1 \delta_x^{s_1} \kappa_\theta}{\kappa}$ where

$$\kappa_\theta = r + (1-r) \left(\beta_{z_2}^* - \frac{\gamma}{\lambda_2} (1-r) \lambda_1 b_\theta \right)$$

$$\kappa_y \equiv (1-r) \left(\beta_{z_2}^* - \frac{\gamma}{\lambda_2} (1-r) \lambda_1 b_y \right)$$

which are function of (b_θ, b_y) . Given κ_θ and κ_y are both function of $(b_\theta, b_y) \in \mathbb{R}^+$, I can show that (b_θ, b_y) solve for the following system of equations:

$$\begin{aligned} b_\theta &= \frac{\lambda_1 \delta_x^{s_1} \kappa_\theta}{\gamma} \\ b_y &= \frac{\alpha_{x_1} \lambda_1 (1-r) \beta_{z_2}^* \frac{\alpha_\omega}{\alpha_\omega + \alpha_{z_2}}}{\gamma \left(\alpha_{x_1} + \alpha_{p_1} + \alpha_{x_1} \frac{1}{\lambda_2} (1-r)^2 \lambda_1^2 \right)} \end{aligned}$$

Furthermore,

$$\beta_\theta = \frac{\lambda_1 \delta_x^{s_1} \kappa_\theta + \left(\lambda_1 \left((1-r) \left(\bar{\delta}_{p_1} + (1-r) \frac{\lambda_1 \gamma b}{\lambda_2} \right) \right) \right) + \lambda_1 \delta_{p_1}^{s_1} \kappa_\theta + 1}{(1 + \lambda_1)}$$

which is also a function of (b_θ, b_y) . As a consequence, $\beta_{p_1}^* = \frac{\bar{\delta}_{p_1}}{\beta_\theta}$ also depends only on (b_θ, b_y) . Finally, notice that

$$\beta_y = \frac{\alpha_{x_1}}{\alpha_{x_1} + \alpha_{p_1}} \left(\frac{\lambda_1 (1-r) \left(\beta_{z_2}^* - \frac{\gamma \lambda_1 (1-r)}{\lambda_2} b_y \right) \frac{\alpha_\omega}{\alpha_\omega + \alpha_{z_2}}}{\kappa} \right)$$

■

Corollary 18 In any linear equilibrium,

- (i) $0 \leq \frac{\beta_y}{\beta_\theta} \leq 1$
- (ii) $\bar{\delta}_{p_1}, \bar{\delta}_{z_2} \geq 0$

Proof. From above, we know that $\beta_{z_1}^* \geq 0$, given

$$\bar{\delta}_{p_1} = \frac{\alpha_{p_1} \left(1 - \frac{\beta_y}{\beta_\theta} \right)}{\alpha_{p_1} \left(1 - \frac{\beta_y}{\beta_\theta} \right) + \alpha_{z_2} \left(1 - \frac{\frac{\beta_y \sigma_{z_2}^2}{\beta_\theta} \sigma_u^2 \gamma^2 + \beta_y^2 \sigma_y^2}{\beta_\theta^2} \right)}$$

where for any $\frac{\beta_y}{\beta_\theta} > 1$, $\beta_{z_1}^* (\bar{\delta}_{p_1}) \geq (\leq) 1$; for any $0 \leq \frac{\beta_y}{\beta_\theta} \leq 1$, $\beta_{z_1}^* (\bar{\delta}_{p_1}) \in [0, 1]$ and for any

$\frac{\beta_y}{\beta_\theta} \leq 0$, $1 - \frac{\frac{\beta_y}{\beta_\theta} \sigma_{z_2}^2}{\sigma_u^2 \gamma^2 + \beta_y^2 \sigma_y^2} \geq 0$, as a consequence, $\beta_{z_1}^* (\bar{\delta}_{p_1}) \geq 0$. It follows that: $0 \leq \frac{\beta_y}{\beta_\theta} = \frac{\left((1-r) \beta_{z_1}^* \frac{\alpha_\omega}{\alpha_\omega + \alpha_{z_1}} \right)}{(r + (1-r) \beta_{z_1}^*)} \leq 1$. As a consequence, we know that $\frac{\beta_y}{\beta_\theta} \in [0, 1]$, $\bar{\delta}_{p_1} \geq 0$ ■

Lemma 8

It is the direct application of (i)

$$\bar{p}_1 = \theta + \frac{\beta_u}{\beta_\theta} u_1 + \frac{\beta_y}{\beta_\theta} y$$

where $\bar{p}_1 \equiv \frac{p_1}{\beta_\theta}$ with precision $\alpha_{\bar{p}_1} = \frac{\beta_\theta^2}{\beta_u^2 \sigma_u^2 + \beta_y^2 \sigma_y^2}$; $\sigma_y^2 \equiv \sigma_\omega^2 + \sigma_{\epsilon_1}^2$ and (ii)

$$\theta | z_2, p_1 \sim N(P_Z, \alpha_{P_Z}^{-1})$$

where $\alpha_{P_Z} = \alpha_{z_2} \frac{1 - \frac{\sigma_{zp}}{\sigma_p^2}}{1 - \rho^2} + \alpha_{\bar{p}_1} \frac{1 - \frac{\sigma_{zp}}{\sigma_z^2}}{1 - \rho^2}$. Furthermore, $\rho^2 = \frac{\left(\frac{\beta_y}{\beta_\theta}\right)^2 \sigma_{z_2}^4}{\sigma_u^2 + \beta_y^2 \sigma_y^2 \sigma_{z_2}^2}$; $\frac{\sigma_{zp}}{\sigma_{z_2}^2} = \frac{\beta_y}{\beta_\theta}$ and

$$\frac{\sigma_{zp}}{\sigma_p^2} = \frac{\frac{\beta_y}{\beta_\theta} \sigma_{z_2}^2}{\frac{\sigma_u^2 + \beta_y^2 \sigma_y^2}{\beta_\theta^2}}$$

Notice that (i) $\lambda_{z_2} \in [0, 1]$ given $1 \geq \frac{\beta_y}{\beta_\theta} \geq 0$ and (ii) $\lambda_{p_1} \in [0, 1]$ given $1 \geq \frac{\frac{\beta_y}{\beta_\theta} \sigma_{z_2}^2}{\sigma_u^2 \gamma^2 + \beta_y^2 (\sigma_{z_2}^2 + \sigma_\omega^2)}$

is always true in equilibrium, which follows from the corollary 2.

Lemma 9

It follows directly from $\lambda_{z_2}, \lambda_{p_1} \in [0, 1]$.

Figure 1

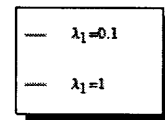
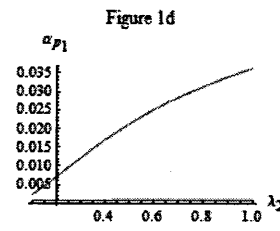
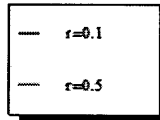
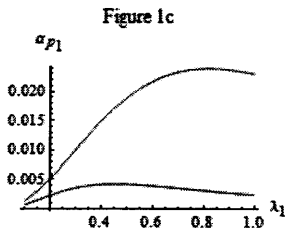
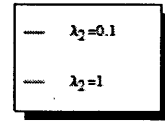
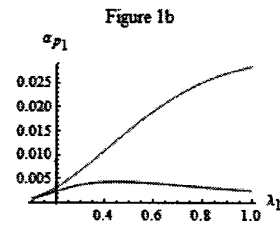
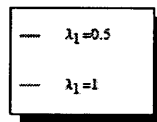
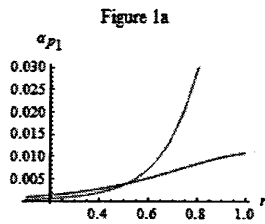
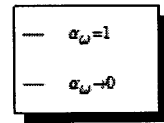
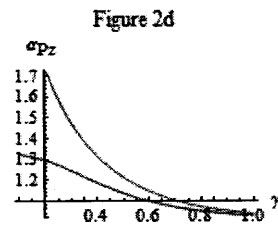
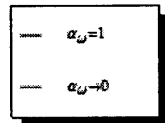
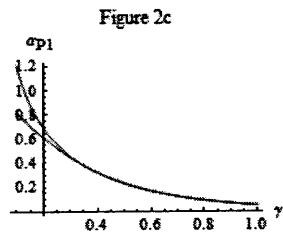
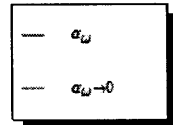
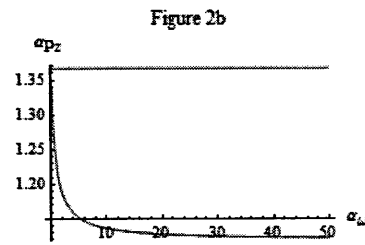
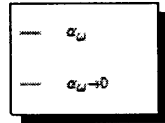
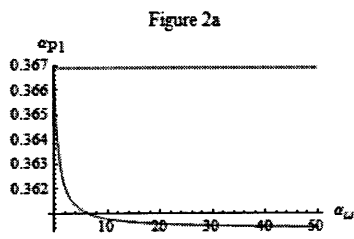


Figure 2



Chapter 2

Institutional Investors' Short-Termism, Trading Frequency and Firm-Level Stock Price Volatility

2.1 Introduction

Investors' short investment horizon is often cited as an important factor affecting stock prices behavior. Recurrently, policy makers and practitioners associate investors' short-termism with instability, volatility and noise in financial markets. The use of information by short-term investors is highlighted as a potential channel through which the horizon of investors leads to volatility and/or the disconnect of a stock price from its fundamental valuation. This idea goes back as far as Keynes' beauty contest analogy of financial markets.¹ An investor with a short

¹Keynes described the strategy of rational investors in financial markets, using an analogy to a newspaper beauty contest. The beauty contest is described as a game in which the players win if they pick the most popular photographs of women. "It is not a case of choosing those [faces] that, to the best of one's judgment, are really the prettiest, nor even those that average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees." (Keynes, *General Theory of Employment Interest and Money*, 1936). Keynes likened the behavior of investors in stock markets to the beauty contest's players as highlighted in the previous quote.

horizon cares about near future price movements and future market participants' beliefs or perceptions of the intrinsic value of the company. The investor's investment decision could be unsupported by information about fundamentals such as a firm's profitability. Referring to the risk related to the myopic behavior of financial institutions, a Member of the Executive Board of the ECB stated:²

"...The combination of the two risks mentioned above - myopic behavior and market concentration - might further fuel the risk of herding behavior... Asset managers might thus become more and more focused on trying to anticipate other managers' expectations, including the central bank short-term behavior, rather than looking at fundamentals." L. Bini Smaghi, 2006.

Although these popular notions have spurred considerable theoretical work, there is little empirical work validating, or rejecting, these notions. This paper attempts to fill this gap.

Assessing the empirical relevance of the relation between the horizon of an investor and stock prices, faces an important challenge. This challenge is associated to the difficulty of mapping these notions to the data. An investor's horizon is a primitive of its preferences and behavior and by its nature, is not observable. This paper constructs an indicator for the investor's horizon using its trading frequency. As such, the measure for the horizon in this paper is less directly connected to the notion of horizon referring to the length of an investor's life-span, as an investor's life-span may not be directly related to an investor frequency of trading. On the other hand, this measure closely relates to the common notion shared in the popular claims that associate short-term investors to short-term price movements' chasers.

"...We think of Berkshire as being a non-managing partner in two extraordinary businesses, in which we measure our success by the long-term progress of the companies rather than by the month-to-month movements of their stocks. In fact, we would not care in the least if several years went by in which there was no trading, or quotation of prices, in the stocks of those companies. If we have good long-

²This quote is taken from the speech of Lorenzo Bini Smaghi, a member of the executive board of the ECB, at a conference on "Dealing with the New Giants", Geneva, 4 May 2006.

term expectations, short-term price changes are meaningless for us..." Berkshire Hathaway, 1999.

Here, the investment horizon refers to the length of time that an investor expects to hold a security or a portfolio. As such, the horizon of an investor is tightly linked to the frequency with which it tends to rebalance its portfolio (its trading frequency). An investor with a shorter horizon –who holds on to a security for a shorter period of time, will trade more frequently.

"...Here's no doubt Warren Buffett [primary shareholder, chairman and CEO of Berkshire Hathaway] is a long-term investor. He looks for outstanding businesses that he can buy at a fair price with the intent of holding them for a long time... — ideally forever" G. Forsythe, 2007.³

However, it is worth noting that while an investment horizon is an unobservable investor's characteristic, its frequency to trade is an equilibrium outcome. Hence, a simple comparison of the portfolio turnover rates across investors and time will confound the horizon with the effect of security and market characteristics. For example, consider two investors who hold different securities and have different portfolio turnover rates. The difference in their portfolio turnover rates may be due to the differences in the investors' investment horizon (what we are interested in) or may be due to the differences in the characteristics of the securities they hold. Similarly, the portfolio turnover rate of investors at different points in time may be different due to aggregate market effects that change across time.

In this paper, I construct a novel measure of the intrinsic frequency of trading for the large US institutional investors as the fixed effect in the investors' trading frequency. I then study how the composition of these fixed effects at the security level impacts stock prices behavior using firm-level panel data.

I use quarterly holding data for the large US financial institutions, also named the 13-F institutions after the report they are required to fill on a quarterly basis, from the Thomson-Reuters Institutional Holdings dataset from 1980 to 2005. I measure the institution's intrinsic

³This quote has been taken from the February 2007 issue of Schwab Investing Insights, a monthly publication for Schwab clients. It has been written by Greg Forsythe, CFA, Senior Vice President, Schwab Equity Ratings, Schwab Center for Financial Research.

trading frequency as the investor's fixed effect in a three way fixed effects model of the investors' change in their positions. In particular, I use information about the quarterly change in an institution's position in a security, measured by the absolute value of the percentage change in the number of shares they are holding in the security. I exploit the three dimensions of the variable: investors, security and time. The institution's fixed effect captures the institutions' intrinsic trading frequency, by controlling for any security and market characteristics, which could influence the investor's change in its position across time and across securities. This new measure captures an institution's trading characteristic and provides a way to compare one institution to another in terms of their trading behavior.

As shown by Gompers and Metrick (2001), the 13-F institutions have experienced a tremendous growth and represent an important share of the US equity markets. The growth in the institutional investor population has been accompanied with an increase in the size of the mutual funds, the brokerage companies and the investment banks. I show that on average, the latter institutions appear to exhibit the highest intrinsic trading frequency. This finding is in line with claims made about the stock markets becoming more short-termist:

"...The culture of short-termism—investing for short-term gains at the expense of long-term accumulation—has taken hold on Wall Street. Managerial capitalism has replaced financial capitalism as holding periods for stocks dropped from eight years in the 1960s to as low as six months today" B. George, Wall Street Journal, 2010.

For each security traded in the NYSE, Nasdaq and Amex and held by a 13-F institution, I then construct an institution's trading frequency index. This index is measured as the weighted average of the trading frequency fixed effect of the investors holding the security. I then explore the relation between the institution's trading frequency index and stock prices movements at the security level. I look at the extent to which security prices respond, on average, to different kinds of information, when held by high as opposed to low trading frequency investors. In particular, does the price of the securities held by high trading frequency investors forecast less the long run cash flow of a company than the securities held by low trading frequency investors? Does the price of the securities held by high trading frequency investors forecast better the long run discount factor relative to the securities held by low trading frequency

investors? Concretely, I estimate the forecasting coefficients of a regression of the total return and the total return on equity on the book to market ratio—a proxy for the price, for different horizons, i.e. from one to thirteen years into the future. The cash flow is proxied by the return on equity. The return on equity can be seen as a normalized measure of the earnings (net income) of a company, defined as the earnings over the book value of the company. Thus, it is related to the company's fundamental because of its link to productivity. On the other hand, the discount rate component, measured by the return, can reflect time-varying risk aversion or investor sentiments.⁴ Forecasting regressions are informative on the extent to which investors react to different types of information. For instance, if the investors react to information about the long run cash flow of a company, then on average the prices should respond to that information and should forecast the cash flow of the company. I use firm level annual data and I explore the difference in the forecasting coefficients for the securities held by high trading frequency investors as opposed to low trading frequency investors.

The results can be summarized as follows. First, consistent with claims made about the volatility and short-termism, I find that securities held by higher trading frequency investors exhibit a higher volatility than securities held by lower trading frequency investors. The return of the securities with a high institution's trading index, i.e. an index in the upper tercile, has a standard deviation 7 percentage points higher than the securities with a low institution's trading index.⁵

However, looking at the forecasting role of the book to market ratio, I then show that the price of a security held by low trading frequency investors mainly forecasts the long run cash flow of a firm, as captured by the profitability of a firm, as opposed to the long run return. Overall, there is no statistical and economic difference between the forecasting role of the prices for the return on equity for securities held by high trading frequency investors as opposed to low trading frequency investors at any horizon. However, when it comes to the long run return, the forecasting role of the book to market ratio differs between high and low trading frequency

⁴Conceptually, using Vuolteenaho (2002) accounting present value identity, the book to market ratio can be decomposed between long run return on equity (cash flow component) and long run discount factor (discount factor component) as well as an error term. Hence, the book to market—a proxy for the price—should reflect a mixture of information about the long run return and the long run return on equity.

⁵This phenomenon is a pervasive feature of the data and holds for weekly, monthly and quarterly returns as well.

securities. The book to market does not seem to forecast long run return for the securities held by high trading frequency investors (with a coefficient of 0.08 and a standard error of 0.07 for the return after 13 years) as opposed to the securities held by low trading frequency investors (with a coefficient of 0.26 and a standard deviation of 0.037).

I then explore the return predictability in more detail. Does the return predictability of the securities held by high trading frequency investors differ as well with respect to other traditional forecasting variables used in the literature? Overall, in terms of the return predictability literature, this study shows that the securities held by investors with different trading characteristics differ in terms of their risk premia. In particular, contrary to the return of the securities held by low trading frequency investors, the return of the securities held by high trading frequency investors, is on average non forecastable by the usual forecasting variables highlighted in the literature, such as the return, the return on equity, the book to market ratio or the leverage. Moreover, I show that the trading frequency index contains additional information regarding the next year return, not embedded in the usual forecasting variables.

Another way to explore this question is to look at the return directly by using the return decomposition framework put forward by Campbell (1991). Conceptually, this decomposition divides the unexpected change in the return on the change in the expectation of future cash flows (Cash flow News) and/or future return (Discount Factor News) using the present value identity. Looking at the return decomposition, I find that the unexpected return movements (with a variance of 0.11) are traced back to the cash flow news (with a variance of 0.095) as opposed to the discount factor news (with a variance of 0.004). Both cash flow and discount factor news are estimated through a panel VAR.

Finally, I show that the lower volatility of the return of the securities held by low trading frequency institutions index is traced back to an immediate under-reaction of the price to cash flow news. The cash flow news have been built using the VAR-return decomposition. Overall, a portfolio of securities held by low trading frequency investors exhibits on average a slower reaction of the return to the cash flow news than a portfolio of the securities held by high trading frequency investors. After distinguishing between good and bad cash flow news, I find that the differential reaction of the return between high and low trading frequency institutions index is associated to the reaction of the return to good news. On average a security reaction

to a 1% cash flow news, is 24% higher for securities held by high trading frequency institutions relative to low trading frequency institutions.

To summarize, using yearly frequency data and the 13-F institutional investors, it seems that short-term investors are more like the dog that barks, [but] does not bite. In other words, they are associated to higher volatility, but on average the volatility is related to the price reflecting the fundamental and to firm-level information about cash flow news. It seems that the prices are aggregating the information about the firm's cash flow well.

It is important to stress the scope of this study as well as some directions for future research. First, given that I use quarterly holdings data for the 13-F institutions, I cannot make any inference regarding the relationship between stock prices and short-termism for day traders or retail investors. It is plausible that the disconnect between the prices and the fundamental valuation does indeed take place with day traders or retail investors. Likewise, short-termism could generate a deviation from the fundamental valuation at higher frequency (for monthly or daily data) and such deviation decreases for annual frequency data. Second, my measure on the investor's idiosyncratic frequency to trade is a proxy for, but does not capture exclusively the investor's horizon. For instance, the trading frequency fixed-effect could confound the investor's horizon together with other investor's intrinsic characteristics making him or her likely to trade more frequently such as its information such as his or her access to information.⁶

Nonetheless, beyond these caveats related to the interpretation of the trading frequency fixed effect, this study highlights a new type of variable in understanding and explaining stock prices. This variable, contrary to usual securities characteristics, is related to the heterogeneity in the investors holding the securities. As such, the results show that the trading frequency index contains novel information to understand stock prices beyond the usual variables such as the size of a security, among others. This new variable embeds an interesting way of looking at asset pricing as it relates not to the characteristics of the firm, but rather to the characteristics of the investors who hold the particular security.

The remainder of the paper is organized as follows. Section 2 provides a brief description of the literature. Section 3 introduces the measure for the horizon of the institutional investors

⁶Thanks to the focus on large U.S. institutional investors, the variations in the fixed-effect are less likely to capture differences in risk aversion or liquidity constraints. These could be more of a constraints among retail investors, which might be more subject to heterogeneity in risk aversion or the funds they have access to.

as well as some descriptive statistics. Section 4 introduces the interaction between stock prices behavior and the average horizon of the institutions holding the security. Section 5 concludes.

2.2 Literature Review

This study builds on and contributes to various existing literatures. A number of studies have highlighted a role for short-term investors in generating financial market inefficiencies. Among others, DeLong, Summers, Shleifer and Waldmann (1990a) and (1990b), emphasize the role of rational risk averse short-term speculators for mispricing generated by irrational traders. Short-term investors might also trade on the basis of overconfidence; Odean (1998) and Barber and Odean (2000), Grinblatt and Keloharju (2009).⁷ In addition to these behavioral approaches, Dow and Gorton (1994) show that investors will use their signals only if it is impounded in the price at which they will sell the asset. Similarly, in Allen, Morris and Shin (2006), Bacchetta and Van Wincoop (2008), Cespa and Vives (2009), Grisse (2009), rational short-lived investors overreact to public signals, as it informs them about future market participants beliefs reflected in the price at which they will liquidate the asset in the future. Directly studying the incidence of the horizon of traders, Froot, Scharfstein and Stein (1992) emphasize a negative relation between the horizon and stock price informativeness. A similar result is obtained in Parsa (2011). The results in the current paper challenge the previous studies as I find that securities held by institutional investors trading more frequently leads to higher volatility, but the volatility is not "disconnected" from the long run security payoff.

More broadly, I show that securities held by institutions trading more frequently are not predictable (at least by the usual variables used in the literature), while the securities held by institutions trading less frequently exhibit substantial predictability and under-reaction. These results relate to the stock price "anomalies" and return predictability literature. The explanation behind the return predictability remains a source of academic debate, as the empirical

⁷In DeLong et al. (1990a), short-lived risk averse speculators are limited in their capacity to arbitrage the mispricing generated by noisy traders due to the unpredictability of the same mispricing, which introduces risk that can not be diversified away in the short-run. Their assumption on the horizon of speculators is at the core of noise traders aptitude to crowd away arbitrage. In DeLong et al. (1990b), impatient rational traders can even generate excess movements in the prices as they try to forecast the beliefs of irrational traders, i.e. positive-feedback traders extrapolating past price trends. Odean (1999) and Barber and Odean (2000), Grinblatt and Keloharju (2009) provide evidence that overconfident investors are likely to trade more frequently.

findings are consistent with either efficiency or mispricing (irrational or rational) (see for example Fama and French (1993), Fama (1998), Campbell and Vuolteenaho (2004), Campbell, Polk, and Vuolteenaho (2009), Barberis, Shleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), Hong and Stein, (1999) and (2007), and Banerjee, Kaniel and Kremer (2009) amongst others). The results in the current work suggest that the understanding of stock return predictability should focus on the low trading frequency institutional investors and the friction that allows the securities held by high and low trading frequency institutions to differ.⁸

The empirical strategy builds on the literature that explores the fundamental components behind the stock price movements, namely the cash flow and the discount factor; Campbell and Shiller (1988a), (1988b), Campbell (1991), Campbell and Ammer (1993). In particular, this study builds on and contributes to the firm-level stock price decomposition literature; see Vuolteenaho (2002), Cohen, Gompers and Vuolteenaho (2002), Campbell and Vuolteenaho (2004), Koubouros, Malliaropulos, and Panopoulou (2005), Hecht and Vuolteenaho (2006), Larrain, and Yogo (2008) and Campbell, Polk, and Vuolteenaho (2009). Conceptually, using Vuolteenaho (2002) accounting present value identity, the book-to-market ratio can be decomposed between long run return on equity (cash flow component) and long run discount factor (discount factor component) as well as an error term. Hence, the book-to-market - a proxy for the price - should reflect a mixture of information about the long run return and the long run return on equity. I explore the two components of the book to market movements as a function of the institutional investors' trading frequency characteristics.

This paper also adds to the vast literature on institutional investors. On one hand, Lakonishok, Shleifer and Vishny (1992), Grinblatt, Titman and Wermers (1995), Wermers (1999, 2000), Nofsinger and Sias (1999) documented a positive contemporaneous relation between institutional investors' buying and stock returns. Daniel, Grinblatt, Titman and Wermers (1997), Gompers and Metrick (2001) highlighted a positive relation between institutional buying and the short-term expected return, where the expected returns are higher (lower) for stocks ex-

⁸One potential explanation for the predictability and under-reaction pattern for the securities held by institutions trading less frequently is related to the underlying characteristics of the securities low trading frequency institutions invest in. Nevertheless, I control for an important set of securities characteristics. An alternative explanation recognizes that institutional investors are facing some constraints (Shleifer and Vishny (1997)) and are unable to take full advantage of stock price differences. As such, the results suggest an institution's phenomenon related to mispricing. In both cases, the understanding is tightly linked to low trading frequency institutions.

periencing a significant institutional buying (selling). These studies consider the institutional investors as a homogenous group. On the other hand, Wermers (1999), Grinblatt and Keloharju (2001), Cohen, Polk and Vuolteenhao (2002) explores a source of heterogeneity in the type of investors (mutual funds, retail investors, institutional investors, and so on) on the equilibrium investors' trading behaviors.⁹ This paper contributes to the previous literature by exploiting the intrinsic trading frequency behavior of institutional investors as a source of heterogeneity among institutional investors, in understanding stock prices' behavior.

Finally, this paper connects to previous studies, which have looked at the portfolio turnover rate of institutional investors and its interaction with financial markets; Gaspar, Massa and Matos (2005), Ke, Ramalingegowda and Yu (2006), Yan and Zhang (2009). Motivated by the effect of the investment horizon of institutional investors, Gaspar, Massa and Matos (2005) use institutional investors portfolio turnover rate and explore the corporate controls market.¹⁰ Ke, Ramalingegowda and Yu (2006), Yan and Zhang (2009) focus on stock prices.¹¹ This paper is related and adds to the previous studies as it uses the institution's trading information. However, by using the fixed-effect trading frequency of the investors, it focuses on the institutions' intrinsic trading characteristic as opposed to their equilibrium trading behavior, which might confound securities and market characteristics.

As such, this study is related to Jin and Kogan (2007), Khan, Kogan and Serafeim (2010). Jin and Kogan (2007) focus on one particular channel behind the mutual funds trading frequency intrinsic to the mutual fund. They use the variation in the portfolio turnover rate of the mutual fund managers, and its interaction with a measure of investor's impatience, defined as the sensitivity of money flows into and out of the fund in response to the short-term performance of the fund. They find that mutual fund managers tend to focus on short horizon investments

⁹Grinblatt and Keloharju (2001) explores a dataset of the shareholdings in FSCD stocks and documents differences in the buy and sell behavior as well as the performance of different types of investors, such as households, foreign investors, financial institutions and insurance companies. Wermers (1999) focuses on the mutual fund industry and provides evidence on the "herding" behavior of mutual funds as well as their impact for the stock prices. Cohen, Polk and Vuolteenhao (2002) study the difference between the trading behaviors of institutional investors as opposed to individual investors in their reaction to cash flow news using the VAR-return decomposition at the firm level.

¹⁰Gaspar, Massa and Matos (2005) show that firms with shareholders having a higher portfolio turnover are more likely to get an acquisition bid, but at a lower premium.

¹¹Yan and Zhang (2009) finds that the trading of institutional investors with a high portfolio turnover rate forecasts future stock returns. Ke, Ramalingegowda and Yu (2006) focus on the reaction to information by institutions with different portfolio turnover rate.

due to the short horizon of their investors (and not the other way around). Their evidence suggests that this behavior may result in abnormal returns as it leads to an inflated demand of short horizon investment opportunities at the expense of longer horizon alternatives. Similar to the findings in Jin and Kogan (2007), I provide evidence that the institution's trading frequency matters for the behavior of stock prices.¹² Nevertheless, this study differs from Jin and Kogan (2007) as I don't focus on one particular channel behind the trading frequency of the institutions. The measure I construct for the institutional investors' trading frequency is a "black box", which captures the component of the institution's turnover explained by the institution's intrinsic characteristics as opposed to the market and/or characteristics of the securities they invest in. Institutional investors can have different horizons for many reasons: different levels of patience (subjective discount factor), liquidity needs, administrative costs, legal restrictions, competitive pressures related to the performance based pay; see Dow and Gorton (1997), Shleifer and Vishny (1997), Bolton, Scheinkman and Xiong (2006). Instead, the measure used in this study, allows me to focus on the whole set of institutional investors and the interaction of their trading frequency with stock prices, as the only information required is the holdings of the investors.

2.3 Data Description and Methodology

In order to study the relation between the investors' trading frequency and the stock prices behavior, (i) I construct an investor-specific measure of the intrinsic frequency of trading; then (ii) I construct a security-specific measure of the composition of the intrinsic trading frequency of the investors holding the security at a given moment in time; Finally, (iii) I use the security level measure constructed in (ii) to see how the aforementioned security-specific characteristic impacts stock market outcomes. In what follows, I describe step by step, each of these points as well as the results on the relation between the investors' trading frequency and stock prices.

¹²Contrary to earlier studies, the main object of interest of this paper and Jin and Kogan (2007) is not on the effects of the demand by institutional investors on stock prices, but rather on the differential response of the stock prices held by institutions with different trading frequency.

2.3.1 Institutional Investor's Trading Frequency Fixed Effect

In order to study the institutional investor's trading frequency, I use information about the quarterly equity holdings of all the institutions provided by the Thomson Reuters Ownership dataset.¹³ The dataset results from the 1978 amendment to the Securities and Exchange Act of 1934 which requires all institutions with greater than \$100 million of securities under discretionary management to disclose their holdings on all their common-stock positions greater than 10,000 shares or \$200,000 on the SEC's form 13F. The institutions included are divided into 5 categories: Banks, Insurance Companies, Investment Companies and Their Managers (e.g. Mutual Funds), Investment Advisors which includes the large brokerage firms and all Others (Pension Funds, University Endowments, Foundations). It reports a total of 4382 managers. The data coverage increased in terms of both the security and manager dimension from a total of 573 managers and 4451 securities in 1980 to 2617 managers and 13125 securities in 2005. The institutional investors represented initially 16% of the market they invested in (\$954 millions) in 1980 but this number increases to an average of 44% (\$17,500 millions) in 2005.¹⁴

For each institution, define s_{ijt} as the number of shares institution i is holding in security j at quarter t and y_{ijt} as the absolute value of the percentage change in the position of institution i in security j at quarter t ¹⁵

$$y_{ijt} = \text{abs} \left(\frac{s_{ijt} - s_{ijt-1}}{\frac{1}{2}(s_{ijt} + s_{ijt-1})} \right) \quad (2.1)$$

y_{ijt} is capturing the frequency of trade of institution i in each security j at quarter t . If an institution i is holding the same number of securities at quarter t and $t - 1$, then $y_{ijt} = 0$. If on average y_{ijt} is bigger for institution i than institution i' , then on average, institution i is rebalancing its portfolio more frequently than institution i' during a given period of time.

¹³The dataset was previously known as the CDA/Spectrum 34 database. The institutions in the sample are also referred to as the 13F institutions in reference to the form they are required to fill at a quarterly basis.

¹⁴Some of this growth is due to an increase in the value of the equity market throughout the sample period, which forced more institutions to fill the 13-F forms as the rising market pushed their portfolio across the nominal threshold level of \$100 million. For more details about the dataset, I refer you to Gompers and Metrick (2001).

¹⁵I am using in the denominator the average number of shares in quarter t and quarter $t - 1$ instead of the number of shares in quarter $t - 1$. The main reason is to keep y_{ijt} from being forced to a missing value when the number of shares moves from 0 to a positive number. However, notice that as the number of shares increase from 0 to a positive number y_{ijt} will be equal to 2. Hence, part of the information is clearly missing as a change of an institution's position is treated differently whether it was holding a positive number at $t - 1$ or 0 at $t - 1$.

y_{ijt} presents an obvious weakness at capturing an institution's intrinsic characteristic. The trading behavior of an investor, captured by the absolute value of the percentage change in the position of institution in a given security at a given quarter, is an equilibrium outcome. As such, it is likely to be the endogenous result of the investor's characteristics such as its investment horizon, as well as of the characteristics of the securities they are investing in and aggregate market effects. As a consequence, it is unclear that this measure will allow one to capture an investor intrinsic characteristic. For instance, two managers identical in every respect except for the securities they are investing in, could have different portfolio turnover rates as the flow of information in the two sets of securities is very different. Ultimately, an investor's trading behavior will confound its characteristics (risk aversion, horizon) as well as the market and the security characteristics.

In addition, an investor's portfolio turnover rate or any measure based directly on the investors' trading information might lead to spurious results when looking at the effect of the investors' trading frequency on the security price behavior. Using again the example above if two otherwise identical managers invest in a set of securities with different levels of volatility, then the investor investing in the more volatile securities might end up with a larger portfolio turnover rate. One could spuriously associate high trading frequency investors with securities that have a higher volatility, as their portfolio turnover rates will be high as well. Thus, the behavior of the portfolio turnover rate might inherit the dynamic characteristics of the securities they are investing in.

Overall, any measure of an investor's trading frequency based solely on the observed change in the investor's positions will carry these concerns. In what follows, I construct a measure, which captures an investor's idiosyncratic tendency to change its position, once any security or market effects have been partialled out. To this end, I estimate by ordinary least squares, for each year $T = 1980, \dots, 2005$ a regression of the form:

$$y_{ijt}^T = a^T + h_i^T + g_{jt}^T + \beta X_{ijt}^T + \varepsilon_{ijt}^T, \quad (2.2)$$

where y_{ijt}^T is the absolute value of the change in the holding of institution i in security j in quarter t of year T , h_i^T is the institution fixed effect; g_{jt}^T is the time-security interaction fixed effect and X_{it}^T controls for the size of the portfolio of investor i as well as the size of each security

in the portfolio of investor i .¹⁶ The estimates of h_i^T in equation (2.2) provide an annual measure of the investor's trading frequency that does not confound any security or time effects. The two latter effects are fully absorbed by the term g_{jt} . I allow the measure of the institution's trading frequency (h_i^T) to change annually in order to capture changes across time that could be driven by an investor characteristics, such as the investment horizon associated to changes in its corporate governance, its objective, its CEO, the regulation or its preferences.

Definition 19 *An investors's intrinsic trading frequency, or "horizon", is defined by the fixed effect in the regression (2.2) :*

$$h_i^T, \forall i \in I \quad (2.3)$$

A larger institution's fixed effect h_i^T , is associated to investors who change their positions more often and hence have a higher idiosyncratic trading frequency. Ultimately, h_i^T provides a measure comparable to the portfolio turnover rate. However, by exploiting the three dimensions of the data (institutions, security and quarter), it combines the changes in an institution security holdings in one churning rate, which summarizes only the trading behavior that results from the institution.

I compared the yearly h_i^T with different alternatives such as the quarterly rolling measure, the 2 years rolling measure as well as the 5 years rolling measure. The quarterly rolling measure is defined by a fixed effect estimated from the first quarter of 1981 using data from the four previous quarters rolling for each quarter until the last quarter of 2005. The 2 years and 5 years rolling measure roll over each year. In each of these two alternatives, the fixed effect for a given year is estimated using data from the second quarter of a 2 and 5 years to the second quarter of the year of interest. I also allowed for time breaks whenever they change their type, e.g. from a bank to a mutual fund. The decision to look mainly at the yearly measure has been made in order to allow changes in the horizon of the investor through time without imposing it happening only at a specific moment such as when they change their type. However, the three measures are highly correlated (above 90%).¹⁷

¹⁶Concretely, the fixed effect measures are computed with respect to the following normalization: $\sum_t \sum_j \delta_{jit} g_{jt} = \sum_i \delta_{jit} b_i = 0$ where $\delta_{jit} = 1$ if y_{jit} is non missing and 0 otherwise.

¹⁷More details are provided in the appendix found at <http://econ-www.mit.edu/grad/sparsa/research>

Descriptive Statistics

As a first reality check on the measure, Table I describes the distribution of the fixed effect estimates in two dimensions: (i) the dispersion across different types of institutions and (ii) the persistence of the fixed effect estimates across time. Panel A summarizes the descriptive statistics—mean, standard deviation, 25% and 75% percentiles—of the fixed effect estimates by type of institutions from 1980 to 1998.¹⁸ The investors are divided into 5 types of institutions: Banks, Insurance Companies, Investment Companies and their Managers (e.g. Mutual Funds), Investment Advisors, and all Others (Pension Funds, University Endowments, Foundations). In line with common priors, the mutual funds and the investment advisers exhibit on average the highest trading frequency fixed effects, respectively 0.063 and 0.177. The group of insurance companies and banks have on average the lowest trading frequency fixed effects, respectively -0.007 and 0.006. It has been shown in previous studies that the growth of the financial institutions came hand in hand with an increase in the presence of the investment advisers, and the mutual funds. This could give support to claims made in the media or among policy makers about Wall street becoming too short-termist, as the institutions dominating it are the institutions with the shortest horizon.¹⁹ Finally, notice that despite the difference in terms of the averages, each type exhibits a high level of within group variability.²⁰

Panel B reports the first to fifth order autocorrelation of the trading frequency fixed effects. One can notice that the fixed effect measure is relatively persistent across time with a first order autocorrelation coefficient of 0.72. The autocorrelation of the fixed effect decays slowly with time (as expected) but it remains above 0.5.

¹⁸The reason I am using only the dataset from 1980 to 1998 is because there has been a mapping error that occurred when TFN integrated data from the former Technimetrics. Many of these institutions were and are still improperly classified as "Others".

¹⁹For instance, "Overcoming Short-termism: A call for a more responsible Approach to Investment and Business Management", September 2009.

²⁰One needs to remain cautious about these results as the mutual funds are considered as a block and report only an aggregated number. As we saw, the largest institutions are dominated by the mutual funds. Hence, it is very likely that the standard deviation is downward-biased specially among the largest institutions.

Portfolio Characteristics

Do investors characterized by a different trading frequency fixed effect differ in terms of the securities they hold in their portfolio? Table II compares the characteristics of the portfolios of both high and low trading frequency institutions. A high and a low trading frequency institution is defined as follows. Each year, I rank each institution according their h_i^T measure and sort them in 5 different groups defined by the quintiles of h . The institutions with a fixed effect lower than the lowest quintile are defined as low trading frequency institutions. On the other hand, the institutions with a fixed effect higher than the upper quintile are defined as high trading frequency institutions.

Table II reports three groups of portfolio characteristics: (i) Market level, (ii) Accounting level and (iii) Others. The market level variables correspond to the size, the volatility, the volume of trade, the turnover as well as the past 3 and 12 months return of the securities (momentum 3 and momentum 12). The accounting level variables include the book to market ratio, the profitability (measured by the Generally Accepted Accounting Principles (GAAP) return on equity), the yield, the earning price ratio, and leverage ratio. Finally, the last group of variables illustrates the concentration of the investor's portfolio at both the security and the industry level and the stock exchange they invest in. Below, I summarize the definition of each variable (for more details on the construction of the variables see the Appendix).

1. Market level: For each end of quarter and security held by an institution, the size of the security is measured as the combined value of all common stock classes outstanding. The volatility is defined as the standard deviation of the daily excess return during the previous year, i.e. from the beginning of quarter $t+1$ year $s-1$, to the end of quarter t year s . The daily excess return is defined as $(r_{it} - r_t^f)$, i.e. in excess of the daily risk free rate. I also look at the decomposition of the volatility between idiosyncratic volatility and the common component. The idiosyncratic risk is defined as the firm specific risk. For each year ending at the end of a quarter, daily excess returns of individual stocks are regressed on the daily Fama-French (1993,1996) three factors: the excess return on a broad market portfolio $(r_{mt} - r_t^f)$, SMB, i.e. small minus big, and HML, i.e. high minus low. The predicted values are used to estimate the common movement and the residuals are used to

estimate the idiosyncratic risk. By definition, the idiosyncratic risk is independent of the common movement. The volume of trade is measured as the sum of the trading volumes during the year preceding a given quarter. The share turnover is measured as the volume divided by the number of shares outstanding at the end of a given quarter. Finally, the 3 and 12 months returns are respectively compounded from monthly returns, recorded from 3 months and 12 months prior to the end of the quarter t .

2. Accounting level: The accounting variables are measured at an annual frequency. For each year, the book-to-market of a firm is defined as the book equity at the end of the previous fiscal year divided by the market equity, i.e. the size, at the end of June of the current year. The profitability is measured as the earning in the current year divided by the book equity at the end of the previous fiscal year. The leverage is measured as the book equity divided by the sum of the book and debt equity. Finally, the Yield and the earning price ratio are respectively measured by the cash dividend of a company at the end of the fiscal year divided by the market equity at the end of June of the same year and the earning value divided by the market equity.
3. Others: I look at the market they are investing in. Finally, I report a measure of the concentration of the investor's portfolio at the security level and industry level. I use the Herfindahl index, which is defined for each institution at each quarter as $\sum_{i=1}^N w_{it}^2$, where w_{it} is the share of the portfolio, they are holding in a given security (industry). A institution with a larger index has a more concentrated portfolio.

Table II reports the average share of the investors' portfolio held in securities with a given characteristic. For all the variables at the market and accounting level, I rank all the securities in the NYSE for each time period, quarterly for the market variables and yearly for the accounting variables, according to the variables of interest. I use the lowest quintile and upper quintile of the given variable, to generate two groups of securities. The securities held by the institutions with a given characteristic x , below the lowest quintile (Low group) and the securities with a given characteristic x above the upper quintile (High group). For instance, consider the size of a security. For each quarter, I defined the lowest and upper quintile according to the size of the securities in the NYSE. I then create two groups of securities based on the two cutoffs: the small

securities and the large securities. I finally report the relative share of an investor's portfolio that is held in small and large securities. I do the same exercise for each of the variables I defined above.

Each row represents a security level characteristic. The first two columns show the portfolio composition of the institutions with a high trading frequency and the last two columns the portfolio composition of the institutions with a low trading frequency. For each group of investors, the first column summarizes the average shares held in securities with a high value and the second column reports the average shares held in securities with a low value.

Table II highlights mainly three differences in the portfolio composition of investors with a different trading frequency: (i) volatility, (ii) turnover and (iii) profitability. In particular, from the second row of the Table II, high trading frequency investors hold 24% of their portfolio on average in securities with a high volatility, while low trading frequency investors hold 10% of their portfolio in securities with a high volatility. The converse happens when looking at the securities with a low volatility. Only 15% of the portfolio of high trading frequency investors is in low volatility shares while the corresponding figure for low trading frequency investors is 21%. This result is a preview of the security level analysis. High trading frequency investors have a higher share in securities with a high turnover (± 0.38) relative to low trading frequency investors (± 0.18). Finally, high trading frequency investors have a lower share in securities with a high profitability (0.007) relative to low frequency investors (± 0.46).

It is important to notice that these patterns are not spurious in the sense that my measure of the intrinsic trading frequency of investors controls for aggregate- and security-level effects, including the volatility of different securities. Nevertheless, this does not mean that these patterns identify a causal relation. Investors with a high fixed effect could prefer securities with a high volatility, or a high turnover as well as the high turnover rate, or volatility is being generated partly by the presence of high trading frequency investors. Finally, apart from these three characteristics, the table does not highlight any other striking difference in the portfolio of investors with different trading frequencies.

2.3.2 A Security Trading Frequency Index

The preceding discussion explained the investor-specific measure of its intrinsic trading frequency. I now proceed to construct a security-specific characteristic, by looking at the composition of the investors that hold a particular security and taking a weighted average of the investors' intrinsic frequency of trade. This seeks to capture the effect of the institutions' trading frequency on the stock price movements.

In order to explore the interaction between the stock price movements and the institutions trading frequency fixed effect, I use information from three sources: (i) The Center for Research in Security Prices (CRSP), (ii) The Standard & Poor's Compustat North America annual research file and (iii) The Thomson Reuters Ownership Data. The set of securities included corresponds to the intersection of these three databases, i.e. the securities that belong to the portfolio of the 13-F financial institutions from the beginning of 1980 to the end of 2005 with both market and accounting information available respectively in the CRSP and the Compustat.

The CRSP provides information on monthly prices, dividends, shares outstanding, and returns for NYSE, AMEX, and Nasdaq securities. The Compustat contains accounting information for most publicly traded U.S. companies. In addition, the one-month Treasury Bill Rate at monthly frequency gives the risk-free interest rate from Ibbotson Associates. I restrict attention to the securities traded in the NYSE, the AMEX and the NASDAQ as well as the securities, which are held by more than 25 institutions, or the institutions represent at least 10% of the shares outstanding of the securities. In other words, the present study focuses on the securities, where the institutional investors as a group hold a non-marginal share of the total ownership. Depending on the year, 30 to 50% of the securities have been dropped from the sample due to these criteria. More details on the data described in this section can be found in the Appendix.

For each 13-F institution, the Thomson Reuters ownership data reports the securities the investor is holding in its portfolio. For each year T and security j held by a group of institutions I_j , I am defining the security j 's trading frequency index at year T as the weighted average of the fixed effect of the institutions in I_j :

$$H_{jT} = \sum_{i \in I_j} \omega_{ijT} h_{iT} \quad (2.4)$$

where $\omega_{ijT} \equiv \frac{s_{ijT}}{\sum_{i \in I_j} s_{ijT}}$, s_{ijT} is the number of shares outstanding of security j held by institution i at year T and h_{iT} is the fixed effect of institution i at year T . The weight, i.e. ω_{ijT} , is capturing the relative importance of investor i for security j at year t , in terms of the number of shares investor i holds relative to the total number of shares the group of institutional investors is holding. It implies that the trading frequency of an investor holding 90% of the shares of a security should have a larger effect than the trading frequency of an investor holding only 10% of the shares of a security. The security's trading frequency index will give more weight to the former investor's fixed effect than to that of the latter. Notice that in order to align the information on the investors with the accounting level information, the measure H in year T is using information from the end of June of year $T - 1$ to the end of June of year T and ω_{ijT} is the average relative size of institution j from the end of June in year $T - 1$ to the end of June in year T .

Definition 20 *The security specific trading frequency index is defined as the weighted average of the trading frequency fixed effect of the investors holding the security given by (2.4):*

$$H_{jT}, \forall j \in J \quad (2.5)$$

H_{jT} maps the institutional investors' trading frequency, i.e. h_i^T , to the security. H is interpreted as the average trading frequency of the population of institutional investors holding the security j at year T . A security j will have a high trading frequency index if, on average, the institutional investors holding the security, are characterized by a short investment horizon, proxied by a large h . Overall, the institutions are weighted by their relative size with respect to the institutions holding the security. As a consequence, the variation in H can be traced back to one of two sources: (i) For a given pool of investors, the investors with a lower value of the fixed effect are holding a higher share of the security. In other words, the high trading frequency investors represent a higher share of the security, i.e. higher weight ω_{ijT} on the high h_{iT} (the high trading frequency investors). (ii) For a given weight, the institutions holding the security have a higher institution's trading frequency. Both sources of variation, translate in a security having higher trading frequency investors than another security or having a higher

trading frequency across time.²¹

In addition, I define a high and low trading frequency group. Each year, I sort all the securities with respect to H_{it} . I construct three groups of securities based on the terciles of H : lowest tercile, L , the middle tercile, M , and the upper tercile, U . Group L is defined as the securities, which have a trading frequency index H in the first tercile. I will name these securities the low trading group. Group M is defined as the securities which have a trading frequency index H in the second tercile. I will name these securities the medium trading group securities. Finally, group U is defined as the securities, which have a trading frequency index in the upper tercile. The securities in the last group are the high trading group securities.

Descriptive Statistics

Before proceeding to the results on the predictability of the firm level return and the return on equity, Table III summarizes the descriptive statistics (mean, standard deviation, 25 as well as 75 percentiles) of the variables and the sample used in the remaining of this paper. In particular, the set of variables used in order of appearance in Table III are: the log of the annual return r_{it} , the log of the Clean-Surplus return on equity e_{it} at the end of the fiscal year. The Clean-Surplus profitability is defined as one plus the Clean-Surplus earnings divided by the last fiscal year book equity. The earnings on the clean-surplus relation X_t for the fiscal year t are given by:

$$X_t = \left[\frac{(1 + R_t) M_{t-1} - D_t}{M_t} \right] B_t - B_{t-1} + D_t$$

It can be seen as the return on the book value adjusted for equity offerings. The accounting clean surplus identity is given by $B_t = B_{t-1} + X_t - D_t$, where B_t is the book value of the company at the end of the fiscal year t and D_t is the dividend. The log of the Generally Accepted Accounting Principles (GAAP) return on equity, the log of the book to market ratio θ_{it} , the log of the leverage ratio, the share turnover of the security, the size of the security (in billions), the institutional ownership (measured as the ratio of the shares outstanding of the

²¹The variation of H_{jt} through time is either the result of : (i) investors selling or buying the security characterized by different horizon, (ii) the investors experiencing a change in their characteristics to trade (which could come from a change in the CEO or a merger), or (iii) both. The variation of H_{jt} across security mainly comes from different securities being held by a population of investors characterized by different horizons at a given moment in time.

security at the held by the 13-F institutions) and the security level institution trading frequency index H_{it} as defined above. More details on the data construction and the restrictions on the sample are given in the Appendix.

Table III reports the descriptive statistics for the securities held on average by high trading frequency investors (in group U) and the securities held on average by low trading frequency investors (in group L). There is a total of 1193 securities and 18348 observations, where 989 securities are in the L group least one year and 1116 securities are U group at least one year. Consistent with table II (characteristics of the portfolio of high and low trading frequency investors), the securities held by high trading frequency fixed effect investors (in group U), have a higher turnover (4.7 as opposed to 11.57 for the low group), a higher return standard deviation (0.37 as opposed to 0.29 for the low group) as well as a lower size (3.7 as opposed to 2.1 billions). Furthermore, they also exhibit a higher return on average (0.2 as opposed to 0.08 for the low group).²² Finally, these two groups of securities seem to be held by the same share of institutional investors on average

2.4 Institutions' Trading Frequency and Forecastability

The starting point of the results is summarized in Figure 1. Consistent with the beliefs of many scholars, policy makers and practitioners, Figure 1 shows that securities held by high trading frequency investors are associated with more volatility. Figure 1 illustrates the ratio of the standard deviation of the annual return of the securities in the three frequency groups relative to the standard deviation of the low trading frequency group. The figure plots the ratio from 1980 to 2005 at a quarterly basis. Securities held by high trading frequency investors exhibit a standard deviation of their return, on average, 7 percentage points higher than the securities held by low trading frequency investors (this difference is statistically significant at the 5% level). Given that the securities' average standard deviation throughout the whole sample is approximately 0.15, this difference is also economically significant. In fact, in some periods in the sample, the standard deviation for the high trading group investors is up to two or three

²²The focus of this paper is not on the difference in the unconditional expected return that the two groups of securities are holding. This is an interesting direction for future return, which is being more exploited in future research. For the purpose of this study, I will take the difference as given in order to estimate the forecasting coefficients more precisely.

times larger than the volatility of securities held by low trading frequency investors. Finally, the R^2 of the regression of the standard deviation on the dummies for the different trading frequency groups reveals that almost 22% of the variation in the volatility across securities is explained by the type of institutions (low and high frequency) holding the securities. Figure 1 also reveals a monotone relationship between the volatility and the trading frequency group.

From this starting point, two questions arise. (i) What are the sources of the this volatility? In order to answer the first question, I study the difference in the forecasting power of the price in forecasting future return and return on equity as a function of the investors holding the securities, i.e. high and low trading frequency investors. The first question leads us to the second question. (ii) Does the return of the securities held by investors who trade more frequently differ in terms of its forecastability? In other words, does the price of the securities held by investors who trade more frequently differ in a systematic way from the price of the securities held by investors who trade less frequently? To answer the second question, I look at the forecastability of the return using a set of usual forecasting variables, which have been highlighted in the literature as being important in forecasting future return. In other words, does the trading frequency characteristics of the investors holding the securities have information about stock market return beyond the usual variables?

2.4.1 Forecastability of the Return and the Return on Equity

What are the sources of this difference in volatility? A first step at understanding the sources of the volatility consists in looking at the predicting power of the security prices in forecasting future return and/or future cash flow (profitability). In other words, do the prices of the securities held by high trading frequency investors forecast long run profitability or long run return? Do they forecast long run profitability less than the securities held by low trading frequency investors? In order to address these two questions, I estimate the following two forecasting regressions of a security's total return (r) and total Clean-Surplus profitability (e) on its book to market ratio at different horizons k using yearly frequency data (the book to

market ratio can be seen as a proxy for the price of a security):²³

$$r_{it \rightarrow t+k} = \alpha_g + \lambda_t + \beta_{rk}^U(U * \theta_{it-1}) + \beta_{rk}^M(M * \theta_{it-1}) + \beta_{rk}^L(L * \theta_{it-1}) + \varepsilon_{it \rightarrow t+k}^r \quad (2.6)$$

$$e_{it \rightarrow t+k} = \alpha_g + \lambda_t + \beta_{ek}^U(U * \theta_{it-1}) + \beta_{ek}^M(M * \theta_{it-1}) + \beta_{ek}^L(L * \theta_{it-1}) + \varepsilon_{it \rightarrow t+k}^e \quad (2.7)$$

where $k = 1$ to 13 years; U stands for high trading frequency institution dummy (upper tercile), M stands for the medium trading frequency institution dummy (middle tercile) and L stands for the low trading frequency institution dummy (lower tercile); $r_{it \rightarrow t+k}$ and $e_{it \rightarrow t+k}$ are defined as the total return and clean-surplus return on equity for the next k years, i.e. $x_{it \rightarrow t+k} = \sum_{s=1}^k x_{it-1+s}$, where the variables are expressed in log terms; α_g is the security group fixed effect with $g \in \{U, M, L\}$ and λ_t is the time fixed effect. The reason I am controlling for the group fixed effect is to alleviate the effect that the difference in the return mean (as seen in table III) could have on the forecasting coefficients. As such, the forecasting coefficients for each group exploits the within trading frequency group's source of variation. Furthermore, notice that the estimated coefficients are generally informative for the market-adjusted return, as well as the excess return, given I cross-sectionally demean all the data in the regressions (2.6) and (2.7), via the introduction of λ_t . The reason I look at the variables cross-sectionally demeaned is to alleviate the cross-sectional dependence that exists at the firm-level panel data. Finally, the choice of 13 years has been made in order to minimize the loss in the data and the number of securities, which naturally arises as k increases. Furthermore, for comparison purposes, the securities included in the sample remain for at least 10 years.²⁴

The main results are illustrated in Figures 2 (a)-(b) as well as Table IV. Figures 2 (a)-(b) plot the forecasting coefficients of the total return on equity (Figure 2(a)) and the total return (Figure 2(b)) as a function of the different horizon (k), where $k = 1$ to 13 years (β_{rk} and β_{ek}). They compare the forecasting coefficients for the securities in the high trading frequency

²³The reason I use the Clean-Surplus return on equity as opposed to the generally accepted accounting principles is because the clean-surplus profitability is linked through Vuolteenaho (2002) accounting-based present value identity to the book to market as well as the return. As such it allows some clarity in the exposition as well as an extra intuition behind the results.

²⁴The results have been estimated for a larger sample with all the securities and they remain similar. I decided for comparison and clarity of exposition to keep the securities, which remain at least 10 years. This affects particularly the coefficients for low k .

investors group (β_k^U) and the low trading frequency investors group (β_k^L). Table IV reports the forecasting coefficients of the return (first three columns) and the return on equity (last three columns) for $k = 1, 4, 7, 10, 13$. The standard errors are reported, in parentheses, using the Driscoll-Kraay non parametric standard errors in the presence of time and cross sectional dependence. The table uses $T^{1/4}$ as the number of lags in the Driscoll-Kraay standard errors. For each variable (the return and the return on equity) Table IV reports the coefficients for the low and the high trading group as well as the difference between the two, i.e. $(\beta_k^U - \beta_k^L)$ (the third column).

From Figure 2 (a), the book to market ratio of the security held by high trading frequency investors forecasts the long run return on equity, where the β_{ek} is monotonically increasing, starts at 0.1 and reaches -0.7 for $k = 13$ (long run). In other words, a 1% increase in the book to market (decrease in price as the book to market is the book value over the market value) is associated with a 0.7% decrease in the return on the *book* equity of the company in 13 years. The forecasting coefficients for the two groups of securities are identical, both from an economic and a statistical point of view, as can be seen from the last three columns of table IV.

On the other hand, from Figure 2 (b), the forecasting role of the price differs for the return regression. For the high trading frequency investors group of securities (group U), β_{rk}^U exhibits a hump shaped curve picking at around $k = 7$ at 0.18 (with a standard error of 0.046) and decreasing to 0.08 at $k = 13$ (with a standard error of 0.068). Conversely, the predictability of the return by the book to market for the securities held by low trading frequency investors steadily increases with k , reaching a coefficient of 0.26 (with a standard error of 0.037) at $k = 13$. From table IV, the difference between the two groups is statistically different from $k = 10$ onwards. Overall, the book to market ratio does not seem to forecast the long run return for the securities held on average by investors who trade more frequently as opposed to the securities held by low trading frequency investors.

Given the differences in the portfolio of the investors with a different trading frequency, one can wonder whether the differences in the return predictability come from the size or the turnover of the securities. As we saw from the descriptive statistics (see Table III), the turnover of the securities in the U group is on average higher, while the size of the securities U is on average lower. In fact, these variables are known from previous studies to affect the

predictability of the return and help understand the cross section of expected stock returns. However, notice that previous literature suggests that small stocks exhibit higher returns, under-react to cash flow news, and the prices forecast better the long run return; Banz (1981), Basu (1983), Vuolteenaho (2002), Cohen et al. (2002). As such, the results should go even more in the direction of Figure 2(a)-(b), given the high trading frequency securities are also smaller. Figures 3 (a)-(b) show β_{rk} and β_{ek} after controlling for the size or the turnover of the securities. Likewise, Figures 4 (a)-(b) illustrate β_{rk} and β_{ek} , after controlling for the size, the turnover, the leverage ratio and the industry the firms belong to.²⁵ In order to control for the size, turnover or the leverage of the firm, I use the following general specification:

$$r_{it \rightarrow t+k} = \alpha_g + \lambda_t + \beta_{rk}^U (U * \theta_{it-1}) + \beta_{rk}^M (M * \theta_{it-1}) + \beta_{rk}^L (L * \theta_{it-1}) + \gamma_1^r X_{it-1} + \gamma_2^r (X_{it-1} * \theta_{it-1}) + \varepsilon_{it \rightarrow t+k}^r$$

$$e_{it \rightarrow t+k} = \alpha_g + \lambda_t + \beta_{ek}^U (U * \theta_{it-1}) + \beta_{ek}^M (M * \theta_{it-1}) + \beta_{ek}^L (L * \theta_{it-1}) + \gamma_1^e X_{it-1} + \gamma_2^e (X_{it-1} * \theta_{it-1}) + \varepsilon_{it \rightarrow t+k}^e$$

where X_{it-1} is the set of control variables (size, turnover, leverage). In order to capture that the book to market ratio of the securities with different characteristics forecast the return differently, I add the interaction term, $(X_{it-1} * \theta_{it-1})$. Given the differences are not statistically significant and the results are very comparable to the Figures 2(a)-(b) as well as Table IV, I did not report the standard errors in a separate table. Overall, the pattern found in Figures 2(a)-(b) persists in Figures 3 and 4 in both magnitude and shape, as well as statistical significance. To summarize, the predictability of the return on equity (firm's fundamental) by the book to market ratio does not seem to differ for the securities held by high as opposed to low trading frequency investors. The book to market ratio mainly forecasts the long run return on equity (where long run means 13 years). On the other hand, securities held by high and low trading frequency investors differ in terms of the predictability of the return. (i) The book to market ratio does not forecast long run return for securities held by high trading frequency investors, while it does for securities held by low trading frequency investors. (ii) Furthermore, the return predictability for the securities held by high trading frequency investors at $t - 1$, has a hump shaped curve picking at year 7, and converging to 0, while it increases steadily for the securities held by low trading frequency investors. This patterns hold after controlling for many securities

²⁵I am using 2 digits for the industries dummies, in order to conserve enough variation in the data.

characteristics such as the size, the turnover, the leverage or the industry. Overall, the book to market ratio mainly forecasts the long run return on equity as opposed to the long run return for the securities held by high trading frequency investors. As such, contrary to claims made by the literature on the investor's short investment horizon, securities held by high trading frequency institutional investors do not seem to be "disconnected" from the fundamental more than the securities held by low trading frequency investors.

In order to get another intuition behind the forecasting regression, consider the accounting present value identity (2.8) introduced by Vuolteenaho (2002). The book to market ratio summarizes a mixture of information about long run profitability and/or long run return. In particular,

$$\theta_{t-1} = \sum_{j=0} \rho^j r_{t+j} - \sum_{j=0} \rho^j e_{t+j} + \sum_{j=0} \rho^j \kappa_{t+j} \quad (2.8)$$

Where κ is the related to the first order Taylor expansion approximation error term. Multiplying by θ_{t-1} and taking an unconditional expectation:

$$E(\theta_{t-1}^2) = E\left(\theta_{t-1}, \sum_{j=0} \rho^j r_{t+j}\right) - E\left(\theta_{t-1}, \sum_{j=0} \rho^j e_{t+j}\right) + E\left(\theta_{t-1}, \sum_{j=0} \rho^j \kappa_{t+j}\right) \quad (2.9)$$

which can be rewritten as

$$1 = \frac{\text{cov}\left(\theta_{t-1}, \sum_{j=0} \rho^j r_{t+j}\right)}{\text{var}\left(\theta_{t-1}\right)} - \frac{\text{cov}\left(\theta_{t-1}, \sum_{j=0} \rho^j e_{t+j}\right)}{\text{var}\left(\theta_{t-1}\right)} + \frac{\text{cov}\left(\theta_{t-1}, \sum_{j=0} \rho^j \kappa_{t+j}\right)}{\text{var}\left(\theta_{t-1}\right)} \quad (2.10)$$

$$= \beta_{LR}^r - \beta_{LR}^e + \beta_{LR}^\kappa \quad (2.11)$$

The book to market ratio must forecast either of these components. More precisely, from the identity, the book to market ratio varies if and only if it forecasts long run profitability $\left(\sum_{j=0} \rho^j e_{t+j}\right)$ or long run returns $\left(\sum_{j=0} \rho^j r_{t+j}\right)$, or the error term $\left(\sum_{j=0} \rho^j \kappa_{t+j}\right)$. From Figures 2 (a)-(b), the book to market for the securities characterized by high trading frequency investors, mainly forecasts long run cash flow (-0.70) as opposed to the long run discount factor (≈ 0.08) at horizons as long as $k = 13$ years. Extrapolating from the trends from the results for 13 years, one can expect that the long run return on equity will be the main component as the

long run discount factor will converge to zero.

2.4.2 Return Forecastability for the securities held by high trading frequency investors

As we just saw, the return in both the short run and the long run of the securities held by high trading frequency investors, is almost non forecastable (the forecastability picks at $k = 10$ at a relatively low value compared to the return on equity). One question, which arises, is: Is the return of the securities held by high trading frequency investors non forecastable by other forecasting variables as well? Table V addresses this question. Table V reports the coefficients of a forecasting regression for the next year return (r_{it}) for the high and low trading frequency groups, β^U and β^L :

$$r_{it} = \alpha_g + \lambda_t + \beta^U(U * z_{it-1}) + \beta^M(M * z_{it-1}) + \beta^L(L * z_{it-1}) + \varepsilon_{it}^r$$

where as above U stands for high trading frequency institution dummy (upper tercile), M stands for the medium trading frequency institution dummy (middle tercile) and L stands for the low trading frequency institution dummy (lower tercile); α_g is the security group fixed effect with $g \in \{U, M, L\}$ and λ_t is the time fixed effect. In particular, I build on previous literature to choose the set of forecasting variables, z_{it-1} , which have been highlighted in the firm-level predictability literature. These variables are the log return, the log of the GAAP return on equity,²⁶ and the log of the book to market. I also use the log of the leverage ratio and the institutional ownership of the firm. Most of these variables have been highlighted by previous studies. Among others the return is capturing the existence of reversal and momentum in the data; see De Bondt and Thaler (1985), Jegadeesh and Titman (1993). High book to market firms have earned a higher average stock return than low book to market firms; Rosenberg, Reid, and Lanstein (1985). The firms with a higher profitability, measured by the GAAP return on equity, have earned higher average stock returns; Haugen and Baker (1996). The firms with a high leverage have higher average returns than firms with low leverage; Bhandari

²⁶In order to keep comparison with previous literature highlighting the forecasting role of the return on equity, I use the GAAP return on equity as opposed to the Clean-Surplus one. Notice however, that they are both highly correlated.

(1988). Institutional ownership forecasts one period return; Wermers (1999).

Each column of Table V represents a subset of the variables just defined, where the common set of variables across all securities is the return, the return on equity and the book to market. The standard errors are in parentheses. The standard errors are also estimated using the Driscoll-Kraay non parametric standard errors to take into account the cross-sectional and time series dependence in the data. The number of lags included are approximately $T^{1/4}$.

From the first row of Table V, one can observe an interesting feature. Even though the next period return is close to non forecastable by most of the variables for the securities held by high trading frequency investors, apart from the return on equity and the book to market (slightly significant), Table V reports interesting features. These features suggest that the trading frequency of the investors relate to a new source of heterogeneity among the securities' return. First, the forecasting coefficients are, except for the leverage ratio, all statistically insignificant for the securities held by high trading frequency investors. Second, from an economic point of view, the coefficients are more important for the securities held by low trading frequency investors. A 1% increase in the return on equity forecasts a 0.4% increase in the next year return for the low trading frequency group, while only forecasting less than 0.2% increase in the next year return for the high trading frequency investors group. Third, from the third row, one can observe that the return exhibits momentum for the securities held by low trading frequency investors and the momentum is statistically significant. Conversely, the securities held by high trading frequency investors show signs of reversal. In the finance literature, momentum in return has been associated to an under-reaction phenomenon; while reversal can be justified by overreaction. However, notice that the reversal for the high trading frequency investors' group of securities is not statistically significant (with a coefficient of -0.018 and a standard error of 0.04). This could hint at a potential explanation of the lower volatility of the return for the securities held by low trading frequency investors, despite the difference in the standard deviation of the return on equity. If securities under react to the news of a company, then the security return is likely to move less and exhibit a lower volatility. This channel should be explored in more detail in future research.

Even though, one cannot accept the null hypothesis of non forecastability of the return by the usual forecasting variables, from this exercise, beyond the low forecastability of the future

return by the book to market ratio (described in section 4.1), we observe a low forecastability of the future return by the "usual suspects".

The Horizon as an extra forecasting variable

Given that return predictability differs for the securities held by different trading frequency investors, this suggests that the trading frequency index has valuable information to understand the cross section of stock prices. For instance, the prices of the securities held by high trading frequency investors could be less forecastable because high trading frequency investors practice arbitrage in the markets. Overall, the high trading frequency investors could have superior information about the securities, which are being embedded in the price as they trade in the security. In general, it will be interesting to explore whether in addition to the "usual suspects", the trading frequency index has information about future stock prices. Table V summarizes the results. Table V reports the coefficients of a forecasting regression for the next year return, where I add the Trading Frequency Index as an additional forecaster. Table V compares the forecasting coefficients for the securities in the high trading frequency investors group (β^U) and the low trading frequency investors group (β^L). The Driscoll-Kraay standard errors are in parentheses. The first column looks at the regression where in addition to the trading frequency index, the return, the return on equity and the book to market are being used. The second column adds the institutional ownership. Finally, the last column adds the leverage ratio.

From table V, one can notice that the trading frequency index of a security helps to forecast next year's return. However, only for the securities held by high trading frequency investors. In particular, a one standard deviation increase in the horizon (approximately 0.10), is only associated to a 0.05% increase in the next year return for the securities held by high trading frequency investors. On the other hand, the trading frequency index forecasts a decrease in the next period return when held by low trading frequency investors. This decrease is neither economically, nor statistically significant. Overall the introduction of the trading frequency index does not change dramatically the picture above. It does not affect the forecastable role of the other variables, such as the return, the return on equity, the book to market and the leverage. It mainly changed the coefficient on the institutional ownership. Before the introduction of the trading frequency index as an additional forecasting variable, the institutional ownership

predicted a decrease in the return, while it predicts an increase in the return once I add the trading frequency index. Notice however, that it still remains statistically insignificant.

This relation between the trading frequency and future return could capture different mechanisms could generate the relation between the horizon and next year return. The horizon could be reflecting information about the long run cash flow which is associated to higher future risk. Or as hypothesized by earlier work, the short horizon investors have better and more precise information about long run cash flow, which the securities tend to under-react to. The result on the predictive role of the horizon of a security, i.e. the average horizon of the investors who are holding the securities, is connected to early studies about the relation between institutional investors and future stock returns, e.g. Nofsinger and Sias (1999), Gompers and Metrick (2001). Most of these studies document a positive relation between institutional ownership and future stock returns. Institutional ownership is measured as the number of shares of a security held by institutional investors relative to the total numbers of shares outstanding. Most of these studies relate the positive relation to either the informational advantage of institutional investors as a group, or to demand pressures. Yan and Zhang (2009) look at the the forecasting role of institutional investors found in the data, comparing investors with high portfolio turnover rate to low portfolio turnover rate. They show that the results are being driven by securities held by high portfolio turnover rate investors. They claim that investors with a high portfolio turnover rate have superior information. Given the data used in this study, it is hard to conclude that higher institutional investors leads to higher next year return for both the securities held by high and low trading frequency investors.

2.4.3 Return Decomposition

So far we have seen that the (i) the book to market ratio does not forecast long run return for securities held by high trading frequency investors, while it does for securities held by low trading frequency investors. More generally (ii) the return predictability differs for the securities held by high trading frequency investors as opposed to low trading frequency investors. Another way to look at it, which will summarize all the information is given by Vuolteenaho (2002) accounting extension of Campbell (1991) VAR return decomposition. In this section, the focus is on the return. I explore the sources of the difference in volatility illustrated in Figure 1.

Variance Decomposition

Campbell (1991) and Vuolteenaho (2002) relate the unexpected return of a security at time t to the changes in the expectation of future cash flow (or cash flow news, $\Delta E_t \sum_{j=0}^{\infty} \rho^j e_{t+j}$) and future return (or discount factor news, $\Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+j}$):

$$r_t - E_{t-1}r_t = \Delta E_t \sum_{j=0}^{\infty} \rho^j e_{t+j} - \Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} + \kappa_t, \quad (2.12)$$

where E_{t-1} is the expectation conditional on all the information as of time $t-1$; the ΔE_t is defined as the change in the expectation from $t-1$ to t , i.e. $E_t - E_{t-1}$; ρ is a positive constant smaller than 1; Furthermore, as earlier, r_t is the log of return, e_t is the clean-surplus log accounting return on equity and κ_t is an approximation error.

The return decomposition in (2.12) originates from the log-linear approximation of a present value identity expressed in (2.8). From the accounting clean surplus identity ($B_t = B_{t-1} + X_t - D_t$), the return on equity (ROE_t) is given by ²⁷:

$$ROE_t = 1 + \frac{X_t}{B_{t-1}} = \frac{B_t + D_t}{B_{t-1}} \quad (2.13)$$

By definition, the return (R_t) is given by:

$$R_t = \frac{M_t + D_t}{M_{t-1}} \quad (2.14)$$

Where M_{t-1} is the market value of the company the previous year. The clean surplus identity states that the book equity this year (B_t) has to be equal to the book equity the previous year (B_{t-1}) plus the earnings (X_t) less the dividends (D_t). The clean surplus return on equity is capturing the return on the book value of the company. Log linearizing (2.13) and (2.14):

$$e_t = b_t - b_{t-1} + \log(1 + \exp(d_t - b_t)) \quad (2.15)$$

²⁷The set of assumptions for the accounting present value identity to hold are given by: (i) the book equity, B , the dividend, D , and the market equity, M are strictly positive; (ii) the difference between the log book equity and the log market value, $b - m$, as well as the difference between the log dividend and the log market value, $d - m$ are stationary; (iii) the earnings, dividends and book equity must satisfy the clean-surplus identity.

$$r_t = m_t - m_{t-1} + \log(1 + \exp(d_t - m_t)) \quad (2.16)$$

Taking the first order Taylor expansion and subtracting the two identities, the dividends drop out of the relation:

$$e_t - r_t = \rho\theta_t - \theta_{t-1} + k_t \quad (2.17)$$

where θ_t is the log of the book to market ratio. Once a relation between the return, the profitability and the book to market ratio has been established, one can solve the relation forward to get the present value identity in (2.8):

$$\theta_{t-1} = \sum_{j=0}^{\infty} \rho^j r_{t+j} - \sum_{j=0}^{\infty} \rho^j e_{t+j} + \sum_{j=0}^{\infty} \rho^j k_{t+j}$$

where I used the assumption that $\lim_{t \rightarrow \infty} \rho^j \theta_j = 0$. The return decomposition is a derivation of the present value identity (2.8) after substituting (2.8) in (2.17) and taking the expectation conditional on information at time $t - 1$. The cash flow news is the percentage change in the price due to the change in the expected cash flow, i.e. the unexpected return that results from the cash flow news at given discount factor news and error terms. The same applies to the discount factor news. Hence, denoting the two return components: (i) $N_{cf,t} \equiv \Delta E_t \sum_{j=0}^{\infty} \rho^j e_{t+j}$ and (ii) $N_{r,t} \equiv \Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+j}$, one can rewrite the relation in (2.12):

$$r_t - E_{t-1}r_t = N_{cf,t} - N_{r,t} + \kappa_t \quad (2.18)$$

If the unexpected return of a security is negative, either future cash flow, i.e. future ROE, must be expected to decrease or future return must be expected to increase, or both. In other words, suppose that the price falls, such that the return decreases, and that the cash flows are constant. This must be accompanied by an increase in the return at a given point as the asset cannot be expected to experience a capital loss forever.

Equation (2.12) has been derived in Vuolteenaho (2002) using an accounting based identity. Contrary to the original Campbell (1991) return decomposition, it expresses the cash flow news component in terms of the accounting return, i.e. ROE. It uses an accounting-based present value formula, substituting for the dividends. The accounting based present value formula helps

alleviate several drawbacks associated to the use of the dividends in the firm-level analysis. In general, the relevance of the use of the dividends in the valuation of stocks is more questionable, as the dividends are often a poor quality variable at the firm-level.²⁸

Vector Autoregressive approach to the return decomposition estimation

Since neither the cash flows news nor the discount factor news are observable, a common practice to follow has been to back them out from a VAR specification. The VAR approach consists in modelling the evolution through time of the return and its forecasting variables using a vector autoregressive system:

$$z_t = \Gamma z_{t-1} + \varepsilon_t \quad (2.19)$$

where the first element of the k -dimensional state variable z_t is the log of return. Under the assumption that the evolution of the variables is well specified, that is, all the information available as of time $t - 1$ is included in z_{t-1} and the error term ε_t is independent of everything known as of time $t - 1$, I estimate the discount factor news component as follows. Let the expected return from the VAR specification be given by:

$$E_t(r_{t+j}) = e_1' \Gamma^j z_t \text{ where } j \geq 1 \quad (2.20)$$

where e_1 is a k -dimensional column vector with the first element equal to one, $[1 \ 0 \ 0 \dots 0]$. Assuming that the transition matrix remains the same for all $j \geq 1$, the unexpected return is given by:

$$\Delta E_t(r_{t+j}) = E_t - E_{t-1}(r_{t+j}) = e_1' \Gamma^j \varepsilon_t \quad (2.21)$$

²⁸Not only do many firms don't pay dividends, but the dividends may be an incomplete measure of cash flows to the shareholders. As shown in Grullon and Michaely (2002), corporate firms in the US have substituted dividend payout with share repurchases and other forms of distributions. This evolution makes the dividends a smaller fraction of the cash flows to shareholders. Furthermore, the firms tend to use the dividends to distribute the permanent component of earnings. In contrast, the firms tend to use repurchases to distribute the transitory component of earnings. This is because the repurchase and issuance policy allows financial discretion, while the dividend policy calls for financial commitment (Lintner, 1956). Consequently, the use of other types of payout and the slow dividends change make the dividends partly obsolete for asset valuation. Finally, from a more theoretical point of view, in a no-friction world such as the one assumed in the Modigliani-Miller model, changes in the dividends will not affect the value of a stock. Miller and Modigliani (1961) argue that the dividend policy is irrelevant: "while corporate managers have large discretion over payout options, such discretion should be irrelevant for stock prices. Rather, stock prices should be driven by "real" behavior—the earnings power of corporate assets and investment policy—and, crucially, not by how the earnings power is distributed."

and the discount factor news is given by:

$$\begin{aligned}
N_{r,t} &= \Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} = \sum_{j=1}^{\infty} \rho^j 1'_1 \Gamma^j \varepsilon_t \\
&= 1'_1 \rho \Gamma (I - \rho \Gamma)^{-1} \varepsilon_t = \lambda' \varepsilon_t
\end{aligned} \tag{2.22}$$

where $\lambda' = 1'_1 \rho \Gamma (I - \rho \Gamma)^{-1}$. The cash flow news is estimated indirectly as a residual.

$$\begin{aligned}
N_{cf,t} + \kappa_t &= \Delta E_t \sum_{j=0}^{\infty} \rho^j e_{t+j} + \kappa_t \\
&= r_t - E_{t-1} r_t + \Delta E_t \sum_{j=1}^{\infty} \rho^j r_{t+j} \\
&= (e'_1 + \lambda') \varepsilon_t
\end{aligned} \tag{2.23}$$

I call this method, the *indirect method*, as the cash flow news is estimated as a residual. The cash flow and discount factor news summarize all the information in the state variables of the VAR.

Following the *indirect method*, if the return is not predictable, λ' is equal to a vector of zeroes and all the variation in the return is attributable to the cash flow news and/or the error term. Furthermore, notice that if the return is unpredictable, the unexpected return is mainly driven by the cash flow news and, from the book to market present value identity, the price forecasts long run cash flow news. As a consequence, understanding the decomposition in the two components not only informs us of the driving force behind the volatility in financial markets, but it is also informative about the long run return and cash flow predictability as well.

There is another method to estimate the cash flow news, which I will call the *direct method*. One can explicitly add the log clean surplus profitability as a state variable and let 1_k be a column vector of size k where the k^{th} variable is one:

$$N_{cf,t} = \Delta E_t \sum_{j=0}^{\infty} \rho^j e_{t+j} = e'_k (I - \rho \Gamma)^{-1} \tag{2.24}$$

This method has the advantage of distinguishing the approximation error components from the cash flow component. Overall, as pointed out in Chen and Zhao (2009), even though the

indirect method has the advantage of not relying on the dynamics of the cash flows, just the one of the expected returns, it will suffer from any misspecification of the expected return. Any misspecification of the expected return will be absorbed by the residuals, i.e. cash flow news. At the limit the cash flow news, could be larger, as it will embed part of the discount factor news. To that purpose, I use and compare both the *direct and the indirect method*.

I estimate a panel VAR, where the state variables included follow the previous section on the return predictability. The variables are the log return as well as the "usual suspects" in predicting the return, i.e. the log of the U.S. GAAP return on equity, the log of the book to market and the institutional ownership. I also use the log of the clean-surplus profitability, in order to estimate the cash flow news directly and I add the trading frequency index. Furthermore, in order to incorporate the results in the previous section in the return decomposition, I allow for the securities held by high frequency investors to have a different transition matrix than the securities held by low trading frequency investors. In particular, the following "Short VAR" specification²⁹, i.e. with only one lag summarizes the specification:

$$\begin{bmatrix} r_{it} \\ e_{it} \\ \theta_{it} \\ io_{it} \\ H_{it} \end{bmatrix} = \begin{pmatrix} \Gamma^S & 0 & 0 \\ 0 & \Gamma^M & 0 \\ 0 & 0 & \Gamma^L \end{pmatrix} \left(\begin{bmatrix} r_{it-1} \\ e_{it-1} \\ \theta_{it-1} \\ io_{it-1} \\ H_{it-1} \end{bmatrix} \otimes d_{it-1}^h \right) + \begin{bmatrix} \varepsilon_{it-1}^r \\ \varepsilon_{it-1}^e \\ \varepsilon_{it-1}^\theta \\ \varepsilon_{it-1}^{io} \\ \varepsilon_{it-1}^H \end{bmatrix} \quad (2.25)$$

$, i = 1, \dots, N \text{ and } t = 1, \dots, T_i$

where d_{it-1}^h is a column vector whose first element is equal to one if the security belongs to the high trading frequency group, whose second element is equal to one if the security belongs to the medium trading frequency group, and whose third element is equal to one if the security belongs to the low trading frequency group at $t - 1$; r_{it} is defined as the log return, θ_{it} the log of the book to market ratio, e_{it} is the log of the GAAP profitability, io_{it} is the institutional ownership and H_{it} is the trading frequency index. Notice that depending on the method used, I add the log of the clean-surplus profitability to the state variable.

²⁹Different specifications with different lags have been considered and the final specification is the one with one lag as the qualitative results remain the same.

Results

What explains the higher volatility of the securities held by high trading frequency investors? From the return decomposition, the variance of the unexpected return is given by:

$$Var(\tau_t - E_{t-1}\tau_t) \approx Var(N_{cf,t}) + Var(N_{r,t}) - 2 * cov(N_{cf,t}, N_{r,t})$$

Given the VAR specification:

$$\begin{aligned} Var(N_{r,t}) &= \lambda' \Sigma \lambda \\ Var(N_{cf,t}) &= (e'_1 + \lambda') \Sigma (e'_1 + \lambda')' \\ cov(N_{cf,t}, N_{r,t}) &= (e'_1 + \lambda') \Sigma \lambda \end{aligned}$$

where Σ is defined as the variance covariance matrix of the error terms in the VAR ($\Sigma = E(\varepsilon_t \varepsilon_t')$).³⁰ Table VII summarizes the variance of the unexpected return (third column) as well as that of each of its components (fourth to fifth column) for the different investor's trading frequency group of securities as well as the total. From the third column of Table VII, the securities held by high trading frequency investors have a higher volatility (variance of 0.111) than the securities held by the low trading frequency group (variance of 0.059). This higher volatility, however is mainly driven by the cash flow news (0.095) component.

This evidence is tightly linked to the results reported in Table V. Security prices held by high trading frequency investors are mainly forecasting future cash flows, measured by the clean-surplus profitability. However, this table brings extra information in the analysis as it internalizes the forecastability of the long run return and cash flow by other variables as well (defined in the VAR). The securities held by high trading frequency investors lack forecastability by the return, return on equity and book to market. As a result, the discount factor news is almost non varying and the covariance term is close to zero.

³⁰Notice that the estimation of the components of the variance decomposition via the VAR requires the transition matrix to remain the same for each securities. In further version of the this paper, I deal with this issue. In terms of the interpretation, one can see the variance decomposition as the decomposition which will follow if a security were to be held by high or low trading frequency investors forever. What really matters is that the return is almost non predictable by highly persistent variables for high trading frequency investors. It makes the discount factor component of the decomposition small relative to the low trading frequency investors' securities. I am working an alternative way to convey this result.

The question then becomes: Why are the returns of the securities held by low trading frequency investors less volatile? Interestingly, one can notice that the variation of the cash flow news is of a similar order of magnitude for both groups of securities. This is because the securities' long run profitability is mainly driven by the book to market ratio as a result of its persistence. As we saw earlier, the predictability coefficient of this variable is of a similar order of magnitude for both groups (around 0.70). The difference comes from the covariance between the two components (cash flow news and discount factor news). In particular, low trading frequency securities have a higher covariance (0.022 as opposed to -0.006) than high trading frequency securities. For low trading frequency securities, on average, the cash flow news is associated with a discount factor news of the same sign. This is related to an underreaction of the return to cash flow news and justifies a lower volatility. This is explored in more detail in the next section.

Underreaction

In this section, I study the extent in which the return of the securities held by low trading frequency investors exhibit a lower volatility as a result of an underreaction. First, I look at the extent the data shows signs of reversal or momentum. I then explore the presence of underreaction from the following regression:

$$r_{it} = a + b^U(U * N_{cf,it}) + b^M(M * N_{cf,it}) + b^L(L * N_{cf,it}) + \varepsilon_{it} \quad (2.26)$$

where again, U , M and L correspond to the dummies for the upper, middle and lower terciles of the trading frequency. An underreaction is associated with a $b < 1$, an overreaction is associated with a $b > 1$ and $b = 1$ corresponds to a one to one reaction. A way to interpret the results is to consider that the b 's are capturing the reaction of the return of a portfolio constructed held only by low, high or medium trading frequency investors.

Table VI reports the coefficients of the regression (2.26). In addition, Panel B reports the coefficients of a regression, in which I differentiate between good and bad news. A good (bad)

news is defined as a positive (negative) cash flow news. In particular,

$$r_{it} = a + b^U(U * N_{cf,it}) + b^M(M * N_{cf,it}) + b^L(L * N_{cf,it}) \\ + \delta^U(U * N_{cf,it}^G) + \delta^M(M * N_{cf,it}^G) + \delta^L(L * N_{cf,it}^G) + \varepsilon_{it}$$

where the superindex G refers to good news. In order to alleviate the attenuation bias problem, which could come from the *indirect method* (as $N_{cf,t}$ also has the error terms and could be subject to the misspecification as explained earlier), I use the *direct method* of estimating the cash flow news. From the present value identity, the stock prices summarize a mixture of information on the long run cash flow and discount factor. If a variable forecasts long run cash flow news on top of the book to market ratio, then the security prices are underreacting to that information. To see this, suppose that there is a change in the expectation of the future cash flows of a company and the price is not fully reacting to that change. Then the return today will react by less than it would have, had the price fully responded to the news and no other variables were forecasting future cash flows. As a consequence, the return will underreact to that information. From the present value relation, this will be associated, on average, with a change of the discount factor in the same direction as the cash flow news term. Ultimately, this generates the decreased predictive power of the price to future cash flow and the underreaction of the return to cash flow news. As the future return will move in the same direction as the cash flow news, the immediate underreaction, by construction, can be seen as a slow reaction of the return to cash flow news. The cash flow news will be fully incorporated to the prices in the long run.

Overall, from Table VIII, the returns of the securities held by low trading frequency investors underreact relatively more to cash flow news than the returns of the securities held by high trading frequency investors. Interestingly, this relative underreaction comes from the reaction of the returns of the securities held by low trading frequency investors to good news only. The reaction to bad news is economically and statistically identical for all groups of securities. On the other hand, the returns held by low trading frequency investors underreact to positive news with a differential effect (δ^L) of -0.25. This underreaction is the source of the lower volatility of the return of the securities held by low frequency investors.

2.5 Conclusion

In this paper, I studied the differential response of the firm-level stock price to cash flow news and/or discount factor news as a function of the trading frequency of institutional investors—a commonly used proxy for the investment horizon.

I construct a measure for the institution's intrinsic trading frequency. I use the absolute value of the percentage change in the number of shares investors are holding in a security. I exploit the three dimensions of the variable: investors, security and time in a three way-fixed effects model and estimate an institution's fixed effect, which captures the institutions' intrinsic trading frequency, by controlling for any security and market characteristics, which could influence the investor's change in its position through time and across securities.

The results are summarized as follows: The predictability of the return on equity (firm's fundamental) by the book to market ratio does not seem to differ for the securities held by high as opposed to low trading frequency investors. The book to market ratio mainly forecasts the long run return on equity (where long run means 13 years). On the other hand, securities held by high and low trading frequency investors differ in terms of the predictability of the return. (i) The book to market ratio does not forecast long run return for securities held by high trading frequency investors, while it does for securities held by low trading frequency investors. (ii) Furthermore, the return predictability for the securities held by high trading frequency investors, has a hump shaped curve picking at year 7, and converging to 0, while it increases steadily for the securities held by low trading frequency investors. This patterns hold after controlling for many securities characteristics such as the size, the turnover, the leverage or the industry. Overall, the book to market ratio mainly forecasts the long run return on equity as opposed to the long run return for the securities held by high trading frequency investors. As such, the results challenge the view that higher frequency of trading—a commonly used proxy for investor short-termism—causes a disconnect between asset prices and fundamentals. Furthermore, I show that the lack of predictability of the return held by high trading frequency investors is a more general feature of the data, after using the "usual suspects" in forecasting future return, such as the return on equity, the book to market, the past return, the leverage or the institutional ownership.

Overall, beyond a specific interpretation of the trading frequency fixed effect, this study

highlights a new type of variable in understanding and explaining stock prices. This variable, contrary to usual securities characteristics, is related to the heterogeneity in the investors holding the securities. As such, the results show that the trading frequency index contains novel information to understand stock prices beyond the usual variables such as the size of a security among others. This new variable embeds an interesting way of looking at asset pricing as it relates not to the characteristics of the firm or but rather to characteristics of the investors who hold the particular security.

Finally, it is worth noting that the present paper is a first attempt to highlight the role of the trading frequency of the investors to explain and understand stock prices. Even though these results suggest a specific role for the horizon of institutional investors, there is a definite need for more studies (both theoretical and empirical) to understand the interaction of the investment horizon of the investors and the behavior of stock prices in financial markets. In particular, given that I use quarterly holdings data for the 13-F institutions, I cannot make any inference regarding the relationship between stock prices and short-termism for day traders or retail investors. Likewise, short-termism could generate a deviation from the fundamental valuation at higher frequency (for monthly or daily data) and such deviation decreases for annual frequency data. Future work should include a better understanding of the relation between the return, information, and the horizon at different frequencies and for different groups of investors (retail investors), where some of the mechanisms highlighted by previous theories could be particularly relevant.

2.6 Appendix: Data Description

Table II uses data from mainly 3 sources : Thomson-Reuters Institutional Holdings (13F), The Center for Research in Securities Prices (CRSP) and the COMPUSTAT annual research file (COMPUSTAT) database. The market level variables use data from the intersection between the Institutional Holdings (13F) Database and the CRSP monthly stock file, which contains monthly prices, shares outstanding, volume of trade, and returns for all publicly traded US stocks. The accounting based variables use the intersection between the Institutional Holdings (13F) Database, the COMPUSTAT annual research file, and the CRSP monthly file. The COMPUSTAT file contains the relevant accounting information for most publicly traded US stocks. Finally, daily factor data, i.e. High Minus Low (HML), stock market excess return, risk-free interest rate (one month treasury bill rate), Small Minus Big (SMB), are downloaded from the Fama-French data source, i.e. Kenneth French's web site at Dartmouth.³¹

I. Market level variables: For each end of quarter and for each security held by an institution,

(a) the size of the security is measured as the combined value of all common stock classes outstanding. If the quarter t size is missing, I ignore it from the analysis for the given quarter.

(b) For each quarter from 1980 to 2005, daily excess returns of individual stocks are regressed on the daily Fama-French three factors (1993,1996):

$$\left(r_{it} - r_t^f\right) = \beta_{i\tau}^m \left(r_{mt} - r_t^f\right) + \beta_{i\tau}^s SMB_t + \beta_{i\tau}^h HML_t + \varepsilon_{it}$$

where r_{it} is the return of security i at time t and r_t^f is the daily risk free rate, the excess return on a market portfolio is $\left(r_{mt} - r_t^f\right)$, SMB, i.e. small minus big, is the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks, and HML, i.e. high minus low, is the difference between the return on a portfolio of high book-to-market ratio stocks and the return on a portfolio of low book-to-market stocks. Finally, t stands for the days and τ stands for the quarter. Each regression uses daily data from the year preceding

³¹http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html. I also thank Kenneth French for making these data available. The risk free rate is constructed by Ibbotson Associates.

the end of the quarter τ , i.e. from the beginning of April 1980 to the end of March 1981 to estimate β' s for the end of the first quarter of 1981.

Define the volatility, the "systemic" risk and the idiosyncratic volatility for each year as the sample analogue:

$$Vol_{\tau} (r_{it} - r_t^f) = Vol_{\tau} \left(\hat{\beta}_{i\tau}^m (r_{mt} - r_t^f) + \hat{\beta}_{i\tau}^s SMB + \hat{\beta}_{i\tau}^h HML \right) + Vol_{\tau} (\hat{\epsilon}_{it})$$

where $Vol_{\tau} (x_{t\tau}) = \sqrt{\frac{1}{T_{\tau}} \sum_{t=1}^{T_{\tau}} (x_{t\tau} - \bar{x}_{\tau})^2}$. The idiosyncratic volatility of a stock is computed as the standard deviation of the regression residuals. To reduce the impact of infrequent trading on idiosyncratic volatility estimates, I require a minimum of 15 trading days in a month for which CRSP reports both a daily return and non-zero trading volume during the year preceding the quarter τ . It represents a loss of 1% of my sample. The volatility for quarter τ is computed using data from the beginning of the quarter in the previous year to the end of the quarter in the current year.

(c) The Volume of trade is simply given by the average of the trading volume during the year preceding the quarter τ . The turnover is measured as the average of the volume of trade divided by the number of shares outstanding during the year preceding the end of the quarter τ . I require at least 6 months with trading volume, and number of shares outstanding data available to be included in the sample.

(d) The momentum measures are constructed using monthly data information on the return for each security. Momentum_3 is the compounded past 3 months return from the beginning of the quarter to the end of the quarter τ . Momentum_12 is the compounded past 12 months return from the beginning of the quarter in the previous year to the end of the quarter of the current year. I discard any security where the return is missing.

The final set of securities includes all the securities where the set of variables defined from (a) to (b) is available.

II. Accounting level variables: At the end of each fiscal year and each security held by an institution,

(a) The book equity uses the total common ordinary equity (COMPUSTAT data item 60) or the liquidation value of the common equity (data item 235) if data item 60 is unavailable. I

add short and/or long-term deferred taxes (data items 35 and 71) to the book equity, if they are available. If both data items 60 and 235 are unavailable, I proxy book equity by the last period's book equity plus earnings less dividends.

(b) The book-to-market is defined as the end of the previous fiscal year book equity divided by the market equity at the end of June of the current year. The market equity is simply given by the size of the firm as defined above. For instance, the book-to-market for 1981 is given by the book equity of the fiscal year 1980 divided by the market equity of the end of June of 1981. If the book equity is missing, I use the liquidation value of the company (Compustat-data item 235) In addition, I add short and/or long-term deferred taxes (Compustat-data items 35 and 71). If both data items 60 and 235 are unavailable, I proxy the book equity by the last period book equity plus the earnings (Compustat-data item 172) less the dividends. If neither book equity nor the earning are available, I estimate the book equity by the previous year book-to-market multiplied by the current year market equity (assuming that the book to market has not changed).

(c) The GAAP return on equity is the earnings constructed by the ratio the net income (data item 172) over the last period's book equity, measured according to the US Generally Accepted Accounting Principles. If the earnings are missing, they are constructed by the change in book equity plus the total dividends. I do not allow the firm to lose more than its book equity. I define the net income as the maximum of the reported net income (or Clean-Surplus net income, if the earnings are not reported) and the negative of the beginning of the period book equity. Thus, the minimum GAAP ROE is truncated to -100%.

(d) I calculate the leverage ratio as the book equity over the sum of the book equity and book debt. The book debt is the sum of debt in current liabilities (data item 34), total long-term debt (data item 9), as well as the preferred stock (data item 130).

(e) I calculate the yield as the total cash dividends over the market equity at the end of June of the current year. The earning price ratio is defined as the net income divided by the market equity at the end of June of the current year. Similar to the GAAP return on equity, I proxy the earnings by the change in the book equity plus the dividends when the earnings are missing.

Similar to the market level variables, the set of securities will be restricted to the securities which have all the variables defined from (a) to (e) available in a given year.

III. 'others' level variables:

(a) The portfolio concentration is measured as the Herfindahl index, i.e. $H_{it} = \sum_i^N (w_{ijt})^2$, where w_{ijt} is the portfolio weight of the security j , institution i is holding at the end of quarter t . The Industry concentration index is given by the same statistics at the industry level instead of the security level, i.e. w_{iJt} is the portfolio weight of industry J in the portfolio of institution i at the end of the quarter t .

(b) Finally, I look at the three main stock exchanges (NYSE, Nasdaq and Amex), each institution invests in.

The forecastability exercise mainly uses the same data. In addition, the annual return is constructed from the CRSP monthly data from the beginning of 1980 to the end of 2005. I annualized return compounding monthly return from the beginning of July $t - 1$ to the end of June of year t . Notice that t stands for the year. All variables are in logarithm. In order to deal with the presence of return close to -1 in the log transformation, I redefine a firm as a portfolio constituted by 10% of the risk free rate and 90% of the firm return as done in Vuolteenaho (2002). The results are not sensitive to that transformation. I let any missing or negative book equity to be zero. In order to deal with values close to -100 percent in the log transformation, I redefine the Book-to-Market as $[0.9 * B_{t-1} + 0.1 * M_t] / M_t$, where B_t is the book value at the end of the fiscal year t and M_t is the market value at the end of June of year t . I redefine the return on equity as 10 percent treasury bill and 90 percent GAAP return on equity for the log transformation. The institutional ownership is defined as the relative number of shares outstanding of the firm held by institutional investors (13-F institutions) over the total number of shares outstanding of the firm. Finally, I impose a series of restrictions on the dataset. All firms must have a December fiscal year $t - 1$, in order to align the accounting variables across all the firms. All firms must have at least one trading day in the month preceding the period t return. They must have at least one trading day a month in the previous 5 years. I drop all the firms with a market equity smaller than 10 millions and a book to market ratio larger than 100 or smaller than $1/100$ at $t - 1$.

Table I: Cross institutions and time description of the fixed effect

Panel A: Descriptive statistics of the fixed effect by institutional investors type

Variables	Obs.	Mean	Std. Dev.	25%	75%
Bank	4610	0.006	0.405	-0.26	0.17
Insurance Companies	1491	-0.007	0.375	-0.27	0.15
Mutual Funds	1384	0.063	0.382	-0.2	0.24
Financial Advisers	12396	0.177	0.445	-0.16	0.48
Others (University Endowments, Pension Funds)	2090	0.044	0.487	-0.29	0.32

Panel B: Persistence of the fixed effect through time

Variables	1 Year Lag	2 Years Lag	3 Years Lag	4 Years Lag	5 Years Lag
Institution Trading Frequency	0.72	0.67	0.64	0.60	0.49

Table I summarizes the descriptive statistics of the fixed effect estimates. Panel A reports, from the left to the right, the number of observations, the mean, the standard deviation and the 25 as well as the 75 percentiles for different types of institution. The types are defined by the classification given by the Thomson Financial Dataset. The statistics are reported for the whole sample from 1980 to 1998 across all institutions within a group (as the classification of the institutions by type have been misreported after 1998). Panel B reports the first to fifth order autocorrelation of the fixed effect using all the institutions in the sample from 1980 to 2005.

Table II: Portfolio Composition of High and Low Trading Frequency Group of Investors

Market	High Trading Frequency Investors			Low Trading Frequency Investors		
	Low	High	Ratio H/L	Low	High	Ratio H/L
Size	0.05	0.56	12.10	0.03	0.75	27.18
Volatility	0.15	0.24	1.58	0.22	0.10	0.47
Volume	0.04	0.60	15.51	0.05	0.69	14.82
Turnover	0.06	0.38	6.46	0.10	0.18	1.87
Momentum 3	0.16	0.28	1.78	0.15	0.19	1.31
Momentum 12	0.13	0.36	2.77	0.13	0.23	1.72

Accounting	High Trading Frequency Investors			High Trading Frequency Investors		
	Low	High	Ratio H/L	Low	High	Ratio H/L
Book to Market	0.37	0.09	0.25	0.40	0.06	0.15
Yield	0.17	0.30	1.71	0.17	0.18	1.06
Profitability	0.04	0.01	0.17	0.02	0.46	19.54
Earning Price Ratio	0.22	0.19	0.88	0.22	0.13	0.60
Leverage	0.01	0.97	2.23	0	0.98	245.78

Others	High Trading Frequency Investors		High Trading Frequency Investors	
NYSE		0.71		0.83
Nasdaq		0.26		0.16
AMEX		0.03		0.02
Industry		0.17		0.16
Concentration		0.08		0.09

Table II reports the average share of the portfolio of the investors in the high trading frequency group and the low trading frequency group in securities with different characteristics. The first three columns report the information for the high trading frequency investors group and the last three columns report the information for the low trading frequency group. For each group of investors, I report the composition of their portfolio for three groups of the securities' characteristics: Market, Accounting and Others. For the Market and Accounting characteristics, I report three numbers, the share held in securities with a low value, a high value of the given characteristics as well as the difference in the shares with a low and high value. For instance, the first line of the three first columns, from the left to the right, gives the average shares of the portfolio of high trading frequency investors in small securities, in large securities and the ratio between these two numbers. A security is said to be small (large) if it belongs to the lower (upper) quintile group in terms of its size (price times number of shares outstanding). The industry and the concentration reports the average Herfindhal Index of the portfolio of the investors in the two groups. Finally, notice that I don't report the standard errors as given the size of the sample all the differences are statistically significant.

Table III : Descriptive Statistics of the state variables

Variables	Investors Trading Frequency							
	Low	High	Low	High	Low	High	Low	High
	Mean		Std. Dev.		25%		75%	
Return	0.08	0.20	0.29	0.37	-0.06	-0.01	0.25	0.41
(Clean-Surplus) Return On Equity	0.10	0.15	0.20	0.31	0.03	0.03	0.19	0.25
(GAAP) Return On Equity	0.09	0.10	0.16	0.23	0.06	0.06	0.15	0.17
Book to Market	-0.36	-0.51	0.55	0.63	-0.73	-0.92	-0.01	-0.14
Leverage	-0.50	-0.53	0.46	0.54	-0.71	-0.73	-0.15	-0.16
Turnover	4.78	11.57	4.51	11.78	2.17	4.56	5.94	14.38
Size (billions)	3.7	2.08	15.1	5.83	0.152	0.213	1.62	1.81
Institutional Ownership	0.43	0.50	0.23	0.22	0.27	0.32	0.58	0.67
Trading Frequency Index	-0.14	0.05	0.06	0.09	-0.17	0.00	-0.09	0.08

Table III summarizes the descriptive statistics for the securities used in the security level analysis by the investors' trading frequency group (High Trading Frequency Group and Low Trading Frequency Group). A security is in the High (Low) Trading Group if the average trading frequency fixed effect of the investors holding the security is in the lowest (highest) tercile group. For each characteristics of the securities, Table III reports the mean, the standard deviation, the 25 as well as the 75 percentiles. The return is the log of the annual compounded return using monthly data from the beginning of July to the end of June of each year. The Clean Surplus return on equity is the return on equity adjusted for equity offering, while the GAAP return on equity is the earnings over the past fiscal year book value of the firm. Both are in log terms. The book to market is the log of the book value for the past fiscal year over the market value at the end of June of each year. The leverage is the log of the book equity over the sum of the book equity and book debt of the firm. The turnover is the ratio of the volume divided by the size of the security at the end of June of each year. The size is the market value of the security, i.e. the price times the number of shares outstanding at the end of June of each year. The institutional ownership is the ratio of the number of shares outstanding held by institutional investors (13-F institutions) over the total number of shares outstanding of the security. Finally, the Trading frequency index is the weighted average of the trading frequency fixed effect of the institutions holding the security. The weights are defined as the ratio of the number shares of the institution j over the total number of shares of the institutions holding the security at a given year (end of June).

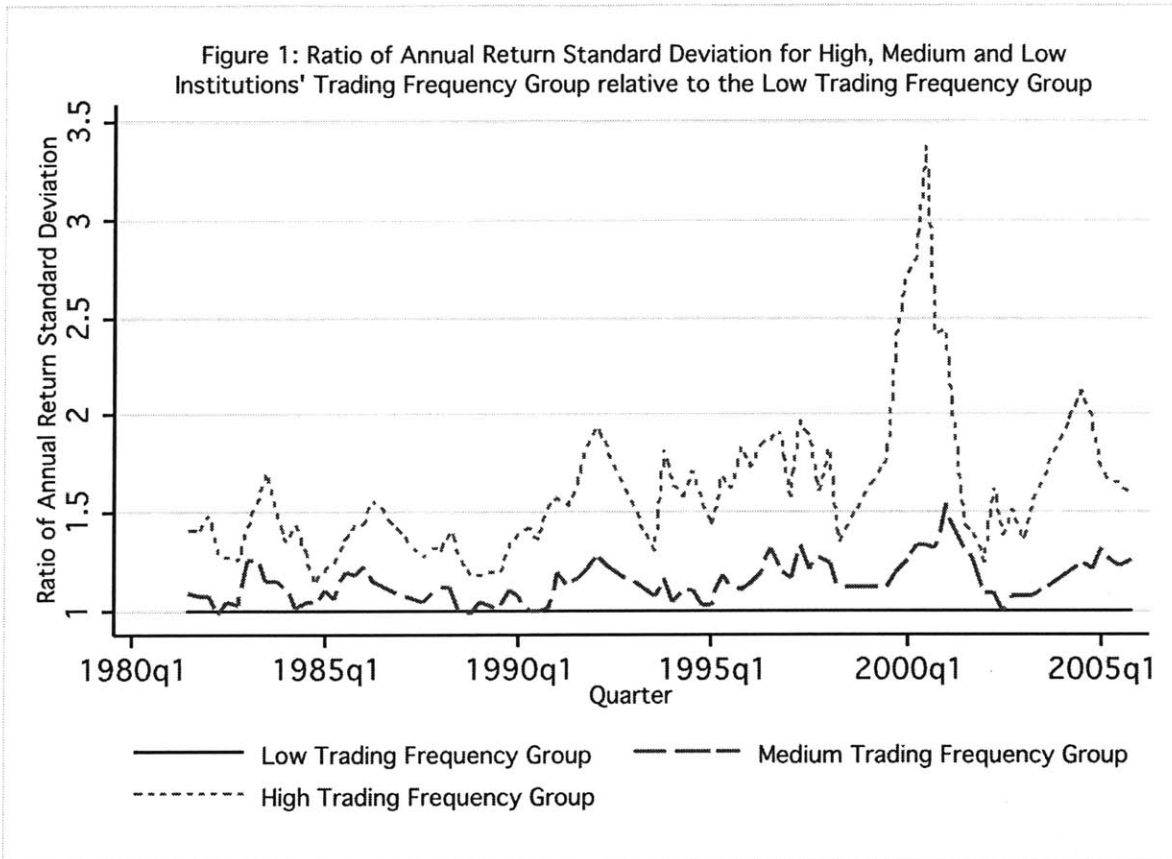


Figure 1 illustrates the ratio of the average standard deviation for the securities held by high, medium and low trading frequency investors relative to the average standard deviation for the securities held by low trading frequency investors across time (quarter). The annual return for each security is using daily compounding. The standard deviation for each security is estimated from the daily frequency annual return for each quarter. I then take the average of the standard deviation for each group of security (Low, Medium, High) and show the ratio of each average with respect to the Low group. Each group is defined with respect to the trading frequency index of the securities at quarter t , i.e. the weighted average fixed effect of the institutions holding the securities at quarter t . I sort and assign each security at a given quarter to a trading frequency group defined by the three terciles of the trading frequency index at quarter t . The difference in the standard deviation is statistically significant as can be seen in the following regression:

$$\text{Standard_Deviation}_{it} = 0.10(=a) + 0.07(=\delta_{\text{HighTradingGroup}}) + 0.016(=\delta_{\text{MediumTradingGroup}}) + \epsilon_{it}$$

(0.0006) (0.008) (0.008)

Where i stands for the group of securities and t the quarter and the standard errors are below in parentheses. Furthermore, the adjusted R^2 is equal to 0.22.

Figure 2(a): Forecasting coefficients of the return on equity at different horizons (k)

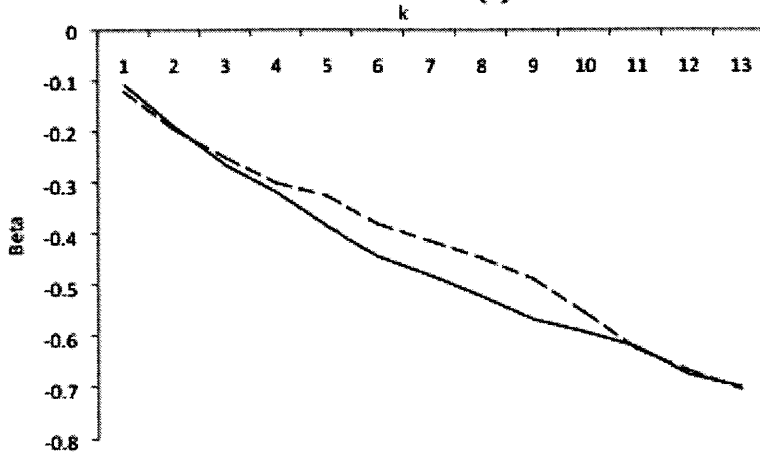
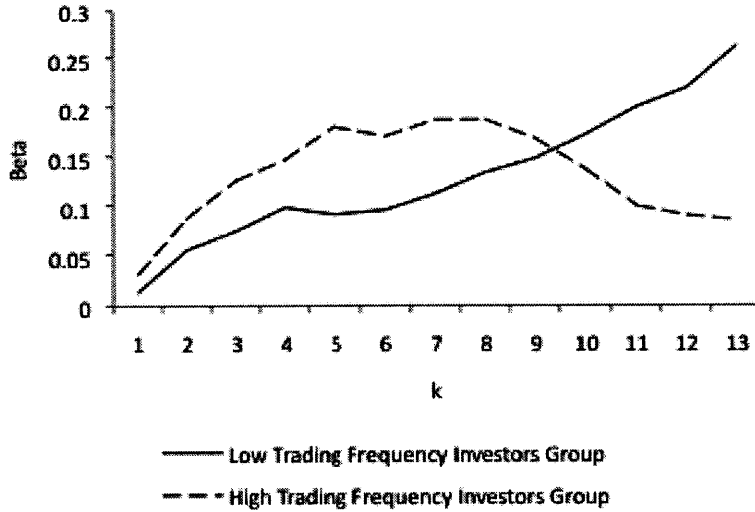


Figure 2(b): Forecasting coefficients of the return at different horizons (k)



Figures 2 (a)-(b) plot the forecasting coefficients of the total return on equity (Figure 2(a)) and the total return (Figure 2(b)) at different horizon (k), where k = 1 to 13 years (β_{rk} and β_{ek}):

$$r_{it,t+k} = a_g + \lambda_t + \beta_{rk}^U (U * \theta_{it-1}) + \beta_{rk}^M (M * \theta_{it-1}) + \beta_{rk}^L (L * \theta_{it-1}) + \varepsilon_{it,t+k}^r$$

$$e_{it,t+k} = a_g + \lambda_t + \beta_{ek}^U (U * \theta_{it-1}) + \beta_{ek}^M (M * \theta_{it-1}) + \beta_{ek}^L (L * \theta_{it-1}) + \varepsilon_{it,t+k}^e$$

Where θ_{it-1} is the log of the book to market of security i for the previous fiscal year; U, M, L are respectively a dummy equal to 1 if the security i at year t-1 belongs to the high, medium or low trading group and zero otherwise; $r_{it,t+k}$ is the total return from t to t+k and $e_{it,t+k}$ is the total Clean-Surplus return on equity from t to t+k. Finally, a_g for $g = \{U, M, L\}$ is the security group dummy and λ_t is the time fixed effect. The figures 2 (a)-(b) compare the forecasting coefficients for the securities in the high trading frequency investors group (β_{rk}^U) and the low trading frequency investors group (β_{rk}^L). The low (high) trading frequency investors group is defined as the securities with a trading frequency index at year t lower (higher) than the trading frequency lowest (highest) tercile. The trading frequency index is defined as the weighted average of the fixed effects of the institutions holding the securities at year t, where the weights are defined as the number of shares institution i holds relative to the total number of shares all the institutions hold in security i. The standard errors and the difference between the forecasting coefficients of the low and high group are given in table IV for clarity purposes. The total number of years (k) has been chosen in order to minimize the loss in the number of securities in the sample.

Table IV: Forecasting Regression Coefficients by Trading Frequency Group for k=1, 4, 7, 10, 13 years and the return as well as the return on equity

	Return			Clean-Surplus Return on Equity		
	Trading Frequency Group			Trading Frequency Group		
	Low	High	Diff High - Low	Low	High	Diff High - Low
$\beta(1)$	0.014 (0.013)	0.032 (0.018)	0.018 (0.016)	-0.11 (0.008)	-0.12 (0.011)	-0.014 (0.012)
$\beta(4)$	0.097 (0.032)	0.14 (0.036)	0.052 (0.044)	-0.321 (0.017)	-0.3 (0.028)	0.021 (0.039)
$\beta(7)$	0.11 (0.028)	0.18 (0.046)	0.077 (0.055)	-0.48 (0.024)	-0.41 (0.029)	0.07 (0.046)
$\beta(10)$	0.17 (0.039)	0.13 (0.044)	-0.033 (0.064)	-0.59 (0.032)	-0.55 (0.04)	0.039 (0.058)
$\beta(13)$	0.26 (0.037)	0.08 (0.068)	-0.18 (0.016)	-0.69 (0.04)	-0.7 (0.053)	-0.005 (0.059)

Table IV reports the forecasting coefficients of the return (first three columns) and the return on equity (last three columns) for k = 1, 4, 7, 10, 13 (β_{rk} and β_{ek}):

$$r_{it \rightarrow t+k} = \lambda_t + \alpha_g + \beta_{rk}^U (U * \theta_{it-1}) + \beta_{rk}^M (M * \theta_{it-1}) + \beta_{rk}^L (L * \theta_{it-1}) + \varepsilon_{it \rightarrow t+k}^r$$

$$e_{it \rightarrow t+k} = \lambda_t + \alpha_g + \beta_{ek}^U (U * \theta_{it-1}) + \beta_{ek}^M (M * \theta_{it-1}) + \beta_{ek}^L (L * \theta_{it-1}) + \varepsilon_{it \rightarrow t+k}^e$$

Where α_g for $g = \{U, M, L\}$ is the security group dummy and λ_t is the time fixed effect, θ_{it-1} is the log of the book to market of security i for the previous fiscal year; U, M, L are respectively a dummy equal to 1 if the security i at year $t-1$ belongs to the high, medium or low trading group and zero otherwise; $r_{it \rightarrow t+k}$ is the total return from t to $t+k$ and $e_{it \rightarrow t+k}$ is the total Clean-Surplus return on equity from t to $t+k$. The standard errors are reported, in parentheses, using the Driscoll-Kraay non parametric standard errors in the presence of time and cross sectional dependence. The table uses $T^{1/4}$ as the number of lags in the Driscoll-Kraay standard errors. For each variables, the return and the return on equity, Table IV reports the coefficients for the low, the high trading group as well as the difference between the two; $\beta_{rk}^U, \beta_{rk}^M, \beta_{rk}^L$ as well as $\beta_{rk}^U - \beta_{rk}^L$ (the third column). The low (high) trading frequency investors group is defined as the securities with a trading frequency index at year t lower (higher) than the trading frequency lowest (highest) tercile. The trading frequency index is defined as the weighted average fixed effect of the institutions holding the securities at year t , where the weights are defined as the number of shares institution i holds relative to the total number of shares all the institutions hold in security i . For the return, notice that the difference becomes statistically significant at 5% from $k = 10$ onwards.

Figure 3(a): Forecasting coefficients of the return on equity at different horizons (k) - Size and Turnover Control

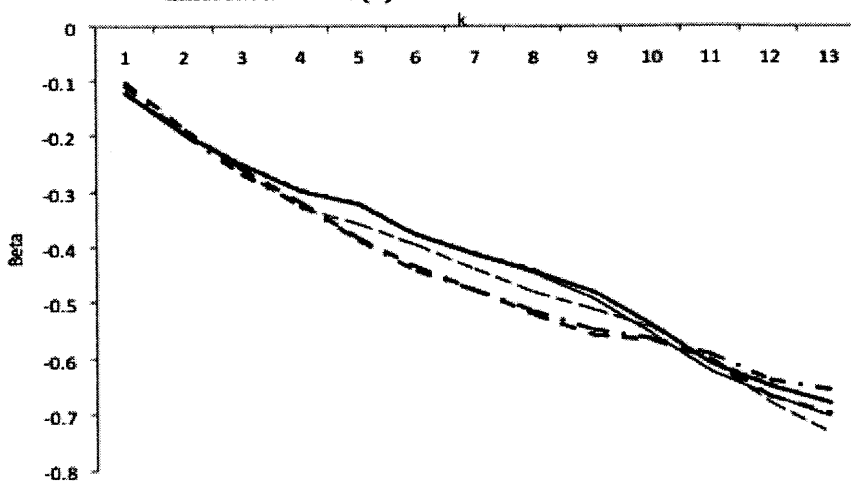
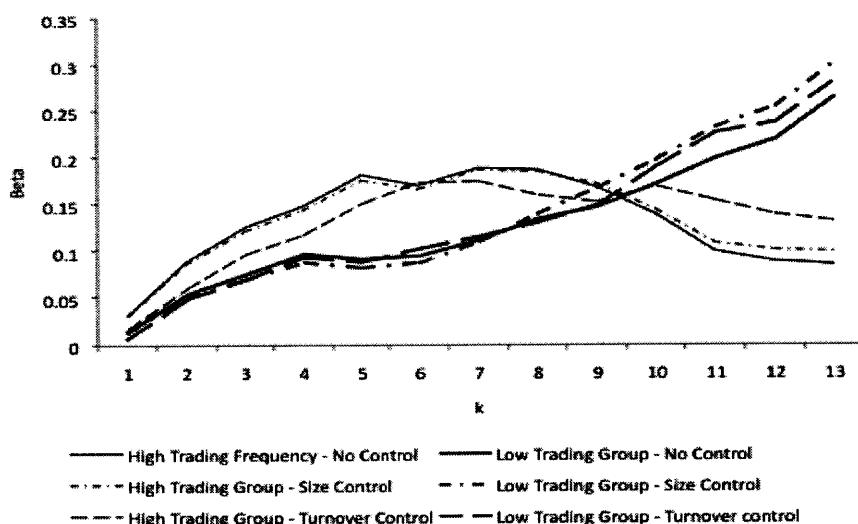


Figure 3(b): Forecasting coefficients of the return at different horizons (k) - Size and Turnover Control



— High Trading Frequency - No Control — Low Trading Group - No Control
 - - - High Trading Group - Size Control - - - Low Trading Group - Size Control
 - - - High Trading Group - Turnover Control - - - Low Trading Group - Turnover control

Figures 3 (a)-(b) plot the forecasting coefficients of the total return on equity (Figure 3(a)) and the total return (Figure 3(b)) at different horizon (k), where k = 1 to 13 years (β_{rk} and β_{ek}) after for controlling for size or turnover:

$$r_{it \rightarrow t+k} = a_g + \lambda_t + \beta_{rk}^U (U * \theta_{it-1}) + \beta_{rk}^M (M * \theta_{it-1}) + \beta_{rk}^L (L * \theta_{it-1}) + \delta_1 (X_{it-1} * \theta_{it-1}) + \delta_2 X_{it-1} + \epsilon_{it \rightarrow t+k}^r$$

$$e_{it \rightarrow t+k} = a_g + \lambda_t + \beta_{ek}^U (U * \theta_{it-1}) + \beta_{ek}^M (M * \theta_{it-1}) + \beta_{ek}^L (L * \theta_{it-1}) + \delta_1 (X_{it-1} * \theta_{it-1}) + \delta_2 X_{it-1} + \epsilon_{it \rightarrow t+k}^e$$

Where θ_{it-1} is the log of the book to market of security i for the previous fiscal year; U, M, L are respectively a dummy equal to 1 if the security i at year t-1 belongs to the high, medium or low trading group and zero otherwise; $r_{it \rightarrow t+k}$ is the total return from t to t+k and $e_{it \rightarrow t+k}$ is the total Clean-Surplus return on equity from t to t+k; X_{it-1} is the size or the turnover of the security. Finally, a_g for $g = \{U, M, L\}$ is the security group dummy and λ_t is the time fixed effect. The figures 3 (a)-(b) compare the forecasting coefficients for the securities in the high trading frequency investors group (β_{rk}^U) and the low trading frequency investors group (β_{rk}^L). The low (high) trading frequency investors group is defined as the securities with a trading frequency index at year t lower (higher) than the trading frequency lowest (highest) tercile. The trading frequency index is defined as the weighted average fixed effect of the institutions holding the securities at year t, where the weights are defined as the number of shares institution i holds relative to the total number of shares all the institutions hold in security i. The total number of years (k) has been chosen in order to minimize the loss in the number of securities in the sample. The Standard errors are omitted for clarity purpose. For each k, the Coefficients are not statistically different at 5% significance level.

Figure 4(a): Forecasting coefficients of the return on equity at different horizons (k) - General Control

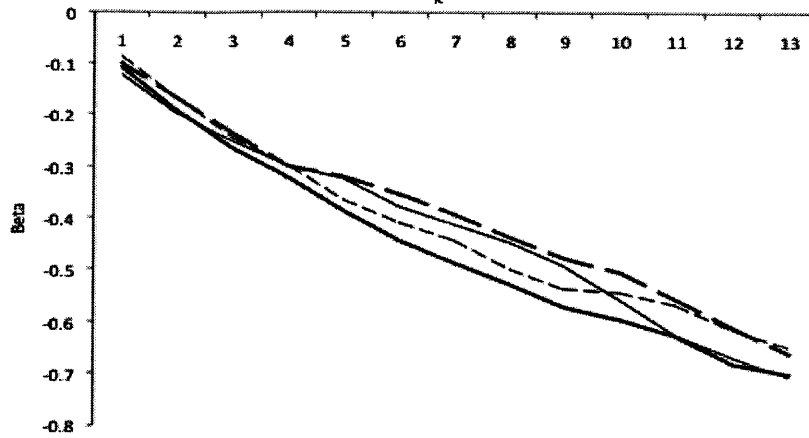
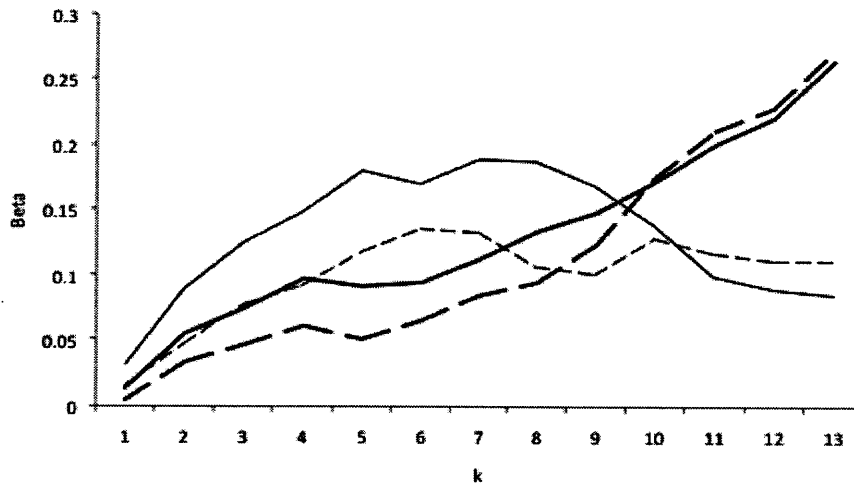


Figure 4(b): Forecasting coefficients of the return at different horizons (k) - General Control



— High Trading Frequency - No Control — Low Trading Group - No Control
 - - - High Trading Group - General Control - - - Low Trading Group - General Control

Figures 4 (a)-(b) plot the forecasting coefficients of the total return on equity (Figure 3(a)) and the total return (Figure 3(b)) at different horizon (k), where k = 1 to 13 years (β_{rk} and β_{ek}) after for controlling for size or turnover:

$$r_{it \rightarrow t+k} = a_g + \lambda_t + \beta_{rk}^U (U * \theta_{it-1}) + \beta_{rk}^M (M * \theta_{it-1}) + \beta_{rk}^L (L * \theta_{it-1}) + \delta_1 (X_{it-1} * \theta_{it-1}) + \delta_2 X_{it-1} + \varepsilon_{it \rightarrow t+k}^r$$

$$e_{it \rightarrow t+k} = a_g + \lambda_t + \beta_{ek}^U (U * \theta_{it-1}) + \beta_{ek}^M (M * \theta_{it-1}) + \beta_{ek}^L (L * \theta_{it-1}) + \delta_1 (X_{it-1} * \theta_{it-1}) + \delta_2 X_{it-1} + \varepsilon_{it \rightarrow t+k}^e$$

Where θ_{it-1} is the log of the book to market of security i for the previous fiscal year; U, M, L are respectively a dummy equal to 1 if the security i at year t-1 belongs to the high, medium or low trading group and zero otherwise; $r_{it \rightarrow t+k}$ is the total return from t to t+k and $e_{it \rightarrow t+k}$ is the total Clean-Surplus return on equity from t to t+k.; X_{it-1} is the size, the turnover, the leverage, the primary exchange market as well as the industries of the security. Finally, a_g for $g = \{U, M, L\}$ is the security group dummy and λ_t is the time fixed effect. The figures 4 (a)-(b) compare the forecasting coefficients for the securities in the high trading frequency investors group (β_{rk}^U) and the low trading frequency investors group (β_{rk}^L). The low (high) trading frequency investors group is defined as the securities with a trading frequency index at year t lower (higher) than the trading frequency lowest (highest) tercile. The trading frequency index is defined as the weighted average fixed effect of the institutions holding the securities at year t, where the weights are defined as the number of shares institution i holds relative to the total number of shares all the institutions hold in security i. The total number of years (k) has been chosen in order to minimize the loss in the number of securities in the sample. The Standard errors are omitted for clarity purpose. For each k, the Coefficients are not statistically different at 5% significance level.

Table V: Return Predictability by investor trading Frequency Group

	(1)		(2)		(3)		(4)	
	Trading Frequency Index Group		Trading Frequency Index Group		Trading Frequency Index Group		Trading Frequency Index Group	
	Low	High	Low	High	Low	High	Low	High
Book to Market(t-1)	0.058 (0.014)	0.04 (0.019)	0.058 (0.014)	0.041 (0.018)	0.056 (0.013)	0.039 (0.019)	0.056 (0.013)	0.041 (0.017)
GAAP return on Equity(t-1)	0.392 (0.048)	0.165 (0.058)	0.393 (0.049)	0.165 (0.058)	0.396 (0.048)	0.166 (0.058)	0.397 (0.048)	0.166 (0.058)
Return(t-1)	0.071 (0.032)	-0.018 (0.04)	0.071 (0.031)	-0.018 (0.04)	0.069 (0.032)	-0.018 (0.04)	0.069 (0.032)	-0.018 (0.04)
Leverage(t-1)			-0.007 (0.016)	0.013 (0.018)			-0.006 (0.016)	0.013 (0.018)
Institution Ownership(t-1)					-0.068 (0.03)	-0.016 (0.042)	-0.067 (0.03)	-0.014 (0.042)
Observations	18348		18348		18348		18348	
Nb. Securities	1193		1193		1193		1193	
Adjusted R ²	0.194		0.195		0.195		0.195	

Table V reports the coefficients of a forecasting regression for the next year return (β^U and β^L) :

$$r_{it} = \lambda_t + \alpha_g + \beta^U (U * z_{it-1}) + \beta^M (M * z_{it-1}) + \beta^L (L * z_{it-1}) + \varepsilon_{it}$$

Where λ_t and α_g ($g = \{U, M, L\}$) are the time and trading frequency group dummies; U, M, L are respectively a dummy equal to 1 if the security i at year $t-1$ belongs to the high, medium or low trading group and zero otherwise; z_{it-1} is defined as the log of the book to market ratio, the log of the return, the log of the return on equity. In addition, I add the log of the leverage ratio and the institutional ownership as potential forecaster of future return as it has been highlighted by the literature. Table V compares the forecasting coefficients for the securities in the high trading frequency investors group (β^U) and the low trading frequency investors group (β^L). The low (high) trading frequency investors group is defined as the securities with a trading frequency index at year t lower (higher) than the trading frequency lowest (highest) tercile. The trading frequency index is defined as the weighted average of the fixed effects of the institutions holding the securities at year t , where the weights are defined as the number of shares institution j holds relative to the total number of shares all the institutions hold in security i . The standard errors are in parentheses. The standard errors are estimated using the Driscoll-Kraay non parametric standard errors to take into account the cross sectional and time series dependence in the data. The number of lags included are approximately $T^{1/4}$.

Table VI: Return predictability by investor trading Frequency Group-Trading Frequency Index as an additional forecaster

	(1)		(2)		(3)	
	Trading Frequency Index Group		Trading Frequency Index Group		Trading Frequency Index Group	
	Low	High	Low	High	Low	High
Trading Frequency Index(t-1)	-0.054 (0.081)	0.478 (0.115)	-0.044 (0.079)	0.51 (0.12)	-0.047 (0.084)	0.507 (0.12)
Book to Market(t-1)	0.06 (0.014)	0.034 (0.02)	0.057 (0.013)	0.036 (0.02)	0.057 (0.013)	0.038 (0.017)
GAAP return on Equity(t-1)	0.395 (0.049)	0.164 (0.057)	0.4 (0.048)	0.164 (0.056)	0.4 (0.049)	0.164 (0.057)
Return(t-1)	0.072 (0.031)	-0.031 (0.041)	0.069 (0.031)	-0.031 (0.041)	0.069 (0.031)	-0.031 (0.042)
Leverage(t-1)					-0.005 (0.017)	0.011 (0.018)
Institution Ownership			-0.074 (0.031)	0.032 (0.043)	-0.073 (0.032)	0.033 (0.044)
Observations	18348		18348		18348	
Nb. Securities	1193		1193		1193	
Adjusted R ²	0.199		0.2		0.2	

Table VI reports the coefficients of a forecasting regression for the next year return (β^U and β^L) :

$$r_{it} = \lambda_t + \alpha_g + \beta^U (U * z_{it-1}) + \beta^M (M * z_{it-1}) + \beta^L (L * z_{it-1}) + \varepsilon_{it}$$

Where λ_t and α_g ($g = \{U, M, L\}$) are the time and trading frequency group dummies; U, M, L are respectively a dummy equal to 1 if the security i at year t-1 belongs to the high, medium or low trading group and zero otherwise; z_{it-1} is defined as the log of the book to market ratio, the log of the return, the log of the return on equity. In addition, I add the log of the leverage ratio and the institutional ownership as potential forecaster of future return as it has been highlighted by the literature. Most importantly, I add the Trading Frequency Index as an addition forecaster. Table V compares the forecasting coefficients for the securities in the high trading frequency investors group (β^U) and the low trading frequency investors group (β^L). The low (high) trading frequency investors group is defined as the securities with a trading frequency index at year t lower (higher) than the trading frequency lowest (highest) tercile. The trading frequency index is defined as the weighted average fixed effect of the institutions holding the securities at year t, where the weights are defined as the number of shares institution j holds relative to the total number of shares all the institutions hold in security i. The standard errors are in parentheses. The standard errors are estimated using the Driscoll-Kraay non parametric standard errors to take into account the cross-sectional and time series dependence in the data. The number of lags included are approximately $T^{1/4}$.

Table VII: Variance Decomposition by Investors' Trading Group

Variance	Nobs-Nsec	Total Variance	Discount Factor News Variance	Cash Flow News Variance	Discount Factor-Cash Flow News Covariance
Low Frequency Std error JK	6093-989	0.059 (0.007)	0.018 (0.008)	0.086 (0.020)	0.022 (0.012)
Medium Frequency Std error JK	6808-1146	0.060 (0.006)	0.009 (0.008)	0.075 (0.017)	0.012 (0.01)
High Frequency Std error JK	6640-1116	0.111 (0.014)	0.004 (0.005)	0.095 (0.014)	-0.006 (0.007)
Total Std error JK	18348-1193	0.077 (0.008)	0.010 (0.005)	0.085 (0.012)	0.009 (0.007)

Table VII summarizes the return decomposition of the securities by trading frequency group. From the left to the right, it shows the number of observations, the total variance, the variance of the discount factor news and the cash flow news and the covariance. The results are illustrated for the low, medium and high trading frequency group. The low (high) trading frequency investors group is defined as the securities with a trading frequency index at year t lower (higher) than the trading frequency lowest (highest) tercile. The trading frequency index is defined as the weighted average of the fixed effects of the institutions holding the securities at year t, where the weights are defined as the number of shares institution i holds relative to the total number of shares all the institutions hold in security i. The standard errors are in parentheses and they are estimated using a Shao Rao Jackknife standard errors.

Table VIII : Underreaction by institution trading group $r(t)=a+bCF(t)$

Panel A: Reaction to Cash Flow News				
Variables	Total	Low Frequency	Medium Frequency	High Frequency
b	0.701	0.665	0.657	0.763
jk s.e.	(0.029)	(0.037)	(0.033)	(0.044)
Panel B: Good Cash Flow News-Bad Cash Flow News				
Variables		Low Frequency	Medium Frequency	High Frequency
b		0.759	0.720	0.776
jk s.e.		(0.022)	(0.039)	(0.04)
b (Good news)		-0.250	-0.158	-0.032
jk s.e.		(0.079)	(0.064)	(0.085)

Table VIII summarizes the coefficients of a regression of the return on the cash flow components of the security:

$$r_{it}=a+b^L*(CF_{it}^*L)+b^U*(CF_{it}^*U)+b^M*(CF_{it}^*M)+\epsilon_{it}$$

Where CF_{it} is the cash flow components estimated using the VAR; U, M, L are respectively a dummy equal to 1 if the security i at year t-1 belongs to the high, medium or low trading group and zero otherwise; r_{it} is the return. The figures 3 (a)-(b) compare the forecasting coefficients for the securities in the high trading frequency investors group (b^U) and the low trading frequency investors group (b^L). The low (high) trading frequency investors group is defined as the securities with a trading frequency index at year t lower (higher) than the trading frequency lowest (highest) tercile. The trading frequency index is defined as the weighted average of the fixed effects of the institutions holding the securities at year t, where the weights are defined as the number of shares institution i holds relative to the total number of shares all the institutions hold in security i. The standard errors are in parentheses and they are estimated using a Shao Rao Jackknife standard errors.

Chapter 3

Institutional Investors' Intrinsic Trading Frequency and the Cross-Section of Stock Returns¹

3.1 Introduction

Heterogeneity among investors is a prevalent feature of financial markets. Investors differ in many dimensions such as their preferences, their types, their constraints, their information, the markets they participate in and their investment horizon. However, depending on the environment, heterogeneity may play little or no role in equilibrium asset prices. For example, in a world with complete markets, diversity in investors' characteristics is irrelevant. In particular, all financial claims can be priced through a representative agent's stochastic discount factor that is uniquely determined by prices and not by the underlying heterogeneity of investors. Rubinstein (1974), Constantinides (1982), Grossman and Shiller (1981), Krusell and Smith (1998) and many others provide conditions under which aggregation, or at least approximate aggregation, obtains even in the presence of heterogeneous agents and incomplete markets. Nevertheless, there are many theoretical models in which heterogeneity of investors is a key determinant of asset prices. Examples include heterogeneity of beliefs (Geneakoplos (2010), Scheinkman and

¹Joint with Fernando Duarte

Xiong (2003)), information (Allen, Morris and Shin (2006), Angeletos, Lorenzoni and Pavan (2010)) and preferences (Constantinides and Duffie (1996), Chan and Kogan (2002)).

On the empirical side, the literature has downplayed the importance of heterogeneity in investors' characteristics as a source of information to understand stock prices. Most leading asset pricing models² ignore heterogeneity, yet successfully match the observed patterns of a wide range of macroeconomic and financial variables. When studying the cross-section of stock returns, a standard approach³ is to use variables that are inherent to the underlying firm—such as size or book-to-market ratio—and not to the type of investor holding the stock.

This paper exploits institutional investors' intrinsic trading frequency as a source of heterogeneity to empirically answer the following question: Do the returns of a given security differ in a systematic way when held by investors with different trading frequency? We find that the answer is yes. We show that, even after controlling for security fixed-effects, time fixed-effects, market volatility, trading volume, liquidity, momentum and exposure to the Fama-French factors, the returns of portfolios held by investors with different intrinsic trading frequency differ significantly. Moving from the first to the last quintile in the distribution of trading frequency—that is, moving from stocks held by investors who trade more frequently to those held by investors who trade less frequently—is associated with an expected gain in returns of 6 percentage points over the next year.

Our results allow us to make two contributions. First, we show that stock holders' characteristics provide information about the cross-sectional distribution of stock returns that is not contained in firm-specific characteristics or aggregate market variables. This is an important finding because it challenges two widely used paradigms in finance: the existence of a representative agent (or in general, of aggregation) and the irrelevance of the identity of stock holders. To understand these two paradigms, consider the net-present value formula for a stock's price:

$$P_t = \sum_{s=t+1}^{\infty} \Lambda_s d_s. \quad (3.1)$$

²Both consumption-based models such as Bansal and Yaron (2004)'s long-run risk model, Barro (2005) and Gabaix (2008)'s rare disasters and Campbell and Cochrane (1999)'s habit-formation, and factor models like the Capital Asset Pricing Model (CAPM).

³Popularized by Fama and French (1993)

According to equation (3.1), if two different investors are not large enough to directly affect the aggregate discount factor Λ_t , and do not have a controlling stake in the firm so that they can not influence the cash flow d_t , then the fact that one of them owns the stock —and not the other— makes no difference in the stock’s price. In contrast, we find that stock prices do depend on at least one intrinsic characteristic of its holder, the trading frequency. Because we control for aggregate and firm-specific variables, and because we can study the subset of institutional investors that are small enough so that they can not affect the aggregate discount factor and do not hold a large enough proportion of stocks to control any firms, we provide evidence that investors’ trading horizon are not acting on prices through Λ_t or d_t . We conclude that heterogeneity across investors is an important dimension of asset prices.

The second contribution is to introduce a new variable, the trading frequency of a stock, that helps predict the cross-sectional distribution of returns. We find that our results are a “pricing anomaly” in the sense that common indicators of systematic risks such as the Fama-French factors do not explain the spread in returns between stocks held by high and low-frequency traders.

To obtain our results, we use the Thomson-Reuters Institutional Holdings dataset to get stock positions for large US financial institutions at a quarterly frequency for the period 1980-2005. Following Parsa (2010) we construct a security-specific trading frequency index by taking the weighted average of the intrinsic trading frequencies of the institutional investors who hold the security, with weights given by the size of the position of each investor. We construct the intrinsic trading frequency of investors by using a fixed-effects model. Concretely, we measure an investor’s change in his position as the absolute value of the percentage change of number of shares in a given security. We estimate a regression of institutions’ turnover of securities on a time fixed effect, a security fixed effect, their interaction, and an institution fixed effect. The institution fixed effect captures the institutions’ intrinsic trading frequencies by controlling for any security and market characteristics which could influence the investor’s change in his position across time and across securities. In this way, changes in institutional holdings due to events like an increase in market-wide volatility or a flow of information at the security level do not in themselves affect our measure of investors’ intrinsic trading frequencies.

To identify systematic differences in stocks with different trading frequencies, we form

portfolios by sorting stocks based on their trading frequency on the previous year. We find that the relation between expected mean returns and trading frequency is monotonically decreasing. This pattern holds within subgroups of securities that are independently sorted on size, book-to-market, liquidity and past performance. In addition, the relationship between trading frequency and returns does not disappear when considering returns that are risk-adjusted by the Fama-French factors, two different measures of liquidity introduced by Sadka (2006) and Pastor and Stambaugh (2003), and the momentum factor of Jegadeesh and Titman (1993).

The remainder of the paper is organized as follows. Section 2 provides a brief description of the literature. Section 3 describes the data as well as the methodology. Section 4 provides the results. Section 5 concludes.

3.2 Literature Review

This paper uses the trading frequency measure developed in Parsa (2010), which is the first paper to suggest the importance of the intrinsic trading frequency to understand properties of asset prices. Parsa (2010) interprets the trading frequency as a measure for short-termism and then studies whether short-termism is associated with excess volatility and a disconnect between prices and fundamentals. In contrast, the present paper studies whether trading frequency, not necessarily interpreted as short-termism, can be used to predict the cross-section of stock returns. The focus of the present paper is on the heterogeneity of investors and stocks —on the cross-sectional aspects— rather than the evolution and relation between volatility, cash flows and fundamentals —the time-series aspects— studied in Parsa (2010). These two papers highlight the usefulness of the measure developed in Parsa (2010) to learn about different aspects of financial markets and the economy.

More generally, this paper connects and contributes to three different strands of the existing literature. First, this paper adds to the vast literature on the relationship between the institutional investors and stock prices. This literature has documented a positive, contemporaneous relation between institutional investors' buying and stock returns; Lakonishok, Shleifer and Vishny (1992), Grinblatt, Titman and Wermers (1995), Wermers (1999, 2000), Nofsinger and Sias (1999). It has also been highlighted that institutional buying is positively related to

short-term expected return, where the expected returns are higher (lower) for stocks experiencing significant institutional buying (selling); see Daniel, Grinblatt, Titman and Wermers (1997), Gompers and Metrick (2001). Most of this literature considers the group of institutional investors to be a homogeneous group. In line with Parsa (2010), this paper contributes to the previous literature by considering the group of institutional investors as a heterogeneous group and by exploiting the heterogeneity among the institutional investors in order to understand stock prices. Thus, this paper contributes to a subset of the literature which explores the heterogeneity of investors. Grinblatt and Keloharju (2001) explores a dataset of the shareholdings in FSCD stocks and documents differences in the buy and sell behavior as well as the performance of different types of investors, such as households, foreign investors, financial institutions and insurance companies. Wermers (1999) focuses on the mutual fund industry and provides evidence on the "herding" behavior of mutual funds as well as their impact on stock prices. Cohen, Polk and Vuolteenaho (2002) study the difference between the trading behavior of institutional investors as opposed to individual investors in their reaction to cash flow news using a VAR-return decomposition at the firm level. In general, the approach in these studies consists of exploring a source of heterogeneity in the type of investors, i.e. mutual funds, retail investors, institutional investors, and so on. In this paper, the heterogeneity is the intrinsic investor trading behavior measured by the trading frequency fixed effect.

Second, this paper is connected to previous studies that have examined the portfolio turnover rate of institutional investors and its interaction with financial markets motivated by the effect of the investment horizon of institutional investors; see Gaspar, Massa and Matos (2005), Ke, Ramalingegowda and Yu (2006), Jin and Kogan (2007), Khan, Kogan and Serafeim (2010), Parsa (2010), Yan and Zhang (2009). Gaspar, Massa and Matos (2005) look at the corporate controls market and show that firms with shareholders having a higher portfolio turnover are more likely to get an acquisition bid, but at a lower premium. Yan and Zhang (2009) find that the trading of institutional investors with a high portfolio turnover rate forecasts future stock returns. This paper is related and adds to the previous studies, as it uses the institution's equity portfolio churning information. However, following Parsa (2010), it focuses on the institutions' intrinsic trading characteristic as opposed to its equilibrium trading behavior to find evidence on the relation between the institution's investment horizon and stock prices. We

exploit the variation in the trading behavior intrinsic to the institution by using the fixed-effect trading frequency of the investors. Furthermore, in contrast to earlier studies, the main focus of this study is on the differential response of the stock prices to the interaction of the trading frequency fixed effect rather than on the effects of the demand by institutional investors on stock prices. In this manner, the study is related to Jin and Kogan (2007) as well as Parsa (2010). Jin and Kogan (2007) use the variation in the portfolio turnover rate of the mutual fund managers and its interaction with a measure of investor impatience, defined as the sensitivity of money flows into and out of the fund in response to the short-term performance of the fund. They find that mutual fund managers tend to focus on short-horizon investments due to the short horizon of their investors (and not the other way around). Their evidence suggests that this behavior may result in abnormal returns as it leads to an inflated demand of short horizon investment opportunities at the expense of longer horizon alternatives. However, Jin and Kogan (2007) differs on several points with respect to this study. Similar to Parsa (2010), the measure we construct for the institutional investors' trading frequency is a "black box", which captures the component of the institution's turnover, which is explained by the institution's intrinsic characteristics as opposed to the market and/or characteristics of the securities in which they invest. Thus, we do not focus exclusively on one particular channel through which the higher trading frequency of the institutions may affect stock prices. Institutional investors can have different horizons for many reasons: different levels of patience (subjective discount factor), liquidity needs, administrative costs, legal restrictions, competitive pressures related to performance-based pay; see Dow and Gorton (1997), Shleifer and Vishny (1997), Bolton, Scheinkman and Xiong (2006). Instead, the measure used in this study allows us to focus on the whole set of institutional investors and the interaction of their trading frequency with stock prices, as the only information required is the holdings of the investors. Similar to the findings in Jin and Kogan (2007), we provide evidence that the institution's trading frequency matters for the behavior of stock prices. Finally, this paper complements Parsa (2010), which focuses on the source of the volatility in stock prices between its cash flow and discount factor component as a function of the trading frequency index. Parsa (2010) highlights that the movements of the prices of the securities held by investors trading more frequently is traced back by the long run cash flow of the securities. In line with the results in Parsa (2010), we demonstrate that

the portfolio of the securities held by investors trading more frequently is closer to their risk adjusted return.

Finally, this paper connects to the literature on the cross sectional behavior of stock returns. This literature has documented a number of empirical patterns unsupported by a standard Capital Asset Pricing Model.⁴ The firm size, the book-to-market ratio (Basu (1983), Fama and French (1993)), the firm's prior performance (Jegadeesh and Titman (1993)) and the liquidity (Pastor and Stambaugh (2003), Sadka (2006)) have each been established as an important dimension in order to understand stock prices. This paper contributes to the previous literature as it underlines a new variable that brings forth information about stock prices, the trading frequency index. However, in contrast to previous work, the role of the trading frequency index in understanding stock prices suggests a new way of looking at asset pricing as it exploits the heterogeneity of the investors characteristics. Not only do we show that the cross-sectional return of the trading frequency portfolio is not explained by their respective market risk or the usual variables (Fama-French factor, liquidity factor, momentum factor), but the dimension of interest is related to a characteristic of the securities, which is embedded in their ownership.

3.3 Data Description and Methodology

In order to study the relationship between the investors' trading frequency and the cross-section of stock returns, (i) We construct an investor-specific measure of the intrinsic frequency of trading; then (ii) we construct a security-specific measure of the composition of the intrinsic trading frequency of the investors holding the security at a given moment in time. Finally, (iii) we use the security level measure constructed in (ii) to study the relationship between the aforementioned security-specific characteristic and the cross-section of stock returns. In what follows, we begin with a brief description of the different data sources. We then describe, step by step, each of the three former points as well as the results on the relationship between the investors' trading frequency and the cross-section stock returns.

⁴The Capital Asset Pricing Model, introduced by Sharpe (1964), Lintner (1965), Mossin (1966), Treynor (1961), implies that the expected stock returns are determined by their level of beta risk through a positive and linear relation.

3.3.1 Data Description

The information used in this study comes mainly from three sources: The Thomson Reuters Ownership Data, the Fama-French factors and the Center for Research in Security Prices (CRSP). In addition, the one-month Treasury Bill Rate at monthly frequency gives the risk-free interest rate from Ibbotson Associates.

In order to study the institutional investors' trading frequencies, we use information about the quarterly equity holdings of all the institutions provided by the Thomson Reuters Ownership dataset.⁵ The dataset results from the 1978 amendment to the Securities and Exchange Act of 1934 which requires all institutions with greater than \$100 million worth of securities under discretionary management to disclose their holdings on all their common-stock positions more than 10,000 shares or \$200,000 on the SEC's form 13F. The institutions included are divided into 5 categories: Banks, Insurance Companies, Investment Companies and Their Managers (e.g. Mutual Funds), Investment Advisors, which includes the large brokerage firms, and all Others (Pension Funds, University Endowments, Foundations). It reports a total of 4382 managers. The data coverage increased in both the securities' and managers' dimensions from a total of 573 managers and 4451 securities in 1980 to 2617 managers and 13125 securities in 2005. The institutional investors represented initially 16% of the market they invested in (\$954 million) in 1980 but this number increased to about 44% (\$17,500 million) in 2005.⁶

The Fama-French and momentum factors are taken from Kenneth French's website at Dartmouth.⁷ The Sadka liquidity measures are described in Sadka (2006). The measure captures non-traded, market-wide, undiversifiable liquidity risk. Finally, the Pastor and Stambaugh (2003) liquidity factor is based on the turnover of the securities.

The monthly market information—i.e. return, price, shares outstanding—about each security is taken from the Center for Research in Security Prices (CRSP). The set of securities included corresponds to the intersection of our two main data sources, i.e. the securities that belong to the portfolio of the 13-F financial institutions and the market information available

⁵The dataset was previously known as the CDA/Spectrum 34 database. The institutions in the sample are also referred to as the 13F institutions in reference to the form they are required to file on a quarterly basis.

⁶Some of this growth is due to an increase in the value of the equity market throughout the sample period, which forced more institutions to file the 13-F forms, as the rising market pushed their portfolios across the nominal threshold level of \$100 million. For more details about the dataset, see Gompers and Metrick (2001).

⁷http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

in the CRSP. We restrict our attention to securities traded in the NYSE, the AMEX and the NASDAQ, provided they are held by more than 25 institutions, or that the institutions hold at least 10% of the shares outstanding. Our sample has 12455 securities represented and a total of 288760 data points.

3.3.2 Methodology

After briefly introducing the dataset used, the remainder of this section describes each step of the methodology. We start with the institution-specific trading frequency measure. Then we construct the security-specific trading frequency measure as the composition of the trading frequency of the institutions holding the security. Finally, we explain the methodology used to study the relationship between the security-specific measure and the cross section of stock returns. The trading frequency measures closely follow Parsa (2011) where more detailed information about the respective measures can be found.⁸

Institutional Investors Intrinsic Trading Frequency

Define s_{ijt} as the number of shares institution i is holding in security j at quarter t . We capture the trading frequency of institution i in each security j at quarter t as the absolute value of the percentage change in the position of institution i in security j at quarter t :⁹

$$y_{ijt} = \text{abs} \left(\frac{s_{ijt} - s_{ijt-1}}{1/2(s_{ijt} + s_{ijt-1})} \right). \quad (3.2)$$

If an institution i is holding the same number of securities at quarter t and $t - 1$, then $y_{ijt} = 0$. If on average y_{ijt} is bigger for institution i than institution i' , then the institution i is rebalancing its portfolio more frequently than institution i' during a given period of time.

In order to construct a measure that captures an investor's idiosyncratic tendency to change his or her position, once any security or market effects have been partialled out, we exploit y_{ijt} 's

⁸More details are provided in the appendix found at <http://econ-www.mit.edu/grad/sparsa/research>.

⁹We are using in the denominator the average number of shares in quarter t and quarter $t-1$ instead of the number of shares in quarter $t-1$. The main reason is to keep y_{ijt} from being forced to be a missing value when the number of shares moves from 0 to a positive number. However, notice that as the number of shares increases from 0 to a positive number y_{ijt} will be equal to 2. Hence, part of the information is clearly missing as a change of an institution's position is treated differently whether it was holding a positive number or 0 at $t - 1$.

three dimensions in a three-way, fixed-effect model. In particular, we estimate by ordinary least squares, for each year $T=1980, \dots, 2005$ a regression of the form:

$$y_{ijt}^T = a^T + h_i^T + g_{jt}^T + \beta X_{ijt}^T + \epsilon_{ijt}^T \quad (3.3)$$

where y_{ijt}^T is the absolute value of the change in the holdings of institution i in security j in quarter t of year T , h_i^T is the institution fixed effect; g_{jt}^T is the time-security interaction fixed effect and X_{ijt}^T controls for the size of the portfolio of investor i as well as the size of each security in the portfolio of investor i .¹⁰ The estimates of h_i^T in equation (3.3) provide an annual measure of the investor's trading frequency that does not confound any security or time effects. The two latter effects are fully absorbed by the term g_{jt} . We allow the measure of the institution's trading frequency (h_i^T) to change annually in order to capture changes across time that could be driven by investor characteristics, such as the investment horizon associated with changes in its corporate governance, its objective, its CEO, the regulation or its preferences. An investor's intrinsic trading frequency is defined by the fixed effect h_i^T in regression (3.3). A larger institution's fixed effect h_i^T is associated with investors who change their positions more often and hence have a higher idiosyncratic trading frequency. Ultimately, h_i^T provides a measure comparable to a portfolio turnover rate. However, by exploiting the three dimensions of the data (institutions, security and quarter), it combines the changes to an institution's security holdings in one churning rate, which summarizes only the trading behavior that results from the institution.

3.3.3 Security Specific Trading Frequency

For each 13-F institution, the Thomson Reuters ownership data reports the securities the investor is holding in his or her portfolio and their respective position in the securities. For each year T , quarter t and security j held by a group of institutions I_j , the security j 's trading frequency index at year T and end of quarter t is defined as the weighted average of the fixed

¹⁰Concretely, the fixed effect measures are computed with respect to the following normalization: $\sum_t \sum_j \delta_{jit} g_{jt} = \sum_i \delta_{jit} h_i = 0$ where $\delta_{jit} = 1$ if y_{jit} is non missing and 0 otherwise.

effects of the institutions in I_j :

$$H_{jtT} = \sum_{i \in I_j} \omega_{ijtT} h_{iT} \quad (3.4)$$

where the weights are $\omega_{ijtT} = \frac{s_{ijtT}}{\sum_{i \in I_j} s_{ijtT}}$, and s_{ijtT} is the number of shares outstanding of security j held by institution i at year T quarter t , and h_{iT} is the fixed effect of institution i at year T . The weight ω_{ijtT} captures the relative importance of investor i for security j at year T and quarter t , in terms of the number of shares investor i holds relative to the total number of shares the group of institutional investors is holding. This implies that the trading frequency of an investor holding 90% of the shares of a security should have a greater effect than the trading frequency of an investor holding only 10% of the shares of a security. The security's trading frequency index will give more weight to the former investor's fixed effect than to that of the latter.

H_{jtT} maps the institutional investors' trading frequency, h_i^T , to the security. H_{jtT} is interpreted as the average trading frequency of the population of institutional investors holding the security j at year T . A security j will have a high trading frequency index if, on average, the institutional investors holding the security are characterized by a short investment horizon, proxied by a large h . Overall, the institutions are weighted by their relative size with respect to the institutions holding the security. As a consequence, the variation in H can be traced back to one of two sources: (i) For a given pool of investors, the investors with a lower value of the fixed effect are holding a higher share of the security. In other words, the high trading frequency investors represent a higher share of the security, i.e. higher weight ω_{ijtT} on the high h_{jT} (the high trading frequency investors). (ii) For a given weight, the institutions holding the security have a higher institution's trading frequency. Both sources of variation, translate into a security having higher trading frequency investors than another security or having a higher trading frequency across time.¹¹ Finally, it is important to note that even though the trading frequency fixed effects at the institution level are orthogonal to any security and market characteristics by construction, there is a correlation between the securities characteristics and the

¹¹The variation of H_{jtT} through time is either the result of : (i) investors selling or buying the security characterized by a different horizon, (ii) the investors experiencing a change in their characteristics to trade (which could come from a change in the CEO or a merger), or (iii) both. The variation of H_{jtT} across security mainly comes from different securities being held by a population of investors characterized by different horizons at a given moment in time.

trading frequency index. This dependence arises from the portfolio selection of the investors, which ultimately defines the weights ω_{ijtT} .

3.3.4 Cross-Section of Expected Stock Return

In order to analyze the effects of trading frequency on the cross-section of expected stock return, we first sort all the securities for each time period into 5 or 10 portfolios based on their measure of trading frequency. For the 5-portfolio case, the portfolios are assembled based on the quintiles in the following way: the first portfolio is the value-weighted portfolio of the 20 percent of the stocks with the lowest trading frequency index the previous year, the second portfolio is the value-weighted portfolio of the 20 percent of the stocks with the next highest trading frequency index the past year, and so on. For the 10-portfolio case, the quintiles are simply replaced by deciles. The main exercise will consist of comparing the average excess return along the trading frequency dimension. Given that the trading frequency fixed effects use all the information for the whole year in which it was estimated, we consider only the trading frequency measure lagged by one year. This assures that we are using exclusively past information in our cross-sectional regression in order to predict the cross section of stock returns. All of the remaining sorting exercises follow the same precept so that an investor could have reproduced our study in real time.

Descriptive Statistics

Table I summarizes the descriptive statistics of the main characteristics of the securities in our sample, which consists of 12,455 securities and 288,760 data points. Table I reports the means and the standard deviations for the excess return, the size, the book-to-market ratio, the past performance, the liquidity and the trading frequency index. For each statistic, we also report a number for two groups of securities, the securities held in the previous year by the low and high trading frequency investors. Notice from the last line that the trading frequency index ranges from -0.18 (for the low trading frequency group of securities) to 0.18 (for the high trading group of securities), while it is close to zero for the full sample, giving a relatively easy benchmark to understand the magnitude of the trading frequency measure. There is more variation within the high trading frequency group than low trading frequency group. Looking

at the column of the mean, one can notice that the high trading frequency securities have lower excess returns, are substantially more liquid and are larger than the low trading group of securities. Interestingly, from the momentum line, one can observe that the securities held by the low trading group of securities also exhibit lower past performance. However, one should notice the substantial difference in the standard deviation across the two groups of securities for the liquidity as well as the size, highlighting a difference in the heterogeneity within the groups in terms of the characteristics of the securities. We will show in the next section that after we control for heterogeneity in all of these dimensions in several ways, the portfolios still show the spread in returns stemming from their different trading horizons.

3.4 Results

In this section, we explore the extent to which ownership matters in explaining differences in expected returns in the cross-section of stock by exploiting the heterogeneity in investors' trading frequency.

3.4.1 Is there a relation between trading frequency and returns?

Figure I illustrates the empirical relation between the realized return and the trading frequency index by reporting the average annualized return for the different trading frequency portfolios. The only difference between Figure I (a) and Figure I (b) is that the number of portfolios formed increased¹² from 5 to 10. Independent of the number of portfolios considered, there is a clear negative relation between the horizon and the realized return. The higher the average trading frequency of the institutional investors holding the security the previous year, the smaller the realized return this year. The spread in the realized returns is economically significant: The low-trading frequency portfolio exhibits an average annualized return of approximately 11 percentage points and the high trading frequency portfolio is exhibiting an average annualized return of approximately 5.4 percentage points.

Table II shows that the relation exists within sub-groups of different types of securities by double-sorting portfolios with respect to their size, book-to-market ratios, liquidity and past

¹²Our results still hold when forming 25 portfolios, although the statistical inference becomes more challenging because, especially in the double sorting, some portfolios end up having a small number of firms.

performance. The double sorting is accomplished as follows: (i) We sort all the securities into five groups based on their trading frequency. (ii) We independently sort all the securities into three groups based on each of the dimensions mentioned above. (iii) We construct fifteen different portfolios for each trading frequency and characteristic combination.

Table II shows that a strategy that consists of buying low and selling high trading frequency securities generates an annual return close to 5 percentage points. This difference is statistically significant at the 5% level, as can be noticed from the t-statistic of the last column. The same pattern is revealed when one looks at stocks divided by any of the other characteristics considered. The difference is the smallest for the group of small securities, which is mainly driven by a higher average return for the high trading group of securities. However, in terms of the statistical significance, the relation remains relatively stable even for the small securities.

The natural next step consists of exploring these spreads and the extent to which it can be explained by the characteristics or the risk exposures of the portfolios, and not by their institutional ownership.

3.4.2 Can we explain trading frequency returns by systematic risks?

The first step in exploring the relation highlighted in Figure I is to explore the results controlling for the Fama-French factors. Figure II reports the mean, annualized excess return of the different trading frequency portfolios as a function of the mean excess return predicted by the standard Fama-French model. For each portfolio p , we run the following time-series regression:

$$R_{p,t} - r_t^f = \alpha_p + (R_{m,t} - r_{f,t})\beta_p^m + SMB_t\beta_p^{SMB} + HML_t\beta_p^{HML} + \varepsilon_t \quad (3.5)$$

$$t = 1, \dots, T \quad (3.6)$$

where $R_m - r_f$ is the excess return on a broad market portfolio, SMB (small minus big) is the difference between the return on a portfolio of small and large stocks, and HML (high minus low) is the difference between the return on portfolios of high and low book-to-market stocks, and the time variable t refers to quarters. The OLS estimates are $\hat{\alpha}_p$ and $\hat{\beta}_p$. Figure II plots $E[R_{p,t} - R_t^f] = \frac{1}{T} \sum_{t=1}^T (R_{p,t} - R_t^f)$ in the y-axis and $E[X_t \hat{\beta}_p] = \frac{1}{T} \sum_{t=1}^T X_t \hat{\beta}_p$ on the x-axis. Each portfolio is represented by a triangle as well as a number that denotes the quintile

of the trading frequency index (increasing from 1 (low trading frequency) to 5 (high trading frequency)). Figure III summarizes the pricing error (alpha) of the different portfolios as a function of the trading frequency index. The average trading frequency within each portfolio ranges from -0.15 for the low trading frequency group to 0.16 for the high trading frequency group.

Figure II shows a discrepancy between realized and predicted returns. This divergence is more pronounced for the low trading group of securities. Overall, the portfolio defined by the low trading group of institutions exhibits a realized return of 12 percentage points, from which approximately 9 percentage points have been accounted for by the model. Figure II suggests that the ownership matters, and it matters specifically for the low trading frequency group of securities. Figure III shows that the pricing error is a linear and monotonically decreasing function of the trading frequency index. As such, the higher the trading frequency of the institutional investors holding a security, the smaller the underlying alphas.

A more econometrically precise picture of Figures II and III is given in Table III. This table reports the characteristics of trading frequency portfolios from the lowest trading frequently portfolio to highest trading frequency portfolio divided into five value-weighted portfolios. The table reports the Fama-French factor sensitivities, i.e. the slope coefficients in the Fama-French, three-factor-model time-series regressions as well as the alphas and the R^2 (from the left to the right). From Table III, one can notice that overall it seems that apart from the low trading group of securities, the model seems to do a fair job from the R^2 point of view. However, the market risk does not help explain the difference in the return, as the coefficients of the different portfolios are roughly constant. The risk-adjusted return from the first column (alpha) shows that the portfolio that shorts the high trading frequency securities and buys the low trading frequency securities earns approximately 4 percentage points on an annual basis. The bottom line from Table III is that there is a substantial risk-adjusted average return from the trading frequency strategy that can be implemented.

Given the particular nature of our portfolios, there are two other dimensions of portfolios highlighted in the literature that could account for our results: liquidity and momentum. More liquid securities are naturally associated with a higher trading frequency index. This high correlation is expected as investors trading more frequently might select a more liquid security.

Conversely, investors trading more frequently might increase the liquidity of the securities they invest in by the activities they engage in. For these two reasons, it is necessary to control for the liquidity of these portfolio to make sure that the results are not completely driven by liquidity risk. Likewise, for the momentum, one could expect that high-trading frequency securities might be more correlated to the momentum factor as high trading frequency investors could potentially care more about the short-term price movements and engage in momentum strategies. Figures IV and V illustrate the results after accounting for the two factors. Specifically, for each portfolio, we estimate:

$$R_{p,t} - r_t^f = \alpha_p + (R_{m,t} - r_{f,t})\beta_p^m + SMB_t\beta_p^{SMB} + HML_t\beta_p^{HML} \quad (3.7)$$

$$+ MOM_t\beta_p^{MOM} + LIQ_t\beta_p^{LIQ} + \varepsilon_t, \quad t = 1, \dots, T, \quad (3.8)$$

where in addition to the variables from (3.5), we have added the liquidity factors based on Sadka (2006) and Pastor and Stambaugh (2003) and the momentum factor. Interestingly, from Figure V one can notice that the introduction of the new factors actually increases the pricing errors. As such, the spread highlighted in Figure II is not confounding these two characteristics. As in Figure III, Figure V shows that the pricing error (alpha) decreases monotonically with the trading frequency.

Table IV summarizes the results for all the cases considered. It reports the statistical significance of the figures just discussed. It compares the estimates of the pricing error, $\hat{\alpha}$, for the regressions (3.5) and (3.8) as well as the simple CAPM model and a model controlling for the long run and short run reversal. The t -statistic is computed using a Newey-West estimator with 3 lags, which is robust to correlation of the error terms across portfolios, within portfolios and across time. Furthermore, we report in the column labeled GRS, the "GRS test statistic" for the hypothesis that all $\hat{\alpha}_p$ are jointly zero. It is simply an F-test adjusted for finite samples and is F-distributed, $F[M, T - M - 1]$, with M and T-M-1 degrees of freedom, where M is the number of factors in X_t . From Table IV, even though the alpha of each portfolio is not statistically significant on its own, the null hypothesis that all the $\hat{\alpha}$ are jointly zero is rejected. Our results show that an investor can earn on average 3.3 percent per year without being exposed to any source of the common systematic risks considered here.

3.4.3 Can we explain trading frequency returns by a trading frequency index?

So far, we have highlighted a relationship between trading frequency and stock returns. We showed that the relationship cannot be accounted for by the usual factors or variables used in the literature. Can this difference be explained by a trading frequency “factor”? In the previous section, Figures III and IV suggest a linear and monotone negative relation between the trading frequency index and the pricing error. In other words, the higher the trading frequency index, the closer the return from its fundamentals or from the return predicted by a standard cross-sectional model.

In order to explore this further, we build a trading frequency factor as the difference between the return of the portfolio of the bottom 20% trading frequency group of securities and the top 20% trading frequency group of securities. We then try to explain the extent to which adding this extra factor helps us account for the pricing error. A first answer to this exercise is summarized in Figure VI and VII. Figure VI illustrates the relation between the realized excess return and the predicted return and Figure VII illustrates the relationship between the pricing error and the predicted return after controlling for the trading frequency factor. In particular, for each portfolio, we estimate:

$$R_{p,t} - r_t^f = \alpha_p + (R_{m,t} - r_{f,t})\beta_p^m + SMB_t\beta_p^{SMB} + HML_t\beta_p^{HML} \quad (3.9)$$

$$+ MOM_t\beta_p^{MOM} + LIQ_t\beta_p^{LIQ} + TF_t\beta_p^{TF} + \varepsilon_t, t = 1, \dots, T. \quad (3.10)$$

where in addition to the variables from (3.8), we have added the trading frequency factor TF as defined above. Figure VI shows that the realized return aligns more naturally with the 45 degree line. The difference between the realized and the predicted excess return is by and large accounted for by the inclusion of the trading factor. This is also reported in Figure VII, which shows that the new pricing errors from a model that internalizes the trading frequency factor are smaller and do not have a systematic correlation with the trading frequency measure. Table V shows the related statistical information. On one hand, even though the $\hat{\alpha}$'s are smaller and they do not exhibit a specific relation with the trading frequency, one can still reject the null of all $\hat{\alpha}$'s being jointly zero. On the other hand, from an economic point of view, the return

an investor will make exploiting the trading frequency difference is now substantially smaller after accounting for the trading frequency return. Hence, adding the trading frequency factor, even though it adds new information, provides a mixed response to the spread in returns of the different trading frequency portfolios.

3.5 Conclusion

In this paper, we show that stock returns are predicted by the intrinsic frequency of trading of its institutional holders. Moving from the first to the last quintile in the distribution of the security-specific trading frequency is associated with an expected gain in returns of 6 percentage points over the next year. The magnitude and predictability of these returns persist or even increase when risk-adjusted by measures of systematic risks such as the Fama-French factors.

The result that stock returns depend on who holds them is at odds with two standard views in finance. The first is that a stock's price is frictionlessly determined by the discounted sum of its dividends. If two institutional investors are not large enough to directly affect the aggregate discount factor and do not have a controlling stake in the firms in which they invest, then the fact that one of them owns the stock —and not the other— should make no difference in the stock's return. The second standard view that is challenged by our results is that of the representative agent whose stochastic discount factor prices any given cash flow. In such an economy, the identity and heterogeneous characteristics of stockowners should provide no information about the cross-section of stock returns.

Another way to state our findings is to interpret them as a “pricing anomaly” in the sense that neither aggregate risk factors nor firm-specific characteristics can explain the spread in returns between stocks held by high and low-frequency traders. An explanation of our results will most likely be found by analyzing the “demand side” instead of the “supply side” of the market, i.e. how traders who demand stocks behave, instead of how firms who supply stocks behave.

Herein lies a limitation of our study: even though the relationship between trading horizon and stock returns is empirically strong and pervasive among different subgroups of stocks, there is no theoretical explanation for why this is the case. The apparent breakdown of the relation

between stock prices and their corresponding discounted sum of dividends and the emphasis on traders' heterogeneity suggests that behavioral explanations in the spirit of Shleifer and Vishny (1997) and Daniel, Hirshleifer and Subrahmanyam (2001), could provide potential explanations of our results. At a minimum, explanations will most likely deviate from complete market, representative agent, frictionless economies.

Table I
Descriptive statistics

The table shows the annualized mean and standard deviation of excess returns $R - R^f$, market capitalization (Size), Book-to-Market ratio (B/M), volume per number of shares outstanding (Liquidity), last quarter's excess returns (Momentum) and Trading Frequency. Excess returns are from CRSP. Market capitalization is measured as price multiplied by the number of shares outstanding reported in CRSP. The trading frequency of a security is constructed following Parsa (2010). The first column shows the statistics for the full sample of securities, while the last two columns show the statistics for stocks in the lowest and highest quintile of the trading frequency distribution, respectively.

	Full sample		Low trading frequency		High trading frequency	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
$R - R^f$	0.0310	0.278	0.0348	0.253	0.0212	0.311
Size	1.62	1.62	1.62	1.62	0.754	2.96
B/M	0.723	0.723	0.723	0.723	0.723	0.723
Liquidity	3.29	3.29	3.29	3.29	5.21	5.21
Momentum	0.0305	0.304	0.0381	0.294	0.0286	0.310
Trading Frequency	-0.0176	0.137	-0.177	0.0646	0.171	0.130

Table II
Mean Returns and *t*-Statistics of Sorted Portfolios

The table shows annualized mean excess returns and the corresponding *t*-statistics of value-weighted portfolios formed by sorting on the characteristics defined in Table I. The first row is a single sort on quintiles of trading frequency for each quarter *t*. The next rows perform a double sort by independently placing each stock into one of five trading frequency quintiles and one of three size, book-to-market, liquidity or momentum groups. Portfolios are formed by grouping stocks that belong to the intersection of two groups. The reported mean returns are the time-series averages of the annualized returns of each portfolio. The column High-Low constructs a zero-investment portfolio by buying the portfolio in the High trading frequency group and shorting the portfolio in the Low frequency group.

	Trading Frequency					
	Low	2	3	4	High	High-Low
Single-sort	0.129 [3.37]	0.112 [2.89]	0.108 [2.62]	0.0939 [2.03]	0.0789 [1.53]	0.0505 [2.18]
<i>Size</i>						
Small	0.143 [3.41]	0.132 [2.75]	0.139 [2.69]	0.105 [1.95]	0.107 [1.98]	-0.0366 [-1.60]
Medium	0.114 [3.23]	0.108 [2.75]	0.101 [2.30]	0.0923 [1.89]	0.0638 [1.21]	-0.0505 [-1.94]
Big	0.111 [3.41]	0.105 [3.21]	0.0878 [2.52]	0.0789 [1.85]	0.0528 [1.07]	-0.0583 [-1.62]
<i>Book-to-Market</i>						
High Growth	0.0843 [2.25]	0.0821 [2.15]	0.0342 [1.12]	0.00407 [0.002]	-0.0454 [-0.392]	-0.130 [-3.42]
Medium	0.0102 [1.03]	0.0244 [1.35]	0.00783 [0.00532]	-0.00807 [-0.0031]	-0.0181 [-0.47]	-0.0283 [0.0001]
High value	0.148 [9.78]	0.132 [5.13]	0.137 [3.35]	0.0639 [2.68]	0.0353 [1.35]	-0.113 [2.53]
<i>Liquidity</i>						
More illiquid	0.0163 [1.95]	0.0133 [1.54]	0.00959 [1.13]	0.00648 [0.70]	0.00597 [0.633]	-0.0104 [-2.90]
Medium	0.0398 [3.53]	0.0225 [2.3]	0.0216 [2.23]	0.021 [2.00]	0.0135 [1.17]	-0.0263 [-5.98]
More liquid	0.125 [6.78]	0.0838 [5.31]	0.0618 [3.95]	0.0449 [2.78]	0.0406 [2.25]	-0.084 [-6.74]
<i>Momentum</i>						
High past returns	0.169 [2.29]	0.121 [1.93]	0.118 [1.36]	0.11 [1.22]	-0.0928 [-1.19]	-0.262 [-2.87]
Medium	0.105 [0.994]	0.077 [1.55]	0.0179 [0.151]	0.101 [0.952]	0.0407 [0.379]	-0.0642 [-0.363]
Low past returns	0.0119 [0.0829]	0.236 [4.23]	0.0972 [1.84]	0.0342 [0.357]	0.0547 [1.03]	0.0428 [1.02]

Table III
**Time-Series Regressions of Returns of Trading
Frequency Portfolios on Fama-French Factors**

The table shows estimates of the intercept, coefficients and R^2 of time series regressions of excess returns on the Fama-French factors. Each row corresponds to one of the portfolios constructed by sorting on trading frequency as explained in Table II. t -statistics are reported in brackets.

	$\hat{\alpha}$	$\hat{\beta}^{MKT}$	$\hat{\beta}^{SMB}$	$\hat{\beta}^{HML}$	R^2
<i>Trading Frequency</i>					
Low	-0.0131 [-0.86]	1.10 [20.9]	-0.312 [-4.72]	0.488 [6.40]	0.926
2	-0.0005 [0.0516]	1.12 [35.8]	-0.113 [-2.89]	0.24 [5.30]	0.965
3	0.0156 [1.66]	1.01 [30.8]	-0.0226 [-0.554]	-0.0587 [-1.25]	0.944
4	0.0195 [1.83]	1.01 [27.6]	0.181 [3.92]	-0.103 [-1.94]	0.918
High	0.0261 [1.59]	0.923 [16.3]	0.302 [4.24]	0.0758 [0.921]	0.796
<i>High-Low</i>	0.0392 [1.51]	-0.007 [-1.96]	0.0246 [5.44]	-0.0165 [-3.17]	0.513

Table IV
Pricing Errors of Different Models
When Pricing Frequency-Sorted Portfolios

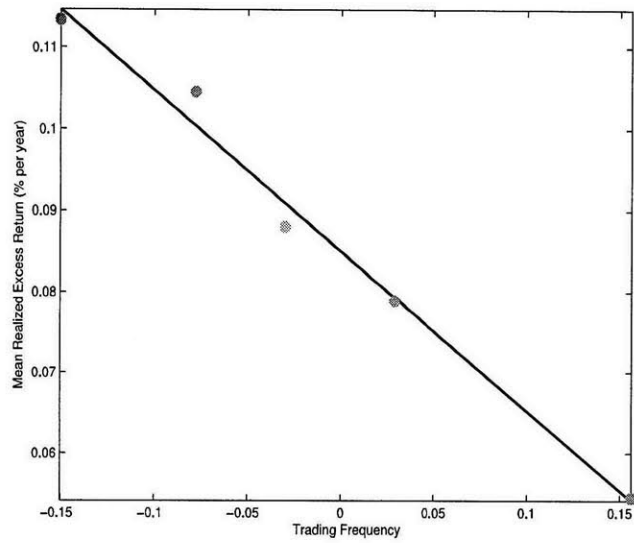
The table shows the performance of different factors when pricing 5, 10 and 25 portfolios constructed by sorting on stock's trading frequency as described in Table II. As factors, we consider the market excess return (CAPM), the Fama-French factors (FF), Jegadeesh and Titman's momentum (UMD), long-term return reversal (Rev) and liquidity factors of Sadka and Pastor/Stambaugh (Liq). All the reported statistics are obtained from regressions of the excess return of the 5, 10 or 25 trading frequency portfolios on the different pricing factors. Mean $|\hat{\alpha}|$ is the average across regressions of the absolute value of the estimate of the intercept in annualized percentage points. GRS is the Gibbons-Ross-Shanken test-statistic (an F-statistic adjusted for finite sample bias) of the null hypothesis that the $\hat{\alpha}$ for all portfolios are jointly zero, for which we also report its p -value. The Mean R^2 is the average value of the R^2 across regressions.

	CAPM	FF	FF+UMD+Liq	FF+UMD+Rev+Liq
5 portfolios				
Mean $ \hat{\alpha} $	0.0324	0.015	0.0221	0.0327
<i>GRS</i>	15.9	15.5	14.2	12.1
<i>p</i> -value	0.00	0.00	0.00	0.00
Mean R^2	0.885	0.910	0.910	0.914
10 portfolios				
Mean $ \hat{\alpha} $	0.0307	0.0161	0.0215	0.0310
<i>GRS</i>	12.3	11.4	8.38	8.1988
<i>p</i> -value	0.00	0.00	0.00	0.00
Mean R^2	0.808	0.831	0.877	0.837
25 portfolios				
Mean $ \hat{\alpha} $	0.0342	0.0215	0.0278	0.0339
<i>GRS</i>	25	25.9	37.5	30.1
<i>p</i> -value	0.00	0.00	0.00	0.00
Mean R^2	0.635	0.685	0.812	0.707

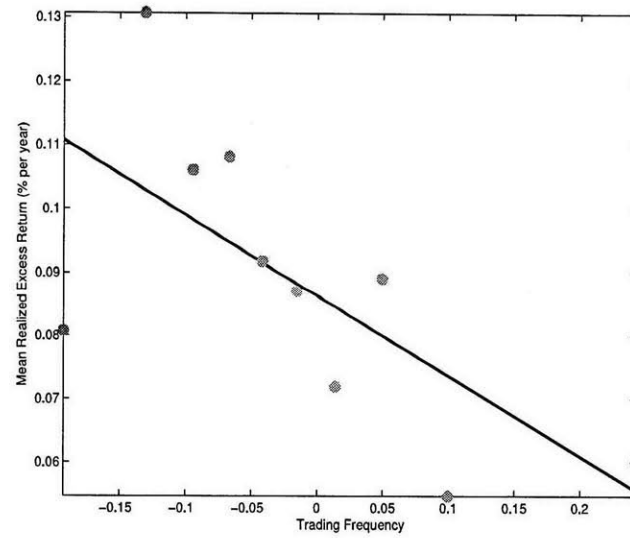
Table V
**Time-Series Regressions of Returns of Trading Frequency Portfolios
on Fama-French and a Trading Frequency Factor**

The table shows estimates of the intercept, coefficients and R^2 of time series regressions of excess returns on the Fama-French factors and a the High-Low portfolio. Each row corresponds to one of the five portfolios constructed by sorting on trading frequency as explained in table II. t -statistics are reported in brackets.

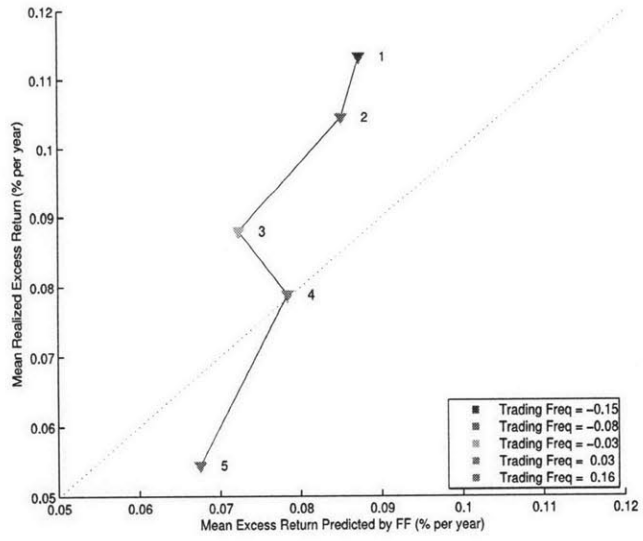
	$\hat{\alpha}$	$\hat{\beta}^{FREQ}$	$\hat{\beta}^{MKT}$	$\hat{\beta}^{SMB}$	$\hat{\beta}^{HML}$	R^2
<i>Trading Frequency</i>						
Low	0.0054 [0.591]	47.2 [13.2]	1.02 [31.8]	-0.0220 [-0.488]	0.294 [6.14]	0.974
2	0.0043 [0.494]	9.91 [2.88]	1.06 [35.8]	-0.0525 [-1.21]	0.199 [4.34]	0.968
3	0.0131 [1.39]	-6.51 [-1.77]	1.01 [30.9]	-0.0626 [-1.35]	-0.0319 [-0.651]	0.945
4	0.0092 [1.11]	-26.3 [-8.13]	1.10 [36.7]	0.0194 [0.475]	0.0053 [0.122]	0.952
High	0.0054 [0.591]	-52.8 [-14.7]	1.01 [31.8]	-0.0220 [-0.488]	0.294 [6.14]	0.938



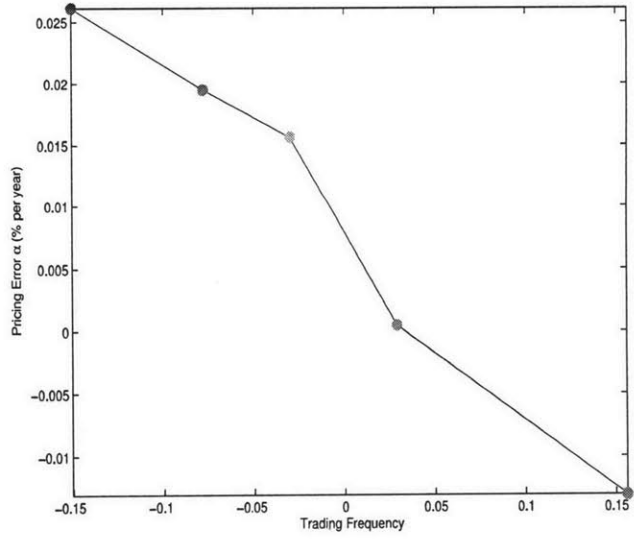
(a) Figure I.a



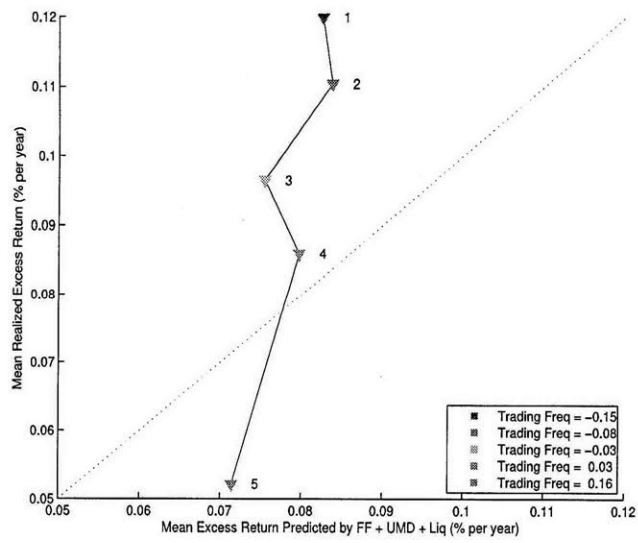
(b) Figure I.b



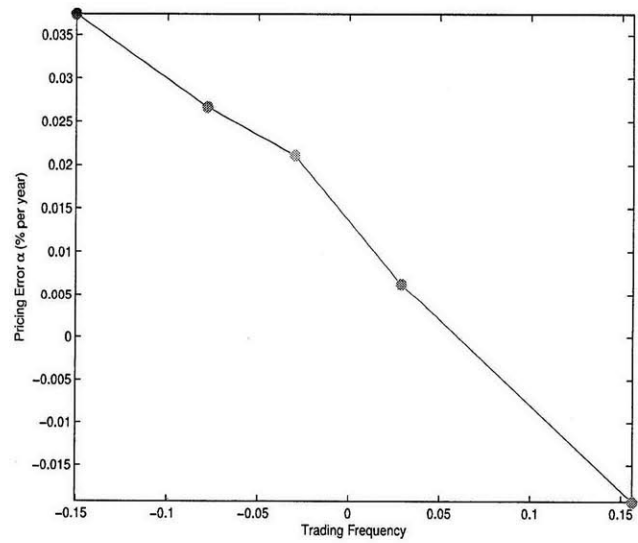
(c) Figure II



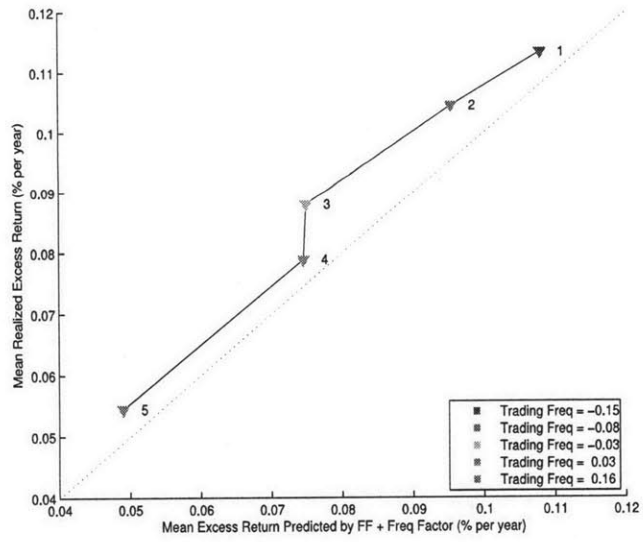
(d) Figure III



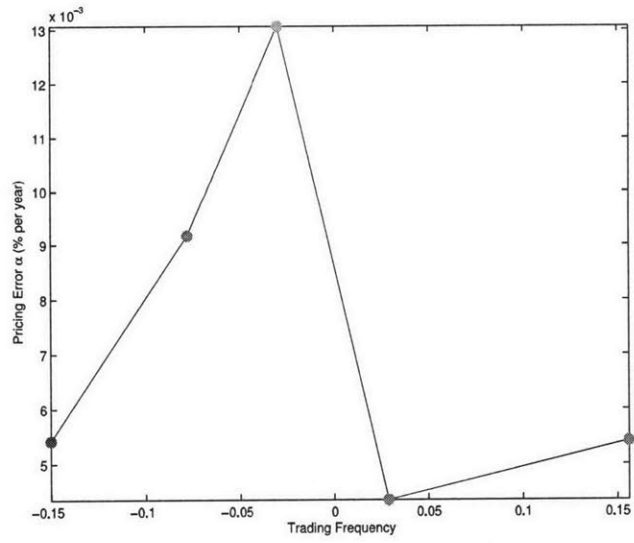
(e) Figure IV



(f) Figure V



(g) Figure VI



(h) Figure VII

Bibliography

L.F. Ackert and B.F. Smith, 1993, Stock price volatility, ordinary dividends, and other cash flows to shareholders, *The Journal of Finance*, vol. 48 (4) pp. 1147-1160.

A. Abel and G. Mailath, 1994, Financing losers in financial markets, *Journal of Financial Intermediation*, vol. 3 pp. 139-165.

A.R. Admati, 1985, A noisy rational expectations equilibrium for multi-asset securities markets, *Econometrica*, vol. 53 pp. 629-657.

T. Adrian and J. Rosenberg, 2006, Stock returns and volatility: pricing the short-run and long-run components of market risk, *Journal of Finance*, vol. 63 (6) pp. 2997-3030.

F. Allen, S. Morris and H.S. Shin, 2006, Beauty contests and iterated expectations, *Review of Financial Studies*, vol. 19 (3) pp. 719-752.

F. Allen, S. Morris, and A. Postlewaite, 1993, Finite bubbles with short sales constraints and asymmetric information, *Journal of Economic Theory*, vol. 61 pp. 206-229.

F. Allen, S. Morris, and H.S. Shin, 2006, beauty contests and iterated expectations, *Review of Financial Studies*, vol. 19 pp. 161-177.

A. Ang and G. Bekaert, 2007, Stock return predictability: Is it there?, *Review of Financial Studies*, vol. 20 (3) pp. 651-707.

G.M. Angeletos and J. La'O, 2008, Dispersed information over the business cycle: Optimal fiscal and monetary policy with dispersed information, MIT mimeo.

G.M. Angeletos and J. La'O, 2009, Incomplete information, higher-order beliefs, and price inertia, *Journal of Monetary Economics*, vol. 56(1), 19-37.

G.M. Angeletos and J. La'O, 2011, Decentralization, coordination, and the origins of fluctuations, MIT mimeo.

- G.M. Angeletos, G. Lorenzoni and A. Pavan, 2010, Beauty contests and irrational exuberance: A neoclassical approach, NBER Working Papers No. 15883.
- P. Bacchetta and E. Van Wincoop, 2008, Higher order expectations in asset pricing, *Journal of Money, Credit and Banking*, vol. 40 (5) pp. 837-866.
- S.G. Badrinath and S. Wahal, 2002, Momentum trading by institutions, *The Journal of Finance*, vol. 57 (6) pp. 2449-2478.
- M. Baker and J. Wurgler, 2007, Investor sentiment in the stock market, *Journal of Economic Perspectives*, vol. 21 (2) pp. 129-152.
- S. Banerjee, R. Kaniel, and I. Kremer, 2009, Price drift as an outcome of differences in higher order beliefs, *Review of Financial Studies*, vol. 22 (9) pp. 3707-3734.
- R. Bansal and A. Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *The Journal of Finance*, Vol. 59 (4) pp. 1481-1509.
- R. Bansal and A. Yaron, 2007, The asset pricing-macro nexus and return-cash flow predictability, working paper.
- R.W. Banz, 1981, The relationship between return and market value of common stocks, *Journal of Financial Economics*, vol. 9 (1) pp. 3-18.
- B.M. Barber and T. Odean, 2000, Trading is hazardous to your wealth: The common stock investment performance of individual investors, *Journal of Finance*, vol. 55 pp. 773-806.
- N. Barberis and R. Thaler, 2003, A survey of behavioral finance, *Handbook of the Economics of Finance*, Chapter 18, pp. 1051-1121.
- N. Barberis, A. Shleifer and R. Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics*, vol. 49 (3) pp. 307-343.
- R.J. Barro, 2005, Rare Events and the Equity Premium, NBER Working Paper No. 11310.
- S. Basu, 1983, The relationship between earnings' yield, market value and return for NYSE common stocks: Further evidence, *Journal of Financial Economics*, vol. 12 (1) pp. 129-156.
- J.A. Bennett, R.W. Sias and L.T. Starks, 2003, Greener pastures and the impact of dynamic institutional preferences, *Review of Financial Studies*, vol. 16 (4) pp. 1203-1238.
- H. Berkman and V.R. Eleswarapu, 1998, Short-term traders and liquidity: a test using Bombay Stock Exchange data. *Journal of Financial Economics*, vol. 47 (3) pp. 339-355.
- B. Biais and P. Bossaerts, 1998, Asset prices and trading volume in a beauty contest, *Review*

of Economic Studies, vol. 65 pp. 307-340.

L.C. Bhandari, 1988, Debt/equity ratio and expected common stock returns: Empirical evidence, *The Journal of Finance*, vol. 43 (2) pp. 507-528.

E. Boehmer and E.K. Kelley, 2009, Institutional investors and the informational efficiency of prices, *Review of Financial Studies*, vol. 22 (9) pp. 3563.

P. Bolton, J.A. Scheinkman and W. Xiong, 2006, Executive Compensation and Short-Termist Behaviour in Speculative Markets, *Review of Economic Studies*, vol. 73(3) pp. 577-610.

J. Boudoukh, M. Richardson, Y.Q. Shen, R.F. Whitelaw, 2007, Do asset prices reflect fundamentals? Freshly squeezed evidence from the OJ market, *Journal of Financial Economics*, vol. 83 (2) pp. 397-412.

Buffet, Warren E. 1999. Berkshire Hathaway Inc.' Owners Manual.

<http://www.berkshirehathaway.com/owners.html>

J.Y. Campbell, 1991, A variance decomposition for stock returns, *The Economic Journal*, vol. 101 (405) pp. 157-179.

J.Y. Campbell and J. Ammer, 1993, What moves the stock and bond markets? A variance decomposition for long-term asset returns, *Journal of finance*, vol. 46 (1) pp 3-37.

J.Y. Campbell and J.H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy*, Vol. 107(2) pp. 205-251.

J.Y. Campbell, S. Giglio and C. Polk, 2010, Hard times, nber.

J.Y. Campbell, M. Lettau, B. Malkiel, Y. Xu, 2001, Have individual stocks become more volatile? An empirical exploration of idiosyncratic risk, *Journal of Finance*, vol. 56 (1) pp. 1-43.

J.Y. Campbell and R.J. Shiller, 1988(a), The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies*, vol. 1 (3) pp. 195-228.

J.Y. Campbell and R.J. Shiller, 1988(b), Stock prices, earnings, and expected dividends, *Journal of Finance*, vol. 43 (3) pp. 661-676.

J.Y. Campbell, C. Polk, T. Vuolteenaho, 2009, Growth or glamour? Fundamentals and systematic risk in stock returns, *Review of Financial Studies*, vol. 23 (1) pp. 305-344.

J.Y. Campbell and T. Vuolteenaho, 2004, Bad beta, good beta, *American Economic Review*, vol. 94 (5) pp. 1249-1275.

G. Cespa and X. Vives, 2009, Dynamic trading and asset prices: Keynes vs. Hayek, IESE Business School Working Paper No.716.

L.K.C. Chan, N. Jegadeesh and J. Lakonishok, 1996, Momentum strategies, *The Journal of Finance*, vol. 51 (5) pp. 1681-1713.

Y. Chan and L. Kogan, 2002, Catching Up with the Joneses: Heterogeneous Preferences and the Dynamics of Asset Prices, *Journal of Political Economy*, vol. 110 pp. 1255-1285.

L. Chen, 2009, On the reversal of return and dividend growth predictability: A tale of two periods, *Journal of Financial Economics*, vol. 92 (1) pp. 128-151.

L. Chen, Z. Da and R. Priestley, 2009, Dividend smoothing and predictability, papers.ssrn.com.

L. Chen and S. Zhao, 2010, What drives stock price movement?, papers.ssrn.com.

L. Chen and X. Zhao, 2009, Return decomposition, *Review of Financial Studies*, vol. 22 (12) pp. 5213-5249.

T. Chordia and L. Shivakumar, 2006, Earnings and price momentum, *Journal of Financial Economics*, vol. 80 (3) pp. 627-656.

J.H. Cochrane, 1991, Volatility tests and efficient markets: A review essay, *Journal of Monetary Economics*, vol. 27 (3) pp. 463-485.

J.H. Cochrane, 1992, Explaining the variance of price-dividend ratios, *Review of Financial Studies*, vol. 5 (2) pp. 243-280.

J.H. Cochrane, 2008, The dog that did not bark: A defense of return predictability, *Review of Financial Studies*, vol. 21 (4) pp. 1533-1575.

R.B. Cohen, C.K. Polk and T. Vuolteenaho, 2009, The price is (almost) right, *Journal of Finance*, vol. 64 (6) pp. 2739-2782.

R.B. Cohen, P.A. Gompers and T. Vuolteenaho, 2002, Who underreacts to cash-flow news? Evidence from trading between individuals and institutions, *Journal of Financial Economics*, vol. 66 (2-3) pp. 409-462.

J. Conrad and G. Kaul, 1998, An anatomy of trading strategies. *Review of Financial Studies*, vol. 11 (3) pp. 489.

G.M. Constantinides, 1982, Intertemporal asset pricing with heterogeneous consumers and without demand aggregation, *Journal of Business*, vol. 55 pp. 253-267.

G.M. Constantinides, and D. Duffie, 1996, Asset pricing with heterogeneous consumers,

Journal of Political Economy, vol. 104 pp. 219-240.

K. Daniel, M. Grinblatt, S. Titman and R. Wermers, 1997, Measuring mutual fund performance with characteristic-based benchmarks, Journal of Finance, vol. 52 (3) pp. 1035-1058.

K. Daniel, D. Hirshleifer and A. Subrahmanyam, 2001, Overconfidence, Arbitrage, and Equilibrium Asset Pricing. The Journal of Finance, vol. 56 pp 921-965.

K. Daniel, D. Hirshleifer, A. Subrahmanyam, 1998, Investor psychology and security market under- and overreactions, Journal of Finance, vol. 53 (6) pp. 1839-1885.

B. De Bondt, F.M. Werner and R. Thaler, 1985, Does the stock market overreact?, Journal of Finance, vol. 40 (3) pp. 793-805.

B. DeLong, A. Shleifer, L. Summers and R. Waldmann, 1990a, Noise trader risk in financial markets, Journal of Political Economy, vol. 98 pp. 703-738.

B. DeLong, A. Shleifer, L. Summers and R. Waldmann, 1990b, Positive feedback investment strategies and destabilizing rational speculation, Journal of Finance, vol. 45 pp. 379-395.

P.J. Dennis and D. Strickland, 2002, Who blinks in volatile markets, individuals or institutions?, The journal of Finance, vol. 57(5) pp. 1923-1949.

D. Diamond and R. Verrecchia, 1981, Information aggregation in a noisy rational expectations economy, Journal of Financial Economics, vol. 9 pp. 221-235.

J. Dow and G. Gorton, 1994, Arbitrage chains, Journal of Finance, vol. 49 (3) pp. 819-49.

J. Dow and G. Gorton, 1997, Noise trading, delegated portfolio management, and economic welfare, Journal of Political Economy, vol. 105 (5) pp. 1024-1050.

F. Duarte and S. Parsa, 2011, Institutional investors' intrinsic trading frequency and the cross-section of stock returns, Mimeo.

E. Fama, 1990, Stock returns, expected returns, and real activity, Journal of finance, vol. 45(4) pp. 1089-1108.

E.F. Fama, 1998, Market efficiency, long-term returns, and behavioral finance, Journal of Financial Economics, vol. 49 pp. 283-306.

E.F. Fama and K.R. French, 1993, Common risk factors in the returns on stocks and bonds, Journal of finance Economics, vol. 33 (1) pp. 3-56.

E.F. Fama and K.R. French, 1993, Multifactor explanations of asset pricing anomalies, Journal of finance, vol. 51 (1) pp. 55-84.

K. Froot, D. Scharfstein, and J. Stein, 1992, Herd on the street: informational inefficiencies in a market with short-term speculation, *Journal of Finance*, vol. 47, 1461-1484.

Forsythe, G. February 27, 2007, Warren Buffett misunderstood, Schwab Investing Insights http://www.schwab.com/public/schwab/research_strategies/market_insight/investing_strategies-/stocks/warren_buffett_misunderstood.html.

D.F. Foster and S. Viswanathan, 1996, Strategic trading when agents forecast the forecasts of others, *Journal of Finance*, vol. 51 pp. 1437-78.

X. Gabaix, 2008, Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance, NBER Working Paper No. 13724.

J.M. Gaspar, M. Massa and P. Matos, 2005, Shareholder investment horizons and the market for corporate control, *Journal of Financial Economics*, vol. 76 (1) pp. 135-165.

J. Geanakoplos, 2010, Solving the present crisis and managing the leverage cycle, Cowles Foundation Discussion Paper No. 1751.

George, Bill, 2010, 'Short-Termism' won't create value, *The Wall Street Journal Europe*, January 12: p.27.

P.A. Gompers and A. Metrick, 2001, Institutional investors and equity prices, *Quarterly Journal of Economics*, vol. 116 (1) pp. 229-259.

M. Grinblatt and M. Keloharju, 2001, What Makes Investors Trade?, *The Journal of Finance*, vol. 56 (2) pp. 589-616.

M. Grinblatt and M. Keloharju, 2009, Sensation seeking, overconfidence and trading activity, *Journal of Finance*, vol. 64 pp. 549-578.

M. Grinblatt, S. Titman and R. Wermers, 1995, Momentum investment strategies, portfolio performance, and herding: A study of mutual fund behavior, *The American Economic Review*, vol. 85 (5) pp. 1088-1105.

C. Grisse, 2009, Higher Order Beliefs and the Comovement of Asset Prices, mimeo.

S. Grossman and R. Shiller, 1981, The determinants of the variability of stock market prices, *The American Economic Review*, vol. 71(2) pp.222-227.

S. Grossman, 1976, On the efficiency of competitive stock markets where traders have diverse information, *Journal of Finance*, vol. 31 pp. 573-585.

B. Han and Q. Wang, 2004, Institutional investment constraints and stock prices, *Dice*

Center for Research in Financial Economics, Working Paper

M. Harrison, and D. Kreps, 1978, Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations, *Quarterly Journal of Economics* XCII, 323-336.

R.A. Haugen and N.L. Baker, 1996, Commonality in the determinants of expected stock returns, *Journal of Financial Economics*, vol. 41 (3) pp. 401-439.

H. He and J. Wang, 1995, Differential information and dynamic behavior of stock trading volume, *Review of Financial Studies*, vol. 8 pp. 919-72.

J. Heaton and D. Lucas, 1999, Stock prices and fundamentals, *NBER Macroeconomics Annual*, vol. 14 pp. 213-242.

P. Hecht and T. Vuolteenaho, 2006, Explaining returns with cash-flow proxies, *Review of Financial Studies*, vol. 19 (1) pp. 159-194.

M. Hellwig, 1980, On the aggregation of information in competitive markets, *Journal of Economic Theory*, vol. 22 pp. 477-498.

C.W. Holden and A. Subrahmanyam, 1996, Risk aversion, liquidity, and endogenous short horizons, *Review of Financial Studies*, vol. 9 (2) pp. 691-722.

H. Hong and T. Lim, J. Stein, 2000, Bad news travels slowly: Size, analyst coverage, and the profitability of momentum strategies, *Journal of Finance*, vol. 55 (1) pp.265-295.

H. Hong and J.C. Stein, 1999, A unified theory of underreaction, momentum trading and overreaction in asset markets, *Journal of Finance*, vol. 54 pp. 2143-2184.

H. Hong and J.C. Stein, 2007, Disagreement and the stock market, *Journal of Economic Perspectives*, vol. 21 (2) pp. 109-128.

J. Huang and J. Wang, 2008, Market Liquidity, Asset Prices and Welfare, *NBER Working Papers* 14058, National Bureau of Economic Research.

N. Jegadeesh and S. Titman, 1993, Returns to buying winners and selling losers: Implications for stock market efficiency, *The journal of Finance*, vol. 48 (1) pp. 65-91.

N. Jegadeesh and S. Titman, 2001, Profitability of momentum strategies: An evaluation of alternative explanations, *The Journal of Finance*, vol. 56 (2) pp. 699-720.

N. Jegadeesh, 1990, Evidence of predictable behavior of security returns, *Journal of Finance* (1990) pp. 881-898.

L. Jin and L. Kogan, 2008, Managerial career concern and mutual fund short-termism,

mimeo.

A. Kajii and S. Morris, 1997a, The robustness of equilibria to incomplete information, *Econometrica*, vol. 65 pp. 1283-1309.

A. Kajii and S. Morris, 1997b, Common p-Belief: The General Case, *Games and Economic Behavior*, vol. 18 pp. 73-82.

J.M. Karpoff and R.A. Walkling, Short-term trading around ex-dividend days: Additional evidence, 1988, *Journal of Financial Economics*, vol. 21 (2) pp. 291-298.

B. Ke, S. Ramalingegowda and Y. Yu, 2006, The effect of investment horizon on institutional investors' incentives to acquire private information on long-term earnings, *ssrn working paper*.

J.M. Keynes, 1936, *The General Theory of Employment, Interest and Money*, Macmillan, London.

M. Khan, L. Kogan and G. Serafeim, 2010, Mutual fund trading pressure: Firm-level stock price impact and timing of SEOs, *papers.ssrn.com*

P. Kondor, 2009, The more we know, the less we agree: higher-order expectations, public announcements and rational inattention, *Working Paper*.

S.P. Kothari and J.A. Shanken, 1992, Stock return variation and expected dividends: A time-series and cross-sectional analysis, *Journal of Financial Economics*, vol. 31 (2) pp. 177-210.

M.S. Koubouros, D. Malliaropoulos and E. Panopoulou, 2005, Long-run cash-flow and discount-rate risks in the cross-section of US returns, *EconWPA*.

A. Kraus and M. Smith, 1989, Market created risk, *Journal of Finance*, vol. 44 pp. 557-569.

P. Krusell, A.A. Smith, 1998, Income and wealth heterogeneity in the macroeconomy, *Journal of Political Economy*, vol. 106 pp.867-896.

A.S. Kyle, 1985, Continuous auctions and insider trading, *Econometrica*, vol. 53 pp. 1315-36.

J. Lakonishok, A. Shleifer and R.W. Vishny, 1992, The impact of institutional trading on stock prices, *Journal of Financial Economics*, vol. 32 (1) pp. 23-43.

O.A. Lamont and C. Polk, 2002, The diversification discount: Cash flows versus returns, *Journal of Finance*, vol. 56 (5) pp. 1693-1721.

- B. Larrain and M. Yogo, 2008, Does firm value move too much to be justified by subsequent changes in cash flow, *Journal of Financial Economics*, vol. 87 pp. 200-226.
- M.A. Lasfer, 1995, Ex-day behavior: Tax or short-term trading effects, *The Journal of Finance*, vol. 50 (3) pp. 875-897.
- W. Lee, C. Jiang, D. Indro, 2002, Stock market volatility, excess returns, and the role of investor sentiment, *Journal of Banking & Finance*, vol. 26 pp. 2277-2299.
- S.F. LeRoy and R.D. Porter, 1981, The present-value relation: Tests based on implied variance bounds, *Econometrica*, vol. 49 (3) pp. 555-574.
- M. Lettau and S. Ludvigson, 2005, Expected returns and expected dividend growth, *Journal of Financial Economics*, vol. 76 (3) pp. 583-626.
- J. Lintner, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics*, vol. 47(1) pp. 13-37.
- L. Menkhoff, 2001, Short-term horizons in foreign exchange? Survey evidence from dealers and fund managers, *Kyklos*, vol. 54 (1) pp. 27-48.
- D. Monderer and D. Samet, 1989, Approximating common knowledge with common beliefs, games and economic behavior, vol. 1 pp. 170-190.
- S. Morris and H. Shin, 1997, Approximate common knowledge and co-ordination: Recent lessons from game theory, *Journal of Logic, Language and Information*, vol. 6 pp. 171-190.
- J. Mossin, 1966, Equilibrium in a capital asset market, *Econometrica*, vol. 34 (4) pp. 768-783.
- J. R. Nofsinger, R.W. Sias, 1999, Herding and feedback trading by institutional and individual investors, *Journal of Finance*, vol. 54 (6) pp. 2263-2295.
- T. Odean, 1999, Do investors trade too much?, *American Economic Review*, vol. 89 pp. 1279-1298.
- S. Parsa, 2010, Institutional investors' short-termism, trading frequency and firm-level stock price volatility, Mimeo.
- S. Parsa, 2011, Short-term traders, learning and informational externalities, mimeo MIT.
- L. Pastor, R.F. Stambaugh, 2003, Liquidity risk and expected stock returns, *Journal of Political Economy*, vol. 111 (3) pp. 642-685.
- B. Rosenberg, K. Reid. and R. Lanstein, 1985, Persuasive evidence of market inefficiency,

Journal of Portfolio Management, vol. 11 (3) pp. 9-17.

M. Rubinstein, 1974, An aggregation theorem for securities markets, Journal of Financial Economics, vol. 1 pp. 225-244.

A. Rubinstein, 1989, The electronic mail game: strategic behavior under almost common knowledge, American Economic Review, vol. 79 pp. 385-391.

R. Sadka, 2006, Momentum and post-earnings-announcement drift anomalies: The role of liquidity risk, Journal of Financial Economics, vol. 80 (2) pp. 309-349.

D. Samet, 1998, Iterated expectations and common priors, games and economic behavior, vol. 24, pp. 131-141.

J.A. Scheinkman and W. Xiong, 2003, Overconfidence and speculative bubbles, Journal of Political Economy, vol. 111 (6) pp. 1183-1219.

W.F. Sharpe, 1964, Capital asset prices: A theory of market equilibrium under conditions of risk, Journal of Finance, vol. 19(3) pp. 425-42.

R.J. Shiller, 1981, Do stock prices move too much to be justified by subsequent changes in dividends?, The American Economic Review, vol. 71 (3) pp. 421-436.

R.J. Shiller, 1985, An unbiased reexamination of stock market volatility: Discussion, The Journal of Finance, vol. 40 (3) pp. 688-689.

R.J. Shiller, 1987, The volatility of stock market prices, Science, New Series, vol. 235 (4784) pp. 33-37.

R.J. Shiller, 1989, Comovements in stock prices and comovements in dividends. The Journal of Finance, vol. 44 (3) pp. 719-729.

A. Shleifer and R.W. Vishny, 1997, The limits of arbitrage, Journal of Finance, vol. 52 (1) pp. 35-55.

R.W. Sias, 1996, Volatility and the institutional investor. Financial Analysts Journal, vol. 52 (2) pp. 13-20.

Smaghi, Lorenzo B. 2007. The growth of pension funds and financial markets: Implications for central banks.

<http://www.ecb.int/press/key/date/2006/html/sp060508.en.html>

J. Stiglitz, 1989, Using tax policy to curb speculative short-term trading, Journal of Financial Services Research, vol. 3 pp. 101-115.

- J. Tirole, 1982, On the possibility of speculation under rational expectations, *Econometrica*, vol.50 pp. 1163-1181.
- R. Townsend, 1978, Market anticipations, rational expectations and bayesian analysis, *International Economic Review*, vol. 19 pp. 481-94.
- R. Townsend, R., 1983, Forecasting the forecasts of others, *Journal of Political Economy*, vol. 91 pp. 546-588.
- J.L. Treynor, 1961, Market value, time, and risk, Unpublished manuscript
- P. Veronesi and L. Pastor, 2003, Stock valuation and learning about profitability, *Journal of Finance*, vol. 58 (5) pp. 1749-1790.
- R. Verrechia, 1982, Information acquisition in a noisy rational expectation economy, *Econometrica* vol. 50
- X. Vives, 1995, Short-term investment and the informational efficiency of the market, *Review of Financial Studies*, vol. 8 (1) pp. 125-160.
- T. Vuolteenaho, 2002, What drives firm-level stock returns?, *The Journal of Finance*, vol. 57 (1) pp. 233-264.
- J. Wang, 1993, A model of intertemporal asset prices under asymmetric information, *Review of Economic Studies*, vol. 60 (2) pp. 249-282.
- R. Wermers, 1999, Mutual fund herding and the impact on stock prices, *Journal of Finance*, vol. 54(2) pp. 581-622.
- R. Wermers, 2000, Mutual fund performance: an empirical decomposition into stock-picking talent, style, transaction costs, and expenses, *Journal of Finance*, vol. 55 (4) pp. 1655-1695.
- X.S. Yan and Z. Zhang, 2009, Institutional investors and equity returns: Are short-term institutions better informed?, *Review of Financial Studies*, vol. 22 (2) pp. 893-924.
- M. Yogo, 2006, A consumption-based explanation of expected stock returns, *The Journal of Finance*, vol. 61 (2) pp. 539-580.
- C. Zhou, 1998, Dynamic portfolio choice and asset pricing with differential information, *Journal of Economic Dynamics and Control*, vol. 22(7) pp. 1027-1051.