Design and Performance of Thomas Telford's Bonar Bridge and Mythe Bridge

by

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BEng, McGill University (2015)

Submitted to the Department of Civil and Environmental Engineering in partial fulfillment of the requirements for the degree of

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Abstract

This paper assesses two cast-iron arch bridges of Thomas Telford (1757-1834) – Bonar Bridge (1810-12) and Mythe Bridge (1824-26) – to draw a broader conclusion about his career in bridge building. The bridges are introduced and Telford's design influences are investigated. While Telford was influenced by theory through the advice of his contemporaries, he was more heavily influenced by experience, especially design precedents, and most of all by his own judgment, which placed great emphasis on both practicality and aesthetics.

The structural performance of the two bridges is assessed and compared. The cast-iron arch's ability to resist vertical loading is the main focus of the analysis, following Heyman's framework for limit analysis of arches. Global equilibrium and graphic statics indicate that the each rib, when acting alone, is insufficient to support asymmetric loading, demonstrating that the secondary members are necessary, and therefore that neither bridge is grossly overdesigned. Deck-stiffening effects are tested following Billington's method, and are found to be negligible. The spandrel bracing members are found to be sufficient apart from the development of tension forces in Bonar Bridge. The later Mythe Bridge performs slightly better in all areas; overall, however, the performance is very similar.

Based on these results, the paper concludes that Telford chose not to refine his design substantially over the course of his career. It is argued that this was a conscious decision, based on the progression of the industry from cast iron towards wrought iron, and that these bridges are significant because they bookend the short golden age of cast iron bridges, of which Telford was the unquestionable master.

Thesis Supervisor: John A. Ochsendorf

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1. Introduction

The early Industrial Revolution works of Thomas Telford (1757 – 1834) continue to define the infrastructure of Britain, even up to the present day. Telford's highways and canals spiderweb across Scotland, England, and Wales, forging connections in a completely unprecedented manner, and literally paving the way for the large-scale development that the country would see throughout the nineteenth century. There is much to be said about his contributions to the design of roads, bridges, canals, tunnels, harbors, and indeed to the profession itself. A few of his most famous works – the ground-breaking suspension bridge over the Menai Straits, the vast and breathtaking Pontcysyllte Aqueduct – are very well-documented. However, his cast-iron bridges are almost entirely overlooked by scholars. These bridges, though smaller in scale than many of his better-known works, represent a real achievement in the early use of cast iron, and had profound implications for the future of bridge design.

Cast iron dominated bridge design for a very short but crucial period in the early nineteenth century. The new material allowed designers to profoundly reconsider the conventions and limitations that had come to be accepted over hundreds of years of masonry bridge construction. This was the era that saw bridge design move from the field of architecture to engineering, challenging physical limits of span and form, when modern bridge design can really be seen appearing over the horizon. Telford was a master of the cast-iron bridge, and over the course of his career he created some of the most beautiful and long-standing cast-iron bridges ever built, many of which still survive today.

This thesis introduces the reader to two of Telford's most significant cast-iron bridges, Bonar Bridge (1810-12) and Mythe Bridge (1824-26). Bonar Bridge was the first in Telford's iconic series of long-span cast-iron bridges, and he chose to reuse its design in many subsequent works, including Craigellachie Bridge (1812-14), which is the oldest of the series still standing. Mythe Bridge, on the other hand, fell at the end of his career, and, although clearly a descendent of Bonar, incorporated some substantial design changes, whose impact has not yet been fully understood. This thesis investigates the design influences behind each bridge, comprising a combination of theory, experience based on past precedent, and, most importantly, Telford's own judgment. Based on the author's visit to the extant Mythe Bridge and Craigellachie Bridge, as well as a careful study of primary and secondary sources, new conclusions can be drawn on the historical importance of the Bonar and Mythe Bridges.

Structural analysis for the various elements of each bridge is used to unpack the behavior and performance of each. The implications of the arched form are considered, as well as the development of stresses in the ribs and spandrel bracing and the load-bearing contributions of the deck. Traditional analysis methods are used, including global equilibrium and graphic statics, taking a lower-bound approach to confirm the bridges' acceptability. By comparing the structural design and performance of the two bridges, it is hoped that this analysis will contribute to the understanding of major developments in cast-iron bridge design over the course of Telford's career. A finding that Mythe Bridge represents a dramatic improvement over Bonar Bridge could indicate that Telford had effectively improved his mastery over the material in the intervening decade and a half. On the other hand, a finding that the two bridges behave very similarly could have implications for Telford's relevance as a leader in the field or for the progression of his career as a whole. In either case, the comparison between the two bridges provides an opportunity to draw a broader conclusion about the role of Telford's cast-iron bridges in the development of bridge design.

2. Background

2.1. Telford's Career

Telford was born in Eskdale, Scotland, in 1757. His formal education ended at the age of fourteen, when he took a job as a mason. This provided him with a practical background that would shape his professional outlook for the rest of his career. He would later say in his autobiography, "I ever congratulate myself upon the circumstances which compelled me to begin by working with my own hands" (Telford 1838). He moved to Edinburgh, where he was deeply impressed by the beauty of the architecture – another early and profound influence – and thence to London in 1782. His career was fully launched in 1787 when he became County Surveyor in Shropshire, in the west of England. It was in this role that his work on cast-iron bridges first began, with Buildwas Bridge in 1795. Europe's first iron arch bridge had been built in 1777-79 by Abraham Darby III at nearby Coalbrookdale. The bridge (Figure 2.1), which features several slender semi-circular iron ribs, with spandrel bracing in the form of smaller circles, was strongly influenced by masonry precedents, and clearly demonstrates the limited state of understanding of the new material. It opened an entirely new world of design, however, and provided inspiration for Telford's first, and Europe's second, cast-iron bridge, at Buildwas (Figure 2.2).

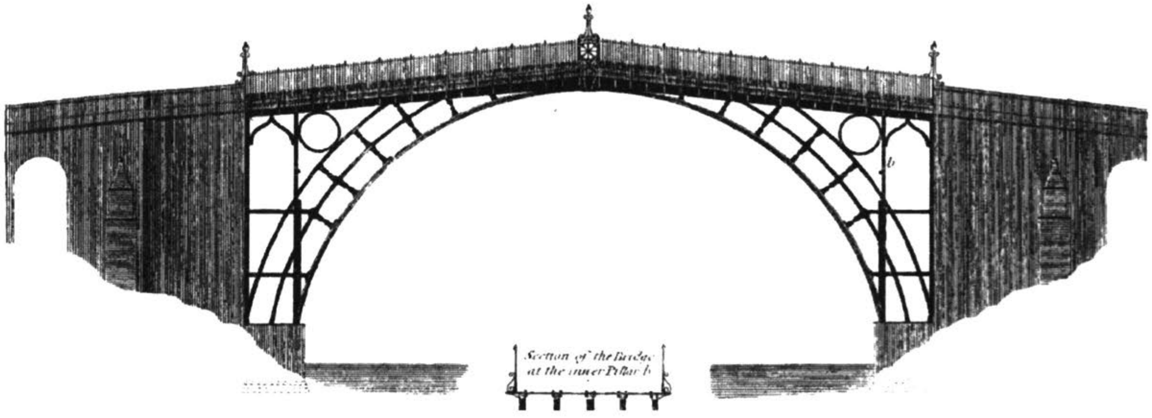

Figure 2.1: The Iron Bridge at Coalbrookdale (1779) (Telford, Bridge 1812)

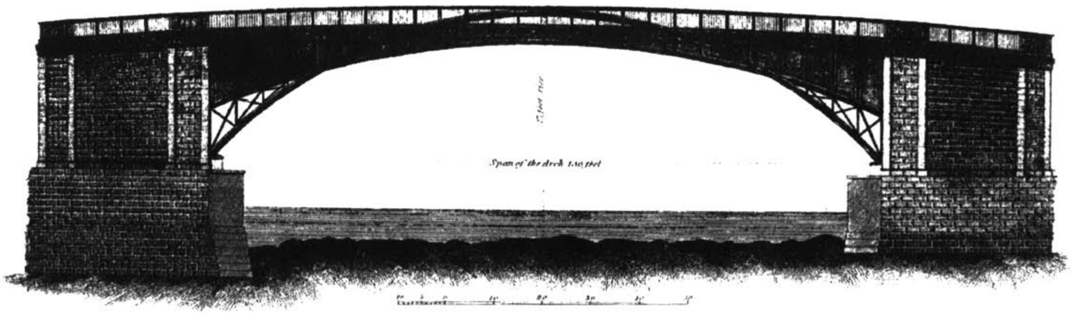

Figure 2.2: Buildwas Bridge (1795) (Telford, Bridge 1812)

Telford's career continued to grow as he designed roads, bridges, canals, tunnels, and harbors, but he did little work with cast iron for the next decade. In 1801, he submitted an astounding proposal for a 600-foot span cast-iron arch bridge to replace the existing London Bridge (Figure 2.3), but after much profitable discussion, it was decided that the abutments required for such a bridge would be prohibitive, and the project was never developed.

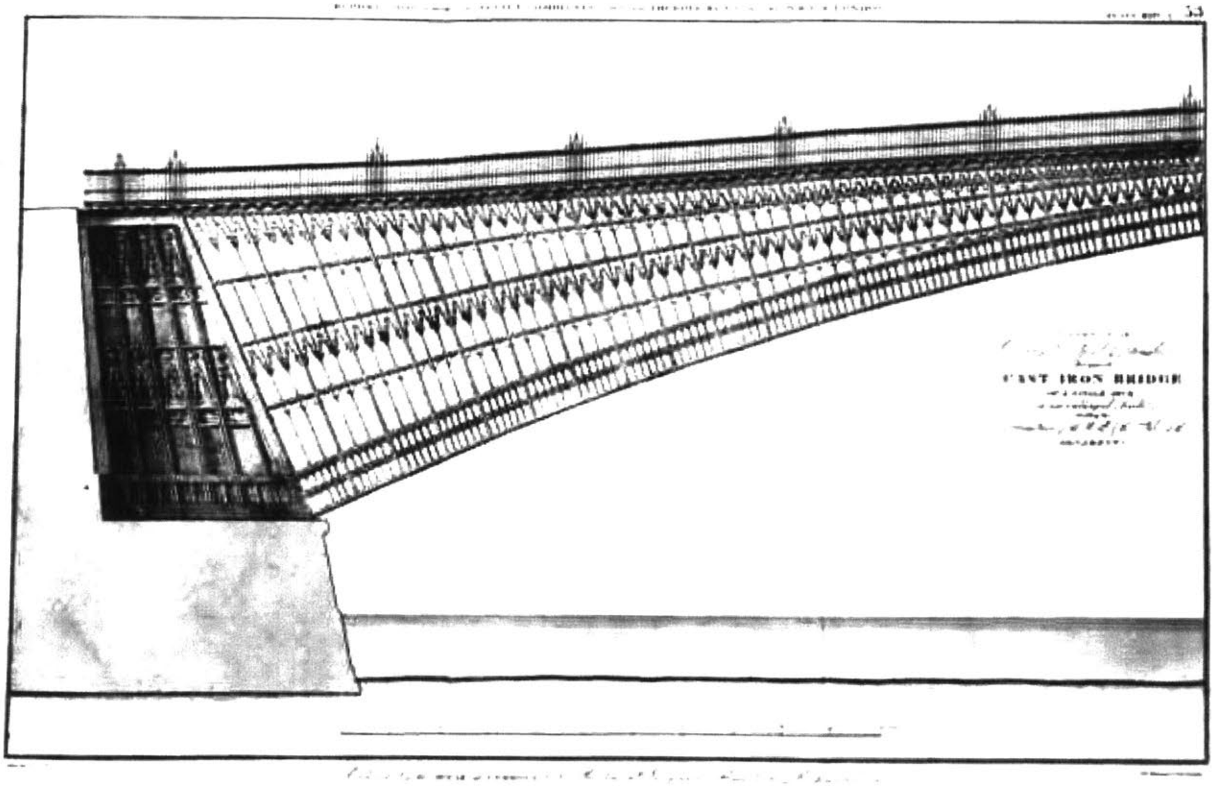

Figure 2.3: Proposed London Bridge design (Lowry 1801)

Throughout the next decade, Telford continued to work with an almost unbelievable prolificacy, establishing himself as the foremost surveyor and engineer in the country. Probably his most notable work of this period was the breathtaking Pontcysyllte Aqueduct (1805), a cast-iron aqueduct in northeast Wales, spanning 1007 ft and carrying the Ellesmere Canal over the River Dee, 70 ft below. However, he did not again experiment with cast-iron bridges until 1810, in the course of his work improving the Scottish Highlands. Bonar Bridge was built across the Kyle of Sutherland in 1810-12, and featured an entirely revolutionary design. Arched ribs span 150 feet, supporting a deck connected to the ribs by "lozenge lattice" spandrel bracing of an airy slenderness, all of cast iron. Telford, always a practical designer, reused this design – even the castings – many times during the rest of his career, including for Craigellachie Bridge (1814), in Speyside, which still stands as the oldest remaining cast-iron bridge in Britain.

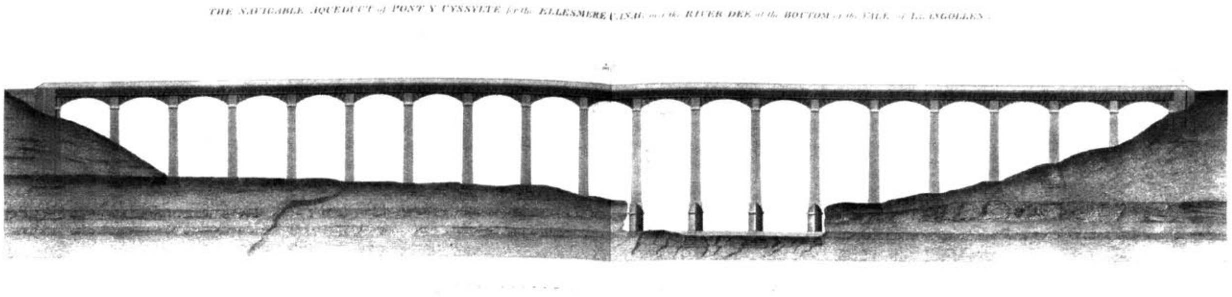

Figure 2.4: Original plan for Pontcysyllte Aqueduct (1805) (Telford 1838)

Although Bonar was a defining and significant work in the history of bridge design, this was not an era of specialization, and Telford continued to devote his considerable energy to roads, canals, harbors, and aqueducts, as well as bridges. His most famous bridge, the Menai Strait Suspension Bridge (Figure 2.5), was built 1818-26 as part of a massive project to form a better mail link between London and Northern Ireland. One of Europe's first suspension bridge when construction began (although the Union Suspension Bridge was built 1819-1820, and was therefore the first to be completed), it stretches 577 ft, supported by wrought iron chains strung between two masonry towers (Paxton 1975). This bridge, which still stands, is the direct ancestor of those of Brunel and the Roeblings, and it places Telford unquestionably in the company of the greatest bridge designers of all time.

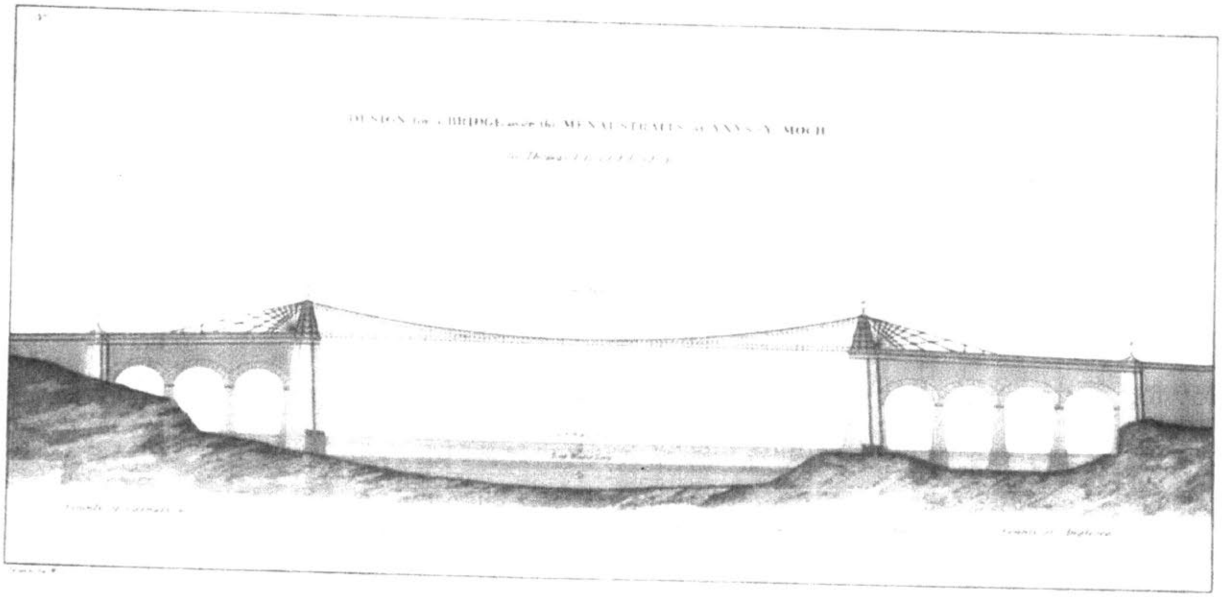

Figure 2.5: Original plan for Menai Strait Bridge *(Papers Relating to the Building of a Bridge over the Menai Strait 1819)*

The innovation of the Menai Suspension Bridge cannot be overstated. It was among the first to truly abandon the compression-based arch form, upon which bridge-builders had relied for millennia, back to the Romans and before. Instead, it made use of the tensile capacity of wrought iron, supporting the bridge in reverse, by hanging it upon towers, rather than resting it upon piers. This was to open a path that would eventually lead to the Golden Gate Bridge and beyond.

Telford continued to work on projects large and small. The same year the Menai Strait Suspension Bridge opened, a 170-foot bridge carrying Mythe Road over the Severn was completed near Tewkesbury, in Gloucestershire. Mythe Bridge is of the same general style of cast-iron arch bridge as Bonar, but with a larger span and lower rise. It still stands as Telford's longest-span cast-iron bridge, and although by and large it has been overlooked in the literature, it represents the definite pinnacle of Telford's career in cast-iron bridge building. It has the longest span he ever attempted in cast iron, and the grace and efficiency of the design make it vastly more attractive and modern-looking than the Bonar series.

Mythe was also constructed near the end of Telford's career. As he grew older, he continued to work on various projects, notably the St. Katharine's Docks in London (1824-28), a massive harbor project to increase London's dock space. He also devoted much energy to the furthering of the civil engineering profession. In 1818, he helped to found the Institution of Civil

Engineers and became its first President, a position he held until his death. He died in 1834, at the age of 77, venerable and well-respected.

Telford's legacy can clearly be seen by his many works across Britain, an impressive number of which are still standing. More importantly, though, he paved the way towards the explosion of engineering of the Industrial Revolution. In Telford's later years, new giants like I.K. Brunel rose to prominence in Britain. Telford resisted the move towards railways advocated by Brunel and others, insisting that canals were more practical, and believing the future to lie in individual steam-powered vehicles, which could run on any road (Glover 2017, 325). This was not to be truly realized for another century, during which the landscape of Britain and whole Western world would be changed by railways and the explosion of industry they allowed. Nevertheless, his works had played a crucial role in the first act of this drama. Telford has been called the first true civil engineer. For three decades, he dominated the industry, overseeing development reaching to the most remote corners of the island. His vision of connecting Britain with well-designed, closely-knit transportation would be developed over the remainder of the 19th century, and would become the foundation upon which Britain would base its success as the greatest Industrial Revolution power.

Telford's cast-iron bridges play a crucial role in this development. At the beginning of his career, bridge design was left mainly to architects. Bridges were sturdy masonry creations, constructed using stone-working techniques and rules of proportion that stretched back to the Romans or earlier. Telford was one of the first of the great designers to recognize that cast iron provided a new and powerful opportunity. He worked to optimize bridge design, enabling him to solve problems that had previously been impossible – spanning new lengths, avoiding fast-moving water and poor soil, connecting remote areas of the country by the most direct routes. Telford was a true civil engineer, marrying his beginnings as a mason with his strong personal aesthetic for results that were both practical and elegant, then went beyond both these things to engineer new solutions. He recognized that cast iron could be used as a new material, to be tested and applied, not merely as a substitute for masonry, but in new ways. The drive towards development and optimization that began with Telford and his contemporaries led directly from cast iron, to wrought iron, to steel design. This was truly the birth of the modern era of bridge design. In less than a century, designers left behind the traditional methods that had dominated bridge design for thousands of years, to embrace entirely new possibilities. The innovations of this period are therefore of immense importance in the history of civil engineering.

Bonar Bridge, by far Telford's best-know cast-iron bridge, was stunning and innovative, but it was still early in his cast-iron bridge career. Mythe Bridge, on the other hand, comes much closer to its end. It is a quieter, less spectacular work, surprisingly modern in approach. By comparing the design and performance of these two bridges, it is possible to trace, not only the development of Telford's ideas and abilities over the course of his career, but also the role of cast-iron bridges in the march towards modernity.

2.2. Literature Review

There is an abundance of primary source material surrounding Telford's work, but much of it deals only cursorily with his cast-iron bridges. His own writings include his autobiography *Life of Thomas Telford, Civil Engineer*, published posthumously in 1838 and edited by John Rickman. The work is incomplete, as he died before finishing it. Mythe is the only cast-iron bridge which he describes in detail, saying, "I deem it to be a good specimen of a large cast-iron arch" (256). The work gives few hints regarding his design process, but does provide excellent insight into his design philosophies. His article "Bridge" in the *Edinburgh Encyclopedia* (1812), meanwhile, gives a history of bridge-building up to and including his own bridges and those of his contemporaries. The article also includes an essay by Telford's protégé, Alexander Nimmo, on the state of structural theory as applied to bridge-building – a valuable indication of general knowledge at the time. Also extant are many reports to the House of Commons, providing designs, structural drawings, cost estimates, and more. Furthermore, the Institution of Civil Engineers has many original letters and notes, particularly surrounding the London Bridge proposal. These primary sources together paint a compelling picture of Telford's character, design philosophy and influences, and the construction of his bridges, but leave substantial gaps regarding his design process and some structural details.

Perhaps it is for this reason that most of the literature concerning Telford takes a broad-brushed approach to his work. He was undoubtedly an important historical figure in the development of British infrastructure and the civil engineering profession, and it is in these contexts that most sources provide an overview of his work. L.T.C. Rolt's biography *Thomas*

Telford (1958) outlines the life and accomplishments of the great man in a historical context. David Billington's *The Tower and the Bridge* (1983) considers his contribution to the profession as one of the first to take on bridge design as a civil engineering challenge, bringing a strong artistic sensibility, but remaining truly a civil engineer, his aesthetic rooted in structural form. Billington regards his career as landmark in shifting bridge design from the realm of architecture to civil engineering, but does no detailed structural analysis of the cast-iron bridges. Ted Ruddock writes extensively on Telford in *Arch Bridges and Their Builders* (1979), skillfully situating Telford's work in the context of his contemporaries. Ruddock does an excellent job of explaining the factors influencing bridge design of the era, but does not extensively investigate the structural behavior of Telford's bridges. More recently, Sangree et al (2015) apply modern topology optimization methods to compare Craigellachie and Mythe Bridges, as a demonstration of the ability of topology optimization tools to facilitate an understanding of form. They argue that Mythe represents a substantial improvement over Craigellachie, based on a visualization of the "optimal" form of the bridge, which resembles Mythe more closely, but without reference to the actual geometry or performance of the two bridges (Carstensen 2017).

The definitive work regarding Telford's cast-iron bridges is Roland Paxton's master's thesis at Heriot-Watt University, Edinburgh: *The Influence of Thomas Telford on the Use of Improved Construction Materials* (1975). It provides a detailed and comprehensive overview of Telford's bridges, including overall dimensions and rough load calculations for each. These calculations only consider the performance of a single rib under uniform loading, and do not assess the deck or spandrel behavior or asymmetric loading. The load calculations provide valuable benchmarks, and the gathering of information is unparalleled in its diligence, but a more advanced assessment is needed to better understand the structural behavior of these bridges.

Generally, the pool of literature surrounding Telford is broad, but shallow when it comes to structural analysis. There exists an immense amount of primary source material, documenting his many works and much of their design and construction. From a historical angle, much has been written about the man himself, the significance of his works, and his impact on the profession. Structural analysis, however, has typically been focused on his more famous works, particularly the Menai Straits Bridge. His cast-iron bridges are treated with intelligence and care by Ruddock and Paxton, who emphasize their significance, but they have not been assessed in depth from a structural standpoint. Almost nothing has been written about Telford since this explosion of work in the late 1970's, and it is time to rediscover his works. This paper provides a structural analysis of two of Telford's unsung masterpieces, Bonar Bridge (1812) and Mythe Bridge (1826). By affording a more thorough understanding of the design and performance of Telford's cast-iron bridges, it attempts to provide fresh insight into the significance of these bridges and the development of Telford's cast-iron bridges over time.

3. Case Studies

The choice of these two bridges is based on their iconic design, as well as the different places they hold in Telford's career. Bonar Bridge was the first in a style that would come to be seen as recognizably very Telford, and he literally copied it for many later bridges, even reusing the same castings. It was near the beginning of his cast-iron bridge-building career, and represented a spectacular innovation compared to his earlier bridges. Mythe Bridge, on the other hand, was near the end of Telford's career. It is his longest-span cast-iron bridge, and incorporates various changes relative to Bonar. It is of interest, therefore, to examine more minutely the differences between these bridges, as they are representative of the course of Telford's career, and, more broadly, the course of cast-iron bridge design.

3.1. Bonar Bridge

Bonar Bridge was designed and built 1810-12. Although well along in Telford's career, this was only his second long-span cast-iron bridge, but it is certainly his most iconic. It was built across the Kyle of Sutherland in the far north of Scotland, and stood until a flood in 1892 destroyed one of the abutments, causing it to collapse (Paxton, Thomas Telford's Cast-Iron Bridges 2007). The bridge spanned 260 ft, and featured a cast-iron arch, with two smaller masonry arches, of 60 and 50 ft respectively, on one side (Sixth Report of the Commissioners for Highland Roads and Bridges 1813). The cast iron arch, which is the focus of this discussion, spanned 150 ft, with a rise of 20 ft and a width of 15 ft, and its design would become the blueprint for all Telford's later cast-iron bridge designs. The No. 2 cast iron was provided by well-regarded Shropshire ironmaster William Hazeldine (Paxton 1975, 251). The bridge cost £9736 at the time (Paxton 2007), or approximately $850,000 in 2017 USD (O'Donoghue and Goulding 2004), and the ironwork weighed 180 British tons (410 kips) (Telford 1838, 685).

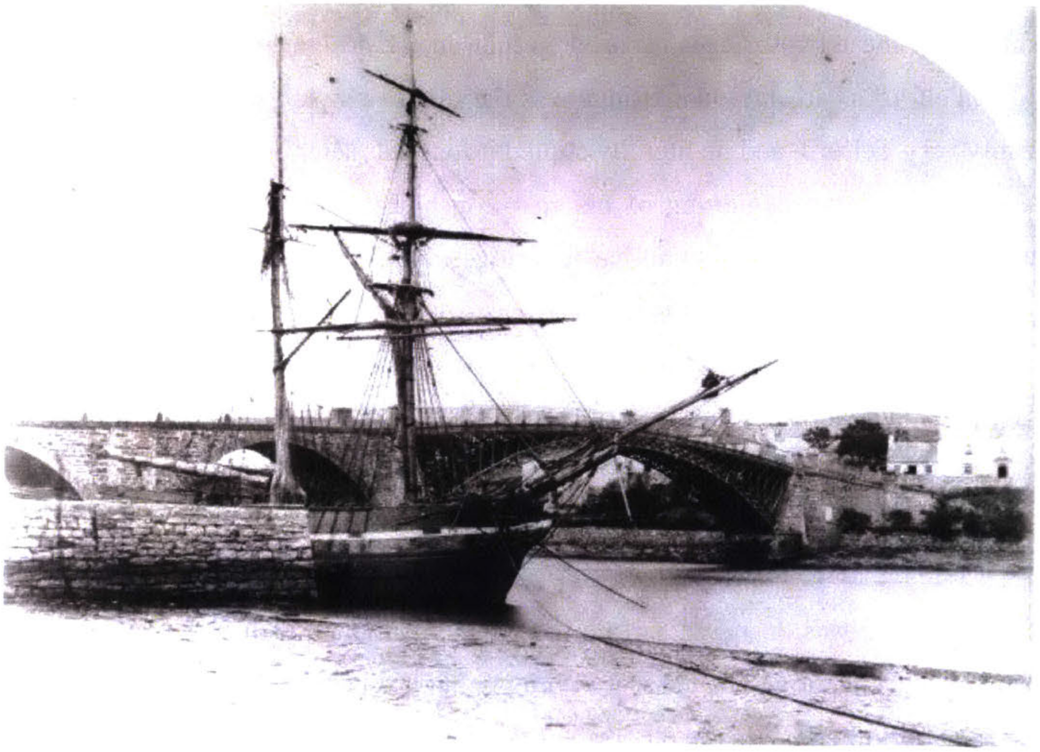

Figure 3.1: Photograph of Bonar Bridge (Old Bonar Bridge n.d.)

3.1.1. Design of Bonar Bridge

As shown in Figure 3.2 below, the main gravity-resisting structural elements of the bridge were as follows:

1. Four arched ribs, of radius 150 feet. Each rib is made up of five individually-cast sections, and each section is perforated to include seven "x" segments. The perforations are of such size that they leave solid sections 6 inches wide in every part of the x.
2. Telford's iconic "lozenge-lattice" spandrel bracing, made up of very thin, cruciform-section braces, having a cross-section of 4 in^2 (Paxton 1975, 251). The lattices are radially oriented, and frame into the rib at the edges of each "x".
3. An arched deck, of radius 740 ft. The deck is made up of cast-iron plates, of thickness 0.75 in, which are bolted together through upturned flanges. The plates rest on deck bearers,

which are 6 in deep and aligned with the ribs, and connected to them via the spandrel bracing.

4. Abutments, radially oriented, such that the ribs spring from cast-iron plates on the abutments.

Figure 3.2: Telford's sketch of Bonar Bridge *(Telford, Life of Thomas Telford, Civil Engineer 1838)*, with labels added by the author

These elements are shared with those of Craigellachie Bridge (1814), which still stands in Speyside, Scotland. The photographs of Craigellachie, with labels consistent with those above, help to better illustrate the design.

Figure 3.3: Vertical components of Craigellachie Bridge, perspective view (Lane 2017)

Figure 3.4: Vertical components of Craigellachie Bridge, closeup view (Lane 2017)

In addition, the bridge incorporates several lateral elements, as seen in Figure 3.5 and Figure 3.6:

5. Connecting plates run laterally between each rib section, connecting each to its neighbor and tying all four ribs together.
6. Covering plates lie across the tops of all the ribs.
7. Each rib is laterally connected at two points through the lower chord of the rib by wrought-iron ties encapsulated in cast-iron tubes to prevent corrosion.
8. Four sets of diagonal bracing tie the deck to the ribs.
9. The spandrels are laterally connected at their midpoints through hollow cast-iron tubes, encapsulating wrought-iron ties.

Figure 3.5: Lateral components of Craigellachie Bridge, perspective view (Lane 2017)

Figure 3.6: Lateral components of Craigellachie Bridge, perspective view closeup (Lane 2017)

Telford was an immensely thorough designer, and his designs also included non-structural elements such as railings, mile-markers, and toll houses. The non-structural elements of Bonar Bridge included:

1. Roadway of crushed stone of width 15 feet, to a depth of 13 inches
2. Cast-iron railings

Telford was evidently pleased with Bonar Bridge, because he used the same design for many later bridges, include Craigellachie Bridge (1814), Betws-y-Coed Bridge (1815), Eaton Hall Bridge (1824), and Holt Fleet Bridge (1828). The design is detailed in the Sixth Report to the Commission for Highland Roads and Bridges, and accompanied by a sketch (Figure 3.7), which is also reproduced in the Atlas to Telford's autobiography (Telford 1838). Interestingly, a different sketch, labelled "Bonar Bridge," (Figure 3.8) appears in the Edinburgh Encyclopedia, but it is not actually the design for Bonar Bridge (Paxton 2017). It features 45 "x"s in the ribs, rather than the actual 35, a secondary brace running along the bridge at the midspan of the spandrel bracing, and does not include the secondary masonry arch included in the true bridge. Given that the article was written in 1812, the same year that Bonar Bridge was completed, this sketch likely represents an earlier idea for the bridge's construction, but has caused some confusion in later sources.

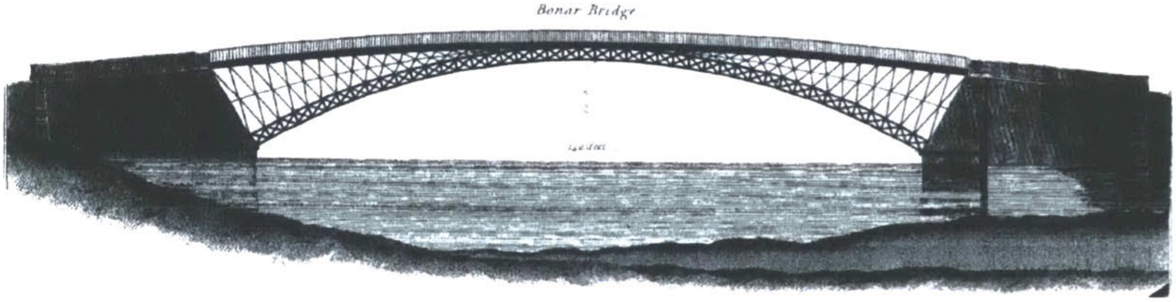

Figure 3.7: Sketch labelled "Bonar Bridge" in the Edinburgh Encyclopedia, differing from the true bridge in various ways (Telford 1812)

Bonar is referenced in nearly every biography of Telford, typically the only one of his cast-iron bridges to earn a mention, as it is his most iconic and recurring design. Structurally, however, Bonar Bridge has been given little treatment in the literature. Paxton performs a brief structural analysis of the deck, and calculates a maximum allowable wheel load of 1.17 kips to rest on one roadway bearer, neglecting any arching action and considering the bearer to be a simply supported

5-foot span, since the bearers are constructed in 5-foot segments. This assumption is oversimplified. He also performs a check using graphic statics, and finds the thrust line to lie within the middle third of the arch (Paxton 1975). The question of asymmetric loading, and the interaction of the various structural components of the bridge, have not been fully investigated for Bonar Bridge. However, reports on the condition of Craigellachie Bridge, by W.W. Lowson (1967), and Eaton Hall Bridge, by John Laing and Son Ltd. (1964), shed light on the behavior of Bonar Bridge, since they are so closely related. These reports provide substantial information regarding the condition of these bridges, including member stresses and material properties, recommendations for repairs, and repairs actually undertaken. There is little detailed structural analysis regarding the behavior of the bridge, however.

3.1.2. Performance of Bonar Bridge and Craigellachie Bridge

Bonar Bridge was destroyed in 1892 when floodwaters swept away its abutments (Hume 1980). Up until that point, however, it was found to be performing adequately, as referenced by the Sixth, Seventh, Eighth, and Ninth Reports of the Commissioners for Highland Roads and Bridges. There is no indication that the bridge suffered any performance issues during its lifetime, until the failure of its abutments. Indeed, in the winter of 1813-14, an ice floe carrying several logs crashed into the bridge, but inflicted no damage (Seventh Report of the Commissioners for Highland Roads and Bridges 1815). In 1826, a boat hit the bridge, and its mast snapped off, leaving the bridge's ironwork intact (Hume 1980). These are both indications of the bridge's robustness.

The history of Craigellachie Bridge (1814) provides further insight into the merits of the Bonar Bridge design, with the benefit of a longer lifetime and more modern analysis. The bridge was assessed by local authorities in 1902, and was found to be in excellent shape, apart from the road plates, which were in need of reinforcement. The ribs showed no defects of any kind. The commissioners specifically noted that, although the spandrel bracing seemed unreasonably slender, it was still in excellent condition, and they could see no reason to doubt its performance. They further noted that the bridge's continued existence in itself constituted "the best possible guarantee for sufficiency" (Lowson 1967, 24). The road plates were cleaned and reinforced with steel sections placed at 9 in intervals, which were then cast in concrete to prevent movement. A roadway of crushed stone, 8 in deep, was then laid, with the intention of preserving the original weight of the structure. After this reconstruction, the bridge stood without further major repairs for another sixty years (Lowson 1967).

In 1963-64, W. A. Fairhurst and Partners conducted a massive reconstruction of Craigellachie Bridge. The bridge had undergone serious loading during World War II, with large numbers of military vehicles passing over it. The road plates had buckled and cracked, and significant corrosion was present. The spandrel bracing was found to be overstressed and, in several cases near the crown of the bridge, braces had actually sheared off, and been repaired by the local blacksmith. In other braces, the morticed joints had worked loose, so that members were no longer continuous (Lowson 1967). These effects are discussed in more detail in Section 5.4.

Substantial testing was carried out and major reconstruction took place. The diagonal bracing was completely replaced. Steel sections were welded together to recreate the cruciform shape of the braces, and four sections were welded at the center of the "x", rather than the original morticed joint (Figure 3.8). All but two of the original deck plates were retained, and a concrete deck was poured. The original handrails were also replaced, as they were in very poor condition (Lowson 1967). Overall, the reconstruction is impressively faithful to the original bridge and is almost indistinguishable from the original design.

Figure 3.8: Closeup view of Craigellachie Bridge lattice, showing new steel sections and connections

Craigellachie was designated a Category A Listed Structure in 1972 (Craigellachie, Old Bridge over River Spey (Telford Bridge) 2017). It was designated as pedestrian only, and a road bridge was built alongside it. Today, it is again in need of renovation, but lack of funds is proving an obstacle (Paxton 2017).

In spite of this, visiting Craigellachie Bridge is as spectacular as ever. The bridge is set against a sheer cliff, with a dramatic rock face rising above its north abutment. It stretches across the beautiful Spey River, beloved by whisky enthusiasts and salmon fishermen, and is accessible by a charming woodland path. Rounding a curve to see it in the distance across a meadow of wildflowers, the modern viewer cannot help but agree with the poet Robert Southey, who wrote:

> "As I went along the road by the side of the water, I could see no bridge. At last I came in sight of something like a spider's web in the air. If this be it, thought I, it will never do! But, presently, I came upon it; and oh! it is the finest thing that ever was made by God or man!" (Smiles 1867, 296).

The first impression is certainly of an impossible sparseness. The elements line up almost perfectly, so that only the outer rib and braces are at first visible, those behind them fading into shadow, so that the bridge appears almost to hang as a two-dimensional image in mid-air. From an oblique angle, however, and especially when viewed from below, the bridge's substantial ironwork forms a jumbled forest. The reconstruction is barely noticeable, even up close. There can be no doubt that this bridge, now the oldest surviving member of its family, deserves to be preserved for generations to come.

3.2. Mythe Bridge

Mythe was one of Telford's last bridges. It was designed and constructed 1823-26 (Mackenzie 1838), across the Severn west of Tewkesbury, in Gloucestershire, where it still stands. It cost a vastly larger £14500 (Telford 1838, 258), or approximately $1,500,000 in 2017 USD (O'Donoghue and Goulding 2004). Like Bonar, Mythe's No. 2 cast iron was cast by Shropshire ironmaster William Hazeldine (Mackenzie 1838).

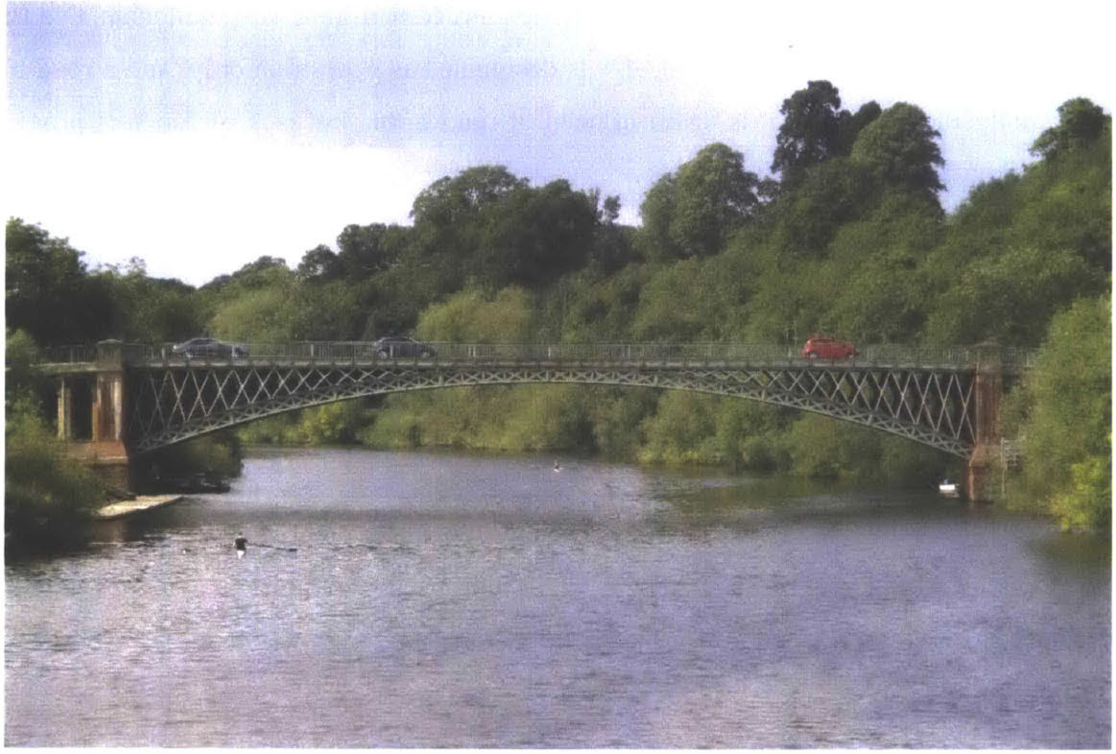

Figure 3.9: Mythe Bridge, full elevation (Lane 2017)

3.2.1. Design of Mythe Bridge

The bridge has a span of 170 ft and a rise of 17 ft, and the main structural components are much as those of Bonar Bridge, as seen in Figure 3.10 and 3.11 below.

1. Six arched ribs, of radius 221 ft, each composed of eight sections.
2. Lozenge-lattice bracing, vertically-oriented, and with a cruciform cross-section of approximately 4 in^2, diminishing to 3.5 in^2 at the ends (Mackenzie 1838). Each pair of braces is joined at midspan by a mortice and tendon joint (Figure 3.12), and also by 1.5 in wrought-iron bolts, as described in (9) below (Mackenzie 1838, 13).
3. Arched deck plate, of radius 17,350 ft, resting on six deck bearers, which are positioned directly over the ribs and connected to them via the spandrel bracing.
4. Abutments, radially-oriented so that the rib springs orthogonally from cast-iron plates, but then vertically oriented above the level of the rib, matching the vertically-oriented spandrel bracing.

5. Six small pointed masonry arches on either side of the abutments, allowing flood water to pass through (Figure 3.14) (Mackenzie 1838, 4).

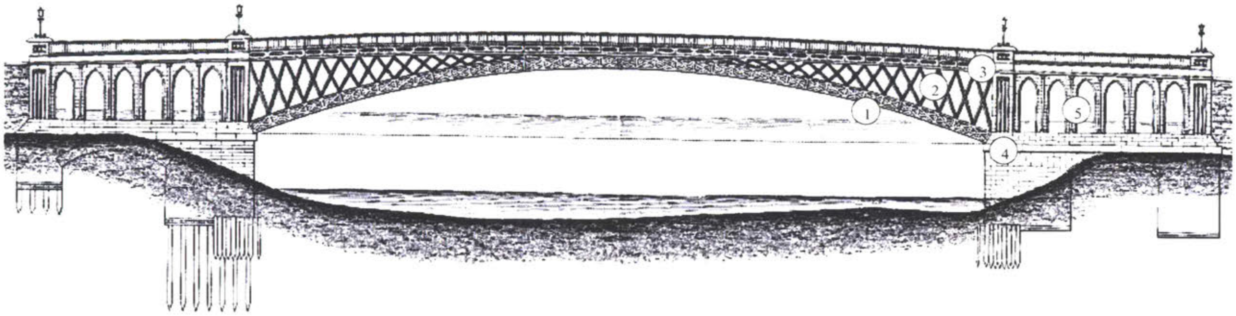

Figure 3.10: Telford's sketch of Mythe Bridge *(Mackenzie 1838)*, with labels added by the author

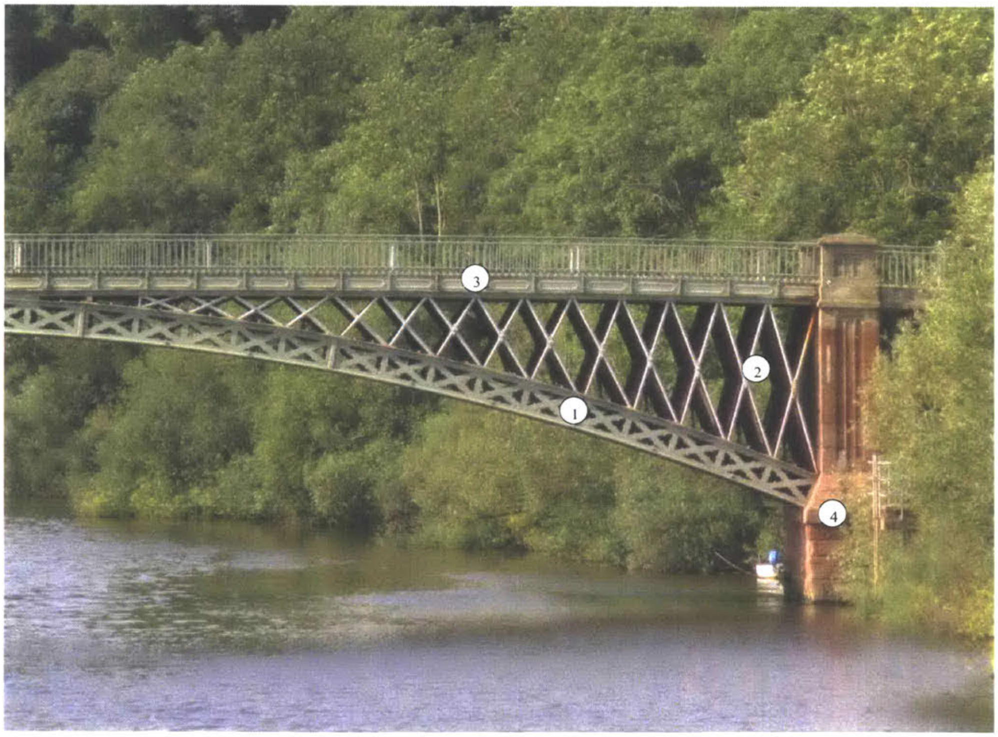

Figure 3.11: Mythe Bridge – half-elevation (Lane 2017)

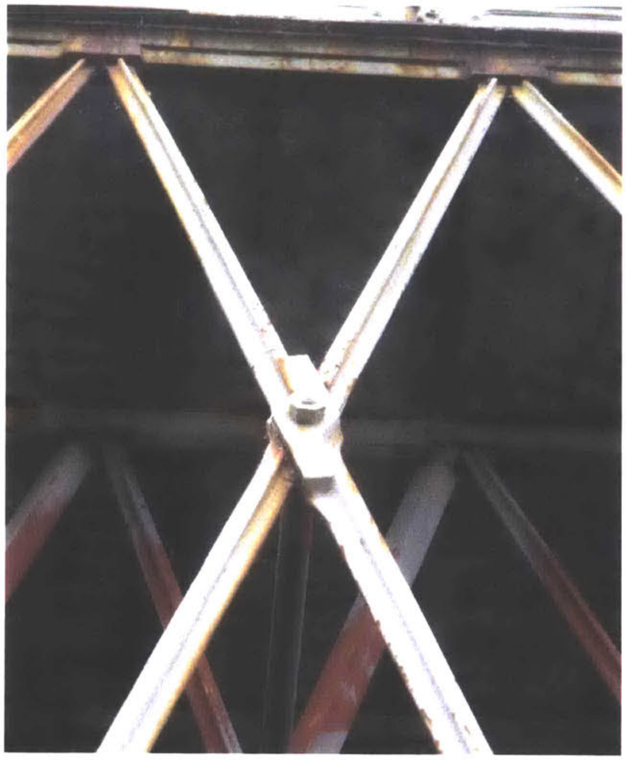

Figure 3.12: Mythe Bridge – lattice bracing midspan connection (Lane 2017)

Figure 3.13: Mythe Bridge – lattice-rib connections, covering plates, external rib (Lane 2017)

Figure 3.14: Mythe Bridge – abutment (Lane 2017)

The lateral elements not shown in Figure 3.10 above include:

6. Connecting plates running laterally between each rib section, connecting each to its neighbor and tying all six ribs together.
7. Covering plates lying across the tops of all the ribs, providing full lateral support to the ribs. The places are connected to the ribs via
8. Diagonal bracing tying the deck to the ribs between the second and third rib plates from the crown. They have a cruciform section, diminishing from 5.5 in^2 at the center to 4 in^2 at the ends (Mackenzie 1838, 13).
9. 1.5 in wrought-iron bolts laterally connected each set of the spandrels at their midpoints, covered in cast-iron tubes to prevent corrosion.
10. The spandrels are laterally connected at their midpoints through hollow cast-iron tubes, encapsulating wrought-iron ties.

Figure 3.15: Mythe Bridge – viewed from below, showing ribs, connecting plates, covering plates, springing plates, and abutments (Lane 2017)

Non-structural elements of the bridge included:

10. A roadway of crushed stone, of width 17 ft (Mackenzie 1838).
11. Pathways on either side, lined with large granite stones, of width 3.5 ft each (Mackenzie 1838).
12. Cast-iron railings, of Telford's own design (Figure 3.16).
13. Toll-houses on either end of the bridge (Figure 3.17). These do not contribute to the bridge design in any way, but they do indicate the level of dedication that Telford showed in his work.

Figure 3.16: Mythe Bridge – original railings, skirting plates, and ribs (Lane 2017)

Figure 3.17: Mythe Bridge toll house, designed by Telford (Lane 2017)

Mythe Bridge was the subject of a report to the ICE by William Mackenzie in 1838, on the basis that it was one of Telford's most important works. The report summarizes the project, including Telford's original communications and drawings, but provides essentially no interpretation thereof. Elsewhere Mythe has been largely overlooked, and no structural analysis appears to exist, apart from Paxton's approximation of the compressive stress in the ribs under uniform loading.

The choice of a cast-iron arch at Tewkesbury was due to the Severn's tendency to flood and the importance of maintaining navigation. Telford was called into the project when work was already underway, because of negotiation problems with the architect and difficulties with the foundations. The original plan had been for three cast-iron arches, but after much trouble placing piers, Telford recommended in 1824 that a single cast-iron arch be substituted (Mackenzie 1838). This difficulty and the necessity of changing the design partway through the project help to account for the substantial increase in price compared to Bonar Bridge.

It is important to consider the motivations behind the choice of the 170-foot span arch, rather than either extending the span and reducing the size of the abutments, or returning to his typical 150-foot span. It seems a conservative choice, especially given that this was late in Telford's career, and he had already achieved far greater feats in iron. Most likely the foundations for the abutments had already been placed, and the county trustees were hesitant to make further changes. Telford's initial report in December 1823 states that "the masonry of the abutment on the Tewkesbury side as far as performed, seems executed in a very proper manner" (Mackenzie 1838, 2). However, in a report from 1824, Telford makes his recommendation that the three cast-iron arches be substituted for a single one, and states "I consider an arch of 170 feet span is sufficient for this purpose" (Mackenzie 1838, 4), which seems to suggest that the span was not already fixed. In addition, he goes on: "The distribution of this flood water-way I propose to be by having six small openings upon each abutment" (Mackenzie 1838, 4), indicating that the abutments had not yet been constructed." In addition, his specifications for the design include requirements not only for the abutments, but for their foundations (Mackenzie 1838).

In short, it would likely have been possible to extend the cast-iron arch on either side, creating a more open waterway to deal with flooding and eliminating the need for more complex masonry work upon challenging soil. Instead, Telford chose a span that was still longer than any he had yet attempted in cast iron, but which failed to be truly revolutionary. Perhaps this was due to the hesitation of the trustees, or a disinclination to change the plans more than necessary. Possibly, having been called in to the project late, Telford did not feel the need to push the boundaries of what he could achieve. It is even possible that the quiet country setting did not appeal to Telford as a backdrop for a great new achievement; Rolt suggested that he chose the sites for his greatest works based on their romance and drama (1958, 123). Whatever the reason, though, it may be postulated that he could have gone farther.

That being said, Mythe Bridge was the first of Telford's cast-iron bridges that actually broke with the 150-foot span introduced at Bonar. For the first time in over a decade, he introduced real innovation in his cast-iron bridge work. The longer span, flatter arch, and vertically-oriented bracing are all departures from the Bonar model. By coming into the work partway through, Telford appears to have been forced to create a new design, which benefited from his experience of the intervening fourteen years. It is certainly not a dramatic departure; as noted above, it fails to be truly revolutionary in any aspect. In spite of this, it is certainly a more modern and aesthetically pleasing bridge, and Telford chose to showcase this design in his autobiography many years later, clearly finding it worthy of celebration.

3.2.2. Performance of Mythe Bridge

Mythe Bridge has been renovated twice in its lifetime. The original roadway and sidewalk of crushed stones were replaced with a concrete deck in 1923. The bridge was listed as a Grade II structure in 1952. Major renovations were then undertaken in 1992 (Mythe Bridge 2017). The external sets of ribs and bracing were left intact to preserve the appearance of the bridge from the river, but many of the internal braces near the abutments were strengthened with large, square steel braces, which have the same connectivity as the original members, but with substantially larger cross-sections. This is an indication that these members were considered to be seeing unacceptable stress levels. Overall, the reconstruction is sympathetic; although the internal bracing is not attractive or true to the original design, it is not easily visible, and unless viewed from directly below, the bridge appears much as it would have done in 1834.

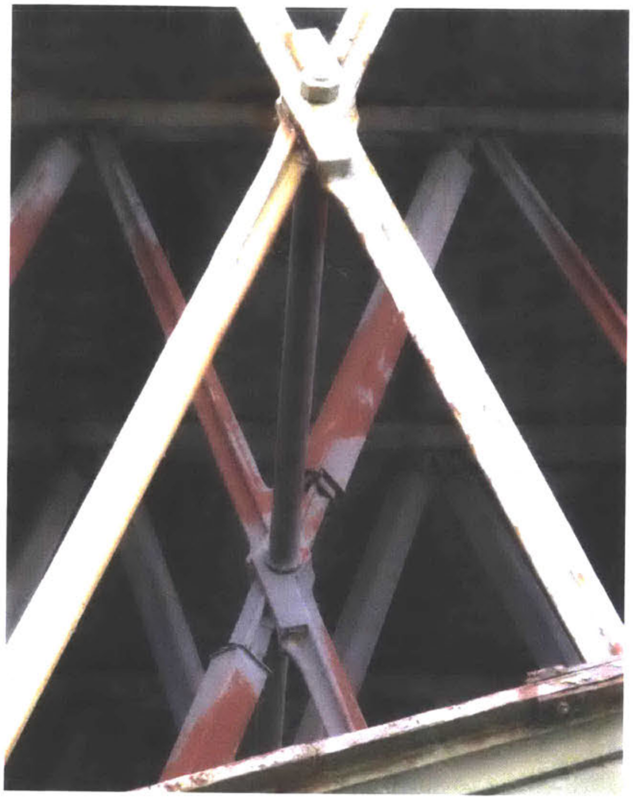

Figure 3.18: Mythe Bridge – bracing mid-span connection, showing original exterior brace and strengthened interior brace, connected by wrought-iron tie encased in cast-iron tube (Lane 2017)

The bridge is still open to traffic, although a 7.5-ton weight limit was imposed in 1990, which was increased to a 17-ton limit in 1992 after reconstruction. It remains in fairly good condition. Several of the ribs have experienced very substantial longitudinal cracking towards the springing plates, as shown in Figure 3.19. This suggests damage due to temperature stress or fatigue, rather than typical loading conditions. There is also a fair amount of corrosion, particularly on the covering plates around the point where the spandrel braces frame into them. This suggests that water collects and follows the lines of the braces to the covering plates. The deck itself has seen some water damage, especially around the connections between the road plates, as shown in Figure 3.20, and along the roadway bearers, as shown in Figure 3.21. Overall, however, it is spectacular that the bridge is still largely original and continues to carry traffic today.

Figure 3.19: Mythe Bridge – longitudinal cracking of rib (Lane 2017)

Figure 3.20: Mythe Bridge – corrosion of covering plates (Lane 2017)

Figure 3.21: Mythe Bridge – deck corrosion along connections between deck plates (Lane 2017)

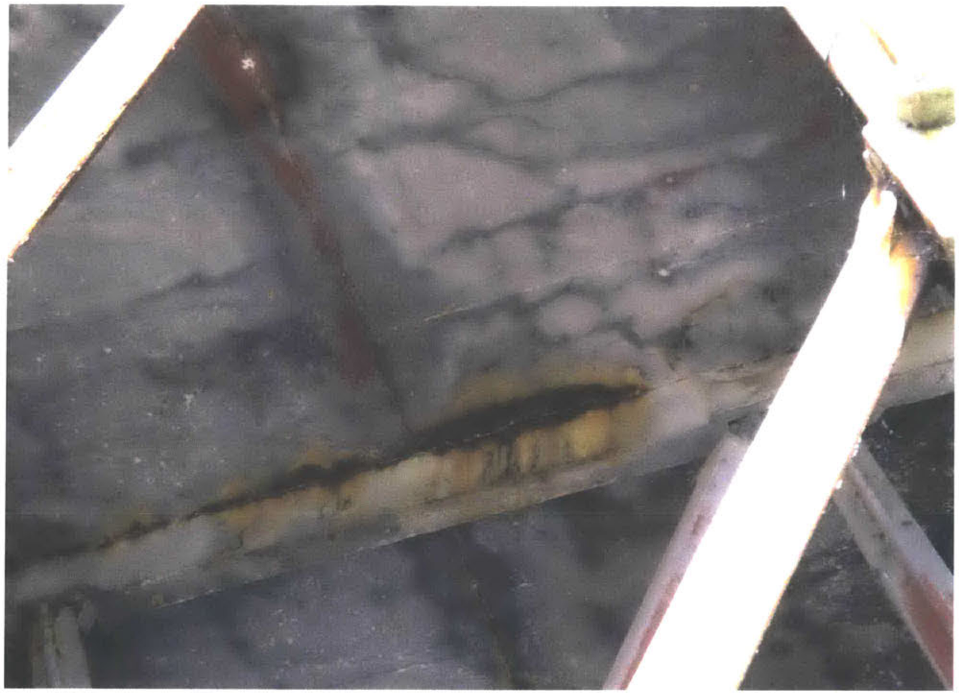

Figure 3.22: Mythe Bridge – deck corrosion along roadway bearer (Lane 2017)

When visiting the bridge, the difference in form compared to Craigellachie is instantly apparent. At Craigellachie, the spectacular setting, the larger rise, and the fact that the bridge is situated so high above the water level combine to make it stand out. Mythe Bridge's setting is much less dramatic. The Severn is a slower river, frequented by narrowboats and rowers, and Mythe stretches gracefully over it, close to the water. An overgrown footpath follows the river up from the town of Tewkesbury and actually passes through one of the arches of the abutment. The bridge is painted a soft green, and the abutments are of a warm-colored stone. The slight ornamentation on the outer ribs and the beautiful original railings help to make the bridge feel airy and gracious. Its gentle arched form is exceptionally pleasing to the eye and feels much more natural than that of Craigellachie, especially with the evenly-apportioned spandrel bracing.

From the road, the bridge is almost entirely unremarkable; apart from the tollhouse at one end and the original ornamental railings, there is little indication that this is a valuable historic landmark. There is a beauty to that – it is easier to remember that this bridge was built, not as a monument, but as a small, functional component of Telford's vast road system. Furthermore, it provides verification of a divide that cannot quite be captured in a drawing, that the experience of Mythe Bridge is vastly more modern than that of Craigellachie or Bonar. That being said, it is essentially of the same form, and it remains to be determined whether Mythe truly represents a substantial structural improvement.

Bonar Bridge and Mythe Bride hold different and unique places in the history of Telford's career and the history of cast iron bridge design. Having introduced the two bridges, and provided a brief overview of their forms and histories, it is now possible to assess their design and performance in more detail. First, though, to identify the roots of each design, it is necessary to go back further.

4. Design Influences

For every leading designer, it is of great interest to establish key sources of inspiration. This is especially true of early structural designers like Telford, who had so much less to rely on in terms of theory and precedent. There were no design codes, not even a broad consensus on the behavior of materials – or indeed an industry that was regulated to the point where material properties could be accurately predicted. There can be no question that structural theory played a role in his designs. Although biographers agree that Telford himself had little theoretical knowledge, he solicited the help of academics who did use what knowledge was available to them. Their advice, particularly on the London Bridge proposal of 1801, had a substantial influence on his cast-iron bridge designs. More importantly, Telford relied on experience gained through his own previous projects, the work of his contemporaries, and extensive experimentation. Finally, design decisions came down to his own judgment. Telford was a designer with a strong belief in his own ability, and his judgement was guided by the two cornerstones of his design philosophy: practicality and aesthetics. Through a combination of all these factors, Telford's designs for cast-iron bridges quickly found their own character, which would give all his cast-iron works a strong similarity in both style and construction. A better understanding of these influences is essential in order to establish the sources of Telford's designs, as well as his own and his contemporaries' understanding of their design, performance, and significance.

4.1. Theory

Telford had almost no academic training, having left school at age fourteen (Rolt 1958), but even had he been afforded a university education, there would have been a limited pool of knowledge available to him. Structural theory was in its infancy at the time, and science had yet to be widely acknowledged as a basis for engineering. The concepts of stress and strain were just beginning to be developed, and were not generally understood or applied, although Euler and others had produced much valuable work on the subject over the course of the 18th century (Troyano 2003).

The theory surrounding arch bridges was likewise just beginning to emerge. Robert Hooke had proposed the idea of the hanging chain in 1675, leading to P. La Hire's arch calculation method, based on compressive force paths through the arch voussoirs, in 1695. This led to Coulomb's study of the arch in 1773, and then to the more widespread application of graphic statics methods for masonry arches beginning with Gerstner's work in 1831 – around the end of Telford's career (Heyman 1972). On the other hand, Navier's important elastic analysis methods, based on material stresses and strains, were used for iron arch bridges beginning in 1826 – again too late to influence Telford's design decisions (Troyano 2003, 280). Up until this point, applications of theory were limited. Thomas Young performed a strength calculation on Rennie's Southwark Bridge in 1817 (Ruddock 1879, 167), investigating the position of the thrust line and the effects of temperature, probably the first strength calculation for an iron arch bridge (Paxton 1975). Tredgold's "Practical Essay on the Strength of Cast Iron" (1822) and Gregory's "Mathematics for Practical Men" (1825) became well-regarded reference books later in Telford's career (Paxton 1975, 116), but the notable similarities in their titles makes it clear that science was still fighting to make its impact on "practical" fields such as bridge-building.

Although Telford was aware of this emerging body of literature on structural theory, he appears never to have familiarized himself with detailed theoretical concepts, or performed any extensive calculations for the design of his bridges. He included a detailed section on theory in his article "Bridge" for the *Edinburgh Encyclopedia* (1812), but it was penned by an apprentice and not himself. However, he turned to theory whenever pushing the boundaries of the field – particularly on two occasions. The first was his proposal for a new London Bridge in 1801, which featured a single-span cast-iron arch bridge. Although the bridge was never built, it was under serious consideration for several months. During this process, the House of Commons required Telford to solicit the advice of mathematicians and scientists on several points. He corresponded at length with many of the most notable experts of the time, from Nevil Maskelyne, the Royal Astronomer, to Charles Bage, architect of the world's first iron-framed building. These correspondences appear to have had a profound effect on Telford's designs for cast-iron arch bridges, and may be shown to have greatly influenced his Bonar Bridge design.

He was again asked to seek theoretical advice when proposing the suspension bridge across the Menai Strait in 1818. Again, this was requested by the House of Commons, in the face of a dramatic and unprecedented design proposal. Telford's reliance on theoretical advice was therefore most significant when he was attempting to convince the public that his boldest proposals were safe and practical. It could be argued that he would not have solicited this advice had he not been required to do so, and certainly he does not seem to have requested further help from theoreticians for his later works. Generally, Telford is reputed to have considered theory to be largely dissociated from the practical reality of building a bridge. “The World has been too long amused with the mathematical discussions of mere theoretic book learned Bridge builders,” he wrote in 1816 (Paxton 1975, 111). He was much more strongly guided by precedent, experimentation, and, ultimately, his own powerful and deeply individual aesthetic. However, structural theory played its role in guiding his design choices, and the influence of “book-learned Bridge builders” on his cast-iron bridge design, particularly through the research for London Bridge, has been under-emphasized in the existing literature on Telford.

4.1.2. London Bridge Academics

Telford’s proposal in 1801 for a 600-foot cast-iron bridge, spanning the Thames with a single arch, was truly revolutionary. Incredibly, with a mere five years’ experience in building in cast iron, he proposed to the House of Commons a slender 600-foot arched bridge, leaping the Thames in a single span, shown in Figure 4.1. The budget for the abutments expected to be necessary for such a design were ultimately found to be prohibitive, and the project was abandoned until Rennie’s more traditional masonry bridge was ultimately erected in 1831. However, the inquiry into the design solicited the input of most of the notable scholars, mathematicians, and ironmasters of the day, and it is perfectly possible to trace their influence on the design, to which he would finally return for Bonar Bridge in 1801.

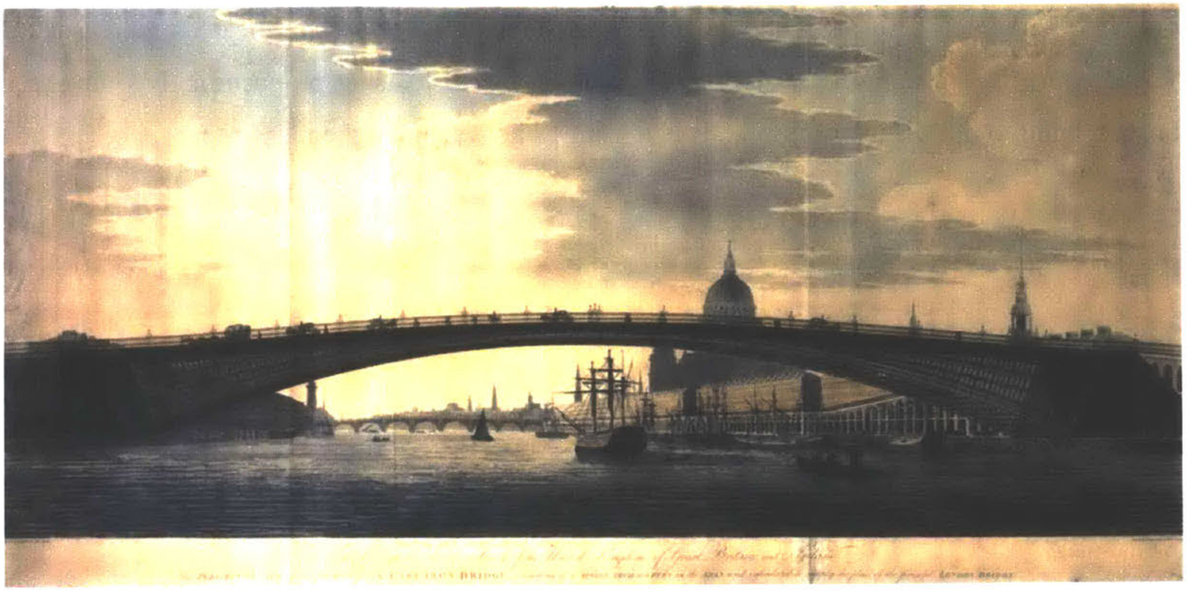

Figure 4.1: Plate of Telford's proposed London Bridge (Lowry 1801)

The House of Commons requested that Telford release an open list of questions, asking his colleagues whether they thought his bridge feasible. Telford's original plans were released with a brief explanation, a couple of sketches, and very little detail as to the dimensions or connections of the bridge, much to the annoyance of his many correspondents. This quite astonishing lack of detail indicates the extent to which this bridge was merely a sketch in Telford's mind. He was still in transition from architect to engineer; from conceptualization to structural design – a transition that was representative of the industry as a whole at this time. Telford's correspondence, with everyone from the Royal Astronomer to professors of philosophy to architects and ironmasters, is indicative of the broad uncertainty of a nascent industry still finding its footing. For this reason, these correspondences are fascinating and pertinent, not only through their lasting effect on Telford's bridge building, but because they mark a place in civil engineering history, when academics and "practical men" alike were turning their attention to the design of the built environment.

The bridge's initial design is instantly recognizable as distinctively Telford's own. It features an arched lower rib, of a quite phenomenally large radius compared with contemporary arches, connected to the deck via radially-oriented ties. Unlike Bonar and later bridges, these ties are not provided in crossed pairs. They are instead braced by three sets of intermediate ribs, the whole creating a vertical frame. There are 13 sets of these frames across the width of the bridge, connected with lateral cross ties and diagonal cross-bracing. The width of the deck increases from the crown to the springing, with the spacing of the ribs increasing proportionally – unlike the constant width of all his later bridges. These features are visible in the plans reproduced in Figure 4.2.

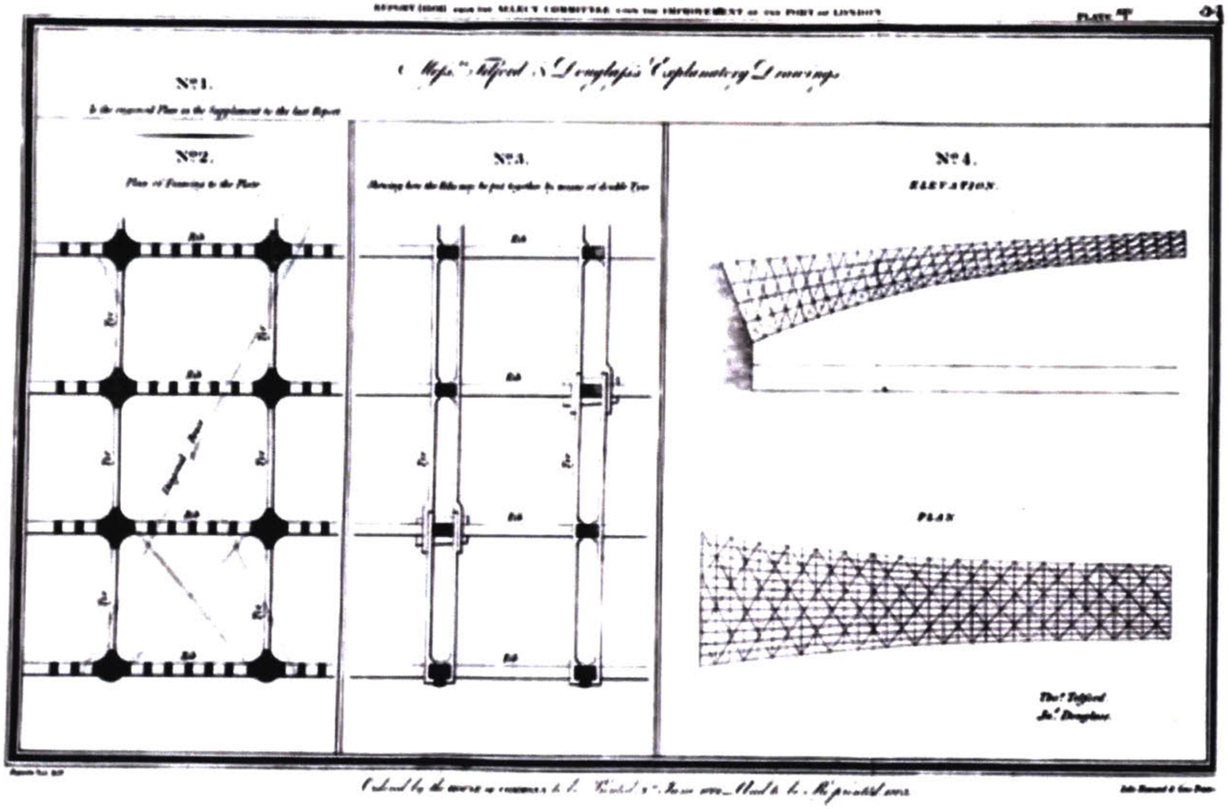

Figure 4.2: Plans for proposed London Bridge (1801)

Each of these design decisions is discussed in the letters to and from notable thinkers of the day, many of which have been preserved in the archives of the Institution of Civil Engineers, and are transcribed and reproduced in Appendix C of this paper, along with a brief list of Telford's correspondents. In general, the responses express amazement and awe at the design, but most conclude that the bridge is feasible.

Telford asked his correspondents whether he could view the bridge as a single frame, and most of his correspondents agreed. "My idea is, that the parts of the bridge should be so firmly connected together, both laterally and vertically, that the whole may be considered as one frame of iron," states Nevil Maskelyne (1732-1811), Astronomer Royal, in his letter of April 14 (see

Appendix C). This view is echoed by William Reynolds (1758-1803), the Shropshire ironmaster (May 14) and John Wilkinson (1728-1808), the iron master at Bradley, near Birmingham (April 14). These correspondences also appear to have effectively emphasized to Telford the necessity of extensive interconnectedness of members within the bridge. This was the guiding overall assumption behind all of Telford's future cast-iron bridge designs, and led to frames that were rigidly connected in the lateral direction.

In the vertical direction, however, he quickly abandoned the rigidly connected frame. The initial plan for London Bridge uses five sets of ribs, stacked vertically, and connected through vertical ties. This appears much more primitive than the single rib, which was Telford's ultimate decision in Bonar Bridge and all subsequent bridges, and speaks to the early tendency to fill in the spandrels with material, harkening back to masonry bridge design. An early sketch of Bonar Bridge also shows the provision of a similar intermediate rib, which was not ultimately included, according to photographs of the bridge (Old Bonar Bridge n.d.). Charles Bage (1751-1822), the architect who created the world's first iron-framed building in 1796, warned Telford "to depend on the lower ribs only for strength; the upper ones being very good in Theory, but in practice good for nothing unless extreme accuracy of workmanship be attained" (April 23).

Telford's original idea seems to have been that it was important to provide these intermediate ribs to stiffen the main arch, "that the pulling up the upper frames may pull up the lower frames, and pulling down the lower frames may pull down the upper frames," as Maskelyne wrote (April 14). From a modern perspective, this is recognizable as the theory behind the deck-stiffened arch bridge. Instead, Telford chose to quickly move away from this approach, going to the other extreme, with exceptionally slender ties connecting the ribs to the bridge in all his later cast-iron bridges, forcing the bridge to rely entirely on the main ribs for strength. This idea is summed up perfectly in a letter from Charles Bage, dated April 12, 1801: "Every rib should be a bridge of itself, and all the joining should be absolutely close. If this can be accomplished, and the abutments so firm as not to yield in any part when the center is struck, each portion of every rib will take its share of the pressure, and the tyes both perpendicular and horizontal will have litterally nothing to do but to keep the ribs from bending, just as you would contrive stays for a very long and slender pillar." Whether or not this advice was a major guiding force behind Telford's later designs, it must have given him reassurance that it was reasonable to rely solely on the main ribs, rather than the ties.

The design of the ribs themselves is different than that of Bonar or later bridges, but the discussion surrounding them hits on many of the considerations that Telford would later adopt. He originally describes the ribs as diminishing in height from the abutments to the crown, rather than the constant section he would choose for all his bridges. He asks Maskelyne whether the benefit of casting the ribs in smaller pieces, to equilibrate cooling time, would reduce the strength because of the increase in the number of connections. Meanwhile, Wilkinson recommends that "an increase of the depth of the rib, and a diminution of the width, would give a superiority in point of strength, and keep within the weight of metal" (April 17) – in more theoretical terms, increasing the moment of inertia of the rib for a given area. A further refinement is suggested by Bage, of adding lower flanges to the outer two ribs, to strengthen them and protect against the possibility of ships' masts striking them (April 12) – an alteration that is in fact used for Telford's later bridges. These are small things, but their agreement with Telford's later work suggests that he was strongly influenced by these ideas.

Overall, these correspondences yielded many important design ideas, which undoubtedly influenced Telford's later work. The importance of redundancies and firm connections, including in the lateral direction, sufficient to essentially create a single, rigid frame, is proposed by Telford and receives strong approbation. The lower rib is shown with the x-shaped section that would be used in his later bridges. Perhaps most interestingly, Telford's iconic cross ties are introduced. They are already indicative of his individual style, with a preference for radially-oriented ties, rather than the vertical ties of Rennie or Jessop (Ruddock 1979, 162). His correspondence with Bage, in particular, discusses the drive to make these ties as slender as possible, another hallmark of Telford's later designs. This led to one of the most substantial breaks with the London Bridge design, the move away from the intermediate ribs and idea of the "vertical frame," towards a single rib, connected to the deck with slender ties, and each rib connected to its neighbors laterally into a horizontal frame.

It is characteristic of Telford that the influence of theory on his designs is shown mainly through the advice of his friends and colleagues. He was well-known for his good relationships with his associates and workmen, and it is evident, by the fact that these letters contain so much theory, while his notebooks contain little, that he was most willing to make use of theory when it was vouched for by men whom he knew and respected. However, in spite of the wealth of advice he received, and his own lack of experience in the field, he does not lose track of his own instincts and sensibilities. Bonar and London Bridge share a unique aesthetic that was already, at the beginning of his bridge-building career, classically Telford.

4.2. Experience

Telford relied heavily on experience, believing it to be the most applicable source of information. Although he was often working at the forefront of the industry, with few direct precedents to guide him, he drew on his own earlier works, the works of his contemporaries, and extensive practical testing, to provide background on which to base his designs.

4.2.1. Personal Experience

Telford had great confidence in his own work, and naturally relied heavily on his own experience. He worked up slowly to long-span cast-iron bridges, and continually referred back to his earlier projects. Longdon-on-Tern Aqueduct, built 1795-1796, was his first project with cast iron. He was most likely involved in the project mainly as an architectural consultant, with the bulk of the work falling to William Reynolds, the ironmaster in charge of the project (Paxton 1975, 73). This allowed him to gain experience with the new material, based on Reynold's extensive knowledge. Telford likely modeled the road plates and bracing members of Bonar Bridge on this aqueduct. The road plates have the same form – thin plates, 5 feet long, bolted together by flanges of 4 inch depth. The bracing members have a cruciform section, as do those of his later bridges, although they have a much larger cross-sectional area of approximately 20 in^2 (Paxton 1975, 205).

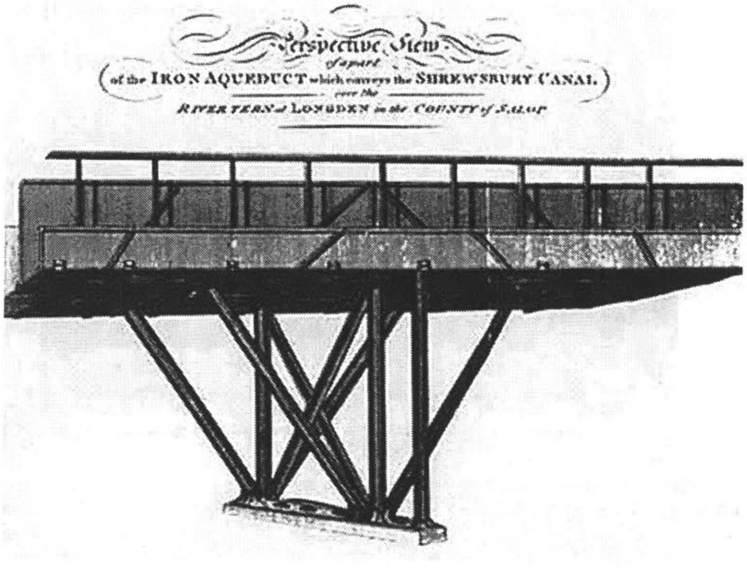

Figure 4.3: Longdon-on-Tern Aqueduct, showing cast-iron plates and bracing *(Shrewsbury and Newport Canal n.d.)*

This early experience was very quickly followed with designs of his own. Pontcysyllte Aqueduct, built 1795-1805, was designed by Telford, and approved by William Jessop. Telford references the aqueduct in his description of Bonar Bridge, saying that this was where he first used the cast-iron connecting plates which he used at Bonar to join the rib segments (Sixth Report of the Commissioners for Highland Roads and Bridges 1813, 37). These connecting plates can be seen in Figure 4.4. This cross-section also shows that Pontcysyllte employs cast-iron ribs, the forerunner of those seen in Bonar and Mythe. Importantly, however, each rib incorporates solid spandrels, very reminiscent of masonry design, rather than the open spandrels with cross-bracing, which is so iconic of Telford's later cast-iron bridges. This makes Pontcysyllte a clear intermediate step in Telford's design progression.

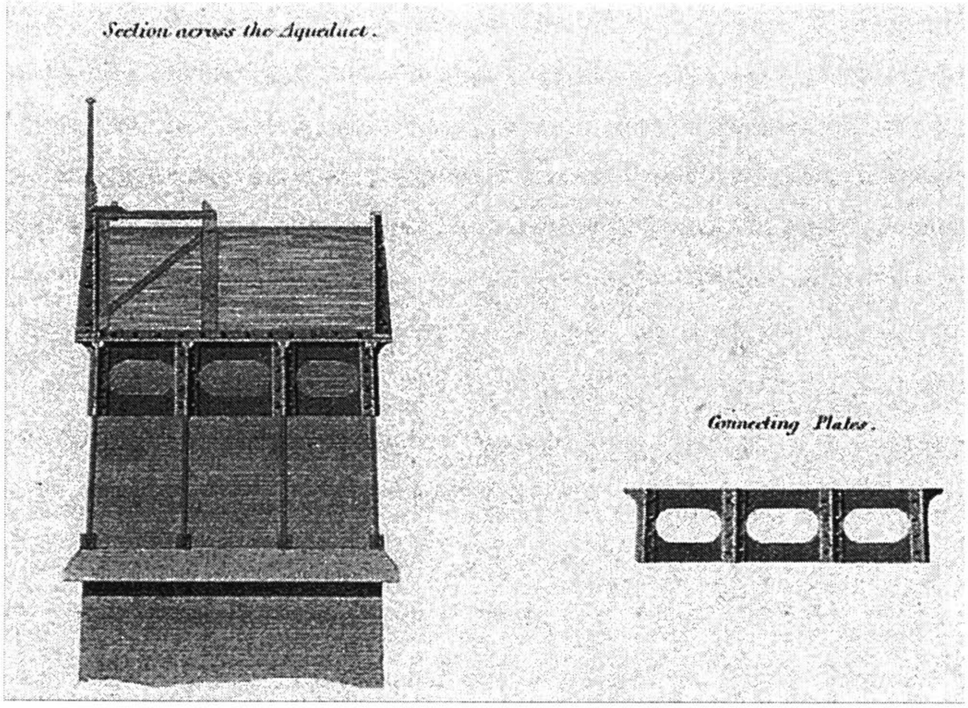

Figure 4.4: Section of Pontcysyllte Aqueduct, showing connecting plates

Buildwas Bridge (1795) represents the next stage of this development. It was Telford's first long-span cast-iron bridge, crossing the Severn River just two miles upstream from Coalbrookdale. Its heavy, boxy appearance is reminiscent of Longdon-on-Tern Aqueduct, which Telford undoubtedly used as a reference. However, it features an arched iron rib as the main structural element and it is clearly already moving away from Iron Bridge's circular spandrel bracing towards Telford's iconic radially-oriented bracing. It also features a secondary arched rib, whose inspiration Telford attributes to the Schaffhausen timber bridge in Switzerland (1758) (Telford 1812). However, this rib developed tension cracks, and he abandoned it after this bridge – a clear example of a design refinement over the early portion of his career.

Of course, Telford was influenced not only by his own works in cast iron, but also by the precedents of earlier designers, and the works of his contemporaries. It is therefore important to identify the extent to which he drew inspiration from these designs.

4.2.2. Design Precedents

4.2.2.1. Roman Arches

Traditional bridges at this time were still substantially influenced by traditional Roman rules of proportion. Masonry precedents, including Telford's own masonry bridges, conform substantially to these rules, so it is reasonable to consider whether this was a major influence behind the overall form of Bonar and Mythe Bridge. In *Roman Bridges* (1993), Colin O'Connor summarizes the Roman arch design guidelines as:

1. Ratio of rib thickness to span (t/L): guideline is t/L = 1/10, lower bound is t/L = 1/20
2. Avoid stones of depth greater than 5 Roman feet, or 4.85 modern feet (1.48m)

The second general guideline is clearly met by both bridges; both have a rib depth of 3 feet. It would be an obvious choice to reduce the depth of the rib because of the superior strength of iron. As for the proportional relationship between rib thickness and bridge span, the guidelines are also well above the actual choice for both bridges, but the implications are more interesting.

In addition to the general rule attributed to the Romans, O'Connor also references Heyman's formulae for minimum thickness for a perfect semicircular masonry arch supporting its self-weight only, which leads to the boundaries plotted in Figure 4.5 below (Heyman 1969). This is the rib thickness below which the thrust line cannot be contained within the rib, leading to hinging and failure.

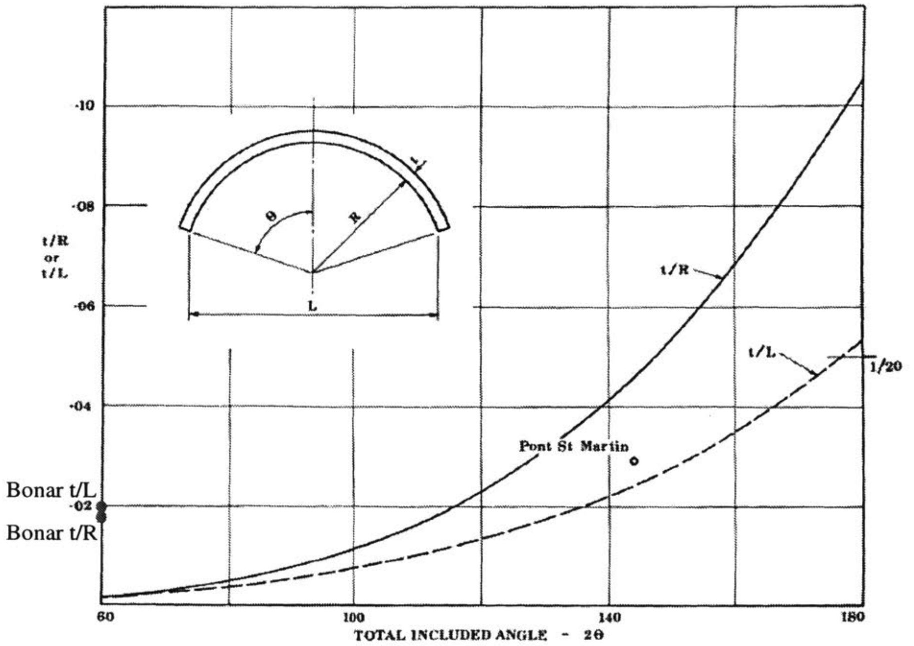

Figure 4.5: Heyman's theoretical minimum rib thickness to radius or span (t/R or t/L) for a perfect semicircular arch, with data point for Bonar Bridge added by the author *(Heyman 1969)*

The relevance of this minimum value depends, of course, on the assumptions involved in its derivation. The first is Heyman's general assumptions for masonry: zero sliding, zero tensile strength, and infinite compressive strength. The calculated minimum is for failure due to instability, rather than insufficient material strength. As Heyman argues in "Iron Arches" (1982), this is a relevant assumption for cast-iron, which has high compressive strength but is weak and experiences brittle failure in tension.

The other important assumption is that the arch sustains its self-weight only. This is of course not the case for Telford's arches – they support the deck as well as any live load moving across it. However, as will be shown in Section 5.3, the bulk of the weight of the bridge comes from the iron frame making up the rib. On the other hand, the live load clearly has a substantial impact on the loading of the arch, especially in asymmetric live loading, as will be demonstrated in Section 5.2. In any case, Heyman's calculations provide a lower-bound reference point.

Bonar Bridge, the earlier of the two bridges in question, and Telford's first long-span cast-iron bridge of this design, has a rib thickness of 3 ft, a span of 150 ft, a radius of 151 ft, and an included angle of 60°. It is therefore right at the lower boundary of the graph included by O'Connor. Already Telford had moved away from the traditional hemispherical arch, towards a smaller, shallower segment. Its proportions fall well below the guidelines quoted above, with a t/L ratio of 1/50, but fit comfortably above the theoretical minimum derived by Heyman, as indicated on the graph; it is well above the acceptable ratio to prevent hinging for a compression-only arch.

Mythe Bridge, on the other hand, also has a rib thickness of 3 ft, but with a span of 170 ft and, even more significantly, a radius of 223 ft, for an included angle of only 45°. Mythe, therefore, is slightly further from the Roman guidelines quoted by O'Connor, with a t/L ratio of 1/57, and an included angle much smaller than the graph shows. For such a small angle, the development of hinging is much less of a concern. Therefore, Mythe is actually much more conservative from a stability standpoint; a conclusion that will be repeated in Section 5.2.1 below. Furthermore, it may be concluded that the Roman rules of proportion, while perhaps providing guidelines, did not have a strong influence on the design of these bridges, and that Telford continued to move away from these standards, towards flatter arches with a smaller included angle.

Indeed, Telford's writings seem to indicate that, right from the beginning of his work with cast iron, he felt that it was important to embrace the possibilities of the new material. He criticized Iron Bridge, whose designers, he said, "had not disengaged their ideas from the usual masonry arch, the form of which is not graceful in iron" (Telford 1838). Iron Bridge's semicircular design is indeed highly reminiscent of the Roman masonry designs. Telford, however, pushed away from the large included angle, so that even his first designs had a flatter arch, and seems to have become more confident in this direction, moving from Bonar's 60° angle to a dramatically lower 45° at Mythe. From an aesthetic standpoint, Bonar still has the look of a masonry arch, with a radius almost equal to its span, a large included angle and correspondingly large rise. Mythe has a much more modern look, with a much flatter arch, of larger radius and smaller included angle. In this, Telford is moving away from the Roman ideal for the masonry arch.

From an aesthetic point of view, therefore, Telford moved away from the traditional masonry precedents, whose influence can still be seen in Bonar Bridge, but is much less dominant in Mythe, whose flatter arch and smaller included angle are substantially more modern. In both bridges, the rib thickness does not appear to have been influenced by traditional Roman proportions for masonry, and instead seems to have been determined more practically, according to the strength of materials.

4.2.2.2. Timber Design

As already mentioned, timber bridge design was clearly an inspiration for Buildwas Bridge. Timber bridges tended to feature spandrel bracing of the kind seen in Telford's bridges; for obvious reasons, these tended to use long, thin members to brace the arches. Telford called the Schaffhausen Bridge "the boldest and most ingeniously constructed wooden bridge on the European continent" (1812, 488), and although he abandoned the Schaffhausen suspending arch after Buildwas, the slender lattice bracing that would become so iconic of his cast iron bridges can clearly find a precedent here. In particular, the diagonally-oriented crossed bracing, as seen in Figure 4.6, is suggestive of Telford's later work.

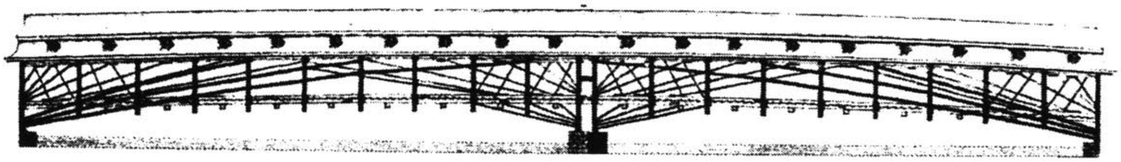

Figure 4.6: Elevation of Schaffhausen Bridge (1758) included in Telford's article (Bridge 1812)

4.2.2.3. Contemporary Iron Bridges

This era of bridge design has rightly been called individualistic. However, Telford was clearly aware of the designs of his contemporaries, and although his own arched bridges have an entirely unique style, it is evident that he was influenced by the triumphs and failures of his peers. It was the Iron Bridge at Coalbrookdale that first attracted him to the use of iron in bridge building, and led him to build his own iron arch at Buildwas. He references Thomas Paine's iron arch in his correspondence regarding the London Bridge. He criticizes both designs – an indication that he was always thinking about how to improve on others' designs.

This was no trivial aim, because with little precedent or theoretical knowledge to guide them, designers tended sometimes toward overdesign, and sometimes toward catastrophe. Staines Bridge (1803) and Yarm Bridge (1805), both built by Thomas Wilson, both failed because of abutments that were not sufficient to take the thrust of the bridge. Staines developed fractures in the iron ribs and had to be taken down, but, worse, Yarm collapsed spectacularly into the river in

1806 before it was even opened (Ruddock 1979, 159). These failures resulted in widespread public distrust of the new material's potential for infrastructure. There was an overall decrease in the number of cast-iron bridges built in England for several years following these collapses, and designers were very conscious of the risks associated with innovation. There can be no doubt that this influenced Telford's designs, and he sought to address the structural issues highlighted by these failures. "It has hitherto arisen from insufficiency of the abutments, that failures in iron bridges have occurred, although mistakenly attributed to the iron-work," he writes in his description of Bonar Bridge (Sixth Report of the Commissioners for Highland Roads and Bridges 1813, 37). He also recognized the importance of designing a bridge that provided the appearance of strength and stability to reassure the public of its reliability (Billington 1983, 39).

One specific design influence was Bristol Bridge (1805), designed by Jessop, which developed unacceptable stresses in the ribs. Telford attributes its failure to residual differential stresses from the cooling of large segments, and this was an influence that led clearly to an important design feature of all of Telford's subsequent cast-iron bridges. He used small segments, and made sure that the ribs, grating plates, and connecting plates were all perforated so as to leave all elements only 6 inches wide, allowing for more even cooling, which he claimed was the first time such a modification had been implemented.

It would appear that, in general, Telford used his contemporaries' work as a guide for what not to do. He was clearly highly aware of the work of other designers, and often applauded their innovations, but he was also shrewdly critical of their flaws, and always chose to follow his own judgment, backed by experience and practical tests.

4.2.3. Experimentation

Telford had a great belief in the importance of experimentation as the basis for understanding material properties. He had an excellent relationship with many notable ironmasters, including William Reynolds, John Wilkinson, and the Walker family. He relied heavily on their advice regarding the capabilities of cast iron, and placed great trust in the quality of the materials they provided. For London Bridge, all three of these sources sent Telford information on the strength of iron, and referred to many experiments they had done to determine its tensile, compressive, and bending strength and behavior.

Telford himself also conducted tests to measure the material properties of iron, investigating how much load a section could sustain, although without a clear understanding of stress and strain. He planned to make a model of his London Bridge proposal, in order to load-test it. Of course, the use of scale models for load testing has limited validity in a case where material stresses are important, since strength of materials is not scalable. This was an important change from masonry design, where, as Heyman notes, failure is almost never due to crushing of the stone, but rather due to instability, which is perfectly scalable. This transition had critical implications for the future of civil engineering, as physical models were no longer practical, and instead theory became increasingly important. However, this was early in the transition, and not only Telford, but his contemporaries also, appear to have placed complete faith in these models. In the correspondences regarding the London Bridge model, Wilkinson advocates strongly for a scale model as "by far the best Mode" to "point out any improvements such were required" (Jones April 11, 1801). Bage cautions Telford that only a model whose members are exactly proportionate can give safe results – again, a masonry holdover (Bage April 12, 1801). The use of scale models was therefore clearly still widely held to be useful, and this was evidently a technique in which Telford placed great confidence. Although it was a mistake to place excessive confidence in the model in ensuring no material failure, it would still have provided useful information about the stability and general behavior of the bridge.

Telford also performed experimentation on specific components of his bridges – a more safely applicable technique. For the Menai Straits Bridge, he tested each individual cable link at twice the load he expected it to take when in use, but well below the load at which experimentation had shown the link would yield. Even more applicable were the load tests he ran on the bridges themselves. Many of Telford's bridges were pre-erected at the foundry, and load-tested there, including Buildwas Bridge. Bonar Bridge was also pre-erected, and it was probably pre-tested, according to Paxton (1975).

4.3. Judgment

More important than theory or even than experimental results, however, was Telford's own judgment. He had a strong belief in his own intuition and experience, which enabled him to push the boundaries of design even into realms that had never before been tested. He was a firm advocate of the knowledge gained by working with a material, and wrote in his autobiography that a designer requires “the practised eye, and the hand which has experience of the kind and qualities of stone, of lime, of iron, of timber, and even of earth, and of the effects of human ingenuity in applying and combining all these substances” (13). He also had a deep-seated aesthetic sensibility, and describes in the same chapter a defining moment when he discovered “architecture appropriated to the purposes of magnificence, as well as utility” (1838, 15). His own designs are simple, graceful, and elegant. All of his writings, and accounts of his works, indicate that these were the driving factors behind his designs: judgment based on practical experience and a desire to create structures that contribute to their surroundings in both usefulness and beauty.

4.3.1. Practicality

The practical benefits of the bridges’ designs are clear. The choice of cast iron in the first place was mainly due to the fact that a larger arch was achievable with cast iron than with masonry. This was important in both cases because it allowed Telford to cross each river with a single arch, rather than several smaller arches, as would have been necessary for masonry. At Bonar, fast-moving water would have been a danger to intermediate piers (Sixth Report of the Commissioners for Highland Roads and Bridges 1813). At Tewkesbury, flooding was a substantial concern, and soil conditions were imperfect, making it impractical to place intermediate foundations (Telford, Life of Thomas Telford, Civil Engineer 1838, 256). Although Telford used these occasions to push the boundaries of design, the fundamental basis of the single arch design was practicality.

The ribs are cast in sections, and the choice of an arc, rather than a parabola, as well as a constant cross-section, meant that a single mold was required to cast each section of each rib. In contrast, Rennie varied the cross-section of his arch, in accordance with the new structural theory that the arch takes more force at its springing than its crown, although the variation was well out of proportion to the variation in stresses (Ruddock 1979). Telford seems to have been aware of this concept, but considered it an “unnecessary refinement” (Paxton 1975), with practicality outweighing theory, as was so often the case for him. Indeed, the rib castings for Bonar were able to be reused again and again, for later bridges of the same dimensions, throughout Telford’s career (Paxton 1975). This is an indication of a broader characteristic of Telford’s bridges. He was perfectly comfortable with using the same design several times over, with few refinements, because it was a practical solution. He was a busy man, with many projects under way at all times throughout his career. By using a design that he had already found to be successful, he could complete projects more quickly and with confidence. This is perhaps the most significant reason for the lack of variation in his cast-iron bridge design later in this career.

4.3.2. Aesthetics

"Something like a spider's web in the air," Southey wrote of Craigellachie Bridge, which has the same form as Bonar. The airiness and grace of Telford's bridges are unparalleled by those of his contemporaries, and throughout his work it is clear that beauty and magnificence were major drivers of his designs. He even wrote a section on "Civil Architecture" for the Edinburgh Encyclopedia, quoting Alison's *Essays on the Principles of Taste* (Ruddock 190). Throughout his career, however, Telford made it clear that his view of a beautiful bridge was one whose form was structurally expressive – which is why Billington cites him as a pioneer in structural art (Billington 1983).

Telford also realized the effect of aesthetics on public acceptance of the new style of bridges. He wrote that it was essential to convey the appearance of strength and stability, for instance by using radially-oriented spandrels, as at Bonar, instead of vertical ones, such as those used by Rennie and Jessop, which he critiqued for giving a bridge a "crippled" appearance (Ruddock 1979, 162). Of course, his own spandrels appear almost impossibly slender by modern standards, a decision again governed almost certainly by aesthetics as much as by weight, in an aspiration towards lightness and simplicity. This highly individual sensibility is evident in both designs.

Guided by this sensibility, as well as by practicality and experience, Telford relied principally on his judgment, even when pursuing new and unprecedented projects. He was always careful to proceed only when he was certain of success, but in spite of the fact that he was often pushing into new territory, he was always confident in his own ability. "It occurred to me...that no very serious difficulty could occur" he wrote of Pontcysyllte Aqueduct, which stretches an unprecedented 1007 ft above a valley floor 70 ft below (1838, 42). His instincts never led him astray, for none of his structures collapsed before their time, and many still stand as a testament to his ability.

This assurance in his own judgment presents a powerful argument for the similarity of the two bridges' designs. In the very early days of his career, Telford refined his bridge design dramatically, making changes to combat problems as they arose; for instance, the cracking of the Schauffhausen arch. Remarkably quickly, however, he settled into his own distinctive style. Bonar

Bridge combined practical advantages with visual appeal, and it appears to have struck Telford as a design that needed no further improvement. He proceeded throughout the rest of his career without varying this design much, adapting it as necessary for various situations, but apparently confident that there was no need to push further away from the design.

The major exception, of course, is the Menai Strait Bridge. Telford actually sketched designs for Menai in 1811 using the same style of cast-iron arch bridge, stretching an incredible 500 feet; however, by the time the bridge actually went ahead in 1826, he chose a revolutionary suspension design instead. For his cast-iron bridges, however, Telford never pushed forward a dramatically new design after Bonar. From a practical point of view, this meant that they were easy to design, manufacture, and erect. It also shows a confidence that he had chosen the best design possible; an independent conviction that was entirely typical of Telford.

5. Structural Analysis of Telford's Bridge Designs

This paper investigates and compares several components of the design of Bonar and Mythe Bridges, and seeks to answer the following questions:

1. What are the implications of the bridges' overall form? How does the change in dimensions between Bonar and Mythe influence load path and structural efficiency?
2. Are the ribs sufficient to withstand all vertical loads alone? In other words, is the addition of the secondary iron an unnecessary addition to the weight and cost of the bridges?
3. Does the deck contribute to the bridge's ability to withstand vertical loading through deck-stiffening effects?
4. Is the spandrel bracing sufficient to transfer vertical loads? How does the change in brace orientation between Bonar and Mythe influence load path and structural efficiency?
5. Globally, how reasonable were Telford's design decisions, and how effective were the designs of these two bridges, in terms of boldness, safety, and efficiency?

5.1. Overall Form

Both Bonar and Mythe Bridge follow the traditional arched form, whose origins are well-established in masonry design. As discussed above, both have ribs that are sections of a large circle, connected via lattice bracing to decks that are sections of an even larger circle. The most immediately noticeable difference between the bridges, however, is their overall dimensions. Mythe Bridge is 20 ft longer, but with a 3 ft lower rise – therefore forming a much shallower arch, with a rise/span of 0.10, compared with Bonar's 0.13. Architecturally, this results in a much more modern-looking bridge. It is less spectacular than the dramatic arch of Bonar or Craigellachie, with smaller abutments and a flatter deck that blends into the landscape of the roadway, much like modern road bridges. Bonar stands out in a different way – it incorporates a steeply inclined road that takes the traveller high above the river, dominating the landscape. This advancement from the more dramatic but less efficient, to the decidedly more modern, is immediately felt when approaching each bridge, and may be expressed through the photographs of Craigellachie Bridge (Figure 5.1) and Mythe Bridge (Figure 5.2) below.

Figure 5.1: Craigellachie Bridge (Lane 2017)

Figure 5.2: Mythe Bridge (Lane 2017)

The structural implications of this difference, however, are also substantial. A shallower arch clearly leads to an increased horizontal thrust on the abutments (Heyman 1995). Telford had been interested in the thrust of an arch for some time. He criticized the bridge at Coalbrookdale, whose arch he considered too weak to sustain the lateral earth pressure on the abutments exerted by the steep walls of the gorge (Telford 1838, 29); indeed, this pressure led to a rise of several inches at the crown of the bridge at Coalbrookdale (Telford 1812, 488). Testing on an iron arch in 1795 confirmed the importance of firm abutments (Paxton 1975, 61). On the other hand, his 1801 design for London Bridge was ultimately scrapped because the immense lateral pressure exerted by the arch would require abutments of vast proportions. This bridge had a proposed span of 600 ft and a rise of 65 ft, for a rise/span ratio of a mere 0.108. The stability of the abutments was a concern voiced by many of Telford's correspondents, and ultimately the cost and disruption of raising such abutments was found to be simply unfeasible. When constructing Bonar, he wrote that "In constructing the Masonry abutments, it is of importance to have a firm foundation, good materials and workmanship, forming altogether a mass of sufficient stability to resist the lateral pressure of the arch" (Sixth Report of the Commissioners for Highland Roads and Bridges 1813, 37). It was clear to him, therefore, that by decreasing the rise, he was increasing the thrust of the arch. Perhaps it is for this reason that the bridge at Bonar takes such a conservative form; it is curious, and rather sad, however, that Telford never again attempted anything approaching the daring shown in his 1801 proposal.

On the other hand, while conservative in radius when compared with Mythe Bridge, Bonar is further from a parabolic shape, the ideal form of an arch under uniform loading. A parabolic arch under uniform load experiences only axial compressive stress, with zero moment, and is thus an ideal arch. Figure 5.3 provides a comparison of the rib dimensions of the two bridges, set against an ideal parabolic arch with the same rise and span, while Figure 5.4 shows the absolute deviation of each from this parabola. It is clear that Bonar deviates more from the ideal shape, so it may be expected that the thrust line varies from the centroid of the rib more than for Mythe. However, it is important to note that, in an era when thrust line analysis was not yet practiced, these bridges still manage to come very close to following the theoretical ideal shape of an arch, and it is expected that they maintain the thrust line well within the section of the shape under uniform load.

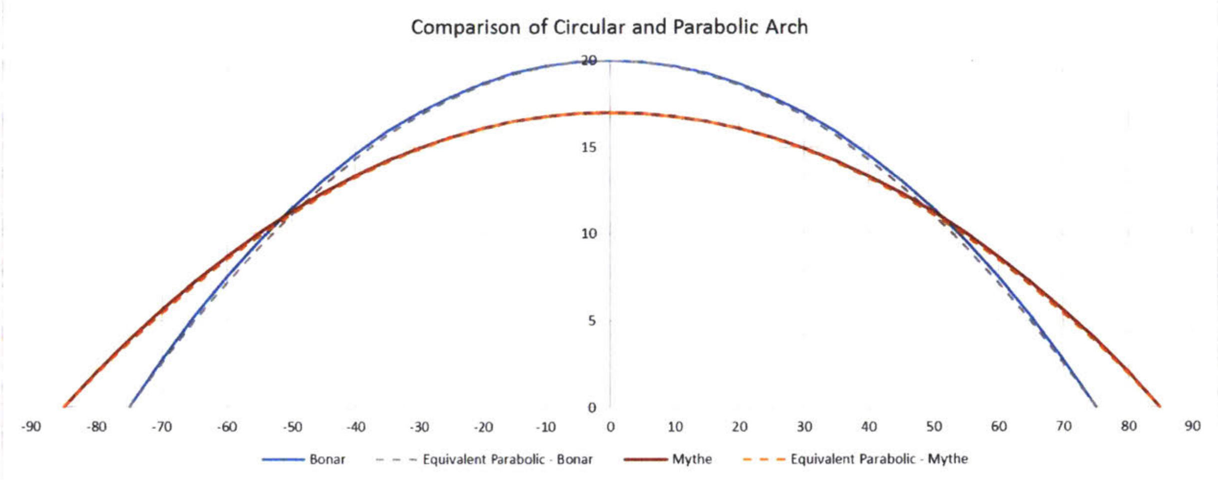

Figure 5.3: Comparison of circular and parabolic arch for Bonar and Mythe Bridge

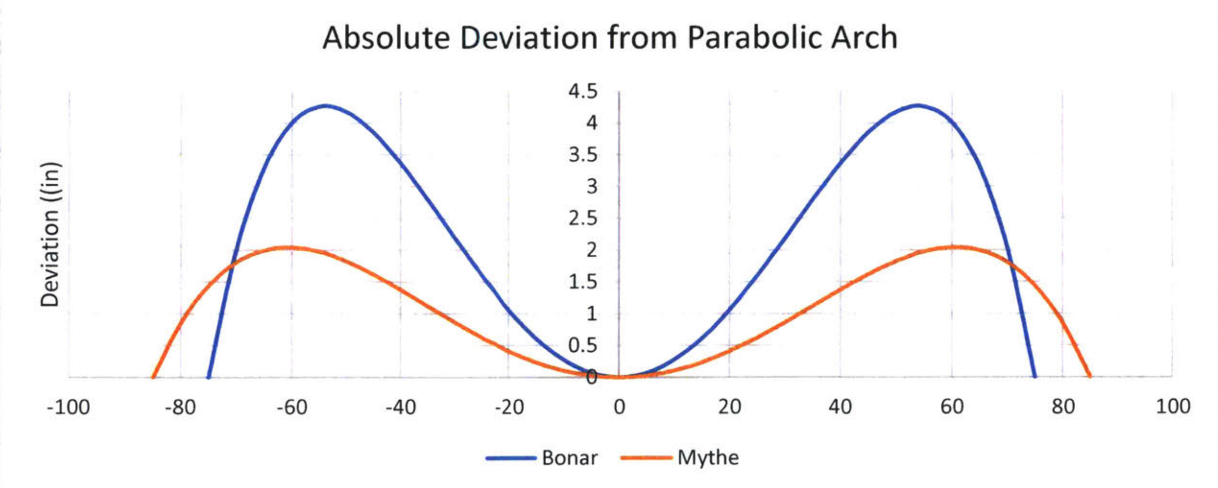

Figure 5.4: Deviation of arches from the parabolic form

These two factors affect the bridge's performance in opposing ways. On the one hand, a shallower arch leads to an increased horizontal thrust. On the other, it conforms more closely to a parabola, the ideal shape of an arch under uniform loading. These two effects tend to counteract one another, but while the thrust has a more substantial effect on the abutments, the shape of the rib has a greater effect on the stresses felt by the arch itself, which is the focus of this analysis. Therefore, the shallower arch of Mythe Bridge is a more efficient shape, when considering the performance of the arch alone, under an assumption of rigid abutments.

5.2. Rib Design

Neglecting the abutments, the ribs are the primary structural element, and Telford evidently envisaged the ribs as taking the bulk of the bridge's loading. However, there are many other structural elements in his design that add to their weight and cost. A relevant lower-bound approach, therefore, is to assess whether the ribs alone could have withstood all vertical loading; in other words, whether the addition of the covering and connecting plates, the spandrel bracing, and the deck were necessary for the performance of each bridge. If the ribs alone can be shown to be sufficient, the bridges are clearly overdesigned for vertical loading; if not, they may still be overdesigned, but less severely so.

Cast iron is a brittle material, with limited tensile capacity, and should not be put into tension, which causes fractures to develop. In addition, based on masonry precedents, designers of this time would have been designing with the intention of maintaining compression in the arch. In "Iron Arches," Jacques Heyman argues that cast-iron bridges can therefore be assessed like masonry structures, and the pure-compression requirement may be imposed (1982). This assumption guides the following analysis.

5.2.1. Global Equilibrium Analysis

The bridges are analyzed under both uniform and asymmetric loading. A first approximation is to consider a single arch as an ideal three-hinged arch and perform a global equilibrium analysis. This assumption may be used as first pass at the problem, but its validity may be questioned based on the rib depth and way in which the rib is connected. A more precise graphic statics analysis follows.

5.2.1.1. Analysis Methodology

Dead loading is determined based on the geometry of each bridge. For Bonar Bridge, a total weight of 606 kips is calculated, of which 399 kips are ironwork, which compares with the 403 kips recorded by Telford (Paxton, Thomas Telford's Cast-Iron Bridges 2007) – an acceptable error of 1%. This load is approximated as a uniformly distributed load of 1.15 kips/ft to an interior rib. For Mythe Bridge, a total weight of 1124 kips is calculated. This results in a dead load of 1.22 kips/ft applied to an interior rib. A live load of 0.6 kips/ft is applied to Bonar and 0.54 kips/ft to

Mythe, following Paxton's calculations and accounting for the difference in tributary area to the rib in each bridge (1975, 388).

Each bridge is analyzed under two loading conditions: first, with both dead and live load applied to the full arch; then, with dead load applied across the whole, and live load applied to only one half of the arch. Under uniform loading, it is expected that the arch will experience mainly compressive stress, and only a small amount of moment, resulting in overall compressive stress. Under asymmetric loading, however, larger moments will be generated, so the stresses in the section will depend on the relative contributions of the dead load and live load. If the bridges are very heavy, the dead load will weigh down the bridge enough that the live load will not cause the bridge to go into tension. If the bridges are light, the moment from the asymmetric live load may be enough to create tension in the rib. It is important to note that this result indicates, not that the bridge will in fact develop unsafe tensile stresses, but that the rib alone is not sufficient to resist loading, and it relies on the secondary members.

Allowable stresses for cast iron can be found in the literature as described below. The cast iron used in these bridges is grey, or hard, cast iron, produced by William Hazeldine. Telford recorded results for the crushing strength, tensile strength, bending strength, and modulus of elasticity in his notebook, quoting the results of various experiments, including those sent to him by William Reynolds and Charles Bage in 1801 (Telford 1838, 686). Sections were also subjected to tensile testing in 1963 by W. A. Fairhurst and Partners during the reconstruction of Craigellachie Bridge (Lowson 1967). Craigellachie, which is close to a copy of Bonar, was cast in 1814 at Hazeldine's foundry (Lowson 1967); Mythe and Bonar were also cast there, so the quality of the iron is likely very similar. Other acceptable ranges based more generally on the literature are quoted by Paxton and Heyman. These results are summarized in Table 5.1 below. As shown, there is a substantial range to the data. Cast iron production at the time was not heavily regulated, so the material differed substantially between foundries, and testing facilities were also imperfect, leading to a wide range of results in early tests. For a conservative result, allowable compressive stress will be taken as 17 ksi, while tensile stress of any magnitude will not be allowed, as discussed above.

Table 5.1: Allowable Stresses in Cast Iron From Literature

	Compressive Strength (ksi)	Tensile Strength (ksi)
Reynolds (1801)	29	-
Telford (1838)	36	42
Lowson (1963)	-	25
Paxton (1975)	17-29	3-8
Heyman (1982)	22	0*
Governing	**17**	**0**

*not because the material cannot sustain any tension, but because it will develop stress fractures

Axial stresses depend on both the compression force and moment in the section, based on the global geometry of the arch and on the rib's cross-sectional area and moment of inertia. The cross-section varies depending on the choice of cut, as shown in Figure 5.5, and has a major impact on the stresses in the rib. Calculations were taken at the maximum and minimum cross-section for each "x" – resulting in a large envelope for stresses, as seen below.

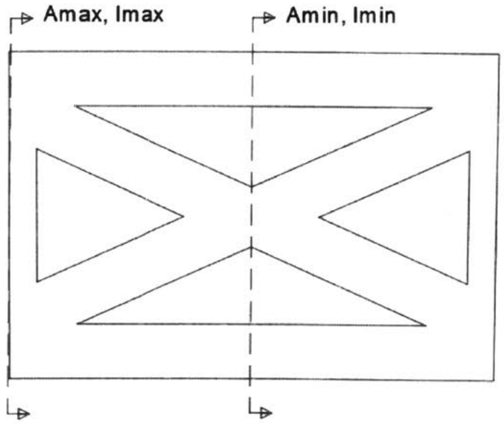

Figure 5.5: Portion of a rib, showing the variation in cross-sectional properties

5.2.1.2. Analysis Results

The results of global equilibrium analysis are summarized in Table 5.2. Under uniform loading, both bridges are safe, experiencing no tension, and low compressive stresses. It is evident that under asymmetric loading, however, neither rib is sufficient. Both bridges experience substantial tension of similar magnitudes, and Mythe experiences compressive stresses very close to ultimate, with an unacceptable factor of safety of only 1.05.

Table 5.2: Global Equilibrium Analysis Results - Rib Stresses for Uniform and Asymmetric Loading

	Bonar Bridge		Mythe Bridge	
	Maximum Stress (ksi)	FS	Maximum Stress (ksi)	FS
Uniform Loading				
Tension	None	N/A	None	N/A
Compression	-6.63	2.56	-9.50	1.79
Asymmetric Loading				
Tension	3.14	0	2.98	0
Compression	-12.1	1.40	-16.2	1.05

Figure 5.6 shows the result of uniform loading for both bridges. As expected, the stresses are entirely compressive, do not vary much across the section, and are well below the allowable compressive stress. It is notable that the stresses are greater in Mythe Bridge, which has a greater weight, but there is more variation in Bonar Bridge, indicating more moment. This follows from the fact that Bonar deviates more from the parabolic shape of the ideal arch. Bonar Bridge actually approaches a zero-stress state, which is a potential concern for the development of tension (Heyman 1982). The large stress envelopes resulting from the changing rib cross-section are very apparent. Note, again, that these are not the stresses actually expected in the ribs, but rather the stresses that would be expected if the ribs were to act alone.

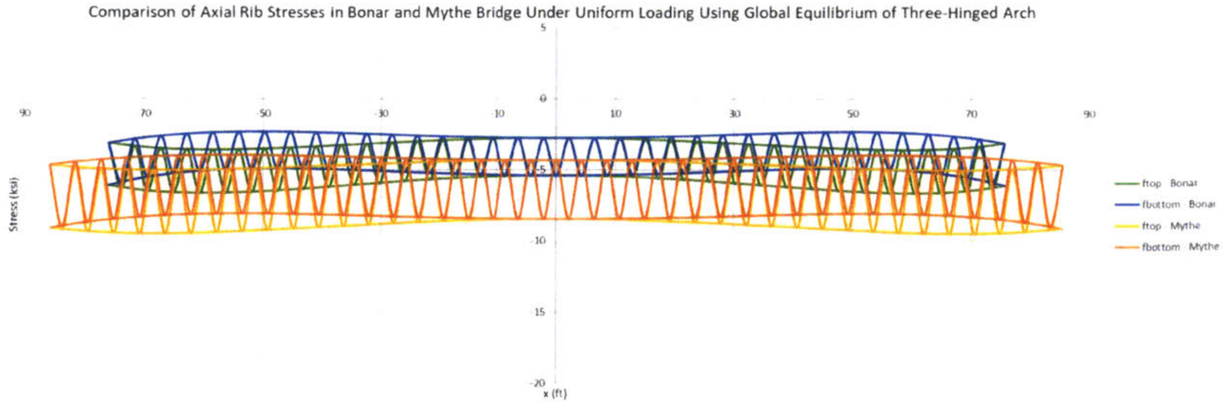

Figure 5.6: Global equilibrium results - uniform loading

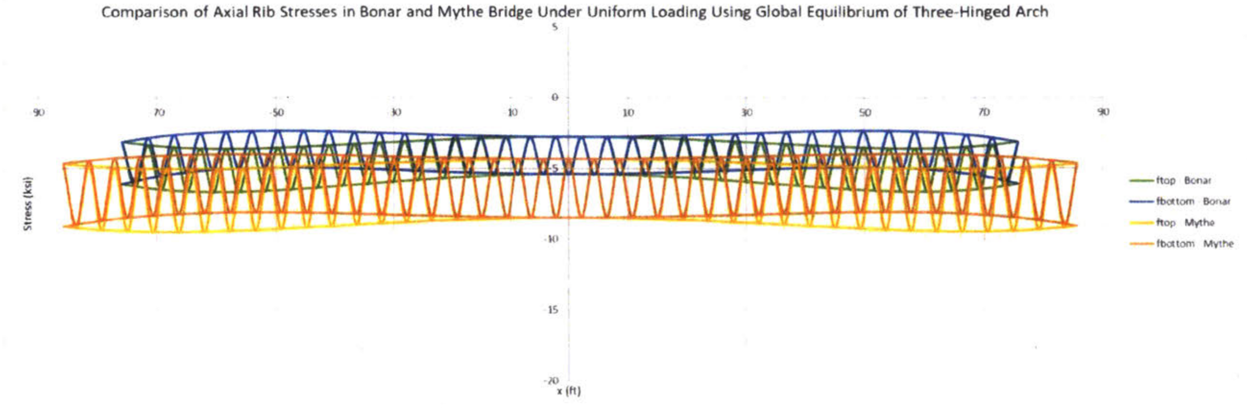

Figure 5.6: Global equilibrium results - uniform loading

The results of asymmetric loading are shown in Figure 5.7 below. The stresses are much higher, due to a substantial increase in moment. In the compressive zone, Mythe Bridge begins to approach unacceptable stresses, due to its substantial weight plus large moments. More critically, it also experiences tension on both sides of the arch. Bonar Bridge, on the other hand, has a low enough weight that it is not in danger in compression, but the large moments it experiences push it even further into the tension zone than Mythe. Its smaller weight is unable to counter the effect of the moment, which is larger than for Mythe because the arch is further from the ideal parabolic shape.

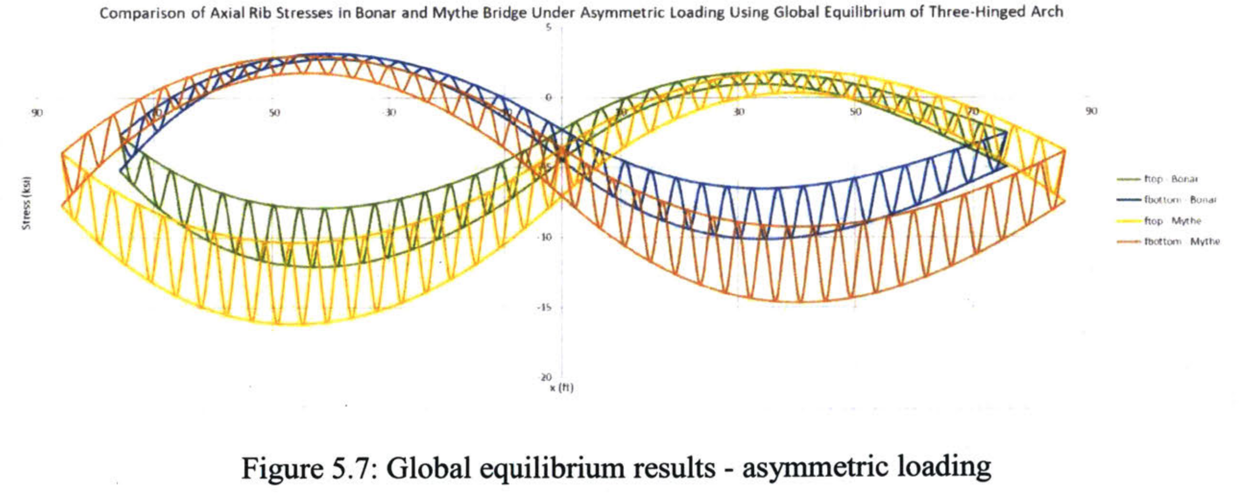

Figure 5.7: Global equilibrium results - asymmetric loading

The global equilibrium analysis therefore indicates that neither bridge is satisfactory using the ribs alone. Under uniform loading, both bridges are adequate, but under asymmetric loading, both bridges experience tension, and Mythe also experiences large compressive stresses. This is an indication that both Bonar and Mythe rely on the secondary framing to resist loading. Overall, Mythe may be said to be more critical, since it experiences both substantial tensile stresses and large compressive stresses. This is a possible indication that that bridge is a more efficient design, thanks to its shape, which is closer to the parabolic, but further investigation is needed.

5.2.1.3. Effect of Support Conditions on Equilibrium Results

Seeing the substantial effect of even small changes in arch shape, an important parameter affecting these results is the choice of hinge location. For an initial assumption, hinges are taken at the center of the section, at the crown and springing points. However, depending on connection conditions, the hinge may be unable to form here. Following Heyman (Iron Arches 1982), the analysis is repeated, forming an upper and lower bound based on hinge location. Figure 5.8 summarizes these three conditions: condition (1) is the max rise/span possible; condition (2) is the average, which was taken above as a first assumption; and condition (3) is the min rise/span.

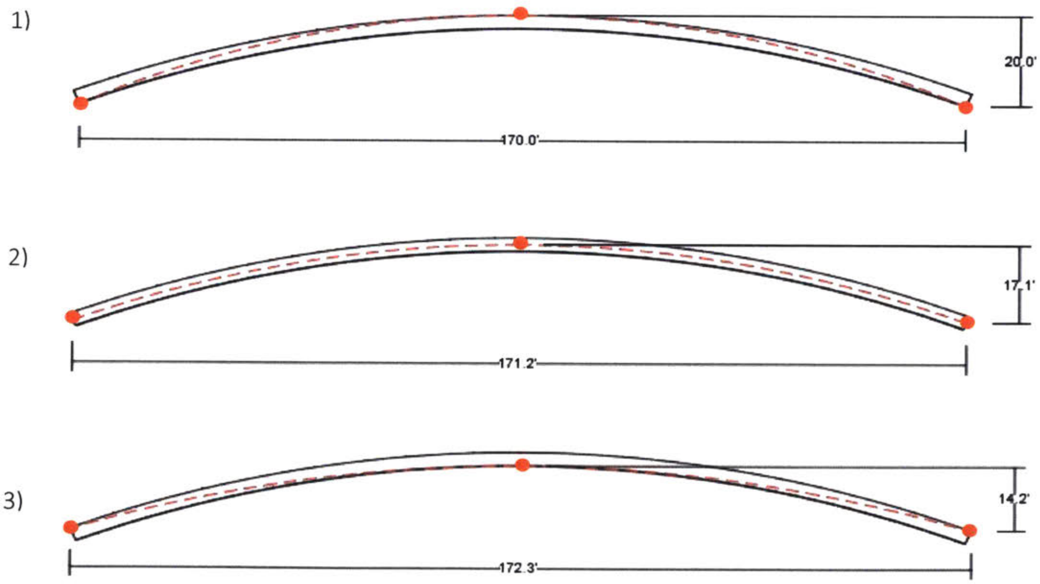

Figure 5.8: Varying assumption of hinge location for Mythe Bridge

The results are shown in Figure 5.9 and Figure 5.10 below. To simplify the graph, an averaged cross-section of 61 in^2 was used for Bonar Bridge, and 68 in^2 for Mythe Bridge. The difference in the cross-sections for the two ribs is due to the fact that Mythe's rib is more solid than Bonar's, although they have the same depth, because the rib's "x"s are more closely spaced.

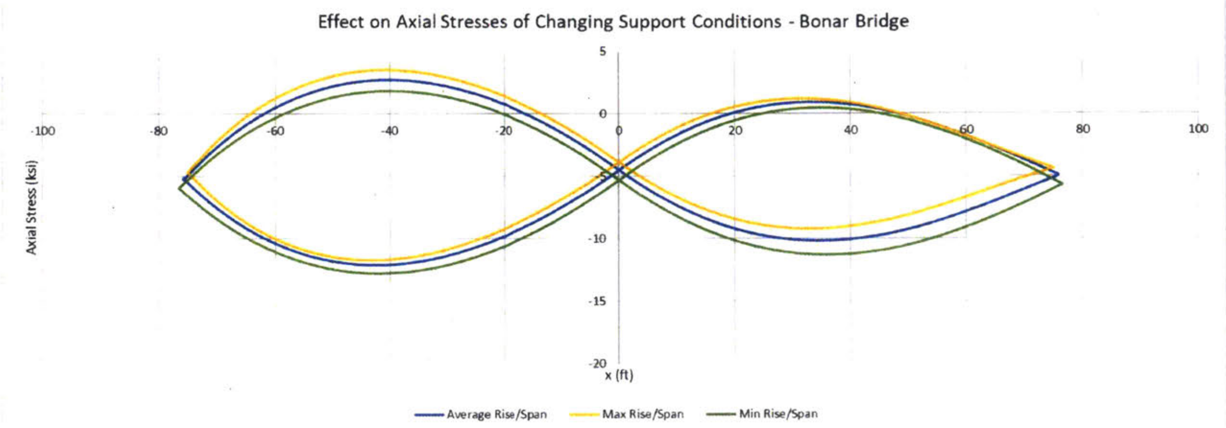

Figure 5.9: Global equilibrium under varying support conditions - Bonar Bridge

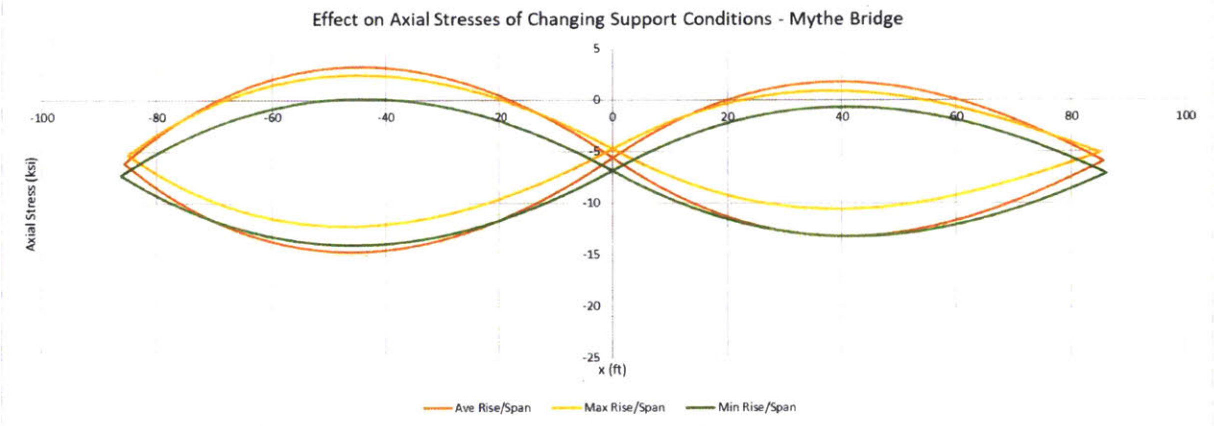

Figure 5.10: Global equilibrium under varying support conditions - Mythe Bridge

It is clear that global equilibrium results depend to a great extent on the assumption of support conditions. Condition (1) results in lower compressive stresses, because of a decrease in axial compressive force. Condition (3) results in higher compressive stresses, because of an increase in axial compressive force. Conditions (1) and (3) represent an upper and lower bound for stresses found using global equilibrium methods. Importantly, for Mythe Bridge, an assumption of condition (1) (upper bound) results in stresses moving well into the tensile region, whereas an assumption of condition (3) (lower bound) is close to a zero-stress configuration. Therefore, it is apparent that the choice of support condition is an important consideration in determining the behavior of the rib under loading, and must be carefully considered.

Site visits indicate that any of these conditions is a reasonable assumption. Full connectivity is maintained for all ribs, both at the springing, and the crown. Following Heyman, therefore, if any of these provides a "safe" configuration, the rib may be said to be safe. The analysis, however, indicates that all three support conditions result in the development of unacceptable tension stresses, so regardless of the assumptions made regarding the supports, the ribs are not sufficient to support the bridge loading alone.

5.2.1.4. Equilibrium Conclusions

All in all, the forgoing analysis indicates that both bridges fail to perform adequately under asymmetric loading, when the rib is considered to act alone. This suggests that the bridges both need the stiffening effect of the secondary members, which connect the ribs, adding substantially to both their axial and bending rigidity. This indication is important, because it shows that Telford did not drastically overdesign these bridges. When comparing them through this analysis, it seems that Mythe is the bolder design – closer to developing unacceptable compression stresses, and approaching similar stresses in the tension zone, still light enough to be affected by asymmetric loads in spite of its more closely-spaced ribs and greater span.

These findings can be compared with the analysis done by W. A. Fairhurst and Partners of Craigellachie Bridge in 1963. The bridge was analyzed by considering the ribs as four elements, tied together and subjected to two 14-ton vehicles, without the composite action of the deck or spandrels. The result was a maximum tensile stress of 3.02 ksi, and a maximum compressive stress of 13.7 ksi. These results compare closely with those calculated above for Bonar Bridge, whose design is extremely similar to that of Craigellachie. However, these are the results for the real bridge, with all secondary members intact, so the tensile stress found by Lowson's analysis is an indication that, not only are the ribs are insufficient to withstand loading alone, they may not be sufficient even with support.

5.2.2. Graphic Statics Analysis

A more precise analysis of the rib behavior may be performed using graphic statics. Plasticity theory indicates that if a structure can find an equilibrium solution, it may be considered the global equilibrium method because it considers the depth of the rib, and the ability of the structure to find multiple configurations.

It is assumed for this analysis that the full rib cross-section is attached to both springing plates and along the length of the rib, and therefore that any configuration can be sustained as long as it falls within the rib. If the rib is not fully connected at any point, loads cannot be transferred through that point, and the configuration is limited. In both bridges, the rib is attached to the springing plate through slotted grooves that the ribs fit into, and by iron cement; there are no bolts tying them together. This may be seen in Figure 5.11. This is a clear indication that compressive stresses will be effectively transferred at the ends of the arch, but tensile stresses will not. Along the length of the arch, bolts are used to completely tie the sections together, so it is reasonable to assume a full section. However, the perforated "x" shape means that load paths are limited, as again stresses cannot be transferred across openings. However, a first assessment may be made using this assumption.

Figure 5.11: Springing plate – Mythe Bridge (Lane 2017)

Under uniform loading, as expected, both bridges perform well. The thrust line fits not only within the section, but within the middle third, for both bridges. Bonar experiences a thrust of 268 kips, while Mythe experiences a substantially larger thrust of 382 kips, in keeping with its larger weight and flatter arch. However, in Mythe Bridge, the thrust line runs almost perfectly along the center of the section, while Bonar experiences more substantial deviation. This is in line with the fact that Bonar deviates further from the ideal parabolic arch, as discussed above. The stresses in the ribs reflect this – Bonar is predicted to experience maximum normal stresses of -297 kips and maximum moments of 147 kips ft, resulting in stresses varying from -2.07 ksi to -7.82 ksi, while Mythe is predicted to experience maximum normal stresses of -420 kips and moments of 95.0 kips ft, resulting in small stress variations between -6.29 and -8.52 ksi. Both bridges experience stresses well within acceptable limits. These results are summarized in Table 5.3.

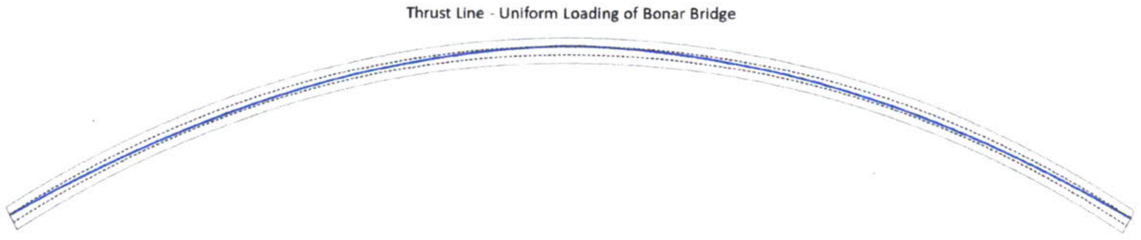

Figure 5.12: Graphic statics – Bonar Bridge under uniform loading, showing thrust line contained within the middle third

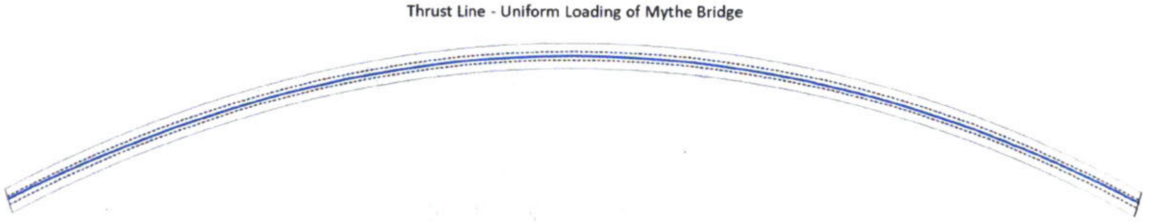

Figure 5.13: Graphic statics - Mythe Bridge under uniform loading, showing thrust line contained within the middle third

Under asymmetric loading, however, both bridges experience difficulties. In both Bonar, (Figure 5.14), and Mythe (Figure 5.15), the thrust line cannot be contained within the section; in other words, the rib cannot withstand asymmetric loading. Bonar performs slightly worse under this analysis, which is in keeping with the results of the global equilibrium analysis.

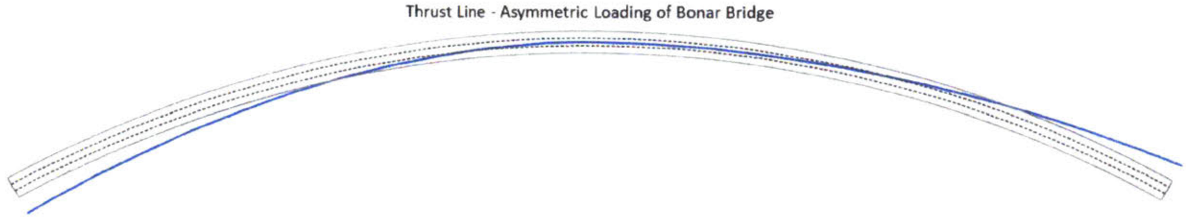

Figure 5.14: Graphic statics - Bonar Bridge under asymmetric loading, showing thrust line exiting the section

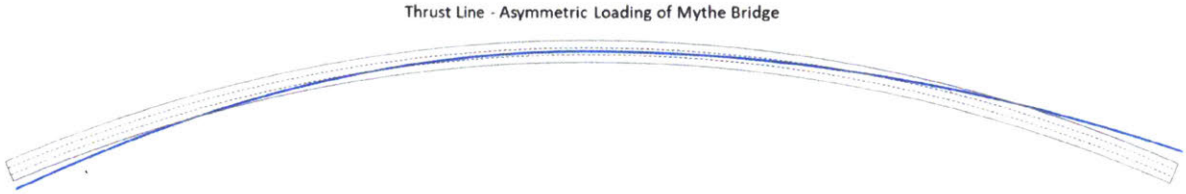

Figure 5.15: Graphic statics - Mythe Bridge under asymmetric loading, showing thrust line exiting the section

The graphic statics approach takes different assumptions than the global equilibrium analysis. It assumes that the rib can develop moment at the edges, rather than the zero-moment assumption of the three-hinged arch. Large moments therefore accumulates towards the edges of the arch, following the increase in eccentricity of the thrust line from the centerline. This results in much larger predicted moments than when the moments are constrained to be zero at the three hinges.

Using graphic statics for asymmetric loading, the maximum normal force in the rib is found to be 256 kips for Bonar Bridge, an acceptable deviation of 6.45% from the global equilibrium analysis result of 240 kips. The maximum moment, however, is found to be a much higher 991 kips ft, compared to 250 kips ft found by global equilibrium analysis. This is due to the large thrust line eccentricity of 3.89 feet at the end of the arch. This means that moment stresses dominate normal stresses, with maximum tensile stresses of 24.9 ksi, vastly different from the 3.14 calculated by global equilibrium, and a wholly unacceptable result. The maximum compressive stress was found to reach a maximum of -36.0 ksi, again much higher than the value of -12.1 ksi predicted by global equilibrium methods, and again an unacceptable result. These results are summarized in Table 5.3.

For Mythe Bridge, under asymmetric loading the maximum normal force of -363 kips again compares well with the -356 kips calculated by global equilibrium methods, for a 1.9% difference. Again, however, extremely large moments of up to 1011 kips ft result in much larger tension and compression forces than the 300 kips ft predicted by global equilibrium analysis. Overall, however, it is notable how extremely similar the results are for the two bridges.

Table 5.3: Graphic Statics Results – Rib Stresses for Bonar and Mythe Bridge

	Bonar Bridge		Mythe Bridge	
	Maximum Stress (ksi)	FS	Maximum Stress (ksi)	FS
Uniform Loading				
Tension	None	NA	None	NA
Compression	-7.82	2.17	-8.52	2.00
Asymmetric Loading				
Tension	24.9	0	22.3	0
Compression	-36.0	0.47	-38.1	0.45

Again, it is important to emphasize that the results found by these analyses are useful for the purposes of comparison, and to provide a benchmark, but do not represent the true stresses experienced by the ribs. The results found for asymmetric loading have little meaning in themselves because the large moments are generated by forces being applied outside the width of the arch – a mechanism that essentially indicates collapse when used for arch design. These result are useful only in showing us the similarity of the values for the two bridges. The predictions for stresses based on the two methods are different enough that neither can be relied upon as accurate predictors. They can, however, be seen as outer benchmarks.

The differences in results found by graphic statics and global equilibrium methods emphasizes the importance of questioning the assumptions of an analysis. By assuming that no moment could be transferred by the springing plate, the global equilibrium analysis predicts vastly lower moments and therefore much less risk of tension developing in the rib. Graphic statics provides a much more conservative result, assuming that large moments can develop towards the edges of the rib. The true springing plate is likely able to transmit some moment, as it is a 3-foot deep plate firmly embedded in the abutment; it is therefore dangerous to assume the three-hinged arch when considering rib stresses.

Both analyses, however, agree that the bridges perform very similarly. The stresses predicted by both methods are remarkably comparable for the two bridges. Both agree that Bonar experiences slightly higher tensile stresses under asymmetric loading, and Mythe experiences slightly higher compressive loading – results that make sense based on the shape and weight of the arches – but both indicate that Mythe's design is not vastly superior.

5.2.3.Rib Design Conclusions

The conclusion of all this analysis, then, is that for both Bonar and Mythe Bridge, the rib alone is adequate to sustain uniform loading, but cannot withstand asymmetric loading. Both global equilibrium and graphic statics agree that for both bridges, the rib is in danger of going into tension when considered alone. Overall, then, this lower-bound analysis concludes that the ribs are not vastly overdesigned; both bridges do rely on the secondary members, as Telford envisaged.

This finding is important, because it indicates that Telford was working close to the edge of what was possible. Despite having no theoretical understanding of tensile and compressive stresses and strains, he was able to construct a rib that was almost, but not quite, sufficient to carry vertical loading, and then to engage secondary members to help with that loading. Telford's work is frequently criticized for being extremely overdesigned, but this result indicates his genuine understanding of the material and the loading the bridge would see.

This achievement is made even more remarkable when situated with respect to Telford's peers at the time. Many bridges had ribs that were either vastly overdesigned, such as Southwark Bridge (Rennie, 1819) or dangerously underdesigned, such as Wilson's Yarm Bridge (1805), whose ribs immediately collapsed when loaded. Telford managed to create ribs that were just slightly below what was needed to carry loading, and then engage the secondary steel, such as the grated covering plates, which were able to provide lateral bracing to the ribs, resist lateral loading from wind and traffic, and increase the effective cross-section of the ribs enough to be able to withstand loading.

5.3. Deck-Stiffening Effects

Having established that the ribs alone are not sufficient to carry vertical loading, but that the addition of the framing provides enough capacity, it is also important to consider whether the deck contributes to the stiffness of the bridge. Telford clearly envisaged the deck as simply sitting on the lozenges, and felt that the ribs, and the secondary steel connecting them, acted together to withstand all loading. Since this has been shown to be sufficient, a substantial increase in the bridge's capacity would be an indication of overdesign. In *Robert Maillart's Bridges* (1979), Billington considers the concept of the deck-stiffened arch. In traditional bridge design, the engineer would expect the arch to take most of the loading for the bridge. Under asymmetric loading, therefore, the arch would experience bending and deflection, as described above. Maillart's contribution, however, was to recognize that a very stiff deck would tie the arch, preventing this deflection, and that therefore the bridge could be designed under the assumption that the deck would take all bending, and the arch all axial forces. He used this principle to create his leading deck-stiffened arches, including the distinctive Schwandbach Bridge design (1933), with a stiff, heavy deck, and a very light, flexible arch.

Billington performs a rigorous consideration of this effect and shows that the contribution of the deck may be roughly assessed by comparing the moment of inertia of the arch (I_A) to that of the deck (I_G). He discusses the way in which a deck that is very stiff, relative to the arch, is able to take most of the bridge loads, allowing for a very light arch and low stresses. Conversely, an arch that is very stiff compared to the deck (no deck-stiffening), relying solely on the arch for strength, allows the deck to be light, also reducing stress, as in typical American designs of the mid-20th century. Maximum stresses result when the arch is about half as stiff as the deck, increasing weight without allowing it to contribute to the strength of the bridge.

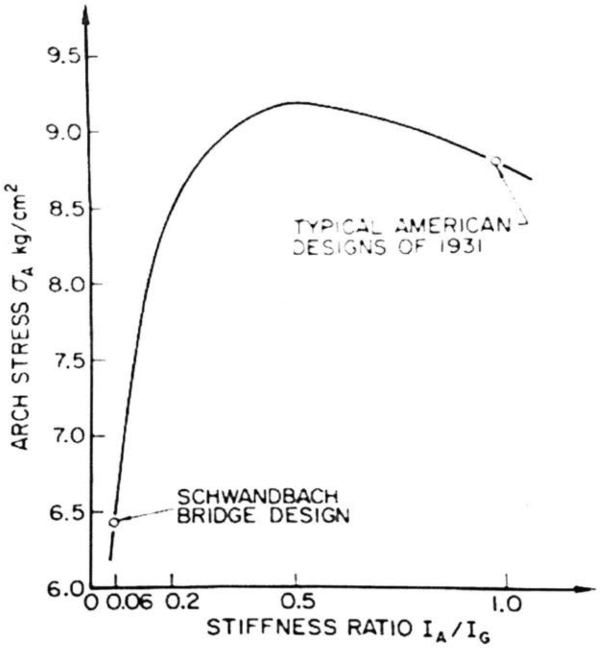

Figure 5.16: Effect on arch stress of the relative moment of inertia of arch and deck girder, from Billington (1979)

Following this logic, this calculation was performed to provide insight into the bridge's behavior. A typical cross-section of each bridge is shown in Figure 5.17. To calculate I_A, the moment of inertia of the ribs alone was taken about the rib centroid, and averaged based on the ribs' varying cross-section. The moment of inertia of the whole frame was then taken, including the ribs, covering plates, and connecting plates, around the centroid of the framing. These were again averaged based on the varying cross-section of each element along the length of the bridge. Calculations may be found in Appendix G.

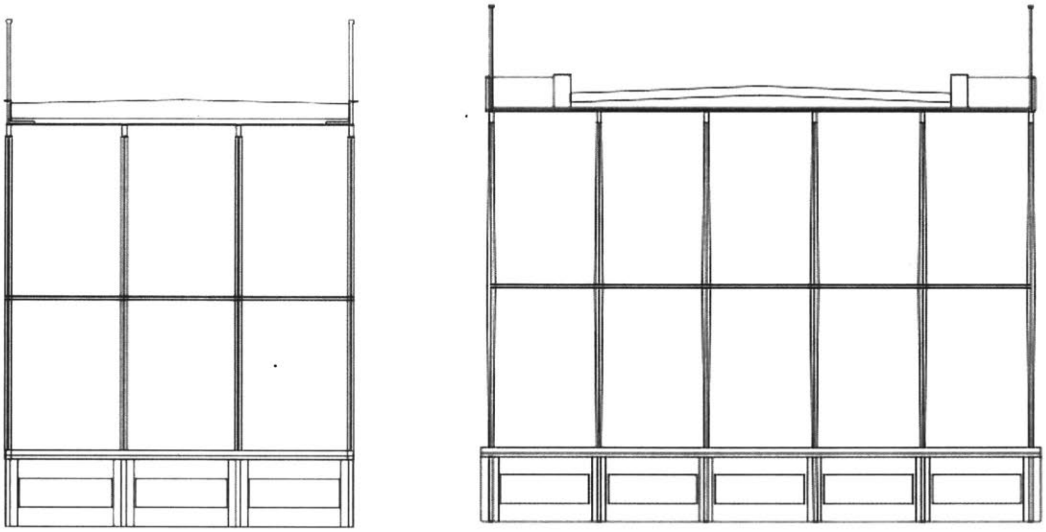

Figure 5.17: Typical cross-section of Bonar Bridge (left) and Mythe Bridge (right) (Lane 2017)

The choice of deck elements contributing to bridge stiffness requires slightly more thought. In Billington's work, the bridges were cast concrete, and therefore the whole deck could be considered to act as a single girder, neglecting the small contribution of the roadway. In Telford's bridges, however, the deck is composed of a very thin cast-iron bearing plate, resting on bearing ribs. In the original designs, a large amount of crushed stone covers the plates, making up the roadway and footpaths; these have since been replaced by pavement. The railings are relatively large and heavy, made of cast iron, anchored in large stone piers at either end of the bridge. An upper bound for deck stiffness could be found taking into account all of these components. However, the crushed stone, which makes up the majority of the deck's cross-section, has a very low modulus of elasticity, taken as 30,000 psi. This is in comparison with the modulus of elasticity of cast iron, which was found during testing of Craigellachie Bridge to be between 16.3 and 16.9 x 10^6 psi. An equivalent section could be found using a scaling ratio of $n = E_1/E_2 = 1/548$, but the result is a negligible section; in other words, the roadway is insufficiently stiff to carry load, when compared with the cast iron.

The railings, meanwhile, are far removed from the center of mass of the deck, so it is possible that they will not be able to contribute substantially to the load-bearing. However, they are sturdy, and the major posts are connected directly to the bearing plate, at every point where the spandrels join the bridge. This provides a direct line of action between the spandrels and the major posts.

This connection may be seen at Craigellachie Bridge in Figure 5.18; although these are new railings, replaced in 1963, the positioning is the same as the original for both Craigellachie and Bonar (Lowson 1967, 28). The minor posts are connected via the skirting plate, and the railing ties together all the posts at the top. In both bridges, the railings are anchored at either end of the bridge by large stone posts. The secure connections between the railings and the deck make it hard to discount their contribution. An upper and lower bound is taken for the deck stiffness, based on whether or not the railings are taken into account.

Figure 5.18: Railing post, skirting plate, and bracing connections (reconstructed) – Craigellachie Bridge (Lane 2017)

This assumption has a substantial influence on the expected stiffness of the deck. The roadway bearing plates are astonishingly thin, only 3/4 in thick for Mythe, and 7/8 in thick for Bonar, and their moment of inertia is extremely low. When the railings are considered, however, their substantial distance from the centroid of the section gives them a large moment of inertia, relative to their size, increasing the stiffness of the deck by an order of magnitude. The stiffness of the bridge elements is summarized in Table 5.4.

Table 5.4: Stiffness of Bridge Elements for Bonar and Mythe Bridge

	I (x 10^3 in^4)	
	Bonar Bridge	Mythe Bridge
Ribs	54.9	81.8
Frame	88.0	141
Deck (no railings)	4.91	5.99
Deck (with railings)	36.9	38.5

These values are then compared for both Bonar and Mythe Bridge, and the results are summarized in Table 5.5 and Table 5.6 respectively.

Table 5.5: IA/IG, Depending on Stiffness Assumptions – Bonar Bridge

Arch Girder	Ribs only	Frame
Deck (no railings)	11.2	17.9
Deck (with railings)	1.49	2.38

Table 5.6: IA/IG, Depending on Stiffness Assumptions – Mythe Bridge

Arch Girder	Ribs only	Frame
Deck (no railings)	13.6	23.6
Deck (with railings)	2.12	3.67

It is evident for both Bonar and Mythe Bridge that the stiffness of the rib dominates the bridge behavior; under all comparisons, the ratio I_A/I_G is greater than 0.5. When considering the frame, compared to the deck without the railings, the proportion is a massive 32.2 for Bonar, and 44.5 for Mythe. Even if the railings are also considered, the ratio for both bridges is more than twice the maximum value of 1.0 shown on the chart, where Billington situates typical 1931 American designs.

This provides another indication that Telford had a fundamental understanding of the behavior of his bridges. The bridge does indeed depend on the rib and framing only and does not engage the deck for further stiffening. It also provides justification for the very light deck; the rib is designed to take all stresses, so the deck is designed to minimize weight, and therefore stresses. In Lowson's article describing the reconstruction of Craigellachie Bridge (1967), he expressed concerns that damage to spandrel bracing resulted in a loss of composite action with the deck; however, it appears that this particular result is not of substantial concern, as the bridge already relies very little on this effect.

However, the very light deck does provoke concerns regarding its own performance. During renovations of Craigellachie Bridge in 1902, it was found that the bearing plates were insufficient, and they were reinforced by placing sections transverse to the bridge, at 9 in intervals. When the bridge was reevaluated in 1963, it was found that there was substantial damage to the bearing plates, which had been cracked in two places, and deflected an average of 3/4 in at the center. The two cracked plates were replaced by stainless steel plates, but the other original plates were returned to use. A concrete slab deck was cast over the plates, to a depth of 11 ¼ in, for increased stiffness (Lowson 1967).

Mythe Bridge was also renovated in 1923, and a concrete deck was added, but the original deck plates remained in place. Inspections in 1990 indicated that the deck was not strong enough for modern loads, and a 17-ton weight limit was enforced (Mythe Bridge 2017). Although clearly both bridges' decks needed reinforcement, suggesting they had been designed close to the minimum possible thickness, they stood without reinforcement for almost a century.

Both bridges are globally well-designed to minimize weight, with a very light deck that does not contribute to the global load-bearing capacity, and a heavy rib frame that takes the bulk of the loading. This is entirely in line with Telford's own vision, and indicates that he had a strong understanding of the bridges' design. The performance issues with the bearing plate thickness indicate that a slightly thicker plate could have been used to avoid local buckling failures; however, it could be argued that these issues are an indication that Telford knew exactly how far he could push his design without triggering collapse.

Finally, it is important to remark that the stiffness ratios are comparable for the two bridges, another indication of the similarity of the bridges' design. The stiffness of each element is virtually unchanged between the two bridges, but Mythe's higher width and more closely-spaced ribs provide a higher value of I_A, while the railings, which are very similar for the two bridges, have a proportionately greater effect on I_G for Bonar Bridge, since it is narrower, and therefore has a lower moment of inertia from the deck itself. These effects work together to give Mythe a substantially higher ratio of I_A/I_G in all comparisons.

The I_A/I_G ratio must be used as a measure of efficiency only with caution. In this case, the deck has been shown, by performance issues, to be close to the smallest possible stiffness. If this is taken as a baseline, a higher I_A/I_G ratio is achieved by increasing the weight of the arch, which actually reduces the efficiency. Billington was considering the problem from the other end; looking at reducing the deck stiffness, on the one hand, or the arch stiffness, on the other, both of which would reduce overall stresses. In this case, both bridges have remarkably thin decks; so the higher I_A/I_G of Mythe Bridge is probably indicative of an excessive arch stiffness, rather than a superior deck lightness. On the other hand, the greater span of Mythe Bridge requires a greater stiffness – a factor that is not taken into consideration by this analysis. The main value in the comparison, therefore, lies in its clear indication that neither bridge relies on deck-stiffening effects.

5.4. Spandrel Design

Given that the deck does not contribute substantially to the load-carrying capacity of the bridge, it is the job of the spandrel bracing to transfer deck loads to the ribs and framing. Telford's lacy "lozenge lattice" bracing is unquestionably the most iconic feature of his cast-iron bridges. From a modern viewpoint, it is difficult to imagine providing bracing of such incredible slenderness – yet the design is clearly effective, as no global spandrel failure is recorded. A check is performed assessing the bracing's ability to carry axial load, to determine whether the bracing is designed with a sufficient factor of safety. A further calculation considers the bridge's performance and its response to the failure of a single brace or two adjacent braces.

The spandrel bracing is designed in pairs of lozenge lattices, which are connected at their half-lengths by mortice and tenon joints. Of each pair, one extends the full length, while the other is actually made up of two segments, which come together at the joint. This braces the lattices at midspan, to help prevent buckling. The lattices are also well braced at midspan in the other direction by wrought-iron ties encased in cast-iron tubes, running laterally across the bridge, which connect each set of lattices. The connection of the braces to the deck and rib is through mortice and tenon connections, with slots in the ribs and road bearers to receive the braces. The configuration of the braces may be seen in Figure 5.19 and Figure 5.20.

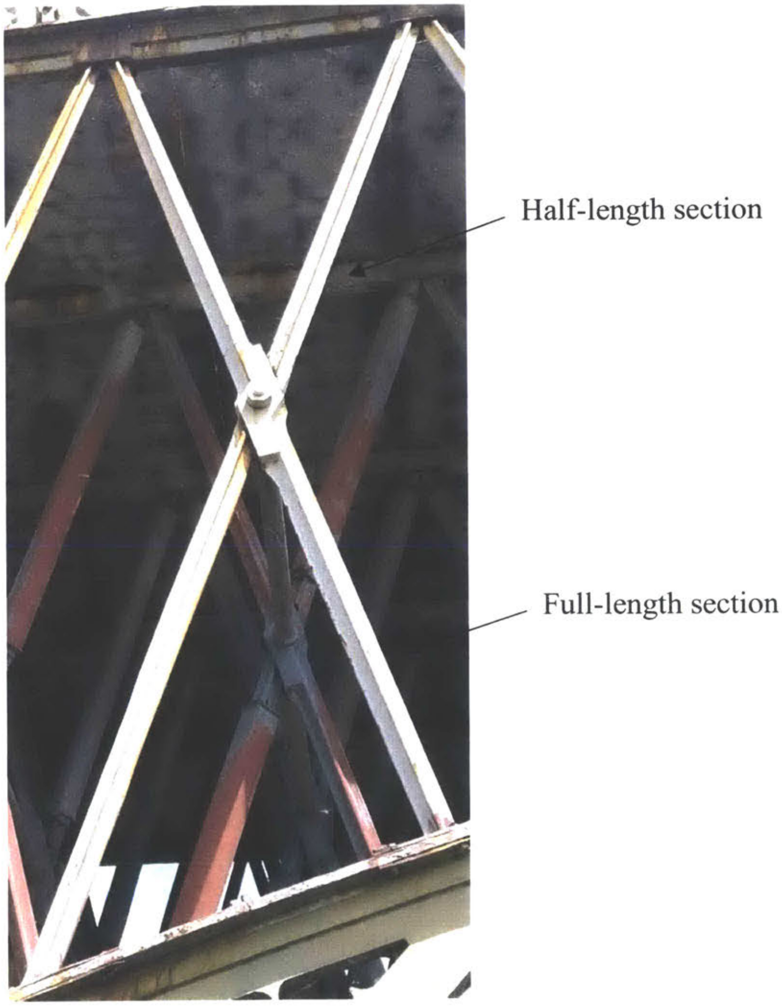

Figure 5.19: Brace construction, showing midspan connection and lateral bracing - Mythe Bridge

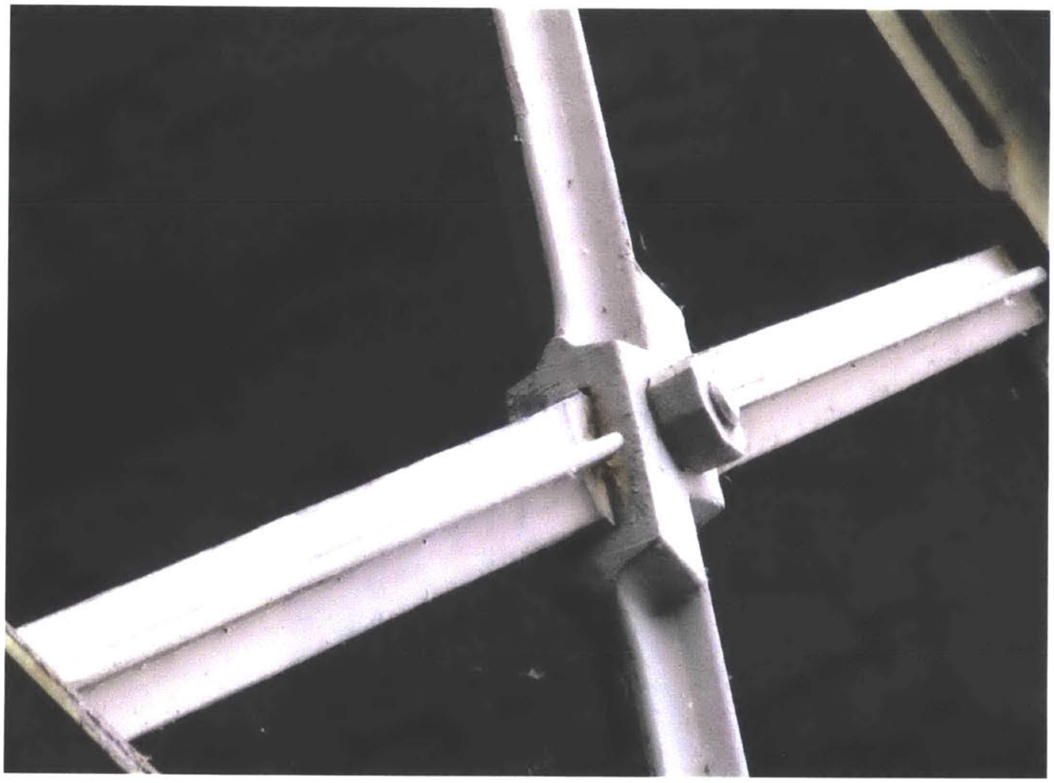

Figure 5.20: Closeup view of midspan connection - Mythe Bridge

Spandrel performance is assessed based on the lattices' ability to withstand tensile and compressive forces and to avoid buckling. It is assumed for this analysis that the spandrels take all vertical loading from the deck down to the ribs, and that the deck does not carry any vertical load. This assumption is slightly conservative, given that the deck is itself an arch, but reasonable based on the flatness of this arch for both bridges, as well as the extreme thinness of the deck plates.

For bridge performance, there is the concern that the joints connecting the pairs of lattices at their midpoints may come undone. This has been recorded at Craigellachie Bridge, and the effect on the spandrel performance is twofold. Without these connections, the unbraced length doubles. In addition, because of the construction of the lattices, only the member that has the full length will remain capable of carrying load; the other, which is actually comprised of two half-length members, will no longer be connected, and therefore a single lattice will have to do the work of two. For the failure of a single brace, these effects will not combine; however, if two adjoining braces both undergo this failure, these effects will combine.

A first check is therefore performed under the assumption of full bracing at midspan, to ensure the loading on the bracing will remain below the critical compressive load for both cross-section failure and buckling. The same calculation is then repeated under the assumption that one joint has failed, leaving a single full-length member, unbraced in the middle, to carry load from the deck to the rib. Finally, the effect of losing two adjacent members is studied, to consider the result if these effects compile.

Acceptable compressive stress for cast iron is taken to range from 17-35 ksi, as shown in Table 5.1 above. The modulus of elasticity was found by W. A. Fairhurst & Partners to range from $16.3\text{x}10^6$ psi – $16.9\text{x}10^6$ psi, based on testing of Craigellachie Bridge (Lowson 1967). Since the iron for all three bridges came from the same ironmaster, William Hazeldine, it is reasonable to use these value for both Bonar and Mythe Bridge calculations. Governing values of $\sigma_{all} = 17$ ksi and $E = 16.3\text{x}10^6$ psi were used. The cross-section of the lattices is 4 in^2 for Bonar Bridge, and taken as 3.5 in^2 for Mythe Bridge – a slightly conservative estimate, since the cross-section tapers from 4 in^2 at the center to 3.5 in^2 at the ends. Full calculations may be found in Appendix H.

The load on the spandrels is due to the load on the tributary area for each rib, applied at the point where each pair of spandrels meets the deck. The design of the bridge is such that its weight is deliberately concentrated in the rib and framing, which means that the deck's dead load is very small compared to the bridge's total dead load. The rest of the load is due to the live load. This results in a small average point load of 5.2 kips for each pair of braces at Mythe Bridge, and 4.6 kips at Bonar. However, the load actually transferred to each spandrel brace depends on the path of the vertical deck loads through the members. The braces are very slender, and may reasonably be modelled as axial-force members – designed to take very little moment, and mainly axial compressive force. In addition, their mortice and tenon connections to the rib and roadway bearers may reasonably be modeled as pin connections, and therefore unable to transmit moment. However, under this assumption, the angle of each lattice has a substantial effect on the load traveling through it. This results in different analysis requirements for Bonar and Mythe Bridges, due to the differences in spandrel orientation.

5.4.1. Spandrel Design of Mythe Bridge

For Mythe Bridge, all the lattices are diagonals, but because of the vertical orientation, each member of a pair has a similar angle. These angles range between 15° and 60° to the vertical, as shown in Figure 5.21. This angle increases the load through all the members, by a reasonable amount in the outer braces, and a large amount in the inner braces. The inner members experience substantially higher loads, but are short enough not to be at risk of buckling.

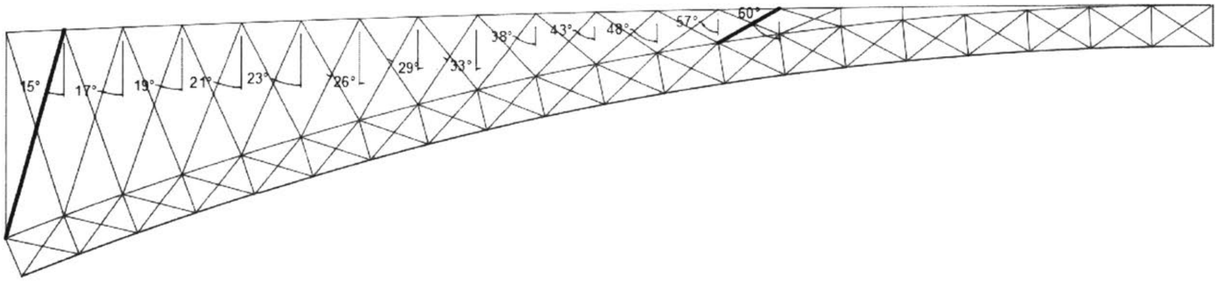

Figure 5.21: Mythe Bridge spandrel orientation, with worst-case spandrels noted

Therefore, there are two clear worst-case pair of braces for buckling – the edge pair, which have the longest length, and which yield a force of -3.03 kips in the left-hand member, and the inner pair, which are the most inclined, and which yield a force of -6.95 kips in the left-hand member. Both these members have very high factors of safety, of 6.77 and 8.56, when compared to the allowable load for buckling and cross-section failure, respectively. This is due to the extremely light deck, as well as the large number of braces, which combine to result in very low loads being transferred through each member.

Even if one member fails, leading to a doubled buckling length, the outer brace holds a very acceptable factor of safety of 1.87. However, if two adjacent members buckle, resulting in a doubled load as well as a doubled buckling length, the factor of safety slips to 1.05, an unacceptable result. This being said, the analysis indicates that it is unlikely that any member would fail, since the working load is so far below the allowable value. Moreover, the high degree of redundancy within the structure suggests that the bridge could likely redistribute the extra forces to find a stable solution in the event of such a failure, especially since, even in this circumstance, the spandrel would theoretically have close to the necessary resistance. This is borne out by the fact that the spandrels have continued to function adequately over a long service life, with no global failure recorded. The three loading scenarios are illustrated in Figure 5.22, and the results are summarized in Table 5.7 below.

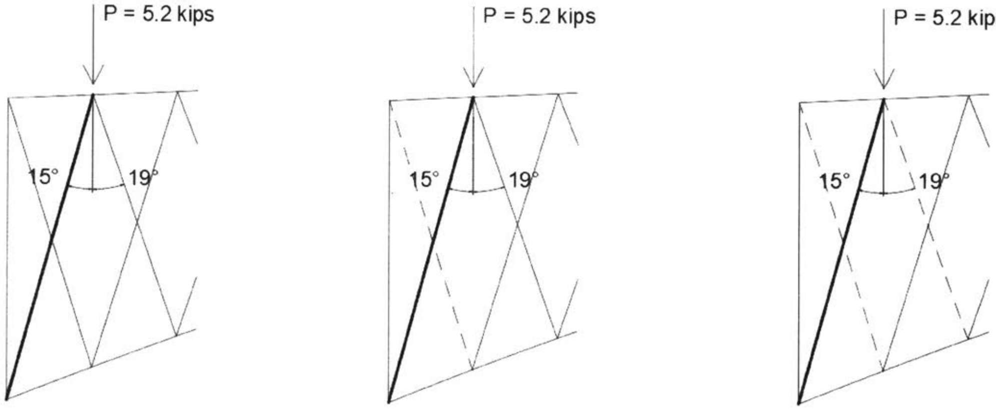

Figure 5.22: Worst-case buckling scenario – braces intact (left), loss of one brace (center), loss of two adjacent braces (right) – Mythe Bridge

Table 5.7: Spandrel Performance - Mythe Bridge

	Maximum load (kips)	Allowable load (kips)	Factor of safety
Outer member – critical for buckling failure	-3.03	-20.5	6.77
Inner member – critical for cross-section failure	-6.95	-59.5	8.56
Joint or member failure – outer member unbraced	-3.03	-5.66	1.87
Failure of two adjacent joints or members – outer member unbraced, load doubled	-5.38	-5.66	1.05

5.4.2. Spandrel Design of Bonar Bridge

Bonar Bridge, in contrast to Mythe, has radially-oriented lattices, creating a more complex loading situation, since the angles of each pair varies so radically. Importantly, for many of the pairs, one member is almost vertical, with as little as a 2° angle, while the other is inclined at a very large angle, up to 49°, as shown in Figure 5.23.

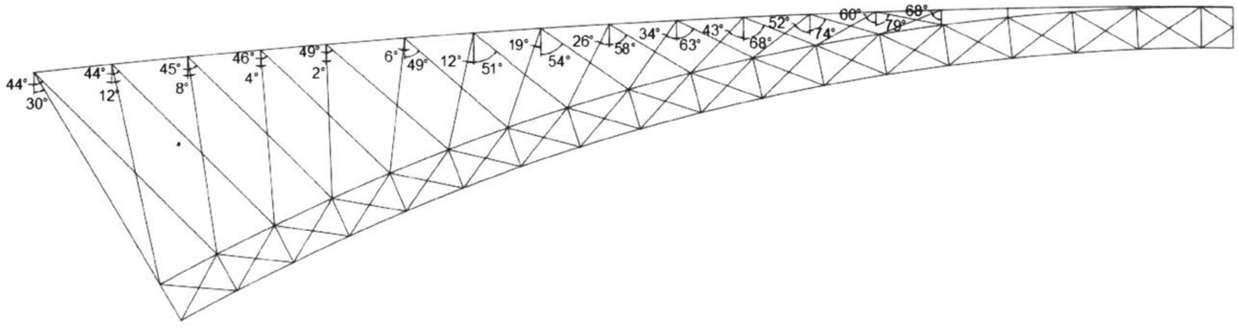

Figure 5.23: Bonar Bridge spandrel orientation

If it is assumed that the deck can take no force, the vertical load is transferred into a large compressive force in the less-inclined member, and, depending on the relative angles of the two members, a small compressive or tensile force in the more-inclined member. If, however, the deck is allowed to take horizontal forces, the contribution of each brace is more difficult to assess.

A reference point is derived by assuming that the deck takes no loading, and therefore each node is a determinate system, with a single vertical load split between the axial loading of two inclined members. This assumption is in keeping with the results derived earlier in this paper, showing that the arched rib is the main structural member of the bridge, and therefore draws most of the load from the deck.

This maximizes the load on the near-vertical members for those near the edges where both diagonals are in the same direction, since additional compression is needed to balance the tension of the more inclined member. In the other pairs, where the diagonals are angled in different directions, the member closest to the vertical takes the bulk of the compressive load, while the more inclined member takes a smaller portion, but still contributes to the compressive resistance. These cases are illustrated in Figure 5.24.

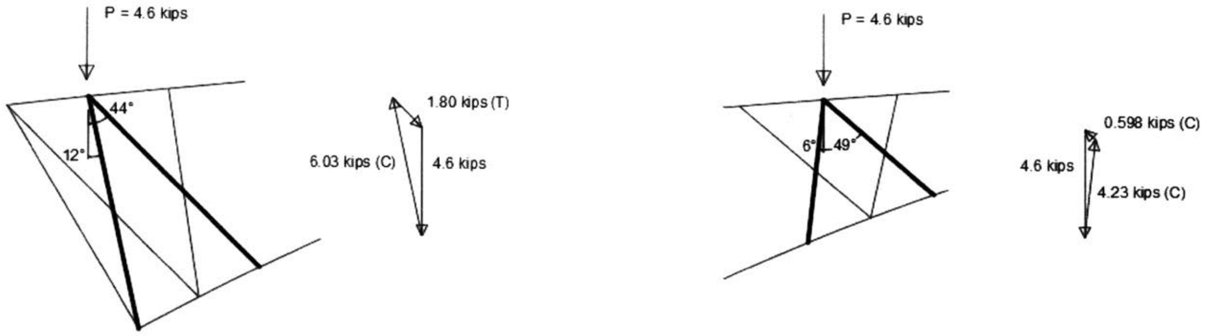

Figure 5.24: Spandrel loading, showing a pair diagonal in the same direction, resulting in tension (left) and a pair diagonal in different directions, resulting in compression (right)

Checking the results for each pair of braces results in the governing values shown in Table 5.8. The critical members are shown in bold and labeled in Figure 5.25. Those members shown by a dotted line experience unacceptable tension forces. Member forces are compared with the allowable values, which are based on buckling as the governing compression condition for member 1, cross-section failure as the governing compression condition for members 3, and the zero-tension assumption as the governing tensile condition for member 2.

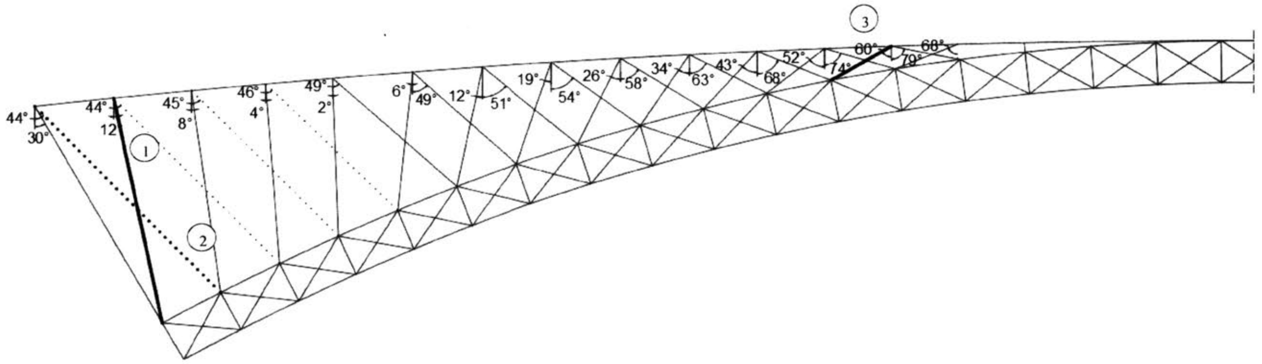

Figure 5.25: Spandrel bracing, with critical members shown in bold and tension members shown with dotted lines - Bonar Bridge

Table 5.8: Spandrel Performance - Bonar Bridge, No Failure

Member No.	Maximum Load (kips)	Allowable Load (kips)	Factor of Safety
1	-6.03	-21.0	3.48
2	+1.80	0 (Tension)	NA
3	-6.88	-68.0	9.88

Clearly, the compression members are well within the allowable limits for compressive member forces, since the deck loads on each member are so low. However, several outer members experience tension. Importantly, the development of tensile forces in the more-inclined braces at the edge of the bridge is considered unacceptable in the present analysis.

Under these analysis assumptions, eight members in each line of bracing (32 in total) will experience small tensile forces, up to +1.80 kips. According to Paxton and others, cast iron is able to withstand tensile forces of this magnitude, but Heyman warns against any tension developing, due to the risk of brittle fracture. At Craigellachie Bridge, however, Lowson reported that instead of brittle failure, these members had simply come unhinged at midspan as a result of fatigue and vibration. This indicates that the members did sustain some tension, and that the joints were the weaker component in the design. Nonetheless, the result of tension in these members is a substantial criticism of the Bonar design, and the most significant way in which the design of Mythe is superior.

A final load case to be considered for Bonar Bridge is the case where the joints have failed or members have snapped, and one brace is forced to support the full compressive load, without any bracing at midspan. The most important concern in this case is the possibility of progressive collapse – if the failure of a single brace or joint causes its partner to be insufficient to withstand the new loading, this could trigger global collapse.

However, at Bonar, unlike at Mythe, the loss of an outer half-member actually results in a lower force in the remaining member sharing that point load, since it will no longer be forced to counter the tensile force. In this case, it must be noted, it is assumed that the deck carries a small horizontal load to allow for equilibrium. If the tension member with the same loading point as the outermost compression member fails, the force in the outermost compression member is therefore reduced from -6.03 kips to -5.32 kips, resulting in an even larger factor of safety of 3.95. However, if the outermost tension member fails, as shown in the second case in Figure 5.26, it results in an increased buckling length for the outermost compression member, which results in a massively decreased allowable load of -5.25 kips. This results in an unacceptable factor of safety of 0.87. If the two half-members adjacent to the outermost compression member both fail (the third case in Figure 5.26), the release in tension helps to somewhat counter the increased buckling length, as discussed above; the factor of safety rises to 0.987 – still an unacceptable result. These results are summarized in Table 5.9.

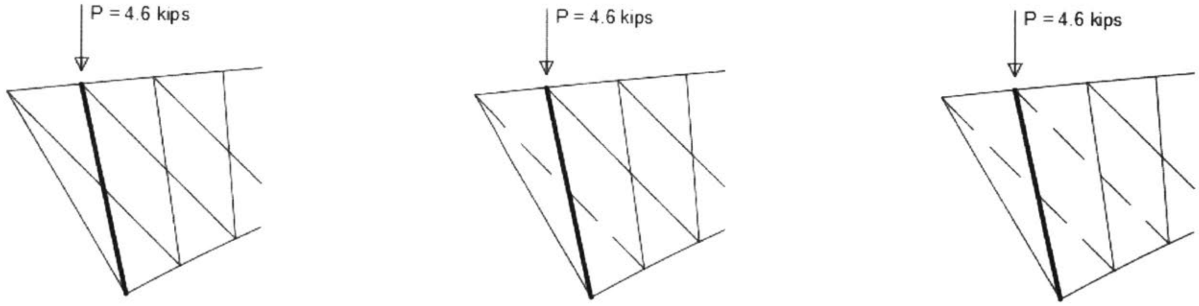

Figure 5.26: Exterior brace buckling scenario – braces intact (left), loss of one brace (center), loss of two adjacent braces (right) – Bonar Bridge

Table 5.9: Spandrel Performance – Bonar Bridge, After Failure

	Maximum Load (kips)	Allowable Load (kips)	Factor of Safety
Failure of outermost tensile member – compressive member unbraced	-6.03	-5.25	0.871
Failure of next tensile member – compressive member takes full load	-5.32	-21.0	3.95
Failure of both outermost and next tensile members	-5.32	-5.25	0.987

In fact, the loss of several adjacent half-members was observed at Craigellachie, but no global failure took place. It is important to account for the high level of redundancy of the bridge, which likely enables it to find an acceptable solution, even if individual members are close to buckling. Moreover, only the compressive member at the outermost edge of the rib is long enough to be in danger; the next member has an allowable force of -8.74, giving it a relatively comfortable factor of safety of 1.45 with the tension member intact, or 1.67 without the tension member. This suggests that global collapse is unlikely; however, the outermost braces are still vulnerable to damage due to buckling. This result is confirmed by Lowson's report, which indicates that the outermost compression members at Craigellachie were grossly overstressed (Lowson 1967, 25).

These results demonstrate several things. For the design scenario, with no failure of the joints, the spandrels at Bonar are somewhat more appropriately designed for compressive loading than those of Mythe. The factor of safety is 3.5, still conservative, but much better than Mythe's larger 7.6. It is clear for both bridges that the spandrel bracing, while extremely slender by modern standards, is actually more than sufficient to transfer compressive loading from the deck to the ribs, due to the very small loads and the large number of braces. This justifies Telford's design decision – the heavy rib, which takes most of the loading on the bridge, allows for a very light deck, which in turn allows for extremely slender spandrel bracing.

However, the major design flaw at Bonar Bridge is the fact that, at the outer edges of the bridge, pairs of spandrels both inclined in the same direction lead to the development of tension forces, if it is to be assumed that no load is transferred through the deck. This is an unacceptable result, and in this way, Mythe Bridge has a much better spandrel bracing design. Moreover, this increases the vulnerability of the outermost members to in-plane buckling, by increasing their compressive load, as well as increasing the risk that these members will become unbraced due to the loss of a tension member. The vulnerability of these members to collapse is a major weakness of Bonar's deisgn. Overall, the spandrels are much better placed to efficiently transfer load in Mythe Bridge. However, if no failures occur, the struts are sufficient to carry the loading in both cases, and the lack of efficiency does not massively impact the performance of the structure overall.

Finally, it must be noted that the assumption that the braces act as axial-force members is unconservative, as it fails to account for the development of shear and moment in the braces, both of which result in unacceptable tensile stresses. This effect was observed directly at Craigellachie by W. A. Fairhurst and Partners. It was found that several of the members near the center of the bridge, which have the greatest incline to the vertical, had actually fractured completely due to the development of shear tensile stresses (Lowson 1967). This is a powerful indication that the zero-tension assumption is a valid one for analysis, but does provide an indication that this truss analysis is very simplified and fails to fully address the brace behavior. Further, Lowson cites vibration as a concern for the braces – another avenue requiring further analysis.

5.5. Structural Analysis Conclusions

Through investigation of the various structural members of both bridges, it is possible to make several conclusions regarding their design and performance. Firstly, both global equilibrium methods and graphic statics predict that the ribs, if unsupported by secondary structure, would develop tensile stresses in both bridges under asymmetric loading. Graphic statics predicts much higher moments than global equilibrium, resulting in much higher stresses for both bridges. This variation indicates the importance of carefully considering support assumptions. Notably, in both analyses, stress levels are very similar between the two bridges, with Bonar seeing slightly higher tensile stresses and Mythe slightly higher compressive stresses. With the addition of the secondary covering plates, however, both bridges see stresses drop to acceptable levels. This is a clear indication that the frame is safe, and that it avoids gross overdesign by requiring the secondary members to be engaged in order to sustain loading.

Telford's vision of the bridge, with ribs connected into a frame that takes all the loading, is borne out by this analysis. It is further justified by the extremely small moment of inertia of the deck's structural members, when compared to the rib's. An I_A/I_G ratio of 2.4 for Bonar and 3.7 for Mythe indicates that neither bridge relies on deck-stiffening effects. Although the deck's extreme thinness poses problems for its own performance, it does not endanger the bridge as a whole. The deck simply rests on the spandrels, sending all loading through them to the rib.

The spandrels, however, experience difficulties in transferring this load. This is the area in which Mythe represents the most substantial improvement over Bonar. In Bonar, extremely long, thin members, often at angles very far from the vertical, develop unacceptable tension stresses, making them vulnerable to failures that could, in turn, induce buckling failure in adjacent members. Mythe Bridge's spandrels appear more than sufficient to carry loading; load is evenly distributed among the spandrels, and no tension is expected to develop. This is true even considering the possibility that one of the joints fails, and the half-length member is no longer able to carry load. However, at Bonar Bridge, if no failure of tension members or joints takes place – and no such failure had been recorded as of 1902 (Lowson 1967, 24) – both bridges demonstrate entirely acceptable factors of safety for compressive loading: 3.5 for Bonar and 6.8 for Mythe.

Overall, structural analysis of the bridges indicates that both are able to sustain the loads for which they were designed. Mythe Bridge provides a smoother flow of forces from the deck, through the spandrels, to the rib, and is therefore more resilient to damage to the spandrels. Apart from this, the bridges demonstrate very similar behavior. Neither bridge is grossly overdesigned in the rib, but for both, the ribs and frame connecting them are by far the strongest part of the structure.

Clearly, the later Mythe Bridge represents a marked improvement structurally from Bonar. Its ribs are less overdesigned, engaging more of its compressive capacity, while its spandrels are less underdesigned, better able to work together to transfer loading to the ribs without risk of failure. Generally, however, the bridges perform very similarly. Both have the same general behavior, make use of the same load paths, and engage the same members to similar extents. Mythe cannot be said to represent any kind of real departure from Bonar. It is simply a newer model, with a few tweaks, a sleeker look, performing slightly better under testing. Its superiority cannot be contested, but it fails to address the fundamental problems of the bridge – the imbalance between the heavy rib and the light deck, and the constraints of the material itself that force the form towards the traditional arched shape relying on compressive stresses. Mythe Bridge is Telford's best cast-iron bridge, and structural analysis shows that he was able to make definite improvements over the course of his career. But it simply doesn't make any new leaps.

6. Conclusion

The interest in studying and comparing Bonar and Mythe Bridges lies in the fact that they bookend the bulk of Telford's career in cast-iron bridge building. The most immediately evident fact, when looking at these two bridges, is that they have a remarkably similar construction. Bonar Bridge was a huge departure from anything that had previously been done, and it was a remarkable leap from Buildwas Bridge, Telford's previous cast-iron bridge. It drew on his research for London Bridge, on his own past projects and experimentation, and, most of all, on his own sense of practicality and aesthetics. But for ten years after this point, Telford's cast-iron bridges were churned out on an assembly line, literally reusing castings to create pre-erected bridges that could be placed at crossings wherever needed. There is the temptation to conclude, therefore, that Telford was not concerned with further optimization, or that he saw this design as the pinnacle of his potential. However, Telford was also a remarkably busy man, and he was, above all, a practical person. Cast-iron bridges took up a very small proportion of his projects, and in all likelihood, he saw this design for what it was – very good, sturdy, attractive – and simply didn't see the point in taking the time and expense to reimagine it. It is reasonable to argue, indeed, that he might never have extended Mythe Bridge the extra 20 feet, had foundations not already been laid, at great expense and difficulty. Telford is perhaps well-characterized, then, as a man capable of vast leaps of imagination and innovation, with truly remarkable works, but disinclined to take such leaps unless he saw them as necessary from a practical standpoint. Having hit upon a successful formula, he chose to repeat it, minimizing time, risk, and expense. This is, perhaps, the true secret behind his ability to achieve truly staggering quantities of work over the course of his career.

Mythe Bridge, then, does not represent a monumental improvement over Bonar. Telford was influenced above all by his own experience and judgment, so the mass of experience he acquired over this period should have allowed him to create a better bridge at Mythe. Instead, he chose to simply tweak an existing design. Mythe shows a moderate improvement in a number of ways. It has a much more modern profile, taking up a smaller arc of a larger circle. In overall form, it moves substantially further from the traditional Roman semicircular arch, much closer to the modern parabolic shape. It is lower, requiring smaller abutments and less disruption to the roadway. It can be shown to be a more efficient design, with the ribs being used closer to their full capacity, the spandrels being stressed more uniformly, and the load paths allowing for a smoother flow of forces. The use of vertically-oriented spandrel bracing, again, moves more towards the modern approach. All of these together indicate that Mythe Bridge represents a considerable improvement over Bonar Bridge's design. But they are still cut very much of the same cloth. After Bonar, Telford made no more dramatic improvements to his cast-iron bridge designs.

Based on the extraordinary vision and creativity he displayed as a designer, there is a strong argument to be made that this was not a failure of ingenuity, but rather a practical choice, by a practical man. Telford recognized the limitations of cast iron; with the Menai Strait Suspension Bridge, he charted out the path of future long-span bridge design, towards tension-based bridges, which relied on material properties that cast iron simply could not provide. He never tried to recreate his London Bridge vision for a massive cast-iron arched bridge. Over the course of his career, he came to realize that, despite his distaste for it, wrought iron provided vastly more opportunities for true innovation in span and form. Telford was the first to truly take the bridge away from the arched form to which it had been tied for millennia. Even as he was breaking these barriers, though, he continued to use cast iron for bridges of an intermediate span. He saw these bridges as elegant and well-constructed. He trusted the ironmasters to provide excellent material, and he trusted his workmen to erect them without issue. They were a practical choice.

Bonar Bridge and Mythe Bridge provide an exceptional, if somewhat unexpected, view of Telford's career. Bonar represents innovation, change, and optimization. It marks an incredible improvement over any of Telford's previous bridges, and points the way forward to a new era of bridge design. It evokes the grand, if slightly awkward, posturing of youth. Mythe, on the other hand, marks the late afternoon of a crowded day, somewhat sleepy, unhurried. It is well-conceived, well-constructed, beautiful, and efficient. It shows signs of experience and the grace of age, a smoothing of the details compared to Bonar. But they are, essentially, of the same spirit. They are of a fleeting generation, made obsolete even as they were perfected, already being overtaken by the new possibilities of an even newer material. But they stand with grace and elegance, in the knowledge that they have played an important part in the march of progress. Telford's cast iron bridges did not undergo substantial optimization between 1810 and 1826. But they did their jobs, and continue to do so, sustaining traffic for decades, while around them bridge design continues to evolve in the quest for innovation that began with the Iron Bridge.

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Appendix A. Original Drawings

Drawings owned by the Institution of Civil Engineers, reproduced with permission

A.1. London Bridge Proposal, 1801

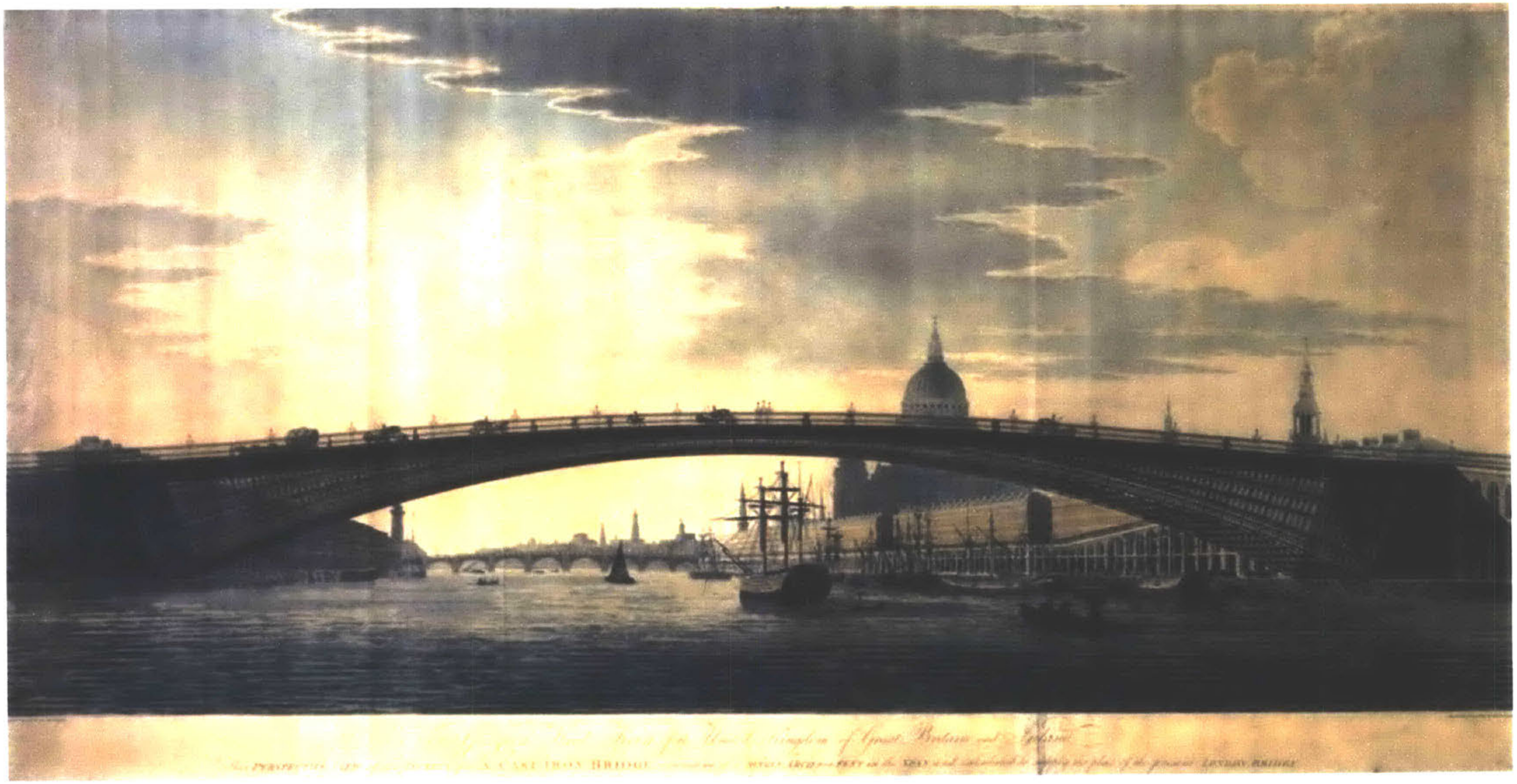


Figure A.1: Plate of Telford's proposed London Bridge *(Thomas Telford 2017)*

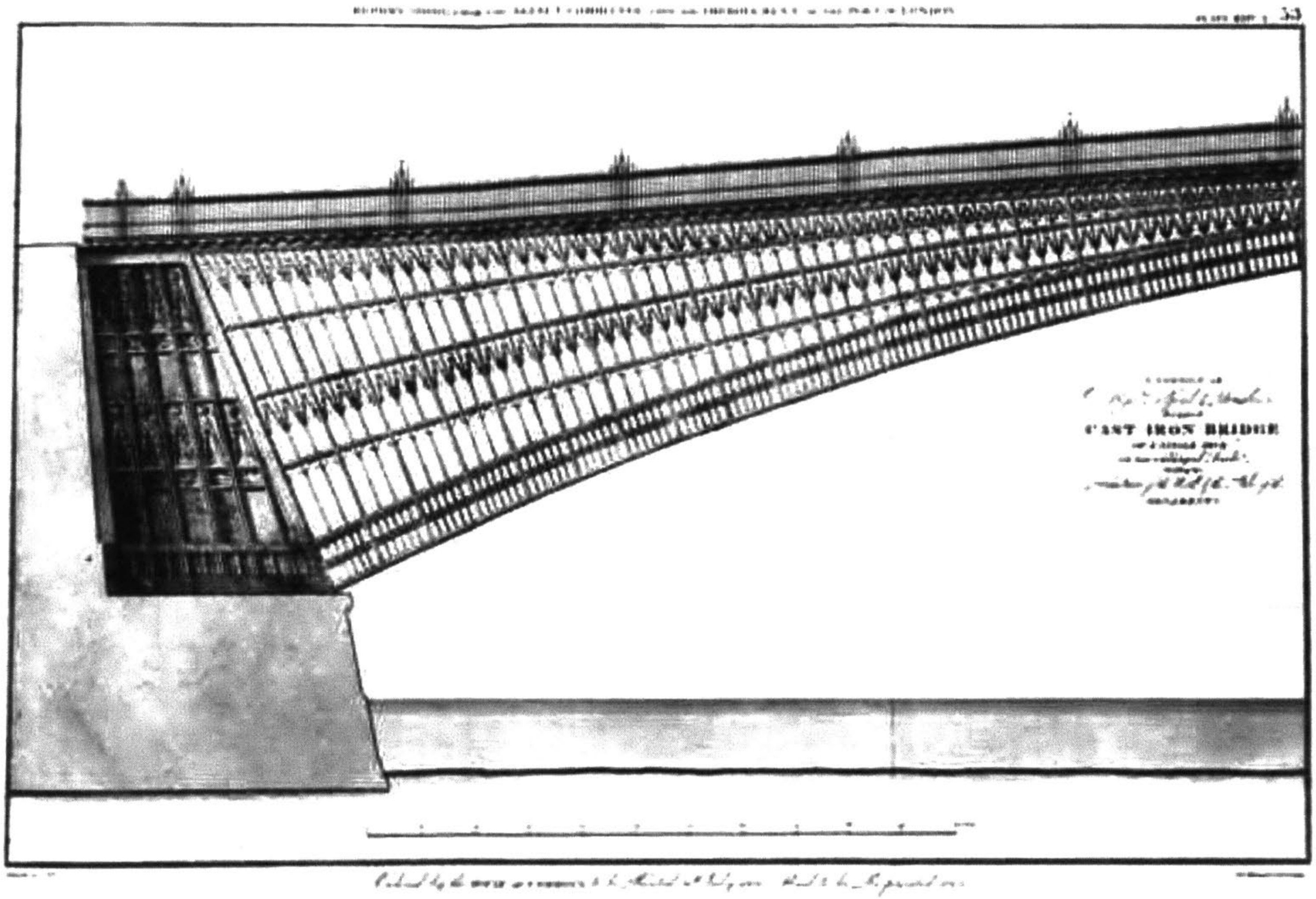


Figure A.2: Partial elevation of London Bridge proposal, 1801 *(Thomas Telford 2017)*

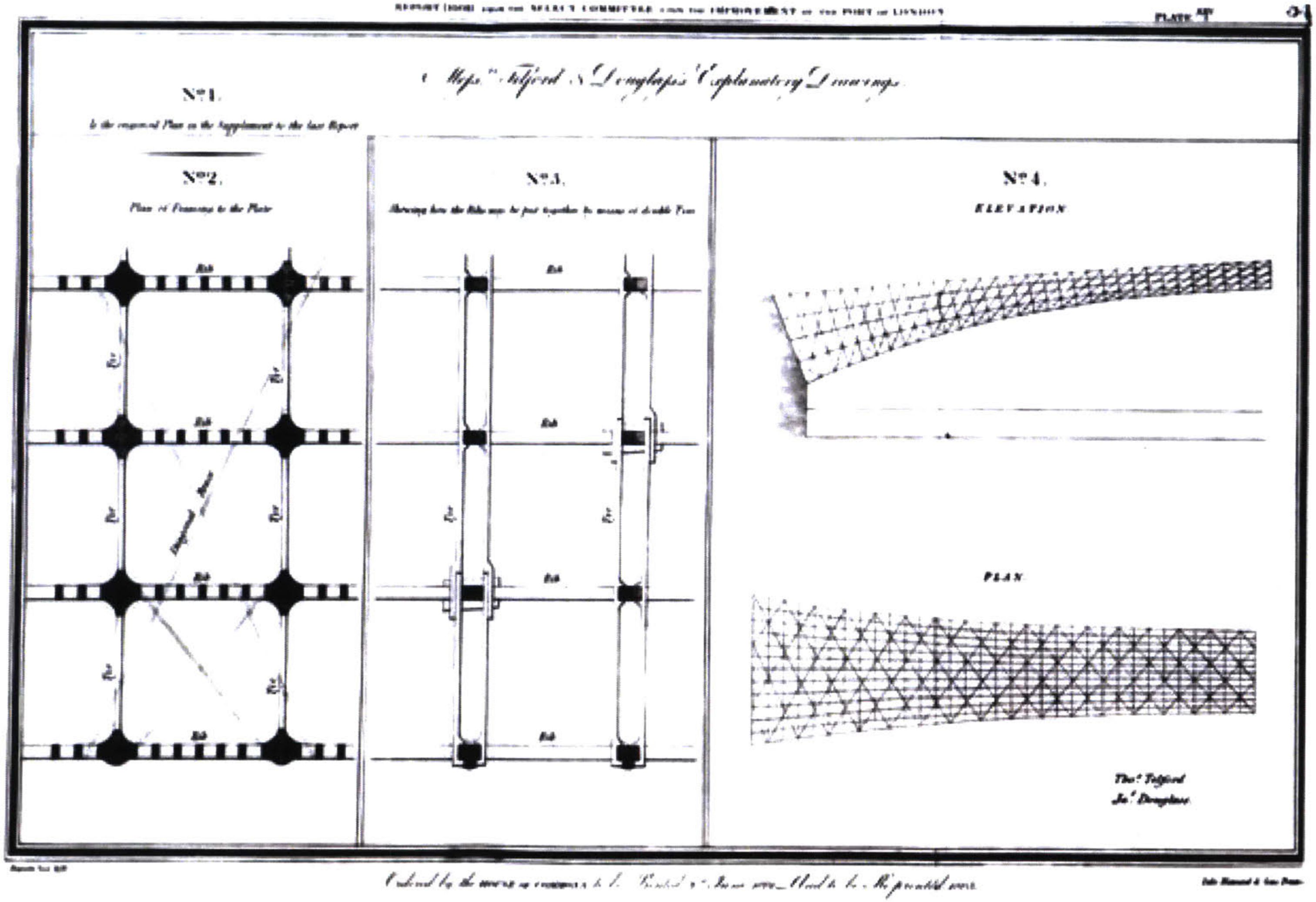


Figure A.3: London Bridge proposal details *(Thomas Telford 2017)*

A.2. Bonar Bridge, 1810-12

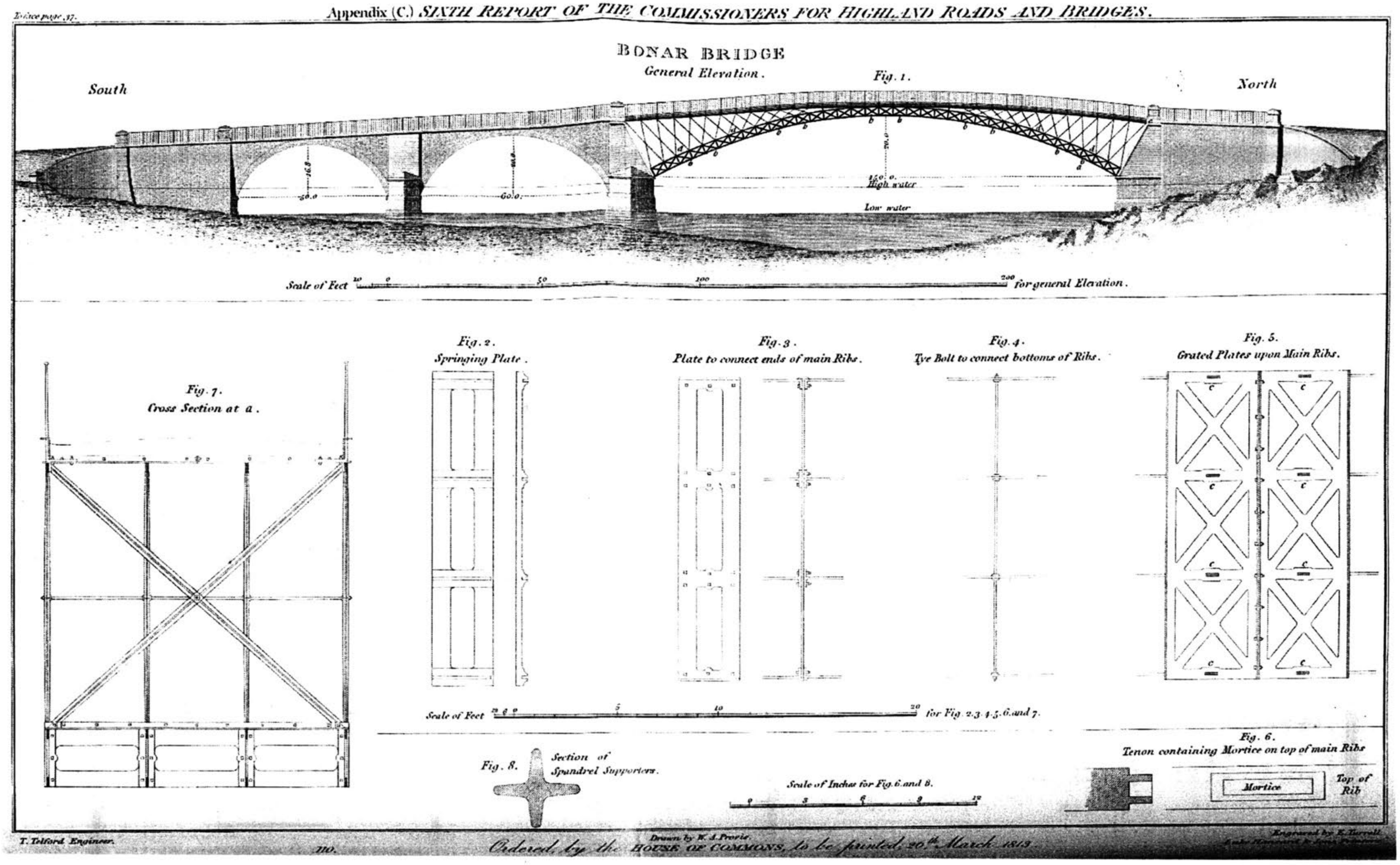


Figure A.4: Elevation, sections, and details for Bonar Bridge *(Sixth Report of the Commissioners for Highland Roads and Bridges 1813)*

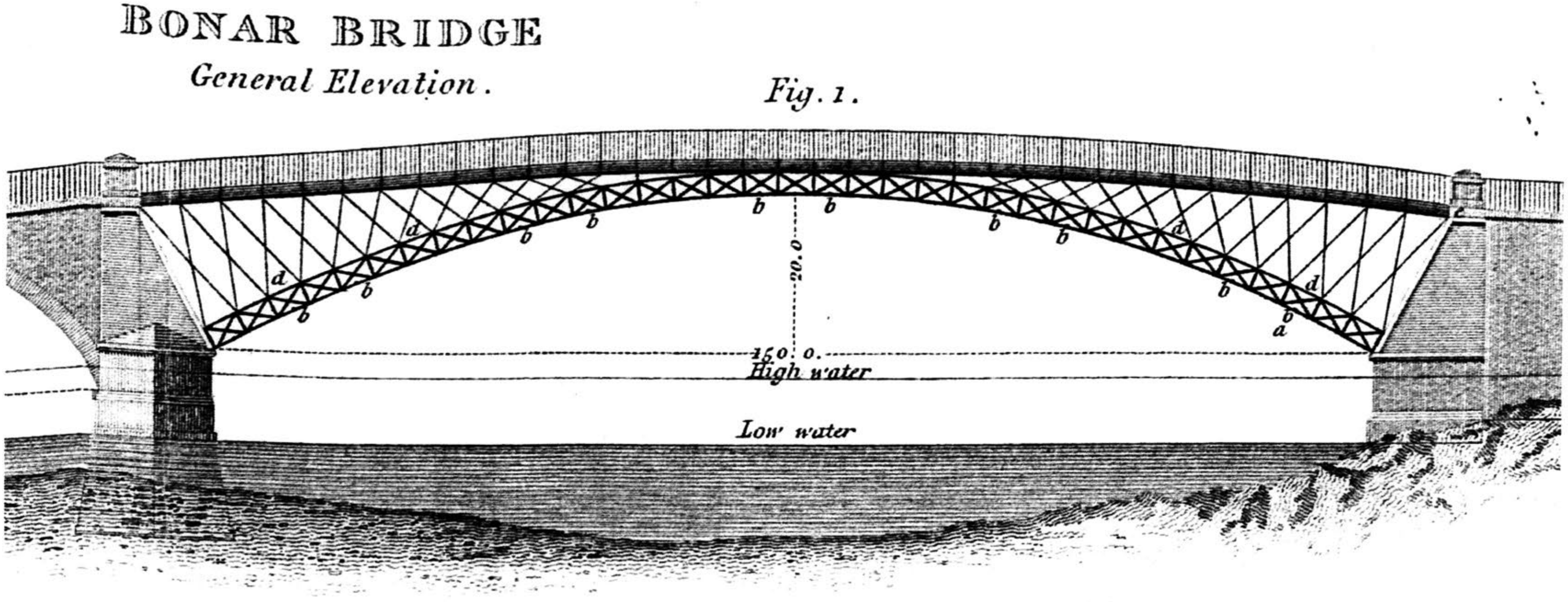


Figure A.5: Close-up, elevation, Bonar Bridge *(Sixth Report of the Commissioners for Highland Roads and Bridges 1813)*

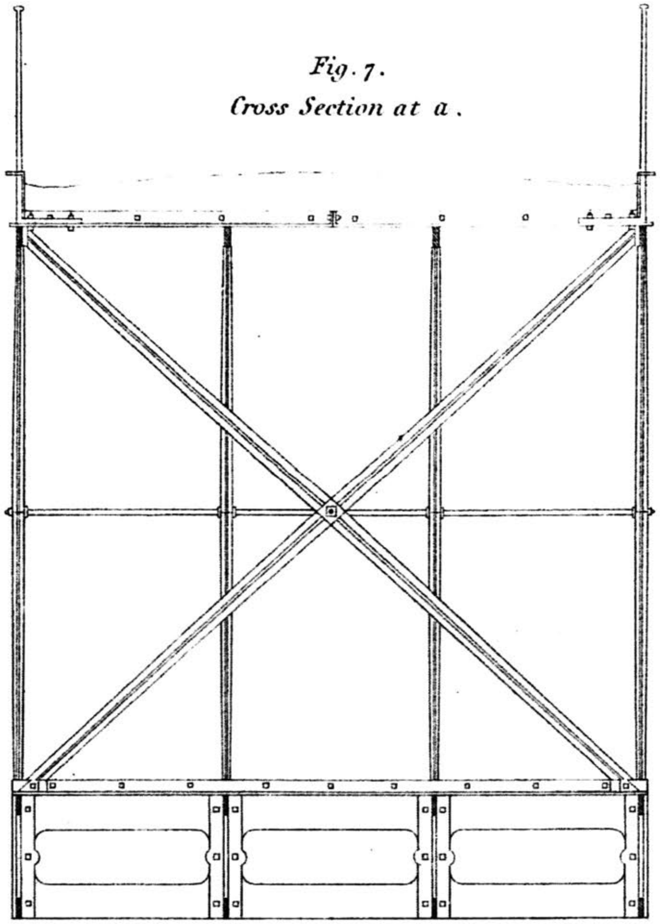


Figure A.6: Close-up, section, Bonar Bridge (Sixth Report of the Commissioners for Highland Roads and Bridges 1813)

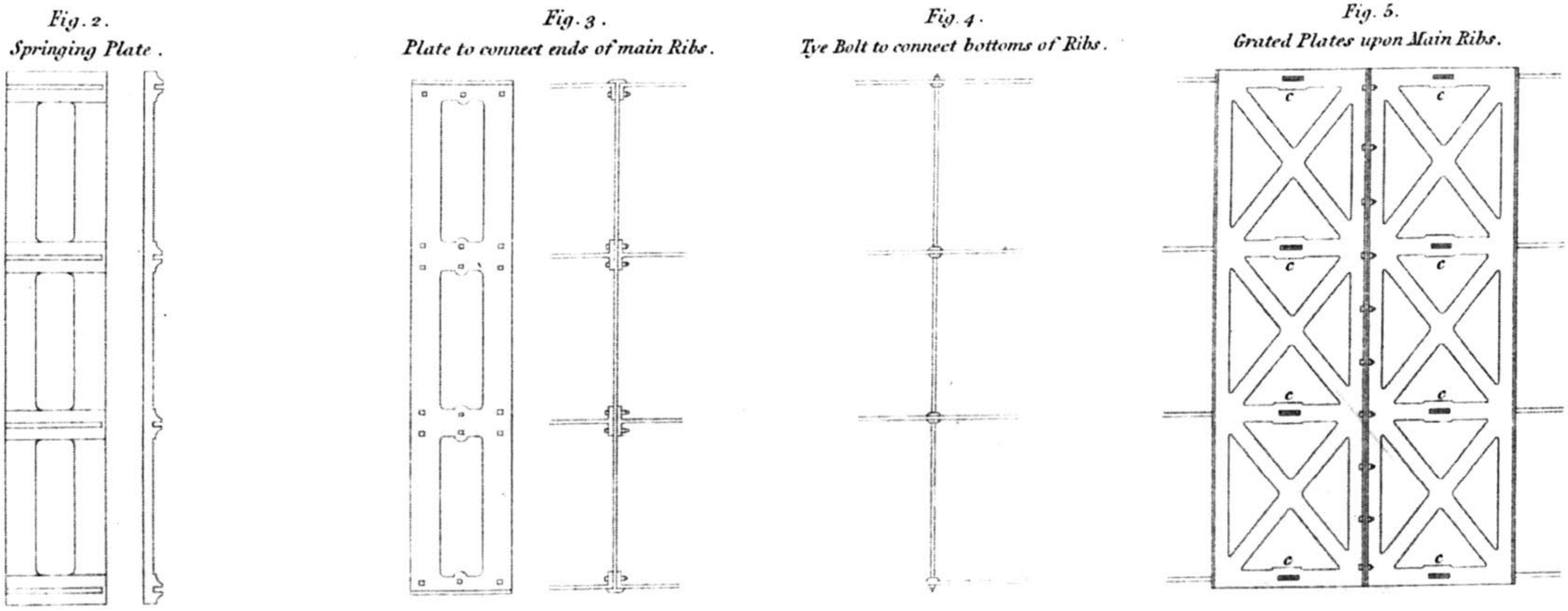


Fig. 2. Springing Plate.

Fig. 3. Plate to connect ends of main Ribs.

Fig. 4. Tye Bolt to connect bottoms of Ribs.

Fig. 5. Grated Plates upon Main Ribs.

Figure A.7: Close-up, details, Bonar Bridge (Sixth Report of the Commisioners for Highland Roads and Bridges 1813)

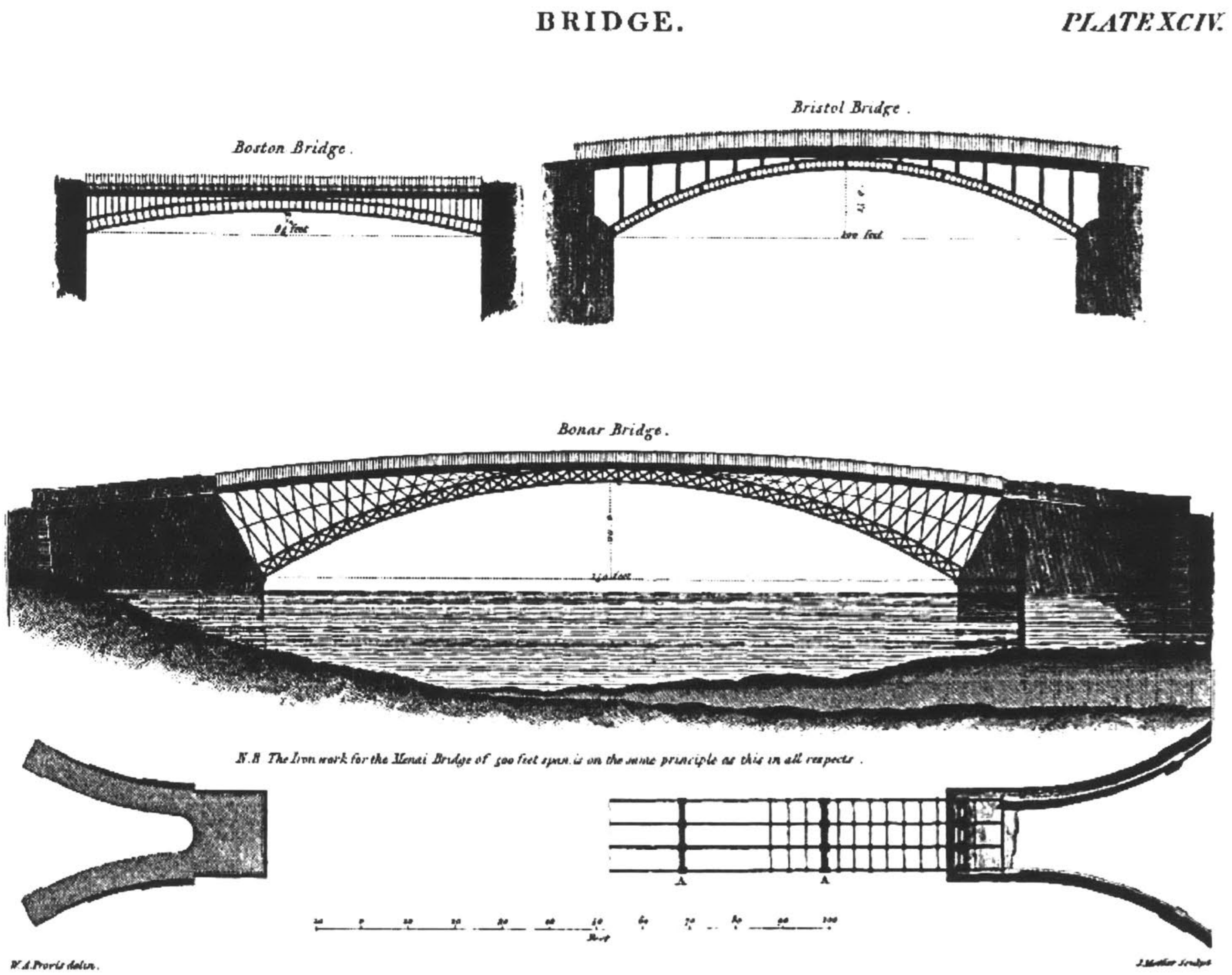


Figure A.8: Telford's drawing, labelled as Bonar Bridge, compared with the bridges of his contemporaries, from the Edinburgh Encyclopedia – not the true Bonar Bridge, however – see Section 3.1 (Telford 1812)

A.3. Maps of Highland Roads and Bridges

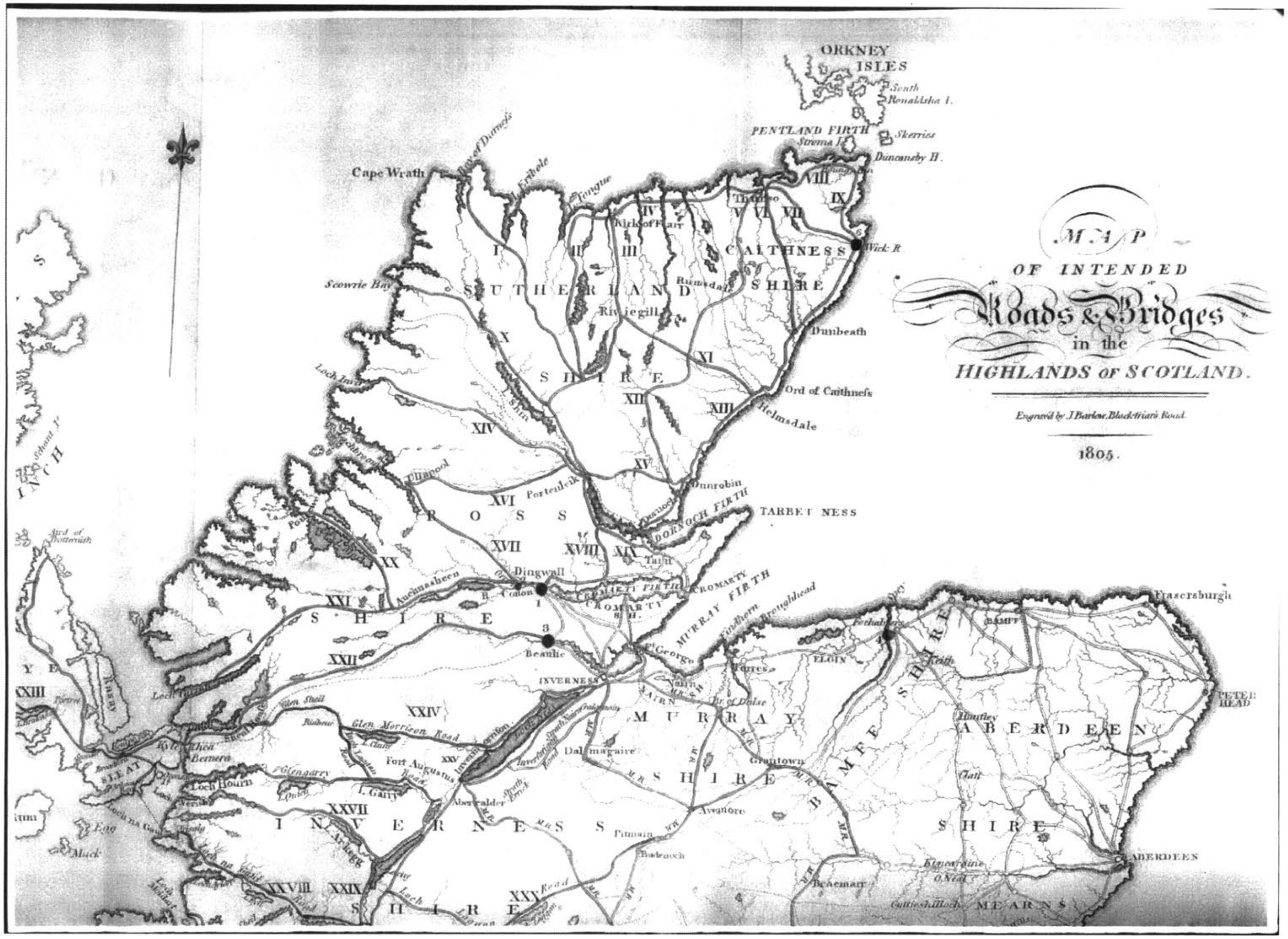


Figure A.9: Map of Telford's Highland works 1/2 – 1805

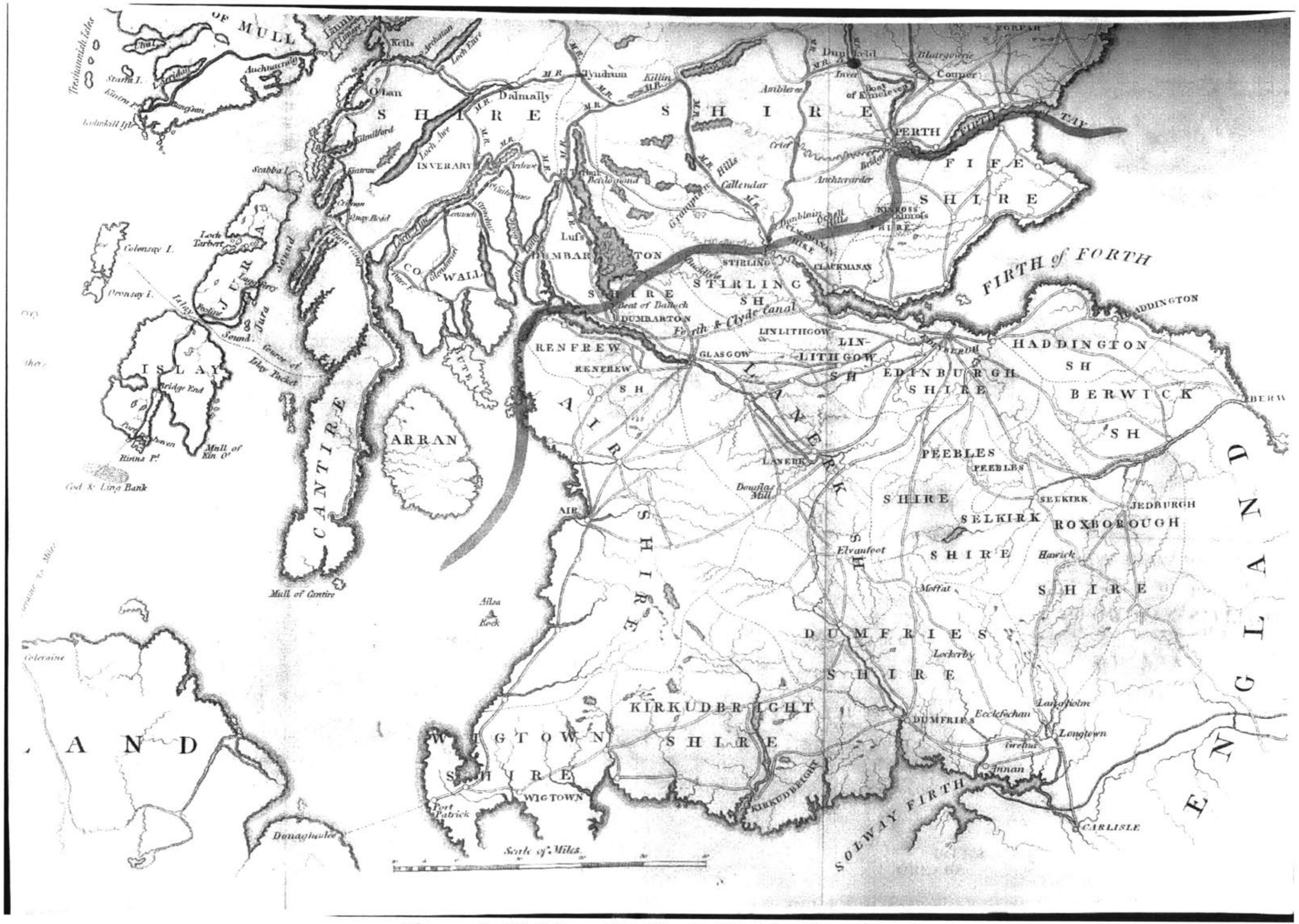


Figure A.10: Map of Telford's Highland works 2/2 – 1805

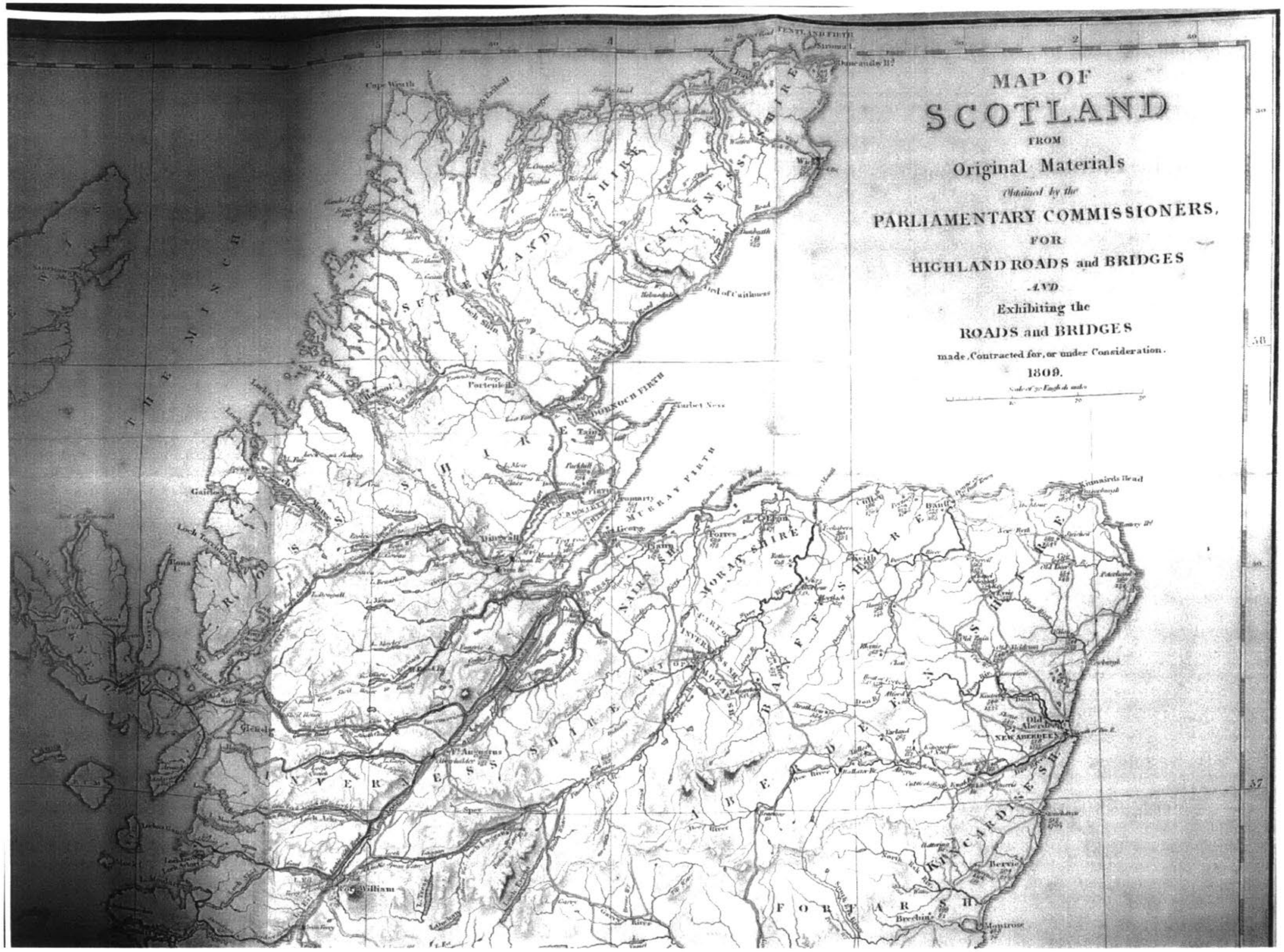


Figure A.11: Map of Telford's Highland works – 1809 *(Fourth Report of the Commissioners for Highland Roads and Bridges 1809)*

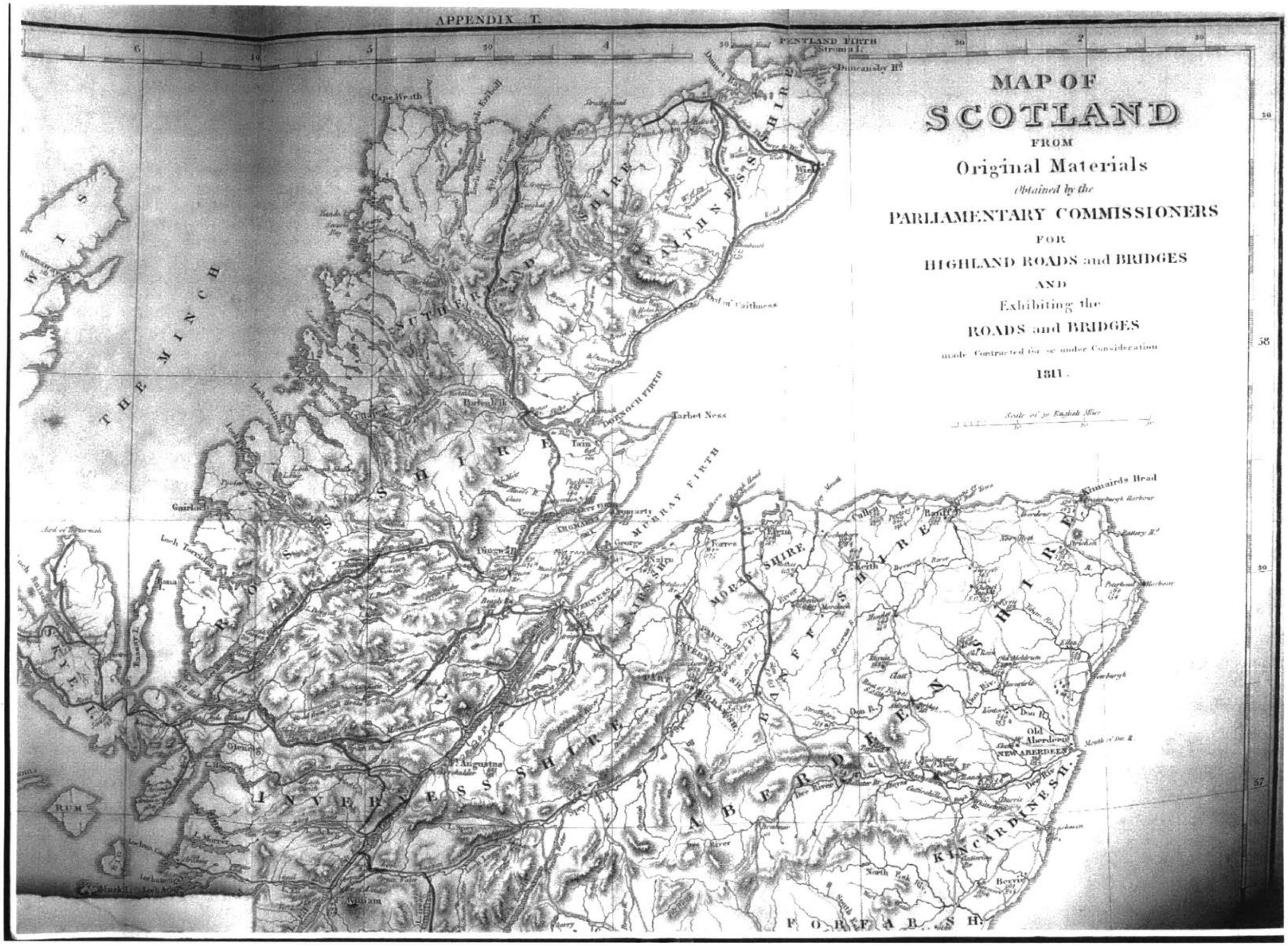


Figure A.12: Map of Telford's Highland works, showing Bonar Bridge contracted for – 1811 *(Fifth Report of the Commissioners for Highland Roads and Bridges 1811)*

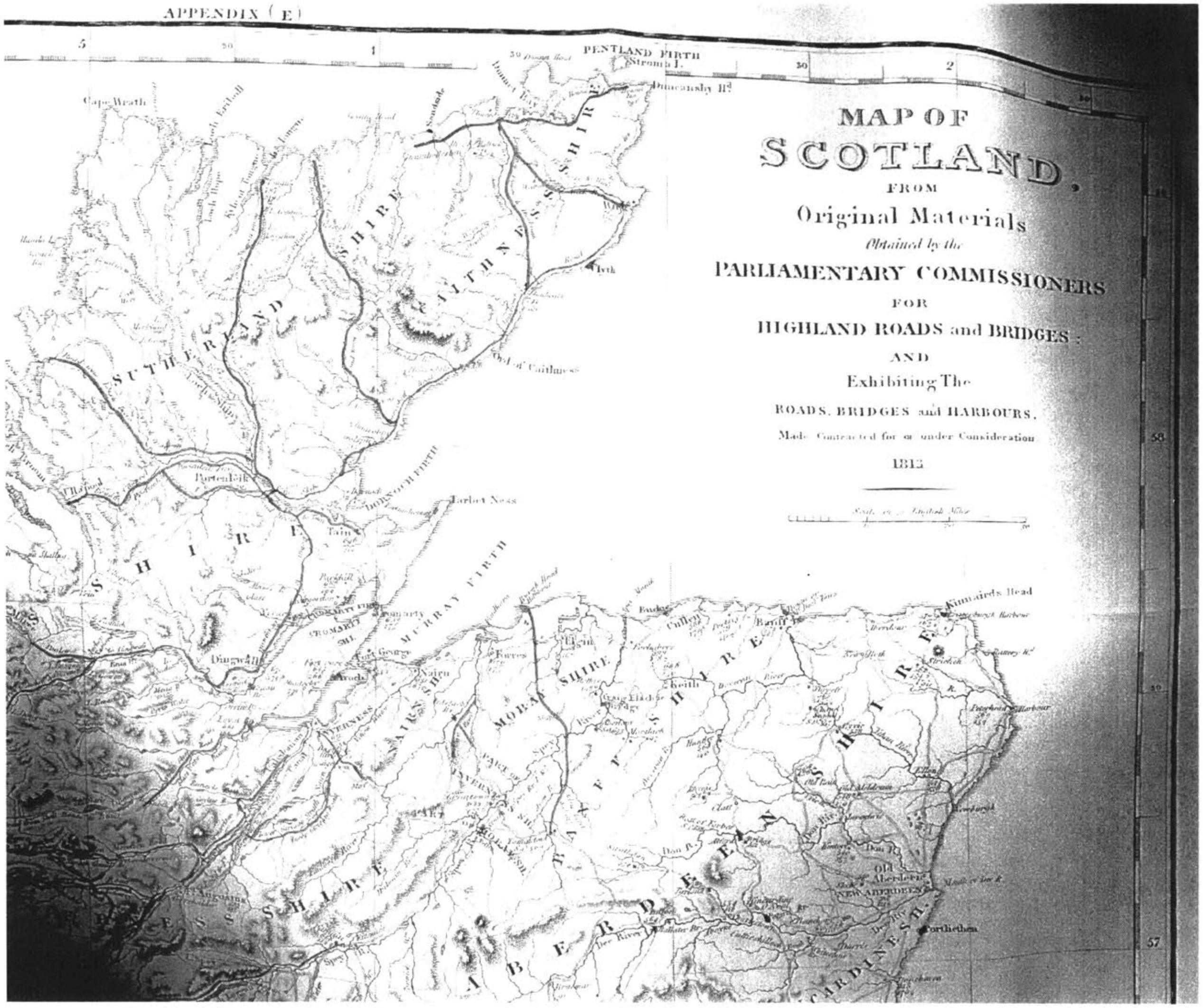


Figure A.13: Map of Telford's Highland works, showing Bonar Bridge completed and Craigellachie Bridge contracted for *(Sixth Report of the Commissioners for Highland Roads and Bridges 1813)*

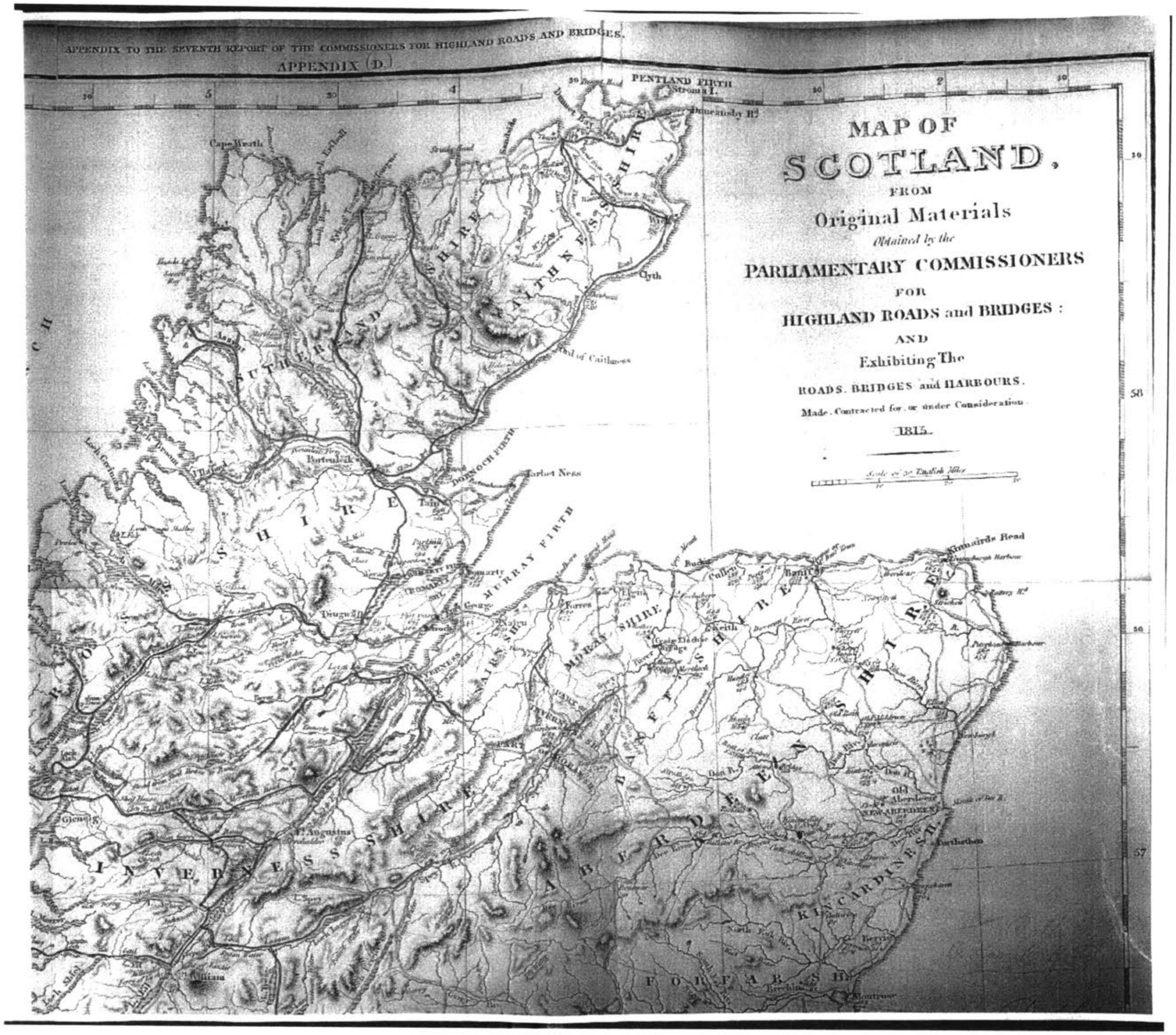


Figure A.14: Map of Telford's Highland works, showing Bonar Bridge and Craigellachie Bridge completed – 1815 *(Seventh Report of the Commissioners for Highland Roads and Bridges 1815)*

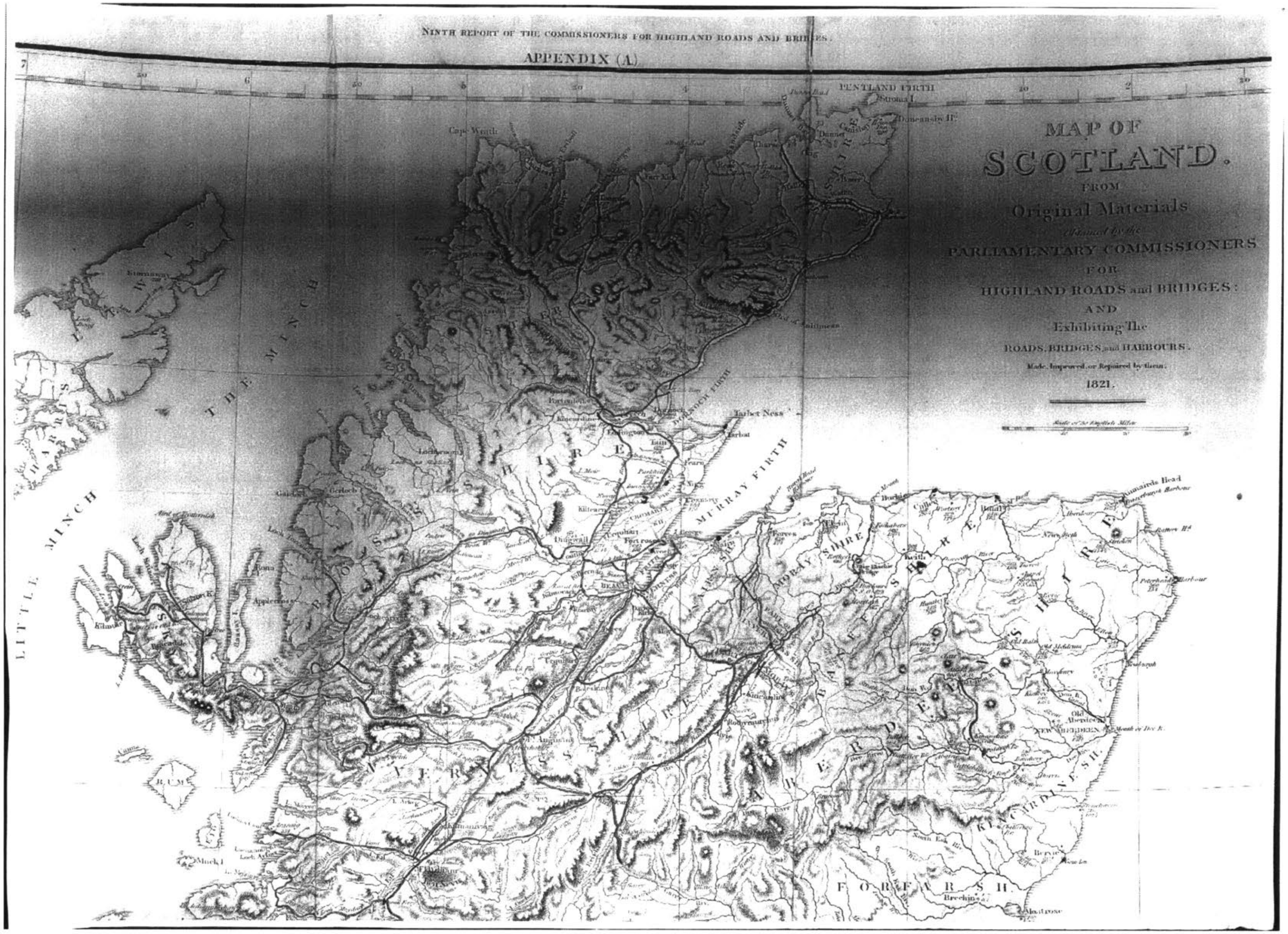


Figure A.15: Map of Telford's Highland works – 1821 *(Ninth Report of the Commissioners for Highlands and Islands 1821)*

A.4. Menai Bridge Proposal, 1811

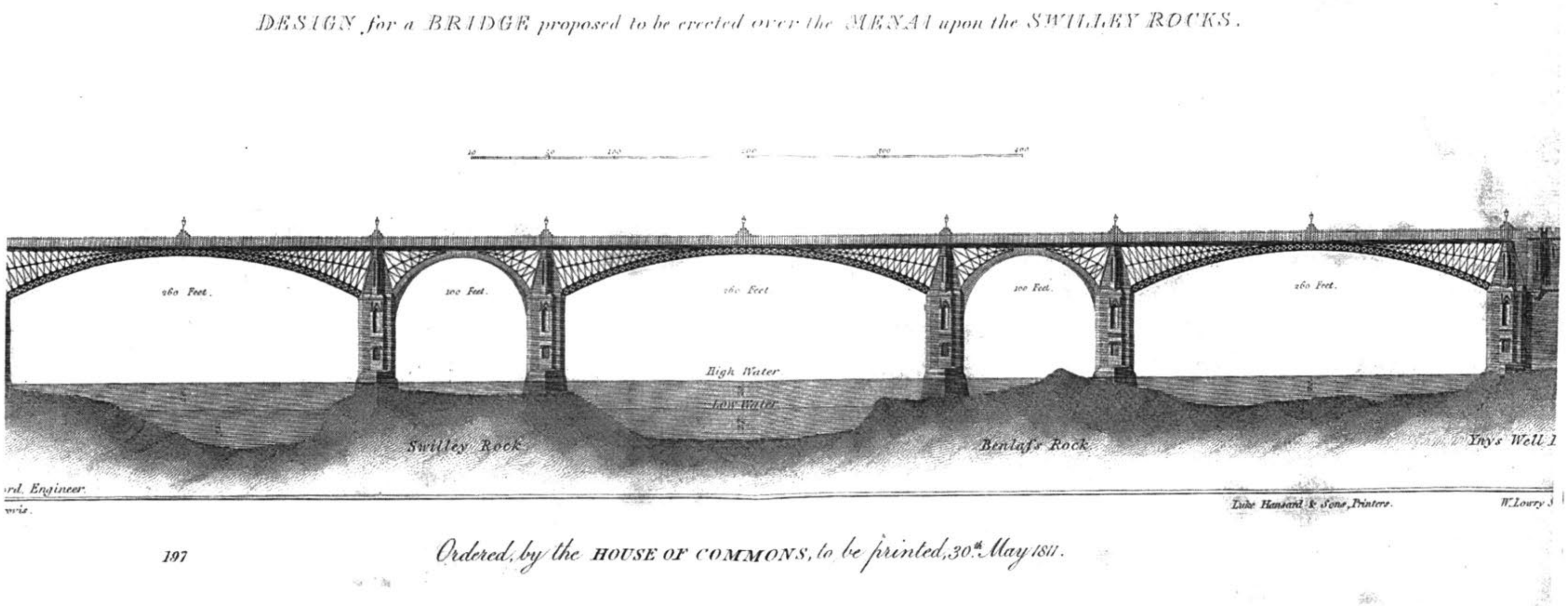


Figure A.16: Proposal for Menai Bridge *(Report from the Committee on Roads to Holyhead 1811)*

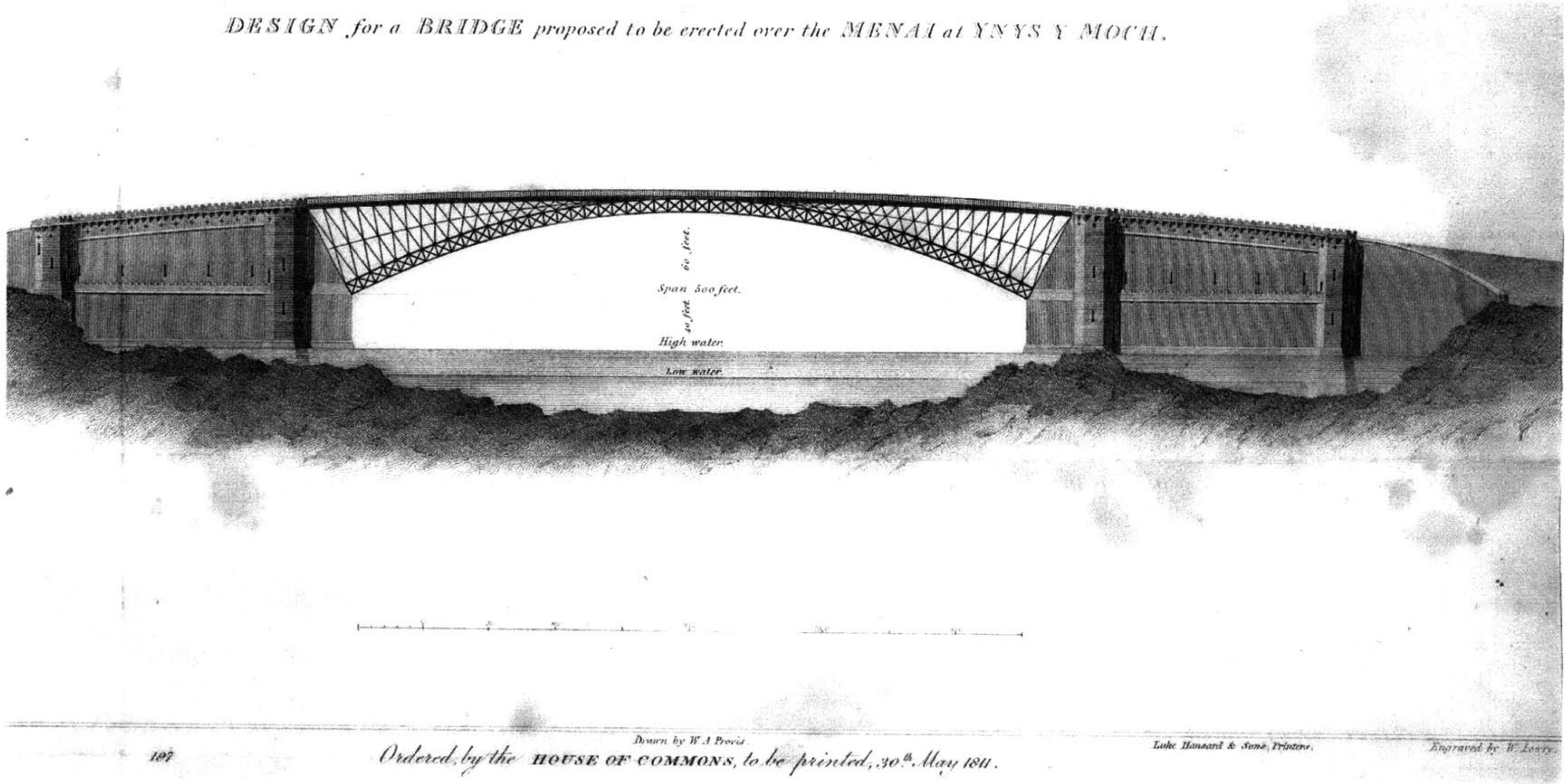


Figure A.17: Proposal for Menai Bridge 1811 *(Report from the Committee on Roads to Holyhead 1811)*

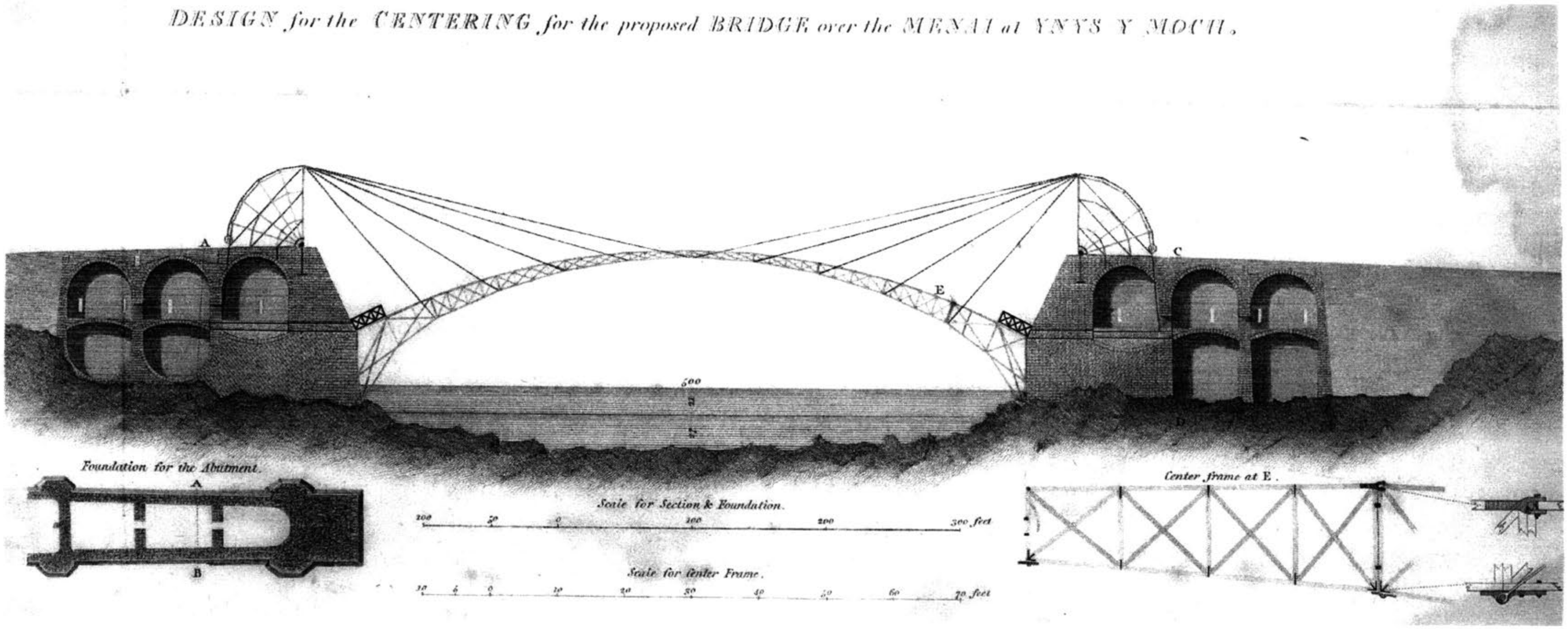


Figure A.18: Design for centering for proposed Menai Bridge *(Report from the Committee on Roads to Holyhead 1811)*

A.5. Mythe Bridge, 1824-26

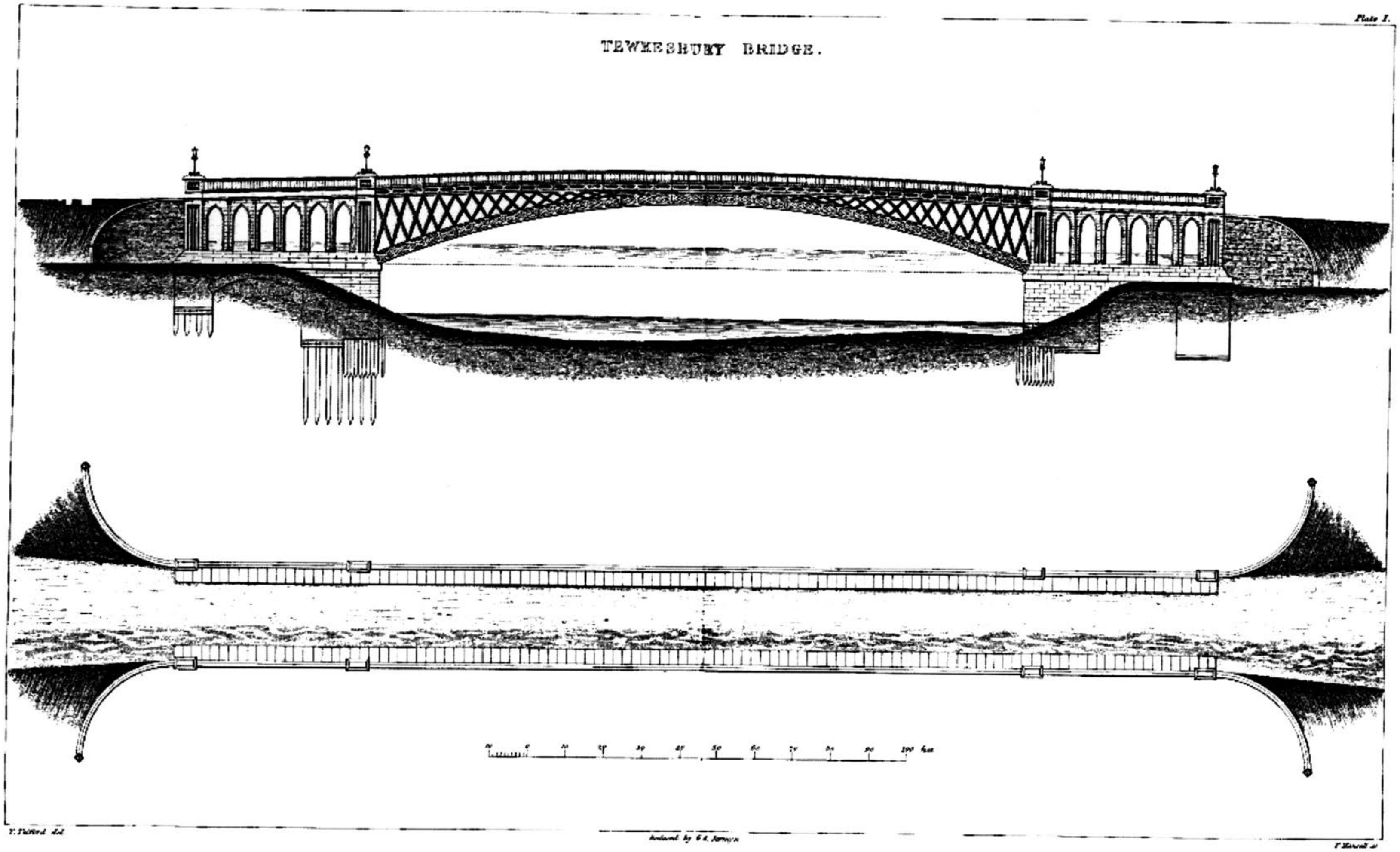


Figure A.19: Telford's original drawings of Mythe Bridge – plan and elevation (Mackenzie)

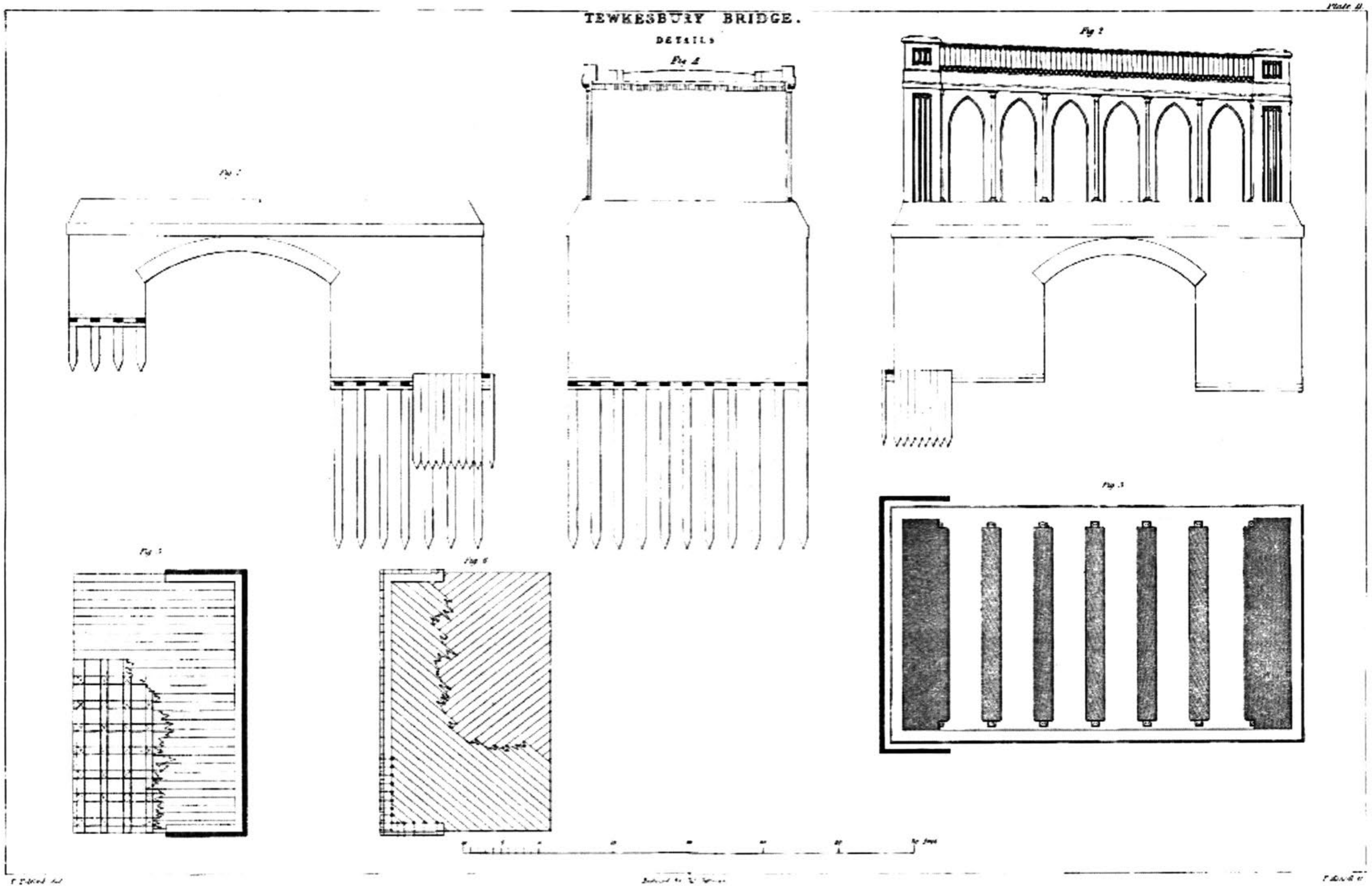


Figure A.20: Telford's original drawings of Mythe Bridge – abutment details (Mackenzie)

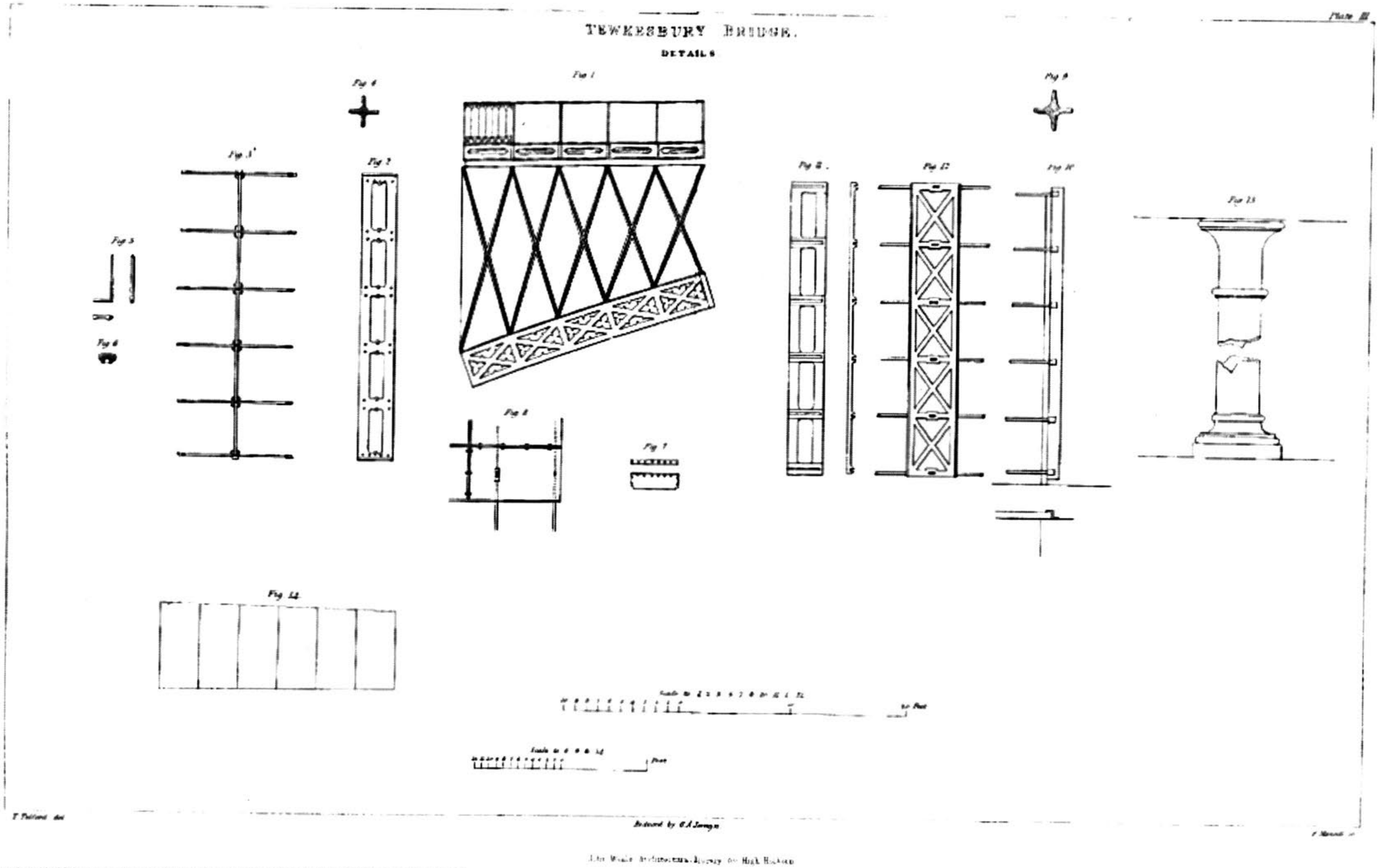


Figure A.21: Telford's original drawings of Mythe Bridge - ironwork details (Mackenzie)

A.6. Galton Bridge, 1829

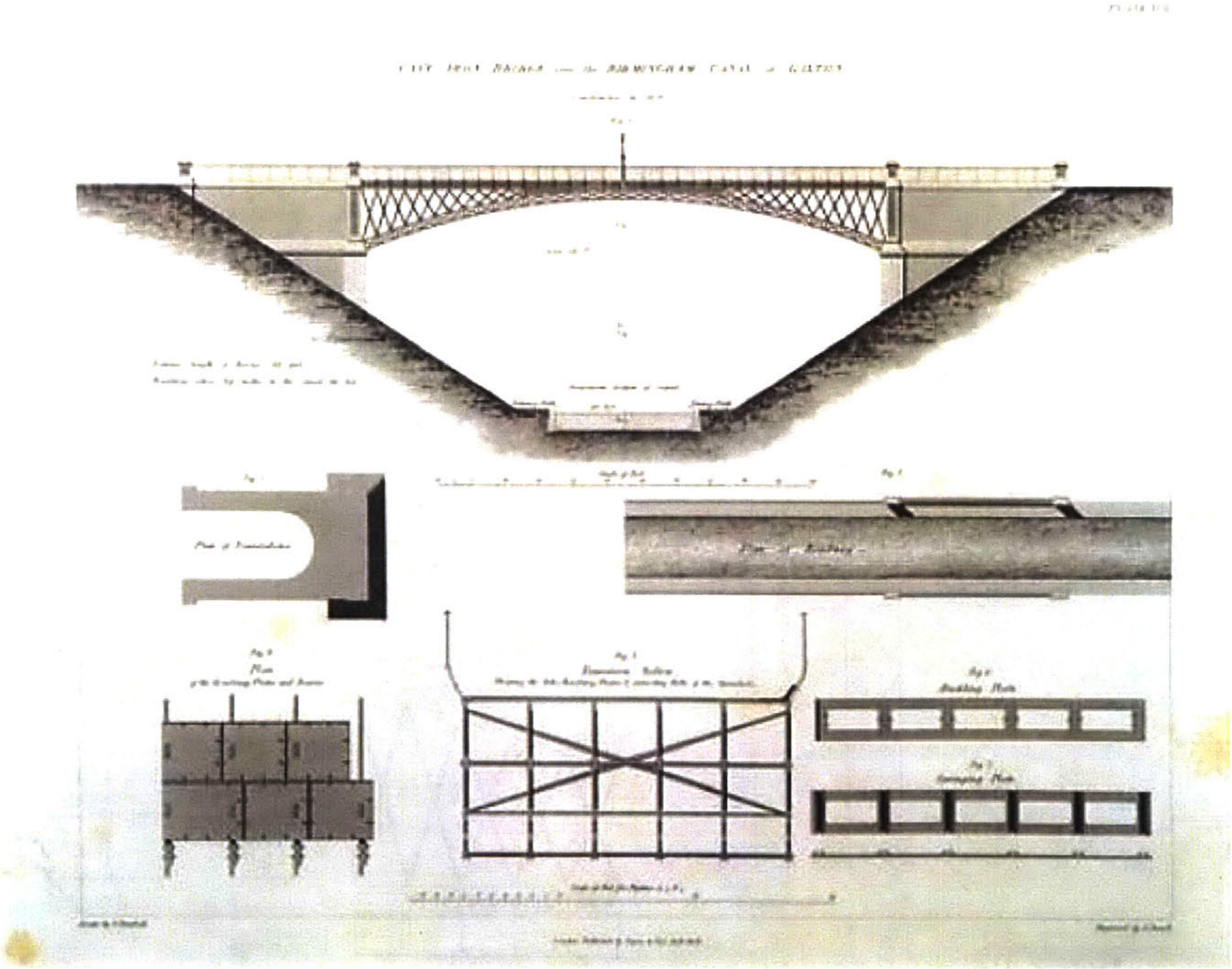


Figure A.22: Galton Bridge plans

Appendix B. Photos of Bridges

B.1. Bonar Bridge

Figure B.1: The First Bonar Bridge 1812-1892, 2012

Figure B.2: Bonar Bridge (Daniell)

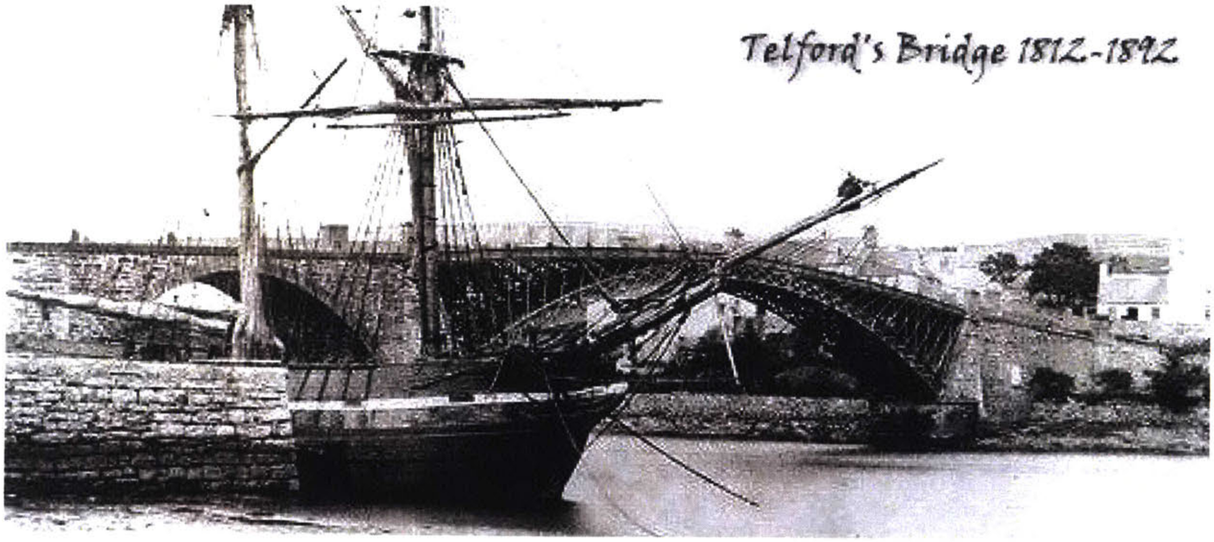


Figure B.3: Telford's Bridge 1812-1892 (Dornoch Links)

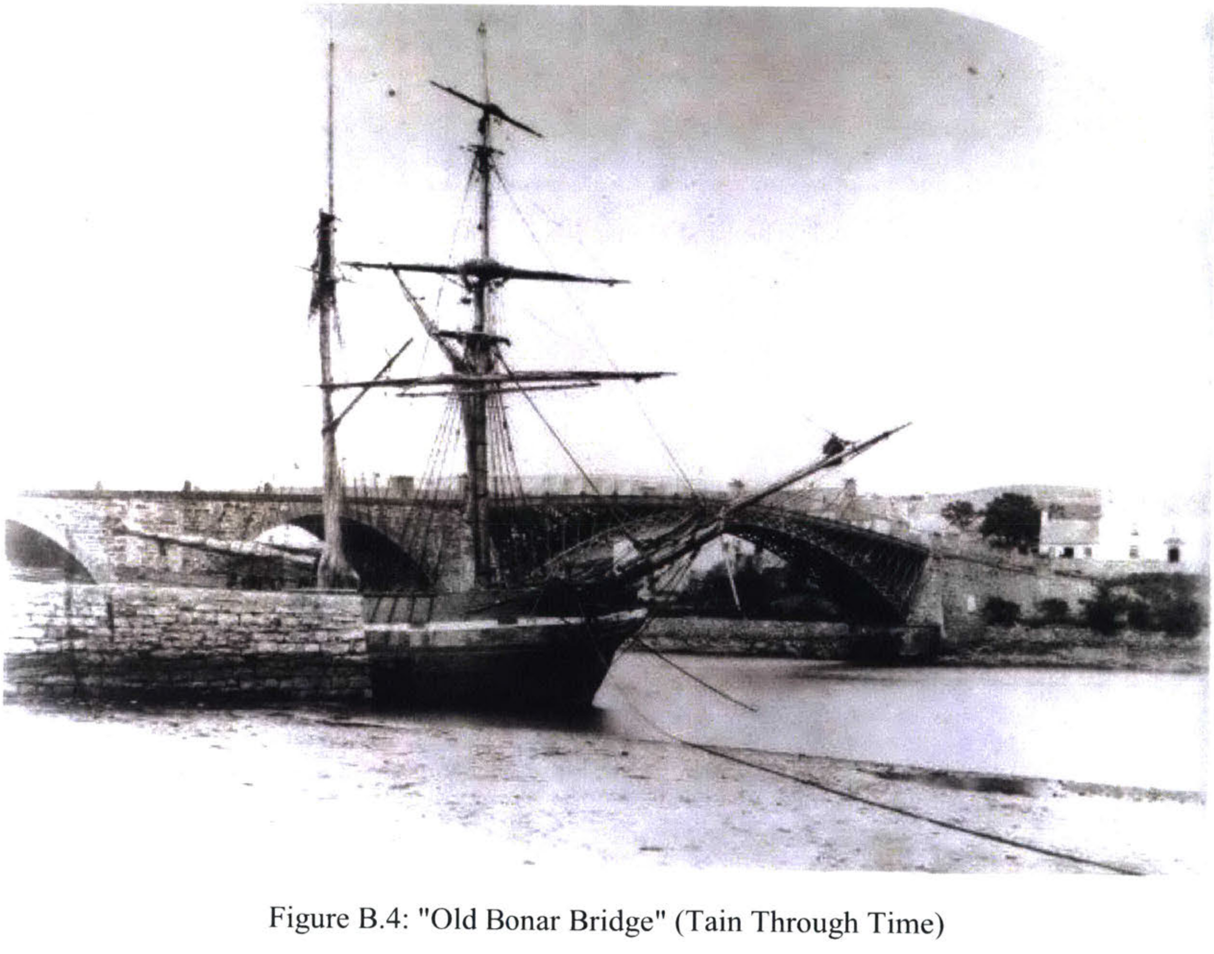
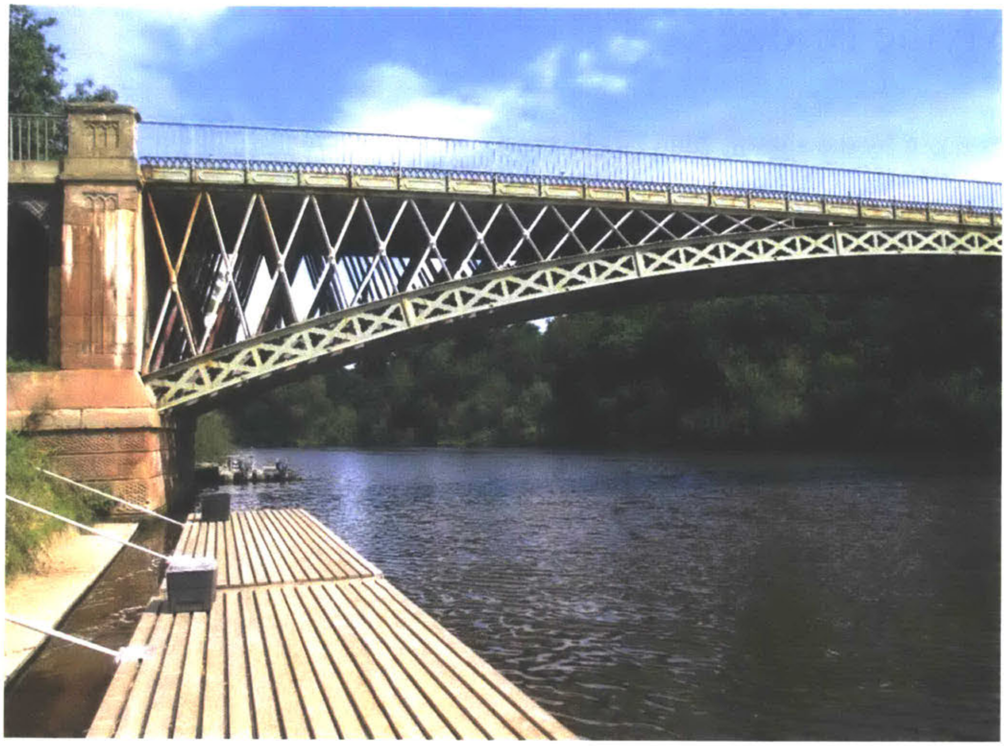


Figure B.4: "Old Bonar Bridge" (Tain Through Time)

B.2. Mythe Bridge

All photos taken by the author, June 2017

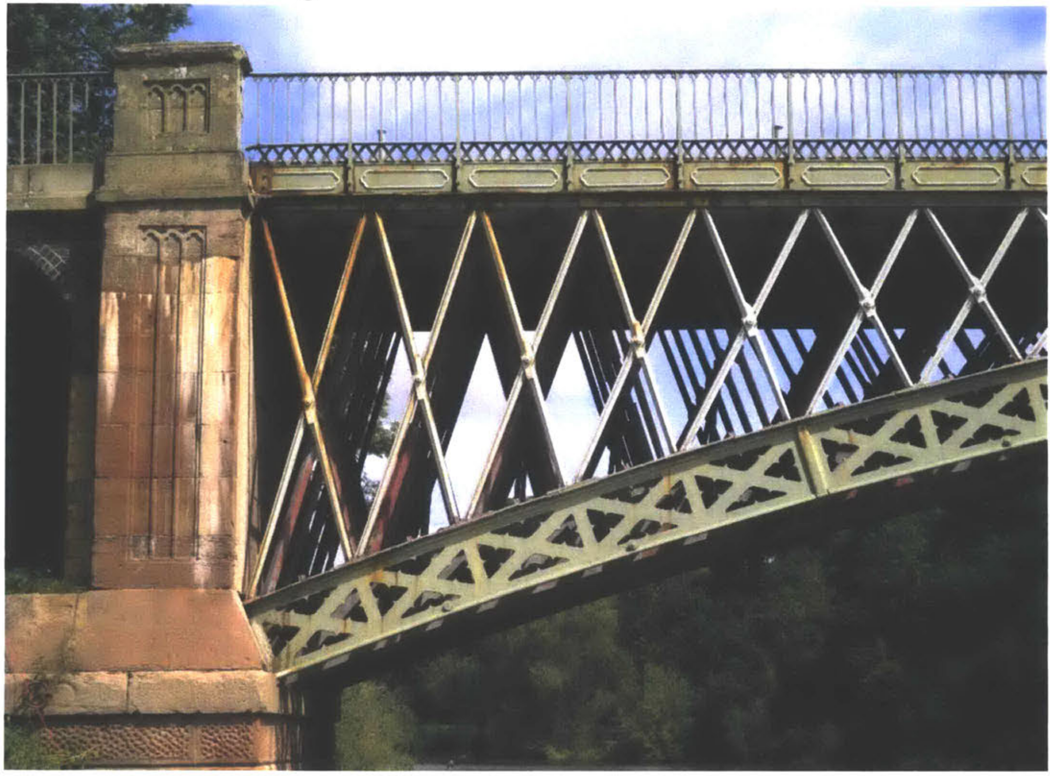
B.2.1. Full Elevation

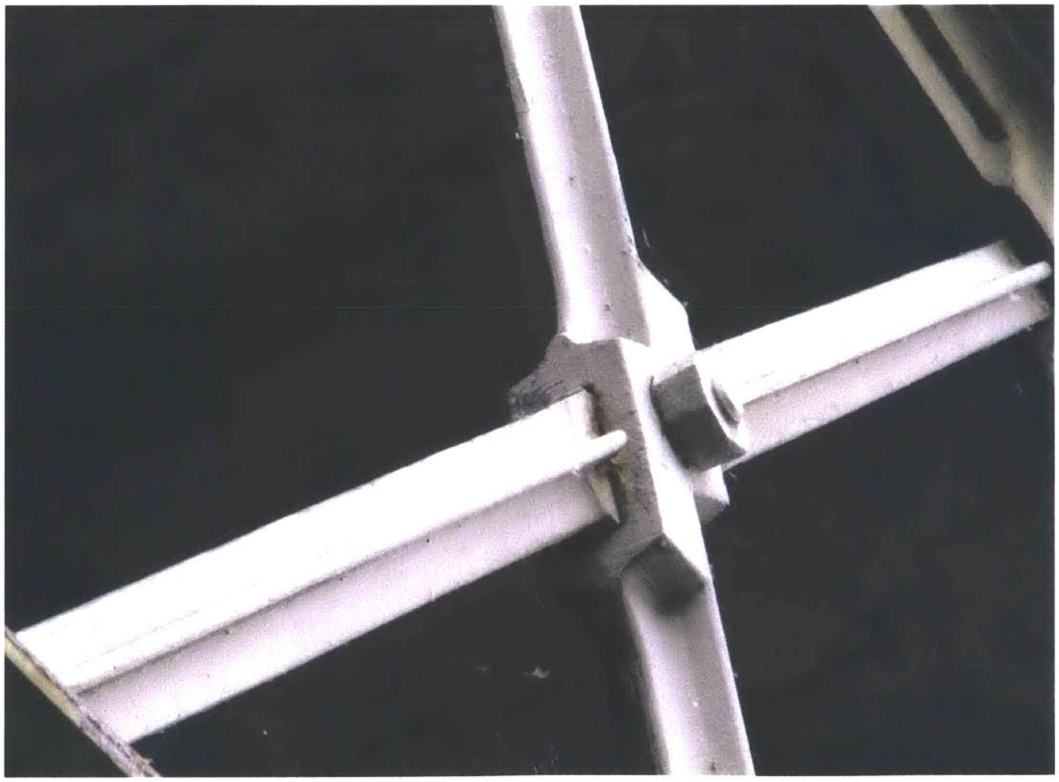


B.2.2. Ribs

B.2.3. Covering Plates

B.2.4. Spandrel Bracing

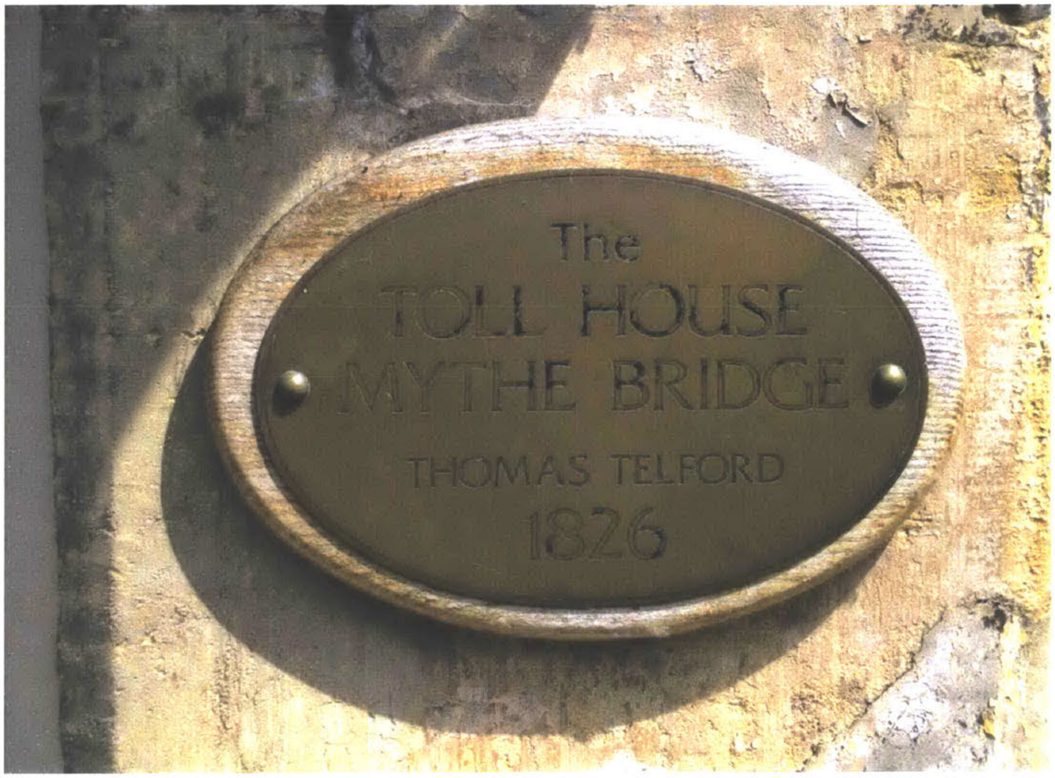




B.2.5. Roadway

B.2.6. Abutments

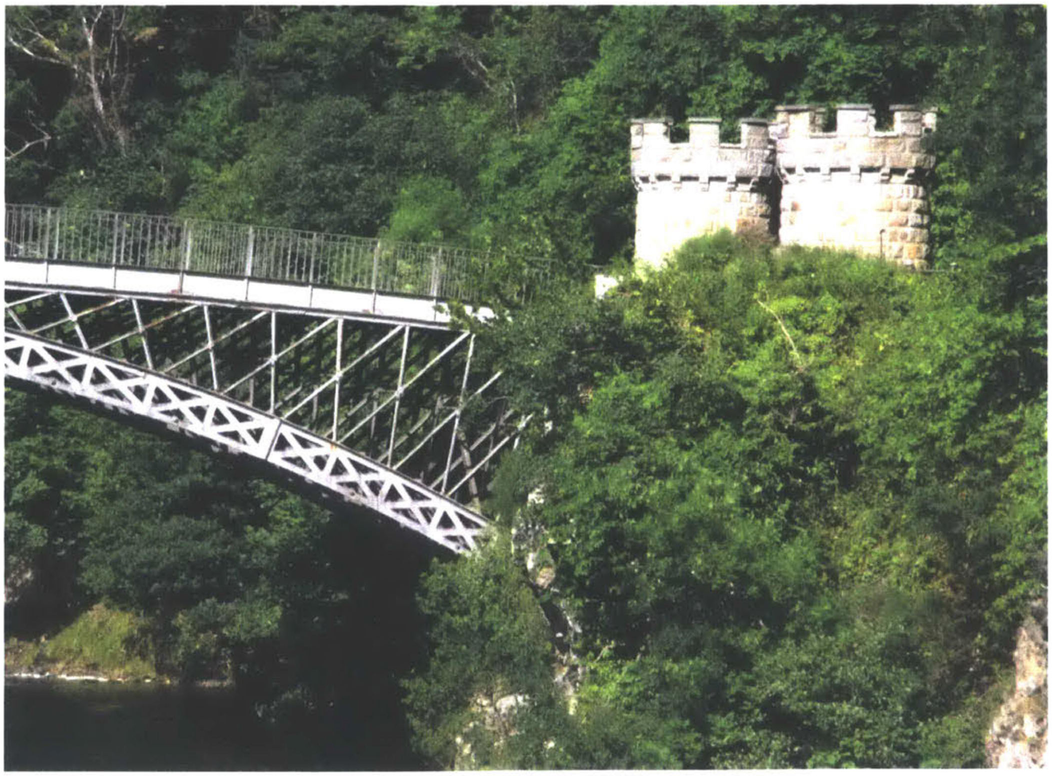
B.2.7. Tollhouse



B.3. Craigellachie Bridge

B.3.1. Full Elevation

B.3.2. Spandrel Bracing

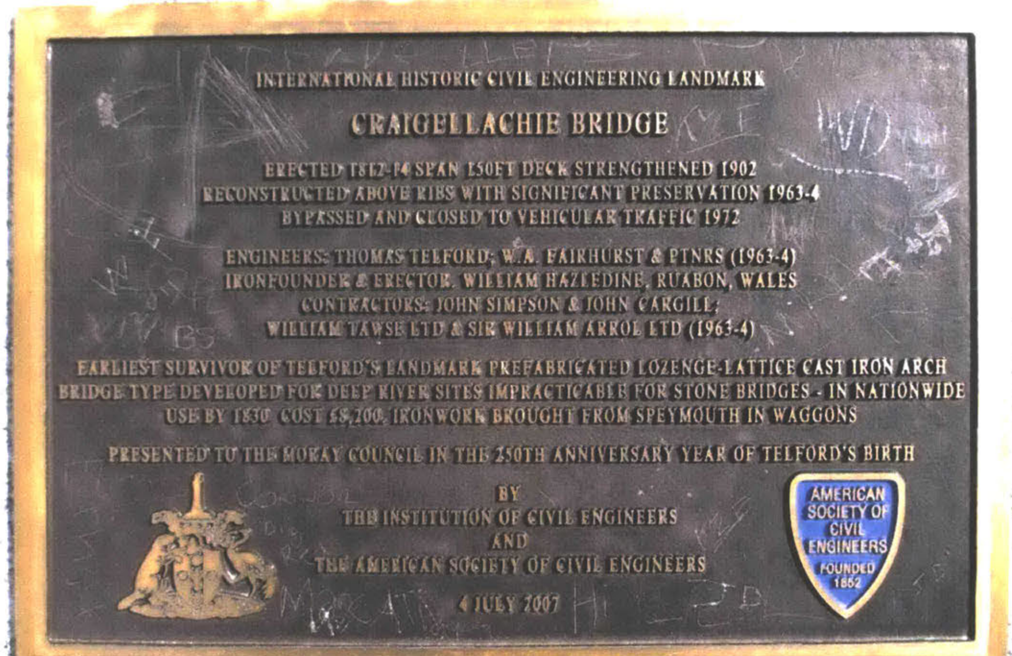


B.3.3. Ribs

B.3.4. Towers

B.3.5. Marks and Plaques

CRAIGELLACHIE BRIDGE
DESIGNED AND BUILT BY
THOMAS TELFORD
1814
RESTORED TO ITS ORIGINAL DESIGN BY
THE COUNTY COUNCILS OF BANFF, MORAY & NAIRN
1964



INTERNATIONAL HISTORIC CIVIL ENGINEERING LANDMARK
CRAIGELLACHIE BRIDGE
ERECTED 1812-14 SPAN 150FT DECK STRENGTHENED 1902
RECONSTRUCTED ABOVE RIBS WITH SIGNIFICANT PRESERVATION 1963-4
BYPASSED AND CLOSED TO VEHICULAR TRAFFIC 1972

ENGINEERS: THOMAS TELFORD; W.A. FAIRHURST & PTNRS (1963-4)
IRONFOUNDER & ERECTOR, WILLIAM HAZLEDINE, RUABON, WALES
CONTRACTORS: JOHN SIMPSON & JOHN CARGILL;
WILLIAM TAWSE LTD & SIR WILLIAM ARROL LTD (1963-4)

EARLIEST SURVIVOR OF TELFORD'S LANDMARK PREFABRICATED LOZENGE-LATTICE CAST IRON ARCH
BRIDGE TYPE DEVELOPED FOR DEEP RIVER SITES IMPRACTICABLE FOR STONE BRIDGES - IN NATIONWIDE
USE BY 1830 COST £8,200, IRONWORK BROUGHT FROM SPEYMOUTH IN WAGGONS

PRESENTED TO THE MORAY COUNCIL IN THE 250TH ANNIVERSARY YEAR OF TELFORD'S BIRTH
BY
THE INSTITUTION OF CIVIL ENGINEERS
AND
THE AMERICAN SOCIETY OF CIVIL ENGINEERS
4 JULY 2007

Appendix C. Letters Regarding London Bridge

Letters owned by the Institution of Civil Engineering, transcribed by the author and reproduced with permission.

Where possible, original spelling, grammar, and punctuation have been preserved.

C.1. List of Letters, Arranged by Correspondent

No.	Sender	Recipient	Date (1801)
1	A.A.	T.T.	May 6
2	Charles Bage	T.T.	April 12
3	Charles Bage	T.T.	April 23
4	William Botfield	T.T.	April 28
5	T.T.	Samuel Gunnell?	May 12
6	Rowland Hunt	T.T.	May 15
7	Charles Hutton	T.T.	February 8
8	Charles Hutton	T.T.	April 26
9	Thomas Jones	T.T.	April 11
10	W. Lowry	T.T.	May 12
11	Nevil Maskelyne	T.T.	March 19
12	Nevil Maskelyne	TT?	March 24
13	Nevil Maskelyne	T.T.	March 31
14	Nevil Maskelyne	T.T.	April 3
15	T.T.	Nevil Maskelyne	April 3
16	Nevil Maskelyne	T.T.	April 14
17	Nevil Maskelyne	T.T.	April 18
18	William Reynolds	T.T.	April 19
19	William Reynolds	T.T.	April 23
20	William Reynolds	T.T.	May 10
21	William Reynolds	T.T.	May 14
22	Abram Robertson	T.T.	March 6

23	Abram Robertson	T.T.	April 3 33
24	Abram Robertson	T.T.	April 9 34
25	Abram Robertson	T.T.	April 21
26	T.T.	Abram Robertson	May 5
27	Samuel Vince	T.T.	April 11
28	Joshua Walker	T.T.	April 14
29	T.T.	Messrs Walker	April 21
30	John Wilkinson	T.T.	April 17
31	John Wilkinson	T.T.	April 21

C.2. Brief List of Relevant People

Name	Letters	Description	Address	Source
A. A.	1	University of Edinburgh	Edinburgh	
Charles Bage (1751-1822)	2,3	Architect of the first iron-framed building - Ditherington Flax Mill, 1796-97	Shrewsbury	(Charles Bage (1751-1822) 2002)
Thomas Jeffrys (1742-1820)	9*	Goldsmith, subscribed for plate of London Bridge	Cockspur, London	(Crippen 1907-1908) (Worms 2004)
William Bancks (?-1804)	9*	Subscribed for plate of London Bridge	Corbyns Hall, Dudley	(Banks 2015)
Thomas Botfield (1738-1801)	4*	Ironmaster	Shropshire	(Belford 2012)
William Botfield	4,18*	Ironmaster, son of Thomas Botfield	Malins Lee	(Records of the Botfield family 1758-1873)
Richard Dearman	20*	Ironmaster	Birmingham	(Reperatory of Arts and Manufactures: Vol V 1796)
Mr & Mrs Elliot	1*		London	
William Fawcett	9*	Subscribed for plate of London Bridge	Liverpool	
Sir Francis Freeling (1764-1836)	31*	Secretary of HM General Post Office (most probable reference)	London	(G. B. Smith 2004)
Samuel Gunnell (Gunnel?)	5^{+},8*, 30*	Committee clerk to the House of Commons	London	(Dawson 2012)

Rowland Hunt (1784-1836)	6	Lord of Manor in Baschurch, Shropshire; asked for a plate of London Bridge	Stone House, Shrewsbury, Shropshire	(Rowland Hunt 2016)
Charles Hutton (1737-1823)	7,8	Mathematician, surveyor	Woolwich	(Hutton, Charles 1911)
Thomas Jones	9,30*	John Wilkinson's nephew, writes on his behalf, continued to run mill after Wilkinson's death	Brymbo, Wrexham	(Dawson 2012)
Richard Kirk (?-1839)	9*	Ironmaster, coal miner, subscribed for plate of London Bridge	Gwersylt, Wrexham	(Eastaugh and Sternal-Johnson 2011)
W Lowry	10	Engraver for Telford's London Bridge plate		(Telford, Thomas (1757-1834) 2002)
Dr Nevil Maskelyne (1732-1811)	11,12,13,14,15^{+},1617,22*,26*	Astronomer Royal	Greenwich	(Howse 1989)
Dr Isaac Milner (1750-1820)	27*	Mathematician at Cambridge	Cambridge	(Isaac Milner n.d.)
Thomas Paine (1737-1809)	28*,29*	American thinker, inventor, and political activist, brought the second iron bridge to Europe in 1787		(Ruddock 1979)
Sir Thomas Pasley (1734-1808)	1*	Admiral of the British Royal Navy		(Crimmin 2004)
John Pasley	1*	Brother of Thomas Pasley		(Smiles 1867)

William Playfair (1759-1823)	1*,26*	Scottish engineer, professor at Edinburgh University	Edinburgh	(Spence 2004)
William Reynolds (1758-1803)	18,19,20,21	Tuckies, Jackfield, Coalport		(Powell 2002) (Ironbridge Gorge Museum Trust 1975)
Richard Reynolds	21*	Father of William Reynolds, partner at ironworks at Ketley		
Joseph Reynolds	21*	Half-brother of William Reynolds, continued with Richard Reynold's ironworks		
Rev Abram Robertson (1751-1826)	22,23,24,25,26^{+}	Savilian professor, Oxford - mathematician, astronomer	New College Lane, Oxford	(Sedgwick 2004)
John Robinson (1739-1805)	1*	Physicist, mathematician, University of Edinburgh	Edinburgh	(Playfair 1815)
Charles Shand	9*	Subscribed for plate of London Bridge	Liverpool	
Count Thyville	21*	Patented lamp for painters, 1801		
Dugald Stewart (1753-1828)	1*,26*	Professor of philosophy and mathematics, University of Edinburgh	Edinburgh	(Stewart, Dugald (1753-1828) 1911)
Samuel Vince (1749-1821)	12*,22*, 27	Clergyman, astronomer, mathematician at Cambridge	Cambridge	(Vince, Samuel n.d.)
Joshua Walker (1750-1812)	5*,28,29^{+}	Family ironworkers	Rotherham	(Taylor 1986) (Samuel Walker of Rotherham 2015)
Joseph Walker (1752-1801)	5*,29^{+}			

Thomas Walker (1756-1828)	5*,29^{+}			
John Wilkinson (1728-1808)	9*,30,31	Ironmaster	Bradley, Birmingham	(Dawson 2012)
Thomas Wright	9*	Subscribed for plate of London Bridge	Bersham, Wrexham	

no mark: author of letter

*referenced in letter

$^{+}$recipient of letter

C.3. The Letters

C.3.1. A.A.

1.

My dear friend,

You will before this have received the official letters from our Professors – From what I can learn they all agree as conceiving the Plan free from any scientific objection. tho' they may differ about details – So far for public sentiment – As for private Sentiments Dugald Stuart says he is lost in astonishment both at the grandeur & the Beauty of the Conception: and Playfair declares openly that when it is upon its Legs there will be nothing like it in the whole Solar System except the Ring of Saturn – Is not this good from an Astronomer? As for Mr Robison He keeps so close at home & is in such infirm health that I have not learnt his private sentiments –

I think you will see in Playfairs Letter that his great wish would be for a little more Height or it is probable that the want of full Plans & Explanation of the Model has prevented them from completely understanding the whole of your Ideas. – You will therefore take their opinions with this allowance.

– As the time of decision is drawing on. my anxieties are increasing. Do not lose a post to tell me any thing you think will be satisfactory. – Every one here would be disappointed if the high Arch were not adopted. – tho' all agree that whether adopted or not it will produce a new One in Architecture –

We are all well- tho Laura was lately seized with an attack of the Jaundice which made us very uneasy – It has however gone off very rapidly. & she is as well as ever. – We are in a few days to leave Prestonfield to our great sorrow. as another Person has taken a Lease of it, which the terms of English Reformers for non-residence has prevented Me – We have however got a very pleasant House in the neighbourhood with a very good garden –

Arrange all your matters for seeing us sometime or other in the Summer. – M^{rs} A. & the Children salute you with constant affection – Believe me always –

Most truly Your Sv^{t}

A.A.

Edin^{h} May 6 1801

– I write this in a Coffee house where my Ears are dinn'd with the noise of military officers. – Say in a word, <u>Can</u> you, or is it agreeable to you, to recommend a <u>Navy</u> Officer a Relation of mine. to Sir Tho' Pasley, either by yourself or this John Pasley? Answer with the same pardon that I ask – If you can, I'll send you particulars soon – my cordial & affn^{t} respects to all in Gower Street & to M^{r}. & M^{rs}. Elliot

C.3.2. Charles Bage

2.

Dear Telford,

Your magnificent bridge requires more consideration than I can make time to bestow on it. What observations I make are for you privately; if they are of any value you will commit them to use; if not they will perish quietly and lie in the grave without an epitaph. The first question suggests to me that the idea of one frame holding itself together should be given up. Because if you take away the benefit of its abutments, it would tear itself to pieces in the center, which instead of the thinnest part, ought to be the thickest. If a cast iron beam of 600 feet long were supported at each end it would break by its own weight unless it were more than 22f 3i deep. The lower edge would tear asunder in the middle. This gives the idea of the great danger of constructing the bridge on such principles that its weight can be applied to tear asunder rather than squeeze together, which must be the case independent of the abutments. It may be useful to you to know that if you have an inch square cast iron bar hanging perpendicularly, a weight suspended to the lower end of 54 Cw^{t}. will break it. Wrought iron of the same size would take 32 tons to break it. I know not what size your ribs are to be – perhaps 6 by 4 the weight of each about 20 tons, the pressure against the abutments perhaps 50 tons, and the force with which the rib endeavours to squeeze itself together in every part of its length, like a pillar supporting a weight, the same; that is 50 tons, which a rib of that size will readily bear. The abutments however will have altogether an immense pressure and require perhaps more deliberate precautions than the bridge itself. Every rib should be a bridge of itself, and all the joining should be absolutely close. If this can be accomplished, and the abutments so firm as not to yield in any part when the center is struck, each portion of every rib will take its share of the pressure, and the tyes both perpendicular and horizontal will have litterally nothing to do but to keep the ribs from bending, just as you would contrive stays for a very long and slender pillar. Supposing the ribs to be put up with absolute perfection, the tyes would have nothing to do, and therefore may be made slender, and may be taken out singly if necessary without danger; so that I think double tyes a useless expense. Besides they will last as long as the ribs which I suppose you have no idea of replacing in parts when they decay The Ribs well bolted together and well supported at the abutments, I think will be sufficient without diagonal braces, which, beside increasing the expence & weight will make it more difficult to put together so well as seems essential. The tyes will do everything – that is they will correct the beginning of irregular pressure and will be employed some in bearing pressure & some in being pulled at each end; but none of them ought to have much to do. I like your horizontal plan the least. I think it deficient both in simplicity and contrivance. I will take the liberty of suggesting the following sketch, which will give more effectual resistance against lateral pressure. With regard to the form of the arches I should recommend them to be slightly elliptical for these reasons. If the ribs had themselves only to support, the Catenarian curve would be advised. The arch of a circle will admit of a [diagram] greater pressure on the sides; but you are circumstanced as to be obliged to weight the sides considerably. Your very mathematical friends however will settle this point exactly. This elegant drawing of 3 curves will show what I mean. The lowest is the catenary; the middle the circle, the other the ellipsis, which I suppose necessary to resist the weight you are forced [diagram] to lay at A.A. .. In answer to the second question, as I suppose the chief lateral pressure to arise from wind, I imagine the catenary to be the best form, the pressure being equal on every part; unless vibration may make a difference; which I do not know. I do not completely understand the 4th question. With whatever force the ribs press against the abutments, with the same force each portion of the rib is pressed through its whole length; because action and reaction are equal. It is useless to consider the 5th and 6th question. whilst every part is good no force can displace the whole that is not sufficient by mere pressure to squeeze cast iron into atoms. Quest' 7 – The smaller the radius of the arch the smaller the dimensions of the ribs may be, but it is better to keep all the iron work above water. Qn: 8. No model of moderate dimensions can be made, the result of which would be satisfactory, because the scantlings if in exact proportion would be too small to be cast well – and if not in exact proportion would afford no safe conclusion. Qn 10. If the mast of a vessel were to strike the middle of a portion of the rib with sufficient force it would break it; but the bridge would remain, and a new piece might be put in. If these portions of rib were 6 by 4 and 10 feet long the stroke to break them must be equal to a pressure of 20 tons. If this is to be dreaded (though I should rather think the mast would be slivered to pieces) the two lower ribs, one on each side might be made stronger. The Bridge in the mass would not be affected.

All the rest of the questions are either out of my line, or virtually answered by some of the foregoing observations. I wish I could do you some good – for besides promoting so grand a national object I should like to see your fame soaring as high as the pinnacle of Pauls. – When you have five minutes leisure I shall be glad to hear from you.

Adieu, yours very truly,
Cha' Bage
Apr 12 1801

3.

M^{r}. Bage

23 April 1801

Mr. Telford

Salopian Coffee House

London

Dear Telford,

I had no intention of poking my nose amongst so many wise snouts as will exhibit themselves before your Committee. they will end themselves up when they smell (?). I have exposed myself entirely to please you, and shall not please you at last, because I propose alterations.

First I dwell on the stability of the abutments.

2dly. I desire the bridge not to be considered as a piece of framing, because merely as such it has little strength. –

3dly. I bid you beware of inaccuracies of workmanship and advise you to throw your ribs over the river without any thing but the cross tyes.

4thly. I desire you to consider the superstructure scaffolding to support the road, & to make it as pretty as you can – And I advise you to depend on the lower ribs only for strength; the upper ones being very good in Theory, but in practice good for nothing unless extreme accuracy of workmanship be attained –

5thly. I let you into the secret that each portion of Rib &c – will act as a pillar – that we know not whether it has been ascertained what weight an Iron pillar will bear - & therefore it is needless to ask what weight will crush the bridge, for that question cannot be answered.

6thly. I represent the importance of weighting each portion of the lower ribs with mathematical accuracy. –

7thly. I make a panegyric on the bridge being wider at the ends than in the middle; and show how the outer ribs standing obliquely will give firmness to resist tempests. –

8thly. I assure them the plan is sublime & fit to be executed. – What more can I do?

Not one syllable would I have said but at your particular request, - I was in Staffordshire when your letters arrived – and I have litterally no time to think upon the subject. -

Let me know how you go on. -

Yours very faithfully

Cha^{s}. Bage

Shrewsbury

Ap 23 1801

C.3.3. William Botfield

4.

Mr. Wm. Botfield

28th April 1801

Mr. Thomas Telford –

Salopian Coffee House

Charing Cross

London

Malinslee April 28th. 1801

<u>Dear Sir</u>

I have your Letters of March 2? and April 16th. but no plans or papers – I should have acknowledged the Receipt sooner & would with pleasure have given you or the Committee any Information in my power – but an sorry to tell you I have sustan'd such a Loss which has prevented me attending to Business for some time. – that is The death of my Father. I remain –

Dr. Sir

Yours truly

Wm. Botfield

C.3.4. Samuel Gunnel

5. [written by TT]

Salopian Coffee house Charing Cross

12th May 1801

Sir.

I think it right to communicate the following information which I have received respecting the strength of Cast Iron – as this information comes from experienced Iron masters, it may be of service to the Committee in their present interrogations.

Extract of a Letter from Messrs. Walkers of Rotherham in Yorkshire dated 14 April 1801

"To crush a piece of Metal of an Inch square would require an immense weight, the resistance of Cast Iron compared to that of stone is far beyond the proportion of 1000 to 1" –

-

Extract from a letter from M. William Reynold of Coalbrookdale dated 23 April 1801 –

The mean results of a variety of experimentation is thus, that a Cube of ¼ of an Inch square of soft or Grey metal resists a pressure of 80 Cwt. before it gives way and crumbles into small pieces, and a similar Cube of what we call Strong Iron, and which I presume is the same you denominate firm Metal resists a pressure of 200 Cwt. before it gives the least way and takes 10 Cwt more to separate its parts

[end of letter missing?]

C.3.5. Rowland Hunt

6.

M. Hunt

15 May 1801

Mr Telford

Salopian Coffee House

Charing Cross

London

N. Mines Place – May 15th 1801

Sir/

I have thought much of your noble London Bridge Print – Please to let me have a copy – <u>tinted</u>: I hope not <u>much</u> coloured strained on Canvass <u>varnished</u> – <u>edged</u> with Border of Black Paper varnished also - & a proper <u>oil</u> gilt Frame (<u>not</u> a burnished Gold one, for that hides an unglazed Print - & thus arranged let it be well packed in a Case - & directed to Me at the <u>Stone House</u> Shrewsbury – but to wait at the Warehouse till called for – the Noble Draw'g is worth this Direction - & I am Your's &

Y. Obed. Svt.

Rowld. Hunt.

C.3.6. Charles Hutton

7.

Dr. Hutton

Thomas Telford Esq
Salopain Coffehouse
Strand

Woolwich, Feb 8, 1801

Dear Sir,

The conversation now concerning your proposed new bridge has so greatly engaged the public attention that there can be no doubt but some record of it will be given in the Magazines and other periodical publications. If these are left so got up only by persons but little or not at all acquainted with the particulars, it will be no wonder if very improper accounts be given, & any? Of the projects. To prevent therefore such improper or unfavorable accounts I take the liberty to suggest to your consideration the propriety of giving such account yourself, as you may think fit & fitted to go abroad to the public.

Should this hint meet your approbation, the bearer of this note, Mr Phillips of St Paul Ch Yd No 71, and proprietor of the Monthly Magazine, is desirous of doing all the honour in his power to the project, by an insertion in that Mag. Of any account or engraved plan it, or both, should may be furnished him. In that case, I have no doubt he will give (in?) the most distinguished (manner?) what you (?) commit to his case.

I am,
Dear Sir,
Your most obedient Serv^{t}
Charles Hutton

8.

Dear Sir,

I called with my report on Friday last at the Salopian, but finding you were gone out of town, put it into the penny post, addressed to Mr Gunnell, & hope it will go safe. If you do not hear by him of its being received, on your coming to town, when you see him, be so good as give me a line to inform me.

Wishing you all success, I am,

D^{r}. S^{r}. Your most obed. Serv^{t}.

Cha. Hutton.

Woolwich

April 26.

C.3.7. Thomas Jones

9.

Mr. Jones

Bradley

11th April 1801

Mr. Thos. Telford

Salopian Coffee House

Charing Cross

London.

Bradley 11th April 1801

Dear Sir,

Mr Wilkinson has your letter of Yesterday, in reply to which, he recd. the parcel safe which you sent containing your Plans, w^{ch}. are now under Consideration & an Answer will be sent in a few Days. He requests me to say that he thinks it will be by far the best Mode if a Model was made to a Scale, which would point out any Improvements such were required - & he wo^{d}. himself undertake to make this Model under your Direction, & take the Iron back after it was done, without any other Charge to the Committee than the actual Expences he might be put to exclusive of the Metal. –

You will put the annexed Names down for your print of the Bridge – Mr. W. will recommend it among his Friends & if I procure any more, I will either give you a Line, or let you know when I next see you –

Mr. Wilkinson has some Thoughts of being in Town at No. 2 Thavies Inn Holborn the latter End of the Month, when he hopes to see you –

I am Dear Sir

Your very obed Serv

Tho Jones

John Wilkinson Esq. – Bradley W. Birmingham
Tho^{s}. Jeffrys Esq. – Cockspur St. London
W^{m}. Bancks Esq. Corbyns Hall – Dudley
M^{r}. Cha^{s}. Shand – Liverpool
M^{r}. W^{m}. Fawcett – D^{o}.
M^{r}. R^{d}. Kirk – Gwersylt Wrexham
Tho Wright – Bersham – D^{o}.
Tho Jones – Brymbo – D^{o}.

all Coloured – or if you think it better send me a proof – or one of each & it will do to give some Friend -

C.3.8. W. Lowry

10.

Mr. Telford

Salopian Coffee House

Charing Cross

M. Lowry

Sir

I hope to get the plate done in a Week or ten days it would have been finished before now but in an attempt to work by Lamp light when I had it almost ready for the Aqua fortis I threw some Oil over it which obliged me to do a great part of it over again – I shall be glad if you can spare the Drawing for a few hours and as I shall be in Westminster tomorrow about 12 OClock I will call for it at your House –

Your Obliged serv^{t}.

W Lowry

May 12th.

C.3.9. Nevil Maskelyne

11.

D$^{r.}$ Maskelyne

Telford Esq., Architect
Salopian Coffee House
Charing Cross

Royal Observatory
Greenwich Thursday
March 19 1801

Sir,

Perhaps Mr Isaac Hawkins (Brown?) has acquainted you that Mr Cavendish & myself would wish to call on you some morning & get a sight of your plan of the iron bridge & the models. I have inquired of Mr Cavendish, by letter, whether it would suit him to attend on you with me next Saturday or Monday Morning about eleven o'clock for that purpose. I shall go to town to morrow & shall there receive Mr Cavendish's answer, to be left with Mr (Gilpin?) Clerk to the Royal Society, at their apartment Somerset Place, & will then inform you of Mr Cavendish's answer. in the meantime be pleased to favor me with a line, either at the bar of the Salopian Coffee House, or with Mr Gilpin, whether either of the will be suitable to you, or if not what other day next week. I shall at all events wish to be favored with a night of the plan & models, even tho' Mr Cavendish should not attend. I am,

Sir,

your most humble Servant,

N. Maskelyne

12.

Greenwich March 24 1801

Dear Sir,

Mr Cavendish declined taking any part in the business, so I went without him & saw the drawings & models & received Mr Telford's explanations, who behaved with great civility on the occasion. I find however that I want to have drawings of the parts, with a minute explanation before me, to enable me to recollect what I saw & to fully comprehend & judge of the construction. These I shall be obliged to Mr Telford for; & I apprehend they will be necessary for other gentle men who wish to judge of the construction. After seeing & considering these, I shall wish to see the model again. I apprehend Mr Vince, who never saw either the plan or model, will want the same information. I remain,

Dear Sir,

Your Obed^{t}. Servant,

N. Maskelyne

13.

Dr Maskelyne
31st March 1801

Telford Esq Engineer
Salopian Coffee House
Charing Cross

Greenwich March 31 1801

Sir,

I desire to know whether it will suit you next Thursday morning, at half past twelve, to favor me with a further view of the model of the new bridge as needed that case I would attend you at that time. I shall also be very glad of a vertical section, or elevation of the bridge, shewing the disposition and relative dimensions of the ribs & perpendicular pieces in detail. I am,

Sir

Your humble servant

N. Maskelyne

14.

Dr Maskelyne
3 April 1801

Thomas Telford Esq, Engineer
Salopian Coffee House
Charing Cross

Greenwich Apr. 3. 1801

Sir,

I request you to inform me whether the plan to the framing of the plate No. 2 is the vertical section, & a detailed plan of No. 3 the elevation? If so, I want a horizontal section of the bridge. The sooner you send me the section I want, the better. Any written explanation or account of the plans, you may prefer to send at the same time, will be agreeable. I shall be ready to inspect the model when it shall be repared. I am,

Sir,

Your humble Servant,

N. Maskelyne

15.
[on overleaf - Telford's hand]

Sir,

Your request 4th April 1801 and with regard to the more detailed acc^{t} of the several parts of the plans you have viewed I try here to observe

I have rec^{d}. yours of the 3^{d}. The plate which is considered as N^{o}. 1 is an Elevation to which the model nearly correspond - The Ribs are composed of frames of Cast Iron, secured in their places by cross Tyes formed with Shoulders between which the frames or pieces of Ribs are introduced. N^{o}. 2 is an horizontal Section corresponding with the Plate, it shows the manner in which the Pieces of Ribs are secured by the cross Ties, also how they can be further secured by means of diagonal braces. The other drawing on the same sheet also marked N°. 2 is likewise an horizontal Section and shows another method of securing the pieces of Ribs in their places by means of double ties, in this mode the Ends of the pieces of Ribs instead of resting or abutting against the Shoulders of the cross Tyes, are made to abut immediately against each other. This last method of working with double Tyes is introduced in order to show how the Bridge may be formed so as each piece of the tyes as well as of the Ribs may be taken out & replaced separately, if any of them should quick decay. In the first method of cross Tyes with Shoulders each piece of the Ribs may be taken out and replaced separately, but there would be some difficulty in taking out the Tyes - the Shoulders of the Tyes against which the pieces of the Ribs abut by being excluded from the action of the line by means of Iron cement, melted Iron, or melted Lead, or other proper substance, would be less likely to decay than any other part of the Bridge - It therefore becomes a consideration whether the working with the single cross Tyes with Shoulders, or with the double Tyes is the most advisable. - I have hitherto only considered No. 1 and 2 as to their horizontal Sections. In the Vertical Section or Elevation, I would make the outside Ribs in the Gothic manner (?) (?) the lightness and elegance of the forms, this being well adapted to the ornamental Iron work and admitting of finishing the top and Railing with such Gothic work & Promenades for accessing the Lamps - but these I want from the other Ribs with diagonals these and the whole of them from the Line of Road way to the lower line of the Arch may be connected together so as to act as frames by means of Notches in the Joints, by cast Iron Straps Wedges or forelocks, or by long diagonal braces crossing over and connecting with the several frames, and instead of the upright Joints being (?) immediately over each other from top to bottom These Frames may be made to join alternately in different places. The frame or Ribs are proposed cast to diminish in height from the abutments to the crown of the Arch and these to be at each a depth as to connect the whole sufficiently from abutment to abutment. [diagram]

No. 3 contains both an horizontal and vertical Section and shows how the Bridge may be composed of separate pieces of Iron cast of (?) lengths - that is to say the Ribs cross Tyes and diagonal braces - the consideration is, whether the advantage of having the Iron work greater lengths is not more than overbalanced by having the main bearings being (?) made to depend on so many (?) joinings and fittings. In this drawing - the Ribs are shown each of them pressing the crown of the arch in the Elevation, and the plan instead of crossing each other as in the plate they are suspended at equal Lengths at the abutment and in the middle. –

I think I also mentioned to you that in the constructing the Bridge there may be some advantage gained by introducing strong Cast Iron chains from several parts of the (?) (?) friction (?) into the body & (?) part of the abutment where they may be fixed to Iron work Cut into the Stone – and when they be examined from time to time by means of (?) made for that purpose.

I hope this information will afford to a tolerably correct notion of the several parts of the Bridge – I shall be glad to attend to any other matter which may occur to you, as soon as the model is put up again I will let you know, and shall be glad to have to honour of showing it to you.

I am –

16.

D^{r}. Maskelyne
14 April 1801

Thomas Telford Esq. Architect
Salopian Coffee House
Charing Cross
Post paid

Greenwich April 14. 1801

Sir,

I received your letter of the 4th with a vertical section of the bridge. In N^{o}. 2 the rib seems to be a series of bars of iron connected together at the ends by the help of the crossties. In the vertical section, each vertical frame is considered as a rib. Are not then the tops of each vertical frame to be considered as the horizontal rib, and to be tied together at the ends with the help of the cross ties? Will not this be impeded by the application of the cross vertical frames at B,B? If so, might not they be applied in the middle part of the horizontal rib?
I should be glad of a cross vertical section, that I may see wherein it differs from the direct vertical section in your letter. Cross braces, like S$^{t.}$ Andrews employs would be useful here, as well as in the vertical section.

I apprehend that it would be necessary that the contiguous iron frames in the vertical section should be strongly connected together laterally, ~~But I see no contrivance for this. This might be effected by hooked contrivances~~ and the contiguous vertical frames, which are over each other, should be also strongly connected together at the horizontal line in which they meet, that the pulling up the upper frames may pull up the lower frames, and pulling down the lower frames may pull down the upper frames. The first may be supposed effected by the application of the cross vertical frames, at the place where they join in a vertical line; but if the joining should be made in the middle of the vertical frames some other contrivance would be requisite instead. Bars of iron sliding over conical grooves decreasing downwards might be used, instead. My idea is, that the parts of the bridge should be so firmly connected together, both laterally and vertically, that the whole may be considered as one frame of iron. Should not you send the same vertical sections and additional information to the other gentlemen as to me? I am,

Sir, Your humble Servant,

N. Maskelyne

17.

Dr. Maskelyne

18 April 1801

Thomas Telford Esq.

Salopian Coffee House

Charing Cross

Greenwich April 18. 1801

Sir,

I received yours of the 15th in answer to mine of the 14th from which I seem to infer that the rib frames, which are disposed parallel to one another according to the length of the bridge, are not connected together by any framed pieces like the rib frames but only by the cross ties shewn in N°. 2. Am I right in this apprehension? I desire to know whether the supposition of the 7th question means that the bottom of the arch is to be ten feet lower or nearer to the river, while the summit of the circular arch remains at the same height as before, & consequently that versed sine of the arch is to be 75 feet instead of 65 feet. I observe that the summit of the arch in the plan is 72 feet above high water mark; and the bottom of the arch 3 feet above the same; & that the height of the arch above its base or the versed sine is 69 feet. On the first page of N°. 2 is there any thing but friction to keep the ends of the ribs, where they join, from separating sideways or according to the length of the bridge? Or is this to be effected by the iron cement?

I am,

Sir

Your humble Servant

N. Maskelyne –

C.3.10. William Reynolds

18.

Tuckies April 19th 1801

My dear Friend,

I am sorry to see by yours of the 16th that you think me long in writing on the subject of the Iron Bridge but I assure you I have not been idle in it, tho' a part of the time since I received the plan & papers I have been prevented by indisposition from attend^{g}. to it or any other business – but I have been mak^{g}. some exp^{s}. on the strength of diff^{t}. Qualities of Iron & the power requisite to squeeze Iron to pieces by an endways pressure w^{ch}. have turned out curious & w^{ch}. I will communicate with the answers I may give to such of the queries as I think myself competent to answer w^{ch}. I will take care to send up before the day appointed & with advertence to what you say –

I am sorry I have succeeded so ill in obtain^{g}. subsciptions for you hav^{g}. only got orders for 4 proofs & 1 color'd plate, most people ask if the Bridge is sure to be erected –

Yours ever

W^{m}. Reynolds

I forwarded immediately M^{r} Botfield's plan's

19.

W. Reynolds

23 April 1801

Thos. Telford Esq^{r}.

Salopian Coffeehouse

or Hos. of Commons

Tuckies Ap^{l} 23rd 1801

My Dear Friend

I have at last tho' somewhat reluctantly given answers to such of the queries sent down as I tho^{t}. myself equal in some degree to do, but I find the whole business so great & involve such a mess of enquiry that I have been obliged to speak with Caution – between us I have no doubt but that if the Bridge be built on the present plan & stands one day it will stand as long as time lasts – but my only fear is from the Abutment on the (?) Side for as this must be all of masonry & dependent on its own strengths for the support it is to give the Bridge I can not help fearing that from the Mass of soft matter in such a number of Joint a little giving way may take place & if but a few inches its ruinous tendency to the whole fabric is at once perceptible – I have been trying my experiments on the diff^{t}. strengths of Cast Iron & did not believe it possible to compress any of the species wth. so small a force as I find it – however there is plenty of resistance to ensure all you wish in the Bridge – but the mean results of a variety of trials turn out thus, that a Cube of ¼ of an Inch square of soft or gray cast Iron resists a pressure of 80 Cw^{t}. before it gives way & crumbles into small pieces – And a similar cube of what we call strong Iron & w^{h}. I presume is the same wh. you denominate firm Metal resists a pressure of 200 Cw^{t}. before it gives the least way & takes 10 Cw^{t}. more to separate its parts – These expt^{s}. tho easily told have cost me a good deal of time & some trouble in the execution, to be sure that the bearings were perfectly accurate & to ascertain precisely the forces required w^{h}. was not done till after many repeated trials – As I must sincerely wish success to so grand a undertaking I hope you will be very careful to avoid all dangers in the outset & if you had abutments of grow^{g}. rock on each side you w^{d}. have more to encounter in the opinion of

Yours ever

W. Reynolds

P.S. I inclose you a press Copy of the answers I have given to the queries which I hope you will be able to read tho it is a very imperfect one, but if laid upon a sheet of white paper I believe it will be legible –

WR

20.

Madley Wood May 10th 1801

My dear friend

I have yours of the 7th Ins^{t}. & am glad to find by it that you approve of what I have said very unwillingly I assure you as I do not think myself competent to such an undertaking –

I am sorry I cannot get you more subscribers to your perspective view w^{h}. I dare say will be beautiful & a good print whether the real Bridge be in Utopia or at London –

Coalbrookdale - 1 set proof –

Ketley Co. - 1 D^{o} –

Rich^{d} Reynolds - 1 D^{o} –

Rich^{d} Dearman - 1 D^{o} –

Will^{m} Reynolds - 1 D^{o} –

D^{o} - 1 Coloured

Jos. Reynolds - 1 proof

I am glad to hear that you think it likely it will go forward, make haste to get an act for it lest our Bridge sh^{d}. fall & throw a damp upon it – My health is better, but walk I can not, when you come into the Country let us see you, your company always does good to

Your friend

W Reynolds

– I believe I ask'd you to enquire about Fitzgerald's Trumpet for increas^{g}. the sound of Guns Pistols etc & to buy me one if I did not I now do it –

21.

William Reynolds

14th May 1801

Thos. Telford Esqr.

Salopian Coffee House

Tuckies Madeley Wood Furnaces

My dear Friend,

In reply to yours of the 8th Inst. on mature consideration I think your plan for the Iron Bridge excellent, & do not fore see the least difficulty in its execution for tho' I do not for from letter or sketch at all comprehend what size or weight your scantlings of Iron will be, yet I imagine be they what they may, as they will all be cast out of a reverbecatory furnace the Pig Iron may be so selected that very little variation in the contraction of the pieces will take place & if any should a (?) may be contrived to adjust them by grinding with very little trouble or expense – the joints being fitted up with a cement of Iron, as to render the whole as of one piece.

I shd. not apprehend the least danger from the expansion or contraction wh. wd. be occasioned by any variation of temperature this Climate affords or indeed any others as from the great span of the arch such a liberty of rising in the Center is given to the arch & at the same time the alteration in height wh. wd. be wanted is so small, that I shd. fear no derangement of the parts in consequence there of.

As to what weight is likely ever to come upon it affecting it except by vibration I shd. think St. Pauls as likely to be affected by a Gnat alightg. upon it - & as to vibration I should think from the superior width of the Ends to the Center there wd. be little danger to be apprehended, if however the road way should happen to be bad & the swinging motion of one of your 9 Ton Waggons become isochronous with the vibration of the Bridge I shd. literally tremble to stand on the center thereof – And this I think the only possible danger arriving from such prodigious extensions of Metal, still there are many preventatives –

And now my friend I think I have answered all your queries as well as I can & I thank you for havg. given me the option of doing it by writing as well as I most sincerely do for your kind attention to Glazebrooke & service therein rendered to my self, & as you have constituted yourself my agent whilst in Town, perhaps I need not apologize for the trouble I am about to give you w^{h}. is that you will have the goodness to convince yourself by ocular demonstration of the utility of Count Thyville's new Patent Lampe & if on inspection you approve them purchase one for me & send it down, it is intended for enlightening the work of Painters sitting in a row at a Deck or Table & perhaps they are so contrived or may be as to throw the light upon 3 or 4 small spaces detached from each other about 3 feet, but as you know how they paint China I need say no more to you as verbi sapi sati except that I am as ever

Yours sincerely

W Reynolds

Are not you happy to think of the Emperors complaisance in leaving us to pick the bone alone

C.3.11. Abram Robertson

22.

Wrights' Coffee House, Soho Square, March 6th. 1801

Dear Sir,

I certainly would have written to you, from Oxford, long before this time, had I not been so severely indisposed as to be incapable of any thing like business. Before my illness I had made upwards of 60 calculations relating to the arch of the Bridge.

Professor Vince from Cambridge is at this House, and as Dr. Maskelyne had written tho him on the subject of the Bridge he is desirous of seeing the drawing and model. We will wait upon you to-morrow at eleven o'clock, when we hope it will be convenient for you to be at home. We have to attend a Board of Longitude at 12 O'Clock.

I am, Sir, your most obedient

humble Servt.

A. Robertson

Mr. Robertson

6th March 1801

23.

M. Robertson
3 April 1801

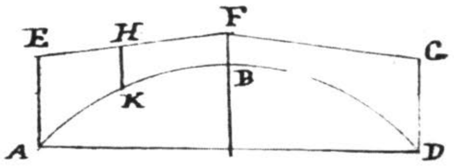
M^{r}. Telford.
Salopian Coffee House.
Charing Cross.
London.

Oxford. April the 3^{d}. 1801.

My dear Sir,

Before I received your Letter this morning I had intended to write to you, and to direct to you the Saalopian Coffee House at a venture. The Parcel containing the Print, Drawings, Queries, &c arrived safe on last Saturday Morning.

Since I had the pleasure of seeing you here I have employed every minute I could spare in calculating concerning the Bridge. I have now filled 20 folio pages with calculations, all tending to ascertain what we considered as the principal points to be determined, and I should be able to bring these to a conclusion, in a few days, if you will be so good as to favour me with an answer to the following Querie.



Supposing AHBD to be the arch of the Bridge, EHFG the roadway, and HK a small portion of the Bridge perpendicular to the horizon: is HK to be equally dense throughout and if it is not in what proportion is the density to increase from the upper part downwards, or from the under part upwards?

As soon as you have informed me on this head I shall be able to calculate the horizontal pressures of the several sections &c. I calculate upon a supposition of a sink of 2 inches in a yard in the road way from the crown of the Bridge; and also upon a supposition that at the crown the road way is to be 15 feet above the under part of the arch at the middle.

While this business is under consideration I should wish to be informed how to direct to you, in case of your leaving the Salopian Coffee House. M^{rs}. R. desires her comp^{n}. and I am, with much respect, your's

sincerely

A. Robertson

24.

Mr. Robertson
Oxford 9th April 1801

Mr. Telford.
Salopian Coffee House.
Charing Cross
London.

New College Lane, April 9th. 1801.

Dear Sir,

On Sunday morning I received your last letter, but before it arrived I had discovered a better method for obtaining what I had in view, than that for which the Querie was put. I now find that if the weight of ten feet across the Bridge at the end, or part next the Abutment, be 340 Tons, the weight of half the Bridge will be 9344.191 Tons, and consequently the weight of the whole Bridge will be 18688.382 Tons. I also find that the pressure against the Abutment at each end, in the direction of the Horizon, will be 20,551.45 Tons.

The weight of 10 feet across the end being 340 Tons, the weight of the remaining part of half the Bridge will be 9004.191 Tons, and the pressure of this portion, in the direction of the Horizon, against the section below it will be 20551.56 Tons.

If I rightly recollect our conversation when I had the pleasure of seeing you here, these were the only particulars of which you wished to be informed previous to the general return of answers to the Questions put by the Committee.

I shall certainly remember to consider the Bridge in the point of view which you recommend.

I am, with much regard, your's

&c. A. Robertson

25.

New College Lane, Oxford, April 21st 1801.

Dear Sir,

I am now drawing up my answers to the Queries, and I wish to have your advice how to send them and to whom. The name of the Clerk to the Committee is written in such a way that I can not make it out clearly; and if I could I should be at a loss by what conveyance to send. My papers, I believe, will make about three Sheets. Information upon this subject will much oblige

yours faithfully.

A. Robertson.

26.

[note by some reader: "To the Rev. M. Robertson Savilian Professor Oxford"]

London 5 May 1801

My dear Sir

It was not till after your Report respecting the Bridge had arrived that I received your letter – I was absent for a Week at Bath, your Report however found the proper person – The others are mostly come to hand, the gentlemen of Cambridge decline for the present, so does M^{r}. Stewart of Edinburgh, M. Playfair has transmitted –

The Committee have not yet met on the business they wait for Dr. Maskelyne whose report is daily expected.

As soon as any further steps have been taken in this business you shall hear from me.

Yours very sincerely

Tho^{s}. Telford

C.3.12. Samuel Vince

27.

Mr. Vince
11 April 1801

Mr. Thos. Telford,

Salopian Coffee House,

Charing Cross,

London

Sir

I received the plans safe, but cannot understand the Construction of the Bridge sufficiently to give any Opinion upon it, or to make any Calculations. A particular Description of a vertical section should have been given. When I come to town I will call and see the Model, from which I shall best understand the Construction, and see how far mathematical Computations can be of any use. I am surprised that any Answer could be expected at such short notice, a Matter of this Kind requires great Deliberation before an Opinion can be formed. I shall be in Town, the first Week of June.

I am $\dot{S}^r$.

Your humb S^t.

S Vince.

Cambridge,

11 April, 1801

Dr. Milner is under the same difficulties with myself.

C.3.13. Messrs Walker

28.

Mes^r^. Walkers
Rortherham
14th April 1801

Mr Telford
Salopian Coffee House
Charing Cross
London

Roth^m^ Ap^l^. 14^h^ 1801

Sir,

Your favour of the 30th Ult^o^ came in course and the Drawings &c in about a week after – It would have given us much pleasure had we been able to have given some information by answering the Queries propos'd by the Select Committee. They are however of such a nature that we have declined entering thereon as we should have wanted much explanation – The subject would have required much investigation, much experiment as well as local Knowledge – We have therefore beg'd permission to decline – The Plate &c shall be return'd to you tomorrow by the Coach, directed to the Salopian Coffee House –

Respecting the expansion of Metal in different States of the Atmosphere, we once made an experiment on an Arch of 105 feet Span which rose on the Crown, 37 In: perpendicular in the middle of the day in Summer – This was Pains' Arch –

To crush a piece of Metal of 1 Inch square would require an immense weight – the resistance of cast Iron compar'd to that of Stone is far beyond the proportion of 1000 to 1 –

Perhaps nothing protects Iron from the effect of Weather more effectually than Paint; nothing we believe has been found superior –

Your Idea of an Arch of 600 feet is a bold stretch – Tom Pains opinion was, that a Bridge upon his System, might be thrown over the Atlantic if Center for erecting it could be fixed – We have however liv'd to see his System of Bridges, as well as of Politick, exploded –

(I am for Self &c)
Sir
Your mo' humble Serv^t^

Jos^{ua} Walker

P.S. An early acknowledgement of the receipt of the Drawings would have been sent had the writer been at home

29.

London 21 April 1801

Gentlemen

I have received yours of the 14th. also the Plan and papers respecting London Bridge; I am sorry it does not suit you convenience to enter upon the subject –

It is only by second hand that I have known of Tom Paine or his Bridge – but I have understood that his Bridge was composed of a series of Barrs of wrought Iron. Therefore this instance of expansion will not apply to another made with Cast Metals – Buildwas Bridge in Shropshire is 130 feet span, and the effect upon it, has not been observable. An Aqueduct 180 feet long, made of Cast Iron, in the same County, has not been visibly affect'd – nor have I heard that Sunderland Bridge has suffered from "expansion" altho' 236 feet span

I am glad that your statement respecting the resistance of Cast Iron, when compared with Stone, corresponds with my Ideas on the subject – and I think it affords a sufficient ground to conclude, that altho' the political Quixote with his usual temerity went far enough beyond the bounds of credibility in straddling across the Atlantic, yet that by the means of this excellent metal, the Span of Arches may be extended considerably farther than by the present practice

I remain –

to Mess. Walker

Rotherham

Copy of a Letter to Mess. Walkers

Rotherham

21st April 1801

C.3.14. John Wilkinson

30.

John Wilkinson
17th April 1801
Oxf^{d} 20th

To M^{r} Tho^{s} Telford
at the Salopian Coffeehouse
Charing Cross
London

Bradley, 17th April 1801

Mr Tho Telford,

Sir,

I have your two favours of 15th and 16th Ins^{t}.

By the annexed copies of the letters to M Gunnell (as you formerly directed) which were sent off prior to receipt of yours, my sentiments may be ascertained upon the subject of the bridge, as far as comes within my own experience, or knowledge.

With respect to the scantlings you speak of, experiments upon them, and many others, tending to show the strength or resistance of iron, on the different planes, might undoubtedly be made, and much requisite information gathered: all this, would however be accomplished with the most facility and propriety at or near the foundry where the model shall be cast, and for which trials, if made here, I shall put the committee to no expence whatever on the metal, taking the whole back, after done with.

Upon the question of size of the model, most certainly the larger the better: nevertheless the scale given by me (see above answer to 8 and 9) I think might serve the purpose, especially as the strength of the metal in full scantlings may be tried separately while the model is preparing, and these scantlings may be made at once of the size you intend them in the bridge at large and the difficulties you suggest (which also strike me) upon the difference of proportionate strength betwixt the model and the large work, smoothed.

The expansion of the bridge to the abuttments would assuredly instead of 90 feet, be better at about half as much more, say 130 or 135 ft, but these are matters more immediately falling within the provinces of yourselves, as architects, than me as a founder. – Of the practicability of the scheme I have no hesitation in forming an opinion in the affirmative; the expence is for consideration.

My nephew T. Jones left this place for Brymbo on Wednesday last, and I am

Sir
Your very obed. Serv
John Wilkinson

PS. I think it necessary further to state, that I have chains here, capable of supporting 50 tons weight, so that the strength of one entire rib of your proposed bridge might be tried: - I conceive that an increase of the depth of the rib, and a diminution of the width, would give a superiority in point of strength, and keep within the weight of metal: - but, finally, it would be on every account desirable, that either yourself, or some other competent judge or judges, should be present, not only as supervisors of the model, but of the various trials independent of it. – How far 3 arches instead of 1, might be preferable, is a point, indeed, best submitted to your own ideas:- piers support any conceivable weight, and at same time lessen the pressure upon the abutments. That I wish well to the scheme you can readily conjecture from the offers I make, and have stated above.

JW.-

/Copy/

Bradley near Birmingham 15th. April 1801

M. Sam. Gunnel
Sir

I beg leave to enclose you such remarks as occur to me relative to Mess^{rs} Telford & Douglas's plan of an iron bridge over the river Thames sent me by order of the Committee upon the further improvement of the port of London. I have only to add that if the Committee should determine upon a model, and would have it done at this place, I would undertake to execute it under the direction of any persons they may appoint, without any further charge than merely such expenses as may absolutely be incurred independent of the metal, which I would furnish provided it was left with me here after such experiments were made as were thought desirable.

I am Sir
Y very obed Serv
/ Signed /
John Wilkinson

<u>Copy continued</u>

answer to questions / 14 April 1801 /

1 to 7. are rather mathematical than practical questions, but I give it as my opinion upon 1. that a bridge upon the proposed plan, if properly put together, would act as one piece, not as separate parts.

8 & 9. It appears to me that in an undertaking like the present it is not only advisable, but absolutely necessary, that a <u>model</u> should be made; which would point out more clearly than can possibly be done by theory, if any, and what alterations would be requisite: - if one upon a scale of ¾ of an inch to the foot, which would be a span of 37 ½ feet, it would answer every purpose, and an opinion could then be formed, and a determination made with a degree of certainty which cannot now be done.

10. 11. & 12 are nautical and architectural questions.

13. It would be most adviseable not to make the bridge of wrought and cast iron combined, but of either separately: - if, of the former, it should be a metal in the medium between hard or white, and soft or grey pig iron: - and if this was properly attended to in the casting, it would be preferable to wrought iron and erected with much less expence.

14. 15. 16. These will be best determined by experiments on the model.

17. A Cement may be made which would become both hard and durable, but as to liquid iron being poured into the joints, it is inadmissible.

18. Lead would not answer so well as a proper cement.

19. 20. These questions would be best solved by observations and experiments upon a model, when it might be ascertained whether any deviation from the present plan would render the bridge more substantial and durable.

21. With respect to any other articles, excepting iron work, I cannot form a judgment: and scaffolding and putting up, included with that, makes it difficult to give any opinion thereon; but, if connected together, I apprehend £20 per Ton will be found much too small a sum.

31.

/ Copy /

Mr Sam^{l}. Gunnell

London

Sir

I beg the favour of reference to my last letter under date of 15 Ins^{t} – since which having reflected much, and also had frequent conversations with practical men upon the subject of an iron bridge of 600 feet span, I am of opinion it is feasible and may be rendered sufficiently secure, admitting the expense upon such a bridge to hold out no impediment of importance, and admitting also that the expansion of the bridge at the abuttments shall be about half as great again as originally proposed, say about 130 @ 135 feet

I am

Sir

Your very obed Serv

/ Signed / John Wilkinson

M^{r} Tho^{s}. Telford

London

Bradley 21st April 1801

Sir

Your letter of yesterday is at hand; and following your wishes therein, I have by this post written as above to M^{r}. Gunnell, under cover to M^{r}. Freeling, which I hope may be satisfactory: - You will observe that I did not venture any remarks upon a 3=arched bridge, <u>to the Committee</u>: those I made were simply in my last private letter <u>to yourself</u>.

I remain

Sir

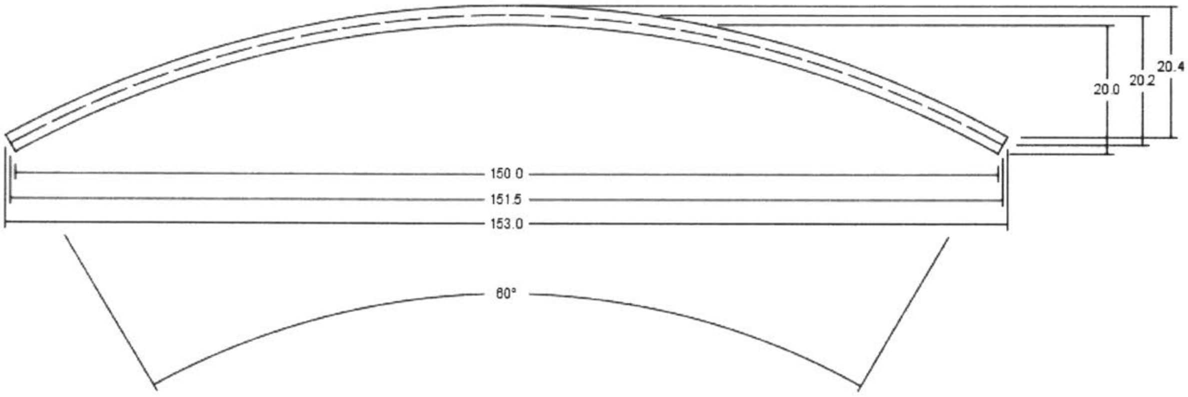
Your very hble Serv^{t}.

John Wilkinson

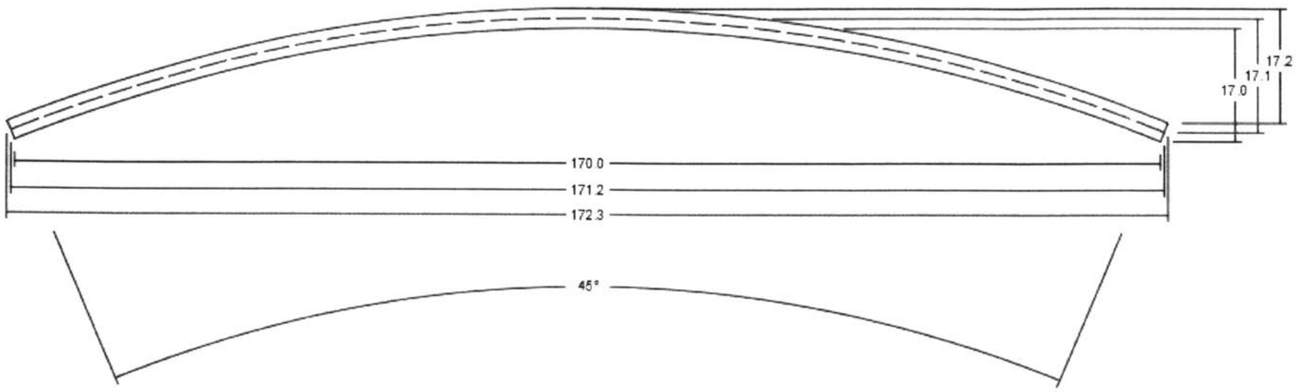
Appendix D. Modelling Bridge Geometry

	Bonar	Mythe
Rib span (interior) (ft)	150	170
Rib rise (interior) (ft)	20	17
Rib radius (interior) (ft)	151	221
Rib depth (ft)	3	3
Rib radius (centerline) (ft)	152	223
Bridge width (ft)	15	24
Number of ribs	4	6
Tributary width per interior rib (ft)	5	4.5

Bonar:



Mythe:



Radius of an arch: $r = \frac{h}{2} + \frac{s^2}{8h}$ (h = rise, s = span)

Equation of semi-circular rib: $-\frac{s}{2} \leq x \leq \frac{s}{2}, 0 \leq y \leq h$

$$y = \sqrt{r^2 - x^2} - (r - h)$$

Global equilibrium calculations: take centerline of rib

Bonar: $y = \sqrt{(152ft)^2 - x^2} - (152ft - 20.2ft)$

Mythe: $y = \sqrt{(223ft)^2 - x^2} - (223ft - 17.1ft)$

Slope of tangent to arch at any point x:

$m(x) = y'(x) = -x(r^2 - x^2)^{-\frac{1}{2}}$

Arch angle at point x:

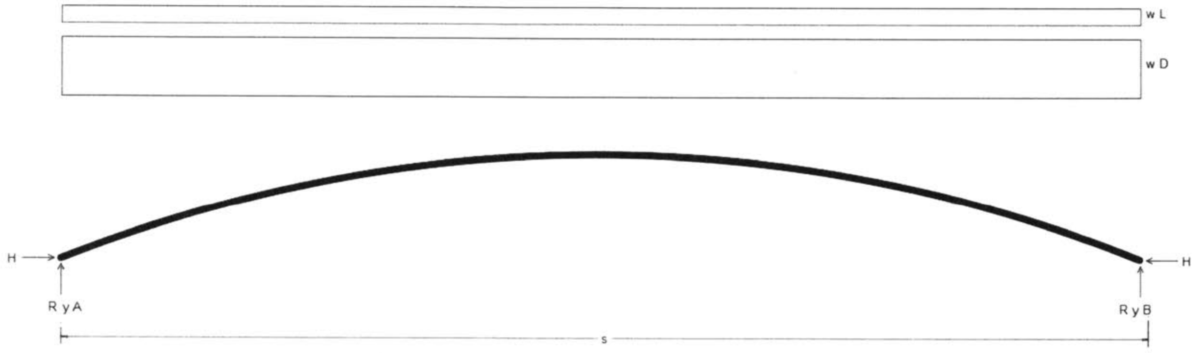
$\alpha(x) = \arctan(m)$

Appendix E. Global Equilibrium Calculations

Three-hinged arch assumption: $M_A = M_B = M_C = 0$

E.1. Force Approximations

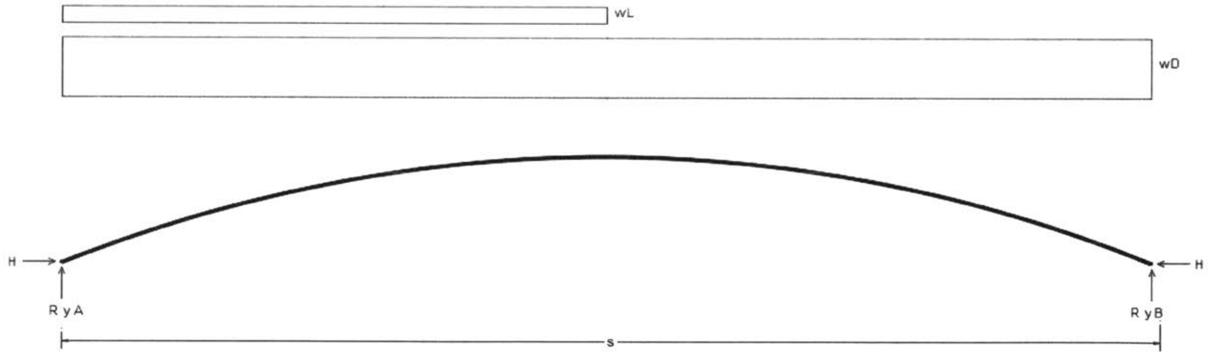
Uniform Loading:



Solve for vertical reactions:

$$R_{yA} = R_{yB} = \frac{1}{2}(w_L + w_D)s$$

Asymmetric Loading:

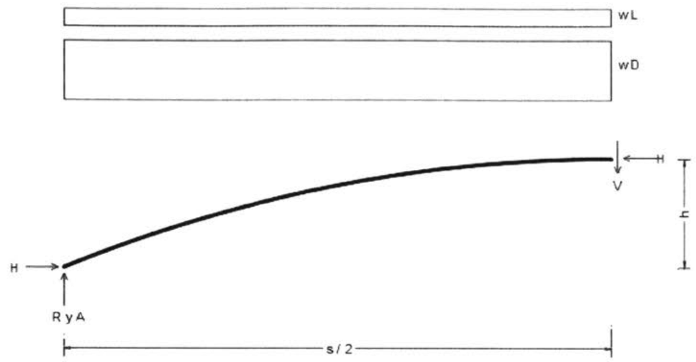


Solve for R_{yB}:

$$\sum M_A = -\frac{1}{8}w_L s^2 - \frac{1}{2}w_D s^2 + R_{yB}s = 0$$

Solve for R_{yA}:

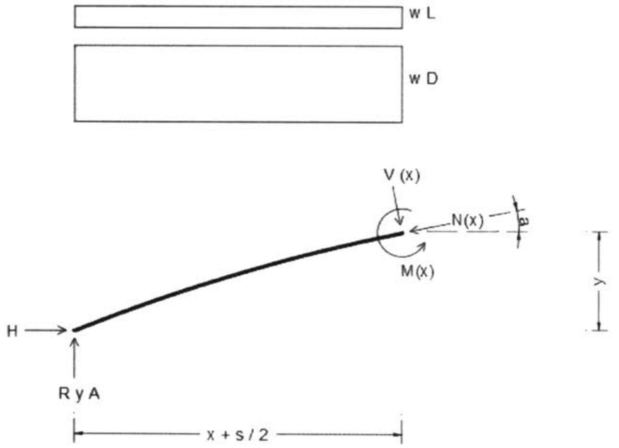
$$\sum F_y = R_{yA} + R_{yB} - \frac{1}{2}w_L s - w_D s = 0$$



Solve for H:

$$\sum M_C = \frac{1}{8}(w_L + w_D)s^2 + Hh - \frac{1}{2}R_{yA}s = 0$$

Uniform loading and asymmetric loading, x < 0:



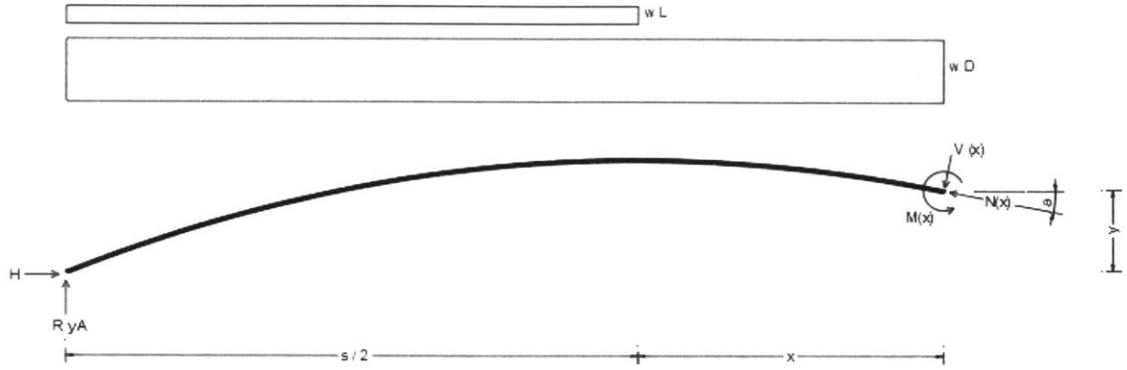
Solve for internal forces at any point along the arch:

$$\sum F_x = H + V(x)\sin\alpha - N(x)\cos\alpha = 0$$

$$\sum F_y = R_{yA} - (w_L + w_D)(x + \frac{s}{2}) - V(x)\cos\alpha - N(x)\sin\alpha = 0$$

$$M(x) = Hy - R_{yA}(x + \frac{s}{2}) + \frac{1}{2}(w_L + w_D)(x + \frac{s}{2})^2$$

Asymmetric loading, x > 0:



Solve for internal forces at any point along the arch:

$$\sum F_x = H - V(x)\sin\alpha - N(x)\cos\alpha = 0$$

$$\sum F_y = R_{yA} - \frac{1}{2}w_L s - w_D(x + \frac{s}{2}) - V(x)\cos\alpha + N(x)\sin\alpha = 0$$

$$M(x) = Hy - R_{yA}(x + \frac{s}{2}) + \frac{1}{2}w_L s\left(x + \frac{s}{4}\right) + \frac{1}{2}w_D(x + \frac{s}{2})^2$$

E.2. Sample Calculation - Bonar Bridge, Uniform Loading

$$R_{yA} = R_{yB} = \frac{1}{2}(0.60\ kips/ft + 1.15\ kips/ft)(151.5\ ft) = 133\ kips$$

$$\sum M_C = \frac{1}{8}\left(0.60\frac{kips}{ft} + 1.15\frac{kips}{ft}\right)(151.5\ ft)^2 + H(20.2\ ft) - \frac{1}{2}(133\ kips)(151.5\ ft) = 0$$

$$H = 249\ kips$$

Max compressive stress occurs at x = -58 ft – f_c = -6.63 ksi

$$y(-58) = \sqrt{(152ft)^2 - (-58\ ft)^2} - (152ft - 20.2ft) = 8.51ft$$

$$\begin{aligned} M(58) = {} & (249\ kips)(8.51\ ft) - (133\ kips)(-58ft + \frac{1}{2}(151.5ft)) \\ & + \frac{1}{2}(0.60\frac{kips}{ft})(151.5ft)\left((-58\ ft) + \frac{1}{4}(151.5\ ft)\right) \\ & + \frac{1}{2}(1.15\frac{kips}{ft})(-58ft + \frac{1}{2}(151.5ft))^2 = 26.6\ kipsft \end{aligned}$$

$$m(58) = y'(x) = -x(r^2 - x^2)^{-\frac{1}{2}} = -(-58ft)((152ft)^2 - (-58\ ft)^2)^{-\frac{1}{2}} = 0.413$$

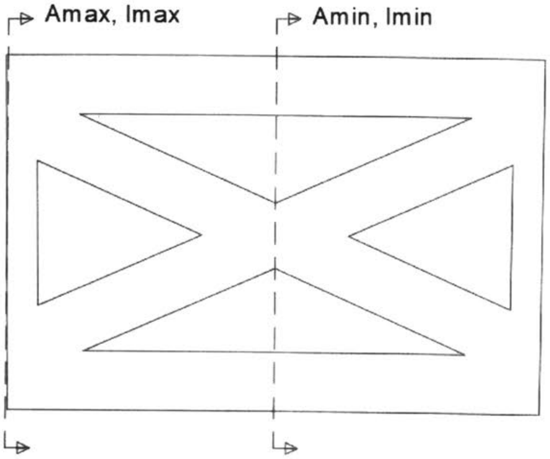
$\alpha(-58) = \arctan(0.413) = 22.4°$

$$\sum F_x = 249\ kips - V\sin(22.4°) - N\cos(22.4°) = 0$$

$$\sum F_y = 133\ kips - \frac{1}{2}\left(0.60\frac{kips}{ft}\right)(\ 151.5ft) - (1.15\frac{kips}{ft})(-58ft + \frac{1}{2}(151.5ft)) - V\cos 22.4° + N\sin 22.4° = 0$$

$N = F_x \cos\alpha + F_y \sin\alpha = -269\ kips\ (C)$

$f_N = \frac{N}{A}$ $\quad f_M = \frac{My}{I}$ $\quad f_{top} = f_N + f_M$ $\quad f_{bottom} = f_N - f_M$



$A_{max} = 90\ in^2$; $A_{min} = 46\ in^2$

$I_{max} = 9720\ in^4$; $I_{min} = 7235\ in^4$

$y_{max} = 3$ ft

max f_c at x = -58 ft

$$f_{Nmax} = \frac{N}{A} = \frac{-269\ kips}{46.1\ in^2} = -6.63\ ksi$$

$$f_{Mmax} = \frac{My}{I} = \pm\frac{(26.6\ kips\ ft)(1.5ft)(12\ in/ft)^2}{(9720\ in^4)} = \pm 0.591\ ksi$$

$f_{top} = -6.63\ ksi - 0.591\ ksi = -7.22\ ksi$

$f_{bottom} = -6.63\ ksi + 0.591\ ksi = -6.05\ ksi$

Appendix F. Graphic Statics

F.1. Bonar Bridge – Uniform Loading

Radius: $r = \frac{rise}{2} + \frac{span^2}{8\,rise}$

Included angle: $\theta = \frac{2\arctan\left(\frac{span}{2}\right)}{radius - rise} = 59.7°$

Divide into 35 voussoirs: each includes an angle of 1.71°

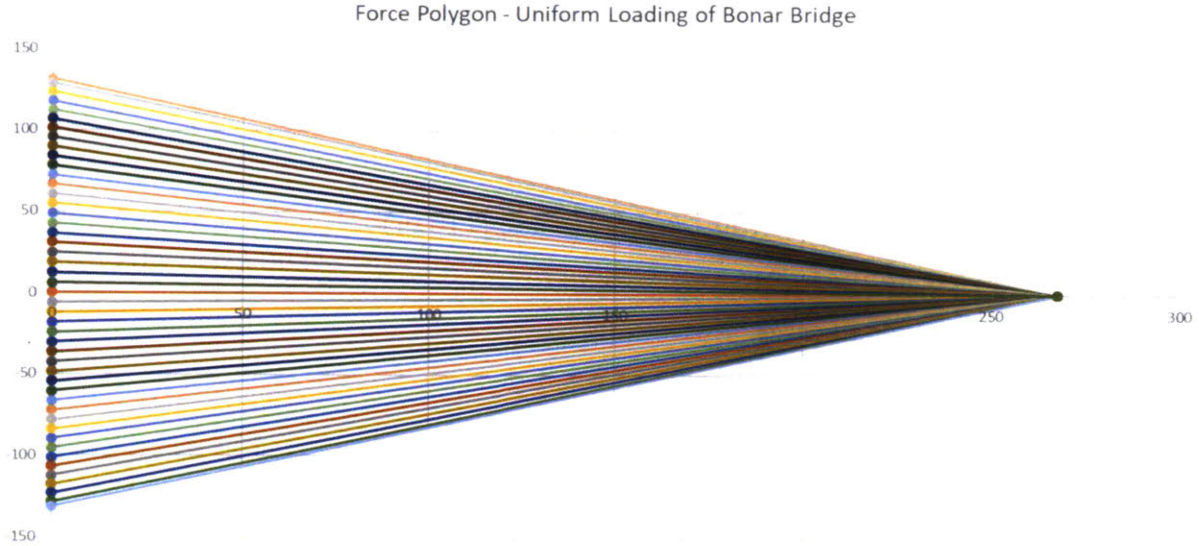
Voussoir boundaries: $\beta = \frac{\theta}{2} + \frac{\pi}{2}$, $x = r\cos\beta$, $y = r\sin\beta - r\sin\beta_{start}$

Load applied by each voussoir (assumed to be applied at the center of the segment):

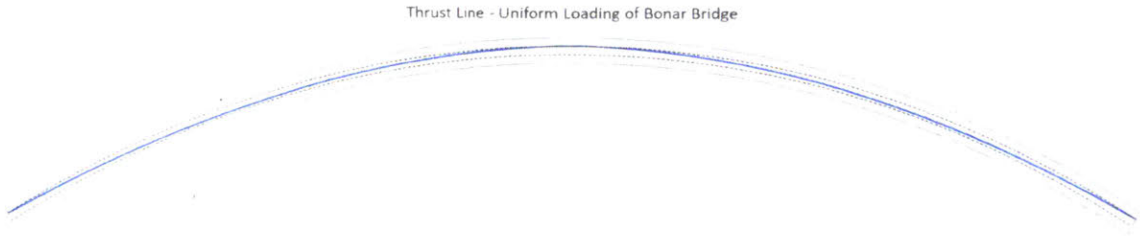
$W_i = w(x_{(i+1)center} - x_{(i-1)center})/2$

These weights used to construct force polygons

At the bridge's crown, horizontal force H only



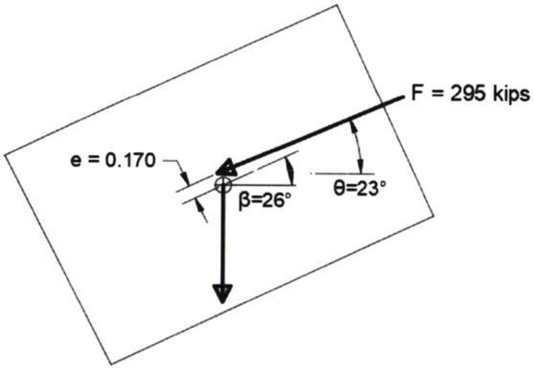
H = 268 kips

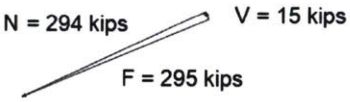


Thrust line contained within the middle third.

Stresses found based on load and eccentricity.

Sample calculation:





$$N = F\cos(\theta - \beta) = -294\ kips$$

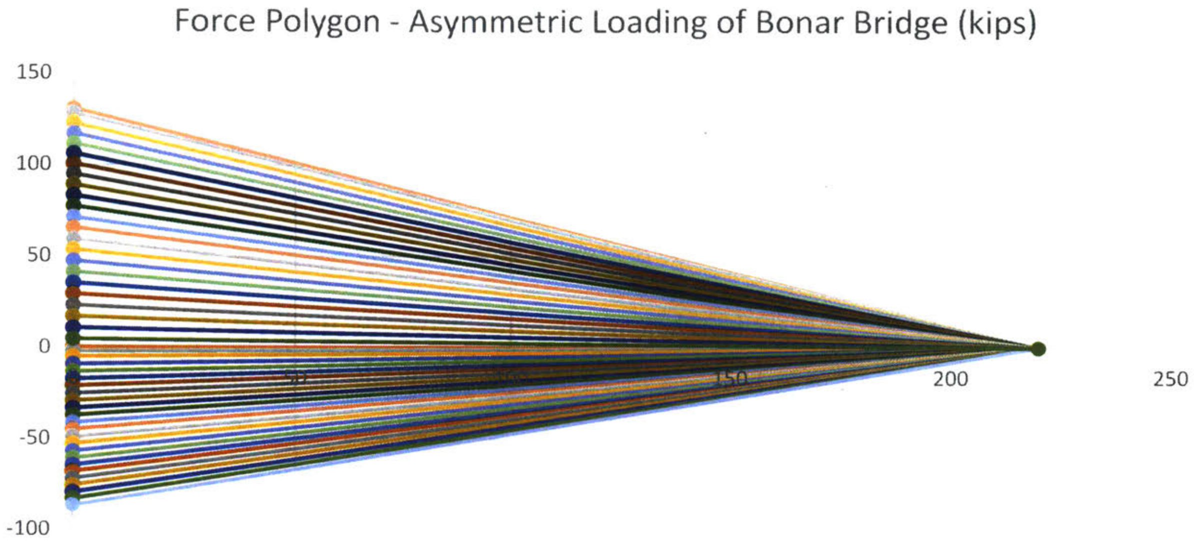
$$V = F\sin(\theta - \beta) = 15.4\ kips$$

$$M = Ne = 50.1\ kip\ ft$$

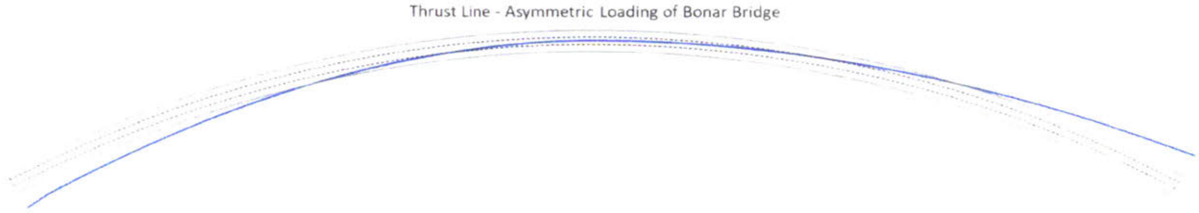
$$\sigma = \frac{N}{A} \pm \frac{My}{I} = \frac{-294\ kips}{46\ in^2} \pm \frac{(50.1\ kip\ ft)\left(12\frac{in}{ft}\right)(18\ in)}{7235\ in^4} = -4.90\ ksi, -7.89\ ksi$$

This procedure repeated for all cases.

F.2. Bonar Bridge – Asymmetric Loading



H = 220 kips



F.3. Mythe Bridge – Uniform Loading

Radius: $r = \frac{rise}{2} + \frac{span^2}{8\,rise}$

Included angle: $\theta = \frac{2\arctan\left(\frac{span}{2}\right)}{radius - rise} = 45.2°$

Divide into 40 voussoirs: each includes an angle of 1.13°

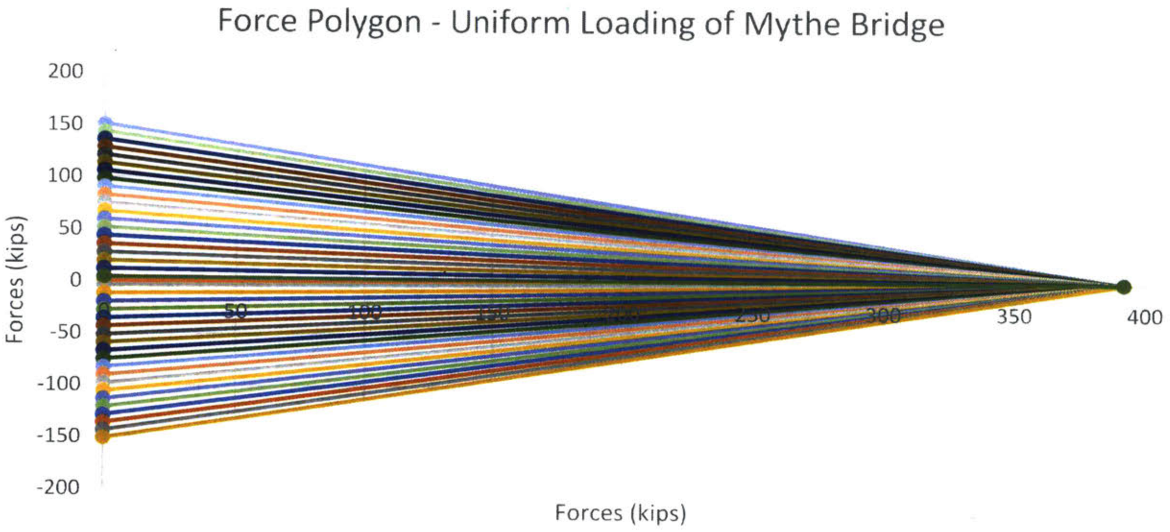
Voussoir boundaries: $\beta = \frac{\theta}{2} + \frac{\pi}{2}$, $x = r\cos\beta$, $y = r\sin\beta - r\sin\beta_{start}$

Load applied to each voussoir (assumed to be applied at the center of the segment):

$W_i = w(x_{(i+1)center} - x_{(i-1)center})/2$

These weights used to construct force polygons

At the bridge's crown, horizontal force H only

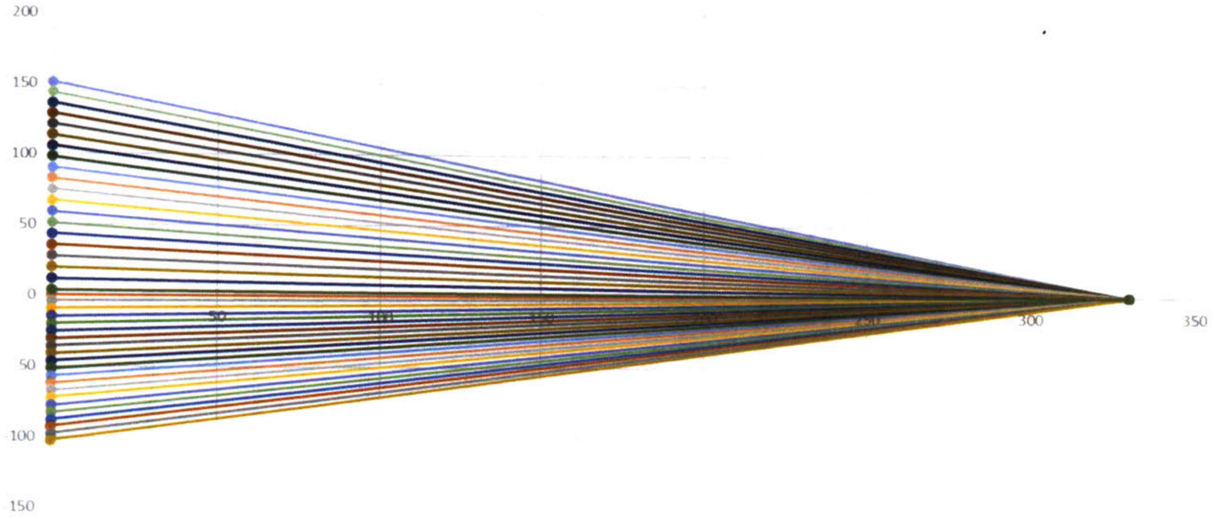


Force Polygon - Uniform Loading of Mythe Bridge

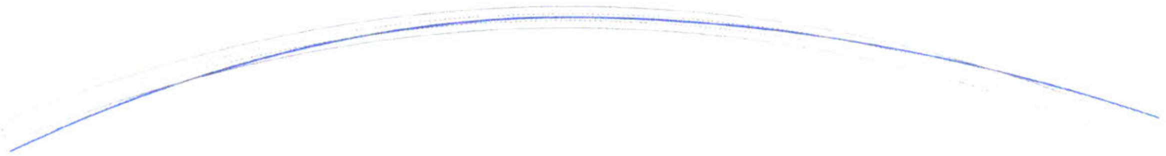
H = 392 kips

Thrust Line - Uniform Loading of Mythe Bridge

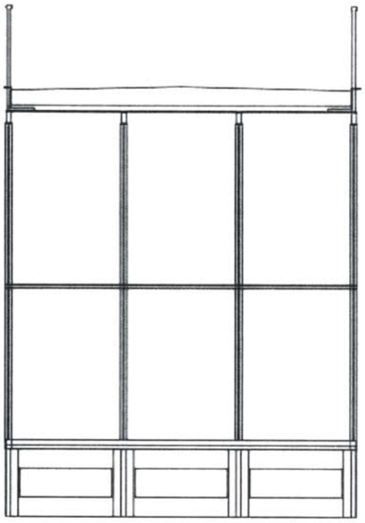
F.4. Mythe Bridge – Asymmetric Loading

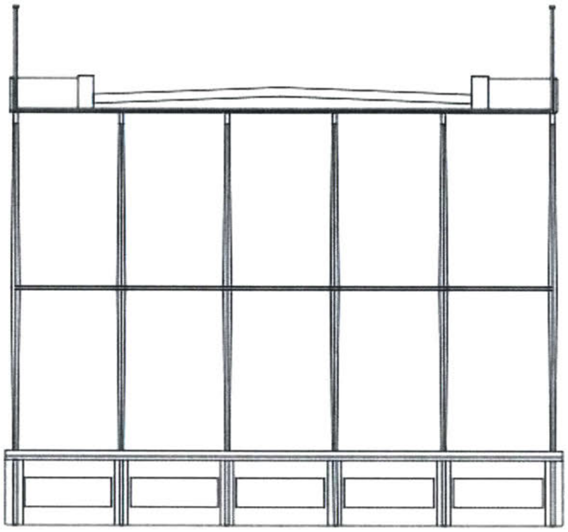

Force Polygon - Asymmetric Loading of Mythe Bridge (kips)

H = 330 kips


Thrust Line - Asymmetric Loading of Mythe Bridge

Appendix G. Deck-Stiffening Calculations





Moment of inertia of a rectangle:

- About its centroid: $I_{cent} = \frac{1}{12}bh^3$
- About a parallel axis: $I_{parallel} = I_{cent} + Ad^2$

Centroid of a shape: $y_{cent} = \frac{A_1y_1 + \cdots + A_ny_n}{A_1 + \cdots + A_n}$

G.1. Deck Stiffness

Taking centroid from the bottom of the bearing plate

G.1.1. Bearing Plate

	Bonar	Mythe
Height (in)	0.875	0.750
Width (in)	180	288
Centroid (in)	0.438	0.375
Area (in^2)	158	216
Ay (in^3)	68.9	81.0
Centroidal moment of inertia (in^4)	10.0	10.1
Moment of inertia about deck centroid (in^4)	1401	858

G.1.2. Bearing Ribs

	Bonar	Mythe
Height (in)	6	6
Width (in)	2.5	2.5
Centroid (in)	-3	-3
Number of ribs	4	6
Area – one rib (in^2)	15	15
Area – all ribs (in^2)	60	90
Ay (in^3)	-180	-270
Centroidal moment of inertia – one rib (in^4)	45	45
Centroidal moment of inertia – all ribs (in^4)	180	270
Moment of inertia about deck centroid (in^4)	2644	2852

G.1.3. Skirting Plates

	Bonar	Mythe
Height (in)	12	12
Width (in)	1.5	1.5
Centroid (in)	6.88	6.75
Number of plates	2	2
Area – one rib (in^2)	18	18
Area – both plate (in^2)	36	36
Ay (in^3)	248	243
Centroidal moment of inertia – one plate (in^4)	216	216
Centroidal moment of inertia – both plates (in^4)	432	432
Moment of inertia about deck centroid (in^4)	865	1127

G.1.4. Railings

Railing (major posts)

	Bonar	Mythe
Height (in)	54	54
Width (in)	2	2
Centroid (in)	27.9	27.8
Width (longitudinal) (in)	2	2
Number of posts	1 per rib segment = 34 over 150 ft	1 per rib segment = 39 over 170 feet
Factor for post frequency, one side	0.0378	0.0382
Area – one post (in^2)	108	108
Area – averaged, two sides (in^2)	8.16	8.26
Ay (in3)	227	229
Centroidal moment of inertia – one post (in^4)	26244	26244
Centroidal moment of inertia – averaged, two sides (in^4)	1983	2007
Moment of inertia about deck centroid (in^4)	6867	7333

Railing (minor posts):

	Bonar	Mythe
Height (in)	54	54
Width (in)	0.625	0.625
Centroid (in)	27.9	27.8
Width (longitudinal) (in)	0.625	0.625
Number of posts	7 per rib segment = 238 over 150 ft	7 per rib segment = 280 over 170 feet
Factor for post frequency, one side	0.0826	0.0836
Area – one post (in^2)	33.8	33.8
Area – averaged, two sides (in^2)	5.58	5.65
Ay (in^3)	155	157
Centroidal moment of inertia – one post (in^4)	8201	8201
Centroidal moment of inertia – averaged, two sides (in^4)	1355	1372
Moment of inertia about deck centroid (in^4)	4695	5013

Railing (hand rail):

	Bonar	Mythe
Height (in)	1.50	1.50
Width (in)	2.50	2.50
Centroid (in)	55.6	55.5
Area – one rail (in^2)	3.75	3.75
Area – two rails (in^2)	7.50	7.50
Ay (in^3)	417	416
Centroidal moment of inertia – one post (in^4)	0.703	0.703

G.1.5. Deck, Neglecting Railings

Centroid: taken from bottom of bearing plate

- Bonar: y = 0.538 in
- Mythe: y = 0.158 in

Total moment of inertia of deck (in^4):

	Bonar	Mythe
Bearing plate	1401	1526
Bearing ribs	2644	3536
Skirting plates	865	932
Total	**4910**	**5994**

G.1.6. Deck, All Elements

Centroid: taken from bottom of bearing plate

- Bonar: y = 3.41 in
- Mythe: y = 3.02 in

Total moment of inertia of deck (in^4):

	Bonar	Mythe
Bearing plate	1401	1526
Bearing ribs	2644	3536
Skirting plates	865	932
Railing (major posts)	6867	7056
Railing (minor posts)	4695	4823
Railing (handrail)	20450	20654
Total	**36922**	**38528**

G.2. Arch Stiffness

G.2.1. Ribs

	Bonar	Mythe
Height (in)	36	36
Width (in)	2.5	2.5
Centroid (in)	-18	-18
Amax - longitudinal	15	15
Atot - longitudinal	10	10
Ratio	0.681	0.681
Number of ribs	4	6
Area – one rib (in^2)	90	90
Area – averaged, all ribs (in^2)	245	368
Ay (in^3)	-4414	-6621
Centroidal moment of inertia – one post (in^4)	9720	9720
Centroidal moment of inertia – averaged, all ribs (in^4)	26486	39729
Moment of inertia about frame centroid (in^4)	54875	81824

G.2.2. Covering Plates

	Bonar	Mythe
Height (in)	2.5	2.5
Width (in)	180	288
Centroid (in)	1.25	1.25
Total area of one segment (longitudinal) (in^2)	69	69
Actual area of segment (longitudinal) (in^2)	39.7	39.7
Ratio	0.578	0.578
Gross area (in^2)	450	720
Area – averaged, two sides (in^2)	260	416
Ay (in^3)	325	520
Centroidal moment of inertia – gross area (in^4)	894070	1430511
Centroidal moment of inertia – averaged (in^4)	516442	826307
Moment of inertia about deck centroid (in^4)	26149	42442

G.2.3. Connecting Plates

	Bonar	Mythe
Height (in)	36	36
Width (in)	180	288
Centroid (in)	-18	-18
Width of one segment (longitudinal) (in^2)	2.5	2.5
Number of segments (longitudinal)	4	7
Ratio	0.00556	0.00858
Gross area (in^2)	6480	10368
Area of all plates, averaged (in^2)	36.0	88.9
Ay (in^3)	-648	-1601
Centroidal moment of inertia – gross area (in^4)	7.00×10^5	11.2×10^5
Centroidal moment of inertia – averaged (in^4)	3888	9606
Moment of inertia about deck centroid (in^4)	6966	17091

G.2.4. Frame, All Elements

Centroid: taken from top of rib

- Bonar: -8.75 in
- Mythe: -8.83 in

Total moment of inertia of arch (in^4):

	Bonar	Mythe
Ribs	54875	81824
Covering plates	26149	42442
Connecting plates	6966	17091
Total	**87990**	**141356**

G.3. Deck vs Arch Stiffness

	I (x 10^3 in^4)	
	Bonar Bridge	Mythe Bridge
Ribs	54.9	81.8
Frame	88.0	141
Deck (no railings)	4.91	5.99
Deck (with railings)	36.9	38.5

Bonar Bridge:

Arch Girder	Ribs only	Frame
Deck (no railings)	11.2	17.9
Deck (with railings)	1.49	2.38

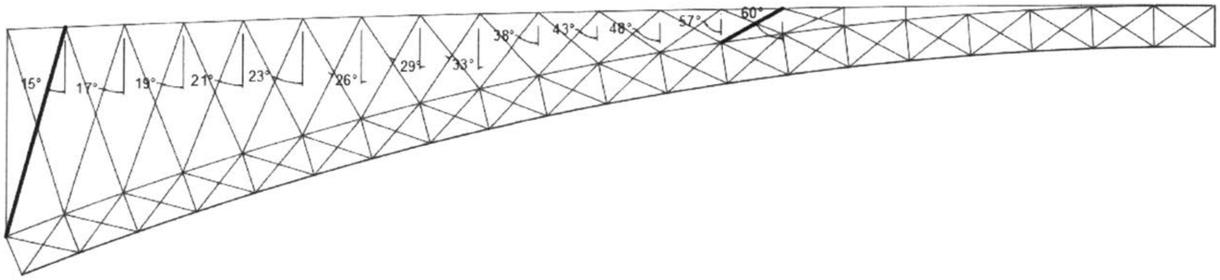
Mythe Bridge:

Arch Girder	Ribs only	Frame
Deck (no railings)	13.6	23.6
Deck (with railings)	2.12	3.67

Appendix H. Spandrel Bracing Calculations

H.1. Mythe Bridge

Spandrel orientation, length, and spacing determined from drawings.

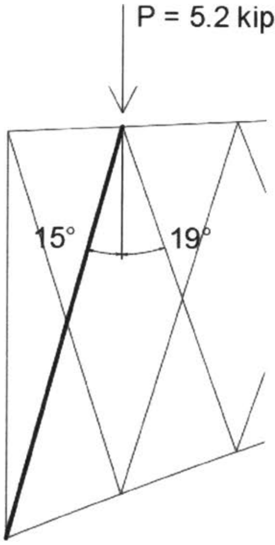


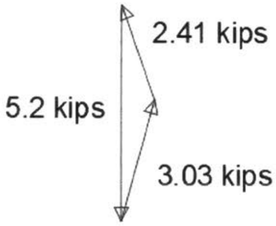
All brace forces calculated for, but worst-case members (shown in bold above) are:

- Edge braces, at risk of buckling
- Center braces, at risk of cross-section failure

H.1.1. Buckling Failure

Brace of interest shown in bold below:





Euler buckling load: braced length = 8.32 ft

$$P = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2(16.3 \times 10^6 psi)(1.27\ in^4)(1000\ kips/lb)}{\left(1.0 * 8.32\ ft * \frac{12in}{ft}\right)^2} = 20.5\ kips$$

Euler buckling load: unbraced length = 15.83 ft

$$P = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2(16.3 \times 10^6 psi)(1.27\ in^4)(\frac{1\ kip}{1000\ lb})}{\left(1.0 * 15.83\ ft * \frac{12in}{ft}\right)^2} = 5.66\ kips$$

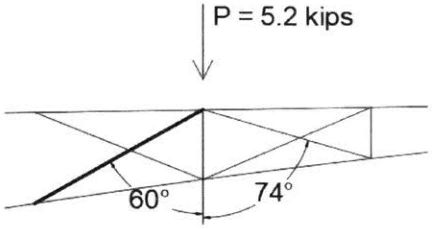
Case 1: no failure, fully braced at midspan: $FS = \frac{20.5}{3.03} = 6.77$

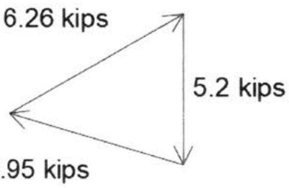
Case 2: failure of one member – same load, unbraced: $FS = \frac{5.66}{3.03} = 1.87$

Case 3: failure of two adjacent members – increased load, unbraced: $FS = \frac{5.66}{5.38} = 1.05$

H.1.2. Cross-Section Failure

Brace of interest shown in bold below:





Cross-section failure: $P = \sigma_{all}A = (17\ ksi)(3.5\ in^2) = 59.5\ kips$

Factor of safety: $FS = \frac{59.5}{6.95} = 8.56$

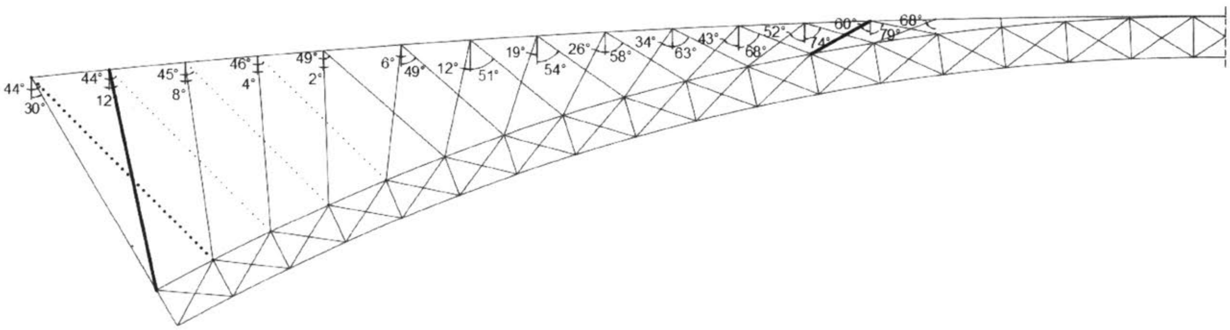
H.1.3. Summary

	Maximum load (kips)	Allowable load (kips)	Factor of safety
Outer member – critical for buckling failure	-3.03	-20.5	6.77
Inner member – critical for cross-section failure	-6.95	-59.5	8.56
Joint or member failure – outer member unbraced	-3.03	-5.66	1.87
Failure of two adjacent joints or members – outer member unbraced, load doubled	-5.38	-5.66	1.05

H.2. Bonar Bridge

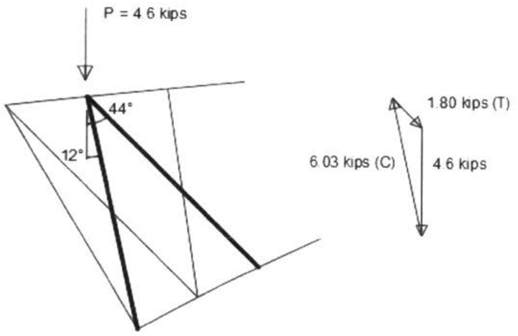
Spandrel lengths and orientations determined from drawings. Spandrels of interest shown in bold below:

- Edge brace (solid), at risk of compressive buckling failure
- Edge brace (dotted), at risk of tension failure
- Center brace, at risk of compressive cross-section failure



H.2.1. Tensile Failure

Edge braces: both members of a pair lean in the same direction – left-hand member develops compression, right-hand member develops tension. Tension members shown by dotted lines above. Worst-case pair shown below.



Max tension force: +1.80 kips. All tension forces are considered unacceptable by this analysis. The four edge members shown by dotted lines above develop tension, and are therefore considered unacceptable.

H.2.2. Buckling Failure

According to the figure above, the most critical member for buckling develops a compressive force of -6.03 kips.

If failure of a single tension member occurs, the unbraced length may be doubled.

If failure of two adjacent members occurs, the unbraced length will double, but the loss of tension force will result in a smaller compressive load of -5.32 kips.

Euler buckling load: braced length = 8.22 ft

$$P = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2(16.3 \times 10^6 psi)(1.27\ in^4)(\frac{1\ kip}{1000\ lb})}{\left(1.0 * 8.22\ ft * \frac{12in}{ft}\right)^2} = 21.0\ kips$$

Euler buckling load: unbraced length = 16.44 ft

$$P = \frac{\pi^2 EI}{(kL)^2} = \frac{\pi^2(16.3 \times 10^6 psi)(1.27\ in^4)(1000\ kips/lb)}{\left(1.0 * 16.4\ ft * \frac{12in}{ft}\right)^2} = 5.28\ kips$$

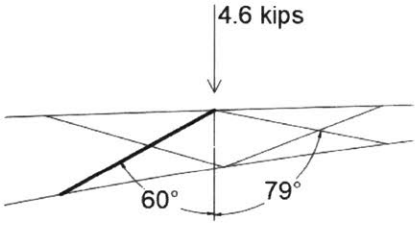
Case 1: no failure, fully braced at midspan: $FS = \frac{21.5}{6.03} = 3.48$

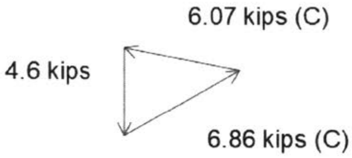
Case 2: failure of one member – same load, unbraced: $FS = \frac{5.25}{6.03} = 0.871$

Case 3: failure of two adjacent members – increased load, unbraced: $FS = \frac{5.25}{5.32} = 0.987$

H.2.3. Cross-Section Failure

Brace of interest shown in bold below:





Cross-section failure: $P = \sigma_{all} A = (17\ ksi)(4.0\ in^2) = 68.0\ kips$

Factor of safety: $FS = \frac{68.0}{6.86} = 9.91$

H.2.4. Summary

Member No.	Maximum Load (kips)	Allowable Load (kips)	Factor of Safety
Exterior brace – buckling failure	-6.03	-21.0	3.48
Exterior brace – tension failure	+1.80	0 (Tension)	NA
Interior brace – cross-section failure	-6.88	-68.0	9.88
Failure of outermost tensile member – compressive member unbraced	-6.03	-5.25	0.871
Failure of next tensile member – compressive member takes full load	-5.32	-21.0	3.95
Failure of both outermost and next tensile members	-5.32	-5.25	0.987