# A SIMPLE MODEL OF CAPITAL MARKET EQUILIBRIUM WITH INCOMPLETE INFORMATION†

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## I. Prologue

The sphere of modern financial economics encompases finance, micro investment theory and much of the economics of uncertainty. As is evident from its influence on other branches of economics including public finance, industrial organization and monetary theory, the boundaries of this sphere are both permeable and flexible. The complex interactions of time and uncertainty guarantee intellectual challenge and intrinsic excitement to the study of financial economics. Indeed, the mathematics of the subject contain some of the most interesting applications of probability and optimization theory. But for all its mathematical refinement, the research has nevertheless had a direct and significant influence on practice.

It was not always thus. Thirty years ago, finance theory was little more than a collection of anecdotes, rules of thumb, and manipulations of accounting data with an almost exclusive focus on corporate financial management. There is no need in this meeting of the guild to recount the subsequent evolution from this conceptual potpourri to a rigorous economic theory subjected to systematic empirical examination.[1] Nor is there a need on this occasion to document the wide-ranging impact of the research on finance practice.[2] I simply note that the conjoining of intrinsic intellectual interest with extrinsic application is a prevailing theme of research in financial economics.

The later stages of this successful evolution has however been marked by a substantial accumulation of empirical anomalies; discoveries of theoretical inconsistencies; and a well-founded concern about the statistical power of many of the test methodologies.[3] Finance, thus finds itself today in the seemingly-paradoxical position of having more questions and empirical puzzles than at the start of its modern development.[4] To be sure, some of the empirical anomalies will eventually be shown to be mere statistical artifacts. However, just as surely, others will not be so easily dismissed.

I see this new-found ignorance in finance as mostly of the useful type that reflects our "...express recognition of what is not yet known, but needs to be known in order to lay the foundation for still more knowledge."[5] Anomalous empirical evidence has indeed stimulated wide-ranging research efforts to make explicit the theoretical and empirical limitations of the basic finance model with its frictionless markets, complete information, and rational, optimizing economic behavior. Although much has been done, this research line is far from closure. Some hold that the paradigm of rational and optimal behavior must be largely discarded if knowledge in finance is to significantly advance. Others believe that most of the important empirical anomalies surrounding the current theory can be resolved within that traditional paradigm.[6] Whichever view emerges as the dominant theme in finance, our understanding of the subject promises to be greatly enriched by these research programs.

Although I must confess to a traditional view on the central role of rational behavior in finance, I also believe that financial models based on frictionless markets and complete information are often inadequate to capture the complexity of rationality in action. For example, in the modern tradition of finance, financial economic organizations are regarded as existing primarily because of the functions they serve and are, therefore, endogenous to the theory. Yet, derived rational behavior in a perfect-market setting rarely provides explicit and important roles for either financial institutions, complicated financial instruments and contracts, or regulatory constraints, despite their observed abundance in the real financial world. Moreover, the time scale for adjustments in the structures of financial institutions, regulations and business practices is wholly different than the one for either adjustment of investor portfolios or changes in security prices. Thus, even if all such structural changes served to accommodate individuals' otherwise unconstrained optimal plans, current (and perhaps, suboptimal) institutional forms can significantly affect rational financial behavior for a considerable period of time.

Consider, for instance, the perfect-market assumption that firms can instantly raise sufficient capital to take advantage of profitable investment opportunities. This specification may be adequate to derive the general properties of investment and financing behavior by business firms on a time scale of sufficiently-long duration. It is, however, almost surely too crude an abstraction for the study of the detailed microstructure of speculative markets. On the time scale of trading opportunities, the capital stock of dealers, market makers and traders is essentially fixed. Entry into the dealer business is neither costless nor instantaneous. Thus, margin and other regulatory capital requirements can place an effective constraint on the number of opportunities that these professionals may undertake at a given point in time. Hence, these institutional factors may cause the short-run marginal cost of capital for these financial firms to vary dramatically over short time intervals. Therefore, to abstract from these factors may be to neglect an order-one influence on the short-run behavior of security prices.

Similarly, models that posit the usual tâtonnement process for equilibrium asset-price formation do not explicitly provide a functional role for the complicated and dynamic system of dealers, market makers and traders observed in the real world. It would, thus, be no surprise that such models generate limited insights into market activities and price formation on this time scale. The expressed recognition of a nontâtonnement process for speculative-price formation is probably only important in studies of very short-run behavior. The limitations of the perfect-market model are not however confined solely to such analyses.

The acquisition of information and its dissemination to other economic units are, as we all know, central activities in all areas of finance, and especially so in capital markets. As we also know, asset pricing models typically assume both that the diffusion of every type of publicly available information takes place instantaneously among all investors and that investors act on the information as soon as it is received. Whether so simple an information structure is adequate to describe empirical asset-price behavior depends on both the nature of the information and the time scale of the analysis. It may, for example, be reasonable to expect rapid reactions in prices to the announcement through standard channels of new data (e.g., earnings or dividend announcements) that can be readily evaluated by investors using generally-accepted structural models. Consider, however, the informational event of publication in a scientific journal of the empirical discovery of an anomalous profit opportunity (e.g., smaller-capitalized firms earn excessive risk-adjusted average returns). The expected duration between the creation of this investment opportunity and its elimination by rational investor actions in the market place can be considerable.

Before results are published, an anomaly must in fact exist for a long enough period of time to permit sufficient statistical documentation.[7] After publication, the diffusion rate of this type of information from this source is likely to be significantly slower than for an earnings announcement. If the anomaly applies to a large collection of securities (e.g., all small stocks), then its "correction" will require the actions of many investors. If an investor does not know about the anomaly, he will not, of course, act to correct it.

Once an investor becomes aware of a study, he must decide whether the reported historical relations will apply in the future. On the expected duration of this decision, I need only mention that six years have passed since publication of the first study on the "small-firm effect" and we in academic finance have yet to agree on whether it even exists. Resolving this issue is presumably no easier a task for investors. Beyond this decision, the investor must also determine whether the potential gains to him are sufficient to warrant the cost of implementing the strategy. Included in the cost are the time and expense required to build the model and create the data base necessary to support the strategy. Moreover, professional money managers may have to expend further time and resources to market the strategy to clients and to satisfy prudence requirements before implementation. If profitable implementation requires regulatory and business practice changes or the creation of either new markets or new channels of intermediation, then the delay between announcement of an anomaly and its elimination by corrective action in the market place can, indeed, be a long one.

Much the same story applies in varying degrees to the adoption in practice of new structural models of evaluation (e.g., option pricing models) and to the diffusion of innovations in financial products (cf. Rogers, 1972 for a general discussion of the diffusion of innovations). Recognition of the different speeds of information diffusion is particularly important in empirical research where the growth in sophisticated and sensitive techniques to test evermore-refined financial-behavioral patterns severely strains the simple information structure of our asset pricing models. To avoid inadvertent positing of a "Connecticut Yankee in King Arthur's Court," empirical studies that use long historical time series to test financial-market hypotheses should take care to account for the evolution of institutions and information technologies during the sample period. It is, for example, common in tests of the weak form of the Efficient Market Hypothesis to assume that real-world investors at the time of their portfolio decisions had access to the complete prior history of all stock returns. When, however, investors' decisions were made, the price data may not have been in reasonably-accessible form and the computational technology necessary to analyze all these data may not even have been invented. In such cases, the classification of all prior price data as part of the publicly-available information set may introduce an important bias against the null hypothesis.

All of this is not to say that the perfect-market model has not been and will not continue to be a useful abstraction for financial analysis. The model may indeed provide the best description of the financial system in the long run.[8] It does, however, suggest that researchers be cognizant of the insensitivity of this model to institutional complexities and explicitly assess the limits of precision that can be reasonably expected from its predictions about the nature and timing of financial behavior. Moreover, I believe that even modest recognition of institutional structures and information costs can go a long way toward explaining financial behavior that is otherwise seen as anomalous to the standard frictionless-market model. To illustrate this thesis, I now turn to the development of a simple model of capital market equilibrium with incomplete information.

## II. Capital Market Equilibrium With Incomplete Information

In this section, we develop a two-period model of capital market equilibrium in an environment where each investor knows only about a subset of the available securities. In subsequent sections, we explore the impact on the structure of equilibrium asset prices caused by this particular type of incomplete information.

There are n firms in the economy whose end-of-period cash flows are technologically specified by:

$$\tilde{C}_k = I_k[\mu_k + a_k\tilde{Y} + s_k\tilde{\varepsilon}_k] \tag{1}$$

where $\tilde{Y}$ denotes a random variable common factor with $E(\tilde{Y}) = 0$ and $E(\tilde{Y}^2) = 1$ and $E(\tilde{\varepsilon}_k) = E(\tilde{\varepsilon}_k \mid \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_{k-1}, \varepsilon_{k+1}, \ldots, \varepsilon_n, Y) = 0$ , $k = 1,\ldots,n$. $I_k$ denotes the amount of physical investment in firm k and $\mu_k$, $a_k$, and $s_k$ represent parameters of firm k's production technology. Let $V_k$ denote the equilibrium value of firm k at the beginning of the period. If $\tilde{R}_k$ is the equilibrium return per dollar from investing in the firm over the period, then $\tilde{R}_k \equiv \tilde{C}_k/V_k$, and

$$\tilde{R}_k = \bar{R}_k + b_k\tilde{Y} + \sigma_k\tilde{\varepsilon}_k \quad , \tag{2}$$

where from (1), $\bar{R}_k = E(\tilde{R}_k) = I_k\mu_k/V_k$; $b_k = a_kI_k/V_k$ and $\sigma_k = s_kI_k/V_k$ , $k = 1,\ldots,n$. By inspection of (2), the structure of returns is like that of the Sharpe (1964) "diagonal" model or the "one-factor" version of the Ross (1976) Arbitrage Pricing Theory (APT) model.

In addition to shares in the firms, there are two other traded securities: a riskless security with sure return per dollar R and a security that combines the riskless security and a forward contract with cash settlement on the observed factor index Y. Without loss of generality, it is assumed that the forward price of the contract is such that the standard deviation of the equilibrium return on the security is unity. Thus, the return on the security can be written as:

$$\tilde{R}_{n+1} = \bar{R}_{n+1} + \tilde{Y} \quad . \tag{3}$$

It is assumed that both this and the riskless security are "inside" securities and therefore, investors' aggregate demand for each of them must be zero in equilibrium.

The model assumes the standard frictionless-market conditions of no taxes, no transactions costs, and borrowing and shortselling without restriction. There are N investors where N is sufficiently large and the distribution of wealth sufficiently disperse that each acts as a price taker.

Each investor is risk averse and selects an optimal portfolio according to the Markowitz-Tobin mean-variance criterion applied to end-of-period wealth. The preference of investor j is represented as:

$$U_j = E(\tilde{R}^j W^j) - \frac{\delta_j}{2W^j} \operatorname{Var}(\tilde{R}^j W^j) \quad , \tag{4}$$

where $W^j$ denotes the value of his initial endowment of shares in the firms evaluated at equilibrium prices; $\tilde{R}^j$ denotes the return per dollar on his portfolio; and $\delta_j > 0$, $j = 1,\ldots,N$.

In addition to an initial endowment of shares, each investor is endowed with an information set described as follows: Common knowledge in all investors' information sets includes the return on the riskless security; the expected return and variance of the forward contract security given in (3); and the basic structure of securities' returns given in (2). However, for any given security $k$, knowledge of the specific parameter values in (2) may not be included in some investors' information sets.

An investor is said to be "informed (know) about security k" if he knows $(\bar{R}_k, b_k, \sigma_k^2)$. All investors who know about security $k$ agree on these parameter values (i.e., conditional homogeneous beliefs). Let $J_j$ denote a collection of integers such that $k$ is an element of $J_j$ if investor $j$ knows about security $k$, $k = 1,\ldots,n + 2$, where security $n + 2$ is the riskless security. Thus, by assumption, $n + 1$ and $n + 2$ are contained in $J_j$, $j = 1,\ldots,N$. If $J_j$ contains all the integers $k = 1,\ldots,n + 2$, then investor $j$'s information set is complete. Although the model does not rule out this possibility for some investors, if all investors' information sets were complete, then the model would reduce to the standard Sharpe-Lintner-Mossin Capital Asset Pricing Model. Therefore, it is assumed that investors generally know only about a subset of the available securities and that these subsets differ across investors.

The key behavioral assumption of the model is that <u>an investor uses security k in constructing his optimal portfolio only if the investor knows about security k</u>. The prime motivation for this assumption is the plain fact that the portfolios held by actual investors (both individual and institutional) contain only a small fraction of the thousands of traded securities available.[9] There are, of course, a number of other factors (e.g., market segmentation and institutional restrictions including limitations on short sales, taxes, transactions costs, liquidity, imperfect divisibility of securities) in addition to incomplete information that in varying degrees, could contribute to this observed behavior.[10] Because this behavior can be derived from a variety of underlying structural assumptions, the formally-derived equilibrium-pricing results are the theoretical analog to reduced-form equations.

As in the Grossman-Stiglitz (1976) single-security model of asymmetric-information trading, the conditional-homogeneous-beliefs assumption posited here ensures that all informed traders in security k have the same information about security k. However, in contrast to their analysis, the issues of gaming between informed and uninformed investors that surround trading in an asymmetric information environment do not arise here because only equally-informed investors trade in each security.

Concern about asymmetric information among investors could be an important reason why some institutional and individual investors do not invest at all in certain securities, such as shares in relatively small firms with few stockholders. However, as is evident from the Grossman-Stiglitz analysis, such concerns about informed traders are not alone adequate to render this polar extreme in behavior as optimal. Therefore, I discuss briefly other types of information cost structures that could lead to the posited behavior in our model. For this purpose, it is useful to think of information costs as partitioned into two parts: (1) the cost of gathering and processing data and (2) the cost of transmitting information from one party to another.

A prime source of information about a particular firm is, of course, the firm itself. Information required by investors overlaps considerably with the information managers require to operate the firm. Hence, the firm's marginal cost for gathering and processing the data needed by investors would seem to be small. Nevertheless, as is evident from the extensive literature on the principal-agent problem and signalling models,[11] the cost of transmitting this information to investors so that they will use it efficiently, can be considerable.

The signalling models are focused on the problem of the firm transmitting to investors specific information such as earnings prospects and investment plans. The types of costs emphasized are the incentive costs necessary to induce managers to transmit information and the costs required to make credible the information they transmit. It is generally assumed in these models that all public ("non- insider") investors receive the same information whether they are currently shareholders or not.

In the Bawa-Klein-Barry-Brown models of differential information in which each investor has the same information set,[12] the focus of analysis is on the price effects from differences in the quality of information across securities (i.e., parameter-estimation risks). In contrast, our model assumes that the quality of information (i.e., the precision of the estimates of $\bar{R}_k, b_k, \sigma_k$) is the same for all securities, and focuses on the price effects from different distributions of that information across investors. Thus, the differential-information models cover the price effects of differences in the depth of investor cognizance among securities, whereas the emphasis here is on the differences in the breadth of investor cognizance.

Although the types of costs underlying the signalling and differential information models would surely be an important part of a more-detailed information-cost structure for our model, there is another type of cost that logically proceeds them: namely, the cost of making investors aware of the firm. That is, for Party A to convey useful information to Party B, requires not only that Party A has a transmitter and sends an accurate message, but also that Party B has a receiver. If an investor does not follow a particular firm, then an earnings or other specific announcement about that firm is not likely to cause that investor to take a position in the firm.[13] If, for each firm, investors must pay a significant "set-up" (or "receiver") cost before they can process detailed information released from time to time about the firm, then this fixed cost will cause any one investor to follow only a subset of the traded securities. Because this fixed cost is a "sunk cost" for existing shareholders, the effective information received by current shareholders, even from a public announcement by the firm, will not be the same as that received by other investors.

The firm is of course, not the only source of information available to investors. There are stock market advisory services, brokerage houses, and professional portfolio managers. However, much the same argument used here for the firm can also be applied to the costs in making investors aware of these sources.[14]

Our background information-cost story fits well with the Arbel-Carvell-Strebel theory of "generic" or "neglected" stocks.[15] In their theory, neglected stocks are ones that are not followed by large numbers of professional analysts on a regular basis. They assume that if the quantity of analysts following a stock is relatively small, then the quality of information available on the stock is relatively low. From this, they conclude that *ceteris-paribus*, equilibrium expected returns on neglected stocks will be larger than on widely-followed stocks. Although our simple model posits no differences in the quality of information across securities, it is clear in our model that if the number of investors that know about security k is relatively small, then security k would fit the definition of a neglected security in the Arbel-Carvell-Strebel model. With this, we close our discussion on the information cost structure underlying the model.

With the structure of the model established, we turn now to the solution of the portfolio selection problem for investor j. If $w_k^j$ denotes the fraction of initial wealth allocated to security k by investor j, then from (2) and (3), the return on the portfolio can be written as:

$$\tilde{R}^j = \bar{R}^j + b^j\tilde{Y} + \sigma^j \tilde{\varepsilon}^j \tag{5}$$

where:

$$b^j \equiv \Sigma_1^n w_k^j b_k + w_{n+1}^j$$

$$\sigma^j \equiv \sqrt{\Sigma_1^n (w_k^j)^2 \sigma_k^2}$$

$$\tilde{\varepsilon}^j \equiv \Sigma_1^n w_k^j \sigma_k \tilde{\varepsilon}_k / \sigma^j \quad .$$

From (2), we have that $E(\varepsilon^j | Y) = E(\varepsilon^j) = 0$. Using the condition that $w_{n+2}^j = 1 - \Sigma_1^{n+1} w_k^j$ and substituting $b^j - \Sigma_1^n w_k^j b_k$ for $w_{n+1}^j$, we can write the variance and expected return on the portfolio as:

$$\text{Var}(\tilde{R}^j) = (b^j)^2 + \Sigma_1^n (w_k^j)^2 \sigma_k^2 \tag{6.a}$$

and

$$\bar{R}^j = R + b^j(\bar{R}_{n+1} - R) + \Sigma_1^n w_k^j \Delta_k \tag{6.b}$$

where

$$\Delta_k \equiv \bar{R}_k - R - b_k(\bar{R}_{n+1} - R) \quad .$$

From (4), the optimal portfolio choice for the investor can be formulated as the solution to the constrained maximization problem:

$$\max_{\{b^j, w^j\}} \quad [\bar{R}^j - \frac{\delta_j}{2} \mathrm{Var}(\tilde{R}^j) - \sum_1^n \lambda_k^j w_k^j] \tag{7}$$

where $\lambda_k^j$ is the Kuhn-Tucker multiplier to reflect the constraint that investor $j$ cannot invest in security $k$ if he does not know about security $k$. That is, $\lambda_k^j = 0$ if $k \in J_j$ and $w_k^j = 0$ if $k \in J_j^c$, the compliment to $J_j$, $k = 1,\ldots,n$. From (6.a) and (6.b), the first-order conditions for (7) can be written as:

$$0 = \bar{R}_{n+1} - R - \delta_j b^j \tag{8.a}$$

$$0 = \Delta_k - \delta_j w_k^j \sigma_k^2 - \lambda_k^j \, , \quad k = 1,\ldots,n \quad . \tag{8.b}$$

From (8.a) and (8.b), the optimal common-factor exposure and portfolio weights can be written as:

$$b^j = [\bar{R}_{n+1} - R]/\delta_j \tag{9.a}$$

$$w_k^j = \Delta_k/(\delta_j \sigma_k^2) \, , \quad k \in J_j \tag{9.b}$$

$$w_k^j = 0 \quad , \qquad k \in J_j^c \tag{9.c}$$

$$w_{n+1}^j = b^j - \sum_1^n w_k^j b_k \tag{9.d}$$

$$w_{n+2}^j = 1 - b^j - \sum_1^n w_k^j (b_k - 1) \tag{9.e}$$

From (8.b) and (9.c), we have that

$$\lambda_k^j = \Delta_k \quad , \quad k \in J_j^c \tag{10}$$

and $\lambda_k^j = 0$ for $k \in J_j$. By inspection of (10), the "shadow cost" of not knowing about security $k$ is the same for all investors.[16]

Having solved for the individual investor optimal demands, we now aggregate to determine equilibrium asset prices and expected returns. To simplify the analysis and focus attention on the effects of incomplete information on equilibrium prices, we make the further assumption that investors have identical preferences and the same initial wealths. (I.e., $\delta_j = \delta$ and $W^j = W$, $j = 1,\ldots,N$ .) Under these conditions, it follows from (9.a) that all investors will choose the same exposure to the common-factor $b^j = b$, $j = 1,\ldots,N$, and that:

$$\bar{R}_{n+1} = R + \delta b \quad . \tag{11}$$

If $D_k (\equiv \sum_1^N w_k^j W^j)$ denotes the aggregate demand for security $k$, then it follows from (9.b) and (9.c) that:

$$D_k = N_k W \Delta_k / \delta \sigma_k^2 \quad , \tag{12}$$

where $N_k$ denotes the number of investors who know about security $k(0 < N_k \leq N)$, $k = 1,\ldots,n$. From (9.d) and (9.e), we have that:

$$D_{n+1} = NWb - \Sigma_1^n D_k b_k \tag{13}$$

and

$$D_{n+2} = NWb - \Sigma_1^{n+1} D_k \tag{14}$$

Let $M \equiv \Sigma_1^N W^j = NW$ denote equilibrium national wealth. If $x_k (\equiv V_k/M)$ is the fraction of the market portfolio invested in security $k$, then, from the equilibrium condition $V_k = D_k$ and (12), we have that:

$$x_k = q_k \Delta_k / \delta \sigma_k^2 \tag{15}$$

where $q_k \equiv N_k/N$ is the fraction of all investors who know about security $k(0 < q_k \leq 1)$, $k = 1,\ldots,n$.[17]

Because the market portfolio is a weighted average of investors' optimal portfolios and because all investors choose the same common-factor exposure b, it follows that the common-factor exposure of the market portfolio is also equal to b. Moreover, by assumption, security $n + 1$ and security $n + 2$ are inside securities and hence, $V_{n+1} = x_{n+1} = 0$ and $V_{n+2} = x_{n+2} = 0$. Thus, $b = \Sigma_1^n x_k b_k$ and $M = \Sigma_1^n V_k$.

From the definition of $\Delta_k$ in (6.b), (11), and (15), we have that the equilibrium expected return on security k can be written as:

$$\begin{aligned} \bar{R}_k &= R + b_k b\delta + \Delta_k \\ &= R + b_k b\delta + \delta x_k \sigma_k^2/q_k \quad , \quad k = 1,\ldots,n \quad . \end{aligned} \tag{16}$$

By substitution for $R_k$, $b_k$, and $\sigma_k$ as defined in (2), into (16) and rearranging terms, we can derive the equilibrium relation between the market value of firm k and the distributional characteristics of its end-of-period cash flow, $(I_k,\mu_k,a_k,s_k)$; the relative size of the investor base who know about the firm, $q_k$; and the aggregate-economy variables, ($\delta$,b,R and M). Namely, we have that for $k = 1,\ldots,n$:

$$V_k = I_k[\mu_k - \delta b a_k - (\delta s_k^2 I_k)/q_k M]/R \quad . \tag{17}$$

Armed with (16) and (17), we now explore the effects of incomplete information on equilibrium expected returns and asset prices.

To facilitate the analysis, let $V_k^*$ and $\bar{R}_k^*$ denote the equilibrium market value and expected return on firm k if all investors were informed about firm k (i.e., $q_k = 1$). If we hold fixed the aggregate-economy variables ($\delta$,b,,R and M), then from (17), we have that:

$$V_k = V_k^* - \delta(1 - q_k)s_k^2 I_k^2/(q_k MR) \quad . \tag{18}$$

By inspection of (18), we have that the market value of firm k will always be lower with incomplete information, and the smaller the investor base, the larger is the difference.

To see the connection between this effect on market price and the shadow cost of incomplete diffusion of information among investors, let $\lambda_k (\equiv \Sigma_1^N \lambda_k^j/N)$ denote the equilibrium aggregate shadow cost (per investor) for security k. From (10), we have that for $k = 1,\ldots,n$:

$$\begin{aligned} \lambda_k &= (N - N_k)\Delta_k/N \\ &= (1 - q_k)\Delta_k \quad . \end{aligned} \tag{19}$$

From (15), we have that in equilibrium, $\Delta_k > 0$ because $x_k > 0$. Hence, from (19), $\lambda_k \geq 0$ with equality holding only if all investors know about security k (i.e., $q_k = 1$).

By definition of $\sigma_k^2$ and $x_k$, $s_k^2 I_k^2 = \sigma_k^2 V_k^2$ and $x_k = V_k/M$. It therefore follows from (15), (18), and (19) that:

$$V_k = V_k^*/[1 + \lambda_k/R] \quad . \tag{20}$$

Note: $\lambda_k$ has dimensions of incremental expected rate of return and R equals one plus the riskless rate of interest. Hence, from (29), the effect of incomplete information on equilibrium price is similar to applying an additional discount rate. Indeed, because $\bar{R}_k = I_k\mu_k/V_k$, we have from (20) that the incremental equilibrium expected return on security k is proportional to its shadow cost: Namely, for $k = 1,\ldots,n$:

$$\bar{R}_k - \bar{R}^*_k = \lambda_k(\bar{R}^*_k/R) \quad . \tag{21}$$

As noted at the outset, in the complete-information case $(q_k = 1,$ $k = 1,\ldots,n)$, the model reduces to the Sharpe-Lintner-Mossin Capital Asset Pricing Model. In that model, all investors hold perfectly-correlated mean-variance efficient portfolios and therefore, the market portfolio is mean-variance efficient in equilibrium. The well-known Security Market Line relation among equilibrium expected returns follows directly from this equilibrium condition.

Because (18), (20), and (21) detail the effects of incomplete information holding fixed the aggregates $b$, $R$ and $M$, they cannot be used directly to compare the aggregate incomplete-information equilibrium with the aggregate complete-information one. We can, however, use them to examine the Security Market Line relation that applies in the incomplete-information equilibrium.

If $\tilde{R}_M$ denotes the return on the market portfolio, then $\tilde{R}_M = \Sigma_1^n x_k\tilde{R}_k$ because $x_{n+1} = x_{n+2} = 0$. From (2) and the condition that $\Sigma_1^n x_k b_k = b$, we have that $\text{Var}(\tilde{R}_M) = b^2 + \Sigma_1^n x_k^2\sigma_k^2$. If we define in standard fashion, the beta of security $k$, $\beta_k$, to be the covariance of the return on security $k$ with the return on the market portfolio divided by the variance of the market return, then we have that, for $k = 1,\ldots,n$:

$$\beta_k = [b_k b + x_k\sigma_k^2]/\text{Var}(\tilde{R}_M) \quad . \tag{22}$$

From (19) and (22), we can rewrite (16) as:

$$\bar{R}_k = R + b_k b\delta + \delta x_k \sigma_k^2 + \lambda_k$$
$$= R + \delta \, \mathrm{Var}(\tilde{R}_M)\beta_k + \lambda_k \quad . \tag{23}$$

If we multiply (23) by $x_k$ and sum from $k = 1$ to $n$, we have that:

$$\bar{R}_M - R = \delta \, \mathrm{Var}(\tilde{R}_M) + \lambda_M \quad . \tag{24}$$

where $\lambda_M \equiv \Sigma_1^n x_k \lambda_k$ is the weighted-average shadow cost of incomplete information over all securities. Substituting for $\mathrm{Var}(\tilde{R}_M)$ from (24) into (23) and rearranging terms, we have that:

$$\bar{R}_k - R = \beta_k(\bar{R}_M - R) + \lambda_k - \beta_k \lambda_M \quad . \tag{25}$$

Using the standard notation, $\alpha_k$, for the discrepancy between a security's equilibrium expected return and the Security Market Line, we have from (25) that:

$$\alpha_k = \lambda_k - \beta_k \lambda_M \quad . \tag{26}$$

Because a necessary and sufficient condition for the market portfolio to be mean-variance efficient with respect to all available securities is that $\alpha_k = 0$, $k = 1,\ldots,n$, it follows from (26) that the market portfolio will not be mean-variance efficient in the incomplete-information model. Indeed, given equilibrium prices, the optimal combination of risky assets for a fully-informed investor $\{w_k^*\}$ is given by (9b), and hence, the difference between this portfolio's holdings and the market portfolio are given by:

$$w_k^* - x_k = \lambda_k / \delta\sigma_k^2 \quad , \quad k = 1,\ldots,n \quad . \tag{27}$$

In the section to follow, we use comparative statics to examine further the equilibrium structure of asset prices and discuss its possible connection with observed empirical anomalies. In Section IV, we examine the effects of incomplete information on firm investment decisions and provide at least a partial basis for the endogeneous determination of $q_k$. Section V addresses some pending issues surrounding the model including the introduction of financial institutions and other types of investors, and touches on some of the reasons why information diffusion need not be complete.

## III. A Cross-Sectional Study of the Equilibrium Structure of Asset Returns

Assuming an equilibrium as derived in Section II, we use comparative statics to analyze cross-sectional differences among expected returns on the available securities. Although theoretical in nature, the analysis uses the same methodological approach commonly employed in empirical studies of equilibrium asset return structures.

Because we are analyzing cross-sectional differences among securities and not comparing the same security in different equilibria, the aggregate variables, $R$, $b$, $\delta$, $\bar{R}_M$, $\mathrm{Var}(R_M)$, are held fixed. By inspection of (16), the four parameters that cause cross-sectional differences in equilibrium expected returns are: $b_k$, the exposure level to the common factor; $x_k$, the relative size of the firm; $\sigma_k^2$, the firm-specific component of the firm's return variance (which also captures the degree of nonsubstitutability of other securities for security $k$); and $q_k$, the relative size of the investor base (i.e., degree of "investor recognition") for security $k$.

Define the elasticity of the expected excess return on security $k$, $\bar{R}_k - R$, with respect to parameter $y$, by $\psi(y) \equiv d \log|\bar{R}_k - R|/d \log(y)$. From (16), we have that:

$$\psi(b_k) = q_k b_k b/(q_k b_k b + x_k \sigma_k^2) > 0 \tag{28.a}$$

$$\begin{aligned}\psi(x_k) &= x_k \sigma_k^2/(q_k b_k b + x_k \sigma_k^2) > 0 \\ &= 1 - \psi_k(b_k)\end{aligned} \tag{28.b}$$

$$\psi(\sigma_k^2) = \psi(x_k) > 0 \tag{28.c}$$

$$\psi(q_k) = -\psi(x_k) < 0 \tag{28.d}$$

By inspection of (28), expected returns will tend to be higher on firms with larger common-factor exposure, larger firm-specific variance, and larger size. Expected returns will tend to be lower on better-known firms with relatively larger investor bases. The magnitudes of these differences will, of course, depend on the parameter values. All shareholders in firms whose returns are highly correlated with the common factor have available a close substitute investment (i.e., security $n + 1$) and hence, different firm sizes or investor bases among such firms will have a small effect on expected returns. Indeed, for firms where $x_k\sigma_k^2/q_k << b_k b$, $\psi(x_k) = \psi(\sigma_k^2) = \psi(q_k) \overset{\sim}{=} 0$ and $\psi(b_k) \simeq 1$. Such is usually taken to be the case in the standard complete-information model with a large capital market where it is assumed that $q_k = 1$ and $x_k << 1$ for $k = 1,\ldots,n$. However, in the model presented here, it is entirely possible for $x_k << 1$ and for $x_k\sigma_k^2/q_k$ to be of a similar order of magnitude to $b_k b$ if $q_k << 1$. That is, it is not the size of the firm <u>relative to national wealth</u>, $(x_k \equiv V_k/NW)$ that matters, but instead, the size of the firm <u>relative to the aggregate wealth of the investors in the firm</u> $(x_k/q_k = V_k/N_kW)$.

To explore this point further, consider the findings that would occur if one tested the Capital Asset Pricing Model in the usual fashion but in an environment described by the model of Section II. The standard test is the Security Market Line relation, $\bar{R}_k - R = \beta_k(\bar{R}_M - R) + \alpha_k$, where the null hypothesis is that $\alpha_k = 0$, $k = 1,\ldots,n$. As Roll (1977) has shown, this is equivalent to the test that the portfolio selected for the market portfolio is mean-variance efficient relative to the securities selected for the sample.

As was shown in (26), $\alpha_k$ will not equal zero for each $k$ in our model, and hence, the CAPM does not obtain. However, more can be said about the nature of the deviation. Consider a cross section of securities with the same parameter values for $q_k$, $\sigma_k^2$ and $x_k$, but different values for the common factor exposure, $b_k$. It follows from (15) and (19) that these securities will all have the same $\lambda_k$ value. From (22), $\beta_k$ will differ across these securities with $\partial\beta_k/\partial b_k > 0$. For this sample, it follows from (26) that:

$$\frac{\partial\alpha_k}{\partial\beta_k} = -\lambda_M < 0 \quad . \tag{29}$$

Thus, if we were to examine a cross section of these securities, we would find that $\alpha_k$ is a systematically decreasing function of $\beta_k$ and that the empirical line in the $(\beta_k, \bar{R}_k)$ plane is "too flat." This finding is consistent with the empirical results reported by Blume and Friend (1973), Black, Jensen and Scholes (1972), and Fama and MacBeth (1973).

By inspection of (22), $\beta_k$ does not depend on $q_k$ and in the empirically-relevant case where $x_k \ll 1$, it is essentially determined by $b_k$. $\lambda_k$ depends only on $x_k, \sigma_k^2$ and $q_k$, and not $b_k$. Hence, in large samples of stocks stratified only by their betas, one could perhaps argue that $E(\lambda_k | \beta_k)$ is the same for all stocks and therefore, equal to $\lambda_M$. In that case, we have from (26) that for the total sample of stocks:[18]

$$E(\alpha_k | \beta_k) = (1 - \beta_k)\lambda_M \quad . \tag{30}$$

Black (1972) develops a modified CAPM with a "zero-beta" factor that predicts that $\alpha_k = (1 - \beta_k)(\bar{R}_z - R)$ where $\bar{R}_z - R$ is the expected excess return on a zero-beta portfolio of securities with positive variance. Our result (30) is consistent with the Black model's prediction if the securities are stratified by beta alone. However, unlike the Black model, our model predicts that alpha depends on other characteristics in addition to market risk. Consider a cross section of securities with the same market risk, $\beta_k$. From (26), we have that for $q_k < 1$:

$$\frac{\partial \alpha_k}{\partial \sigma_k^2} = \delta(1 - q_k)x_k/q_k \tag{31.a}$$

$$\frac{\partial \alpha_k}{\partial x_k} = \delta(1 - q_k)\sigma_k^2/q_k \tag{31.b}$$

$$\frac{\partial \alpha_k}{\partial q_k} = - \delta x_k \sigma_k^2/q_k^2 \quad . \tag{31.c}$$

Among securities with the same market risk, we have from (31.a), that firms with larger firm-specific variance will have larger alphas. This finding is consistent with empirical findings [cf. Friend, Westerfield, and Granito (1978)] that expected returns seem to depend on both market risk and total variance.

Among firms with the same total volatility and the same relative degree of investor recognition, we have from (31.b), that the relatively larger firms will have larger alphas. Although the analysis is cross sectional, this result is suggestive of significantly downward-sloping demand curves for some securities.

The prediction of (31.b) appears to conflict with the empirical findings of Banz (1981), Reinganum (1981), Schwert (1983) and others that smaller firms tend to have larger alphas. They also report that this size effect is considerably weaker for the post-war period. Moreover, Reinganum (1983), Keim (1983), Roll (1983), and Constantinides (1984) find that the small-firm effect seems to apply only in the early part of the month of January and suggest that year- end tax selling is a possible explanation.

The effect of size on alpha in (31.b) is a conditional prediction for a cross section of securities with the same $\sigma_k^2$ and $q_k$, whereas the cited empirical evidence is not. Therefore, the appropriate comparative-static analysis for comparison with the empirical results on size is $d\alpha_k/dx_k = \partial\alpha_k/\partial x_k + (\partial\alpha_k/\partial\sigma_k^2)d\sigma_k^2/dx_k + (\partial\alpha_k/\partial q_k)dq_k/dx_k$. As an empirical matter, smaller firms tend to have larger total variances and smaller correlations of their returns with the general market (i.e., $d\sigma_k^2/dx_k < 0$). Smaller firms generally have many fewer shareholders than larger firms, and hence, one would expect $dq_k/dx_k$ to be strongly positive. From (31.c), our model predicts that more-widely known firms with larger investor bases will have lower alphas. Thus, it is possible, (but surely not required in our model), that $d\alpha_k/dx_k < 0$ even though $\partial\alpha_k/\partial x_k > 0$.

The empirical findings of Barry and Brown (1984) and Arbel, Carvell and Strebel (1983) can be interpreted as evidence consistent with the predictions (31) of our model. Barry and Brown analyze returns on samples of New York Stock Exchange stocks stratified by beta, size and the period of time since listing. They use the Fama-MacBeth (1973) procedure to control for interactions among the variables, and find strong interactions between the effects of size and listing period and between size and beta. Their analysis confirms the previously-discussed Banz-Reinganum-Keim results for the size effect itself. However, they also report that "Controlling for size, beta and the interactions between the variables, there is a listing effect, and this effect persists in the post-war period as well as when the month of January is excluded from the data" (p. 292). Addressing the effect of the interaction between size and listing period on excess returns, Barry and Brown (p. 293) find that "In fact, this interaction effect is more significant than the size effect per se."

Although surely not the expressed intent of Barry and Brown, if their listing period is viewed as a positively-associated proxy variable for our $q_k$, then their findings are consistent with our (31c). Because among smaller firms especially, $x_k$ is likely to be inversely related to both $q_k$ and $\sigma_k^2$, the strong effect on excess returns from the interaction of size and listing period is also consistent with (31a) and (31c). In (31b), the effect of size _per se_ on $\alpha_k$ is small if either $\sigma_k^2$ is small or $q_k$ large.

In examining the evidence for their theory of "neglected" stocks, Arbel, Carvell and Strebel (1983) study the returns on samples of stocks stratified by beta, size and the degree of institutional investor holdings of the stock. For the period 1971-1980, they report (p. 5) that on a risk-adjusted basis, there is a strong negative relation between excess returns and the degree of institutional holdings and that this effect persists even when controlled for size. Moreover, they claim that there is no systematic small-firm effect if one controls for institutional holdings and beta. As noted in Section II, a small $q_k$ in our model would correspond to a neglected stock in their model. Thus, their measure of institutional concentration is a positively-associated proxy variable for $q_k$ and their findings are consistent with (31).

From (26), the widely-held stocks with $q_k \simeq 1$ will have $\alpha_k \simeq -\beta_k\lambda_M$, and therefore, for this top-tier of stocks, the qualitative predictions of our model and the Black model will coincide. For even the largest of firms, $x_k << 1$. From (31), one would thus expect that important cross-sectional differences among securities' expected returns from factors other than market risk, will tend to be concentrated among lesser-known stocks with small investor bases (i.e., $q_k << 1$).

To provide a sense of the potential magnitudes of these non-market-risk parameters on equilibrium returns, we present a few numerical examples. A typical annual variance rate for stocks is $(b_k^2 + \sigma_k^2 =)$ 0.16. If half of that variance is attributed to the common factor, then $\sigma_k^2 = 0.08$. Empirical studies of aggregate risk aversion[19] suggest that $\delta = 2$ is reasonable. It follows from (15) and (19) that $\lambda_k = .16(1 - q_k)x_k/q_k$.

Representative of the largest and best-known stocks, General Electric, Ford, and IBM have market values per shareholder of approximately $53,000, $27,000 and $97,000, respectively. If there are about 45 million investors in the aggregate, then, under the hypotheses of our model, the corresponding $q_k$ values computed from the number of shareholders of these companies are .011, .007, and .017. It is readily apparent that if only individual investors held these stocks, then the implied $\lambda_k$ values for these large well-known stocks would be enormous. Such naive calculations, however, vastly overstate the $\lambda_k$ for these stocks. First, both the total variance and the fraction attributed to firm-specific variance are substantially smaller on these stocks than on a typical stock. Second, and much more important, a large fraction of their shares are held by financial institutions (e.g., mutual funds, commingled trusts and pension funds) and hence, from an economic perspective, the effective number of individual shareholders in such firms is many times larger than the reported number. Therefore, from these data alone, one cannot reliably assess whether the $\lambda_k$ values for such stocks are of any significance.

Consider, however, a relatively-small, hypothetical stock with a $200 million market value and no important institutional shareholders. If the aggregate market value of all stocks were $2 trillion, then $x_k = .0001$, a tiny fraction of the total market. Nevertheless, if this firm had 22,500 shareholders (a substantial number for a listed firm of this size), then the implied shadow cost $\lambda_k$ would exceed .03 and the equilibrium expected return would be 300 basis points larger than in the corresponding complete-information model with $q_k = 1$. Thus, one cannot reject out-of-hand, the possibility that the incomplete diffusion of information among investors has an empirically-significant impact on equilibrium expected returns, and especially so, for smaller firms with little institutional following.

To underscore the point, we compute the market value and market value per shareholder for a sample of 1387 firms for which data on the number of shareholders were available on the Compustat tapes. The firms were ranked on total market value and assigned to one of ten groups with the same number of stocks in each group. For December 31, 1985, the market value of each group as a fraction of the total and the average market value per shareholder for each group as a fraction of the average for the sample, are presented in Table 1.

Table 1: Distributions of Market Value and Market Value

Per Shareholder (1387 firms)

December 31, 1985

| Group | Market Value of Group/ Market Value of Sample (%) | Cumulative Market Value (%) | Market Value per Shareholder/ Average for Sample | Indicated $q_k/q_{10}$ |
|---|---|---|---|---|
| 1 | 0.11 | 0.11 | 0.161 | .02 |
| 2 | 0.33 | 0.44 | 0.371 | .03 |
| 3 | 0.69 | 1.13 | 0.545 | .04 |
| 4 | 1.21 | 2.34 | 0.793 | .05 |
| 5 | 1.97 | 4.31 | 0.972 | .06 |
| 6 | 3.24 | 7.55 | 1.127 | .09 |
| 7 | 5.28 | 12.83 | 1.257 | .13 |
| 8 | 8.65 | 21.48 | 1.490 | .18 |
| 9 | 15.05 | 36.53 | 1.348 | .34 |
| 10 | 63.47 | 100.00 | 1.957 | 1.00 |

As in Barry and Brown (1984, p. 289, Table 1), the distribution of firm sizes in our Table 1 is very skewed with the top 10 percent of stocks accounting for over 60 percent of the market value of all firms and the top two groups almost 80 percent. In contrast, the distribution of market value per shareholder is more symmetric with the average holdings in the fifth group close to the average for the sample.

From the earlier discussion of whether smaller firms would have larger alphas, a sufficient condition for $d\alpha_k/dx_k < 0$ is that $d(x_k/q_k)/dx_k \leq 0$. If market value per shareholder is accepted as a good empirical proxy for $x_k/q_k$, then inspection of Table 1 suggests that as an empirical matter, the opposite is the case with $d(x_k/q_k)dx_k > 0$. However, as noted for the examples of General Electric, Ford, and IBM, these numbers have a significant upward bias for larger firms that have a disproportionally-larger fraction of institutional shareholders.

Even with this bias, the rate of increase in market value per shareholder is substantially smaller than for $x_k$ itself. That is, although the aggregate market value of firms in the top group is almost 600 times that of the bottom group, the corresponding market value per shareholder comparison is only 12 times. This finding, of course, does no more than quantify the well-known fact that smaller firms tend to have fewer shareholders than larger firms. Since in our model, $[x_k/(V_k/N_k)]/[x_j/(V_j/N_j)] = q_k/q_j$, these ratios are also presented in Table 1 using the $q$ for the top group as "numeriare." These measures are rather crude estimates for the parameters specified in the model. Nevertheless, they do provide some indication that the $q_k$ values for the lower 80 percent of firms are considerably smaller than for the large firms. Because it is differences in $q_k$ which provide all the differences between our model and the complete-information CAPM, these preliminary results provide a reasonable basis for further empirical study of the model.

## IV. Firm Behavior and The Determination of Its Investor Base

In the development of our model, the information sets of investors $\{J_j\}$ are treated as part of their initial endowments and hence, the distribution of the $q_k$ values across firms is exogeneously specified. In the previous section, we used comparative statics analysis to study cross-sectional differences among firms' equilibrium expected returns. In this section, we examine the investment behavior of an individual firm and the role of the firm in determining the size of its investor base, $N_k = q_k N$. Throughout the analysis, it is assumed that the management of the firm makes its decisions in the best interests of the current shareholders and that this interest is best served by maximizing the current market value of the firm.

In Section II, the focus of our discussion on information cost structures was on the fixed costs that an investor must pay before detailed substantive information about the firm can be processed into a portfolio decision. Investors that are not aware of the firm in this sense will not become stockholders of the firm. If an increase in the size of the firm's investor base is in the best interests of the current stockholders, then management should expend resources of the firm to induce investors who are not currently shareholders to incur the necessary costs of becoming aware of the firm.

From (16) and (28.d), we have that:

$$\frac{\partial \bar{R}_k}{\partial q_k} = - \bar{R}_k \delta x_k \sigma_k^2 / q_k^2 \tag{32}$$

and from the identity, $V_k \equiv I_k \mu_k / \bar{R}_k$, we have from (32) that:

$$\frac{\partial V_k}{\partial q_k} = V_k \delta x_k \sigma_k^2 / q_k^2 \quad . \tag{33}$$

Ceteris paribus, from (32) and (33), an increase in the relative size of the firm's investor base will reduce the firm's cost of capital and increase the market value of the firm. Thus, in our model, managers of the firm have an incentive to expand the firm's investor base. As is evident from (32) and (33), the magnitude of the effect will be greatest for lesser-known firms (with small $q_k$) and for firms with large firm-specific variances.

It is, of course, possible for the investor base to increase without the firm spending anything. For example, a newspaper or other mass media story about the firm or its industry that reaches a large number of investors who are not currently shareholders, could induce some of this number to incur the set-up costs and follow the firm. Having done so, in our model, these investors would evaluate the detailed substantive information about the firm, become new shareholders, and the value of the firm would rise.[20]

It should be stressed that the current shareholders may already know all the information contained in such stories. Nevertheless, if the form of the prior public releases of the information did not capture widespread attention among investors who do not follow the stock and if the new form does, then the firm's investor base will increase and the stock price will rise. Thus, our model is consistent with the observation that stock price sometimes reacts to a broad and widely-circulated report about the firm, even when all the substantive information in the report has been previously announced.[21]

Although not mutually exclusive, the techniques used and resources expended by the firm to expand its investor base are logically separable from those used to provide substantive information that existing shareholders can use to evaluate their portfolio position in the firm. Thus, our model provides a rationale for expenditures on advertising about the firm that is targeted for investors and on public relations designed to generate stories about the firm in the financial press. In the standard financial-equilibrium models, there is no purpose for expenditures that increase the visibility of the firm in the investment community without providing new and meaningful information for investor evaluation of the firm.

Although not directly an incomplete-information issue, the existence of prudent-investing laws and traditions as well as other regulatory constraints, can also rule out investment in the firm by some investors. The effect of these constraints on investor behavior is captured in our model because these investors act as if they did not know about the firm. Thus, the firm can also expand its investor base by spending resources to make the firm an eligible investment for these investors (e.g., to have the firm listed for trading on a national exchange or to have its securities rated by outside agencies).

As these examples suggest, the process of inducing investors who are not currently shareholders, to follow the firm's securities is not unlike that used to market the firm's products. It is, therefore, reasonable to assume that the cost structure for the firm to expand its investor base will be similar to other marketing cost functions. Let $F(N_k)$ denote the cost to firm $k$ of creating an investor base of $N_k$ shareholders where we posit $F(0) = 0$.[22] We assume that marginal cost, $dF/dN_k \equiv F'(N_k)$ is strictly positive and that marginal cost increases with the number of investors already informed (i.e., $F''(N_k) > 0$). Since the model of Section II posits that the total investor population, $N$, is very large, we further assume for analytical convenience that $F'(N) = \infty$.

Facing this cost function, managers of firm $k$ will choose the amount of resources to spend on expanding the investor base so as to maximize $[V_k - F(N_k)]$. From (17), the optimal $N_k$ will satisfy:

$$F'(N_k) = \delta s_k^2 I_k^2 / R N_k^2 W \quad . \tag{34}$$

From the strict convexity of $F$ and $F'(N) = \infty$, (34) has a unique solution with $0 < N_k < N$. From this convexity and (34), it follows that the optimal-size investor base is an increasing function of the firm-specific component of the variance, measured in terms of end-of-period, dollar cash flows. The effect of macro variables on the optimal base is that $N_k$ increases with greater investor risk aversion, decreases with larger per capita wealth, and decreases with the interest rate.

Suppose the management of firm $k$ contemplates increasing the scale of investment in the firm. As in the cross-sectional analysis in (28), let $\psi(I_k)$ denote the elasticity of the equilibrium expected excess return, $\bar{R}_k - R$, with respect to a change in $I_k$. From (17) and $\bar{R}_k = I_k \mu_k / V_k$, we have that:

$$\psi(I_k) = \left( \frac{\bar{R}_k}{R} \right)(\sigma_k^2 x_k / q_k)/(q_k b_k b + \sigma_k^2 x_k) \quad . \tag{35}$$

By inspection, the expected return required by current investors to hold shares in firm $k$ increases with $I_k$. Moreover, from the discussion in Section III, the magnitude of $\psi(I_k)$ can be significant for firms with relatively small investor bases $(q_k << 1)$ and relatively larger firm-specific variances. Such firms will face significantly downward-sloping investor demand curves that should be taken into account in their investment-financing decisions. Thus, our model provides characteristics for determining whether a downward-sloping demand curve or perfect substitution is a better descriptor for a firm's stock price response to changes in the supply of its shares.

From (33) and (34), if management decides to expand the size of the firm, then it should also increase its investor base. The management thus chooses $I_k$ and $N_k$ so as to maximize $[V_k - I_k - F(N_k)]$. From (17), the first-order condition with respect to $I_k$ can be written as:

$$I_k = N_k W[\mu_k - R - \delta ba_k]/2\delta s_k^2 \qquad , \tag{36}$$

where it is understood that $I_k = 0$ if $[\mu_k - R - \delta ba_k] < 0$. From (36), the optimal quantity of investment is smaller than in the corresponding complete-information model because $N_k < N$. The optimal $N_k$ is derived from (34) which by substituting for $I_k$ from (36), can be written as:

$$F'(N_k) = W[(\mu_k - R - \delta ba_k)/s_k]^2/4\delta R \qquad . \tag{37}$$

Thus, (36) and (37) complete the general equilibrium specification of our model by providing an endogeneous determination of the level of production and the size of the investor base for each firm.

Our analysis suggests that expansion in the firm's investor base and increases in investment by the firm will tend to coincide. If exogeneous events cause the investor base of the firm to expand [either directly or by shifting downward the marginal cost function, $F'(N_k)$], then the firm's cost of capital will fall and it will be optimal for the firm to increase its investment. If other events lead the firm to increase investment, then generally, the firm will also want to increase its investor base. In either case, to provide the additional capital for new investment will often require the firm to issue new securities.

If the firm underwrites its own securities using, for example, a rights offering to its existing security holders with a subscription price substantially below market, then the firm is likely to succeed in raising the new capital needed for investment.[23] However, in our model, such a rights offering is no more likely to increase the size of the firm's investor base than the payment of a stock dividend or a stock split. Thus, the new securities will largely be held in the portfolios of the firm's current security holders.

If, instead, the firm undertakes a negotiated underwriting through an investment bank with broad distribution capabilities, then the firm can use the underwriting to <u>both</u> raise new capital <u>and</u> increase its investor base. Indeed, if the latter is a particularly important objective, then the bank and the terms of the deal can be chosen so as to maximize the number of new shareholders who buy the securities. If the underwriter succeeds in inducing new investors to purchase and follow the firm's stock, then the benefits to the issuing firm can exceed simply the placement of the new securities in other-than-existing shareholders' portfolios. These new investors become part of the base to support secondary-market trading in all the firm's securities as well as future primary offerings.

Thus, our model provides a framework for evaluating the benefits and costs of negotiated underwritings versus rights offerings. At least a part of the underwriting costs can be treated as expenditures for expanding the investor base of the firm. This framework is entirely consistent with models that use asymmetric information and reputation to examine this issue. For example, the reputation of the investment bank is certainly a possible explanation for its ability to expand the firm's investor base at a lower cost than if the firm attempted to do so on its own.

Because the benefits are likely to be greatest for lesser-known firms with large firm-specific variances, perhaps the best class of underwritings to examine for these effects is the closely-held firm undertaking an initial public offering. As indicated in earlier discussion, wide-spread publicity about a firm or industry can either make investors more receptive to finding out about the firm if the publicity is favorable, or less receptive if it is not. Although the firm and its underwriter can use public relations activities as an attempt to stimulate favorable media coverage, such attempts are likely to be marginal in generating media reports and influencing their content. Nevertheless, these reports can significantly change the cost of increasing the firm's investor base, either lowering it with favorable reports or raising it with unfavorable ones.

If, at a point in time, several articles appear in the financial press indicating that initial public offerings generally are a fertile area for investors to investigate, then the cost of a successful initial offering is likely to decline for all firms and the number of such public offerings will rise. Favorable media coverage of an industry will have similar effects, but more selectively. Thus, our model is consistent with both macro "waves" of initial public offerings and the "bunching" of these offerings among specific industries.

Many of the investor and firm actions shown to be consistent with our model can also arise in models of nonrational behavior. In such models, media coverage, public relations and other investor marketing activities could play an important causal role in creating and sustaining speculative bubbles and fads among investors.[24] Although important in our "rational" model, these activities are not viewed as primal factors in either the determination of the investment and financing decisions by firms, or the determination of security prices. In our model, expanded media coverage of a firm, industry or other sector of the economy, is stimulated by changes in the same economic fundamentals that cause firms to change their plans and investors to reassess their portfolios. Advertising that initially attracts investor attention to a firm is assumed to leave that firm's investor base unchanged if the underlying fundamentals do not justify a change.[25]

The analysis has focused on the equilibrium determination of expenditures by the firm to expand the breadth of investor cognizance about the firm and thereby, increase the size of its investor base. In an expanded model with differences in the depth of investor cognizance across firms, it would pay for some firms to expend resources to improve the quality or precision of information available to investors. Thus, much the same analysis could be applied to provide an endogeneous determination of the firm's estimation risk in the Bawa-Klein-Barry-Brown models.

## IV. Pending Issues and Extensions of the Model

As stated at the outset, the manifest objective of our analysis is to show that a reconciling of finance theory with significant empirical violations of the complete-information, perfect-market model need not require a radical departure from that paradigm. The especially-simple structure of the model was selected with that expressed purpose in mind. The highly-specialized assumptions of the model are, of course, the source of the unambiguous and strong conclusions about the structure of equilibrium security prices and the behavior of firms. A careful examination of the robustness of these conclusions should therefore be undertaken before giving more substantive empirical consideration to the model. Although such an examination is beyond the scope of this paper, we provide in this closing section, some preliminary observations on the likely effects of perturbing the simplifying assumptions of the model.

The important equation (26), $\alpha_k = \lambda_k - \beta_k \lambda_M$, still obtains if the model is generalized to allow an arbitrary variance-covariance structure among security returns. However, the shadow cost $\lambda_k^j$ will depend on which other securities are contained in investor j's information set. Hence, unlike in (10), the shadow cost will not be the same for all investors who do not know about security k. It is, moreover, possible that $\lambda_k^j$ can be negative, which implies that investor j would short-sell security k if he knew about it. If, however, the aggregate $\lambda_k$ is a decreasing function of $q_k$, then the fundamental comparative statics results derived here will be robust. Judging by analogy from the work of Errunza and Losq (1985) on mildly-segmented markets, I believe that reasonably-general conditions on the covariance structure can be derived to ensure that $\partial\lambda_k/\partial q_k < 0$.

It seems reasonable to assume that the marginal cost of finding out about security k would be lower for an investor who already knows about other securities (perhaps in the same industry) whose returns are strongly correlated with the return on security k. However, the diversification benefits from finding out about security k will also be reduced for that investor. In the context of our underlying information-cost story, it is, thus, ambiguous how the equilibrium distribution of $q_k$ would be affected between the diagonal and more-general version of the model.[26]

Introducing costly shortsales or their outright prohibition in the model would render the portfolio actions of investors who know about security k and who would like to short-sell it indistinguishable from those who simply do not know about the security. The prohibition of short sales will increase the likelihood that an investor will not incur the necessary set-up cost to become informed about security k, and therefore, institutional constraints against short sales will tend to accentuate the effects derived in our model. The reason is that the <u>ex-ante</u> information that no action can be taken if the <u>ex-post</u> information received would otherwise dictate a short sale, lowers the expected payoff from expending the resources to find out about a security. In effect, this restriction is equivalent to the investor giving away <u>ex-ante</u> a put option on the security. Thus, the "lost-opportunity" cost from prohibition of short sales will cause a larger reduction in expected benefits for those stocks with larger nonsystematic variances, $\{\sigma_k^2\}$.

In practice, many institutional investors are prohibited by charter or general prudence laws from short selling. Even for those who can undertake such transactions, the costs of short sales are low only for the larger and more-widely-held stocks. As it happens, these are also the stocks on which options and futures (with their less-costly short selling facilities) are available. Hence, the effect of costly short sales on $q_k$ and equilibrium security prices may be more pronounced for smaller and less-widely-held securities.

As in the standard complete-information CAPM, our model assumes that individual investors form their portfolios by direct transactions in individual firms' securities, and therefore, provides no explicit recognition of institutional investors. The central issue surrounding the introduction of financial intermediaries (such as investment companies) into our model is whether they enjoy sufficiently increasing returns-to-scale to vitiate the fixed set-up cost information story underlying the assumption that investors only know about a small subset of the available securities. If $K$ individual investors pool their resources under a common portfolio manager, then they can in effect reduce the fixed cost (per investor) of finding out about a security in proportion to $(K - 1)/K$.[27] There is surely little doubt that important benefits accrue to larger-scale investment units by reducing information and transactions costs as well as providing more-skilled portfolio management. This well-founded observation need not, however, imply that the limiting case obtains where $K$ is large enough that the institutional investor (and hence, its clients) become informed about all available securities. To make this case requires among other things,[28] that the financial institution can without cost make investors aware of its presence and convince them to buy its securities. Such an assumption is, of course, inconsistent with the basic information-cost story underlying our model. Contrary to this argument for complete information, there is ample empirical evidence that real-world portfolio-management firms spend considerable resources on marketing and sales to increase assets under management.[29]

As discussed in Section II, individual investors' direct holdings of individual stocks tend to be highly concentrated. Many of these investors do, of course, have substantial indirect holdings of stocks through ownership or beneficial claims on institutional portfolios such as mutual funds, pensions, trusts and profit-sharing plans. However, the number of such indirect holdings by each individual investor (e.g., the number of mutual funds held by one investor) is also small. Even for the largest of institutions, the number of individual stocks held in a single institutional portfolio represent only a small fraction of the total number of securities available.

The empirical evidence also suggests considerable overlap in the selected individual securities held in different institutional portfolios. Moreover, the relative fractions of traded securities contained in the aggregate of institutional portfolios appear to differ from their relative fractions in the market portfolio. These facts support the view that even after accounting for both direct and indirect holdings, any given investor effectively knows about only a small fraction of the total number of securities available in the market place.

To extend the model to explicitly include financial intermediaries, we need simply posit that for each institutional portfolio, its manager knows about some securities and does not invest in those he does not know about.[30] Each individual investor in turn knows about some individual stocks and some institutional portfolios and confines his investments to them.[31] The equilibrium analysis of the extended model would follow the one presented here with the aggregate $\{q_k\}$ for individual stocks determined from both the direct and indirect holdings of individual investors.

Our model assumes "active" investors who select portfolios that are mean-variance efficient relative to the securities contained in their information sets. The model can be extended to include investors who follow specific "passive" portfolio strategies, without causing significant changes in its substantive conclusions.

A "market indexer" is an investor who makes no judgments about stock market value and passively holds the market portfolio. By definition, such investors do not know the expected return on the market portfolio although they may know its long-run historical average. If $\Delta N$ denotes the number of indexers, then the equilibrum aggregate demand for security $k$ by this group is given by $x_k \Delta NW$. From (9.b) and (9.c), the aggregate demand for security $k$ can be written as $D_k = N_k W \Delta_k / \delta\sigma_k^2 + x_k \Delta NW$. In equilibrium, $x_k = D_k/NW$. It follows that the equilibrium $x_k$ is given by (15) where $q_k = N_k/(N - \Delta N)$ is the fraction of the <u>active</u>-investor population that knows about security $k$. The effect of the indexers on equilibrium prices is as if they were an active-investor population with fraction $q_k$ of the group knowing about security $k$. That is, the equilibrium is the same as one with only active investors where the number that know about security $k$ is given by $N_k' \equiv N_k + N_k \Delta N/(N - \Delta N)$ and $q_k = N_k'/N$.

If, as may be the case in practice, passive market-indexers hold only a subset of the market portfolio and if this subset tends to be focused on larger and more widely-held stocks, then indexers can accentuate the results derived here by raising the effective $q_k$ for those stocks and lowering

$q_k$ for the excluded ones. Similar "tilting" effects would obtain for investors who select a small number of individual stocks with the strategy that they are chosen so as to minimize the "residual" variance of the portfolio return with reference to the market index.

Another example of a passive strategy is the "naive" APT investor who assumes that with the single common-factor return process given in (2) and a very large number of securities, n, the optimal portfolio behavior is to simply combine a security with maximum positive correlation with the common factor and the riskless security. In our model, such investors will only hold the forward contract on the common factor in combination with the riskless security. This strategy is formally equivalent to the optimal behavior for an active investor who does not know about any of the individual securities (i.e., $\lambda_k^j > 0$ and $w_k^j = 0$ for $k = 1,\ldots,n$). Thus, investors who follow this passive strategy are already accommodated within the analysis presented here.

Perhaps the most-controversial conclusion of our model is that less well-known stocks of firms with smaller investor bases tend to have relatively larger expected returns than in the comparable complete-information model. If such stocks can be easily identified and if accurate estimates of alphas can be acquired at low cost, then professional money managers could improve performance by following a mechanical investment strategy tilted toward these stocks. If a sufficient quantity of such investments were undertaken, then this "extra" excess return would disappear. This prospect is surely a modulating force on the empirical magnitude of these anomalous returns. However, as discussed in our "Prologue," the time frame over which such corrective action takes place can be considerable and even in the long run, it may not be complete.[32] Moreover, the profits from such strategies are bounded and at least in part, illusionary.

Consider the hypothetical example of the $200 million firm discussed in Section III. Suppose that the equilibrium expected return is 13 percent in our model and 10 percent in the comparable complete-information-model. The effect of this extra-return requirement on share price depends on its anticipated persistence over time as discussed in footnote 20. If the expected duration of this extra-return is one year, then the effect on current market price is about 3 percent or a $6 million undervaluation. If it applies in perpetuity, then the market price will be 23 percent below its complete-information value, or a $46 million undervaluation. Even at this extreme, the aggregate dollar amount involved is, for example, less than a change of a nickel a share in the price of Exxon stock. Many large institutional investors such as registered investment companies are normally restricted from purchasing more than 5 percent of the outstanding stock of a company. Thus, the total maximum potential gain to the institutional investor is between $0.3 and $2.3 million and not $6 to $46 million. The latter is only relevant for a takeover bid, which, as an empirical matter, typically requires a substantial premium over market price. Moreover, given our analysis, it is perhaps realistic to assume that an attempt to buy or sell 5 percent of such a company will have an appreciable effect on price. Indeed, the largest feasible position that can be acquired without significantly affecting price may be considerably smaller than the statutory limit for the institution. Thus, a rational manager of a large institutional portfolio may not take a position in a security with a seemingly-large incremental expected <u>rate</u> of return if the incremental expected <u>dollar</u> return from the position would be too small to justify the additional information, monitoring and general nuisance costs.

In summary, financial markets dominated by rational agents may nevertheless produce anomalous behavior relative to the perfect-market model. Institutional complexities and information costs may cause considerable variations in the time scales over which different types of anomalies are expected to be eliminated in the market place. Whether or not the specific information inefficiency posited can be sustained in the long run, the model may nevertheless provide some intermediate insights into the behavior of security prices.

FOOTNOTES

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1 Elements of this evolution are described in Merton (1983, 1987, forthcoming).

2 The impact of efficient market theory, portfolio selection, risk analysis and contingent-claim pricing theory on money management and capital budgeting procedures is evident from even a casual comparison of current practices with those of twenty years ago. Financial research has also influenced legal proceedings such as appraisal cases, rate of return hearings for regulated industries, takeover rules, and "prudent-person" laws governing behavior of fiduciaries. The role of finance theory in the current wave of financial innovations is well documented in numerous articles in the financial press. See also Van Horne (1985).

3 Cf., "Symposium on Some Anomalous Evidence on Capital Market Efficiency," Journal of Financial Economics, June-September 1978; Black (1986), Summers (1986) and R.C. Merton (1987).

4 Black (1986) describes many of the puzzles and attributes them to noise. Indeed, he predicts that, "...research will be seen as a process leading to reliable and relevant conclusions only very rarely, because of the noise that creeps in at every step." (p. 530)

5 So writes the sociologist of science, R.K. Merton (1987), who calls this particular form of ignorance, "specified ignorance." Noting its common occurrence across both time and scientific fields, he points out that "...as the history of thought, both great and small, attests, specified ignorance is often a first step toward supplanting that ignorance with knowledge."

6 For various views on this issue, see the papers in Hogarth and Reder (1986). From these papers, it is evident that finance, and especially, speculative markets, provides a potentially-rich "strategic research site" (see R.K. Merton, 1987) for psychologists and sociologists as well as economists.

7 In a slightly different context, Miller (1986, p. S467) writes: "That we abstract from all these stories in building our models is not because the stories are uninteresting, but because they may be too interesting and thereby distract us from the pervasive market forces that should be our principal concern."

8 In tests of market efficiency, it is typically assumed that actual investors in real time knew (or should have known) the model and the statistical results derived by the researcher. As is well known, the large volatility of stock prices often requires a very long time series before a statistically-significant, estimated mean can be derived. Perhaps such studies should report the length of time before the discovered anomaly exceeded generally-accepted confidence intervals.

9 Cf. Friend and Blume (1975) and Blume and Friend (1978,1986).

10 Cf. Amihud and Mendelson (1986); Brennan (1970); Constantinides (1984); Errunza and Losq (1985); Levy (1978); and Miller (1977).

11 See Bhattacharya (forthcoming) for an extensive review.

12 Klein and Bawa (1977); Barry and Brown (1984, 1985, 1986).

13 Although larger volatility and lower substitutability among equities will surely make the derived effects greater for equity securities than debt, the same idea applies to all securities. For example, a bond trader who responds quickly to interest rate news by trading U.S. Treasury bonds, may not be willing to trade GNMA mortgage-backed bonds unless he has borne the set-up costs necessary to understand the effect on price of the prepayment feature of these bonds.

14 As with the firm, information from brokerage or investment services will only influence an investor's decisions if he knows about the source and has incurred the set-up cost to properly calibrate the information. Similarly, the investor knows about only a small number of money management institutions and if he does not know about an institution, he will not invest with it.

15 See Arbel and Strebel (1982); Arbell, Carvell and Strebel (1983); and especially, Arbel (1985, p. 5); and Strebel and Carvell (1987, Chapter 1).

16 The shadow cost is measured in units of expected return. Given the return structure (2) and (3), equation (10) for the shadow cost applies not only for the mean-variance criterion, but for all concave utility maximizers.

17 If we maintain the assumption of identical preferences but allow for a non-uniform distribution of wealth among investors, then $q_k$ becomes the fraction of national wealth owned by investors who know about security $k$.

18 As an empirical matter, the complete market portfolio is not observable. Black, Jensen and Scholes (1972) use an equally-weighted portfolio of the

securities in the sample as their market surrogate. Under these conditions, the claim that $E[\lambda_k | \beta_k] = \lambda_M$ is reinforced because $\lambda_M = \Sigma_1^n \lambda_k/n$ .

19 Cf. Friend and Blume (1975) and Mehra and Prescott (1985, p. 154).

20 A proper development of this conclusion requires a dynamic version of the model. In such a model, current (informed) shareholders of firm $k$ would have expectations about the future time path of $q_k$. If a favorable story implies an upward revision in those anticipations, then the price should rise immediately, even if there is a time lag before the newly-informed investors take positions. Similarly, an unfavorable story implying a reduction in the anticipated growth in the investor base should cause an immediate price decline. As in the other analyses, this effect is likely to be most important for smaller and lesser-known firms.

21 In this case, the "new" information that affects price is the report itself (through its anticipated effect on the investor base) and not the substantive information contained in the report which is already known by all those currently trading in the stock.

22 If the firm already has $N_k'$ investors in its base, then we adjust the cost function to $F(N_k) - F(N_k')$ with the choice constraint $N_k \geq N_k'$ . The cost function is likely to vary across firms and in particular, firms with larger $s_k$ will probably have a higher cost $F(N_k)$ for every $N_k$.

23 For deep-in-the-money rights, arbitrageurs, if necessary, will ensure exercise by buying rights and shorting stock. These transactions will not cause an increase in the investor base, since purchasers of the shorted stock are current shareholders.

24 The media would surely play a key role in creating self-fulfilling prophecies, Merton (1948). Although our model provides a rational explanation for the response in stock price to these activities, it does not, as an empirical matter, rule out nonrational behavior. In principle, one could, however, test the fads hypothesis by determining whether the price changes are transient or permanent, provided, of course, that the half-lives of fads are not too long.

25 As in the marketing literature generally, we would expect that in a micro-micro description of information transmission to investors, firms and their agents would take account of cognitive psychological factors such as Tversky-Kahneman (1981) "framing" in selecting the form and medium to present information. Our model assumes that systematic attempts to exploit cognitive errors among investors will not have material effects.

26 Much the same discussion would apply to extending the model to include multiple factors in the context of the Arbitrage Pricing Model.

27 The issue of increasing-returns-to-scale arises frequently in the analysis of information production. For example, the same question raised here applies to the Grossman-Stiglitz (1976) model as well.

28 For example, one must also assume no important intra-organizational information-transmission or agency-type costs that would limit the size or the number of employees of the financial institution.

29 Cf. Pamela Sebastian, "Mass Marketing Becomes Driving Force," Wall Street Journal, December 8, 1986, p. 47. Marketing costs are significant for both retail and institutional (e.g., pension plan sponsors) sales.

30 Institutional managers may avoid some stocks and limit the size of investment in others for other reasons such as monitoring costs, liquidity, prudence, and insider or "five-percent" rules.

31 An investor may "know about" a particular institutional portfolio and not another, because a salesman has contacted him about one and not the other or because of institutional restrictions, e.g., his employer may have a defined-contribution pension plan that limits his investment allocations to selected institutional portfolios.

32 If there are significant marketing costs associated with educating investors about the benefits of a "neglected-stock" fund, then money managers may be reluctant to undertake that task because, for such a generic product, they may not be able to charge fees sufficient to recover these costs in a competitive system. The issue is analogous to the "free-rider" problem of a public good.

BIBLIOGRAPHY

Amihud, Yakov and Haim Mendelson, "Asset Pricing and the Bid-Ask Spread." Journal of Financial Economics 17 (December 1986), 223-249.

Arbel, Avner, "Generic Stocks: An Old Product in a New Package." Journal of Portfolio Management 11 (Summer 1985), 4-12.

___________ and Paul Strebel, "The Neglected and Small Firm Effects." The Financial Review (November 1982), 201-218.

___________, Steven Carvell and Paul Strebel, "Giraffes, Institutions and Neglected Firms." Financial Analysts Journal (May/June 1983), 2-8.

Banz, Rolf, "The Relationship Between Return and Market Value of Common Stocks." Journal of Financial Economics 9 (March 1981), 3-18.

Barry, Christopher and Stephen Brown, "Differential Information and the Small Firm Effect." Journal of Financial Economics 13 (July 1984), 283-294.

___________ and ___________, "Differential Information and Security Market Equilibrium." Journal of Financial and Quantitative Analysis 20 (December 1985), 407-422.

___________ and ___________, "Limited Information as a Source of Risk." Journal of Portfolio Management 12 (Winter 1986), 66-72.

Bhattacharya, Sudipto, "Financial Markets and Incomplete Information: A Review of Some Recent Developments," in Sudipto Bhattacharya and George Constantinides, eds. Frontiers of Financial Theory, (Totowa, NJ: Littlefield and Adams, forthcoming 1987).

Black, Fischer, "Capital Market Equilibrium with Restricted Borrowing." Journal of Business 45 (July 1972), 444-455.

___________, "Noise." Journal of Finance 41 (July 1986), 529-543.

___________, Michael Jensen and Myron Scholes, "The Capital Asset Pricing Model: Some Empirical Findings" in Michael Jensen, ed., Studies in the Theory of Capital Markets (New York: Praeger 1972), 79-121.

Blume, Marshall and Irwin Friend, "A New Look at the Capital Asset Pricing Model." Journal of Finance 28 (March 1973).

___________ and ___________, The Changing Role of the Individual Investor: A Twentieth Century Fund Report (New York: John Wiley & Sons, 1978).

_______________ and ____________, "Recent and Prospective Trends in Institutional Ownership and Trading of Exchange and OTC Stocks." Rodney L. White Center for Financial Research Working Paper, The Wharton School of the University of Pennsylvania, 1986.

Brennan, Michael, "Taxes, Market Valuation and Corporate Financial Policy." National Tax Journal 23 (December 1970), 417-427.

Constantinides, George, "Optimal Stock Trading with Personal Taxes: Implications for Prices and the Abnormal January Returns." Journal of Financial Economics 13 (March 1984), 65-89.

Errunza, Vihang and Etienne Losq, "International Asset Pricing Under Mild Segmentation: Theory and Test." Journal of Finance 40 (March 1985), 105-124.

Fama, E. and J. MacBeth, "Risk and Return and Equilibrium: Empirical Tests." Journal of Political Economy 81 (May 1973), 607-635.

Friend, Irwin and Marshall Blume, "The Demand for Risky Assets." American Economic Review 65 (December 1975), 900-922.

_____________, Randolph Westerfield and Michael Granito, "New Evidence on the Capital Asset Pricing Model." Journal of Finance 33 (June 1978), 903-917.

Grossman, Sanford and Joseph Stiglitz, "Information and Competitive Price Systems." American Economic Review 66 (May 1976), 246-253.

Hogarth, Robin and Melvin Reder, eds., "The Behavioral Foundations of Economic Theory." Journal of Business 59, Part 2 (October 1986).

Keim, Donald, "Size Related Anomalies and Stock Market Seasonality: Further Empirical Evidence." Journal of Financial Economics 12 (June 1983), 13-32.

Klein, Roger and Vijay Bawa, "The Effect of Limited Information and Estimation Risk on Optimal Portfolio Diversification." Journal of Financial Economics 5 (August 1977), 89-111.

Levy, Haim, "Equilibrium in an Imperfect Market: A Constraint on the Number of Securities in a Portfolio." American Economic Review 68 (September 1978), 643-658.

Mehra, Rajnish and Edward Prescott, "The Equity Premium: A Puzzle." Journal of Monetary Economics 15 (          1985), 145-161.

Merton, Robert C., "Financial Economics" in E. Cary Brown and Robert M. Solow, eds., Paul Samuelson and Modern Economic Theory (New York: McGraw-Hill, 1983), 105-140.

_______________, "On the Current State of the Stock Market Rationality Hypothesis" in Stanley Fischer and Rudiger Dornbusch, eds., Macroeconomics and Finance: Essays in Honor of Franco Modigliani (Cambridge: MIT Press, 1987).

_______________, "Capital Market Theory and the Pricing of Financial Securities" in Benjamin Friedman and Frank Hahn, eds., Handbook of Monetary Economics (Amsterdam: North-Holland, forthcoming).

Merton, Robert K., "The Self-Fulfilling Prophecy." Antioch Review (Summer 1948), 193-210.

_______________, "Three Fragments from a Sociologist's Notebook: Establishing the Phenomenon, Specified Ignorance, and Strategic Research Materials." The Annual Review of Sociology (1987).

Miller, Edward, "Risk, Uncertainty and Divergence of Opinion." Journal of Finance 32 (September 1977), 1151-1168.

Miller, Merton, "Behavioral Rationality in Finance: The Case of Dividends" in Hogarth and Reder (1986), 451-468.

Reinganum, Marc, "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings' Yields and Market Values." Journal of Financial Economics 9 (March 1981), 19-46.

_______________, "The Anomalous Stock Market Behavior of Small Firms in January: Empirical Tests for Year-End Tax Effects." Journal of Financial Economics 12 (June 1983), 89-104.

_______________ and Janet Smith, "Investor Preference for Large Firms: New Evidence on Economies of Size." Journal of Industrial Economics 32 (December 1983), 213-227.

Rogers, Everitt M., Diffusions of Innovations (New York: Free Press, 1972).

Roll, Richard, "A Critique of the Asset Pricing Theory's Test: Part I." Journal of Financial Economics 4 (March 1977), 129-176.

_______________, "Vas ist Das? The Turn of the Year Effect and the Return Premia of Small Firms." Journal of Portfolio Management 9 (Winter 1983), 18-28.

Ross, Stephen, "The Arbitrage Theory of Capital Asset Pricing." Journal of Economic Theory 13 (December 1976), 341-360.

Schwert, G. William, "Size and Stock Returns, and Other Empirical Regularities." Journal of Financial Economics 12 (June 1983), 3-12.

Sharpe, William, "Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk." Journal of Finance 19 (September 1964), 425-442.

Strebel, Paul and Steven Carvell, In the Shadows of Wall Street (Englewood: Prentice-Hall 1987).

Summers, Lawrence, "Does the Stock Market Rationally Reflect Fundamental Values?" Journal of Finance 41 (July 1986), 591-601.

Tversky, Amos and Daniel Kahneman, "The Framing of Decisions and the Psychology of Choice." Science 211 (30 January 1981), 453-458.

Van Horne, James, "Of Financial Innovations and Excesses." Journal of Finance 40 (July 1985), 621-631.