# EXPERIMENTS ON THE STABILITY OF AN AXISYMMETRIC WATER JET

by

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## ABSTRACT

The experiments described herein are concerned with the behavior of a circular jet of water emerging into a region concentric with the jet. Several variations in the geometry of this axisymmetric configuration were investigated, with the aim being the correlation and clarification of the seemingly disparate results of previous experiments.

Flows were considered in the entire range from zero velocity at the jet exit up to velocities giving fully turbulent flow. Visual observations were made, supplemented by still photographs.

The onset of instability, and the Reynolds number at which fully turbulent flow is present, are determined experimentally.

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# I. INTRODUCTION

An observation which is fundamental to the field of Fluid Mechanics is that at high enough velocities a fluid flow of fixed geometry and physical properties may undergo transition from a laminar to a turbulent flow. For laminar flow, the principles of the conservation of mass, momentum, and energy, applied to a continuum region, give equations describing the exact velocity at every point and time under consideration in the region. For turbulent flow, in addition to the mean velocity at a point there exists a fluctuating component of velocity, so that the exact velocity cannot be found. Furthermore, the distribution of mean velocity in a turbulent region will not coincide with the exact velocity distribution for a laminar flow.

The mathematical theory most commonly proposed to analyze the onset of turbulence is the application of small perturbations in velocity to the velocity of a laminar flow. The resulting equation leads to characteristic value problems which can be satisfied by perturbations of certain frequency and wavelength, and the boundary conditions of which describe the geometry of the flow. For sufficiently low Reynolds numbers, the solutions indicate that all disturbances decrease exponentially in amplitude with time, and the flow is said to be stable. Above a certain critical Reynolds number, at least some frequency and wavelength perturbations do not decrease but rather increase exponentially in amplitude. In this case the flow is said to be unstable, and it is proposed that turbulence is the eventual result of this growth of infinitesimal disturbances to a laminar flow. The Reynolds number at which turbulence, or even the growth of disturbances, is first observed does not coincide with the critical Reynolds number, and in fact may differ from it by an order of magnitude. A further complication is that the characteristic value problem referred to is described by a fourth order nonlinear differential equation, the Orr-Sommerfeld equation.

## Some Previous Work

The phenomena of the instability of a fluid jet is one of the earliest to have been noticed in Fluid Mechanics, having been observed in some studies which antedated Osborne Reynolds' classic experiments on the laminar-turbulent transition in pipe flows. Attempts to account for the behavior of lighting-gas flames were made by Le Comte (11)* in 1858. Tyndall (25), in 1867, showed that flickering and unsteady motion was not necessarily due to the presence of the flame, but that the fluid of the jet itself was sensitive to disturbances. In these cases, the effects noted were

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* Numbers in parentheses correspond to references listed in the Bibliography.

seen to have been caused by sound waves of particular frequencies. Rayleigh (17) also examined this phenomenon, along with several other fluid stability problems, and formulated the equations for the stability of frictionless flows. In a series of experiments on air jets made visible with cigarette smoke, Brown (3,4) examined the dependence of apparent instabilities in the flow on orifice size, jet exit velocity, and the frequency of the exciting disturbances. After theoretical analyses of the transition to turbulence of laminar flat-plate boundary layers had been made by Tollmein (27) and Schlichting (21), and substantiated by the experiments of Schubauer and Skramstad (22), investigators attempted to apply the theory of hydrodynamic stability to jet flows. In the flat-plate case, the Orr-Sommerfeld equation had been solved to give the critical Reynolds number for viscous flow; that is, the Reynolds number below which disturbances of all wavelengths decay. Several analyses have also been made for the case of the two-dimensional jet (6,10,12). The critical Reynolds number for such a flow is predicted to be about four. For the axisymmetric case, the Orr-Sommerfeld equation remains unsolved for the critical Reynolds number. It may be noted here that while the Orr-Sommerfeld equation is the usual starting point for stability analyses, it itself presents some difficulties. The instability theory postulates that the mean velocity be parallel, so that the component of mean velocity normal to the axis of the jet be zero. Also, the mean velocity is to be considered a function only of the dimension perpendicular to the axis. In a jet, however, the velocity depends on the distance from the orifice -- decreasing with distance due to entrainment of the surrounding fluid by the jet. Furthermore, experimental results point to critical Reynolds numbers low enough so that the associated jet flows are not parallel, since jet spreading is greatest at the lower Reynolds numbers. An additional difficulty with the stability theory, pointed out by Chanaud and Powell (5) is that in theory, the disturbance is applied instantaneously all along the infinite length of the parallel jet, whereas as shown by Brown (3,4), the disturbance exists for a long time but is located mainly near the orifice.

## Recent Experiments

The circularly symmetric geometry is especially difficult to consider analytically, yet is much easier to treat experimentally than the two-dimensional rectangular case, since the problem of end effects in a thin rectangular channel, often chosen for two-dimensional studies, is not present. Such effects were noticed in some previous experiments (1,5). In particular, an axially-symmetric jet is an interesting flow for experimental study and has been chosen as the configuration for several experiments on hydrodynamic stability. Experiments on the stability of axisymmetric jets were performed by Viilu (28) and Reynolds (18). Viilu found the critical Reynolds number to be about 11, based on observations of a dyed water jet emerging into a large beaker. Reynolds was concerned mainly with the range of Reynolds numbers above those investigated by Viilu. Finally, in an unpublished experimental investigation, Sajben (20) examined the behavior of a jet of mercury by observing the dimple formed on the free surface by directing a mercury jet upward at it. The Reynolds number at which the dimple first moved erratically was considered the critical Reynolds number, and was determined to be about 600.

## Present Experiment

The present experiment was undertaken to reconcile the above experimental results, and to more fully investigate the behavior of axisymmetric jets, perhaps as a basis for further theoretical analyses. Flows were considered in the entire range from zero velocity at the jet exit up to velocities giving fully turbulent flow; the range of Reynolds numbers was from 0 - 800, based on jet orifice diameter. The onset of instability was determined experimentally by making visual observations supplemented by still photographs.

## II. APPARATUS

The apparatus was constructed to allow for changes in the geometry of the flow, only axisymmetric flows being considered (see Figure 1). The jet was formed by the efflux of dyed water from a nozzle into a cylindrical plexiglass tank, 3½" in diameter, concentric with the jet. The orientation of the nozzle was either pointing down through the free surface into the tank, or up from the bottom of the tank. In the latter configuration, each nozzle was mounted in a plastic rod which could be threaded into the base of the tank.

Concentric with the nozzle and the inner tank was an outer plexiglass cylinder, 5½" in diameter. The outflow from the tank was thus between the annulus formed by the two plexiglass walls, so that the level of the free surface could be kept constant.

Dyed water was supplied to the jet through a polyethylene tube leading from the reservoir. This was a 250 ml graduate with an exit at the bottom, suspended to give a static head of about six feet. The head was sufficient to supply the jet with the desired flow without need for a pump, the vibrations of which would have imposed unwanted disturbances on the flow. The tank-and-nozzle unit was suspended on soft springs, with the period of the spring-mass system being about 1 second, thus reducing the effect of the ambient vibrations.

Nozzles were made of glass or from hypodermic needles. Glass nozzles were made by heating and drawing 7 mm glass tubing (see Figure 2). The tubing was rotated and guided by bearings to keep its axis straight while it was being heated. Weights were fixed to the bottom end of the tubing to draw it as soon as it was soft. The length of tubing being heated by the torch and the amount of weight used determined the tip diameter and shape of the nozzles. For example, heating the tubing for a short distance and using a heavy weight produced a nozzle with a long, small-diameter tip.

After the nozzles were drawn they were ground square by hand and inspected through a forty-power microscope. To measure the inside diameter at the nozzle exit, a microscope having a moveable carriage was used. The nozzle was moved relative to cross-hairs on the eyepiece, and the amount of motion read to ± 0.0002" on the carriage micrometer.

The outside diameter of the nozzle was also measured in this way. It was necessary for fully developed Poiseuille flow at the orifice to have a constant internal diameter for at least forty diameters in the nozzle. By measuring the outside diameter, and assuming no variation in wall thickness of the nozzle while the outer diameter stayed the same, the constancy of the nozzle internal diameter could be checked without breaking off the tip. Nozzle inside diameters ranged from 0.008" at the tip to 0.030".

When it was found that hypodermic needles gave results identical with those obtained from the glass nozzles, the former were used wherever possible due to their greater resistance to breaking. However, when small diameter nozzles were required, glass nozzles were again employed. Occasionally, minute particles of dirt lodged in the tips of the finer nozzles, and it proved easier to remove them from the glass nozzles.

The jet fluid was dyed red using a sodium-hydroxide-phenolphthalein solution, greatly diluted. The specific gravity of the solution used was 1.0004 using the following proportions:

| | | |
|---|---|---|
| ½ cc Phenolphthalein | (2.1 gm/cc) | 1.1 gm |
| 3 ml 1.0N NaOH | (1.04 gm/ml) | 3.12 gm |
| 2997 ml $H_2O$ (dist.) | (1.0 gm/ml) | 2997.0 |
| 3000 ml solution | | 3001.2 gm |

# III. EXPERIMENTAL RESULTS

The Reynolds number was calculated as shown in the appendix. For Reynolds numbers less than ten, the dyed jet emerging from the tank was perfectly straight and decidedly laminar. A disturbance applied to the tank wall did not affect the behavior of the jet. Even jolting the nozzle caused only a small break in the dyed stream which quickly closed again; after a moment the result of this disturbance could not be detected anywhere in the flow.

Flows with Reynolds numbers greater than ten behaved in two ways. Either the jet was straight, as in the case for Reynolds numbers less than ten, or else a large-amplitude sinuosity of the dyed portion of the jet could be seen. (It must be remarked that the "jet" consisted not only of the visible dyed portion, representing the efflux from the nozzle, but also of fluid which had been entrained by the dyed portion by means of fluid "friction", or viscosity.) The dyed core comprised either a regular sine-like curve of constant or increasing amplitude, or else was helical with constant or increasing diameter.

Above Reynolds numbers of 100, the axis of the jet was again straight as evidenced by the dyed portion. Increasing the Reynolds numbers led to "patches of turbulence" which interrupted the otherwise laminar core. These patches grew in size and frequency until fully turbulent flow was achieved.

The presence of turbulence in the core or central portion of the jet near the nozzle was first observed near Reynolds number of 100. These observations coincided with the "straightening" of the jet axis. At first, the interface between the dyed and clear fluid was observed to consist of ripples which moved with a phase velocity higher than the fluid velocity at the center of the jet. These ripples travelled along the entire length of the jet, growing slowly in amplitude with distance. As the Reynolds numbers increased, the amplitude of these ripples increased. At still higher Reynolds numbers, these ripples were replaced by the patches of turbulence referred to above.

The jet was not considered turbulent while the so-called ripples were present, although these certainly indicated instability. The occurrence of patches of turbulence was taken as the "onset of turbulence", and this occurred after Reynolds numbers of about 135.

The occurrence of any of the phenomena described depended only on Reynolds number, and not on nozzle diameter or on orientation; that is, whether the jet was pointing up or down.

A plot of "jet behavior" versus Reynolds numbers (see Figure 3) illustrates the various modes of behavior of the jet. It is seen that for Reynolds numbers less than ten, only straight jets occurred. For Reynolds numbers greater than ten, those flows for which the "wavelength" of the sinuous jet axis or the "pitch" of the helical axis were recorded, show an increase of this dimension with the Reynolds number. Above a Reynolds number of 100 the axis was again straight. This does not represent an "increase in the wavelength", as there is no curvature of the axis for Reynolds numbers greater than 100. This diagram also shows the phenomena occurring above Reynolds number 100. The transitional Reynolds number of 135 was inferred from this plot.

## Photographs

The photographs in Figure 4 show the jet in some of the sinuous configurations exhibited as transition between laminar and fully turbulent flow occurred. After a number of regular periods, the undulations degenerated into what may properly be called a turbulent flow several thousand nozzle diameters downstream.

Figure 4a shows the jet at Reynolds number 11.2. Although it is barely above the maximum Reynolds number for which only straight, stable jets can occur, its sinuosity is pronounced. Figure 4b (Reynolds number 32.2) shows the increase in wavelength with increasing Reynolds number. Figure 4c shows a jet of high Reynolds number (192) which remained straight until far downstream became turbulent.

Close-up photographs of the jet taken with an electronic flash-gun at 1/1000 of a second are shown in Figure 5. In Figure 5a, ripples at the edge of the dyed portion can be seen. In Figure 5b, the jet has already become turbulent; the appearance of the patches of turbulence can be seen. Figure 5c, taken an inch and a half farther downstream, shows the growth of such turbulent patches. Figure 5d, a jet in which turbulent mixing of the jet and the surrounding fluid is very great, was taken one and a half inches from the nozzle.

## IV. DISCUSSION OF RESULTS

It has been shown that the transition between laminar and turbulent flow does not occur suddenly, and that in undergoing transition from a straight, laminar jet to a fully turbulent jet, a whole range of flows is encompassed. These helical and sinuous modes are not turbulent in the true sense, as they are quite regular and show a trend to greater wavelengths at higher Reynolds numbers. Yet at a given Reynolds number the wavelengths still vary considerably, so that it can not be predicted from this plot exactly what the jet will look like. For example, two flows of Reynolds number 15 and 50 both exhibit wavelengths of 5; in another case, three flows of Reynolds number about 30 show wavelengths ranging from 2½ to 7. If the jet is considered to be unstable when it takes on these curved forms, one would not expect to get a one-to-one correspondence between wavelength of the undulations and Reynolds number; instead, several "characteristic modes" might be expected for each Reynolds number.

The sinuosity may be due not to an instability of the jet itself, but to effects of the secondary flow. This is the flow induced in the tank by the entrainment, by the jet, of the originally still water there. In a tank of finite volume, the entrained fluid is replaced by fluid from near the walls of the tank, so the flow is as shown in Figure 6. To investigate this possibility, the tank was used with only the 5½" diameter cylinder. The amplitudes were all higher, for the same nozzle diameters and Reynolds numbers, so that the amplitude of sinuosity apparently depended on the tank diameter. It would not be expected that the amplitude would increase without limit with the diameter, for then, in a truly infinite sea of fluid an extremely large diameter would be predicted. However, the secondary flow decreases as the volume it occupies increases -- that is, the effects of the walls dimish and a truly "infinite sea" is approached. Then in the limit there would be no "secondary flow" -- only the dyed jet and the surrounding entrained fluid, with all fluid flowing in the same direction as that at the axis.

A physical, and even mathematical explanation for the phenomena leading to the onset of turbulence may be had by recourse to the stability theory. By comparison with the plane two-dimensional case (see Schlichting (21), Chapter XVI), the ripples would correspond to disturbances imposed on the laminar flow; these disturbances grow with distance along the jet for a given Reynolds number flow. But the perturbations to the flow due to these disturbances increase much more rapidly with jet Reynolds numbers. As the jet Reynolds number is increased, the amplitude of the perturbations is no longer small enough to be considered a linear perturbation, and the result is turbulence occurring close to the nozzle exit, as was observed.

## Comparison with Other Experiments

It would be fruitful to compare the above results with the results of the recent experiments mentioned in the Introduction. The sinuous modes of behavior of the jet may be the result of the growth of disturbances as described by Viilu (28). In his case, jet stability was examined by observing the growth of such disturbances as occurred when the table on which his experiment was resting was tapped with a pencil. In the present case, only the effects of background disturbances, which might be considered "infinitesimal", were considered. Still, the coincidence of these results makes it appear that 10 is the Reynolds number at which an axisymmetric jet becomes unstable.

Reynolds (18) has also noted large-amplitude sinuousity similar to that described herein, and has commented on the difficulty of obtaining a straight flow at Reynolds numbers immediately above ten.

It was not possible to obtain a correlation of the present experiments with Sajben's work. As mentioned earlier, his observation of the dimple at the free surface of a mercury jet led him to conclude that instability first occurred at a Reynolds number of about 600. An attempt was made to use this surface-detection method on the water jet. The dimple was seen to move only when a "turbulent patch" hit the surface but a very large turbulent patch caused the simple to disappear altogether. It would be expected that the high surface tension of mercury might keep a dimple at the free surface of that liquid stable at a higher Reynolds number than in the case of water; perhaps, then, surface tension accounts for this difference. In any case, it can be concluded that the surface detection method may be useful as an indication of turbulence but that it does not provide a suitable means of detecting the onset of instability of a jet.

## Suggestions for Further Research

1. The secondary flow could be examined to determine its effect on the behavior and stability of the jet. Such an investigation might include the use of streak photographs of small particles suspended in the flow. The velocity distribution throughout the tank could then be determined. The results of using a large tank to approximate an "infinite sea" would also be interesting. The liquid in the tank should stand for several days to allow it to come to rest, and thermal currents could be reduced by insulating the tank and providing a cover for it.

2. A determination could be made of the dependence of stability on the frequency and amplitude of the exciting disturbance. This would be a useful guide to theoretical analyses of the stability

of an axisymmetric jet. Whether or not the disturbance need be rotationally symmetric (as indicated by Lew (12)) could also be examined. A method similar to that used by Chanaud and Powell (5) for the two-dimensional jet might be used here.

3. Another investigation could involve the use of an imposed secondary flow surrounding the jet and parallel to it, so that velocity profiles as shown in Figure 7 would be obtained.

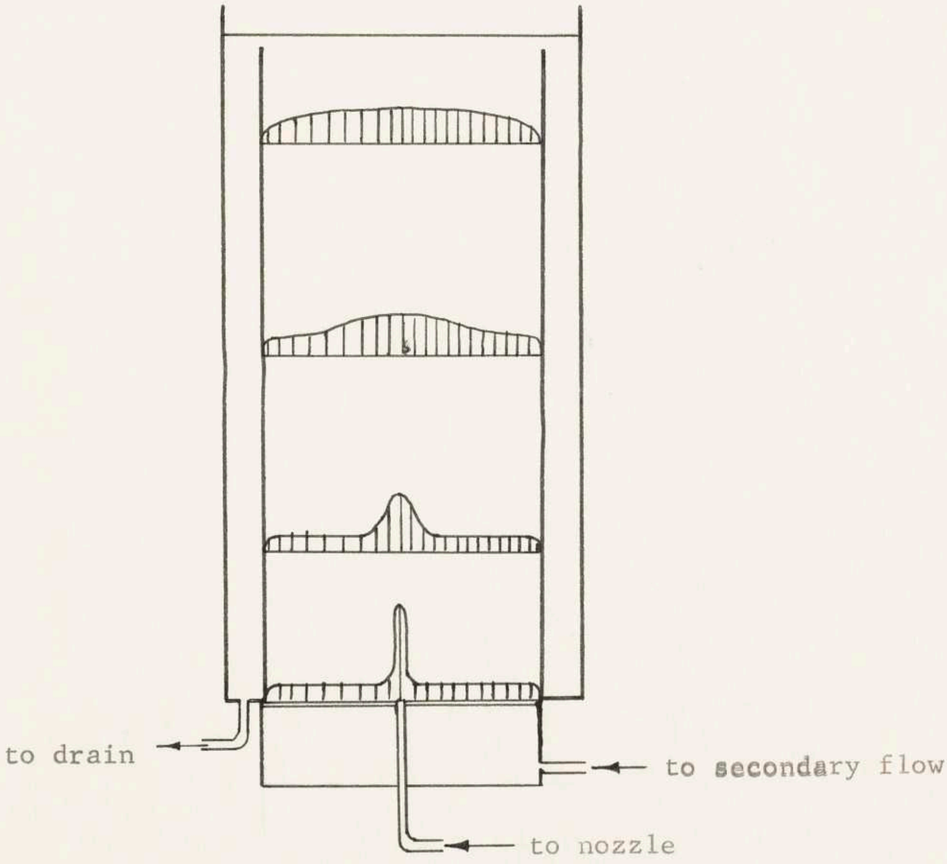


Figure 7: Imposed Secondary Flow Pattern

## V. CONCLUSIONS

The critical Reynolds number of instability of an axisymmetric jet is 10. Above this Reynolds number large-amplitude sinuosity may occur, the wavelength tending to increase with the Reynolds number. Above a Reynolds number of 100, the jet is again straight; at higher Reynolds numbers, ripples occur at the interface between the dyed and the clear fluid. These ripples become "patches of turbulence" at about a Reynolds number of 135. Thus the jet is turbulent close to the nozzle at a Reynolds number of 135 or greater. The occurrence of these phenomena depends only on Reynolds number; not on nozzle diameter or jet orientation.

## APPENDIX: CALCULATIONS AND DISCUSSION OF ERROR

To calculate the Reynolds number based on nozzle diameter, mean velocity at the nozzle, and kinematic viscosity at the temperature of the fluid, the volume flow rate was determined for each run and related to the nozzle exit diameter and viscosity, as shown below:

$$Re \quad \frac{V_{mean}\, d_{jet}}{\nu}$$

$$V_{mean} = \frac{Q}{A} = \frac{Q}{(\pi/4)d_j^{\,2}}$$

$$Re = \frac{Q}{(\pi/4)d_j^{\,2}} \quad \frac{d_j}{\nu}$$

$$Re = \frac{K(T)Q}{d_j}$$

The quantities measured were volume flow rate, nozzle exit diameter, and water and ambient temperature. Viscosity was obtained from the Handbook of Chemistry and Physics (29).

The errors involved in each of these measurements must be considered. The flow rate was obtained using direct measurements only; that is, by reading the change in level of the graduated reservoir during a known time interval. The error involved in such a reading can be considered as that due to uncertainty in elapsed time between accurately measured volumes. These volumes are read from the scribed lines on a 250 ml graduate. In the case of a very slowly falling level, it may be possible to determine only within three or four minutes when it is that the water level is just at the center of a scribed line. Yet, even if the error is as large as the time it takes for the level to cross one of these lines, this maximum error in one level reading would be less than the ratio of the thickness of the line to the actual drop in level during the run. Thus, for a drop in level of 4 ml, which corresponds to the smallest interval, the error would be $\frac{0.008''}{0.160''}$ or 5%. For a larger drop in level the error would be correspondingly less. A similar analysis would apply to a rapidly dropping level.

The error involved in determining nozzle diameter is plus or minus 0.0002" regardless of the diameter of the nozzle. For the smallest nozzle (0.008"), considering that two readings are involved, the error would thus be

$$\pm \frac{0.0002}{0.0080} \times \sqrt{2}$$

$$= \frac{1}{40} \times \sqrt{2}$$

$$= .035$$

$$= 3.5\%$$

Finally, the temperature of the jet was read to the nearest degree Centrigrade. As the graph of the constant K versus temperature is linear, the percentage error involved in K is the same as that involved in T; i.e., plus or minus $\frac{1}{2}^{\circ}$. At room temperature, the error is $\frac{1}{2}^{\circ}/25^{\circ}$ or 2%.

Thus, the total root mean square error due to errors in volume flow rate, nozzle diameter, and temperature is less than 6½%.

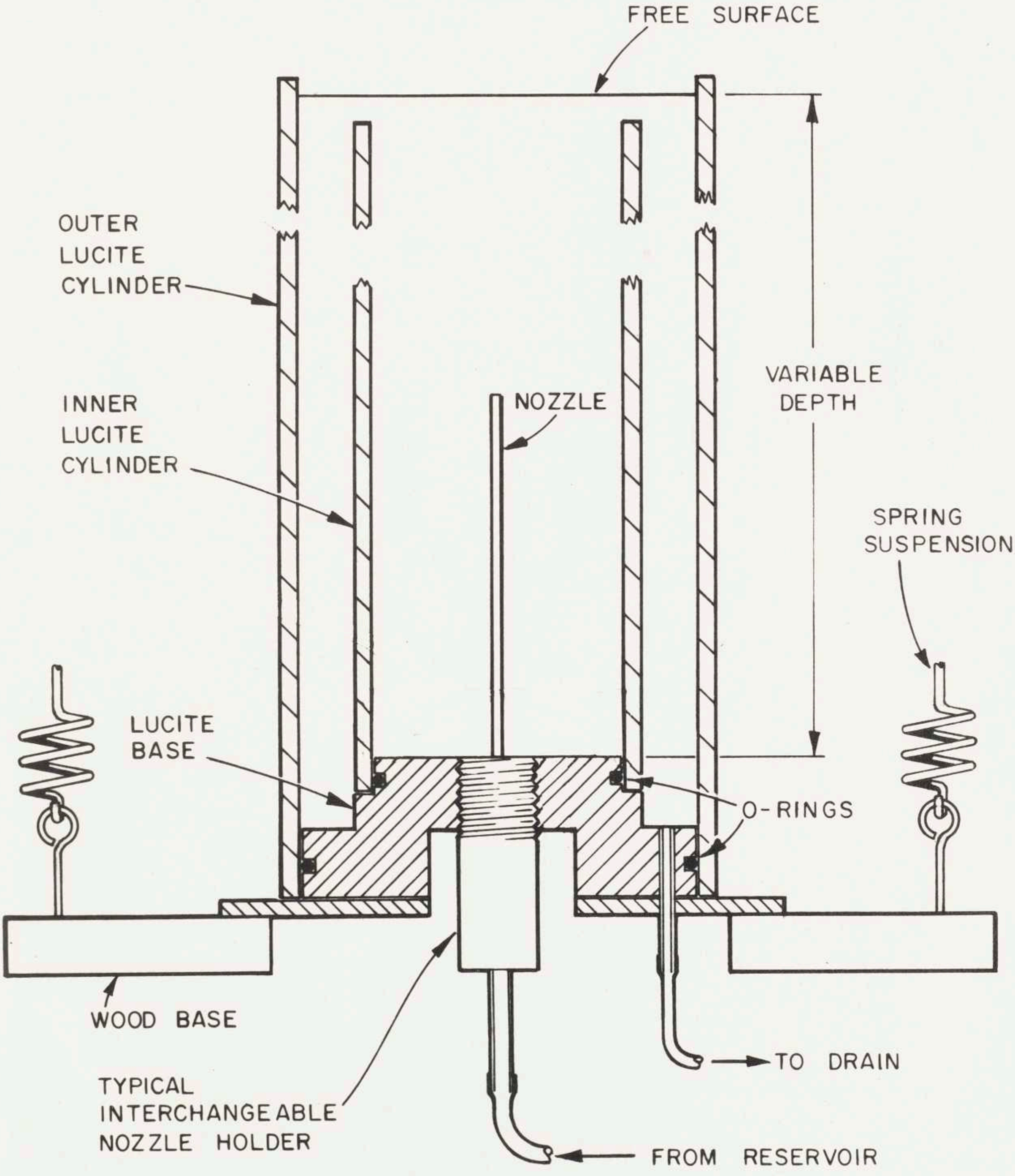
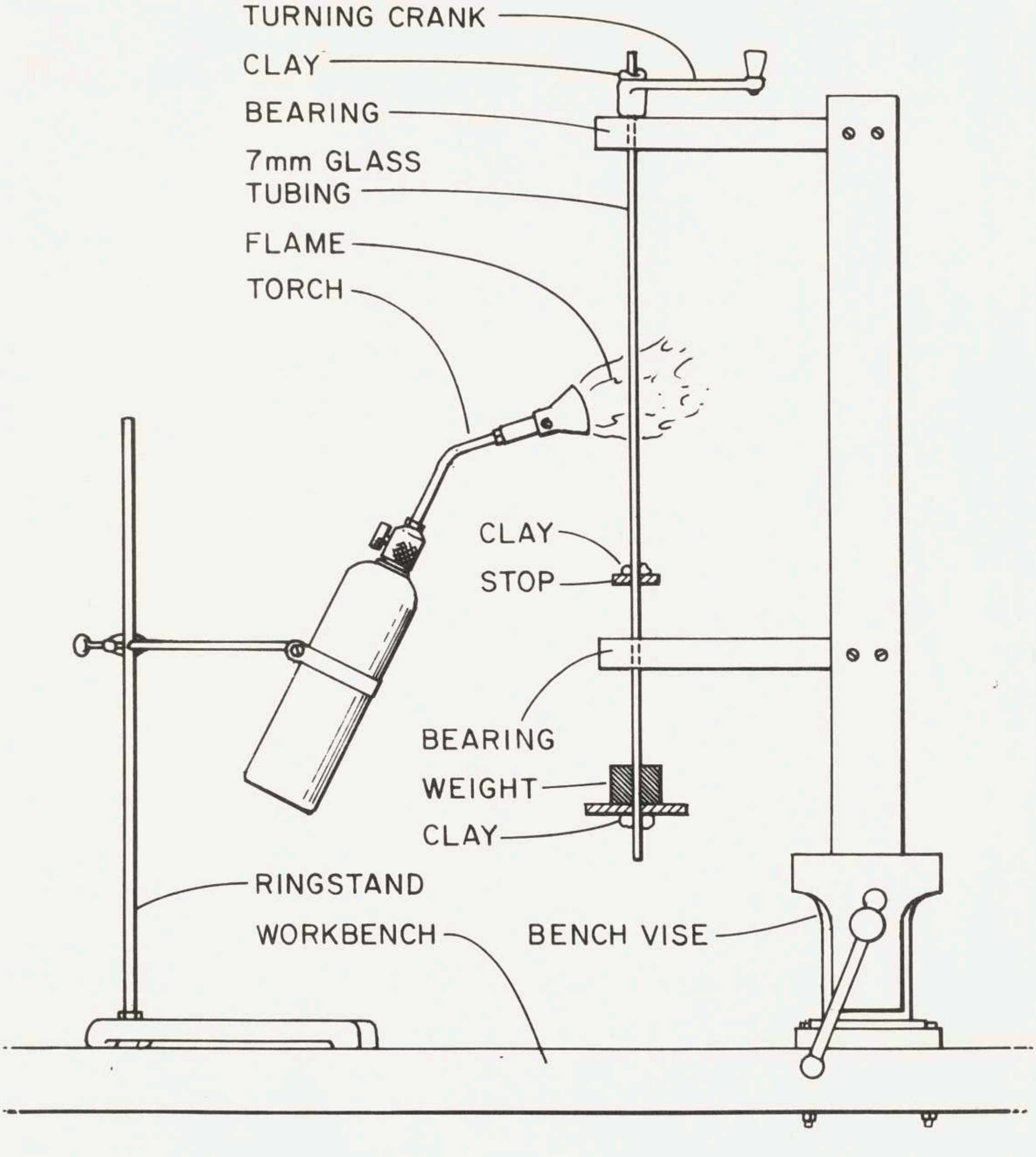


Figure 1: Apparatus



DEVICE FOR MAKING GLASS NOZZLES

Figure 2

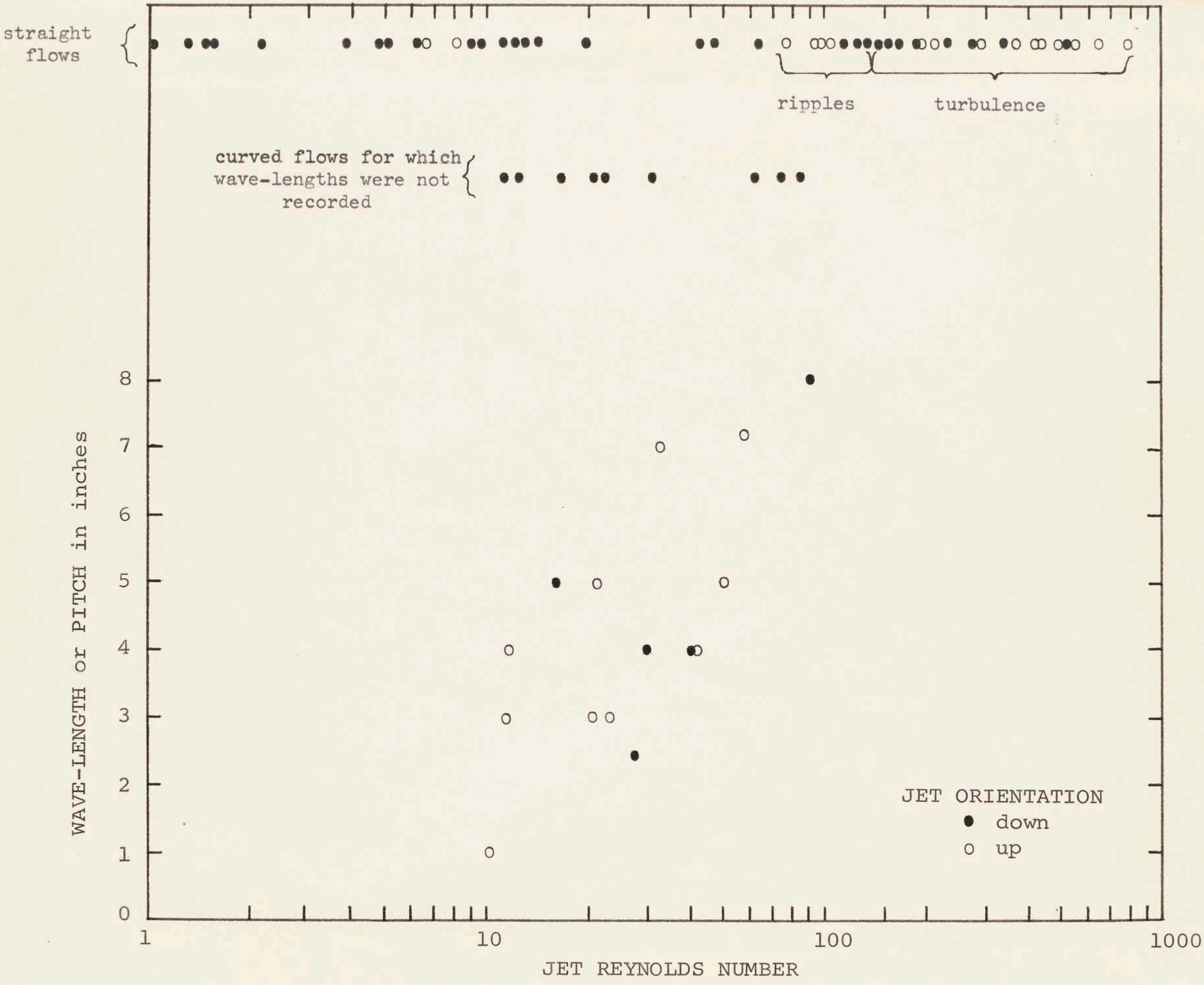
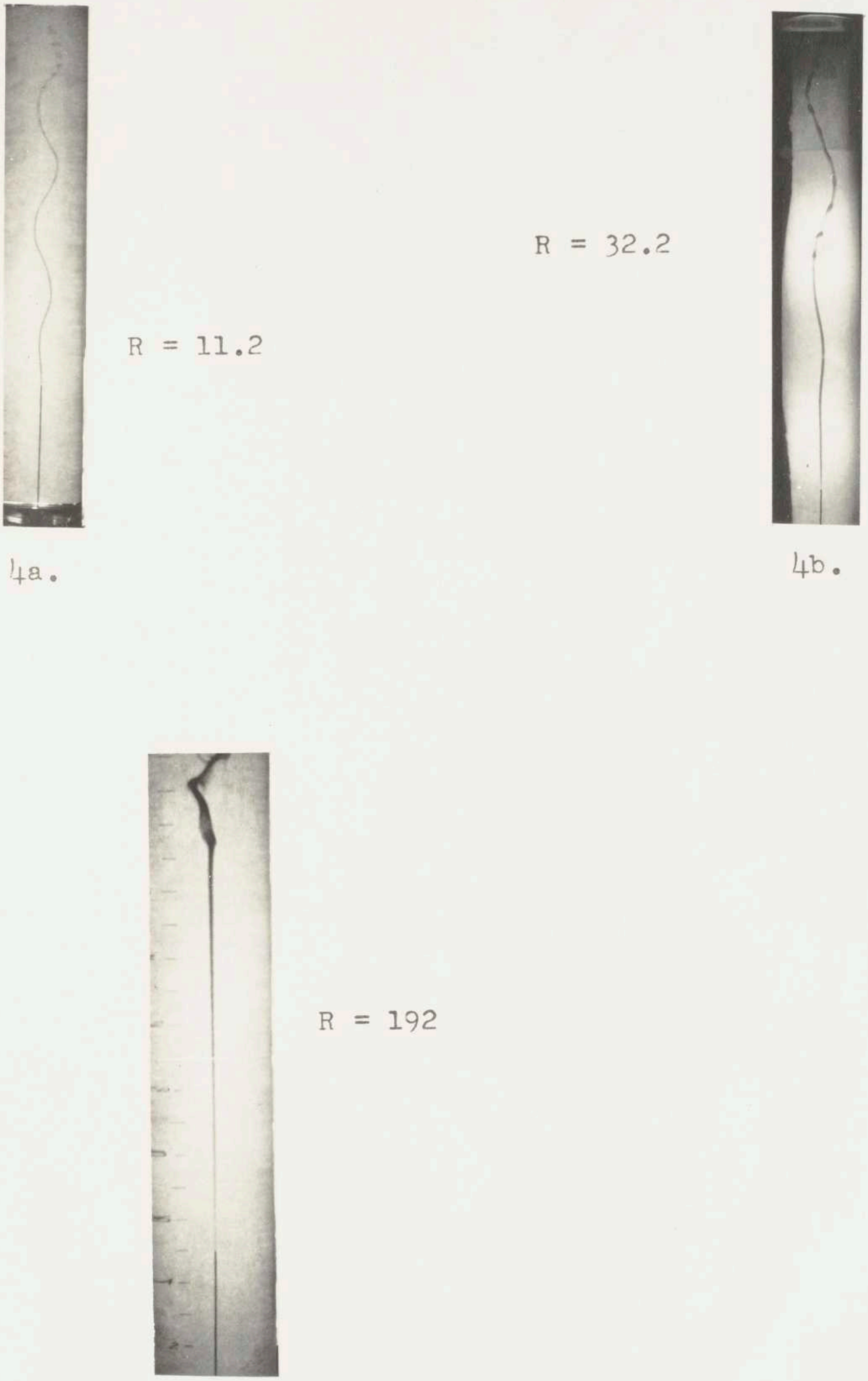

straight flows
ripples
turbulence
curved flows for which wave-lengths were not recorded
WAVE-LENGTH or PITCH in inches
0
1
2
3
4
5
6
7
8
1
10
100
1000
JET REYNOLDS NUMBER
JET ORIENTATION
● down
○ up

fig. 3



R = 11.2

4a.

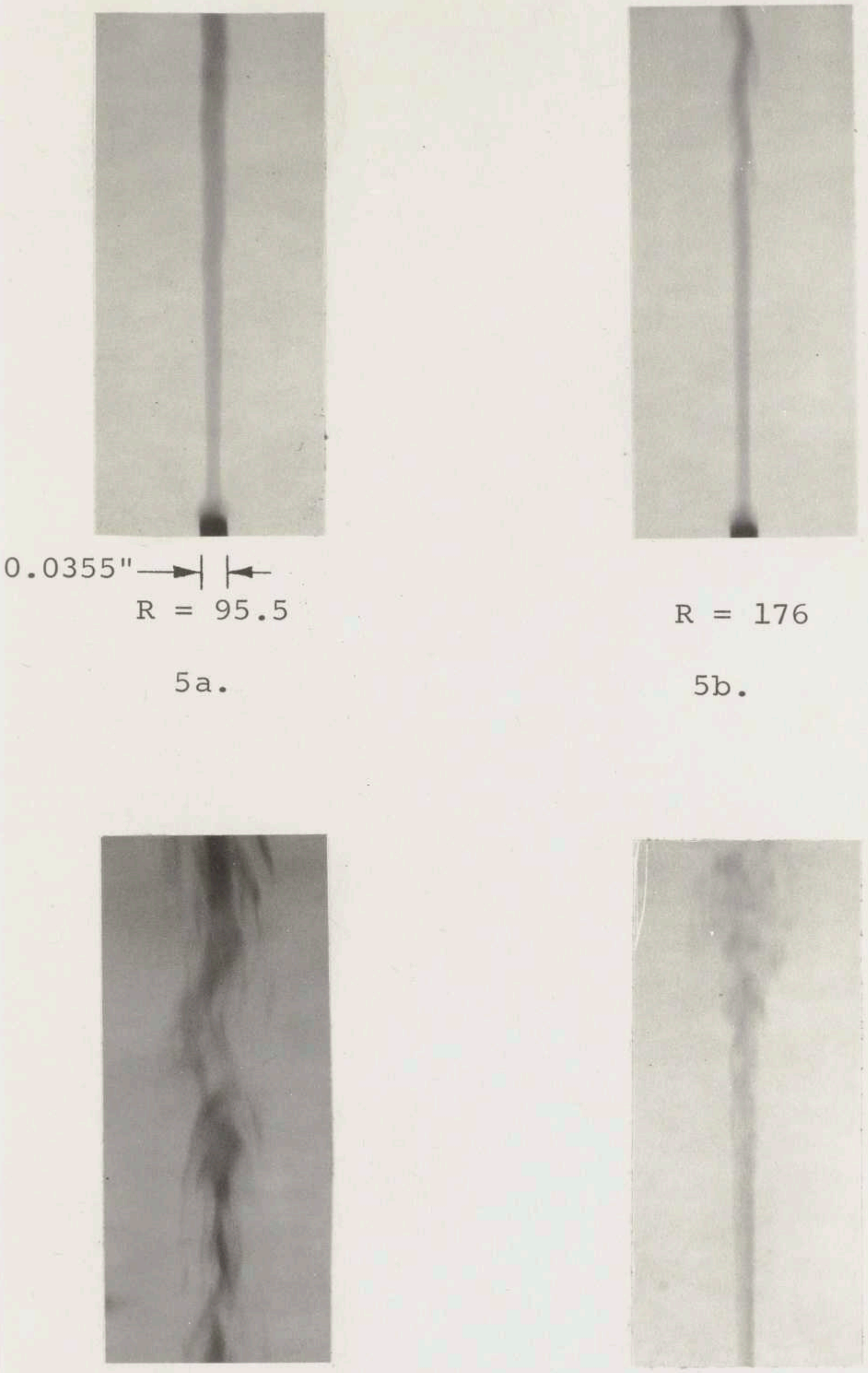
R = 32.2

4b.

R = 192

4c.

Figure 4: Photographs of Jet



R = 95.5

5a.

R = 176

5b.

R = 211

5c.

R = 411

5d.

Figure 5: Close-up Photographs of Jet

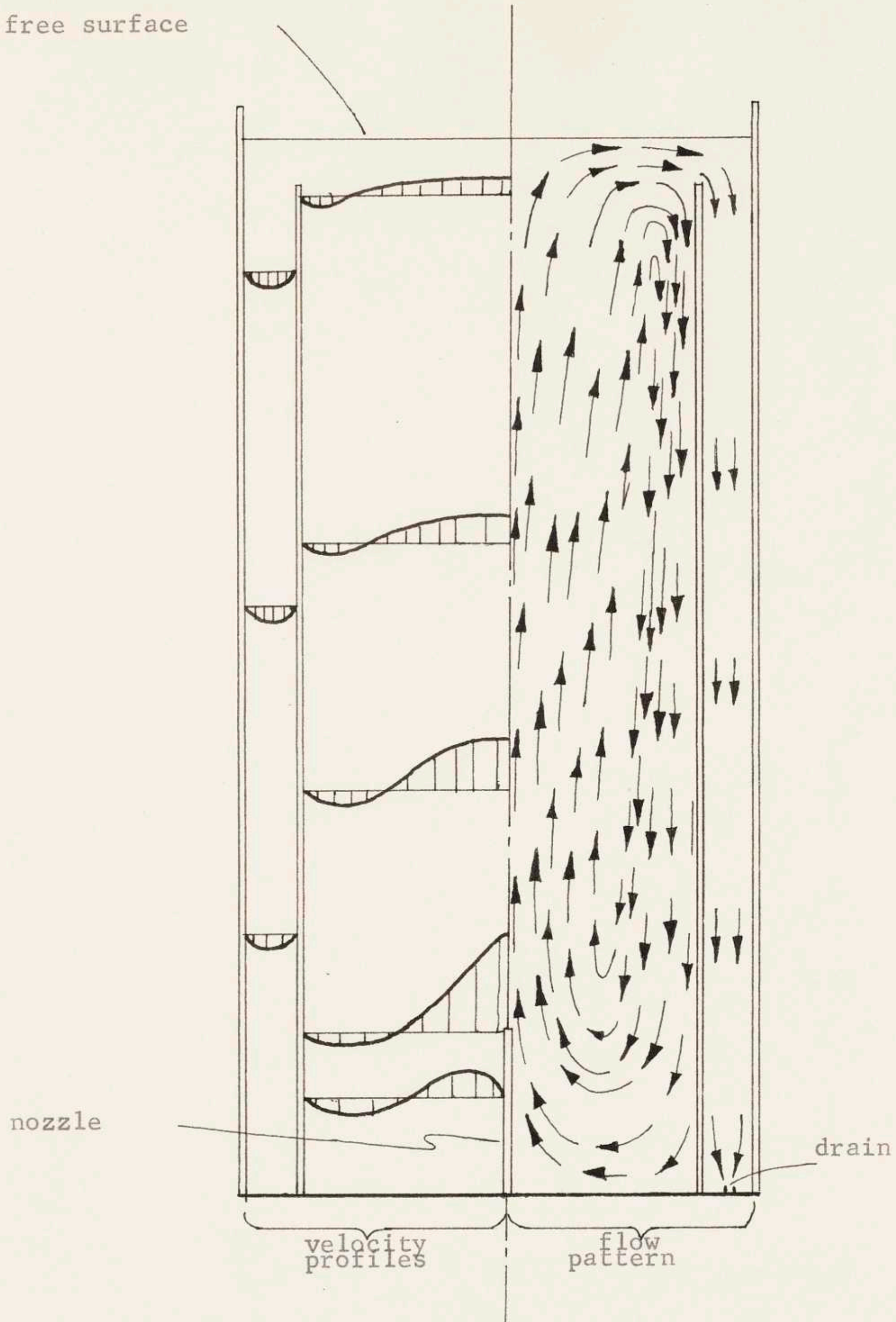


Figure 6: Induced Secondary Flow Pattern

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