

I. IONIZATION OF CESIUM AT SURFACES
II. THE ENERGY DISTRIBUTION FOR
ELECTRONS IN A THERMIONIC DIODE
PLASMA CANNOT BE TRULY MAXWELLIAN

W. B. NOTTINGHAM

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
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Cannot Be Truly Maxwellian

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I. Ionization of Cesium at Surfaces

Abstract

Indications from voltage-current curves of thermionic power converters suggest increased ionization at, or very near, cesium-coated surfaces by low-energy electrons. Experiments have been conducted to study this phenomenon quantitatively for a low cesium concentration. The tube contained three indirectly heated tantalum surfaces surrounded by a shield. These heated surfaces, E1, E2, and E3, were tantalum tubes located at the three corners of an equilateral triangle. The extension of a line from the corner E1 perpendicular to the base of the triangle formed by the line E2 and E3 served to locate the position of a fine-wire ion collector. Electrons were accelerated from surfaces E2 and E3 to bombard E1. Ions produced at or near E1 were then accelerated toward the ion collector to serve as a measure of the ion production. Results of this experiment indicate that no appreciable ion current was produced in cesium vapor controlled by a $T_{Cs} = 350^\circ K$, by electrons having an energy less than 3.89 volts. The square root of the ion-to-electron current ratio was a linear function of the surface electron energy in excess of the ionization potential. Although not accurately determined, the ionization coefficient is given by $P_i = P_1(V - V_1)$ with $P_1 = 20 \text{ cm}^{-1} \text{ v}^{-1}$ for the standard condition of $3.54 \times 10^{16} \text{ atoms/cm}^3$.

II. The Energy Distribution for Electrons in a Thermionic Diode Plasma Cannot Be Truly Maxwellian

Abstract

For the refractory emitters, such as tungsten, tantalum, and molybdenum, operated in an energy-conversion diode, the ratio of the emitter temperature to the cesium temperature strongly influences the operating properties. If this ratio exceeds 3.2, an ion-rich sheath usually forms at the emitter surface. Electrons are accelerated in the plasma space by this injection potential. An analysis made here shows that at the open-circuit condition, the energy distribution of the electrons at the plasma edge of the emitter sheath cannot be a true Maxwellian over the entire range in electron energy. Instead, it is thought to be made up of two quasi-Maxwellians. The low-energy electrons are trapped and may have a high average energy close to that of the injection potential. The untrapped electrons will have an electron temperature equal to that of the emitter and their density will be that associated with an apparent or fictitious density many times that of the actual density.

A theory has been developed to describe quantitatively the relations that must be satisfied for the description above to apply. From typical numerical results that are applicable to specific diode configurations it is concluded that if the emitter-to-cesium temperature ratio exceeds 3.6, volume ionization is not required for a stable sheath. In the range 3.2-3.6 surface ionization is usually insufficient, and it is necessary to have volume ionization to sustain the sheath. Further studies will be required to establish the mechanism. It may be direct ionization by high-energy electrons, or ionization from excited states of the cesium atoms.

I. IONIZATION OF CESIUM AT SURFACES

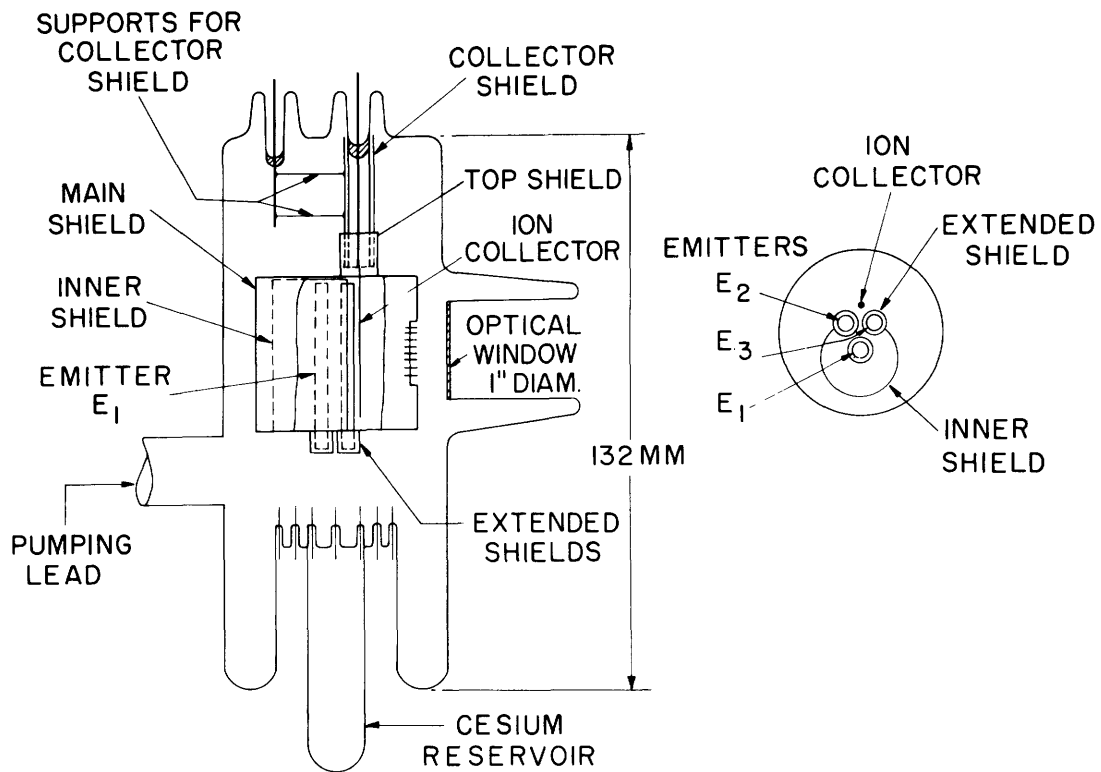
A. INTRODUCTION

A detailed analysis¹ has been made of the voltage-current curves shown by Houston² which seems to indicate that ions are produced near the electron collecting surface by low-energy electrons. This study indicates that the converter current is limited by electron space charge if it is operated at a relatively high cesium concentration defined approximately by a temperature relation requiring that the emitter temperature T_1 be less than $2.7 T_{Cs}$. The small number of ions that is produced by thermal ionization will tend to trap in the space between the emitter and the collector. Unless the surface potential of the collector is made very negative, an electron space-charge sheath forms at the collector, and thus prohibits the loss of ions to that surface. Electrons are accelerated into the collector surface by the potential drop across the collector sheath. When the potentials are adjusted so that the electrons that drift into this sheath are accelerated across it with an energy of approximately 2 volts, a new supply of ions seems to become available and the fully ignited mode of diode operation develops. Even though the difference in potential across this collector sheath is much smaller than the ionization potential of cesium, the existence of this sheath sets up a very strong field to accelerate ions away once they are produced. It was therefore postulated that some mechanism exists for the production of positive ions with low-energy electrons.

The experiment to be described here was designed to study ionization in the neighborhood of electron collecting surfaces, even though the field conditions that apply in a thermionic diode could not be duplicated. The results to be shown in some detail give no indication of ionization effects produced by low-energy electrons near the surfaces of cesium-coated electron collectors. Some effects are found that indicate the need for further study of the ionization process.

B. DESCRIPTION OF EXPERIMENTAL TUBE

Even though the choice of Pyrex (Corning 7740) and conventional tungsten lead wires limited the maximum cesium reservoir temperature that could be used safely, the tube was designed for construction with these materials. Figure 1 shows the general features. The main power leads and supports for the various parts in the tube came through a system of leads at one end; whereas on the opposite end, an insulated and guarded ion-collector lead passed through the glass. This structure was used in order to minimize surface leakage. It did not eliminate it entirely. Three of the principal elements in the tube were tantalum cylinders, placed at three corners of an equilateral triangle. Each of these cylinders contained an internal heater so that its temperature could be controlled, although no provision was made for its accurate determination. With these cylinders identified as E1, E2, and E3,



DIMENSIONS: MAIN SHIELD - 39.5 DIAM. x 39 MM HIGH
 INNER SHIELD - 19MM DIAM.
 EMITTERS - 1/8 IN. DIAM.
 ION COLLECTOR 0.010 IN. DIAM.
 CENTER OF E₁ TO ION COLLECTOR - 9MM
 E₁, E₂, E₃ EQUALLY SPACED - 6MM
 CENTER TO CENTER

Fig. 1. Essential features of tube construction.

the base of the triangle may be described as the line joining E2 and E3. A perpendicular line drawn from tantalum tube E1 through this base and extended a short distance outside of the triangle served to locate the axis of the ion collector. Fully surrounding these four elements just mentioned is the pillboxlike structure with an internal additional guard as shown to serve as a means of eliminating all currents that might flow either to or from the ion-collector wire by paths outside of this structure.

The design shown was dictated by the concept that the use of a small wire ion collector would minimize photoelectric and thermionic effects that might otherwise be so prominent as to mask the ionization effects. There were at least three principal methods of operation. The first depended on the use of surface E1 as the electron collector, with E2 and E3 serving as electron emitters. Ions produced near the surface of E1 would be in a weak, accelerating field that would return the ions toward E2 and E3, and a certain fraction of them would pass on through to be collected at the ion collector. The yield of ions thus produced will be shown in detail in a later section of this report.

A second method of operation uses surface E1 as the electron emitter and surfaces E2 and/or E3 as electron collectors. Ions produced at those surfaces will be accelerated partly toward the electron emitter E1, and partly toward the ion collector.

A third method of operation permits the use of any of the heated surfaces E1, E2, and E3 as the electron emitter, and the internal surface of the pillbox guard as the electron collector. Again, ions produced near this surface can be collected on the small wire collector.

C. CIRCUIT ARRANGEMENT

The circuit used in these studies is shown in Fig. 2. The cesium temperature was controlled by the monitoring of an appendix containing cesium liquid. The temperature was maintained and measured by means of a thermocouple. The tube as a whole was operated in an oven at a temperature approximately 50°C higher than the liquid cesium temperature T_{Cs} . In the immediate neighborhood of the collector lead, an external heater was installed to maintain the glass in this area at a still higher temperature.

The fixed biases used in the operation of the circuit were generally supplied by batteries, and the value of the potential was accurately known. The variable potential was generally supplied from a calibrated potentiometer so that its value was easily adjustable and accurately established. The vacuum-tube electrometer was supplied with an external biasing potential in order to buck out the relatively large leakage current and the thermionic current that was generally a constant background after all temperatures had become stabilized. The ion current which might not exceed 10 per cent of the background current could nevertheless be measured reproducibly

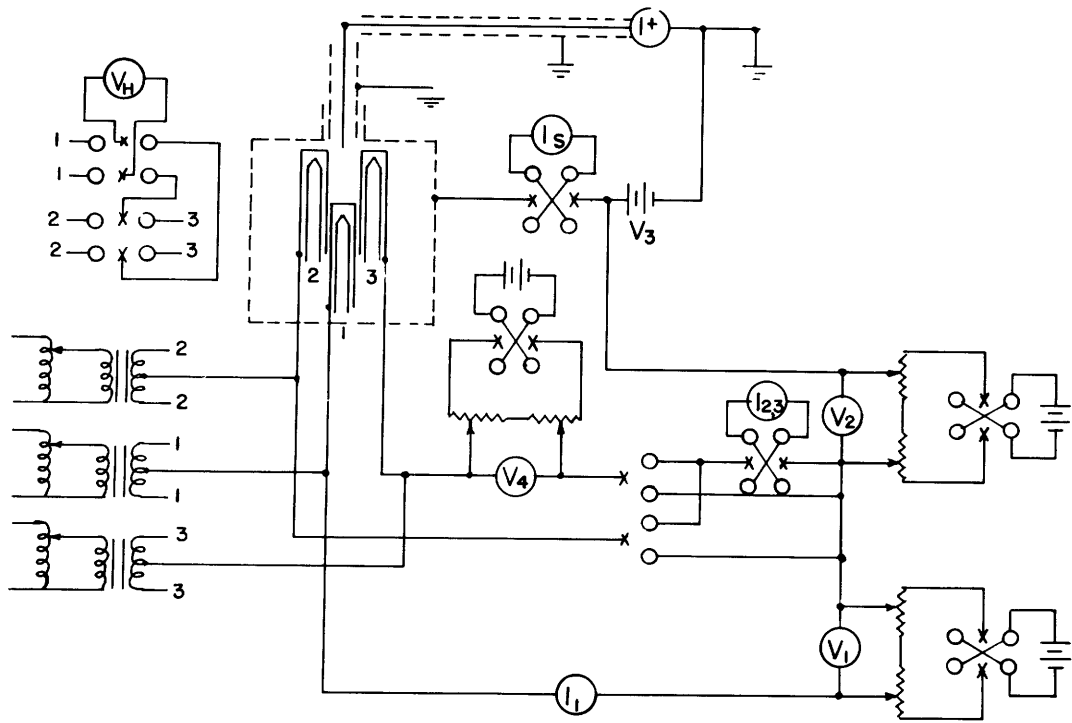


Fig. 2. Circuit diagram.

with all of the accuracy demanded in this study. An ion current of less than 1 per cent of the background could be clearly identified.

D. THEORETICAL ANALYSIS

Inspection of the experimental results indicated strongly that the ionization that was observed to take place near the electron collecting surface could quite possibly be accounted for by the ionization of those cesium atoms that happen to be bombarded by electrons of sufficient energy for ionization just before the electrons were absorbed on the collector surface. The analysis which will be worked out in this section indicates that, if an ideal experiment could be performed, a plot of $(i_+/i_-)^{1/2}$ as a function of applied voltage should be linear with an intercept on the axis occurring for the applied voltage that delivers electrons to the surface with 3.89 ev energy. All observations of the ion-current-to-electron-current ratio plotted in this manner do yield straight lines with an intercept voltage as predicted.

The analysis begins with the assumption that the "ionization probability" is a linear function of the electron energy in excess of the ionization potential. This statement is represented by

$$P = (V - V_i) P_1. \quad (1)$$

In this expression V is the electron energy in volts, V_i is the ionization potential (3.89 ev), and P_1 is the slope of the probability curve; and therefore P_1 is the value of the ionization probability when the energy of the electrons exceeds the ionization potential by exactly 1 volt. The definitions used in the finding of P are such that P^{-1} is the average distance of electron travel (mean-free path) for the production of a single ion in a gas of number density 3.54×10^{16} atoms/cm³. This is the number density of an ideal gas at a pressure of 1 Torr and a temperature of 273°K.

The ion production in a short distance dx over which the electron energy is V may be expressed by the relation

$$di_+ = i_- P_1 P_{Cs} \frac{273}{(T_{Cs} T)^{1/2}} (V - V_i) dx. \quad (2)$$

The relation between V and the applied voltage V_1 can best be expressed in terms of the symbols illustrated in the motive diagram of Fig. 3 and the following equation

$$V = V_1 + \phi_{23} + V_s - \phi_1 - \int_0^x \left(\frac{dV}{dx} \right) dx. \quad (3)$$

In the absence of space charge, the depth of the potential minimum V_s is zero, and the gradient of the potential will be given by a solution to Laplace's equation in which the potential is given by the relative positions of the surface potential of the collector with respect to that of the emitter. The total difference of potential in any case that

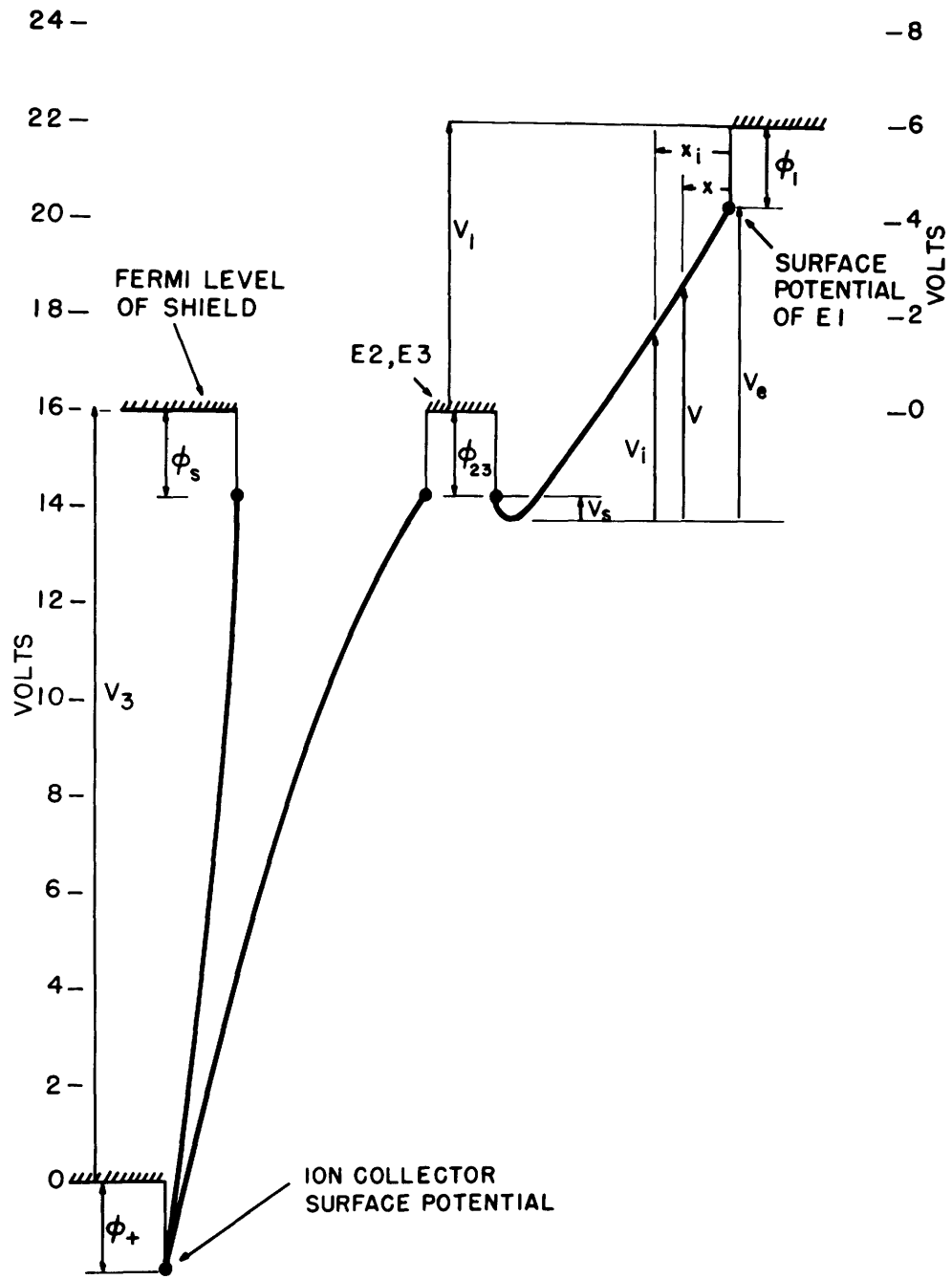


Fig. 3. Motive diagram for E1 receiving electrons from E2 and E3. Ionization energy V measured from space-charge minimum V_s negative to emitters E2 and E3. Maximum electron energy V_e . Applies directly to data of Fig. 4.

represents the maximum energy with which electrons bombard the surface is V_e , and is given by

$$V_e = V_1 + \phi_{23} + V_s - \phi_1. \quad (4)$$

Under the experimental conditions analyzed in this report, the gradient (dV/dx) is taken as a constant, and is represented by the symbol γ .

$$\gamma = \left(\frac{dV}{dx} \right) \quad [\text{assumed independent of } x]. \quad (5)$$

At a certain small distance x away from the collector surface, the electron energy will be precisely that of the ionization potential. The combination of the equations above serves to define this value of x as

$$x_i = \frac{V_e - V_i}{\gamma}. \quad (6)$$

The total ion production may now be written

$$i_+ = i_- P_1 P_{Cs} \frac{273}{(T_{Cs} - T)^{1/2}} \frac{(V_e - V_i)^2}{2\gamma}. \quad (7)$$

Before this equation can be compared with experiment, two more factors must be introduced. The first is the relation between the electron current that is observed and the electron current that is actually effective in producing ionization; the latter is represented by i_- . In the practical tube used for these experiments, the electron current that passed from an emitter to a collector within the pillbox could not be measured with certainty, and therefore the true ionizing current was always less than the observed electron current by a factor α . The same considerations must be taken into account in connection with the observed ion current, since not all of the ions could be collected on the small wire, and therefore, again, the observed current is less than the true total rate of production of ions by a factor β . The introduction of these two quantities and additional substitutions permit us to write

$$\left(\frac{I_+}{I_-} \right)^{1/2} = \left[\frac{\alpha\beta P_1 P_{Cs} 273}{2\gamma (T_{Cs} - T)^{1/2}} \right]^{1/2} (V_1 + \phi_{23} + V_s - \phi_1 - V_i). \quad (8)$$

This equation suggested that the best way to analyze the data was to compute the square root of the ion-current electron-current ratio as a function of the applied voltage V_1 . If these observed results turn out to yield a straight line as is suggested by this equation, then the slope of the line is given by the square root of the quantities within the bracketed factor of Eq. 8.

E. EXPERIMENTAL RESULTS RELATIVE TO ION PRODUCTION

Five sets of data are shown graphically in Fig. 4. These correspond to the electron bombardment of surface E1 by electrons emitted from surfaces E2 and E3 as the temperature of E1 was raised. The heater voltage for each is shown. The slopes of the corresponding linear plots remain nearly constant except for that for the highest temperature. Inspection of Eq. 8 suggests that if γ increased with the temperature of the surface, the slope of the curve could be expected to decrease. In that equation, the temperature T represents the temperature of the gas and not the temperature of the surface. The mean-free path at the pressures used was quite long, and therefore any small change in the temperature of the surface would have made only a secondary effect with regard to the cesium-atom density in that neighborhood. As the temperature of the surface was increased, the thermionically emitted electrons were held in a strong retarding field. This increased density resulted in an intensification of the voltage gradient in the immediate neighborhood of the surface, thereby giving an increase in the value of γ . The change in slope shown in Fig. 4 is attributed to that cause.

Note that there is a systematic change in the location of the intercept of these lines on the voltage axis. The two sets of low-temperature data are represented by a single line that intersects at 3.74 volts. This is 0.15 volt lower than might have been expected unless one takes into account the fact that a small space-charge minimum in front of the two electron emitters E2 and E3 could have an effect that is the equivalent of an electron source of slightly greater work-function than that of the electron receiver E1. As the receiver temperature is raised by increasing the heater voltage to 2.5, a still lower receiver work-function displaces the curve to still lower voltages. At a heater voltage of 3, this tendency is reversed, and the reduction in cesium coverage can account for an increase in the work-function and a further displacement. At this heater value the intercept is precisely 3.89. At the highest temperature used, the best straight line for representing the data intercepts at this same value. It is not unlikely that this line should have been drawn with a slightly greater slope corresponding to an additional increase in the work-function of E1. The main points illustrated by the data of Fig. 4 are:

1. The linear relations indicate that the ionization coefficient P seems to rise linearly with voltage in excess of the ionization potential for a range of approximately 1.5 volts.
2. The onset of ionization takes place so near the ionization potential that there is no evidence for unusual ionization effects near surfaces.
3. The fact that the slopes of the curves do not increase as the temperature is increased indicates that the coefficient P_1 does not change appreciably with the temperature, since the small change in slope can reasonably well be accounted for by an increase in the average gradient near the ionization surface because of space-charge effects.

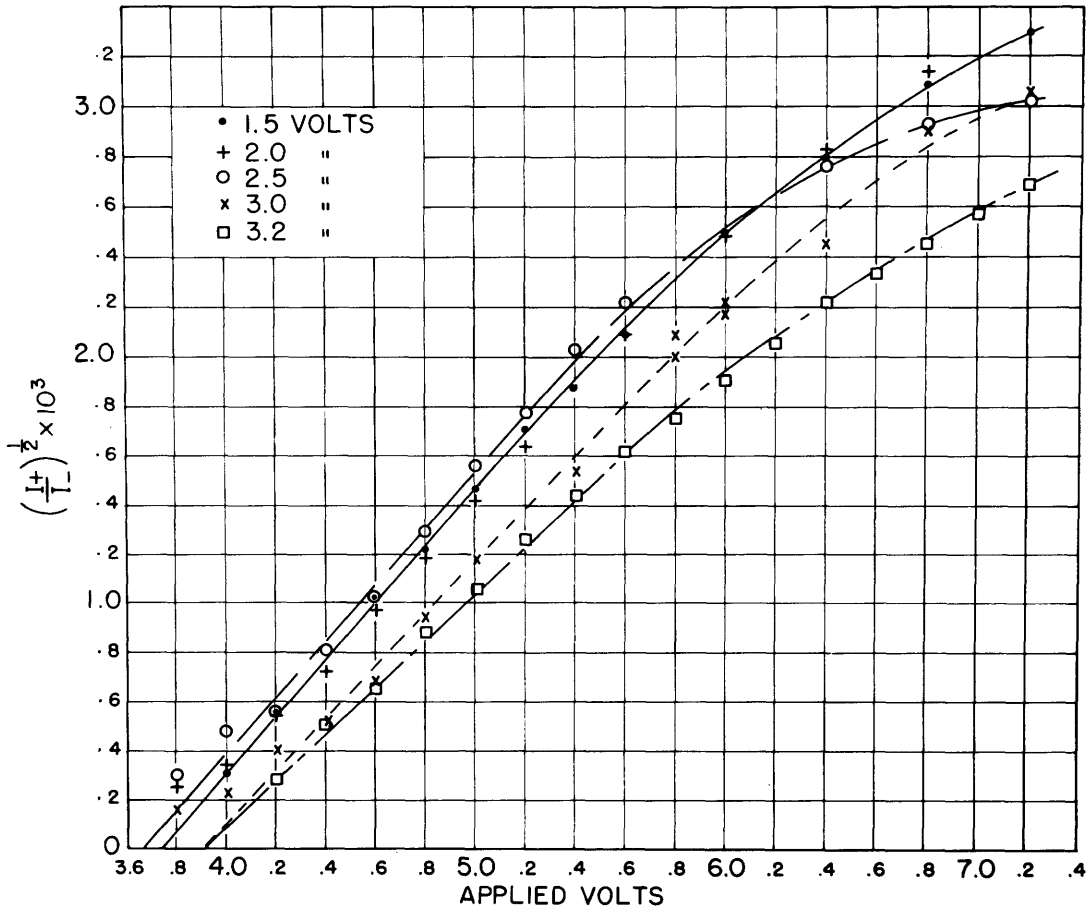


Fig. 4. Square root of ratio of ion current I_+ received at the wire collector to electron current I_- received by E1 from E2 and E3 as a function of applied potential V_1 . Heaters in E2 and E3 constant at 1.5 volts. Heater in E1 varied.

4. It must be pointed out that the gradient γ is not precisely constant but is expected to increase with applied voltage. Compensating effects take place because of the geometry involved, and therefore in the analysis no correction was made for this dependence of γ on applied voltage. For the experimental condition related to the results of Fig. 4, an average value of 16 volts per centimeter for γ seems appropriate. The cesium temperature was 346°K , corresponding to a pressure of 8.5×10^{-5} Torr. The product of $(\alpha\beta)$, 0.035, is suggested by the geometrical configuration. It is here that the largest uncertainty in the calculations to follow is to be found. The average slope of the lines in Fig. 4 is 1.4×10^{-3} . The value of P_1 computed from the coefficient in Eq. 8 is, then, $20 \text{ cm}^{-1} \text{ volt}^{-1}$. The estimated value used in previous analyses³ was $35 \text{ cm}^{-1} \text{ volt}^{-1}$, and the fact that the present value of 20 is so near in order of magnitude indicates that the experimental results are consistent with the concept that ionization in the space near the electron collector that was used shows nothing unusual, such as increased ionization at low electron energy, or a change in P_1 with surface temperature.

Additional data are shown in Fig. 5. The electron emitter was E1. It bombarded the two surfaces E2 and E3. The applied voltage shown in the graph is the difference in potential between the emitter E1 and the more positive electron collectors E2 and E3. Throughout the experiment the shield was maintained 2 volts positive with respect to the emitter E1. The ion collector was maintained 16 volts negative with respect to the shield. With the heated surfaces E1, E2, and E3 all operated with a heater voltage of 1.5 volts, the line shown intersects the axis at the ionization potential of 3.89. An increase in emitter temperature can very well develop additional space charge in its immediate neighborhood. This in effect increases the energy of the electrons that arrive at the ionization surface for a given applied potential. It is therefore expected that the observed displacement of the curves should take place. In addition, an increase in work-function of the emitter, as well as the influence of space charge, results in still more shift, as is shown by the curve associated with the heater voltage of 3 volts. The linear relation does not follow accurately to the very lowest values but shows some ionization in excess of that characteristic of the expected relation if all electrons were emitted with negligible kinetic energy. The details concerning this apparent excess ionization near the onset have not been evaluated in every respect. The over-all picture is essentially the same as that described in its relation to Fig. 4.

Geometrical considerations lead to the assumption that a larger value of $(\alpha\beta)$, 0.07, would be a reasonable approximation. Also, the average potential gradient should be somewhat different and is taken as 8.5 volts/cm for the value of γ . The average slope of the curves in Fig. 5 is approximately 1.98×10^{-3} , and the computed value of P_1 becomes $22 \text{ cm}^{-1} \text{ volt}^{-1}$ for these data.

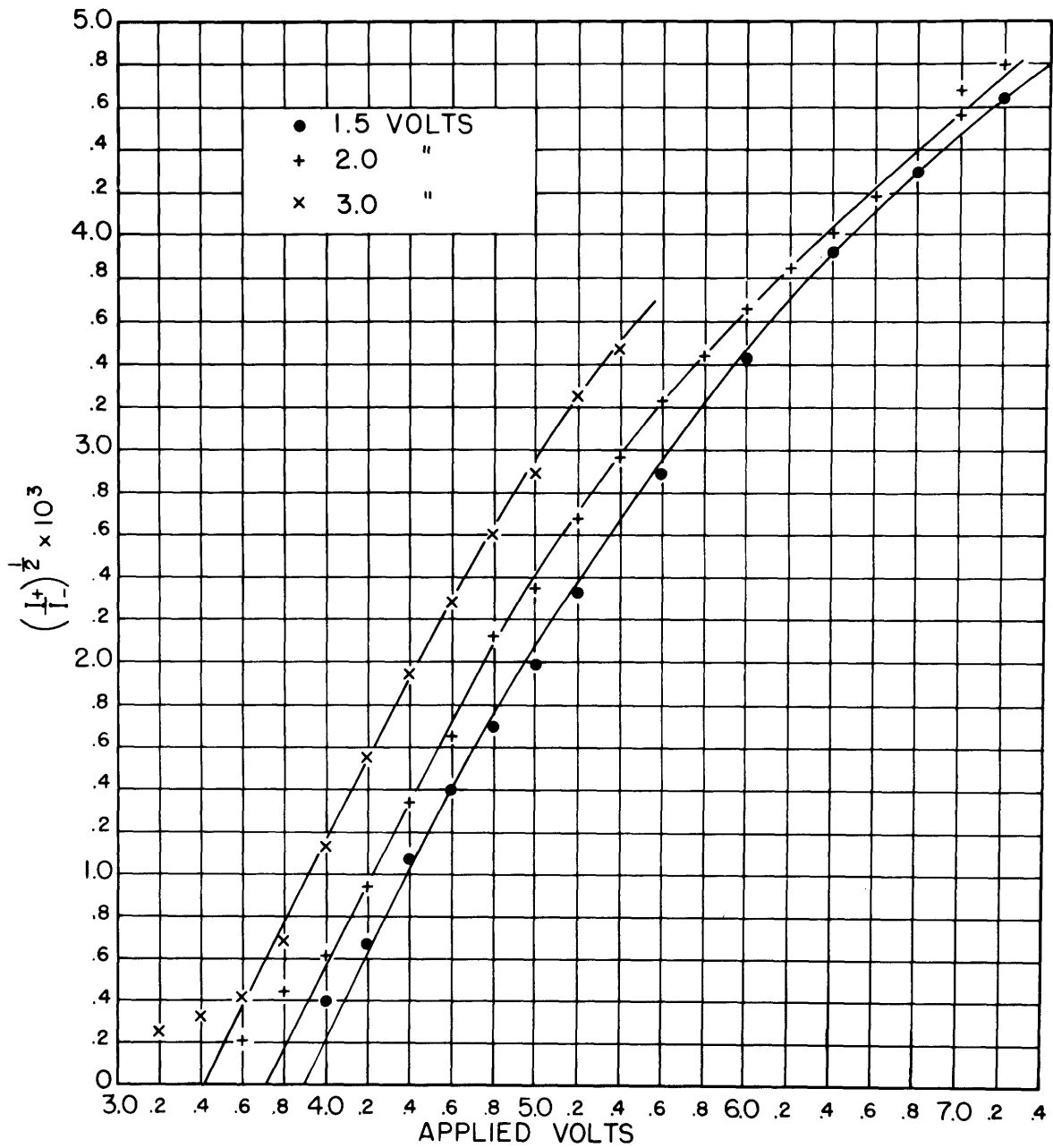


Fig. 5. Square root of ratio of ion current I_+ received at the wire collector to electron current I_- received at E2 and E3 as a function of the applied potential V_1 with E1 negative to the shield 2 volts for all values of V_1 .

F. THERMIONIC EMISSION FROM THE ION COLLECTOR

The collector wire was 10-mil tungsten which had not been especially heat-treated to a very high temperature and therefore undoubtedly had a very thin layer of tungsten oxide on its surface. The reaction of the cesium with this oxide seemed to produce an area of very low work-function. It is assumed that this work-function was not far from 1.4 ev. This wire received radiation from the various heated surfaces E1, E2, and E3. When their temperatures were less than approximately 525^oK, the thermionic emission from the wire was negligible; but as the surfaces were raised to higher temperatures, a very noticeable thermionic emission was obtained. With surface temperatures close to 700^oK, the thermionic emission was approximately 10⁻⁶ amp. This emission is consistent with the assumption that the equilibrium temperature of the ion collector was as high as 525^oK. This thermionic current was very constant, and at all levels it was possible to balance it out to high accuracy and thus distinguish the arrival of ions and measure their arrival rate, even though it represented a very small fraction of the total current flowing in the circuit.

G. CONCLUSIONS

In spite of the fact that these experiments have revealed nothing new regarding ionization effects near surfaces in the presence of cesium vapor, they do establish the applicability of conventional concepts in this respect. As a long-term research project, additional studies need to be made under conditions that are more like those encountered in thermionic energy converters, that is, higher temperatures, higher pressures, and much stronger ion accelerating fields. The evidence obtained by investigation of the ignition potential in completed plasma diodes can best be explained in terms of some new, unexplored mechanisms for ionization with low-energy electrons. One of these, of course, would be the surface excitation of a sufficient number of cesium atoms through contact with the hot surface to provide excited atoms for ionization by low-energy electrons.

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II. THE ENERGY DISTRIBUTION FOR ELECTRONS IN A THERMIONIC DIODE PLASMA CANNOT BE TRULY MAXWELLIAN

A. INTRODUCTION

After many years of investigation of low-voltage arcs, the concept that there exists at the surface of the electron emitter a strong excess of positive ions beyond that required for electron space-charge neutralization is well established by experiment. To formulate an exact solution to the problem of describing the transition in potential from the surface of the emitter to the plasma potential at the boundary between the emitter sheath and the plasma region proper is nearly impossible for a variety of reasons. Some of these reasons are: 1) The need to solve simultaneous differential equations, including specifically Poisson's equation, in a space that accelerates ions toward the electron emitter surface and retards and reflects the electrons that approach the sheath region from the plasma. 2) Even with an idealized emitter surface of uniform work-function under the zero field condition, it is difficult to know the value of the potential difference between the emitter Fermi level and the surface barrier, since the electric field that accelerates the electrons into the plasma space cannot be known before the complete solution of the problem. 3) Details concerning ion production both at the heated emitter surface and in the plasma space are not known with the desired accuracy. 4) Ions are lost from the plasma space by being accelerated through the emitter sheath to the emitter surface and also are lost by recombination with low-energy electrons, as well as being lost to other diode elements such as the electron collector. 5) Details concerning atom excitation by electron bombardment, imprisonment of radiation, and ionization from the excited state are unknown. From this enumeration of some of the important controlling factors, it is evident that even though a computer could be programmed to solve some of the problems related to the creation and maintenance of an ion-rich emitter sheath, it would be practically impossible to establish the necessary boundary conditions that are applicable to the actual performance of a plasma thermionic diode.

The theory presented in this report depends on the application of a number of simplifying assumptions recognized at the outset to be incorrect in detail but thought to be sufficiently near the truth to warrant calculations based on them followed by an experimental evaluation of the conclusions drawn.

B. GENERAL DISCUSSION OF THE EMITTER-SHEATH PROBLEM APPLICABLE TO THE "OPEN-CIRCUIT" CONDITION

Theory¹ and experiment² show that even under the open-circuit condition a strong ion-rich emitter sheath can exist. The experimental condition for which this situation is most likely to occur depends on the ratio of the emitter temperature T to the cesium condensation temperature T_{Cs} . If this ratio exceeds approximately 3.6, then an injection

sheath at open circuit may be expected. If the ratio falls between 2.7 and 3.2, the thermal ionization at the emitter surface is inadequate to generate an ion-rich sheath, even though such a condition can be developed through the means of volume ionization by the application of an external potential that drives the surface potential of the collector positive with respect to that of the emitter. Ion production then fails to maintain the sheath at open circuit. For intermediate ratios of the temperatures mentioned between 3.2 and 3.6, examples of self-maintenance at open circuit may be found after a fully developed plasma has been established by the previous application of an externally applied potential. The existence of an emitter sheath at open circuit can then be established only by a probe experiment.

Under the open-circuit condition, the net current across any boundary must be zero. Our discussion implies that the diode is "ideal" in that boundary conditions can be described as being one-dimensional. In practice, this situation can be realized only with a parallel plane configuration of sufficient area to minimize edge effects, or with a configuration in which suitable guard rings are applied to accomplish the same purpose. The motive diagram applicable to this idealized situation is shown in Fig. 1. In this diagram the displacement of the Fermi level of the collector relative to that of the emitter to establish the open-circuit condition is shown as V_{oc} . The potential of the plasma space relative to the Fermi level of the emitter is shown as V_p . The emitter and the collector barriers relative to their respective Fermi levels are shown as ϕ_1 and ϕ_2 . The emitter and collector temperatures are T_1 and T_2 . The difference in potential between the barrier level and the plasma at the emitter surface is ${}_pV_1$. According to this code, the sheath potential difference at the collector is ${}_pV_2$. The sheath thicknesses are difficult to define with precision, since there is no absolutely sharp line at which the gradient of the field in the plasma is precisely zero. This would correspond to the region in which the space charge is precisely zero and continues so across the plasma space from the indicated location at S_1 away from the emitter surface over to the location that is S_2 away from the collector surface. In both cases, within this sheath region, the positive ion space charge exceeds that of the electrons to an ever increasing extent as the space position at x_1 , for example, decreases towards zero. The total excess positive charge in this space is equalized precisely by the negative surface charge on the emitter and thus establishes an average surface field which when balanced against the mirror-image field reduces the barrier height at the emitter from the zero-field work-function value to the one actually existing and shown to have the value ϕ_1 .

Figure 2 presents a detail of the emitter-sheath potential distribution expanded in distance compared to that of Fig. 1. Here the mirror-image motive function for zero field is illustrated schematically by the dotted line that comes asymptotically to the energy value ${}_o\phi_1$ away from the emitter Fermi level. The scaling is such that 99.9 per cent of the total of the zero-field motive is achieved in a distance of

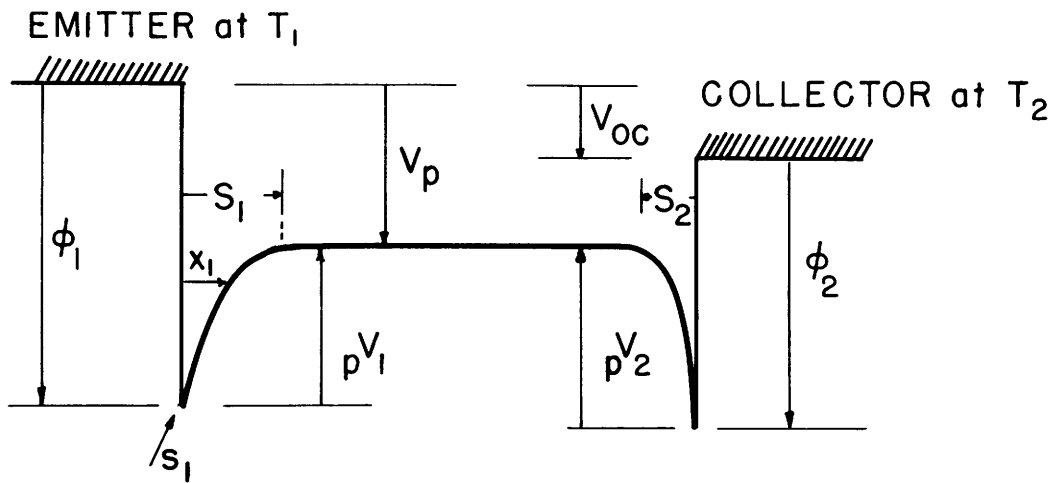


Fig. 1. Motive diagram of plasma diode at open circuit with positive ion shields.

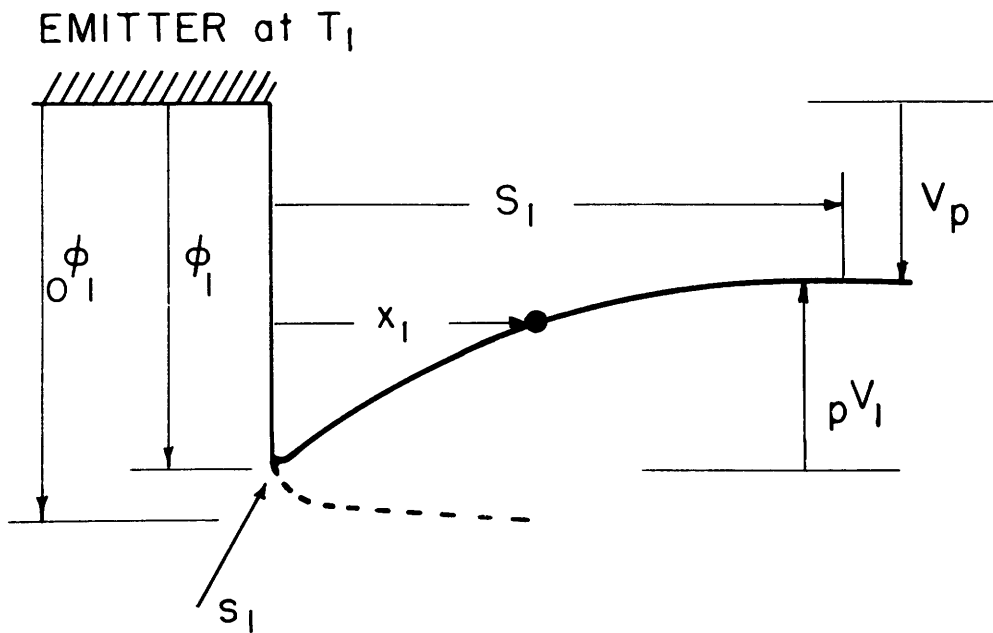


Fig. 2. Expansion of emitter sheath to show more detail.

approximately 10^{-7} m. The strong surface field created by the presence of uncompensated positive charges reduces the actual barrier to ϕ_1 . The gradient of the motive at this point is zero and the distance from the emitter surface to the location of the limiting barrier is likely to be of the order of 10^{-9} m, or less. These dimensions are very small compared with the sheath thickness S_1 which is dependent in a complex way on the fractional ionization existing in the plasma space near the boundary of the sheath. In nearly all cases of practical interest, the sheath thickness, nevertheless, is small compared with the electron mean-free path.

Under the open-circuit condition, the net current flowing across any boundary set up between the emitter and the collector must be zero. Special attention should be paid to the current components across the plane at s_1 , where the motive is ϕ_1 negative with respect to the emitter Fermi level. An electron current passes out of the emitter with an energy distribution corresponding to a half-Maxwellian distribution characterized by the temperature of the emitter and a density that is dependent on the temperature and on the value of ϕ_1 . Were it not for the current component provided by the ions that are accelerated from the plasma space across the sheath and neutralized at the emitter surface, the electron current crossing the barrier at s_1 from the plasma region into the emitter would have to be precisely equal, energy level by energy level, to that emitted from the emitter surface described above as a half-Maxwellian. Because of the ion current present, the incoming electron current must exceed the outgoing electron current by an amount precisely equal to the ion arrival rate. Ion generation takes place either at the surface of the emitter or in the plasma space, or both. A very large fraction of the ions generated at the emitter surface never leaves the emitter sheath, since in order to do so they must have a thermally generated kinetic energy associated with the direction of motion perpendicular to the sheath which exceeds the value pV_1 . Some ions will have this kinetic energy and will therefore contribute a balancing current to the ions that are found in the plasma space near the sheath boundary and in their normal motion sweep across the boundary toward the emitter. Since the net ion current may be either in or out of the sheath region and since this current is nearly always small compared with the electron current in and out at the surface barrier s_1 , the ion current will be neglected and the fact that the net current across the barrier must be zero will be identified by the presence at this point s_1 of the complete Maxwellian, having a net current of zero because of the exact equality of current flow associated with each half-Maxwellian, namely, the outgoing group and the incoming group.

In the field shown by Fig. 2, the outgoing group of electrons will be accelerated through the difference in potential pV_1 before they are randomized in energy and direction, since the distance S_1 is generally very short compared with an electron mean-free path. In the steady state of open-circuit voltage, the net current across any boundary that cuts the plasma region must be zero which, because of the perfect

compensation of positive and negative space charge, implies that the electric field will average to zero and that the positive ion current passing in one direction will be precisely balanced by the positive ion current crossing in the opposite direction. Furthermore, no matter what the actual energy distribution of the electrons in the plasma is, the electron current passing in one direction must be precisely equal to the electron current passing in the opposite direction. Not only must this equality exist for the total current in order to establish a drift current of zero, it must apply energy level by energy level in order that there be no net transfer of energy. Clearly, in the practical diode in which there is a temperature difference between the emitter and the collector, there will be some energy flow. The contribution to the energy flow under the condition of open-circuit voltage carried by the electrons may be considered to be vanishingly small.

The location of the surface potential at the collector corresponding to open-circuit voltage depends on the balancing of three current components. These are: 1) The delivery of an ion current that is due to those ions that drift into the collector sheath and are accelerated into the collector surface by the difference in potential V_p . 2) The electron emission from the collector depends on its temperature and the barrier height ϕ_2 . The third component is the photo electrons which generally contribute an exceedingly small current, and are accelerated into the plasma space. To achieve zero current at this boundary, the applied potential V_{oc} must be adjusted so that the surface potential is located at such an energy level that the necessary electron current, which in spite of the strong retarding potential V_p , can pass over the barrier into the collector. By operating the collector at a sufficiently low temperature T_2 and thus eliminating thermionic emission, the electron current removal from the plasma space at this surface can be made very small compared with the removal already described as it is related to the emitter surface.

Were it not for the interaction of the emitted electrons with the atoms, ions, and electrons in the plasma, the emitted group of electrons could pass out of the emitter, be accelerated by the difference in potential V_p , reflect perfectly back from the barrier at the collector sheath, and return to the emitter. As this group of electrons passes through the emitter sheath on the reverse trip, it would be slowed down and arrive at the barrier limit located at s_1 with precisely the same energy distribution that it had upon emission, except that the component of momentum associated with the x direction of motion would have been reversed. In this way, a complete Maxwellian distribution is maintained at this point even though at no other point between the surface of the emitter and the surface of the collector did this group of electrons constitute a complete Maxwellian.

This imaginary situation is not realistic in that electrons upon being injected into the plasma space from the emitter can change direction and interact with plasma atoms, ions, and electrons to become trapped in energy levels between the surface

potential at s_1 and the plasma potential which is positive with respect to the surface potential by the amount $\phi_p V_1$. A steady-state situation must be visualized in which electrons that momentarily occupy energy levels negative with respect to ϕ_1 transfer to levels positive with respect to this reference at exactly the same rate at which electrons momentarily in the trapped states acquire energy by interaction with other electrons to place them in the untrapped states which are negative with respect to ϕ_1 . As far as this picture goes, the one requirement that must be placed on this steady-state distribution of electrons in momentum space and corresponding to a region in volume space near the sheath edge at S_1 , is that those electrons in energy levels negative with respect to ϕ_1 must have precisely the required density and energy distribution at the location s_1 to complete the perfect Maxwellian demanded for no net current and no net energy transfer across this boundary.

The second requirement (not yet mentioned in detail) is that the total electron density including those in the trapped states and the untrapped states, just described, must be precisely equal to the steady-state ion density in the plasma near the boundary of the emitter sheath. If these electrons that acquire their energy by injection across the emitter sheath difference in potential $\phi_p V_1$ maintain a large fraction of that energy in spite of their randomization in direction, then it may be anticipated that they will appear to have a Maxwellian distribution characterized by a very high temperature so that their average energy may be approximated by

$$\bar{\epsilon} \doteq q \phi_p V_1 \doteq \frac{3}{2} kT_t = \frac{3}{2} \bar{V}_t q. \quad (1)$$

In this equation $\bar{\epsilon}$ is the average energy per electron, T_t is the apparent temperature of the electrons in the trapped space even though the quasi-Maxwellian distribution must be deficient in high-energy electrons because those present belong to the group that are not in the trapped states. The symbol \bar{V}_t represents the voltage equivalent of this temperature and is defined as

$$\bar{V}_t \equiv \frac{kT_t}{q}. \quad (2)$$

Here, k is Boltzmann's constant, and q is the electron charge. If the approximation represented by Eq. 1 is at all realistic, then the maximum possible value of \bar{V}_t will be

$$\left[\bar{V}_t \right]_{\text{Max}} \doteq \frac{2}{3} \phi_p V_1. \quad (3)$$

Consistent with this background of general considerations, some specific relations may be worked out in detail.

C. EQUATIONS FORMULATED TO REPRESENT INTERDEPENDENT RELATIONS

The first relation of interest is the expression for the electron density of a complete Maxwellian to be found at the emitter surface located at s_1 and negative with respect to the emitter Fermi level an amount ϕ_1 , as illustrated in Fig. 2. Two forms of this expression are given as follows:

$$n_s = 4.83 \times 10^{21} T_1^{3/2} e^{-\frac{\phi_1}{V_1}} \text{ electrons/m}^3 \quad (4)$$

$$\ln n_s = 49.94 + \frac{3}{2} \ln T_1 - \frac{\phi_1}{V_1} . \quad (5)$$

The electron concentration, associated with the half-Maxwellian moving from the emitter toward the emitter sheath, will be one half of that given by Eq. 4, which is applicable to the complete Maxwellian and includes those that are returning from the plasma to the emitter. The average velocity in the outgoing direction of this half-Maxwellian before being accelerated across the plasma space is given by

$$\bar{v}_s = \left(\frac{2qV_1}{\pi m} \right)^{1/2} . \quad (6)$$

Since the emitter sheath is generally very small in comparison with a mean-free path, it is of interest to have an expression for the average velocity of a half-Maxwellian distribution of electrons after they have been accelerated through a specified difference in potential. The exact expression for this average velocity is best presented in terms of dimensionless parameters defined by the two following equations.

$$z^2 = \frac{o^p_x{}^2}{2mkT_1} = \frac{o^p_x{}^2}{2mqV_1} \quad \text{at the starting point.} \quad (7)$$

Here, o^p_x is the x component of momentum at a given energy level before acceleration, and m is the electron mass.

$$p\psi_1 = \frac{pV_1}{V_1} . \quad (8)$$

The average velocity of a Maxwellian group after being accelerated through a difference in potential pV_1 is given by

$$p\bar{v}_x = \left(\frac{2qV_1}{m} \right)^{1/2} \left[\frac{2}{\sqrt{\pi}} \int_0^\infty (z^2 + p\psi_1)^{1/2} e^{-z^2} dz \right] . \quad (9)$$

It is not obvious that this equation can be integrated explicitly and therefore it has been integrated by means of a computer program. The result of this integration has been compared with an empirical formula which deviates only 5 per cent at $p\psi_1 = 0.5$, and less than 2 per cent for all values greater than 3.5. This formula is

$$p\bar{v}_x = \left(\frac{2qV_1}{\pi m} \right)^{1/2} (1 + \pi p\psi_1)^{1/2} \quad (10)$$

$$p\bar{v}_x = 5.93 \times 10^5 V_1^{1/2} \left(\frac{1}{\pi} + p\psi_1 \right)^{1/2} = 3.35 \times 10^5 \bar{V}_1^{1/2} (1 + \pi p\psi_1)^{1/2}. \quad (11)$$

Since current continuity is required, the electron density associated with a single stream of electrons moving away from the emitter can be evaluated by taking one half of those computed by Eq. 4 after dividing it by the velocity ratio expressed by

$$\frac{p\bar{v}_x}{\bar{v}_s} = (1 + \pi p\psi_1)^{1/2}. \quad (12)$$

Since under the open-circuit condition there will be a double stream of electrons occupying energy states negative with respect to ϕ_1 , the actual electron density associated with this group will be that given by Eq. 4 divided by the velocity ratio expressed by Eq. 12.

It has been assumed from the beginning that the energy distribution of the electrons in the plasma space near the boundary of the emitter sheath is not a true Maxwellian, but can be thought of very superficially as a combination of two quasi-Maxwellians, one applicable only to the energy states that are not trapped by the sheath potential. The question then arises as to how these two groups should be matched one to the other. The concept followed here is that within a small region in volume space there will be a certain total number of electrons per unit volume. This total number establishes the total number of representative points to be assigned to energy levels best described by their distribution in momentum space. Since kinetic energy will be associated with a particular value of the momentum, there will be a spherical surface in momentum space corresponding to the demarcation that identifies all representative points that are in the trapped levels as separate from those that have energy enough to escape from the region. The simplifying assumption made is that the density of representative points at this demarcation line shall be the same whether it is approached through the low-energy group of trapped electrons or through the higher-energy group associated with those that may possibly escape and, therefore, are classified as not being trapped. This statement concerning the method used to join the two distribution functions clearly represents an assumption, which, it is thought, will lead to a valid result and serves a simplification to describe phenomena, the details of which are largely impossible to ascertain.

The suitability of these assumptions must be judged in terms of an experimental evaluation of the conclusions that are based on them.

The equation that represents the fraction of a Maxwell-Boltzmann distribution of representative points in momentum space lying between momentum p and $p + dp$ is:

$$f(p) dp = \frac{4\pi p^2 e^{-\frac{p^2}{2mkT}}}{(2\pi mkT)^{3/2}} dp. \quad (13)$$

If use is made of the dimensionless parameter given in Eq. 8 and the additional one defined by Eq. 14, Eq. 15 will represent a generalized expression derived from Eq. 13:

$$\zeta_1^2 = \frac{p^2}{2mkT_1} = \frac{p^2}{2mq\bar{V}_1} \quad (14)$$

$$f(\zeta_1) d\zeta_1 = \frac{4}{\sqrt{\pi}} \zeta_1^2 e^{-\zeta_1^2} d\zeta_1. \quad (15)$$

The first use that will be made of these equations will be to determine an electron density which will be called a "fictitious density" expressed by the symbol n_f . This density is called a fictitious density because it corresponds to the electron concentration that would have been required if the plasma electrons had a true Maxwellian distribution characterized by the temperature of the emitter, and had a sufficiently high density so that the high-energy ones that are not trapped would exactly balance the electron density that must exist at the surface of the emitter represented in Fig. 2 by the position s_1 . The first step in this analysis comes from the equation given as

$$\frac{-n_s}{(1 + \pi p \psi_1)^{1/2}} = n_f \frac{4}{\sqrt{\pi}} \int_{\zeta_1 = p \psi_1}^{\infty} \zeta_1^2 e^{-\zeta_1^2} d\zeta_1. \quad (16)$$

Equation 16 represents the following situation. If a complete Maxwellian distribution of electrons characterized by the temperature T_1 existed in the plasma space with the concentration n_f , then the half-Maxwellian associated with this group after they have been retarded as they pass over the emitter sheath toward the emitter surface would arrive at the surface with precisely the correct energy distribution and density of charge that would balance the half-Maxwellian of emitted electrons.

Inspection of Eq. 16 shows that this fictitious density n_f could be obtained directly from a knowledge of the emitter temperature and its work-function through the application of Eq. 4 and the definitions given in Eqs. 8 and 14. A start toward the solution can be made by writing the following equation:

$$n_f = \frac{n_s}{(1 + \pi p \psi_1)^{1/2} \left[\frac{2 p \psi_1^{1/2}}{\sqrt{\pi}} e^{-p \psi_1} + \frac{2}{\sqrt{\pi}} \int_{p \psi_1}^{\infty} e^{-\zeta_1^2} d\zeta_1 \right]} \quad (17)$$

Note that this expression is exact in the sense that the two densities are equal only if the injection potential represented in its dimensionless symbol $p \psi_1$ is zero. For all finite values of this parameter, the fictitious density exceeds the electron density n_s . A series expansion is available³ that represents the integral quite accurately. This expansion is given as

$$\frac{2}{\sqrt{\pi}} \int_{p \psi_1}^{\infty} e^{-\zeta_1^2} d\zeta_1 = \frac{e^{-p \psi_1}}{p \psi_1^{1/2} \sqrt{\pi}} \left(1 - \frac{1}{2 p \psi_1} + \frac{3}{4 p \psi_1^2} - \frac{15}{8 p \psi_1^3} + \dots \right) \quad (18)$$

For values of $p \psi_1$ of 3 or greater, the approximate relation is accurate enough to give final answers that deviate less than 0.7 per cent from an exact answer as obtained from Eq. 17. With Eq. 18 substituted in Eq. 17, the final relationship that is useful for making this calculation is

$$n_f = \frac{n_s}{\left(1 + \frac{1}{\pi p \psi_1} \right)^{1/2} \left[2 p \psi_1 + 1 - \frac{1}{2 p \psi_1} + \frac{3}{4 p \psi_1^2} - \frac{15}{8 p \psi_1^3} + \dots \right] e^{-p \psi_1}} \quad (19)$$

As a numerical example to indicate the importance of this relation, the substitution in Eq. 19 of a value of $p \psi_1 = 10$ shows that the fictitious density n_f is 1040 times the true density at the surface given by Eq. 4.

The next step in the calculation involves the relation between the fictitious density n_f and a second fictitious density n'_f which will satisfy the requirement that in the momentum space at the borderline energy, the density of representative points infinitesimally spaced above this line and below it shall be the same. The direct application of Eq. 13 serves to establish this relation, and it is

$$n'_f = n_f \left(\frac{V_t}{V_1} \right)^{3/2} e^{-p V_1} \left(\frac{1}{V_1} - \frac{1}{V_t} \right) \quad (20)$$

This equation depends upon the choice of the momentum at which matching takes place and is given by

$$\frac{p_m^2}{2m} = q p V_1 \quad (21)$$

Again, there is the problem of relating the actual density of trapped electrons to the fictitious density n_f' . The relation involved here is

$$n_t = n_f' \frac{4}{\sqrt{\pi}} \int_0^{p\psi_t^{1/2}} \zeta_t^2 e^{-\zeta_t^2} d\zeta_t. \quad (22)$$

The dimensionless parameter is defined as

$$\zeta_t^2 = \frac{p^2}{2mkT_t}. \quad (23)$$

As a first step in the integration of Eq. 22, it may be written

$$n_t = n_f' \left[-\frac{2 p\psi_t^{1/2} e^{-p\psi_t}}{\sqrt{\pi}} + \frac{2}{\sqrt{\pi}} \int_0^{p\psi_t^{1/2}} e^{-\zeta_t^2} d\zeta_t \right]. \quad (24)$$

If the relation between the electron temperature of the trapped group of electrons and the sheath potential were correctly given by Eq. 3, then the value of $p\psi_t$ in Eq. 24 should be 1.5. The symbol j is used for the generalized value which might be as large as 5. If tabular data are used for the integral of Eq. 24 and the quantity in the square brackets is worked out numerically, two values thus obtained are 0.61 and 0.74 for 1.5 and 2. If the value of $p\psi_t$ of 1.5 is taken, the density of trapped electrons will be given as

$$n_t = 1.48 n_f p\psi_1^{3/2} e^{-p\psi_1}. \quad (25)$$

The corresponding equation for the value of $p\psi_t$ of 2 is

$$n_t = 1.94 n_f p\psi_1^{3/2} e^{-p\psi_1}. \quad (26)$$

The generalized form of the relation that establishes the numerical coefficients of 1.48 and 1.94 in Eqs. 25 and 26, respectively, is given as

$$F(j) = j^{-3/2} e^j \left[P(j)^{1/2} - \frac{2j^{1/2} e^{-j}}{\sqrt{\pi}} \right]. \quad (27)$$

In this equation the symbol $P(j)^{1/2}$ represents the probability integral expressed by the second term in Eq. 24. The two values of this integral found in a published table³ are 0.9168 and 0.9545.

The equation by which the density of trapped electrons may be computed is

$$n_t = \frac{-n_s F(j) p \psi_1^{3/2} e^{-p \psi_1}}{(1 + \pi p \psi_1)^{1/2} \left[\frac{2 p \psi_1^{1/2}}{\sqrt{\pi}} e^{-p \psi_1} + 1 - P(p \psi_1)^{1/2} \right]} \quad (28)$$

The density of the untrapped electrons is

$$n_p = \frac{-n_s}{(1 + \pi p \psi_1)^{1/2}} \quad (29)$$

Equation 29 results from the application of Eq. 12. The total density of the plasma electrons at the boundary of the sheath is then the sum of these two expressions, and is given by

$$-n_p = -n_s (1 + \pi p \psi_1)^{-1/2} \left[1 + \frac{F(j)}{\frac{2}{\sqrt{\pi} p \psi_1} + \frac{e^{p \psi_1}}{p \psi_1^{3/2}} (1 - P(p \psi_1)^{1/2})} \right] \quad (30)$$

Inspection of Eqs. 28 and 29 shows that for $p \psi_1$ of zero there are no trapped electrons and the density of the untrapped electrons given by Eq. 29 is equal to the emission density n_s . Equation 30 is useful over the range of values of $p \psi_1$ less than 4. For values greater than 4, the expression shown in Eq. 18 is more satisfactory to use and results in

$$\frac{-n_p}{-n_s} = R_j = (1 + \pi p \psi_1)^{-1/2} \left[1 + \frac{\sqrt{\pi} F(j)}{\frac{2}{p \psi_1} + \frac{1}{p \psi_1^2} - \frac{1}{2 p \psi_1^3} +} \right] \quad (31)$$

Here the ratio of the plasma density of electrons to the surface density is R_j . This ratio depends on the choice of j and on the value of $p \psi_1$. The two values of $\sqrt{\pi} F(j)$ for (3/2) and 2 are 2.63 and 3.46.

D. FRACTIONAL IONIZATION AT THE SHEATH BOUNDARY AND THE EMITTER

With the help of either Eq. 30 or Eq. 31 and the recognition that the ion density n_p must be exactly equal to the electron density $-n_p$ at the sheath boundary, the fractional ionization at that point can be expressed in terms of the usual parameters of the problem: the emitter temperature, the emitter work-function, and the cesium temperature. The atom density in the immediate neighborhood of the emitter may be expressed with sufficient accuracy by

$$\ln n_a = 76.85 - \frac{8910}{T_{Cs}} - \ln T_{Cs} - \frac{1}{2} \ln T_1. \quad (32)$$

In this equation the density is expressed in atoms/m³. The surface electron density is expressed by Eq. 5 and these two when combined with Eq. 30 or Eq. 31 gives the fractional ionization at the sheath boundary:

$$\ln f_p^{-1} = 26.91 - \frac{8910}{T_{Cs}} - \ln T_{Cs} - 2 \ln T_1 + \frac{\phi_1}{V_1} - \ln R_j. \quad (33)$$

Equation 33 is formulated in a manner to use the R_j value from either Eq. 30 or Eq. 31 and depends on the value of $F(j)$ computed by Eq. 27. Over the working range, $\ln R_j$ differs very little from 1 and it is therefore a small contribution when compared with the other terms in Eq. 33. In order to evaluate R_j , some means must be available to estimate ${}_p\psi_1$. The following equation serves to help in this process.

$${}_p\psi_1 = \ln f_p^{-1} - \frac{1}{2} \ln \frac{T_1}{T_{Cs}}. \quad (34)$$

The method of procedure is to carry through the computation according to Eq. 33 with an estimated value of $\ln R_j$ between 0.5 and 1.0 that is determined by the degree of ion richness expected. After $\ln f_p^{-1}$ has been approximated by Eq. 33, it may be used in Eq. 34 to find a close approximation to ${}_p\psi_1$. This value may now be used in Eq. 30 or 31 to get a more accurate evaluation of R_j and its corresponding logarithm. If this value differs appreciably from the estimate, then the small correction can be incorporated and a better value of ${}_p\psi_1$ obtained.

E. NUMERICAL RESULTS

It has been our purpose in the previous sections to develop equations that are the consequence of the concept that the electron energy distribution in a plasma diode is not truly Maxwellian, since some of the electrons are in energy states that are trapped whereas others are not trapped. The untrapped electrons can proceed against the retarding forces in the emitter sheath and still enter the emitter to balance the outgoing flow of electrons from the emitter. It is postulated that there is an emitter sheath potential ${}_pV_1$ and that there is a relation between this injection potential and the average energy of the quasi-Maxwellian distribution of the trapped electrons. With these generalized conditions, it has been possible to find an expression for the fractional ionization and the corresponding ion and electron density at the sheath boundary which depend on the basic parameters of the problem as expressed in Eq. 33. Furthermore, Eq. 34 provides the means for the determination of the corresponding emitter sheath potential in dimensionless units.

There are two classes of plasma-diode operating conditions which are of interest

for the application of these equations. These conditions can best be described in terms of the ratio of the emitter temperature to the cesium temperature. If this ratio exceeds 3.6, diode operation at open circuit does not demand an appreciable ion production in the plasma space. The ion loss rate to the emitter is satisfactorily equalized by the ion production rate at the emitter surface. This condition is illustrated by the results of the computations which are summarized in Table 1 and entered in line 1 and line 3. The ion loss rate and the ion delivery rates are expressed in terms of ions per square meter during a unit time of one second.

The second example of diode operation with an emitter sheath at open circuit is associated with a temperature ratio in excess of 3.2 and less than 3.6. These conditions are illustrated in lines 2 and 4 of Table 1. In these two examples the ion density and the electron density at the sheath edge are considerably greater, although the fractional ionization is comparable. Because of the fact that the ions must be delivered against an ion repelling field, the delivery rate can account for only 2 per cent of the loss rate. Although detailed experimental results under these operating conditions are not readily available, such data as do exist seem to indicate that even with a temperature ratio of 3.2, an electron injection sheath is maintained and ion production in the plasma space by electron impact processes is able to make up the 98 per cent of the ion loss rate and thus maintain an injection sheath even under the open-circuit condition.

F. CONCLUSIONS

There are two experimental methods that may be used to assist in the evaluation of the new concepts proposed in this report. One method depends on the construction and operation of plasma diodes in which probe measurements can be used to evaluate the plasma space potential and the electron energy distribution. A preliminary study of this kind has been made by Shaver²; according to his results, at open circuit an injection sheath is present and the trapped electrons seem to have a very high electron temperature. With the techniques used for his study it was impossible to evaluate the energy distribution of the untrapped electrons. The conclusion that may be drawn here, then, is that when and if improved probe measurements are made, an evaluation of the energy distribution of both the trapped and the untrapped electrons should be attempted.

A second method of investigation depends on a detailed study of voltage current curves in diodes. The results reported by Steele⁶ would lend support to the concept that the trapped electrons may be characterized by a quasi-Maxwellian at relatively high temperature, while the untrapped electrons seem to have been well characterized by the temperature of the emitter used in his experiments. Other voltage-current curves supplied for study by Hatsopoulos and Kitrilakis⁴ seem to indicate the presence of an injection sheath when applied voltages are both negative and positive with respect

Table 1. Summary of Typical Calculations.

	Emitter	T_1	T_{Cs}	$\frac{T_1}{T_{Cs}}$	ϕ_1	f_p	$p\psi_1$	$\ln(\text{Loss})$	$\ln(\text{Del})$	Surface ionization enough	Ref.
1	W	2050	440	4.66	4.3	1.45×10^{-4}	8.1	42.64	42.40	yes	Shaver ²
2	W	1809	537	3.37	3.1	2.7×10^{-4}	7.6	47.42	43.50	no	TEE ⁴
3	Ta	2270	573	3.95	4.1	3.7×10^{-9}	18.7	36.74	36.47	yes	Houston ⁵
4	Ta	1810	573	3.16	3.2	5.9×10^{-5}	9.17	46.26	42.00	no	Houston ⁵

to the open-circuit value. With the applied voltage approximately 0.5 volt more negative than the open-circuit voltage, the reverse current attributed to ions is observed to be as much as 1000 times greater than the electron emission current from the collector expected under the specified operating conditions. Even if it were assumed that temperature ionization of the cesium atoms at the emitter operated to deliver its full supply of ions without any emitter sheath barrier to retard them, only 2 per cent of the observed back current can be accounted for in this manner. The only mechanism that seems to be available to account for the very large reverse current is that the electrons, after injection in the plasma space, have ionization capability through either of two processes. They may ionize directly if there is a sufficient number with energy in excess of the ionization potential, or they may ionize excited atoms some of which are produced by thermal excitation at the emitter surface and others excited by electron bombardment. The ionization from the excited states should have a high probability and be accomplished by lower energy electrons.

If these concepts concerning the electron energy distribution and electron density in a plasma diode are supported by experiment and if, in addition, improved data on excitation and ionization cross sections become available, it may be possible to understand the performance of cesium plasma diodes and establish on a fundamental basis the interdependence of the various operating parameters. These include: 1) the emitter temperature, 2) its work-function in the presence of cesium, 3) the cesium condensation temperature, and 4) the diode spacing.

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