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Cite this article as: Amelia Musselman, Valerie M. Thomas, Dima Nazzal, Dimitri J. Papageorgiou, Aranya Venkatesh and Dharik S. Mallapragada, The impact of development priorities on power system expansion planning in sub-Saharan Africa, Energy Systems https://doi.org/10.1007/s12667-021-00433-z

Noname manuscript No.
(will be inserted by the editor)

The Impact of Development Priorities on Power System Expansion Planning in Sub-Saharan Africa

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Abstract Sub-Saharan Africa faces unique barriers to electricity development due to the large proportion of the population that is un-electrified and the prevalence of rural populations. Typically, power system expansion planning models assume all potential consumers can be immediately electrified. This assumption is unrealistic in sub-Saharan Africa, where electrification will likely be a gradual process over a number of years. Furthermore, since a large proportion of the population in sub-Saharan Africa is located in rural regions, the prioritization of these regions may impact how the grid develops. In this research, we develop a multi-period optimization model for power generation and transmission system expansion planning in sub-Saharan Africa. In contrast to existing models, which assume full electrification, we consider a variety of electrification policies and analyze the impact of varying the electrification rate and policy on the cost and resources selected for power system expansion. We test our model on a case study of Rwanda. We find that varying the year in which full electrification is reached has a larger impact on cost and generation capacity than varying the electrification policy does, although, when urban and rural regions are considered equitably, more rooftop solar is built. Varying the electrification policies has a larger impact on transmission expansion than on generation expansion and this impact is amplified when starting from zero initial system capacity rather than the original Rwanda system. Additionally, a sensitivity analysis shows that tightening the bounds on CO_{2eq} emissions has a large impact on the generation portfolio and cost.

Keywords

Africa, electricity development, electrification, power system expansion planning

1 Introduction

Goal 7 of the United Nations' Sustainable Development Goals is to guarantee universal energy access by 2030 [44]. However, according to the International Energy Agency, in sub-Saharan African, growth in energy access

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has barely kept up with population growth, and 530 million people, over 30% of the projected population, are expected to be without electricity access in 2030 [18]. When electricity from the centralized system is unreliable or unavailable, businesses [23] and critical industries such as healthcare and government services are forced to rely on diesel generators and other individual power sources for backup generation. Such barriers severely limit economic growth and development.

There are a few key factors that make sub-Saharan Africa unique compared to other regions when considering power system expansion planning. For one, unlike many recently developed regions in which electrification has occurred largely through urbanization, both the urban and rural populations in sub-Saharan Africa continue to grow [17]. Thus, it is important to consider distributed as well as centralized electrification options. Additionally, although many countries have set goals of reaching full electrification within the next 5-10 years [17, 38], development funds are limited, and it is important to consider how to optimally allocate these funds if full electrification cannot be reached in the near future.

In this study, we develop a mathematical model for power system expansion planning in sub-Saharan Africa. To allow for gradual development and analyze the impact of different development priorities, we consider four different electrification metrics and vary the time frame in which full electrification is reached. We also consider both centralized and distributed resources and distinguish between rural and urban populations. We test our model on a case study of Rwanda.

1.1 Background

There have been numerous technical reports and energy reviews for Africa, analyzing the current situation as well as projecting electricity development pathways and setting electrification goals. The International Energy Agency [17, 18] presents a broad overview of the energy situation in Africa in its Africa Energy Outlook reports. McKinsey & Company's Brighter Africa report [12] gives predictions for electricity demand growth in Africa to 2040 based on desired growth in gross domestic product and experience with more developed emerging markets about the percent increase necessary to achieve such growth. The International Renewable Energy Agency [25] developed a power planning tool for electricity development in Southern Africa and used this tool to analyze a development scenario with an emphasis on renewables. Bazilian et al. [7] develop five demand scenarios for sub-Saharan Africa and determine that a much greater level of capacity will be required to fully satisfy demand than projected by previous studies.

Specifically addressing Rwanda, the African Development Bank Group [2] gives an overview of the energy situation in Rwanda and lays out plans and requirements for future development, using least cost analysis estimates. The Technical Assistance Facility for the SE4All Initiative [38, 39, 40, 41] has written a series of draft reports outlining electricity development and financing strategies for Rwanda. Safari [34] and Bimenyimana et al. [8] provide overviews of the state of energy demand and production in Rwanda.

Demand for electricity is tightly linked to income and economic development policies. Ahlborg and Hammar [3] find government policies and priorities to be the key driver for electrification, while limited funds and technical capabilities are a barrier. In Kenya, Fobi et al. [14] show that as electrification proceeds to increasingly rural populations, the demand of newly electrified households is lower than previously electrified households, hindering the economic capacity for further electrification. In contrast, Wolfram et al. [45] show that in Brazil, the combination of aggressive electrification policies and financial support for low-income households contributed to successful centralized grid electrification. Osunmuyiwa and Kalfagianni [28] show that state-level policies, institutions, and income have driven differential adoption of renewable energy in Nigeria. Nock et al. [26], using a multi-objective optimization approach, show that emphasis on equality results in lower electricity consumption and a higher electrification rate. Trotter et al. [43], also using multi-objective optimization, show that desire for electricity self-sufficiency significantly affects electricity system development.

Many researchers have considered decentralized or distributed options as a way to reach rural areas more quickly and cost-effectively than expansion of the centralized power grid. Alfaro and Miller [4] compare the costs of four different decentralized electricity technologies in Liberia to rural Liberians' stated and demonstrated willingness to pay. Brent and Rogers [9] perform a sustainability assessment on a proposed wind and solar off-grid system with storage in South Africa. Camblonga et al. [10] study factors impacting electricity development through village-centered micro-grids in Senegal. Zeyringer et al. [46] develop an optimization model to select between grid extension and distributed photovoltaic (PV) options in Kenya. Ohiare [27] and Sanoh et al. [36] compare the cost of off-grid, mini-grid, and centralized grid electrification options in Nigeria and Senegal, respectively. Ilskoga et al. [16] propose electricity co-operatives as a solution for rural electrification, citing an electricity co-operative in Tanzania as a successful example. Levin and Thomas compare centralized and decentralized infrastructure options at various demand levels [19, 20] and consider financing mechanisms for each of these [21]. In contrast, Rose et al. [32] use a system-level optimization model with unit commitment to show that grid-connected solar PV could displace distributed diesel generators in Kenya.

Power system planning models specifically for Africa have been developed. Sanoh et al. [35] develop a simple linear program to make country-level capacity planning decisions for the whole of Africa. Heinrich et al. [15] modify the MARKAL energy systems model to include emissions taxes and demand uncertainty, solving for a case study of South Africa. Panos et al. [30] also use MARKAL, combined with an econometric model, to analyze two electricity development scenarios for sub-Saharan Africa. Ekholm et al. [13] use a similar model, adding constraints on the cost of capital, to analyze the effect on generation mix, electricity cost, and CO_2 emissions. Carvallo et al. [11] use a mixed integer linear optimization model to evaluate options for low-carbon electricity development in Kenya.

Very few studies have considered the option of leaving demand unmet in power system planning. In a case study of a region in India with electricity shortages, Balachandra and Chandru [6] compare the cost of load lost to adding generation capacity and determine the threshold at which it becomes economical to build new capacity rather than letting demand go unmet. Trotter et al. [5] consider electrification rates and inequality across regions using a multi-objective optimization approach, showing the additional cost to fully meet demand. Afful-Dadzie et al. [1] consider budget constraints in Ghana, using a multi-stage stochastic optimization approach.

1.2 Approach Overview

We develop a multi-period linear optimization model for power generation and transmission system expansion planning in sub-Saharan Africa. In contrast to previous studies, which assume full electrification, we allow for gradually increasing electrification rates over time and directly consider the electrification rates of urban and rural areas in different regions. Part of the challenge of considering partial electrification is determining how to electrify the country in a way that is both fair and implementable. If we only consider the electrification rate for the country as a whole, some regions, especially rural regions, may be left with little or no access. Alternatively, if we require the electrification rate for both urban and rural areas of every region to be the same, this may force underdeveloped regions to develop very rapidly in an unsustainable and costly way while not requiring any further development in regions in which this level is already met. We analyze the costs and system development decisions across four different electrification policies and a range of electrification rates.

To model different electrification policies, we parameterize the fraction of demand to be met in each year of the time horizon, using the fraction of demand met as a proxy for the electrification rate. We define four different metrics for the fraction of demand met. The first metric measures the average fraction of demand met across the entire country, the second measures the average fraction of demand met by area type (urban or rural), the third measures the average fraction of demand met by region, the fourth measures the average fraction of demand met by area type within each region. Comparing development under these four different metrics allows us to evaluate the trade-offs between electrifying the country in the least costly way (by only considering the total electrification rate for the whole country) and ensuring all areas receive access to electricity (by enforcing a minimum electrification rate for each region and/or urban and rural area). Furthermore, by individualizing electrifications rates for each region, we allow for current electrification levels to be considered to allow for steady development that is region-specific. We define four different constraint options based on these metrics, constraining the fraction of demand met to be above the minimum set in all cases. Additionally, we index the fraction of demand met by time to allow for a gradual increase in electrification rates over the time horizon. This approach provides insight into the costs and generation mix that result from different choices regarding equity and the time frame for full electrification. To the best of our knowledge, this study is the first to address the implications of varying the time frame for full electrification as well as equity across the urban-rural spectrum and different regions.

Previous studies have either modeled Africa at a very high level, considering country-level decisions for one or more countries, or modeled decisions for a single, smaller region. We optimize across a set of regions within a country, differentiating also between urban and rural demand within each region. We simultaneously optimize transmission and generation expansion planning and consider both centralized and distributed generation options to allow customers to be electrified without being connected to the transmission system, an important option for rural customers. Additionally, unlike other studies we have seen for developing countries, we account for both long term capacity expansion decisions and short term energy output and transmission decisions by using a representative day for each year in the time horizon, similar to what was done for the U.S. in Mai et al. [22].

We test our model on a case study of Rwanda over a thirty-year time horizon. We chose Rwanda in part because data availability was better for Rwanda than for the other countries in Africa that we initially considered. Additionally, Rwanda has set aggressive electrification targets. Specifically, Rwanda has set a goal of reaching 100% household electrification by 2025, although as of 2014, household electrification rates were only at 22% [38]. More recent sources have cited electrification rates in Rwanda to be somewhere between 34%-51% in 2017 [31, 33, 42]. Rwanda is also working to develop the natural gas reserve at Lake Kivu, which it shares with the Democratic Republic of the Congo. We analyze how the country can eventually, if not by 2025, reach this goal of complete electrification.

Finally, we perform a sensitivity analysis to test the impact of stronger emissions restrictions on electricity development and to see how the system would develop differently if built from the ground-up rather than from the existing Rwanda system. We also test the sensitivity of the results to the cost of different resource types.

2 Model

In this study, we develop a power system expansion planning model for regions within sub-Saharan Africa. We model two levels of timesteps to capture both strategic and operational planning decisions. Specifically, we optimize over T years, considering a representative hourly profile for each year and each region in order to capture hourly variations in load and solar. Construction and retirement decisions are made on an annual basis, whereas electricity production decisions are made on an hourly basis over the representative hourly time horizon. We parameterize the minimum fraction of demand that must be met in each year and vary this parameter to compare different electrification trajectories. Additionally, we consider four constraint variants on where the fraction of demand is met to represent different electrification priorities.

We model the problem as a linear program. Although it may be more realistic to make capacity decisions discrete, the computational burden is too heavy to do so given the size of our problem. However, it has been shown that when building multiple units of generation or transmission assets, the linearization of investment variables leads to a small error but significant computational benefits [29].

2.1 Definitions

The index sets used in the model are given in Table 1. We distinguish between urban and rural areas, which we call *region type*, within each region. All regions include a rural part and an urban part, although the demand for one of these could be 0 if the region was entirely urban or entirely rural. We do not geographically distinguish between urban and rural areas within each region due to a lack of data availability. However, we do distinguish between urban and rural demand within each region.

We do not index any of the storage parameters or variables by type because we only consider one type of storage. Specifically, we only consider batteries paired with rooftop solar systems since it is typical for these systems to be sold as a unit. The model could, however, be easily extended to consider additional storage types.

Table 1: Index sets used in the model.

Set	Definition
$\mathcal{G}$	all generator types
$\mathcal{G}^C$	centralized generator types
$\mathcal{G}^D$	distributed generator types (assumed to include rooftop solar)
$\mathcal{G}^{\text{dis}}$	dispatchable generator types
$\mathcal{G}^{\text{var}}$	generator types for which availability depends on the weather and thus varies by time
$\mathcal{H}$	set of consecutive hours used for hourly decisions within each year ($\mathcal{H} := \{1, \dots, H\}$)
$\mathcal{L}$	transmission line types
$\mathcal{R}$	region types (0 for rural, 1 for urban)
$\mathcal{RT}$	all resource types
$\mathcal{RT}^{\text{deplete}}$	non-renewable resource types
$\mathcal{RT}^{\text{renew}}$	renewable resource types
$\mathcal{T}$	years in the optimization time horizon ($\mathcal{T} := \{1, \dots, T\}$)
$\mathcal{V}$	regions in the country, before accounting for urban and rural components

The parameters used in the model are given in Table 2. Values in parentheses at the end of the parameter descriptions indicate the units. Where no value is given the parameter is unitless.

Table 2: Parameters used in the model.

Parameter	Definition
α	factor governing the relationship between rooftop solar capacity and associated battery capacity (h)
H^{annual}	number of times the hourly time horizon occurs in a year. For example, if H is 24, H^{annual} will be 365
$c_{gt}^{\text{Const-gen}}$	generator capital cost for generator type g in year t (\$/MW)
$c_{it}^{\text{Const-trans}}$	capital cost for a new transmission line of type i in year t (\$/(MW·km))
c_r^{dist}	distribution cost for region type r, before losses (only applies to centralized resources) (\$/MWh)
c_{gt}^{gen}	fuel cost after losses for generator type g in year t (0 if not applicable) (\$/MWh)
$c_g^{\text{OM-gen-dis}}$	annual (fixed) operations and maintenance cost for generation type g (\$/(MW·year))
$c_g^{\text{OM-gen-var}}$	variable operations and maintenance cost for generation type g (\$/MWh)
$c_i^{\text{OM-trans}}$	annual operations and maintenance costs for transmission lines of type i (\$/(MW·km·year))
$\text{Cap}_{vj}^{\text{resource}}$	resource potential at each period for renewable resource type j in region v, before losses (MW)
CO_{2eq_g}	CO_2-equivalent emissions per unit energy output from generation type g (tons/MWh)
$\text{CO}_{2\text{eq}_t}^{\max}$	maximum allowed $\text{CO}_{2\text{eq}}$ emissions from all plant types in period t (tons)
CT_g^{gen}	construction time to build a generator of type g (years)
CT_i^{trans}	construction time to build a transmission line of type i (years)
d_{vrth}	energy demand in region v, region type r, in year t and hour h (MWh)
$\delta_{v_1 v_2}$	distance between regions v_1 and v_2. Note that $\delta_{v_1 v_2} = \delta_{v_2 v_1}$ (km)
E_{vj}^{resource}	total energy available across the time horizon from depletable resource type j in region v (MWh)
$f_g^{\text{gen-dis}}$	capacity factor of generator $g \in \mathcal{G}^{\text{dis}}$
$f_{vgh}^{\text{gen-var}}$	fraction of peak capacity of generator $g \in \mathcal{G}^{\text{var}}$ in region v that is available at hour h
$f_t^{\text{elec-tot}}$	minimum total fraction of demand to be met in year t
$f_{vt}^{\text{elec-reg}}$	minimum fraction of demand to be met in region v in year t. When $t = 0$ this is the initial fraction of demand met in region v
f_{vg}^{peak}	peak output for region v divided by peak output across all regions for generator type $g \in \mathcal{G}^{\text{var}}$
$f_g^{\text{ramp}+}$	max ramp up rate for dispatchable generators as a fraction of generation capacity
$f_g^{\text{ramp}-}$	max ramp down rate for dispatchable generators as a fraction of generation capacity
ℓ_i^{trans}	transmission losses per distance for line type i (km^{-1})
γ	annual discount rate
η^{dist}	distribution efficiency
η_g^{gen}	efficiency of generator type g
$\eta^{\text{store}+}$	efficiency when charging storage
$\eta^{\text{store}-}$	efficiency when discharging storage
LT_g^{gen}	lifetime of generator type g, starting when construction is completed (years)
$\tau^{\text{store}+}$	time to fully charge storage unit (h)
$\tau^{\text{store-}}$	time to fully discharge storage unit (h)

The variables used in the model are given in Table 3. The following subscripts are used throughout to index the sets to which the parameter applies: v for region, r for region type, g for generator type, i for transmission line type, t for year, and h for hour. v_1v_2 indicates transmission between regions v_1 and v_2. Values in parentheses at the end of the variable descriptions indicate the units. All variables are restricted to $\mathbf{R}_+$.

Table 3: Decision variables used in the model.

Variable	**Definition**
p^{gen}_{vrgth}	energy output (MWh)
p^{store}_{vrth}	energy in storage at the end of the hour (MWh)
$p^{\text{store}+}_{vrth}$	energy increase in storage before losses (MWh)
$p^{\text{store}-}_{vrth}$	energy extracted from storage before losses (MWh)
$p^{\text{trans}}_{v_1v_2ith}$	energy transmitted from region v_1 to v_2 before losses (MWh)
$p^{\text{use-}C}_{vrth}$	energy consumed from centralized resources after losses (MWh)
$p^{\text{use-}D}_{vrth}$	energy consumed from distributed resources (MWh)
z^{gen}_{vrgt}	installed generation capacity. When $t=0$ this is an input parameter (MW)
$z^{\text{gen}+}_{vrgt}$	generation capacity for which construction begins at the beginning of the year For $t \in \{-CT^{\text{gen}}_g+1,\dots,0\}$ this is an input parameter (MW)
$z^{\text{gen}-}_{vrgt}$	generation capacity retired at the beginning of the year (MW)
z^{store}_{vrt}	installed storage capacity (MWh)
$z^{\text{trans}}_{v_1v_2it}$	available transmission capacity. When $t=0$ this is an input parameter (MW)
$z^{\text{trans}+}_{v_1v_2it}$	new transmission capacity for which construction is started at the beginning of the year For $t \in \{-CT^{\text{trans}_i}+1,\dots,0\}$ this is an input parameter (MW)

Some variables are indexed from years less than 1, as indicated. For these indices, the variables are actually input parameters representing the initial state of the system. The decision space is over $t \in \mathcal{T}$ for all variables.

2.2 Objective

We seek to minimize the discounted cost of electrifying the population over time,

$$\begin{aligned}\min \sum_{t=1}^{T} \frac{1}{(1+\gamma)^t} \Bigg(&\sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}}\sum_{g\in\mathcal{G}} (c^{\text{Const-gen}}_{gt} z^{\text{gen}+}_{vrgt} + c^{\text{OM-gen-dis}}_{g} z^{\text{gen}}_{vrgt} \\ &\qquad + H^{\text{annual}} \sum_{h=1}^{H} (c^{\text{OM-gen-var}}_{g} + c^{\text{gen}}_{gt}) p^{\text{gen}}_{vrgth}) \\ &+ \sum_{\substack{v_1,v_2\in\mathcal{V}:\\ v_1<v_2}} \sum_{i\in\mathcal{L}} \delta_{v_1v_2} (c^{\text{Const-trans}_i}_{t} z^{\text{trans}+}_{v_1v_2it} + c^{\text{OM-trans}}_{i} z^{\text{trans}}_{v_1v_2it}) + H^{\text{annual}} \sum_{v\in\mathcal{V}}\sum_{r\in\mathcal{R}}\sum_{h=1}^{H} c^{\text{dist}}_{r} \frac{p^{\text{use-}C}_{vrth}}{\eta^{\text{dist}}} \Bigg). \end{aligned} \tag{1}$$

The objective includes generation, transmission, and distribution costs. We assume distribution costs are only incurred for centralized resources. The only storage units we include are batteries paired with rooftop solar, so the cost of storage is included in the cost of the rooftop solar systems.

2.3 Constraints

The electrification cost is minimized subject to the following constraints.

2.3.1 Power Balance and Availability Constraints

Since we are primarily interested in the expansion decisions and only model hourly power output to ensure generation and transmission capacities are sufficient to meet the demand requirements, we model power balance as a simple network flow. Sun et al. [37] have shown that for national scale generation and transmission expansion, the network flow representation provides very similar results as more detailed power flow models, although it slightly ($< 1\%$ for a U.S. case study) underestimates transmission capacity and total system costs.

We have different power balance constraints for centralized and distributed resources because we assume distributed resources are disconnected from the transmission system. We assume that centralized resources can only be built in urban regions, but energy from these resources may be distributed in urban or rural regions. This assumption is without loss of generality because our model only distinguishes between region locations, not between the locations of urban and rural areas within a region. Distributed resources may be built in urban or rural regions, but energy from these resources must be consumed in the same region that it is produced.

The constraints governing power balance of centralized resources are,

$$\sum_{g\in\mathcal{G}^C} p^{\text{gen}}_{v_2 1 gth} + \sum_{v_1\in\mathcal{V}}\sum_{i\in\mathcal{L}} \max(0, 1-\ell^{\text{trans}}_i \delta_{v_1 v_2}) p^{\text{trans}}_{v_1 v_2 ith} = \sum_{v_1\in\mathcal{V}}\sum_{i\in\mathcal{L}} p^{\text{trans}}_{v_2 v_1 ith} + \sum_{r\in\mathcal{R}} \frac{p^{\text{use-}C}_{v_2 rth}}{\eta^{\text{dist}}} \quad \forall\ v_2\in\mathcal{V}, t\in\mathcal{T}, h\in\mathcal{H}. \tag{2}$$

For the index r, 0 corresponds to rural and 1 corresponds to urban regions, so the left-hand side considers centralized generation only in urban regions. On the right-hand side, we divide $p^{\text{use-}C}_{v_2rth}$ by η^{dist} to get the energy used prior to distribution losses.

The constraints for distributed resources are similar except we do not allow for transmission between regions, and we consider storage for rooftop solar,

$$\sum_{g\in\mathcal{G}^D} p^{\text{gen}}_{vrgth} + \eta^{\text{store}-} p^{\text{store}-}_{vrth} = p^{\text{use-}D}_{vrth} + p^{\text{store+}}_{vrth}\ \forall\ v\in\mathcal{V}, r\in\mathcal{R}, t\in\mathcal{T}, h\in\mathcal{H}. \tag{3}$$

We do not divide $p^{\text{use-}D}_{vrth}$ by η^{dist} because the distribution efficiency only applies to centralized resources that are distributed within a region. We assume that distributed resources are close enough to the demand that distribution costs and losses are negligible.

The energy transmitted cannot exceed transmission capacity,

$$p^{\text{trans}}_{v_1v_2ith} + p^{\text{trans}}_{v_2v_1ith} \le z^{\text{trans}}_{v_1v_2it}\cdot(1\text{ hr}) \qquad \forall\ \{v_1,v_2\in\mathcal{V}: v_2>v_1\}, i\in\mathcal{L}, t\in\mathcal{T}, h\in\mathcal{H}, \tag{4}$$

$$p^{\text{trans}}_{v_1v_2ith} = 0 \qquad \forall\ \{v_1,v_2\in\mathcal{V}: v_2=v_1\}, i\in\mathcal{L}, t\in\mathcal{T}, h\in\mathcal{H}. \tag{5}$$

Transmission lines are bi-directional (i.e. $z^{\text{trans}}_{v_1v_2it} = z^{\text{trans}}_{v_2v_1it}$ and $z^{\text{trans+}}_{v_1v_2it} = z^{\text{trans+}}_{v_2v_1it}$), so we only define z^{trans} and $z^{\text{trans+}}$ for $v_2 > v_1$. However, since energy transmitted is directed, we include both $p^{\text{trans}}_{v_1v_2ith}$ and $p^{\text{trans}}_{v_2v_1ith}$ in Constraints 4. We multiply by 1 hour to convert the transmission line capacity to MWh for the hour.

For dispatchable generation types the generation capacity constraints are,

$$p^{\text{gen}}_{vrgth} \le f^{\text{gen-dis}}_g z^{\text{gen}}_{vrgt}\cdot(1\text{ hr})\ \forall\ v\in\mathcal{V}, r\in\mathcal{R}, g\in\mathcal{G}^{\text{dis}}, t\in\mathcal{T}, h\in\mathcal{H}. \tag{6}$$

We also have max ramp-up and ramp-down constraints for dispatchable generators,

$$p^{\text{gen}}_{vrgt1} - p^{\text{gen}}_{vrgtH} \leq f^{\text{ramp+}}_{g} z^{\text{gen}}_{vrgt} \cdot (1 \text{ hr}) \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}^{\text{dis}}, t \in \mathcal{T}, \tag{7a}$$

$$p^{\text{gen}}_{vrgth} - p^{\text{gen}}_{vrgt(h-1)} \leq f^{\text{ramp+}}_{g} z^{\text{gen}}_{vrgt} \cdot (1 \text{ hr}) \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}^{\text{dis}}, t \in \mathcal{T}, h \in \{2, \ldots, H\}. \tag{7b}$$

$$p^{\text{gen}}_{vrgtH} - p^{\text{gen}}_{vrgt1} \leq f^{\text{ramp}-}_{g} z^{\text{gen}}_{vrgt} \cdot (1 \text{ hr}) \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}^{\text{dis}}, t \in \mathcal{T}, \tag{8a}$$

$$p^{\text{gen}}_{vrgt(h-1)} - p^{\text{gen}}_{vrgth} \leq f^{\text{ramp}-}_{g} z^{\text{gen}}_{vrgt} \cdot (1 \text{ hr}) \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}^{\text{dis}}, t \in \mathcal{T}, h \in \{2, \ldots, H\}. \tag{8b}$$

We assume that the hourly horizon within each year is cyclic in Constraints (7a) and (8a). That is, within each year, the value for the final hour of the representative day is used as the previous state when constraining the ramp rate for the first hour of the representative day.

For non-dispatchable generation types, we multiply both by the fraction of maximum potential that would be available at peak times and the fraction of this value that is available at the given time,

$$p^{\text{gen}}_{vrgth} \leq f^{\text{peak}}_{vg} f^{\text{gen-var}}_{vgh} z^{\text{gen}}_{vrgt} \cdot (1 \text{ hr}) \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}^{\text{var}}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{9}$$

In this case both of the fractions f^{peak}_{vg} and $f^{\text{gen-var}}_{vgh}$ depend on the region because the effectiveness of the installed generators depends on how strong the resource is in that region. As for transmission, we multiply by 1 hour to convert the generation capacity to MWh for the hour in both (6) and (9).

Similarly, we cannot store more than the current storage capacity,

$$p^{\text{store}}_{vrth} \leq z^{\text{store}}_{vrt} \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{10}$$

The only type of storage considered is batteries paired with rooftop solar systems, so we have an additional requirement that we can only increase storage up to the amount that is generated from rooftop solar in each period,

$$p^{\text{store+}}_{vrth} \leq p^{\text{gen}}_{vrgth} \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, g = RS, t \in \mathcal{T}, h \in \mathcal{H}. \tag{11}$$

where RS indicates the generation type is rooftop solar.

We also cannot extract more from storage than is available at the current period,

$$p^{\text{store}-}_{vrth} \leq p^{\text{store}}_{vrth} \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{12}$$

Energy balance constraints govern changes in storage over time,

$$p^{\text{store}}_{vrt1} = p^{\text{store}}_{vrtH} + \eta^{\text{store+}} p^{\text{store+}}_{vrt1} - p^{\text{store}-}_{vrt1} \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, \tag{13a}$$

$$p^{\text{store}}_{vrth} = p^{\text{store}}_{vrt(h-1)} + \eta^{\text{store+}} p^{\text{store+}}_{vrth} - p^{\text{store}-}_{vrth} \ \forall \ v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{13b}$$

Similarly to the ramping constraints for dispatchable generators, we assume that the hourly horizon within each year is cyclic in Constraints (13a).

Additionally, the change of energy in storage in each hour is restricted by the maximum charge/discharge rate,

$$\frac{p^{\text{store+}}_{vrth}}{1 \text{ hr}} \leq \frac{z^{\text{store}}_{vrt}}{\tau^{\text{store+}}} \; \forall \; v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}, \tag{14}$$

$$\frac{p^{\text{store-}}_{vrth}}{1 \text{ hr}} \leq \frac{z^{\text{store}}_{vrt}}{\tau^{\text{store-}}} \; \forall \; v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{15}$$

2.3.2 Construction, Retirement, and Capacity Constraints

To ensure that generators have been retired by the end of their lifetime, we require that at least as many generators have been retired by period t as have started construction by period $t - LT^{\text{gen}}_g - CT^{\text{gen}}_g$,

$$z^{\text{gen}}_{vrg0} + \sum_{t=-CT^{\text{gen}}_g+1}^{\tau - LT^{\text{gen}}_g - CT^{\text{gen}}_g} z^{\text{gen+}}_{vrgt} \leq \sum_{t=1}^{\tau} z^{\text{gen}-}_{vrgt} \; \forall \; v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}, \tau \in \{LT^{\text{gen}}_g + 1, \ldots, T\}. \tag{16}$$

In Constraints (16), we assume that existing generation capacity will be retired LT^{gen}_g periods after the start of the time horizon. The sum over $z^{\text{gen+}}_{vrgt}$ includes generators that have started but not completed construction at the start of the time horizon.

Generation capacity in each year is the capacity in the previous year adjusted by the capacity built and retired,

$$z^{\text{gen}}_{vrgt} = z^{\text{gen}}_{vrg(t-1)} + z^{\text{gen+}}_{vrg(t-CT^{\text{gen}}_g)} - z^{\text{gen}-}_{vrgt} \; \forall \; v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}, t \in \mathcal{T}. \tag{17}$$

Since we only consider storage that is associated with rooftop solar, the storage capacity is directly proportional to rooftop solar capacity,

$$z^{\text{store}}_{vrt} = \alpha \cdot z^{\text{gen}}_{vrgt} \; \forall \; v \in \mathcal{V}, r \in \mathcal{R}, g = RS, t \in \mathcal{T}. \tag{18}$$

Since the lifetime of transmission lines is typically longer than our time horizon, we do not allow transmission lines to retire. Thus, the transmission capacity only changes as transmission is constructed,

$$z^{\text{trans}}_{v_1 v_2 it} = z^{\text{trans}}_{v_1 v_2 i(t-1)} + z^{\text{trans+}}_{v_1 v_2 i(t-CT^{\text{trans}}_i)} \; \forall \; \{v_1, v_2 \in \mathcal{V} : v_2 > v_1\}, i \in \mathcal{L}, t \in \mathcal{T}. \tag{19}$$

For renewable resources, the maximum installed capacity is constrained by the regional resource potential,

$$\sum_{r \in \mathcal{R}} \sum_{g \in \mathcal{G}^j} \frac{z^{\text{gen}}_{vrgt}}{\eta^{\text{gen}}_g} \leq \text{Cap}^{\text{resource}}_{vj} \; \forall \; j \in \mathcal{RT}^{\text{renew}}, v \in \mathcal{V}, t \in \mathcal{T}. \tag{20}$$

The max potential capacity, $\text{Cap}^{\text{resource}}_{vj}$, may be infinite for some resource types. For centralized resources, capacity may only be constructed in the urban region, so we set the capacity in rural regions to 0,

$$z^{\text{gen}}_{v0gt} = 0 \; \forall \; v \in \mathcal{V}, g \in \mathcal{G}^C, t \in \mathcal{T}. \tag{21}$$

For depletable resources, the availability is limited by the fuel reserve, so rather than directly constraining the capacity, we constrain the total energy output across the time horizon by the energy available from the corresponding fuel type in each region,

$$H^{\text{annual}} \sum_{r \in \mathcal{R}} \sum_{g \in \mathcal{G}^j} \sum_{t=1}^{T} \sum_{h=1}^{H} \frac{p^{\text{gen}}_{vrgth}}{\eta^{\text{gen}}_g} \leq E^{\text{resource}}_{vj} \; \forall \; j \in \mathcal{RT}^{\text{deplete}}, v \in \mathcal{V}, \tag{22}$$

where $\mathcal{G}^j$ is the set of generators that use resource type j.

2.3.3 Emissions Constraints

We constrain the $\text{CO}_{2\text{eq}}$ emissions from energy generation for each year,

$$H^{\text{annual}} \sum_{g \in \mathcal{G}} \text{CO}_{2\text{eq}_g} \sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}} \sum_{h=1}^{H} p^{\text{gen}}_{vrgth} \leq \text{CO}^{\max}_{2\text{eq}_t} \; \forall \; t \in \mathcal{T}. \tag{23}$$

2.3.4 Electricity Consumption Constraints

Energy consumption cannot exceed demand,

$$p^{\text{use-}C}_{vrth} + p^{\text{use-}D}_{vrth} \leq d_{vrth} \; \forall \; v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{24}$$

Furthermore, because we never want to unelectrify previously electrified populations, we require that the energy used in each region is non-decreasing across years,

$$p^{\text{use-}C}_{vrth} + p^{\text{use-}D}_{vrth} \geq p^{\text{use-}C}_{vr(t-1)h} + p^{\text{use-}D}_{vr(t-1)h} \; \forall \; v \in \mathcal{V}, r \in \mathcal{R}, t = \{2, \ldots, T\}, h \in \mathcal{H}. \tag{25}$$

In the first year, the average electricity usage in each region must at least meet current electrification levels,

$$\frac{\sum_{r \in \mathcal{R}} (p^{\text{use-}C}_{vr1h} + p^{\text{use-}D}_{vr1h})}{\sum_{r \in \mathcal{R}} d_{vr1h}} \geq f^{\text{elec-reg}}_{v0} \; \forall \; v \in \mathcal{V}, h \in \mathcal{H}. \tag{26}$$

We do not have urban and rural values for initial electrification rates, so we use the total electrification rate for each region. We assume this constraint holds for every hour because a population is not truly electrified if its electricity demand can only be met part of the time.

2.3.5 Electrification Goals

We consider four variations of constraints on the minimum fraction of demand met in each time period. These constraints distinguish our model from typical power system planning models by allowing us to compare electrification policies when demand is not fully met.

In the first variant, the fraction of demand met for the entire country must be at least the minimum specified,

$$\frac{\sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}} (p^{\text{use-}C}_{vrth} + p^{\text{use-}D}_{vrth})}{\sum_{v \in \mathcal{V}} \sum_{r \in \mathcal{R}} d_{vrth}} \geq f^{\text{elec-tot}}_t \; \forall \; t \in \mathcal{T}, h \in \mathcal{H}. \tag{27}$$

In the second, the fraction of demand met for both urban and rural regions across the country must simultaneously be at least the minimum specified,

$$\frac{\sum_{v\in\mathcal{V}}(p^{\text{use-}C}_{vrth} + p^{\text{use-}D}_{vrth})}{\sum_{v\in\mathcal{V}} d_{vrth}} \geq f^{\text{elec-tot}}_{t} \;\forall\; r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{28}$$

In the third, the fraction of demand met in each region must be at least the minimum specified for that region, simultaneously across all regions,

$$\frac{\sum_{r\in\mathcal{R}}(p^{\text{use-}C}_{vrth} + p^{\text{use-}D}_{vrth})}{\sum_{r\in\mathcal{R}} d_{vrth}} \geq f^{\text{elec-reg}}_{vt} \;\forall\; v \in \mathcal{V}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{29}$$

Finally, in the fourth constraint variant, the fraction of demand met for both the urban and rural areas in each region must be at least the minimum specified for the region, simultaneously across all regions and region types,

$$\frac{p^{\text{use-}C}_{vrth} + p^{\text{use-}D}_{vrth}}{d_{vrth}} \geq f^{\text{elec-reg}}_{vt} \;\forall\; v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}. \tag{30}$$

We only include one of these four constraint variants at a time in our computational experiments. Constraint (27) gives the model the flexibility to select which demand is met in the most economically efficient way. The other constraint variants sacrifice some amount of economic efficiency in order to promote various measures of fairness. Constraints (27) and (28) are only directly comparable with Constraints (29) and (30) if $f^{\text{elec-reg}}_{vt} = f^{\text{elec-tot}}_{t} \;\forall\; v \in \mathcal{V}$. In this case, Constraint (28) and (29) are more restrictive than Constraint (27), but neither one of these constraint variants is necessarily more restrictive than the other. Also in this case, Constraint (30) is the most restrictive, ensuring that all urban and rural regions across the country are electrified equally.

2.3.6 Variable Bounds and Type Constraints

Finally, we restrict all variables to be non-negative,

$$\begin{aligned}
p^{\text{gen}}_{vrgth} &\geq 0 \;\forall\; v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}, t \in \mathcal{T}, h \in \mathcal{H}, && (31)\\
p^{\text{store}}_{vrth},\; p^{\text{store+}}_{vrth},\; p^{\text{store-}}_{vrth} &\geq 0 \;\forall\; v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}, && (32)\\
p^{\text{trans}}_{v_1 v_2 ith} &\geq 0 \;\forall\; v_1, v_2 \in \mathcal{V}, t \in \mathcal{T}, h \in \mathcal{H}, && (33)\\
p^{\text{use-}C}_{vrth},\; p^{\text{use-}D}_{vrth} &\geq 0 \;\forall\; v \in \mathcal{V}, r \in \mathcal{R}, t \in \mathcal{T}, h \in \mathcal{H}, && (34)\\
z^{\text{gen}}_{vrgt},\; z^{\text{gen+}}_{vgt},\; z^{\text{gen}-}_{vgt} &\geq 0 \;\forall\; v \in \mathcal{V}, r \in \mathcal{R}, g \in \mathcal{G}, t \in \mathcal{T}, && (35)\\
z^{\text{trans}}_{v_1 v_2 it},\; z^{\text{trans+}}_{v_1 v_2 it} &\geq 0 \;\forall\; \{v_1, v_2 \in \mathcal{V} : v_2 > v_1\}, t \in \mathcal{T}. && (36)
\end{aligned}$$

We do not need to explicitly enforce non-negativity for z^{store}_{vrt} since it is a fixed proportion of z^{gen}_{vrRSt}.

3 Rwanda Case Study

We test our model on a case study of Rwanda. We use the 30 districts of Rwanda, shown shaded by initial average hourly demand for each district in Figure 1, as our regions in consideration, further distinguishing between urban and rural demand within each of these districts.

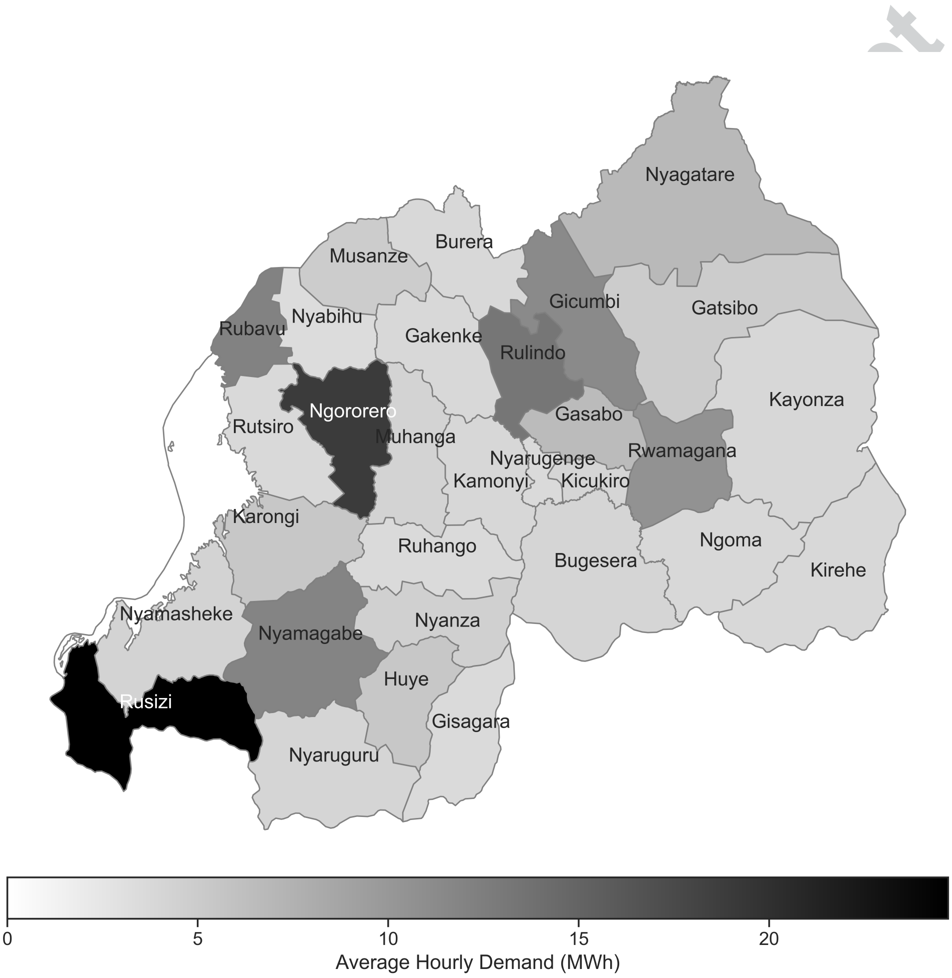


Fig. 1: Map of the districts in Rwanda shaded by average hourly demand. Base map created using a shapefile from Map Library [24].

The generator types considered along with their corresponding resource types, dispatchability, and scale (centralized or distributed) are shown in Table 4.

Table 4: Generator types considered and corresponding resource types, dispatchability, and scale.

Generator Type	Fuel Source	Dispatchability	Scale
Diesel	Diesel	Dispatchable	Centralized
Diesel 100 kW (Diesel Ind)	Diesel	Dispatchable	Distributed
Heavy Fuel Oil (HFO)	Diesel	Dispatchable	Centralized
Open Cycle Gas Turbine (OCGT)	Natural Gas	Dispatchable	Centralized
Combined Cycle Gas Turbine (CCGT)	Natural Gas	Dispatchable	Centralized
Peat	Peat	Dispatchable	Centralized
Hydroelectric	Hydro	Dispatchable	Centralized
Small Hydroelectric	Hydro	Dispatchable	Distributed
Biomass	Biomass	Dispatchable	Centralized
Geothermal	Geothermal	Dispatchable	Centralized
Utility Solar Photovoltaic (PV)	Solar	Variable	Centralized
Rooftop PV with 2h Battery	Solar	Variable	Distributed

Although the main purpose of the model is to inform long-term capacity planning decisions, it is important to capture fluctuations in demand and solar patterns that may affect these decisions. For this reason, we model two different time intervals: a year and an hour. Capacity planning decisions are made on an annual basis, and energy generation decisions are made on an hourly basis over a representative subset of hours. We use a 30-year time horizon with one 24-hour representative day each year (so $H^{\text{annual}} = 365$). Rwanda is very close to the equator, so there is no need to capture seasonal variations. The 30-year time horizon starts in 2015. That is, year 0, the starting state, corresponds to year 2015, and we optimize from years 2016 to 2045. We consider cases where 100% electrification is reached in years 2020, 2025, 2030, 2035, 2040, and 2045 for each of the four electrification constraint variants, giving a total of 24 cases. In each case, we start from the initial electrification rate, either across the country or by region, depending on the model variant, and increase $f_t^{\text{elec-tot}}$ or $f_{vt}^{\text{elec-reg}}$ linearly until it reaches 1 in the specified year.

Details on the data used are available in the online supplementary information (SI) document.

4 Results

We program the model in Python and solve with Gurobi. We analyze the impact of varying the year at which 100% electrification is reached and the metric for the fraction of demand. Whenever results for the four variants of the fraction of demand met (electrification goal) constraints are shown in a figure,"Total" indicates Constraints (27) are used, "UR Total" indicates Constraints (28) are used, "Regional" indicates Constraints (29) are used, and "UR Regional" indicates Constraints (30) are used. We focus on the results for 2025, which is the year in which Rwanda would like to reach full electrification [38], and 2045, which is the final year in our time horizon.

4.1 Electrification Cost

Figure 2 shows the objective value (discounted total cost) across the years considered for full electrification. The cost monotonically decreases as the year at which 100% electrification is reached increases. The cost to reach full electrification by 2045 is 40-50% less, depending on the model variant, than the cost to reach full electrification by 2025. The costs for the UR Total and UR Regional models are higher than the costs for the Total and Regional models, as expected. Even though the UR Total constraint is generally less restrictive than the UR Regional constraint, the cost for the UR Regional model is slightly lower than for the UR Total model. This result is possible because the values for $f_{vt}^{\text{elec-reg}}$ and $f_t^{\text{elec-tot}}$ are based on current electrification rates in Rwanda and don't satisfy $f_{vt}^{\text{elec-reg}} = f_t^{\text{elec-tot}}\ \forall\ v \in \mathcal{V}$. (See SI for details). For the same reason, in 2020, the cost for the Regional model is slightly lower than for the Total model, but for later electrification years, the Total model has lower costs, as expected.

Part of the decrease in cost in Figure 2 is due to discounting. Annual costs prior to discounting are shown in the SI.

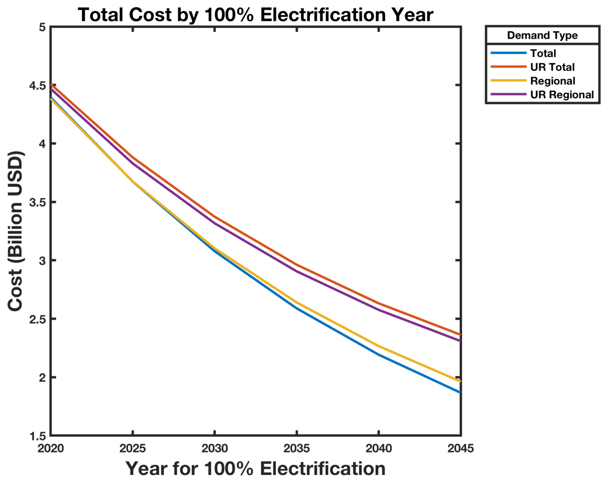


Fig. 2: Discounted total cost to reach full electrification by the given year.

4.2 Generation Capacity and Output

Figure 3 shows the generation capacity and energy output over the time horizon for the Total demand constraint model when 100% electrification is reached by 2025 or 2045. In subfigures 3(c) and 3(d), the total energy output for the country, averaged across the hours in the representative day, is shown for each year of the time horizon. In subfigures 3(e) and 3(f), the total energy output for the country is shown for each hour of the representative day every fifth year (we skip by five years in this case to more clearly show the hourly fluctuations). There is a cyclic pattern in the energy output due to the 24-hour load curve used.

The system starts off with more centralized generation, but by the end of the time horizon, over two-thirds of generation capacity comes from distributed resources. The proportion of energy output from distributed resources is closer to half, primarily due to the variable output from solar rooftop systems, which is compensated for by variation in output from individual (100 kW) diesel systems. In the initial system, large hydroelectric plants make up the majority of generation capacity, but new hydroelectric generation that is added all comes from small plants. Of the centralized resources, increase in geothermal capacity is the greatest.

We see that the capacity mix and energy output at the end of the time horizon is similar for the 2025 full electrification case as for the 2045 full electrification case, but the buildup of resources is smoother, occurring more gradually over time in the 2045 case. In both cases, the total energy output is slightly less than half of the total capacity due to low efficiencies and variability of some resources combined with load variability. For the cases when full electrification is reached between 2025 and 2045, the results gradually move from being more similar to the 2025 results to more similar to the 2045 results. The results for the Regional, UR Regional, and UR Total models exhibit similar trends.

When the year in which 100% electrification is reached is fixed, varying the metric for the fraction of demand met has a relatively small impact on the overall generation capacity mix and energy output. We do observe that the total generation capacity and energy output from distributed resources is slightly higher in the UR model variants, as expected, although even for the Total model a large proportion of capacity comes from distributed resources. Additionally, by comparing the generation capacity across the four model variants for individual generator types, we see that there actually are significant differences in capacity, but relative to the total generation capacity across all types, these differences are small and tend to decrease by the end of the time horizon. Capacity and energy output plots for the four electrification goal constraint options are shown in the SI.

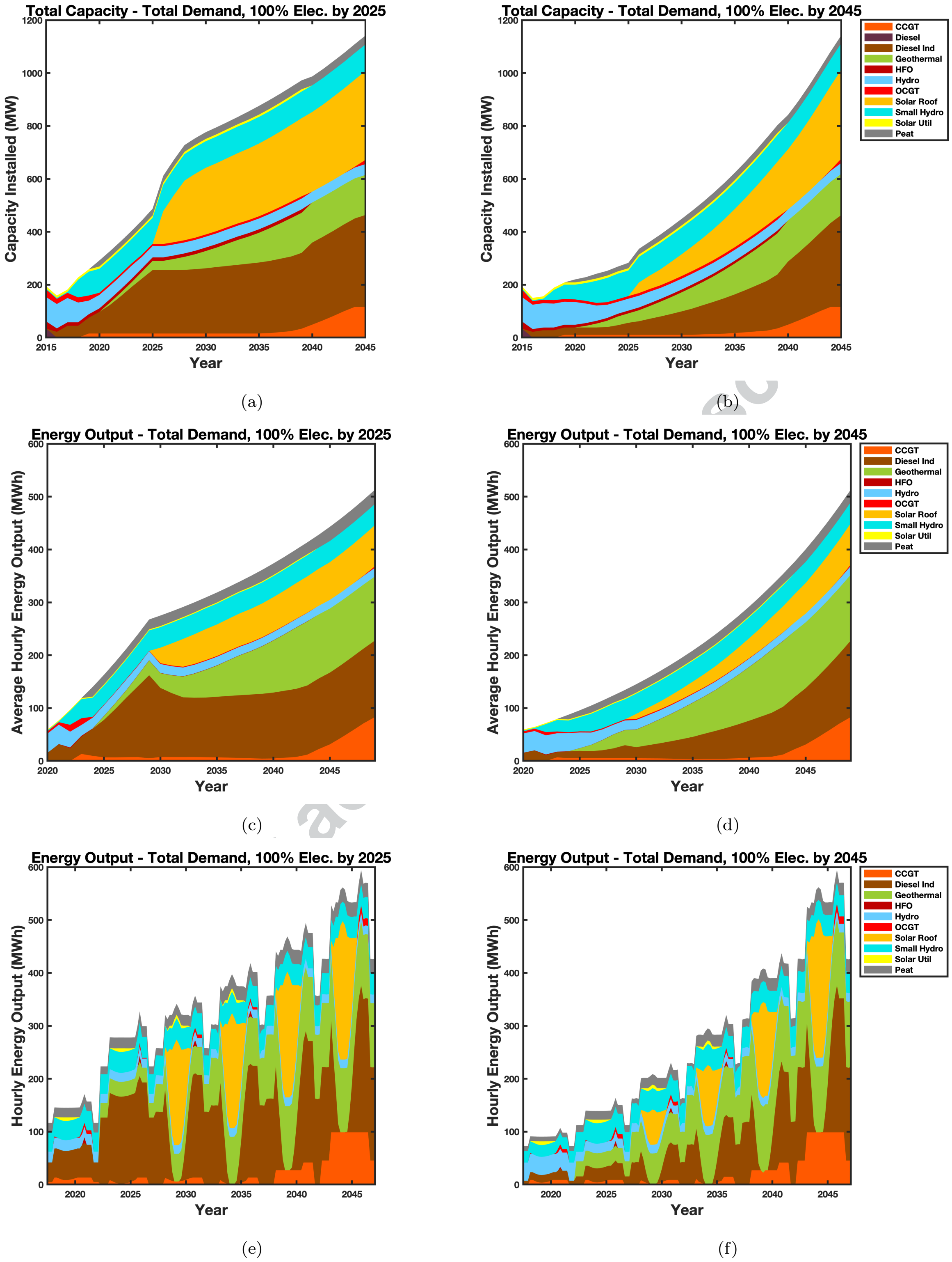
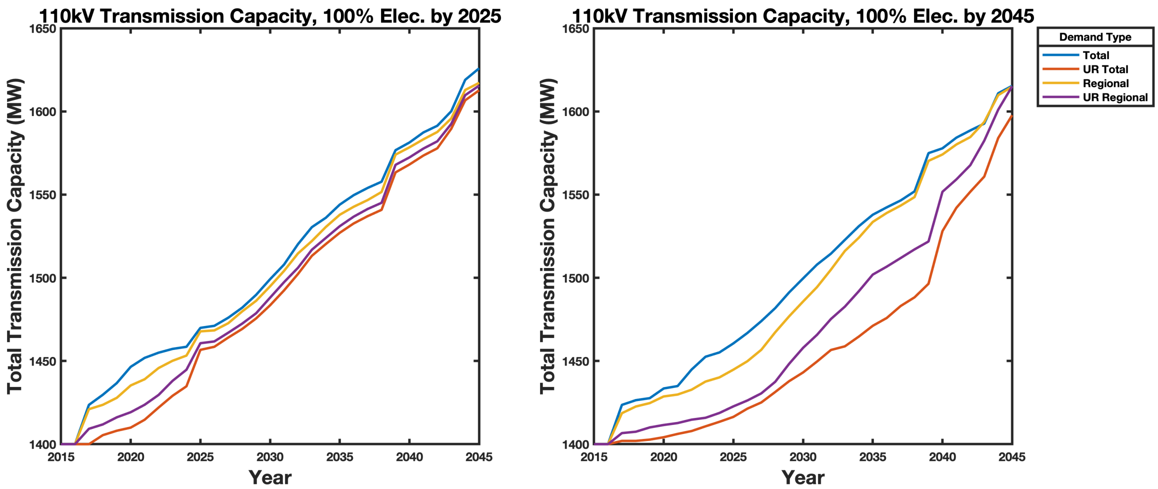


Fig. 3: Generation capacity mix ((a) and (b)), average energy output ((c) and (d)), and hourly energy output for over the representative day every fifth year ((e) and (f)) for the Total demand constraint model when 100% electrification is reached by 2025 ((a), (c), and (e)) or 2045 ((a), (d), and (f)). The oscillation in the energy output in ((e) and (f)) is because a representative day is used to model hourly decisions in each year.

4.3 Transmission Capacity

Figures 4 shows the total transmission capacity over time for the 110kV line for 100% electrification years 2025 and 2045. There is a greater difference between the four model variants when full electrification is reached by 2045 than when it is reached by 2025. The total transmission capacity for the total demand constraint variant is similar in both cases, but for the other model variants, the transmission capacity is built up more slowly when full electrification is reached by 2045 than when it is reached by 2025. Between 100% electrification years 2025 and 2045, the results gradually move from being more similar to the 2025 results to more similar to the 2045 results. The 30 kV transmission capacity is constant at 450 MW throughout time horizon in all cases because the 110kV lines are cheaper to build per unit of capacity, so it is uneconomical to build any 30 kV lines.



(a) (b)

Fig. 4: Total transmission line capacity over time for the 110 kV line for 100% electrification years (a) 2025 and (b) 2045.

In Figure 5 we focus in on one of the years in the time horizon to show how the transmission system is different between the four model variants when 100% electrification is reached by 2045. On these maps, the thick red 110kV lines and the blue 30kV lines were all present in the initial system (shown in the SI). These lines are the same across the four model variants since transmission line retirements are not considered. There are significant differences between the four model variants in the new transmission lines built, most notably between the Regional and UR Total models, however, these differences are overshadowed by the capacity of the initial transmission system. This observation is the main motivation for part of our sensitivity analysis, discussed in Section 5.1.

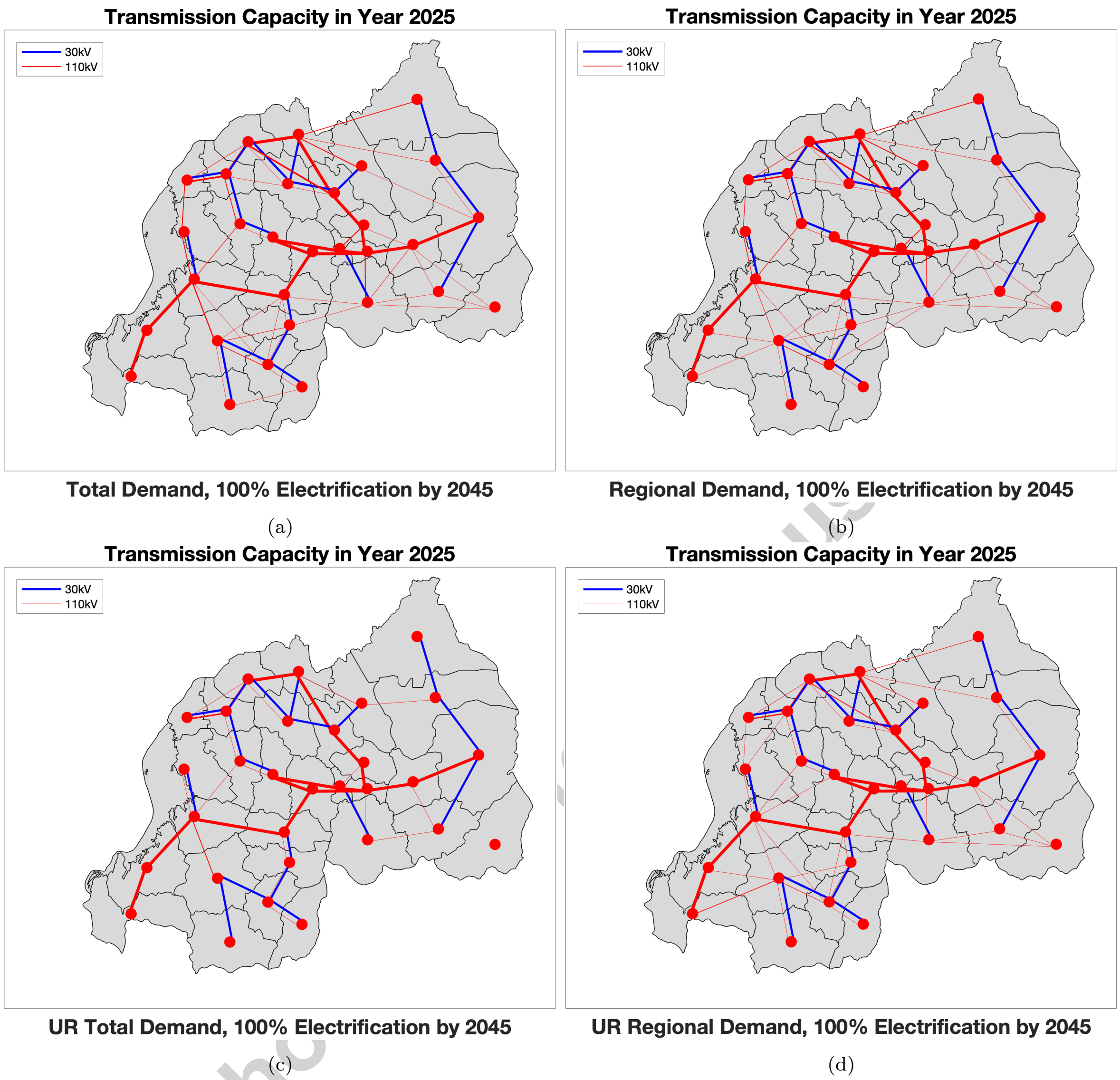


Fig. 5: Map of the transmission system in year 2025 for the (a) Total, (b) Regional, (c) UR Total, and (d) UR Regional demand met constraints when 100% electrification is reached by 2045. Blue lines are 30 kV, red lines are 110 kV. The line thickness indicates the installed capacity on a logarithmic scale. Map created with Matlab using shapefile from Map Library [24].

5 Sensitivity Analysis

In the original results, we observed that the increase in transmission capacity is relatively small compared to the total initial transmission capacity, which may be limiting the diversity in solutions between the different demand constraints. In order to test how the power system would develop if it were not constrained by existing infrastructure, we optimize a power system built from scratch, with zero initial transmission and generation capacity. We compare this "greenfield" system to its "brownfield" counterpart to analyze the impact of the current system on development trajectories.

We also observed that, in the initial results, the CO_{2eq} emissions constraint was not tight. To test the impact of stronger environmental regulations, we solve the model with CO_{2eq} emissions held constant.

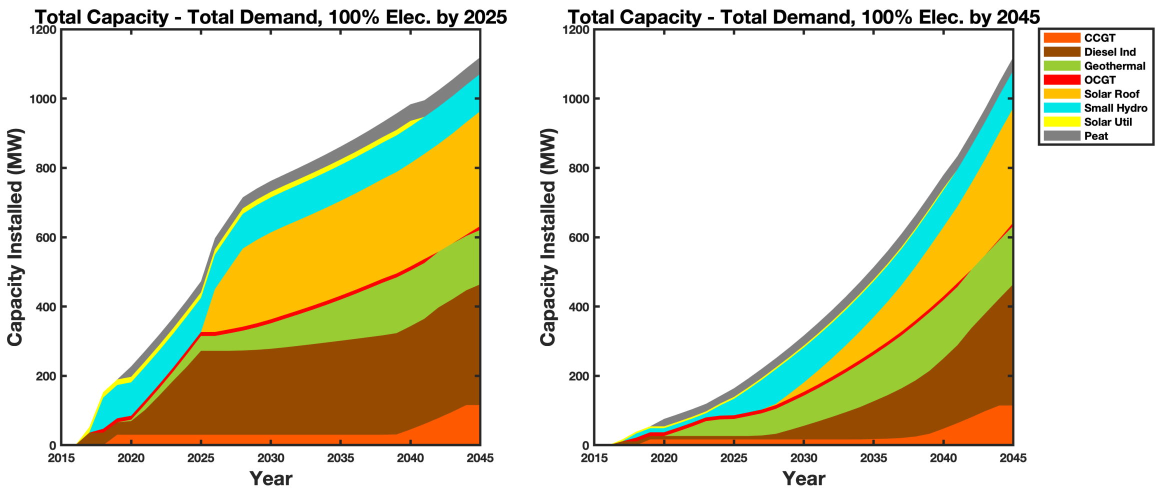
Finally, to test the impact of cost on the balance of centralized and distributed resources that are selected, we solve the model first with all of the costs for centralized generators halved then with all of the costs for distributed generators halved.

5.1 Greenfield Analysis

We re-optimize, starting from an initial system with no transmission or generation capacity. In this case, we set the fraction of demand met in the first year (2016) to zero since transmission lines and most generator types take at least a year to construct. While this is unrealistic, it allows us to understand how historical investments in the power system impact future trajectories of power system evolution. We keep the fraction of demand met requirements for the rest of the time horizon the same as in the brownfield analysis.

The costs by electrification year are similar to the results when starting from the current Rwanda system with a few notable differences. The cost for the UR Regional model is the highest in all cases whereas previously the UR Total model had the highest cost. There is a larger difference in cost between the Regional model and Total model (with the Regional model costing more) and a smaller difference between the Regional model and UR Total model (with the Regional model costing less) than previously observed. The cost to reach full electrification by 2025 is 3-8% lower and by 2045, 24-34% lower, depending on the model variant, than in the brownfield analysis. Note that for the first year, the operational costs of the greenfield analysis are 0 since we assumed 0% of the demand is met in the first year. The capital costs in the first year, however, are higher than in the brownfield analysis for model variants in which full electrification is reached within the first 15 years since entirely new infrastructure must be built to meet this demand. Cost values and plots for the greenfield analysis are show in the SI.

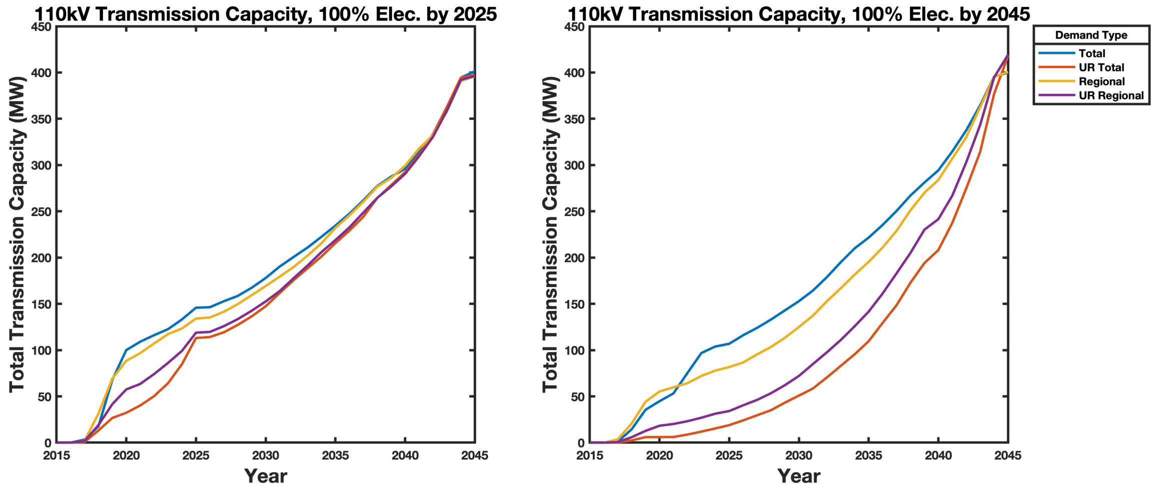
For the generation capacity, we don't build any (large) diesel, HFO, or (large) hydroelectric plants in the greenfield system. This finding is consistent with the brownfield analysis in which these generation types either stayed at initial levels or were retired. The generation mix is otherwise overall similar to the brownfield analysis, with other generator types accounting for the lack of the three unbuilt types in similar proportions as previously observed. Generation capacity plots for Total demand constraint model are shown in Figure 6. Additional generation and energy output plots are included in the SI.



(a) (b)

Fig. 6: Generation capacity mix for the Total demand constraint model when 100% electrification is reached by (a) 2025 or (b) 2045 in the greenfield analysis.

The total transmission capacity is much lower, about a quarter of the capacity at the end of the time horizon, than when starting from the original Rwanda system, as shown in Figure 7.



(a)

(b)

Fig. 7: Total transmission line capacity over time for the 110 kV line for 100% electrification years (a) 2025 and (b) 2045, starting from zero initial capacity.

The largest differences between the different demand constraints also occur in the transmission system. Figure 8 shows the transmission system for the four different model variants in 2025 when 100% electrification is reached by 2045, starting from zero initial capacity. Only 110 kV lines are built in this case. The transmission system is by far the sparsest for the UR Total Demand model, indicating a higher reliance on distributed resources in rural regions. The transmission system for the Total Demand model is somewhat less dense than for the Regional and UR Regional models and has some higher capacity transmission lines. All of the lines that are included in the transmission system for the UR Total model are included in that of the Total model. For the other three model variants, there are some differences in the lines chosen. Compared to the transmission system when starting from the original Rwanda system, there are more lower capacity lines, rather than the high capacity lines that already existed in the original Rwanda system. Also there are no 30 kV lines, as there were in the original system.

Note that distributed resources comprise a large fraction of the overall generation capacity in both the greenfield and the brownfield analysis. These resources give the model the flexibility to meet rural demand without building transmission lines, as observed for the UR Total Demand case. There are many regions in which only distributed resources are built. However, these resources are not sufficient to meet demand for every region, which is why more transmission lines are built for model variants in which we require the minimum fraction of demand to be met in every region.

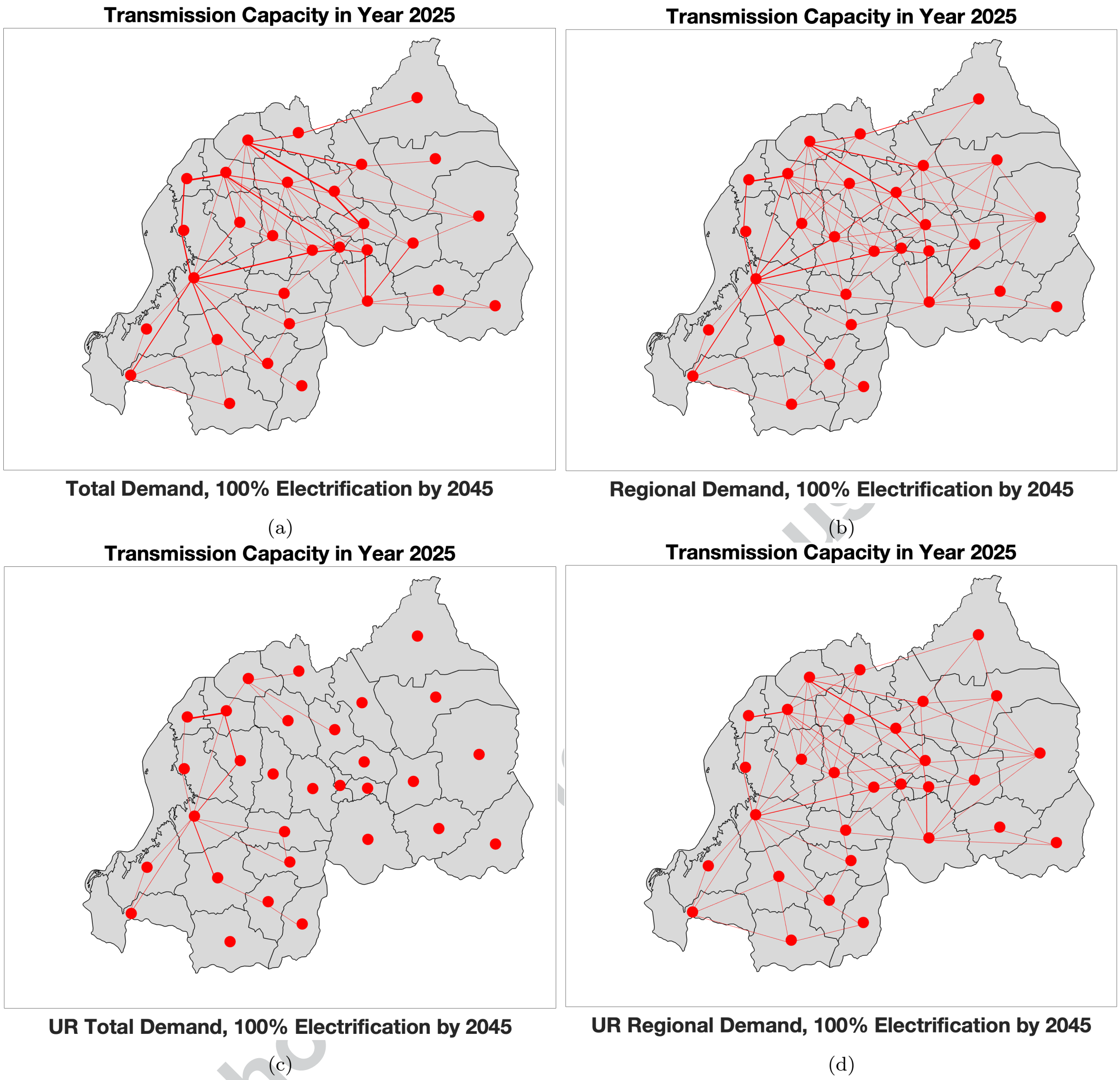


Fig. 8: Map of the transmission system in year 2025 for the (a) Total, (b) Regional, (c) UR Total, and (d) UR Regional demand met constraints when 100% electrification is reached by 2045, starting from zero initial capacity. Only 110 kV (red) lines are built. The line thickness indicates the installed capacity on a logarithmic scale. Map created with Matlab using shapefile from Map Library [24].

5.2 Tighter CO_2-Equivalent Emissions Restrictions

To explore drivers for renewable and distributed generation, we analyze the results when CO_{2eq} emissions are not allowed to exceed current levels as electrification rates are increased. Other parameters are kept at the values in the original Rwanda system case.

Figure 9 shows the objective value (discounted total cost) across the years considered for full electrification, with tighter CO_{2eq} emissions restrictions. The cost to reach full electrification by 2025 is 12-13% higher, and by

2045 around 6% higher, than in the original case. This finding suggests that there is a cost to limiting emissions levels while increasing electrification rates. The difference in costs between the four demand constraint variants are very similar to the original Rwanda system case. Additional details are given in the SI.

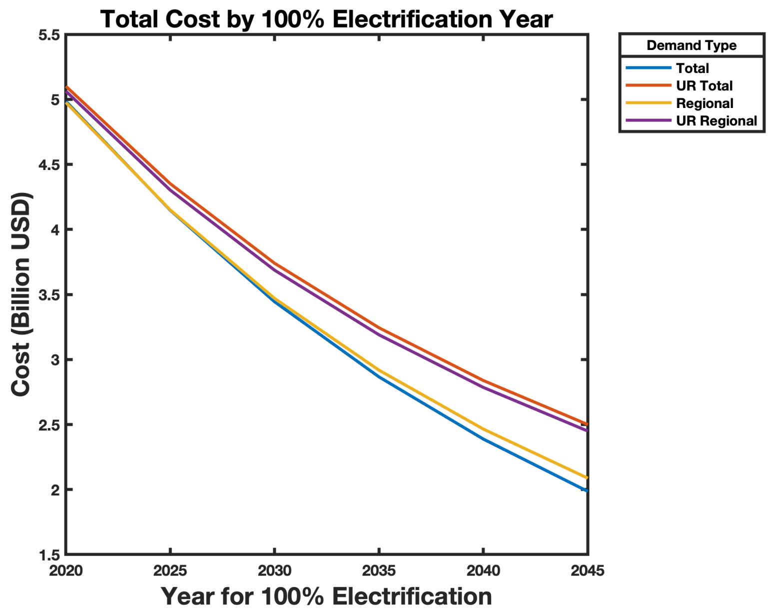


Fig. 9: Discounted total cost to reach full electrification by the given year with tighter CO_{2eq} emissions restrictions.

Figure 10 shows the generation capacity and energy output over time for the Total demand constraint model when 100% electrification is reached by 2025 or 2045, with tighter CO_{2eq} emissions constraints. The total installed capacity is much higher, around 600 MW higher by the end of the time horizon, smaller in earlier years, than when starting from the original Rwanda system. Peat is almost entirely eliminated, and individual diesel units are reduced. These resources are replaced with higher levels of solar and geothermal. Higher total capacity is necessary because solar resources are variable and uncontrollable. Total energy output levels are similar, but with a higher percent of output coming from low-carbon resources, as expected. Generation levels from diesel are much lower, and CCGT generation is shifted and geothermal generation increased to balance solar fluctuations. The differences between the four model variants are still relatively small but are shown in the SI.

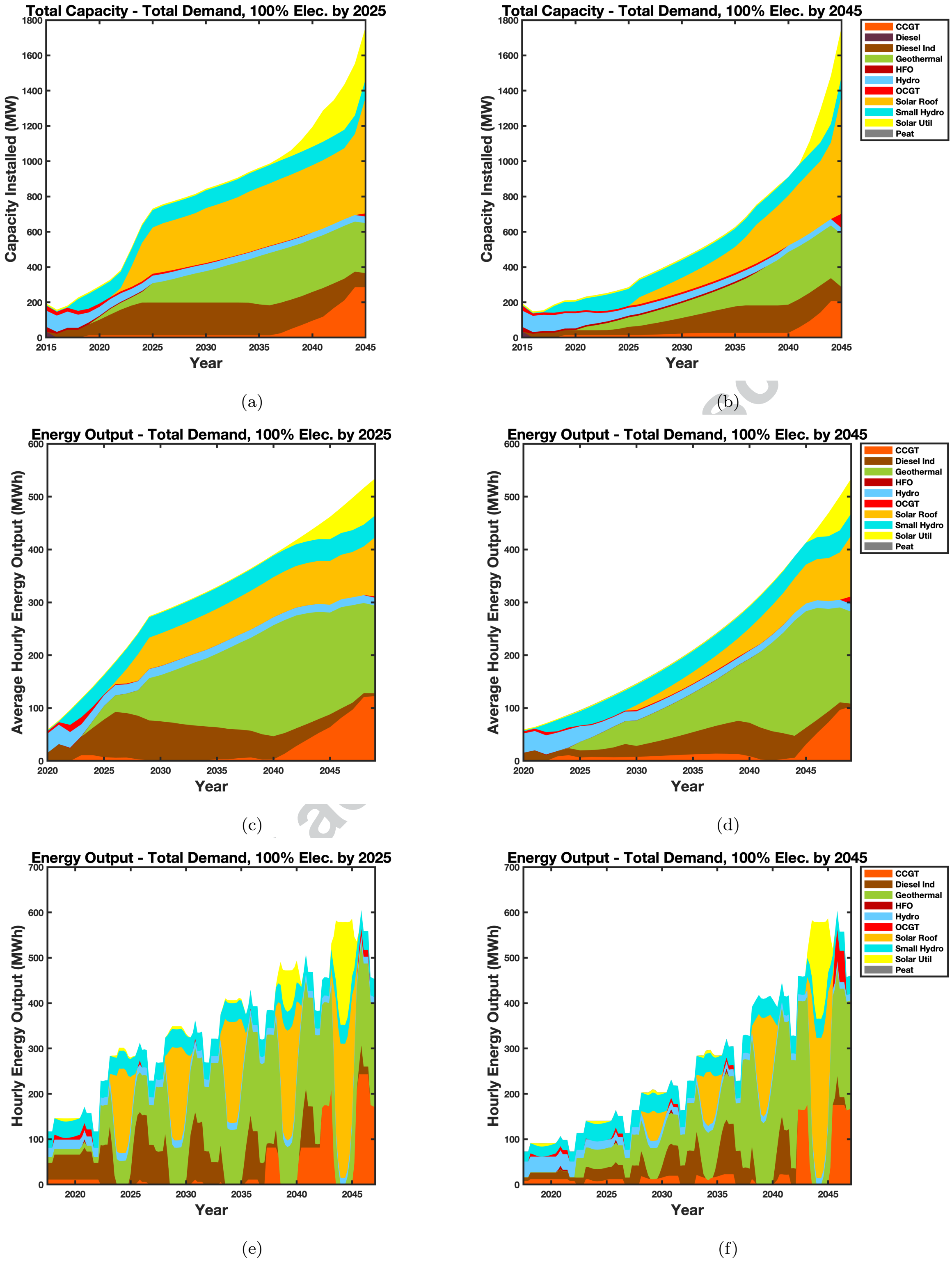


Fig. 10: Generation capacity mix ((a) and (b)), average energy output ((c) and (d)), and hourly energy output for over the representative day every fifth year ((e) and (f)) for the Total demand constraint model with tighter CO_{2eq} emissions restrictions when 100% electrification is reached by 2025 ((a), (c), and (e)) or 2045 ((a), (d), and (f)). The oscillation in the energy output in ((e) and (f)) is because a representative day is used to model hourly decisions in each year.

There are differences in the transmission system between model variants and between the original model and the model with tighter CO_{2eq} emissions restrictions. However, these differences are small relative to the transmission differences between the original system and the zero initial capacity system. Transmission maps and total transmission plots are shown in the SI.

5.3 Half Centralized or Distributed Generation Costs

A high level of distributed resources were selected in the original generation mix. In order to test the impact of cost on the balance of centralized and distributed resources selected, we re-solve the model with the costs for each of these halved. That is, we solve the model first with the cost of all centralized generation resources, specifically utility diesel, HFO, OCGT, CCGT, peat, utility hydroelectric, biomass, geothermal, and utility solar PV, halved, then with the cost of all distributed generation resources, specifically 100 kW individual diesel, small hydroelectric, and rooftop PV, halved.

Halving the cost of centralized resources causes an overall cost decrease of between 5-13%. The total cost decrease from halving the costs of distributed resources is much greater, between 37-45%. Cost plots are shown in the SI.

The generation capacity and energy output is almost identical for the case when centralized generation costs are halved as in the original results, so these results are left to the SI. The generation capacity and energy output for the Total model when 100% electrification is reached by 2025 or 2045 and distributed generation costs are halved are shown in Figure 11. Although there is slightly less generation from centralized resources in this case, as expected, overall the generation portfolio is still very similar to the resulting portfolio with the original costs. Some noticeable differences are: CCGT capacity is lower throughout the time horizon and does not increase as much at the end of the horizon as in the original results, OCGT is almost all retired immediately, and the total capacity of 100 kW diesel and rooftop solar is larger than before.

For both variants, the total transmission plots follow a similar trend to the original case, but the total transmission capacity is slightly lower, between 40-70 MW, by the end of the time horizon. The total transmission capacity at the end of the time horizon is around 10-20 MW lower when the costs of centralized resources are halved than when the costs of distributed resources are halved. As observed previously, the transmission capacity is highest for the Total model, followed by the Regional model, and lowest for the UR Total model, in general. There are some differences in where transmission lines are built when comparing the reduced cost cases to the original model and to each other, but overall, the layout of the transmission system is similar to the original results. Transmission plots and maps are shown in the SI.

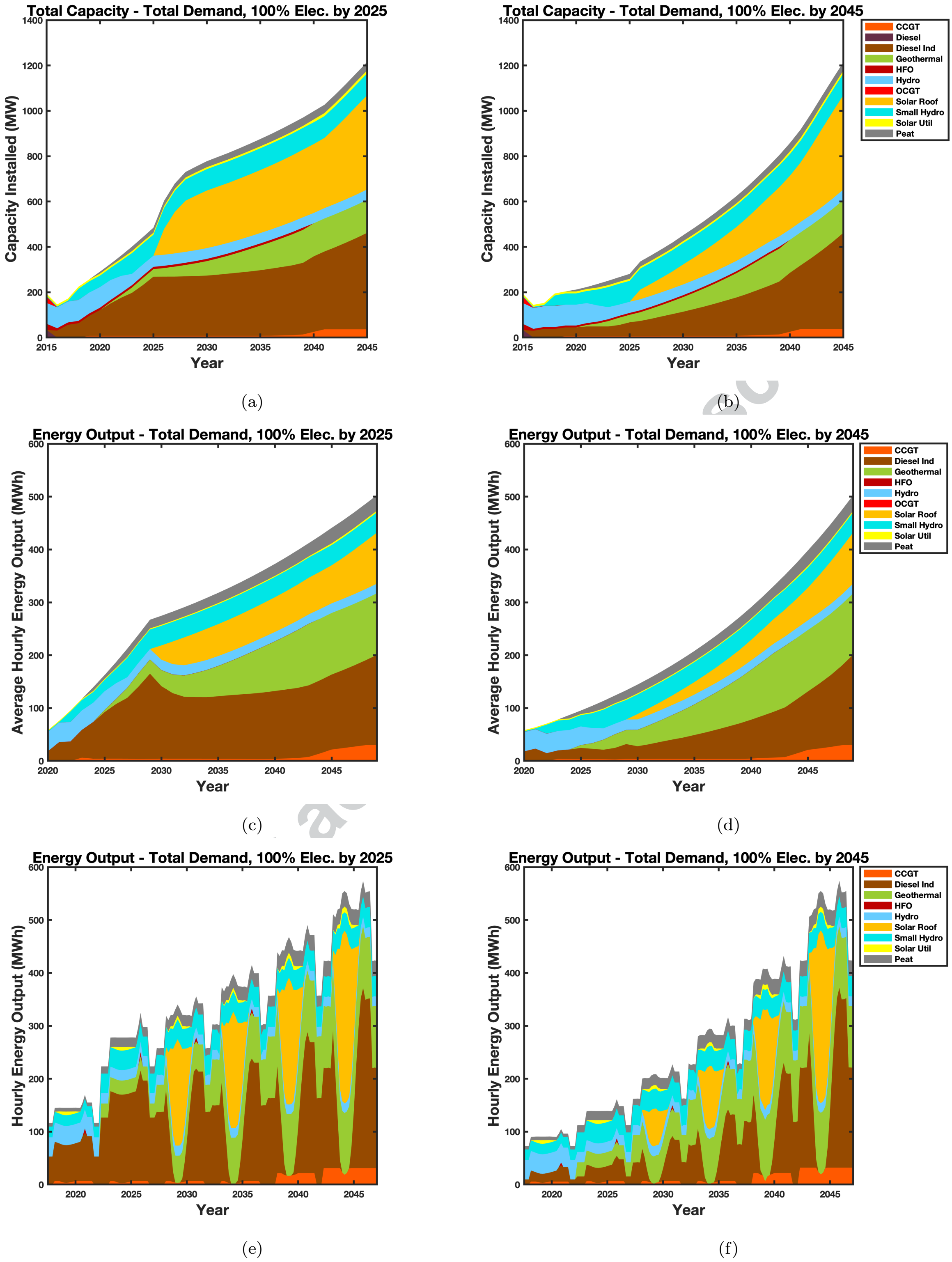


Fig. 11: Generation capacity mix ((a) and (b)), average energy output ((c) and (d)), and hourly energy output for over the representative day every fifth year ((e) and (f)) for the Total demand constraint model when distributed generation costs are halved and 100% electrification is reached by 2025 ((a), (c), and (e)) or 2045 ((a), (d), and (f)). The oscillation in the energy output in ((e) and (f)) is because a representative day is used to model hourly decisions in each year.

6 Discussion and Conclusion

In this work we develop a model for electricity development planning in sub-Saharan Africa. To study the impact of various electrification policies on power system expansion planning, we model four different options for constraining the fraction of demand met and vary the year in which 100% electrification is reached. Unlike most research on electrifying developing countries, our model includes both strategic and operational planning decisions. Specifically, our model includes decisions on how to build and operate the power system including generators, storage units, and transmission lines. We distinguish between centralized and distributed resource options as well as urban and rural regions.

We test our model on a case study of Rwanda. We compare the costs and system development across 24 different electrification cases (four model variants across six electrification trajectories). For the Rwanda case study, the year in which 100% electrification is reached affects both the cost and the installed generation capacity across the time horizon. However, the total installed capacity by the end of the time horizon is similar across all cases, as is the generation mix. That is, although varying the year in which full electrification is reached affects when generators are built, the results converge over time once 100% electrification is reached. There is a 40-50% decrease in cost between the cases when 100% of electrification is reached by 2025 and 2045. The cost to electrify earlier is higher since the effective demand is larger. The change in the total cost is also affected by the capital cost, which is heavily discounted when paid in later years. The differences in cost across the four demand constraint variants start very small and increase as the year for 100% electrification is increased.

We find that varying the year in which 100% electrification is reached has a greater impact on the generation capacity and energy output than varying the measure of the fraction of demand met. We do observe a larger amount of solar rooftop construction and generation in the UR variants of the demand constraint, which makes sense since rooftop solar is a distributed resource, more suited to meet demand in rural regions. For transmission, there is a larger difference between demand constraint variants when full electrification is reached later in the time horizon. However, even in this case, the differences in the development of the transmission system are relatively small compared to the overall transmission capacity. To test how the results would differ if the transmission system were not already largely developed, we re-run the model starting from zero initial system capacity. In this case, there are still only small differences in generation capacity across the four demand constraint variants, but significant differences in transmission system development.

We also test the effect of restricting CO_{2eq} emissions to not exceed current levels. Tightening this constraint has a strong impact on generation, compared to the results for the original Rwanda system. Besides resulting in a different the mix of resources for generation capacity and energy output, tightening the CO_{2eq} emissions constraint leads to much higher total capacity. The difference in generation capacity and output between the four different demand constraints is still small relative to the overall capacity, but there are large system differences when varying the year for 100% electrification. The total transmission capacity required for this case is also larger than when starting from the initial Rwanda system. These increased capacity requirements lead to higher costs, demonstrating the importance of considering emissions reduction goals in conjunction with electrification goals when the development budget is limited. If such tight emissions restrictions were required to be satisfied within a limited budget, one option for achieving both goals would be to decrease the development rate. The cost to electrify decreases fairly rapidly as the year for full electrification increases, such that even these very strict CO_{2eq} emissions restrictions could be achieved for the same cost or less as in the original results by delaying the year to reach 100% electrification by at most four years. Alternatively, since the cost for the Total and Regional models is less than for the UR Total and UR Regional models, one of these variants could be chosen to allow for more rapid development while still satisfying emissions restrictions.

We test the impact of varying the cost of centralized and distributed resources on the balance of these two types of resources by halving the cost of each. Reducing the cost of centralized resources has almost no impact on the generation mix and a relatively small impact on cost and transmission capacity. Reducing the cost of distributed resources has a large impact on cost, but still a relatively small impact on the generation portfolio and transmission system development. It is interesting that the total transmission is slightly more in the case when distributed generation costs are halved. One possible explanation is that since fewer centralized generation types are built in this case, more transmission is required from the areas in which centralized generation still is built. The difference in transmission capacity is very small between the two cases though, and compared to the results with the original costs, both cases of halved generation cost result in slightly reduced transmission capacity.

Overall, we conclude that the generation mix is fairly robust to variations in the demand metric and generation cost, but not to changes in emissions restrictions. Thus, when developing a generation expansion plan, emissions targets should be carefully considered. Variations in the demand constraints have a larger effect on transmission system expansion, especially with a less developed initial system. One option for decision makers to develop a more sustainable and equitable power system expansion plan is to first decide on a desired emissions target, after which a rough mix of which generation resources to use can be determined. The decision maker can then compare the cost to electrification rate trade-offs to determine how aggressively to increase electrification. The model can then be run for that electrification year to determine the optimal transmission system, given the decision maker's preferred demand metric. The results will also specify the location and capacity of new generation construction over time.

One of the challenges of modeling electricity development in sub-Saharan Africa is the lack of data availability. In our initial survey of electrification data for sub-Saharan Africa, we considered five neighboring countries: Burundi, the Democratic Republic of the Congo, Rwanda, Tanzania, and Uganda. Of these, Rwanda was the country for which data availability was the best, which was part of the reason we chose it for our initial analysis. However, our model could directly be applied to other countries in sub-Saharan Africa or other regions where the electricity system is under-developed.

Through compiling data from a variety of resources and making approximations where necessary, we have gained a detailed understanding of the existing electricity system in Rwanda (details included in the SI), which we hope will be useful for others as well. Additionally, as new and better data becomes available for Rwanda or other developing countries, our models could be re-run understand system development options as electrification priorities and emissions regulations evolve.

Acknowledgements This work was supported by ExxonMobil through the Georgia Institute of Technology. This work was performed in part under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

The authors would like to thank the three anonymous reviewers whose feedback helped to improve the quality of this paper. We would like to thank Paul Rugambwa who provided us with some relevant resources and helped us form our initial understanding of the electrification environment in Rwanda. We would also like to thank Todd Levin for answering questions on his previous modeling efforts, which provided a starting point for our model.

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