COMMODITY SWAPS

by

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ABSTRACT

Commodity swaps are in the stage of development that interest rate swaps were at the beginning of this decade. Even so their rate of growth promises to outstrip the corresponding rate of growth of interest rate swaps. Interest rate swaps have grown enormously and evolved in many directions in this decade. They have indeed become a mainstay of risk management techniques.

Commodity swaps and interest rate swaps are alike in many respects. The one major obstacle to the development of commodity swaps, namely regulation by the Commodity Futures Trading Corporation, seems to have been circumvented for the time being. This favorable circumstance has brought a number of new players into the field. Thus this area seems poised for a take off. One of the desiderata is the development of an analytical framework for commodity swaps.

This thesis considers commodity swaps in a framework similar to interest rate swaps. A mathematical model for pricing a commodity swap is described. Further extensions such as caps and floors are considered next. A hedging strategy for caps on oil prices is simulated using historical oil futures prices (West Texas Intermediate on the New York Mercantile Exchange). Finally, some possible directions for future developments in this field are explored.

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CHAPTER I

INTRODUCTION

One of the salient features of the development of the financial services industry in the past decade has been the proliferation of new financial instruments. This can be traced in part to increasing deregulation, availability of computing power and communications, and the increasing globalization of business. One area that has been particularly active has been that of risk management techniques.

The need for such techniques has been fostered by the increasing risk in the financial markets. The causes for such a rise in risk in the past fifteen years have been classified as follows (1).

(1) High and volatile inflation and nominal interest rates.

(2) Growth and persistence of government deficits.

(3) Exchange rate volatility associated with current account imbalances.

(4) Desegmentation of capital markets.

(5) Shifts in global portfolio preferences.

Among the new risk management instruments, swaps have been an outstanding success. Interest rate swaps and currency swaps have become very popular and this popularity has spawned a number of other derivative products such as caps, floors and collars to mention only a few. The rationale for the existence of these products is that they allow a corporation to manage its exposure to interest rate risk or currency risk and thus free its energies to deal

with its primary non-financial business. Another group of products that can help corporations manage their exposure to commodity price risk would be commodity swaps and allied products.

As commodity swaps are, in principle, very similar to interest rate swaps, we shall describe the mechanics of the mature interest rate swap market in some but not exhaustive detail since many expositions of this topic including books are available (2). We will attempt to tie in the commodity swap market into this framework. Then we shall describe a model that would allow us to price such commodity swaps and formulate appropriate hedging strategies. The next step would be the extension of this model to price caps and floors on commodity swaps and appropriately hedge these instruments as well. This model can then be simulated using historical data for prices of the commodity and the hedging instruments. Finally we can explore the direction in which further developments could follow.

CHAPTER II

INTEREST RATE SWAPS

Since their introduction in the early eighties, interest rate swaps whether in a single currency (typically dollar) or cross-currency has grown explosively so that the current volume in 1988 was, as estimated by the International Swap Dealers Association, of the order of a trillion dollars as measured by the notional principal outstanding (3). In its simplest incarnation two institutional borrowers in a quid-pro-quo agreement decide to take on each other's interest payments. This involves swapping the payment streams on their borrowing (not the loan principal)from fixed to floating-rate or the other way around in one particular currency.

The original impetus toward swaps was an arbitrage opportunity because investors in the fixed income instruments are more credit sensitive than investors in floating rate instruments mainly because of the short-term nature of the latter. Thus a lower credit rating is a detriment when it comes to borrowing in the fixed rate capital markets. The swap essentially uses this disparity perceived of as an arbitrage opportunity while it is in fact an example of a risk return correlation. This aspect of the plain vanilla swap can be shown by the following example.

If a AAA credit rated institution is assumed to be able to borrow fixed-rate loans at a 11% rate of interest but is interested in a floating-rate loan. The counterparty is another institution

whose credit rating is lower than AAA and can borrow fixed-rate loans at 12.5% interest rate and is somehow trying to lower this cost. Since the credit rating sensitivity is lower in the floating-rate capital markets, we can assume that the AAA institution would be able to raise money at LIBOR (London Interbank Offered Rate) plus 0.25% while the lower-rated counterparty can raise money at LIBOR + 0.75%. Thus we have picked the AAA institution to have a 150 basis point advantage in the fixed rate market while enjoying a considerably less, only a 50 basis point advantage in the floating rate market.

Prior to entering into this swap, the two institutions take out loans for the same notional principal in the markets where each has a comparative advantage. Thus the AAA institution borrows in the fixed rate market and the counterparty in the floating rate market and they remain responsible for these throughout the life of the swap and the loan. The swap could then be a contract in which the AAA institution agrees to pay LIBOR to the counterparty in return for 11.20% and they could use this to pay off their 11% liability and thus achieve LIBOR minus 0.20% financing. From the vantage point of the counterparty, we have LIBOR payments which pay part of their LIBOR plus 0.75% debt. This 0.75% differential added to their 11.20% swap obligation leads to fixed rate loan of 11.95% which is considerably less than the 12.5% they would have had to pay. Thus both parties have benefited.

If this example looks contrived because the numbers picked above are notional, the following data from the Federal Reserve Bulletin shows the quality spread in US fixed rate corporate bond market for borrowers rated Baa vs Aaa (4).

MONTH	1982	1983	1984	1985
JAN	1.92	2.15	1.45	1.18
FEB	1.91	1.94	1.51	1.10
MAR	2.24	1.88	1.42	1.13
APR	2.32	1.78	1.50	1.28
MAY	2.38	1.63	1.46	1.43
JUN	2.11	1.63	1.50	1.46
JUL	2.19	1.24	1.71	1.46
AUG	2.61	1.13	1.76	1.45
SEP	2.69	1.18	1.69	- -
OCT	2.61	1.21	1.31	- -
NOV	2.62	1.20	1.19	- -
DEC	2.31	1.18	1.27	- -

In 1982 the year with the widest spread, the spread between BBB and AAA rated borrowers was even more dramatic reaching up to 500 basis points (5).

WHY INTEREST RATE SWAPS?

Initially such attractive arbitrage opportunities brought many institutional players into the market for swaps. Now they are used by the top 1000 corporations along with other players (6). Investment banks and commercial banks are generally the intermediaries. It cannot be denied that some institutional

investors engage in interest rate swaps for purposes of pure speculation.

The importance of swaps is reflected by the fact that 30 to 40% of all capital market transactions now involve an interest rate or a currency swap (7). Similarly, as early as in 1985, 75% of of the Euro dollar bond issues were swapped (8). That the swap market has become an integral part of corporate financing strategy becomes obvious when General Electric Capital completed a $1.2 billion five-year bond issue in March 1989, it made a point of announcing that the issue had not been swapped. While it is obvious that cost savings can be effected by combining bond issues with a swap, the source of this cost saving over single transactions bears investigation.

We have already seen how simple arbitrage (so called) across the fixed and floating-rate market could lead to savings. The catch in such an argument is that in an efficient market the very act of taking advantage of this arbitrage opportunity should soon eliminate it. This means that a swap transaction, in effect, increases the demand for funds in the lower interest rate market and diminishes the demand for funds in the higher interest rate market leading to a narrowing of the differential in the two markets till all arbitrage opportunities are eliminated.

This argument is borne out by the facts. The importance of financial arbitrage has gone down steadily since the inception of the swaps. Spreads which were common originally have been narrowed

considerably by the very act of taking advantage of the arbitrage opportunity. If we define swap spread as the fixed spread minus the floating spread, data indicate that swap spreads had shrunk from about 100 basis points in 1983 to 7 to 15 basis points by 1986. Similarly the corresponding fees charged by the intermediary had fallen from 30 - 40 basis points to 3 -8 basis points (6). In fact some issuers find it more cost effective to approach the floating rate market directly rather than engage in a bond/swap transaction (9). This implies that there must be other factors driving the continuing expansion of the swaps markets.

The next possibility is one of savings tied to transaction costs associated with managing interest rate exposures. Thus if we consider a scenario of falling interest rates with expectations of further declines in which a company wants to refinance long-term fixed debt. One way to do it would be call the outstanding bonds (assuming the call provision to be operable) and then borrow at the new floating rate. An alternative would be to engage in an interest rate swap and exchange fixed-rate for floating. The latter could be preferred because of lower transaction costs. Of course, individual cases would have various cost savings, if any.

Another possibility is the availability of tax and regulatory arbitrage. This is even more powerful when the interest rate swap is accompanied by a currency swap as well. This allows an institution to take advantage of differing tax codes and regulatory statutes in various countries. This has become practical only due to

the increasing globalization of markets. Previously, dollar denominated bonds generally originated in the US and had to comply with US regulatory provisions and the buyers moreover were subject to the US tax laws. Now the issuer could go to the Eurodollar market or to any country that offers preferential regulatory or tax treatment and then swap the debt into the domestic market at fixed rate or floating rate and still enjoy the original benefits. This sort of arbitrage is unlikely to disappear till the emergence of a uniform regulatory and tax structure around the world (10).

Finally, the swaps markets contributed to the completion and integration of financial markets by providing opportunities to fill in some of the gaps. For example, until recently there was a gap in the markets left by the unavailability of forward rate agreements (FRAs). Since an interest rate swap could be thought of as a series of FRAs, a swap could be used in place of an FRA. Thus one could synthetically fill in the gap in the market (10a).

There are other benefits to the interest rate swap than just cost effectiveness. These can be summarized as follows:

1) The major advantage that has been a driving force behind swaps has been the fact that credit risk exposure marked under a deal is typically a small fraction of the notional principal. Of course this exposure is still considerable compared to the fees involved. Still, one makes very efficient use of interbank lines and the potential number of participants is increased greatly.

2) This again is a bookkeeping device that is very advantageous to banks who are major players in this game. The deal is entirely off-balance-sheet like other forward positions and may occasionally be noted in the various footnotes. FASB requires corporations to footnote swaps only if they have a "material effect" on the balance sheet. While the banks are required to monitor their swap and other forward exposures as contingency commitments, they are not currently required to make any reserve or capital allocations against them. This is being changed even as we write but the full changes will take effect only in 1992.

3) Since the swap is distinct from the lending, the decision to issue debt can be separated from the interest rate structure decision. This becomes a source of flexibility. Thus one can borrow money and then adjust the rate. In practice the swap might even be done before the debt if it is feared that the rates are rising and the borrowing can not be done quickly enough. The firm can cancel a swap prior to funding maturity. It can swap only a percentage of funding or enter into a swap which will not begin for a specified period of time. Moreover the firm can continuously readjust its debt portfolio without recourse to capital markets.

4) This once again impinges on the convenience issue. Since a swap is independent of the loan, the required paperwork is not great which allows for deals to be done rapidly and even ahead of the loan itself as noted above.

THE EVOLUTION OF INTEREST RATE SWAPS

We have seen how the interest rate swaps initiated in 1982 offered many advantages in the volatile interest rate environment. We shall now see how this market has come of age and how it has spawned other derivative products. In the category of maturation of the product, we have seen swaps progressing from a custom-made, circumstance-specific product to a fairly standardized forms. Innovation has come about because competition has lowered profits possible from "plain vanilla" swaps as we have already noted.

The earliest variety swaps were ones in which an investment bank or a commercial bank acted as an intermediary and matched two parties which desired the two separate legs of the transaction. This extended even to the amount of the notional principal even though it was an easy enough task to find multiple parties to match the principal amount as well. Thus the expertise needed was one of problem solving and a large client base rather than any large amounts of capital. This meant that the investment banks took the lead in this area. The only commercial bank that played a role was Morgan Guaranty.

With growing activity in this field, efforts began towards standardization of the product. An industry group, International Swap Dealers Association, ISDA was created to aid in this process particularly after a landmark court decision in Homestead vs Life

Savings held that a swap was null and void because there was no binding contract. ISDA came up with a Code of Standard Wording, Assumptions, and Provisions for Swaps that serves as the de facto industry standard (9).

Moreover, it was realized that it made no sense to look for counterparties for each and every transaction and the intermediaries started taking positions. They could then look for counterparties at a later time or match it internally with an offsetting position or just hedge the position using various financial instruments. This naturally meant that the capital commitment became higher and that commercial banks took the leading role in swap transactions. With the continuation of this trend, one could expect a homogeneous swap market with high volumes, low margins turning into a capital-intensive area favoring the commercial banks (11).

However the commercial banks are not in favor of complete standardization of the swaps which could lead to their becoming tradeable on exchanges much like financial futures or options. Such a move would favor the investment banks which would no longer have to carry the long-term transactions on their books (12). They would moreover be able to dominate the market because of their expertise in trading securities.

In the area of product innovation in interest rate risk management we have reverse swaps, swaptions, caps, floors and collars, contingent swaps, callable swaps, extendable and

retractable swaps, swaps against financial futures, against private placements, against tax-exempt securities, zero coupon swaps and finally floating rate against floating rate swaps (13). Most of these can be readily understood. As to the genealogy of these products, we shall content ourselves with just an overview of some of the interesting products.

Almost immediately after the arrival of the swaps on the market, the idea of the reverse swap came about when the parties wanted to revert to their original positions. Next came the choice of various floating rate benchmarks. While LIBOR was the most common, prime rate or certificate of deposit composite etc. could also be used. These differing standards led to the floating-rate to floating-rate swap. The first such deal that was developed was the LIBOR-prime swap. The rationale for such a deal could vary a lot based on whether matching asset and liability exposures in the two floating bases. One could think of such a move to swap a proportion of one's exposure in one floating index to other indices as a means of portfolio diversification. As long as the two indices are not perfectly positively correlated, one can reduce risk.

Some of the other products are fairly self-explanatory. Interest rate caps are provided as protection against an unexpectedly steep rise in rates. One could provide an interest rate collar by buying a cap and simultaneously selling a floor. It is possible to structure this transaction so that there is no upfront fee. Another way to protect oneself against movement of interest

rate beyond a window is to buy a swaption or an option to engage in a swap. A commonly used form of a swaption is an option to terminate the swap. Other more recent forms are call and put swaptions on bank debt and swaptions on corporate debt introduced in January 1988 (14).

OTHER FEATURES OF THE INTEREST RATE SWAP MARKET

The two other areas of interest that should be mentioned are interrelated. The first is the area of credit risk and attendant regulatory problems. The second is the secondary market in swaps which while fairly liquid still suffers from problems related to credit risk.

Credit risk has assumed a new dimension because the swap market saw its first defaults earlier this year in Britain. The local government authorities of Hammersmith & Fulham, Harlow and Blackburn defaulted on their swap obligations. Apparently they lacked the legal authority to enter into these speculative swaps. The investment bank that arranged the deals, Salomon Brothers, had not considered the possibility that a western governmental body would not make good on its commitments. These local governments were apparently enticed by the front-end fees. When the market moved against them they were, having spent the money, unable to pay up. The notional principal involved has been estimated to be between 3 to 10 billion pounds. It is possible that to avoid recurrence, the British parliament will enact legislation retroactively legalizing these swaps (15).

It was precisely this kind of scenario that has led the Federal Reserve to include interest rate swap exposure in determining capital adequacy requirements. These new regulations do not take effect until 1992 as already mentioned. One effect of this proposed regulation is that it biases the market in favor of the investment banks that do not fall under the supervision of the Fed. There is also nothing to prevent corporations from over-leveraging themselves by means of swaps or banks from exposing themselves to interest rate risk. In 1983, Continental Illinois unwound a large portion of its swaps portfolio to cushion its massive losses. While it achieved its short-term goal, its balance sheet was left open to interest rate risk (12).

Another way to avoid credit risk is being pioneered by Manufacturers Hanover. This innovation is called the marked-to-market swap. The basic idea is very simple. The main risk as we have seen is the party on the losing side of the swap (i.e. the party that took the floating side in a rising interest rate environment or the party that took fixed in a falling interest rate environment) might go broke. This scenario is further complicated by the different bankruptcy laws in the various countries from which the swap parties are drawn. A way to protect oneself is to turn the swaps from their forward contract like character to one resembling a futures contract.

A forward contract is an agreement to buy or sell a commodity or fulfill a financial obligation at a set time in the future. This contract is secured by nothing more than the general creditworthiness of the parties. If the market conditions should adversely impact on one of the parties, that party is under no obligation to post a margin or collateral to guarantee adherence to the contract on the settlement date. If the contract had been a futures contract, an intermediary, the clearinghouse would have been involved and made sure that adequate collateral is posted on an ongoing basis. One could similarly withdraw one's profits on a daily basis.

Manufacturers Hanover's Marked-to-Market swap is reviewed every six months. If the client agreed to pay 10% fixed and the market has moved to 9%, he is charged a marked-to-market payment and in return his coupon payments are reduced to the current market rate of 9%. The structure of the swap remains the same but long-term exposure that will be subject to capital requirements has been turned into a series of short-term exposures which the regulators look upon more favorably and are not subject to capital requirements (15).

The problem with these marked to market or other collateralized swaps is that they require a degree of administrative sophistication and cash flow flexibility that may not be available to the average corporate borrower. What happens is that swap is made more secure at the expense of imposing uneven cash flows on the

parties. This might be something that a highly leveraged corporation might find unpalatable. In any case, one needs a way to quantify and incorporate credit risk into the structure of swaps.

Another feature that emphasizes the maturity of the interest rate swap market is the existence of a secondary market for swaps. This market has now become fairly liquid with the main players being the commercial and investment banks. They have assumed the role of the counterparty to most transactions. Instead of undoing a swap by entering into the reverse of the first one with a different customer and thus doubling their credit risk they have the option of trading it on the secondary market. However before the swaps could become a truly tradeable instrument the issue of credit risk that we dealt with at length would have to be resolved.

CHAPTER III

COMMODITY SWAPS

A commodity swap is structured very similarly to an interest rate swap except that here we are talking about a deal indexed to the price of a commodity, typically oil rather than LIBOR (16). The mechanics of a fully offset oil swap look like this. An oil producer wants to lock in a set price for its oil. It goes to a swap dealer, let us say a bank which agrees to pay the producer a set price over a period of time. In return the producer pays the bank the average market price of oil. In the other leg of the swap, a consumer wants to lock in the price it will pay for oil and once again goes to the bank to arrange this transaction. The consumer pays the bank a set price over a period of time and in return the bank pays the consumer the average market price for the same period of time. The banks profit is the difference between the price it receives from the consumer and the price it pays the producer. In the simple case where the time period for both legs of the swap is the same it has merely matched up the needs of the producer and the consumer and earned a profit for doing so (17).

The area of commodity swaps is currently at the position interest rate swaps were in the early eighties. Transaction volume is only of the order of a few billion dollars but the market is poised for a major takeoff. The field is dominated by a handful of financial institutions. Chase Manhattan, Bankers Trust and Banque Paribas among the commercial banks and Salomon Brothers representing the

investment banks. Shearson Lehman Hutton has just got into the act as well.

One of the major reasons for the sudden activity in this field is the recent ruling by the CFTC that commodity swaps do not constitute commodity futures and hence will not be regulated by the CFTC provided certain conditions are met. Prior to this ruling, most transactions were carried out offshore, typically in London. According to David Yeres, former counsel to the chairman of the CFTC, "Commodity swaps have progressed faster and further in the two months since the CFTC policy statement than did interest rate swaps in two years when they began" (3). We shall come back to the regulatory stance later.

GENESIS OF COMMODITY SWAPS:

The early commodity swaps that were considered were oil swaps as means of restructuring LDC debts. The problem with LDC debts had arisen because the loans had been made originally with the idea that the price of oil would stay artificially high. When the oil prices plunged, a number of third world countries experienced cash flow problems and could not meet interest payments on their debt let alone repay the principal. With an oil swap, they could lock in a set price for the oil and thus use it to reschedule their debts. Although this was all well in theory, there was no agreement on the price of oil and the scheme fell apart.

In the middle eighties, financial institutions got commodity swaps going between commercial producers and consumers rather than sovereign governments. Initially they were of the type already described, i.e. a fully offset swap in which the bank acts merely as an intermediary matching up the needs of the two parties. There were two problems with this approach. One was that it was difficult to find two counterparties whose needs matched perfectly. The other was that consumers usually want to buy on a downtick in the market and producers want to sell on an uptick in the market. Thus the intermediaries have had to take one side of the transaction and hedge their exposure and then look for the other side of the transaction to offset their position.

DIFFERENCES BETWEEN COMMODITY AND INTEREST RATE SWAPS:

Up to now we have seen the ways in which the commodity swaps mirror the interest rate swaps. However there are a couple of major differences. The first factor is the multitude of hedging tools that are available in interest rates including thirty year treasury bonds. In case of oil one is limited to the oil futures, typically the West Texas Intermediate (WIT) futures traded on the New York Mercantile Exchange (NYMEX). A corollary to this is that while the term structure for interest rates is observable for periods up to thirty years, forward prices on most commodities is not observable past a few months. Thus forwards on say oil prices must be created synthetically.

The second point is that in the case of term structure interest rates, one can calculate the implied forward rates based an assumption of no arbitrage. In the case of commodities this arbitrage argument becomes a little more complicated. If the forward price is higher than the spot price plus storage and insurance costs and the interest on the capital, an arbitrage profit could be locked in. If the forward price is lower than spot price plus the interest, the reverse arbitrage would require borrowing the commodity and selling it at spot and repaying the commodity at maturity and locking in an arbitrage profit. However in case of commodities like oil borrowing is not possible and backwardation (backwardation for storable commodities is not the same as for, say, interest rates) is frequently observed. Our hedging strategy must take this fact into account as well.

FUTURES ROLL STRATEGY

A simple numerical example given below will clarify what the roll strategy involves and it will be generalized in the mathematical appendix and then the model will be expanded to swaps as well.

Consider a simple two period situation. Current time is time 0 and we have two future periods time 1 and time 2. Futures contracts are available just one period out. For simplicity we shall assume a zero rate of interest and treat the futures as though they were forwards by ignoring the marked to market process. Let S(i) and F(i) denote the spot and the future prices at time i. This means

that time i we pay S(i) for immediate delivery and F(i) for delivery at time i+1.

If our requirement was such that we needed one unit of oil at time 1 and one unit at time 2 and the price structure was as follows (not observable at time 0 but from a later vantage point). This is a simplistic example merely to demonstrate the general idea. A more rigorous treatment is found in Appendix I.

S(0) = \$18	F(0) = \$19
S(1) = \$20	F(1) = \$21
S(2) = \$22	

If we did nothing fancy and bought one unit of oil at time 1 and one unit at time 2, our price would be \$20 + \$22 = \$42. If instead we bought two futures contracts at time 0 for \$19 apiece. At time 1 we take delivery on the futures and use 1 unit for our needs and sell the other at spot for \$20 and buy another future contract for \$21. At time 2, we merely take delivery and use the oil for our needs. The cashflows are shown below

Time 0	\$0
Time 1	2 x \$19 - \$20 = \$18
Time 2	\$21
Total	\$39

Thus we have reduced our cost oil from \$42 to \$39 in a predictable way. Note however that we have made an assumption that the premium of future price over the spot price remains the

same. We can show that the spot price does not matter as we have been able to lock in the price at time 1 and time 2. To lock in the price at time 1, we merely had to buy the future. If we denote the one period forward as Q(1), then Q(1) = F(0). The two period forward Q(2) is constructed by rolling over one period futures. If we denote d(i) to be the slope of the forward price curve at time i, we see that,

$$\begin{aligned} Q(2) &= (F(0) - S(1)) + F(1) \\ &= S(0) + (F(0) - S(0)) + (F(1) - S(1)) \\ &= S(0) + d(0) + d(1) \end{aligned}$$

We had assumed that d(i) was constant and in this numerical example equal to $1. Thus Q(2) works out to be $20. Our cost as before is Q(1) + Q(2) = $19 + $20 = $39. Thus if slope were not constant we have converted price risk into slope risk. In the mathematical appendix, slope is considered to be a mean-reverting process and the model is built up.

PRICING THE SWAP

In pricing the swap, the floating part is generally tied to an index such as spot oil prices. The fixed amount can be calculated as the average amount, adjusted for the time value of money, that the dealer can expect to break even depending on the cost of hedging. In practice one adds on charges for credit risk, agency costs and variance from the expected cost. Another problem is that while the time period for the hedging instrument, say the oil futures, is monthly, the swap settlement is typically made every six months. This creates the need to average the claims over the longer time

period. All in all one can calculate the outlays involved in synthetically creating the floating side by rolling over the futures as described. Thus knowing the fixed payments, we can calculate the expected value of the breakeven point and it can be termed the averaging claim. This scheme provides only an indication of the upper boundary of the cost. In practice, if an offsetting leg is entered into, the market risk is taken care off and the hedging costs are no longer significant.

OTHER DERIVATIVE INSTRUMENTS:

As in the case of interest rate swaps, the party that has the floating part, can opt for a cap which limits the ceiling beyond which payments would not float or a floor below which the floating would not fall. These caps and floors can be thought of as simple options on the floating side and priced using a Black-Scholes option valuation formula using the averaging claim as the underlying.

The Black-Scholes model needs the volatility of the underlying as a necessary input. This was accomplished by calculating the value of the underlying based on the daily futures quotes on the WTI futures on the NYMEX. Another consideration was choosing the appropriate time interval to calculate the volatility. Data going back to over a year from the present (summer of 1989) were conveniently available. These data are shown in Appendix II as six monthly futures quotes obtained every business day. The volatility number derived would not only help in the simulation of the same historic data but also be meaningful in quoting prices in the near

future. Appropriate values for the mean reverting process are chosen based on econometric analysis of the futures data. Thus one gets a value for the averaging claim based on the daily futures prices. Assuming 252 business days per year, one could come up with an annual volatility number for the underlying averaging claim.

The Black-Scholes model as used (run with an arbitrary but realistic strike price of $18/barrel for oil) not only calculates the values of the caps and the floors but also the various corresponding sensitivities. Namely the option deltas or the sensitivity of the price of the option to changes in the value of the underlying or gamma, the sensitivity of the delta to changes in the value of the underlying, i.e. a second derivative.

HEDGING STRATEGY:

A way to check the internal self-consistency of the model above is to check the possibility of hedging one of the options, say a cap, using the Cox, Ross, Rubinstein binomial model in which an option is equivalent to a combination of delta times the underlying and a bond (this is synonymous with lending or borrowing). If initially one starts out with a portfolio of the option, the underlying and bond with total value of zero (that is perfectly hedged), it should be possible to constantly readjust the hedge to leave the value of the portfolio close to zero. This can be seen below.

If V is the value of the portfolio at any given time, C is the value of the cap, n is the value of the delta of the cap, A is the value

of the underlying averaging claim and B is the value of the bond in the portfolio and r is the daily rate of interest on the bond. We can see that:

At t=0 $V = C(t_0) - n_0A(t_0) + B_0 = 0$

Which gives

$$B_0 = n_0A(t_0) - C(t_0)$$

At t=1 $V = C(t_1) - n_0A(t_1) + B_0(1+r)$

$$= C(t_1) - n_1A(t_1) + (n_1 - n_0)A(t_1) + B_0(1+r)$$

$$= C(t_1) - n_1A(t_{1)} + B_1$$

Thus we have a recursive formula for B_t. At settlement date

At t=T $V = C(T) - n_{T-1}A(T) + B_{T-1}(1+r)$

These values of the total portfolio should all be ideally zero. As shown in Appendix III under the column val, these values starting out from zero fluctuate between minus 4 cents to plus 5 cents. This is a fair indication that the model is selfconsistent.

FUTURE POSSIBILITIES:

One can imagine the commodity swap market evolving in ways similar to that of the interest rate swap market both in terms of product maturation as well as product evolution. While it would be commercially viable to have caps, floors, collars or floating to floating commodity swaps indexed to various market indicators. While it is easy to foresee the rise of commodity swaptions or

perhaps marked to market commodity swaps, there are other possibilities as well.

One could think of commodity LIBOR swaps. These could be used by firms issuing commodity indexed securities who have no commodity exposure. The securities would appeal to buyers who want the commodity exposure and are attracted to the issuer because of credit considerations. These hybrid deals have to cope with two problems. One is regulatory and the other is demand. The first commodity linked deal was a bond offering from SOHIO in 1986 which paid back principal plus a kicker if the value of oil rose above a certain level. There was a flurry of activity following this but the market never took off. With the increasing popularity of commodity swaps and sophistication of the investors hybrid deals might find a niche. Secondary markets in commodity swaps seem precluded for the time being by regulators.

REGULATORY POLICY:

The CFTC has ruled that commodity swaps have to follow minimum guidelines designed to keep these swaps from looking too much like futures. Swaps must carry individually tailored terms as well as provisions that bar the completed swap from being sold to another party without permission from the counterparty. The deals can be undertaken only in conjunction with a company's line of business and can't be sold to the public. As for commodity linked deals, the significant restriction is that the implied option portion of the deal cannot account for more than 40% of the value of the

security. The chief effect of these regulations would be ensure that commodity swaps are never as liquid as futures. This is a facet that appeals to some players (3).

CONCLUSIONS:

It seems possible that commodity swaps could grow to rival interest rate swaps in terms of scope and activity. It would be very interesting to follow the developments in this field in the next few years. As has been noted, oil is a commodity whose trading volume is second only to that of money.

APPENDIX I

MATHEMATICAL MODELS

At first we shall consider the futures roll strategy and then extend the treatment to swaps. We begin by assuming that there are only two liquid markets, a spot market and a one month futures contract. One starts off with a long position in the futures and at the end of the month it is offset at spot price and rolled over into the next month's future. This is pretty much as we described in the numerical example. If we index months by the variable m and suppose that the forward agreement is initiated at month zero for settlement at month T. We further assume that full futures price change occurs at the end of the month. If we denote the following:

S(m) = Spot price at month m

F(m) = One month out futures price at month m

N(m) = Long futures position at month m

P(T) = Forward price for settlement at month T

CF(m) = Cash flow on the hedged position at month m

Then we have:

$$CF(m) = \begin{cases} 0, & m=0 \\ N(m-1)(S(m) - F(m-1)), & 0<m<T \quad (1) \\ N(T-1)(S(T) - F(T-1)) + P(T) - S(T), & m=T \end{cases}$$

The present value of the net cash flows discounted at the continuously compounded rate of interest, r, is denoted V(T)

$$V(T) = \sum_{m=0}^{T} R^{-m} CF(m)$$

$$V(T) = \sum_{m=1}^{T} R^{-m} N(m-1)(S(m) - F(m-1)) + R^{-T} (P(T) - S(T)) \qquad (2)$$

where $R = e^{r/12}$.

The relationship between the futures price and the spot price can be assumed to be linear. If we define D(m) to be the adjusted spread between cash and futures prices which for the time being is considered to be independent of S(m)s. We can also define a constant C that measures the systematic impact of a change in the spot price on the futures price or in other words,

$C = Cov[F(m),S(m)]/Var[S(m)]$ and

$$F(m) = CS(m) + D(m) \qquad (3)$$

In any situation where arbitrage possibilities are available in both the long and short directions (i.e. it is possible to borrow the commodity for shorting etc.), one could expect

$$C = e^{(r+h)/12}$$

where h is the proportional holding cost rate (continuosly compounded) on the spot commodity. Thus one can see that D(m) then represents a deviation from the arbitrage controlled cash vs futures price relationship. Alternatively, if we look at the case where C = 1, it represents the special case where the forward price curve shifts in parallel without regard to arbitrage considerations. In this case D(m) measures the simple futures to cash spread which has no systematic relationship to the spot

price. In markets where perfect arbitrage is not possible, we can estimate C from time series data for spot and futures prices by means of regression analysis.

We choose the futures hedge amounts N(m) so as to immunize the value of V(T) to variations in spot prices S(m) at times m=1 through T, i.e in the future. Substituting equation 3 into equation 2, we get

$$V(T) = \sum_{m=1}^{T} R^{-m} N(m-1)(S(m) - CS(m-1) - D(m-1))) + R^{-T}(P(T) - S(T)) \quad (4)$$

Differentiating V(T) with respect to each of the unknown S(m)s for times 1 through T and equating the derivatives to zero yields

$$N(m-1) - CN(m)/R = 0, \qquad m=1,\ldots,T-2$$

$$N(T-1) = 1, \text{ or in general}$$

$$N(m) = (C/R)^{(T-m-1)}, \qquad m=0,\ldots,T-1 \quad (5)$$

Substituting equation 5 into equation 4, we get

$$V(T) = R^{-T}\{(P(T) - C^{T}S(0)) - \sum_{m=1}^{T} C^{-(T-m)} D(m-1)\} \quad (6)$$

Equation 6 shows the hedging condition that no terms relating to spot prices in the future are present and the only terms that are uncertain are those relating to the adjusted spreads the D(m)s. If further we denote as Q(T), the breakeven value of P(T), i.e. the value that makes V(T) go to zero, we have

$$Q(T) = C^{T}\{S(0) + \sum_{m=1}^{T} C^{-m} D(m-1)\} \quad (7)$$

If the adjusted spreads D(m)s were known with certainty, we can lock in the forward price Q(T) with certainty as well. Moreover if we interpret C as the carry factor, the first term in equation 7 is the usual forward price dictated by arbitrage conditions. The whole equation then can be seen as the deviations in D(m)s because of lack of perfect arbitrage where the forward has been constructed synthetically by a futures roll strategy.

Next we consider the fact D(m)s can, in general, be approximated only. We have indeed moved away from spot risk into spread risk. To characterize the spread risk, we can make an assumption that D(m)s follow a simple mean reverting process.

$$D(m) - D(m-1) = k(U - D(m-1)) + e(m) \qquad (8)$$

Where e(m)s are uncorrelated and identically distributed with,

$$E[e(m)] = 0 \text{ and } Var[e(m)] = W^2, \quad m=0,....T. \qquad (9)$$

The parameter U represents the long-run average spread toward which the D(m)s tend to revert. The parameter k determines the speed of reversion. For the special case k=0, it implies that D(m)s follow a random walk process with no mean reversion tendency. The special case k=1 implies that D(m)s themselves are random draws around a mean of U. In general, k is between these two extremes and can be estimated econometrically.

If we substitute the model for D(m) into equation 7, we get

$$Q(T) = C^T[S(0) + \sum_{m=1}^{T} C^{-m}\{kU \sum_{j=1}^{m-1} (1-k)^{j-1} + (1-k)^{m-1} D(0)\}] +$$

$$C^T[\sum_{m=1}^{T} C^{-m} \sum_{j=1}^{m-1} (1-k)^{m-1-j} e(j)] \qquad (10)$$

The second term in equation 10 can be rearranged to give:

$$Q(T) = C^T[S(0) + \sum_{m=1}^{T} C^{-m}\{kU \sum_{j=1}^{m-1} (1-k)^{j-1} + (1-k)^{m-1} D(0)\}] +$$

$$C^T[\sum_{m=1}^{T-1} e(m) \sum_{j=m+1}^{T} C^{-j} (1-k)^{j-(m+1)}] \qquad (11)$$

Now we can get the expectation value and variance of Q(T)

$$E[Q(T)] = C^T[S(0) + \sum_{m=1}^{T} C^{-m}\{kU \sum_{j=1}^{m-1} (1-k)^{j-1} + (1-k)^{m-1} D(0)\}]$$

$$Var[Q(T)] = W^2 C^{2T} \sum_{m=1}^{T-1} [\sum_{j=m+1}^{T} C^{-j} (1-k)^{j-(m+1)}]^2$$

Now we have formulae which will calculate the expectation value and the variance of the synthetic forward provided estimates of C, k and U and the initial values of the spot price and the spread are available.

Now we can extend this analysis to the case of a swap. A swap is treated as a portfolio of forward contracts, one for each payment date and each written at the same forward price. The value of a swap with N monthly payments in return for a fixed payment P, denoted X(N), is shown below.

$$X(N) = \sum_{t=1}^{N} V(t), \text{ with } V(t) \text{ defined as in equation 2} \qquad (12)$$

Substituting the value for V(t) from equation 6, we get

$$X(N) = \sum_{t=1}^{N} R^{-t}\{(P - C^t S(0)) - \sum_{m=1}^{t} C^{t-m} D(m-1)\}$$

$$= AP - \sum_{t=1}^{N} (C/R)^t \{S(0) + \sum_{m=1}^{t} C^{-m} D(m-1)\} \qquad (13)$$

Where $A = \sum_{t=1}^{N} R^{-t} = (1-R^{-N})/(R-1)$

The first term, AP in equation 13 represents the present value of the stream of fixed payments while the second term is the present value of the outlays made to create the synthetic forwards or the floating side using the futures roll strategy. The breakeven value for the fixed side is denoted Z(N) and can be figured out by the fact that X(N) is zero when P is equal to Z(N). Thus from equation 13 we get,

$$Z(N) = A^{-1} \sum_{t=1}^{N} (C/R)^{t} \{S(0) + \sum_{m=1}^{t} C^{-m} D(m-1)\} \qquad (14)$$

Now as before we can use our formulation of D(m) as a mean reverting process to get the expectation value of Z(N) and its variance.

$$E[Z(N)] = \sum_{t=1}^{N} (C/R)^{t} \{S(0) + \sum_{m=1}^{t} C^{-m} ((1-k)^{m-1} D(0) + kU \sum_{j=1}^{m-1} (1-k)^{j-1})\}$$

$$Var[Z(N)] = (W/A)^{2} \sum_{t=2}^{N} \{ \sum_{m=t}^{N} C^{-m} (1-k)^{m-t} \sum_{i=m}^{N} (C/R)^{i} \}^{2}$$

Once again what we need to know are C, k, U and the initial values for spot and the spread to calculate the breakeven value for the swap. Of course, in practice this is only an indication to work from.

APPENDIX II
OIL FUTURES DATA (WTI ON THE NYMEX)

17.690000	17.480000	17.350000	17.230000	17.130000	17.050000
17.850000	17.660000	17.520000	17.400000	17.300000	17.220000
17.820000	17.610000	17.470000	17.360000	17.270000	17.200000
17.390000	17.170000	17.030000	16.910000	16.820000	16.750000
17.310000	17.140000	17.020000	17.200000	16.830000	16.770000
16.750000	16.630000	16.510000	16.410000	16.340000	16.280000
16.630000	16.470000	16.340000	16.250000	16.190000	16.130000
16.590000	16.400000	16.280000	16.180000	16.090000	16.020000
17.210000	17.040000	16.940000	16.830000	16.750000	16.680000
16.950000	16.770000	16.650000	16.560000	16.480000	16.410000
17.350000	17.280000	17.200000	17.110000	17.030000	16.960000
17.250000	17.420000	17.320000	17.210000	17.120000	17.040000
17.180000	17.030000	16.890000	16.770000	16.660000	16.560000
17.010000	16.880000	16.740000	16.620000	16.510000	16.410000
17.090000	16.960000	16.840000	16.730000	16.630000	16.540000
17.040000	16.970000	16.860000	16.760000	16.670000	16.590000
16.710000	16.660000	16.580000	16.500000	16.430000	16.370000
16.960000	16.910000	16.800000	16.690000	16.590000	16.500000
16.940000	16.840000	16.740000	16.640000	16.540000	16.460000
16.820000	16.730000	16.650000	16.550000	16.450000	16.360000
16.940000	16.840000	16.750000	16.650000	16.550000	16.460000
17.140000	17.050000	16.960000	16.860000	16.760000	16.670000
17.140000	17.060000	16.970000	16.870000	16.770000	16.670000
17.280000	17.210000	17.120000	17.020000	16.920000	16.820000
17.660000	17.560000	17.450000	17.330000	17.220000	17.120000
17.420000	17.360000	17.280000	17.170000	17.070000	16.970000
17.130000	17.090000	17.040000	16.940000	16.840000	16.740000
17.110000	17.110000	17.060000	16.970000	16.880000	16.790000
16.750000	16.730000	16.710000	16.630000	16.550000	16.460000
16.670000	16.660000	16.610000	16.510000	16.410000	16.310000
16.610000	16.590000	16.530000	16.440000	16.350000	16.260000
16.450000	16.400000	16.340000	16.260000	16.170000	16.080000
16.700000	16.660000	16.570000	16.490000	16.410000	16.330000
16.600000	16.490000	16.400000	16.310000	16.220000	16.140000
16.460000	16.360000	16.290000	16.210000	16.130000	16.060000
15.920000	15.810000	15.760000	15.710000	15.660000	15.610000
15.780000	15.710000	15.660000	15.610000	15.560000	15.510000
16.010000	15.930000	15.870000	15.810000	15.750000	15.690000
15.720000	15.670000	15.630000	15.590000	15.550000	15.510000
15.670000	15.570000	15.540000	15.510000	15.480000	15.450000
15.500000	15.420000	15.380000	15.340000	15.310000	15.280000
15.590000	15.530000	15.490000	15.450000	15.410000	15.380000
15.370000	15.350000	15.310000	15.280000	15.250000	15.230000
15.450000	15.410000	15.380000	15.360000	15.340000	15.330000
15.490000	15.450000	15.420000	15.400000	15.380000	15.370000
16.000000	15.950000	15.930000	15.900000	15.860000	15.840000
16.290000	16.180000	16.150000	16.120000	16.090000	16.080000

15.590000	15.480000	15.460000	15.430000	15.400000	15.390000
15.680000	15.570000	15.550000	15.520000	15.490000	15.480000
16.000000	15.880000	15.810000	15.770000	15.730000	15.710000
16.280000	16.120000	15.990000	15.920000	15.860000	15.820000
16.580000	16.270000	16.140000	16.070000	16.010000	15.970000
16.490000	16.310000	16.190000	16.120000	16.070000	16.160000
16.090000	16.370000	16.290000	16.230000	16.180000	16.150000
16.860000	16.760000	16.690000	16.630000	16.590000	16.560000
16.960000	16.820000	16.740000	16.670000	16.620000	16.580000
17.030000	16.920000	16.850000	16.790000	16.740000	16.700000
17.100000	16.980000	16.920000	16.860000	16.810000	16.770000
16.980000	16.860000	16.800000	16.740000	16.690000	16.650000
17.080000	16.970000	16.900000	16.830000	16.770000	16.720000
17.080000	16.980000	16.910000	16.840000	16.780000	16.730000
17.010000	16.900000	16.830000	16.760000	16.700000	16.650000
16.790000	16.680000	16.610000	16.540000	16.490000	16.450000
16.810000	16.690000	16.630000	16.570000	16.520000	16.480000
17.060000	16.950000	16.880000	16.810000	16.750000	16.710000
16.880000	16.760000	16.680000	16.610000	16.550000	16.510000
17.890000	17.740000	17.620000	17.510000	17.410000	17.320000
18.090000	17.950000	17.820000	17.700000	17.600000	17.510000
18.120000	17.980000	17.880000	17.780000	17.690000	17.610000
18.300000	18.210000	18.110000	18.010000	17.930000	17.860000
18.370000	18.290000	18.180000	18.080000	18.000000	17.930000
18.520000	18.550000	18.440000	18.340000	18.260000	18.190000
18.030000	18.140000	18.050000	17.950000	17.870000	17.800000
17.740000	17.980000	17.910000	17.830000	17.770000	17.710000
18.360000	18.270000	18.190000	18.130000	18.070000	18.010000
18.300000	18.240000	18.150000	18.090000	18.030000	17.970000
18.400000	18.340000	18.210000	18.210000	18.150000	18.090000
18.600000	18.550000	18.460000	18.390000	18.330000	18.270000
18.270000	18.220000	18.150000	18.090000	18.040000	17.990000
17.980000	17.970000	17.890000	17.840000	17.800000	17.760000
17.990000	17.940000	17.890000	17.840000	17.790000	17.750000
17.140000	17.130000	17.120000	17.090000	17.070000	17.050000
17.370000	17.400000	17.400000	17.370000	17.340000	17.310000
17.200000	17.300000	17.350000	17.350000	17.340000	17.320000
17.390000	17.460000	17.470000	17.440000	17.410000	17.380000
17.740000	17.750000	17.730000	17.690000	17.640000	17.590000
17.540000	17.630000	17.630000	17.600000	17.560000	17.520000
17.460000	17.610000	17.650000	17.630000	17.600000	17.570000
17.490000	17.640000	17.700000	17.700000	17.690000	17.670000
17.480000	17.620000	17.710000	17.730000	17.720000	17.700000
17.490000	17.660000	17.730000	17.730000	17.730000	17.710000
17.660000	17.870000	17.960000	17.970000	17.970000	17.950000
17.760000	17.900000	17.950000	17.950000	17.940000	17.920000
17.430000	17.630000	17.710000	17.750000	17.760000	17.760000
17.450000	17.700000	17.840000	17.910000	17.950000	17.980000
17.170000	17.700000	17.870000	17.960000	18.000000	18.030000
17.350000	17.520000	17.660000	17.740000	17.780000	17.820000
17.400000	17.560000	17.660000	17.730000	17.760000	17.790000

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17.540000	17.760000	17.880000	17.940000	17.960000	17.980000
17.430000	17.670000	17.780000	17.840000	17.880000	17.900000
17.510000	17.710000	17.810000	17.860000	17.890000	17.910000
17.580000	17.720000	17.770000	17.790000	17.800000	17.800000
17.650000	17.830000	17.890000	17.920000	17.930000	17.930000
17.500000	17.680000	17.760000	17.800000	17.810000	17.820000
17.260000	17.370000	17.410000	17.430000	17.430000	17.430000
17.370000	17.450000	17.470000	17.480000	17.470000	17.460000
17.330000	17.400000	17.430000	17.440000	17.440000	17.440000
17.070000	17.120000	17.130000	17.150000	17.150000	17.150000
16.730000	16.810000	16.780000	16.800000	16.800000	16.800000
16.430000	16.570000	16.620000	16.650000	16.660000	16.670000
16.830000	16.930000	17.000000	17.040000	17.050000	17.060000
16.530000	16.620000	16.650000	16.680000	16.690000	16.700000
16.660000	16.780000	16.820000	16.870000	16.890000	16.900000
16.450000	16.570000	16.630000	16.700000	16.740000	16.770000
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16.370000	16.590000	16.770000	16.880000	16.960000	17.010000
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15.430000	15.610000	15.780000	15.900000	15.990000	16.050000
15.160000	15.320000	15.450000	15.570000	15.680000	15.750000
14.940000	15.140000	15.270000	15.380000	15.480000	15.550000
15.090000	15.300000	15.410000	15.510000	15.610000	15.680000
15.360000	15.550000	15.690000	15.810000	15.920000	16.000000
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14.760000	14.940000	15.070000	15.220000	15.370000	15.490000
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15.700000	15.890000	15.950000	16.020000	16.080000	16.120000
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16.050000	16.100000	16.160000	16.220000	16.270000	16.310000
15.960000	16.040000	16.090000	16.200000	16.260000	16.310000
16.160000	16.250000	16.350000	16.450000	16.510000	16.560000
16.150000	16.240000	16.340000	16.430000	16.490000	16.540000
16.310000	16.440000	16.550000	16.640000	16.690000	16.740000
16.080000	16.200000	16.300000	16.400000	16.460000	16.510000
15.610000	15.650000	15.800000	15.950000	16.030000	16.090000
15.260000	15.360000	15.470000	15.600000	15.680000	15.740000
15.030000	15.180000	15.310000	15.450000	15.540000	15.610000
15.260000	15.490000	15.610000	15.730000	15.820000	15.880000

15.870000	16.060000	16.170000	16.330000	16.420000	16.480000
15.680000	15.860000	15.980000	16.110000	16.200000	16.260000
15.710000	15.860000	15.940000	16.000000	16.070000	16.110000
15.760000	15.930000	16.070000	16.170000	16.250000	16.300000
15.520000	15.670000	15.830000	16.080000	16.170000	16.230000
15.610000	15.810000	15.950000	16.120000	16.220000	16.270000
15.490000	15.700000	15.810000	15.970000	16.070000	16.120000
15.470000	15.660000	15.780000	15.930000	16.010000	16.050000
15.570000	15.710000	15.820000	15.940000	16.000000	16.020000
15.730000	15.850000	15.950000	16.060000	16.120000	16.140000
15.760000	15.910000	15.990000	16.090000	16.120000	16.130000
15.700000	15.780000	15.850000	15.850000	15.850000	15.850000
15.680000	15.720000	15.790000	15.790000	15.790000	15.790000
15.310000	15.310000	15.410000	15.450000	15.450000	15.450000
15.340000	15.370000	15.450000	15.450000	15.450000	15.450000
15.220000	15.250000	15.310000	15.340000	15.380000	15.420000
15.270000	15.330000	15.430000	15.470000	15.510000	15.550000
15.180000	15.190000	15.230000	15.260000	15.290000	15.320000
15.080000	15.090000	15.110000	15.140000	15.170000	15.200000
14.790000	14.800000	14.820000	14.850000	14.880000	14.910000
14.240000	14.250000	14.300000	14.350000	14.400000	14.450000
14.160000	14.200000	14.260000	14.340000	14.410000	14.480000
14.480000	14.450000	14.460000	14.520000	14.580000	14.640000
14.180000	14.050000	14.050000	14.080000	14.130000	14.180000
14.490000	14.310000	14.310000	14.340000	14.410000	14.480000
14.560000	14.430000	14.420000	14.440000	14.510000	14.580000
15.400000	15.140000	15.060000	15.030000	15.050000	15.100000
14.900000	14.690000	14.590000	14.560000	14.590000	14.640000
14.570000	14.360000	14.260000	14.240000	14.270000	14.320000
14.730000	14.380000	14.280000	14.240000	14.260000	14.300000
15.040000	14.370000	14.240000	14.180000	14.200000	14.240000
14.560000	14.350000	14.270000	14.240000	14.260000	14.290000
14.650000	14.400000	14.330000	14.320000	14.340000	14.360000
14.180000	13.940000	13.860000	13.860000	13.880000	13.900000
14.200000	13.970000	13.900000	13.900000	13.920000	13.940000
14.140000	13.840000	13.760000	13.750000	13.770000	13.790000
14.110000	13.780000	13.680000	13.660000	13.660000	13.680000
13.920000	13.630000	13.540000	13.520000	13.530000	13.550000
13.370000	13.020000	12.960000	12.950000	12.970000	13.010000
13.060000	12.740000	12.660000	12.660000	12.690000	12.730000
13.070000	12.870000	12.850000	12.900000	12.950000	13.000000
12.600000	12.460000	12.490000	12.570000	12.650000	12.730000
12.660000	12.370000	12.350000	12.420000	12.500000	12.580000
12.940000	12.650000	12.620000	12.680000	12.750000	12.830000
13.550000	13.200000	13.120000	13.120000	13.160000	13.210000
14.110000	13.740000	13.610000	13.550000	13.560000	13.590000
14.210000	13.800000	13.630000	13.560000	13.560000	13.590000
14.920000	14.440000	14.240000	14.160000	14.160000	14.190000
15.210000	14.670000	14.450000	14.380000	14.380000	14.410000
14.610000	14.040000	13.790000	13.710000	13.710000	13.740000
15.400000	14.820000	14.570000	14.490000	14.500000	14.540000

14.600000	14.460000	14.380000	14.380000	14.410000	14.460000
14.370000	14.360000	14.360000	14.410000	14.460000	14.520000
13.170000	13.360000	13.360000	13.410000	13.470000	13.540000
13.260000	13.260000	13.310000	13.380000	13.440000	13.510000
13.410000	13.450000	13.500000	13.570000	13.630000	13.700000
13.680000	13.690000	13.720000	13.790000	13.850000	13.920000
13.850000	13.870000	13.910000	13.980000	14.040000	14.110000
13.580000	13.560000	13.590000	13.660000	13.710000	13.760000
13.440000	13.410000	13.410000	13.480000	13.540000	13.600000
13.750000	13.680000	13.650000	13.710000	13.770000	13.830000
13.900000	13.750000	13.740000	13.790000	13.850000	13.910000
14.040000	13.800000	13.770000	13.780000	13.820000	13.860000
14.060000	13.910000	13.910000	13.920000	13.970000	14.010000
13.800000	13.770000	13.780000	13.830000	13.880000	13.930000
13.860000	13.860000	13.850000	13.890000	13.930000	13.970000
13.980000	13.950000	13.940000	13.970000	14.000000	14.030000
13.950000	13.870000	13.830000	13.870000	13.900000	13.930000
14.270000	14.140000	14.050000	14.050000	14.090000	14.110000
13.900000	13.790000	13.690000	13.750000	13.770000	13.790000
13.670000	13.550000	13.410000	13.430000	13.450000	13.470000
13.290000	12.860000	12.720000	12.700000	12.700000	12.700000
13.600000	12.910000	12.780000	12.740000	12.740000	12.740000
12.980000	12.790000	12.770000	12.770000	12.770000	12.770000
13.860000	13.750000	13.690000	13.690000	13.690000	13.690000
13.970000	13.910000	13.850000	13.850000	13.850000	13.850000
15.030000	14.910000	14.850000	14.850000	14.850000	14.850000
14.920000	14.830000	14.820000	14.840000	14.860000	14.880000
15.320000	15.140000	15.070000	15.050000	15.050000	15.050000
15.610000	15.350000	15.260000	15.210000	15.190000	15.180000
15.640000	15.390000	15.300000	15.250000	15.230000	15.220000
15.340000	15.010000	14.940000	14.900000	14.880000	14.870000
15.460000	15.130000	15.030000	14.980000	14.960000	14.950000
15.740000	15.460000	15.360000	15.300000	15.270000	15.250000
15.440000	15.200000	15.100000	15.040000	15.010000	14.990000
15.840000	15.600000	15.460000	15.380000	15.340000	15.320000
16.060000	15.720000	15.540000	15.430000	15.380000	15.360000
16.000000	15.500000	15.280000	15.160000	15.090000	15.050000
16.370000	15.890000	15.670000	15.540000	15.470000	15.420000
16.350000	15.740000	15.430000	15.260000	15.170000	15.100000
16.720000	16.050000	15.660000	15.500000	15.410000	15.340000
16.310000	15.820000	15.430000	15.310000	15.230000	15.170000
17.730000	16.150000	15.750000	15.530000	15.430000	15.360000
16.300000	15.890000	15.620000	15.500000	15.410000	15.350000
16.490000	16.070000	15.820000	15.700000	15.630000	15.580000
16.610000	16.220000	15.990000	15.890000	15.820000	15.770000
16.950000	16.530000	16.250000	16.120000	16.040000	15.970000
16.970000	16.460000	16.140000	15.960000	15.860000	15.770000
16.770000	16.240000	15.910000	15.730000	15.630000	15.550000
17.240000	16.690000	16.250000	16.000000	15.840000	15.720000
17.360000	16.760000	15.380000	16.180000	16.050000	15.950000
17.080000	16.580000	16.280000	16.100000	15.990000	15.910000

17.420000	16.890000	16.540000	16.320000	16.200000	16.110000
17.550000	16.950000	16.590000	16.380000	16.260000	16.180000
17.720000	17.070000	16.700000	16.480000	16.350000	16.260000
17.690000	17.030000	16.680000	16.470000	16.340000	16.250000
18.140000	17.390000	16.970000	16.710000	16.540000	16.420000
18.170000	17.480000	17.060000	16.800000	16.640000	16.530000
18.480000	17.680000	17.160000	16.840000	16.630000	16.500000
18.950000	18.080000	17.550000	17.210000	16.990000	16.850000
19.260000	18.710000	18.340000	18.110000	17.950000	17.830000
19.280000	18.710000	18.220000	17.940000	17.690000	17.520000
18.860000	18.390000	17.880000	17.620000	17.400000	17.230000
17.330000	16.900000	16.720000	16.550000	16.410000	16.290000
17.540000	17.140000	16.970000	16.800000	16.660000	16.540000
18.120000	17.610000	17.340000	17.150000	17.000000	16.880000
17.720000	17.190000	16.900000	16.710000	16.550000	16.440000
17.740000	17.170000	16.870000	16.670000	16.510000	16.400000
17.290000	16.810000	16.590000	16.400000	16.240000	16.110000
17.030000	16.600000	16.400000	16.230000	16.080000	15.960000
17.510000	17.190000	16.990000	16.810000	16.650000	16.510000
17.730000	17.370000	17.140000	16.900000	16.720000	16.560000
17.530000	17.050000	16.770000	16.520000	16.320000	16.150000
17.380000	16.890000	16.580000	16.310000	16.110000	15.930000
17.700000	17.210000	16.900000	16.610000	16.370000	16.160000
17.490000	16.990000	16.690000	16.420000	16.190000	15.990000
17.390000	16.920000	16.640000	16.410000	16.210000	16.030000
17.110000	16.570000	16.330000	16.150000	15.980000	15.820000
17.590000	17.000000	16.670000	16.430000	16.230000	16.060000
17.420000	16.810000	16.450000	16.190000	15.980000	15.800000
18.250000	17.470000	16.980000	16.680000	16.430000	16.220000
18.330000	17.600000	17.120000	16.830000	16.590000	16.390000
18.560000	17.730000	17.270000	16.960000	16.720000	16.520000
18.590000	17.630000	17.140000	16.870000	16.630000	16.430000
17.840000	17.390000	17.110000	16.890000	16.700000	16.560000
17.830000	17.390000	17.110000	16.880000	16.680000	16.530000
18.070000	17.580000	17.260000	17.020000	16.810000	16.650000
18.140000	17.650000	17.330000	17.070000	16.840000	16.660000
18.150000	17.660000	17.360000	17.100000	16.880000	16.710000
18.280000	17.790000	17.460000	17.200000	16.980000	16.810000
18.720000	18.120000	18.740000	17.740000	17.460000	17.230000
18.560000	17.930000	17.520000	17.240000	17.010000	16.830000
18.670000	18.090000	17.720000	17.440000	17.220000	17.040000
18.300000	17.790000	17.440000	17.180000	16.970000	16.810000
18.540000	18.100000	17.800000	17.550000	17.370000	17.230000
18.520000	18.080000	17.770000	17.520000	17.340000	17.200000
18.500000	18.090000	17.760000	17.500000	17.310000	17.160000
19.030000	18.630000	18.200000	17.900000	17.650000	17.460000
19.290000	18.800000	18.360000	18.010000	17.730000	17.510000
19.770000	19.190000	18.720000	18.330000	18.020000	17.780000
19.840000	19.240000	18.780000	18.410000	18.110000	17.860000
20.330000	19.480000	18.910000	18.510000	18.180000	17.930000
19.510000	19.340000	18.840000	18.470000	18.170000	17.920000

20.050000	19.490000	19.080000	18.740000	18.450000	18.190000
20.040000	19.460000	19.060000	18.720000	18.430000	18.170000
20.150000	19.580000	19.180000	18.820000	18.510000	18.250000
20.530000	20.000000	19.580000	19.170000	18.810000	18.490000
19.910000	19.330000	18.910000	18.520000	18.180000	17.890000
20.200000	19.610000	19.190000	18.780000	18.450000	18.160000
20.810000	20.170000	19.700000	19.290000	18.930000	18.620000
20.190000	19.500000	19.030000	18.600000	18.240000	17.930000
19.950000	19.240000	18.720000	18.290000	17.930000	17.620000
20.420000	19.650000	19.060000	18.560000	18.170000	17.850000
19.990000	19.170000	18.530000	18.010000	17.610000	17.290000
19.990000	19.170000	18.530000	18.010000	17.610000	17.290000
20.020000	19.210000	18.700000	18.290000	17.990000	17.750000
20.590000	19.720000	19.100000	18.580000	18.240000	17.960000
20.630000	19.740000	19.080000	18.540000	18.140000	17.830000
20.640000	19.740000	19.080000	18.560000	18.160000	17.850000
20.290000	19.380000	18.680000	18.200000	17.830000	17.540000
20.690000	19.860000	19.160000	18.650000	18.270000	17.980000
21.220000	20.290000	19.530000	18.950000	18.520000	18.190000
21.510000	20.340000	19.580000	18.980000	18.540000	18.200000
22.610000	20.810000	19.860000	19.180000	18.690000	18.320000
24.650000	21.280000	20.060000	19.340000	18.800000	18.390000
21.320000	20.170000	19.490000	19.000000	18.620000	18.330000
20.610000	19.380000	18.720000	18.250000	17.900000	17.660000
21.410000	19.860000	18.990000	18.430000	18.040000	17.760000
21.190000	19.530000	18.690000	18.140000	17.780000	17.520000
20.920000	19.340000	18.540000	18.030000	17.690000	17.450000
20.420000	19.340000	18.760000	18.400000	18.130000	17.910000
20.660000	19.450000	18.880000	18.520000	18.260000	18.040000
19.800000	18.720000	18.180000	17.880000	17.670000	17.490000
20.100000	19.140000	18.580000	18.300000	18.090000	17.920000
20.580000	19.550000	18.880000	18.460000	18.190000	17.970000
20.020000	19.060000	18.450000	18.090000	17.850000	17.650000
19.440000	18.480000	18.000000	17.710000	17.530000	17.370000
19.760000	18.750000	18.220000	17.870000	17.650000	17.460000
19.520000	18.530000	18.060000	17.770000	17.600000	17.450000
20.090000	18.820000	18.190000	17.830000	17.600000	17.430000
20.510000	18.990000	18.240000	17.830000	17.570000	17.390000
20.700000	19.180000	18.430000	18.020000	17.750000	17.560000
20.200000	18.800000	18.180000	17.780000	17.530000	17.360000
20.600000	18.740000	18.150000	17.780000	17.530000	17.360000
20.930000	18.610000	17.830000	17.480000	17.270000	17.110000
19.040000	18.160000	17.710000	17.460000	17.280000	17.130000
19.480000	18.640000	18.260000	18.000000	17.800000	17.620000
19.520000	18.720000	18.360000	18.130000	17.940000	17.770000
19.950000	18.960000	18.480000	18.150000	17.900000	17.700000
19.900000	18.910000	18.390000	18.050000	17.820000	17.630000
19.810000	18.940000	18.450000	18.090000	17.840000	17.650000
20.140000	19.350000	18.890000	18.570000	18.370000	18.220000
20.520000	19.740000	19.290000	18.960000	18.740000	18.580000
20.460000	19.710000	19.290000	18.940000	18.700000	18.510000

19.670000	18.960000	18.570000	18.290000	18.080000	17.920000
20.000000	19.180000	18.770000	18.440000	18.230000	18.070000
19.880000	18.850000	18.410000	18.140000	17.940000	17.790000
19.260000	18.250000	17.780000	17.530000	17.360000	17.230000

APPENDIX III
SIMULATION RESULTS

day	stk	vol	fowv	tex	gam	del	avg	bond	val	cap
0	18.000	0.192	15.439	1.000	0.132	0.418	15.439	5.643	-0.000	0.807
1	18.000	0.191	15.600	0.996	0.132	0.438	15.600	5.958	-0.002	0.870
2	18.000	0.191	15.596	0.992	0.133	0.436	15.596	5.933	-0.005	0.863
3	18.000	0.191	15.193	0.988	0.132	0.381	15.193	5.105	0.003	0.692
4	18.000	0.191	15.225	0.984	0.133	0.384	15.225	5.150	-0.001	0.699
5	18.000	0.191	14.785	0.980	0.129	0.325	14.785	4.280	0.010	0.538
6	18.000	0.190	14.654	0.976	0.127	0.307	14.654	4.015	0.008	0.492
7	18.000	0.190	14.553	0.972	0.126	0.293	14.553	3.808	0.006	0.457
8	18.000	0.190	15.160	0.968	0.133	0.370	15.160	4.986	0.026	0.654
9	18.000	0.190	14.920	0.964	0.131	0.337	14.920	4.493	0.027	0.564
10	18.000	0.190	15.428	0.960	0.135	0.404	15.428	5.520	0.040	0.747
11	18.000	0.189	15.501	0.956	0.136	0.412	15.501	5.654	0.037	0.771
12	18.000	0.189	15.055	0.952	0.133	0.351	15.055	4.732	0.048	0.595
13	18.000	0.189	14.925	0.948	0.132	0.332	14.925	4.454	0.046	0.546
14	18.000	0.189	15.057	0.944	0.134	0.348	15.057	4.697	0.044	0.586
15	18.000	0.189	15.115	0.940	0.135	0.354	15.115	4.796	0.040	0.601

16	18.000	0.188	14.933	0.937	0.133	0.329	14.933	4.413	0.040	0.534
17	18.000	0.188	15.039	0.933	0.134	0.341	15.039	4.605	0.037	0.565
18	18.000	0.188	15.015	0.929	0.134	0.337	15.015	4.536	0.034	0.551
19	18.000	0.188	14.923	0.925	0.133	0.323	14.923	4.332	0.032	0.516
20	18.000	0.188	15.021	0.921	0.135	0.334	15.021	4.508	0.029	0.543
21	18.000	0.187	15.220	0.917	0.137	0.360	15.220	4.898	0.028	0.607
22	18.000	0.187	15.221	0.913	0.137	0.359	15.221	4.880	0.025	0.602
23	18.000	0.187	15.366	0.909	0.139	0.377	15.366	5.167	0.023	0.650
24	18.000	0.187	15.648	0.905	0.140	0.415	15.648	5.764	0.024	0.756
25	18.000	0.186	15.518	0.901	0.140	0.395	15.518	5.461	0.022	0.698
26	18.000	0.186	15.314	0.897	0.139	0.366	15.314	5.004	0.022	0.615
27	18.000	0.186	15.373	0.893	0.140	0.372	15.373	5.109	0.018	0.631
28	18.000	0.186	15.077	0.889	0.137	0.330	15.077	4.468	0.021	0.522
29	18.000	0.185	14.941	0.885	0.135	0.309	14.941	4.170	0.020	0.473
30	18.000	0.185	14.907	0.881	0.135	0.303	14.907	4.079	0.016	0.458
31	18.000	0.185	14.748	0.877	0.132	0.280	14.748	3.744	0.015	0.407
32	18.000	0.185	14.990	0.873	0.137	0.311	14.990	4.207	0.016	0.474
33	18.000	0.184	14.821	0.869	0.134	0.287	14.821	3.847	0.015	0.419
34	18.000	0.184	14.759	0.865	0.132	0.277	14.759	3.701	0.012	0.396
35	18.000	0.184	14.359	0.861	0.122	0.224	14.359	2.946	0.020	0.292

36	18.000	0.184	14.273	0.857	0.119	0.212	14.273	2.774	0.018	0.270
37	18.000	0.183	14.441	0.853	0.124	0.231	14.441	3.047	0.017	0.303
38	18.000	0.183	14.289	0.849	0.120	0.211	14.289	2.758	0.015	0.266
39	18.000	0.183	14.243	0.845	0.118	0.203	14.243	2.657	0.013	0.252
40	18.000	0.182	14.092	0.841	0.113	0.184	14.092	2.388	0.012	0.220
41	18.000	0.182	14.190	0.837	0.116	0.194	14.190	2.523	0.010	0.235
42	18.000	0.182	14.061	0.833	0.111	0.177	14.061	2.293	0.009	0.207
43	18.000	0.182	14.162	0.829	0.115	0.187	14.162	2.431	0.007	0.223
44	18.000	0.181	14.205	0.825	0.116	0.190	14.205	2.477	0.004	0.227
45	18.000	0.181	14.643	0.821	0.130	0.242	14.643	3.243	0.013	0.318
46	18.000	0.181	14.873	0.817	0.136	0.271	14.873	3.674	0.013	0.372
47	18.000	0.180	14.241	0.813	0.117	0.189	14.241	2.504	0.037	0.223
48	18.000	0.180	14.330	0.810	0.120	0.198	14.330	2.630	0.035	0.237
49	18.000	0.180	14.547	0.806	0.127	0.223	14.547	2.994	0.035	0.279
50	18.000	0.180	14.652	0.802	0.130	0.234	14.652	3.166	0.032	0.299
51	18.000	0.179	14.797	0.798	0.135	0.252	14.797	3.425	0.030	0.330
52	18.000	0.179	15.002	0.794	0.140	0.278	15.002	3.821	0.030	0.379
53	18.000	0.179	14.978	0.790	0.140	0.273	14.978	3.744	0.026	0.368
54	18.000	0.178	15.364	0.786	0.149	0.327	15.364	4.574	0.033	0.478

55	18.000	0.178	15.389	0.782	0.149	0.329	15.389	4.604	0.029	0.481
56	18.000	0.178	15.507	0.778	0.152	0.345	15.507	4.854	0.026	0.515
57	18.000	0.177	15.579	0.774	0.153	0.354	15.579	4.998	0.023	0.534
58	18.000	0.177	15.474	0.770	0.152	0.336	15.474	4.725	0.020	0.492
59	18.000	0.177	15.547	0.766	0.153	0.345	15.547	4.870	0.016	0.511
60	18.000	0.176	15.563	0.762	0.154	0.346	15.563	4.883	0.012	0.511
61	18.000	0.176	15.496	0.758	0.153	0.334	15.496	4.695	0.009	0.482
62	18.000	0.175	15.315	0.754	0.150	0.304	15.315	4.249	0.008	0.419
63	18.000	0.175	15.350	0.750	0.151	0.307	15.350	4.300	0.004	0.424
64	18.000	0.175	15.573	0.746	0.156	0.340	15.573	4.805	0.004	0.490
65	18.000	0.174	15.394	0.742	0.153	0.310	15.394	4.352	0.002	0.426
66	18.000	0.174	16.166	0.738	0.163	0.431	16.166	6.307	0.044	0.706
67	18.000	0.174	16.352	0.734	0.163	0.460	16.352	6.781	0.042	0.781
68	18.000	0.173	16.451	0.730	0.164	0.475	16.451	7.026	0.038	0.820
69	18.000	0.173	16.691	0.726	0.162	0.512	16.691	7.657	0.038	0.931
70	18.000	0.172	16.765	0.722	0.162	0.523	16.765	7.841	0.033	0.961
71	18.000	0.172	17.016	0.718	0.159	0.562	17.016	8.512	0.033	1.090
72	18.000	0.172	16.661	0.714	0.165	0.503	16.661	7.535	0.039	0.892
73	18.000	0.171	16.578	0.710	0.167	0.488	16.578	7.288	0.035	0.843
74	18.000	0.171	16.865	0.706	0.164	0.535	16.865	8.071	0.036	0.982

75	18.000	0.170	16.835	0.702	0.166	0.529	16.835	7.973	0.031	0.958
76	18.000	0.170	16.950	0.698	0.165	0.546	16.950	8.277	0.027	1.012
77	18.000	0.169	17.133	0.694	0.162	0.575	17.133	8.776	0.025	1.106
78	18.000	0.169	16.874	0.690	0.168	0.531	16.874	8.038	0.026	0.954
79	18.000	0.169	16.660	0.687	0.171	0.494	16.660	7.416	0.025	0.837
80	18.000	0.168	16.661	0.683	0.172	0.493	16.661	7.398	0.020	0.829
81	18.000	0.168	15.997	0.679	0.172	0.376	15.997	5.530	0.054	0.533
82	18.000	0.167	16.253	0.675	0.175	0.418	16.253	6.222	0.055	0.628
83	18.000	0.167	16.256	0.671	0.176	0.417	16.256	6.203	0.050	0.622
84	18.000	0.166	16.332	0.667	0.177	0.428	16.332	6.396	0.045	0.646
85	18.000	0.166	16.550	0.663	0.178	0.465	16.550	7.011	0.044	0.736
86	18.000	0.165	16.484	0.659	0.179	0.452	16.484	6.791	0.039	0.698
87	18.000	0.165	16.529	0.655	0.180	0.458	16.529	6.899	0.034	0.710
88	18.000	0.164	16.616	0.651	0.181	0.472	16.616	7.133	0.030	0.743
89	18.000	0.164	16.645	0.647	0.182	0.476	16.645	7.197	0.024	0.748
90	18.000	0.163	16.661	0.643	0.183	0.477	16.661	7.223	0.019	0.748
91	18.000	0.163	16.890	0.639	0.182	0.517	16.890	7.905	0.018	0.853
92	18.000	0.162	16.876	0.635	0.183	0.514	16.876	7.842	0.012	0.837
93	18.000	0.161	16.705	0.631	0.186	0.480	16.705	7.293	0.010	0.744

94	18.000	0.161	16.883	0.627	0.185	0.512	16.883	7.825	0.007	0.823
95	18.000	0.160	16.927	0.623	0.186	0.519	16.927	7.942	0.001	0.837
96	18.000	0.160	16.733	0.619	0.189	0.481	16.733	7.312	-0.000	0.732
97	18.000	0.159	16.724	0.615	0.191	0.477	16.724	7.257	-0.006	0.719
98	18.000	0.159	16.720	0.611	0.192	0.475	16.720	7.218	-0.012	0.708
99	18.000	0.158	16.921	0.607	0.191	0.512	16.921	7.848	-0.014	0.799
100	18.000	0.157	16.843	0.603	0.194	0.495	16.843	7.570	-0.019	0.751
101	18.000	0.157	16.866	0.599	0.195	0.498	16.866	7.620	-0.025	0.753
102	18.000	0.156	16.801	0.595	0.197	0.484	16.801	7.383	-0.030	0.713
103	18.000	0.155	16.925	0.591	0.197	0.506	16.925	7.772	-0.035	0.765
104	18.000	0.155	16.816	0.587	0.200	0.483	16.816	7.380	-0.040	0.702
105	18.000	0.154	16.475	0.583	0.201	0.412	16.475	6.221	-0.033	0.540
106	18.000	0.153	16.525	0.579	0.203	0.420	16.525	6.355	-0.039	0.553
107	18.000	0.153	16.500	0.575	0.204	0.413	16.500	6.234	-0.045	0.534
108	18.000	0.152	16.231	0.571	0.200	0.356	16.231	5.314	-0.043	0.423
109	18.000	0.151	15.912	0.567	0.189	0.291	15.912	4.278	-0.038	0.312
110	18.000	0.150	15.762	0.563	0.182	0.260	15.762	3.790	-0.040	0.264
111	18.000	0.150	16.130	0.560	0.200	0.327	16.130	4.879	-0.034	0.365
112	18.000	0.149	15.805	0.556	0.185	0.261	15.805	3.840	-0.028	0.263
113	18.000	0.148	15.983	0.552	0.195	0.292	15.983	4.330	-0.030	0.305

114	18.000	0.147	15.827	0.548	0.188	0.259	15.827	3.809	-0.033	0.255
115	18.000	0.146	15.630	0.544	0.175	0.220	15.630	3.196	-0.034	0.202
116	18.000	0.146	15.714	0.540	0.181	0.231	15.714	3.377	-0.038	0.215
117	18.000	0.145	15.916	0.536	0.194	0.266	15.916	3.928	-0.040	0.258
118	18.000	0.144	15.975	0.532	0.199	0.274	15.975	4.058	-0.045	0.267
119	18.000	0.143	15.702	0.528	0.181	0.218	15.702	3.189	-0.042	0.194
120	18.000	0.142	15.533	0.524	0.167	0.185	15.533	2.676	-0.044	0.154
121	18.000	0.141	15.409	0.520	0.156	0.161	15.409	2.309	-0.046	0.127
122	18.000	0.140	15.055	0.516	0.123	0.108	15.055	1.514	-0.040	0.076
123	18.000	0.139	14.765	0.512	0.095	0.074	14.765	1.002	-0.037	0.046
124	18.000	0.138	14.580	0.508	0.077	0.055	14.580	0.731	-0.038	0.032
125	18.000	0.137	14.714	0.504	0.087	0.063	14.714	0.854	-0.039	0.038
126	18.000	0.136	14.990	0.500	0.110	0.087	14.990	2.162	0.910	0.056
127	18.000	0.135	15.442	0.496	0.154	0.143	15.442	3.024	0.919	0.103
128	17.864	0.135	15.002	0.492	0.121	0.097	15.138	2.324	0.922	0.062
129	17.728	0.135	14.213	0.488	0.059	0.036	14.486	1.443	0.943	0.018
130	17.592	0.135	14.000	0.484	0.051	0.029	14.408	1.349	0.942	0.014
131	17.456	0.135	13.659	0.480	0.035	0.019	14.203	1.197	0.943	0.008
132	17.321	0.135	13.871	0.476	0.056	0.032	14.550	1.396	0.944	0.016

133	17.186	0.135	13.765	0.472	0.055	0.031	14.579	1.385	0.943	0.015
134	17.051	0.135	14.394	0.468	0.128	0.096	15.343	2.379	0.962	0.058
135	16.916	0.135	14.074	0.464	0.104	0.070	15.158	1.992	0.962	0.039
136	16.781	0.135	14.358	0.460	0.152	0.119	15.577	2.755	0.969	0.074
137	16.646	0.135	14.652	0.456	0.203	0.190	16.006	3.892	0.979	0.134
138	16.511	0.135	14.455	0.452	0.195	0.172	15.944	3.607	0.974	0.117
139	16.376	0.135	14.002	0.448	0.151	0.112	15.626	2.668	0.980	0.066
140	16.242	0.135	13.842	0.444	0.145	0.104	15.600	2.536	0.977	0.059
141	16.107	0.135	13.922	0.440	0.172	0.133	15.816	2.997	0.977	0.080
142	15.972	0.135	13.789	0.437	0.170	0.128	15.817	2.919	0.973	0.075
143	15.838	0.135	13.832	0.433	0.194	0.155	15.994	3.351	0.971	0.095
144	15.703	0.135	13.512	0.429	0.164	0.116	15.809	2.743	0.971	0.065
145	15.570	0.135	12.998	0.425	0.105	0.061	15.428	1.889	0.980	0.029
146	15.436	0.135	12.607	0.421	0.070	0.035	15.171	1.501	0.983	0.015
147	15.303	0.135	12.353	0.417	0.054	0.025	15.050	1.350	0.983	0.010
148	15.170	0.135	12.461	0.413	0.077	0.038	15.291	1.552	0.983	0.016
149	15.037	0.135	12.809	0.409	0.143	0.086	15.773	2.312	0.992	0.042
150	14.903	0.135	12.527	0.405	0.115	0.063	15.624	1.947	0.992	0.028
151	14.770	0.135	12.370	0.401	0.107	0.056	15.600	1.846	0.990	0.025
152	14.637	0.135	12.341	0.397	0.120	0.064	15.704	1.970	0.989	0.028

153	14.504	0.135	12.076	0.393	0.095	0.046	15.572	1.692	0.989	0.019
154	14.371	0.135	12.024	0.389	0.103	0.051	15.653	1.763	0.987	0.021
155	14.239	0.135	11.804	0.385	0.086	0.039	15.565	1.582	0.987	0.015
156	14.107	0.135	11.658	0.381	0.080	0.035	15.551	1.518	0.986	0.013
157	13.974	0.135	11.570	0.377	0.082	0.036	15.596	1.528	0.985	0.013
158	13.842	0.135	11.549	0.373	0.095	0.042	15.708	1.634	0.984	0.016
159	13.709	0.135	11.455	0.369	0.097	0.043	15.746	1.639	0.983	0.016
160	13.577	0.135	11.226	0.365	0.077	0.031	15.649	1.460	0.983	0.011
161	13.445	0.135	11.073	0.361	0.069	0.027	15.628	1.394	0.982	0.009
162	13.313	0.135	10.696	0.357	0.037	0.012	15.383	1.168	0.984	0.004
163	13.182	0.135	10.602	0.353	0.037	0.012	15.420	1.166	0.984	0.003
164	13.051	0.135	10.405	0.349	0.028	0.009	15.355	1.113	0.984	0.002
165	12.919	0.135	10.355	0.345	0.032	0.010	15.436	1.132	0.984	0.003
166	12.789	0.135	10.132	0.341	0.022	0.006	15.344	1.078	0.985	0.002
167	12.658	0.135	9.946	0.337	0.016	0.004	15.288	1.049	0.985	0.001
168	12.528	0.135	9.644	0.333	0.008	0.002	15.116	1.012	0.985	0.000
169	12.400	0.135	9.191	0.329	0.002	0.000	14.791	0.990	0.986	0.000
170	12.271	0.135	9.053	0.325	0.001	0.000	14.782	0.990	0.986	0.000
171	12.143	0.135	9.095	0.321	0.002	0.000	14.952	0.993	0.987	0.000

172	12.015	0.135	8.746	0.317	0.001	0.000	14.732	0.988	0.987	0.000
173	11.886	0.135	8.807	0.313	0.001	0.000	14.921	0.991	0.988	0.000
174	11.758	0.135	8.760	0.310	0.001	0.000	15.002	0.991	0.988	0.000
175	11.628	0.135	9.072	0.306	0.009	0.002	15.444	1.017	0.989	0.000
176	11.499	0.135	8.679	0.302	0.002	0.000	15.180	0.995	0.989	0.000
177	11.370	0.135	8.377	0.298	0.001	0.000	15.007	0.991	0.990	0.000
178	11.242	0.135	8.302	0.294	0.001	0.000	15.060	0.991	0.990	0.000
179	11.113	0.135	8.232	0.290	0.001	0.000	15.119	0.992	0.990	0.000
180	10.985	0.135	8.053	0.286	0.000	0.000	15.067	0.991	0.991	0.000
181	10.858	0.135	7.983	0.282	0.000	0.000	15.125	0.992	0.991	0.000
182	10.731	0.135	7.620	0.278	0.000	0.000	14.889	0.992	0.991	0.000
183	10.604	0.135	7.532	0.274	0.000	0.000	14.928	0.992	0.992	0.000
184	10.478	0.135	7.367	0.270	0.000	0.000	14.889	0.992	0.992	0.000
185	10.352	0.135	7.232	0.266	0.000	0.000	14.880	0.993	0.993	0.000
186	10.227	0.135	7.047	0.262	0.000	0.000	14.820	0.993	0.993	0.000
187	10.103	0.135	6.651	0.258	0.000	0.000	14.548	0.993	0.993	0.000
188	9.980	0.135	6.405	0.254	0.000	0.000	14.425	0.994	0.994	0.000
189	9.858	0.135	6.363	0.250	0.000	0.000	14.506	0.994	0.994	0.000
190	9.736	0.135	6.064	0.246	0.000	0.000	14.327	0.995	0.995	0.000
191	9.615	0.135	5.942	0.242	0.000	0.000	14.327	0.995	0.995	0.000

192	9.494	0.135	5.977	0.238	0.000	0.000	14.484	0.995	0.995	0.000
193	9.370	0.135	6.137	0.234	0.000	0.000	14.767	0.996	0.996	0.000
194	9.246	0.135	6.278	0.230	0.000	0.000	15.033	0.996	0.996	0.000
195	9.121	0.135	6.202	0.226	0.000	0.000	15.081	0.997	0.997	0.000
196	8.994	0.135	6.386	0.222	0.000	0.000	15.392	0.997	0.997	0.000
197	8.867	0.135	6.384	0.218	0.000	0.000	15.517	0.997	0.997	0.000
198	8.742	0.135	6.009	0.214	0.000	0.000	15.267	0.998	0.998	0.000
199	8.614	0.135	6.228	0.210	0.000	0.000	15.614	0.998	0.998	0.000
200	8.488	0.135	5.910	0.206	0.000	0.000	15.422	0.999	0.999	0.000
201	8.364	0.135	5.744	0.202	0.000	0.000	15.380	0.999	0.999	0.000
202	8.243	0.135	5.208	0.198	0.000	0.000	14.966	0.999	0.999	-0.000
203	8.122	0.135	5.101	0.194	0.000	0.000	14.980	1.000	1.000	-0.000
204	8.000	0.135	5.064	0.190	0.000	0.000	15.064	1.000	1.000	-0.000
205	7.878	0.135	5.053	0.187	0.000	0.000	15.175	1.001	1.001	0.000
206	7.756	0.135	5.011	0.183	0.000	0.000	15.256	1.001	1.001	-0.000
207	7.634	0.135	4.801	0.179	0.000	0.000	15.167	1.001	1.001	0.000
208	7.513	0.135	4.646	0.175	0.000	0.000	15.133	1.002	1.002	0.000
209	7.391	0.135	4.641	0.171	0.000	0.000	15.249	1.002	1.002	0.000
210	7.269	0.135	4.572	0.167	0.000	0.000	15.303	1.003	1.003	0.000

211	7.146	0.135	4.496	0.163	0.000	0.000	15.349	1.003	1.003	0.000
212	7.024	0.134	4.408	0.159	0.000	0.000	15.384	1.003	1.003	0.000
213	6.902	0.134	4.236	0.155	0.000	0.000	15.334	1.004	1.004	0.000
214	6.781	0.134	4.151	0.151	0.000	0.000	15.370	1.004	1.004	0.000
215	6.659	0.134	4.073	0.147	0.000	0.000	15.415	1.005	1.005	0.000
216	6.537	0.134	3.950	0.143	0.000	0.000	15.413	1.005	1.005	0.000
217	6.414	0.134	3.925	0.139	0.000	0.000	15.510	1.005	1.005	0.000
218	6.293	0.134	3.717	0.135	0.000	0.000	15.424	1.006	1.006	0.000
219	6.172	0.134	3.548	0.131	0.000	0.000	15.376	1.006	1.006	0.000
220	6.053	0.134	3.320	0.127	0.000	0.000	15.267	1.007	1.007	0.000
221	5.933	0.134	3.275	0.123	0.000	0.000	15.342	1.007	1.007	0.000
222	5.815	0.134	3.062	0.119	0.000	0.000	15.247	1.007	1.007	0.000
223	5.694	0.134	3.169	0.115	0.000	0.000	15.475	1.008	1.008	0.000
224	5.573	0.134	3.089	0.111	0.000	0.000	15.516	1.008	1.008	0.000
225	5.449	0.134	3.204	0.107	0.000	0.000	15.755	1.009	1.009	0.000
226	5.324	0.134	3.066	0.103	0.000	0.000	15.741	1.009	1.009	0.000
227	5.199	0.134	3.026	0.099	0.000	0.000	15.828	1.009	1.009	0.000
228	5.072	0.134	2.961	0.095	0.000	0.000	15.889	1.010	1.010	0.000
229	4.945	0.134	2.847	0.091	0.000	0.000	15.902	1.010	1.010	0.000
230	4.820	0.134	2.674	0.087	0.000	0.000	15.854	1.011	1.011	0.000

231	4.694	0.134	2.577	0.083	0.000	0.000	15.883	1.011	1.011	0.000
232	4.567	0.134	2.498	0.079	0.000	0.000	15.932	1.011	1.011	0.000
233	4.441	0.134	2.328	0.075	0.000	0.000	15.887	1.012	1.012	0.000
234	4.313	0.134	2.263	0.071	0.000	0.000	15.949	1.012	1.012	0.000
235	4.185	0.133	2.167	0.067	0.000	0.000	15.982	1.013	1.013	0.000
236	4.057	0.133	2.032	0.063	0.000	0.000	15.974	1.013	1.013	0.000
237	3.928	0.133	1.949	0.060	0.000	0.000	16.021	1.013	1.013	0.000
238	3.799	0.133	1.817	0.056	0.000	0.000	16.018	1.014	1.014	0.000
239	3.668	0.133	1.725	0.052	0.000	0.000	16.057	1.014	1.014	0.000
240	3.539	0.133	1.553	0.048	0.000	0.000	16.014	1.015	1.015	0.000
241	3.404	0.132	1.548	0.044	0.000	0.000	16.143	1.015	1.015	0.000
242	3.275	0.132	1.294	0.040	0.000	0.000	16.018	1.015	1.015	0.000
243	3.145	0.132	1.178	0.036	0.000	0.000	16.033	1.016	1.016	0.000
244	3.015	0.131	1.055	0.032	0.000	0.000	16.040	1.016	1.016	0.000
245	2.883	0.131	0.942	0.028	0.000	0.000	16.058	1.017	1.017	0.000
246	2.751	0.130	0.808	0.024	0.000	0.000	16.057	1.017	1.017	0.000
247	2.620	0.130	0.665	0.020	0.000	0.000	16.045	1.017	1.017	0.000
248	2.487	0.128	0.547	0.016	0.000	0.000	16.060	1.018	1.018	0.000
249	2.354	0.127	0.413	0.012	0.000	0.000	16.059	1.018	1.018	0.000

250	2.222	0.124	0.271	0.008	0.000	0.000	16.049	1.019	1.019	0.000
251	2.088	0.117	0.138	0.004	0.000	0.000	16.050	0.000	1.019	0.000

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