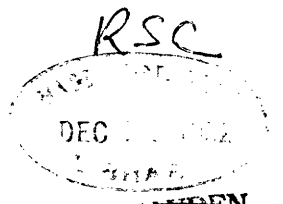


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CROSSCORRELATION FUNCTIONS OF AMPLITUDE-DISTORTED  
GAUSSIAN SIGNALS

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TECHNICAL REPORT NO. 216

MARCH 26, 1952

RESEARCH LABORATORY OF ELECTRONICS  
MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
CAMBRIDGE, MASSACHUSETTS

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Abstract

This report treats the crosscorrelation functions of gaussian signals. It is shown that the crosscorrelation function of two such signals taken after one of them has undergone a nonlinear amplitude distortion is identical except for a factor of proportionality to the crosscorrelation function taken before the distortion. Possible practical applications of this property are indicated. A number of correlation functions associated with some of the most common types of distortion are computed.

# CROSSCORRELATION FUNCTIONS OF AMPLITUDE-DISTORTED GAUSSIAN SIGNALS

## I. Introduction

Considerable attention has recently been given to the application of statistical methods to communication problems (1). Some of these problems involve stationary random noise signals possessing gaussian distributions of amplitudes. It is sometimes desirable to determine the autocorrelation and crosscorrelation functions of such signals after they undergo nonlinear amplitude distortion in terms of the original correlation functions. Analysis based on the ergodic hypothesis and on some well-known properties of the gaussian variable can be applied (2, 3, 4).

It is the purpose of this report to present some of the results of this analysis which do not seem to have been noted yet in the literature and to indicate where they may be found useful. We begin by outlining the technique of correlation analysis of distorted gaussian signals.

## II. Correlation Function Analysis of Distorted Gaussian Signals

Suppose that we are dealing with a signal  $f(t)$  which is generated by a stationary random process. If we consider an ensemble of such time functions, generated by the same process, the amplitude at an arbitrary time  $t_1$  can be treated as a random variable  $x$ . Associated with this random variable  $x$  are a certain probability density  $p(x)$ , a mean value  $\bar{x}$ , and a mean-square value  $\sigma_x^2$ . Let us assume that the distribution of amplitudes is normal with a zero mean value and a unity mean-square value. We then have

$$p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad (1)$$

$$\sigma_x^2 = \int_{-\infty}^{\infty} x^2 p(x) dx = 1 \quad (2)$$

$$\bar{x} = \int_{-\infty}^{\infty} x p(x) dx = 0 \quad (3)$$

where the bar over a variable is used to denote its statistical average. We shall call such a signal a gaussian signal.

Let  $g(t)$  be another gaussian signal. Then this will also be the case with the delayed signal  $g(t+\tau)$ . If  $y$  is the random variable associated with  $g(t+\tau)$ , then the probability density of  $y$  will be  $p(y)$ .

Now,  $f(t)$  and  $g(t)$  will be, in general, coherent functions of time, and therefore there will be some dependence between the variables  $x$  and  $y$ . This means that the mean of the product (or covariance) of the two variables, which we shall call  $r$ , is

nonzero. Writing  $p(x, y; \tau)$  for the density of the joint distribution of  $x$  and  $y$ , where  $\tau$  is inserted to indicate clearly the dependence, we have

$$r = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy p(x, y; \tau) dx dy. \quad (4)$$

Since we are dealing with stationary processes, the ensemble average and the time average can be equated, that is

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy p(x, y; \tau) dx dy = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t) g(t+\tau) dt. \quad (5)$$

Now, the right-half member of Eq. 5 is by definition (ref. 1, p. 10, Eq. 45) the cross-correlation function  $\phi_{fg}(\tau)$  of the two signals  $f(t)$  and  $g(t)$ . With this in mind Eq. 5 can be rewritten as

$$\phi_{fg}(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} xy p(x, y; \tau) dx dy \quad (6)$$

that is

$$\phi_{fg}(\tau) = r. \quad (7)$$

It is a property of the two-dimensional normal distribution that the joint probability density of  $x$  and  $y$  depends only on their covariance and is given explicitly by the expression (ref. 5, p. 395, Eq. 29.6)

$$p(x, y; \tau) = \frac{1}{2\pi \sqrt{1-r^2}} e^{-(x^2+y^2-2rxy)/[2(1-r^2)]}. \quad (8)$$

In Eq. 8,  $r$  is written without an indication of the time functions with which the variables  $x$  and  $y$  are associated in order to simplify the notation.

Let the signal  $f(t)$  be passed through a device whose output, at any time  $t_1$ , is some single-valued function  $U$  of the amplitude of  $f(t)$  at that instant, defined for all values of  $f(t)$ . We shall term such a device an amplitude-distorting device and  $U$  will be referred to as the distortion function. If  $F(t)$  is the resulting time function at the output, we have

$$F(t_1) = U [f(t_1)]. \quad (9)$$

If  $g(t)$  is passed through a similar device with a characteristic  $V$ , instead of  $U$ , the resulting output  $G(t)$  is related to  $g(t)$ , at any instant  $t_1$  by

$$G(t_1) = V [g(t_1)]. \quad (10)$$

It is not difficult to see that

$$G(t_1 + \tau) = V [g(t_1 + \tau)] . \quad (11)$$

The amplitudes of  $F(t)$  and  $G(t+\tau)$ , at an arbitrary time  $t_1$ , can then be thought of as random variables  $U(x)$  and  $V(y)$ , respectively, with  $x$  and  $y$  as defined above. Figure 1 shows a schematic illustrating the notation.

The crosscorrelation function of  $F(t)$  and  $G(t)$  can be written, by definition, as

$$\phi_{FG}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T F(t) G(t+\tau) dt. \quad (12)$$

Despite distortion, the processes are still stationary, so that the time and the ensemble averages are equal, that is

$$\phi_{FG}(\tau) = \overline{U(x) V(y)}. \quad (13)$$

But

$$\overline{U(x) V(y)} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x) V(y) p(x, y; \tau) dx dy. \quad (14)$$

Substituting Eq. 8 for  $p(x, y; \tau)$ , we finally obtain

$$\phi_{FG}(\tau) = \frac{1}{2\pi\sqrt{1-r^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U(x) V(y) e^{-(x^2+y^2-2rxy)/[2(1-r^2)]} dx dy. \quad (15)$$

Equation 15 permits us to express the crosscorrelation function of two distorted gaussian signals in terms of their crosscorrelation function before distortion, provided the distortion characteristics are specified. In the case of  $U = V$  and  $f = g$ , that is, when we are dealing with the input  $f(t)$  and the output  $F(t)$  of a single device, one can compute the autocorrelation function  $\phi_{FF}(\tau)$  after distortion in terms of the original autocorrelation function  $\phi_{ff}(\tau)$ .

The normalizations of the mean and of the mean-square values of the distributions are not really essential; they do, however, considerably simplify computations. A nonzero mean value will introduce additive constants and error-function type integrals in the formulas, while a different mean-square value condition will call for the multiplication of results by a constant factor.

The same method can thus be extended to any gaussian distribution. In addition, it will also cover correlation functions of higher orders, provided all the covariances are given.

Note, that if  $V$  and  $U$  comprise a periodic time dependence one can compute the

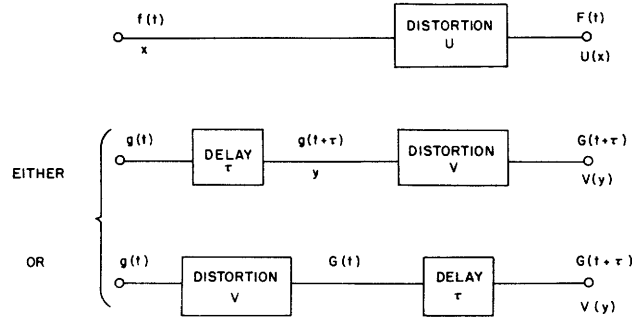


Fig. 1

Schematic explaining the notation.

integral in Eq. 12 and take the time average of the result to obtain the correlation function (6).

In general, it appears desirable to leave the results in terms of correlation functions, since the transformation to power spectra will involve an additional step and the explicit knowledge of the original correlation function.

### III. A Theorem Regarding Crosscorrelations

For two gaussian signals, the crosscorrelation function taken after one of them has undergone nonlinear amplitude distortion is identical, except for a factor of proportionality, to the crosscorrelation function taken before the distortion.

This means, for example, that the limiting of one signal in amplifiers, no matter how severe, has no effect on the shape of the crosscorrelation curve, provided the other signal is undistorted.

We next proceed to prove this, by considering simply a special case of Eq. 15, in which  $F = f$ . It follows that  $U(x) = x$ . For this case Eq. 15 becomes

$$\phi_{fG}(\tau) = \frac{1}{2\pi\sqrt{1-r^2}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x V(y) e^{-(x^2+y^2-2rxy)/[2(1-r^2)]} dx dy. \quad (16)$$

Since

$$\int_{-\infty}^{\infty} x e^{-x^2/[2(1-r^2)] + [ry/(1-r^2)]x} dx = r\sqrt{2\pi(1-r^2)} y e^{r^2 y^2/[2(1-r^2)]} \quad (17)$$

Eq. 16 may be simplified to read

$$\phi_{fG}(\tau) = \frac{r}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y V(y) e^{-y^2/2} dy. \quad (18)$$

The integral above is seen to depend only on the distortion characteristic  $V$  and is

independent of  $r$ . Remembering that  $r = \phi_{fg}(\tau)$ , we observe that for a given distortion characteristic  $V$  the ratio  $\phi_{fG}(\tau)/\phi_{fg}(\tau)$  is constant for all values of  $\tau$ . Let us denote this constant by  $K_v$ . We then have

$$K_v = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} y V(y) e^{-y^2/2} dy. \quad (19)$$

Equation 18 can now be rewritten in the form

$$\phi_{fG}(\tau) = K_v \phi_{fg}(\tau). \quad (20)$$

Equation 20 is the mathematical expression of the stated theorem. We see that it applies to nonlinear distortions of a very general character and that no restrictions have been imposed on the frequency range of the signals considered.

In particular, if the input and output of a distorting device are crosscorrelated, the result will be proportional to the autocorrelation of the input signal, that is

$$\phi_{gG}(\tau) = K_v \phi_{gg}(\tau). \quad (21)$$

It also follows directly from Eq. 21, that for the systems discussed above, the order of correlation between input and output is of no importance, that is

$$\phi_{gG}(\tau) = \phi_{GG}(\tau). \quad (22)$$

In comparing the input autocorrelation with the input-output crosscorrelation, it is convenient to consider normalized quantities. We shall thus define a normalized coefficient  $k_v$ , characteristic of any system  $V$ , as

$$k_v = K_v \left[ \frac{\overline{y^2}}{V^2(y)} \right]^{1/2}. \quad (23)$$

It will be seen that  $k_v$  is the correlation coefficient (ref. 5, p. 265, Eq. 21.2.8) of the two variables  $y$  and  $V(y)$ .

If the time notation is used, we have

$$k_v = \frac{\phi_{fG}(\tau_1)}{\phi_{fg}(\tau_1)} \left[ \frac{\phi_{gg}(0)}{\phi_{GG}(0)} \right]^{1/2} \quad (24)$$

where the first ratio can be taken at any time  $\tau_1$ . The values of  $k_v$  for several specific distorting devices have been computed.

#### IV. Power Series Devices

It is useful to consider a family of distortion functions defined as follows:

$$V_1(x) = \begin{cases} a_i x^i & x > 0 \\ b_i x^i & x < 0 \end{cases} \quad i = 0, 1, 2, \dots \quad (25)$$

The first member of this family,  $V_0$ , represents a perfect peak clipper; the second,  $V_1$ , the linear rectifier; the third,  $V_2$  (if  $b_2 = 0$ ), the square law rectifier; and so on.

Given a certain range  $x_1 < x < x_2$ , we can approximate the distortion characteristic  $V$  by a series of  $V_i$ , so that by suitably choosing the coefficients  $a_i$ ,  $b_i$

$$V(x) = \sum_i V_i(x) \quad x_1 < x < x_2. \quad (26)$$

Applying the result of Eq. 20 to the distortion function  $V$  of Eq. 26, we find

$$\phi_{fG}(\tau) = \phi_{fg}(\tau) \sum_{i=0, 1, 2, \dots} K_{V_i} [a_i - (-1)^i b_i] \quad (27)$$

where

$$K_{V_i} = \begin{cases} \frac{2^{i/2}}{\sqrt{2\pi}} \left(\frac{i}{2}\right)! & \text{for } i \text{ even} \\ \frac{i!}{\left(\frac{i-1}{2}\right)! 2^{(i+1)/2}} & \text{for } i \text{ odd.} \end{cases} \quad (28)$$

Values of the factor  $K_{V_i}$  for  $i$  from 0 to 8 are given in Table 1.

Table 1

Values of  $K_{V_i}$  for  $i = 0, 1, 2, \dots, 8$ .

$i$	$K_{V_i}$
0	$1/\sqrt{2\pi} = 0.399$
1	$1/2 = 0.500$
2	$\sqrt{2/\pi} = 0.798$
3	$3/2 = 1.50$
4	$8/\sqrt{2\pi} = 3.20$
5	$15/2 = 7.50$
6	$24/\sqrt{2\pi} = 9.60$
7	$35/2 = 17.50$
8	$384/\sqrt{2\pi} = 153.5$

## V. Linear Rectification

The characteristic of a linear rectifier is shown in Fig. 2. The distortion function  $V_1(x)$  is defined as follows

$$V_1(x) = \begin{cases} a_1 x & x \geq 0 \\ b_1 x & x \leq 0 \end{cases} \quad (29)$$

where  $a_1$  and  $b_1$  can be positive, negative or zero.

The crosscorrelation function of two rectified signals can be computed in terms of the original crosscorrelation function  $\phi_{fg}(\tau)$ . We have

$$\phi_{FG}(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V_1(x) V_1(y) p(x, y; \tau) dx dy. \quad (30)$$

Substituting for the expressions in Eq. 30 from Eqs. 29 and 8, we arrive at

$$\begin{aligned} \phi_{FG}(\tau) = & \frac{1}{2\pi \sqrt{1-r^2}} \left\{ (a_1^2 + b_1^2) \int_0^{\infty} \int_0^{\infty} xy e^{-(x^2+y^2-2rxy)/[2(1-r^2)]} dx dy \right. \\ & \left. - 2a_1 b_1 \int_0^{\infty} \int_0^{\infty} xy e^{-(x^2+y^2+2rxy)/[2(1-r^2)]} dx dy \right\}. \end{aligned} \quad (31)$$

Let us denote the first of these integrals by  $H(r)$ , that is

$$H(r) = \frac{1}{2\pi \sqrt{1-r^2}} \int_0^{\infty} \int_0^{\infty} xy e^{-(x^2+y^2-2rxy)/[2(1-r^2)]} dx dy. \quad (32)$$

On carrying out the integration, we find

$$H(r) = \frac{1}{2\pi} \left[ \sqrt{1-r^2} + \frac{r\pi}{2} + r \tan^{-1} \frac{r}{\sqrt{1-r^2}} \right]. \quad (33)$$

Thus remembering Eq. 7, we obtain

$$\phi_{FG}(\tau) = (a_1^2 + b_1^2) H[\phi_{fg}(\tau)] - 2a_1 b_1 H[-\phi_{fg}(\tau)]. \quad (34)$$

It can be verified that for  $a_1 = b_1 = 1$ ,  $\phi_{FG}(\tau)$  reduces to  $\phi_{fg}(\tau)$ .

The crosscorrelation function of a rectified signal with a nonrectified signal follows immediately from Eq. 27. We obtain

$$\phi_{fG} = \frac{1}{2}(a_1 + b_1) \phi_{fg}. \quad (35)$$

If we normalize the rms value of the output power to unity, so that

$$a_1^2 + b_1^2 = 1 \quad (36)$$

and in addition let

$$\frac{b_1}{a_1} = \tan \theta \quad (37)$$

Eq. 35 can be rewritten in the form

$$\phi_{fG}(\tau) = \phi_{fg}(\tau) \cos \left( \theta - \frac{\pi}{4} \right). \quad (38)$$

This means that

$$k_v = \cos \left( \theta - \frac{\pi}{4} \right). \quad (38a)$$

Figure 3 illustrates Eq. 38.

There are several types of distortion not of the power series type which are of practical importance. Two of these, symmetrical peak clipping and symmetrical center clipping, are considered below.

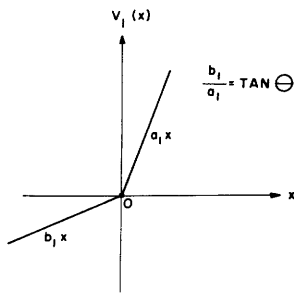


Fig. 2

The characteristic of a linear rectifier.

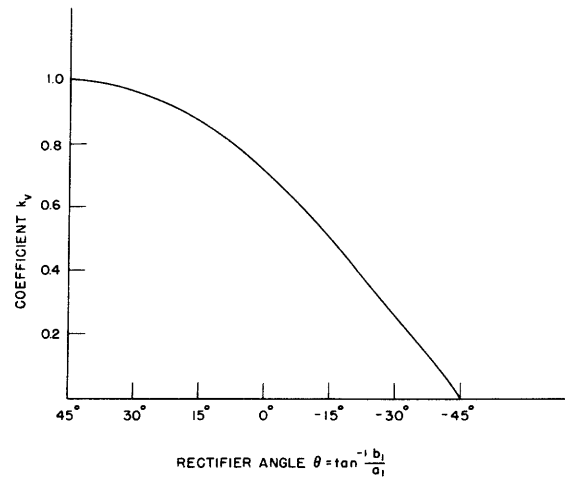


Fig. 3

Correlation coefficient  $k_v$  as a function of the rectifier angle  $\theta$ .

## VI. Symmetrical Center Clipping

The amplitude characteristic of a symmetrical center clipper is shown in Fig. 4. If we call the distortion function  $V$ , it can be defined as follows

$$V(x) = \begin{cases} \beta(x-h) & x > h \\ 0 & |x| < h \\ \beta(x+h) & x < -h. \end{cases} \quad (39)$$

If the output rms value is normalized to unity,  $\beta$  and  $h$  are related by the condition

$$\frac{\beta^2}{\sqrt{2\pi}} \int_h^\infty (x-h)^2 e^{-x^2/2} dx = \frac{1}{2}. \quad (40)$$

On integration one obtains

$$2\beta^2 \left[ \frac{1}{2}(1 - \alpha_h)(1 + h^2) - \frac{he^{-h^2/2}}{\sqrt{2\pi}} \right] = 1 \quad (41)$$

where  $\alpha_h$  is the well-tabulated probability integral given by the expression

$$\alpha_h = \frac{1}{\sqrt{2\pi}} \int_{-h}^h e^{-x^2/2} dx. \quad (42)$$

The crosscorrelation of an undistorted signal with a center-clipped signal can be computed from Eq. 18

$$\phi_{fG}(\tau) = \frac{\phi_{fg}}{\sqrt{2\pi}} \left[ \int_h^\infty y \beta(y-h) e^{-y^2/2} dy + \int_{-\infty}^{-h} y \beta(y+h) e^{-y^2/2} dy \right]. \quad (43)$$

This turns out to be

$$\phi_{fG}(\tau) = \beta(1 - \alpha_h) \phi_{fg}(\tau). \quad (44)$$

The correlation coefficient  $k_V$  computed on the basis of Eqs. 41 and 44 is plotted in Fig. 5, as a function of  $h$ .

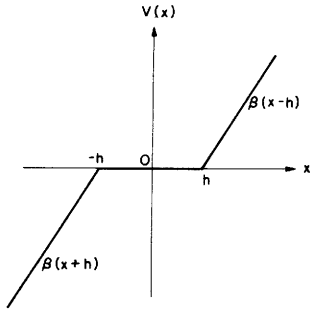


Fig. 4

The characteristic of a linear center clipper.

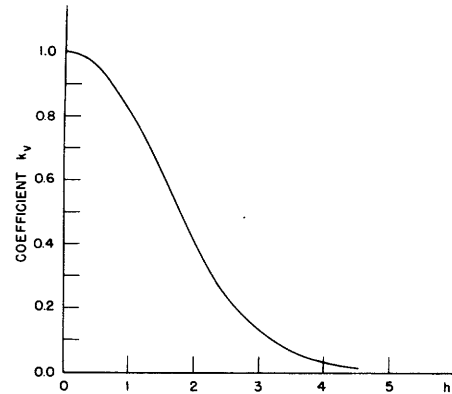


Fig. 5

Correlation coefficient  $k_V$  as a function of the clipping level for a symmetrical center clipper.

## VII. Symmetrical Peak Clipping

We shall consider the abrupt peak clipping whose characteristic is shown in Fig. 6 and the gradual clipping with the characteristic shown in Fig. 7. The distortion function for the abrupt clipping is taken to be

$$V(x) = \begin{cases} c & x > d \\ \frac{c}{d}x & x < d \\ -c & x < -d. \end{cases} \quad (45)$$

Van Vleck (ref. 2, Eq. 41, p. 35) gives the autocorrelation function of the output of this clipper for the case  $c = d$ . One can immediately extend this result to the crosscorrelation of two identically clipped signals. Using the notation introduced above, we have

$$\phi_{FG}(\tau) = \frac{c^2}{d^2} \left\{ \phi_{fg} a_d / \sqrt{2} + \sum_{n=3, 5, \dots}^{\infty} \frac{\phi_{fg}^n}{n!} \left[ H_{n-2}(d) e^{-d^2/2} \right]^2 \right\} \quad (46)$$

where  $H_n(y)$  is the Hermitian polynomial of order  $n$ . That is

$$H_n(y) = \left[ (-1)^n \frac{d^n}{dy^n} e^{-y^2/2} \right] e^{y^2/2}. \quad (47)$$

In the limiting case of "extreme" clipping when the output signal of each clipper is two-valued, following Van Vleck, we obtain the compact relation (ref. 2, Eq. 42, p. 35)

$$\phi_{FG}(\tau) = \frac{2}{\pi} c^2 \sin^{-1} \phi_{fg}(\tau). \quad (48)$$

Figure 8 illustrates Eq. 48 for the case in which the input crosscorrelation is given by  $\phi_{fg} = e^{-a|\tau|}$ . It is interesting to notice that the slope of  $\phi_{FG}$  is infinite at the origin.

The crosscorrelation of a distorted signal with an undistorted signal is obtained by substituting our particular  $V$  in Eq. 19. Since the distortion is symmetrical, we have

$$\frac{1}{2} K_V = \frac{c}{d} \int_0^d y y e^{-y^2/2} dy + c \int_d^{\infty} y e^{-y^2/2} dy. \quad (49)$$

This leads to

$$\phi_{fG}(\tau) = \frac{c}{d} a_d \phi_{fg}(\tau). \quad (50)$$

If it is desired to normalize the crosscorrelation of clipped signals to be unity at  $\tau = 0$ ,  $c$  and  $d$  must be related by

$$\frac{1}{\sqrt{2\pi}} \int_0^d \frac{c^2}{d^2} x^2 e^{-x^2/2} dx + \frac{c^2}{\sqrt{2\pi}} \int_d^{\infty} e^{-x^2/2} dx = \frac{1}{2} \quad (51)$$

that is,

$$2c^2 \left[ -\frac{1}{d} \frac{e^{-d^2/2}}{\sqrt{2\pi}} + \frac{1}{d^2} \frac{1}{2} a_d + \frac{1}{2} (1 - a_d) \right] = 1 \quad (52)$$

where  $a_d$  is defined by Eq. 42.

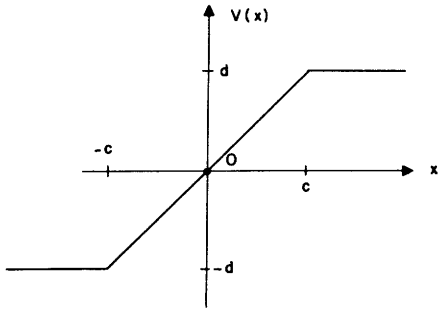


Fig. 6  
The characteristic of an abrupt peak clipper.

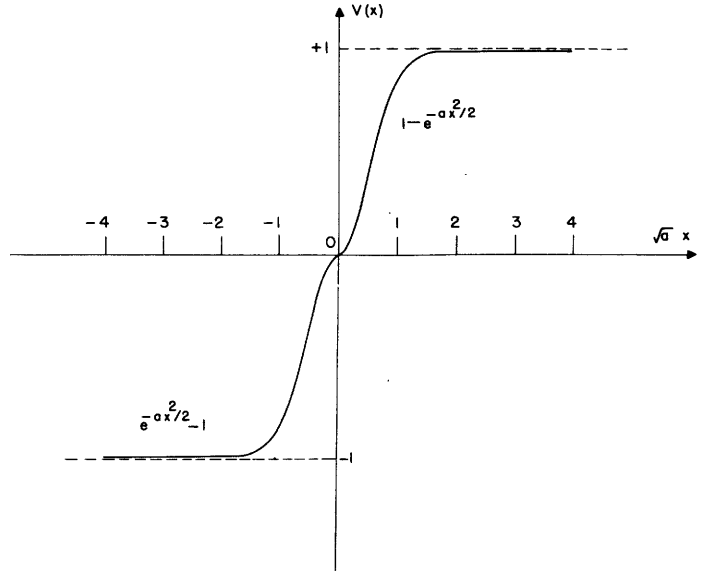


Fig. 7  
The characteristic of a gradual peak clipper.

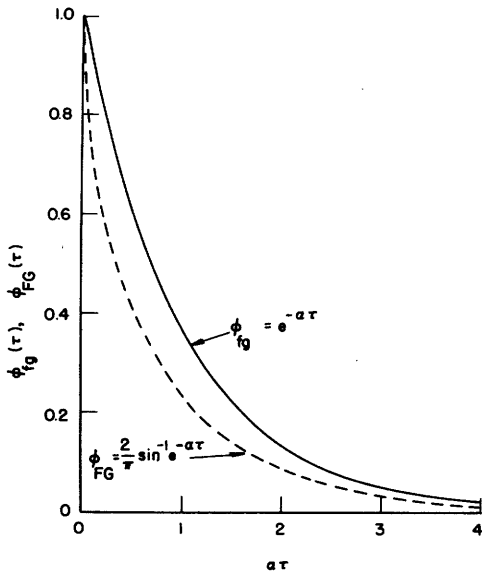


Fig. 8  
Correlation functions before and after extreme clipping.

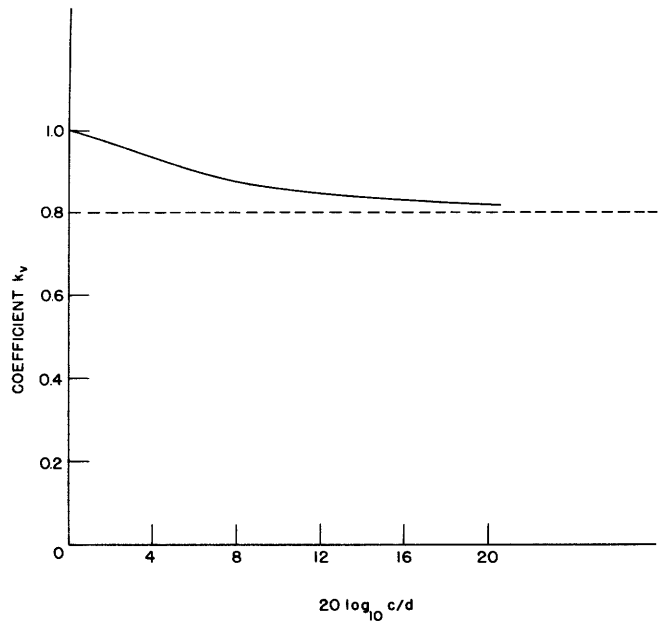


Fig. 9  
Correlation coefficient  $k_v$  as a function of the peak clipper slope.

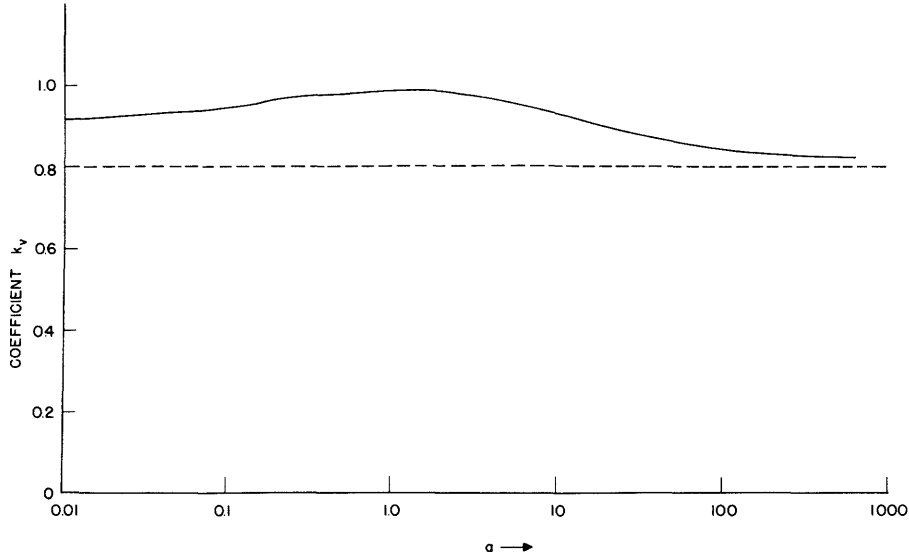


Fig. 10  
Correlation coefficient  $k_v$  of a gradual clipper as a function of the exponent  $a$ .

On the basis of Eqs. 50 and 52 the normalized coefficient  $k_v$  is plotted in Fig. 9.

In some cases it is of interest to consider gradual rather than abrupt clipping. Van Vleck computes the autocorrelation for a distortion function  $V(x) = c \tanh(x/c)$ . His result is (ref. 2, Eq. 12, p. 20)

$$\phi_{FF} = \phi_{ff} + \frac{1}{2}(-2\phi_{ff}) + \frac{1}{4}\left(5\phi_{ff} + \frac{2}{3}\phi_{ff}^3\right) + \dots \quad (53)$$

Equations 46 and 53 are most useful when we are dealing with amplifiers. A distortion characteristic which may be suitable for analyzing actual clippers is shown in Fig. 7. It is defined as follows

$$V(x) = \begin{cases} 1 - e^{-(a/2)x^2} & x > 0 \\ e^{-(a/2)x^2} - 1 & x < 0 \end{cases} \quad a > 0. \quad (54)$$

This definition is convenient because the autocorrelation function of the output can be found in a closed form. The flat region near the origin must be remembered. It extends approximately from  $x = -(1/\sqrt{a})$  to  $x = (1/\sqrt{a})$  and may be made as small or as large as we please.

The expression for the autocorrelation of the output is

$$\phi_{FF}(\tau) = 2 \int_0^\infty \int_0^\infty \left(1 - e^{-ax^2/2}\right) \left(1 - e^{-ay^2/2}\right) [p(x, y; \tau) - p(x, -y; \tau)] dx dy. \quad (55)$$

If we write  $r$  for  $\phi_{ff}(\tau)$  for the sake of simplicity, this leads to

$$\phi_{FF}(\tau) = \frac{2}{\pi} \left[ \sin^{-1} r - \frac{2}{\sqrt{1+a}} \sin^{-1} \frac{r}{\sqrt{1+a(1-r^2)}} + \frac{1}{\sqrt{1+2a+a^2(1-r^2)}} \sin^{-1} \frac{r}{1+a(1-r^2)} \right]. \quad (56)$$

We notice that the first term is the "extreme" clipper term of Eq. 48, and that the two other terms are correction terms which decrease as  $a$  increases.

$K_v$  is easily found from Eq. 19 and is given by

$$\begin{aligned} K_v &= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} y (1 - e^{-ay^2/2}) e^{-y^2/2} dy \\ &= \sqrt{\frac{2}{\pi}} \frac{1}{1 + \frac{1}{a}}. \end{aligned} \quad (57)$$

It follows that

$$k_v = \sqrt{\frac{2}{\pi}} \frac{1}{1 + \frac{1}{a}} \left[ \frac{1}{1 + \frac{1}{\sqrt{1+2a}} - \frac{2}{\sqrt{1+a}}} \right]^{1/2}. \quad (58)$$

The curve of  $k_v$  is plotted in Fig. 10.

#### VIII. Gaussian Correlator

On the basis of the relation expressed by Eq. 20, a simplified correlator can be designed capable of handling gaussian signals. One of the signals can be deliberately peak clipped. It should be, of course, the one which is not corrupted by noise. One can then exchange the problem of continuously multiplying two signals, which often presents difficulties, for the problem of gating one signal with another. In addition, it might also be advantageous if only two fixed amplitudes of the signal need be stored in the delay circuits and if the limitations of amplitude range in the unclipped signal channel can be overcome by boosting the clipped signal level with the resulting high sensitivity.

#### Acknowledgment

The author wishes to express his indebtedness to Professor J. B. Wiesner under whose supervision he worked at the Research Laboratory of Electronics and to Professor Y. W. Lee who gave so generously of his time in guiding the preparation of this report. He also wishes to thank his colleagues Messrs. C. Desoer, B. Howland and R. F. Schreitmueller for their valuable comments and suggestions.

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