

**CONSTRAINED AND UNCONSTRAINED GROWTH:
APPLYING THE AVRAMI EQUATION TO
THE PRODUCTION OF MATERIALS**

By:

Marianna B. See

SUBMITTED TO
THE DEPARTMENT OF MATERIALS SCIENCE AND ENGINEERING
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
BACHELOR OF SCIENCE IN MATERIALS SCIENCE AND ENGINEERING

AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

JUNE 2013

© 2013 Marianna B. See.

All rights reserved.

*The author hereby grants to MIT permission to reproduce
and to distribute publicly paper and electronic copies of
this thesis document in whole or in part in any medium now
known or hereafter created.*

Signature of Author: _____

Department of Materials Science and Engineering

May 3, 2013

Certified by: _____

Thomas W. Eagar
Professor of Materials Engineering and Engineering Systems
Thesis Supervisor

Accepted by: _____

Jeffrey C. Grossman
Carl Richard Soderberg Associate Professor of Power Engineering
Chairman, Committee for Undergraduate Students

Constrained and Unconstrained Growth:
Applying the Avrami Equation to
the Production of Materials

by

Marianna B. See

Submitted to the Department of Materials Science and Engineering
on May 2, 2013 in Partial Fulfillment of the
Requirements for the Degree of Bachelor of Science
in Materials Science and Engineering

ABSTRACT

Production of materials which are limited by the amount available on the earth's surface follow a growth curve similar to the Avrami equation which governs the process of nucleation and growth. This thesis will analyze whether the product curve follows not only the same path but the same steps as the Avrami model: initially slow growth during an introductory period, accelerated growth during market acceptance, and declining growth following market saturation. This thesis will use two materials, steel and aluminum, as a case study to further understand the applicability of the Avrami model to production forecasts of materials available in finite or limited amounts.

The aim of this project was to provide producers of various materials a model to use to predict when it would be profitable to invest in and enter a market and when not to do so. The framework developed provides a well-behaved model for the initial two stages, introduction and market acceptance, and forecasts the transition point between those two stages. However, due to lack of current data as neither aluminum nor steel have reached market saturation, a fit for the final stage and a forecast for the transition from market acceptance to market saturation has not yet been determined.

Thesis Supervisor: Thomas W. Eagar

Title: Professor of Materials Engineering and Engineering Systems

Table of Contents

Abstract.....	2
List of Tables.....	4
List of Figures.....	5
Acknowledgements.....	6
1. Introduction.....	7
2. Model for Material Production.....	10
2.1 Model: Explanation.....	10
2.2 Model: Assumptions.....	15
2.3 Primary Usage.....	16
3. Aluminum Production Curve.....	18
3.1 US History of Aluminum Production.....	18
3.2 World History of Aluminum Production.....	23
3.3 Fit to Model Discussion.....	25
4. Steel Production Curve.....	28
4.1 US History of Steel Production.....	30
4.2 World History of Steel Production.....	37
4.3 Fit to Model Discussion.....	40
5. Growth Analysis	43
5.1 Determine end of introductory phase/beginning of market acceptance.....	43
5.2 Determine end of market acceptance/beginning of market saturation.....	45
6. Conclusion.....	47
7. Next Steps.....	49
8. References.....	51
Appendix A.....	53

List of Tables

Table 3-1: U.S. Metal Supply.....	22
Table 4-1: Survey of Continuous Casting Output (1973-1977).....	34

List of Figures

Figure 2.1:	Homogeneous nucleation and growth as described by the Avrami equation.....	12
Figure 2.2:	Sigmoid Function.....	17
Figure 3.1-1:	Events affecting US aluminum production between 1900-1960.....	19
Figure 3.1-2:	Comparison of US aluminum supply.....	21
Figure 3.1-3:	Events affecting US aluminum production between 1960-2010.....	23
Figure 3.2:	Events affecting world aluminum production between 1900-2010.....	25
Figure 3.3-1:	Overview of US aluminum production.....	26
Figure 3.3-2:	Overview of world aluminum production.....	27
Figure 4.1-1:	Events affecting US steel production between 1880-1910.....	30
Figure 4.1-2:	Events affecting US steel production between 1910-1946.....	31
Figure 4.1-3:	The diffusion of continuous casting (1960-1980).....	33
Figure 4.1-4:	Events affecting US steel production between 1946-1980.....	35
Figure 4.1-5:	Events affecting US steel production between 1980-2010.....	37
Figure 4.2-1:	Events affecting US vs. world steel production (1900-1970).....	38
Figure 4.2-2:	Events affecting world steel production between 1970-2010.....	40
Figure 4.3-1:	Overview of US steel production.....	41
Figure 4.3-2:	Overview of world steel production.....	42
Figure 5.1-1:	Historic production growth for aluminum and steel.....	44
Figure 5.1-2:	5 year standard deviation of historic production growth for aluminum and steel...	44
Figure 7.1:	Sigmoid function for recycling.....	50

Acknowledgements

Professor Thomas Eagar has played an instrumental role in the creation of this thesis, helping me draft the premise of this paper as well as providing an informative discussion and assistance with historical events and modeling.

1. Introduction

The production and consumption of materials is constrained by the size of the world. The process of producing and consuming a material exhibits a progression from nearly exponential initial growth following the discovery of the material to a period of steady growth followed by a plateau as the market saturates. Many processes, even those with complex learning curves, follow a mathematical model known as the Avrami equation. In studying the nucleation and growth of phase changes, Avrami predicted that a material undergoing a phase transformation would follow a characteristic s-shaped, or sigmoidal, profile: initially slow to nucleate, quick to grow, and then growth slows as the growing material impinges upon itself.

It is an hypothesis of this thesis that the finite nature of material markets should cause that material's production curve to follow the same three-stage sigmoidal profile set by the Avrami model. A material's introductory phase, the initial understanding of its uses, parallels the initial slow nucleation because there is little constraint of the amount of the material available worldwide. The quick growth of the second stage corresponds to the efficient production resulting from increasing demand. And the diminishing growth rate that culminates in a final plateau is comparable to industry approaching the limit of the finite amount of materials available or saturated markets.

This study provides a framework for understanding the world demand for production and consumption of finite materials. By using steel and aluminum as case studies for materials, this study examines whether or not materials in constrained and unconstrained growth will follow the sigmoid function derived from the Avrami equation. Analysis of world demand using the Avrami equation will allow suppliers to

predict future markets for material production and consumption. From this analysis, a methodology can be extrapolated to determine the end of the introductory phase, the beginning of market acceptance, the end of this fast growth phase, and the beginning of market saturation. This will give suppliers the time to plan for ramping up or scaling down production facilities, reduce overstocks or shortages, and increase worldwide economic efficiency.

The market for finite materials, however, is not a perfect market unaffected by world events (wars, depressions, national politics, etc.). This study will use primary source data to account for historical market forces that may have affected U.S. production of materials as well as world production of materials. These forces may shift the production and recycling curves away from the sigmoid function. Analyzing these events and their effects on the U.S. market and world market, however, will provide the industry insight into which variables do or do not affect materials production, offering another tool with which to predict future markets.

Because of steel and aluminum's considerable history as structural materials, ample data on consumption of both metals is available and can be used to determine patterns in materials usage. This study begins by studying the production curves for both steel and aluminum and using this analysis to derive a model charting the stages of the materials' consumption in the market: market introduction, fast growth, and market saturation. This model for confined materials should fit the sigmoid function – the shape of the Avrami equation.

This study continues by examining the U.S. production curves for steel and aluminum, and looking to historical events to explain fluctuations from the sigmoid

function. This study then looks at worldwide demand for both steel and aluminum, again concentrating on variables that could cause variations from the predicted model.

2. Model for Material Production

The finiteness of our world constrains the production and consumption of materials. This constraint makes the process of producing, consuming, and even recycling a material follow a similar growth model: the Avrami Equation. While there are many historical factors that affect the production of materials, this thesis will begin by introducing an economic model for the growth of material production and matching the model to historical data to determine if the Avrami Equation can be used to predict when nucleation, fast growth, and market saturation will occur. Explanations for noted deviations due to historical events will follow.

2.1 - Model: Explanation

The Avrami Equation describes how solids transform from one phase or state of matter to another. The transformation of the phases follows a sigmoidal profile where “the new phase sets in at a rate first slow, then faster, then finally toward the end of the transformation, slow again.”¹ This changing growth pattern results from the changing composition of the transforming material.

Phase change can only occur when enough germ nuclei are present in the initial material and in the early stages of transformation (Fig. 2.1a). Avrami predicts that the number of growth nuclei present, however, is proportional to “time and their respective volumes.”² The “time needed to establish a steady-state distribution”³ of germ nuclei of

¹ Melvin Avrami, “Kinetics of Phase Change I. General Theory,” *Journal of Chemical Physics* 7 (1939): 1103.

² Melvin Avrami, “Kinetics of Phase Change II. Transformation-Time Relations for Random Distribution of Nuclei,” *Journal of Chemical Physics* 8 (1940): 217.

³ George W. Scherer, “Models of Confined Growth,” *Cement and Concrete Research* 42 (2012): 1256.

critical size is called the “induction period” and can be attributed for the initially slow growth rate of phase transformation.

After enough germ nuclei of critical size have formed, however, Avrami hypothesizes that the material transitions into an acceleratory period of rapid phase transformation as seen in the steep rise of the S-shaped transformation-time curve.⁴ This accelerated growth period corresponds both to an increase in the surface area of the precipitates as they grow into particles and consume the old phase and the continuation of germ nuclei forming in the initial phase (Fig. 2.1b).

The material transitions into its final stage of phase transformation as it nears transformation completion. This occurs when there is little untransformed material for germ nuclei to form in, causing the production of new particles to slow, and existing particles begin to impinge on one another, forming a boundary where growth stops (Fig. 2.1c).⁵ The inability of the material to form new germ nuclei or to grow its existing particles is reflected in the growth rate of transformed material – a sharp decline from the rapid growth of the intermediate phase.

⁴ Avrami, “Kinetics of Phase Change II,” 218. The transition from initially slow to rapid growth “does not require any new phenomenon, such as nucleation of a new phase or a change in growth mechanism.” Rather, the S-shape of the transformation time curve is an “automatic consequence of the constant radial growth of the product, which leads to a volume increasing roughly cubically with time.” Scherer, “Models of Confined Growth,” 1256.

⁵Clinton DeW. Van Sielen, “Random Nucleation and Growth Kinetics,” *Physical Review B: Condensed Matter* 54 (1996): 845.

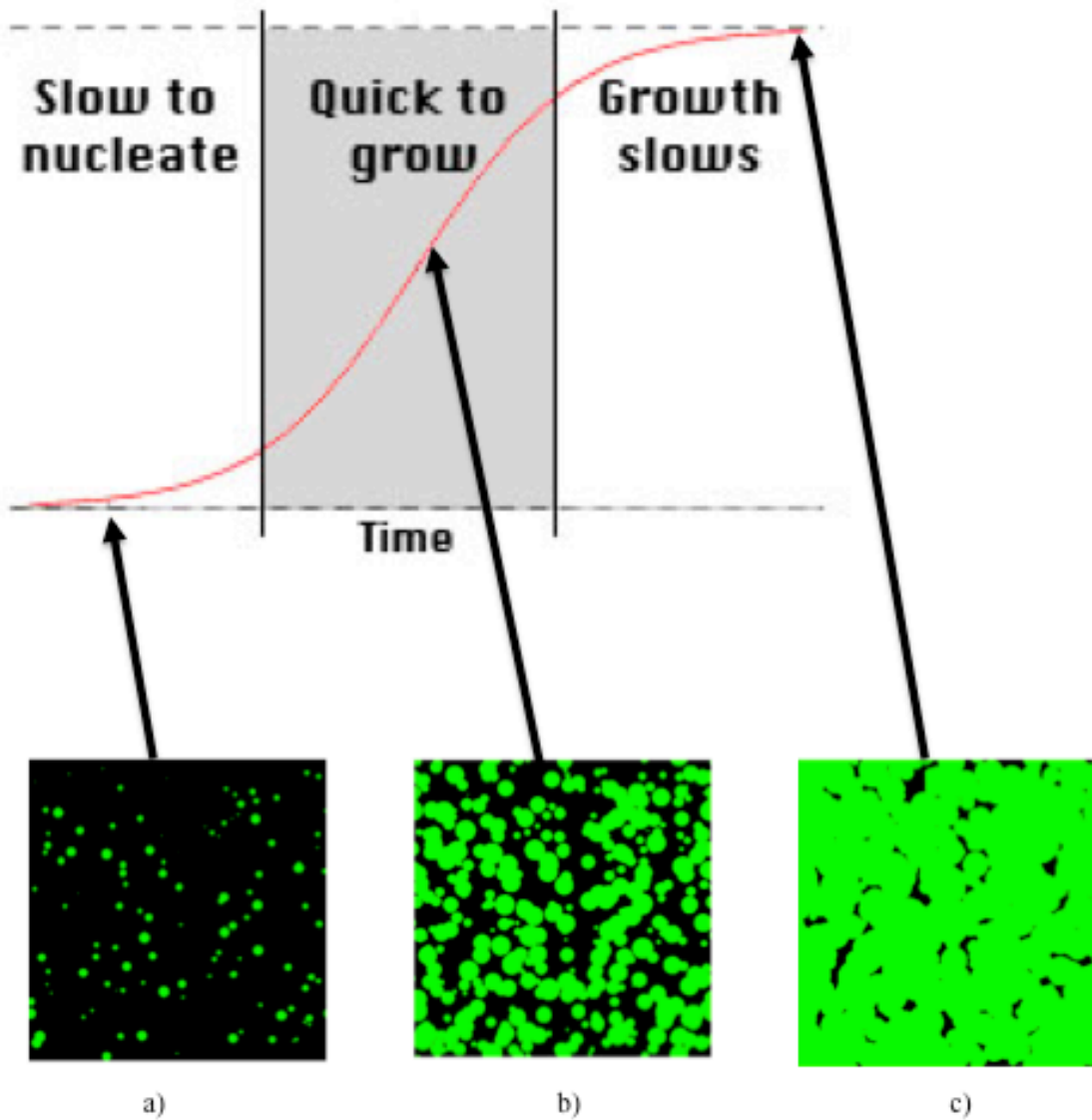


Fig. 2.1: Homogeneous nucleation and growth as described by the Avrami equation. A) Initially the rate of growth is slow as enough nuclei need to form at a critical size before growth can occur. B) The growth rate increases as the surface of the precipitates increase, once transformation has begun. C) When precipitates start to impinge the rate of growth slows down as the available surface decreases.

The Avrami Model follows the equation:

$$y = 1 - \exp^{-kt^n}$$

with y equaling the fraction of the material that has been transformed, k representing a parameter that is a function of nucleation rate and growth rate, and n is a positive constant typically called the “Avrami constant.” If we multiply this equation by the total size of the market m , the ultimate amount of recoverable resource, then we can develop an understanding of the penetration of a metal in the market place and how much will be produced annually y^* .

$$y^* = [1 - \exp^{-kt^n}]^* m$$

We can develop a better physical understanding of what the parameters k and n mean in this equation with respect to the production growth of finite materials by looking to the Bass Model, a diffusion model which describes the process of how new products get adopted as an interaction between users and potential users.⁶ The Bass Model makes several key assumptions that are in line with the Avrami model such as⁷:

- *The market potential (y^*) remains constant.*
- *The marketing strategies supporting the innovation do not influence the adoption process:* marketing variables such as price, advertising, and selling effort do not affect consumers decisions surrounding adoption.
- *The customer decision process is binary (adopt or not adopt):* multistage decision processes (awareness → interest → adoption → word of mouth) are relaxed into one stage.
- *Uniform mixing:* everyone can come into contact with everyone else.

⁶ Juanjuan Zhang. Verbal correspondence, March 7, 2013.

⁷ Gary L. Lilien, Arvind Rangaswamy, and Arnaud De Bruyn, “The Bass Model: Marketing Engineering Technical Note,” *Principles of Marketing Engineering* (2007): 1.

- *Imitation always has a positive impact:* the model allows only for interactions between innovators and noninnovators who favor the innovation

The alignment of the assumptions of the Bass model and the assumptions we make in the Avrami model (further discussed below) as well as the alignment of the products under consideration (metals) with the products the Bass model best fits allows us to apply the parameters of the Bass Model to the Avrami model to give us a better physical understanding of it. The Bass Model fits best for durable goods and long-term services, products with a higher perceived risk (radical innovations), and initial or single purchase products.⁸

The Bass Model also charts out a sigmoidal curve for the diffusion of innovation.

$$y^* = \frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p}e^{-(p+q)t}} m$$

Again, y^* is the annual amount of material produced, m is the ultimate amount of recoverable resource, and t is time. The new parameters introduced, p and q , are the ones we will apply to the Avrami model. Parameter p is the coefficient of innovation or external influence while parameter q is the coefficient of imitation or internal influence of a new product or process.

When looking at the Avrami model, we can see that parameter k is equivalent to $p+q$ or the speed of diffusion. Parameter n is equivalent to q/p or the relative strength of the forms of influence. When using a regressive model of non-linear least squares (see Appendix A for example fit curves), a more reliable non-parametric method of estimation of the Bass Model, we will see that the Avrami model is more sensitive to changes in the

⁸ Juanjuan Zhang. Verbal correspondence, March 7, 2013.

value of k . This tells us that it matters less *how* a technology is diffused, rather it matters more *how fast* a technology is diffused. This is indicative of a technology that controls a competitive field – it is more important that a company hear of the innovation and implement it as quickly as possible, it matters less how they heard of it. Also, as more people adopt a technology, those who haven't yet adopted are reassured about the validity of the innovation and are therefore more likely to adopt.

It also makes sense that in the regressive model, the value of n is less than 1. This means that q is greater than p , which makes sense for a innovation in industrial durables as opposed to in consumer durables. For consumer durables, we would expect p (external effects) to matter more and impact the shape of the curve more. For industrial durables, however, it makes sense that the imitation or word-of-mouth factor affects the curve more.⁹

2.2 - Model: Assumptions

This equation makes several basic assumptions and simplifications, of which the first two can easily be applied to the market for constrained materials.

1. Throughout the entire transformation process, nucleation occurs randomly and homogeneously in the entire untransformed portion of the material.¹⁰
During the discovery and production periods, innovation of applications and processes to develop the material occur randomly throughout the world.
2. Growth ceases upon impingement.¹¹
Production growth rate slows as it reaches the constraining limit. The market is limited by the number

⁹ Joshi, Amit, "Product Life Cycle and The Bass Model," Strategic Marketing Management, University of Central Florida. Lecture.

¹⁰ Avrami, "Kinetics of Phase Change II," 213.

¹¹ Avrami, "Kinetics of Phase Change I," 1108.

of people in the world, the need for buildings, and the constrained size of the universe.

Avrami's third assumption proves more difficult to apply to the market for constrained materials. While Avrami assumes that growth occurs anisotropically during the transformation process, markets cannot be so easily simplified. When Avrami makes this assumption of anisotropic growth, he is dealing with a constant sized material. Markets for materials, however, are not constant sized but in fact expand as countries develop and can produce the material, the world's population grows, and as products become more sophisticated and land becomes more scarce (therefore increasing the need for taller buildings).

3. Growth occurs at the same rate in all directions.¹²
Markets grow isotropically - the market size changes within a studied timeframe.

2.3 - Primary Usage

Introduction: When a new material, such as steel or aluminum, is introduced to the market, there is an introductory phase during which the material must develop acceptance within the market. Uses for the material must be discovered and manufacturing processes for the material's applications must be developed. The growth rate for this introductory stage is slow, just as Avrami predicted an initially slow growth rate due to the time required for a significant number of new phase nuclei to form and begin growing.

Market Acceptance: Once the market accepts the new material, the growth rate will increase significantly. Avrami explains that, during this intermediate period, the

¹² Ibid, 1105.

transformation is rapid – the nuclei grow into particles and consume the old phase while nuclei continue to form in the remaining initial phase. Similarly, once the uses have been determined for a material, the production rate escalates both as demand for the product increases and better and more efficient manufacturing processes are discovered. The increasing production rate of a material can also result from discovery of additional applications or expansion of user industries.

Market Saturation: In this third stage, the market has matured with regard to the material. Again a parallel can be drawn with Avrami’s explanation of transformation kinetics. When transformation nears completion, there is little space in which nuclei can form so the production of new particles begins to slow. As well, as the particles become larger the initial nuclei begin to impinge on one another, forming a boundary where growth stops, further slowing overall growth. Production growth similarly slows due to one of three causes: industry has reached as many customers as it could, used up all the available material, or is producing at maximum efficiency.¹³

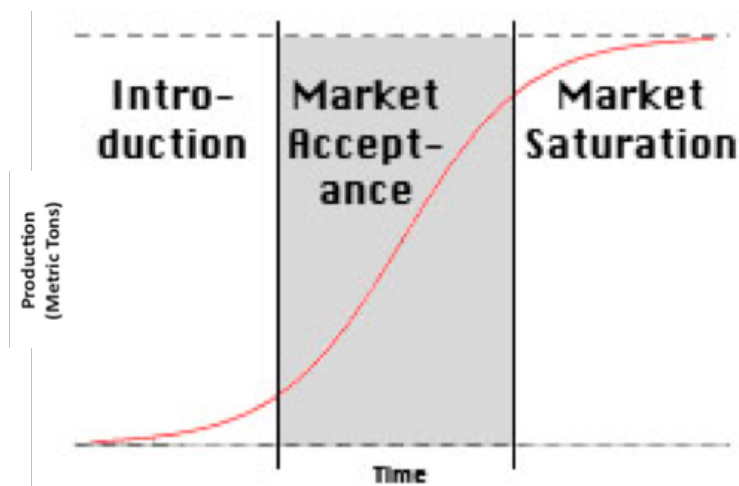


Fig. 2.2: Sigmoidal function divided into three sections – initial stage of slow nucleation, middle stage of quick growth, and final stage of decayed growth rate.

¹³ Paul Crompton, “The Diffusion of New Steelmaking Technology,” *Resources Policy* 27 (2001): 89.

3. Aluminum Production Curve

With uses ranging from construction to consumer durables to packaging to transportation, aluminum is the most abundant metal to be found in the earth's crust, making up 8.1% by weight of the Earth's solid surface.¹⁴ There are three stages of production: bauxite mining, alumina production, and finally reduction of alumina to aluminum. The latter two steps have remained essentially unaltered since the late nineteenth century when Charles Martin Hall discovered the commercial production method for aluminum.¹⁵ The process for producing aluminum is so energy intensive that the main exporters of aluminum are those that built their industries on the basis of cheap hydro and gas energy while major importers lack both essential energy supplies.¹⁶

3.1 - US History of Aluminum Production

Based along the Little Tennessee River, the Pittsburgh Production Company had the access to the required energy source, but still had the chore of creating a need and a demand for aluminum. Production of aluminum remained at a moderate, if low, level throughout the early twentieth century, until the start of WWII caused an almost instant ramp-up in production (Fig. 3.1-1). WWI did not affect aluminum production because the material was still in a growth stage and its uses had yet to be determined. Annually during WWI, over 20 million tons of steel were produced compared to the less than a thousand tons of aluminum.¹⁷

¹⁴ "Aluminum," Los Alamos National Laboratory, accessed March 19, 2013, <http://periodic.lanl.gov/13.shtml>.

¹⁵ Douglas W. Woods and James C. Burrows, *The World Aluminum-Bauxite Market: Policy Implications for the United States* (New York: Praeger Publishers, 1980), 67.

¹⁶ Sterling Brubaker, *Trends in the World Aluminum Industry* (Baltimore: The Johns Hopkins Press, 1967), 41.

¹⁷ Thomas Eagar. Personal communication. October 30, 2012.

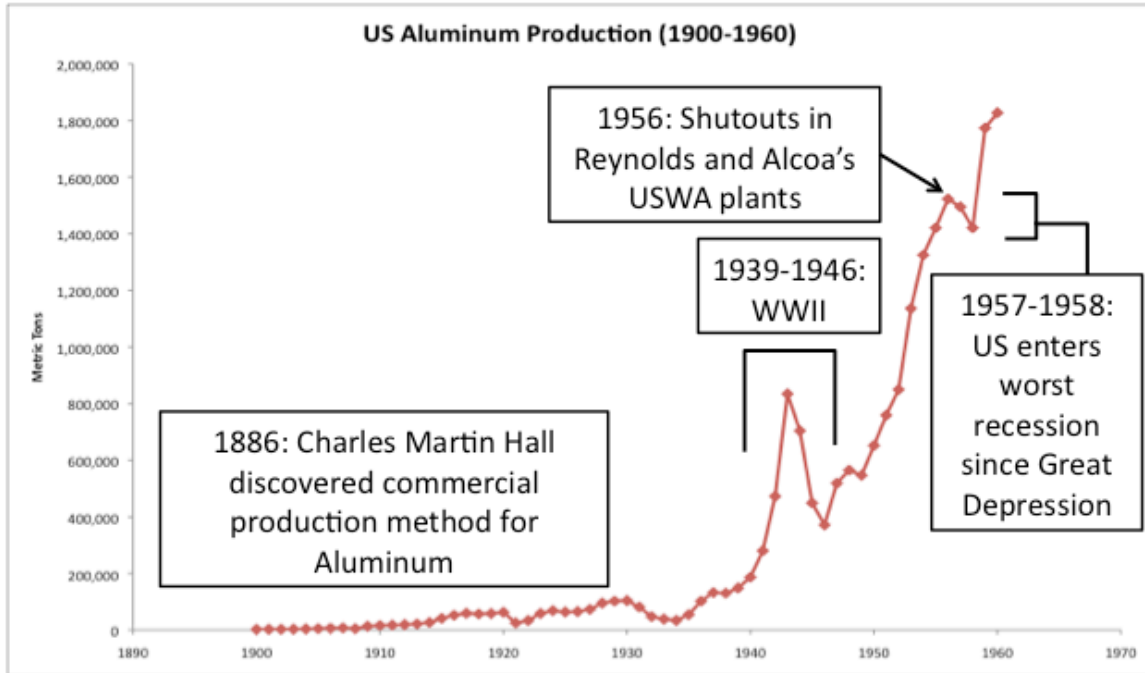


Fig. 3.1-1. Events affecting US aluminum production between 1900-1960.

The almost instantaneous ramp-up of production for WWII was made possible by the public projects built during the Great Depression, such as the Hoover Dam, which provided the hydroelectric energy necessary to ramp up aluminum production. The aluminum produced during the 1939-1946 WWII period was then used for lightweight aircrafts, among other military uses. The end of the war meant the end of this period of high production levels for the aluminum industry and production growth rates declined to more subdued levels.

The 1950's signaled the coming of tumultuous times for the US aluminum industry, though not as tumultuous as for the US steel industry. In 1956, United Steelworkers of America (which absorbed the debt-ridden, 20,000 member Aluminum Workers of America, CIO, in 1944)¹⁸ organized shutdowns of both Reynolds and Alcoa

¹⁸ George D. Smith, *From Monopoly to Competition: The Transformations of Alcoa, 1888-1986* (New York: Cambridge University Press, 1988), 290.

plants, forcing a reduction in US aluminum production that year.¹⁹ Aluminum union members, however, were more quiescent in comparison to steel union workers, which helped dampen the volatility that the steel industry experienced.²⁰ In 1957, the US's economy slid into its worst slump since the Great Depression, spurring a recession that lasted until 1958.²¹ Despite the financial and labor relation setbacks, the aluminum industry handled the disruptions with care, preventing the industry from experiencing the same volatility the steel industry did during the same period.

The following decade did bring a period of uninterrupted growth, but during the 1970's and 80's aluminum once again experienced an unstable market. This period was characterized by excess capacity due to the growth of hydroelectric facilities in other countries and their entry into the aluminum market as well as the saturation of key markets. The oil crisis of 1973 also caused a decline in production, showing just how dependent the production of aluminum was on energy.²² In 1980, Congress approved the Northwest Power Act which, for aluminum companies, meant not only rate increases but also long-term power sales contracts.²³ Despite these rising rates, 1980 was the greatest year of primary aluminum production in US history.²⁴ This peak was followed by several sharp changes in production levels as the volatility of the 1970s and 80s continued into the 1990s. The continuation of the instability into the 1990s was due both to US policy and to the fall of the former Soviet Union at the end of the Cold War. In 1990, the US government passed the Clean Air Act, forcing electric utilities to reduce sulfur dioxide

¹⁹ Ibid, 292.

²⁰ Ibid, 295.

²¹ Ibid, 295.

²² Thomas Eagar. Personal communication. October 30, 2012.

²³ "Aluminum," Northwest Power and Conservation Council, accessed March 19, 2013, <http://www.nwcouncil.org/history/aluminum>.

²⁴ William T. Choate and John A.S. Green, "Modeling the Impact of Secondary Recovery (Recycling) on U.S. Aluminum Supply and Nominal Energy Requirements" *TMS* (2004), 913.

emissions. The costs “incurred by electric utilities in making these emissions reductions were in turn passed on to primary aluminum manufacturers. Aluminum producers [also had to] invest large sums of capital into advanced scrubber systems in order to satisfy the Clean Air requirements.”²⁵ These high costs forced aluminum producers to reevaluate production levels with respect to demand, as excess now cost more per metric ton. The fall of the former Soviet Union in 1991 also reversed the growth trend of production as, after the USSR opened, it dumped aluminum onto the world market: saturating key markets, causing global excess capacity, and forcing aluminum producers internationally to cut back on production.²⁶

Until this point, the proportion of the US’s metal supply stemming from primary production of aluminum had remained dominant to secondary production or imports. During the 1990s, however, there was a “time period of several years, best exemplified by the 1999 data, when the total supply was almost exactly balanced between the three” before the trend was reversed and imports and secondary production grew while primary production declined (Fig. 3.1-2).²⁷

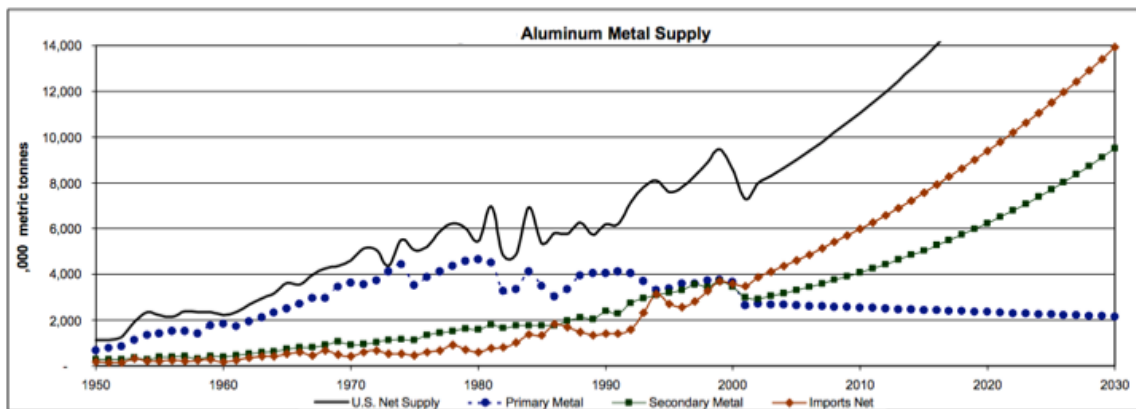


Fig. 3.1-2. Comparison of US aluminum supply by primary production, secondary production, and imports.

²⁵ “Primary Production of Aluminum,” Highbeam Business, accessed March 13, 2013, <http://business.highbeam.com/industry-reports/metal/primary-production-of-aluminum>.

²⁶ Thomas Eagar. Personal communication. October 30, 2012.

²⁷ Choate, “Modeling the Impact of Secondary Recovery (Recycling) on U.S. Aluminum Supply and Nominal Energy Requirements,” 913.

Between 1980 and 2000, primary production fell by half while imports grew four fold and secondary recovery grew from 23 percent of the US metal supply to 32 percent (Table 3-1).

Table 3.1 – U.S. Metal Supply – Total Aluminum Production and its Distribution Between Components

Year	Metric Tons (tonne)	Primary Production	Imports	Secondary Recovery
1980	6,833,000	68.1%	8.9%	23.0%
1990	7,863,000	51.5%	18.1%	30.4%
1999	11,154,000	33.9%	33.0%	33.1%
2000	10,699,000	34.2%	33.5%	32.2%
2002	9,500,000	28.5%	40.8%	30.7%

One factor in the decline of primary production was the strike put on by 3,000 workers at five Kaiser Aluminum plants in 1995.²⁸ Other factors were the economic downturn, energy shortfalls due to the West Coast energy crisis, and resultant price increase in the Pacific Northwest, occurring in 2000. Aluminum production took an indirect hit with the tragedy of September 11, 2001, when the aerospace industry came to a screeching halt and the sudden decrease in demand of aluminum to build commercial airliners led to overcapacity.²⁹ Finally, the aluminum industry took yet another hit with the global financial crisis of 2008, which affected the transportation industry, construction, consumer purchase patterns, and machinery and equipment purchases among other industries – all industries heavily involving the use of aluminum.

²⁸ “Kaiser Aluminum Says Plants to Operate Despite Strike,” *The Seattle Times*, February 21, 1995, accessed March 19, 2013,

<http://community.seattletimes.nwsourc.com/archive/?date=19950221&slug=2106293>.

²⁹ “Primary Production of Aluminum,” accessed March 13, 2013.

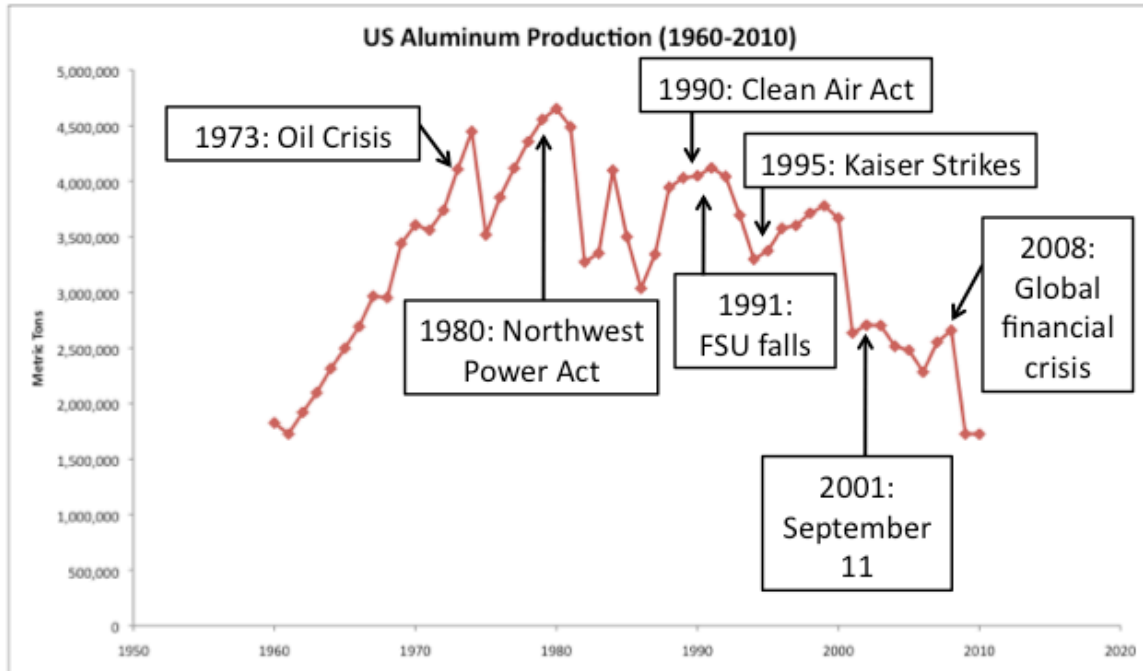


Fig. 3.1-3. Events affecting US aluminum production between 1960-2010.

There is “little optimism that [US] smelters ever will resume full production.”³⁰ The turn of the millennium has been characterized by increases in prices for raw materials, volatile electricity prices, and worldwide competition with other, often lower-cost production and supply, foreign production companies.³¹

3.2 – World History of Aluminum Production

Prior to the late 1930s, virtually all aluminum was produced in Europe and North America, with both sharing a comparable magnitude of production. In the late 1930s, however, the Japanese empire and the USSR entered the market. With the US hit by the Great Depression and Europe and the Communist-bloc position preparing for war, production in Europe moved well ahead of US production levels. During WWII,

³⁰ “Aluminum,” Northwest Power and Conservation Council, accessed March 19, 2013.

³¹ Ibid.

however, US production growth quickly surpassed all other countries, only to decline with everyone after the war was over.³²

Prior to the 1950s, the size of the aluminum market was too small to support more than a few efficiently sized production companies.³³ But, by 1950, demand for aluminum was growing and production levels followed.³⁴ Late in the decade, appearance of excess capacity combined with diminished restrictions on trade pushed producers to seek out foreign markets as outlets for their metals.³⁵ The capacity expansion and entry activity of the post-war period has “gradually reduced industry concentration.”³⁶ Despite the removal of monopolies, however, the distribution of international production continues to maintain a “rough tendency toward conformity with consumption if viewed on a continental basis.”³⁷ In 1964, the Americas consumed 47 percent of the world’s primary aluminum used and produced about 51 percent of it. In comparison, Europe consumed 26 percent but produced 20 percent of the world’s primary aluminum used and the Communist-bloc position also consumed and produced an equally balanced share.³⁸

The oil crisis of 1973 signaled another down-turn in production growth, as producers got scared by threats of limited access to the energy required to run their plants.³⁹ Within a few short years, however, this trend was reversed and production was growing yet again. A more substantial disruption occurred with the uncertainty surrounding the fall of the former Soviet Union. The aluminum market suffered both from the uncertainty of Soviet actions and from the amount of aluminum the Soviets

³² Brubaker, *Trends in the World Aluminum Industry*, 34.

³³ Woods and Burrows, *The World Aluminum-Bauxite Market*, 67.

³⁴ Brubaker, *Trends in the World Aluminum Industry*, 34.

³⁵ *Ibid.*, 46.

³⁶ Woods and Burrows, *The World Aluminum-Bauxite Market*, 72.

³⁷ Brubaker, *Trends in the World Aluminum Industry*, 37.

³⁸ *Ibid.*

³⁹ Thomas Eagar. Personal communication. October 30, 2012.

dumped on the market, causing a global overcapacity and forcing producers to stem production. Another disruption in production growth came with the Alcan Aluminum Strikes of 1995 in Canada. Again, this only briefly disrupted the upward growth trend and the industry was quickly back on track. The growth curve for production of aluminum continued on a smooth, exponential upward trend until the recent global financial crisis in 2008, which forced production into a decline as industries such as construction and transportation suddenly froze.

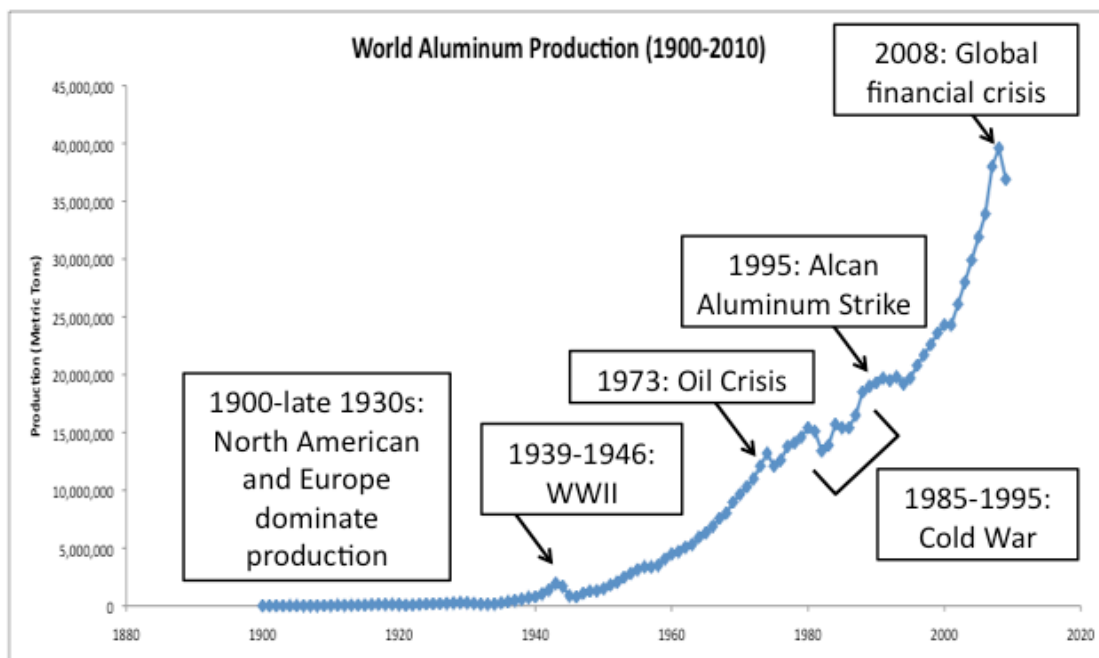


Fig. 3.2. Events affecting world aluminum production between 1900-2010.

3.3 – Fit to Model Discussion

The US aluminum production data fit the Avrami model very well, showing a distinct introductory period (the reticence to adopt the new metal shown by its lack of usage during WWI), followed by a period of rapid growth showing its acceptance by the market (in WWII it was used to make lightweight airplanes – the military recognized the material’s benefits and accepted the material). The past several decades have been

characterized by a period of volatility, but averaged they indicate a plateau indicative of the market saturation period.

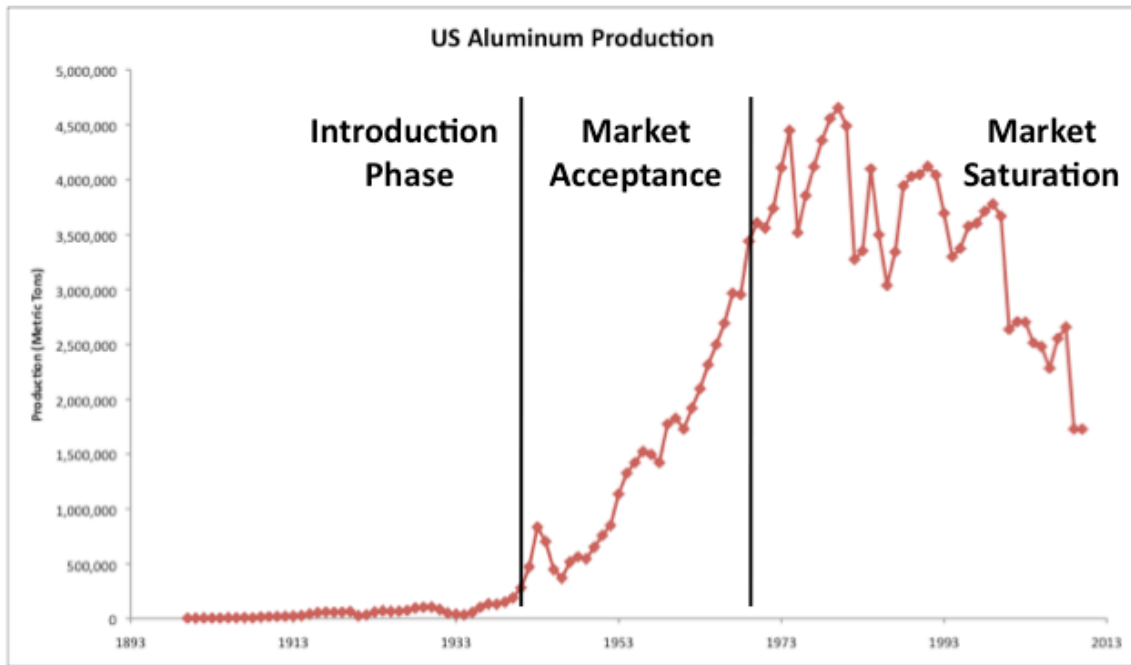


Fig. 3.3-1 Overview of US aluminum production.

The world aluminum production data, however, indicate that the world aluminum market has only gone through two stages of the Avrami model: introduction and market acceptance. Similar to the US production model, the world production data indicate an introductory period prior to WWII. After WWII, production grows exponentially but never appears to approach an asymptote (the market size) and plateau into the market saturation stage.

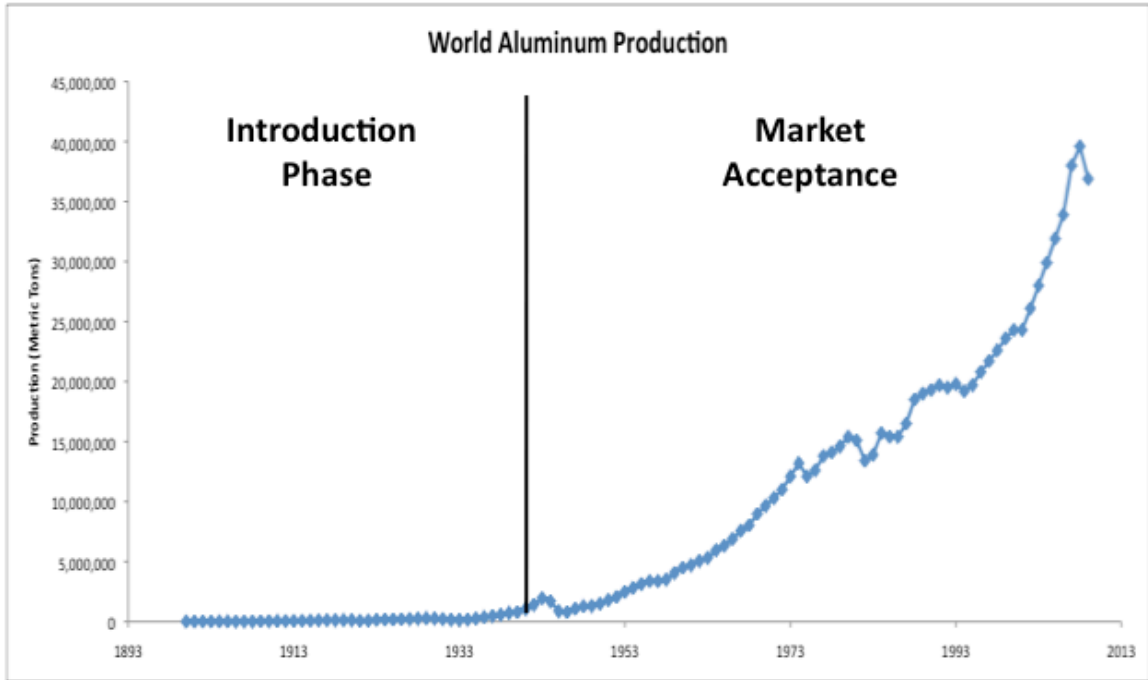


Fig. 3.3-2. Overview of world aluminum production.

4. Steel Production Curve

Steel has the largest market volume of any metal currently produced, representing over 95% of the market. As a structural material, steel is “used in extremely large quantities in bridges, buildings, ships, transportation, vehicles, and the like” due to its “favorable combination of strength and toughness.”⁴⁰ This widespread use is made possible only through technological developments in the production process. Steel production dates back millennia, revered as a skilled art but limited by the production abilities of an individual. In 1855, however, Henry Bessemer relieved that limitation with the invention of the Bessemer process for making steel.⁴¹ Bessemer’s mechanization of the refining process not only decreased the production time for steel but also increased the material’s toughness. This harder steel was ideal for rail-making, a highly demanded item during the Civil War. Bessemer’s invention “sparked mechanical improvements throughout the industry, prompting steel masters to integrate all stages of the production process.”⁴² The Bessemer process was succeeded by the open-hearth method, developed just two years later, which greatly lowered the cost of and time required for steel production and resultantly increased annual steel production.

The demand for steel to construct these items is inelastic – changes in price do not strongly affect the quantity of steel demanded. This means that no integrated producer who controls a large enough share of the market for his actions to affect others can gain by increasing his output. Consumers will still demand the same amount of steel;

⁴⁰ Thomas Eagar, “The Future of Metals,” *Welding Journal* (1991), 69.

⁴¹ Daniel Madar, *Big Steel: Technology, Trade, and Survival in a Global Market* (Vancouver: UBC Press, 2009), 1.

⁴² “Iron and Steel Industry,” Oxford Reference, accessed February 8, 2013, <http://www.oxfordreference.com/view/10.1093/acref/9780195082098.001.0001/acref-9780195082098-e-0795?rskey=ok36uQ&result=2&q=steel%20industry>.

producers, however, will have to reduce their prices due to the oversupply in the market. The demand inelasticity of steel, therefore, gives producers an incentive to collude to set production levels.⁴³ While demand for steel may not be affected by changes in price, steel's role as an intermediate good does makes it subject to the demand for its end products. When demand for durable goods – the bridges, buildings, ships, and vehicles – falls during “cyclical downturns in the economy, demand for steel falls as well.”⁴⁴

When looking at a graph of raw steel production, fluctuations in the curves will somewhat reflect these variations in demand. Integrated steel producers' ability to vary production to meet these variable demand levels, however, is limited by the high capital and fixed costs inherent in the steel industry and the “fact that integrated steelmakers' technology requires continuous operation.”⁴⁵ Other causes of fluctuations in the curve are policy changes, implementations, and world events relevant to the steel industry.

Steel production in the United States will be more sensitive to events due to its smaller scale of production relative to global production. This sensitivity, as shown in the large rises and falls found in Fig. 4.1-1 through Fig. 4.1-5, will make it difficult to fit the US production curve to the proposed model. Still, evaluating the types of events and policies that can cause those fluctuations will be a good exercise for drawing trends and making predictions of fluctuations.

⁴³ Madar, *Big Steel*, 46-47.

⁴⁴ *Ibid*, 47.

⁴⁵ *Ibid*, 48.

4.1 - US History of Steel Production

The United States established its first Bessemer converter in 1864 and completed its first open-hearth furnace just four years later. These establishments allowed the United States to increase steel rail production from none before the Civil War to 115,000 tons in 1873, one-eighth of all American rail production.⁴⁶ Following the end of the Civil War, American steel production continued to ramp-up at an astonishing pace. Between 1880 and the turn of the twentieth century, steel production increased from 1.25 million tons to more than 10 million tons.⁴⁷ By 1910, the US was producing more than 24 million tons, more than any other country.⁴⁸

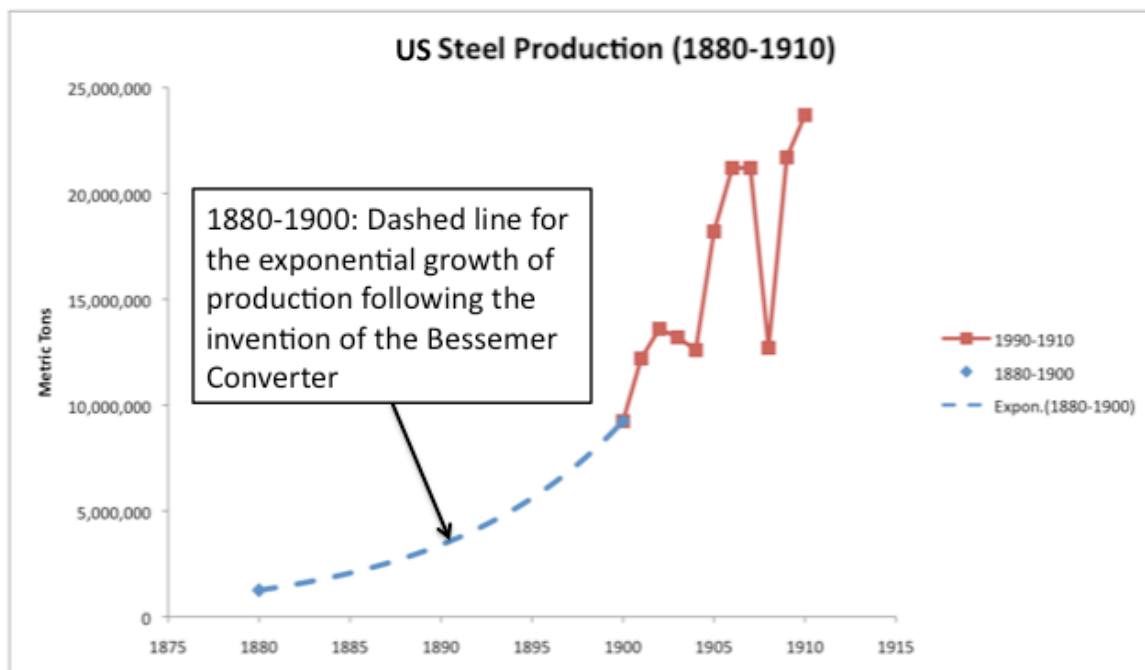


Fig. 4.1-1. Events affecting US steel production between 1880-1910.

⁴⁶ "Iron and Steel Industry," History, accessed February 8, 2013, <http://www.history.com/topics/iron-and-steel-industry>.

⁴⁷ Ibid.

⁴⁸ Ibid.

US steel production experienced another jump in production in 1914 with the start of World War I. In times of war, demand for steel weapons and military vehicles increases exponentially, resulting in a corresponding increase in production as suppliers listen to the market and its needs. This severe increase in demand remained steady throughout the war, but dropped off with the signing of the Treaty of Versailles in 1919, which ended the war. The market then returned to normal, predictable production levels until 1929 when the Great Depression began and demand for and ability to produce steel suddenly decreased. Production levels remained low throughout the financial crisis, until they again experienced a sharp upturn with the start of World War II in 1939.⁴⁹ Again, demand levels, and therefore productions levels, remained steadily high throughout the war until peace was declared in 1946, when production again dropped off.

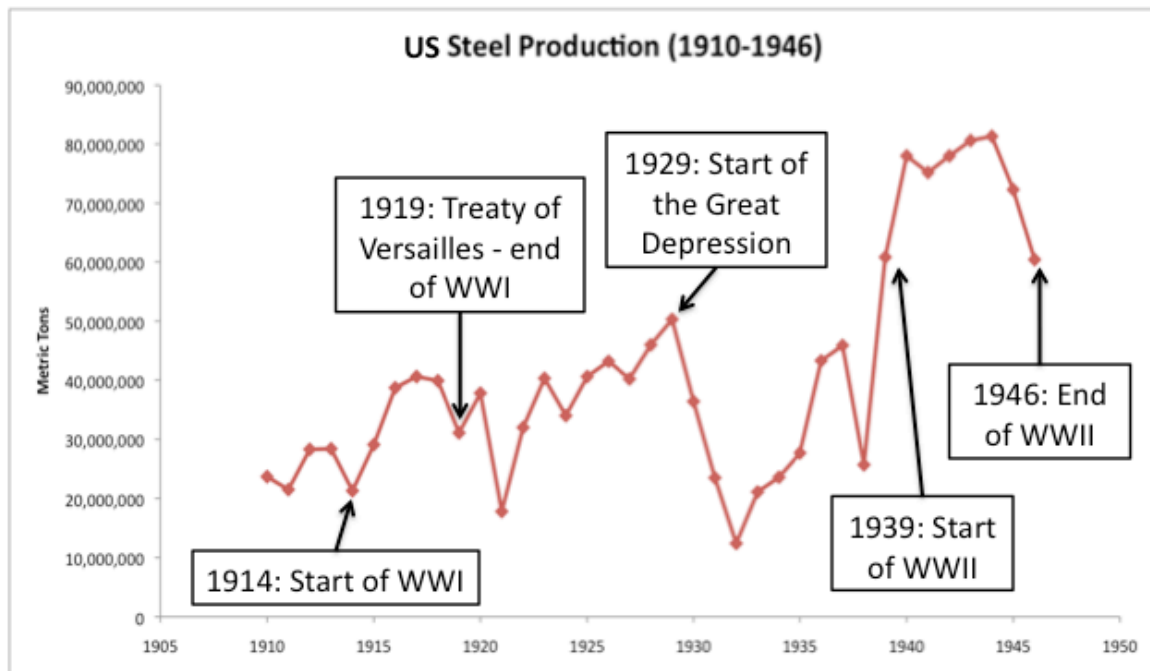


Fig. 4.1-2. Events affecting US steel production between 1910-1946.

⁴⁹ “Since the beginning of the war in 1939, expanding war-industries needs have been responsible for a continued stimulus in the production of steel ingots and castings; consequently, a new record has been established each year.” Harold E. Carmony and H.L. Cullen, “Iron and Steel Scrap Year 1943,” *Minerals Yearbook 1943*, 1945, Bureau of Mines, 525.

Following the peace agreements that ended WWII, the steel industry faced a series of turbulent decades rife with policy changes, labor strikes, and technological innovations. Between 1946 and 1956, United Steel Workers Union members struck five times, causing delays in production.⁵⁰ These strikes and their affect on production are reflected in the massive fluctuations in production in Fig. 5.1-4. In 1959, another strike, the Great Steel Strike, kept production suppressed, causing buyers to turn to foreign imports instead.⁵¹ Just three years later, the steel industry faced another big loss when President Kennedy faced down U.S. steel over price increases in 1962. At that point, steel was the dominant industrial constraining cost and therefore controlled the economy – this loss signified a change in the winds.

Production throughout the 1950s was also affected by a heavy investment in additional capacity. This was done, however, by adding open-hearth furnaces which increased volume but not efficiency. They became “a liability in the 1960s, when the growth of steel demand fell short of industry expectations and when Japan began gaining significant export advantages from modernization and scale.”⁵² Internationally, steelmakers adopted the new continuous casting and basic oxygen furnace (BOF) technologies, recognizing it’s extreme benefits – considerable energy savings, reduced yield loss, improved labor productivity, improved quality of steel, reduced pollution, and

⁵⁰ “The United Steelworkers Union was founded in violence in 1942 and hostility remained its trademark. There were steel strikes in 1946, 1948, 1952, 1955, and 1956. Some lasted a few days. Some a few months. At least three involved full-scaled confrontations with the governments. Harry Truman even seized mills in 1952, forcing strikers back to work.” John Strohmeier, “The Demise of Big Steel,” Alicia Patterson Foundation Fellowship Program. 1984. APF Reporter Vol 7 #4.

⁵¹ “The Iron and Steel industry experienced the longest strike in its history. Although consumers anticipated the strike and built up large stocks of steel, the 116-day work stoppage caused shortages in many industries. Workers were idled in the automotive, railroad, construction, and other fields, curtailing the output of many finished products.” James C. O. Harris, “Iron and Steel,” *Minerals Yearbook Metals and Minerals (except fuels) 1959*. 1960. Bureau of Mines, 571.

⁵² Madar, *Big Steel*, 53-54.

reduced capital costs.⁵³ While steelmakers worldwide recognized these benefits, the US steelmakers lagged in their adoption of the innovative technologies due to the large sunk costs they incurred from building more open-hearth facilities just as BOF technology was becoming available (see Fig 4.1-3 and Table 4.1). While two major integrated steel companies, McLouth and National Steel, did replace their ingot casting with continuous casting, other US steel companies remained reticent to follow suit.

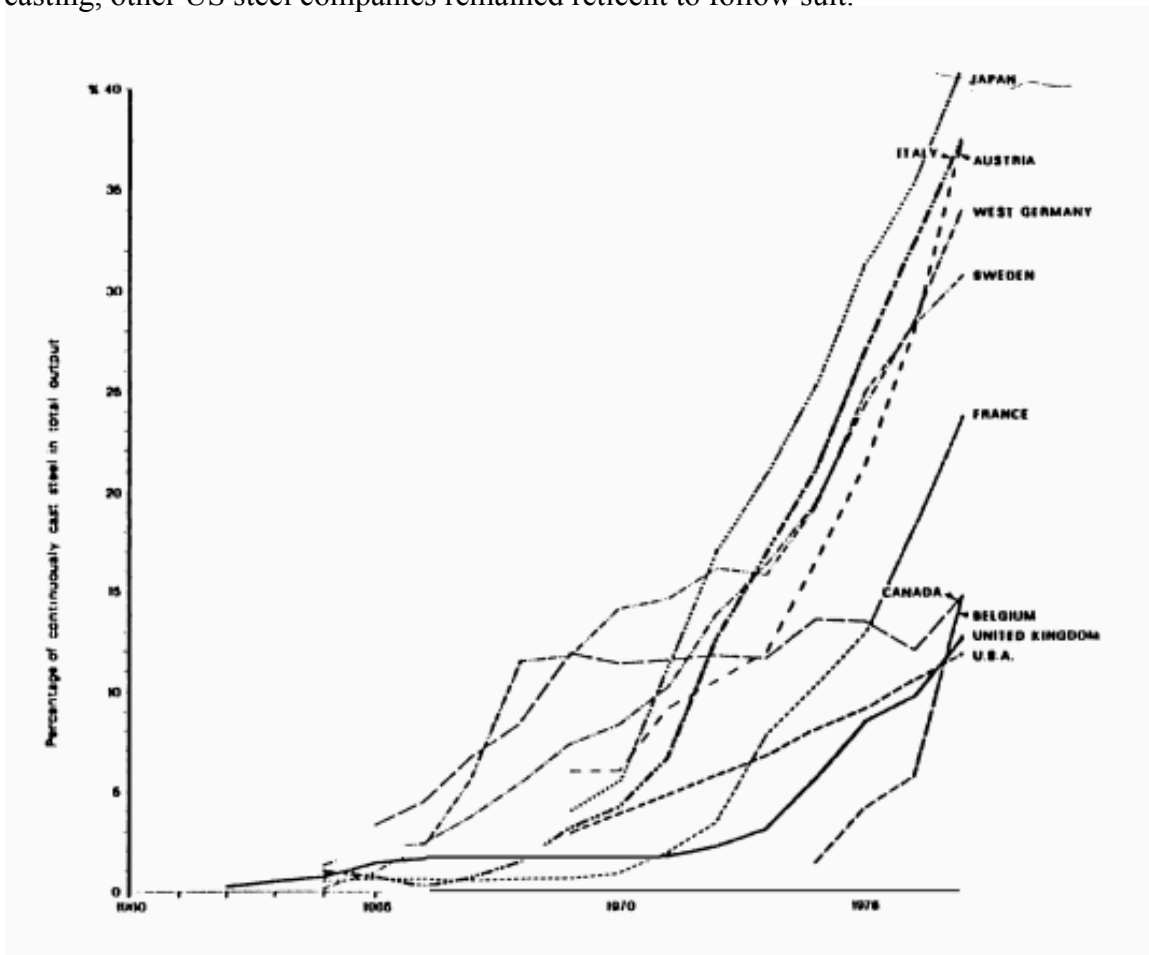


Fig. 4.1-3. The diffusion of continuous casting (1960-1980).⁵⁴

⁵³ Office of Technology Assessment, *Benefits of Increased Use of Continuous Casting By the U.S. Steel Industry: A Technical Memorandum*, (Washington: Government Printing Office, 1979), 3.

⁵⁴ *Ibid*, 25.

Table 4.1. Survey of Continuous Casting Output (1973-1977)

	1973		1974		1975		1976		1977	
	Tons	%	Tons	%	Tons	%	Tons	%	Tons	%
Germany	8,057	16.3	10,337	19.4	9,013	24.3	12,014	23.3	13,272	34.0
France	1,845	7.3	2,756	10.2	2,771	12.9	4,212	--	5,209	23.6
Italy	3,375	18.1	5,165	21.7	5,904	27.0	7,559	32.2	7,984	38.5
U.K.	832	3.3	1,232	5.5	1,686	8.5	2,120	9.4	2,590	12.6
U.S.	9,270	6.8	10,772	8.1	9,653	9.1	12,246	10.5	13,350	11.8
Canada	1,551	11.6	1,873	13.8	1,735	13.3	1,582	12.0	1,992	14.7
USSR	6,558	6.3	7,355	5.4	9,729	6.9	11,729	8.1	12,200	8.3
Japan	24,716	20.7	29,411	25.1	31,808	31.1	37,733	35.1	41,807	40.8

Source: Iron and Steelmaker, 1978.

To compound matters, despite weak demand and revenue, American steelmakers sought to maintain capacity despite slackening markets as they saw international steelmakers' gain from scale economies (a result of BOP adoption) and from a sharp drop in transportation costs.⁵⁵ This resulted in financial weakness, which made it even more difficult for American steelmakers to modernize, forcing them to modernize in a piecemeal fashion. Spreading scarce capital over extensive facilities, however, meant that steelmakers were unable to realize strong returns.

Steelmakers did get slight aid when, in 1971, the US devalued the dollar, forcing domestic prices lower compared to imports. Other US policies, however, added further financial difficulties for steelmakers. Large wage-rate increases were implemented in the 1970s, raising hourly pay by 68 percent between 1972 and 1977 but only resulting in a 3 percent increase in production.⁵⁶

The disadvantages created by US policy –victories by labor unions and JFK– coupled with US steelmakers' efforts to ramp up production to compete with international imports despite the slackening markets, resulted in a volatile rate of production throughout the 1960's and early 1970's. Annual production quantities began

⁵⁵ "Iron and Steel Industry," History, accessed February 8, 2013.

⁵⁶ Madar, *Big Steel*, 54.

to level out, however, in the late 1970's as a result of the balancing of beneficial policies and technological adoptions (such as the devaluation of the dollar (which made domestic prices lower compared to foreign imports), instillation of new small, non-integrated steel plants which used scrap fed electric furnaces ("mini-mills") which processed their steel by the more efficient continuous casting method, and minimal adoption of continuous casting by integrated steel companies) and detrimental policies (such as large wage-rate increases). The volatility of the 1960s followed by the leveling out of the 1970s can be seen below in Fig. 4.1-4.

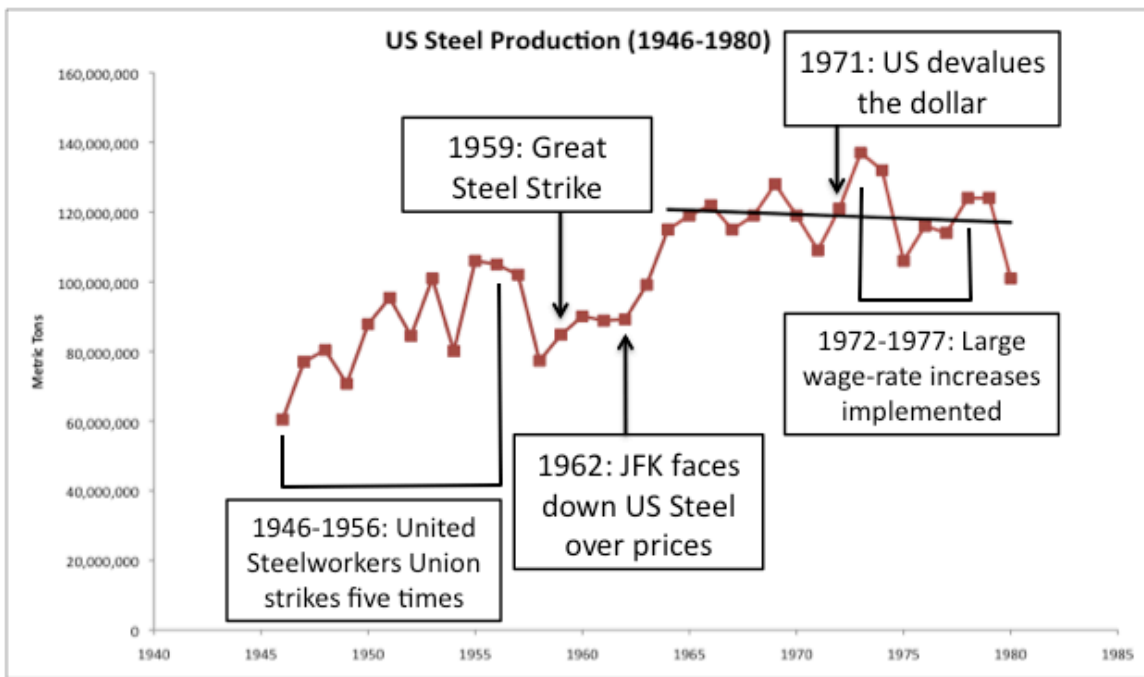


Fig. 4.1-4. Events affecting US steel production between 1946-1980.

Similar to the period just discussed, the 1980's through mid-2000's experienced an initially volatile period followed by a fairly consistent period of production. The early 1980's saw a sharp decline in US steel production, caused by the elimination of import

quotas on specialty steels in 1980,⁵⁷ and the Paul Volcker Depression, which lasted from 1979-1982 and decreased inflation, therefore making foreign products more desirable as the US dollar increased in value. These two events, coupled with the opening of steel mills in the third world, caused US steel exports to drop drastically and even affected domestic sales as cheaper imports began to look more attractive. Other causes of the production decline were the decline of demand as major construction projects begun in the 1950s, notably expressways and other large infrastructure, were completed and the consolidation and line abandonment in the railway industry (previously one of the steel industry's biggest customers). Indirect trade also negatively affected US steel production, as foreign steel content in manufactured imports lowered the consumption of domestic steel.⁵⁸

Beginning in 1982, however, annual production quantities began to level out as the new BOF and continuous casting technologies became more widely adopted. Between 1982 and 1995, worker productivity “more than doubled from 10.1 man-hours per tonne of finished steel to 3.9 man-hours. Some facilities were able to produce a tonne in one man-hour. As related advantages, synchronized continuous production allows mills to reduce inventories, eliminate lead times, and respond more quickly to customer orders.”⁵⁹ The open-hearth process took 24 hours for molten steel to refine while BOF only takes one hour, this innovative process gave steelmakers a 24-fold production efficiency improvement. Not only was the new process faster, but also continuous casting decreased yield loss – steelmakers went from shipping 65 percent of the pounds of steel

⁵⁷ These quotas were instituted on June 14, 1976. D.H. Desy, “Iron and Steel,” *Minerals Yearbook Metals and Minerals 1980*, 1980. Bureau of Mines, 437.

⁵⁸ *Ibid*, 48.

⁵⁹ *Ibid*, 29.

they poured to 95 percent. This increased efficiency led to a flattening of the production curve as production levels were as high as technology would allow them.

The high levels of efficiency, however, did lead to worldwide excess capacity as steelmakers produced steel faster than it was demanded or could be consumed. In the late 2000s, production dropped off dramatically as inventories sat in warehouses and steelmakers had to stop producing as they had nowhere to put their new steel.

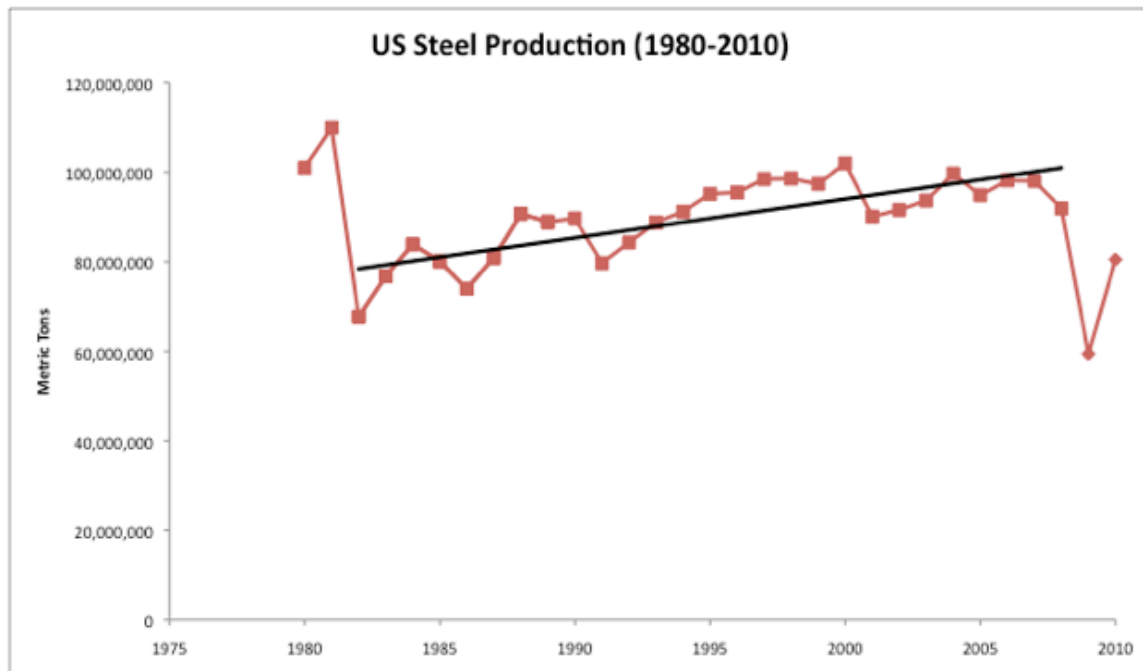


Fig. 4.1-5. Events affecting US steel production between 1980-2010.

4.2 - World History of Steel Production

Annual production levels of steel in the US and worldwide were well matched from the late 1800's through the mid-1940's (the end of World War II). Following the peace agreements that ended the war, however, the two production curves diverge, with worldwide production leaving the US in the dust. This failure by the US to continue to match world production levels was a direct result of domestic integrated producers' loss

of their technological lead in the postwar decades.⁶⁰ Instead of striving to stay ahead of the proverbial curve and investing in industry, American managers sat on their laurels and tried to milk their old equipment, built in the early 1900's, for the highest profits possible. While US integrated steel producers continued to work with old technologies, other steel producers worldwide readily embraced the new basic oxygen furnace (BOF) and continuous casting processes which promised to increase efficiency and reduce yield loss. With the adoption of these technological innovations, international steel producers increased their yield from 65 percent to 95 percent and only had to wait one hour, instead of the previous 24, for their molten steel to settle. Between their ability to “get more out of what they poured” and the increased efficiency in production of steel accelerated by BOF processing, international steel producers handily outstripped American steelmakers, rising to produce almost four times as much steel annually as their American counterparts within twenty years following WWII.⁶¹

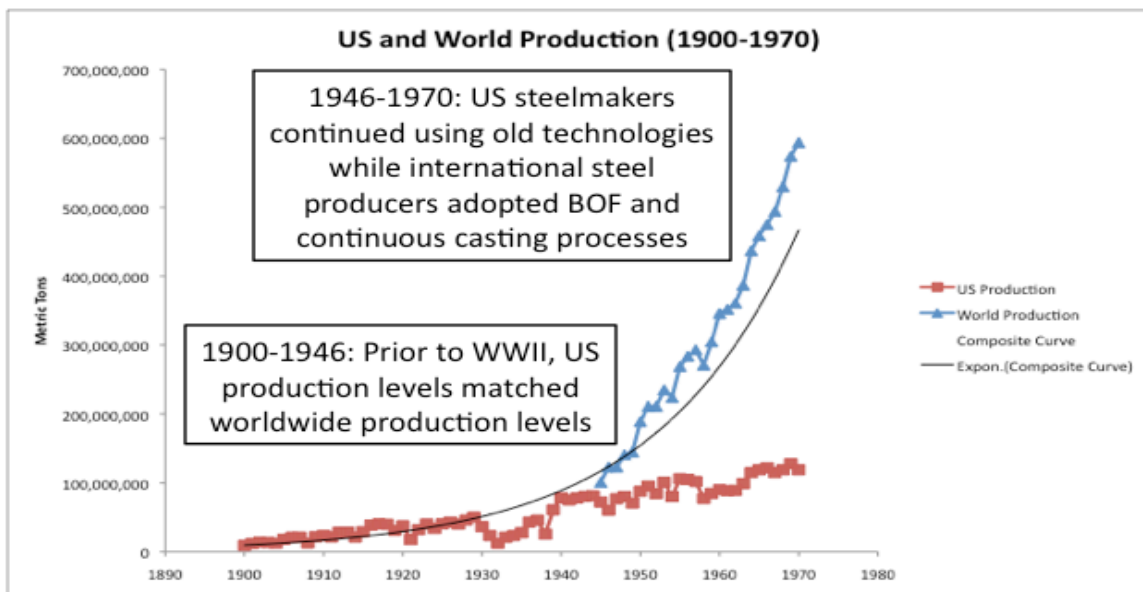


Fig. 4.2-1. Events affecting US vs. world steel production (1900-1970).

⁶⁰ Michael L. Dertouzos, Richard K. Lester, and Robert M. Solow, *Made in America: Regaining the Productive Edge* (Cambridge: The MIT Press, 1989), 279.

⁶¹ Thomas Eagar. Personal communication. October 30, 2012.

The 1970s, however brought a flattening of the worldwide production curve. Many factors were responsible for distorting the traditional sigmoid shape, chief among which were the market shocks caused by the oil crisis of 1973 (caused by the US production peak in 1971) and the 1979 energy crisis (caused by the Iranian Revolution).⁶² Many other factors limited growth in the 1975-2000 period, including efficiency and yield increases leading to excess capacity. Another market shock, the opening of the Former Soviet Union in December 1991, also contributed to the flattening of the production curve. The 1990s were characterized by an oversupply of steel in the world market, weak prices, and talk of global overcapacity – a direct result of the opening of the FSU.⁶³

By 2000, however, the previous scare of overcapacity was erased when “rapidly growing domestic markets in China and India began devouring industrial commodities, including steel.”⁶⁴ By the turn of the millennium, not only were excess inventories used up, but production also began to grow again. Mid-2008, however, saw another downturn as the global financial crisis suddenly “froze credit markets and frightened consumers, collapsing the demand for durable goods such as automobiles and appliances – industries that are two prime steel customers.”⁶⁵ As economies have been slow to recover since the crisis, so has the steel market, but production levels have slowly been growing as they overcome the excess capacity of the 1980s and 1990s.

⁶² Ibid.

⁶³ Madar, *Big Steel*, 47.

⁶⁴ Ibid.

⁶⁵ Ibid, 48.

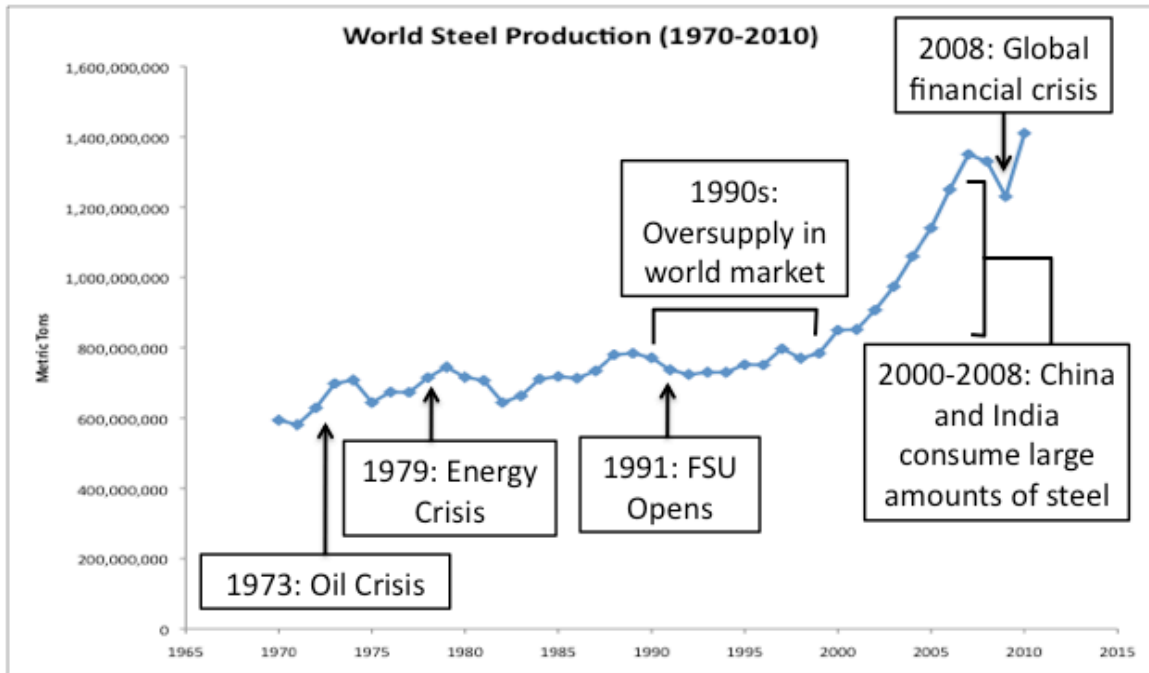


Fig. 4.2-2. Events affecting worldwide steel production (1970-2010).

4.3 – Fit to Model Discussion

When we look at the production curve for the US steel industry as a whole (Fig. 4.3-1), we can see elements of the Avrami model. Initially, there is limited growth of production due to a general level of uncertainty and risk associated with the adoption of new technologies such as the Bessemer process. As steelmakers gained confidence in the technology and more adopted it, efficiency increased and production levels grew, signaling a period of accelerated growth and market acceptance at the start of WWI. Accelerated growth continued throughout the twentieth century until the early 1960's, when steel production transitioned into a period of market saturation as the industry had reached the point of producing at maximum efficiency.

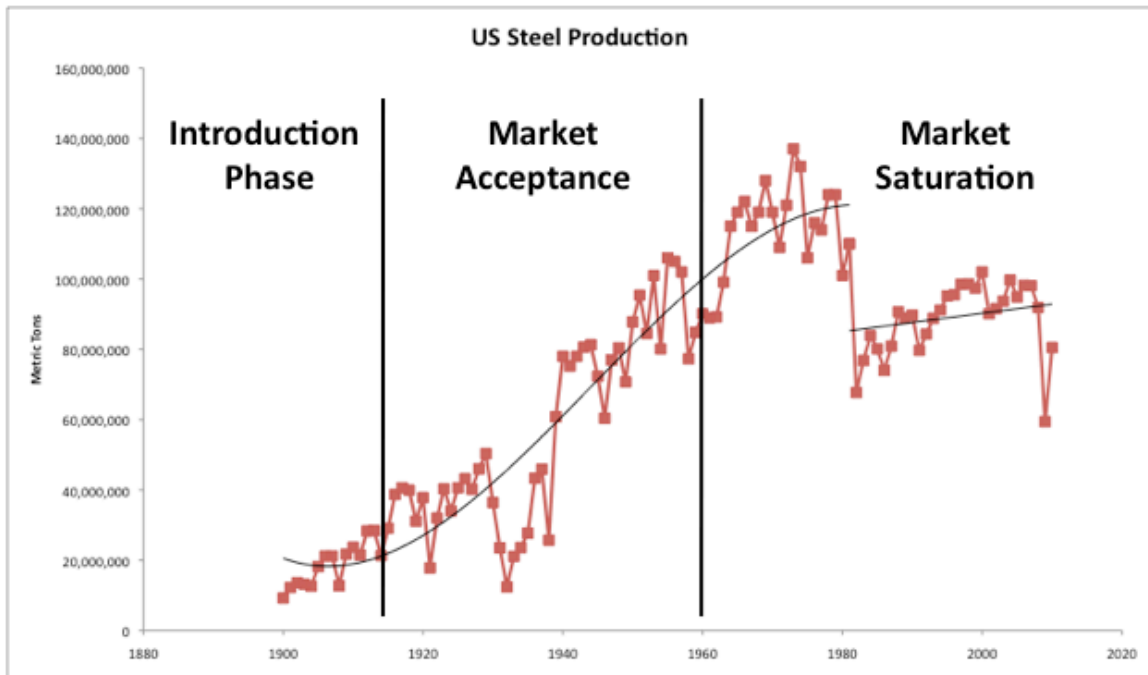


Fig. 4.3-1. Overview of US steel production.

World production also fits the initial stages of the Avrami model, but shows a more distorted sigmoidal curve (Fig. 4.3-2). Similar to US production, there was a period of initial timidity to first develop and then accept new technology for steel production resulting in slow growth at first. Between the 1940s and 1960s, however, the steel industry transitioned into a period of market acceptance and production growth accelerated. The development of new, more efficient technologies, which lead to a global oversupply as steelmakers produced steel faster than the world could consume it at the time, caused a plateau from the late 1960's through the turn of the millennium, distorting the sigmoid. As rapidly growing domestic markets, such as China and India, began consuming steel rapidly, the sigmoid returned to be on track. The worldwide steel industry is still in a period of market acceptance and therefore rapid production growth.

With many countries rapidly developing and therefore consuming large amounts of steel to build their infrastructure, the steel industry has not yet transitioned to market saturation.

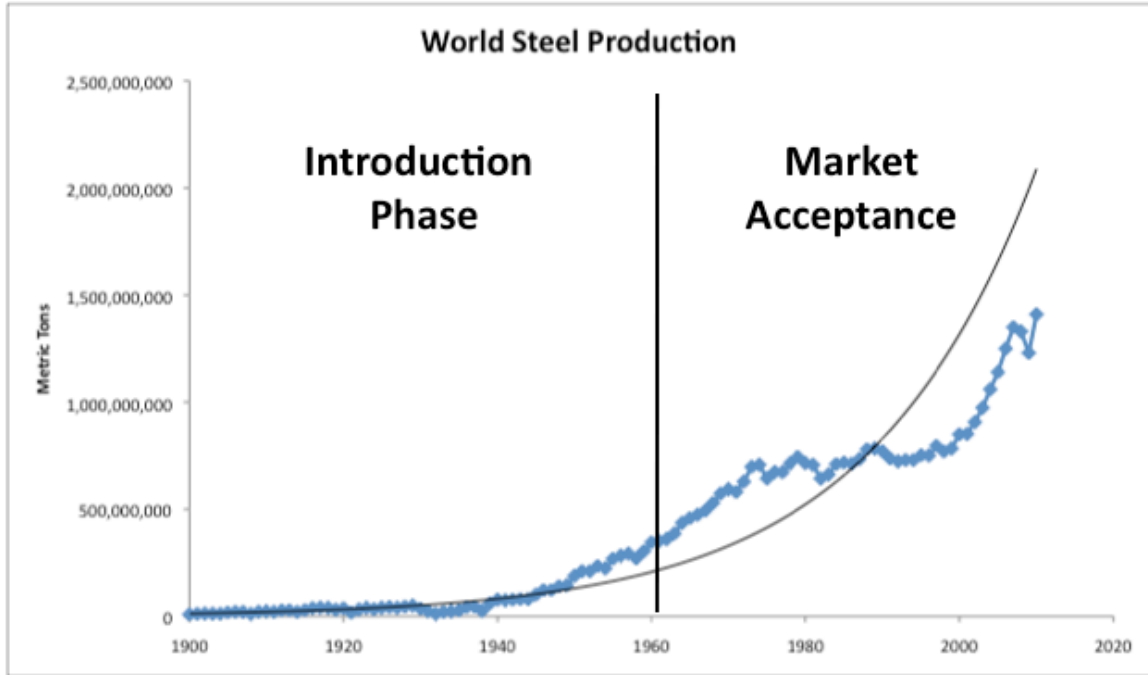


Fig. 4.3-2. Overview of worldwide steel production.

5. Growth Analysis

5.1 – Determine End of Introductory Phase/Beginning of Market Acceptance

Use of the Avrami Equation to model the production of constrained materials allows for the prediction of market acceptance and the fast-growth phase for the material in question. This, in turn, gives suppliers the time to plan for ramping up or scaling down production facilities and increase material extraction, reduce overstocks or shortages, and increase worldwide economic efficiency. It gives industry a “heads-up” about an emerging material.

Below are two proposed methods to predict the coming of the fast-growth phase.

1. Growth Curves

The first proposed method utilizes the following production growth equation:

$$\text{Production Growth} = \frac{\text{Production in Year Two} - \text{Production in Year One}}{\text{Production in Year One}} \times 100$$

Charting the production growth curves of both steel and aluminum provides an insight into when the switch from the introductory to fast-growth phase will occur. During the introduction phase, production growth is erratic. This high variability, however, disappears and constant production growth appears with the transition to market acceptance. The highlighted transition to constant production growth in Fig. 2.2 correlates with the marked increase in production seen for both aluminum and steel in Fig. 5.1-1.

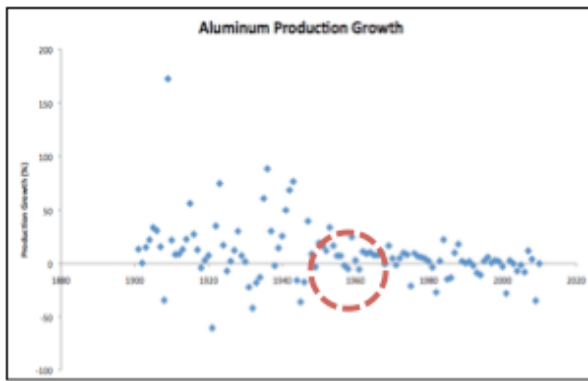


Fig. 5.1-1a: Historic production growth for aluminum. The circle highlights the transition from high variability to constant production growth.

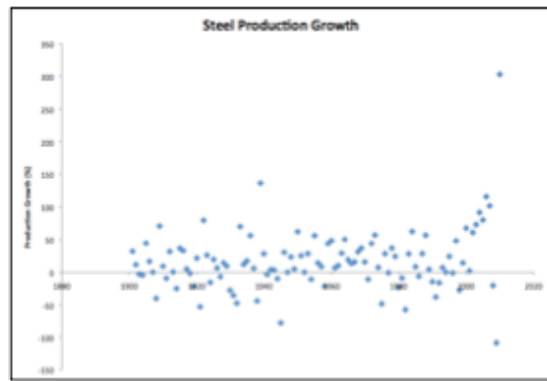


Fig. 5.1-1b: Historic production growth for steel. There is no clear transition from high variability to constant production growth with this analysis model.

2. 5 Year Standard Deviation Growth Curves

The second proposed method is a more precise continuation of the first. This method utilizes the difference in the spread in the introductory phase vs. the fast-growth phase by taking the standard deviation between a year and the four previous years. This more distinctly highlights the evolution from introduction to market acceptance. Again, this turning point corresponds to the marked increase in production seen for both aluminum and steel in Fig. 5.1-2.

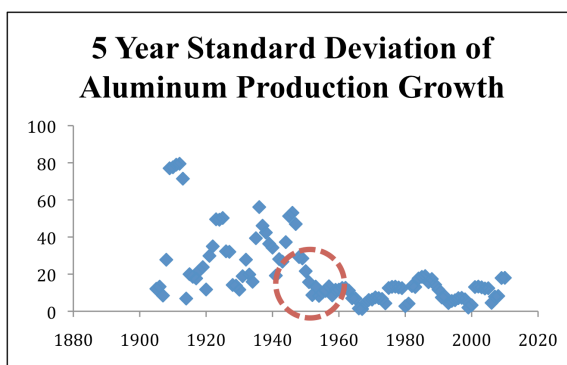


Fig. 5.1-2a: 5 year standard deviation of historic production growth for aluminum. The circle highlights the transition from high variability to constant production growth.

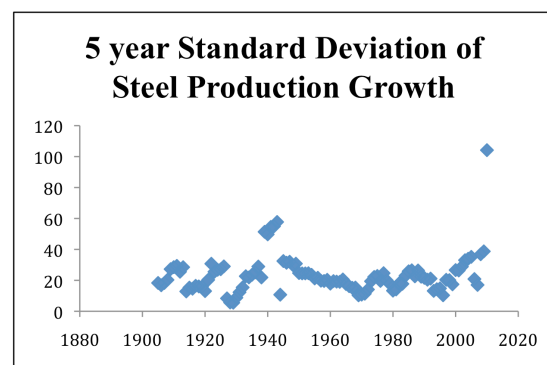


Fig. 5.1-2b: 5 year standard deviation of historic production growth for steel. No visible transition exists.

If no historical data is available to potential producers to aid them in making the decision to whether to enter the market or not, it is still possible to make assumptions and decisions via forecasting. By following these steps, potential producers can make educated decisions about whether to enter a market or not⁶⁶:

1. Select analogous products or markets (products with data past the inflection point of the sigmoid). If none, then use the average of a range of products or markets.
2. Obtain parameter for p and q for these products through:
 - Previous analyses in marketing

OR

 - Regression analysis
3. Estimate potential m for the new innovation. Do this using management judgment and/or survey research.
4. Conduct scenario analysis. For each analogous product plug p , q , and m into the Avrami model to obtain the adoption forecast. Explore the sensitivity of the forecasts to changes in parameters.

5.2 – Determine End of Market Acceptance/Beginning of Market Saturation

As shown in Fig. 3.3-1, 3.3-2, 4.3-1, and 4.3-2, the Avrami model applies for the initial two stages of the sigmoid (introduction and market acceptance) but fails in forecasting the transition into the final stage of market saturation. Therefore, there is currently no method to predict this transition.

⁶⁶ Joshi, Amit, “Product Life Cycle and The Bass Model,” Strategic Marketing Management, University of Central Florida. Lecture.

Currently, the best prediction for this transition can be made by forecasting the inflection point of the sigmoid and estimating that the transition occurs shortly thereafter. Research predicts that a product transitions into the “maturity phase” of product life cycle, after which adoption rates decline, once cumulative production is approximately equal to half the ultimate amount of recoverable resource.⁶⁷ This is, however, assuming a symmetric or deterministic market – a constant sized system that stays constant. In a non-symmetric or non-deterministic market, the inflection point, the indicator after which producers should no longer consider entering the market because the market saturation period is quickly approaching, will not occur when cumulative production reaches 50 percent of the total possible amount. In our constantly expanding universe, however, the inflection point will likely occur before the 50 percent mark.⁶⁸

⁶⁷ Padua Research. “The Aggregate Approach to Innovation Diffusion: The Bass Model.” *Institution Repository for University of Padua Research Works*, 11.

⁶⁸ Thomas Eagar. Personal communication, March 19, 2013.

6. Conclusion

The Avrami model is a sigmoidal function that relates to constrained growth. In a case study of steel and aluminum, both materials which are constrained by the amount of that metal available to be mined and produced, we have seen that it is possible to model the initial introductory and market acceptance stages, but not the final market saturation stage. In the last stage, the model falls apart because the market size is not constant; instead it's constantly growing and shrinking due to wars, policy changes, and energy crises.⁶⁹

The Avrami model can be useful to determine when a metal producer should enter a market as the model fits the production curve for the first two stages. However, the model is not useful in determining when a producer should get out of the market because the market is saturated. As stated before, this is due to the inconstancy of the size of the market. As well, for metals that invoke such patriotic feelings as steel or aluminum, used heavily to create military vehicles, armor, and weapons, laws aimed at protecting the industries and keeping a thriving domestic business also distort the production curve and cause the Avrami model to fail. Trade protection in industries such as the US steel industry “rewards poor performance, reduces incentives to innovate, and frustrates the normal Schumpeterian process of creative destruction.”⁷⁰

Social, economic, and political externalities therefore throw off the model for the final stage of production of finite materials. Methods to help producers determine the transition from introduction to market acceptance have been suggested, but desires to continue production of materials revered as insurers of safety, policies to protect the

⁶⁹ Ibid.

⁷⁰ Stefanie Lenway, Randall Morck, and Bernard Young, “Rent Seeking, Protectionism and Innovation in the American Steel Industry” *The Economic Journal* 106 (1996), 410.

continued production, and the corresponding economic downturns and resulting domestic backlash all factor into the volatility of the final stage and therefore the failure of the model.

When looking at the respective aluminum (Fig. 4.3-1 and 4.3-2) and steel (Fig. 5.3-1 and 5.3-2) production curves, it is apparent that aluminum has more variability and steel is better behaved in the early stages. This could be the result of a technology that's advancing at a pace where productivity is increasing two to three fold in just 20-30 years – forcing companies to consider whether plant that lasts for 50 years is worth the investment. The steel industry is a long-term industry, longer at least than the aluminum – this accounts for its “better” behavior in terms of the Avrami model in its early stages of production. The longer-term an industry and the more consideration a company or country must have about investing in establishing or expanding its industry, the better the production of that industry will fit to the Avrami model.

7. Future Work

Several key observations and questions are drawn from this study. These observations merit further study and the implication from potential future work may lead to aiding the efficiency of the world economy.

In the currently suggested Avrami model, the market size for metals has been assumed to be constant. However, with growing population and development of previously undeveloped countries, the market size is expanding and it is doing so quickly. This makes the assumption of there being a ceiling on the number of potential consumers, and therefore on the amount of annual production, and that the ceiling is fixed over the entire diffusion process (sigmoid curve) false. From a theoretical perspective “there exists little rationale for a static adopter population,” and there is a need to develop another version of the Avrami model with a dynamic, as opposed to the current static, ceiling for consumers and corresponding production. This would permit the consumption ceiling and therefore production ceiling to change over the sigmoid curve and adjust over time for rising population and development of the third world.

Recycling also must be accounted for in future studies of the Avrami model. As more and more material is recycled, less of the primary material is mined from the earth’s core. The current model is based on the assumption that there is a finite amount of material available in the earth’s core, but if material is being re-used, then there is more material available than is originally accounted for. The recycling curve is similar in shape to, therefore undergoing the same stages as, the production curve for a material. The primary difference between the two curves, however, arises from the recycling curve sitting below the production curve and lagging in time. The recycling curve sits below the

production curve because not all material put into production will be recycled. Some material is non-recyclable, some gets dumped, and some is used in a capacity of continual use, never needing to be thrown away or recycled. The time lag results from the fact that, while the material may have production capacity for virgin product, it is unlikely that recycling technology can be immediately implemented due to usage delays. Comparing these curves by looking at variables such as time lag between material usage and recycling or the amount recycled of one material versus the other will help develop the relationship between materials usage and recycling volume.

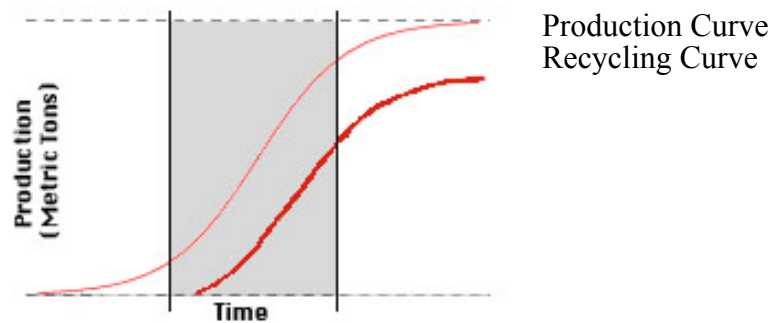


Fig. 7.1: Sigmoidal function for both production and recycling curves. Recycling curve has time lag and sits below production curve.

Analysis of the growth trends of recycling will give industry a better understanding of the market. And which historical events have the largest impact. This will help industry be able to predict and adjust for similar future events allowing it to operate in the world market more efficiently.

Another useful study would be to model production of metals by the country producing that metal. This investigation could look deeper into adoption patterns and determine if developing countries adopt innovations more quickly than countries already producing that material. It could also determine if different cultures have different coefficients of imitation or innovation.

8. References

Note: Most sources have been referenced in the footnotes section. It should be noted that the data for the amount of material produced worldwide and specifically in the U.S. was obtained from the United States Geological Survey and the U.S. Bureau of Mines through their Minerals Yearbooks and previous incarnations of the same publication.

Avrami, Melvin. "Kinetics of Phase Change I. General Theory." *Journal of Chemical Physics* 7 (1939): 1103-1112.

Avrami, Melvin. "Kinetics of Phase Change II. Transformation-Time Relations for Random Distribution of Nuclei." *Journal of Chemical Physics* 8 (1940): 212-224.

Carmony, Harold E., and H. L. Cullen. "Iron and Steel Scrap Year 1943," *Minerals Yearbook 1943*. (1945) Bureau of Mines, 517-540.

Choate, William T., and John A.S. Green. "Modeling the Impact of Secondary Recovery (Recycling) on U.S. Aluminum Supply and Nominal Energy Requirements." *TMS* (2004), 913-918.

Crompton, Paul. "The Diffusion of New Steelmaking Technology," *Resources Policy* 27 (2001): 87-95.

Dertouzos, Michael L., Richard K. Lester, and Robert M. Solow. *Made in America: Regaining the Productive Edge*. Cambridge: The MIT Press, 1989.

Eagar, Thomas. "The Future of Metals." *Welding Journal* (1991): 69-72.

Eagar, Thomas. Personal communication. October 30, 2012.

Eagar, Thomas. Personal communication. March 19, 2013.

Harris, James C. O. "Iron and Steel." *Minerals Yearbook Metals and Minerals (except fuels) 1959*. (1960) Bureau of Mines, 571-608.

Highbeam Business. "Primary Production of Aluminum." Accessed March 13, 2013. <http://business.highbeam.com/industry-reports/metal/primary-production-of-aluminum>.

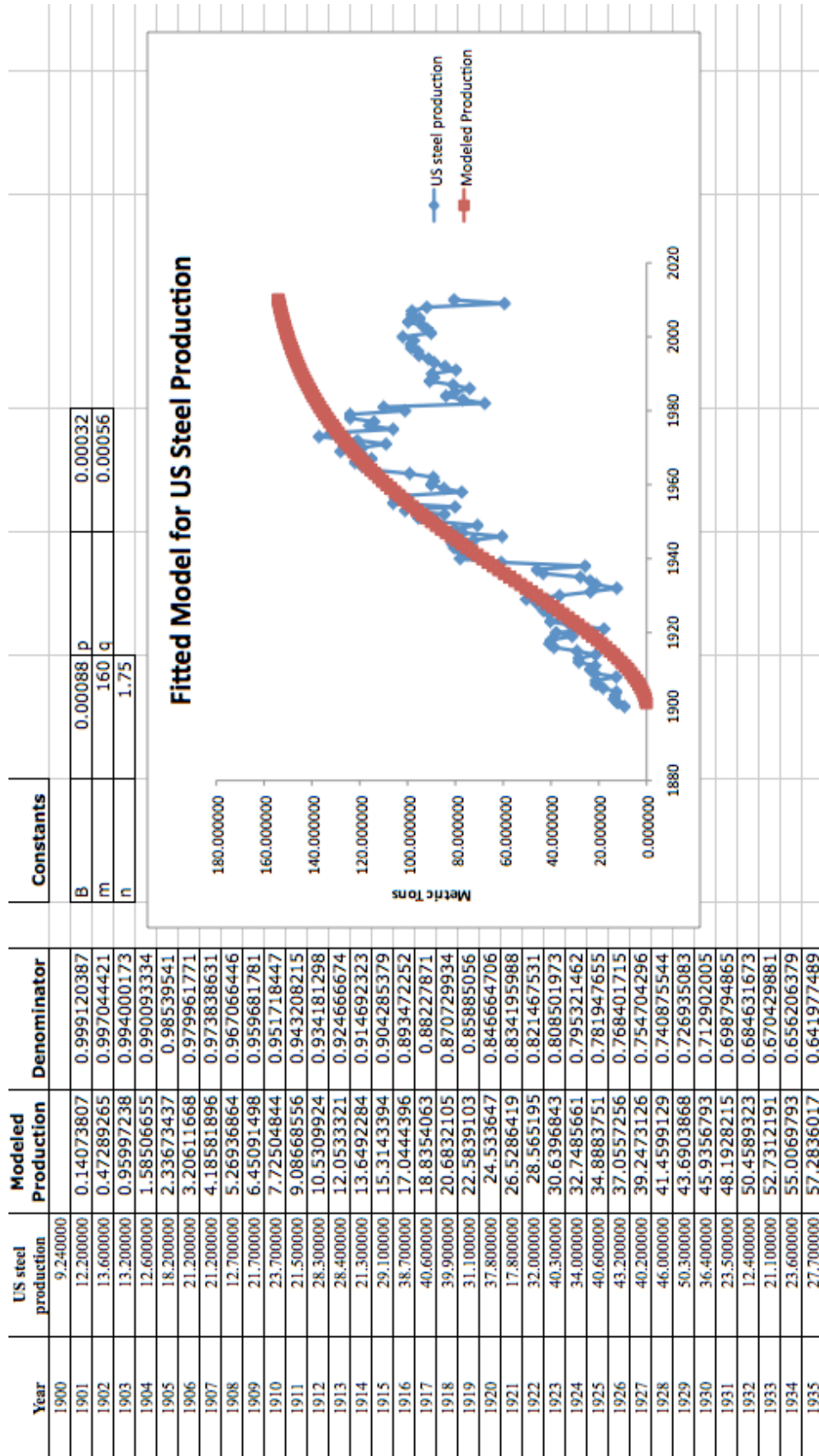
History. "Iron and Steel Industry." Accessed February 8, 2013. <http://www.history.com/topics/iron-and-steel-industry>.

Joshi, Amit, "Product Life Cycle and The Bass Model," Strategic Marketing Management, University of Central Florida. Lecture.

Juanjuan Zhang. Personal communication, March 7, 2013.

- “Kaiser Aluminum Says Plants to Operate Despite Strike.” *The Seattle Times*, February 21, 1995. Accessed March 19, 2013.
<http://community.seattletimes.nwsources.com/archive/?date=19950221&slug=2106293>.
- Lenway, Stefanie, Randall Morck, and Bernard Young. “Rent Seeking, Protectionism and Innovation in the American Steel Industry.” *The Economic Journal* 106 (1996): 410-421.
- Lilien, Gary L., Arvind Rangaswamy, and Arnaud De Bruyn. “The Bass Model: Marketing Engineering Technical Note.” *Principles of Marketing Engineering* (2007): 1-8.
- Los Alamos National Laboratory. “Aluminum.” Accessed March 19, 2013.
<http://periodic.lanl.gov/13.shtml>.
- Madar, Daniel, *Big Steel: Technology, Trade, and Survival in a Global Market*. Vancouver: UBC Press, 2009.
- Northwest Power and Conservation Council. “Aluminum.” Accessed March 19, 2013.
<http://www.nwcouncil.org/history/aluminum>.
- Office of Technology Assessment. *Benefits of Increased Use of Continuous Casting By the U.S. Steel Industry: A Technical Memorandum*. Washington: Government Printing Office, 1979.
- Oxford Reference. “Iron and Steel Industry.” Accessed February 8, 2013.
<http://www.oxfordreference.com/view/10.1093/acref/9780195082098.001.0001/acref-9780195082098-e-0795?rskey=ok36uQ&result=2&q=steel%20industry>.
- Padua Research. “The Aggregate Approach to Innovation Diffusion: The Bass Model.” *Institution Repository for University of Padua Research Works*, 11.
- Scherer, George W. “Models of Confined Growth.” *Cement and Concrete Research* 42 (2012): 1252-1260.
- Strohmeyer, John. “The Demise of Big Steel.” Alicia Patterson Foundation Fellowship Program. (1984) APF Reporter Vol 7 #4.
- Van Sichen, Clinton DeW. “Random Nucleation and Growth Kinetics.” *Physical Review B: Condensed Matter* 54 (1996): 845-848.
- Woods, Douglas W., and James C. Burrows. *The World Aluminum-Bauxite Market: Policy Implications for the United States*. New York: Praeger Publishers, 1980.

Appendix A



Year	US steel production	Modeled Production	Denominator	Constants												
1900	9.240000	0.16951014	0.998940562	<table border="1"> <tr> <td>B</td> <td>0.00106</td> <td>p</td> <td>0.0004</td> </tr> <tr> <td>m</td> <td>160</td> <td>q</td> <td>0.00066</td> </tr> <tr> <td>n</td> <td>1.65</td> <td></td> <td></td> </tr> </table>	B	0.00106	p	0.0004	m	160	q	0.00066	n	1.65		
B	0.00106	p	0.0004													
m	160	q	0.00066													
n	1.65															
1901	12.200000	0.53137751	0.996678891													
1902	13.600000	1.03577729	0.993526392													
1903	13.200000	1.66172734	0.989614204													
1904	12.600000	2.39582089	0.985026119													
1905	18.200000	3.22817408	0.979823912													
1906	21.200000	4.15090004	0.974056875													
1907	21.200000	5.15737865	0.967766383													
1908	12.700000	6.24185557	0.960988403													
1909	21.700000	7.39920104	0.953754993													
1910	23.700000	8.6247552	0.94609528													
1911	21.500000	9.91422385	0.938036101													
1912	28.300000	11.2636056	0.929602465													
1913	28.400000	12.6691394	0.920817879													
1914	21.300000	14.1272657	0.91170459													
1915	29.100000	15.6345974	0.902283766													
1916	38.700000	17.1878978	0.892575639													
1917	40.600000	18.7840626	0.882599609													
1918	39.900000	20.4201073	0.87237433													
1919	31.100000	22.0931555	0.861917778													
1920	37.800000	23.8004309	0.851247307													
1921	17.800000	25.5392505	0.840379684													
1922	32.000000	27.3070183	0.829331135													
1923	40.300000	29.1012214	0.818117366													
1924	34.000000	30.9194257	0.806753589													
1925	40.600000	32.7592733	0.795254542													
1926	43.200000	34.6184793	0.783634504													
1927	40.200000	36.4948302	0.771907311													
1928	46.000000	38.3861817	0.760086364													
1929	50.300000	40.2904571	0.748184643													
1930	36.400000	42.2056459	0.736214713													
1931	23.500000	44.1298031	0.724188731													
1932	12.400000	46.0610473	0.712118455													
1933	21.100000	47.9975604	0.700015248													
1934	23.600000	49.9375863	0.687890085													
1935	27.700000															

