

Essays in Macroeconomics: Liquidity and Taxation

by

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MSc Economics, Fundação Getulio Vargas (2007)

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Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

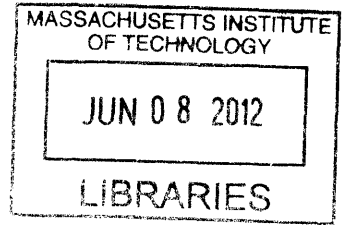
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Abstract

This thesis consists of three independent chapters on the Macroeconomics of Liquidity and Taxation.

The first chapter studies how concerns about future funding difficulties and liquidity dry ups influence investment decisions. In an environment with financial frictions, investors need to take liquidity management into account when deciding between different investment alternatives and when designing financial arrangements with other fund providers. Their decisions affect both idiosyncratic and aggregate exposure to shocks and fluctuations. When shocks occur to external liquidity sources, such as changes in the cash-flows that support mortgage-backed securities or other non-corporate assets, these are transmitted through financial arrangements towards the real sector. The anticipation of these shocks and its reflection in asset prices influence project selection and change the pattern of fluctuations, creating additional comovement across sectors of economy and different assets. Likewise, the anticipation of variations in the internal liquidity of firms, resulting from shocks to their productivity, changes their choice of projects. For moderate liquidity scarcity, the effect through project choice is shown to lead to the dampening of these underlying productivity shocks; while for more severe shortages, amplification emerges. Despite the possibility of excess exposure to risk being generated endogenously, equilibrium allocations are constrained efficient. Policy implications are then discussed in light of this result.

The second chapter focuses on the auxiliary role of taxes in helping smooth income fluctuations. From a mechanism design perspective, it studies the characterization of the constrained optimal allocation in an economy with endowment fluctuations which are private information, where agents are also able to trade assets unobservably. In this environment, production and aggregate savings can be manipulated by a planner through the use of capital taxation. Using this instrument, the planner is capable of affecting prices on the unobservable trades. In this environment, the constrained optimal allocation can be implemented in a simple decentralized way, which takes the form of a bond market economy with capital taxes. The chapter provides conditions ensuring that an untaxed economy would fail to achieve an efficient allocation. The essential element for a possible Pareto improvement is a wedge which is introduced between the returns on capital and the market return on bonds. Around the undistorted economy the sign of a welfare improving wedge depends centrally on the covariance of asset holdings and marginal utility in the cross-section of the population. The covariance term represents the redistributive impact of a combination of price changes and lump-sum revenue rebates: a bond price increase affects agents negatively in proportion to their asset holdings, while the rebate increases their individual welfare in proportion to average savings. A necessary condition for the optimal tax is also presented. This condition takes that redistributive effect into account, in addition to the combined consequences that each price change has on revenues across periods.

The third paper is product of joint work with Plamen T. Nenov. Its central concern are the effects of increased uncertainty on financial stability. By studying a debt roll-over coordination

game with dispersed information and a market-determined liquidity scenario, it describes conditions under which an improvement in the precision of individual information about financial institutions' fundamentals leads to greater financial stability. For the limiting case of arbitrarily precise private information, that condition obtains a simple form in terms of payoff elasticities. Conversely, we characterize when an increase in uncertainty leads to a higher frequency of debt runs and show how this deleterious effect is amplified through the deterioration of prices for liquidated assets.

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Chapter 1

Liquidity Scarcity, Project Selection, and Volatility

Abstract

The severe contraction that followed the recent financial crisis highlighted the exposure of the real sector to financial markets and the volatility in credit conditions. Unreliability of future funding influences the way in which firms balance risks when choosing investment projects and designing financial arrangements. This chapter studies the behavior of project choice in an environment with financial frictions and its consequences for the aggregate behavior of the economy. I focus on responses to fluctuations in the external supply of liquidity and in the liquidity created by the entrepreneurial projects themselves. When shocks occur to external liquidity sources, such as changes in the cash-flows that support mortgage-backed securities or other non-corporate assets, these are transmitted through financial arrangements towards the real sector. The anticipation of these shocks and its reflection in asset prices influence project selection and change the pattern of fluctuations, creating additional comovement. Likewise, the anticipation of variations in the internal liquidity of firms, resulting from shocks to their productivity, changes their choice of projects. For moderate liquidity scarcity, the effect through project choice is shown to lead to the dampening of these underlying productivity shocks; while for more severe shortages, amplification emerges. Despite the possibility of excess exposure to risk being generated endogenously, allocations are constrained efficient. Policy implications are then discussed in light of this result.

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1.1 Introduction

The recent financial crisis was followed by one of the sharpest credit contractions since the Great Depression. Major drops were experienced in syndicated lending, down by 79% of its peak volume ((Ivashina and Scharfstein 2010)), and in industrial and commercial loans by U.S. commercial banks, which dropped by approximately a quarter from the Oct/2008 peak to the Oct/2009 bottom¹. Concerns about a market freeze in commercial paper also led to an unconventional intervention, with the creation of the Commercial Paper Funding Facility by the Federal Reserve System at the height of the crisis. This special purpose vehicle, by acting as buyer of last resort in the commercial paper market, eventually held up to approximately U\$350 billion in commercial paper ((Adrian, Kimbrough, and Marchioni 2011)). Shock-waves of the crisis were felt across multiple sectors of the economy and the severe recession that followed highlighted the exposure of the real sector to financial factors and to the volatility in credit conditions. A few important questions emerge. First, how can the financial system be made more resilient, to prevent other such crises from emerging? Second, how does the anticipation of unreliability in future funding affect decisions of non-financial firms regarding their exposure to both real and financial risks? Last, is this exposure excessive, creating a case for future intervention?

The elusive answer to the first of these questions has attracted a number of important contributions². The present chapter attempts to address the remaining set of questions. To do so, it is necessary to study an environment in which unreliable financial conditions and fluctuations in asset markets matter for real economy activity. Also, one in which agents in the real sector face trade-offs in their exposure to the different risks involved in production and its financing.

I build on the framework of (Holmström and Tirole 1998; Holmström and Tirole 2001), which provide a model environment in which liquidity conditions affect investment, asset prices and output. There, however, investment prospects are fixed. I extend their baseline model to incorporate the choice over different investment projects and to endogenize the economy's response to a set of shocks which includes asset return fluctuations, productivity volatility and financial distress possibilities.

There are three time periods. Investments on projects are made over the first two and they only mature, generating revenues, on the third one. Projects differ in how their productivity, costs and capacity to attract external funding respond to shocks. Entrepreneurs and lenders design financial contracts to cover the random costs of these projects and need to take into account constraints that arise from both sides of the agreement. As a consequence, decisions regarding project selection and financial arrangements are intertwined.

From the entrepreneurial side, only a limited share of the cash flows generated can be cred-

¹Board of Governors of the Federal Reserve System, Release H.8

²Some examples include (Adrian and Shin 2009; Acharya, Mehran, and Thakor 2010; Brunnermeier 2009; Curdia and Woodford 2010; Diamond and Rajan 2001; Farhi and Tirole 2011b; Geanakoplos 2009; Gertler, Kiyotaki, and Queralto 2011; Hanson, Kashyap, and Stein 2010; Kurlat 2010; Lorenzoni 2008; Shleifer and Vishny 2010; Stein 2011)

ibly committed to the repayment of other agents, i.e., there is limited pledgeability of output. Consequently, a project's potential to guarantee the funding necessary for its own completion is compromised and there is limited *internal liquidity*. Since financial needs of projects might exceed available internal liquidity, there is a demand for pre-arranged transfers of resources from lenders.

From the lenders' side, limited commitment constrains their promises to transfer resources in the future to help fund the project. As a result, other assets available in the economy play a role in this arrangement, as they can serve as collateral and help back reinvestment promises. These assets serve as *external liquidity* and are demanded as part of the optimal financial contract. Important practical examples of non-corporate assets which are either held directly by firms for contingent liquidation or back funding delivery promises include cash, sovereign bonds and mortgage-backed securities.

Jointly, the availability of internal and external liquidity in the economy determine its aggregate liquidity conditions and asset prices. In turn, these asset prices influence optimal financial contracts and project choices. Through these interactions, endogenous project selection and general equilibrium effects are key determinants of the behavior of the aggregate economy and its responses to shocks.

My first main result originates from an application in which I study the choice over projects which differ in the volatility of their capacity to generate output and revenues. As only a fraction of this output is pledgeable, internal liquidity drops in case of a negative productivity shock and financial needs, which need to be backed by external assets, increase. The opposite occurs when positive productivity shocks hit projects. The optimal financial contract specifies which project is chosen, under which conditions it is completed, downsized or terminated, as a well as all relevant transfers and the asset acquisitions that are necessary for their backing. The possibility of controlling the exposure to productivity risk, by choosing among different projects, is shown to work in this environment as an imperfect substitute for external asset purchases.

When the economy features a single risk-less asset that can be used for backing transfer promises, its price signals its scarcity and determines how liquidity-constrained entrepreneurs are in equilibrium. Project selection and financial contract design work together in ensuring that pledgeable resources are available in the states where they are the most valuable. When asset prices are low, entrepreneurs purchase enough of these assets to be constrained only in states with low productivity. Therefore, choosing projects with lower volatility helps move resources to those states. However, when assets are sufficiently scarce and prices are high, entrepreneurs find themselves constrained even in states with higher productivity. The relative value of resources across those different states determines in which direction they want to bias project choice. As prices increase and entrepreneurs become more liquidity constrained, they choose projects with higher volatility, to make sure they have resources to finance ongoing investments at least in the situations in which the project is the most productive. Therefore, the deterioration of aggregate liquidity conditions leads to the choice of riskier projects, showing that endogenous project selection can be a powerful determinant of

aggregate volatility.

I then turn to the consequences of fluctuations which are driven by changes in the values of non-corporate assets, i.e., by shocks to external liquidity. Some examples of central relevance given recent events include the possibility of a drop in house prices leading to a collapse in mortgage-backed securities or sudden changes in the value of sovereign bonds. In the model studied, such fluctuations are introduced as variations in payouts from a set of trees which are in fixed supply. Contingent claims are traded, serve as external liquidity and are backed by these trees. Asset trades are sufficiently sophisticated and allow for positions that include but are not restricted to the holding of risk-less claims. I study how shocks to the payouts of these trees are transmitted towards corporate investment policies and also how endogenous project selection, by generating additional comovement of entrepreneurial output and asset values, can work as an amplification mechanism for these shocks.

In this setting, liquidity premia³ are always higher for assets that pay out in states where tree output scarcer. Additionally, completion rates for entrepreneurial projects and their final output are always non-decreasing in the trees' output. Since, claims on trees play the role of a financial input in an entrepreneurial sector which is liquidity constrained, a lower payout from them is transmitted towards entrepreneurial output whenever there is a shortage of internal liquidity. This is a natural transmission mechanism and generates some output comovement on its own.

When project choice is introduced in this environment, an additional degree of comovement arises endogenously. Whenever internal liquidity falls short of the necessary costs of investment, investment opportunities and external assets payouts are complementary. Therefore, a project that offers these opportunities in future states in which external liquidity is more plentiful and, consequently, cheaper to acquire in advance is preferred by entrepreneurs. As a result, the entrepreneurial sector biases its investment towards projects that comove positively with the trees' output and ends up being endogenously more exposed to the factors which determine that level.

A third set of results relates to constrained efficiency in the environments studied. Despite the possibility of additional exposure to risk and amplification of fluctuations emerging endogenously through project selection, all outcomes are constrained Pareto efficient. Therefore, a planner that does not have advantages in the creation of liquidity nor in its contingent reallocation across firms cannot improve the overall efficiency of production nor increase welfare. This generates a characterization of which classes of policies cannot lead to improvements. Examples of such policies are the ones which ban projects deemed excessively risky, mandate minimum liquid asset holding levels or preclude the use of risky assets as part of financial arrangements. On the other hand, that does not imply the inexistence of policies that could lead to improvements; but if they do exist, they need to rely on an governmental advantage in the creation of liquid assets⁴, on its

³Liquidity premia are present as assets might sell above their value for consumption purposes. They are defined as a ratio of asset prices to their expected payouts.

⁴As in (Holmström and Tirole 1998), which discusses how exclusive ("regalian") enforcement powers give the public sector a unique opportunity to create liquidity backed by its ability to tax citizens in the future.

greater flexibility in reallocating resources after realizations of aggregate states of the economy or on its capacity of improving the underlying contractual environment.

The chapter also includes a series of additional results. First, a general model is introduced. A few closed-form criteria for project selection in this environment are analyzed. For instance, as a consequence of these frictions, there is an important departure from standard net-present-value criteria largely used in corporate practice. Output generated in a given period is treated differently and needs to be decomposed according to its shares which can be credibly pledged to outsiders and the one that needs to be claimed by entrepreneurs. Also, given credit constraints, optimal leverage determination plays a key role. After this analysis, specialized environments are proposed, to illustrate the different aggregate consequences of the interactions between liquidity scarcity and project selection. The central conclusions from most of these have been reported in the previous paragraphs. The last environment studies the consequences of enriching the set of assets trades in the economy with endogenous choice of output volatility. It shows that although allocations change in interesting ways, the main qualitative conclusions regarding incentives for the amplification or dampening of productivity fluctuations are robust to these more sophisticated trades and are, thus, not a consequence of the single risk-less asset assumption initially made.

Related Literature- The present chapter is related to different strands of economic literature. As anticipated, it is the most closely related to the literature on liquidity asset pricing which follows from (Holmström and Tirole 1998; Holmström and Tirole 2001). My focus, however, is on the joint determination of the exposure to real and financial risks which occurs when real investments have to be selected and financed by arrangements which need to take into account the frictions that arise from both sides of a relationship. This focus brings into light interplays between technological and financial decisions, as well as their aggregate consequences.

Difficulties in securing future funding and the need to manage liquidity buffers also seem to be a growing concern in corporate practice. A dramatic increase in corporate liquid holdings has been observed in the last few decades, through a steep growth in the cash-to-asset ratio of U.S. industrial firms, which more than doubled in the 1980-2006 period ((Bates, Kahle, and Stulz 2009)). Indeed, that study also reports that the average corporation has enough cash to withdraw all of its debt and that a common measure of leverage which nets out cash holdings, the net debt ratio⁵, has suffered a substantial secular decline. The picture becomes even more impressive when we take into account additional instruments for liquidity hoarding beyond cash. For instance, (Campello, Giambona, Graham, and Harvey 2011) report results of a survey which shows that the average firm has credit line access amounting to 24% of the value of their total assets, about twice the volume of cash they additionally hold⁶.

Some recent empirical papers have also studied the behavior of the mix between cash and credit

⁵Defined as debt minus cash, divided by book assets.

⁶It is worth noting a significant discrepancy in cash ratios in (Bates, Kahle, and Stulz 2009) and (Campello, Giambona, Graham, and Harvey 2011), due to sampling among firms with different characteristics.

lines and highlighted the importance of covenants in determining the availability of these pre-committed funds⁷. In particular, (Acharya, Almeida, and Campello 2010) discusses the importance of aggregate risk in triggering covenant violations and reducing the amount of resources available for covering corporate expenses. The recent financial crisis has also provided rich data on the interaction between liquidity dry-ups and the responses in corporate investment policy, employment and financial management⁸. The current chapter focuses on how production decisions and financial arrangements anticipate these possibilities and, especially, on the consequences on the aggregate behavior of economy in face of real and financial shocks.

It is thus also related to another set of papers which have addressed the broad issue of project selection, or investment composition, in environments with financial frictions. For example, (Matsuyama 2004; Matsuyama 2007a; Matsuyama 2007b), which study deterministic aggregate implications of imperfect credit markets, such as credit cycles, leapfrogging, aggregate demand spill-overs, reverse international capital flows and traps. Or (Aghion, Angeletos, Banerjee, and Manova 2010), which studies the choice between a low volatility, but financially exposed investment, versus a more volatile short-term investment, across economies with borrowing constraints of different severity. It shows that the determination of investment composition can help account for empirical patterns in levels of cross-country growth rates and their volatility. The present work differs from these papers in studying how the joint selection of projects and financial arrangements respond to the scarcity of aggregate liquidity and the importance of this mechanism in determining the pattern of fluctuations in the economy. Its conclusions add a new perspective to this broad set of macroeconomic consequences of imperfect financial markets.

One of the central results of the chapter regards the emergence of the choice of riskier projects in economies which face severe liquidity scarcity. I identify a form of risk-seeking behavior on entrepreneurial decisions. To the best of my knowledge, it significantly differs from previously known channels, such as an agency problem leading to asset substitution and risk-shifting ((Jensen and Meckling 1976)) and non-convexities in the entrepreneur's value function derived from a combination of credit constraints and occupational choice ((Vereshchagina and Hopenhayn 2009)). The key driver of this risk-seeking mechanism lies in the partial pledgeability of output and on the way through which output fluctuations move pledgeable resources across states of the world. These resources are useful for backing the financing of investment, substitute for costly asset hoarding and are especially valuable when investment is more productive. Unlike in the risk-shifting literature, the contracts between lender and borrower that I study offer sufficient state contingency and the choice for riskier projects is an ex ante decision on which both lender and borrower agree as the best response to the constraints and environment they face.

⁷Data on credit line availability and drawdowns have typically been hard to obtain. A few recent papers such as (Sufi 2009) and (Acharya, Almeida, and Campello 2010) made progress in its obtainment and analysis.

⁸See, for instance, (Almeida, Campello, Laranjeira, and Weisbenner 2009; Campello, Giambona, Graham, and Harvey 2011; Ivashina and Scharfstein 2010).

The chapter is also related to the literature on optimal risk management⁹, but takes a less common approach by studying how investment decisions interact with financial arrangements; and also by doing that in a general equilibrium environment. The first of these elements is present in a recent paper by (Almeida, Campello, and Weisbach 2011)(ACW), which studies investment and risk management when future financing involves frictions. Its main insight is that the possibility of future financing shortfalls leads to investment in projects with earlier payouts and lower risk exposure. Some key distinctions are responsible for generating different analysis and complementary results between our papers. In ACW, investment opportunities are independent across periods. The potential for funding shortages on an upcoming decreasing returns to scale investment opportunity creates a form of risk-aversion¹⁰ and, without temporal dependence in productivity across projects, biasing is always towards safer projects. In the environment I study in Section 1.4.1, the same project is financed sequentially, which naturally introduces an inter-temporal dependence in investment productivity. A more volatile project, while more severely affected by shocks on the downside, generates more pledgeable output, which backs its own financing, exactly in the situations in which reinvestment is more productive. This mechanism is at the heart of the emergence of the form of risk-seeking behavior which is identified in that section. Other sources of complementarity lie in the study of the aggregation of multiple firms in general equilibrium on the current chapter, which is essential for its focus on aggregate consequences, and also on the presence a richer set of macroeconomic shocks.

Last, it can also be related to the literature on financial development and volatility, when different comparative statics on the magnitude of the underlying frictions and the availability of non-corporate assets are conducted. For instance, the result linking liquidity scarcity, if interpreted as low financial development, to the choice of riskier projects is consistent with the finding that less developed countries specialize in riskier sectors ((Koren and Tenreyro 2007)), which is difficult to reconcile with models based on optimal portfolio choice approaches in face of a mean and volatility trade-off.

Structure- The rest of the chapter is organized as follows. Section 1.2 proposes the general model. Section 1.3 studies incentives driving project choice in this environment. Section 1.4 discusses the interactions and aggregate consequences in specialized environments. Section 1.5 proves the constrained Pareto optimality result and policy consequences, while section 1.6 concludes. All proofs omitted from the main text are in the appendix.

⁹For instance, (Froot, Scharfstein, and Stein 1993; Leland 1998; Holmström and Tirole 2000; Rampini and Viswanathan 2010).

¹⁰As first illustrated by (Froot, Scharfstein, and Stein 1993).

1.2 The Model

The central features of the model are the presence of a set of agents with a menu of investment opportunities, entrepreneurs, and a set of agents without these opportunities, who act as lenders. They design a financial contract subject to constraints from both sides of the arrangement: limited pledgeability from the entrepreneur side and limited commitment from lenders. The presence of random costs before the completion of projects occurs creates a need to ensure the availability of resources for those situations, generating a rationale for liquidity insurance and management. Entrepreneurs try to make sure they have resources in situations in which they can be used productively, for salvaging a project under distress or for taking advantage of investment opportunities. Limited commitment constrains liquidity insurance by lenders and creates a role for asset purchases from third parties in enabling some limited insurance. The markets for these assets are potentially incomplete, to allow for potential difficulties in fully state-contingent liquidity trades.

Time and uncertainty

Time is described by $t = 0, 1, 2$. There is a single good in each period, which can be used for both consumption and investment.

The state of the world is fully described by $\omega \in \Omega$, where Ω is a finite set with $\#\Omega$ elements. All uncertainty is realized at time $t = 1$ and $\pi : \Omega \rightarrow [0, 1]$ is a probability mass function.

Agents

Entrepreneurs - The economy is populated by a continuum measure one set of identical entrepreneurs, indexed by $j \in [0, 1]$. They are the only agents in the economy with access to a menu of investment technologies, soon to be described. Each one has initial net worth A at $t = 0$ and no endowment in future periods. They are risk neutral, with utility given by $U(c_0, c_1, c_2) = E[c_0 + c_1 + c_2]$.

Lenders/consumers - There is a continuum of agents without direct access to investment opportunities, but with large endowments in the first two periods, A_0^L and A_1^L and no endowments in the last period. We assume that the measure of this set is strictly greater than one, so there are more lenders than entrepreneurs available. They are also risk neutral and also evaluate consumption streams according to $U(c_0, c_1, c_2) = E[c_0 + c_1 + c_2]$. The large endowment assumption ensures that scarcity of resources does not limit investment, leaving the determination of scale to be a consequence contractual frictions and not resource scarcity. Lenders are not able to commit to payments at $t = 1, 2$.

Assets

There are K assets in fixed supply $L \in \mathbb{R}_+^K$, which are initially held by lenders. The payout vector at state ω is given by $z(\omega) \in \mathbb{R}_+^K$. To emphasize the role as stores of value and not as physical inputs, let us assume that these resources are only available for consumption at $t = 2$. Additionally,

let the payoff matrix have full rank K and $K \leq \#\Omega$. Therefore, there are no redundant assets and, for each asset k , $z_k(\omega) > 0$ for at least some $\omega \in \Omega$.

These assets are traded at prices $q \in \mathbb{R}_{++}^K$ at time zero, with q_k representing the price of asset k . For simplicity, there is no market for such assets at $t = 1$. Given the ability to pledge payoffs from assets in financial contracts and common preferences, this assumption is innocuous.

This general formulation nests the case in which there are only real assets that can be purchased with the purpose of backing promises of transfers across agents, as well as an economy in which a complete set of Arrow-Debreu state contingent financial securities can be traded.

Definition 1.1. A **liquidity premium** on asset k is defined as the excess payment made for this asset at $t = 0$ relative to its expected output, that is, $\frac{q_k}{E[z_k]} - 1$.

Projects

Entrepreneurs choose from a menu of projects. These are described by the choice of $\gamma \in \Gamma$, where Γ is a compact subset of \mathbb{R}^n . Each of these projects involves a constant-returns-to-scale technology that generates $\rho_1(\omega, \gamma)$ units of output per-unit of investment if brought to completion. Investment is made at time $t = 0$ and output becomes available at $t = 2$. However, due to a contractual friction, only $\rho_0(\omega, \gamma) < \rho_1(\omega, \gamma)$ can be pledged to lenders. This friction can be motivated using a moral hazard problem, limited commitment or other distortions. The set-up cost of these projects is $\phi(\gamma)$ per-unit at $t = 0$ ¹¹.

Project choice can be narrowly interpreted as a technological decision, describing different ways to produce a final good or as alternative investment possibilities in different sectors of an economy. More broadly, it can also involve choices over different costly actions that can be taken by management during the implementation of a single enterprise which lead to changes in its returns, costs and responses to risks.

The projects involve a time-to-build component and might suffer additional cost shocks at $t = 1$, which make projects require essential reinvestment before completion occurs. These reinvestment shocks are denoted by $\rho(\omega, \gamma)$. Each unit of project γ will only be brought to completion and deliver output at $t = 2$ if an additional amount $\rho(\omega, \gamma)$ of resources is invested in the intermediate period, $t = 1$. An incomplete unit does not generate any output.

Partial continuation at any state contingent scale $x(\omega) \in [0, 1]$ is possible. That means that if entrepreneurs face liquidity shortages that render them unable to fully continue the project, a downsizing possibility exists. By downsizing the projects to a fraction $x(\omega)$ of their initial scale, total and pledgeable returns can still be collected for the relevant share of completed units.

Financial Contract

¹¹Given constant returns to scale, assuming that projects have different set-up costs is equivalent to normalizing this cost to be one and scaling all relevant returns and liquidity shocks by a multiplicative factor of $\phi(\gamma)^{-1}$. The additional function is left for convenience in the applications that follow.

At the beginning of period $t = 0$, each entrepreneur competitively offers a financial contract to be accepted by a single lender. The contract specifies $\{I, \{x(\omega)\}_{\omega \in \Omega}, \gamma, a\}$, where I is an investment scale, $\{x(\omega)\}_{\omega \in \Omega}$ is the fully-state-contingent continuation policy, γ is the project chosen and $a \in \mathbb{R}^K$ is the portfolio of external assets held by the entrepreneur-lender pair as part of the financial arrangement. This contract also determines time and state contingent transfers $\tau = \{\tau^0, \tau^1(\omega), \tau^2(\omega)\}$ from the lender to the entrepreneur. Given limited commitment, the lender can walk away at $t = 1$, losing rights to any payoffs from the project or external assets that are held as part of the financial arrangement. Since lenders lose access to the payoffs from assets in case they do not deliver the specified transfers to entrepreneurs, external assets play the role of collateral in the financial arrangement.

Taking as given an outside option of $\underline{\tau}$, lender participation at $t = 0$ requires

$$\underline{\tau} \geq E \left[\tau^0 + \tau^1(\omega) + \tau^2(\omega) \right]. \quad (1.1)$$

The lender commitment problem, imposes an interim participation constraint for each $\omega \in \Omega$ at $t = 1$ of the form

$$0 \geq \tau^1(\omega) + \tau^2(\omega). \quad (1.2)$$

That means that, in order for the lender not to walk away from the contract at $t = 1$ when state $\omega \in \Omega$ is realized, the sum of continuation transfers from the lender to the entrepreneur has to be non-positive.

Feasibility of the plan and entrepreneurial consumption $(c^{0,E}, c^{1,E}, c^{2,E})$ requires

$$\tau^0 + A = \phi(\gamma) I + q \cdot a + c^{0,E}, \quad (1.3)$$

which means that transfers from lenders plus initial entrepreneurial wealth need to cover the costs of investment, portfolio purchases and any entrepreneurial consumption,

$$\tau^1(\omega) = \rho(\omega, \gamma) x(\omega) I + c^{1,E}(\omega), \text{ for each } \omega \in \Omega, \quad (1.4)$$

that is, transfers from lenders need to cover any additional project costs at $t = 1$ plus any entrepreneurial consumption at that stage and, last,

$$\rho_1(\omega, \gamma) x(\omega) I + \tau^2(\omega) = c^{2,E}(\omega), \text{ for each } \omega \in \Omega, \quad (1.5)$$

total output generated by the project plus any additional transfers equal entrepreneurial consumption at $t = 2$.

The cases of interest are the ones in which $\tau^2(\omega) < 0$, indicating that there is repayment from entrepreneurs to lenders. These repayments are bounded by limited pledgeability, which imposes

that $-\tau^2(\omega)$ needs to be covered by pledgeable income from the project and the assets held,

$$\rho_0(\omega, \gamma) x(\omega) I + z \cdot a \geq -\tau^2(\omega). \quad (1.6)$$

Entrepreneurs therefore solve

$$\max_{c^E, \tau, I, \{x(\omega)\}_{\omega \in \Omega}, \gamma, a} E \left[c^{0,E} + c^{1,E} + c^{2,E} \right] \quad (1.7)$$

subject to constraints (1.1)-(1.6).

The timing of consumption and the transfers between lenders and entrepreneurs are not particularly interesting in this environment, given perfect substitution in consumption. As a consequence, the study of the environment and allocations can be much simplified once we work in terms of surpluses from investment, which are defined below. Additionally, there are two simplified formulations of the entrepreneur's problem, which do not depend on these elements, and are justified through the use of Lemma 1.1, which follows shortly.

Definition 1.2. We define the **total unit surplus** of an investment and portfolio plan as

$$B_1(\omega; q; \gamma, x, \hat{a}) \equiv \rho_1(\omega, \gamma) x(\omega) - \rho(\omega, \gamma) x(\omega) - \phi(\gamma) - [q - z(\omega)] \cdot \hat{a}.$$

The **pledgeable unit surplus** is

$$B_0(\omega; q; \gamma, x, \hat{a}) \equiv \rho_0(\omega, \gamma) x(\omega) - \rho(\omega, \gamma) x(\omega) - \phi(\gamma) - [q - z(\omega)] \cdot \hat{a}.$$

The **non-pledgeable component** of investment is

$$B_{1-0}(\omega, \gamma) \equiv [\rho_1(\omega, \gamma) - \rho_0(\omega, \gamma)] x(\omega).$$

The total surplus, $B_1(\omega; q; \gamma, x, \hat{a})$, is simply the final output per unit generated by the investment at $t = 2$, taking into account the completion rate $x(\omega)$, plus the payout from the portfolio of assets $z(\omega) \cdot \hat{a}$ net of all opportunity costs of generating this value. By investing and buying a portfolio at $t = 0$, entrepreneurs and lenders forgo $\phi(\omega) + q \cdot \hat{a}$ units of consumption. At $t = 1$, an additional $\rho(\omega) x(\omega)$ are spent to ensure completion. Given identical and linear preferences, consumption in any period is evaluated at a one-to-one rate by the lenders or entrepreneurs. The pledgeable unit surplus, $B_0(\omega; q; \gamma, x, \hat{a})$, is analogously defined, with pledgeable output replacing total output. Finally the non-pledgeable component of investment, a wedge $B_{1-0}(\omega, \gamma)$, is simply the difference between total output and total pledgeable output per unit. These surpluses are useful in simplifying the entrepreneurs' problem, dropping all the determination of transfers, and simplifying the set of constraints, as done below.

Lemma 1.1. *Whenever the entrepreneur's problem admits a solution, the optimal entrepreneurial*

choice relative to investment, continuation decision and asset purchases, $\{I, \{x(\omega)\}_{\omega \in \Omega}, \gamma, a\}$, solves

$$\max_{\{I, \{x(\omega)\}_{\omega \in \Omega}, \gamma, \hat{a}\}} E_{\omega} [B_1(\omega; q; \gamma, x, \hat{a})] I \quad (1.8)$$

s.t.

$$E[B_0(\omega; q; \gamma, x, \hat{a})] I + A \geq -t \quad (1.9)$$

$$z(\omega) \cdot \hat{a}I + \rho_0(\omega, \gamma) x(\omega) I \geq \rho(\omega, \gamma) x(\omega) I, \text{ for each } \omega \in \Omega \quad (1.10)$$

and

$$\max_{\{I, \{x(\omega)\}_{\omega \in \Omega}, \gamma, \hat{a}\}} E_{\omega} [B_{1-0}(\omega; \gamma, x)] I \quad (1.11)$$

s.t. (1.9) and (1.10). Where we define \hat{a} from $a \equiv \hat{a}I$, so that it acts as a normalization of the portfolio by the investment scale.

According to Lemma 1.1, entrepreneurs can be thought of as solving either one of two problems. The first one is the maximization of total surplus subject to the constraints to be explained in detail momentarily. The second equivalent formulation leads to the maximization of non-pledgeable benefits, that have to be consumed by entrepreneurs, subject to the same two constraints.

The first constraint (1.9) is derived from a combination of feasibility constraints and the participation constraint for the lender. It determines that investment is limited by the entrepreneur's capacity to generate pledgeable surplus and initial net worth A . The cases of interest are the ones in which despite its efficiency, investment is limited by difficulties in generating sufficient pledgeable surplus. Whenever projects are sufficiently productive, entrepreneurs would like to lever up by pledging all that is possible from the project to outsiders. As such, (1.9) is a leverage constraint, which pins down the maximum scale of investment given initial entrepreneurial net worth and the choices made regarding project selection, continuation policies and asset purchases.

The second set of constraints (1.10) follow from the lack of commitment from lenders. They can be interpreted as liquidity constraints in the following way. On the left-hand side there are the two sources of pledgeable income that entrepreneurs can rely on to ensure that they get funding for continuation. The $z(\omega) \cdot a$ term is the payout from assets acquired that are external to the project, therefore external liquidity. The second term, $\rho_0(\omega, \gamma) x(\omega)$, is the pledgeable output that can be still generated by the project if a continuation share $x(\omega)$ is guaranteed. On the right-hand side, there are the resource requirements from the project in state $\omega \in \Omega$ at time $t = 1$.

Whenever $\rho_0(\omega, \gamma) \geq \rho(\omega, \gamma)$, the project alone offers enough pledgeable income (internal liquidity) to guarantee full continuation of all units, even without reliance on the payoffs from the assets. Pledgeable income is sufficient to ensure sufficient financing from the lender and the project is said to be *self-refinancing*. Those are the states against which entrepreneurs would typically like to borrow to finance investment at $t = 0$.

On the other hand, whenever $\rho_0(\omega, \gamma) < \rho(\omega, \gamma)$, pledgeable income from the project itself does

not fully cover the additional cost at $t = 1$. Project γ is not self-refinancing in state ω and is said to be under *financial distress*. In the absence of any external assets, lenders would be unwilling to transfer any additional amounts to fund continuation of the project. This possibility is responsible for generating a demand for external liquidity. The purchases of external assets can be either interpreted as entrepreneurial savings towards those states or as the acquisition of collateral to enable transfers from lenders in state ω at $t = 1$ and, consequently, insurance of continuation possibilities.

Allocations and Equilibrium

Let Σ be the space in which entrepreneurial decisions towards investment, portfolio and continuation decisions lie with $\sigma = \{I, \{x(\omega)\}_{\omega \in \Omega}, \gamma, \hat{a}\} \in \Sigma$ being the typical element¹². In all examples analyzed, Σ can be made compact to ensure the existence of solutions to the entrepreneurs' problem¹³.

We then define an allocation as a mapping from the set of entrepreneurs to their decision space Σ . This definition is purposely leaving out specifics of the borrower-lender relationship, which determine the timing of consumption and all potential transfers. Whenever an equilibrium under the definition to follow exists, these elements can be easily obtained.

Definition 1.3. An allocation is mapping $\sigma : [0, 1] \rightarrow \Sigma$ such that each coordinate is Lebesgue measurable over $[0, 1]$. An allocation naturally defines a probability measure F_σ over Σ , that is, a distribution of entrepreneurs over their decision space.

Definition 1.4. A competitive equilibrium consists of an allocation σ , an outside option for lenders $\underline{\tau}$ and asset prices $q \in \mathbb{R}_{++}^k$, so that:

1. For every entrepreneur $j \in [0, 1]$, $\sigma(j)$ is a solution to the entrepreneur's problem given asset prices and the outside option of lenders $\underline{\tau}$.
2. For each asset $k \in \{1, \dots, K\}$,

$$q_k = E[z_k(\omega)] \text{ and } \int a_k dj \leq L_k, \quad (1.12)$$

or

$$q_k > E[z_k(\omega)] \text{ and } \int a_k dj = L_k. \quad (1.13)$$

3. The presence of excess lenders drives their outside option $\underline{\tau}$ to zero.

The definition of a competitive equilibrium requires entrepreneurial maximization, allowing for indifference between several equilibrium strategies. It is important that it allows for ex post heterogeneity, which emerges in some of the applications studied. Even with indeterminacy at the

¹²Note that $\Sigma \equiv \mathcal{I} \times [0, 1]^{\#\Omega} \times \Gamma \times \mathcal{A}$, where $\mathcal{I} \subset \mathbb{R}$ is the space of allowed scales and $\mathcal{A} \subset \mathbb{R}^K$ is the space of allowed asset holdings

¹³ $x(\omega)$ and γ belong to compact sets and a and I can be restricted to lie in sufficiently large closed intervals without loss of generality.

individual level, aggregates are uniquely defined in these cases. Market clearing conditions take into account that consumers are willing to hold any amount of assets as long as their prices equal their expected payouts (as in condition 1.12). Otherwise, when a liquidity premium emerges for any given asset, this has to be held exclusively by entrepreneurs as part of financial arrangements (as in condition 1.13).

1.3 Project Choice

The two frictions introduced have the potential to drive up asset prices and change the costs of ensuring reinvestment in the different states of the world. Additionally, pledgeable and non-pledgeable income offer different benefits to entrepreneurs. Pledgeable income, can be promised to lenders, helping raise more funds to finance the project's costs, increasing leverage possibilities. However, entrepreneurs can only consume any non-pledgeable resources generated by the projects, as these cannot be credibly transferred to other agents. As projects differ in their liquidity requirements, pledgeable income and non-pledgeable income, all these factors are taken into account in the optimal choice of projects.

In this section, we analyze general criteria for the choice of projects in this environment. Given the constant-returns-to-scale property of the production function and the linearity of the entrepreneurs problem, an optimal project is one that offers the highest shadow value on entrepreneurial wealth or, equivalently, one which has the highest Lagrange multiplier associated to the leverage constraint. Under complete markets, that shadow value is a ratio of expected non-pledgeable benefits to the net liquidity costs of the project, properly weighted by the prices for liquidity delivery in all states of the world. That multiplier can also be interpreted as the product of a leverage ratio and the non-pledgeable returns on investment.

1.3.1 Project choice under complete markets

The central assumption for this section is that the set of assets is composed of a full set of Arrow-Debreu securities and that entrepreneurs are allowed to short those, as long as there is sufficient pledgeable income to back this sale. When entrepreneurs are borrowing constrained, they invest all their net worth in the project and pledge all that is possible to lenders. As such, they consume only the non-pledgeable component $B_{1-0}(\omega, \gamma) = [\rho_1(\omega, \gamma) - \rho_0(\omega, \gamma)]x(\omega)$. To simplify the expressions derived, I introduce the wedge between total and pledgeable income per unit $\rho_{1-0}(\omega, \gamma) \equiv \rho_1(\omega, \gamma) - \rho_0(\omega, \gamma)$.

Under this situation, the entrepreneur's problem for a **fixed** project γ can be written as

$$\max_{\{I, \{x(\omega)\}_{\omega \in \Omega, \gamma, \hat{a}}\}} \sum_{\omega} \pi(\omega) \rho_{1-0}(\omega, \gamma) x(\omega) I \quad (1.14)$$

s.t.

$$a_\omega \geq (\rho(\omega, \gamma) - \rho_0(\omega, \gamma)) x(\omega) I, \text{ for each } \omega \in \Omega, \quad (1.15)$$

$$A - \sum \pi(\omega) (\rho(\omega, \gamma) - \rho_0(\omega, \gamma)) x(\omega) I - \sum_{\omega} (q(\omega) - \pi(\omega)) a_\omega - \phi(\gamma) I = 0, \quad (1.16)$$

$$I \geq x(\omega) I \geq 0. \quad (1.17)$$

Taking the necessary first-order conditions for an optimum while treating $x(\omega) I$ as a single choice variable, we obtain

$$x(\omega) I : \pi(\omega) \rho_{1-0}(\omega, \gamma) - (\lambda \pi(\omega) + \mu(\omega)) (\rho(\omega, \gamma) - \rho_0(\omega, \gamma)) \begin{cases} = \eta_{x(\omega)I} > 0, & \text{if } x(\omega) = 1, \\ = \eta_{x(\omega)I} = 0, & \text{if } x(\omega) \in (0, 1), \\ < 0, \eta_{x(\omega)I} = 0, & \text{if } x(\omega) = 0. \end{cases} \quad (1.18)$$

$$a_\omega : \mu(\omega) = \lambda (q(\omega) - \pi(\omega)), \quad (1.19)$$

and

$$I : \phi(\gamma) \lambda = \sum_{\omega} \eta_{x(\omega)I}, \quad (1.20)$$

where $\mu(\omega)$, λ and $\eta_{x(\omega)I}$ are respectively the multipliers on constraints 1.15, 1.16 and $I \geq x(\omega) I$.

A few insights emerge from these conditions. First, after optimization, only a subset of states enter the expression for the marginal value of wealth to the entrepreneur (λ). These are the states in which entrepreneurs strictly prefer to fully continue the project and which are associated to a multiplier $\eta_{x(\omega)I} > 0$, represents the shadow benefit of a scale expansion in a given state. Let this subset be denoted by $\Omega_+(\gamma, q)$. States in this subset are all the states in which,

$$\pi(\omega) \rho_{1-0}(\omega, \gamma) - \lambda q(\omega) (\rho(\omega, \gamma) - \rho_0(\omega, \gamma)) > 0,$$

that is, states in which the private benefit from completion outweighs the opportunity cost in terms of liquidity consumption necessary for continuation, $(\rho(\omega, \gamma) - \rho_0(\omega, \gamma))$, properly weighted by the price $q(\omega)$. This condition is naturally satisfied for all states in which financial distress does not occur, as both $\rho_{1-0}(\omega, \gamma) > 0$ and $\rho(\omega, \gamma) - \rho_0(\omega, \gamma) < 0$.

The shadow value of entrepreneurial wealth can be rewritten as

$$\lambda^*(\gamma, q) = \frac{\sum_{\omega \in \Omega_+(\gamma, q)} \pi(\omega) \rho_{1-0}(\omega, \gamma)}{\phi(\gamma) + \sum_{\omega \in \Omega_+(\gamma, q)} q(\omega) (\rho(\omega, \gamma) - \rho_0(\omega, \gamma))}. \quad (1.21)$$

Given linearity of the entrepreneur's problem, the value obtained from investing in project γ and choosing optimal continuation policies and portfolios is given by $\lambda^*(\gamma, q) A$. Project choice is then a matter of choosing the project $\gamma^* \in \Gamma$ which is associated to the highest multiplier $\lambda^*(\gamma, q)$. From equation 1.21, we can study which characteristics make a project more desirable. For instance,

fixing all other elements, an increase in the non-pledgeable benefits has that effect. Alternatively, a project with a lower requirement of expensive liquidity ($\rho(\omega, \gamma) - \rho_0(\omega, \gamma) > 0$ in states associated to high $q(\omega)$) is reduced. Both $t = 1$ costs $\rho(\omega, \gamma)$ and pledgeable income $\rho_0(\omega, \gamma)$ enter the denominator and are weighted by the price of the relevant Arrow-Debreu security. The set-up cost at $t = 0$ also consumes net worth and enters additively the denominator of the shadow value of wealth. This multiplier increases in a state price whenever the project is a net liquidity supplier in that state and decreases in prices whenever the project is in financial distress in that event but still taken to completion.

Notice that pledgeable and non-pledgeable income are treated significantly differently according to this project selection criterion. This procedure, based on the shadow value of pledgeable income, also reflects a departure from a net-present-value criterion, which indicates how projects should be optimally chosen in a frictionless environment. The essential distinction are different roles played by pledgeable and non-pledgeable income of the project. While pledgeable income is evaluated at the same prices as costs at $t = 1$, since they enter the same liquidity constraints, non-pledgeable income enters in the numerator, as in a rate of return calculation.

Notice also that $\lambda^*(\gamma^*, q) = \frac{E_\omega[B_1(\omega; q; \gamma^*, x^*, \hat{a}^*)]}{E[-B_0(\omega; q; \gamma^*, x^*, \hat{a}^*)]}$, which gives rise to a leverage interpretation. Given the leverage constraint of the form $E[B_0(\omega; q; \gamma, x, \hat{a})]I + A \geq 0$, under the optimal policy, entrepreneurs lever up their net worth by a factor of $\frac{I}{A} = \frac{1}{E[-B_0(\omega; q; \gamma, x^*, \hat{a}^*)]}$ and can reap all the social benefits from completion of the project. All elements in the denominator, which include set-up costs, additional costs at $t = 1$ and pledgeable income can be viewed in light of the effects they have on leverage of the entrepreneurial net worth and, therefore, on the determination of the scale of the project.

1.3.2 Project choice under incomplete markets

Under incomplete markets, in the entrepreneur's problem, constraint

$$A - \sum \pi(\omega) (\rho(\omega, \gamma) - \rho_0(\omega, \gamma)) x(\omega) I - \sum_{k \leq K} [q_k(\omega) - E(z_k)] a_k - \phi(\gamma) I = 0 \quad (1.22)$$

replaces constraint (1.16).

The first-order conditions (1.18) and (1.20) are unchanged. The conditions relative to asset purchases become

$$\sum_{\omega} \mu(\omega) z_k(\omega) = \lambda(q_k - E(z_k)), \quad (1.23)$$

for each asset k . On the left-hand side, we see the benefits of relaxing liquidity constraints which is a product of the relevant Lagrange multipliers and the asset returns on the different states. That benefit term is equalized to the term on the right-hand side, the cost of tightening the leverage constraint, that arises from purchasing an asset which features prices that are above its expect payouts. That naturally implies that $\frac{\mu^*(\omega; \gamma^*, q)}{\pi(\omega) \lambda^*(\omega; \gamma^*, q)} + 1$ works as a stochastic discount factor, for

each entrepreneur j and for every project $\gamma^*(j)$ that is selected in equilibrium. Despite all agents having linear preferences, this stochastic discount factor can be above unity, given the presence of a stochastic liquidity premium $\frac{\mu^*(\omega; \gamma^*, q)}{\pi(\omega) \lambda^*(\omega; \gamma^*, q)}$ which is reflected on asset prices.

Additionally, it follows from (1.20) and (1.23) that

$$\lambda = \frac{\sum \eta_{x(\omega)} I}{\phi(\gamma)} = \frac{\sum_{\omega} \mu(\omega) z_k(\omega)}{q_k - E(z_k)}$$

indicating a trade-off between the two possible uses of pledgeable income. Entrepreneurs might use pledgeable income to expand scale, which leads to a shadow benefit of $\frac{\sum \eta_{x(\omega)} I}{\phi(\gamma)}$, or alternatively, to purchase more assets for liquidity insurance purposes, for a shadow benefit of $\frac{\sum_{\omega} \mu(\omega) z_k(\omega)}{q_k - E(z_k)}$ which is obtained from relaxing the liquidity constraints.

Again, given linearity of the entrepreneur's problem, the criterion for project selection is one of choosing the investment prospect that leads to the highest shadow value for entrepreneurial wealth or, equivalently, on pledgeable income generated by the project.

A stronger characterization can be obtained when the economy features a single risk-less asset. In that case, the optimality condition for the asset purchase can be reduced to

$$\sum_{\omega} \mu(\omega) = \lambda(q - 1),$$

indicating that the purchase of the only asset available helps relax all the liquidity constraints. Given asset prices and a project chosen, we can partition the set of states in three disjoint sets: $\Omega_+(q, \gamma)$, the set of states in which entrepreneurs strictly prefer to fully continue and exhibit a multiplier $\eta_{x(\omega)} I > 0$ indicating a gain from a scale increase; $\Omega_p(q, \gamma)$, the set of states with partial continuation and $\eta_{x(\omega)} I = 0$; and last, $\Omega_0(q, \gamma)$, representing states in which the entrepreneur would prefer to fully terminate the project.

For all states in which partial continuation occurs and the liquidity constraint binds, it is easy to find and interpret the multiplier on that constraint. Simple algebraic manipulation allows us to write

$$\pi(\omega) \left\{ \frac{\rho_{1-0}(\omega, \gamma)}{\rho(\omega, \gamma) - \rho_0(\omega, \gamma)} - \lambda \right\} = \mu(\omega) > 0.$$

In those states, the binding liquidity constraint imposes that $x(\omega) I = \frac{a}{\rho(\omega, \gamma) - \rho_0(\omega, \gamma)}$. Notice a leverage effect in place, as a units of asset payouts used to ensure completion at state ω can generate $\frac{a}{\rho(\omega, \gamma) - \rho_0(\omega, \gamma)}$ completed project units. In any of those states, payoffs from assets have an opportunity cost: if they were simply pledged to outsiders and the project were fully terminated, they would generate an expected $\pi(\omega) a$ units of fully pledgeable income. That has a shadow value of $\lambda \pi(\omega) a$ to entrepreneurs. Completion of the project to the maximum extent allowed by the liquidity constraint consumes some net worth, since in a distress state $\rho(\omega, \gamma) - \rho_0(\omega, \gamma) > 0$. On the other hand, it enables a total non-pledgeable benefit of $\frac{\rho_{1-0}(\omega, \gamma)}{\rho(\omega, \gamma) - \rho_0(\omega, \gamma)}$ to be collected by the entrepreneur, if that state is reached. Therefore, the shadow value of the liquidity constraint

in states where entrepreneurs choose to partially continue up to the point in which the liquidity constraint binds is the levered non-pledgeable component of income $(\frac{\rho_{1-0}(\omega, \gamma)}{\rho(\omega, \gamma) - \rho_0(\omega, \gamma)})$, net of the opportunity cost of the pledgeable income dissipated (λ), all of which multiplied by the probability of state ω .

Notice that for all states with full continuation

$$a \geq [\rho(\omega, \gamma) - \rho_0(\omega, \gamma)] I.$$

This constraint can only bind for a single state: the one with the largest financial shortfall $\rho(\omega, \gamma) - \rho_0(\omega, \gamma)$. Let that state be called $\tilde{\omega}$. For all other states with full continuation, liquidity constraints are slack and $\mu(\omega) = 0$. As a consequence of the existence of a single non-entrepreneurial asset used for liquidity management purposes, for a fixed project γ , there is a single state $\tilde{\omega}$ that can have both a positive shadow value on scale increases and a binding liquidity constraint. We can write further that

$$\phi(\gamma) \lambda = \sum_{\Omega_+} \pi(\omega) \{ \rho_{1-0}(\omega, \gamma) - \lambda(\rho(\omega, \gamma) - \rho_0(\omega, \gamma)) \} - (\rho(\tilde{\omega}, \gamma) - \rho_0(\tilde{\omega}, \gamma)) \mu(\tilde{\omega}),$$

indicating the shadow value of wealth used in a increase in scale as being the sum over all states with full continuation of the private benefit net of liquidity opportunity costs minus the shadow value of the tightening of the liquidity constraint on $\tilde{\omega}$. Also,

$$\lambda(q-1) = \sum_{\Omega_p} \pi(\omega) \left\{ \frac{\rho_{1-0}(\omega, \gamma)}{\rho(\omega, \gamma) - \rho_0(\omega, \gamma)} - \lambda \right\} + \mu(\tilde{\omega}).$$

A purchase of external liquidity, in the form of the single risk-less asset, helps relax all the binding relevant liquidity constraints, both in the states with partial continuation, as in the single full continuation state with a binding liquidity constraint. Indeed, using $\hat{a} = a/I$, the normalized asset holdings, it is possible to rewrite the shadow value of entrepreneurial wealth as

$$\lambda^*(q, \gamma) = \frac{\sum_{\Omega_+} \pi(\omega) \rho_{1-0}(\omega, \gamma) + \sum_{\Omega_p} \pi(\omega) \frac{\rho_{1-0}(\omega, \gamma)}{\rho(\omega, \gamma) - \rho_0(\omega, \gamma)} \hat{a}}{\phi(\gamma) + (q-1) \hat{a} + \sum_{\Omega_p} \pi(\omega) \hat{a} + \sum_{\Omega_+} \pi(\omega) (\rho(\omega, \gamma) - \rho_0(\omega, \gamma))}.$$

The interpretation of $\lambda^*(q, \gamma)$ is similar to the case with complete markets. The shadow value of the entrepreneurial wealth is a ratio of private benefits collected, both in states with full continuation and in states with partial continuation (in which a leverage on liquidity, $\frac{\rho_{1-0}(\omega, \gamma)}{\rho(\omega, \gamma) - \rho_0(\omega, \gamma)} \hat{a}$, term emerges), relative to all costs of the project implementation and risk-less asset purchases in terms of pledgeable income. As stated before, project selection boils down to choosing $\text{argmax}_\gamma \lambda^*(q, \gamma)$.

Once a significant departure from a standard Arrow-Debreu benchmark is acknowledged in the selection of projects, a remaining question concerns whether it leads to significant macroeconomic consequences. That question is addressed in the next section, which illustrates the macroeconomic

effects of the interactions between liquidity scarcity and endogenous exposure to risks through project selection and optimal financial arrangements.

1.4 Macroeconomic Consequences

In this section, I specialize the general model into particular cases to analyze the aggregate consequences of the interactions between liquidity scarcity and project selection. In the first environment, entrepreneurs face *ex ante* and *ex post* credit rationing and choose projects which differ in the volatility of their output. As a consequence of partial pledgeability of output, these projects also differ in their ability to guarantee their own financing in future events. In this environment, the main source of fluctuations is in the corporate sector itself and works through variations in the internal liquidity of projects. Therefore, it is useful for understanding how aggregate liquidity scarcity interact with corporate liquidity fluctuations in determining the endogenous degree of volatility that firms face in the economy.

In the second environment, the fluctuations studied arise from the supply of external liquidity, which is stochastic. There, shocks which are external to the entrepreneurial sector, such as a housing market collapse, are transmitted towards it through their impacts on financial arrangements. I then introduce project choice, in which some projects are allowed to co-vary more strongly or weakly with the factors behind fluctuations in external liquidity, and show that project selection responses lead to additional endogenous comovement. Behind this comovement results lies a complementarity between projects, which might need additional investments before completion, and assets payouts that back reinvestment promises. This environment illustrates how this complementarity is responsible for biasing project choice in a direction which makes corporate investment and output covary strongly with external factors that determine aggregate liquidity conditions of the economy. Due to this positive comovement, fluctuations are also intensified in this set up.

Finally, I go back to a variant of the first environment and introduce a more complex set of instruments for liquidity distribution. As a consequence, financial arrangements and project choices change, highlighting a complementarity between different forms of contingent liquidity and different projects. Strong forms of specialization might emerge, despite the initial homogeneity of entrepreneurs. Although this leads to changes in the allocations and the possible specialization of firms with the introduction of richer asset trades, the qualitative results regarding dampening or amplification of productivity fluctuations remain similar. For example, for sufficient liquidity scarcity there is still amplification at an aggregate level. Therefore, this environment illustrates that amplification of fluctuations in economies with severe liquidity scarcity is robust to the introduction of more sophisticated financial arrangements.

1.4.1 Environment 1: Liquidity Scarcity and Volatility, with a single risk-less asset

When output is partially pledgeable to outsiders, variations in how much output can be generated, such as the ones caused by productivity shocks, change the volume of resources that can be used to finance both the project set-up and continuation in case of distress. The impact on the latter is of particular importance. As such, productivity shocks have the potential to change how much internal liquidity is available across states of the world and the depth of the financial shortfalls that need to be covered by external liquidity. When there are negative shocks to total output that can be produced, pledgeable output which is useful for ensuring external financing is reduced and creates more difficulties for funding the continuation of the project. The opposite is true for a shock that leads to an increase in total and pledgeable output.

By choosing among projects with different levels of volatility in productivity, entrepreneurs alter their liquidity needs across states of the world and, indirectly, the value which external assets have in their financial arrangements. Therefore, project selection interacts with liquidity management. If projects differ in their output volatility, the choice over this variable is of particular importance.

In this section, we study an environment in which there is a single risk-less asset that can be held as a buffer of external liquidity¹⁴. The equilibrium price of this asset is shown to be of central importance for the joint determination of which projects are selected, the quantity of the asset that is purchased and which continuation policies are implemented. In particular, while for low prices, entrepreneurs choose full continuation and low volatility of productivity, once prices are higher, partial continuation emerges and high volatility might be chosen.

The intuition for this mechanism is that by controlling volatility, entrepreneurs move pledgeable resources across states of the world and this works as an imperfect substitute for asset purchases. When asset prices are low, entrepreneurs are only constrained in a low productivity state. Therefore, at the margin, it is worth to choose projects with less volatility and relax that constraint. On the other hand, when acquiring assets is too expensive, entrepreneurs purchase less of these and end up liquidity constrained in multiple states. Pledgeable resources can then be the most valuable at the margin in states with higher productivity, but binding funding needs. Choosing higher volatility, in that case, helps move resources to those states.

Uncertainty- There are four states of nature. Financial needs at $t = 1$ are given by an aggregate shock which belongs to $\{0, \rho\}$, with $\rho > 0$. Their realization is ρ with probability π_ρ . Additionally, the aggregate determinants of the productivity of projects belong to $\{g, b\}$ and occur with respective probabilities of π_g and $\pi_b \equiv 1 - \pi_g$. In the g (good, higher productivity) states each unit of each project is capable of delivering its highest possible output, while in the b (bad, lower productivity) states it is capable of delivering a lower output. Productivity and reinvestment need shocks are assumed to be independent. Therefore, the state of the world is fully described by $\omega \in \Omega \equiv \{g, b\} \times \{0, \rho\}$.

¹⁴The case in which trades on external liquidity can be made state-contingent is studied later. This example is particularly useful for its simplicity and for contrasting its results with what is achieved when richer contracts for external liquidity trades are feasible. Formally, if one wants a justification for the absence of contracts for deliveries of risk-less assets at $t = 1$ across firms, we could resort to spatial separation or commitment problems.

Projects- There is a continuum of projects which differ in their initial set-up costs and in the magnitude of their productivity fluctuations. Formally, there is a compact set of projects indexed by $\gamma \in \Gamma \equiv [\underline{\gamma}, \bar{\gamma}] \in \mathbb{R}_{++}$. The project specific parameter γ measures the dispersion of output of a given project across the aggregate productivity states. Let

$$\rho_1(\omega, \gamma) = \begin{cases} \rho_1 + \frac{\gamma}{\pi_g} & , \text{ for } \omega \in \{g\} \times \{0, \rho\} \\ \rho_1 - \frac{\gamma}{\pi_b} & , \text{ for } \omega \in \{b\} \times \{0, \rho\} \end{cases}.$$

Notice that, conditional on full completion, all units of all projects have the same expected output of ρ_1 , but differ in their variance, which is increasing in γ . Therefore, the higher γ , the more volatile the output of a project is. In particular, the extreme project $\bar{\gamma}$ is the one that is the most adversely affected by the realization of an event with low productivity, while also the one that is the most positively affected by a high productivity event.

Assume that there exists a baseline, lowest cost project, γ_0 with $\phi(\gamma_0) = 1$. The output process of this project provides the benchmark level of fluctuations, around which amplification and dampening are defined. A project involving $\gamma > \gamma_0$ features more output fluctuation than the baseline project and is said to lead to *amplification*. Analogously, a project with $\gamma < \gamma_0$ fluctuates less than the benchmark project and is said to lead to *dampening*. Let $\phi(\gamma)$ be C^2 and strictly convex.

Let us assume that limited pledgeability is caused by an agency problem, with a severity which does not vary across states of nature. That means that a constant, state independent, private benefit $\rho_{1-0} > 0$ per continued unit of the project has to be offered to entrepreneurs in order to ensure diligent behavior. As a consequence, pledgeable output will move one-to-one with total output and $\rho_0(\omega, \gamma) = \rho_1(\omega, \gamma) - \rho_{1-0}$, for each state ω and project γ . This assumption is chosen for two reasons. The first is that private benefits are not made pro-cyclical or counter-cyclical in themselves, so that all incentives guiding project choice are entirely financial and related to liquidity costs. The second is that it generates greater tractability by allowing the entrepreneurs' problem to take an average-cost formulation, in which dependence on total output produced, ρ_1 , disappears.

Assets- There is a single risk-less asset that pays out a certain unit of consumption in all states $\omega \in \Omega$ at $t = 2$.

Additional Assumptions- The following set of assumptions about parameters in the production functions is made:

A1 $0 < \rho_0 < \rho < \rho_1$,

A2 $\rho + \frac{\bar{\gamma}}{\pi_b} < 1 + \pi_\rho \rho$,

A3 $\rho_1 > \frac{1}{1 - \pi_\rho}$,

A4 $\rho_0 + \frac{\bar{\gamma}}{\pi_g} < \rho$.

Assumption A1 ensures that financial distress occurs when the refinancing shock is ρ . Assumption A2, that it is optimal to fully continue even the project that is most adversely affected by a negative productivity shock if there is no premium on the risk-less asset. Jointly, A1 and A2 imply finite leverage. Assumption A3 implies that entrepreneurs are willing to undertake the project even when unable to continue in the distress states. Assumption A4 ensures that not even the project that is the most positively affect by a high productivity realization becomes self-financing in the (g, ρ) state.

Analysis

The purchase of the existing asset will enable entrepreneurs to simultaneously relax the two relevant liquidity constraints, which are

$$a + \left(\rho_0 - \frac{\gamma}{\pi_b} \right) x(b, \rho) I \geq \rho x(b, \rho) I \quad (1.24)$$

and

$$a + \left(\rho_0 + \frac{\gamma}{\pi_g} \right) x(g, \rho) I \geq \rho x(g, \rho) I, \quad (1.25)$$

since $\{(g, \rho), (b, \rho)\}$ are the two states in which financial distress occurs¹⁵. The first term on the left-hand side of both constraints is the level of purchases of risk-less assets (a) or, alternatively, how much externality was acquired at $t = 0$. The second term is the amount of internal liquidity available after the realization of the productivity shock is learned, for a continuation at scale $x(\omega) I$. To back financing, given the absence of commitment from lenders, the sum of those two terms needs to cover the required disbursement of ρ for the $x(\omega) I$ units of the project that should be taken to completion.

Output shocks create and destroy internal liquidity across states of nature, as seen in second term on the left-hand side of the liquidity constraints (1.24) and (1.25). Therefore, the negative productivity shock state (b) always involves a more stringent liquidity constraint than the positive state (g). For any level of asset purchases, continuation in the b state needs to be weakly lower than in the positive state, g . Therefore, productivity shocks induce supply shocks on aggregate liquidity and represent a force towards higher liquidity premia on the asset. By choosing less productive projects that involve dampening (lower γ), entrepreneurs can shift internal liquidity to the b state, where it is scarcer. This provides a rationale for why output shocks can lead to incentives for the dampening of fluctuations.

However, there is another force in place. The return on liquidity hoarding towards a given state is also influenced by the productivity shocks. A unit of liquidity in a state ω where a liquidity

¹⁵Assumption A4 and $x(\omega) I > 0$ imply that $a \geq 0$ from 1.25. As a consequence, the liquidity constraints relative to all states in which the reinvestment shock is 0 are always slack.

constraint binds enables the completion of project units that have a social surplus of $\rho_1(\omega, \gamma) - \rho$. From the liquidity constraints, the completion of each of these requires $\rho - \rho_0(\omega, \gamma)$ units of external liquidity in that state. Notice that a multiplier or leverage effect is in place: a unit of liquidity brought into state ω can enable the production of $\frac{1}{\rho - \rho_0(\omega, \gamma)}$ units of output, which create a non-pledgeable benefit of $\frac{\rho_1 - \rho}{\rho - \rho_0(\omega, \gamma)}$.

When the aggregate productivity shock is more favorable, both total, $\rho_1(\omega, \gamma)$, and pledgeable outputs, $\rho_0(\omega, \gamma)$, are higher for all projects. Completion becomes more valuable in the g state, given that a unit of the project completed delivers more social surplus. The multiplier effect on external liquidity is larger, meaning that each completed unit offers the entrepreneur more pledgeable income on which she will be able to lever up at $t = 1$. These effects are in place as long as constraint (1.25) binds, which occurs when external liquidity is still necessary at the margin in the higher output state.

These two combined generate a higher potential return on liquidity in the high productivity state and provide a rationale for the choice of projects that offer more liquidity in the g state. Those are the projects with higher γ , potentially involving amplification.

The proposition below helps understand the results that follow, by characterizing optimal decisions regarding asset holding and continuation policies, when entrepreneurs are restricted to an arbitrary fixed project.

Proposition 1.1. *For any fixed project $\gamma \in \Gamma$, there exist two cutoffs, $\underline{q}(\gamma)$ and $\bar{q}(\gamma)$, such that*

1. For $1 \leq q \leq \underline{q}(\gamma)$, full continuation in all states is optimal. Asset holdings are exactly sufficient to ensure full continuation in the $\omega=(b, \rho)$ state, with $\frac{a}{I} = \left(\rho - \rho_0 + \frac{\gamma}{\pi_b}\right)$. Constraint (1.24) holds with equality and constraint (1.25) is slack.
2. For $\underline{q}(\gamma) \leq q \leq \bar{q}(\gamma)$, an optimal policy features full continuation in the (g, ρ) state and partial continuation in the (b, ρ) state. Asset holdings are pinned down by $\frac{a}{I} = \left(\rho - \rho_0 - \frac{\gamma}{\pi_g}\right)$. Constraints (1.24) and (1.25) hold with equality.
3. For $q \geq \bar{q}(\gamma)$, it is optimal to set $a = 0$ and fully terminate the project in the distress states.

Whenever the costs of liquidity hoarding are relatively low, it is optimal to purchase enough assets to guarantee full continuation even in the worst possible state of nature. In that situation, entrepreneurs are only effectively liquidity constrained in the (b, ρ) -state¹⁶.

In this full continuation regime, there are incentives towards dampening, as the choice of a project with lower volatility reduces the need for asset hoarding. This can be seen in constraint (1.24), which is relaxed with the choice of lower γ . The forces pushing towards amplification are absent, given that the liquidity constraint for the (g, ρ) state does not bind.

¹⁶As $x(b, \rho) = x(g, \rho) = 1$, it follows that $\left(\rho_0 + \frac{\gamma}{\pi_g}\right)I + a > \left(\rho_0 - \frac{\gamma}{\pi_b}\right)I + a = \rho I$, implying that constraint (1.25) is slack.

However, once the liquidity premium is sufficiently high, it is optimal to switch to a policy of limited liquidity hoarding. In that situation, resorting to partial liquidation in case reinvestment needs coincide with low productivity helps reduce asset purchases, economizing on the use of expensive external liquidity. Since both liquidity constraints bind, there are forces in place both in the direction of dampening (relaxing constraint (1.24) by choosing a project with lower volatility) and of amplification (relaxing constraint (1.25) by choosing a project with higher volatility) if project choice is permitted. Which one dominates depends on the costs of asset hoarding.

Finally, for every project, there are sufficiently high prices leading to optimality of full liquidation in case of distress. Enabling insurance through the accumulation of stores of value becomes too expensive from the entrepreneurs' perspective. Later, when equilibrium conditions are taken into account, prices cannot rise beyond the point in which entrepreneurs stop demanding liquidity.

This general behavior remains similar once project choice is incorporated: there will be three relevant regimes for the solution of the individual problem as a function of asset prices. A first one with full continuation in all states, a second with full continuation only in case of high productivity shock and a third with full termination in case of distress. Within each one of these, which project is optimal can be determined by a first-order condition. Determining actual cut-offs for the switch between these regimes requires comparisons across solutions and either closed-form examples or a computational approach. Nonetheless, the essential qualitative properties can be proved without resorting to those. They are summarized by the proposition below.

Proposition 1.2. *The solution to the individual problem features three regions, delimited by the prices \underline{q} and \bar{q} . The following properties hold:*

1. For $1 < q < \underline{q}$, there exists a unique optimal project, which features dampening and full continuation. In this region, optimal project choice is a decreasing function of q .
2. For $\underline{q} < q < \bar{q}$, there exists a unique optimal project with full continuation in the (g, ρ) state and partial continuation in the (b, ρ) state. Project choice is an increasing function of q and, for sufficiently high q within this range, there is amplification.
3. For $q > \bar{q}$, there exists a unique optimal project, involving full termination. It is the lowest cost project, γ_0 .
4. For $q = 1$, γ_0 and full continuation are optimal.
5. At the thresholds \underline{q} and \bar{q} , two projects with respective different continuation policies are optimal.

Figure 1-1 below illustrates the main results from Proposition 1.2, regarding continuation shares, asset purchases and project choice as a function of the price of the risk-less asset's.

When prices are interpreted as a partial equilibrium measure of the intensity of the external liquidity scarcity, a characterization of the behavior of the equilibria emerges. If liquidity is plentiful

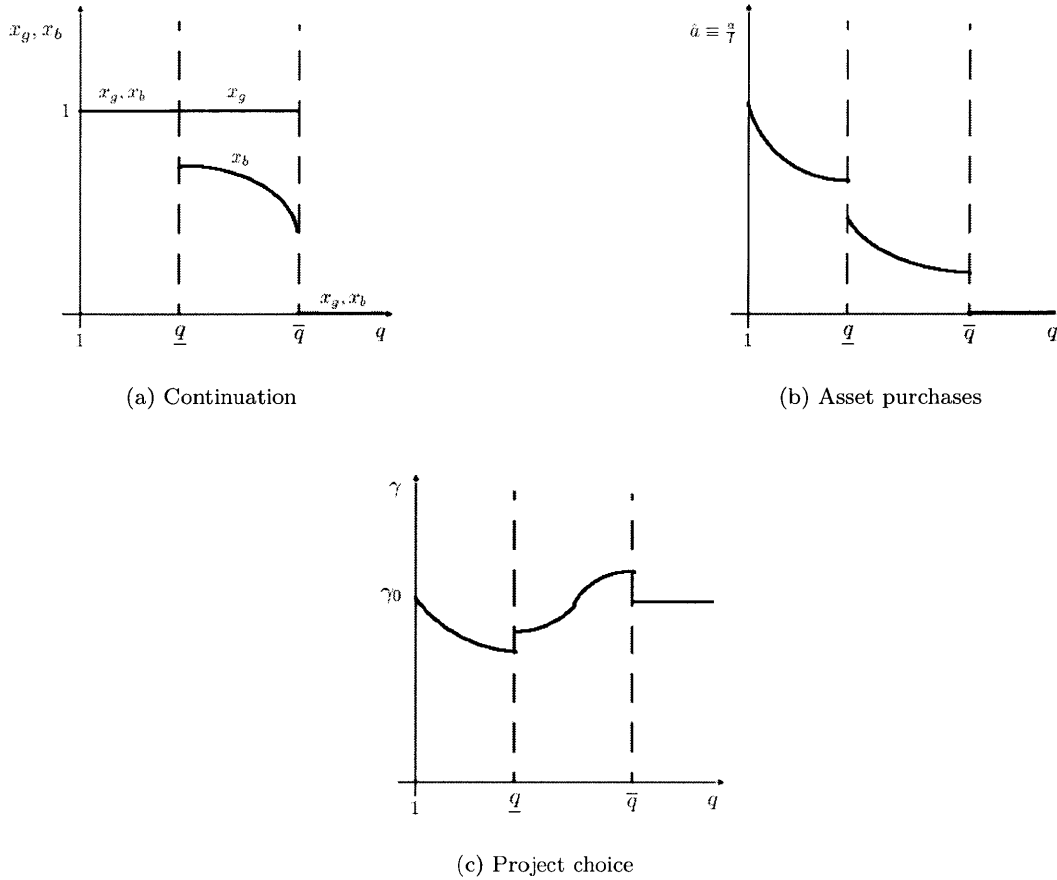


Figure 1-1: Optimal entrepreneurial decisions, as functions of asset price (liq. premium)

enough, no premium is present ($q = 1$) and, as a consequence, continuation decisions and project choice are efficient: full continuation always occurs and the most productive project, γ_0 , is chosen by all entrepreneurs.

As liquidity becomes moderately scarce, the economy moves to the behavior described in Proposition 1.2, part 1. The relevant binding liquidity constraint is in the low productivity state and, as a consequence, entrepreneurial choice over projects takes the relaxation of that constraint into account. That leads to projects that, although less productive due to the higher cost, fluctuate less and dampen the underlying shock. As such, they require less hoarding of liquidity, even for full continuation. In these economies, no financial crises involving termination of projects are ever observed.

If liquidity is more severely scarce, entrepreneurs opt to sacrifice continuation in the worst state of nature as in part 2 of Proposition 1.2. Asset purchases are just enough to ensure full continuation in the best financial distress state. By choosing projects that fluctuate more, entrepreneurs can save on the costly asset hoarding necessary to ensure this level of continuation, but end up sacrificing

efficient continuation in the worst financial distress state. When liquidity is scarce enough, as signaled by a high liquidity price, the gains from moving internal liquidity to the high productivity state to economize on external liquidity become large. The losses in terms of continuation in the bad productivity state are more than offset and projects leading to amplification end up being preferred.

Eventually, with sufficiently high prices, it becomes optimal for entrepreneurs not to buy any assets and to choose the most efficient project γ_0 again. Conditional on the policy of full termination in case of distress, there is no role for the relaxation of liquidity constraints through the choice of any other projects.

Moving back towards general equilibrium, we can construct an aggregate demand for assets from the behavior of the individual demand. Aggregation helps smooth out the discontinuities around the two thresholds, \underline{q} and \bar{q} , by exploiting the indifference of entrepreneurs between different projects and associated continuation policies. Together, the aggregate demand for assets and the inelastic supply of external liquidity pin-down the asset price and liquidity premium, q . A sketch of the equilibrium determination is provided in Figure 1-2.

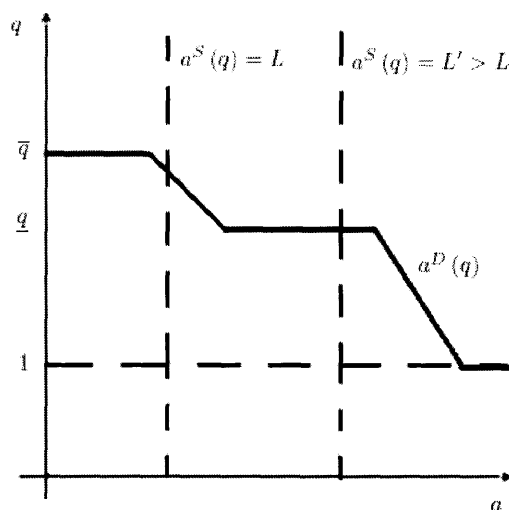


Figure 1-2: Sketch of equilibrium determination for economies with different levels for the supply of stores of value, L and L' . In the flat parts of the aggregate asset demand, two projects are optimal and the supply of risk-less assets determines the fraction of entrepreneurs that chooses each one the possible optimal policies, with their different underlying project choices and continuation decisions.

Changes in the liquidity supply, L , or in the pledgeable component of income, ρ_0 , have similar qualitative effects. Both can be thought of as measures of financial development and a reduction in either moves equilibrium asset prices towards higher levels. By doing so, the economy moves within and across the regions described in the partial equilibrium analysis.

Economies with severe financial underdevelopment experience higher fluctuations of output and severe financial crises, with the discontinuation of many projects. Economies with moderate financial development experience dampened fluctuations relative to first-best, but at cost in terms of lower productivity in entrepreneurial projects.

1.4.2 Environment 2: Fluctuations in non-corporate assets; Transmission and Synchronization

When financial conditions matter for economic activity, an important source of fluctuations lies outside the corporate sector itself. Liquidity conditions in the economy fluctuate as the value of assets that back promises of reinvestment in the economy change. The sudden drop in the prices of mortgage-backed securities in mid-2007 and the ensuing contraction of credit and investment highlight the practical importance of this particular channel.

In the framework proposed here, these fluctuations occur through the realization of different payouts for the assets available. A variation in external liquidity can be captured by the introduction of a mass L of trees, which produce a stochastic payout, or fruit. Suppose the output from these trees can be high (z_u) or low (z_d), with $z_u > z_d$. I allow for the contingent trading of this output through assets, to be described shortly. As these assets back financial arrangements and enable some insurance for the continuation of projects, competition for them might drive their prices above their expected output, creating liquidity premia.

In the first section to follow, I study the transmission of the fluctuation in the value of these assets into output from the entrepreneurial projects. In the presence of aggregate distress, the value of tree output determines how much reinvestment the economy as a whole can afford. Asset prices, investment scale and continuation policies are jointly determined in equilibrium. In this environment, premia on assets are always decreasing in the realization of tree output, while continuation shares of projects under distress and, consequently, entrepreneurial output are increasing. In this sense, shocks to the trees' capacity to generate fruit are transmitted towards the entrepreneurial sector and influence its capacity to take investment under distress to completion. In itself, this creates comovements across sectors of the economy, in which one sector is the provider of liquidity (trees) and another sector is the one that needs that liquidity in its financial arrangements (entrepreneurial projects).

I then show that if entrepreneurs face a choice of exposure to the same risks that drive tree output, they endogenously choose to increase their exposure to these risks. This is done in section 1.4.2.2, the synchronization result. The intuition underlying the mechanism is that when reinvestment shocks are proportional to the output that can be generated, projects that comove positively with the trees will have more plentiful, and therefore, cheaper external liquidity as complements. As a consequence, this economy creates an even stronger comovement across sectors than the transmission mechanism alone.

1.4.2.1 Transmission

We specialize the structure of the general model to the following particular case.

Uncertainty- Assume that financial distress and the realization of the trees' output are independent. The economy features four relevant states of nature: $\Omega = \{u, d\} \times \{\rho, 0\}$. Projects might suffer an aggregate refinancing shock $\rho > 0$ with a probability $\pi_\rho \in [0, 1]$. Otherwise, no additional costs have to be paid to ensure continuation of investments towards completion. Additionally, total external liquidity, derived from the fruits of the trees can take two realizations in $\{z_u, z_d\}$ with $z_u > z_d$. Let π_u be the probability of the high realization and $\pi_d \equiv 1 - \pi_u$. Independence of external liquidity shocks and refinancing needs is imposed and the probability of each realization of the state of the world $\pi(\omega)$ is naturally defined as the product of the relevant marginal probabilities.

Project- Suppose there is a single project available in the economy, by setting $\Gamma = \{1\}$, and normalize its cost to unit, $\phi(1) = 1$. Both total output per unit completed and the pledgeable component do not vary across states of nature. That means $\rho_1(\omega, 1) = \rho_1$ and $\rho_0(\omega, 1) = \rho_0$. When the aggregate reinvestment need shock happens, firms find themselves under financial distress. In that case, they are required to pay an additional $\rho > \rho_0$ to bring each unit of the project to completion. Otherwise, no additional cost has to be paid and firms are not under financial distress.

Assets- Let two assets be traded¹⁷ which are contingent on the output of the tree: u (d) is a claim on all the output of a tree contingent on it being revealed to be z_u (z_d) and is traded at price q_u (q_d) at $t = 0$. Therefore, asset u pays out z_u in states $\omega \in \{(u, \rho), (u, 0)\}$ and 0 otherwise. Similarly, asset d pays out z_d in $\omega \in \{(d, \rho), (d, 0)\}$ and 0 in other states. Let a_u and a_d be the quantities of these claims purchased as part of a financial arrangement between an entrepreneur and a lender. Both of these assets are available in fixed supply L .

Parameter Assumptions- We make the following assumptions about the parameters of the production function:

A1 $0 < \rho_0 < \rho < \rho_1$

A2 $\rho < \frac{1}{1-\pi_\rho}$

A3 $\rho_1 > \frac{1}{1-\pi_\rho}$

Assumption A1 ensures that financial distress occurs when the refinancing shock is ρ . Assumption A2 guarantees that it is optimal to fully continue the project if liquidity premia are sufficiently close to zero. Jointly, A1 and A2 imply finite leverage. Assumption A3 implies that entrepreneurs are willing to undertake the project even if unable to continue in the distress states.

¹⁷Trade in these two assets is sufficient to fully span both all contingencies of the trees output and to allow independent continuation decisions in all distress states.

Analysis

We proceed in the following way. First, we analyze the solution to the entrepreneurs problem and describe it graphically. Then, equilibrium conditions are imposed over that graphic description. Different regimes are possible for the behavior of the equilibrium and the total availability of liquidity in the economy determines which one holds. Finally, the key proposition of the section, concerning the transmission mechanism, is analyzed.

In both states that do not involve additional investment needs, projects are self-financing at $t = 1$ and offer excess liquidity. It is easily shown that liquidity constraints cannot bind in those states and that full continuation under those contingencies is optimal. Therefore, without loss of generality one can restrict attention to policies that set $x(\omega) = 1$ for $\omega \in \{(u, 0), (d, 0)\}$. The two possibly binding liquidity constraints that entrepreneurs face are given by

$$z_i a_i + \rho_0 x(i, \rho) I \geq \rho x(i, \rho) I, \text{ for } i = u, d. \quad (1.26)$$

The minimum purchase of these contingent assets that needs to be made to enable continuation is obtained by solving for an equality in the conditions (1.26). By proceeding this way, one obtains the minimal amount of asset purchases necessary to enable continuation of a share $x(i, \rho)$ of investment, denoted as $\hat{a}_i(x) = \frac{(\rho - \rho_0)x(i, \rho)}{z_i}$, which can be plugged into the entrepreneur's problem to write it as

$$\max_{I, x(\omega)} E[B_1(\omega; q; \gamma, x, \hat{a}(x))] I \quad (1.27)$$

s.t.

$$A + E[B_0(\omega; q; \gamma, x, \hat{a}(x))] I \geq 0, \quad (1.28)$$

where again B_1 and B_0 represent, respectively, the total and the pledgeable surpluses from investment as defined in section 1.2¹⁸.

A more complete description of the equilibrium behavior is offered in the appendix. Figure 1-3 describes the key elements of the characterization. In a graphical representation of the entrepreneurs' problem, there are four main regions, with the liquidity premia on both assets being the key elements for determining optimal continuation policies.

Around the lower-left corner of Figure 1-3 is found the region with low liquidity premia on both assets, in which full continuation in all states is optimal. When premia are sufficiently close to zero, liquidity hoarding is relatively inexpensive and entrepreneurs choose to fully insure against distress shocks. Thus, a policy of full continuation described by $(x_u, x_d) = (1, 1)$ is optimal.

The region above it, marked with $(x_u, x_d) = (1, 0)$, displays the area in the liquidity premium space in which full termination in the (d, ρ) state and full continuation in the (u, ρ) state is optimal.

¹⁸In the entrepreneurs' problem: $E[B_1(\omega, \gamma, q, x, \hat{a}(x))] = (1 - \pi_r) \rho_1 + \pi_r (\rho_1 - \rho) [\pi_u x(u, \rho) + \pi_d x(u, \rho)] + \sum_{i=u, d} (q_i - \pi_i z_i) \frac{(\rho - \rho_0)x(i, \rho)}{z_i} - 1$. $E[B_0(\omega, \gamma, q, \hat{a}(x))]$ can be analogously obtained by substituting ρ_0 for ρ_1 in that expression.

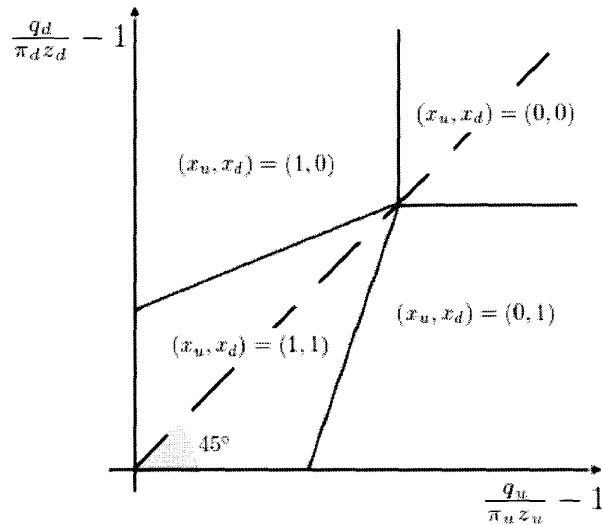


Figure 1-3: Optimal Continuation Policy Regions, as function of liquidity premia.

There, the premium in the d asset is sufficiently high while the premium on asset u is relatively low. The frontier between these two areas, described by a segment, is the locus of points where entrepreneurs would be willing to partially liquidate in case of a combination of low output and high liquidity needs. This will be an important object in the characterization of the equilibrium.

Analogously, the region in the lower-right corner is the one in which the premium on asset u would be sufficiently high as to induce termination, while the premium on asset d is not. As liquidity is scarcer in the (d, ρ) state than on (u, ρ) , it will not be a relevant region once equilibrium conditions are taken into account. Finally, the rectangular region in the upper-right corner describes the area in which both liquidity premia are so high that it is optimal for entrepreneurs to fully liquidate in both financial distress states, choosing continuation shares $(x_u, x_d) = (0, 0)$.

Aggregate liquidity scarcity in a distress state has two possible equilibrium consequences. It might constrain the scale of investment, by creating an equilibrium liquidity premium which ensures that liquidity insurance becomes sufficiently costly and consumes a fraction of the initial entrepreneurial net worth, as seen in equation (1.28). This way it would reduce the average entrepreneurial leverage and the scale of the average project. Alternatively, it might drive a liquidity premium to such a high level that entrepreneurs become indifferent regarding liquidity insurance for a given state of the world and some termination happens in equilibrium. In the brief description to follow, liquidity scarcity in one state of the world might be responsible for limiting the aggregate scale of investment. If that is not the worst possible state in terms of aggregate liquidity supply, all states with worse conditions will involve some termination of investment.

Equilibrium with a positive asset payout in both states imposes that full termination in a given

state can never be uniquely optimal. Therefore, the border segments between the four regions represented in the previous figure are the loci where equilibrium prices have to lie. As formalized in the appendix, possible equilibrium price and allocation behavior can be described by the numbered (i)-(v) points and segments displayed in Figure 1-4.

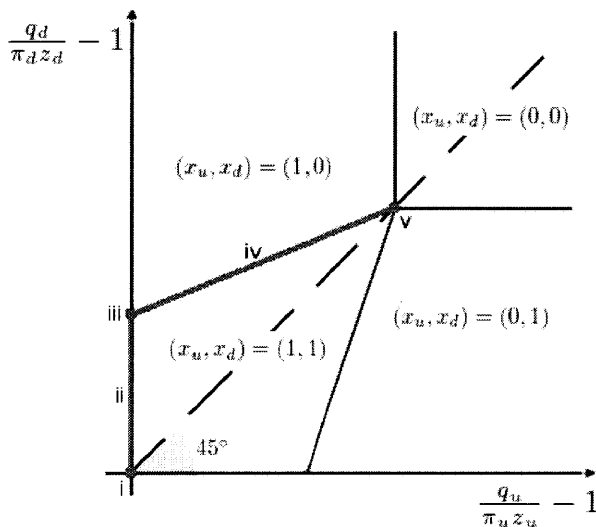


Figure 1-4: Loci of possible equilibria: premia and policies.

At point (i), liquidity even in the state with the most severe degree of scarcity, (d, ρ) , is sufficient to allow for full insurance of a scale of investment which is the highest possible. Liquidity premia on both assets are zero and the scale of investment is only constrained by entrepreneurial net worth, not by costs of asset hoarding. In all alternative cases to follow, liquidity premia are a feature of the equilibrium.

At point (ii), a liquidity premium in asset d alone is sufficient to increase the cost of liquidity insurance, crowding out investment in scale while still guaranteeing full continuation in all states. However, if the required premium is too high, as at point (iii) and above, entrepreneurs become indifferent regarding liquidation in that state. For all points above (iii), such as loci (iv) and (v), scarcity in the (d, ρ) state no longer limits aggregate investment scale and liquidity crises with termination of projects happen on that state.

For sufficiently low Lz_d , changes in the supply of the scarcer liquidity induced by either a change in the number of trees, L , or in the payout in low output states, z_d , change aggregate availability of insurance for that state and, as a consequence, the share of projects that face termination. This equilibrium reduction frees up entrepreneurial net worth, which is spent on an increase in the scale of the average project, as opposed to being spent on costly liquidity insurance. For very low tree output, it is then possible the aggregate liquidity constraint also binds in the higher liquidity

availability distress state and that a liquidity premium emerges on asset u as well as on d , as in regions (iv) and (v).

Region (iv) involves full continuation in the (u, ρ) state under all policies. There, in equilibrium, aggregate investment scale is such that there is just enough liquidity in that state as to enable continuation of all projects. Last, at point (v), liquidity premia on both assets are equalized and set to the maximum that entrepreneurs are willing to pay to enable insurance of continuation possibilities. Entrepreneurs are just indifferent between hoarding any asset or not being capable of withstanding financial shocks at $t = 1$.

Notice that liquidity premia are always higher in the d -contingent asset, as illustrated by the fact that all possible equilibria have prices that lie on or above the 45-degree line. Higher liquidity availability in states with higher tree output is always translated into lower premia on the u -contingent asset than on the d -contingent one.

Additionally, in regions (iii)-(v), some aggregate liquidation of projects occurs when financial distress happens. In these areas, aggregate continuation of investment under financial distress at $t = 1$ is constrained by the limited availability of external liquidity in the economy. Therefore, higher returns from the trees are always translated into strictly higher continuation shares in the state involving $z_u > z_d$ once the economy is in any of these regimes, in which liquidity is scarce enough. These elements are the essence of Proposition 1.3.

Proposition 1.3. *In the competitive equilibrium of this economy:*

1. Liquidity premia are higher for the asset contingent on low tree output: $\frac{q_d}{\pi_d z_d} \geq \frac{q_u}{\pi_u z_u}$.
2. Continuation shares are higher in the high output state than in the low output state: $x(u, \rho) \geq x(d, \rho)$, with a strict inequality if some termination ever occurs.

Output from the trees serves both as a consumption good itself and as backer of the liquidity reserves held by the entrepreneurs. Resources from the trees are not consumed directly in the production process (as fruits are only available at $t = 2$), but the holding of claims on trees helps guarantee reinvestment for projects under distress. At $t = 1$, news about more availability of fruits in the future enable more continuation of projects if distress happens.

Whenever the aggregate economy is ex post liquidity-constrained and partial continuation of projects occurs in equilibrium, negative shocks to the trees' output imply strictly lower continuation shares and lower output from the investment projects in the economy. In this sense, shocks to the sector of the economy which is a net supplier of liquidity (trees) are naturally transmitted towards the sector of the economy which holds it. In this environment, unlike in the one to follow in the next subsection where which project choice is endogenous, that transmission and comovement of output only works through the aggregate distress states.

Notice that the economy has always enough resources at $t = 1$ to enable full continuation of all projects, given the large consumer endowment assumption. The output from trees is not

necessary as a physical input in the entrepreneurial projects (it is not even available yet when distress happens), but the claims it backs are an essential input into their financing plans.

The economy analyzed was assumed to be closed, but the use of international liquidity can be easily incorporated. As pointed out by (Caballero and Krishnamurthy 2001; Caballero and Krishnamurthy 2003) and (Holmström and Tirole 2011), even with relatively plentiful international liquidity, specific economies might be constrained in their capacity to access foreign financial markets, especially due to limited capacity to generate internationally pledgeable income or tradable goods.

As such, the supply of aggregate liquidity can also incorporate some foreign component and shocks to the trees can also include fluctuations in international liquidity itself or in an economy's ability to access those markets. Therefore, the mechanism for the transmission of liquidity shocks into output fluctuations highlighted can also work across different countries, when there are international markets for liquid assets. This is specially clear when one central country provides a less financially developed economy with liquid stores of value.

In addition to these transmission effects, there might be incentives in place for entrepreneurs to select more pro-cyclical investment projects, synchronizing entrepreneurial output with external liquidity cycles, as the next example highlights.

1.4.2.2 Synchronization

The structure of the economy is mostly the same as in the previous section, with one important distinction. Now entrepreneurs can choose how intensely the output of the projects covaries with the output of the trees. In the case under study, pledgeable output and potential reinvestment shocks are proportional to total output that a project can generate in a given state. That production structure is interpreted as a sequential investment problem in which in the first stage ($t = 0$) entrepreneurs choose projects that give them investment opportunities at $t = 1$ which come in different ways with the output from trees.

Uncertainty, Assets and Parameter assumptions- Same as in Section 1.4.2.1.

Projects- There is a continuum of projects $\Gamma = [-\pi_u, \pi_d]$ and each entrepreneur can choose any $\gamma \in \Gamma$. Total output of project γ in state ω , conditional on full completion, is given by

$$\rho_1(\omega, \gamma) = \rho_1 n(\omega, \gamma). \quad (1.29)$$

Analogously, pledgeable output is $\rho_0(\omega, \gamma) = \rho_0 n(\omega, \gamma)$ and the costs of completion under the financing shock are $\rho(\omega, \gamma) = \rho n(\omega, \gamma)$, for $\omega \in \{u, d\} \times \{\rho\}$.¹⁹ The mapping from $\gamma \in [-\pi_u, \pi_d]$ into $n(\omega, \gamma)$ is given by

$$n(\omega, \gamma) = \begin{cases} 1 + \frac{\gamma}{\pi_u}, & \text{for } u \text{ states} \\ 1 - \frac{\gamma}{\pi_d}, & \text{for } d \text{ states} \end{cases}$$

¹⁹Notice that in all states in which the financing shock does not happen, i.e. in $\omega \in \{u, d\} \times \{0\}$, $\rho(\omega, \gamma) = 0$.

Notice that $E_\omega [n(\omega, \gamma)] = 1$. We assume there exists a baseline benchmark, a "neutral investment" $\gamma = 0$, which has the lowest possible cost per unit, $\phi(0) = 1$, and that $\phi : \Gamma \rightarrow \mathbb{R}_+$ is twice continuously differentiable and weakly convex, so that

$$\phi'(\gamma) \begin{cases} \geq 0, & \text{if } \gamma \geq 0, \\ \leq 0, & \text{if } \gamma \leq 0, \end{cases}$$

and $\phi''(\gamma) \geq 0$.

This production structure can be motivated in the following way. An initial investment at $t = 0$ generates a fixed expected number of profitable investment opportunities ("ideas") at $t = 1$. The cost of taking advantage each one of these opportunities is determined by an aggregate shock: it can be ρ with probability π_ρ or zero with probability $1 - \pi_\rho$.

The output from each implemented project is not fully pledgeable, as incentives have to be provided for diligent entrepreneurial behavior. Each investment opportunity which is implemented generates a total output of ρ_1 , of which only a component ρ_0 is pledgeable to outsiders.

Whenever the cost of additional investment is zero, these investment opportunities are self-financing at $t = 1$. Otherwise, entrepreneurs need to have assets in place to back investment, since $\rho > \rho_0$, and pledgeable income from projects themselves is not sufficient to finance all costs of investing at $t = 1$.

Project $\gamma = 0$, the neutral investment, provides entrepreneurs with one investment opportunity in each state of nature. Importantly, for this neutral project, the emergence of these opportunities is independent from the realization of the trees' output. At a higher cost, entrepreneurs might choose projects that give them investment opportunities which comove more positively or negatively with the trees' output. That is, entrepreneurs might choose within a menu of projects that differ by offering these opportunities in a more strongly pro-cyclical or more strongly anti-cyclical way, where the cycle is defined relative to the aggregate shock causes high or low tree output. Thus, a higher γ biases the distribution of ideas towards being more strongly related to the cycle from the trees, but keeps the expected number of ideas constant. For instance, setting $\gamma = \pi_d$ makes all ideas appear in the states where tree output is learned to be at its highest possible value, Lz_u .

Besides the constant number of expected investment opportunities, all projects also share the same structure per-unit in terms of refinancing costs at $t = 1$, total and pledgeable returns. They differ however on the set-up cost and, as a consequence, also on their returns per-unit of the consumption good invested. In an environment without liquidity premia, $\gamma = 0$ would dominate all other investment possibilities, due to its lower $\phi(\gamma)$.

Analysis

The central synchronization result is summarized by the proposition below:

Proposition 1.4. *Suppose the set-up cost function $\phi(\gamma)$ is strictly convex. Then, in any equilibrium with a premium differential, i.e. where $\frac{q_d}{\pi_d z_d} > \frac{q_u}{\pi_u z_u}$, there is a unique optimal project $\gamma^* > 0$, which features biasing of investment opportunities towards the high tree output states.*

In this environment, there exists a complementarity between investment that leads to profitable ideas at $t = 1$ and liquidity that allows the implementation of these ideas when they turn out not to be self-financing. Once a liquidity premium differential emerges, reflecting the relative scarcity of liquidity delivered in the event of low tree output, projects that offer investment opportunities in a more strongly pro-cyclical way will have as complements a cheaper form of liquidity.

As such, there will be incentives for this synchronization of liquidity needs and liquidity supply, which is made possible by choosing a project with $\gamma > 0$. Despite having a lower return per-unit of the good invested in the technology itself, pro-cyclical projects offer an advantage, as the portfolios that enable their continuation involve the use of cheaper, pro-cyclical liquidity.

In the previous section, output from the entrepreneurial sector covaried with the trees output only once partial continuation occurred. Under those circumstances, completion rates exhibited a cyclical behavior and this was inherited by the entrepreneurial output. This transmission mechanism only worked through the financial distress states.

The present example highlights that once more strongly pro-cyclical projects are chosen at $t = 0$, their output covaries positively with the trees' return. This happens not only on the financial distress states, but even in the states where projects turn out to be self-financing. This comovement does not depend on a lower completion rate as in the previous example, but on an ex ante preference towards projects that require more plentiful liquidity as complements.

1.4.3 Environment 3: Liquidity Scarcity and Volatility with multiple assets

In this section, I study the result of the introduction of a more complete set of assets in the economy described in section 1.4.1, in which exposure to output fluctuations is endogenous. The assets introduced allow entrepreneurs to purchase external liquidity in a way which is contingent on the realization of the productivity states.

A complementarity between specific assets and projects is shown to exist. As a consequence, in economies with severe liquidity scarcity and high premia, entrepreneurs specialize in two projects and holdings of a single asset. One of these projects has strong output volatility, while the other has mild volatility. In case of aggregate financial distress, one of these projects is fully taken to completion, while the other is fully terminated. In that sense, these economies feature extreme levels of partial insurance and crises are always associated to failures in a large set of firms.

Additionally, these economies feature aggregate amplification of productivity fluctuations due to two forces. First, firms which hold liquidity which is contingent on positive productivity shocks are willing to pay high costs to economize on those, which they can do by choosing highly volatile projects. Second, there are more of these highly volatile firms than firms which choose to have

projects with low levels of productivity fluctuations.

The environment is a special case of the general model proposed and is summarized by the following:

Uncertainty, Projects and Parameter assumptions- Same as in 1.4.1.

Assets- Now, unlike in Section 1.4.1, there are two assets that can be traded. Assets g and b , which pay out respectively in the high and low productivity contingencies. Asset g pays out 1 unit of consumption in states $\omega \in \{(g, \rho), (g, 0)\}$ and 0 otherwise. Asset b pays out 1 unit of consumption in states $\omega \in \{(b, \rho), (b, 0)\}$ and 0 otherwise. Both assets are in fixed supply L .

As in the previous section, these two assets are sufficient to fully span the subspace of states of the world in which external liquidity is essential for continuation. That means that there entrepreneurs can independently move liquidity into each state where projects are under financial distress and external assets are essential for continuation. As a consequence, equilibrium outcomes are the same that would be achieved if asset markets were complete, with the presence of four state contingent securities, while we avoid indeterminacy problems on asset holdings.

Analysis

We first analyze the asset pricing consequences of this enriched set of assets when project choice is shut down, i.e., when all entrepreneurs are constrained to managing the baseline γ_0 project. In section 1.4.2.1, the consequences of the shocks to external liquidity, through the output of the tree, generated all pricing and equilibrium characterization results. It is useful to contrast the behavior of the optimal continuation policies when productivity shocks induce shocks to internal liquidity, as they now do, with that case.

As illustrated in Figure 1-5, the two effects that the productivity shock have on the possible prices of external liquidity are present. Following the same reasoning from the previous examples, the competitive equilibrium of a given economy has to lie in regions (i)-(v). Liquidity in the b state is always less plentiful than in the g , so if asset g carries a liquidity premium, the scarcer asset b will also need to carry one.

However, locus (iv), the set of points in which entrepreneurs are indifferent regarding hoarding liquidity to withstand the refinancing shock in the presence of low productivity, crosses the 45°line. Therefore, liquidity in the g asset might be valued at a premium above that embedded in asset b . This reflects the higher return on liquidity hoarding in the high productivity state, which had already emerged in the single-asset environment of Section 1.4.1.

Once project choice is available under the same assumptions as in the previous section, a few new features emerge. First, it is no longer the case that entrepreneurs can be indifferent between choosing a full continuation or a full termination policy, as in point (v) in Figure 1-5.

If this were the case, a joint deviation in choosing a project with a different degree of intrinsic volatility and holding only one of the contingent assets would dominate these options. That is due to the fact that project choice exhibits a complementarity with liquidity purchase decisions:

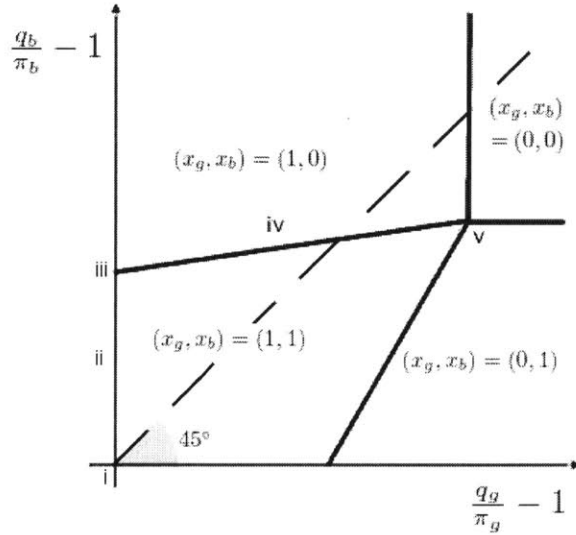


Figure 1-5: Optimal continuation policy regions, as functions of liquidity premia. For a fixed project γ .

a project that which involves amplified fluctuations leads to a higher willingness to pay for the pro-cyclical form of liquidity (asset g). The reverse is true for projects involving dampening, which display a higher reservation value for the b contingent asset.

The graph representing the optimal continuation policy as a function of liquidity premia on the two assets displays a typical behavior as the one in Figure 1-6. Behind each of the possibly optimal policies, there lies an associated optimal project choice.

Due to the complementarity between project choice and liquidity portfolio decisions, a region in which policies leading to continuation in only one of the two financial distress states dominate both full continuation and partial continuation emerges. It illustrated by locus (vi) in Figure 1-6.

Given that the economy has a set of assets which is rich enough to allow for the contingent allocation of external liquidity, equilibria in this region lead to the emergence of fully specialized firms: one with a project which fluctuates more severely, featuring amplification, and which is insured against financial distress only in the event of high aggregate productivity and another that fluctuates less, featuring dampening, and is insured against financial distress only in low aggregate productivity states.

The fact that locus (vi) lies below the 45-degree line has important consequences for project choice: in this region, firms choosing the pro-cyclical continuation policies favor projects leading to amplification and, given that they face a higher liquidity premium on the relevant asset, have stronger incentives for choosing projects away from the baseline γ_0 than do firms which choose less-cyclical projects and anti-cyclical continuation policies.

Indeed, that can be clearly seen from the entrepreneur's problem for fixed continuation policies.

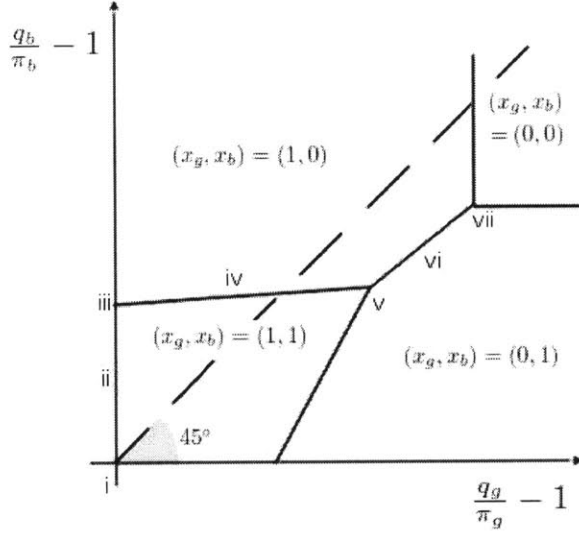


Figure 1-6: Optimal continuation policies, conditional on optimal policy choice, as functions of liquidity premia.

An entrepreneur that fully continues on state (g, ρ) and all non-distress states, but fully terminates on state (b, ρ) needs only to purchase asset u , not b . The relevant binding liquidity constraint is then

$$a_u = \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) I,$$

implying that asset purchases have to be just enough to cover the gap between refinancing needs, ρI , and pledgeable income in that state, $\left(\rho_0 + \frac{\gamma}{\pi_g} \right) I$. It can then be easily shown that the entrepreneur's problem, subject to this fixed continuation policy, can be rewritten as

$$\min_{\gamma_{10}} c_{10}(q_g, \gamma),$$

where

$$c_{10}(q_g, \gamma) \equiv \frac{\phi(\gamma) + \pi_\rho \pi_g \left(\rho - \frac{\gamma}{\pi_g} \right) + (q_g - \pi_g) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right)}{(1 - \pi_\rho) + \pi_\rho \pi_g}$$

is an average cost per project unit completed project. An interior optimum for project choice immediately implies that

$$\phi'(\gamma_{10}) = \pi_\rho + \left(\frac{q_g}{\pi_g} - 1 \right).$$

Fixing the continuation policy, entrepreneurial incentives regarding project choice take into account the possibility of saving on the relevant asset, by choosing projects with a different level of productivity fluctuations. The liquidity premium on asset g , $\left(\frac{q_g}{\pi_g} - 1 \right)$, naturally appears in the

expression.

An analogous procedure shows that the optimal project choice for a policy of full termination in distress state (g, ρ) and full continuation in all other states is given by the first-order condition

$$\phi'(\gamma_{01}) = -\pi_\rho - \left(\frac{q_b}{\pi_b} - 1 \right),$$

which takes into account the liquidity premium on asset b , the relevant one for this policy. The fact that the locus of indifference between these policies lies below the 45-degree line immediately implies that incentives for entrepreneurs holding the pro-cyclical form of liquidity (the g asset, with a higher premium) to deviate from the baseline γ_0 are stronger.

Despite the possible specialization of firms in holding a single form of liquidity and choosing a complementary project, which could not happen under the single-asset environment, the main results regarding amplification and dampening are maintained. Therefore, the emergence of amplification or dampening of fluctuations at the aggregate level is not a direct consequence of the absence of sophisticated contingent arrangements, but a response to the value to liquidity scarcity in accordance to its value across different states of the world.

For a sufficiently high supply of liquidity, the economy features dampening of the underlying shock. That is, a single project with $\gamma < \gamma_0$ is chosen by all entrepreneurs, as in locus (ii).

Once liquidity shortages are more severe, entrepreneurs specialize into two different projects. In regions (vi) and (vii), where policies $(x_g, x_b) = (1, 0)$ and $(x_g, x_b) = (0, 1)$ coexist, there is aggregate amplification. Entrepreneurs running the pro-cyclical projects, involving amplification, face higher incentives for deviating from γ_0 , given that by doing so they save on a form of liquidity with higher equilibrium premium. Additionally, in equilibrium, there is a higher mass of these projects than of the project involving dampening.

These results can be summarized by the proposition below.

Proposition 1.5.

1. An equilibrium regime in which a pro-cyclical ($\gamma_{10} > \gamma_0$) project with associated continuation decisions $(x_g, x_b) = (1, 0)$ and an anti-cyclical project ($\gamma_{01} < \gamma_0$) with associated continuation decisions $(x_g, x_b) = (0, 1)$ coexist occurs for sufficiently scarce external liquidity, L .
2. Assume that $\phi(\gamma)$ is strictly convex and symmetric around γ_0 and that project choice is interior. Then, in regimes (vi) and (vii), there is aggregate amplification: $\|\gamma_{10} - \gamma_0\| > \|\gamma_{01} - \gamma_0\|$ and a higher share of entrepreneurs choose γ_{10} and $(x_g, x_b) = (1, 0)$.

In both environments, with a single asset or with two contingent assets, there are in place two mechanisms for the equilibrium determination of prices, liquidity premia and project choice.

The first mechanism is mostly a supply-contraction effect: a shock to productivity reduces internal liquidity and makes liquidity scarcity more severe in the low productivity states. Aggregate

liquidity constraints are always tighter in these states, in the sense that if any continuation share is feasible in that state, it is also feasible in the higher productivity distress state.

On the opposite direction, there is an effect on the entrepreneurs' willingness to pay for liquidity: a demand-side effect. When the economy is constrained in all distress states, liquidity in states with higher productivity is more valuable than liquidity in states with lower productivity. Higher productivity makes a unit of liquidity have a higher multiplier effect into units of the project salvaged and also increases the total output of each one of these units. Therefore, when scarcity is sufficient to make prices be driven by this demand-side effect, the liquidity premium is higher on the asset that offers payments conditional on the high productivity events.

For moderate liquidity shortfalls, the scarcity effect dominates and influence project choice towards costly dampening. Once these shortfalls are more severe, as they are likely to be in economies with less financial development, termination of projects becomes more common and projects with more intrinsic volatility become more desirable.

The main consequence from a richer asset structure is the emergence of specialized firms. With sufficiently severe liquidity scarcity, all projects face the threat of termination in equilibrium, but external liquidity is efficiently used for salvaging projects with the best prospects in a given state of the world. In a distress state with lower productivity, projects involving dampening offer an advantage in their productivity, as they suffer less from a negative shock. On the other hand, the opposite is true in a distress state with higher productivity: projects with higher volatility have an advantage when relatively better shocks occur. Projects are also chosen in a complementary way to the form of liquidity that entrepreneurs choose to hold. This insight goes beyond the model with homogenous entrepreneurs, which endogenously specialize, and would also apply to examples that involve some original heterogeneity in exposure to productivity fluctuations. Specialization in the use of liquidity is a response to financial frictions and, once present, introduces a feedback into technological decisions that affect the economy's behavior under aggregate risk. Partially insured firms have higher incentives to manipulate their business-cycle exposure to economize only on the few assets they need to buy.

One might be surprised by the stark level of specialization of firm holdings of liquidity, which have a corner solution behavior. In equilibrium, projects are either fully insured or fully uninsured against financial distress shocks in a given state. This is a direct consequence of the constant-returns-to-scale assumption: if a set of firms holds a marginal unit of liquidity and is responsible for driving its price above the willingness to pay of firms with different projects, then these firms will also be the holders of all infra-marginal units. The insights about the complementarity between project choice and liquidity holdings and some degree of specialization in liquidity holdings should extend to economies with decreasing returns to scale, where less extreme forms of partial insurance would emerge.

It is also worth noting that the same allocation could be implemented without state contingent trades, as long as multi-project firms or financial intermediation (a lender with multiple en-

trepreneurs) are allowed. This way, external liquidity can be ex post allocated in the most efficient way.

1.5 Constrained Pareto Optimality and Policy

I first ask the question of whether a planner which is subject to the same constraints as private agents regarding bilateral contracts, limited pledgeability, limited commitment and asset market incompleteness, but which can choose contracts, projects and reallocate assets across the whole economy, can create a Pareto improvement over the original allocation. As previously anticipated, the answer is negative. Therefore, conditional on other frictions, a planner which has no advantages in the creation or distribution of liquidity over the private sector²⁰ cannot improve project choices or asset allocations.

Definition 1.5. An allocation is constrained Pareto optimal if there is no other set of pairwise financial arrangements (transfers, project choice, continuation decisions and asset holdings) and transfers of pledgeable resources at $t = 0$ that respects feasibility and constraints on pairwise financial arrangements (constraints 1.2 to 1.6), which Pareto dominates it²¹.

Proposition 1.6. *Every competitive equilibrium of the economies described in Section 1.2 is constrained Pareto optimal.*

The constrained efficiency result highlights that incentives regarding project choice and asset holdings are properly aligned, not only at the lender and entrepreneur level, but also relative to the rest of the agents in the economy. When agents decide on their project choice, continuation decisions and asset holdings, they have internalized all impacts on other agents. For example, in section 1.4.1, when agents decide to dampen or amplify productivity fluctuations to relax their relevant liquidity constraints, they purchase less risk-less assets and free up this valued collateral to be used by other agents.

The use of external assets for backing transfers between a pair of agents excludes other agents from using these, but that effect is properly reflected in equilibrium asset prices and, as a consequence, asset reallocation at $t = 0$ can never lead to a Pareto improvement. Most importantly, there are no fire-sale externalities in the interim stage ($t = 1$) which a planner could address. Those are present in similar models in which constrained efficiency fails, such as (Lorenzoni 2008), (Shleifer and Vishny 1992) or (Stein 2011).

The constrained optimality result in this environment generates a clear policy message related to which policies have the potential of creating improvements in this environment and which ones

²⁰These advantages would be such as being able to issue assets agents cannot or use instruments for the contingent delivery of liquidity across firms which are more complete than the ones allowed by the original asset markets.

²¹Given the continuum of agents, for a Pareto improvement, a strict utility increase is required for a positive mass of agents.

do not. For instance, no policy distorting or mandating project choice alone can generate a Pareto improvement over the original allocation. That includes banning the choice of the riskiest projects in an economy as the one described in section 1.4.1. The unintended consequences of such policy, in economies in which it effectively constrains project selection, would be a drop in the value of external assets, thus hurting their initial holders, and a potential increase in the number of firms that have to discontinue projects in case of financial distress, as other firms are not allowed to economize on their use of external assets by choosing more volatile projects.

Analogously, another policy that cannot lead to any Pareto improvement is one that forces the exclusive use of risk-less assets in backing financial arrangements. Although it could reduce the transmission of external financial shocks into the corporate sector and eliminate any endogenous increase in exposure the sources of such shocks generated by project choice, it leads to the wasting of valuable resources that fail to pay out in all contingencies. It therefore leaves the entrepreneurial sector excessively exposed to financial needs that could at least be satisfied in some instances.

On the other hand, the constrained optimality result does not imply that there is no room for policy at all in this environment. It does however determine clearly conditions which any improving policy needs to satisfy. Any such policy needs to feature an advantage from some central authority in creating or distributing liquidity which is not shared by the private sector. One example is the use of exclusive taxation powers that could make a bigger share of resources in the economy pledgeable, as suggested by (Holmström and Tirole 1998). That relies on the assumption that the government has some enforcement power, such as non-pecuniary penalties for tax evasion, which is not shared by other agents in the economy. Another set of policies that could be welfare improving have to do with changes the legal and corporate governance environments, which determine the shares of resources which are pledgeable or not in the economy. An increase in enforcement abilities of private agents induced by policy can lead to a Pareto improvement²².

1.6 Conclusion

The present chapter analyzed project choice in an environment with financial frictions, with particular emphasis on its aggregate consequences. Given unreliability of future funding, which stems from a limited commitment problem from lenders, real investment decisions and financial policies are intertwined. Arrangements for future investment have to be backed by transferable cash flows from projects (internal liquidity) or external assets that play a role as collateral (external liquidity). Therefore, shocks to these two influence the economy's fundamentals and create business cycles. As project selection is endogenous, in a general equilibrium environment, it interacts with asset prices and is a key determinant of the pattern of fluctuations of the aggregate economy.

In the environments studied, different forms of endogenous increase in exposure to risk can be

²²It is essential, however, to compensate the initial holders of external liquidity for a possible decrease in the rents they derived from the presence of premia on those assets.

driven by project selection. When there is severe scarcity of non-corporate assets which work as essential inputs for financial arrangements, endogenous project selection is biased towards riskier projects. Essentially, the economy is sufficiently constrained so that resources in the downside, or the worst states of nature, become less valuable than resources in situations in which constraints are less stringent but still present. The use of higher volatility emerges as a natural instrument for dealing with these constraints and transferring resources to where they are the most productive.

Alternatively, fluctuations might also originate from crunches in the value of these non-corporate assets. Under these conditions, there is not only a natural financial transmission mechanism that translates reductions in payouts of non-corporate assets into lower investment and output from corporations, but also an additional comovement effect that emerges in anticipation of these fluctuations, through project selection. The fundamental cause of this additional comovement is a complementarity between asset payouts that enable future investment and specific projects that deliver these investment opportunities.

All outcomes in the economies studied are constrained efficient. Project choice and financial policies respond adequately to the constraints faced by agents in the economy. There are no gains for a planner in distorting asset allocations, as prices properly reflect the assets' scarcity and their shadow value. Nor are there gains in targeting project choices or financial policies. Any gains from policy can only come from natural advantages in the creation of liquidity, in its contingent reallocation in ways that are not allowed by the original assets or in generating improvements in the fundamentals of the contracting environment.

An interesting extension would include the study of situations in which project selection cannot be perfectly contracted upon, so that financial policies and asset purchases play an additional role in providing incentives for project selection. A question to be addressed in that environment is whether asset prices would still properly reflect their value for society or whether an intervention through their reallocation or taxation could then lead to an improvement. Another interesting extension would include the study of overlapping projects, so that their liquidity demand could create spill-overs across the different vintages of investment. Whether constrained efficiency would survive in that environment is also an interesting open question.

1.7 Appendix

A. General Lemmas

Proof. (1.1) - If the problem admits a solution $\tilde{c}, \tilde{t}, \tilde{\sigma}$, it admits a solution $\hat{c}, \hat{t}, \hat{\sigma}$ in which entrepreneurial consumption at $t = 1$ and $t = 2$ is as low as possible (as much anticipation as possible is done), in which

$$\begin{aligned}\hat{c}_2(\omega) &= [\rho_1(\omega, \tilde{\gamma}) - \rho_0(\omega, \tilde{\gamma})] \tilde{x}(\omega) \tilde{I}, \\ \hat{c}_1(\omega) &= 0,\end{aligned}$$

It follows that

$$\rho_0(\omega, \tilde{\gamma}) \tilde{x}(\omega) \tilde{I} + z \cdot \tilde{a} = -\hat{t}_2(\omega)$$

and

$$\hat{t}_1(\omega) = \rho(\omega, \tilde{\gamma}) \tilde{x}(\omega) \tilde{I}.$$

It suffices to set $\hat{t}_0 = \tilde{t}_0 + E[\tilde{t}_2(\omega) + \tilde{t}_1(\omega) - \hat{t}_2(\omega) - \hat{t}_1(\omega)]$ and $\hat{c}_0 = \tilde{c}_0 + E[\tilde{c}_2(\omega) + \tilde{c}_1(\omega) - \hat{c}_2(\omega) - \hat{c}_1(\omega)]$. No constraint is violated and the same value for the objective function is achieved.

When we restrict attention to solutions with these properties, the problem can be simplified into the one of finding an optimal c_0 and σ , in which all consumption in other periods and transfers can be substituted away:

$$\max_{c_0 \geq 0, \sigma} c_0 + E[[\rho_1(\omega, \gamma) - \rho_0(\omega, \gamma)] x(\omega) I]$$

s.t.

$$E[\rho_0(\omega, \gamma) x(\omega) I - \rho(\omega, \gamma) x(\omega) I - [q - E(z)] \cdot a] - A - c_0 \geq -\underline{t},$$

$$z(\omega) \cdot \hat{a} I + \rho_0(\omega, \gamma) x(\omega) I \geq \rho(\omega, \gamma) x(\omega) I, \text{ for each } \omega \in \Omega$$

This problem can only admit a solution when the first constraint binds and has a shadow-value $\lambda^* \geq 1$. In both the $\lambda^* = 1$ and $\lambda^* > 1$ possible cases $c_0 = 0$ is part of an admissible solution. Therefore, σ needs to solve (1.11). Also, the constraint can be added to the objective function and simplifications carried out to write the problem as (1.8). □

Lemma 1.2. *Whenever $\rho_1(\omega, \gamma) > \rho_0(\omega, \gamma) > 0$, for all (ω, γ) , one can restrict attention to policies with full continuation, $x(\omega) = 1$, in the states in which financial distress does not occur, i.e., for all $\omega \in \Omega$ with $\rho(\omega, \gamma) < \rho_0(\omega, \gamma)$.*

Proof. Suppose σ^* is an optimal plan. For a contradiction, assume that in some state ω in which financial distress does not happen, $x_{\sigma^*}(\omega) < 1$. Then a plan σ' which coincides with σ^* , except for setting $x_{\sigma'}(\omega) = 1$, leads to a strictly greater value for the objective function, without violating any of the constraints. □

B. Proofs of results in Section 1.4.1

To simplify notation, let $x_g \equiv x(g, \rho)$ and $x_b \equiv x(b, \rho)$. Using Lemma 1.2, we restrict attention to strategies setting $x(i, 0) = 1$, for $i = g, b$.

Now, $E[B_1(\omega; q; \gamma, x, \hat{a})]$ can be written as

$$E[B_1(\omega; q; \gamma, x, \hat{a})] = \left\{ \rho_1(1 - \pi_\rho) + \pi_\rho \left[\pi_g \left(\rho_1 + \frac{\gamma}{\pi_g} \right) x_g + \pi_b \left(\rho_1 - \frac{\gamma}{\pi_b} \right) x_b \right] \right\} \\ - \{ \pi_\rho \rho (\pi_g x_g + \pi_b x_b) + \phi(\gamma) \} - (q - 1) \hat{a}$$

and $E[B_0(\omega; q; \gamma, x, a)]$ is defined analogously.

Substitution of the leverage constraint into the objective function and the definition of $\hat{a} \equiv \frac{a}{I}$ can transform the entrepreneurs problem in

$$\max_{\gamma, x, \hat{a}} \frac{\rho_1 - c(q; \gamma, x, \hat{a})}{c(q; \gamma, x, \hat{a}) - \rho_0} A \quad (1.30)$$

s.t.

$$\hat{a} + \left(\rho_0 + \frac{\gamma}{\pi_g} \right) x_g \geq \rho x_g \quad (1.31)$$

$$\hat{a} + \left(\rho_0 - \frac{\gamma}{\pi_b} \right) x_b \geq \rho x_b, \quad (1.32)$$

in which

$$c(q; \gamma, x, \hat{a}) \equiv \frac{\left\{ \phi(\gamma) + \pi_\rho \left[\pi_g x_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b x_b \left(\rho + \frac{\gamma}{\pi_b} \right) \right] \right\} + (q - 1) \hat{a}}{(1 - \pi_\rho) + \pi_\rho [\pi_g x_g + \pi_b x_b]} \quad (1.33)$$

is similar to an average cost function, where the equivalence of productivity shocks learned at $t = 1$ and refinancing shocks becomes clear.

Lemma 1.3. *The value function defined in*

$$c(q; \gamma) \equiv \min_{x, a'} c(q; \gamma, x, a') \\ \text{s.t.} \quad (1.31); (1.32)$$

for $q \geq 1$ is bounded from below by $c(q = 1; \gamma) = \phi(\gamma) + \pi_\rho \rho \geq \phi(\gamma_0) + \pi_\rho \rho$ and from above by $\frac{\phi(\gamma)}{1 - \pi_\rho}$.

Proof. Since $\hat{a} \geq 0$, $c(q; \gamma)$ is increasing and will have a minimum at $c(q = 1; \gamma)$. At that point $x_g = x_b = 1$ and \hat{a} set to satisfy (1.32) with equality are optimal, leading to $c(q = 1; \gamma) = \phi(\gamma) + \pi_\rho \rho \geq \phi(\gamma_0) + \pi_\rho \rho$. Finally, since $x_g = x_b = \hat{a} = 0$ is always feasible and leads to $c(q; \gamma, x, \hat{a}) = \frac{\phi(\gamma)}{1 - \pi_\rho}$, it follows that $c(q; \gamma) \leq \frac{\phi(\gamma)}{1 - \pi_\rho}$. □

Proof. (Proposition 1.1) We can solve the entrepreneur's problem by minimizing (1.33) subject to (1.31) and (1.32). The FOCs are

$$x_g : \frac{\pi_\rho \pi_g \left(\rho - \frac{\gamma}{\pi_g} \right)}{D^*} - c^*(q; \gamma) \frac{\pi_\rho \pi_g}{D^*} + \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) \mu_g \pi_g \begin{cases} \leq 0 & , \text{ if } x_g = 1 \\ = 0 & , \text{ if } x_g \in (0, 1) \\ \geq 0 & , \text{ if } x_g = 0 \end{cases} \quad (1.34)$$

$$x_b : \frac{\pi_\rho \pi_b \left(\rho + \frac{\gamma}{\pi_b} \right)}{D^*} - c^*(q; \gamma) \frac{\pi_\rho \pi_b}{D^*} + \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) \mu_b \pi_b \begin{cases} \leq 0 & , \text{ if } x_b = 1 \\ = 0 & , \text{ if } x_b \in (0, 1) \\ \geq 0 & , \text{ if } x_b = 0 \end{cases} \quad (1.35)$$

$$\hat{a} : \frac{(q-1)}{D^*} - \mu_g \pi_g - \mu_b \pi_b \begin{cases} = 0 & , \text{ if } \hat{a} > 0 \\ \geq 0 & , \text{ if } \hat{a} = 0 \end{cases} \quad (1.36)$$

where $\mu_g \pi_g$ and $\mu_b \pi_b$ are the multipliers on constraints (1.31) and (1.32) and D^* is the denominator of the average cost evaluated at the point studied. Additionally,

$$\left[\hat{a} - \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) x_g \right] \mu_g = 0 \quad (1.37)$$

$$\left[\hat{a} - \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) x_b \right] \mu_b = 0 \quad (1.38)$$

We study possible solutions through three mutually exclusive cases: x_b being 1, 0 or interior.

i) $x_b = 1$. If that is the case, Constraint (1.32) implies that constraint (1.31) is slack. Thus, $\mu_g = 0$. Therefore, given A2 and the previous lemma, the FOC for x_g holds with the "<" inequality, meaning that $x_g = 1$. Additionally,

$$q - 1 = D^* \mu_b \pi_b, \quad (1.39)$$

which implies that

$$\pi_\rho \pi_b \left(\rho + \frac{\gamma}{\pi_b} \right) - c^*(q; \gamma) \pi_\rho \pi_b + \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) (q - 1) \leq 0. \quad (1.40)$$

Assuming $x_g = x_b = 1$ is a solution also leads to $c^*(q; \gamma) = \phi(\gamma) + \pi_\rho \rho + (q - 1) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right)$. Therefore, expression 1.40 is violated for sufficiently large q .

ii) $x_b = 0$.

$\hat{a} > 0$ would imply a slack liquidity constraint (1.32) and $\mu_b = 0$. The combination of A2' and Lemma (1.3) implies that this cannot be the case or 1.35 would lead to a contradiction. Therefore, $\hat{a} = 0$, which also forces $x_g = 0$ from the liquidity constraint (1.31). Note that $x_g = x_b = 0$ as a solution leads to $c^*(q; \gamma) = \frac{\phi(\gamma)}{1 - \pi_\rho}$.

iii) x_b is interior.

$\hat{a} > 0$ from (1.32). $\mu_g > 0$ is necessary otherwise for the same reasons as before, 1.34 would lead to a contradiction.

From (1.35) and the usual combination of A2' and the lemma, $\mu_b > 0$. Therefore, (1.32) and (1.31) imply that $x_b = x_g \frac{\left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right)}{\left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right)}$.

Therefore there are 3 regimes to consider: full continuation ($x_g = x_b = 1$), full termination upon distress ($x_g = x_b = 0$) and some continuation with $\hat{a} = \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) x_g = \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) x_b$ and an interior x_b .

To better characterize thresholds of transitions between the three behaviors, it is useful to solve two auxiliary, restricted, optimization problems. Problem 1 assumes that both liquidity constraints bind as in

$$c_{x_g, \kappa x_g}^*(q; \gamma) \equiv \min_{x_g \in [0, 1]} \frac{\left\{ \phi(\gamma) + \pi_\rho \left[\pi_g x_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b x_g \kappa(\gamma) \left(\rho + \frac{\gamma}{\pi_b} \right) \right] \right\} + (q - 1) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) x_g}{(1 - \pi_\rho) + \pi_\rho \left[\pi_g x_g + \pi_b x_g \kappa(\gamma) \right]}, \quad (1.41)$$

with $\kappa(\gamma) \equiv \frac{(\rho - \rho_0 - \frac{\gamma}{\pi_g})}{(\rho - \rho_0 + \frac{\gamma}{\pi_b})}$ being such that $x_b = x_g \kappa(\gamma)$. The FOC for x_g is

$$\begin{aligned} \pi_\rho \left[\pi_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b \kappa(\gamma) \left(\rho + \frac{\gamma}{\pi_b} \right) \right] - \pi_\rho c_{x_g, \kappa x_g}^*(q; \gamma) [\pi_g + \pi_b \kappa(\gamma)] \\ + (q-1) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) \begin{cases} \leq 0 & , \text{if } x_g = 1 \\ = 0 & , \text{if } x_g \in [0, 1] \\ \geq 0 & , \text{if } x_g = 0 \end{cases} \end{aligned} \quad (1.42)$$

Notice that whenever $x_g = 0$ is a solution, $c_{x_g, \kappa x_g}^*(q; \gamma) = \frac{\phi(\gamma)}{1 - \pi_\rho}$. Therefore, by substituting in that value, a threshold where indifference between following $(x_g, x_b) = (1, \kappa(\gamma))$ and full termination occurs is defined implicitly, as in

$$\begin{aligned} \pi_\rho \left[\pi_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b \kappa(\gamma) \left(\rho + \frac{\gamma}{\pi_b} \right) \right] + (\bar{q}(\gamma) - 1) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) \\ = \pi_\rho [\pi_g + \pi_b \kappa(\gamma)] \frac{\phi(\gamma)}{1 - \pi_\rho}. \end{aligned} \quad (1.43)$$

For any $q < \bar{q}(\gamma)$, $c_{1, \kappa}^*(q; \gamma) < \frac{\phi(\gamma)}{1 - \pi_\rho}$ and $(x_g, x_b) = (1, \kappa(\gamma))$ dominates both full termination and any other policy involving an interior x_b .

Problem 2 assumes that $x_g = 1$ and optimizes on x_b , assuming that the liquidity constraint only binds in the bad state. It is written as

$$c_{1, x_b}^*(q; \gamma) \equiv \min_{x_b \in [0, 1]} \frac{\left\{ \phi(\gamma) + \pi_\rho \left[\pi_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b x_b \left(\rho + \frac{\gamma}{\pi_b} \right) \right] \right\} + (q-1) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) x_b}{(1 - \pi_\rho) + \pi_\rho [\pi_g + \pi_b x_b]}$$

and has a FOC given by

$$\pi_\rho \pi_b \left(\rho + \frac{\gamma}{\pi_b} \right) - \pi_\rho \pi_b c_{1, x_b}^*(q; \gamma) + (q-1) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) \begin{cases} \leq 0 & , \text{if } x_b = 1 \\ = 0 & , \text{if } x_b \in [0, 1] \\ \geq 0 & , \text{if } x_b = 0 \end{cases}$$

Let $c_{11}(q; \gamma) = \phi(\gamma) + \pi_\rho \rho + (q-1) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right)$, be the cost associated to full continuation. An interior solution for x_b will be admissible on Problem 2 iff

$$c_{1, x_b}^*(q; \gamma) = c_{11}(q; \gamma) = \left(\rho + \frac{\gamma}{\pi_b} \right) + (\pi_\rho \pi_b)^{-1} (q-1) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right).$$

This defines implicitly another threshold as $\underline{q}(\gamma)$ to the left of which full continuation is preferred to any strategy with $x_g = 1$ and interior x_b . At this threshold, any policy with partial continuation in x_b is equivalent. In particular, $(x_g, x_b) = (1, 1)$ and $(x_g, x_b) = (1, \kappa(\gamma))$ lead to the same value for the objective function. To the right of this threshold, $x_b = 1$ cannot happen in the solution and regime (i) is not possible.

Finally, notice that $c_{11}(q; \gamma)$ crosses the cost of termination upon distress $\frac{\phi(\gamma)}{1 - \pi_\rho}$ at the point where

$$c_{11}(\bar{q}(\gamma); \gamma) = \frac{\phi(\gamma)}{1 - \pi_\rho} = \rho + (\pi_\rho)^{-1} (\bar{q}(\gamma) - 1) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right).$$

$c_{11}(q; \gamma) < \frac{\phi(\gamma)}{1-\pi_\rho}$, if q is less than the value $\bar{q}(\gamma)$ implicitly defined above. It is then easily verified by comparing their two implicit definitions that $\underline{q}(\gamma) < \bar{q}(\gamma)$ and it follows that at $c_{11}(\underline{q}(\gamma); \gamma) = c_{1,x_b}^*(\underline{q}(\gamma); \gamma) < \frac{\phi(\gamma)}{1-\pi_\rho}$, which proves that $\underline{q}(\gamma) < \bar{q}(\gamma)$. \square

Proof. Proposition 1.2:

For a fixed project, the last proposition has shown that at least one of $(x_g, x_b) = (1, 1)$, $(x_g, x_b) = (1, \kappa(\gamma))$ or $(x_g, x_b) = (0, 0)$ solves the optimization problem, with the first one being strictly preferred for a low price range, the second for an intermediate price range and the third for high prices.

One can therefore solve the entrepreneur's problem in two stages. One optimizes on the project for each of these three continuation policies of interest. Then, one optimizes over the three policies.

Let us define cost functions for fixed policies and projects as

$$\begin{aligned} c_{11}(q; \gamma) &\equiv \phi(\gamma) + \pi_\rho \rho + (q-1) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) \\ c_{1\kappa}(q; \gamma) &\equiv \frac{\left\{ \phi(\gamma) + \pi_\rho \left[\pi_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b \kappa(\gamma) \left(\rho + \frac{\gamma}{\pi_b} \right) \right] \right\} + (q-1) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right)}{(1-\pi_\rho) + \pi_\rho [\pi_g + \pi_b \kappa(\gamma)]} \\ c_{00}(q; \gamma) &\equiv \frac{\phi(\gamma)}{1-\pi_\rho} \end{aligned}$$

Analogously, we define the value functions of the first stage of this optimization, which minimizes cost for a fixed continuation policy, as in

$$\begin{aligned} c_j(q) &\equiv \min_{\gamma} c_j(q; \gamma), \\ &\text{for } j = 11, 1\kappa, 00 \end{aligned}$$

First, notice that $c_{00}(q) = \frac{\phi(\gamma_0)}{1-\pi_\rho}$, as the problem is trivially solved at the lowest cost project γ_0 .

Additionally, $c_{11}(q) \geq \phi(\gamma_0) + \pi_\rho + (q-1)(\rho - \rho_0)$, which is the cost of full continuation if shocks were not present in the most efficient project. This crosses $c_{00}(q) = \frac{\phi(\gamma_0)}{1-\pi_\rho}$ at \hat{q} implicitly defined in

$$\underbrace{\rho}_{\rho} + \frac{(\hat{q}-1)}{\pi_\rho} \underbrace{(\rho - \rho_0)}_{\rho - \rho_0} = \frac{\phi(\gamma_0)}{1-\pi_\rho}. \quad (1.44)$$

As a consequence $c_{11}(q) > c_{00}(q)$ for all $q > \hat{q}$.

Now, $c_{1\kappa}(q) \leq c_{1\kappa}(q; \gamma_0)$. $c_{1\kappa}(q; \gamma_0)$ crosses $c_{00}(q)$ at $\bar{q}(\gamma_0)$ implicitly defined in

$$\underbrace{\frac{\left[\pi_g \left(\rho - \frac{\gamma_0}{\pi_g} \right) + \pi_b \kappa(\gamma_0) \left(\rho + \frac{\gamma_0}{\pi_b} \right) \right]}{[\pi_g + \pi_b \kappa(\gamma_0)]}}_{\text{weighted average}} + \frac{1}{\pi_\rho} (\bar{q}(\gamma_0) - 1) \underbrace{\frac{\left(\rho - \rho_0 - \frac{\gamma_0}{\pi_g} \right)}{[\pi_g + \pi_b \kappa(\gamma_0)]}}_{\text{weighted harmonic mean}} = \frac{\phi(\gamma_0)}{1-\pi_\rho}. \quad (1.45)$$

As a consequence, $c_{1\kappa}(q) < c_{00}(q)$ for all $q < \bar{q}(\gamma_0)$.

We compare the terms with braces in equation (1.45) versus equation (1.44). The first term in braces in (1.45) is a weighted average of terms with mean ρ , with a higher weight put in the lower term. Therefore, it is less than ρ . The second term in braces can be rearranged to make clear that it is the probability weighted harmonic mean of $\left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right)$ and $\left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right)$ which is less than the expectation $(\rho - \rho_0)$. As a consequence, $\hat{q} < \bar{q}(\gamma_0)$. This proves that for some intermediate range of prices, $c_{1\kappa}(q) < c_{11}(q)$.

From the Proposition 1.1, at $q = 1$, for any project, full continuation dominates $(1, \kappa)$ and $(0, 0)$. Therefore, $c_{11}(q) < c_{1\kappa}(q)$ and $c_{11}(q) < c_{00}(q)$ for q sufficiently close to 1.

It is also possible to show that crossings occur only once, as using an Envelope Theorem and the inequality involving the arithmetic and harmonic means,

$$\begin{aligned} c'_{11}(q) &= \left(\rho - \rho_0 + \frac{\gamma_{11}^*}{\pi_b} \right) > \rho - \rho_0 > 0 \\ c'_{1\kappa}(q) &= \frac{\left(\rho - \rho_0 - \frac{\gamma_{1\kappa}^*}{\pi_g} \right)}{(1 - \pi_\rho) + \pi_\rho [\pi_g + \pi_b \kappa (\gamma_{1\kappa}^*)]} \\ &\in (0, \rho - \rho_0) \\ c'_{00}(q) &= 0. \end{aligned}$$

As a consequence, there exist two thresholds \bar{q} and \underline{q} , delimiting areas of optimality for the three continuation policies.

Next, we prove the behavior of the optimal policy regarding γ in each of the three regions.

i) For the region with $x_g = x_b = 1$, $\gamma_{11}^*(q)$ satisfies the first-order condition $\phi'(\gamma_{11}^*) = -\frac{(q-1)}{\pi_b}$. Therefore, $\gamma_{11}^*(q) \leq \gamma_0$ and, given convexity of ϕ , it is an decreasing function of q .

ii) For the region with $x_g = 1$ and $x_b = \frac{(\rho - \rho_0 - \frac{\gamma}{\pi_g})}{(\rho - \rho_0 + \frac{\gamma}{\pi_b})}$, it can be shown that $\frac{\partial^2 c_{1\kappa}}{\partial \gamma \partial q} < 0$, indicating that the solution $\gamma_{1\kappa}^*(q)$ it a increasing function of q .

iii) For the region with full termination, it has already been argued that the optimal choice for is $\gamma_{00}^*(q) = \gamma_0$.

Last, it is necessary to show that for sufficiently high q between the \underline{q} and \bar{q} , amplification is optimal. Notice that at $\bar{q}(\gamma_0)$ this will be the case, as

$$c_{1\kappa}(\bar{q}(\gamma_0); \gamma_0) = c_{00}(\bar{q}(\gamma_0); \gamma_0).$$

However,²³

$$\frac{\partial c_{1\kappa}(\bar{q}(\gamma_0); \gamma)}{\partial \gamma} \Big|_{\gamma_0} < 0,$$

which implies that once project choice is incorporated, $c_{1\kappa}(\bar{q}(\gamma_0)) < c_{00}(\bar{q}(\gamma_0))$.

Finally, I show that at $\bar{q}(\gamma_0)$, all projects $\gamma < \gamma_0$ are dominated by the choice of γ_0 and termination upon distress. For that purpose, note that

$$c_{1\kappa}(\bar{q}(\gamma_0); \gamma) > \frac{\left\{ \phi(\gamma_0) + \pi_\rho \left[\pi_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b \kappa(\gamma) \left(\rho + \frac{\gamma}{\pi_b} \right) \right] \right\} + (\bar{q}(\gamma_0) - 1) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right)}{(1 - \pi_\rho) + \pi_\rho [\pi_g + \pi_b \kappa(\gamma)]},$$

since $\phi(\gamma) > \phi(\gamma_0)$. The term on the right-hand side is at least as great as $c_{00}(\bar{q}(\gamma_0); \gamma_0)$ iff

$$\frac{\left[\pi_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b \kappa(\gamma) \left(\rho + \frac{\gamma}{\pi_b} \right) \right]}{[\pi_g + \pi_b \kappa(\gamma)]} + \frac{1}{\pi_\rho} (\bar{q}(\gamma_0) - 1) \frac{\left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right)}{[\pi_g + \pi_b \kappa(\gamma)]} \geq \frac{\phi(\gamma_0)}{1 - \pi_\rho}.$$

²³Writing $c_{1\kappa}(\bar{q}(\gamma_0); \gamma_0) = \frac{\left\{ \phi(\gamma_0) \left(\rho - \rho_0 - \frac{\gamma_0}{\pi_g} \right)^{-1} + \pi_\rho \left[\pi_g \left(\rho - \frac{\gamma_0}{\pi_g} \right) \left(\rho - \rho_0 - \frac{\gamma_0}{\pi_g} \right)^{-1} + \pi_b \left(\rho + \frac{\gamma_0}{\pi_b} \right) \left(\rho - \rho_0 + \frac{\gamma_0}{\pi_b} \right)^{-1} \right] \right\} + (q-1)}{(1 - \pi_\rho) \left(\rho - \rho_0 - \frac{\gamma_0}{\pi_g} \right)^{-1} + \pi_\rho \left[\pi_g \left(\rho - \rho_0 - \frac{\gamma_0}{\pi_g} \right)^{-1} + \pi_b \left(\rho - \rho_0 + \frac{\gamma_0}{\pi_b} \right)^{-1} \right]}$

and using $c_{1\kappa}(\bar{q}(\gamma_0); \gamma_0) = c_{00}(\bar{q}(\gamma_0); \gamma_0)$ is helpful for showing this inequality.

The LHS is decreasing in γ and equality is reached at $\gamma = \gamma_0$, from the definition of $(\bar{q}(\gamma_0) - 1)$. Therefore, $c_{1\kappa}(\bar{q}(\gamma_0); \gamma) > c_{00}(\bar{q}(\gamma_0); \gamma_0)$ and no policy involving dampening of fluctuations can be optimal. As a consequence, the optimal project choice is given by some $\gamma_{1p}^*(\bar{q}(\gamma_0)) > \gamma_0$. From monotonicity of $\gamma_{1p}^*(q)$, this will also hold for higher levels of q . □

C. Proofs of Results from Section 1.4.2

Proofs and Equilibrium Characterization for Section 1.4.2.1

A note on notation: I will make use of the Lemma 1.2 and restrict attention to $x(\omega)$ only for the financial distress states. To simplify notation, x_u and x_d will denote $x(u, \rho)$ and $x(d, \rho)$.

Given assumptions A1-A2, constraint 1.9 will always bind. Otherwise, no solution would exist, as the surplus of a policy of continuation if and only if financial distress does not occur goes to infinity as leverage goes to infinity. Therefore, one can substitute the constraint into the objective function to write it as $-\frac{E[B_1(\omega; q; \gamma, x, a)]}{E[B_0(\omega; q; \gamma, x, a)]}A$.

One should also note that with liquidity premia in place, over-hoarding of liquidity is dominated, i.e., entrepreneurs need only to hoard enough state-contingent liquidity to set the liquidity constraints (1.10) to hold with equality. Liquidity is only valuable as long as it is useful for enabling a decision $x(\omega)$. Therefore, one can restrict attention to

$$a_i(x, I) = \frac{(\rho - \rho_0)x(i, \tau)I}{z_i}.$$

Thus, problem 1.27 can be re-written, as

$$\max_x \frac{\rho_1 - c(q; x)}{c(q; x) - \rho_0} A,$$

where

$$c(q; x_u, x_d) \equiv \frac{1 + \pi_\rho \rho [\pi_u x_u + \pi_d x_d] + \sum_{i=u,d} (q_i - \pi_i z_i) \frac{(\rho - \rho_0)x_i}{z_i}}{(1 - \pi_\rho) + \pi_\rho [\pi_u x_u + \pi_d x_d]} \quad (1.46)$$

represents an average cost per project unit completed. Given that each unit completed generates the same social surplus and there are constant returns to scale, the entrepreneurs' problem can be written in terms of minimizing this average cost function.

Let the value function of the cost minimization problem be written as

$$c^*(q) \equiv \min_{0 \leq x_u, x_d \leq 1} c(q; x_u, x_d), \quad (1.47)$$

and notice that for its partial derivatives

$$\frac{\partial c}{\partial x_i} \propto \pi_\rho \rho + \left(\frac{q_i}{\pi_i z_i} - 1 \right) (\rho - \rho_0) - \pi_\rho c^*(q). \quad (1.48)$$

The solution to the individual problem can be represented in terms of 4 regions, as seen in Figure 1-3. We first demonstrate the propositions below.

Lemma 1.4. *Let q_0 be the price vector free of liquidity premia, in which $q_i = \pi_i z_i$, for $i = u, d$. At this level, a policy of full continuation leads to the lowest possible equilibrium level of $c(q; x)$, $c^*(q_0) = 1 + \pi_\rho \rho$.*

Proof. Notice that A1 implies that,

$$c(q_0, 0, 0) = \frac{1}{1 - \pi_\rho} \geq c(q_0; x) \geq c(q_0; 1, 1) = 1 + \pi_\rho \rho,$$

where inequalities are strict for any x with an interior x_i component. Since $c(q, x)$ is increasing in q , and equilibrium requires $q \geq q_0$, this is the lowest value that $c(q, x)$ can achieve. \square

Lemma 1.5. *The cost function is bounded by $\frac{1}{1 - \pi_\rho}$, i.e., $c^*(q) \leq \frac{1}{1 - \pi_\rho}$. And for sufficiently high q , in a vector sense, a policy of full termination in case of financial distress is optimal.*

Proof. For a policy that leads to full termination in case of distress, $c(q; 0, 0) = \frac{1}{1 - \pi_\rho}$. Since that policy is always feasible, the first part of the lemma follows. For the second part, notice that the first-order relation 1.48 has a single negative component, the third one, which is bounded at $c^*(q) = \frac{1}{1 - \pi_\rho}$. Since the second term grows unbounded in q_i , for sufficiently high liquidity premia, the sign of the expression becomes positive, meaning that it is optimal to set the continuation shares $x(\omega, \rho)$ to the corner level of 0. \square

By continuity, for sufficiently low liquidity premia, a policy of full continuation is optimal. We proceed to determine thresholds where partial or total continuation becomes optimal.

Lemma 1.6. *Indifference between a policies $(x_u, x_d) = (1, 1)$ and $(x_u, x_d) = (1, 0)$ is given by a straight line in the liquidity premium, $\left(\frac{q_u}{\pi_u z_u}, \frac{q_d}{\pi_d z_d}\right)$ -space, with a slope lower than one. Above that line termination in $(x_u, x_d) = (1, 0)$ is preferred to $(x_u, x_d) = (1, 1)$ and the opposite is true below it.*

Indifference between full continuation and full termination in (u, ρ) is given by another line, with a slope above 1. To the right of it, $(x_u, x_d) = (1, 0)$ is preferred to $(x_u, x_d) = (1, 1)$ and the opposite is true for $\left(\frac{q_u}{\pi_u z_u}, \frac{q_d}{\pi_d z_d}\right)$ combinations that lie to the left of it.

These two indifference loci cross over the 45° line, at the point where $\pi_\rho \rho + \left(\frac{q_i}{\pi_i z_i} - 1\right)(\rho - \rho_0) - \pi_\rho \frac{1}{1 - \pi_\rho} = 0$, for $i = u, d$.

Proof. The first indifference is reached in the locus in which

$$c(q, 1, 1) = c(q, 1, 0).$$

That implies

$$\begin{aligned} 1 + \pi_\rho \rho + \sum_{i=u,d} \pi_i \left(\frac{q_i}{\pi_i z_i} - 1\right) (\rho - \rho_0) &= \rho + \left(\frac{q_d}{\pi_d z_d} - 1\right) \frac{(\rho - \rho_0)}{\pi_\rho} \\ 1 - (1 - \pi_\rho) \rho + \pi_u \left(\frac{q_u}{\pi_u z_u} - 1\right) (\rho - \rho_0) &= \left(\frac{1 - \pi_\rho \pi_d}{\pi_\rho}\right) \left(\frac{q_d}{\pi_d z_d} - 1\right) (\rho - \rho_0). \end{aligned}$$

Which means that indifference is a locus of the form

$$\left(\frac{q_d}{\pi_d z_d} - 1\right) = a + b \left(\frac{q_u}{\pi_u z_u} - 1\right),$$

with $a \equiv \frac{1-(1-\pi_\rho)\rho}{(\rho-\rho_0)} \left(\frac{1-\pi_\rho\pi_d}{\pi_\rho} \right)^{-1} > 0$ and $b \equiv \frac{\pi_\rho\pi_u}{1-\pi_\rho\pi_d} = \frac{\pi_\rho\pi_u}{(1-\pi_\rho)+\pi_\rho\pi_u} < 1$. It crosses the 45-degree line at

$$\begin{aligned} \left(\frac{q_d}{\pi_d z_d} - 1 \right) &= \left(\frac{q_u}{\pi_u z_u} - 1 \right) = \frac{a}{1-b} \\ &= \frac{1-(1-\pi_\rho)\rho}{(\rho-\rho_0)} \frac{\pi_\rho}{1-\pi_\rho}. \end{aligned}$$

Since $c(q, 1, 1)$ is strictly increasing in $\left(\frac{q_d}{\pi_d z_d} \right)$ while $c(q, 0, 0)$ does not depend on it, this locus divides the space in regions where $c(q, 1, 1) < c(q, 0, 0)$, which lies below it, and where the opposite is true, which lies above it.

An analogous procedure leads to the locus of indifference between termination in (u, ρ) , given full continuation in (d, ρ) being described by

$$\left(\frac{q_u}{\pi_u z_u} - 1 \right) = a' + b' \left(\frac{q_d}{\pi_d z_d} - 1 \right),$$

where $b' \equiv \frac{\pi_\rho\pi_d}{(1-\pi_\rho)+\pi_\rho\pi_d} < 1$ and $a' \equiv \frac{1-(1-\pi_\rho)\rho}{(\rho-\rho_0)} \left(\frac{1-\pi_\rho\pi_u}{\pi_\rho} \right)^{-1}$. It crosses the 45° line at the same point, which is their single intersection. In an analogous way to the first part, this locus also divides the space in a region of dominance of $(0, 1)$ over $(0, 0)$, to the right of it, and the opposite to its left. □

Lemma 1.7. $\frac{q_u}{\pi_u z_u} \leq \bar{q}$, for \bar{q} implicitly defined in $\pi_\rho\rho + (\bar{q} - 1)(\rho - \rho_0) - \pi_\rho \frac{1}{1-\pi_\rho} = 0$, is necessary and sufficient for $c(q; 1, 0) \leq c(q; 0, 0)$. Analogously, $\frac{q_d}{\pi_d z_d} \leq \bar{q}$ is necessary and sufficient for $c(q; 0, 1) \leq c(q; 0, 0)$.

Proof. Indeed, $c(q; 1, 0) \leq c(q; 0, 0) \iff \rho + \left(\frac{q_u}{\pi_u z_u} - 1 \right) \frac{(\rho-\rho_0)}{\pi_\rho} \leq \frac{1}{1-\pi_\rho} \iff \frac{q_u}{\pi_u z_u} \leq \bar{q}$. □

Characterization of optimal continuation decisions

Notice also that given linearity of the entrepreneurs' problem, at the thresholds of indifference between the four policies in the corners of the continuation possibilities square²⁴, any convex combination of these extreme optimal policies is also optimal. The combination of the lemmas above, justifies the characterization of the entrepreneurs' optimal continuation choice as functions of liquidity premia which is represented in Figure 1-3 in the main text.

Characterization of Equilibria

Proposition 1.7. *The following four statements hold:*

1. Equilibrium prices cannot lie at any point $\left(\frac{q_u}{\pi_u z_u}, \frac{q_d}{\pi_d z_d} \right)$ where $(x_u, x_d) = (1, 1)$ is not in the set of optimal policies.
2. If in region where $(1, 1)$ is the unique optimal policy, there cannot be a liquidity premium on a_u , i.e., equilibrium $\frac{q_u}{\pi_u z_u} = 1$.
3. Equilibrium prices cannot lie in the segment where $(1, 1)$ and $(0, 1)$ are the only optimal policies.

²⁴ $(x_u, x_d) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$

4. Therefore, equilibria can only lie in loci (i)-(v) as depicted in Figure 1-4.

Proof. (1) Equilibria cannot lie in the interior of dominance regions of (0,0), (0,1) and (1,0) as aggregate entrepreneurial demand for an asset carrying a liquidity premium would be zero, which contradicts market clearing. The same reasoning holds for the segments of indifference between (0,0) and (1,0) and between (0,0) and (0,1).

(2) If in this region, aggregate entrepreneurial demand for a_u is $\frac{(\rho-\rho_0)I^*}{z_u}$ and for a_d is $\frac{(\rho-\rho_0)I^*}{z_d}$. Market clearing in a_d requires

$$\frac{(\rho-\rho_0)I^*}{z_d} \leq L,$$

which given $z_u > z_d$ forces market clearing in a_u to be satisfied with a strict inequality, implying that $\frac{q_u}{\pi_u z_u} = 1$.

(3) If this were the case aggregate demand for a_u would be strictly less than aggregate demand for asset a_d . Since supply of both assets is given by L , this would again imply that $\frac{q_u}{\pi_u z_u} = 1$, which is a contradiction of the necessary indifference condition. □

Proofs of results from 1.4.2.2

Again we make use of Lemma 1.2 to restrict attention to policies that always lead to full continuation when the projects are self-financing. To make notation less cumbersome, we will define $n_u(\gamma)$ and $n_d(\gamma)$ for the number of investment opportunities offered by project γ in the states of nature involving u and d payout from the tree.

In a similar manner as in the previous section, it is possible to rewrite the entrepreneur's problem in an average-cost-minimization form. First, one writes the minimum level of asset purchases as function of $\{I, \gamma, x_u, x_d\}$. Therefore,

$$a_i = (\rho - \rho_0) x_i n(\omega, \gamma) I \tag{1.49}$$

can be used to write the entrepreneur's problem as

$$\min_{\gamma; 0 \leq x_u, x_d \leq 1} c(q; \gamma, x_u, x_d) \tag{1.50}$$

where

$$c(q; \gamma, x_u, x_d) \equiv \frac{\phi(\gamma) + \pi_\rho \rho [\pi_u x_u n_u(\gamma) + \pi_d x_d n_d(\gamma)] + \sum_{i=u,d} (q_i - \pi_i z_i) \frac{(\rho-\rho_0)x_i n_i(\gamma)}{z_i}}{1 - \pi_\rho + \pi_\rho [\pi_u x_u n_u(\gamma) + \pi_d x_d n_d(\gamma)]}. \tag{1.51}$$

We can define a change of variables to make the cost minimization problem 1.50 even more in line with Problem 1.47, which was extensively analyzed in the previous section. We set

$$\tilde{x}_u \equiv x_u n_u(\gamma)$$

and

$$\tilde{x}_d = x_d n_d(\gamma)$$

as the relevant choice variables, so that problem 1.50 for a fixed project becomes

$$c(q; \gamma) \equiv \min_{\tilde{x}_u, \tilde{x}_d} \frac{\phi(\gamma) + \pi_\rho \rho [\pi_u \tilde{x}_u + \pi_d \tilde{x}_d] + \sum_{i=u,d} (q_i - \pi_i z_i) \frac{(\rho - \rho_0) \tilde{x}_i}{z_i}}{1 - \pi_\rho + \pi_\rho [\pi_u \tilde{x}_u + \pi_d \tilde{x}_d]}$$

s.t.

$$\begin{aligned} 0 &\leq \tilde{x}_u \leq n_u(\gamma), \\ 0 &\leq \tilde{x}_d \leq n_d(\gamma). \end{aligned}$$

Proof. Proposition 1.4:

Using the change of variables suggested above, one can write the entrepreneurs' problem as

$$\max_{\tilde{x}, \gamma} E[B_1(q; \tilde{x}, \gamma)] I \tag{1.52}$$

s.t.

$$A + E[B_0(q; \tilde{x}, \gamma)] I \geq 0$$

$$\begin{aligned} n_u(\gamma) &\geq \tilde{x}_u \geq 0 \\ n_d(\gamma) &\geq \tilde{x}_d \geq 0. \end{aligned}$$

Here

$$\begin{aligned} E[B_1(q; \tilde{x}, \gamma)] &= \rho_1 [1 - \pi_\rho + \pi_\rho (\pi_u \tilde{x}_u + \pi_b \tilde{x}_d)] \\ &\quad - \phi(\gamma) - \pi_\rho \rho (\pi_u \tilde{x}_u + \pi_b \tilde{x}_d) - \sum_{i=u,d} (q_i - \pi_i z_i) \frac{(\rho - \rho_0) \tilde{x}_i}{z_i} \end{aligned}$$

and analogously for $E[B_0(q; \tilde{x}, \gamma)]$ with ρ_0 replacing ρ_1 .

Let μ be the multiplier associated to the leverage constraint and μ_i , for $i = u, d$, be the multipliers associated to the constraints of the form $n_i(\gamma) \geq \tilde{x}_i$. Then, the first-order condition on \tilde{x}_i is given by

$$\frac{\partial E[B_1]}{\partial x_i} + \mu \frac{\partial E[B_0]}{\partial x_i} - \mu_i \leq 0, \text{ with } = \text{ for } x_i > 0,$$

where $\frac{\partial E[B_1]}{\partial x_i} + \mu \frac{\partial E[B_0]}{\partial x_i} = \pi_\rho \pi_i (\rho_1 + \mu \rho_0) - (1 + \mu) \left(\rho \pi_\rho \pi_i + \pi_i \left(\frac{q_i}{\pi_i z_i} - 1 \right) (\rho - \rho_0) \right)$.

Optimization on γ is associated with

$$\frac{\partial E[B_1]}{\partial \gamma} + \mu \frac{\partial E[B_0]}{\partial \gamma} - \mu_u n'_u - \mu_d n'_d \begin{cases} \geq 0 & , \text{ if } \gamma = \bar{\gamma} \\ = 0 & , \text{ if } \gamma \in (\underline{\gamma}, \bar{\gamma}), \\ \leq 0 & , \text{ if } \gamma = \underline{\gamma} \end{cases}$$

where the left-hand side becomes

$$-(1 + \mu) \phi'(\gamma) - \frac{\mu_u}{\pi_u} + \frac{\mu_d}{\pi_d}.$$

Note that

$$\frac{q_i}{\pi_i z_i} < \frac{q_{-i}}{\pi_{-i} z_{-i}} \implies \frac{\mu_i}{\pi_i} > \frac{\mu_{-i}}{\pi_{-i}}.$$

Therefore,

$$\begin{aligned} \frac{q_u}{\pi_u z_u} &< \frac{q_d}{\pi_d z_d} \implies \phi'(\gamma) > 0 \\ \implies \gamma &> 0. \end{aligned}$$

Uniqueness in project choice follows from the observation that, even with multiple solutions for the entrepreneurs' problem, constant returns to scale implies that each one of the solutions achieves the same optimal value $-\frac{E[B_1(q; \bar{x}, \gamma)]}{E[B_0(q; \bar{x}, \gamma)]} A = \mu A$. As such, they also have to share all μ_i and the same unique solution to the project choice problem.

Last, we rule out $\frac{q_u}{\pi_u z_u} > \frac{q_d}{\pi_d z_d}$ as this would generate $n_d > n_u$ for every individual entrepreneur and $\tilde{x}_d > \tilde{x}_u$. Since $Lz_u > Lz_d$, this is not compatible with asset market equilibrium. \square

Proofs of Section 1.4.3

As before, we simplify notation by using Lemma 2 and restriction attention to the characterization of $x_g \equiv x(g, \rho)$ and $x_b \equiv x_b(b, \rho)$. Given the presence of the two assets described in the text, we can restrict attention to strategies setting

$$a_g = \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) x_g I$$

and

$$a_b = \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) x_b I,$$

since asset purchases are only valuable as long as they help relax a liquidity constraint in one the states. Again, we might work with the minimization of modified average cost functions of the form

$$c(q; x, \gamma) \equiv \frac{\phi(\gamma) + \pi_\rho \left[\pi_g x_g \left(\rho - \frac{\gamma}{\pi_g} \right) + \pi_b x_b \left(\rho + \frac{\gamma}{\pi_b} \right) \right] + (q_g - \pi_g) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) x_g + (q_b - \pi_b) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) x_b}{(1 - \pi_\rho) + \pi_\rho [\pi_g x_g + \pi_b x_b]} \quad (1.53)$$

or with a formulation as

$$\max_{I, x(\omega), \gamma} E[B_1(\omega; q; \gamma, x, \hat{a}(x))] I \quad (1.54)$$

s.t.

$$A + E[B_0(\omega; q; \gamma, \hat{a}(x))] I \geq 0.$$

Lemma 1.8. *With non-degenerate project choice, no optimal investment plan will ever feature an interior solution for continuation shares x_g or x_b .*

Proof. Suppose some optimal σ^* features an interior x_i^* for i equal to g or b . Then, $\sigma'_{x_i=1}$ and $\sigma'_{x_i=0}$ which coincide with σ^* except for, respectively, setting x_i to 1 or 0 will lead to the same value for the objective function as the original optimal strategy σ^* . Common optimality and constant returns to scale would

mean that the Lagrangian associated to Program 1.54, $\mathcal{L}(q; \sigma)$ would feature the same multiplier μ^* for the leverage constraint for $\sigma = \sigma^*, \sigma'_{x_i=1}, \sigma'_{x_i=0}$. However, either $\mathcal{L}_\gamma(q; \sigma'_{x_i=1}) > \mathcal{L}_\gamma(q; \sigma^*) > \mathcal{L}_\gamma(q; \sigma'_{x_i=0})$ or $\mathcal{L}_\gamma(q; \sigma'_{x_i=1}) < \mathcal{L}_\gamma(q; \sigma^*) < \mathcal{L}_\gamma(q; \sigma'_{x_i=0})$ hold showing that the use of one of the corner x_i and an associated re-optimization over the project γ can lead to an improvement in Program 1.54 contradicting the optimality of σ^* . □

Proof. (Proposition 1.5) First, we show that for some sufficiently high prices on the state contingent assets, entrepreneurs would prefer to specialize in two projects, one leading to amplification and another one to dampening, and to be fully exposed to the risks of financial distress in one of the productivity states, while fully insured in the other one.

A equilibrium with fully specialized firms requires that for

$$\begin{aligned} c_{10}(q_g, \gamma) &\equiv \frac{\phi(\gamma) + \pi_\rho \pi_g \left(\rho - \frac{\gamma}{\pi_g} \right) + (q_g - \pi_g) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right)}{(1 - \pi_\rho) + \pi_\rho \pi_g} \\ c_{01}(q_b, \gamma) &\equiv \frac{\phi(\gamma) + \pi_\rho \pi_b \left(\rho + \frac{\gamma}{\pi_b} \right) + (q_b - \pi_b) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right)}{(1 - \pi_\rho) + \pi_\rho \pi_b} \\ c_{11}(q, \gamma) &\equiv \{ \phi(\gamma) + \pi_\rho \rho \} + (q_g - \pi_g) \left(\rho - \rho_0 - \frac{\gamma}{\pi_g} \right) + (q_b - \pi_b) \left(\rho - \rho_0 + \frac{\gamma}{\pi_b} \right) \\ c_{00}(q; \gamma) &\equiv \frac{\phi(\gamma)}{1 - \pi_\rho} \end{aligned}$$

$$\min_\gamma c_{10}(q_g, \gamma) = \min_\gamma c_{01}(q_b, \gamma) \leq \min_\gamma c_{11}(q, \gamma), \min_\gamma c_{00}(q; \gamma). \quad (1.55)$$

Additionally, let

$$\gamma_i(q) \equiv \operatorname{argmin}_\gamma c_i(q; \gamma), \text{ for } i \in \{11, 01, 10, 00\}.$$

Notice that $\gamma_{00}(q) = \gamma_0$.

I show that there exist prices that satisfy condition 1.55.

Let $c_i(q) \equiv \min_\gamma c_i(q, \gamma)$. Notice that

$$\frac{\partial c_{10}}{\partial q_g}, \frac{\partial c_{01}}{\partial q_b} > 0, \quad (1.56)$$

since no project is ever self-financing in a financial distress state. Then,

$$c_{10}(q_g) = c_{01}(q_b) \quad (1.57)$$

defines a path in (q_g, q_b) -space which is strictly increasing. That represents the locus of asset prices that would lead to indifference between policies that involve to full continuation and full termination in opposing states of the world. Given 1.56, for a sufficiently high $\tilde{q} = (\tilde{q}_g, \tilde{q}_b)$ pair, $c_{10}(\tilde{q}_g) = c_{01}(\tilde{q}_b) = c_{00}(\tilde{q}) = \frac{\phi(\gamma_0)}{1 - \pi_\rho}$.

We show that at \tilde{q} , $c_{11}(\tilde{q}) > c_{00}(\tilde{q})$. Suppose towards a contradiction that $c_{11}(\tilde{q}) \leq \frac{\phi(\gamma_0)}{1 - \pi_\rho}$, which implies

$$\rho + \frac{(\tilde{q}_g - \pi_g) \left(\rho - \rho_0 - \frac{\gamma_{11}(\tilde{q})}{\pi_g} \right) + (\tilde{q}_b - \pi_b) \left(\rho - \rho_0 + \frac{\gamma_{11}(\tilde{q})}{\pi_b} \right) + \phi(\gamma_{11}(\tilde{q})) - \phi(\gamma_0)}{\pi_\rho} \leq \frac{\phi(\gamma_0)}{1 - \pi_\rho}$$

and therefore either

$$\left(\rho - \frac{\gamma_{11}(\tilde{q})}{\pi_g} \right) + \frac{(\tilde{q}_g - \pi_g) \left(\rho - \rho_0 - \frac{\gamma_{11}(\tilde{q})}{\pi_g} \right) + \phi(\gamma_{11}(\tilde{q})) - \phi(\gamma_0)}{\pi_\rho \pi_g} \leq \frac{\phi(\gamma_0)}{1 - \pi_\rho}$$

or

$$\left(\rho + \frac{\gamma_{11}(\tilde{q})}{\pi_b} \right) + \frac{(\tilde{q}_b - \pi_b) \left(\rho - \rho_0 + \frac{\gamma_{11}(\tilde{q})}{\pi_b} \right) + \phi(\gamma_{11}(\tilde{q})) - \phi(\gamma_0)}{\pi_\rho \pi_b} \leq \frac{\phi(\gamma_0)}{1 - \pi_\rho}.$$

This, in turn, implies that $c_{10}(\tilde{q}_g, \gamma_{11}(\tilde{q})) \leq c_{00}(\tilde{q})$ or $c_{01}(\tilde{q}_b, \gamma_{11}(\tilde{q})) \leq c_{00}(\tilde{q})$. Since neither $c_{10}(\tilde{q}_g, \gamma)$ nor $c_{01}(\tilde{q}_b, \gamma)$ feature $\gamma_{11}(\tilde{q})$ as a critical point, re-optimization around γ ensures that $c_{10}(\tilde{q}_g) < c_{00}(\tilde{q})$ or $c_{01}(\tilde{q}_b) < c_{00}(\tilde{q})$, achieving the desired contradiction.

By lowering q_g and q_b away from \tilde{q} along the path defined by (1.57), one achieves points where policies with $x_i = 1$ and $x_{-i} = 0$ are strictly preferred to full termination and partial termination. Eventually, a lower bound \tilde{q} where equality (and indifference) with respect to $c_{11}(q)$ is reached. Above this lower bound, all demand for assets comes from these extreme policies reaching the value functions c_{01} and c_{11} .

From the definition of $B_0(\omega; q; \gamma, x)$ and $c(q; \gamma, x)$, the leverage constraint can also be written as

$$I = \{(1 - \pi_\rho) + \pi_\rho [\pi_g x_g + \pi_b x_b]\}^{-1} (c(q; \gamma, x) - \rho_0)^{-1} A.$$

Demand for assets in this price path (1.57) can be described by

$$\begin{aligned} a_g^D(q) &= \left(\rho - \rho_0 - \frac{\gamma_{10}(q)}{\pi_g} \right) I_{10}(\tilde{q}) \\ a_b^D(q) &= \left(\rho - \rho_0 + \frac{\gamma_{01}(q)}{\pi_b} \right) I_{01}(\tilde{q}), \end{aligned}$$

where the aggregate scales $I_{10}(\tilde{q})$ and $I_{01}(\tilde{q})$ solve

$$\begin{aligned} I_{10}(\tilde{q}) &= \{(1 - \pi_\rho) + \pi_\rho \pi_g\}^{-1} (c_{10}(q) - \rho_0)^{-1} A_{10} \\ I_{01}(\tilde{q}) &= \{(1 - \pi_\rho) + \pi_\rho \pi_b\}^{-1} (c_{01}(q) - \rho_0)^{-1} A_{01} \end{aligned}$$

and A_{01} and A_{10} represent the distribution of entrepreneurs (and their net worth) across these two policies and, besides positiveness, have to satisfy $A_{01} + A_{10} = A$ if $q < \tilde{q}$ and $A_{01} + A_{10} \leq A$ if $q = \tilde{q}$ (as a full termination policy is also optimal at $q = \tilde{q}$). Notice that A_{10} and A_{01} will be able to adjust freely to make sure that $\frac{a_g^D}{a_b^D} = 1$, since the supply of external liquidity does not vary across states of the world.

Equilibrium with $L = 0$ is trivially constructed by setting $q = \tilde{q}$ and $A_{10} = A_{01} = 0$. For higher L , prices might adjust downward and asset demands will increase continuously, as long as $q \geq \tilde{q}$, the price point at which the policy involving $x_g = x_b = 1$ becomes relevant for the equilibrium.

(PART 2)

With interior project choice, the optimal decisions associated to the $(x_g, x_b) = (1, 0)$ and $(x_g, x_b) = (0, 1)$ policies are given by first-order conditions

$$\begin{aligned} \phi'(\gamma_{10}(q)) &= \pi_\rho + \left(\frac{q_g}{\pi_g} - 1 \right) \\ \phi'(\gamma_{01}(q)) &= -\pi_\rho - \left(\frac{q_b}{\pi_b} - 1 \right). \end{aligned}$$

Using symmetry of ϕ , I will show that whenever indifference (1.57) holds liquidity premia are higher in the a_g asset, so that $\frac{q_g}{\pi_g} > \frac{q_b}{\pi_b}$ and the rest of the proposition follows. First, notice that

$$c_{10}(q_g) \equiv \frac{\phi(\gamma_{10}(q_g)) + \pi_\rho \pi_g \left(\rho - \frac{\gamma_{10}(q)}{\pi_g} \right) + (q_g - \pi_g) \left(\rho - \rho_0 - \frac{\gamma_{10}(q)}{\pi_g} \right)}{(1 - \pi_\rho) + \pi_\rho \pi_g}$$

and

$$c_{01}(q_b) \equiv \frac{\phi(\gamma_{01}(q_b)) + \pi_\rho \pi_b \left(\rho + \frac{\gamma_{01}(q_b)}{\pi_b} \right) + (q_b - \pi_b) \left(\rho - \rho_0 + \frac{\gamma_{01}(q_b)}{\pi_b} \right)}{(1 - \pi_\rho) + \pi_\rho \pi_b}.$$

If $\frac{q_g}{\pi_g} \leq \frac{q_b}{\pi_b}$ was true in the locus defined by (1.57), then

$$\|\phi'(\gamma_{10}(q_g))\| \leq \|\phi'(\gamma_{01}(q_b))\|,$$

which would imply, given symmetry, that

$$\phi(\gamma_{10}(q_g)) \leq \phi(\gamma_{01}(q_b)).$$

That cannot be true, otherwise both

$$\frac{\phi(\gamma_{10}(q))}{(1 - \pi_\rho)} \leq \frac{\phi(\gamma_{01}(q_b))}{(1 - \pi_\rho)}$$

and

$$\left(\rho - \frac{\gamma_{10}(q)}{\pi_g} \right) + \frac{\left(\frac{q_g}{\pi_g} - 1 \right) \left(\rho - \rho_0 - \frac{\gamma_{10}(q)}{\pi_g} \right)}{\pi_\rho} < \left(\rho + \frac{\gamma_{01}(q_b)}{\pi_b} \right) + \frac{\left(\frac{q_b}{\pi_b} - 1 \right) \left(\rho - \rho_0 + \frac{\gamma_{01}(q_b)}{\pi_b} \right)}{\pi_\rho}.$$

As $c_{10}(q_g)$ and $c_{01}(q_b)$ are weighted averages involving, respectively, the terms on the left-hand side and right-hand side of the inequalities above, this would lead to a contradiction. Therefore, $\frac{q_g}{\pi_g} > \frac{q_b}{\pi_b}$ when (1.57) holds, implying that individual amplification is higher for the entrepreneurs choosing to be pro-cyclical, $(x_g, x_b) = (1, 0)$, as in $\|\phi'(\gamma_{10}(q_g))\| > \|\phi'(\gamma_{01}(q_b))\|$.

Another force for aggregate amplification is in place since, in equilibrium in this regime,

$$\begin{aligned} L = a_g^D \quad (q) &= \left(\rho - \rho_0 - \frac{\gamma_{10}(q)}{\pi_g} \right) I_{10}(\tilde{q}) \\ L = a_b^D \quad (q) &= \left(\rho - \rho_0 + \frac{\gamma_{01}(q)}{\pi_b} \right) I_{01}(\tilde{q}), \end{aligned}$$

which implies

$$I_{10}(\tilde{q}) > I_{01}(\tilde{q}),$$

more investment is made in the pro-cyclical projects. □

D. Constrained Efficiency (Section 1.5)

The planner has asset reallocation, net worth redistribution at $t = 0$ and project choice as possible instruments, but is subject to the same constraints on the initial bilateral arrangements and resource feasibility.

Before proceeding to the proof, it is necessary to add some notation that was not required for the rest of the text. Let A_t^L represent the aggregate endowment of lenders/consumers at period t . For simplicity, let it not depend on the aggregate state. Also, let $C_t^L(\omega)$ be the aggregate consumer/lender consumption at time t and state $\omega \in \Omega$.

The proof use is similar to the usual proof of the first welfare theorem and its version available in (Holmström and Tirole 2011). However, it exploits the fact that complete markets for external assets are not necessary in the environment studied, given equivalence between consumption across different periods.

Proof. (Constrained Pareto Optimality - Proposition 1.6) - Suppose there is another set of pairwise financial arrangements, that leads to a Pareto improvement over allocation σ . Let $\tilde{\sigma}_j = \{\tilde{I}_j, \{\tilde{x}_j(\omega)\}_{\omega \in \Omega}, \tilde{\gamma}_j, \tilde{a}_j\}$ and $\tilde{\tau}_j = \{\tilde{\tau}_j^0, \tilde{\tau}_j^1(\omega), \tilde{\tau}_j^2(\omega)\}$ be the financial contract decisions (scale, continuation, project choice, asset holdings and transfers) involved, as indexed by entrepreneur j and the associated lender. From the feasibility of investment and consumption for entrepreneur j we have

$$\tilde{\tau}_j^0 + A_j = \phi(\tilde{\gamma}_j) \tilde{I}_j + \tilde{c}_j^{0,E} \quad (1.58)$$

$$\tilde{\tau}_j^1(\omega) = \rho(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) I + c_j^{1,E}(\omega) \quad (1.59)$$

$$z(\omega) \cdot \tilde{a}_j + \rho_1(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) \hat{I} + \tilde{\tau}_j^2(\omega) = \tilde{c}_j^{2,E}(\omega) \quad (1.60)$$

That leads to the following level of utility being achieved by entrepreneur j :

$$E[\rho_1(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) - \rho(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) - \phi(\tilde{\gamma}_j)] \tilde{I} + \tilde{A}_j + E\left[\sum_t \tilde{\tau}_j(\omega)\right] + E[z(\omega) \cdot \tilde{a}_j].$$

In order to satisfy interim incentive compatibility of a lender associated to entrepreneur j , this new allocation needs to satisfy, for each $\omega \in \Omega$,

$$\tau_j^1(\omega) + \tau_j^2(\omega) \leq 0$$

and, given limited pledgeability, it also needs to satisfy

$$-\tilde{\tau}_j^2(\omega) \leq z(\omega) \cdot \tilde{a}_j + \rho_0(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) \tilde{I}_j. \quad (1.61)$$

The last two constraints, combined with non-negativity of entrepreneurial consumption, imply that

$$\rho(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) \tilde{I}_j \leq z(\omega) \cdot \tilde{a}_j + \rho_0(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) \tilde{I}_j.$$

As a consequence, any allocation implemented by the planner also needs to satisfy the same liquidity constraints that entrepreneurs face. From (1.58), (1.59) and (1.61) and non-negativity of entrepreneurial consumption, we have that $A_j + E[\sum_t \tilde{\tau}_j(\omega)] + E[\rho_0(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) - \rho(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) - \phi(\tilde{\gamma}_j)] \tilde{I}_j + E[z(\omega) \cdot \tilde{a}_j] \geq 0$.

Implementation of the decision $\tilde{\sigma}_j$, if feasible, under the original competitive equilibrium would lead to a value $E[\rho_1(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) - \rho(\omega, \tilde{\gamma}_j) \tilde{x}_j(\omega) - \phi(\tilde{\gamma}_j)] \tilde{I}_j + A - (q - z(\omega)) \cdot \tilde{a}_j$ for the entrepreneur (from Lemma 1). There are three possibilities to consider. If $\tilde{A}_j + E[\sum_t \tilde{\tau}_j(\omega)] < A - q \cdot \tilde{a}_j$, the plan $\tilde{\sigma}$ would have been feasible under the competitive equilibrium for entrepreneur j , but would have failed to make the leverage constraint bind, being dominated by the equilibrium plan. That leads to a contradiction of a possible improvement. In the case in which $\tilde{A}_j + E[\sum_t \tilde{\tau}_j(\omega)] = A$, the $\tilde{\sigma}(j)$ plan would also have been

feasible, while no strict gains can be made for entrepreneur j and the leverage constraint would hold with equality $A + E [B_0 (\omega; q; \tilde{\gamma}_j, \tilde{x}_j, \tilde{a}_j)] I = 0$. Therefore, for strict gains to be possible for entrepreneur j , we need $\tilde{A}_j + E_\omega [\sum_t \tilde{\tau}_j (\omega)] > A - q \cdot \tilde{a}_j$, which implies that the leverage constraint under the competitive equilibrium is violated by plan $\tilde{\sigma}$, or $A + E [B_0 (\omega; q; \tilde{\gamma}_j, \tilde{x}_j, \tilde{a}_j)] I < 0$. As a consequence, improvements for entrepreneurs require $A + \int E_\omega [B_0 (\omega; q; \tilde{\gamma}_j, \tilde{x}_j, \tilde{a}_j)] \tilde{I}_j dj \leq 0$, with strict inequality if a positive mass of entrepreneurs is made better-off.

Under the previous equilibrium, average consumer/lender utility was given by $C_0^L + E [C_1^L (\omega) + C_2^L (\omega)] = \sum_t A_t^L + q \cdot L$, as they did not participate in the surplus of investment, but could consume their endowments and the value of of assets sold. Another necessary condition for a Pareto improvement over the original allocation follows, with $C_0^L + E [C_1^L (\omega) + C_2^L (\omega)] \geq \sum_t A_t^L + q \cdot L$, and a strict inequality being necessary if a positive mass of consumer/lenders is made better-off. Combining the two, we obtain the necessary condition for a Pareto improvement over the initial allocation

$$C_0^L + E [C_1^L (\omega) + C_2^L (\omega)] - \int E [B_0 (\omega; q; \tilde{\gamma}_j, \tilde{x}_j, \tilde{a}_j)] \tilde{I}_j dj > A + \sum_t A_t^L + q \cdot L. \quad (1.62)$$

Feasibility at aggregate levels at $t = 0$ and $t = 1$ requires

$$\begin{aligned} C_0^L + C_0^E + \int \phi (\tilde{\gamma}_j) I_j dj &\leq A_0^L + A \\ C_1^L (\omega) + C_1^E (\omega) + \int \rho (\omega, \tilde{\gamma}_j) \tilde{I}_j dj &\leq A_1^L \end{aligned}$$

Since only the pledgeable component of output can be transferred to lenders, lender consumption at $t = 2$ is bounded by

$$C_2^L (\omega) \leq \int \rho_0 (\omega, \tilde{\gamma}_j) \tilde{x}_j (\omega) I_j dj + A_2^L + z (\omega) \cdot L.$$

Weighting the three previous constraints by their event probabilities and adding them up, we get

$$C_0^L + E [C_1^L (\omega) + C_2^L (\omega)] - E \left[\int [(\rho_0 (\omega, \tilde{\gamma}_j) - \rho (\omega, \tilde{\gamma}_j)) \tilde{x}_j (\omega) - \phi (\tilde{\gamma}_j)] dj \right] \leq A + \sum_t A_t^L + E [z (\omega) \cdot L].$$

Finally, the constrained planner cannot create any of the assets, given the underlying lack of commitment of consumers, which forces $\int \tilde{a}_j dj \leq L_k$. Multiplying each of these constraints by its positive price q_k from the competitive equilibrium and adding them to the previous inequality, we obtain a reversal of 1.62 and the desired contradiction. □

Chapter 2

Constrained Optimality and Taxation in a Dynamic Hidden Information Economy

Abstract

This chapter studies the characterization of the constrained optimal allocation in an economy with hidden dynamic endowment shocks, in which agents are also able to trade bonds unobservably. Production is monitorable and might be controlled or distorted by a planner, which is also capable of indirectly affecting prices on the unobservable trades. The constrained optimal allocation can be implemented in a simple decentralized way, which takes the form of a bond market economy with taxes. Conditions for the sub-optimality of the untaxed economy and for an optimal tax rule are offered. Around the untaxed economy, the sign of a welfare improving tax on capital depends on the covariance between marginal utility and asset holdings in the cross-section of agents. This expression can be extended into an optimal tax condition, which also takes into account effects that tax changes have on revenues across periods.

2.1 Introduction

This chapter addresses the issue of constrained efficiency in economies with stochastically evolving hidden endowments and non-monitorable private asset trades. It discusses how the manipulation of the production side of the economy might help the provision of insurance to risk averse agents. Despite the lack of knowledge about each agent's asset position at any moment in time, the capital stock and public debt levels are informative about the aggregate savings of the economy, influence the interest rates on the unobservable trades and are valuable policy instruments. The constrained optimal allocation is shown to be implementable in a simple decentralized way, with the introduction of a wedge that separates the interest rate at which agents borrow and lend from the investment cost perceived by firms. A condition for verifying the suboptimality of an untaxed bond economy is provided. It depends crucially on the covariance between marginal utility (or consumption) and asset holdings in the cross-section of the population at each period. This condition can also be extended into an optimal tax formula in which additional terms based on the induced variation in revenues appear.

The starting point of the analysis is an economy with idiosyncratic income shocks, as in the Bewley-Aiyagari¹ class of models. Papers in that tradition typically assume an exogenously incomplete asset market structure through which agents self-insure, while the present chapter takes a mechanism design approach.²

Along the mechanism design tradition, (Allen 1985) shows that, while in a pure hidden information moral hazard problem³ with controlled consumption, some risk-sharing between the principal and agent is possible, that is no longer true once the agent has unmonitored access to credit markets at the same interest rates faced by the principal. (Cole and Kocherlakota 2001) extend that analysis to an environment in which agents can only save hiddenly through the same linear technology available to the principal, without the ability to borrow.

In that situation, under a few additional conditions, the constrained optimal allocation is equivalent to the allocation that would be achieved by allowing the agents to borrow and lend freely at the same rate of return of the linear technology. As a consequence, the welfare level of that is achieved in the constrained optimal arrangement is the same which follows from the decentralized trading of a single riskless bond. This result can be interpreted as providing foundations for the apparent *ad hoc* incompleteness of insurance instruments common in the Bewley-Aiyagari framework.

¹As in Bewley(undated) , (Bewley 1983), (Bewley 1986) and (Aiyagari 1994).

²Along the first tradition,(Aiyagari 1995) analyzes the taxation of savings in a model with heterogeneity, random evolution of labor endowments and limited possibilities for the taxation of labor income. A planner needs to raise resources to finance a stream of public goods. It demonstrates that positive capital taxes are an useful instrument in that environment, even in a steady state. This result, a consequence of incomplete markets and dynamic heterogeneity, is at odds with the Chamley-Judd result ((Chamley 1986);(Judd 1985)) of the sub-optimality of asymptotic taxation of capital, which holds under a general set of conditions, as explored in (Atkeson, Chari, and Kehoe 1999).

³A pure hidden information moral hazard as defined by Allen (1985) is equivalent to an environment with unobservable income fluctuations.

However, the assumptions that agents can directly access the most efficient saving technology without detection, or that their technology has a rate of return that cannot be targeted by the planner even through indirect means, is central for the equivalence and constrained optimality results. More generally, the taxation of markets in the presence of externalities, incomplete contingent claims or private information might lead to potential welfare gains, as first studied by (Greenwald and Stiglitz 1986) and further formalized by (Geanakoplos and Polemarchakis 2008), (Bisin, Geanakoplos, Gottardi, Minelli, and Polemarchakis 2001) and (Citanna, Polemarchakis, and Tirelli 2006) among others.

Therefore, capital taxation can potentially help relax incentive compatibility constraints and have non-trivial redistributive consequences. Additionally, taxation helps decouple the shadow interest rate perceived by the planner from the rate on savings that agents observe on the market. (Doepke and Townsend 2006) have provided a computational example in which, whenever those rates are sufficiently different, the equivalence of self-insurance and the second-best with unmonitored savings breaks down⁴. The present paper starts with an environment similar to the one studied by (Cole and Kocherlakota 2001), but allows the manipulation of aggregate savings and the rates at which agents can save, and analyzes which channels make these instruments helpful for the provision of insurance against income fluctuations.

In another related paper, (Golosov and Tsyvinski 2007) have studied a dynamic Mirrlees economy⁵ in which agents have non-monitorable access to a retrade market. In such environment, agents have hidden information about their labor productivities and might engage in deviations that both misreport their types and use the asset market to reallocate the consumption streams they receive. They show that the competitive allocation involving private insurance is constrained inefficient and provide examples in which a simple tax or subsidy on capital can be welfare improving, with the sign of the optimal distortion depending on the nature of the skill process involved.

In this chapter, I analyze a simple dynamic structure which involves both asymmetric information about endowment realizations and non-monitorable access to credit markets. The aggregate savings technology is monitorable and can be targeted by a planner. The distortion of the aggregate technology gives an additional instrument to this planner, beyond the use of any revelation games about the endowment shocks. Three different arrangements for the provision of insurance are investigated: self-insurance through a riskless bond market, competitive insurance, both subject to the presence of fiscal policy, and public insurance provision. Both competitive insurance and public insurance provision are shown to be equivalent to the competitive equilibrium of a self-insurance economy with the appropriately chosen fiscal policy.

The underlying form of moral hazard, which consists of hidden information about endowment

⁴Due to different modeling assumptions regarding the savings deviation possibilities, this computational result does not translate directly to the environment studied in this chapter. Incentive compatibility will require that transfer stream have to have the same NPV at market prices, which does not occur in (Doepke and Townsend 2006).

⁵The standard Mirrlees problem involves private information about labor productivity, but fully observable labor income.

shocks, allows a simple characterization of the set of allocations that can be supported in the economy. The presence of unobserved consumption and trades on financial markets constrains the incentive compatible transfer streams not to have different net present values at the market interest rate, which is faced at the margin by all agents. The manipulation of such rate is an important instrument for achieving the constrained optimal allocation and can be done with a simple linear tax. By controlling agents' marginal rates of substitution, and possibly allowing it to differ from the marginal rate of transformation, the planner can manipulate pecuniary effects and achieve some redistribution and insurance between agents that end up with different endowment histories, even if no redistribution seems to occur at market prices. Therefore, in terms of welfare ranking, self-insurance with appropriately chosen fiscal policies dominates *laissez-faire* self-insurance, which in its turn dominates autarky.

Given the equivalence results, there is a very simple way to implement the constrained optimal allocation through decentralization. As the constrained optimal allocation is a competitive equilibrium with taxes of an incomplete markets economy, it suffices to choose taxes appropriately to maximize the welfare function. The study of the constrained optimal allocation in this hidden information economy with unobserved trades is reduced to a taxation exercise: a non-standard Ramsey problem, with the presence of dynamic evolution of heterogeneity and availability of anonymous lump-sum transfers. In contrast to the richer Mirrlees environment studied by (Golosov and Tsyvinski 2007), in the economy with pure endowment fluctuations which this chapter studies, an appropriately chosen fiscal policy not only improves welfare, but fully implements the constrained optimal allocation. No exogenous restrictions are imposed on the tax instruments, but personalized transfers are restricted by the presence of asymmetric information and hidden savings possibilities.

Guided by the equivalence results, I provide conditions for the presence of welfare improving tax interventions, or for the sub-optimality of the untaxed economy. These conditions rest centrally on the covariance between marginal utility and asset holdings in the self-insurance economy without taxes.

Two computational examples are provided. In the first example, the stochastic process for the endowments is composed of independent and identically distributed shocks. The technology is linear and its returns are set to simply offset intertemporal discounting. As a consequence, agents with high initial income save, while agents with lower income borrow. Given that pattern, a welfare improving positive tax on capital allows for redistribution across agents. This redistribution occurs as agents with higher expected discounted wealth save more and are responsible for a larger share of the funding of a lump-sum rebate. The second example highlights that capital subsidies might be optimal, although under less realistic conditions which generate a positive covariance between savings and marginal utility. In that example, a higher initial income is associated with bad news about the discounted value of wealth. As a consequence of the positive covariance, an optimal negative tax on the formation of capital is identified.

The remainder of chapter is organized as follows. Section 2 describes the fundamentals of the

economy, Section 3 presents the different institutional frameworks for the provision of insurance, Section 4 discusses the main results obtained, Section 5 presents two examples in which the tax wedge has opposite signs, and Section 6 concludes.

2.2 Model

Time is discrete, finite and indexed by $t = 1, 2, \dots, T$. A single consumption good is present in each period. This good can be converted into capital at a rate of one-to-one. Regarding notation, x_t denotes the current value of variable x at time t and x^t denotes the history vector of all values of x_t up to time t .

Households - There is a continuum unit mass of ex ante identical households. At time 1, each one receives a random endowment θ_1 of the consumption good, drawn from some finite set $\Theta \in \mathbb{R}_{++}$ according to the probability distribution $\pi_1(\theta_1)$. At period t , they learn the value of the current period's endowment, drawn from the same set Θ with probabilities $\pi_t(\theta_t|\theta^{t-1})$. Households have preferences defined over consumption streams according to

$$U(c^t) = E \left[\sum_{t=1}^T \beta^{t-1} u(c_t) \right],$$

for some $\beta \in (0, 1)$ and $u : (\underline{c}, \bar{c}) \rightarrow \mathbb{R}$, where $u \in C^2$, $u'(c) > 0$ and $u''(c) < 0$. To ensure interiority of the solutions to saving problems, we assume Inada Conditions $\lim_{c \rightarrow \underline{c}} u'(c) = +\infty$ and $\lim_{c \rightarrow \bar{c}} u'(c) = 0$ and use natural borrowing constraints. We assume a law of large number holds, so that the mass of agents that suffered income shock history θ^{t+s} after θ^t is given by $\pi_{t+s}(\theta^{t+s}|\theta^t)$.

Production - There is a representative firm for each period $t > 1$, producing consumption goods at time t from capital K_{t-1} purchased and installed at time $t-1$ according to a weakly concave production function $y_t = F(K_{t-1})$. Assume full depreciation and that $F(K_{t-1})$ is continuously differentiable and $F(0) = 0$. Each firm behaves competitively and has access to the asset market. For simplicity, assume the firms' profits, π_t^{prod} , are directed to the government, which can rebate it back to agents.⁶

Markets - At each period t , there exists a financial market in which riskless bonds are traded at price q_t , for a repayment of 1 unit of consumption in the following period. There are also a

⁶This is without loss of generality, as given the absence of initial heterogeneity and private information about initial asset positions, the government can use lump-sum taxation to capture any profits generated by the firms.

markets for consumption goods and capital. For ease of notation, let us also define the prices

$$q_{t,t} = 1,$$

$$q_{t,t+s} = \prod_{s'=0}^{s-1} q_{t+s'}, \text{ for } s > 1$$

the cost at period t of buying one unit of consumption at $t + s$.

Information - Each household's endowment is private information, as are asset positions and consumption. The transformation of consumption goods into capital is assumed to be observable and can be distorted or controlled by a planner.

Government - The government can tax the transformation of consumption goods into capital at $t < T$ at any rate $\tau_t \in (-\infty, \infty)$. This way, a firm that wants to have K_t units of capital installed in the beginning of period $t + 1$, has to buy $K_t(1 + \tau_t)$ units of goods at t with a future cost of $K_t \frac{(1+\tau_t)}{q_t}$ at $t + 1$. This raises revenues $\tau_t K_t$ at t . Additionally, the government might issue a value $q_t B_t$ of bonds and rebate the proceeds uniformly to the agents at t . The total lump sum rebate at t , γ_t , equals the net change in the government's position in financial markets, plus the revenue collected and profits earned. At T , the government receives profits from production and uses lump-sum taxation γ_T to repay the outstanding debt. Therefore, the budget balance conditions for the government are given by

$$q_1 B_1 = \gamma_1 - \tau_1 K_1$$

$$q_t B_t = \gamma_t - \tau_t K_t - \pi_t^{prod} + B_{t-1}$$

$$\gamma_T = -B_{T-1} + \pi_T^{prod}.$$

A fiscal policy is a tuple $p = (B_t, \gamma_t, \tau_t)_{t=1}^T$.

Contracts - A dynamic mechanism determines distributions of transfers $T_t(h^t)$ and action recommendations $\hat{a}_t(h^t)$ after public histories h^t . Without loss of generality, one can restrict attention to mechanisms that are direct and induce obedience from the agents ((Myerson 1986); (Doepke and Townsend 2006)). These involve public histories containing only messages from the household about its current endowment shock θ_t and a recommended savings decision \hat{a}_t from the mechanism back to the household. Additionally, at each moment in time, the household will have a private information set including all information on the public history plus all actual previous endowment realizations and private actions.

A direct mechanism allows the agent to choose strategies that specify report functions $\hat{\theta}_t(h^t, a^{t-1}, \theta^{t-1})$ and actions $a_t(h^t, a^{t-1}, \theta^t, \hat{a}_t)$, conditioning on the relevant private histories at the moment the decision is taken.

We will restrict attention to deterministic mechanisms. On those, the transfers and recommendations evolve deterministically given the reports chosen. Therefore, there is a one-to-one mapping between public histories h^t and sequences of reports $\hat{\theta}^t$. Additionally, it is possible to describe a strategy σ in the deterministic mechanism as simply the specification of $\hat{\theta}_t(\theta^t) : \Theta^t \rightarrow \Theta$ and $a_t(\theta^t) : \Theta^t \rightarrow \mathbb{R}$ for each $t \in \{1, \dots, T\}$, $\theta^t \in \Theta^t$. Let Σ define the space of all possible report and saving strategies.

The expected utility of playing strategy $\sigma \in \Sigma$, given the contract $\Gamma \equiv (T_t, \hat{a}_t)_{t=1}^T$ and prices $q \equiv \{q_t\}_{t=1}^{T-1}$, is defined as

$$V(\sigma, \Gamma, q) = E \left[\sum_{t=1}^T \beta^{t-1} u \left(c_t \left(\theta^t, \sigma, \Gamma, q \right) \right) \right],$$

where

$$\begin{aligned} c_1(\theta^1, \sigma, \Gamma, q) &= \theta_1 - q_1 a_1^\sigma(\theta^1) + T_1(\hat{\theta}_1^\sigma(\theta^1)), \\ c_t(\theta^t, \sigma, \Gamma, q) &= \theta_2 + a_{t-1}^\sigma(\theta^{t-1}) + T_t(\hat{\theta}_t^\sigma(\theta^t)) - q_t a_t^\sigma(\theta^{t-1}), \\ a_T &= 0. \end{aligned}$$

Incentive compatibility then requires that given the truthful and obedient strategy, $\sigma^{TT} \in \Sigma$, which sets

$$\hat{\theta}_t(\theta^t) = \theta_t, \forall t, \theta^t,$$

(truth-telling) and

$$a_t(\theta^t) = \hat{a}_t(\hat{\theta}_t(\theta^t)), \forall t, \theta^t$$

(obedience), the following set of inequalities holds

$$\begin{aligned} V(\sigma^{TT}, \Gamma, q) &\geq V(\sigma, \Gamma, q), \\ \forall \sigma &\in \Sigma. \end{aligned}$$

For notation simplicity, let us write $c_t(\theta^t, \Gamma, q)$ for $c_t(\theta^t, \sigma^{TT}, \Gamma, q)$ suppressing the dependence on the truthful and obedient strategy σ^{TT} .

2.3 Environments

It is possible to analyze the outcomes of different institutional arrangements for the provision of insurance on the economy outlined above. One can focus on the self-insurance through incomplete markets, competitive private insurance or public insurance. The first one is an environment where households have only access to a bond market and can achieve some insurance by solving an optimal

savings problem. Additionally, firms invest in capital and are able to physically transfer resources across periods. Also, the government chooses a fiscal policy. The second framework adds a layer of competitive insurance firms, which need to offer incentive compatible contracts to the households. It is worth noting that incentive compatibility requires both truthful reporting of endowment shocks and obedience on the choice of savings. The third and last framework analyzed is a planning problem, in which a planner controls production and offers contracts to the households, subject to the need for incentive compatibility. The solution to this problem describes the set of constrained optimal allocations.

2.3.1 Self-insurance through Incomplete Markets with taxes

The first important benchmark to have in mind is the analysis of outcomes of self-insurance through the use of a single riskless asset, as in the Bewley-Aiyagari tradition. This is also commonly referred to as the permanent income environment. Here, we add the possibility of use of a tax on the representative firm's purchase of goods for investment.

On that framework, each household, after observing the value of the initial income shock chooses its bond holdings to solve

$$\max_{a_t(\theta^t)} E \left[\sum_{t=1}^T \beta^{t-1} u(c_t) \mid \theta_1 \right], \quad (2.1)$$

s.t.

$$\begin{aligned} c_t(\theta^t) &= \theta_t(\theta^t) - q_t a_t(\theta^t) + \gamma_t + a_{t-1}(\theta^{t-1}), \\ a_0 &= 0, a_T = 0. \end{aligned}$$

The representative firm of period t solves a standard profit maximization problem, described by

$$\max F(K_{t-1}) - \frac{K_{t-1}(1 + \tau_{t-1})}{q_{t-1}}. \quad (2.2)$$

The government is taken to be the first-mover and announces the commitment to a policy p , describing capital taxes $\tau = \{\tau_t\}_{t=1}^{T-1}$, bond issues $B = \{B_t\}_{t=1}^{T-1}$ and lump-sum rebates $\gamma = \{\gamma_t\}_{t=1}^T$.

A competitive equilibrium with taxes is given by the definition below:

Definition 1. A competitive equilibrium with taxes in the single riskless asset self-insurance framework is defined by a policy $p = (B_t, \gamma_t, \tau_t)_{t=1}^T$, asset holdings $a_t(\theta^t)$ and consumption $c_t(\theta^t)$ for each $t \leq T, \theta^t \in \Theta^t$, capital levels $\{K_t\}_{t=1}^{T-1}$, bond prices $q = \{q_t\}_{t=1}^{T-1}$ and profits $\{\pi_t\}_{t=2}^T$ such that:

(i) asset holdings and consumption decisions solve the household's problem described above, taking q and the government's policy p as given;

(ii) each representative production firm chooses K_{t-1} to solve its profit maximization problem and pays out dividends π_t^{prod} , that equal its profit at t . The representative firm that produces at t

issues $\frac{(1+\tau_{t-1})K_{t-1}}{q_{t-1}}$ bonds at $t - 1$ to finance its capital purchases;

(iii) there is market clearing in goods at each t :

$$\begin{aligned} E \left[c_1 \left(\theta^1 \right) \right] &= E \left(\theta_1 \right) - K_1, \text{ for } t = 1 \\ E \left[c_t \left(\theta^t \right) \right] &= E \left(\theta_t \right) + F \left(K_{t-1} \right) - K_t, \text{ for } 1 < t < T \\ E \left[c_T \left(\theta^T \right) \right] &= E \left(\theta_T \right) + F \left(K_{T-1} \right), \text{ for } t = T \end{aligned}$$

(iv) there is market-clearing in the bond market at each $t < T$;

$$E \left[a_t \left(\theta^t \right) \right] = B_t + \frac{(1 + \tau_t) K_t}{q_t}.$$

(v) the government's budget balance is respected.

2.3.2 Competitive Private Insurance

Insurance firms offer exclusive contracts $\Gamma = (T_t, \hat{a}_t)_{t=1}^T$ competitively and receive the rights to all lump-sum rebates from the government to households. These contracts are signed before any uncertainty is resolved, so that there is no initial adverse selection problem. All insurance firms maximize their dividends π_T^{ins} at T and there is free-entry of firms. They finance themselves on the financial market, borrowing the capital necessary to pay out any transfers at t . Let d_t^j denote the debt issues of firm j for payment in period $t + 1$.

Each insurance firm then solves

$$\min_{\Gamma \equiv (T_t, \hat{a}_t)_{t=1}^T} E \left[\sum_{t=1}^T (q_{t,T})^{-1} T_t \left(\theta^t \right) \right] \quad (2.3)$$

s.t.

$$\begin{aligned} V \left(\sigma^{TT}, \Gamma, q \right) &= \underline{U}, \\ V \left(\sigma^{TT}, \Gamma, q \right) &\geq V \left(\sigma, \Gamma, q \right). \end{aligned}$$

Notice that since firms finance themselves through borrowing in the bond market, given a contract $\Gamma^j = (T_t^j, \hat{a}_t^j)_{t=1}^T$ firm j needs to finance $E_\theta \left[T_1^j \left(\theta^1 \right) \right] - \gamma_1$ at period 1 for a mass one of contracts sold. This is done by issuing $d_1^j = \frac{E[T_1^j(\theta^1)] - \gamma_1}{q_1}$ bonds and owing that amount at the beginning of period $t = 2$. At any future period t , an insurance firm j brings in, for each mass one

of contracts sold, a net debt in the total value of

$$d_{t-1}^j = E_\theta \left[\sum_{t'=1}^{t-1} (q_{t',t})^{-1} \left(T_{t'}^j(\theta^{t'}) - \gamma_{t'} \right) \right]$$

and needs to finance an additional $E \left[\left(T_t^j(\theta^t) - \gamma_t \right) \right]$. For that, it needs to issue

$$d_t^j = E_\theta \left[\sum_{t'=1}^t (q_{t',t+1})^{-1} \left(T_{t'}^j(\theta^{t'}) - \gamma_{t'} \right) \right]$$

bonds at t .

Definition 2. A competitive equilibrium with taxes and private insurance is defined as a policy p , a contract $\Gamma_j \equiv \left(T_t^j, \hat{a}_t^j \right)_{t=1}^T$ and market share $\xi_j > 0$ for each active firm j^7 , an utility level \underline{U} , bond prices $q = \{q_t\}_{t=1}^T$ and capital levels $\{K_t\}_{t=1}^{T-1}$ such that:

(i) given the policy p and prices q , the contract $\Gamma_j \equiv \left(T_t^j, \hat{a}_t^j \right)_{t=1}^T$ solves the insurance firm's problem above, which requires incentive compatibility;

(ii) insurance firms earn zero profits;

(iii) agents choose the firm offering them the highest utility \underline{U} ;

(iv) each representative production firm chooses K_{t-1} to solve its profit maximization problem and pays out dividends π_t^{prod} that equal its profits at t .

(v) there is market-clearing in goods at every t ;

(vi) there is market clearing in the asset market at $t < T$;

$$E_{\xi_j, \theta} \left[\hat{a}_t^j(\theta^t) \right] = E_{\xi_j} \left[d_t^j \right] + B_t + \frac{(1 + \tau_t) K_t}{q_t},$$

where the expectations of asset holdings are taken with respect to both market shares and income realizations θ^t .

(vii) the government's budget is balanced.

2.3.3 Public Insurance Provision

The third framework analyzes the problem of a benevolent planner that can control production and the public sector, chooses consumption distributions through the choices of conditional transfers and is constrained by the existence of hidden access to a credit market and the necessity of incentive compatibility.

⁷The market share is only necessary for condition (vi), since there is some indeterminacy on the contract regarding whether firm holds debt or agents hold debt.

Since the planner controls and finances production and can thus control the net supply of assets, the equilibrium condition in the hidden bond market is simply reduced to no agent wanting to change their savings at the market prices q . Even though the planner does not control q directly, whenever all agents' marginal rates of substitution are equated in a feasible allocation, that common marginal rate would clear the hidden asset market by avoiding any re-trade gains for agents.

The planner solves the problem of choosing a contract Γ that determines the constrained optimal allocation described below:

Definition 3. The constrained optimal allocation $(c_t(\theta^t, \Gamma), K_t)_{t=1}^T$ is given by the solution to

$$\max_{\Gamma, K_t, q} E \left[\sum_{t=1}^T \beta^{t-1} u \left(c_t \left(\theta^t, \Gamma, q \right) \right) \right], \quad (2.4)$$

s.t.

$$V \left(\sigma^{TT}, \Gamma, q \right) \geq V \left(\sigma, \Gamma, q \right), \quad (2.5)$$

and

$$\begin{aligned} E \left[c_1 \left(\theta^1, \Gamma \right) \right] &= E[\theta_1] - K_1, \\ E \left[c_t \left(\theta^t, \Gamma \right) \right] &= E[\theta_t] + F(K_{t-1}) - K_t \\ E \left[c_T \left(\theta^T, \Gamma \right) \right] &= E[\theta_T] + F(K_{T-1}). \end{aligned} \quad (2.6)$$

2.4 Results

The first result obtained is much in the spirit of the original (Allen 1985) result. Since agents have uncontrolled access to a credit market and no informative signal can be observed about their endowment shocks, the presence of any transfer stream with a strictly higher net present value at market prices would ensure that all agents would report to have had the endowment history that leads to such stream. Additionally, every agent should always be expected to be saving optimally. As a consequence, we show, in Proposition 1 below, that incentive compatibility of contracts can be reduced to a condition about the constancy of the net present value (NPV) of transfer streams at market prices and Euler equations for savings decisions.

Proposition 1. Incentive compatibility

$$\begin{aligned} V \left(\sigma^{TT}, \Gamma, q \right) &\geq V \left(\sigma, \Gamma, q \right), \\ \forall \sigma &\in \Sigma. \end{aligned}$$

can be reduced to an Euler equation for savings on the equilibrium path

$$u' \left(c_t \left(\theta^t, \Gamma, q \right) \right) = \frac{\beta}{q_t} E \left[u' \left(c_{t+1} \left(\theta^{t+1}, \Gamma, q \right) \right) \mid \theta^t \right], \forall t < T, \forall \theta^t \in \Theta^t,$$

and all transfer streams having the same NPV at market prices, which implies

$$\sum_{t=1}^T q_{1,t} T_t \left(\theta^t \right) = \sum_{t=1}^T q_{1,t} T_t \left(\tilde{\theta}^t \right), \forall \theta^t, \tilde{\theta}^t \in \Theta.$$

Proof. Appendix. □

Proposition 1 allows a very simple characterization of the set of incentive compatible contracts, given the interest rates the agents perceive for any reallocation of their resources. Transfers and asset holdings have to respect inter-temporal optimality from the agents' point-of-view and no transfers of wealth across households with different histories is possible at the prevailing interest rates. Another direct consequence of the NPV condition is that transfer stream might not depend on the endowment realization at the last period, θ_T , as

$$q_{1,T} T_T \left(\theta^T \right) = \sum_{t=1}^T q_{1,t} T_t \left(\tilde{\theta}^t \right) - \sum_{t=1}^{T-1} q_{1,t} T_t \left(\theta^t \right),$$

where the right-hand side does not depend on θ_T . The impossibility of controlling household consumption severely limits what competitive insurance firms can provide to agents, to the point of rendering any private insurance provision useless, as indicated by Proposition 2 below.

Proposition 2. Any consumption distribution and capital accumulation $(c_t(\theta^t, \Gamma, q), K_t)_{t=1}^T$ that can be achieved in a competitive equilibrium with private insurance firms and policy p , can also be achieved through the same policy p as a competitive equilibrium in the self-insurance framework.

As a consequence of incentive compatibility, private insurance firms cannot make payment streams with different net present values to households with distinct reported histories. Therefore, the zero profit condition for insurance firms guarantees that, for every reported history, households receive a stream of transfers with the same NPV of the government rebates they were initially entitled to. An incentive compatible private insurance contract then has to satisfy

$$u' \left(c_t \left(\theta^t, \Gamma \right) \right) = \frac{\beta}{q_t} E \left[u' \left(c_{t+1} \left(\theta^{t+1}, \Gamma \right) \right) \mid \theta^t \right], \\ \forall t < T, \forall \theta^t \in \Theta^t$$

and

$$\begin{aligned}\sum_{t=1}^T q_{1,t} T_t(\theta^t) &= \sum q_{1,t} \gamma_t, \\ c_t(\theta^t, \Gamma, q) &= \theta_t(\theta^t) - q_t \hat{a}_t(\theta^t) + \hat{a}_{t-1}(\theta^{t-1}) + T_t(\theta^t), \\ a_0 &= 0, a_T = 0.\end{aligned}$$

There is indeterminacy in the contract Γ^j that solves these equations, as a change in transfers $T_t(\theta^t), T_{t+s}(\theta^{t+s}|\theta^t)$ that does not affect the net present of the stream and a change in mandated asset holdings $\{\hat{a}_{t'}(\theta^{t'})\}_{t'=1}^{t+s-1}$ that compensates for it as to keep the same consumption stream leaves indifferent both the household and insurance firm, which borrow and lend at the same rates. In other words, a change in the timing of transfers from the insurance firms to households is exactly offset by a change in their asset holdings path.

However, any contract Γ^j that solves the equations above leads to the household-firm pair having the same net aggregate position in the asset market, which equals $\hat{a}_t^j(\theta^t) - d_t^j$ at time t , as the insurance firm finances any transfers in excess of the rebate γ_t with borrowing.

The conditions for optimality in the self-insurance framework are given by

$$\begin{aligned}u'(c_t^*(\theta^t, q)) &= \frac{\beta}{q_t} E[u'(c_{t+1}^*(\theta^{t+1}, q)) | \theta^t], \\ \forall t < T, \forall \theta^t \in \Theta^t\end{aligned}$$

$$\begin{aligned}c_t^*(\theta^t) &= \theta_t - q_t a_t^*(\theta^t) + a_{t-1}^*(\theta^{t-1}) + \gamma_t \\ a_0 &= a_T^* = 0.\end{aligned}$$

Comparing both systems, it is possible to verify that $a_t^*(\theta^t) = \sum_{s=1}^t (q_{s,t+1})^{-1} [\gamma_t - T_t^j(\theta^t)] + \hat{a}_t(\theta^t) = -d_t^j + \hat{a}_t(\theta^t)$ in the solution. Consumption at every period t is unchanged for every type $\theta^t \in \Theta^t$. Equilibria on goods and asset markets for the equivalent self-insurance economy at the same consumption and capital allocation as the private insurance follow from this observation.

Therefore, fixing policy a p , for every equilibrium consumption distribution and capital values in the competitive insurance environment, there is a competitive equilibrium in the self-insurance framework with the same consumption and capital allocation.

In this environment, competitive insurance firms end up not providing any valuable services for households, are redundant and any arbitrarily small operational cost would justify their shut down and the use of a pure riskless bond market as a Pareto superior alternative. Indeed, Proposition 3 below shows that there are even stronger arguments for decentralized institutional arrangements, since a riskless bond economy, once taxes are chosen appropriately, can implement the constrained optimal consumption path.

Proposition 3. (Decentralization) If a constrained optimal allocation is obtained as the solution to Problem 2.4, it can be achieved as a competitive equilibrium with a riskless bond market and taxes for some policy p .

Proof. Appendix. □

The proof is constructive. The optimal capital tax or subsidy is the difference between the marginal rate of substitution between consumption at t and $t + 1$ faced by all agents in the solution to Problem 2.4 and the marginal rate of transformation of the consumption good between periods. Due to the impossibility of taxing asset holdings or monitoring consumption, the planner uses distortions in the aggregate access to the saving technology to ensure that such a wedge is actually in place, separating returns on both sides of the credit market and manipulating prices as perceived by households.

Other elements of the fiscal policy could be used in multiple ways, as long as the total rebate to agents has the appropriate net present value using bond market prices $\{q_t\}_{t=1}^{T-1}$ and the participation of the government in bond markets respects its budget constraints. In the decentralized implementation, the government has to roll over assets relative to profits and tax revenue net of previous rebates to agents. Since there are multiple ways of implementing lump-sum rebating to agents while keeping the same net-present value, there are also different fiscal policies consistent with the constrained optimal allocation.

Using the decentralized implementation, it is possible to conclude that under quite general conditions, that is, except if a tight condition on the covariance between asset holdings and consumption holds, there is the presence of a wedge in the constrained optimal allocation. Typically, a planner has incentives to distort interest rates in the direction that benefits households with histories that lead to higher marginal utility of consumption. The covariance between asset holdings and marginal utility is the key statistic to look at in order to determine in which direction a wedge should go for small deviations away from the untaxed economy, as suggested by (Chamley 2001) for different dynamic environments and, in an earlier reference, by (Diamond 1975) for a static multi-person environment.

To illustrate this point, it is useful to write a Planner's problem in a primal form, as in

$$W_0(K_0 = 0) \equiv \max_{q, \gamma, K} V_0(\theta^0, q, \gamma) \tag{2.7}$$

s.t.

$$\begin{aligned} q_t E[a_t(q, \gamma)] - E[a_{t-1}(q, \gamma)] - \gamma_t + F(K_{t-1}) &= K_t \\ \bar{a}_0 &= 0, K_0 = 0 \\ a_T &= 0, K_T = 0, \end{aligned}$$

where the objective function $V_0(\theta_0, q, \gamma)$ is the agents' indirect utility function from the optimal savings problem, evaluated before the realization of any uncertainty, given prices q and lump-sum transfer stream γ . The program above aggregates the productive side of the economy and the government's accounts, allowing the planner to choose lump-sum transfers and consumer bond prices, subject to resource feasibility. Wedges are defined implicitly as a function of the marginal productivity of period t 's capital and the inverse of bond prices.

Assumption: $E[a_t(q, \gamma)]$ is C^1 for each $t \in \{1, \dots, T\}$.

Proposition 4. Around an untaxed economy where $\tau_t = 0, \forall t \in T$, i.e., $q_t = [F'(K_t)]^{-1}, \forall t \in T$, first-order welfare gains from distorting asset prices exist unless

$$\begin{aligned} Cov(u'(c_t), a_t) &= 0, \\ \forall t &\in \{1, \dots, T\}. \end{aligned}$$

Proof. We look at a perturbation around a point (q^0, γ^0, K^0) in the direction of a vector $v = (\Delta q, \Delta \gamma, \Delta K)$ described below.

For a chosen time period $k \in \{1, \dots, T\}$, the perturbation sets

$$\begin{aligned} \Delta q_k &= 1 \\ \Delta \gamma_k &= E[a_{t-1}]. \end{aligned}$$

For all $t \in \{1, \dots, T\}$, such that $t \neq k$,

$$\Delta q_t = \Delta \gamma_k = 0.$$

To make sure it has no first-order impact on the constraint set around $\tau = 0$, set

$$\Delta K_t = q_t E \left[\frac{\partial a_t}{\partial q_k} + \frac{\partial a_t}{\partial \gamma_k} E[a_t] \right], \text{ for each } t \in \{1, \dots, T\}.$$

This direction of perturbation has no first-order effects on constraint set, since $q_t^0 = [F'(K_t^0)]^{-1}$.

Then,

$$\begin{aligned} \frac{dW_0}{dv} &= \frac{\partial V_0}{\partial q_k} + \frac{\partial V_0}{\partial \gamma_k} E[a_k] \\ &= -\beta^{k-1} E[u'(c_k) a_k] + \beta^{k-1} E[u'(c_k)] E[a_k] \\ &= -\beta^{k-1} Cov(u'(c_k), a_k). \end{aligned}$$

□

Once away from a zero-tax allocation, further changes in asset prices have not only the redistributive effect identified by the covariance between marginal utilities and asset holdings but have

also effects on the tax revenue, which can be raised to finance lump-sum rebates. This is, indeed, an essential trade-off that appears when characterizing the optimal tax policy using a primal approach. The necessary first-order conditions of Problem 2.7 are

$$q_k \quad : \quad \frac{\partial V_0}{\partial q_k} + \lambda_k E[a_k] + \sum_{t=1}^{T-1} (\lambda_t q_t - \lambda_{t+1}) \frac{\partial E[a_t]}{\partial q_k} = 0 \quad (2.8)$$

$$\implies \beta^{k-1} E[-u'(c_k(\theta^s)) a_k^*(\theta^t)] + \lambda_k E[a_k] + \sum_{t=1}^{T-1} (\lambda_t q_t - \lambda_{t+1}) \frac{\partial E[a_t]}{\partial q_k} = 0$$

$$\gamma_k \quad : \quad \frac{\partial V_0}{\partial \gamma_k} - \lambda_k + \sum_{t=1}^{T-1} (\lambda_t q_t - \lambda_{t+1}) \frac{\partial \bar{a}_t}{\partial \gamma_k} = 0 \quad (2.9)$$

$$\implies E[u'(c_k)] - \lambda_k + \sum_{t=1}^{T-1} (\lambda_t q_t - \lambda_{t+1}) \frac{\partial \bar{a}_t}{\partial \gamma_k} = 0$$

$$K_t \quad : \quad -\lambda_t + F'(K_t) \lambda_{t+1} = 0 \quad (2.10)$$

These equations can be combined to write

$$-\beta^k Cov(u'(c_k), a_k) + \sum \lambda_{t+1} \tau_t \left[\frac{\partial \bar{a}_t}{\partial q_k} + \frac{\partial \bar{a}_t}{\partial \gamma_t} \bar{a}_t \right] = 0 \quad (2.11)$$

and

$$-\lambda_t + F'(K_t) \lambda_{t+1} = 0. \quad (2.12)$$

Equation (2.11) provides characterization of the essential trade-off between using lump-sum rebates or a tax wedge on capital accumulation. The first term is the redistributive effect from increasing the price of a bond and rebating the revenue, while the remainder is the resource cost that this reform introduces, as individuals change their asset demands in response to it. These resources are evaluated at the shadow cost to the planner, which is given by production prices, as seen from equation 2.12.

The presence of private information and dynamically evolving heterogeneity, which is shown to be equivalent to endogenously incomplete markets, adds welfare improving capital taxation possibilities, even when lump-sum taxes are available. This contrasts with the fact that without dynamic endowment shocks and hidden side-trades, the availability of lump-sum taxes typically results in an even stronger case against capital taxation, extending the zero-taxation benchmark result to all periods.

2.5 Examples

In this section, I provide two two-period examples, discussing the direction of the welfare improving tax wedge found. In both, I compute a local solution to

$$\max_{\{T_t(\theta^t)\}_{t, \theta^t \in \Theta^t, \bar{T}}} E \left[\sum_{t=1}^T \beta^{t-1} u \left(\theta_t \left(\theta^t \right) + T_t \left(\theta^t \right) \right) \right], \quad (2.13)$$

s.t.

$$q_t u' \left(\theta^t + T_t \left(\theta^t \right) \right) - \beta E \left[u' \left(\theta^{t+1} + T \left(\theta^{t+1} \right) \right) | \theta^t \right] = 0, \\ \forall t < T, \theta^t \in \Theta^t.$$

$$\sum_{t=1}^T q_{1,t} T_t \left(\theta^t \right) = \bar{T}, \\ \forall \theta^T \in \Theta^T.$$

and the resource constraint

$$K_0 = 0, K_2 = 0, \\ F(K_{t-1}) - K_1 = E \left[T_t \left(\theta^t \right) \right], \text{ for } t \leq T.$$

Problem 2.13 is equivalent to Problem 2.4, with the substitution of the equivalent incentive compatibility constraints and the use of zero asset holdings for agents on the unobservable asset markets, which is without loss of generality⁸.

2.5.1 Example 1

For the first example, let us assume $u(c)$ belongs to the constant relative risk aversion class with a coefficient of relative risk aversion of $\rho \equiv 3$, under the parametrization $u(c) = \frac{c^{1-\rho}}{1-\rho}$. The intertemporal discount factor is $\beta = 0.95$ and technology is linear with a rate of return $R = \beta^{-1}$. The underlying process for endowments is independent and identically distributed with $\theta_t \in \{1, 1.5\}$ and equal probabilities.

Given this structure, in the self-insurance economy without the tax wedges, agents with $\theta_1 = 1$ are borrowing, while agents with $\theta_1 = 1.5$ are saving. Aggregate savings and capital are positive. Since the distribution of endowment shocks is i.i.d., agents that suffered the high endowment shock at $t = 1$ are intertemporally richer and end up consuming more at the initial period. Therefore, on

⁸If (Γ, K_t, q) are compatible with a feasible allocation, setting $\tilde{a}(\theta^t) = 0$ and $\tilde{T}(\theta^t) = T(\theta^t) - q_t \hat{a}_t(\theta^t) + \hat{a}_{t-1}(\theta^{t-1})$ which transfers the asset holdings to the government leads to the same allocation.

the untaxed economy $Cov(u'(c_1), a_1^*) < 0$ which generates the possibilities of improvements from taxation.

Indeed, in the optimum obtained $q = 1.14$ which leads to an implicit tax wedge of $\tau = 0.2$ on the installation of capital. The welfare gain from the introduction of the wedge is in the order of 0.3% in terms of the expected utility of agents, which given the CRRA assumption is equivalent to the same welfare impact of an increase of 0.15% in the consumption at each state.

The main element that leads to the presence of the tax wedge can be interpreted as the correlation between savings and marginal utility which is caused purely by agents with better histories choosing to buy more assets, as the transition probabilities for the endowment process are the same for all types.

2.5.2 Example 2

All fundamentals are the same as before, except for the stochastic process for the endowment. Now, there are two equiprobable initial shocks $\theta_1 \in \{1 - \varepsilon, 1 + \varepsilon\}$ for small positive ε and two possible final shocks $\theta_2 = \{0.2, 1.8\}$. However, most of the effects of the endowment shocks are informational: agents with $\theta_1 = 1 - \varepsilon$ learn that they are more likely to receive high endowment in the following period, while agents with initially higher endowment learn they are more likely to receive $\theta_2 = 1.8$ in the next period. The transition probabilities are $\pi(1.8|1 - \varepsilon) = 0.9$ and $\pi(1.8|1 + \varepsilon) = 0.1$.

In the untaxed self-insurance economy, there is a positive covariance of marginal utility and savings and $c(\theta_1 = 1 - \varepsilon) > c(\theta_1 = 1 + \varepsilon)$, indicating that agents with good news about their future endowments (low current endowment) consume more at $t = 1$ in anticipation of the higher likelihood of high endowment at $t = 2$.

As expected given the positive covariance, the local optimum found has a negative wedge, which is equivalent to a capital subsidy of $\tau = -0.28$. This leads to an increase in terms of the expected utility of 1%, which, given CRRA preferences, is the same increase that would be achieved if an increase in consumption of 0.5% happened in every state.

In this example, agents with initially (marginally) higher endowment at $t = 1$ have bad news about their future endowments and end up saving more in anticipation. Despite their initially higher wealth, they consume less and are the agents with the highest marginal utility at $t = 1$. To redistribute towards these agents, the planner moves intertemporal prices towards increasing the returns on savings q_1^{-1} faced by consumers. That can be implemented by implementing a capital subsidy $\tau_1 < 0$, which decreases q_1 .

2.6 Conclusion

In the constrained optimal problem, the fictitious planner can choose the contract offered to all agents and controls production subject to incentive compatibility and feasibility. As agents can

retrade away from any consumption stream that the planner could offer, incentive compatibility takes a simple and interesting form. As in (Allen 1985) and (Cole and Kocherlakota 2001), transfer streams offered to households cannot take different net present values at market prices and intertemporal smoothing is required. However, as the planner chooses all contracts in the economy at the same time, the interest rate for any retrades is endogenous.

Due to the simple form of incentive compatibility present in the model, the solution to the planner's problem can also be implemented in a simple decentralized way. It suffices to induce a wedge, which can be introduced through a tax or subsidy on capital installation, separating the financial costs of investment, as perceived by firms, from the interest rates at which agents could reallocate their consumption streams. Therefore, properties of the mechanism design problem and allocations can also be studied through the characterization of economies with incomplete financial markets and simple taxes on firms.

As no redistribution can occur at market prices, insurance becomes a matter of choosing the appropriate prices to manipulate the pecuniary effects they have on ex ante welfare. The redistributive effects of a tax change at each period, around the untaxed economy, are given by the covariance between marginal utility and asset holdings in the cross-section of the population. In an economy with a constant-returns-to-scale production function, a very clear result emerges: it suffices to have a non-zero covariance in any period for the constrained suboptimality of the zero-tax system.

As a final remark, while the feasibility of the introduction of a wedge between the returns on savings faced by the planner and the ones faced by agents are central to the results, adding another concave productive technology, even a non-monitorable backyard technology, is expected to make the analysis more involved, but not revert the equivalence and suboptimality results.

2.7 Appendix: Proofs

Proof. (Proposition 1)

Necessity:

Assume towards a contradiction that some history $\bar{\theta}^T$ receives a transfer stream with a higher NPV at prices q then every other history $\theta^T \in \Theta^T$, with strict inequality for some history that happens with positive probability. Then, set a deviating strategy $\hat{\sigma}$ according to which the agent always reports $\hat{\theta}_t(\theta^t) = \theta_t(\bar{\theta}^T)$, consumes $c_t(\theta^t, \Gamma, q)$ for $t < T$ and consumes the maximum possible c_T . At T and history θ^T , this agent has received, in terms of consumption at $t = 1$,

$$\sum_{t=1}^T q_{1,t} T_t(\theta^t(\bar{\theta}^T)) \geq \sum_{t=1}^T q_{1,t} T_t(\theta^t(\theta^T))$$

while consumed

$$\sum_{t=1}^{T-1} q_{1,t} c_t(\theta^t, \Gamma, q).$$

The availability of resources for consumption at history θ^T can be found through

$$\sum_{t=1}^{T-1} q_{1,t} \left[T_t \left(\theta^t \left(\bar{\theta}^T \right) \right) - c_t \left(\theta^t, \Gamma, q \right) + \theta_t \left(\theta^t \right) \right] + q_{1,T} T_T \left(\theta^T \left(\bar{\theta}^T \right) \right) \quad (2.14)$$

$$\begin{aligned} &\geq \sum_{t=1}^{T-1} q_{1,t} \left[T_t \left(\theta^t \left(\theta^T \right) \right) - c_t \left(\theta^t, \Gamma, q \right) + \theta_t \left(\theta^t \right) \right] + q_{1,T} T_T \left(\theta^T \left(\theta^T \right) \right), \\ & q_{1,T-1} q_{T-1} \left[a_{t-1} \left(\theta^{t-1}, \hat{\sigma}, \Gamma, q \right) \right] + q_{1,T} T_T \left(\theta^T \left(\bar{\theta}^T \right) \right) \end{aligned} \quad (2.15)$$

$$\begin{aligned} &\geq q_{1,T-1} q_{T-1} \left[a_{t-1} \left(\theta^{t-1}, \sigma^{TT}, \Gamma, q \right) \right] + q_{1,T} T_t \left(\theta^t \left(\theta^T \right) \right), \\ & q_{1,T} \left[a_{T-1} \left(\theta^{T-1}, \hat{\sigma}, \Gamma, q \right) + T_T \left(\theta^T \left(\bar{\theta}^T \right) \right) \right] \\ &\geq q_{1,T} \left[a_{T-1} \left(\theta^{T-1}, \sigma^{TT}, \Gamma, q \right) + T_t \left(\theta^T \left(\bar{\theta}^T \right) \right) \right] \end{aligned} \quad (2.16)$$

With $q_{1,T} > 0$, this implies to

$$c_T \left(\theta^T, \Gamma, \hat{\sigma}, q \right) \geq c_T \left(\theta^T, \Gamma, \sigma^{TT}, q \right),$$

for each $\theta^T \in \Theta^T$, with at least one inequality strict for a history that has positive probability. Therefore,

$$V \left(\hat{\sigma}, \Gamma, q \right) > V \left(\sigma^{TT}, \Gamma, q \right)$$

and the desired contradiction is reached. That implies that all transfer streams have the same NPV

$$\begin{aligned} \sum_{t=1}^T q_{1,t} T_t \left(\theta^t \left(\theta^T \right) \right) &= \sum_{t=1}^T q_{1,t} T_t \left(\theta^t \left(\tilde{\theta}^T \right) \right), \\ \forall \theta^T, \tilde{\theta}^T &\in \Theta^T. \end{aligned} \quad (2.17)$$

Now, let us reach the first-order condition for savings. Obedience along the equilibrium path requires

$$u \left(\theta_t + T_t \left(\theta^{t-1} \right) - q_t \hat{a}_t \left(\theta^t \right) + \hat{a}_{t-1} \left(\theta^{t-1} \right) \right) + \beta E \left[V_{t+1} \left(\Gamma, \sigma^{TT}, q, \theta^{t+1} \right) \mid \theta^t \right] \geq V_t \left(\Gamma, \sigma, q, \theta^t \right), \quad \forall \sigma \in \Sigma.$$

In particular, for a continuation strategy that involves a deviation setting $a_t \left(\theta^t \right) = a_t \left(\theta^t \right) + \varepsilon$ after history θ^t and reverts back to truth-telling and obedience not to be profitable it is necessary and sufficient that

$$u' \left(c_t \left(\theta^t, \Gamma, q \right) \right) = \frac{\beta}{q_t} E \left[u' \left(c_{t+1} \left(\theta^t, \Gamma, q \right) \right) \mid \theta^t \right]. \quad (2.18)$$

Now, we prove the sufficiency part of the main statement. If

$$\sum_{t=1}^T q_{1,t} T_t (\theta^t (\theta^T)) = \sum_{t=1}^T q_{1,t} T_t (\theta^t (\tilde{\theta}^T)),$$

$$\forall \theta^T, \tilde{\theta}^T \in \Theta^T.$$

and

$$u' (c_t (\theta^t, \Gamma, q)) = \frac{\beta}{q_t} E [u' (c_{t+1} (\theta^t, \Gamma, q)) | \theta^t],$$

$$\forall t < T, \theta^t \in \Theta^t,$$
(2.19)

then

$$V (\sigma^{TT}, \Gamma, q) \geq V (\sigma, \Gamma, q), \forall \sigma \in \Sigma.$$

We start from any strategy $\sigma \in \Sigma$ and we might split the problem in two parts: a revelation part and a distribution of consumption or savings part. Take a strategy $\sigma' \in \Sigma$ that reports shocks in the same way as σ , but also saves optimally. As a consequence,

$$V (\sigma', \Gamma, q) \geq V (\sigma, \Gamma, q).$$

Let

$$T \equiv \sum_{t=1}^T q_{1,t} T_t (\theta^t (\theta^T)) = \sum_{t=1}^T q_{1,t} T_t (\theta^t (\tilde{\theta}^T)),$$

$$\forall \theta^T, \tilde{\theta}^T \in \Theta^T,$$

which implies

$$T = \sum_{t=1}^T q_{1,t} [c_t (\theta^t, \sigma, \Gamma, q) - \theta^t],$$

$$\forall t \leq T, \theta^t \in \Theta^t$$

Then $c_t (\theta^t, \sigma, \Gamma, q)$ solves

$$\max_{c_t (\theta^t, \sigma, \Gamma, q)} E [\beta^{t-1} u (c_t)]$$

$$s.t.$$

$$T + q_{1,t} \theta_t \geq \sum q_{1,t} c_t (\theta^t, \sigma, \Gamma, q).$$
(2.20)

The solution is characterized by the necessary and sufficient FOC (2.19) and (2.20). The set of equations is also satisfied by $c_t (\theta^t, \sigma^{TT}, \Gamma, q)$ and has a unique solution due to the strict concavity

of $u(\cdot)$. Therefore, it follows that

$$V(\sigma^{TT}, \Gamma, q) = V(\sigma', \Gamma, q) \geq V(\sigma, \Gamma, q).$$

□

Proof. (Proposition 3) The constrained optimal allocation can be described by the functions $c_t^*(\theta^t) = \theta_t + T_t^*(\theta^t)$, plus the choice of capital path $\{K_t^*\}_{t=1}^{T-1}$. In that allocation, all agents have their MRS between consumption at t and consumption at $t+1$ equalized, so that no further trades would be possible

$$\frac{1}{q_t} = \frac{u'(c_t(\theta^t))}{\beta E[u'(c_{t+1}(\theta^{t+1})) | \theta^t]}, \forall t < T, \theta^t \in \Theta^t.$$

To find a policy p , first set

$$\begin{aligned} (1 + \tau_t) &\equiv q_t F'(K_t^*), \\ \gamma_t &= 0, \forall t < T, \\ q_t B_t &= \sum_{s=1}^{t-1} q_{s,t}^{-1} (-F(K_{s-1}^*) + K_s^*) - \tau_t K_t^*, \end{aligned} \tag{2.21}$$

and

$$\begin{aligned} \gamma_T &\equiv \sum_{s=1}^T q_{s,T}^{-1} T_s^*(\theta_t) \equiv q_{1,T}^{-1} \bar{T} \\ &= \sum_{s=1}^T q_{s,T}^{-1} (c_t^*(\theta^t) - \theta_t), \end{aligned}$$

which does not depend on θ^T due to incentive compatibility.

Let $\{a_t(\theta^t, q, \gamma)\}_{t, \theta^t \in \Theta^t}$ be the solution to Problem 2.1, the individual savings problem.

Now, it is necessary to verify that a competitive equilibrium with the same consumption distribution will be achieved.

i) The individual savings problem leads to the same consumption allocation as the planner's problem.

It is characterized by the solution to

$$u'(c_t(\theta^t)) = \frac{\beta}{q_t} E[u'(c_{t+1}(\theta^{t+1})) | \theta^t], \forall t < T, \theta^t \in \Theta^t \tag{2.22}$$

and

$$\begin{aligned} a_0 &= 0 \\ c_t(\theta^t) &= \theta_t + a_{t-1}(\theta^{t-1}) - q_t a_t(\theta^t), \\ c_T(\theta^T) &= \theta_T + a_{T-1}(\theta^{T-1}) + \gamma_T. \end{aligned}$$

It is possible to isolate each $a_t(\theta^t)$ and write the set of constraints above as a single present value version

$$\begin{aligned} \sum_{t=1}^T q_{1,t} [c_t(\theta^t) - \theta_t] &= q_{1,T} \gamma_T \\ &= \bar{T}. \end{aligned} \tag{2.23}$$

Since u is strictly concave the solution to 2.22 and 2.23 is unique. As, by construction $q_{1,T} \gamma_T = \bar{T}$, the distribution of consumption which solves this system is the initial $\{c_t^*(\theta^t)\}_{t, \theta^t \in \Theta^t}$ that solves the constrained Pareto optimality problem. We can iterate forward using $a_0 = 0$ and $a_t(\theta^t) = q_t^{-1}(\theta_t + a_{t-1}(\theta^{t-1}) - c_t^*(\theta^t))$ to find the solution of the savings problem terms of asset sequence $\{a_t(\theta^t, q, \gamma)\}_{t, \theta^t \in \Theta^t}$.

ii) Representative firm maximization and profits. The representative firm of time t solves

$$\max_{K_t} F(K_t) - q_t^{-1} (1 + \tau_t) K_t.$$

The solution is characterized by the FOC

$$F'(K_t^*) = q_t^{-1} (1 + \tau_t),$$

which coincides with (2.21) and leads to profits

$$\pi_{t+1}^{prod} = F(K_t^*) - q_t^{-1} (1 + \tau_t) K_t^*,$$

generated at period $t + 1$ when production occurs.

iii) Market Clearing in Goods at each t . At $t = 1$, from the feasibility of the constrained optimal allocation

$$\begin{aligned} K_0^* &= 0, K_T^* = 0, \\ F(K_{t-1}^*) + E[\theta_t] - K_t^* &= E[c_t^*(\theta^t)], \text{ for } t < T. \end{aligned}$$

Since the consumption distribution is the same, as well as $\{K_t^*\}_{t=1}^{T-1}$ market clearing follows trivially from feasibility.

iv) Market Clearing in Bonds at t ,

$$\begin{aligned} a_t(\theta^t, q, \gamma) &= \sum_{s=1}^t q_{s,t}^{-1} (\theta_s - c_s^*(\theta^s)) \\ E[a_t(\theta^t, q, \gamma)] &= \sum_{s=1}^t q_{s,t}^{-1} E[(\theta_s - c_s^*(\theta^s))] \\ &= \sum_{s=1}^t q_{s,t}^{-1} [K_s^* - F(K_{s-1}^*)]. \end{aligned}$$

That implies

$$E[a_t(\theta^t, q, \gamma)] = B_t + \frac{(1 + \tau_t) K_t}{q_t},$$

where $\frac{(1+\tau)K_t}{q_t}$ are the bond issues necessary to cover the financing needs of the representative firm that produces at $t + 1$.

v) Government's Budget. We have to verify that

$$q_1 B_1 = -\tau_1 K_1 \tag{2.24}$$

$$q_t B_t = -\tau_t K_t - \pi_t^{prod} + B_{t-1}, \tag{2.25}$$

$$\gamma_T = -B_{T-1} + \pi_T^{prod} \tag{2.26}$$

are respected for the proposed allocation. 2.24 is trivially satisfied. Substituting for B_{t-1} and π_t^{prod} on 2.25,

$$\begin{aligned} q_t B_t &= -\tau_t K_t - F(K_{t-1}^*) + q_t^{-1} (1 + \tau_{t-1}) K_{t-1}^* \\ &\quad + q_{t-1}^{-1} \left[\sum_{s=1}^{t-1} q_{s,t-1}^{-1} (-F(K_{s-1}^*) + K_s^*) - \tau_{t-1} K_{t-1}^* \right] \\ &= -\tau_t K_t - F(K_{t-1}^*) + q_{t-1,t}^{-1} K_{t-1}^* + \sum_{s=1}^{t-2} q_{t-1}^{-1} q_{s,t-1}^{-1} (-F(K_{s-1}^*) + K_s^*) \\ &= -\tau_t K_t + \sum_{s=1}^{t-1} q_{s,t}^{-1} (-F(K_{s-1}^*) + K_s^*), \end{aligned}$$

as it was necessary to verify.

Finally, we verify 2.26,

$$\begin{aligned} \gamma_T &= q_{T-1}^{-1} \tau_{T-1} K_{T-1} - q_{T-1}^{-1} \sum_{t=1}^{T-1} q_{t,T-1}^{-1} (-F(K_{t-1}^*) + K_t^*) + \pi_T^{prod} \\ &= \sum_{t=1}^{T-1} q_{t,T}^{-1} (F(K_{s-1}^*) - K_s^*) + F(K_{T-1}^*), \end{aligned}$$

which implies

$$q_{1,T}\gamma_T = \sum_{t=1}^T q_{1,t} [F(K_{t-1}) - K_t].$$

We can use the individual present value budget constraint, obtained as (2.23) to write

$$\sum_{t=1}^T q_{1,t} [c_t^*(\theta^t) - \theta_t] = \sum_{t=1}^T q_{1,t} E [c_t^*(\theta^t) - \theta_t] = \sum_{t=1}^T q_{1,t} [F(K_{t-1}^*) - K_t^*],$$

which holds as a consequence of period-by-period feasibility (2.6) of the initial $(c_t^*(\theta^t), K_t)_{t, \theta^t \in \Theta^t}$ \square

Proof. (Proposition 4)

First, we might write the individual saving problem with taxes in recursive form as

$$\begin{aligned} V_t(\theta^t, a_{t-1}, q, \gamma) &= \max u(\theta_t + \gamma_t + a_{t-1} - q_t a_t) + \beta E [V_{t+1}(\theta^{t+1}, a_t, q, \gamma) | \theta^t] \\ V_T(\theta^T, a_{T-1}, q, \gamma) &= \theta_T + a_{T-1} + \gamma_T \end{aligned}$$

So,

$$\begin{aligned} \frac{\partial V_t}{\partial q_s} &= \begin{cases} 0, & \text{if } s < t \\ -u'(c_t(\theta^t)) a_t^*, & \text{if } s = t \\ \beta E \left[\frac{\partial V_{t+1}(\theta^{t+1}, a_t, q, T)}{\partial q_s} | \theta^t \right] \end{cases} \\ \frac{\partial V_t}{\partial \gamma_s} &= \begin{cases} 0, & \text{if } s < t \\ u'(c_t(\theta^t)), & \text{if } s = t \\ \beta E \left[\frac{\partial V_{t+1}(\theta^{t+1}, a_t, q, T)}{\partial \gamma_s} | \theta^t \right] \end{cases} \\ \frac{\partial V_0}{\partial q_s} &= \beta^{s-1} E [-u'(c_s(\theta^s)) a_s^*(\theta^t)] \\ \frac{\partial V_0}{\partial \gamma_s} &= \beta^{s-1} E [u'(c_s(\theta^s))] \end{aligned}$$

\square

Chapter 3

Runs on debt and the role of transparency

(joint with Plamen T. Nenov)

Abstract

Financial instability is often characterized by increased uncertainty, debt rollover difficulties and asset liquidation at depressed prices. The present chapter studies a debt roll-over coordination game with dispersed information and a market-determined liquidity scenario. We describe conditions under which an improvement in the precision of individual information about financial institutions' fundamentals leads to greater financial stability. For the limiting case of arbitrarily precise private information that condition obtains a simple form in terms of payoff elasticities. Conversely, we characterize when an increase in uncertainty leads to a higher frequency of debt runs and show how this deleterious effect is amplified by the deterioration of prices for liquidated assets.

Key words: transparency, stress tests, living wills, bank runs, fire sales, global games
JEL Codes: E44, G01

3.1 Introduction

Financial crises are commonly associated with an increase in uncertainty about asset quality¹. The episode of 2007-2009 is no exception, given the substantial increase in uncertainty about the quality of various asset-backed securities combined with opacity of financial firms' portfolios². A leading narrative for the cause of financial crises, including the most recent one, relates the resulting lower quality of information about portfolios to the incentives of depositors and short-term lenders to run on them by refusing to roll over their demand deposits or loans³. This triggers asset sales in markets with limited absorption capacity, leading to fire sale effects⁴, which, in turn, deepen the crisis by spreading the runs onto healthier financial institutions, leading to a potential collapse of the whole financial system. Such a view of the nature of financial crises would call for a policy response aimed at increasing transparency of financial intermediaries' balance sheets.

This chapter theoretically evaluates whether deterioration in information quality can indeed lead to a higher failure rate of financial intermediaries and examines how fire sales can amplify such shocks. We study a model of the financial system with the risk of coordination failures by short-term lenders that leads to bank runs and failure. In the model, short-term lenders make rollover decisions but face a complementarity in their actions through the ability of their respective financial intermediary to withstand a run and bring long-term projects to completion. However, short-term lenders have imperfect information about the quality of their respective financial institution, which makes them uncertain about the actions of other lenders but also directly affects their own expected payoffs conditional on bank survival or failure. Additionally, an asset market in which banks liquidate assets creates a systemic link between the rollover decisions of short-term lenders in different institutions.

We derive conditions, under which more precise information by lenders improves financial stability, by reducing the likelihood of rollover difficulties. If differences in financial intermediaries' portfolios lead to differences in the "upside" risk associated with a higher probability of repayment of long-term lenders conditional on bank survival, then increasing the uncertainty about banks' portfolios lowers bank failure rates. In this case, lenders effectively become more optimistic about the value of each bank's promises conditional on survival. Conversely, if portfolio differences mostly drive differences in "downside" risk associated with lower repayment of long-term relative to short-term lenders in the case of bank failure, more precise information improves financial stability. In this case, higher uncertainty about portfolio qualities increases the likelihood of intermediary failures. In the limiting case of arbitrarily precise information, we show that the condition boils down

¹(Mishkin 1991)

²(Flannery, Kwan, and Nimalendran 2010).

³See, for example, (Gorton 2010). As (Gorton 2010) notes about previous banking crises: "The problem was that no one outside the banking system knew which banks were the weak banks, which banks were risky. Even other banks might not have known. Without knowing which specific banks were the riskiest, depositors were cautious and withdrew their cash from all banks."

⁴(Shleifer and Vishny 2011).

to a simple rule relating the relative sensitivities of the payoffs from rollover and running with respect to changes in bank fundamentals.

After clarifying when increases in transparency can be beneficial for improving financial stability we proceed to examine how limited demand for liquidated assets interacts with shocks to portfolio uncertainty. In particular, the adverse effects of increases in portfolio uncertainty on bank failure are amplified endogenously through asset markets. The dependence of a bank's ability to survive a run of short-term creditors on asset prices creates a downward sloping supply curve for bank assets, which combined with an elastic demand curve, leads to fire sales and amplification. Thus, increases in uncertainty can have important systemic effects.

The issues of transparency, complexity, and quality of information in the context of the recent financial crisis have attracted much recent interest both among academics, aiming to understand the causes of the crisis, as well as among policymakers. Two particularly important recent papers on this topic are (Dang, Gorton, and Holmstrom 2009) and (Caballero and Simsek 2012). The first paper examines the effects of asymmetric information on aggregate liquidity provision. It shows that more information actually reduces welfare as it reduces trade between agents. Debt contracts are optimal for the provision of liquidity as they minimize the incentives of agents to become privately informed and maximizes trade. Therefore, increasing transparency in this setting or increasing agents' incentives to acquire more precise information reduces welfare. The second paper shows how complexity, defined as a financial intermediary's uncertainty about its cross-exposure and counterparty risk amplifies the effect of shocks in the financial system and interacts with secondary asset markets to create fire sale events. In that context, reduction in uncertainty has a beneficial impact for the financial system. Our work complements this growing literature by studying how changes in the quality of individual information can influence the coordination problem implicit in any run or roll over crisis episode. In this way, it clarifies when increasing portfolio transparency, for example, through stress testing is desirable as a tool for improving financial stability. Additionally, it provides an analysis of fire sales in intermediation models featuring coordination as a central element, showing how fire sales can endogenously amplify uncertainty shocks in these models and exacerbate any deleterious effect of uncertainty.

The chapter is also related to models of bank runs ((Diamond and Dybvig 1983)) and the effect of idiosyncratic information on the coordination problem of depositors ((Goldstein and Pauzner 2005) and (Rochet and Vives 2004)). However, it differs in its modeling approach and in the focus of our analysis, which studies the effects of quality of information and transparency, particularly in the presence of an asset market. The model is similar to (Morris and Shin 2004), who examine the effect of coordination risk on the price of corporate debt has a different focus and conclusions. In its emphasis on the effect of information on equilibrium behavior in a coordination game, the chapter is closely related to the work by (Morris and Shin 2002) and (Angeletos and Pavan 2007) who consider the welfare effects of more precise public and private information in economies with strategic complementarities.

The chapter is most closely related to recent work by (Moreno and Takalo 2011), who investigate the effect of transparency on a model with bank runs and find that increasing transparency has an *unambiguously* negative effect on the probability of bank runs in their framework. In contrast to them, we find that this effect is ambiguous and that increasing transparency during a financial crisis may in fact *reduce* the probability of bank runs and the severity of a crisis. Additionally, we examine the amplification effect that fire sales have on increases in uncertainty in this framework.

The chapter is organized as follows. Section 3.2 studies the basic model of a financial system subject to the risk of run by short-term lenders. Section 3.3 presents the main result of this work, which states conditions under which transparency reduces the probability of bank runs. Section 3.4 extends the model by including an asset market that determines liquidity conditions and shows the amplification effect that fire sales have on shocks to information quality in the model. Section 1.6 provides a discussion of optimal transparency policy in our framework, as well as brief concluding remarks.

3.2 Model with exogenous liquidity conditions

3.2.1 Set-up

The economy lasts for two periods: $t = 1, 2$. In this economy there exists a set of financial intermediaries, which are indexed by $i \in [0, 1]$. Each of these has originally invested in an independent project, that delivers random returns and only fully matures at $t = 2$.

Each intermediary has a relationship with a continuum unit measure of lenders, which hold claims on the project. These claims are in the form of short-term debt, which gives each lender the option to roll it over or not. Formally, a debt holder has two actions available at $t = 1$. She can either refuse ($a = 0$) or agree ($a = 1$) to roll over the current debt. Because of the mismatch in the maturity structure of assets and liabilities, intermediaries are potentially illiquid: too many refusals to roll over their current debt lead to failure at $t = 1$.

For each intermediary, there are three relevant events: failure at $t = 1$ due to rollover difficulties, failure at $t = 2$ due to project failure, or successful completion at $t = 2$. The probabilities of these events are influenced by the strength of the bank's fundamentals, $\theta_i \in [0, 1]$, with $\theta_i \sim U[0, 1]$, and by a measure of aggregate market liquidity, l . The index l identifies the facility in liquidating a limited volume of the project's assets, generating revenues for the payment of agents who refuse to roll over, or in finding other sources of funding.⁵

There is imperfect information about the fundamentals of projects. Each debt holder j of intermediary i , receives a signal $\theta_{ij} = \theta_i + \eta_{ij}$, where $\eta_{ij} \sim U[-\epsilon, \epsilon]$, with $\epsilon > 0$ and relatively small. This particular structure is helpful in disciplining both beliefs about fundamentals and about the information obtained by other agents and their likely actions.

⁵For the model in this section l is held fixed. However, in Section 3.4 we allow for a feedback from bank's asset liquidations in to l to capture fire sale events.

We proceed to the formal description of the production technology held by intermediaries. There is limited reversibility at $t = 1$, which means that the technology is only capable of generating a limited amount of resources without leading to early termination. Formally, each intermediary fails at $t = 1$ if the proportion of agents choosing to allow the rollover of its debt (x) is less than the cut-off $g(\theta_i, \Gamma)$. This cut-off is jointly affected by fundamentals (θ_i) and by a vector $\Gamma \equiv (R, K, l)$. This vector specifies both aggregate liquidity conditions (l) and elements of the previously designed debt contract, soon to be described. Therefore, failure occurs whenever $x \leq g(\theta_i, \Gamma)$. The cut-off function $g(\theta_i, l)$ is continuously differentiable and naturally satisfies $\frac{\partial g(\theta_i, \Gamma)}{\partial \theta} \leq 0$, $\frac{\partial g(\theta_i, \Gamma)}{\partial l} \leq 0$, and $\frac{\partial g(\theta_i, \Gamma)}{\partial K} \geq 0$, where K is the payoff that a lender collects if she refuses to roll over and the bank does not fail (see Table 3.1 below).

If the intermediary fails at $t = 1$, lenders who choose to roll over receive a liquidation payoff $\chi_{a=1}(\theta)$ and lenders who refuse to roll over receive a payoff $\chi_{a=0}(\theta)$. $\chi_{a=1}(\theta)$ and $\chi_{a=0}(\theta)$ are continuously differentiable and satisfy $\frac{d\chi_{a=0}}{d\theta} - \frac{d\chi_{a=1}}{d\theta} < 0$, that is, the higher the bank's fundamentals the lower the net payoff difference across lender types.

A project that survives to period $t = 2$ succeeds and generates a return R for each remaining debt holder with probability $p(\theta)$. It fails to deliver any output with probability $(1 - p(\theta))$. These payoffs as functions of relevant events are represented in Table 3.1.

	Refuse	Accept
Project Fails at $t = 1$	$\chi_R(\theta)$	$\chi_A(\theta)$
Project Fails at $t = 2$	K	0
Project is successful	K	R

Table 3.1: Payoffs

We assume that $\chi_{a=1}(\theta) \leq \chi_{a=0}(\theta) \leq K < R$. Since what ultimately determines a lender's decisions is the difference in payoffs between the two available actions, we define $k(\theta) = \chi_{a=0}(\theta) - \chi_{a=1}(\theta) > 0$ as the net payoff from running in case of bank liquidation. Given the properties of $\chi_{a=1}$ and $\chi_{a=0}$ it follows that $k(\cdot)$ is continuously differentiable and decreasing in θ . The assumption of a 0 payoff for agents that roll over when the project fails at $t = 2$ can be obtained from normalizations and does not lead to any loss of generality. Then, the net payoff from rolling over versus running is given by:

$$\pi(\theta, x, \Gamma) = \begin{cases} -k(\theta), & \text{if } x < g(\theta, \Gamma) \\ p(\theta)R - K, & \text{if } x \geq g(\theta, \Gamma) \end{cases} \quad (3.1)$$

or

$$\pi(\theta, x, \Gamma) = -k(\theta) \mathbf{I}_{x < g(\theta, \Gamma)} + (1 - \mathbf{I}_{x < g(\theta, \Gamma)}) (p(\theta)R - K) \quad (3.2)$$

It is represented in Figure 3-1.

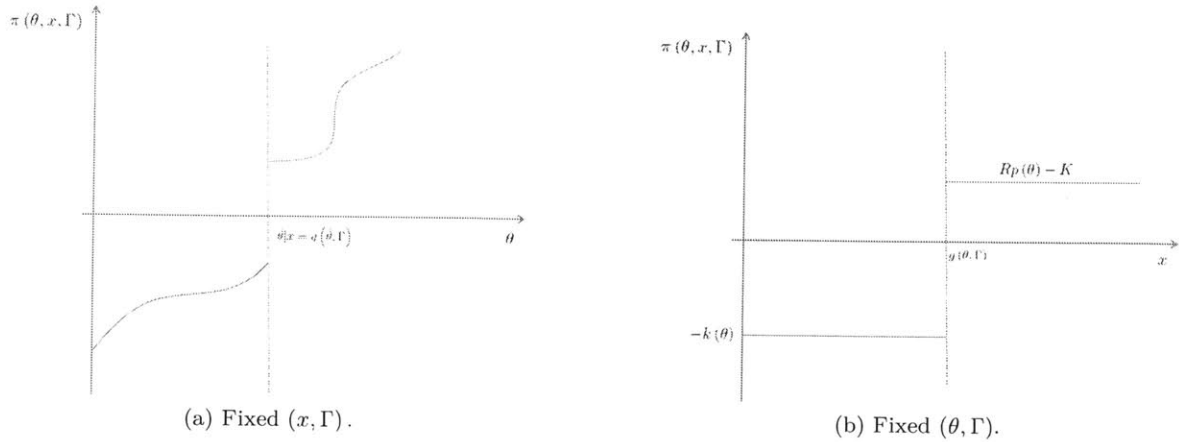


Figure 3-1: Net payoff from rollover decision, $a = 1$.

3.2.2 Dominance regions

We make the following set of assumptions about the properties of the underlying economy.

- A1.** There exists $\underline{\theta}$, such that for all $\theta < \underline{\theta}$ and all Γ , $g(\theta, \Gamma) = 1$.
- A2.** There exists $\bar{\theta}$, such that for all $\theta > \bar{\theta}$ and all Γ , $g(\theta, \Gamma) = 0$ and $Rp(\theta) - K > 0$.
- A3.** $\underline{\theta} > \epsilon$ and $(1 - \bar{\theta}) > \epsilon$.

(A1) ensures the existence of a lower dominance region, i.e., a region in which fundamentals are so weak that, in a perfect information benchmark, refusing to roll over debt becomes a strictly dominant action. (A2) ensures that an upper dominance regions exists: for sufficiently high fundamentals rollover is a strictly dominant action. In principle, both dominance regions can be made arbitrarily small as long as l assumption (A3) is satisfied. We impose assumption (A3) to ensure that whenever a signal $\theta_{ij} \in [\underline{\theta}, \bar{\theta}]$ is received, the support of the posterior about the θ fundamental is bounded away from zero and one. This avoids unnecessary complications in the Bayesian updating process that would emerge close to the extremes of the unit interval.

3.2.3 Examples

In this section, we provide two model environments that satisfy the conditions described previously. These illustrate different illiquid investment technologies, with specific failure thresholds, which when funded with short-term liabilities might lead to liquidity crises and inefficient liquidation.

Example 3.1. A bank holds a set of assets, including a resellable portfolio and intangibles with limited transferability, that jointly offer a return of R with probability $p(\theta)$. Of this set, $a(\theta)$ are liquid assets that can be sold at $t = 1$ at a price of l , where $\theta \in [0, 1]$ parametrizes the strength of the bank's portfolio. Note that $a(\theta)$ is a smooth increasing function of θ . The bank has a measure

one of short-term lenders, who, in case of refusal to roll over, receive payoff K as long as the bank has not failed. Otherwise, they receive $0 < k(\theta) < K$, where $k(\theta)$ is decreasing in θ . If they roll over, they receive R in case the project is successful and 0 otherwise. The bank pays off creditors that refuse to roll over by selling liquid assets from its portfolio. The bank fails whenever it runs out of liquid assets given the demand of creditors for repayment, i.e., $K(1-x) \geq l \cdot a(\theta)$ which implies $g(\theta, \Gamma) \equiv 1 - \frac{l}{K}a(\theta)$. Note that $g_\theta < 0$, $g_l < 0$, and $g_K > 0$.

Example 3.2. Similarly to Example 1, each bank has a productive asset that pays off R with probability $p(\theta)$ at $t = 2$ and delivers a $t = 1$ cash-flow of $f(\theta)$ ⁶, and a measure a of liquid assets that can be sold at $t = 1$ at a price of l . Note that $f(\theta)$ is a smooth increasing function of θ . The bank has a measure one of short-term lenders that choose to roll over their short-term debt at $t = 1$ up until $t = 2$. If they refuse to roll over they receive payoff K as long as the bank has not failed, otherwise they receive $0 < k(\theta) < K$, where $k(\theta)$ is decreasing in θ . If they roll over they receive R in case the project is successful and 0 otherwise. The bank pays off creditors that refuse to roll over by selling liquid assets from its portfolio. The bank fails whenever it runs out of liquid assets given the demand of creditors for repayment, i.e. $K(1-x) \geq f(\theta) + l \cdot a$, which defines the threshold $g(\theta, \Gamma) \equiv 1 - \frac{f(\theta)+l \cdot a}{K}$. Note that $g_\theta < 0$, $g_l < 0$, and $g_K > 0$.

3.2.4 Equilibrium

We first study the properties of the Bayesian Nash Equilibrium in the rollover game, taking as given aggregate liquidity conditions. This allows us to characterize outcomes and the extent of inefficient intermediary failure in a partial equilibrium setting. In Section 3.4.1, we study the consequences from the endogenous determination of aggregate liquidity conditions.

We define a Bayesian Nash Equilibrium for the rollover game, in which the vector Γ is taken as given, as follows.

Definition 3.1. A Bayesian Nash Equilibrium (BNE) for the rollover game consists of a strategy $a : \Theta \rightarrow \{0, 1\}$ and a fraction $x : \Theta \rightarrow [0, 1]$ of lenders that roll over at $t = 1$ s.t.

1. $a(\theta_i)$ solves $a = 1$ if $E_{\theta|\theta_i}[\pi(\theta, x(\theta), \Gamma)] > 0$, $a = 0$ if $E_{\theta|\theta_i}[\pi(\theta, x(\theta), \Gamma)] < 0$ and $a \in \{0, 1\}$ if $E_{\theta|\theta_i}[\pi(\theta, x(\theta), \Gamma)] = 0$;
2. $x(\theta) = E_{\theta_i|\theta}[a(\theta_i)]$.

We focus on equilibria involving cutoff strategies such that $a(\theta_i) = 1$ iff $\theta_i \geq \theta^*$. Equilibria in cut off strategies can be characterized by making use of Proposition 3.1 below, which ensures the existence of a unique equilibrium threshold.

⁶Allowing for negative cash-flows, interpreted as additional financial distress resource requirements, is necessary to allow the existence of a lower dominance region.

Proposition 3.1. *Every BNE in cutoff strategies of this economy can be described by a unique threshold θ^* which solves*

$$\int_0^1 \pi(\theta^* + \epsilon - 2\epsilon \cdot x, x, \Gamma) dx = 0. \quad (3.3)$$

Proof. In the Appendix. □

The proof of Proposition 3.1 follows the intuition for uniqueness results in global games ((Morris and Shin 2001)). A lender at the cutoff θ^* has Laplacian beliefs⁷ about the fraction of other lenders who roll over. Since that lender is indifferent between rolling over and running, her expected net payoff given these posterior beliefs equals zero, which gives Equation (3.3). Lenders who observe lower signals than θ^* are more pessimistic about both the bank's fundamentals and the expected fraction of other lenders who roll over and, in turn, prefer to run. The opposite holds for lenders who observe signals greater than θ^* .

A related way to characterize equilibria is to look at the threshold, θ^f , at which banks fail. Given an equilibrium with a cutoff θ^* , the share of agents willing to roll over debt is a smooth function of the state, as given by

$$x(\theta, \theta^*(\Gamma)) = \begin{cases} 1 & , \theta > \theta^*(\Gamma) + \epsilon, \\ \frac{\theta - (\theta^*(\Gamma) - \epsilon)}{2\epsilon} & , \theta \in [\theta^*(\Gamma) - \epsilon, \theta^*(\Gamma) + \epsilon], \\ 0 & , \theta < \theta^*(\Gamma) - \epsilon. \end{cases}$$

Therefore, θ^f satisfies

$$\frac{\theta^f - (\theta^* - \epsilon)}{2\epsilon} = g(\theta^f, \Gamma), \quad (3.4)$$

or, equivalently

$$\theta^f = 2\epsilon g(\theta^f, \Gamma) + \theta^* - \epsilon. \quad (3.5)$$

A lender that observes a signal θ^* believes that $\theta \sim U[\theta^* - \epsilon, \theta^* + \epsilon]$. Since such a lender is indifferent between rolling over or not, we have that

$$E_{\theta^*} [k(\theta) | \theta < \theta^f] \cdot Pr_{\theta^*}(\theta < \theta^f) + K Pr_{\theta^*}(\theta > \theta^f) = E_{\theta^*} [Rp(\theta) | \theta \geq \theta^f] Pr_{\theta^*}(\theta > \theta^f), \quad (3.6)$$

in which both expectations and probabilities are taken with respect to the posterior belief of the agent that received the cut off signal. The left-hand side represents the payoffs of refusing to roll over, while the right-hand side represents the payoffs from rolling over debt. Expressions (3.4) and (3.6) jointly determine two equilibrium cutoffs: a failure state and a rollover trigger.⁸ Substituting

⁷That is, agents with $\theta_{i,j} = \theta^*$ believe $x(\theta, \Gamma)$ to be uniformly distributed on the $[0, 1]$ interval.

⁸Note that from equation (3.5) it follows that $\theta^f < \theta^* \iff g(\theta^f, \Gamma) < \frac{1}{2}$.

for θ^* in equation (3.6), we get the following equation for the failure cutoff θ^f :

$$\int_{\theta^f}^{\theta^f + 2\epsilon(1-g(\theta^f, \Gamma))} \frac{1}{2\epsilon} Rp(\theta) d\theta = \int_{\theta^f - 2\epsilon g(\theta^f, \Gamma)}^{\theta^f} \frac{1}{2\epsilon} k(\theta) d\theta + K(1 - g(\theta^f, \Gamma)). \quad (3.7)$$

Note that $g(\theta^f, \Gamma)$ has an important interpretation as a probability. If we consider the probability that a lender observing the threshold signal θ^* assigns to his bank failing we have that

$$Pr_{\theta^*}(\theta < \theta^f) = \int_{\theta^* - \epsilon}^{\theta^f} \frac{1}{2\epsilon} d\theta = \int_{\theta^f - 2\epsilon g(\theta^f, \Gamma)}^{\theta^f} \frac{1}{2\epsilon} d\theta = g(\theta^f, \Gamma) \quad (3.8)$$

This probability is distinct from the prior probability that a bank fails,

$$Pr(\theta < \theta^f) = \theta^f. \quad (3.9)$$

In fact, the two probabilities move in opposite directions with θ^f with the former decreasing in θ^f and the latter increasing in θ^f . This distinction will be important for the discussion of the effects of transparency in Section 3.3 below.

Additionally, note that in a limit economy, where $\epsilon \rightarrow 0$, the two strategic and failure thresholds converge, so that $\theta_{\epsilon=0}^* = \theta_{\epsilon=0}^f$. They are determined from

$$0 = [Rp(\theta_{\epsilon=0}^f) - K](1 - g(\theta_{\epsilon=0}^f, \Gamma)) - k(\theta_{\epsilon=0}^f)g(\theta_{\epsilon=0}^f, \Gamma), \quad (3.10)$$

which can be interpreted in the following way. The agent who receives the cut off signal knows the type of his intermediary to be arbitrarily close to $\theta_{\epsilon=0}^*$. However, there is still residual uncertainty about where she ranks in the distribution of posteriors about such fundamental, so that strategic uncertainty about the action of other agents is still present in the limit. That agent therefore believes that the probability of failure of his intermediary is $g(\theta_{\epsilon=0}^*, \Gamma)$, according to equation (3.8), so that the term on the right-hand side is simply the net payoff difference between rolling over debt, which has an expected payoff of $Rp(\theta_{\epsilon=0}^*)(1 - g(\theta_{\epsilon=0}^*, \Gamma))$, and refusing to do so, which has an expected payoff of $K(1 - g(\theta_{\epsilon=0}^*, \Gamma)) + k(\theta_{\epsilon=0}^*)g(\theta_{\epsilon=0}^*, \Gamma)$.

3.3 Understanding the role of noise

How does the quality of lenders' information about the institution's portfolio affect their rollover decisions? When does more transparency decrease the probability of bank failure and can it ever increase it? In this section we address these issues. We look at the effect of private information precision on the failure threshold, θ^f in our general framework. The following proposition provides an answer to this question for the case of small amounts of idiosyncratic uncertainty.

Proposition 3.2. *Consider the above model of investment financing with short-term debt and let*

θ^f be the threshold of fundamentals for which failure occurs at $t = 1$. Then $\frac{\partial \theta^f}{\partial \epsilon} > 0$ iff

$$\begin{aligned} (1 - g(\theta^f, \Gamma)) \{Rp(\theta^f + 2\epsilon(1 - g(\theta^f, \Gamma))) - E_{\theta^*} [Rp(\theta) | \theta > \theta^f]\} < \\ < g(\theta^f, \Gamma) \{k(\theta^f - 2\epsilon g(\theta^f, \Gamma)) - E_{\theta^*} [k(\theta) | \theta < \theta^f]\} \end{aligned} \quad (3.11)$$

Furthermore, $\lim_{\epsilon \rightarrow 0} \left(\frac{\partial \theta^f}{\partial \epsilon}\right) > 0$ iff

$$(1 - g(\theta_{\epsilon=0}^f, \Gamma))^2 R \cdot p'(\theta_{\epsilon=0}^f) < g(\theta_{\epsilon=0}^f, \Gamma)^2 \|k'(\theta_{\epsilon=0}^f)\| \quad (3.12)$$

Proof. See Appendix. □

Proposition 3.2 gives a clear condition under which more transparency lowers the liquidation threshold. What is the intuition for this condition? First, note that lenders have rational expectations about the bank failure cutoff θ^f , but are uncertain whether their bank is a failing or a surviving bank. Consider the payoff of a lender at the strategic threshold θ^* . Such agent is indifferent between running and rolling over, as seen in equation (3.6). A marginal increase in signal uncertainty has two countervailing effects for such a lender. On the one hand, it increases the payoff from rolling over conditional on bank survival, as the lender now expects her bank to have a higher expected type θ when it survives. Nevertheless, it also increases the payoff from running conditional on bank failure, as the lender expects the bank to have a lower type θ conditional on failure.

If the expected increase in the payoff from rolling over given the increase in uncertainty is lower than the expected increase in the payoff from running, in order for a lender to be indifferent, she must rationally expect the failure probability to be lower. The probability of bank failure, is simply given by the probability that an insufficient fraction of lenders roll over, as Figure 3-2 shows, i.e.,

$$Pr_{\theta^*}(\theta < \theta^f) = Pr_{\theta^*}(x(\theta) < g(\theta^f, \Gamma)). \quad (3.13)$$

Since an indifferent lender has Laplacian beliefs about the fraction of other lenders that roll over, that probability is just $g(\theta^f, \Gamma)$. Therefore, in order for a lender to be indifferent, she must rationally expect the failure cutoff θ^f to increase. Conversely, if the expected increase in the payoff from rolling over given the increase in uncertainty is higher than the expected increase in the payoff from running, in order for a lender to be indifferent, she must rationally expect the failure probability for the bank to be higher.

In other words, as idiosyncratic uncertainty increases, if the expected increase in payoffs from rolling over is lower than the expected increase in payoffs from running, then the marginal failing bank must be able to withstand a run by more lenders, as more lenders are better off running, and vice versa. That means that a bank at the failure threshold should have stronger fundamentals,

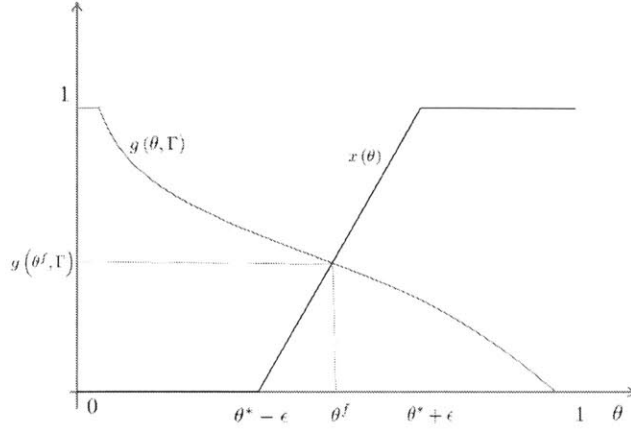


Figure 3-2: Bank failure threshold

indicating that the equilibrium θ^f should increase.⁹

It is even clearer to see the two countervailing effects when looking at the condition for the limiting case with $\epsilon \rightarrow 0$. From equation (3.12) we have that $\lim_{\epsilon \rightarrow 0} \left(\frac{\partial \theta^f}{\partial \epsilon} \right) > 0$ iff

$$\frac{(1 - g(\theta_{\epsilon=0}^f, \Gamma)) R \cdot p'(\theta_{\epsilon=0}^f)}{g(\theta_{\epsilon=0}^f, \Gamma) \|k'(\theta_{\epsilon=0}^f)\|} < \frac{g(\theta_{\epsilon=0}^f, \Gamma)}{(1 - g(\theta_{\epsilon=0}^f, \Gamma))} \quad (3.14)$$

Combining this with the condition for determination of the failure cutoff in the limiting case as $\epsilon \rightarrow 0$ from equation (3.10), we get that

$$\frac{(1 - g(\theta_{\epsilon=0}^f, \Gamma)) R \cdot p'(\theta^f)}{g(\theta_{\epsilon=0}^f, \Gamma) \|k'(\theta^f)\|} < \frac{g(\theta_{\epsilon=0}^f, \Gamma)}{(1 - g(\theta_{\epsilon=0}^f, \Gamma))} = \frac{R \cdot p(\theta_{\epsilon=0}^f) - K}{k(\theta_{\epsilon=0}^f)} \quad (3.15)$$

or equivalently

$$\frac{\eta_{Survival}(\theta_{\epsilon=0}^f)}{\eta_{Failure}(\theta_{\epsilon=0}^f)} < \frac{g(\theta_{\epsilon=0}^f, \Gamma)}{1 - g(\theta_{\epsilon=0}^f, \Gamma)} = \frac{Pr_{\theta^*}(\theta_{\epsilon=0} < \theta^f)}{1 - Pr_{\theta^*}(\theta_{\epsilon=0} < \theta^f)} \quad (3.16)$$

⁹It is important to note that in the case with uniform prior, studied here, changes in the variance of idiosyncratic noise lead to changes in the posterior variance without changing the posterior mean. This is not the case with more general priors, where changes in the variance of the idiosyncratic signals would change other moments of the posterior distribution, as well. Furthermore, unlike the uniform case, with a more general prior, changes in the variance of idiosyncratic signals also affect the uncertainty about the actions of others of the marginal agent observing a signal at the strategic cutoff. Therefore, the uniform prior case serves as an important benchmark where increases in the variance of idiosyncratic noise only affect the posterior uncertainty about the bank's fundamentals without affecting the uncertainty about the actions of other. Decomposing the effects of changes in payoff uncertainty and uncertainty about others' actions is then an important issue that arises when one considers the case of a more general prior. Understanding the effects of both types of uncertainty would be an interesting question to pursue in future research.

where $\eta_{Survival}(\theta) = \frac{R \cdot p'(\theta)\theta}{R \cdot p(\theta) - K}$ and $\eta_{Failure}(\theta) = \left\| \frac{k'(\theta)\theta}{k(\theta)} \right\|$ are the elasticities of the payoff from rolling over with respect to θ conditional on bank survival and the payoff from running conditional on bank failure, respectively, evaluated at $\theta = \theta_{\epsilon=0}^f$. Then a marginal increase in uncertainty about bank quality will increase the failure threshold if the ratio of the sensitivity of the payoff from rolling over conditional on survival to the sensitivity of the payoff from running conditional on failure is less than the bank failure odds ratio that a lender who observes the cutoff signal assigns to his banks. If portfolio differences mostly lead to differences in the “upside” risk associated with a higher probability of repayment of long term lenders conditional on bank survival, then increasing the uncertainty about individual portfolios lowers bank failure threshold, as lenders effectively become more optimistic about the value of each bank’s promises. If, however, portfolio differences mostly drive differences in “downside” risk associated with lower repayment of long-term relative to short-term lenders in the case of bank failure and restructuring, then increasing transparency is what lowers bank failure.

3.4 The effects of market liquidity and short-term debt

As discussed in Section 3.3, under some conditions increases in lender uncertainty about bank portfolio quality may increase the likelihood and magnitude of banking crises. In this section we examine the systemic effects of changes in market liquidity and excessive reliance on short-term debt on the size of a banking crisis. The former effect is of particular interest given the potential role of fire sales for the exacerbation of financial crises ((Shleifer and Vishny 2011), (Duffie 2010)).

Proposition 3.3. θ^f is a decreasing function of l and an increasing function of K .

Proof. See Appendix. □

As Proposition 3.3 shows, a decrease in market liquidity affects bank failure adversely: it increases the fragility of each bank by lowering its ability to survive a run by short-term lenders. A similar result obtains for increases in short-term debt obligations, K . Therefore, whenever market liquidity responds to asset liquidation volumes, a crisis contagion mechanism emerges. In that case, it can have important amplification effects of shocks to lender uncertainty. We turn next to the study of such amplification effects.

3.4.1 Endogenous Liquidation Value - an Example

To illustrate our ideas about how market liquidity may endogenously respond to bank failure we first describe an extension of Example 1. Consider a case in which liquidity conditions are given by a price, l , which is determined endogenously in an asset market. Asset supply is determined by the liquidation required to repay short-term lenders who choose not to roll over debt. On the other side of the market, there is an asset demand which is not perfectly elastic, indicating some limited absorption capacity by other market participants.

Asset demand is then given by

$$l = h(a) \tag{3.17}$$

with $h'(\cdot) < 0$. Asset supply, on the other hand is given by

$$\begin{aligned} a^s &= \int_0^1 \max \left\{ a(\theta), \frac{(1-x)K}{l} \right\} d\theta = \\ &= \int_0^{\theta^f} a(\theta) d\theta + \frac{K}{l} \int_{\theta^f}^{\theta^* + \epsilon} \left(1 - \frac{\theta - (\theta^* + \epsilon)}{2\epsilon} \right) d\theta = \\ &= \int_0^{\theta^f} a(\theta) d\theta + \epsilon(1 - g(\theta, \Gamma)) a(\theta^f). \end{aligned}$$

Therefore,

$$l = h \left(\int_0^{\theta^f} a(\theta) d\theta + \epsilon(1 - g(\theta, \Gamma)) a(\theta^f) \right), \tag{3.18}$$

which implies that l is a decreasing function of θ^f and ϵ . In the next section, we look at a general relation between l and θ^f motivated by this example and endowed with these properties.

3.4.2 General equilibrium determination of liquidity conditions

We assume that the locus $H(\theta^f, \Gamma, \epsilon, l) \equiv h(\theta^f, \Gamma, \epsilon) - l = 0$ describes equilibrium conditions in an asset market, with $\frac{\partial H}{\partial \theta^f} < 0$ and $\frac{\partial H}{\partial \epsilon} < 0$. As discussed in the previous section, these properties arise because of limited absorption capacity on the demand side of an asset market, which would lead to a fire sale effect. We first provide a definition for an equilibrium in the model augmented with an asset market.

Definition 3.2. An equilibrium for the rollover game augmented with an asset market consists of a bank failure cutoff θ^f and an asset price l s.t.

1. θ^f satisfies:

$$\psi(\theta^f, \Gamma) \equiv \int_{\theta^f}^{\theta^f + 2\epsilon - 2\epsilon g(\theta^f, \Gamma)} \frac{1}{2\epsilon} R p(\theta) d\theta - \int_{\theta^f - 2\epsilon g(\theta^f, \Gamma)}^{\theta^f} \frac{1}{2\epsilon} k(\theta) d\theta - K(1 - g(\theta^f, \Gamma)) = 0 \tag{3.19}$$

2. l satisfies an asset market equilibrium condition:

$$H(\theta^f, \Gamma, \epsilon, l) = 0 \tag{3.20}$$

The next result clarifies the conditions under which an endogenous asset price leads to a fire sale and an amplification of the effect of increasing uncertainty in lenders beliefs. First, we characterize the equilibrium effect of ϵ in the next proposition.

Proposition 3.4. Let $\frac{\partial \theta^f}{\partial \epsilon}$ denote the general equilibrium effect of ϵ . Then

$$\frac{\partial \theta^f}{\partial \epsilon} = \frac{\frac{\partial \theta^f}{\partial \epsilon} |_{\psi} + \frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \epsilon} |_H}{1 - \frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \theta^f} |_H} \quad (3.21)$$

Proof. See Appendix. □

Therefore, as long as $\frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \theta^f} |_H < 1$, we can write equation (3.21) as

$$\frac{\partial \theta^f}{\partial \epsilon} = \sum_{i=0}^{\infty} \left(\frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \theta^f} |_H \right)^i \left(\frac{\partial \theta^f}{\partial \epsilon} |_{\psi} + \frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \epsilon} |_H \right). \quad (3.22)$$

The condition $\frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \theta^f} |_H < 1$ holds if the asset market equilibrium condition, H , is sufficiently flat in (θ, l) . That occurs as long as the demand function is not too inelastic, so that variations in the liquidation threshold generate moderate price changes and the term $\sum_{i=0}^{\infty} \left(\frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \theta^f} |_H \right)^i$ converges. We can interpret the equilibrium change in the failure threshold in the following way.

First, notice that $\frac{\partial \theta^f}{\partial \epsilon} |_{\psi}$ is the direct impact of the increased noise on the failure threshold, taking l as fixed, as given by the equilibrium of the game described in Section 3.2.4. In addition to that effect, as the asset market equilibrium condition, $H(\cdot) = 0$, potentially depends on the support of noise as well, an increase in noise has a direct impact on l through the asset market, given by $\frac{\partial l}{\partial \epsilon} |_H$. In a first iteration, that should lead to a change in the cut-off of $\frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \epsilon} |_H$.

However, any increase in the failure cut-off leads to more assets being sold and is reflected into a lower price in asset markets, which feeds back towards a bigger change in the failure cutoff. The i -th feedback interaction gives rise to the term $\left(\frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \theta^f} |_H \right)^i$, which adds to the direct consequences of an increase in noise, so that any direct impact is amplified by a factor of $\sum_{i=0}^{\infty} \left(\frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \theta^f} |_H \right)^i = \frac{1}{1 - \frac{\partial \theta^f}{\partial l} |_{\psi} \frac{\partial l}{\partial \theta^f} |_H}$.

Therefore, provided that the feedback effect is bounded, any direct impact of an increase in uncertainty in financial fragility is amplified through asset markets in a loop, in which more debt rollover crises lead to lower liquidation prices, which lead to rollover difficulties, which in turn lead to more liquidation and further price depression. Indeed, the amplification mechanism discussed in this section applies not only to decreases in the precision of noise but to any effects of exogenous variables on financial fragility. For example, an increase in the return upon success R leads to a reduction in the failure threshold, which is endogenously amplified through a positive impact on asset prices l .

As an additional thought exercise, we can imagine a decrease in the precision of information that applies only to a limited (positive measure) set of intermediaries. Under the conditions previously discussed, that leads to an increase in the failure thresholds for those institutions. In turn, this leads to more asset liquidation among this group, which impacts negatively market prices for the liquidated asset. The effects of this reduction in prices impact all intermediaries in the economy

and generate a feedback loop. Once the repercussions are intermediated by and amplified through asset markets, they are no longer restricted to the original set affected by the lower precision of signals and propagate to the whole set of intermediaries in the economy.

3.5 Policy Discussion and Concluding Remarks

In this chapter, we study how information quality affects short-term lenders' debt rollover decisions and bank failure. When liquidation payoffs are sufficiently sensitive to the bank fundamentals, relative to the sensitivity of rollover payoffs, a decrease in the precision of individual information leads to more frequent rollover crises. An endogenous asset market can serve as an amplification mechanism for this reaction, as rollover difficulties lead to more asset liquidation, lowering prices and precipitating further deterioration of liquidity conditions. Therefore, affecting the level of bank portfolio transparency can be an important policy tool for dealing with runs and systemic events by short-term lenders but whether this entails increasing or decreasing transparency depends on the effect of a bank's portfolio quality on lenders' payoffs.

Examining the relative sensitivities of liquidation versus rollover payoffs conditional on survival in order to understand the role of transparency is ultimately a matter for empirical investigation. However, considering the nature of banks balance sheets, we conjecture that the former should dominate the latter substantially. Debt is limited in its upside conditional on full repayment and banks have equity buffers to absorb losses and protect debt holders from losses. On the other hand, whenever a bank goes bankrupt and its equity is wiped out, debt holders become residual claimants and bank asset quality together with being the first in line for the proceeds from liquidation starts mattering considerably.

Consider, for simplicity, the extreme case where rollover payoffs do not depend on the quality of the bank's portfolio. Therefore, increases in uncertainty affect only the expected payoffs from liquidation and hence affects adversely bank failure probabilities. In that case, a policy that aims to reduce uncertainty about portfolio quality such as a stress test clearly helps stabilize the financial system. There is indeed a common view the stress tests of major U.S. financial institutions of 2009, the so called Supervisory Capital Assessment Program (SCAP), helped stabilize the financial system by providing information about bank portfolios to financial markets ((Peristian, Morgan, and Savino 2010)). This reduced uncertainty, decreased bank CDS premia and increased stock prices prompting banks to seek new equity financing from financial markets ((Greenlaw, Kashyap, Schoenholtz, and Shin 2012)). In fact, the success of the 2009 stress tests for stabilizing the financial system has made annual stress tests of important financial institutions an import component of the Dodd-Frank act.

However, stress tests may not be informative enough regarding the payoffs to debt holders in case of bank failure, as they primarily provide information about future capital shortfalls, which can serve as a low precision signal about portfolio quality and an even lower precision signal about repayments

conditional on liquidation. Another possible policy, which is substantially more informative and aims to reduce uncertainty precisely regarding liquidation payoffs is the so called “Living Will” requirement mandated by the Dodd-Frank act and which regulators of financial intermediaries in the US and Europe have recently begun implementing. A “Living Will” effectively forces a financial intermediary to disclose how its liquidation payoffs would look like conditional on insolvency and failure. In particular, banks are required to produce information on winding up trading books and to arrange potential buyers of their assets in case of failure ((FT 2011)). In the context of our work, this requirement can be rationalized as aiming to stabilize the financial system by reducing the incentives of short term lenders to run on banks during a crisis episode.

Nevertheless, a thorough welfare evaluation of these and other policy interventions requires the addition of a contracting stage in the environment we study, as our analysis was conducted with a fixed pay-off structure for the debt-repayment game, taking previously determined contracts as given. This additional stage can help shed light on the relevant ranges for pay-off parameters and the magnitude of the essential comparative statics and interactions we study. In the presence of an endogenous asset liquidation market and miscoordination in rollover decisions, individual contracts and strategies do lead to externalities. For example, the presence of fire sales and the increased financial fragility that more short-term debt creates as shown by Proposition 3.3 imply that there would be a fire sale externality in short-term debt contracts similarly to (Stein 2012). As such, properly designed policies targeting contracts, asset markets and taxing pay-offs from investment decisions can lead to welfare gains.

3.6 Appendix

Proof of Proposition 3.1

Proof. Let us first define $x(\theta, \tilde{\theta})$ as the fraction of lenders that roll over debt for a bank of type θ when lenders follow a strategy with cutoff $\tilde{\theta}$. Secondly, we define $v(\theta_i, \tilde{\theta}, \Gamma) \equiv \frac{1}{2\epsilon} \int_{\theta_i - \epsilon}^{\theta_i + \epsilon} \pi(\theta, x(\theta, \tilde{\theta}), \Gamma) d\theta$ as the expected net payoff from rollover for a lender that observes a signal θ_i and expects other lenders to follow cutoff strategies with cutoff $\tilde{\theta}$.

We first show several important properties of $v(\theta_i, \tilde{\theta}, \Gamma)$. Firstly, $v(\theta_i, \tilde{\theta}, \Gamma)$ is continuous in θ_i as $\pi(\theta, x(\theta, \tilde{\theta}), \Gamma)$ is bounded and the limits of integration are continuous functions of θ_i . Secondly, the function $\tilde{v}(\tilde{\theta}, \Gamma) \equiv v(\tilde{\theta}, \tilde{\theta}, \Gamma)$ is continuous, non-decreasing in $\tilde{\theta}$ and strictly increasing for $\tilde{\theta} > \underline{\theta}$. To see this, note first that $x(\theta, \tilde{\theta})$ is continuous in $\tilde{\theta}$ and $\pi(\theta, x, \Gamma)$ is continuous in x , so $\pi(\theta, x(\theta, \tilde{\theta}), \Gamma)$ is continuous in $\tilde{\theta}$. It is also bounded and the limits of integration are continuous in $\tilde{\theta}$, so $\tilde{v}(\tilde{\theta}, \Gamma)$ is continuous in $\tilde{\theta}$. Lastly, let $\tilde{\theta}_1 < \tilde{\theta}_2$. Then, for $\tilde{\theta}$ we have that $x(\theta, \tilde{\theta}) = 1 - \frac{\theta - (\tilde{\theta} - \epsilon)}{2\epsilon}$ for $\theta \in [\tilde{\theta} - \epsilon, \tilde{\theta} + \epsilon]$ or inverting this function, $\theta = \tilde{\theta} - \epsilon + 2\epsilon(1 - x)$. Hence, we can do a change of variables in the integral $\tilde{v}(\tilde{\theta}, \Gamma)$ and rewrite it as

$$\tilde{v}(\tilde{\theta}, \Gamma) = \frac{1}{2\epsilon} \int_0^1 (2\epsilon) \pi(\tilde{\theta} + \epsilon - 2\epsilon \cdot (1 - x), (1 - x), \Gamma) dx \quad (3.23)$$

or equivalently

$$\tilde{v}(\tilde{\theta}, \Gamma) = \int_0^1 \pi(\tilde{\theta} + \epsilon - 2\epsilon \cdot x, x, \Gamma) dx \quad (3.24)$$

Then, looking at $\bar{v}(\bar{\theta}_2, \Gamma) - \bar{v}(\bar{\theta}_1, \Gamma)$ we have $\int_0^1 [\pi(\bar{\theta}_2 + \epsilon - 2\epsilon \cdot x, x, \Gamma) - \pi(\bar{\theta}_1 + \epsilon - 2\epsilon \cdot x, x, \Gamma)] dx$. Note, however, that $\pi(\theta, x, \Gamma)$ is weakly increasing in θ , $\forall x$. Therefore, $\bar{v}(\bar{\theta}_2, \Gamma) - \bar{v}(\bar{\theta}_1, \Gamma) \geq 0$, $\forall \bar{\theta}_1 < \bar{\theta}_2$. Furthermore, note that for $\underline{\theta} < \bar{\theta}_2$ this holds with strict inequality.

Given the above properties, $\bar{v}(\bar{\theta}, \Gamma) > 0$ and $\bar{v}(\underline{\theta}, \Gamma) < 0$, it follows that

$$\bar{v}(\theta^*, \Gamma) = 0 \quad (3.25)$$

has a unique solution $\theta^*(\Gamma)$ and $\theta^*(\Gamma) \in (\underline{\theta}, \bar{\theta})$. Note that $\theta^*(\Gamma)$ describes the cutoff for an equilibrium in cutoff strategies iff

$$v(\theta_i, \theta^*(\Gamma), \Gamma) > 0, \quad \forall \theta_i > \theta^*(\Gamma) \quad (3.26)$$

and

$$v(\theta_i, \theta^*(\Gamma), \Gamma) < 0, \quad \forall \theta_i < \theta^*(\Gamma). \quad (3.27)$$

Notice that $\frac{\partial v(\theta_i, \theta^*(\Gamma), \Gamma)}{\partial \theta_i} = \frac{1}{2\epsilon} [\pi(\theta_i + \epsilon, x(\theta_i + \epsilon, \theta^*(\Gamma)), \Gamma) - \pi(\theta_i - \epsilon, x(\theta_i - \epsilon, \theta^*(\Gamma)), \Gamma)] \geq 0$, as π is increasing in (θ, x) . Using

$$x(\theta, \theta^*(\Gamma)) = \begin{cases} 1 & , \theta > \theta^*(\Gamma) + \epsilon \\ \frac{\theta - (\theta^*(\Gamma) - \epsilon)}{2\epsilon} & , \theta \in [\theta^*(\Gamma) - \epsilon, \theta^*(\Gamma) + \epsilon] \\ 0 & , \theta < \theta^*(\Gamma) - \epsilon \end{cases} \quad (3.28)$$

and $\pi(\theta, x, \Gamma) = -k(\theta) \mathbf{I}_{x < g(\theta, \Gamma)} + (1 - \mathbf{I}_{x < g(\theta, \Gamma)}) (p(\theta)R - K)$, at $\theta^*(\Gamma)$, we obtain a strict inequality since

$$\left. \frac{\partial v(\theta_i, \theta^*(\Gamma), \Gamma)}{\partial \theta_i} \right|_{\theta_i = \theta^*} = \frac{1}{2\epsilon} (p(\theta^* + \epsilon)R - K - k(\theta^*)) > 0$$

That inequality is also strict in a neighborhood of θ^* , from the continuity of $x(\theta, \theta^*(\Gamma)) - g(\theta, \Gamma)$, which guarantees that there exists a neighborhood of θ^* , in which $\frac{\partial v(\theta_i, \theta^*(\Gamma), \Gamma)}{\partial \theta_i}$ is continuous. As a consequence, (3.26) and (3.27) follow.

Therefore, the solution to:

$$\int_0^1 \pi(\theta^* + \epsilon - 2\epsilon \cdot x, x, \Gamma) dx = 0 \quad (3.29)$$

describes the unique cutoff for the equilibrium in cutoff strategies. □

Proof of Proposition 3.2

Let

$$\psi(\theta^f, \Gamma) \equiv \int_{\theta^f}^{\theta^f + 2\epsilon - 2\epsilon g(\theta^f, \Gamma)} \frac{1}{2\epsilon} R p(\theta) d\theta - \int_{\theta^f - 2\epsilon g(\theta^f, \Gamma)}^{\theta^f} \frac{1}{2\epsilon} k(\theta) d\theta - K(1 - g(\theta^f, \Gamma)) \quad (3.30)$$

so that $\psi = 0$ implicitly defines θ^f . We can then compute

$$\begin{aligned} \frac{\partial \psi}{\partial \theta^f} &= \frac{R}{2\epsilon} [p(\theta^f + 2\epsilon - 2\epsilon g(\theta^f, \Gamma)) - p(\theta^f)] - g_{\theta} [R p(\theta^f + 2\epsilon - 2\epsilon g(\theta^f, \Gamma)) - K + k(\theta^f - 2\epsilon g(\theta^f, \Gamma))] \\ &\quad - \frac{1}{2\epsilon} [k(\theta^f) - k(\theta^f - 2\epsilon g(\theta^f, \Gamma))] > 0 \end{aligned}$$

and so $\lim_{\epsilon \rightarrow 0} \frac{\partial \psi}{\partial \theta^f} = Rp'(\theta^f)(1-g) - k'(\theta^f)g - g_\theta [Rp(\theta^f) - K + k(\theta^f)] > 0$. Similarly,

$$\begin{aligned} \frac{\partial \psi}{\partial \epsilon} &= \frac{1}{\epsilon} \left[(1-g(\theta^f, \Gamma)) Rp(\theta^f + 2\epsilon - 2\epsilon g(\theta^f, \Gamma)) - \int_{\theta^f}^{\theta^f + 2\epsilon - 2\epsilon g(\theta^f, \Gamma)} \frac{1}{2\epsilon} Rp(\theta) d\theta \right] \\ &\quad - \frac{1}{\epsilon} \left[g(\theta^f, \Gamma) k(\theta^f - 2\epsilon g(\theta^f, \Gamma)) - \int_{\theta^f - 2\epsilon g(\theta^f, \Gamma)}^{\theta^f} \frac{1}{2\epsilon} k(\theta) d\theta \right] \end{aligned}$$

or

$$\begin{aligned} \frac{\partial \psi}{\partial \epsilon} &= \frac{1}{\epsilon} (1-g(\theta^f, \Gamma)) \{ Rp(\theta^f + 2\epsilon(1-g(\theta^f, \Gamma))) - E[Rp(\theta) | \theta > \theta^f] \} - \\ &\quad - \frac{1}{\epsilon} g(\theta^f, \Gamma) \{ k(\theta^f - 2\epsilon g(\theta^f, \Gamma)) - E[k(\theta) | \theta < \theta^f] \} \end{aligned}$$

By the implicit function theorem, $\frac{\partial \theta^f}{\partial \epsilon} = -\frac{\frac{\partial \psi}{\partial \epsilon}}{\frac{\partial \psi}{\partial \theta^f}}$ and so $\frac{\partial \theta^f}{\partial \epsilon} > 0$ iff

$$(1-g(\theta^f, \Gamma)) \{ Rp(\theta^f + 2\epsilon(1-g(\theta^f, \Gamma))) - E[Rp(\theta) | \theta > \theta^f] \} < g(\theta^f, \Gamma) \{ k(\theta^f - 2\epsilon g(\theta^f, \Gamma)) - E[k(\theta) | \theta < \theta^f] \}$$

Furthermore, we have that $\lim_{\epsilon \rightarrow 0} \frac{\partial \psi}{\partial \epsilon} = (1-g(\theta^f, \Gamma))^2 Rp'(\theta^f) + g(\theta^f, \Gamma)^2 k'(\theta^f)$, leading to

$$\lim_{\epsilon \rightarrow 0} \left(\frac{\partial \theta^f}{\partial \epsilon} \right) = -\frac{(1-g(\theta^f, \Gamma)) Rp'(\theta^f) + g(\theta^f, \Gamma) k'(\theta^f)}{Rp'(\theta^f)(1-g) - k'(\theta^f)g - g_\theta [Rp(\theta^f) - K + k(\theta^f)]}$$

Therefore, $\lim_{\epsilon \rightarrow 0} \left(\frac{\partial \theta^f}{\partial \epsilon} \right) > 0$ iff $(1-g(\theta^f, \Gamma))^2 Rp'(\theta^f) + g(\theta^f, \Gamma)^2 k'(\theta^f) < 0$.

Proof of Proposition 3.3

Using the function ψ defined by equation (3.30), we have that

$$\frac{\partial \psi}{\partial l} = -g_l [Rp(\theta^f + 2\epsilon - 2\epsilon g(\theta^f, l)) - K + k(\theta^f - 2\epsilon g(\theta^f, \Gamma))] > 0$$

which by the implicit function theorem and given that $\frac{\partial \psi}{\partial \theta^f} > 0$, implies that $\frac{\partial \theta^f}{\partial l} < 0$. Similarly, we have that

$$\frac{\partial \psi}{\partial K} = -(1-g(\theta^f, \Gamma)) - g_\kappa [Rp(\theta^f + 2\epsilon(1-g(\theta^f, \Gamma))) - K + k(\theta^f - 2\epsilon g(\theta^f, \Gamma))] < 0$$

which implies that $\frac{\partial \theta^f}{\partial K} > 0$.

Proof of Proposition 3.4

We have

$$\begin{aligned} \psi(\theta^f, \gamma, \epsilon) &= 0 \\ H(\theta^f, \gamma, \epsilon) &= 0 \end{aligned}$$

The linearized system is given by

$$\begin{bmatrix} \psi_\theta & \psi_l \\ H_\theta & H_l \end{bmatrix} \begin{bmatrix} d\theta^f \\ dl \end{bmatrix} = - \begin{bmatrix} \psi_\epsilon \\ H_\epsilon \end{bmatrix} d\epsilon.$$

Therefore, the partial derivatives are given by

$$\begin{bmatrix} \frac{\partial \theta^J}{\partial \epsilon} \\ \frac{\partial \theta}{\partial \epsilon} \end{bmatrix} = \frac{1}{\psi_l H_\theta - \psi_\theta H_l} \begin{bmatrix} \psi_\epsilon \\ H_\epsilon \end{bmatrix} \begin{bmatrix} H_l & -\psi_l \\ -H_\theta & \psi_\theta \end{bmatrix} d\epsilon.$$

As a consequence,

$$\frac{\partial \theta^J}{\partial \epsilon} = \frac{H_l \psi_\epsilon - \psi_l H_\epsilon}{\psi_l H_\theta - \psi_\theta H_l} = -\frac{\frac{\psi_\epsilon}{\psi_{\theta J}} - \frac{\psi_l H_\epsilon}{\psi_\theta H_l}}{1 - \frac{\psi_l H_\theta}{\psi_{\theta J} H_l}} = \frac{\frac{\partial \theta^J}{\partial \epsilon} |_\psi + \frac{\partial \theta^J}{\partial l} |_\psi \frac{\partial l}{\partial \epsilon} |_H}{1 - \frac{\partial \theta^J}{\partial l} |_\psi \frac{\partial l}{\partial \theta^J} |_H}.$$

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