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Citation	Chen, H., J. Miao, and N. Wang. "Entrepreneurial Finance and Nondiversifiable Risk." Review of Financial Studies 23.12 (2010) : 4348-4388.
As Published	http://dx.doi.org/10.1093/rfs/hhq122
Publisher	Oxford University Press
Version	Final published version
Accessed	Mon Jul 24 22:44:01 EDT 2017
Citable Link	http://hdl.handle.net/1721.1/65557
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Detailed Terms	

Entrepreneurial Finance and Nondiversifiable Risk

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We develop a dynamic incomplete-markets model of entrepreneurial firms, and demonstrate the implications of nondiversifiable risks for entrepreneurs' interdependent consumption, portfolio allocation, financing, investment, and business exit decisions. We characterize the optimal capital structure via a generalized tradeoff model where risky debt provides significant diversification benefits. Nondiversifiable risks have several important implications: More risk-averse entrepreneurs default earlier, but choose higher leverage; lack of diversification causes entrepreneurial firms to underinvest relative to public firms, and risky debt partially alleviates this problem; and entrepreneurial risk aversion can overturn the risk-shifting incentives induced by risky debt. We also analytically characterize the idiosyncratic risk premium. (*JEL* G11, G31, E20)

Lack of diversification is one of the defining characteristics of entrepreneurship. Numerous empirical studies have documented that (i) active businesses account for a large fraction of entrepreneurs' total wealth; and (ii)

We thank Yakov Amihud, Patrick Bolton, John Cox, Mariacristina De Nardi, Paolo Fulghieri (the editor), Dirk Hackbarth, Bob Hall, Chris Hennessy, Glenn Hubbard, Larry Kotlikoff, Debbie Lucas, Stew Myers, Morten Sorensen, Anne Villamil, Susan Woodward, and seminar participants at Baruch, Boston University Economics and SMG, Colorado, Columbia, George Washington University, Kansas, Michigan, MIT, Conference on Financial Innovation: 35 Years of Black/Scholes and Merton at Vanderbilt University, NYU Stern, Peking University, UIUC Economics, SHUFE SOF, WFA 2009, NBER, and Gerzensee Summer Symposium for comments. We are especially grateful to the anonymous referee for constructive comments that have significantly improved the exposition of the article. Jianjun Miao gratefully acknowledges financial support from the Research Grants Council of Hong Kong under project number 643507. Part of this research was conducted when Jianjun Miao was visiting the Hong Kong University of Science and Technology. The hospitality of this university is gratefully acknowledged. Send correspondence to Hui Chen, MIT Sloan School of Management, Cambridge, MA 02142; telephone: (617) 324-3896. E-mail: huichen@mit.edu.

doi:10.1093/rfs/hhq122 Advance Access publication November 8, 2010

entrepreneurial firms tend to have highly concentrated ownership.[1] Concentrated ownership is a natural way to provide proper incentives for the entrepreneur, as implied by standard agency and informational asymmetry theories.[2] Accordingly, the main sources of financing for private businesses are inside equity and outside debt (see Heaton and Lucas 2004 and Robb and Robinson 2009 for U.S. evidence, and Brav 2009 for UK evidence).

Motivated by both the empirical evidence and micro-theory, we provide a first dynamic incomplete-markets model that explicitly incorporates the effects of nondiversifiable risk on the valuation and intertemporal decision making (investment, financing, business exit) for an entrepreneurial firm. We achieve this objective by unifying a workhorse dynamic capital structure model (e.g., Leland 1994) with models of incomplete-markets consumption smoothing/ precautionary saving (e.g., Friedman 1957; Hall 1978; Deaton 1991) and dynamic consumption/portfolio choice (e.g., Merton 1971). We show that nondiversifiable business risk generates quantitatively significant effects on dynamic capital budgeting, financing, business exits, and valuation of entrepreneurial firms. The model also provides a range of novel empirical predictions.

What determines the optimal amount of debt to issue? Due to market incompleteness, the diversification benefit of risky debt becomes a key factor in addition to the standard tradeoff between tax benefits and costs of financial distress. This role of risky debt has been studied in earlier papers. For example, Zame (1993) argues that risky debt has the advantage in helping complete the markets, and Heaton and Lucas (2004) provide the first model of the diversification benefits of risky debt for entrepreneurial firms in a static setting, and analyze the interactions among capital budgeting, capital structure, and portfolio choice for the entrepreneur.

We take the insight of Heaton and Lucas to a dynamic setting, and incorporate business exit (cash-out), outside equity, investment/project choice, and tax considerations for the entrepreneurial firm. These features not only make the model more realistic, but highlight the impact of market incompleteness on a wide range of firm decisions. For example, like default, the option to cash out also helps complete the markets and can have large effects on firms' financing choices. Moreover, we provide analytical characterization of

[1] For example, Moskowitz and Vissing-Jorgensen (2002) find that about 75% of all private equity is owned by households for whom it constitutes at least half of their total net worth. Heaton and Lucas (2004) document that in the Survey of Small Business Finances, the principal owner of a firm holds on average 81% of the firm's equity, and the median owner wholly owns the firm. Other empirical studies include Gentry and Hubbard (2004), Berger and Udell (1998), Cole, Wolken, and Woodburn (1996), and Petersen and Rajan (1994).

[2] Leland and Pyle (1977) and Myers and Majluf (1984) argue that debt often dominates equity in settings with asymmetric information because debt is less information-sensitive. Jensen and Meckling (1976) suggest that managers with low levels of ownership may exert less effort. Bitler, Moskowitz, and Vissing-Jorgensen (2005) provide evidence that agency considerations play a key role in explaining why entrepreneurs on average hold large ownership shares.

the capital budgeting/hurdle rate, capital structure tradeoff, and endogenous exit decisions.

We consider a risk-averse entrepreneur with access to an illiquid nontradable investment project. The project requires a lump-sum investment to start up, and generates stochastic cash flows that bear both systematic and idiosyncratic risks. Like a consumer, the entrepreneur makes intertemporal consumption/saving decisions and allocates his liquid wealth between a riskless asset and a diversified market portfolio (as in Merton 1971). Like a firm, the entrepreneur also makes investment/capital budgeting, financing, and exit decisions.

If he chooses to take on the project, the entrepreneur sets up a firm with limited liability (e.g., a limited liability company [LLC]; an S corporation), which makes debt nonrecourse. Moreover, the LLC or S corporation allows the entrepreneur to face single-layer taxation for his business income. In normal business times, the entrepreneur uses business income to service the firm's debt. If the firm's revenue falls short of servicing its debt, the entrepreneur may still find it optimal to use his personal savings to service the debt in order to continue to the firm's operation. However, when revenue becomes sufficiently low, the entrepreneur defaults on the debt, which triggers inefficient liquidation, as in classic tradeoff models of corporate finance. If the firm does sufficiently well, he might choose to incur the transaction and other costs (such as taxes), repay the debt in full, and realize the capital gains by selling the firm to cash out. After exiting from his business (cash-out or default), the entrepreneur becomes a regular household and lives on only his financial wealth.[3] Cash-out and default allow the entrepreneur to achieve diversification benefits. These business exit decisions are essentially (nontradable) American-style options on the illiquid project and take the form of endogenous double-threshold policies.

Importantly, the entrepreneur's business income and wealth accumulation are endogenously affected by the firm's capital budgeting, leverage, and business exit decisions. While he can hedge the systematic component of his business risks using the market portfolio, he cannot diversify the idiosyncratic risk. Therefore, the entrepreneur faces incomplete markets, and the idiosyncratic risk exposure will affect his interdependent consumption, investment, financing, and business exit decisions. Such nondiversifiable idiosyncratic risk makes entrepreneurial finance distinct from the standard textbook treatment of corporate finance, and can sometimes overturn the predictions of standard finance theory on firm valuation, financing choices, and agency problems.

While we use the entrepreneurial firm as the motivating example, our framework also applies to public firms with concentrated managerial ownership.

[3] This one-time entrepreneurship assumption does not affect the model's key economic mechanism in any significant way.

Corporations in many countries are run by controlling shareholders (e.g., founders or founders' families/heirs) who have significant cash-flow equity rights in the firms.[4] The lack of investor protection in some countries (La Porta, de Silanes, Shleifer, and Vishny 1998) and (concentrated) ownership structure (via dual-class shares, pyramid-ownership structures, and cross-ownership) entrench underdiversified controlling shareholders and managers, and hence make our model of managerial (entrepreneurial) decision making empirically relevant. Albuquerque and Wang (2008) provide a general-equilibrium incomplete-markets asset pricing model with both controlling shareholders and outside investors. Their focus is on equilibrium asset pricing, which is rather different from the focus of this article.

The main results of the model are the following. First, on capital structure, our framework provides a generalized dynamic tradeoff model, where in addition to the standard tradeoff between tax benefits of debt and costs of financial distress/agency (as in Leland 1994), risky debt also provides diversification benefits. This is because risky debt helps reduce the entrepreneur's exposure to idiosyncratic business risk by enabling risk sharing in the default states. Hence, he rationally chooses more debt and hence higher leverage for the firm. The options of default and cash-out in our model have important feedback effects on the capital structure and pricing of credit risk. Our analysis also suggests that the natural measure of leverage for entrepreneurial firms is private leverage, defined as the ratio of the public (market) value of debt to the private (subjective) value of firm. This private leverage captures the diversification benefits of risky debt and highlights the tradeoff between inside equity and outside debt.

The diversification benefits of debt are large. Even without any tax benefit of debt, the entrepreneurial firm still issues a significant amount of debt. The diversification benefits also lead to a seemingly counterintuitive prediction: More risk-averse entrepreneurs prefer higher leverage. On the one hand, higher leverage increases the risk of the entrepreneur's equity stake within the firm. On the other hand, higher leverage implies less equity exposure to the entrepreneurial project, making the entrepreneur's overall portfolio (including both his private equity in the firm and his liquid financial wealth) less risky. This overall portfolio composition effect dominates the high leverage effect within the firm. The more risk-averse the entrepreneur, the stronger the need to reduce his firm risk exposure, therefore the higher the leverage.

Second, due to market incompleteness, the entrepreneur will demand an idiosyncratic risk premium when valuing the firm. We derive an analytical formula for this idiosyncratic risk premium, the key determinants of which

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4 See La Porta, Lopez-De-Silanes, and Shleifer (1999) for an early cross-country study; Claessens, Djankov, and Lang (2000) for East Asian countries; and Faccio and Lang (2002) for western European countries.

are risk aversion, idiosyncratic volatility, and the sensitivity of entrepreneurial value of equity with respect to cash flow. Quantitatively, we show that ignoring the idiosyncratic risk premium can lead to substantial upward bias in firm valuation. One consequence of this bias is that the conventionally used leverage, which does not account for idiosyncratic risk premium, substantially underestimates the leverage of entrepreneurial firms. Survey evidence by Graham and Harvey (2001) shows that smaller firms are less likely to use the capital asset pricing model (CAPM) than larger ones. Our theory suggests that small firms are likely run by underdiversified owners/managers who may demand an idiosyncratic risk premium.

Naturally, the idiosyncratic risk premium also affects investment decisions. We extend the standard law-of-one-price–based capital budgeting approach (net present value [NPV] and adjusted present value [APV] analysis) to account for nondiversifiable risk and incomplete markets. We show that market incompleteness leads to underinvestment for entrepreneurial firms relative to otherwise identical public firms, especially for projects with high idiosyncratic risk. Risky debt can partially alleviate the underinvestment problem by improving diversification.

Third, unlike for the public firm where equityholders have risk-seeking incentives when risky debt is in place (Jensen and Meckling 1976), the entrepreneur may prefer to invest in projects with low idiosyncratic volatility due to his precautionary motive, provided that the firm is not in deep financial distress. This result holds even for very low risk aversion. Our model thus provides a potential explanation for the lack of empirical and survey evidence on asset substitution and risk-shifting incentives.

Fourth, on option valuation, our model extends the Black-Scholes-Merton option pricing methodology to account for the impact of idiosyncratic risk under incomplete markets on (nontradable) option valuation. The standard dynamic replicating portfolio argument no longer applies, and options can be valued using only utility-based certainty equivalent methodology as we do here. Idiosyncratic volatility now has two opposing effects for option valuation. In addition to the standard positive convexity effect, as in Black-Scholes-Merton, the entrepreneur's precautionary saving motive under incomplete markets implies a negative relation between option value and idiosyncratic volatility, *ceteris paribus*.

The nondiversifiable risk and concentrated wealth in the business make the entrepreneur value his equity less than do diversified investors. Thus, compared with a firm owned by well-diversified investors, the entrepreneur defaults earlier on the firm's debt and cashes out earlier on his business. Black and Scholes (1973) and Merton (1974) make the observation that equity is a call option on firm assets, and hence is convex in the firm's cash flows (under complete markets). In our model, inside equity, while also a call option on the entrepreneurial firm's asset, is not necessarily globally convex in the underlying cash flows. When the entrepreneur's risk aversion and/or idiosyncratic volatility are sufficiently high, the entrepreneur's precautionary saving demand can make his private value of equity concave in cash flows.

Our model generates a rich set of empirical predictions. Consider two otherwise identical firms, one public and one private.[5] First, the private firm will have higher leverage due to diversification arguments. Second, while the standard tradeoff model (e.g., Leland 1994) predicts that leverage decreases with volatility for the public firm, leverage for the private firm might increase with idiosyncratic volatility due to the diversification benefits for entrepreneurs. Third, while the complete-market option pricing analysis suggests that higher idiosyncratic volatility defers the exercise of real options in public firms, our model predicts that more idiosyncratic risk makes the private firm have higher default thresholds and lower cash-out thresholds, hence implying a shorter duration to be private. Finally, our model predicts that even with the same amount of debt, the private firm will have more default risk and thus a higher credit spread.

In a related study, Pastor, Taylor, and Veronesi (2009) model the decision of entrepreneurs to go public as the tradeoff between diversification benefits and the costs of losing private control. While their paper studies an all-equity private firm's decision to go public, we analyze the entrepreneur's decisions on investment, debt/equity financing, consumption-saving, and business exit (cash-out and default). Another related paper is by Herranz, Krasa, and Villamil (2009), who analyze the impact of idiosyncratic risk on consumption, capital structure, and default, among other issues. They also find that more risk-averse owners default more.

Our article contributes to the dynamic capital structure literature[6] and the real options literature[7] by highlighting the role of incomplete markets in entrepreneurs' financing and investment decisions. Unlike Miao and Wang (2007), who analyze a real options model under incomplete markets and all equity financing, we integrate an incomplete-markets real options model with dynamic corporate finance. Our model also contributes to the incomplete-markets consumption smoothing/precautionary saving literature by extending the precautionary saving problem based on constant absolute risk aversion (CARA) utility (see Merton 1971; Caballero 1991; Kimball and Mankiw 1989; Wang 2003, 2006) to allow the entrepreneur to reduce his idiosyncratic risk exposure via financing and exit (i.e., cash-out and default) strategies.

5 The usual heterogeneity and endogeneity argument/critique applies. In reality, the ownership structure is obviously endogenous.

6 See Black and Cox (1976); Fischer, Heinkel, and Zechner (1989); and recent developments of Goldstein, Ju, and Leland (2001); Hackbarth, Miao, and Morellec (2006); and Chen (2010).

7 See Brennan and Schwartz (1985) and McDonald and Siegel (1986) for seminal contributions on real options. See Abel and Eberly (1994) for a unified analysis of investment under uncertainty. See Dixit and Pindyck (1994) for a textbook treatment on the real options approach to investment.

1. Model Setup

1.1 Investment Opportunities

An infinitely lived risk-averse entrepreneur has a take-it-or-leave-it project at time 0, which requires a one-time investment I.[8] The project generates a stochastic revenue process $\{y_t : t \geq 0\}$ that follows a geometric Brownian motion (GBM):

$$dy_t = \mu y_t dt + \omega y_t dB_t + \epsilon y_t dZ_t, \quad y_0 \text{ given}, \tag{1}$$

where μ is the expected growth rate of revenue, and B_t and Z_t are independent standard Brownian motions that provide the sources of market (systematic) and idiosyncratic risks of the private business, respectively. The parameters ω and ϵ are the systematic and idiosyncratic volatility of revenue growth. The total volatility of revenue growth is

$$\sigma = \sqrt{\omega^2 + \epsilon^2}. \tag{2}$$

As we will show, the three volatility parameters ω, ϵ, and σ have different effects on the entrepreneur's decision making.

In addition, the entrepreneur has access to standard financial investment opportunities (Merton 1971).[9] The entrepreneur allocates his liquid financial wealth between a risk-free asset that pays a constant rate of interest r and a diversified market portfolio with return R_t satisfying

$$dR_t = \mu_p dt + \sigma_p dB_t, \tag{3}$$

where μ_p and σ_p are the expected return and volatility of the risky asset, respectively, and B_t is the standard Brownian motion introduced in Equation (1). Let

$$\eta = \frac{\mu_p - r}{\sigma_p} \tag{4}$$

denote the Sharpe ratio of the market portfolio, and let $\{x_t : t \geq 0\}$ denote the entrepreneur's liquid (financial) wealth process. The entrepreneur invests the amount ϕ_t in the market portfolio and the remaining amount $x_t - \phi_t$ in the risk-free asset.

1.2 Entrepreneurial Firm

If the entrepreneur decides to invest in the project, he runs it by setting up a limited-liability entity, such as an LLC or an S corporation. The LLC or S corporation allows the entrepreneur to face single-layer taxation for his business

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8 In the Internet Appendix (available on the authors' websites), we also model the investment decision as a real option.

9 It is straightforward to consider entering the labor market as an alternative to running an entrepreneurial business, which provides an endogenous opportunity cost of taking on the entrepreneurial project. Such an extension does not change the key economics of our article in any significant way.

income and makes the debt nonrecourse. We may extend the model to allow for personal guarantee of debt, which effectively makes debt recourse to varying degrees. The entrepreneur finances the initial one-time lump-sum cost I via his own funds (internal financing) and external financing. In the benchmark case, we assume that the only source of external financing is debt.[10] One interpretation of the external debt is bank loans. The entrepreneur uses the firm's assets as collateral to borrow, so that the debt is secured.

We assume that debt is issued at par and is interest-only (consol) for tractability reasons. Let b denote the coupon payment of debt and F_0 denote the par value of debt. Debt is priced competitively by diversified lenders. We further assume that debt is issued at only time 0 and remains unchanged until the entrepreneur exits. Allowing for dynamic capital structure before exit will not change the key economic tradeoff that we focus on: the impact of idiosyncratic risk on entrepreneurial financing decisions.

After debt is in place, at any time $t > 0$, the entrepreneur has three choices: (1) continuing his business; (2) defaulting on the outstanding debt, which leads to the liquidation of his firm; (3) cashing out by selling the firm to a diversified buyer.

While running the business, the entrepreneur receives income from the firm in the form of cash payments (operating profit net of coupon payments). Negative cash payments are interpreted as cash injections by the entrepreneur into the firm. Notice that trading riskless bonds and the diversified market portfolio alone cannot help the entrepreneur diversify the idiosyncratic business risk. He can sell the firm and cash out, which incurs a fixed transaction cost K. The default timing T_d and cash-out timing T_u are not contractible at time 0. Instead, the entrepreneur chooses the default/cash-out policy to maximize his own utility after he chooses the time-0 debt level. Thus, there is an inevitable conflict of interest between financiers and the entrepreneur. The choices of default and cash-out resemble American-style put and call options on the underlying nontradable entrepreneurial firm. Since markets are incomplete for the entrepreneur, we cannot price the entrepreneur's options using the standard dynamic replication argument (Black-Scholes-Merton).

At bankruptcy, the outside lenders take control and liquidate/sell the firm. Bankruptcy *ex post* is costly, as in standard tradeoff models of capital structure. We assume that the liquidation/sale value of the firm is equal to a fraction α of the value of an all-equity (unlevered) public firm, $A(y)$. The remaining fraction $(1-\alpha)$ is lost due to bankruptcy costs. We also assume that absolute priority is enforced, and abstract away from any *ex post* renegotiation between the lenders and the entrepreneur.

[10] See Petersen and Rajan (1994), Heaton and Lucas (2004), and Brav (2009) for evidence that debt is the primary source of financing for most entrepreneurial firms. In Section 6, we introduce external equity as an additional source of financing.

Before the entrepreneur can sell the firm, he needs to retire the firm's debt obligation at par F_0. We make the standard assumption that the buyer is well diversified. He will optimally relever the firm, as in the complete-markets model of Leland (1994). The value of the firm after sale is the value of an optimally levered public firm, $V^*(y)$.

After the entrepreneur exits his business (through default or cash-out), he "retires" and lives on his financial income. He then faces a standard complete-markets consumption and portfolio choice problem.

1.2.1 Taxes. We consider a simple tax environment. The entrepreneurial firm pays taxes on its business profits at rate τ_e. When $\tau_e > 0$, issuing debt has the benefit of shielding part of the entrepreneur's business profits from taxes. For a public firm, the effective marginal tax rate is τ_m. Unlike the entrepreneurial firm, the public firm is subject to double taxation (at the corporate and individual levels), and τ_m captures the net tax rate (following Miller 1977). Finally, τ_g denotes the tax rate on the capital gains upon sale. Naturally, higher capital gains taxes will delay the timing of cash-out.

1.2.2 Entrepreneur's Objective. The entrepreneur derives utility from consumption $\{c_t : t \geq 0\}$ according to the following time-additive utility function:

$$\mathbb{E}\left[\int_0^\infty e^{-\delta t} u\left(c_t\right) dt\right], \tag{5}$$

where $\delta > 0$ is the entrepreneur's subjective discount rate and $u(\cdot)$ is an increasing and concave function. The entrepreneur's objective is to maximize his lifetime utility by optimally choosing consumption (c_t), financial portfolio (ϕ_t), and whether to start his business. If he starts his business, he also chooses the financing structure of the firm (coupon b), and the subsequent timing decisions of default and cash-out (T_d, T_u).

2. Model Solution

In Section 2.1, we report the complete-markets solution for firm value and financing decisions when the firm is owned by diversified investors. Then, we analyze the entrepreneur's consumption/saving, portfolio choice, default, and initial investment and financing decisions. The complete-markets solution of Section 2.1 serves as a natural benchmark for us to analyze the impact of nondiversifiable idiosyncratic risk on entrepreneurial investment, financing, and valuation.

2.1 Complete-markets Firm Valuation and Financing Policy

Consider a public firm owned by diversified investors. Because equityholders internalize the benefits and costs of debt issuance, they will choose the firm's debt policy to maximize *ex ante* firm value by trading off the tax benefits of debt against bankruptcy and agency costs. The results in this case are well-known.[11] In Appendix A, we provide the after-tax value of an unlevered public firm $A(y)$ in Equation (A19), and the after-tax value of a public levered firm $V^*(y)$ in Equation (A21).

Next, we turn to analyzing the entrepreneur's decision problem under incomplete markets.

2.2 Entrepreneur's Problem

The significant lack of diversification invalidates the standard finance textbook valuation analysis for firms owned by diversified investors. As a result, the standard two-step complete-markets (Arrow-Debreu) analysis[12] (first value maximization and then optimal consumption allocation) no longer applies. This nonseparability between value maximization and consumption smoothing has important implications for real economic activities (e.g., investment and financing) and the valuation of entrepreneurial firm–related financial claims.

We solve the entrepreneur's problem by backward induction. First, we summarize the entrepreneur's consumption/saving and portfolio choice problem after he retires from his business via either cashing out or defaulting on debt. This "retirement-stage" optimization problem is the same as in Merton (1971), a dynamic complete-markets consumption/portfolio choice problem. Second, we solve the entrepreneur's joint consumption/saving, portfolio choice, and default decisions when the entrepreneur runs his private business. Third, we determine the entrepreneur's exit decisions (cash-out and default boundaries) by comparing his value functions just before and after retirement. Finally, we solve the entrepreneur's initial (time-0) investment and financing decisions taking his future decisions into account.

Conceptually, our model setup applies to any utility function $u(c)$ under technical regularity conditions. For analytical tractability, we adopt the CARA utility throughout the remainder of the article.[13] That is, let $u(c) = -e^{-\gamma c}/\gamma$, where $\gamma > 0$ is the coefficient of absolute risk aversion, which also measures precautionary motive. We emphasize that the main results and insights of our article (the effect of nondiversifiable idiosyncratic shocks on investment timing) do not rely on the choice of this utility function. As we show below, the driving force of our results is the precautionary savings effect, which is captured by utility functions with convex marginal utility such as CARA (see

[11] For example, see Leland (1994); Goldstein, Ju, and Leland (2001); and Miao (2005).

[12] Cox and Huang (1989) apply this insight to separate intertemporal portfolio choices from consumption in continuous-time diffusion settings.

[13] The CARA utility specification proves tractable in incomplete-markets consumption-saving problems with labor income. Kimball and Mankiw (1989), Caballero (1991), Svensson and Werner (1993), and Wang (2006) have all adopted this utility specification in various precautionary saving models. Miao and Wang (2007) use this utility specification to analyze a real options exercising problem when the decision maker faces uninsurable idiosyncratic risk from his investment opportunity.

Leland 1968; Kimball 1990). While CARA utility does not capture wealth effects, it helps reduce the dimension of our double-barrier free-boundary problem, which makes the problem much more tractable compared with constant relative risk aversion (CRRA) utility.

2.2.1 Consumption/saving and Portfolio Choice after Exit. After exiting from his business (via either default or cash-out), the entrepreneur no longer has any business income, and lives on his financial wealth. The entrepreneur's optimization problem becomes the standard complete-market consumption and portfolio choice problem (e.g., Merton 1971).

The entrepreneur's wealth follows

$$dx_t = (r(x_t - \phi_t) - c_t)\,dt + \phi_t\left(\mu_p dt + \sigma_p dB_t\right). \tag{6}$$

The consumption and portfolio rules[14] are given by

$$\overline{c}(x) = r\left(x + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{7}$$

$$\overline{\phi}(x) = \frac{\eta}{\gamma r \sigma_p}. \tag{8}$$

2.2.2 Entrepreneur's Decision Making While Running the Firm. Before exit, the entrepreneur's financial wealth evolves as follows:

$$\begin{aligned} dx_t &= (r(x_t - \phi_t) + (1-\tau_e)(y-b) - c_t)\,dt \\ &\quad + \phi_t\left(\mu_p dt + \sigma_p dB_t\right), \quad 0 < t < \min(T_d, T_u). \end{aligned} \tag{9}$$

The principle of optimality implies that the entrepreneur's value function $J^s(x, y)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} \delta J^s(x,y) = \max_{c,\phi}\; & u(c) + \left(rx + \phi\left(\mu_p - r\right) + (1-\tau_e)(y-b) - c\right) J^s_x(x,y) \\ & + \mu y J^s_y(x,y) + \frac{(\sigma_p \phi)^2}{2} J^s_{xx}(x,y) + \frac{\sigma^2 y^2}{2} J^s_{yy}(x,y) \\ & + \phi \sigma_p \omega y J^s_{xy}(x,y). \end{aligned} \tag{10}$$

[14] An undesirable feature of CARA utility models is that consumption and wealth could potentially turn negative. Cox and Huang (1989) provide analytical formulas for consumption under complete markets for CARA utility with nonnegativity constraints. Intuitively, requiring consumption to be positive increases the entrepreneur's demand for precautionary savings (to avoid hitting the constraints in the future), which will likely strengthen our results (such as diversification benefits of outside risky debt).

The first-order conditions for consumption c and portfolio allocation ϕ are as follows:

$$u'(c) = J_x^s(x, y), \tag{11}$$

$$\phi = \frac{-J_x^s(x,y)}{J_{xx}^s(x,y)}\left(\frac{\mu_p - r}{\sigma_p^2}\right) + \frac{-J_{xy}^s(x,y)}{J_{xx}^s(x,y)}\frac{\omega y}{\sigma_p}. \tag{12}$$

We summarize the solution for consumption/saving, portfolio choice, default trigger y_d, and cash-out trigger y_u in the following theorem.

Theorem 1. The entrepreneur exits his business when the revenue process $\{y_t : t \geq 0\}$ reaches either the default threshold y_d or the cash-out threshold y_u, whichever comes first. Prior to exit, for given liquid wealth x and revenue y, he chooses his consumption and portfolio rules as follows:

$$\overline{c}(x,y) = r\left(x + G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{13}$$

$$\overline{\phi}(x,y) = \frac{\eta}{\gamma r \sigma_p} - \frac{\omega}{\sigma_p} y G'(y), \tag{14}$$

where $G(\cdot)$ solves the ordinary differential equation:

$$rG(y) = (1-\tau_e)(y-b) + (\mu - \omega\eta)yG'(y) + \frac{\sigma^2 y^2}{2}G''(y) - \frac{\gamma r \epsilon^2 y^2}{2}G'(y)^2, \tag{15}$$

subject to the following (free) boundary conditions at y_d and y_u:

$$G(y_d) = 0, \tag{16}$$

$$G'(y_d) = 0, \tag{17}$$

$$G(y_u) = V^*(y_u) - F_0 - K - \tau_g\left(V^*(y_u) - K - I\right), \tag{18}$$

$$G'(y_u) = (1-\tau_g)V^{*\prime}(y_u). \tag{19}$$

The complete-markets firm value $V^*(y)$ is defined in Equation (A21), and the value of external debt $F_0 = F(y_0)$ is given in Equation (C6).

Equation (13) states that consumption is equal to the annuity value of the sum of financial wealth x, certainty equivalent wealth $G(y)$, which is the risk-adjusted subjective value of the private business, and two constant terms capturing the effects of the expected excess returns and the wedge $\delta - r$ on consumption. Equation (14) gives the entrepreneur's portfolio holding, where the first term is the standard mean-variance term, as in Merton (1971), and the second term gives the entrepreneur's hedging demand as he uses the market portfolio to dynamically hedge the entrepreneurial business risk.

The differential Equation (15) provides a valuation equation for the certainty equivalent wealth $G(y)$ from the entrepreneur's perspective. In the CAPM, only systematic risk demands a risk premium. Since the systematic volatility of revenue growth is ω, the risk-adjusted expected growth rate of revenue in the CAPM is

$$\nu = \mu - \omega\eta. \tag{20}$$

If we drop the last nonlinear term in Equation (15), the differential equation becomes the standard pricing equation: equating the instantaneous expected return of an asset under the risk-neutral measure (right-hand side) to the risk-free rate (left-hand side). The last term in Equation (15) captures the additional discount due to nondiversifiable idiosyncratic risk. Intuitively, the higher the risk-aversion parameter γ or the idiosyncratic volatility of revenue ϵy, the larger the discount on $G(y)$ due to idiosyncratic risk. The next section provides more detailed analysis on the impact of idiosyncratic risk on $G(y)$.

Equation (16) comes from the value-matching condition for the entrepreneur's default decision. It states that the private value of equity $G(y)$ upon default is equal to zero. Equation (17), often referred to as the smooth-pasting condition, is the optimality condition for the entrepreneur in choosing default.

Now we turn to the cash-out boundary. Because the entrepreneur pays the fixed cost K and triggers capital gains when cashing out, he naturally has incentive to wait before cashing out. However, waiting also reduces his diversification benefits, *ceteris paribus*. The entrepreneur optimally trades off tax implications, diversification benefits, and transaction costs when choosing the timing of cashing out. The value-matching condition in Equation (18) states that the private value of equity upon cashing out is equal to the after-tax value of the public firm value after the entrepreneur pays the fixed cost K, retires outstanding debt at par F_0, and pays capital gains taxes. The smooth-pasting condition in Equation (19) ensures that the entrepreneur optimally chooses his cash-out timing.

2.2.3 Initial Financing and Investment Decisions.

Next, we complete the model solution by endogenizing the entrepreneur's initial investment and financing decision. The entrepreneurial firm has two financial claimants: inside equity (entrepreneur) and outside creditors. The entrepreneur values his ownership at a certainty equivalent value $G(y)$. Diversified lenders price debt in competitive capital markets at $F(y)$, which does not contain the idiosyncratic risk premium because outside investors are fully diversified. Thus, the total *private value* of the entrepreneurial firm is

$$S(y) = G(y) + F(y). \tag{21}$$

We may interpret $S(y)$ as the total value that one needs to pay to acquire the entrepreneurial firm by buying out the entrepreneur and the debtholders.

We show that, at time 0, the optimal coupon b maximizes the private value of the firm:

$$b^* = \underset{b}{\operatorname{argmax}}\ S\left(y_0; b\right). \tag{22}$$

This result arises from the entrepreneur's utility maximization problem stated in Equation (B11). Note the conflicts of interest between the entrepreneur and external financiers. After debt is in place, the entrepreneur will no longer maximize the total value of the firm $S(y)$, but his private value of equity $G(y)$. Theorem 1 captures this conflict of interest between the entrepreneur and outside creditors.

The last step is to determine whether the entrepreneur wants to undertake the project. He makes the investment and starts up the firm at time 0 if his lifetime utility with the project is higher than that without the project. This is equivalent to the condition $S\left(y_0\right) > I$.

We may interpret our model's implication on capital structure as a generalized tradeoff model of capital structure for the entrepreneurial firm, where the entrepreneur trades off the benefits of outside debt financing (diversification and potential tax implications) against the costs of debt financing (bankruptcy and agency conflicts between the entrepreneur and outside lenders). The natural measure of leverage from the entrepreneur's point of view is the ratio between the public value of debt $F(y)$ and the private value of firm $S(y)$,

$$L(y) = \frac{F(y)}{S(y)}. \tag{23}$$

We label $L(y)$ as private leverage to reflect the impact of idiosyncratic risk on the leverage choice. The entrepreneur's preferences (e.g., risk aversion) influence the firm's capital structure. The standard argument that since shareholders can diversify for themselves, diversification plays no role in the capital structure decisions of public firms, is no longer valid for entrepreneurial firms.

Our discussions have focused on the parameter regions where the entrepreneur first establishes his firm as a private business and finances its operation via an optimal mix of outside debt and inside equity. For completeness, we now point out two special cases. First, when the cost of cashing out is sufficiently small, it can be optimal for the entrepreneur to sell the firm immediately ($y_u = y_0$). The other special case is when asset recovery rate is sufficiently high, or the entrepreneur is sufficiently risk averse, so that he raises as much debt as possible and defaults immediately ($y_d = y_0$). In our analysis below, we focus on parameter regions that rule out these cases of immediate exit.

3. Risky Debt, Endogenous Default, and Diversification

We now investigate a special case of the model that highlights the diversification benefits of risky debt. For this purpose, we shut down the cash-out option by setting the cash-out cost K to infinity, making the cash-out option worthless.

We use the following (annualized) baseline parameter values: risk-free interest rate $r = 3\%$, expected growth rate of revenue $\mu = 4\%$, systematic volatility of growth rate $\omega = 10\%$, idiosyncratic volatility $\varepsilon = 20\%$, market price of risk $\eta = 0.4$, and asset recovery rate $\alpha = 0.6$. We set the effective marginal Miller tax rate τ_m to 11.29% as in Graham (2000) and Hackbarth, Hennessy, and Leland (2007).[15] In our baseline parametrization, we set $\tau_e = 0$, which reflects the fact that the entrepreneur can avoid taxes on his business income completely by deducting various expenses. Shutting down the tax benefits also allows us to highlight the diversification benefits of debt. Later, we consider the case where $\tau_e = \tau_m$, which can be directly compared with the complete-markets model. We set the entrepreneur's rate of time preference $\delta = 3\%$, and consider three values of the risk-aversion parameter $\gamma \in \{0, 1, 2\}$. Finally, we set the initial level of revenue $y_0 = 1$.

3.1 Private Value of Equity $G(y)$ and Default Threshold

Figure 1 plots the private value of equity $G(y)$ and its derivative $G'(y)$ as functions of y. The top and the bottom panels plot the results for $\tau_e = 0$ and $\tau_e = \tau_m$, respectively. When $\tau_e = 0$, the entrepreneur who is risk neutral ($\gamma \to 0$, which is effectively the same as having complete markets) issues no debt, because there are neither tax benefits ($\tau_e = 0$) nor diversification benefits ($\gamma \to 0$). Equity value is equal to the present discounted value of future cash flows (the straight-dash line shown in the top-left panel). A risk-averse entrepreneur has incentive to issue debt in order to diversify idiosyncratic risk. The entrepreneur defaults when y falls to y_d, the point where $G(y_d) = G'(y_d) = 0$. When $\tau_e = \tau_m$, the entrepreneurial firm issues debt to take advantage of tax benefits in addition to diversification benefits. The bottom two panels of Figure 1 plot this case.

The derivative $G'(y)$ measures the sensitivity of private value of equity $G(y)$ with respect to revenue y. As expected, the private value of equity $G(y)$ increases with revenue y, i.e., $G'(y) > 0$. Analogous to Black-Scholes-Merton's observation that firm equity is a call option on firm assets, the entrepreneur's private equity $G(y)$ also has a call option feature. For example, in the bottom panels of Figure 1 ($\tau_e = \tau_m$), when γ approaches 0 (complete-markets case), equity value is convex in revenue y, reflecting its call option feature.

Unlike the standard Black-Scholes-Merton paradigm, neither the entrepreneurial equity nor the firm is tradable. When the risk-averse entrepreneur cannot fully diversify his project's idiosyncratic risk, the global convexity of $G(y)$ no longer holds, as shown in Figure 1 for cases where $\gamma > 0$. The entrepreneur

[15] We may interpret τ_m as the effective Miller tax rate, which integrates the corporate income tax, individual's equity, and interest income tax. Using Miller's formula for the effective tax rate, and setting the interest income tax at 0.30, corporate income tax at 0.31, and the individual's long-term equity (distribution) tax at 0.10, we obtain an effective tax rate of 11.29%.

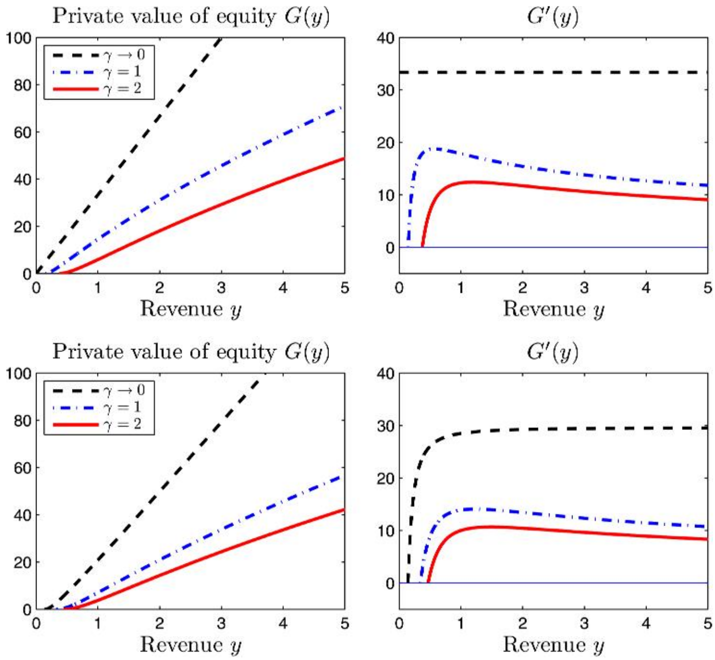

Figure 1
Private value of equity $G(y)$: debt financing only
The top and bottom panels plot $G(y)$ and its first derivative $G'(y)$ for $\tau_e = 0$ and $\tau_e = \tau_m$, respectively. We plot the results for two levels of risk aversion ($\gamma = 1, 2$) alongside the benchmark complete-market solution ($\gamma \to 0$).

now has precautionary saving demand to partially buffer against the project's nondiversifiable idiosyncratic shocks. This precautionary saving effect induces concavity in $G(y)$. When revenue y is large, the precautionary saving effect is large due to high idiosyncratic volatility ϵy, and the option (convexity) effect is small because the default option is further out of the money. Therefore, the precautionary saving effect dominates the option effect for sufficiently high y, making $G(y)$ concave in y. The opposite is true for low y, where the convexity effect dominates.

The precautionary saving effect also causes a more risk-averse entrepreneur to discount cash flows at a higher rate. For a given level of coupon b, the entrepreneur values his inside equity lower (smaller $G(y)$), and thus is more willing to default and walk away. Moreover, a more risk-averse entrepreneur also has a stronger incentive to diversify idiosyncratic risk by selling a bigger share of his firm, which implies a larger coupon b, a higher default threshold, and a higher debt value, *ceteris paribus*. The two effects reinforce each other. Figure 1 confirms that $G(y)$ decreases and the default threshold y_d increases with risk aversion γ.

Table 1
Capital structure of entrepreneurial firms: debt financing only

	Coupon b	Public debt F_0	Private equity G_0	Private firm S_0	Private leverage (%) L_0	Credit spread (bp) CS	10-Yr default probability (%) $p_d(10)$
Panel A: $\tau_e = 0$							
$\gamma \to 0$	0.00	0.00	33.33	33.33	0.0	0	0.0
$\gamma = 1$	0.31	8.28	14.39	22.68	36.5	72	0.4
$\gamma = 2$	0.68	14.66	5.89	20.55	71.3	166	12.1
Panel B: $\tau_e = \tau_m$							
$\gamma \to 0$	0.35	9.29	20.83	30.12	30.9	75	0.3
$\gamma = 1$	0.68	14.85	7.02	21.86	67.9	159	9.5
$\gamma = 2$	0.85	16.50	3.77	20.27	81.4	213	22.3
Panel C: $\tau_e = \tau_m, y_d = b$							
$\gamma \to 0$	0.06	1.94	27.73	29.67	6.5	21	0.0
$\gamma = 1$	0.12	3.67	16.95	20.62	17.8	36	0.2
$\gamma = 2$	0.45	10.94	7.53	18.47	59.2	109	19.8

This table reports the results for the setting where the entrepreneur has access to only debt financing and no option to cash out. We consider two business income tax rates ($\tau_e = 0$ or $\tau_e = \tau_m$ (11.29%)) and three levels of risk aversion. The case "$\gamma \to 0$" corresponds to the complete-markets (Leland) model. The remaining parameters are reported in Section 3. Panels A and B report results for the case of optimal default. Panel C reports results for the case of liquidity-induced default. All the results are for initial revenue $y_0 = 1$.

3.2 Capital Structure for Entrepreneurial Firms

First, we consider the special case where risky debt offers only diversification benefits for the entrepreneur and has no tax benefits ($\tau_e = 0$). Then, we incorporate the tax benefits of debt into our analysis.

Panel A of Table 1 provides results for the entrepreneurial firm's capital structure when $\tau_e = 0$. If the entrepreneur is very close to being risk neutral ($\gamma \to 0$), the model's prediction is the same as the complete-market benchmark. In this case, the standard tradeoff theory of capital structure implies that the entrepreneurial firm will be entirely financed by equity (since debt provides no benefits). The risk-neutral entrepreneur values the firm at its market value 33.33.

For $\gamma = 1$, the entrepreneur issues debt $F_0 = 8.28$ in market value with coupon $b = 0.31$, and values his nontradable equity at $G_0 = 14.39$, giving the private value of the firm $S_0 = 22.68$. The drop in S_0 is substantial (from 33.33 to 22.68, or about 32%) when increasing γ from zero to one. This drop in S_0 is mainly due to the risk-averse entrepreneur's discount of his nontradable equity position for bearing nondiversifiable idiosyncratic business risks. The default risk of debt contributes little to the reduction of S_0 (the 10-year cumulative default probability rises from 0 to 0.4% only).

In Section 2, we introduced the measure of leverage for entrepreneurial firms: private leverage L_0, given by the ratio of public debt value F_0 to private value of the firm S_0. Private leverage L_0 naturally arises from the entrepreneur's maximization problem and captures the entrepreneur's tradeoff between private value of equity and public value of debt in choosing debt coupon policy. For $\gamma = 1$, the private leverage ratio is 36.5%.

With a larger risk-aversion coefficient $\gamma = 2$, the entrepreneur borrows more ($F_0 = 14.66$) with a higher coupon ($b = 0.68$). He values his remaining nontradable equity at $G_0 = 5.89$, and the implied private leverage ratio $L_0 = 71.3\%$ is much higher than 36.5%, the value for $\gamma = 1$. The more risk-averse entrepreneur takes on more leverage, because he has a stronger incentive to sell more of the firm to achieve greater diversification benefits. With greater risk aversion, default is more likely (the 10-year cumulative default probability is 12.1%), and the credit spread is higher (166 basis points over the risk-free rate).

Next, we incorporate the effect of tax benefits for the entrepreneur into our generalized tradeoff model of capital structure for entrepreneurial firms. For comparison with the complete-markets benchmark, we set $\tau_e = \tau_m = 11.29\%$. Therefore, the only difference between an entrepreneurial firm and a public firm is that the entrepreneur faces nondiversifiable idiosyncratic risk.

The first row of Panel B of Table 1 gives the results for the complete-markets benchmark. Facing positive corporate tax rates, the public firm wants to issue debt, but is also concerned with bankruptcy costs. The optimal tradeoff for the public firm is to issue debt at the competitive market value $F_0 = 9.29$ with coupon $b = 0.35$. The implied initial leverage is 30.9%, and the 10-year cumulative default probability is tiny (0.3%).

Similar to the case with $\tau_e = 0$, an entrepreneur facing nondiversifiable idiosyncratic risk wants to issue more risky debt to diversify these risks. The second row of Panel B shows that the entrepreneur with $\gamma = 1$ borrows 14.85 (with coupon $b = 0.68$), higher than the level for the public firm. The private leverage more than doubles to 67.9%. Not surprisingly, the entrepreneur faces a higher default probability, and the credit spread of his debt is also higher. With $\gamma = 2$, debt issuance increases to 16.50, and private leverage increases to 81.4%.

In our model, the entrepreneur has "deep pockets." He chooses the optimal default strategy and might voluntarily inject cash into the firm to service its debt. However, in practice the entrepreneur may be liquidity constrained and have no external funds to cover the firm's debt service even if it is in his interest to do so. Assume that the entrepreneur is forced to default whenever the project revenue cannot cover the coupon payment, or equivalently when the firm's debt service coverage ratio, the ratio between revenue and debt service, just falls below one. We refer to such default as liquidity-induced default, where the default threshold is $y_d = b$.

In Panel C of Table 1, we present the results for the case with liquidity-induced default. Compared with Panel B (with optimal default), the entrepreneur defaults earlier and has higher default risk for the same coupon. His inability to finance debt service when the coverage ratio is below one lowers the option value of default and reduces his debt capacity. Both private leverage and private firm value are hence smaller. As in the previous setting, the optimal coupon and leverage increase significantly with risk aversion in the case

Table 2
Comparison of capital structures

	10-Yr default probability (%) $p_d(10)$	Public debt F_0	Equity value G_0	Firm value S_0	Financial leverage (%) L_0	Credit spread (bp) CS
$\gamma = 2$ ($b = 0.85$, $y_d = 0.47$)	22.3	16.50	3.77	20.27	81.4	213
$\gamma = 2$ ($b = 0.85$, $y_d = 0.85$)	77.0	16.47	1.64	18.11	90.9	214
$\gamma = 2$ ($b = 0.45$, $y_d = 0.45$)	19.8	10.94	7.53	18.47	59.2	109
Public ($b = 0.85$, $y_d = 0.47$)	22.3	16.50	11.10	27.60	59.8	213
Public ($b = 0.85$, $y_d = 0.35$)	9.8	17.71	11.56	29.26	60.5	178
Public ($b = 0.35$, $y_d = 0.14$)	0.3	9.29	20.82	30.11	30.9	75

This table first compares a liquidity-constrained entrepreneur with a deep-pocket entrepreneur, and then compares a private firm owned by a risk-averse entrepreneur with a public firm. There is no option to cash out. We assume $\tau_e = \tau_m$, while the rest of the parameters are reported in Section 3. All the results are for initial revenue $y_0 = 1$.

of liquidity-induced default, which shows that our key results on the diversification benefit of risky debt are robust to the different ways default is triggered.

3.3 Comparison of Capital Structure Decisions

To further demonstrate the important role of idiosyncratic risk and default decisions in determining the capital structure of entrepreneurial firms, we make two comparisons and discuss three experiments, as illustrated in Table 2.

First, we compare a liquidity-constrained entrepreneur with a deep-pocket entrepreneur in Rows 1–3 of Table 2. Rows 1 and 3 are taken from Panels B and C, respectively, of Table 1. Suppose the liquidity-constrained entrepreneur faces the coupon rate $b = 0.85$, which is the optimal level for the deep-pocket entrepreneur. The liquidity-constrained entrepreneur defaults at the threshold $y_d = b = 0.85$. Compared with Row 1, this early default lowers equity value from 3.77 to 1.64 and raises the 10-year default probability from 22.3% to 77%. But it lowers debt value very little because the recovery value of debt is significantly higher. Anticipating higher default probability, the liquidity-constrained entrepreneur will issue less debt by reducing the coupon rate from 0.85 to 0.45 (see Row 3). Comparing liquidity-induced default and optimal default (with endogenously chosen coupon), we see that liquidity constraint lowers private equity value, firm value, and leverage (compare Rows 1 and 3).

Next, we compare a deep-pocket entrepreneur with a public firm. First, we consider an econometrician who has correctly identified the entrepreneurial firm's debt coupon $b = 0.85$ and default threshold $y_d = 0.47$, but does not realize that the entrepreneur's subjective valuation $G(y; b, y_d)$ is lower than the corresponding public equity value $E(y; b, y_d)$ due to nondiversifiable idiosyncratic risk. As Row 4 of Table 2 shows, the econometrician assigns the entrepreneur's equity with a value at $E_0 = 11.10$ instead of the subjective valuation $G_0 = 3.77$, thus obtaining a leverage ratio of 59.8%, substantially lower than the entrepreneur's private leverage $L_0 = 81.4\%$. The large difference between the private and market leverage ratios highlights the economic significance of taking idiosyncratic risk into account. Simply put, standard corporate finance methodology potentially underestimates the leverage of entrepreneurial firms.

Second, we highlight the effect of different default decisions for a deep-pocket entrepreneur and a public firm. The public and the entrepreneurial firms have significantly different leverage decisions because both debt issuance and default decisions on debt (given the same level of debt coupon outstanding) are different. To see the quantitative effects of endogenous default decisions on leverage, we hold the coupon rate on outstanding debt fixed. That is, consider a public firm that has the same technology/environment parameters as the entrepreneurial firm. Moreover, the two firms have the same debt coupons ($b = 0.85$).

As Row 5 shows, facing the same coupon $b = 0.85$, the public firm defaults when revenue reaches the default threshold $y_d = 0.35$, which is lower than the threshold $y_d = 0.47$ for the entrepreneurial firm. Intuitively, facing the same coupon b, the entrepreneurial firm defaults earlier than the public firm because of the entrepreneur's aversion to nondiversifiable idiosyncratic risk. The implied shorter distance-to-default for the entrepreneurial firm translates into a higher 10-year default probability (22.3% for the entrepreneurial firm versus 9.8% for the public firm) and a higher credit spread (213 basis points for the entrepreneurial firm versus 178 basis points for the public firm). Defaulting optimally for the public firm raises its value from $S_0 = 27.60$ to $S_0 = 29.26$. When the public firm chooses optimal debt, it raises firm value further to $S_0 = 30.11$. In addition, it issues less debt (with a smaller coupon) than the deep-pocket entrepreneur, as reported in the last row of Table 2. As a result, the public firm has a lower leverage ratio than the entrepreneurial firm.

The last two comparisons help explain the differences in leverage ratios between the entrepreneurial firm and the public firm. First, fixing both the coupon and the default threshold, the entrepreneur's subjective valuation (due to nondiversifiable risks) has significant impact on the implied leverage ratio. Ignoring subjective valuation can lead one to substantially underestimate the entrepreneurial firm's leverage. Second, facing the same coupon, the entrepreneurial firm defaults earlier than the public firm, which reduces the value of debt and lowers the leverage ratio. Third, diversification motives make the entrepreneur issue more debt than the public firm, which further raises the leverage ratio of the entrepreneurial firm. While the numerical results are parameter specific, the analysis provides support for our intuition that the entrepreneur's need for diversification and the subjective valuation discount for bearing nondiversifiable idiosyncratic risk are key determinants of the private leverage for an entrepreneurial firm.

4. Cash-out Option as an Alternative Channel of Diversification

We now turn to a richer and more realistic setting where the entrepreneur can diversify idiosyncratic risk through both the default and cash-out options. The

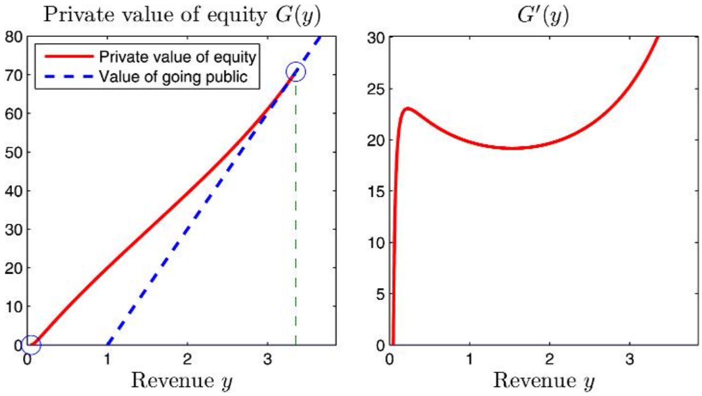

Figure 2
Private value of equity $G(y)$: debt financing and cash-out option
The left panel plots the private value of equity $G(y)$ as a function of revenue y, and the right panel plots its first derivative.

entrepreneur avoids the downside risk by defaulting if the firm's stochastic revenue falls to a sufficiently low level. When the firm does well enough, the entrepreneur may want to capitalize on the upside by selling the firm to diversified investors.

In addition to the baseline parameter values from Section 3, we set the effective capital gains tax rate from selling the business $\tau_g = 10\%$, reflecting the tax deferral advantage.[16] We set the initial investment cost for the project $I = 10$, which is 1/3 of the market value of project cash flows. We choose the cash-out cost $K = 27$ to generate a 10-year cash-out probability of about 20% (with $\gamma = 2$), consistent with the success rates of venture capital firms (Hall and Woodward 2010).

4.1 Cash-out Option: Crowding Out Debt

Figure 2 plots the private value of equity $G(y)$ and its first derivative $G'(y)$ for an entrepreneur with risk aversion $\gamma = 1$ when he has the option to cash out. The function $G(y)$ smoothly touches the horizontal axis on the left and the dashed line denoting the value of cashing out on the right. The two tangent points give the default and cash-out thresholds, respectively. For sufficiently low values of revenue y, the private value of equity $G(y)$ is increasing and convex because the default option is deep in the money. For sufficiently high values of y, $G(y)$ is also increasing and convex because the cash-out option is deep in the money. For revenue y in the intermediate range, neither default nor cash-out option is deep in the money, and the precautionary saving motive may be large enough to induce concavity. As shown in the right panel of Figure 2,

16 In Appendix D.1, we investigate the effects of different capital gains taxes.

Table 3
Capital structure of entrepreneurial firms: debt financing and cash-out option

	Coupon b	Public debt F_0	Private equity G_0	Private firm S_0	Private leverage (%) L_0	10-Yr default prob (%) $p_d(10)$	10-Yr cash-out prob (%) $p_u(10)$
Panel A: $\tau_e = 0$							
$\gamma \to 0$	0.00	0.00	33.33	33.33	0.0	0.0	0.0
$\gamma = 1$	0.12	3.61	19.36	22.97	15.7	0.0	9.2
$\gamma = 2$	0.43	10.36	10.01	20.36	50.9	2.2	20.4
Panel B: $\tau_e = \tau_m$							
$\gamma \to 0$	0.35	9.29	20.83	30.12	30.9	0.3	0.0
$\gamma = 1$	0.55	12.45	9.57	22.02	56.5	4.2	12.3
$\gamma = 2$	0.66	13.68	6.24	19.92	68.7	10.1	23.3

This table reports the results for the setting where the entrepreneur has access to both public debt financing and cash-out option to exit from his project. We report results for two business income tax rates ($\tau_e = 0$ or $\tau_e = \tau_m$(11.29%)). The rest of the parameters are reported in Section 4. All the results are for initial revenue $y_0 = 1$.

$G'(y)$ first increases for low values of y, then decreases for intermediate values of y, and finally increases again for high values of y.

Table 3 provides the capital structure information of an entrepreneurial firm with both cash-out and default options. Again we consider the two cases $\tau_e = 0$ and $\tau_e = \tau_m$.

When markets are complete, with $\tau_e = 0$, there is no reason for the firm to issue debt or go public. Thus, the optimal leverage and the cash-out probability will both be zero. When τ_e is positive, the firm's cash-out option is essentially an option to adjust the firm's capital structure (recall that there is no diversification benefit for public firms). In this case, given our calibrated fixed cost K, the 10-year cash-out probability is essentially zero, and hence this option value is close to zero for the public firm. Therefore, we expect that the bulk of the cash-out option value for entrepreneurial firms will come from the diversification benefits, not from the option to readjust leverage.

For a risk-averse entrepreneur, the prospect of cashing out lowers the firm's incentive to issue debt. When $\tau_e = 0$ and $\gamma = 1$, debt coupon falls from $b = 0.31$ for the firm with only the default option to 0.12 when the cash-out option is added, and the private leverage ratio at issuance falls from $L_0 = 36.5\%$ to 15.7%. The 10-year default probability is close to zero, but the 10-year cash-out probability is 9.2%, which is economically significant. For more risk-averse entrepreneurs (e.g., $\gamma = 2$), the private leverage ratio is 50.9%, smaller than 71.3% for the setting without the cash-out option. While a higher tax rate τ_e does increase the amount of debt the firm issues, the impact of the cash-out option is qualitatively similar to the no-tax case. Thus, given the opportunity to sell his business to public investors, the entrepreneur substitutes away from risky debt and relies more on the future potential of cashing out to diversify his idiosyncratic risk.

Our analysis is under the assumption that debt is priced in the public market. We have also computed the private value of debt if the lenders are

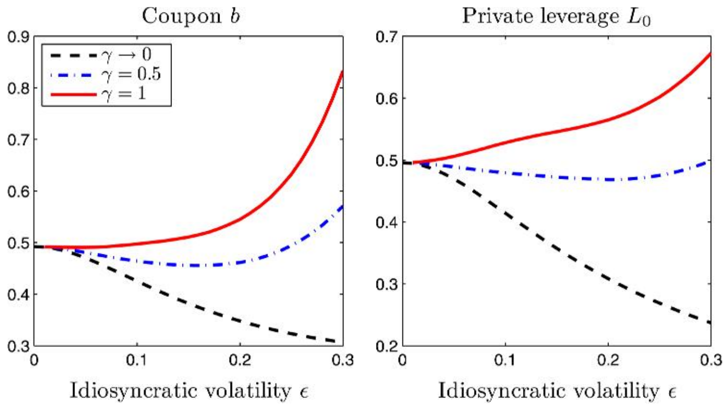

Figure 3
Comparative statics—optimal coupon and private leverage with respect to idiosyncratic volatilities ϵ: the case of debt and cash-out option
The two panels plot the optimal coupon b and the corresponding optimal private leverage L_0 at $y_0 = 1$. In each case, we plot the results for two levels of risk aversion ($\gamma = 0.5, 1$) alongside the benchmark complete-market solution ($\gamma \to 0$).

underdiversified and/or if debt is actually held by the entrepreneur.[17] While nondiversifiable risk does lower the debt value from the perspective of under-diversified investors, the difference from the value of public debt is small. This suggests that even when lenders are underdiversified, issuing risky debt still provides significant diversification benefits for the entrepreneur. Intuitively, this is because in normal times lenders have significantly less exposure to firm-specific risks compared with the entrepreneur.

4.2 Idiosyncratic Risk, Leverage, and Risk Premium

We now turn to the impact of idiosyncratic volatility on leverage and the risk premium for equity. Figure 3 shows its effect on leverage. In complete-markets models, an increase in (idiosyncratic) volatility ϵ raises default risk, hence the market leverage ratio and the coupon rate for the public firm decrease with idiosyncratic volatility. By contrast, risk-averse entrepreneurs take on more debt to diversify their idiosyncratic risk when ϵ is higher. For $\gamma = 1$, both coupon and leverage become monotonically increasing in ϵ. This result implies that the private leverage ratio for entrepreneurial firms increases with idiosyncratic volatility even for mild risk aversion.

For public firms, the risk premium for equity is determined by the firm's systematic risk. For entrepreneurial firms, both systematic and idiosyncratic risks matter for the risk premium. Without loss of generality, we decompose the entrepreneur's risk premium into two components: the systematic risk

[17] We thank the referee for recommending this exercise. The results are reported in the Internet Appendix.

premium $\pi^s(y)$ and the idiosyncratic risk premium $\pi^i(y)$. Rearranging Equation (15) gives

$$\pi^s(y) = \eta\omega \frac{G'(y)}{G(y)} y = \eta\omega \frac{d \ln G(y)}{d \ln y}, \tag{24}$$

$$\pi^i(y) = \frac{\gamma r}{2} \frac{\left(\epsilon y G'(y)\right)^2}{G(y)}. \tag{25}$$

The systematic risk premium $\pi^s(y)$ defined in Equation (24) takes the same form as in standard asset pricing models. It is the product of the (market) Sharpe ratio η, the systematic volatility ω, and the elasticity of $G(y)$ with respect to y, where the elasticity captures the impact of optionality on the risk premium.[18]

Unlike $\pi^s(y)$, the idiosyncratic risk premium $\pi^i(y)$ defined in Equation (25) *directly* depends on risk aversion γ and $\left(\epsilon y G'(y)\right)^2$, the conditional (idiosyncratic) variance of the entrepreneur's equity $G(y)$. The conditional (idiosyncratic) variance term reflects the fact that the idiosyncratic risk premium $\pi^i(y)$ is determined by the entrepreneur's precautionary saving demand, which depends on the conditional variance of idiosyncratic risk (Caballero 1991; Wang 2006).

We examine the behavior of these risk premiums in Figure 4. The entrepreneur's equity is a levered position in the firm. When the firm approaches default, the systematic component of the risk premium $\pi^s(y)$ behaves similarly to the standard valuation model. That is, the significant leverage effect around the default boundary implies that the systematic risk premium diverges to infinity when y approaches y_d. When the firm approaches the cash-out threshold, the cash-out option makes the firm value more sensitive to cash flow shocks, which also tends to raise the systematic risk premium.

The idiosyncratic risk premium $\pi^i(y)$ behaves quite differently. Figure 4 indicates that the idiosyncratic risk premium is small when the firm is close to default, and it increases with y for most values of y. The intuition is as follows. The numerator in Equation (25) reflects the entrepreneur's precautionary saving demand, which depends on the conditional idiosyncratic variance of the changes in the certainty equivalent value of equity $G(y)$ and risk aversion γ. Both the conditional idiosyncratic variance and $G(y)$ increase with y. When y is large, the conditional idiosyncratic variance rises fast relative to $G(y)$, generating a large idiosyncratic risk premium.

[18] Despite this standard interpretation for the systematic risk premium, it is worth pointing out that $\pi^s(y)$ also indirectly reflects the nondiversifiable idiosyncratic risk that the entrepreneur bears, and risk aversion γ indirectly affects $\pi^s(y)$ through its impact on $G(y)$.

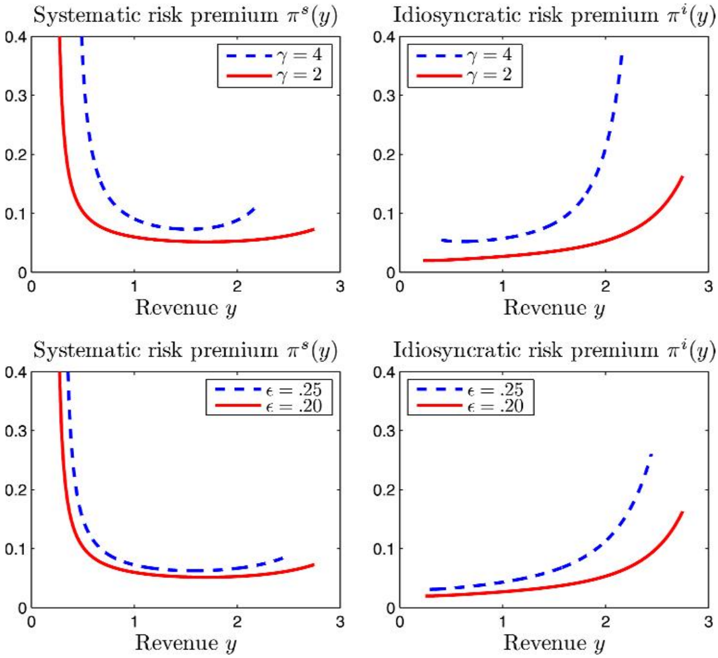

Figure 4
Systematic and idiosyncratic risk premium
This figure plots the systematic and idiosyncratic risk premium for entrepreneurial firms. The top panels plot the results for two levels of risk aversion ($\gamma = 2, 4$). The bottom panels plot the results for two levels of idiosyncratic volatility ($\epsilon = 0.20, 0.25$). We assume $\gamma = 2$ when changing ϵ, and $\epsilon = 0.2$ when changing γ.

5. Idiosyncratic Risk and Investment

So far, we have focused on the effects of idiosyncratic risk on financing decisions. In this section, we explore how idiosyncratic risk influences an entrepreneurial firm's investment decisions.[19] We analyze two aspects of investment decisions. First, we examine the cutoff rule for taking on the investment at time $t = 0$, i.e., the breakeven investment cost I that makes the entrepreneur indifferent between undertaking the project or not. The standard NPV analysis no longer applies due to nondiversifiable risk and incomplete markets. Second, we study the entrepreneur's incentives for risk shifting when choosing among projects with different degrees of idiosyncratic risk (after debt is chosen).

5.1 Project Choice: Breakeven Investment Cost

As shown in Section 2, the entrepreneur will invest in a project at $t = 0$ only if the total value of the entrepreneurial firm $S(y_0)$ is greater than the one-time lump-sum cost I. In the full model where the entrepreneur has access to debt

[19] We thank the referee for suggesting deepening our analysis on the impact of financing on investment decisions.

Table 4
Idiosyncratic risk, risk aversion, and investment decisions

	Optimal leverage			No leverage		
	$\epsilon = 0.15$	$\epsilon = 0.20$	$\epsilon = 0.25$	$\epsilon = 0.15$	$\epsilon = 0.20$	$\epsilon = 0.25$
$\gamma \to 0$	33.33	33.33	33.33	33.33	33.33	33.33
$\gamma = 1.0$	25.64	23.00	20.88	25.13	22.19	19.82
$\gamma = 2.0$	23.09	20.38	18.36	22.38	19.26	16.89

This table reports the breakeven investment cost I for different levels of entrepreneurial risk aversion γ and idiosyncratic volatility ϵ in the case with debt financing and cash-out option ($\tau_e = 0$).

financing and a cash-out option, the entrepreneur triggers capital gains taxes (with the tax base being the firm's investment cost I) when he exercises his cash-out option. Therefore, private firm value $S(y_0)$ depends on the investment cost I via the potential capital gains tax. Technically, this makes finding the breakeven investment cost $I^* = S(y_0|I^*)$ a fixed-point problem.

We compute the breakeven cost I^* for the case with debt financing and cash-out option for various values of risk aversion γ and idiosyncratic volatility ϵ. Two sets of results are reported in Table 4, one with optimal debt financing, the other under the assumption of no risky debt. We focus on the case $\tau_e = 0$, which better highlights the diversification benefit of debt. With complete markets ($\gamma \to 0$), the entrepreneur will neither issue debt nor cash out. The breakeven investment cost is thus simply equal to the present value of the perpetual revenue flow y_t, which is independent of idiosyncratic volatility.

When markets are incomplete, either under optimal leverage or no leverage, the breakeven investment cost falls as the entrepreneur becomes more risk averse and/or when the idiosyncratic volatility of the project becomes higher. For example, under optimal leverage with $\epsilon = 0.15$, those projects with investment costs between 25.64 and 33.33 will be rejected by an entrepreneur with risk aversion $\gamma = 1$, but accepted by an otherwise identical yet fully diversified manager. As γ or ϵ increases, the difference in the breakeven costs between the entrepreneur and a diversified manager gets even bigger, leading to more projects being turned down by the entrepreneur. Intuitively, higher risk aversion and higher idiosyncratic volatility raise the idiosyncratic risk premium that the entrepreneur demands for holding the firm and hence lower the cutoff level for the investment cost I. Moreover, comparing the case under optimal leverage and under no leverage, we see that the ability to issue risky debt raises the breakeven investment costs, hence making the entrepreneur more willing to invest. This effect is again stronger for higher risk aversion and higher idiosyncratic volatility.

To summarize, our results show that idiosyncratic risk and incomplete markets generate underinvestment for risk-averse entrepreneurs (relative to public firms). The underinvestment problem is more severe for more risk-averse entrepreneurs or projects with higher idiosyncratic volatility. Importantly, risky debt helps alleviate this underinvestment problem by improving diversification.

5.2 Project Choice: Asset Substitution Versus Risk Sharing

Jensen and Meckling (1976) point out that there is an incentive problem associated with risky debt: After debt is in place, managers have incentives to take on riskier projects to take advantage of the option type of payoff structure of equity. However, there is limited empirical evidence in support of such risk-shifting behaviors.[20] One possible explanation is that managerial risk aversion can potentially dominate the risk-shifting incentives. Our model provides a natural setting to investigate these two competing effects quantitatively.

We consider the following project choice problem. Suppose the risk-averse entrepreneur can choose among a continuum of mutually exclusive projects with different idiosyncratic volatilities ϵ in the interval $[\epsilon_{\min}, \epsilon_{\max}]$ *after debt is in place*. Let F_0 be the market value of existing debt with the coupon b. The entrepreneur then chooses idiosyncratic volatility $\epsilon^+ \in [\epsilon_{\min}, \epsilon_{\max}]$ to maximize his own utility. As shown in Section 2, the entrepreneur effectively chooses ϵ^+ to maximize his private value of equity $G(y_0)$, taking the debt contract (b, F_0) as given. Let this maximized value be $G^+(y_0)$.

In a rational expectations equilibrium, the lenders anticipate the entrepreneur's *ex post* incentive of choosing the level of idiosyncratic volatility ϵ^+ to maximize $G(y_0)$, and price the initial debt contract accordingly in competitive capital markets. Therefore, the entrepreneur *ex ante* maximizes the private value of the firm, $S(y_0) = G^+(y_0) + F_0$, taking the competitive market debt pricing into account. We solve this joint investment and financing (fixed-point) problem.

Figure 5 illustrates the solution of this optimization problem. We set $\epsilon_{\min} = 0.05$ and $\epsilon_{\max} = 0.35$. When $\gamma \to 0$, the entrepreneur chooses the highest idiosyncratic volatility project with $\epsilon_{\max} = 0.35$. The optimal coupon is 0.297. In this case, the entrepreneur effectively faces complete markets. The Jensen and Meckling (1976) argument applies because the market value of equity is convex and the *risk shifting* problem arises. When the entrepreneur is risk averse, he demands a premium for bearing the nondiversifiable idiosyncratic risk, which tends to lower his private value of equity $G\,(y_0)$. When this effect dominates, the entrepreneur prefers projects with lower idiosyncratic volatility. For example, for $\gamma = 1$, the entrepreneur chooses the project with $\epsilon_{\min} = 0.05$, with the corresponding optimal coupon 0.491. Even when the degree of risk aversion is low (e.g., $\gamma = 0.1$, which implies an idiosyncratic risk premium of 2 basis points for $\epsilon = 0.05$, or 20 basis points for $\epsilon = 0.20$), we still find that the risk-aversion effect dominates the risk-shifting incentive.

From this numerical example, we find that in our model, even with low risk aversion, the precautionary saving incentive tends to dominate the asset substitution incentive in normal times (risk shifting will still be important when the firm is sufficiently close to default, where the entrepreneur's value function becomes convex). Our argument applies to public firms as well, provided that (i) managerial compensation is tied to firm performance; and (ii) managers

[20] See Andrade and Kaplan (1998) and Graham and Harvey (2001), among others.

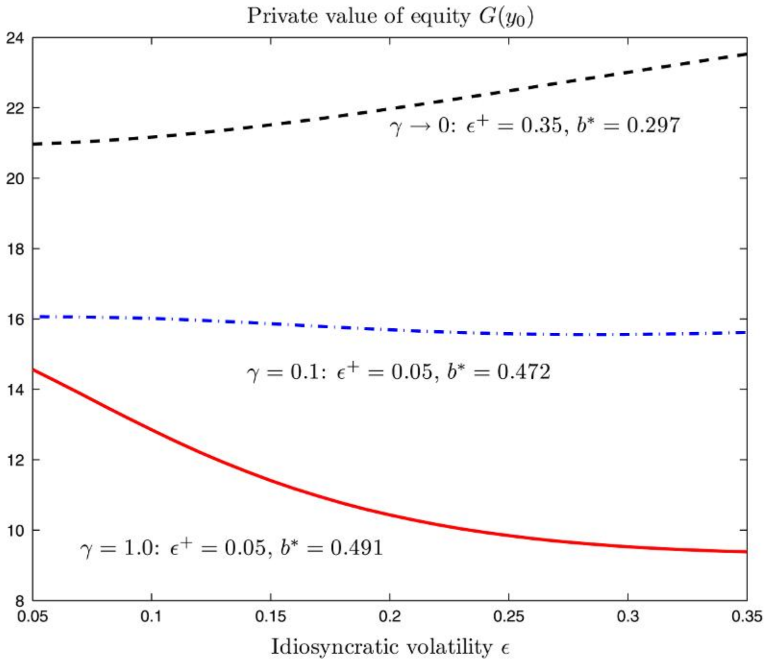

Figure 5
Private equity value as a function of idiosyncratic volatility after optimal debt is in place
This figure plots the private value of equity for different choices of idiosyncratic volatility ϵ after debt issuance. The coupon is fixed at the optimal value corresponding to given risk aversion. We assume $\epsilon_{\min} = 0.05$, $\epsilon_{\max} = 0.35$. The remaining parameters are the same as in Table 3.

are not fully diversified, behave in their own interests, and are entrenched. Thus, the lack of empirical evidence for asset substitution may be due to the nondiversifiable idiosyncratic risk faced by risk-averse decision makers.

6. External Equity

While debt is the primary source of financing for most entrepreneurial (small-business) firms, high-tech startups are often financed by venture capital (VC), which often use external equity in various forms as the primary source of financing. This financing choice particularly makes sense when the liquidation value of firm's assets is low (e.g., computer software firms). Hall and Woodward (2008) provide a quantitative analysis for the lack of diversification of entrepreneurial firms backed by venture capital. In this section, we extend the baseline model of Section 1 by allowing the entrepreneur to issue external equity at $t = 0$, and study the effect of external equity on the diversification benefits of risky debt.

If it is costless to issue external equity, a risk-averse entrepreneur will want to sell the entire firm to the VC right away. We motivate the costs of issuing external equity through the agency problems of Jensen and Meckling (1976). Intuitively, the more concentrated the entrepreneur's ownership, the better incentive alignment he achieves (Berle and Means 1932; Jensen and Meckling 1976). Let ψ denote the fraction of equity that the entrepreneur retains and hence $1-\psi$ denote the fraction of external equity. Consider the expected growth rate of revenue μ in Equation (1). We capture the incentive problem of ownership in reduced form by making μ an increasing and concave function of the entrepreneur's ownership ψ ($\mu'(\psi) > 0$ and $\mu''(\psi) < 0$). Intuitively, the concavity relation suggests that the incremental value from incentive alignment becomes lower as ownership concentration rises, *ceteris paribus*.

More specifically, we model the growth rate μ as a quadratic function of the entrepreneur's ownership ψ, $\mu(\psi) = -0.02\psi^2 + 0.04\psi + 0.03$, with $\psi \in [0, 1]$. This functional form implies that the maximum expected growth rate is 5%, when the entrepreneur owns the entire firm ($\psi = 1$), while the lowest growth rate is 3%, when the entire firm is sold ($\psi = 0$). For simplicity, we rule out dynamic adjustments of ψ. Once ψ is chosen, the expected growth rate μ will remain constant thereafter.

After external debt (with coupon b) and equity (with share $1-\psi$ of the firm ownership) are issued at $t = 0$, the entrepreneur's optimal policies, including consumption/portfolio rule and default/cash-out policies, are summarized in the following theorem.

Theorem 2. The entrepreneur exits from his business when the revenue process $\{y_t : t \geq 0\}$ reaches either the default threshold y_d or the cash-out threshold y_u, whichever occurs first. When the entrepreneur runs his firm, he chooses his consumption and portfolio rules as follows:

$$\overline{c}\,(x,y) = r\left(x + \psi G\,(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{26}$$

$$\overline{\phi}\,(x,y) = \frac{\eta}{\gamma r \sigma_p} - \frac{\psi\omega}{\sigma_p} y G'\,(y)\,, \tag{27}$$

where $G(\cdot)$ solves the free boundary problem given by the differential equation:

$$rG(y) = (1-\tau_e)\,(y-b) + \nu y G'(y) + \frac{\sigma^2 y^2}{2} G''(y) - \frac{\psi\gamma r\epsilon^2 y^2}{2} G'(y)^2, \tag{28}$$

subject to the following (free) boundary conditions at y_d and y_u:

$$G(y_d) = 0, \tag{29}$$

$$G'(y_d) = 0, \tag{30}$$

$$\psi G(y_u) = \psi V^*(y_u) - F_0 - K - \tau_g \left(\psi V^*(y_u) - K - (I - (1-\psi) E_0) \right), \tag{31}$$

$$\psi G'(y_u) = (1-\tau_g) \psi V^{*\prime}(y_u) . \tag{32}$$

The complete-markets firm value $V^*(y)$ is defined in Equation (A21), the value of external debt $F_0 = F(y_0)$ is given in Equation (C6), and the value of external equity $E_0 = E(y_0)$ is given in Equation (C10).

Equation (28) shows how the partial ownership ψ affects the entrepreneur's private value of equity. A more concentrated inside equity position (higher ψ) raises the last nonlinear term, which raises the idiosyncratic risk premium that the entrepreneur demands. The ownership ψ also affects the boundary conditions at cash-out. The value-matching condition in Equation (31) at the cash-out boundary states that, upon cashing out, the entrepreneur's ownership is worth a fraction ψ of the after-tax value of the public firm value net of (i) the amount required to retire outstanding debt at par F_0; (ii) fixed costs K; and (iii) capital gains taxes. The smooth-pasting condition in Equation (32) also reflects the effects of partial ownership.

Finally, at time $t = 0$, the entrepreneur chooses debt coupon b and initial ownership ψ to maximize the private value of the firm $S(y)$, which now has three parts: inside equity (entrepreneur's ownership), diversified outside equity, and outside debt:

$$S(y) = \psi G(y) + (1-\psi) E_0(y) + F(y). \tag{33}$$

The results are reported in Table 5. If the entrepreneur is risk-neutral, he will clearly prefer to keep 100% ownership. In this case, all the equity in the firm is privately held, and the private leverage is 0 if $\tau_e = 0$, or 33.6% when $\tau_e = \tau_m$. When $\tau_e = 0$, an entrepreneur with $\gamma = 1$ lowers his ownership to 67%, which reduces the growth rate to 4.78% (about a 0.2% drop). However, the coupon rises from 0 to 0.43, and private leverage rises from 0 to 34.6%. The 10-year default and cash-out probabilities rise from 0 to 0.6% and 8.9%, respectively. When $\gamma = 2$, the ownership drops further to 62%, while the coupon rises to 0.52, and private leverage rises to 41.7%. The 10-year cash-out probability also rises to 12.0%. In other words, a more risk-averse entrepreneur actively uses all three channels (outside equity, outside debt, and cash-out option) to diversify his idiosyncratic risk exposure.

The results are similar when $\tau_e = \tau_m$. When $\gamma = 1$, the coupon rises from 0.55 to 0.66, and private leverage rises from 33.6% to 48.8%. Such an increase in demand for debt due to diversification is economically sizeable, especially considering that the increase is partially offset by the reduced tax benefit of debt due to lower expected growth rates. When $\gamma = 2$, the ownership drops to 65%, while private leverage rises further to 51.2%. Notice that while the

Table 5
Capital structure of entrepreneurial firms: external debt/equity and cash-out option

	Ownership ψ	Coupon b	Public debt F_0	Public equity $(1-\psi)E_0$	Private equity ψG_0	Private firm S_0	Private leverage (%) L_0	Default prob (%) $p_d(10)$	Cash-out prob (%) $p_u(10)$
Panel A: $\tau_e = 0$									
$\gamma \to 0$	1.00	0.00	0.00	0.00	50.00	50.00	0.0	0.0	0.0
$\gamma = 1$	0.67	0.43	11.49	10.22	11.52	33.22	34.6	0.6	8.9
$\gamma = 2$	0.62	0.52	13.06	10.87	7.42	31.35	41.7	2.2	12.0
Panel B: $\tau_e = \tau_m$									
$\gamma \to 0$	1.00	0.55	15.23	0.00	30.07	45.30	33.6	0.4	0.0
$\gamma = 1$	0.69	0.66	16.00	8.50	8.26	32.76	48.8	3.8	11.3
$\gamma = 2$	0.65	0.68	15.93	9.50	5.67	31.10	51.2	6.0	15.4

This table reports the results for the setting where the entrepreneur has access to public debt/equity financing and cash-out option. We assume $\mu(\psi) = -0.02\psi^2 + 0.04\psi + 0.03$, while the remaining parameters are reported in Section 4. All the results are for initial revenue $y_0 = 1$.

coupon rises from 0.66 to 0.68, the initial value of debt actually falls slightly. This is because both the default and cash-out probability are higher for higher γ, which reduces the expected life of the debt and offsets the effect of a higher coupon on debt value.

The optimal ownership predicted by the model (between 60% and 70%) is low compared with the data (81% on average, according to Heaton and Lucas 2004). One possible explanation is that the agency costs of external equity we consider are small, and raising the agency costs will increase the degree of ownership concentration as well as the amount of debt the entrepreneur issues. The results in Table 5 thus confirm the robustness of our finding: Entrepreneurial firms still have sizeable demand for risky debt and cash-out options for diversification purpose even when external equity is available.

Our model is also applicable to publicly traded firms where managers have significant wealth exposures due to their concentrated equity positions in firms. It shows that the interaction between ownership structure and capital structure is potentially quantitatively important. Our analysis implies that private leverage rather than public leverage is the relevant measure of capital structure for public firms where managers are underdiversified and have significant discretion. Using publicly available data to construct public leverage may potentially misrepresent the managerial tradeoff between equity and debt. One consequence is that credit rating agencies might underestimate the leverage and default probability of these firms considerably, and hence might issue credit ratings that are too high for such firms.

7. Concluding Remarks

Entrepreneurial investment opportunities are often illiquid and nontradable. Entrepreneurs cannot completely diversify away project-specific risks for reasons such as incentives and informational asymmetry. Therefore, the standard law-of-one-price–based valuation/capital structure paradigm in corporate finance cannot be directly applied to entrepreneurial finance. An entrepreneur acts both as a producer making dynamic investment, financing, and exit decisions for his business project, and as a household making consumption/saving and portfolio decisions. The dual roles of the entrepreneur motivate us to develop a dynamic incomplete-markets model of entrepreneurial finance that centers around the nondiversification feature of the entrepreneurial business.

Besides studying the financing and investment decisions for entrepreneurs and underdiversified managers, our modelling framework can also be used to value the stock options of underdiversified executives or to analyze how these executives make capital structure and investment decisions. See Carpenter, Stanton, and Wallace (2010) for a recent study on the optimal exercise policy for an executive stock option and implications for firm costs.

We have taken a standard optimization framework where the entrepreneur's utility depends on only his consumption. While our model is also applicable to public firms where managers are not diversified and sufficiently entrenched, we ignore managerial incentives (e.g., being an empire builder), which could be significant in determining capital structure decisions in public firms (Zwiebel 1996; Morellec 2004). A significant fraction of entrepreneurs view the nonpecuniary benefits of being their own bosses as a large component of rewards. It has also been documented that less risk-averse (see Gentry and Hubbard 2004; De Nardi, Doctor, and Krane 2007) and more confident/optimistic individuals are more likely to self-select into entrepreneurship.

Market incompleteness is taken exogenously in our model. Real-world capital structure decisions of entrepreneurial firms likely reflect agency frictions and informational asymmetries leading markets to be endogenously incomplete and ownership to be concentrated. For example, DeMarzo and Fishman (2007), DeMarzo and Sannikov (2006), and DeMarzo, Fishman, He, and Wang (2009) derive optimal recursive contracts in settings where entrepreneurs face dynamic moral hazard issues. The optimal contracts derived in these papers can be implemented via concentrated insider ownership and credit lines. Our model also does not feature endogenous financial constraints. Moral-hazard–based contracting models (such as those mentioned above) naturally generate endogenous financial constraints. We view endogenously incomplete markets as a complementary perspective and an important direction to extend our article, which can have fundamental implications such as promotion of entrepreneurship and contract design.

Appendix

A. Market Valuation and Capital Structure of a Public Firm

Well-diversified owners of a public firm face complete markets. Given the Sharpe ratio η of the market portfolio and the risk-free rate r, there exists a unique stochastic discount factor (SDF)

$(\xi_t : t \geq 0)$ (see Duffie 2001) satisfying

$$d\xi_t = -r\xi_t dt - \eta\xi_t dB_t, \quad \xi_0 = 1. \tag{A1}$$

Using this SDF, we can derive the market value of the unlevered firm, $A(y)$, the market value of equity, $E(y)$, and the market value of debt, $D(y)$. The market value of the firm is equal to the sum of equity value and debt value:

$$V(y) = E(y) + D(y). \tag{A2}$$

Under the risk-neutral probability measure Q, we can rewrite the dynamics of the revenue y in Equation (1) as follows:

$$dy_t = \nu y_t dt + \omega y_t dB_t^Q + \epsilon y_t dZ_t, \tag{A3}$$

where ν is the risk-adjusted drift defined by $\nu \equiv \mu - \omega\eta$, and B_t^Q is a standard Brownian motion under Q satisfying $dB_t^Q = dB_t + \eta dt$.

A.1 Valuation of an Unlevered Public Firm

Throughout the appendix, we derive our results assuming that there is a flow operating cost z for running the project. The operating cost z generates operating leverage, and hence the option to abandon the firm has positive value. The results reported in this article are for the case $z = 0$. Appendix D.2 provides results for the case $z > 0$.

We start with the after-tax unlevered firm value $A(y)$, which satisfies the following differential equation:

$$rA(y) = (1 - \tau_m)(y - z) + \nu y A'(y) + \frac{1}{2}\sigma^2 y^2 A''(y). \tag{A4}$$

This is a second-order ordinary differential equation (ODE). We need two boundary conditions to obtain a solution. One boundary condition describes the behavior of $A(y)$ when $y \to \infty$. This condition rules out speculative bubbles. To ensure that $A(y)$ is finite, we assume $r > \nu$ throughout the article. The other boundary condition is related to abandonment. As in the standard option exercise models, the firm is abandoned whenever the cash flow process hits a threshold value y_a for the first time. At the threshold y_a, the following value-matching condition is satisfied:

$$A(y_a) = 0, \tag{A5}$$

because we normalize the outside value to zero. For the abandonment threshold y_a to be optimal, the following smooth-pasting condition must also be satisfied:

$$A'(y_a) = 0. \tag{A6}$$

Solving Equation (A4) and using the no-bubble condition and boundary conditions (A5) and (A6), we obtain

$$A(y) = (1 - \tau_m)\left[\left(\frac{y}{r-\nu} - \frac{z}{r}\right) - \left(\frac{y_a}{r-\nu} - \frac{z}{r}\right)\left(\frac{y}{y_a}\right)^{\theta_1}\right], \tag{A7}$$

where the abandonment threshold y_a is given in

$$y_a = \frac{r-\nu}{r}\frac{\theta_1}{\theta_1 - 1}z, \tag{A8}$$

where

$$\theta_1 = -\sigma^{-2}\left(\nu - \sigma^2/2\right) - \sqrt{\sigma^{-4}\left(\nu - \sigma^2/2\right)^2 + 2r\sigma^{-2}} < 0. \tag{A9}$$

A.2 Valuation of a Levered Public Firm

First, consider the market value of equity. Let y_d be the corresponding default threshold. After default, equity is worthless, in that $E(y) = 0$ for $y \leq y_d$. This gives us the value-matching condition $E(y_d) = 0$. Before default, equity value $E(y)$ satisfies the following differential equation:

$$rE(y) = (1 - \tau_m)(y - z - b) + \nu y E'(y) + \frac{1}{2}\sigma^2 y^2 E''(y), \ y \geq y_d. \tag{A10}$$

When $y \to \infty$, $E(y)$ also satisfies a no-bubble condition. Solving this ODE and using the boundary conditions, we obtain

$$E(y; y_d) = (1 - \tau_m)\left[\left(\frac{y}{r - \nu} - \frac{z + b}{r}\right) - \left(\frac{y_d}{r - \nu} - \frac{z + b}{r}\right)\left(\frac{y}{y_d}\right)^{\theta_1}\right]. \tag{A11}$$

Equation (A11) shows that equity value is equal to the after-tax present value of profit flows minus the present value of the perpetual coupon payments plus an option value to default. The term $(y/y_d)^{\theta_1}$ may be interpreted as the price of an Arrow-Debreu security contingent on the event of default. The optimal default threshold satisfies the smooth-pasting condition,

$$\left.\frac{\partial E(y)}{\partial y}\right|_{y=y_d} = 0, \tag{A12}$$

which gives

$$y_d^* = \frac{r - \nu}{r}\frac{\theta_1}{\theta_1 - 1}(z + b). \tag{A13}$$

After debt is in place, there is a conflict between equityholders and debtholders. Equityholders choose the default threshold y_d to maximize equity value $E(y; y_d)$.

The market value of debt before default satisfies the following differential equation:

$$rD(y) = b + \nu y D'(y) + \frac{1}{2}\sigma^2 y^2 D''(y), \ y \geq y_d. \tag{A14}$$

The value-matching condition is given by

$$D(y_d) = \alpha A(y_d). \tag{A15}$$

We also impose a no-bubble condition when $y \to \infty$. Solving the valuation equation, we have

$$D(y) = \frac{b}{r} - \left[\frac{b}{r} - \alpha A(y_d)\right]\left(\frac{y}{y_d}\right)^{\theta_1}. \tag{A16}$$

For a given coupon rate b and default threshold y_d, using Equation (A2), we may write the market value of the levered firm value $V(y; y_d)$ as follows:

$$V(y; y_d) = A(y) + \frac{\tau_m b}{r}\left[1 - \left(\frac{y}{y_d}\right)^{\theta_1}\right] - (1 - \alpha) A(y_d)\left(\frac{y}{y_d}\right)^{\theta_1}. \tag{A17}$$

Equation (A17) shows that the levered market value of the firm is equal to the after-tax unlevered firm value plus the present value of tax shields minus bankruptcy costs.

While y_d^* is chosen to maximize $E(y)$, coupon b is chosen to maximize *ex ante* firm value $V(y)$. Substituting Equation (A13) into Equation (A17) and using the following first-order condition,

$$\frac{\partial V(y_0)}{\partial b} = 0, \tag{A18}$$

we obtain the optimal coupon rate b^* as a function of y_0. We also verify that the second-order condition is satisfied.

Now consider the special case without operating cost ($z = 0$). First, from Equation (A7), the value of an unlevered public firm becomes

$$A(y) = (1-\tau_m)\left[\frac{y}{r-\nu} - \frac{y_a}{r-\nu}\left(\frac{y}{y_a}\right)^{\theta_1}\right]. \tag{A19}$$

For a levered public firm, we have an explicit expression for the optimal coupon:

$$b^* = y_0 \frac{r}{r-\nu}\frac{\theta_1 - 1}{\theta_1}\left(1 - \theta_1 - \frac{(1-\alpha)(1-\tau_m)\theta_1}{\tau_m}\right)^{1/\theta_1}. \tag{A20}$$

Substituting Equations (A13) and (A20) into Equation (A17), we obtain the following expression for $V^*(y)$, the firm value when debt coupon is optimally chosen:

$$V^*(y) = \left[1 - \tau_m + \tau_m\left(1 - \theta_1 - \frac{(1-\alpha)(1-\tau_m)\theta_1}{\tau_m}\right)^{1/\theta_1}\right]\frac{y}{r-\nu}. \tag{A21}$$

Notice that this firm value formula applies only at the moment of debt issuance and will be equal to firm value when the entrepreneur cashes out.

B. Proof of Theorems 1 and 2

Theorem 1 is a special case of Theorem 2. Thus, we prove the results in only the general case where the entrepreneur has partial ownership ψ of the firm.

After exit (via default or cashing out), the entrepreneur solves the standard complete-markets consumption/portfolio choice problem (Merton 1971). The entrepreneur's value function $J^e(x)$ is given by the following explicit form:

$$J^e(x) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(x + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right]. \tag{B1}$$

Before exit, the entrepreneur faces incomplete markets. Using the principle of optimality, we claim that the entrepreneur's value function $J^s(x, y)$ satisfies the HJB Equation (10). The first-order conditions for consumption c and portfolio allocation ϕ are given by Equations (11–12).

We conjecture that $J^s(x, y)$ takes the following exponential form:

$$J^s(x, y) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(x + \psi G(y) + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right]. \tag{B2}$$

As shown in Miao and Wang (2007), $G(y)$ is the entrepreneur's certainty equivalent wealth per unit of the entrepreneur's inside equity of the firm. Under this conjectured value function, it is easy to show that the optimal consumption rule and the portfolio rule are given by Equations (26) and (27), respectively. Substituting these expressions back into the HJB Equation (10) gives the differential Equation (28) for $G(y)$.

We now turn to the boundary conditions. First, consider the lower default boundary. Since equity is worthless at default, the entrepreneur's financial wealth x does not change immediately after default. In addition, the entrepreneur's value function should remain unchanged at the moment of default. That is, the following value-matching condition holds at the default boundary $y_d(x)$:

$$J^s(x, y_d(x)) = J^e(x). \tag{B3}$$

In general, the default boundary depends on the entrepreneur's wealth level. Because the default boundary is optimally chosen, the following smooth-pasting conditions at $y = y_d(x)$ must be satisfied:[21]

$$\left.\frac{\partial J^s(x,y)}{\partial x}\right|_{y=y_d(x)} = \left.\frac{\partial J^e(x)}{\partial x}\right|_{y=y_d(x)} \tag{B4}$$

$$\left.\frac{\partial J^s(x,y)}{\partial y}\right|_{y=y_d(x)} = \left.\frac{\partial J^e(x)}{\partial y}\right|_{y=y_d(x)}. \tag{B5}$$

These two conditions equate the marginal value of wealth and the marginal value of revenue before and after default.

At the instant of cashing out, the entrepreneur retires debt at par, pays fixed cost K, and sells his firm for $V^*(y)$ given in Equation (A17). We assume that the shares owned by existing equity holders are converted one-for-one into the shares of the new firm. Then the entrepreneur pays capital gains taxes on the sale. His wealth x_{T_u} immediately after cashing out satisfies

$$x_{T_u} = x_{T_u-} + \psi V^*\left(y_{T_u}\right) - F_0 - K - \tau_g\left(\psi V^*\left(y_{T_u}\right) - K - (I - (1-\psi)E_0)\right). \tag{B6}$$

The entrepreneur's value function at the payout boundary $y_u(x)$ satisfies the following value-matching condition:

$$J^s(x, y_u(x)) = J^e\left(x + \psi V^*(y_u(x)) - F_0 - K - \tau_g\left(\psi V^*(y_u(x)) - K - (I-(1-\psi)E_0)\right)\right). \tag{B7}$$

The entrepreneur's optimality implies the following smooth-pasting conditions at $y = y_u(x)$:

$$\left.\frac{\partial J^s(x,y)}{\partial x}\right|_{y=y_u(x)} = \left.\frac{\partial J^e\left(x + \psi V^*(y) - F_0 - K - \tau_g\left(\psi V^*(y) - K - (I-(1-\psi)E_0)\right)\right)}{\partial x}\right|_{y=y_u(x)} \tag{B8}$$

$$\left.\frac{\partial J^s(x,y)}{\partial y}\right|_{y=y_u(x)} = \left.\frac{\partial J^e\left(x + \psi V^*(y) - F_0 - K - \tau_g\left(\psi V^*(y) - K - (I-(1-\psi)E_0)\right)\right)}{\partial y}\right|_{y=y_u(x)}. \tag{B9}$$

Using the conjectured value function (B2), we show that the default and cash-out boundaries $y_d(x)$ and $y_u(x)$ are independent of wealth. We thus simply use y_d and y_u to denote the default and cash-out thresholds, respectively. Using the value-matching and smooth-pasting conditions (B3–B5) at y_d, we obtain Equations (29) and (30). Similarly, using the value-matching and smooth-pasting conditions (B7–B9) at y_u, we have Equations (31) and (32).

Finally, we characterize the entrepreneur's investment and financing decision at $t = 0$. Let $\underline{x}$ denote the entrepreneur's endowment of financial wealth. If the entrepreneur chooses to start his business, his financial wealth x_0 immediately after financing is

$$x_0 = \underline{x} - (I - F_0 - (1-\psi)E_0). \tag{B10}$$

At time 0, the entrepreneur chooses a coupon rate b and equity share ψ to solve the following problem:

$$\max_{b,\psi} J^s\left(\underline{x} + F_0 + (1-\psi)E_0 - I, y_0\right), \tag{B11}$$

[21] See Krylov (1980), Dumas (1991), and Dixit and Pindyck (1994) for details on the smooth-pasting conditions.

subject to the requirement that outside debt and equity are competitively priced, i.e., $F_0 = F(y_0)$, and $E_0 = E_0(y_0)$. In Appendix C, we provide explicit formulas for $F(y)$ and $E_0(y)$.

The entrepreneur will decide to launch the project if his value function from the project (under the optimal capital structure) is higher than the value function without the project,

$$\max_b J^s\left(\underline{x} + F_0 + (1-\psi)E_0 - I, y_0\right) > J^e\left(\underline{x}\right). \tag{B12}$$

C. Market Values of Outside Debt and Equity

When the entrepreneur neither defaults nor cashes out, the market value of his debt $F(y)$ satisfies the following ODE:

$$rF(y) = b + \nu y F'(y) + \frac{1}{2}\sigma^2 y^2 F''(y), \; y_d \le y \le y_u. \tag{C1}$$

At the default trigger y_d, debt recovers the fraction α of after-tax unlevered firm value, in that $F(y_d) = \alpha A(y_d)$. At the cash-out trigger y_u, debt is retired and recovers its face value, in that $F(y_u) = F_0$. Solving Equation (C1) subject to these boundary conditions gives

$$F(y) = \frac{b}{r} + \left(F_0 - \frac{b}{r}\right)\overline{q}(y) + \left[\alpha A(y_d) - \frac{b}{r}\right]\underline{q}(y), \tag{C2}$$

where

$$\overline{q}(y) = \frac{y^{\theta_1} y_d^{\theta_2} - y^{\theta_2} y_d^{\theta_1}}{y_u^{\theta_1} y_d^{\theta_2} - y_u^{\theta_2} y_d^{\theta_1}}, \tag{C3}$$

$$\underline{q}(y) = \frac{y^{\theta_2} y_u^{\theta_1} - y^{\theta_1} y_u^{\theta_2}}{y_u^{\theta_1} y_d^{\theta_2} - y_u^{\theta_2} y_d^{\theta_1}}. \tag{C4}$$

Here, θ_1 is given by Equation (A9) and

$$\theta_2 = -\sigma^{-2}\left(\nu - \sigma^2/2\right) + \sqrt{\sigma^{-4}\left(\nu - \sigma^2/2\right)^2 + 2r\sigma^{-2}} > 1. \tag{C5}$$

Equation (C2) admits an intuitive interpretation. It states that debt value is equal to the present value of coupon payments plus the changes in value when default occurs and when cash-out occurs. Note that $\overline{q}(y)$ can be interpreted as the present value of a dollar if cash-out occurs before default, and $\underline{q}(y)$ can be interpreted as the present value of a dollar if the entrepreneur goes bankrupt before cash-out. Using $F_0 = F(y_0)$, we have that the initial debt issuance is given by

$$F_0 = \frac{b}{r} - \left(\frac{b}{r} - \alpha A(y_d)\right)\frac{\underline{q}(y_0)}{1 - \overline{q}(y_0)}. \tag{C6}$$

Similarly, for the outside equity claim, we have the following valuation equation:

$$rE_0(y) = (1-\tau_e)(y - z - b) + \nu y E_0'(y) + \frac{1}{2}\sigma^2 y^2 E_0''(y), \; y_d \le y \le y_u, \tag{C7}$$

subject to the following boundary conditions:

$$E_0(y_u) = V^*(y_u), \tag{C8}$$

$$E_0(y_d) = 0. \tag{C9}$$

Solving the above valuation equation, we have that the value of outside equity $E_0(y)$ is given by

$$\begin{aligned} E_0(y) = {} & (1-\tau_e)\left(\frac{y}{r-\nu} - \frac{z+b}{r}\right) + \left[V^*(y_u) - (1-\tau_e)\left(\frac{y_u}{r-\nu} - \frac{z+b}{r}\right)\right]\overline{q}(y) \\ & - (1-\tau_e)\left(\frac{y_d}{r-\nu} - \frac{z+b}{r}\right)\underline{q}(y). \end{aligned} \tag{C10}$$

The initial outside equity issuance E_0 is then given by $E_0 = E_0(y_0)$.

D. Capital Gains Taxes and Operating Leverage

First, we analyze the case where the capital gains tax is zero. Then, we extend the baseline model to allow for operating leverage.

D.1 Effects of Capital Gains Taxes

In the presence of capital gains taxes with $\tau_g = 10\%$, the benefit from cash-out falls. Table D1 shows that the 10-year cash-out probability decreases, and the entrepreneur takes on more debt in order to diversify idiosyncratic risk. However, the quantitative effects are small in our numerical example. We may understand the intuition from the value-matching condition (18). At the cash-out threshold y_u, when $\psi = 1$, the entrepreneur obtains less value $\left(1 - \tau_g\right) V^* \left(y_u\right)$, but enjoys tax rebate $\tau_g \left(K + I\right)$. Thus, these two effects partially offset each other, making the effect of capital gains taxes small. Clearly, if the cash-out value is sufficiently large relative to the cash-out and investment costs, then the effect of the capital gains tax should be large.

D.2 Effects of Operating Leverage

How does operating leverage affect an entrepreneurial firm's financial leverage? Intuitively, operating leverage increases financial distress risk, and thus should limit debt financing. The top panel of Table D2 confirms this intuition for the complete-markets case (the limiting case with $\gamma \to 0$).

Table D1
Capital structure of entrepreneurial firms: capital gains taxes

	Coupon b	Public debt F_0	Private equity G_0	Private firm S_0	Private leverage (%) L_0	Credit spread (bp) CS	10-Yr default probability (%) $p_d(10)$	10-Yr cash-out probability (%) $p_u(10)$
Panel A: $\tau_e = 0$, $\tau_g = 0$								
$\gamma = 1$	0.11	3.20	19.95	23.14	13.8	32	0.0	12.3
$\gamma = 2$	0.42	10.11	10.36	20.47	49.4	115	1.9	24.1
Panel B: $\tau_e = \tau_m$, $\tau_g = 0$								
$\gamma = 1$	0.54	12.29	9.92	22.22	55.3	138	3.9	15.5
$\gamma = 2$	0.66	13.57	6.47	20.04	67.7	186	9.8	26.8

This table reports the results for the setting where there are no capital gains taxes ($\tau_g = 0$). We report results for two business income tax rates ($\tau_e = 0$ or $\tau_e = \tau_m (11.29\%)$) and two levels of risk aversion ($\gamma = 1, 2$). The remaining parameters are reported in Section 4. All the results are for initial revenue $y_0 = 1$.

Table D2
The effects of operating leverage: the case of debt financing and cash-out option

	Coupon b	Public debt F_0	Private equity G_0	Private firm S_0	Private leverage (%) L_0	Credit spread (bp) CS	10-Yr default probability (%) $p_d(10)$	10-Yr cash-out probability (%) $p_u(10)$
Panel A: $\gamma \to 0$								
$z = 0.2$	0.35	8.03	16.73	24.76	32.4	132	2.2	0.0
$z = 0.4$	0.33	6.72	13.40	20.12	33.4	194	6.2	0.0
Panel B: $\gamma = 1$								
$z = 0.2$	0.59	10.94	6.34	17.28	63.3	237	14.1	13.1
$z = 0.4$	0.62	9.41	3.98	13.39	70.3	356	28.4	13.5
Panel C: $\gamma = 2$								
$z = 0.2$	0.73	11.95	3.57	15.53	77.0	315	26.9	23.7
$z = 0.4$	0.84	10.48	1.57	12.05	86.9	503	50.6	22.3

This table reports the results for the setting where there is operating leverage. We report results for two levels of operating cost ($z = 0.2, 0.4$). The remaining parameters are reported in Section 4. All the results are for initial revenue $y_0 = 1$.

As the operating cost z increases from 0.2 to 0.4, the 10-year default probability rises from 2.2% to 6.2%, and the firm issues less debt. On the other hand, equity value also decreases because operating costs lower the operating profits. As a result, the effect on financial leverage ratio is ambiguous. In our numerical examples, this ratio increases with operating costs.

Our analysis above shows that risky debt has important diversification benefits for entrepreneurial firms. This effect may dominate the preceding "crowding-out" effect of operating leverage. In Table D2, as z increases from 0.2 to 0.4, an entrepreneur with $\gamma = 1$ raises debt with increased coupon payments from 0.59 to 0.62. However, the market value of debt decreases because both the 10-year default probability and the cash-out probability increase with z. The private equity value also decreases with z, and this effect dominates the decrease in debt value. Thus, the private leverage ratio rises with operating costs. This result also holds true for a more risk-averse entrepreneur with $\gamma = 2$. Notice that the more risk-averse entrepreneur relies more on risky debt to diversify risk. The 10-year default probability increases substantially from 26.9% to 50.6% for $\gamma = 2$, but the 10-year cash-out probability decreases from 23.7% to 22.3%.

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