Infrared Topological Plasmons in Graphene

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We propose a two-dimensional plasmonic platform—periodically patterned monolayer graphene—which hosts topological one-way edge states operable up to infrared frequencies. We classify the band topology of this plasmonic system under time-reversal-symmetry breaking induced by a static magnetic field. At finite doping, the system supports topologically nontrivial band gaps with mid-gap frequencies up to tens of terahertz. By the bulk-edge correspondence, these band gaps host topologically protected one-way edge plasmons, which are immune to backscattering from structural defects and subject only to intrinsic material and radiation loss. Our findings reveal a promising approach to engineer topologically robust chiral plasmonic devices and demonstrate a realistic example of high-frequency topological edge states.

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Time-reversal-symmetry ($T$) breaking, a necessary condition for achieving quantum Hall phases [1,2], has now been successfully implemented in several bosonic systems, as illustrated by the experimental observation of topologically protected one-way edge transport of photons [3,4] and phonons [5]. More generally, two-dimensional (2D) $T$ broken topological bosonic phases have been proposed in a range of bosonic phases, spanning photons [6], phonons [7,8], magnons [9], excitons [10], and polaritons [11]. The operating frequency of these systems is typically small, however—far below terahertz—limited by the spectral range of the $T$-breaking mechanism. For example, the gyromagnetic effect employed in topological photonic crystals is limited by the Larmor frequency of the underlying ferrimagnetic resonance, on the order of tens of gigahertz [3]. In phononic realizations, the attainable gyranomal frequencies limit operation further still, to the range of kilohertz [12]. Towards optical frequencies, proposals of dynamic index modulation [13] and optomechanical coupling [14] are promising but experimentally challenging to scale to multiple coupled elements [15–17].

Recently, Jin et al. [18] pointed out that the well-known magnetoplasmons of uniform 2D electron gases [19,20] constitute an example of a topologically nontrivial bosonic phase hosting unidirectional edge states. However, as the topological gap exists only below the cyclotron frequency $\omega_c$, the spectral operation remains limited to low frequencies. In this Letter, we show that by suitably engineering the plasmonic band structure of a periodically nanostructured 2D monolayer graphene, see Fig. 1(a), the operation frequency of topological plasmons [21] can be raised dramatically, to tens of terahertz, while maintaining large-gap–midgap ratios even under modest $B$ fields. Bridging

FIG. 1. Two-dimensional topological plasmonic crystal under magnetically induced $T$ breaking. (a) Schematic of triangular antidot lattice in graphene. Under an external magnetic field $B = B\hat{z}$, a finite lattice supports topologically protected one-way edge plasmons. (b) Band-folded plasmon dispersion in uniform graphene ($d = 0, B \neq 0$).

(c) Spectral amplitudes ($\omega_{pl}, \omega_e$) in $(\alpha, B, \epsilon_1)$ parameter space.
ultrathin and topological band gaps.

Graphene distinguishes itself as an ideal platform for topological plasmonics in three key aspects: first, it supports large, tunable carrier densities \( n \approx 10^{11} - 10^{14} \text{ cm}^{-2} \) \([27-29]\), or equivalently, large, tunable Fermi energies \( E_F = h v_F / \sqrt{2m} \) (Fermi velocity, \( v_F \approx 9.1 \times 10^7 \text{ cm s}^{-1} \) \([30]\)); second, it exhibits an ultrafast, tunable Drude mass \( m^* = eB / v_F^2 \) (e.g., at \( E_F = 0.2 \text{ eV} \), \( m^* / m_e \approx 4\% \)), allowing ultrahigh cyclotron frequencies \( \omega_c \equiv eB / cm^* \approx 34 \omega_F \) up to the terahertz range \([32-34]\); and third, high-quality graphene can exhibit exceptionally long intrinsic relaxation times \( 1/\gamma \), extending into the picosecond range \([35,36]\). These properties enable topological plasmonics in three key aspects: first, it supports large, tunable carrier densities \( n \); second, it exhibits an ultrasmall, tunable Drude mass \( m^* \equiv eB / v_F^2 \); and third, it supports large, tunable carrier densities \( n \).

Material \([38]\).

We explore the band topology of 2D plasmons in periodically structured graphene under magnetic-field induced \( T \) breaking. Figure 1(a) illustrates our design: a triangular antidot lattice of periodicity \( a \) and antidot diameter \( d \) is etched into a suspended sheet of graphene. The domain \( \Omega \) in Eqs. (1) is then the torus defined by the rhombic unit cell of Fig. 2(a). Band folding splits the eigenindex \( \nu \) into a band index \( n = 1, 2, \ldots \) and a crystal wave vector \( k \) restricted to the hexagonal Brillouin zone \((BZ) \) of Fig. 2(b). Accordingly, the eigenvectors assume the Bloch form \( \hat{U}_{nk}(r) = u_{nk}(r)e^{ikr}, \) with periodic component \( u_{nk} \equiv (\omega_F \phi_n)^{\nu} \).

First, we consider the simple but instructive \( d = 0 \) scenario, i.e., the uniform sheet, see Fig. 1(b). This “empty lattice” captures the essential impact of band folding: by folding the uniform sheet plasmon dispersion, \( \omega(k) = \sqrt{2 \pi a \omega_F k + \omega_c^2} \), over the hexagonal BZ, threefold Dirac-like point degeneracies arise between the \( n = 1, 2, \ldots \) bands at the \( \mathbf{K} \) (and \( \mathbf{K}' \)) point. For \( B = 0 \), the lattice’s \( C_{6v} \) symmetry guarantees that twofold-degenerate Dirac points remain between the \( n = 1 \) and \( 2 \) bands even when \( d \neq 0 \).

The uniform-sheet Dirac point plasmon frequency, \( \omega_K \equiv \sqrt{\omega_c^2 + \omega_0^2} \) with \( \omega_0^2 = \sqrt{2 \pi a \omega_F |\mathbf{K}|} \) and \( |\mathbf{K}| = 4\pi / 3a \), along with the cyclotron frequency \( \omega_c \), then define the

\[
\begin{align*}
\sigma_2 &\equiv \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \text{ a Pauli matrix, } \\
\omega_F &\equiv E_F / \hbar \text{ the Fermi “frequency,” and } \alpha \equiv e^2 / \pi \hbar \text{ a prefactor of graphene’s intraband conductivity } \tau a_0 \omega_c^{-1}. \text{ Conceptually, Eqs. (1)}
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characteristic frequencies of the problem and are indicated in Fig. 1(b). By applying a finite $B$ field to the $d \neq 0$ system, the Dirac point degeneracy is split, inducing a gap linearly proportional to $\omega_c$. As a result, topological plasmons with both high frequency and sufficient topological gap require simultaneously large $\omega_0^0$ and $\omega_c$.

The parameter space involved in simultaneously maximizing $\omega_0^0$ and $\omega_c$ is illustrated in Fig. 1(c). The monotonic $E_F$-dependence of the two characteristic frequencies is opposite, highlighting an inherent trade-off between the operating frequency and the gap size. In addition, the accessible parameter space is restricted by several constraints, indicated by gray regions in Fig. 1(c): first, intrinsic Drude loss estimated at $\gamma/2\pi \sim 1$ THz smears out the gap region, necessitating $\omega_c \gtrsim \gamma$; second, interband dispersion is non-negligible when $\omega_K \gtrsim \omega_F$ [52,53], eventually introducing significant loss through Landau damping; and third, Landau quantization of the charge carriers ultimately invalidates a semiclassical description [54,55] when $E_F \lesssim E_L \equiv v_F \sqrt{2heB/c}$ (the first Landau level), or equivalently, when $\hbar \omega_c \lesssim \frac{1}{4} E_L$, see Supplemental Material [38]. Overall, we find that an experimentally favorable region exists for Fermi energies $E_F \approx 0.2$–$0.3$ eV, periodicities $a \approx 100$–$600$ nm, and magnetic fields $B \approx 2$–$8$ T.

Next, we turn to the nanostructured system, settling on a stronger field to the BZ [61], e.g., at $B \approx 400$, and magnetic fields $600$–$8$ T. Particle–antiparticle pairs, indicated by gray regions in Fig. 1(c): first, and 2 bands, an implicit degeneracy exists at $\Gamma$ between the $\bar{n}$th and $n$th band topology for the definition of associated gap Chern numbers. Specifically, the total Chern number of positive (+) and negative (−) frequency bands is $C_{\pm} \equiv \sum_{n=1}^{\infty} C^{(\pm n)}$. In uniform graphene $C_{\pm} = \pm \text{sgn} B$ [18]. Since Chern numbers can be annihilated or created (pairwise) only under band closings, this result holds in nanostructured graphene as well; cf. the finite band gap separating positive and negative bands. With this in mind, we define the $n$th gap Chern number $\tilde{C}_n$, associated with the gap immediately below the $n$th band as

\[
\tilde{C}^{(n)} \equiv \sum_{n'=\pm \infty}^{n-1} C^{(n')} = -\text{sgn} B + \sum_{n'=1}^{n-1} C^{(n')},
\]

specializing to positive-frequency gaps at the last equality. For lattice terminations adjacent to vacuum, bulk-edge correspondence then requires that the number of left minus right propagating topological edge states equal $\tilde{C}^{(n)}$ [65].

These considerations predict the existence of single-mode one-way edge states in the first and second gaps when $B \neq 0$ and multimode one-way edge states in the
FIG. 3. Plasmonic one-way edge states at lattice terminations. (a) Edge termination of the 2D crystal. (b) Projected 1D BZ and its high symmetry points. (c) Projected bulk bands (blue) and topologically protected one-way plasmonic edge states (red) along k_x for B = 0, 4, and 8 T, with associated gap Chern numbers C^{(n)} (green). (d) Typical mode profiles of edge states in real space at B = 8 T; band association is indicated by colored markers in (c).

gap between the n = 3, and 4 bands at B = 4 and 8 T, cf. Fig. 2(c). We confirm these predictions in Fig. 3 by numerically calculating the edge states supported by a broad ribbon (20 unit cells wide) extended along x with the particular edge termination of Fig. 3(a). The bulk states are folded into the projected 1D BZ, k_x ∈ (−π/a, π/a), see Fig. 3(b), due to breaking of Bloch periodicity along y. Additionally, edge states emerge: they are identified and postselected from the ribbon spectrum by their edge confinement and bulk-gap habitation (in emulating single- and multime modes). They connect upper and lower bulk bands, in the band gaps, consistent with the obtained gap Chern numbers. They are nontopological; states at k_x ≠ B/2, with associated gap Chern numbers. They propagate to the right, consistent with the sign (chirality) of C^{(n)} ≠ 0. They are topologically protected from backscattering only in the complete band gap: above it, any defect may scatter them to either bulk or counterpropagating edge states. The low-frequency C^{(1)} = −1 gap hosts edge states entirely analogous to the edge magnetoplasmon of the uniform sheet—an edge-state parallel of the bulk dispersion-agreement (γ ∝ k/λ) between the n = 1 band and the uniform sheet. In contrast, the high-frequency (≈15 THz) edge state in the C^{(2)} = −1 gap results directly from band engineering, and is a qualitatively new type of edge magnetoplasmon. Finally, a multimode triple of edge states appears in the C^{(4)} = −3 gap. Though the gap is comparatively small, it can be widened by tuning a/d. Figure 3(d) illustrates the sharp spatial Bloch mode confinement of the edge states, ∣φ_yk∥(x)∣, for a few select n and k_x at B = 8 T. The degree of confinement correlates positively with the size of the topological band gap, i.e., implicitly with B, paralleling the uniform 2D electron gas [20].

The edge states can be efficiently excited by nearby point sources, as demonstrated in Fig. 4: a y-polarized dipole near the edge, emitting in the gap center (14.6 THz) of the n = 1 and 2 bands, excites the edge plasmon at k_x = 0 (for computational details, see Supplemental Material [38]). In the absence of intrinsic material loss, the edge state propagates unidirectionally to the right with constant amplitude as seen in Figs. 4(b)–4(d). Topological protection ensures that even structural defects, such as the sharp trench in Fig. 4(d), are traversed without backscattering. The increased edge confinement with mounting magnetic field is exemplified by Figs. 4(b)–4(c).

The edge state’s topological nature does not shield it from intrinsic material or radiation loss. While the latter is negligible, owing to the strongly confined and electrostatic nature of graphene plasmons [cf. the nearly vertical light cone in Fig. 2(c)], the former can be appreciable, as in all plasmonic systems. Finite relaxation γ is readily incorporated in Eqs. (1) by the substitution ω_c → ω_c + iγ. This introduces an imaginary spectral component, Im ω_c = −1/2γ (1 + ξ_c ω_c/Re ω_c^0) for γ ≪ Re ω_c^0. This impacts the propagation of edge states in two aspects: first, it blurs the gap region, allowing small but finite loss-induced coupling between edge and bulk states (see Supplemental Material [38]); second, states exhibit a finite lifetime, or, equivalently, finite propagation length k/γ, as illustrated in Fig 4(e). Strategies to reduce the relative impact of intrinsic loss include reducing the lattice constant a, increasing E_F, or maximizing the edge state group velocity by structural design (see Supplemental Material [38]).

In conclusion, we have demonstrated the band topology of 2D plasmons in periodically patterned graphene under a T-breaking magnetic field. Multiple sets of topologically
protected one-way edge plasmons corresponding to nontrivial gap Chern numbers are discovered. Their operating frequencies can be as high as tens of terahertz, i.e., in the far-infrared regime. They can be experimentally verified by terahertz near-field imaging [66,67] and Fourier transform infrared spectroscopy [60]. Our findings suggests a new direction in the synthesis of high-frequency T\(\text{\text{-}}\)broken topological bosonic phases, and can be directly extended to nonmagnetic schemes based on valley polarization [68,69].

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[21] The emerging branches of topological plasmonics encompass both T\(\text{-}\)invariant [22–24] and T\(\text{-}\)broken [25,26] systems: our focus is the plasmonic analogue of the quantum anomalous Hall effect (i.e., T\(\text{-}\)broken and Z topology).
A nonretarded, intraband approach is adopted; this neglects lower the effective electron density instating a monotonic reduction of $\xi_1(\Gamma)$ with increasing $d/a$.

In the uniform sheet ($d = 0$) the shift at $\Gamma$ is exactly $\omega_c$ [63], corresponding to $\xi_1(\Gamma) = 1$; finite antidots ($d \neq 0$) lower the effective electron density instating a monotonic reduction of $\xi_1(\Gamma)$ with increasing $d/a$.